UNITS AND MEASUREMENTS

"When you can measure what you are speaking about and express it in numbers, you known something about it, but when you cannot measure it and express it in number, your knowledge is of a meagre and unsatisfactory kind, but you have scarcely, in your thoughts, advanced to the stage of science whatever the matter may be"

Physical Quantities

All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities. For example, length, mass, temperature, speed, force, electric current, etc. Physical quantities are of two types – fundamental and derived.

(i) Fundamental quantities

The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called fundamental quantities. Seven fundamental or base quantities these are mass, length, time, electric current, temperature, luminous intensity and amount of substance.

(ii) Derived quantities

The physical quantities whose defining operations are based on other physical quantities are called derived quantities. For example, velocity, acceleration, force, momentum, etc.

Measurement

The measurement of a physical quantity is the process of comparing this quantity with a standard amount of the physical quantity of the same kind, called its unit. To express the measurement of a physical quantity, we need to know *two* things:

(i) The *unit* in which the quantity is measured.

(ii) The *numerical value* or the *magnitude* of the quantity i.e., the number of times that unit is contained in the given physical quantity.

$\therefore$ Numerical value of the physical quantity × size of the unit = *nu*

Thus the numerical value (n) is inversely proportional to the size (u) of the unit.

$$n \propto \frac{1}{u} \qquad \text{or} \qquad nu = \text{constant}$$

If n_1 and n_2 are numerical values for a physical quantity corresponding to the units u_1 and u_2, then

$$n_1 u_1 = n_2 u_2$$

Physical Unit

The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.

Desirable characteristics of a physical unit

1. It should be *well–defined.*
2. It should be of *convenient size, i.e.*, neither too small nor too large in comparison with the measurable physical quantity.
3. It should not change with time.
4. It should be easily *reproducible.*
5. It should be *imperishable* or *indestructible.*
6. It should not be affected by the change in physical conditions such as pressure, temperature, etc.

7. It should be *internationally acceptable.*
8. It should be easily *accessible.*

Fundamental units

The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called fundamental units.

Derived units

All the other physical units which can be expressed in terms of the fundamental units are called derived units.

System of units

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.

Some of the commonly used systems of units are as follows:

(i) ***cgs system.*** It was set up in France. It is based on *centimeter, gram and second* as the fundamental units of length, mass and time respectively.

(ii) ***fps system.*** It is a British system based on *foot, pound and second* as the fundamental units of length, mass and time respectively.

(iii) ***mks system:*** It is also a French system based on *metre, kilogram and second* as the fundamental units of length, mass and time respectively.

(iv) ***SI: The international system of units.*** SI is the abbreviation for *"Systeme Internationale d' Unites',* which is French equivalent for *international system of units.* It is a modernized and extended form of the metric systems like cgs and mks systems.

Table: Basic SL Quantities and Units

Sr. No.	Basic physical quantity	Basic unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Quantity of matter	mole	mol

Table: Supplementary SI Units

Sr. No.	Supplementary Quantity	Basic unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

The *seven* **basic SI units** are defined as follow:

(i) **Mere (m).** It is the SI unit of length. *One metre is defined as the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second.*

(ii) **Kilogram (kg).** It is the SI unit of mass. *One kilogram is the mass of prototype cylinder of platinum–iridium alloy (whose height is equal to its diameter)* preserved at the International Bureau of Weights and Measures, at Severs, near Paris.

(iii) **Second (s):** It is the SI unit of time. *One second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium – 133 atom.*

(iv) **Ampere (A).** It is the SI unit of electric current. *One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible cross–section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

(v) **Kelvin (K).** It is the SI unit of temperature. *One Kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.* The triple point of water is the temperature at which ice, water and water vapour co–exist.

(vi) **Candela (cd).** It is the SI unit of luminous intensity. *One candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency* 540×10^{12} *hertz and* that has a radiant intensity of 1/683 watt per steradian in that direction.

(viii) **Mole (mol.)** *One mole is that amount of a substance which contains as many elementary entities as there are atoms in 0.012 kg of carbon – 12 isotope.*

The *two* **supplementary SI Units** are defined as follows:

(a) **Radian (rad).** *It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.* If an arc of length ds subtends an angle dθ at the centre O of a circle of radius r, figure, then $d\theta = \frac{ds}{r}$ *radian*

It is known that π radian = 180°; 1 radian = $\frac{180^o}{\pi} = \frac{180 \times 7^o}{22} = \frac{630^o}{11} = 57.7^o$

Also 1° (degree of arc) = 60' (minute of arc) and 1' (minute of arc) = 60" (seconds of arc).

(b) **Steradian (sr).** *It is defined as the solid angle subtended at the centre of a square by a surface of the sphere equal to area to that of a square, having each side equal to the radius of the sphere.* If an area dΔ of a spherical surface subtends a solid angle dΩ at the centre of the sphere of radius r, figure, then

$$d\Omega = \frac{dA}{r^2} \text{steradian}$$

Coherent System

It is a system of units based on a certain set of fundamental units from which all derived units can be obtained by simple multiplication or division without introducing any numerical factor. For example, mks system is a coherent system of units in mechanics.

Advantages of SI over other systems of units:

(i) ***SI is a coherent system of units:*** All derived units can be obtained by simple multiplication or division of fundamental units without introducing any numerical factor.

(ii) ***SI is a rational system of units:*** It uses only one unit for a given physical quantity. For example, all forms of energy are measured in joule. On the other hand, in mks system, the mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt hour.

(iii) ***SI is a metric system:*** The multiples and submultiples of SI units can be expressed as powers of 10.

(iv) ***SI is an absolute system of units:*** It does not use gravitational units. The use of 'g' is not required.

(v) ***SI is an internationally accepted system of units.***

Table: Prefixes for powers of ten

Multiple	Prefix	Symbol	Sub–multiple	Prefix	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

A. Practical unit for measuring small distances:

(i) ***Femi:*** It is the small practical unit of distance used for measuring nuclear sizes. It is also called femtometre.

$$\textbf{1 fermi = 1fm} = 10^{-15}\ \textbf{m}$$

The radius of a proton is 1.2 fermi.

(ii) ***Angstrom:*** It is used to express wavelength of light.

$$\textbf{1 angstrom = 1Å} = 10^{-10}\ \textbf{m} = 10^{-8}\textbf{cm}$$

(iii) ***Nanometre:*** It is also used for expressing wavelength of light.

$$\textbf{1 nanometre = 1nm} = 10^{-9}\ \text{m}$$

(iv) ***Micron:*** It is the unit of distance defined as micrometre.

$$\textbf{1 micron = 1 μm} = 10^{-6}\textbf{m}$$

B. Practical units used for measuring large distances:

(i) ***Light year.*** *It is the distance traveled by light in vacuum in one year.*

$$1 \text{ light year} = \text{Speed of light in vacuum} \times 1 \text{ year}$$
$$= 3 \times 10^{8}\ \text{ms}^{-1} \times 365.25 \times 24 \times 60 \times 60 \times \text{s}$$

$\therefore$ **1 light year = 1y = 9.467×10^{15} m**

Light year is used in astronomy to measure distances of nearby stars. *alpha centauri*, the nearest star outside the solar system is 4.3 light years away from the earth.

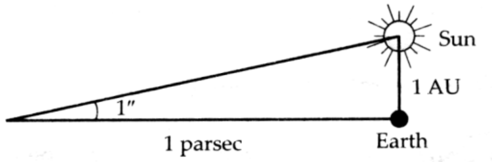
(ii) ***Astronomical unit:*** *It is defined as the mean distance of the earth from the sun.* It is used in astronomy to measure distances of planets.

$$\textbf{1 astronomical unit = 1 AU} = 1.496 \times 10^{11}$$

(iii) **Parsec (parallactic second):** It is the largest practical unit of distance used in astronomy. *It is defined as the distance at which an arc of length 1 astronomical unit subtends an angle of 1 second of arc.*

As $\theta = \dfrac{l}{r}$ $\quad\therefore\quad$ $r = \dfrac{l}{\theta}$

$$1 \text{ parsec} = \frac{1\text{ AU}}{1''} = \frac{1.496\times10^{11}\text{m}}{\frac{1}{3600}\times\frac{\pi}{180}\text{rad}} = 3.08\times10^{16}\text{ m}$$



1 parsec = 3.08×10^{16} m = 3.26ly

Relations between astronomical unit, light year and parsec

1 ly = 6.3×10^4 AU

1 parsec = 3.26 ly

Clearly, **1 parsec > 1 ly > 1 AU**

C. Practical units for measuring areas

(i) ***Barn.*** It is used for very small areas, such as nuclear cross–sections. **1 barn = 10^{-28} m^2**

(ii) ***Acre.*** It is used for measuring large areas. **1 acre = 4047 m^2**

(iii) ***Hectare.*** It is also used for measuring large areas. **1 hectare = 10^4 m^2**

D. Practical units used for measuring large masses:

1 tonne or 1 metric ton = 1000 kg

1 quintal = 100 kg

1 slug = 14.57 kg

1 pound = 1 lb = 0.4536 kg

1 Chandra Shekher limit = 1 CSL = 1.4 times the mass of the sun.

CSL is the largest practical unit of mas.

E. practical units used for measuring very small masses:

Atomic mass unit. It is defined as 1/12th of the mass of one ${}^{12}_{6}C$ atom.

1 atomic mass unit = 1 amu = 1 u = 1.66×10^{-27} kg

The mass of a proton or a neutron is of the order of one amu.

Another unit of small mass is carat, where 1c = 200 mg

F. Practical units used for measuring time:

(i) ***Solar day.*** It is the time taken by the earth to complete one rotation about its own axis w.r.t. the sun.

(ii) ***Sedrial day.*** It is the time taken by earth to complete one rotation about its own axis w.r.t. a distant star.

(iii) ***Solar year.*** It is the time taken by the earth to complete one revolution around the sun in its orbit.

1 solar year = 365.25 average solar days

= 366.25 sedrial days

(iv) ***Tropical year.*** The year in which there is total solar eclipse is called tropical year.

(v) ***Leap year:*** The year which is divisible by 4 and in which the month of February has 29 days is called a leap year.

(vi) ***Lunar month.*** It is the time taken by the moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days

(vii) ***Shake.*** It is the smallest practical unit of time 1 shake = 10^{-8} s

G. **Practical units used for measuring pressure:**

1 bar = 1 atmospheric pressure = 10^5 Nm^{-2} = 10^5 pascal (Pa) = 76 cm of Hg column

= 760 mm of Hg column

But, 1 torr = 1 mm of Hg column

$\therefore$ 1 bar = 760 torr

H. **Two more units retained for general use are**

1 curie (Ci) = 3.7×10^{10} disintegrations/sec

1 roentgen (R) = 2.58×10^{-4} C/kg

Subjective Assignment – I

Q.1 Determine the number of light years in one metre.

Q.2 What is the number of electrons that would weight 1 kg? Mass of an electron is 9.11×10^{-31} kg.

Q.3 The radius of gold nucleus is 41.3 fermi. Express its volume in m^3.

Q.4 Convert an acceleration of 2 km h^{-2} into cm s^{-2}.

Q.5 The Young's modulus of steel is 1.9×10^{11} Nm^{-2}. Express it in dyne cm^{-2}.

Q.6 The density of a material is 0.8 g cm^{-3}. Express it in SI units.

Q.7 Express the average distance of the earth from the sun in (i) light year and (ii) parsec.

Answers

1.	1.057×10^{-16} ly	2.	1.1×10^{30}	3.	2.95×10^{-40} m^3
4.	0.0154 cm s^{-2}	5.	1.9×10^{12} dyn cm^{-2}	6.	800 kg m^{-3}
7.	(i) 1.58×10^{-5} ly, (ii) 4.86×10^{-6} parsec				

Order of magnitude

The order of magnitude of a physical quantity is that power of 10 which is closest to its magnitude. It gives an idea about how big or how small a given physical quantity is. To determine the order of magnitude of a number N, we first express it as $N = n \times 10^x$.

If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

If $n > 5$, then x + 1 will be the order of magnitude of N.

Measure number N	Expressed in nearest power of 10	Order of magnitude
8	0.8×10^1	1
49	4.9×10^1	1
52	0.52×10^2	2
555	0.555×10^3	3
999	0.999×10^3	3
1001	1.001×10^3	3
753000	0.753×10^6	6
0.135	1.35×10^{-1}	– 1
0.05	5×10^{-2}	– 2
0.99	0.99×10^0	0

Subjective Assignment – II

Q.1 Write the order of magnitude of the following measurements:

(i) 25,710,000 m (ii) 0.00000521 kg

Q.2 Express 1 light year in terms of metres. What is its order of magnitude?

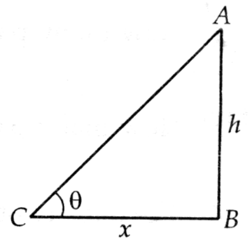
Answers

1. (i) 7, (ii) – 5
2. 9.467×10^{15}m = 0.9467×10^{16}m, 16

Indirect Methods for Measuring Large Distances

Triangulation method for the height of an accessible object. Let AB = h be the height of the tree or the tower to be measured. Let C be the point of observation at distance x from B. Place a sextant at C and measure the angle of elevation,

$$\angle ACB = \theta.$$



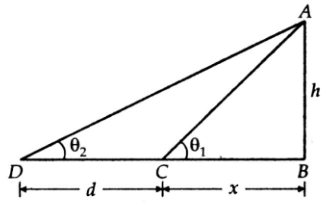
From right ΔABC, we have

$$\tan\theta = \frac{AB}{CD} = \frac{h}{x}$$

or height, $h = x \tan\theta$

Knowing the distance x, the height h can be determined.

Triangulation method for the height of an inaccessible object. Let AB = h be the height of the mountain to be measured. By using a sextant, we first measure the angle of elevation of its peak from a point C on the ground. Let it be θ_1. Move the sextant to another position D such that CD = d. Again measure the angle of elevation, $\angle ADB = \theta_2$.



In rt. ΔABC, $\cot\theta_1 = \dfrac{CB}{AB} = \dfrac{x}{h}$

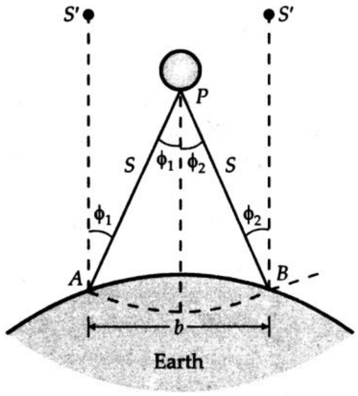
In rt. ΔABD, $\cot\theta_2 = \dfrac{DB}{AB} = \dfrac{d+x}{h}$

$$\therefore \quad \cot\theta_2 - \cot\theta_1 = \frac{d+x}{h} - \frac{x}{h} = \frac{d}{h} \quad \text{or} \quad h = \frac{d}{\cot\theta_2 - \cot\theta_1}$$

Knowing d, the height h can be determined.

Parallax. *Parallax is the apparent shift in the position of an object with respect to another when we shift our eye sidewise.* The closer object always appears to move in the direction opposite to that of our eye.

Distance of the moon or any planet. To measure the distance S of the moon or a far away planet P, we observe it simultaneously from two different positions (observatories) A and B on the earth, separated by a large distance AB = b. We select a distant star S' whose position and direction can be taken approximately same from A and B.



Now $\angle PAS' = \phi_1$ and $\angle PBS' = \phi_2$ are measured from two observatories at the same time. As b << S, so we can take AB as an arc of length b.

$$\text{Now} \quad \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S}$$

$$\therefore \quad S = \frac{b}{\theta}$$

where $\theta = \angle APB = \phi_1 + \phi_2$, is the parallactic angle.

NOTE: *The parallax method is used for measuring distances of nearby stars only.*

Reason: As the distance of star increases, the parallax angle decreases, and a great degree of accuracy is required for its measurement. Keeping in view the practical limitation in measuring the parallax angle, the maximum distance of a star we can measure by parallax method is limited to 100 light years.

Intensity method: This is a spectroscopic method based on **inverse square law of intensity.** According to this law, *the intensity of illumination at a point is inversely proportional to the square of the distance from the source of light.* Here we assume that the intrinsic brightness of all the starts is same. We compare the intensity I_1 of the faint image of a far away star taken on a photographic plate with the intensity I_2 of the bright image of a nearby star. Let r_1 and r_2 be the respective distances of these two stars.

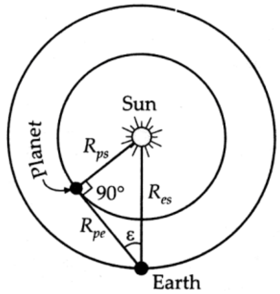
From inverse square law of intensity,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{or} \quad r_1 = r_2\left[\frac{I_2}{I_1}\right]^{1/2}$$

knowing the distance r_2 of the nearby star, the distance r_1 of the far away star can be determined. This method is useful for measuring distances of stars which are more than 100 light years away from the earth.

Inferior planets: *The planets which are closer to the sun than the earth are called inferior planets.* Mercury and Venus are the inferior planets.

Superior planets. *The planets which are farther from the sun than the earth are called superior planets.* Jupiter, Saturn, Uranus, Neptune and Pluto are the superior planets.



Copernicus method. Copernicus assumed circular orbits for the planets. The angle formed at the earth between the earth–planet direction and earth–sun direction is called the *planet's elongation.* As shown in figure, let

R_{ps} = distance of the planet from the sun

R_{pe} = distance of the planet from the earth

R_{es} = distance of the earth from the sun

ε = planet's elongation

When the elongation attains its maximum value and the planet appears farthest from the sun, the angle subtended by the sun and the earth at the planet is 90^o. Then from the right angle triangle shown in the figure, we find that

$$\frac{R_{ps}}{R_{es}} = \sin\varepsilon$$

Hence the distance of the planet from the sun is $R_{ps} = \sin\varepsilon . R_{es} = \sin\varepsilon . \text{AU}$

where R_{es} is the average distance of the earth from the sun and is called *astronomical unit* (AU.)

Distance of a superior planet

By knowing the distance of any planet from the sun, we can determine the distance of any superior planet. For this purpose we use **Kepler's third law of planetary motion.** This law states *the square of the period (T) of revolution of a planet round the sun is proportional to cube of the semi–major axia (a) of the orbit i.e.,*

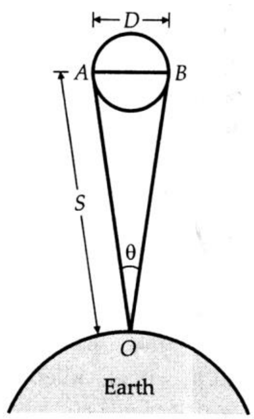
$$T^2 \propto a^3$$

If T_1 and T_2 are periods of revolution of two planets, and a_1 and a_2 are their respective semi–major axes, then

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2} \qquad \text{or } a_2 = a_1\left(\frac{T_2}{T_1}\right)^{2/3}$$

Knowing the values of a_1, T_1 and T_2, we can find out a_2.

Diameter of the moon



Let AB = D be the diameter of the moon (or planet) which is to be measured by an observer O on the earth. A telescope is focused on the moon and the angle AOB subtended by it on the point O of the earth is found.

As $$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{AB}{OB} = \frac{D}{S} \quad \text{or} \quad D = S\theta$$

Linear diameter = Distance × angular diameter

Knowing S and θ, D can be determined.

Laser method: The word laser stands for *light amplification by stimulated emission of radiation.* Laser is a source of very intense, highly monochromatic (or one wavelength) and highly directional beam of light. A laser beam is sent towards the moon and its reflected pulse is received. If t is the time elapsed between the instants the laser beam is sent and received back, then the distance of the moon from the earth is given by

$$S = \frac{c \times t}{2}$$

where $c = 3 \times 10^8\ \text{ms}^{-1}$, is the speed of light.

Radar method: The word RADAR stands for *radio detection and ranging.* A radar can be used to measured accurately the distance of a nearby planet (such as Venus). Here radio waves are sent from a transmitter which after reflection from the planet are detected by the receiver. By measuring the time interval (t) between the instants the radio waves are sent and received, the distance of the planet can be determined as

$$S = \frac{c \times t}{2}$$

where c is the speed of the radiowave. This method can also be used to determined the distance of an aeroplane.

Sonar method: The word SONAR stands for *sound navigation and ranging.* On a sonar, ultrasonic waves (sound waves having frequencies greater than 20, 000 Hz) are transmitted through the ocean. They are reflected by the submerged rock or submarines and received by the receiver. By measuring the time delay t of the receipt of the echo, the distance S of the submerged rock or submarine can be determined as

$$S = \frac{v \times t}{2}$$

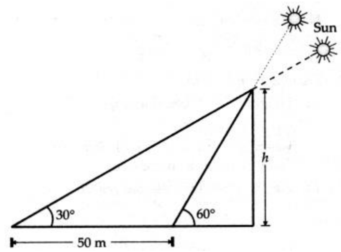
where v is the speed of the ultrasonic waves in water.

Subjective Assignment – III

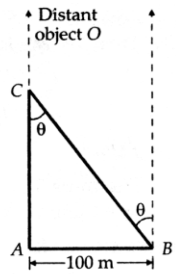
Q. 1 Calculate the angle of (a) 1^o (degree) (b) 1' (minute of arc or arcmin) and (c) 1" (second of arc or arc second) in radians.

Use $360^o = 2\pi$ rad, $1^o = 60'$ and $1' = 60''$.

Q.2 The shadow of a tower standing on a level plane is found to be 50m longer when sun's altitude is 30^o then when it is 60^o. Find the height of the tower.



Q.3 A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC upto B, a distance of 100m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle $\theta = 40^o$ (θ is known as 'parallax'). Estimate the distance of the tower C from his original position A.



Answers

1. (a) 1.745×10^{-2} rad, (b) $\approx 2.91 \times 10^{-4}$ rad, (c) $\approx 4.85 \times 10^{-6}$ rad
2. 43.3 m
3. 119 m

Indirect Methods for Measuring Small Distances

Atomic radius by Avogadro's hypothesis: Atoms are spherical in shape. So when a large number of atoms are packed together, some empty spaces are left between them. According to *Avogadro's hypothesis, the actual volume occupied by the atoms in one gram of a substance is two–third of the volume of one gram of the substance.* Let

M be the molecular mass of a substance. Then M grams of the substance will contain N (Avogadro number) of atoms.

$\therefore$ Number of atoms in 1 gram = N/M

If r be the radius of each atom, then volume of atoms in one gram = $\frac{N}{M}.\frac{4}{3}\pi r^3$. Let V be the actual volume occupied by molecules in 1 gram of the substance. They by

Avogadro's hypothesis, $\frac{N}{M}.\frac{4}{3}\pi r^3 = \frac{2}{3}V$

If ρ is the density of the substance, then $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$

For 1 gram of the substance, $\rho = \frac{1}{V}$ or, $v = \frac{1}{\rho}$

$\therefore$ $\frac{N}{M}.\frac{4}{3}\pi r^3 = \frac{2}{3}.\frac{1}{\rho}$ or $r = \left[\frac{M}{2\pi N\rho}\right]^{1/3}$

Thus, the radius of an atom of the substance can be determined.

Size of molecule of oleic acid

Oleic acid is a soapy liquid with large molecular size. We dissolve 1 cm^3 of oleic acid in 20 cm^3 of alcohol and then redissolve 1 cm^3 of this solution in 20 cm^3 of alcohol. Then the concentration of oleic acid is 1/400 cm^3 in 1 cm^3 of alcohol. We then determine the approximate volume of each drop (V cm^3). Now pour *n* drops of the solution on the surface of water taken in a broad vessel. We stretch the film carefully. As the alcohol evaporates, a very thin film of oleic acid is left on water surface. We measure the area A of the film using a graph paper.

Volume of n drops of the solution = nV cm^3

Amount of oleic acid in this solution = $\frac{nV}{400}cm^3$

Thickness of the oil film,

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}} = \frac{nV}{400\,A}\text{ cm}$$

Assuming that the film has one molecular thickness, then t will be approximately the size or diameter of a molecule of oleic acid. The value of t is found to be of the order of 10^{-9}m.

Subjective Assignment – IV

Q.1 The radius of a muonic hydrogen atom is 2.5×10^{-13}m. What is the total atomic volume in m^3 of a mole of such hydrogen atoms?

Q.2 A drop of olive oil of radius 0.25 mm spreads into a circular film of radius 10 cm on the water surface. Estimate the molecular size of olive oil.

Q.3 If the size of a nucleus ($\simeq 10^{-15}$ m) is scaled upto the tip of a sharp pin ($\simeq 10^{-5}$ m), what roughly is the size of atom?

Answers

1. 3.94×10^{-14} m^3 2. 2.08×10^{-7} cm 3. 1 m

Measurement of Mass & Weight

Mass: *The mass of a body is the quantity of matter contained in it.* It is a basic property of matter. It does not depend on the temperature, pressure or location of the body in space. The SI unit of mass is *kilogram* (kg).

Weight: *The weight of a body is the force with which a body is pulled towards the centre of the earth.* It is equal to the product of the mass (m) of the body and the acceleration due to gravity (g) of the earth on body.

Thus $W = mg$

As the value of 'g' changes from place to place, so the weight of a body is different at different places. The SI unit of weight is *Newton* (N).

Inertial mass: *The mass of a body which determines its inertia in translatory motion is called its inertial mass.* It is defined by Newton's second law of motion and is equal to the ratio of the external force applied on the body to the acceleration produced in it. By Newton's second law,

$$F = m_i a \qquad \text{or} \qquad m_i = \frac{F}{a}$$

Here m_i is the inertial mass of the body which can be measured by using an inertial balance.

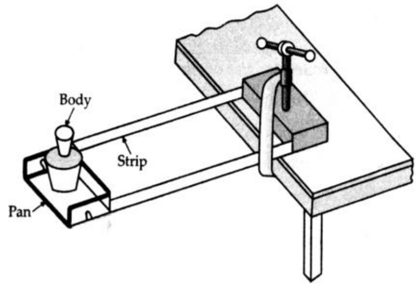
Gravitational mass

The mass of a body which determines the gravitational pull acting upon it due to the earth is called its gravitational mass. It is defined by Newton's law of gravitation. According to this law, the force of gravitation of the earth on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \text{ or } m_g = \frac{FR^2}{GM}$$

Here m_g is the gravitational mass of the body which can be measured by using a physical balance.

Measurement of inertial mass



The inertial mass of a body is measured by using a device called *inertial balance.* As shown in figure, it consists of a long strip of metal.

One end of the strip is clamped to a table. The other end of the strip carries a pan in which body whose inertial mass is to be measured is kept. When the strip vibrates horizontally, its inertia comes into play and not the force of gravity of the earth. It is found that *period of vibration* T *is directly proportional to the square root of the inertial mass m of the body.*

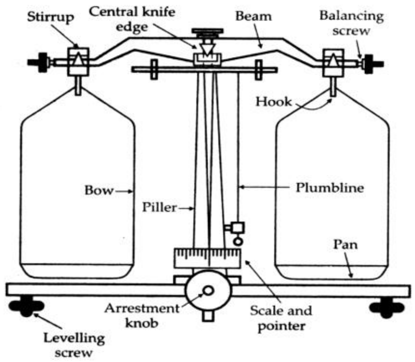
Thus $T \propto \sqrt{m}$ or $T^2 \propto m$

Let m_1 and m_2 be inertial masses of two objects and T_1 and T_2 be their corresponding periods of vibration, then

$$\frac{m_2}{m_1} = \frac{T_2^2}{T_1^2} \qquad \text{or} \qquad m_2 = m_1 \cdot \frac{T_2^2}{T_1^2}$$

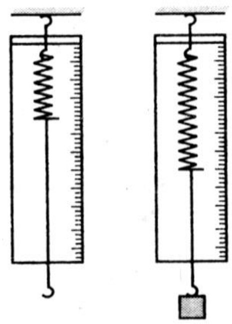
If m_1 is a standard mass, then unknown mass m_2 can be determined.

Gravitational mass by a physical balance: A physical balance works on the *principle of moments i.e.*, when an object is balanced under the action of several forces acting in the same plane, the sum of the clockwise moments is equal to the sum of anticlockwise moments. The essential parts of a physical balance are shown in figure.



The object to be weighed is placed in the left pan and the standard weights are adjusted till the beam becomes horizontal. In this condition, the gravitational force on the object is equal to the gravitational force on the standard weights. Hence the gravitational mass of the object is equal to that of the standard weights (canceling of effect of g).

Gravitational mass by a spring balance: In a spring balance, the gravitational force on a body stretches the spring and we measure the elongation of the spring. The elongation depends on gravitational force which, in turn, is proportional to the gravitational force which, in turn, is proportional to the gravitational mass of the body. Thus the elongation of the spring gives a measure of the gravitational mass. For this we first calibrate the spring balance by using standard masses and measuring the elongations. The calibration of spring balance and the mass measurement should be done at same place because the value of g varies from place to place.



Measurement of time: According to Einstein, *"Time is simply what a clock reads. Any phenomenon that repeats itself after equal intervals of time can be used as a time standard."* Examples of such a phenomenon are

1. Beating of human heart.
2. Oscillations of a pendulum.
3. Rotation of the earth about its own axis.
4. Revolution of the earth around the sun.
5. Vibrations of a quartz crystal in quartz wristwatch.
6. period of vibration of cesium – 133 atom.

   Of the above examples, the period of vibration of cesium – 133 atom serves as the most accurate standard of time.

Some techniques used for measuring time intervals over different ranges:

1. ***Electrical oscillators:*** In India, a.c. is supplied at a frequency of 50 Hz. A motor running synchronously with this a.c. can be used to provide a time scale.

2. ***Electronic oscillators:*** A junction transistor can be used to produce oscillations of very high frequency which can be used to measure small time intervals.

3. ***Quartz clocks:*** A quartz crystal shows piezoelectric effect. If fluctuating pressure is applied across a pair of its parallel faces, an oscillatory emf is developed across another pair of perpendicular faces and vice versa. The oscillations produced can be used to measure time intervals.

4. ***Atomic clocks:*** These clocks are based on periodic vibrations taking place within the atoms. The first cesium atomic clock was set up in 1964.

5. ***Decay of elementary particles.*** The life of many elementary particles varies from 10^{-16} to 10^{-24} s. By making use of their decay times, very small time intervals can be measured.

6. ***Radioactive dating:*** This technique is used to measure long time intervals by finding the ratio of the number of radioactive atoms that have undergone decay to the number of atoms left undecayed. Carbon dating is used to estimate the age of fossils, whereas uranium dating is used to estimate the age of the rocks.

Subjective Assignment – V

Q.1 Consider a white dwarf and a neutron star each of one solar mass. The radius of the white dwarf is same as that of the earth ($\simeq$ 6400 km) and the radius of the neutron star is 10 km. Determine the densities of the two types of the stars. Take mass of the sun = 2.0×10^{30} kg.

Q.2 Assume that the mass of a nucleus is given by M = A m_p, where A is the mass number and radius of a nucleus $r = r_0 A^{1/3}$, where r_0 = 12f. Estimate the density of nuclear matter in kg m^{-3}.

Given $m_p = 1.67 \times 10^{-27}$ kg.

Q.3 The average life of an Indian is 56 years. Find the number of times the human heart beats in the life of an Indian, if the heart beats once in 0.8 s.

Answers

1. 1.822×10^9 kg m^{-3}, 4.77×10^{17} kgm^{-3} 2. 2.3×10^{17} kg m^{-3}
3. 2.2×10^9 times

Seven Dimensions of the World

All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities. We call these fundamental quantities as the *seven dimensions of the world,* which are denoted with square brackets [ ].

Dimension of length = [L]

Dimension of mass = [M]

Dimension of time = [T]

Dimension of electric current = [A]

Dimension of thermodynamic temperature = [K]

Dimension of luminous intensity = [cd]

Dimension of amount of substance = [mol]

Dimensions of a physical quantity

The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely. For example,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length} \times \text{breadth} \times \text{height}}$$

$\therefore$ Dimensions of density

$$= \frac{[M]}{[L][L][L]} = [ML^{-3}] = [M^1 L^{-3} T^o]$$

Hence the dimensions of density are ‘1’ in mass, ‘–3’ in length and ‘0’ in time.

Dimensional formula

The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.

Example: The dimensional formula of the volume is $[M^oL^3T^o]$ and that of momentum is $[MLT^{-1}]$.

Dimensional equation

The equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the given physical quantity.

Example

The dimensional equation of force is $[\text{Force}] = [MLT^{-2}]$

The dimensional equation for pressure is $[\text{Pressure}] = [ML^{-1}T^{-2}]$

Dimensional Formulae and SI units of Some Physical Quantities

SN.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
1	Area	Length × breadth	$L \times L = L^2 = [M^0L^2T^0]$	m^2
2	Volume	Length × breadth × height	$L \times L \times L = [M^0L^3T^0]$	m^3
3	Density	Mass/Volume	$\frac{M}{L^3} = [ML^{-3}T^0]$	kg m^{-3}
4	Speed or Velocity	Distance/Time	$\frac{L}{T} = [M^0LT^{-1}]$	ms^{-1}
5	Acceleration	Change in velocity/Time	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$	ms^{-2}
6	Momentum	Mass × velocity	$M \times LT^{-1} = [MLT^{-1}]$	kg ms^{-1}
7	Force	Mass × acceleration	$M \times LT^{-2} = [MLT^{-2}]$	N
8	Work	Force × distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	J
9	Energy	Amount of work	$[ML^2T^{-2}]$	J
10	Power	Work/Time	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$	W
11	Pressure	Force/Area	$\frac{ML^1T^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Pa or Nm^{-2}
12	Moment of force or torque	Force × $\perp_r$ distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	Nm
13	Gravitational constant ‘G’	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{MLT^{-2}L^2}{M \times M} = [M^{-1}L^3T^{-2}]$	Nm^2 kg^{-2}
14	Impulse of a force	Force × time	$MLT^{-2} \times T = [MLT^{-1}]$	Ns

15	Stress	Force/Area	$\dfrac{MLT^{-2}}{L^2}=[ML^{-1}T^{-2}]$	Nm^{-2}
16	Strain	$\dfrac{\text{Change in dimension}}{\text{Original dimensional}}$	$[M^0L^0T^0]$ (dimensionless)	–
17	Coefficient of elasticity	Stress/Strain	$\dfrac{ML^{-1}T^{-2}}{1}=[ML^{-1}T^{-2}]$	Nm^{-2}
18	Surface tension	Force/Length	$\dfrac{MLT^{-2}}{L}=MT^{-2}=[ML^0T^{-2}]$	Nm^{-1}
19	Coefficient of viscosity	$\dfrac{\text{Force}\times\text{distance}}{\text{Area}\times\text{velocity}}$	$\dfrac{MLT^{-2}\times L}{L^2\times LT^{-1}}=[ML^{-1}T^{-1}]$	Nm^{-2} s or Pa s
20	Angle	Arc/ Radius	$\dfrac{L}{L}=1=[M^0L^0T^0]$(dimensionless)	rad
21	Angular velocity	Angle/ Time	$\dfrac{1}{T}=T^{-1}=[M^0L^0T^{-1}]$	rad s^{-1}
22	Angular acceleration	Angular velocity/ Time	$\dfrac{T^{-1}}{T}=T^{-2}=[M^0L^0T^{-2}]$	rad s^{-2}
23	Moment of inertia	Mass × (distance)^2	$ML^2 = [ML^2T^0]$	kg m^2
24	Radius of gyration	Distance	$L = [M^0L^1T^0]$	m
25	Angular momentum	Mass × velocity × radius	$M \times LT^{-1} \times L = [ML^2T^{-1}]$	kg m^2 s^{-1}
26	T–ratios (sin θ, cos θ, tan θ)	Length/ Length	$\dfrac{L}{L}=1=[M^0L^0T^0]$ (dimensionless)	–
27	Time period	Time	$T = [M^0L^0T]$	s
28	Frequency	1/ Time period	$\dfrac{1}{T}=T^{-1}=[M^0L^0T^{-1}]$	s^{-1} or Hz
29	Planck's constant 'h'	$\dfrac{E}{v}=\dfrac{\text{Energy}}{\text{Frequency}}$	$\dfrac{ML^2T^{-2}}{T^{-1}}=[ML^2T^{-1}]$	Js
30	Relative density	$\dfrac{\text{Density of substance}}{\text{Density of water at }4^\circ C}$	$\dfrac{ML^{-3}}{ML^{-3}}=1=[M^0L^0T^0]$ (dimensionless)	–
31	Velocity gradient	Velocity/ Distance	$\dfrac{LT^{-1}}{L}=T^{-1}=[M^0L^0T^{-1}]$	s^{-1}
32	Pressure gradient	Pressure/ Distance	$\dfrac{ML^{-1}T^{-2}}{L}=[ML^{-2}T^{-2}]$	Pa m^{-1}
33	Force constant	Force/ Displacement	$\dfrac{MLT^{-2}}{L}=MT^{-2}=[ML^0T^{-2}]$	Nm^{-1}

Subjective Assignment – VI

Q.1 Name of physical quantities whose dimensional formulae are as follows:

(i) ML^2T^{-2} (ii) ML^2T^{-3} (iii) MT^{-2}

(iv) $ML^{-1}T^{-1}$ (v) $ML^{-1}T^{-2}$

Q.2 Deduce the dimensional formulae for the following physical quantities:

(i) Gravitational constant (ii) Power (iii) Young's modulus

(iv) Coefficient of viscosity (v) Surface tension (vi) Planck's constant

Q.3 Deduce the dimensional formulae of the following physical quantities:

(i) Heat (ii) Specific heat (iii) Latent heat (iv) Gas constant

(v) Boltzmann's constant (vi) Coefficient of thermal conductivity

(vii) Mechanical equivalent of heat

Answers

1. (i) Work, (ii) Power,. (iii) Surface tension or force constant,
   (iv) Coefficient of viscosity, (v) Pressure or stress
2. (i) $[M^{-1}L^{3}T^{-2}]$, (ii) $[ML^{2}T^{-3}]$, (iii) $[ML^{-1}T^{-2}]$,
   (iv) $[ML^{-1}T^{-1}]$, (v) $[MT^{-2}]$, (vi) $[ML^{2}T^{-1}]$
3. (i) $[ML^{2}T^{-2}]$, (ii) $[L^{2}T^{-2}K^{-1}]$, (iii) $[L^{2}T^{-2}]$,
   (iv) $[ML^{2}T^{-2}K^{-1}\ mol^{-1}]$, (v) $[ML^{2}T^{-2}K^{-1}]$, (vi) $[MLT^{-3}K^{-1}]$, (vii) $[M^{0}L^{0}T^{0}]$

Different Types of Variables and Constants

On the basis of dimensions, we can classify quantities into *four* categories:

(1) **Dimensional variables:** *The physical quantities which possess dimensions and have variable values are called dimensional variables.*

Examples: Area, Volume, Velocity, Force, etc

(2) **Dimensionless variables:** *The physical quantities which have no dimensions but have variable values are called dimensionless variables.*

Examples: Angle, Specific gravity, Strain, etc

(3) **Dimensional constants:** *The physical quantities which possess dimensions and have constant values are called dimensional constants.*

Examples: Gravitational constant, Planck's constant, etc

(4) **Dimensionless constants:** *The constant quantities having no dimensions are called dimensionless constants.*

Examples: π, e, etc

Applications of Dimensional Analysis

The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis. Following are the three main uses of dimensional analysis:

1. To convert a physical quantity from one system of units to another.
2. To check the correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

To convert a physical quantity from one system of units to another

If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are the corresponding numerical values, then $Q = n_1u_1 = n_2u_2$

Let M_1, L_1 and T_1 be the sizes of fundamental units of mass, length and time in one system; and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity Q be $M^a L^b T^c$, then

$$u_1 = M_1^a\ L_1^b\ T_1^c \qquad \text{and} \qquad u_2 = M_2^a\ L_2^b\ T_2^c$$

$$\therefore \qquad n_1\left[M_1^a\ L_1^b\ T_1^c\right] = n_2\left[M_2^a\ L_2^b\ T_2^c\right] \qquad \text{or} \qquad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

This equation can be used to find the numerical value in the second or new system of units.

Example:

Let us convert one joule into erg. Joule is SI unit of energy and erg is the CGS unit of energy. Dimensional formula of energy is ML^2T^{-2}.

$\therefore$ $a = 1,. b = 2, c = -2$

SI	**CGS**
$M_1 = 1\text{kg} = 1000\text{ g}$	$M_2 = 1\text{ g}$
$L_1 = 1\text{m} = 100\text{ cm}$	$L_2 = 1\text{ cm}$
$T_1 = 1\text{s}$	$T_2 = 1\text{s}$
$n_1 = 1$ (joule)	$n_2 = ?$ (erg)

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^2\left[\frac{1}{1}\right]^{-2} = 1 \times 10^3 \times 10^4 = 10^7$$

$\therefore$ 1 joule = 10^7 erg

Note:

- The above conversion technique is applicable to only absolute systems of units. The gravitational or other practical units must be first converted into absolute units before using the above technique.

Subjective Assignment – VII

Q.1 The value G in CGS system is 6.67×10^{-8} dyne cm^2 g^{-2}. Calculate the value in SI units.

Q.2 Find the value of 60 J per min on a system that has 100g, 100 cm and 1 min as the base units.

Q.3 In CGS system, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}. Find its value in SI units. Given 1 J = 10^7 erg.

Q.4 If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass?

Q.5 Young's modulus of steel is 19×10^{10} Nm^{-2}. Express it in cgs units.

Q.6 Convert a power of one mega watt on a system whose fundamental units are 10 kg, 1 dm and 1 minute.

Q.7 When one metre, one kg and one minute are taken as fundamental units, the magnitude of a force is 36 units. What is the value of this force on cgs system?

Q.8 the normal duration of 1 year Science, Physics practical period in Indian colleges is 100 minutes. Express this period in micro centuries. 1 micro century = $10^{-6} \times 100$ years.

Answers

1.	$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}	2.	2.16×10^6 new units of power
3.	5.67×10^{-8} Js^{-1}m^{-2} K^{-4}	4.	10^4 kg
5.	19×10^{11} dyne/cm^2	6.	2.16×10^{12} new units of power
7.	10^3 dyne	8.	1.90 micro century

Principle of homogeneity of dimensions

According to this principle, a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same. This principle is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared. Thus, velocity can be added to velocity but not to force.

To check the dimensional correctness of a physical equation

For this purpose we make use of the principle of homogeneity of dimensions. If the dimensions of all the terms on the two sides of the equation are same, then the equation is dimensionally correct.

Example:

Let us check the dimensional accuracy of the equation of motion, $s = ut + \frac{1}{2}at^2$

Dimensions of different terms are

$$[s] = [L]$$
$$[ut] = [LT^{-1}]\,[T] = [L]$$
$$\left[\frac{1}{2}at^2\right] = [LT^{-2}]\,[T^2] = [L]$$

As all terms on both sides of equations have same dimensions, so the given equation is dimensionally correct.

Note:

- A dimensionally correct equation need not be actually a correct equation, but a dimensionally inconsistent equation must be wrong. The equation of motion: $s = ut + at^2$ is dimensionally correct but numerically it is wrong.
- Principle of homogeneity of dimensions is the consistency test for any equation. If an equation fails this test, it is proved wrong. But if the equation passes this consistency test, it is not necessarily proved right. Why?

Reason: The consistency test for any equation is the principle of homogeneity of dimensions, i.e., an equation is correct, when dimensions/powers of M, L, T on one side of the equation are equal to their respective values/powers on the other side of the equation. When power of even one of M, L, T fails to match, the equation is wrong.

However, if an equation passes this consistency test, it may still not be right. This is because mere dimensional correctness of an equation does not ensure its physical correctness. For example, work = torque is dimensionally correct, but not physically correct. Work is a scalar which is equivalent to energy. Torque is a vector, which represents turning effect of a force.

Subjective Assignment – VIII

Q.1 The distance x traveled by a body in time t which starts from the position x_0 with initial velocity u_0 and has uniform acceleration a, is given by $x = x_0 + u_0t + \frac{1}{2}at^2$. Check the dimensionally consistency of this equation.

Q.2 Check whether the following equation is dimensionally correct. $\frac{1}{2}mv^2 = mgh$

Q.3 Check the correctness of the equation $FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ where F is the force acting on a body of mass m and S is the distance moved by the body when its velocity changes from u to v.

Q.4 Check the correctness of the relation $\tau = I\alpha$, where τ is the torque acting on a body, I is the moment of inertia and α is angular acceleration.

Q.5 Check the dimensional consistency of the following equations:

(i) de–Broglie wavelength, $\lambda = h/mv$ (ii) Escape velocity, $v = \sqrt{\frac{2\,GM}{R}}$

Q.6 Check by the method of dimensions whether the following equations are correct:

(i) $E = mc^2$ (ii) $T = 2\pi\sqrt{\frac{l}{g}}$

(iii) $v = \sqrt{\frac{P}{\rho}}$, where v = velocity of sound, P = pressure and ρ = density of medium

(iv) $v = \frac{1}{2l}\sqrt{\frac{T}{m}}$, where v = frequency of vibration, l = length of the string, T = tension in the string and m = mass per unit length.

Q.7 By the method of dimensions, test the accuracy of the equation: $\delta = \frac{mgl^3}{4b\,d^3\,Y}$

where δ is the depression produced in the middle of a bar of length l, breadth b and depth d, when it is loaded in the middle with mass m. Y is the Young's modulus of the material of the bar.

Q.8 Find the dimensions of a/b in the equation: $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

Q.9 Find the dimensions of a × b in the relation: $P = \frac{b - x^2}{at}$; where P is power, x is distance and t is time.

Q.10 The Vander Wall's equation for a gas is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Determine the dimensions of a and b. Hence write the SI units of a and b.

Q.11 In the equation: $y = a \sin(\omega t - kx)$, t and x stand for time and distance respectively. Obtain the dimensional formula for ω and k.

Q.12 Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments:

(i) $\frac{3}{16}mv^2$ (ii) $\frac{1}{2}mv^2 + ma$

Q.13 The number of particles crossing a unit area perpendicular to X–axis in unit time is given by

$$n = -D\frac{n_2 - n_1}{x_2 - x_1}$$

where n_1 and n_2 are number of particles per unit volume for the values of x meant to be x_1 and x_2. Find the dimensions of the diffusion constant D.

Q.14 The velocity v of a particle depends upon time t, according to the equation $v = a + bt + \dfrac{c}{d+t}$. Write the dimensions of a, b, c and d.

Answers

1.	Correct	2.	Correct	3.	Correct	4.	Correct
5.	(i) Correct (ii) Correct						
6.	(i) Correct (ii) Correct (iii) Correct (iv) Correct			7.	Correct	8.	$L^{-1/2}T^{2}$
9.	$M^{-1} L^{2} T^{2}$	10.	$[a] = [ML^{5}T^{-2}]$, $[b] = [L^{3}]$, $a = kg\ m^{5}s^{-2}$, $b = m^{3}$				
11.	T^{-1}, L^{-1}	12.	(i) acceptable (ii) not acceptable			13.	$[L^{2}T^{-1}]$
14.	$[LT^{-1}]$, $[LT^{-2}]$, $[L]$, $[T]$						

To derive the relationship among physical quantities

By making use of the homogeneity of dimensions, we can derive an expression for a physical quantity if we know the various factors on which it depends.

Example

Let us derive an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r.

Let $F \propto m^{a}\ v^{b}\ r^{c}$ or $F = K\ m^{a}\ v^{b}\ r^{c}$ …. (1)

where K is a dimensionless constant. Writing the dimensions of various quantities in equation (1), we get

$$[MLT^{-2}] = 1\ [M]^{a}\ [LT^{-1}]^{b}\ [L]^{c} \quad \text{or} \quad M^{1}L^{1}T^{-2} = M^{a}\ L^{b+c}\ T^{-b}$$

Comparing the dimensions of similar quantities on both sides, we get

$a = 1$ $\quad$ $b = 2$

$b + c = 1$ $\quad$ $2 + c = 1, \Rightarrow c = -1$

$-2 = -b$

From equation (1), we get

$$F = K\ mv^{2}\ r^{-1} = K\ \frac{mv^{2}}{r}.$$

This is the required expression for the centripetal force.

Subjective Assignment – IX

Q.1 Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation 'T' of the simple pendulum depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity 'g' at the place. Derive the expression for its time period using method of dimensions.

Q.2 The velocity 'v' of water waves depends on the wavelength 'λ', density of water 'ρ' and the acceleration due to gravity 'g'. Deduce by the method of dimensions the relationship between these quantities.

Q.3 Assuming that the mass M of the largest stone that can be moved by a flowing river depends upon 'v' the velocity, 'ρ' the density of water and on 'g', the acceleration due to gravity. Show that M varies with the sixth power of the velocity of flow.

Q.4 The velocity of sound waves 'v' through a medium may be assumed to depend on:
(i) the density of the medium 'd' and (ii) the modulus of elasticity 'E'
Deduce by method of dimensions the formula for velocity of sound. Take dimensional constant K = 1.

Q.5 The frequency 'v' of vibration of a stretched string depends upon:
(i) its length 'l', (ii) its mass per unit length 'm' and
(iii) the tension 'T' in the string.
Obtain dimensionally an expression for frequency 'v'.

Q.6 A planet moves around the sun in nearly circular orbit. Its period of revolution 'T' depends upon:
(i) radius 'r' of orbit (ii) mass 'M' of the sun and (iii) the gravitational constant G
Show dimensionally that $T^2 \propto r^3$.

Q.7 Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid 'ρ', its average speed 'v' and coefficient of viscosity 'η'. Given that N_R is also directly proportional to 'D' (the diameter of the pipe), show from dimensional considerations that $N_R \propto \dfrac{\rho\, v\, D}{\eta}$

Q.8 Derive the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate off flow of liquid depends on
(i) the coefficient of viscosity 'η' of the liquid (ii) the radius 'r' of the pipe and
(iii) the pressure gradient (p/l) along the pipe. Take K = π/8.

Q.9 The period of vibration 'T' of a tuning fork depends on the length 'l' of its prong, density 'd' and Young's modulus 'Y' of its material. Deduce an expression for the period of vibration on the basis of dimensions.

Q.10 The frequency 'v' of an oscillating drop may depend upon radius 'r' of the drop, density 'ρ' of the liquid and surface tension 'S' of the liquid. Establish an expression for 'v' dimensionally.

Q.11 The escape velocity 'v' of a body depends upon (i) the acceleration due to gravity 'g' of the planet and (ii) the radius of the planet 'R'. Establish dimensionally the relationship between v, g and R.

Q.12 A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation (T) in terms of radius of star (R), mean density of fluid (ρ) and universal gravitational constant G.

Q.13 Taking velocity, time and force as the fundamental quantities, find the dimensions of mass.

Q.14 If density ρ, acceleration due to gravity g and frequency v are the basic quantities, find the dimensions of force.

Q.15 If the velocity of light c, acceleration due to gravity g and atmospheric pressure p are the fundamental quantities, find the dimensions of length.

Answers

1.	$T = 2\pi\sqrt{\dfrac{l}{g}}$	2.	$v = K\sqrt{\lambda g}$	4.	$v = \sqrt{\dfrac{E}{d}}$
5.	$v = \dfrac{K}{l}\sqrt{\dfrac{T}{m}}$	8.	$V = \dfrac{\pi r^4 p}{8\eta l}$	9.	$T = Kl\sqrt{\dfrac{d}{Y}}$
10.	$v = K\sqrt{\dfrac{S}{\rho r^3}}$	11.	$v = K\sqrt{gR}$	12.	$T = K\dfrac{1}{\sqrt{\rho G}}$
13.	$[FTV^{-1}]$	14.	$[\rho g^4 v^{-6}]$	15.	$\left[\dfrac{c^2}{g}\right]$

Limitations of the method of dimensions

1. The method does not give any information about the dimensionless constant K.
2. It fails when a physical quantity depends on more than three physical quantities.
3. It fails when a physical quantity (e.g., $s = ut + 1/2\ at^2$) is the sum or difference of two or more quantities.
4. If fails to derive relationships which involve trigonometric, logarithmic or exponential functions.
5. Sometimes, it is difficult to identify the factors on which the physical quantity depends. The method becomes more complicated when dimensional constants like G, h, etc. are involved.
6. It gives no information whether a physical quantity is a scalar or vector.

Significant figures

The significant figures are normally those digits in a measured quantity which are known reliably or about which we have confidence in our measurement plus one additional digit that is uncertain.

Rules for determining the number of significant figures

(i) All non zero digits are significant. So 13.75 has four significant figures.

(ii) All zeros between two non–zero digits are significant. Thus 100.05 km has five significant figures.

(iii) All zeros to right of a non–zero digit but to the left of an understood decimal point are not significant. For example, 86400 has three significant figures.

(iv) All zeros to the right of a non–zero digit but to the left of a decimal point are significant. For example, 6487.00 has six significant figures.

(v) All zeros to the right of a decimal point are significant. So 161 cm, 161.0 cm and 161.00 cm have three, four and five significant figures respectively.

(vi) All zeros to the right of a decimal point but to the left of a non–zero digit are not significant. So 0.161 cm and 0.0161 cm, both have three significant figures. Moreover, zero conventionally placed to the left of the decimal point is not significant.

(vii) The number of significant figures does not depend on the system of units. So 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.

Scientific Notation

In scientific notation, a number is expressed in the power of 10 as $a \times 10^b$, where a is the number between 1 and 10, and b is any positive or negative exponent of 10. The decimal point is written after the first digit.

Example

$3.500\text{m} = 3.500 \times 10^2 \text{ cm} = 3.500 \times 10^3 \text{ mm} = 3.500 \times 10^{-3} \text{ km}$

The power of 10 is not relevant to the determination of significant figures. Each of the above numbers has four significant figures.

Rules for rounding off a measurement

(i) If the digit to be dropped is smaller than 5, then the preceding digit is left unchanged.

(ii) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(iii) If the digit to be dropped is 5 followed by non–zero digits, then the preceding digit is increased by 1.

(iv) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(v) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

Arithmetic operations with significant figures

1. Significant figures in the sum or difference of two numbers

In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places.

2. Significant figures in the product or quotient of two numbers

In multiplication or division, the final result should be reported to the same number of significant figures as that of the original number with minimum number of significant figures.

Subjective Assignment – X

Q.1 State the number of significant figures in the following:

(i) 2.000 m, (ii) 5100 kg (iii) 0.050 cm

Q.2 Round off the following number as indicated:

(i) 18.35 upto 3 digits (ii) 143.45 upto 4 digits (iii) 18967 upto 3 digits

(iv) 12.653 upto 3 digits (v) 248337 upto 3 digits (vi) 321.135 upto 5 digits

(vii) 101.55×10^6 upto 4 digits (viii) 31.325×10^{-5} upto 4 digits

Q.3 Add 7.21, 12.141 and 0.0028, and express the result to an appropriate number of significant figures.

Q.4 Subtract 4.27153 from 6.807 and express the result to an appropriate number of significant figures.

Q.5 Subtract 2.5×10^{-6} from 4.0×10^{-4} with due regard to significant figures.

Q.6 Solve the following and express the result to an appropriate number of significant figures:

(i) Add 6.2g, 4.33g and 17.456 g. (ii) Subtract 63.54 kg from 187.2 kg

(iii) $75.5 \times 125.2 \times 0.51$ (iv) $\dfrac{2.13 \times 24.78}{458.2}$

(v) $\dfrac{2.13 \times 10^{-4} \times 1.81 \times 10^{7}}{0.4463}$

Q.7 Each side of a cube is measured to be 7.203m. What are the total surface area and the volume of the cube to appropriate significant figures?

Q.8 The radius of a sphere is 1.41 cm. Express its volume to an appropriate number of significant figures.

Q.9 The length and the radius of a cylinder measured with vernier calipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.

Q.10 The mass and radius of the earth are 5.975×10^{24} kg and 6.37×10^{6}m respectively. Calculate the average earth's density to correct significant figures. Take $\pi = 3.142$

Q.11 5.74g of a substance occupies 1.2 cm^3. Express its density keeping significant figures in view.

Answers

1.	(i) four: 2, 0, 0, 0	(ii) four: 5, 1, 0, 0	(iii) two: 5, 0		
2.	(i) 18.4	(ii) 143.4	(iii) 1.90×10^4	(iv) 12.7	
	(v) 248000	(vi) 321.14	(vii) 101.6×10^6	(viii) 31.32×10^{-5}	
3.	19.35	4. 2.535	5. 4.0×10^{-4}		
6.	(i) 28.0 g	(ii) 123.7 kg	(iii) 4800	(iv) 0.115	(v) 8.64×10^3
7.	311.3 m^2, 373.7 m^3		8. 11.7 cm^3	9. 43.7 cm^3	
10.	5.52×10^3 kg m^{-3}		11. 4.8g cm^{-3}		

Accuracy and Precision

1. **Accuracy:** *It refers to the closeness of a measurement to the true value of the physical quantity.* It indicates the relative freedom from errors. As we reduce the errors, the measurement becomes more accurate.

2. **Precision:** *It refers to the resolution or the limit to which the quantity is measured.* Precision is determined by the least count of the measuring instrument. *The smaller the least count, greater is the precision.* If we repeat a particular measurement of a quantity a number of times, then the precision refers to the closeness of the set of values so obtained.

Error in Measurement

The error in a measurement is equal to the difference between the true value and the measured value of the quantity.

$$\text{Error} = \text{True value} - \text{Measured value}$$

Different type of errors

1. **Constant errors**

The errors which affect each observation by the same amount are called constant errors. Such errors are due to the faulty calibration of the scale of the measuring instrument. Such errors can be eliminated by measuring the same physical quantity by a number of different methods, apparatus or technique.

2. **Systematic errors**

The errors which tend to occur in one direction, either positive or negative, are called systematic errors. These errors may be of the following types:

(i) ***Instrumental errors:*** These errors occur due to the inbuilt defect of the measuring instrument. For example, wearing off the metre scale at one end, zero error in a vernier calipers (zero of the vernier scale may not coincide with the zero of main scale) etc.

(ii) ***Imperfections in experimental technique:*** *These errors are due to the limitations of the experimental arrangement.* For example, error due to radiation loss in calorimetric experiments, error due to buoyancy of air when we weigh a body in air.

(iii) ***Personal errors:*** *The errors arise due to individuals' bias, lack of proper setting of apparatus or individual's carelessness in taking observations without observing proper precautions, etc.* For example, when an observer (by habit) holds his head towards right, while reading a scale, he introduces some error due to *parallax.*

(iv) ***Errors due to external causes:*** These errors arise due to the change in external conditions like pressure, temperature, wind, etc. For example, the expansion of a scale due to the increase in temperature.

3. Random errors

The errors which occur irregularly and at random, in magnitude and direction, are called random errors. Such errors occur by chance and arise due to slight variation in the attentiveness of the observer while taking the readings or because of slight variations in the experimental conditions.

The random errors are sometimes called the **'chance errors.'** For example, when the same person repeats the same observation, he may get different readings every time. Random errors often follow the well known **'Gaussian Law of Normal Distribution'**. According to this law:

(i) In any measurement x, probability of an error (+ Δx) is same as probability of an error (– Δx)

(ii) In any measurement, small magnitudes of error are more likely than larger magnitudes of error.

The Gaussian law of Normal Distribution is represented graphically in figure.

From the facts stated above and nature of graphs, we come to the conclusion that *arithmetic mean* of a large number of observations is likely to be closer to its 'true value' than any of the individual observations.

Random errors have almost equal chances for both positive and negative errors. Hence the *arithmetic mean* of a large number of observations can be taken as the true value of the measured quantity.

4. Least count error

This error is due to the limitation imposed by the least count of the measuring instrument. It is an uncertainty associated with the resolution of the measuring instrument. *The smallest division on the scale of the measuring instrument is called its least count.*

5. Gross errors or mistakes

These errors are due to either carelessness of the person or due to improper adjustment of the apparatus. No corrections can be applied for gross errors.

For example:

(i) Reading an instrument without setting it properly.

(ii) Taking the observations wrongly without caring for the sources of errors and the precautions.

(iii) Recording the observations wrongly.

(iv) Using wrong values of the observations in calculations.

These errors can be minimized only when the observer is sincere and mentally alert.

Note:

- Least count errors are random errors but within a limited size; they occur with both random and systematic errors.

- The accuracy of measurement is related to the systematic errors but its precision is related to the random errors, which include least count error also.

Absolute Error, Relative Error and Percentage Error

The arithmetic mean of all measurements can be taken as the true value of the measured quantity.

If a_1, a_2, a_3, … a_n be the n measured values of a physical quantity, then its true value, $\bar{a}$ is given by the arithmetic mean,

$$\bar{a} \text{ or } a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i.$$

(i) Absolute error

The magnitude of the difference between the true value of the quantity measured and the individual measured value is called absolute error. If we take arithmetic mean $\bar{a}$ as the true value, then the absolute errors in the individual measured values will be

$$\Delta a_1 = \bar{a} - a_1 \qquad \Delta a_2 = \bar{a} - a_2$$

$$\Delta a_3 = \bar{a} - a_3 \qquad \Delta a_n = \bar{a} - a_n$$

(ii) Mean or final absolute error

The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error. It is given by

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

Thus the final result of the measure of a physical quantity can be expressed as $a = \bar{a} \pm \Delta\bar{a}$. Clearly, any measured value of a will be such that $\bar{a} - \Delta\bar{a} \le a \le \bar{a} + \Delta\bar{a}$

(iii) Relative error

The ratio of the mean absolute error to the true value of the measured quantity is called relative error.

$$\text{Relative error, } \delta a = \frac{\Delta \bar{a}}{a}$$

(iv) Percentage error

The relative error expressed in percent is called percentage error.

$$\text{Percentage error} = \frac{\Delta \bar{a}}{a} \times 100\%$$

Note:

- The unit of absolute error is same as that of the quantity being measured.
- It is the relative error or the percentage error and not the absolute error which truly indicates the accuracy of a measured.

Subjective Assignment – XI

Q.1 The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m, and 2.48m. Find the average length and the percentage error.

Q.2 In successive measurements, the readings of the period of oscillation of a simple pendulum were found to be 2.63s, 2.56s, 2.42s, 2.71s and 2.80s in an experiment. Calculate (i) mean value of the period of oscillation (ii) mean absolute error (iii) relative error (iv) percentage error and (v) express the result in proper form.

Q.3 In an experiment, refractive index of glass was observed to be 1.45, 1.56, 1.54, 1.44, 1.54 and 1.53. Calculate (i) Mean value of refractive index; (ii) Mean absolute error; (iii) Fractional error; (iv) Percentage error. Express the result in terms of absolute error and percentage error.

Answers

1. 2.48 ± 0.01 m, 0.40 %
2. (i) 2.62s, (ii) 0.11s, (iii) 0.04, (iv) 4%, (v) (2.62 ± 0.11)s, $(2.62 \pm 4\%)$s
3. (i) 1.51, (ii) ≈ 0.04, (iii) 0.03, (iv) 3%, $\mu = 1.51 \pm 0.04$, $\mu = 1.51 \pm 3\%$

Combination of Errors

(i) ***Error in the sum of two quantities:*** Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then

Measured value of $A = A \pm \Delta A$

Measured value of $B = B \pm \Delta B$

Consider the sum, $Z = A + B$

The error ΔZ in Z is then given by

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$
$$= (A + B) \pm (\Delta A + \Delta B)$$
$$= Z \pm (\Delta A + \Delta B) \quad \text{or} \quad \Delta Z = \Delta A + \Delta B$$

Hence the rule: The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

(ii) ***Error in the difference of two quantities:*** Consider the difference,

$$Z = A - B$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$
$$= (A - B) \pm \Delta A \mp \Delta B$$
$$= Z + \Delta A \mp \Delta B$$

For error ΔZ to be maximum, ΔA and ΔB must have the same sign, therefore

$$\Delta Z = \Delta A + \Delta B$$

Hence the rule: The maximum error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

(iii) ***Error in the product of two quantities***: Consider the product,

$$Z = AB$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\,\Delta B \pm B\Delta A \pm \Delta A.\,\Delta B$$

Dividing L.H.S. by Z and R.H.S. by AB, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

As $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small quantities, their product term can be neglected. The maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule: The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

(iv) **Error in the division or quotient:** Consider the quotient,

$$Z = \frac{A}{B}$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta A} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$= \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

or $$Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$[\because (1 + x)^n \simeq 1 + nx, \text{ when } x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

As the terms $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small, their product term can be neglected. The maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule: The maximum fractional error in the quotient of two quantities is equal to the sum of their individual fractional errors.

(v) **Error in the power of a quantity:** Consider the n^{th} power of A,

$$Z = A^n$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)^n = A^n \left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$= Z\left(1 \pm n\frac{\Delta A}{A}\right)$$

$[\because (1+x)^n \simeq 1 + nx, \text{ when } x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A} \qquad \text{or} \qquad \frac{\Delta Z}{Z} = n.\frac{\Delta A}{A}$$

Hence the rule

The fractional error in the nth power of a quantity is n times the fractional error in that quantity.

General rule

If $Z = \dfrac{A^p\, B^q}{C^r}$, then maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

The percentage error in Z is given by

$$\frac{\Delta Z}{Z}\times 100 = p\frac{\Delta A}{A}\times 100 + q\frac{\Delta B}{B}\times 100 + r\frac{\Delta C}{C}\times 100$$

$\Rightarrow$ $Z\% = p \times A\% + q \times B\% + r \times C\%$

Subjective Assignment – XII

Q.1 Two resistances $R_1 = 100 \pm 3\Omega$ and $R_2 = 200 \pm 4\Omega$ are connected in series. What is their equivalent resistance?

Q.2 The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.

Q.3 The initial and final temperatures of a water bath are $(18 \pm 0.5)^oC$ and $(40 \pm 0.3)^oC$. What is the rise in temperature of the bath?

Q.4 The resistance R = V/1, where $V = 100 \pm 5V$ and $I = 10 \pm 0.2$ A. Find the percentage error in R.

Q.5 If the errors involved in the measurements of a side and mass of a cube are 3% and 4% respectively, what is the maximum permissible error in the density of the material?

Q.6 The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits.

Q.7 The error in the measurement of radius of a sphere is 2%. What would be the error in the volume of the sphere?

Q.8 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed?

Q.9 The length, breadth and height of a rectangular block of wood were measured to be:

$l = 12.13 \pm 0.02$ cm; $b = 8.16 \pm 0.01$ cm; $h = 3.46 \pm 0.01$ cm

Determine the percentage error in the volume of the block.

Q.10 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. What is the accuracy in the determination of g?

Q.11 A physical quantity x is calculated from the relation $x = \frac{a^2 b^3}{c\sqrt{d}}$. If percentage error in a, b, c, d are 2%, 1%, 3% and 4% respectively, what is the percentage error in x?

Q.12 For the estimation of Young's modulus: $Y = \frac{4Mg}{\pi d^2}.\frac{L}{l}$

for the specimen of a wire, following observations were recorded: L = 2.890, M = 3.00, d = 0.082, g = 9.81, l = 0.087. Calculate the maximum percentage error in the value of Y and mention which physical quantity causes maximum error.

Q.13 Find the relation error in Z if $Z = \frac{A^4 B^{1/3}}{CD^{3/2}}$

Q.14 If two resistors of resistances $R_1 = (4 \pm 0.5)\Omega$ and $R_2 = (16 \pm 0.5)\Omega$ are connected (i) in series and (ii) in parallel; find the equivalent resistance in each case with limits of percentage error.

Q.15 It is required to find the volume of a rectangular block. A vernier caliper is used to measure the length, width and height of the block. The measured values are found to be 1.37 cm, 4.11 cm and 2.56 cm respectively. Calculate correctly, the volume of the block.

Q.16 Two clocks are being tested against a standard clock located in a national laboratory. At 12 : 00 : 00 noon by the standard clock, the readings of the two clocks are:

	Clock I	Clock II
Monday	12 : 00 : 05	10 : 15 : 06
Tuesday	12 : 01 : 15	10 : 14 : 59
Wednesday	11 : 59 : 08	10 : 15 : 18
Thursday	12 : 01 : 50	10 : 15 : 07
Friday	11 : 59 : 15	10 : 14 : 53
Saturday	12 : 01 : 30	10 : 15 : 24
Sunday	12 : 01 : 19	10 : 15 : 11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer?

Q.17 In an experiment in determining the density of a rectangular block, the dimensions of the block are measured with a vernier caliper with a least count of 0.01 cm and its mass is measured with a beam balance of least count 0.1g, l = 5.12 cm, b = 2.56 cm, t = 0.37 cm and m = 39.3 g. Report correctly the density of the block.

Q.18 The nth division of main scale coincides with (n + 1)th division of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

Q.19. The pitch of a screw gauge is 1mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47^{th} division on circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in proper significant figures.

Answers

1.	$(300 \pm 7)\Omega$	2.	31.4 ± 0.6 cm	3.	$(22 \pm 0.8)^oC$
4.	7%	5.	13%	6.	(19.0 ± 1.5) sq. cm
7.	6%	8.	8%	9.	0.58%
10.	3%	11.	$\pm$ 12%	12.	3.95%, diameter
14.	(i) $20\Omega \pm 5\%$, (ii) $3.2\ \Omega \pm 20.625\%$			15.	$(14.4 \pm 02)cm^3$
16.	prefer clock II	17.	(8.1 ± 0.3) g cm^{-3}	18.	$\frac{a}{n+1} unit$
19.	2.6 cm^2				

NCERT Exercise

Q.1 Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to .... m^3

(b) The surface area of cube of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to .... $(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers ..... m in 1 s.

(d) The relative density of lead is 11.3. Its density is .... g cm^{-3} or .... kg m^{-3}

Q.2 Fill in the blanks by suitable conversion of units:

(a) 1 kg m^2 s^{-2} = ... g cm^2 s^{-2} (b) 1 m = ... light year (c) 3 ms^{-2} = ... km h^{-2}

(d) G = 6.67×10^{-11} Nm^2 kg^{-2} = ... cm^3 s^{-2} g^{-1}

Q.3 A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 1 kg m^2 s^{-2}. Suppose, we employ a system of units in which the unit of mass equals α kg, the unit of length equal β m and the unit of time is γ s. Show that a calorie has a magnitude of 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.

Q.4 Explain this statement clearly: (i) To call a dimensionless quantity 'large' or 'small' is meaningless without specifying a standard for comparison. (ii) In view of this, reword the following statements, wherever necessary.

(a) Atoms are very small objects (b) A jet plane moves with great speed

(c) The mass of Jupiter is very large

(d) The air inside this room contains a large number of molecules

(e) A proton in much more massive than an electron

(f) The speed of sound is much smaller than the speed of light.

Q.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between sun and the earth in terms of the new unit, if light takes 8 min and 20 sec. to cover the distance?

Q.6 Which of the following is the most precise device for measuring length? (a) a Vernier callipers with 20 divisions on the sliding scale, coinciding with 19 main scale divisions (b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale (c) an optical instrument that can measure length to within a wave length of light.

Q.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is his estimate on the thickness of hair?

Q.8 (a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?

(b) A screw gauge has pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gauge arbitrarily by increasing the number of divisions on the circular scale?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Q.9 The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector screen arrangement?

Q.10 State the number of significant figures in the following:

(a) 0.007 m^2 (b) 2.64×10^{24} kg (c) 0.2370 g cm^{-3} (d) 6.320 J

(e) 6.032 N m^{-2} (f) 0.0006032 m^2

Q.11 The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Q.12 The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures.

Q.13 A physical quantity P is related to four observables a, b, c and d as follows: $P=\dfrac{a^3b^2}{\left(\sqrt{c}\,d\right)}$. The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Q.14 A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion:

(i) $y=a\sin\dfrac{2\pi t}{T}$ (ii) $y=a\sin v\,t$ (iii) $y=\dfrac{a}{T}\sin\dfrac{t}{a}$ (iv) $y=\dfrac{a}{\sqrt{2}}\left[\sin\dfrac{2\pi t}{T}+\cos\dfrac{2\pi t}{T}\right]$

Q.15 A famous relation in Physics relates moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A body recalls the relation almost correctly but forgets where to put

the constant c. He writes $m=\dfrac{m_0}{(1-v^2)^{1/2}}$. Guess where to put the missing c?

Q.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Q.17 One mole of an ideal gas at NTP occupies 22.4 litre (molar volume). What is ratio of molar volume to atomic volume of a mole of hydrogen? Take size of H_2 to be 1Å. Why is this ratio so large?

Q.18 Explain this common observation clearly. If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the moon, the stars etc). seem to the stationary.

Q.19 The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11}$ m. However even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

Q.20 The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of par sec? How much parallax would this star show when viewed from two locations of the earth six months apart in its orbit around the sun?

Q.21 Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Q.22 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

(a) the total mass of rain–bearing clouds over Indian during the Monsoon

(b) the mass of an elephant

(c) the wind speed during a storm

(d) the number of strands of hair on your head,

(e) the number of air molecules in your class room

Q.23 The sun is a hot plasma (ionised matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At such high temps, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be? In the range of densities of solids, liquids or gases? check if your guess is correct from the following data: mass of sun = 2.0×10^{30} kg; radius of the sun = 7.0×10^8 m

Q.24 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.27" of arc. Calculate the diameter of Jupiter?

Q.25 A man walking briskly in rain with speed v must slant his unmbrella forward making an angle (with the vertical). A student derives the following relation between θ and v: $\tan \theta = v$ and checks that the relation has a correct limit: as $v \to 0$, $\theta \to 0$ as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Q.26 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02s. What does this imply for the accuracy of the standard cesium clock in measuring a time interval of 1s?

Q.27 Estimate the average atomic mass density of a sodium atom, assuming its size to be 2.5Å. Compare it with density of sodium in its crystalline phase (970 kg m^{-3}). Are the two densities of the same order of magnitude? If so, why?

Q.28 The unit of length convenient on nuclear scale is a Fermi, 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation: $r = r_0 A^{1/3}$, where r is radius of the nucleus and r_0 is a constant equal to 1.2f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with average mass density of sodium atom in (4.67×10^3 kg/m^3)

Q.29 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Q.30 A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and located objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0s. What is the distance of the enemy submarine? (Speed of sound in water = 14.50 ms^{-1})

Q.31 The farthest objects in our universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the earth. These objects (known as quasers) have many puzzling features, which have yet not been satisfactorily explained. What is the distance in km of a quasar from which light take 3.0 billion years to reach us?

Answers

1.	(a) 10^{-6}, (b) 1.26×10^4, (c) 5, (d) 11.3, 11.2×10^3				
2.	(a) 10^7, (b) 1.053×10^{-16}, (c) 3.888×10^4, (d) 6.67×10^{-8}			6.	(c)
9.	94.1	10.	(a) 1, (b) 3, (c) 4, (d) 4, (e) 4, (f) 4	11.	8.72 m^2, 0.0855 m^3
12.	(a) 2.3 kg, (b) 0.02 g	13.	$\pm$ 13%, 3.8	14.	(ii) and (iii)
15.	$m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$	16.	3.154×10^{-7} m^3	17.	7.1×10^4
19.	3.1×10^{16} m	20.	1.323 parsec, 1.512 sec	23.	1.392×10^3 kg/m^3
24.	1.429×10^5 km	25.	No, $\tan\theta = v^2/rg$	26.	10^{-12} sec
27.	4.67×10^3 kg/m^3	28.	2.29×10^{17} kg m^{-3}, 4.9×10^{13}	29.	3.84×10^8 m
30.	55825 m	31.	2.84×10^{22} km		

Objective Assignment

Multiple Choice Questions with One Correct Answer

Q.1 Pressure depends on distance as $P = \dfrac{\alpha}{\beta}\exp\left(\dfrac{-\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzmann's constant and θ is temperature. The dimensions of β are

(a) $M^0L^0T^0$ (b) $M^{-1}L^{-1}T^{-1}$ (c) $M^0L^2T^0$ (d) $M^{-1}L^{-1}T^2$

Q.2 A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for

	Δl	ΔT	n
(a)	5 mm	0.2 sec	10
(b)	5 mm	0.2 sec	20
(c)	5 mm	0.1 sec	10
(d)	1 mm	0.1 sec	50

Q.3 A student performs an experiment to determine the Young's modulus of a wire, exactly 2m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of $\pm$ 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of $\pm$ 0.01 mm. Take g = 9.8 m/s^2 (exact). The Young's modulus obtained from the reading is

(a) $2.0 \pm 0.3) \times 10^{11}$ N/m^2 (b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2 (d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

Q.4 A wire has a mass 0.3 $\pm$ 0.003 g, radius 0.5 $\pm$ 0.005 mm and length 6 $\pm$ 0.06 cm. The maximum percentage error in the measurement of its density is

(a) 1 (b) 2 (c) 3 (d) 4

Q.5 Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	**Length of the pendulum (cm)**	**No. of oscillations (n)**	**Total time for (n) oscillations (s)**	**Time period (s)**
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students I, II and III respectively,

(a) $E_{III} = 0$ (b) E_I is maximum (c) $E_I = E_{II}$ (d) E_{II} is maximum

Q.6 The dimensions of length are expressed as $G^x c^y h^z$ whre G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(a) x = (1/2), y = (1/2) (b) x = (1/2), z =(1/2) (c) y = (–3/2),z=(1/2) (d) y=(1/2), z=(3/2)

Q.7 Which of the following represents the correct dimensions of the coefficient of viscosity?

(a) $[ML^{-1}T^{-2}]$ (b) $[ML^{-2}T^{-2}]$ (c) $[ML^{-1}T^{-1}]$ (d) $[MLT^{-1}]$

Q.8 Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale is 35. The diameter of the wire is

(a) 3.32 mm (b) 3.73 mm (c) 3.67 mm (d) 3.38 mm

Q.9 In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half–a–degree (= 0.5^o), then the least count of the instrument is

(a) one minute (b) half minute (c) one degree (d) half degree

Q.10 The time period T of a small drop of liquid (due to surface tension) depends on density ρ, radius r and surface tension S. The relation is

(a) $T \propto \left(\frac{\rho r^3}{S}\right)^{1/2}$ (b) $T \propto \rho r S$ (c) $T \propto \frac{\rho r}{S}$ (d) $T \propto \frac{S}{\rho r}$

Q.11 In an experiment, on the measurement of g, using a simple pendulum, the time period was measured with an accuracy of 0.2% while the length was measured with an accuracy of 0.5%. The percentage accuracy in the value of g thus obtained is

(a) 0.7% (b) 0.1% (c) 0.25% (d) 0.9%

Q.12 The difference in the length of a mean solar day and a sidereal day is about

(a) 1 minute (b) 4 minutes (c) 15 minutes (d) 56 minutes

Q.13 Gravitational mass is proportional to gravitational

(a) field (b) force (c) intensity (d) all of these

Q.14 SONAR emits which of the following waves?

(a) radio (b) light (c) ultrasound (d) none of these

Q.15 Which of the following pair does not have similar dimensions?

(a) stress and pressure (b) angle and strain

(c) tension and surface tension (d) Planck's constant and angular momentum

Q.16 The dimensions of universal gravitational constant are

(a) $[M^{-1}L^3T^{-2}]$ (b) $[ML^2T^{-1}]$ (c) $[M^{-2}L^3T^{-2}]$ (d) $[M^{-2}L^2T^{-1}]$

Q.17 According to Newton, the viscous force acting between liquid layers of area A and velocity gradient $\Delta v/\Delta x$ is given by $F = -\eta A\frac{\Delta v}{\Delta x}$, where η is constant called coefficient of viscosity. The dimensional formula of η is

(a) $[ML^{-2}T^{-2}]$ (b) $[M^0L^0T^0]$ (c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$

Q.18 The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are

(a) [L], [LT] and $[LT^{-2}]$ (b) $[LT^{-2}]$, [L] and [T]

(c) $[L^2]$, [T] and $[LT^{-2}]$ (d) $[LT^{-2}]$, [LT] and [L]

Q.19 P represents radiation pressure, c represents speed of light and S represents radiation energy striking per unit area per sec. The non–zero integers x, y, z such that $P^x S^y c^z$ is dimensionless, are

(a) x = 1, y = 1, z = 1 (b) x = – 1, y = 1, z = 1 (c) x = 1, y = – 1, z = 1 (d) x = 1, y = 1, z = – 1

Q.20 If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be

(a) velocity if a = 1, b = 0, c = – 1
(b) acceleration if a = 1, b = 1, c = – 2
(c) force if a = 0, b = – 1, c = – 2
(d) pressure if a = 1, b = – 1, c = – 2

Q.21 Percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and speed will be

(a) 8% (b) 2% (c) 12% (d) 10%

Q.22 Force F is given by $F = at + bt^2$, where t is time. What are the dimensions of a and b?

(a) $[MLT^{-3}]$ and $[MLT^{-4}]$
(b) $[MLT^{-1}]$ and $[MLT^0]$
(c) $[MLT^{-3}]$ and $[MLT^4]$
(d) $[MLT^{-4}]$ and $[MLT^1]$

Q.23 The dimensional formula of the constant a in van der Waal's gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ is}$$

(a) $[ML^3T^2]$ (b) $[ML^2T^{-1}]$ (c) $[ML^5T^{-2}]$ (d) $[ML^5T^{-3}]$

Q.24 In a system of units, the units of mass, length and time are 1 quintal, 1 km and 1 h respectively. In this system 1 N force will be equal to

(a) 1 new unit (b) 129.6 new units (c) 125.7 new units (d) 10^3 new units

Q.25 The angle subtended by a coin of radius 1 cm held at a distance of 80 cm from your eyes is

(a) 1.43^o (b) 0.72^o (c) 0.0125^o (d) 0.025^o

Q.26 A cube has a side of length 1.2×10^{-2}m. Calculate its volume.

(a) 1.7×10^{-6} m^3 (b) 1.73×10^{-6} m^3 (c) 1.0×10^{-6} m^3 (d) 1.732×10^{-6} m^3

Q.27 Out of the following pairs, which one does not have identical dimensions

(a) Moment of inertia and moment of a force
(b) Work and torque
(c) Angular momentum and Planck's constant
(d) Impulse and momentum

Q.28 Out of the following four dimensional quantities, which one qualifies to be called a dimensional constant?

(a) Acceleration due to gravity
(b) Surface tension of water
(c) Weight of a standard kilogram mass
(d) The velocity of light in vacuum.

Q.29 $[ML^2T^{-2}]$ are dimensions of

(a) force (b) moment of force (c) momentum (d) power

Q.30 The dimensions of Planck's constant are

(a) $[M^2L^2T^{-2}]$ (b) $[MLT^{-2}]$ (c) $[ML^2T^{-2}]$ (d) $[ML^2T^{-1}]$

Q.31 In the relation, y = r sin(ωt + kx), the dimensional formula for kx or ωt is same as

(a) r/ω (b) r/y (c) ωt/r (d) yr/ωt

Q.32 If L = 2.331 cm, B = 2.1 cm, then L + B = ?

(a) 4.431 cm (b) 4.43 cm (c) 4.4 cm (d) 4 cm

Q.33 If error in radius is 3%, what is error in volume of sphere?
(a) 3% (b) 27% (c) 9% (d) 6%

Q.34 Parsec is the unit of
(a) time (b) distance (c) frequency (d)angular momentum

Q.35 Length cannot be measured by
(a) Fermi (b) Debye (c) Micron (d) Light year

Q.36 Which of the following is a dimensionless quantity?
(a) Strain (b) Stress
(c) Specific heat (d) Quantity of heat

Q.37 Turpentine oil is flowing through a tube of length l and radius r. The pressure difference between the two ends of the tube is P. the viscosity of oil is given by $\eta = \frac{P(r^2 - x^2)}{4vl}$, where v is the velocity of oil at a distance x from the axis of the tube. The dimensions of η are
(a) $[M^0L^0T^0]$ (b) $[MLT^{-1}]$ (c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$

Q.38 The ratio of the dimensions of Planck constant and that of moment of inertia is the dimensions of
(a) time (b) frequency (c) angular momentum (d) velocity

Q.39 The time dependence of a physical quantity p is given by $p = p_0 \exp(-\alpha t^2)$, where α is a constant and t is the time. The constant α
(a) is dimensionless (b) has dimensions $[T^{-2}]$
(c) has dimensions $[T^2]$ (d) has dimensions of p

Q.40 The frequency of vibration f of a mass m suspended from a spring of spring constant k is given by a relation $f = am^x k^y$; where a is a dimensionless constant. The values of x and y are
(a) $x=\frac{1}{2}, y=\frac{1}{2}$ (b) $x=-\frac{1}{2}, y=-\frac{1}{2}$ (c) $x=\frac{1}{2}, y=-\frac{1}{2}$ (d) $x=-\frac{1}{2}, y=\frac{1}{2}$

Q.41 A certain body weighs 22.42 g and has a measured volume of 4.7 cc. The possible errors in the measurement of mass and volume are 0.01 g and 0.1 cc. The maximum error in the density will be
(a) 22% (b) 2% (c) 0.2% (d) 0.02%

Q.42 Which of the following is true for the solid angle?
(a) $\delta\omega = \frac{\delta A \cos\theta}{r^2}$ (b) $d\omega = \frac{\delta A \cos^2\theta}{r}$
(c) $\delta\omega = \frac{\delta A \cos\theta}{r^3}$ (d) $\delta\omega = \frac{\delta A \cos^2\theta}{r^3}$

Q.43 The significant figures of the number 6.0023 are
(a) 1 (b) 5 (c) 4 (d) 2

Multiple Choice Questions with One or More than One Correct Answers

Q.44 The dimensions of the quantities of the quantities in one (or more) of the following pairs are the same. Identity the pairs(s)
(a) Torque and work (b) Angular momentum and work
(c) Energy and Young's modulus (d) Light year and wavelength

Q.45 The pairs of physical quantities that have the same dimensions are

(a) Reynold number and coefficient of friction (b) Curie and frequency of a light wave

(c) Latent heat and wavelength (d) Planck's constant and torque

Answers

1	c	2	d	3	b	4	d	5	c
6	b, c	7	c	8	d	9	a	10	a
11	d	12	b	13	b	14	c	15	c
16	a	17	d	18	b	19	c	20	d
21	a	22	a	23	c	24	b	25	a
26	a	27	b	28	d	29	b	30	d
31	b	32	c	33	d	34	b	35	b
36	a	37	d	38	b	39	b	40	d
41	b	42	a	43	b	44	a, d	45	a, b

Objective Assignment – II

Q.1 The number of significant figures in 0.06900 is

(a) 5 (b) 4 (c) 2 (d) 3

Q.2 The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

(a) 663.821 (b) 664 (c) 663.8 (d) 663.82

Q.3 The mass and volume of a body are 4.237g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is

(a) 1.6048 g cm^3 (b) 1.69 g cm^{-3} (c) 1.7 g cm^{-3} (d) 1.695 g cm^{-3}

Q.4 The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give

(a) 2.75 and 2.74 (b) 2.74 and 2.73 (c) 2.75 and 2.73 (d) 2.74 and 2.74

Q.5 The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm respectively. The area of the sheet in appropriate significant figures and error is

(a) 164 $\pm$ 3 cm^2 (b) 163.62 $\pm$ 2.6 cm^2 (c) 163.6 $\pm$ 2.6 cm^2 (d) 163.62 $\pm$ 3 cm^2

Q.6 Which of the following pairs of physical quantities does not have same dimensional formula?

(a) work and torque (b) Angular momentum and Planck's constant

(c) Tension and surface tension (d) Impulse and linear momentum

Q.7 Measure of two quantities along with the precision of respective measuring instrument is

A = 2.5 ms^{-1} $\pm$ 0.5 ms^{-1}, B = 0.10 s $\pm$ 0.01 s. The value of AB will be

(a) (0.25 $\pm$ 0.08) m (b) (0.25 $\pm$ 0.5) m (c) (0.25 $\pm$ 0.05) m (d) (0.25 $\pm$ 0.135) m

Q.8 You measure two quantities as A = 1.0 m $\pm$ 0.2 m, B = 2.0 $\pm$ 0.2 m. We should report correct value for $\sqrt{AB}$ as:

(a) 1.4m $\pm$ 0.4 m (b) 1.41m $\pm$ 0.15 (c) 1.4m $\pm$ 0.3 m (d) 1.4m $\pm$ 0.2 m

Q.9 Which of the following measurements is most precise?

(a) 5.00 mm (b) 5.00 cm (c) 5.00 m (d) 5.00 km

Q.10 The mean length of an object is 5 cm. Which of the following measurements is most accurate?

(a) 4.9 cm (b) 4.805 cm (c) 5.25 cm (d) 5.4 cm

Q.11 Young's modulus of steel is 1.9×10^{11} N/m^2. When expressed in CGS units of dynes/cm^2, it will be equal to (1N = 10^5 dyne, 1 m^2 = 10^4 cm^2)

(a) 1.9×10^{10} (b) 1.9×10^{11} (c) 1.9×10^{12} (d) 1.9×10^{13}

Q.12 If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula

(a) $(P^1 A^{-1} T^1)$ (b) $(P^2 A^1 T^1)$ (c) $(P^1 A^{-1/2} T^1)$ (d) $P^1 A^{1/2} T^{-1})$

Q.13 On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct:

(a) y = a sin 2 πt/T (b) y = a sin *v*t (c) $y=\frac{a}{T}\sin\left(\frac{t}{a}\right)$ (d) $y=a\sqrt{2}\left(\sin\frac{2\pi t}{T}-\cos\frac{2\pi t}{T}\right)$

Q.14 If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

(a) (P – Q)/R (b) PQ – R (c) PQ/R (d) (PR – Q^2)/R (e) (R + Q)/P

Q.15 Photon is quantum of radiation with energy E = h*v* where *v* is frequency and h is Planck's constant. The dimensions of h are the same as that of

(a) Linear impulse (b) Angular impulse (c) Linear momentum (d) Angular momentum

Q.16 If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?

(a) Mass of electron (m_e) (b) Universal gravitational constant (G)

(c) Charge of electron (e) (d) Mass of proton (m_p)

Q.17 Which of the following ratios express pressure?

(a) Force/Area (b) Energy/Volume (c) Energy/Area (d) Force/Volume

Q.18 Which of the following are not a unit of time?

(a) Second (b) Parsec (c) Year (d) Light year

Q.19 A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) (are) true?

[IIT JEE, 2010]

(a) Error ΔT in measuring T, the time period, is 0.05 seconds

(b) Error ΔT in measuring T, the time period, is 1 second

(c) Percentage error in the determination of g is 5%

(d) Percentage error in the determination of g is 2.5%

Q.20 A student measures the distance traversed in free fall of a body, initially at rest, in a given time. He uses this data to estimate g, the acceleration due tot gravity. If the maximum percentage errors in measurement of the distance and the time are e_1 and e_2 respectively, the percentage error in the estimate of g is **[AIPMT Mains, 2010]**

(a) $e_2 - e_1$ (b) $e_1 + 2e_2$ (c) $e_1 + e_2$ (d) $e_1 - 2e_2$

Q.21 Choose the correct statement (s):

(a) A dimensionally correct equation may be correct

(b) A dimensionally correct equation may be incorrect

(c) A dimensionally incorrect equation may be correct

(d) A dimensionally incorrect equation may be incorrect

Q.22 The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

[IIT JEE, 2011]

(a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

Q.23 A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading: 0 mm. Circular scale reading: 52 divisions. Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is

[AIEEE, 2011]

(a) 0.052 cm (b) 0.026 cm (c) 0.005 cm (d) 0.52 cm

Q.24 The density of a material in CGS system of units is 4 g/cm^3. In a system of units in which unit of length is 10 cm and unit of mass is 100g, the value of density of material will be

[AIPMT Mains, 2011]

(a) 0.4 (b) 40 (c) 400 (d) 0.04

Comprehension Type Questions

Whether a given relation/formula is correct or not can be checked on the basis of the *principle of homogeneity of dimensions.* According to this principle, only that formula is correct, in which the dimensions of the various terms on one side of the relation are equal to the respective dimensions of these terms on the other side of the relation. With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions:

Q.25 The distance travelled by a body in nth second is given by $S_{nth} = u + \frac{a}{2}(2n-1)$ where u is initial velocity and a is acceleration. The dimensions of S_{nth} are

(a) L (b) LT^{-1} (c) LT^{-2} (d) $L^{-1}T$

Q.26 In the equation y = A sin (ωt – kx), the dimensional formula of ω is

(a) $[M^0L^0T^{-1}]$ (b) $[M^0L^{-1}T^0]$ (c) [L] (d) [T]

Q.27 In the above equation, the dimensional formula of k is

(a) $[M^{-1}LT^{-1}]$ (b) $[M^0L^1T^0]$ (c) $[M^0L^{-1}T^0]$ (d) $[M^0L^{-1}T^{-1}]$

Q.28 Power P is related to distance x and time t as $P=\frac{b-x^2}{at}$. The dimensional formula of b is

(a) $[M^0LT^{-2}]$ (b) $[M^0L^2T^2]$ (c) $[M^0L^2T^{-2}]$ (d) $[M^0L^2T^0]$

Q.29 In the same equation, the dimensional formula of a is

(a) $[M^{-1} L^0 T^2]$ (b) $[ML^0 T^2]$ (c) $[ML^{-1} T^{-2}]$ (d) $[M^{-1} L^1 T^{-2}]$

Compression – II

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of measurement and vice–versa. In addition or subtraction, the number of decimal places in the result should equal the smallest number of decimal places in any term in the operation.

In multiplication and division, the number of significant figures in the product or in the quotient is the same as the smallest number of significant figures in any of the factors. With the help of the compression given above, choose the most appropriate alternative for each of the following questions:

Q.30 The area enclosed by a circle of diameter 1.06 m with correct number of significant figures is

(a) 0.88 m^2 (b) 0.883 m^2 (c) 1.88 m^2 (d) 0.882026 m^2

Q.31 The circumference of the circle of diameter 1.06 m with correct number of significant figures is

(a) 3.33 m (b) 3.33142 m (c) 3.3 m (d) 3m

Q.32 Subtract 2.6×10^4 from 3.9×10^5 with due regard to significant figures

(a) 3.64×10^5 (b) 3.7×10^5 (c) 3.6×10^5 (d) 3.65×10^6

Q.33 The correct number of significant figures in 0.0006032 is

(a) seven (b) six (c) eight (d) four

Q.34 Add 3.8×10^{-6} to 4.2×10^{-5} with due regard to significant figures

(a) 4.6×10^{-5} (b) 4.6×10^{-6} (c) 4.58×10^{-5} (d) 4.580×10^{-5}

Assertion and Reason Type Questions

The following questions consist of two statements each, printed as **Assertion and Reason.** While answering these questions, you are required to choose any one of the following four responses.

A. If both, assertion and reason are true and the reason is the correct explanation of the assertion.

B. If both, assertion and reason are true but reason is not a correct explanation of the assertion.

C. If assertion is true but the reason is false

D. If both, assertion and reason are false

Q.35 **Assertion:** Rate of flow of a liquid represents velocity of flow.

Reason: The dimensions of rate of flow are $[M^0 L^1 T^{-1}]$

Q.36 **Assertion:** Parallax method cannot be used for measuring distances of stars more than 100 light years away.

Reason: Because parallax angle reduces so much that it cannot be measured accurately.

Q.37 **Assertion:** Energy cannot be divided by volume.

Reason: Because dimensions of energy and volume are same.

Q.38 **Assertion:** When radius of a sphere is measured with an error of $\pm$ 1%, error in the calculation of its volume is $\pm$ 3%.

Reason: Because, $V = \frac{4}{3}\pi r^3$ and $\frac{\Delta V}{V} = \pm 3\left(\frac{\Delta r}{r}\right)$

Q.39 **Assertion:** Out of three measurements l = 0.7m; l = 0.70 m and l = 0.700 m, the last one is most accurate.

Reason: In every measurement, only the last significant digit is not accurately known.

Q.40 **Assertion:** The dimensional formula of surface energy is $[M^1 L^2 T^{-2}]$

Reason: Surface energy is potential energy of free surface.

Q.41 **Assertion:** Distance travelled in nth second has the dimensions of velocity.

Reason: Because it is the distance travelled in one (particular) second.

Q.42 **Assertion:** If error in measurement of distance and time are 3% and 2% respectively, error in calculation of velocity is 5%.

Reason: Velocity = $\frac{distance}{time}$

Multiple Choice Questions

Q.43 The centimeter scale on a vernier caliper is divided into eight equal parts. Ten vernier divisions are contained in one cm. Calculate vernier constant

(a) 0.1 cm (b) 0.01 cm (c) 0.025 cm (d) 0.001 cm

Q.44 The vernier constant of a vernier calipers is 0.1 mm and it has a zero error of 0.04 cm. While measuring diameter of a rod, the main scale reading is 1.2 cm and 5th vernier division is coinciding with any scale division. The corrected diameter of the rod is

(a) 1.21 cm (b) 1.21 mm (c) 1.29 mm (d) 1.29 cm

Q.45 What is the use of a thin strip at the back of vernier calipers?

(a) for measuring internal diameter of a beaker (b) for measuring depth of a cylinder

(c) for measuring diameter of a hollow cylinder (d) none of these

Q.46 When the two jaws of a vernier caliper are in touch, zero of vernier scale lies to the right of zero of main scale and coinciding vernier division is 3. If vernier constant is 0.1 mm, the zero correction is

(a) – 0.03 cm (b) + 0.03 cm (c) – 0.03 mm (d) + 0.03 mm

Q.47 The circular scale of a crew gauge has 200 divisions. When it is given 4 complete rotations, it moves through 2 mm. The least count of screw gauge is

(a) 0.25×10^{-2} cm (b) 0.25×10^{-3} cm (c) 0.001 cm (d) 0.001 mm

Q.48 While measuring diameter of a wire using a screw gauge, the main scale reading is 7 mm and zero of circular scale is 35 division above the reference line. If screw gauge has a zero error of – 0.003 cm, the correct diameter of wire is (given least count = 0.001 cm)

(a) 0.735 cm (b) 0.732 cm (c) 0.738 cm (d) 7.38 cm

Q.49 A spherometer has 200 equal divisions marked on the periphery of its disc. On giving five complete rotations, the central screw advances by 5 mm. Least count of spherometer is

(a) 0.001 cm (b) 0.001 mm (c) 0.0005 cm (d) 0.0005 mm

Q.50 When a screw gauge is completely closed, zero of circular scale is 4 divisions below the reference line of graduation. If least count of screw gauge is 0.001 cm, the zero correction is

(a) – 0.004 cm (b) + 0.004 cm (c) – 0.004 mm (d) + 0.004 mm

Q.51 Two spherometers A and B have the same pitch. A has 100 divisions on periphery of its circular disc and B has 200 divisions on periphery of its circular disc. Then

(a) Both A and B have same least count (b) L.C. of A is twice the L.C. of B

(c) L.C. of A is half the L.C. of B (d) Nothing can be said

Q.52 Choose the correct statement(s):

(a) All quantities may be represented dimensionally in terms of the base quantities.

(b) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities

(c) The dimension of a base quantity in other base quantities is always zero

(d) The dimension of derived quantity is never zero in any base quantity.

Q.53 A student measured the thickness of a glass slip using a spherometer with least count 0.001 cm. The correct listing is

(a) 0.23 cm (b) 0.234 mm (c) 2.34 mm (d) 0.234 cm

Q.54 Which of the following sets cannot enter into the list of fundamental quantities in any system of units?

(a) length, mass and velocity (b) length, time and velocity

(c) mass, time and velocity (d) length, time and mass

Q.55 A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then

(a) $n \propto$ size of u (b) $n \propto u^2$ (c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Q.56 Suppose a quantity x can be dimensionally represented in terms of M, L and T, that is, $[x] = M^a L^b T^c$. The quantity mass

(a) can always be dimensionally represented in terms of L, T and x

(b) can never be dimensionally represented in terms of L, T and x

(c) may be represented in terms of L, T and x if a = 0

(d) may be represented in terms of L, T and x if $a \neq 0$

Q.57 A dimensionless quantity

(a) never has a unit (b) always has a unit (c) may have a unit (d) does not exist

Q.58 A unitless quantity

(a) never has a non zero dimension (b) always has a non zero dimension

(c) may have a non zero dimension (d) does not exist

Q.59 $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$

The value of n is

(a) 0 (b) – 1 (c) 1 (d) none of these

Q.60 The dimensions $ML^{-1}T^{-2}$ may correspond to

(a) work done by a force (b) linear momentum

(c) pressure (d) energy per unit volume

Answers

1.	b	2.	c	3.	c	4.	d	5.	a	6.	c	7.	a
8.	d	9.	a	10.	a	11.	c	12.	d	13.	bc	14.	ae
15.	bd	16.	abd	17.	ab	18	bd	19.	c	20.	d	21.	a, b, d
22.	c	23.	a	24	b	25.	b	26.	a	27.	c	28.	d
29.	a	30.	b	31.	a	32.	c	33.	d	34.	a	35.	d
36.	a	37.	d	38.	a	39.	a	40.	a	41.	a	42.	b
43.	c	44.	a	45.	b	46.	a	47.	b	48.	c	49.	c
50.	a	51.	b	52.	a, b, c	53.	d	54.	b	55.	d	56.	d
57.	c	58.	a	59.	a	60.	c, d						

Higher Order Subjective Assignment

Q.1 Find the dimensions of (a) linear momentum, (b) frequency and (c) pressure

Q.2 Find the dimensions of

(a) angular speed ω (b) angular acceleration α

(c) torque τ and (d) moment of interia I

Some of the equations involving these quantities are $\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$, $\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$, $\tau = F.r$ and $I = mr^2$. The symbols have standard meanings.

Q.3 Find the dimensions of (a) electric field E, (b) magnetic field B and (c) magnetic permeability μ_0. The relevant equations are $F = qE$, $F = qvB$, and $B = \frac{\mu_0 I}{2\pi a}$ where F is force, q is charge, v is speed, I is current, and a is distance.

Q.4 Find the dimensions of (a) electric dipole moment p and (b) magnetic dipole moment M. The defining equations are p = q.d and M = IA where d is distance, A is area, q is charge and I is current.

Q.5 Find the dimensions of Planck's constant h from the equation E = hv where E is the energy and v is the frequency.

Q.6 Find the dimensions of (a) the specific heat capacity c, (b) the coefficient of linear expansion α and (c) the gas constant R. Some of the equations involving these quantities are

$Q = mc(T_2 - T_1)$, $l_t = l_0[1 + \alpha(T_2 - T_1)]$ and PV = nRT.

Q.7 Taking force, length and time to be the fundamental quantities find the dimensions of

(a) density (b) pressure (c) momentum and (d) energy

Q.8 Suppose the acceleration due to gravity at a place is 10 m/s^2. Find its value in cm/(minute)^2.

Q.9 Let x and a stand for distance. Is $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ dimensionally correct?

Q.10 The height of mercury column in a barometer is a Calcutta laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data: Specific gravity of mercury = 13.6. Density of water = 10^3 kg/m^3, g = 9.8 m/s^2 at Calcutta. Pressure = hρg in usual symbols.

Q.11 Express the power of a 100 watt bulb in CGS unit.

Q.12 The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How many microcenturies did you sleep yesterday?

Q.13 The surface tension of water is 72 dyne/cm. Convert it in SI unit.

Q.14 The kinetic energy K of a rotating body depends on its moment of inertia I and its angular speed ω. Assuming the relation to be $K = kI^a\omega^b$ where k is a dimensionless constant, find a and b. Moment of inertia of a sphere about its diameter is $\frac{2}{5}Mr^2$.

Q.15 Theory of relativity reveals that mass can be converted into energy. The energy E so obtained is proportional to certain powers of mass m and the speed c of light. Guess a relation among the quantities using the method of dimensions.

Q.16 Let I = current through a conductor, R = its resistance and V = potential difference across its ends. According to Ohm's law, product of two of these quantities equals the third. Obtain Ohm's law from dimensional analysis. Dimensional formulae for R and V are $ML^2 I^{-2} T^{-3}$ and $ML^2 T^{-3} I^{-1}$ respectively.

Q.17 The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m. Guess the expression for its frequency from dimensional analysis.

Q.18 Test if the following equations are dimensionally correct:

(a) $h = \frac{2S\cos\theta}{\rho r g}$ (b) $v = \sqrt{\frac{P}{\rho}}$ (c) $V = \frac{\pi P r^4 t}{8\eta l}$ (d) $v = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$;

where h = height, S = surface tension, ρ = density, P = pressure, V = volume, η = coefficient of viscosity, v = frequency and I = moment of inertia.

Answers

1. (a) MLT^{-1}, (b) T^{-1}, (c) $ML^{-1}T^{-2}$
2. T^{-1}, (b) T^{-2}, (c) ML^2T^{-2}, (d) ML^2
3. (a) $MLT^{-3}I^{-1}$, (b) $MT^{-2}I^{-1}$, (c) $MLT^{-2}I^{-2}$
4. (a) LTI, (b) L^2I
5. ML^2T^{-1}
6. (a) $L^2T^{-2}K^{-1}$, (b) K^{-1}, (c) $ML^2T^{-2}K^{-1}(mol)^{-1}$
7. (a) $FL^{-4}T^2$, (b) FL^{-2}, (c) FT, (d) FL
8. 36×10^5 cm(minutes)^2
9. no
10. 10×10^4 N/m^2, 10×10^5 dyne/cm^2
11. 10^9 erg/s
12. 1.9 microcenturies
13. 0.072 N/m
14. a = 1, b = 2
15. $E = kmc^2$
16. V = IR
17. $\frac{k}{L}\sqrt{\frac{F}{m}}$
18. all are dimensionally correct