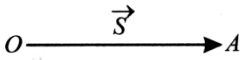
VECTORS

The physical quantities in physics are divided into two groups *viz. scalars and vectors*. Those physical quantities which have magnitude only are called scalars e.g., mass, length, temperature, speed etc.

A physical quantity that has both magnitude and direction and also obeys the laws of vector addition is called a vector.

A vector is represented *graphically* by a straight line with an arrowhead as shown in figure. The length of line (OA) represents the magnitude of the vector (on suitable scale) and the arrowhead indicates its direction.



Discussion: The following points are worth noting:

(i) The magnitude of a vector is a scalar and is always positive.

(ii) The magnitude of a vector is also called *modulus* of the vector and is represented by enclosing the vector symbol between two vertical lines. For example, the modulus of displacement vector $\vec{S}$ will be represented as $|\vec{S}|$.

(iii) Vectors can be added, subtracted and multiplied. *However, division of a vector by another vector is not a valid operation in vector algebra.* It is because the division of a vector by a direction is not possible.

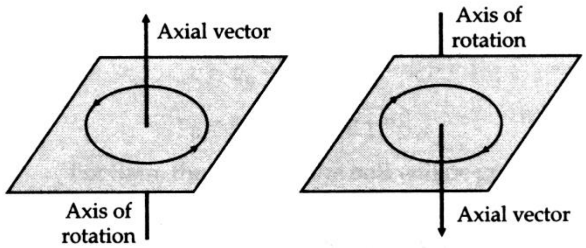
Broadly speaking, vectors are of two types:

Polar vectors: *The vectors which have a starting point or a point of application are called polar vectors.*

Axial vectors: *The vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors.*

Example:

Angular velocity, torque, angular momentum, etc. are axial vectors. As shown in figure, axial vector will have its direction along its axis of rotation depending on its anticlockwise or clockwise rotational effect.



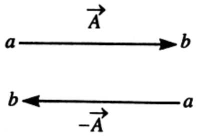
Some Definitions in Vector Algebra

(i) Equal vectors

The two vectors are said to be equal if they have the same magnitude and the same direction.

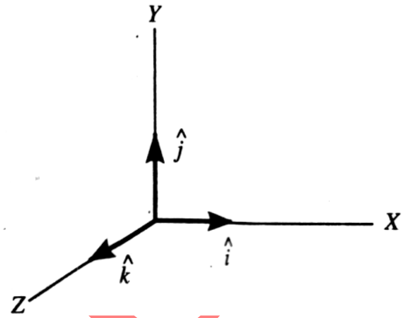
(ii) Negative of a vector

A vector is said to be negative vector of a given vector if its magnitude is the same as that of the given vector but its direction is opposite.



(iii) Unit vector

A unit vector is a vector that has a magnitude of 1 and has the same direction as that of the given vector. *A unit vector of the given vector is a vector of unit magnitude and has the same direction as that of the given vector.* Any vector $\vec{A}$ can be written as: $\vec{A} = A\hat{A}$



Here A is the magnitude of $\vec{A}$ and $\hat{A}$ is the unit vector whose magnitude is 1 and direction is the same as that of $\vec{A}$. In rectangular coordinate system, these unit vectors are called $\hat{i}$, $\hat{j}$ and $\hat{k}$; they point respectively along positive X, Y and Z axes.

(iv) Zero vector

A vector that has zero magnitude is called zero vector or null vector. A zero vector or null vector is represented by $\vec{0}$. Example

(a) The velocity vector of a stationary object is zero vector.

(b) The acceleration vector of an object moving with uniform velocity is a zero vector.

(c) The position vector of the origin of the coordinate axes is a zero vector.

(v) Fixed Vector:

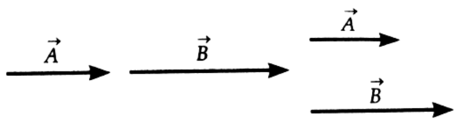
The vector whose initial point is fixed is called a fixed vector or a localized vector. For example, the position vector of a particle is a fixed vector because its initial point lies at the origin.

(vi) Free vector:

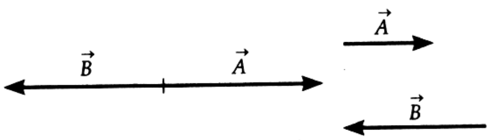
A vector whose initial point is not fixed is called a free vector or a non–localized vector. For example, the velocity vector of a particle moving along a straight line is a free vector.

(vii) Collinear vectors

The vectors which either act along the same line or along parallel lines are called collinear vectors. Two collinear vectors having the same direction ($\theta = 0^o$) are called *like* or *parallel vectors*. Two collinear vectors having the opposite directions ($\theta = 180^o$) are called *unlike* or *antiparallel vectors.*



(*a*) Like vectors



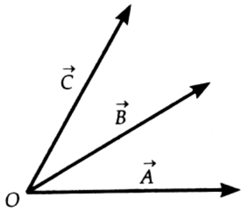
(*b*) Unlike vectors

(viii) Coplanar vectors

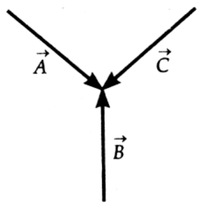
The vectors which act in the same plane are called coplanar vectors.

(ix) Co–initial vectors

The vectors which have the same initial point are called co–initial vectors. In figure, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are co–initial vectors.



Co–initial vectors

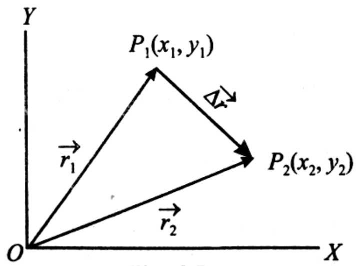


Co–terminus vectors

(X) **Co–terminus vectors:**

The vectors which have the common terminal point are called co–terminus vectors. In figure, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are co–terminus vectors.

1. **Position vector:** *The* ***position vector*** *of an object is the vector from the origin O of the coordinate system to the position of the object.*



2. **Displacement vector:** Displacement is a vector quantity and is called displacement vector. Displacement vector is a vector that points from object's initial position to its final position and whose magnitude is equal to the straight line distance between the two points.

Position vector, $\overrightarrow{OP_1} = x_1\hat{i} + y_1\hat{j}$

Position vector, $\overrightarrow{OP_2} = x_2\hat{i} + y_2\hat{j}$

$\therefore$ Displacement vector, $\overrightarrow{P_1P_2}$ = Final position – Initial position

$$= \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

$$= (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\therefore \quad \overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

Composition of vectors

The resultant of two or more vectors is that single vector which produces the same effect as the individual vectors together would produce. The process of adding two or more vectors is called composition of vectors.

As the vectors have both magnitude and direction, so they cannot be added by using ordinary rules of algebra. Vectors can be added *geometrically*. The following three *laws of vector addition* can be used to add two or more vectors having any inclination to each other.

(i) Triangle law of vector addition for adding two vectors.

(ii) Parallelogram law of vector addition for adding two vectors.

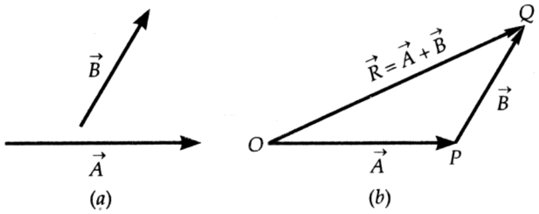
(iii) Polygon law of vector addition for adding more than two vectors.

(i) **Triangle Law of Vector Addition**

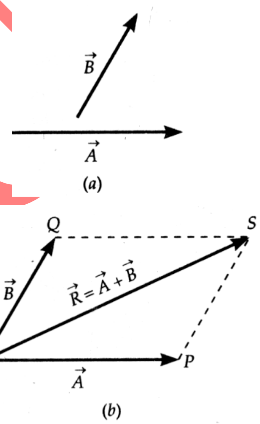
If two vectors acting simultaneously at a point are represented in magnitude and direction by two sides of a triangle taken in the same order, then third or closing side of the triangle taken in the opposite order represents their resultant in magnitude and direction.

Here vectors $\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by the two sides of a triangle taken in the same order. The third or closing side of the triangle taken in the opposite order represents their resultant $\vec{R}$ in magnitude and direction.

$$\therefore \qquad \vec{R} = \vec{A} + \vec{B}$$



(ii) **Parallelogram Law of Vector Addition:** *If two vectors acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through that point represents their resultant in magnitude and direction.*



$\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by the adjacent sides of the parallelogram. Then diagonal of the parallelogram passing through point O represents their resultant $\vec{R}$ in magnitude and direction.

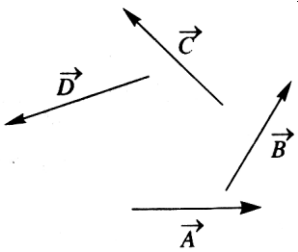
The resultant vector $\vec{R}$ is represented in magnitude and direction from the tail of the first vector to the tip of the last vector. $\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{OQ}$ or $\vec{R} = \vec{A} + \vec{B}$

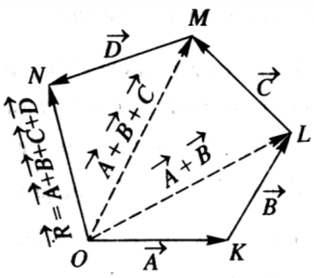
(iii) **Polygon law of vector addition**

If a number of vectors acting simultaneously at a point are represented in magnitude and direction by the sides of a polygon taken in the same order, then closing side of the polygon taken in opposite order represents the resultant in magnitude and direction.

Suppose we wish to add four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$, as shown in figure. Draw vector $\overrightarrow{OK} = \vec{A}$. Move vectors $\vec{B}, \vec{C}$ and $\vec{D}$ parallel to themselves so that the tail of $\vec{B}$ touches the head of $\vec{A}$, the tail of $\vec{C}$ touches the head of $\vec{B}$ and the tail of $\vec{D}$ touches the head of $\vec{C}$, as shown in figure. According to the polygon law, the closing side ON of the polygon taken in the reverse order represents the resultant $\vec{R}$. Thus

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$





Proof: We apply triangle law of vector addition to different triangles of the polygon shown in figure.

In Δ OKL, $\quad \overrightarrow{OL} = \overrightarrow{OK} + \overrightarrow{KL} = \vec{A} + \vec{B}$

In ΔOLM, $\quad \overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM} = \vec{A} + \vec{B} + \vec{C}$

In ΔOMN, $\quad \overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

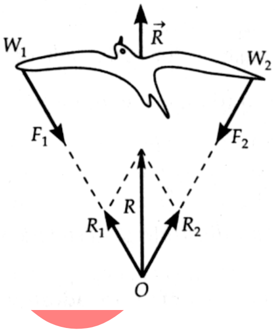
or $\qquad \vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

This proves the polygon law of vector addition.

Examples based on Composition of Vectors

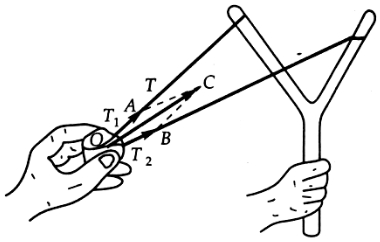
Ex.1 *Is the flying of a bird an example of composition of vectors? Explain.*

Flight of a bird: When a bird flies, it pushes the air with forces F_1 and F_2 in the downward direction with its wings W_1 and W_2. The lines of action of these two forces meet at point O. In accordance with Newton's third law of motion, the air exerts equal and opposite reactions R_1 and R_2. According to the parallelogram law, the resultant R of the reactions R_1 and R_2 acts on the bird in the upward direction and helps the bird to fly upward.



Ex.2 *Is the working of a sling based on the parallelogram law of vector addition? Explain.*

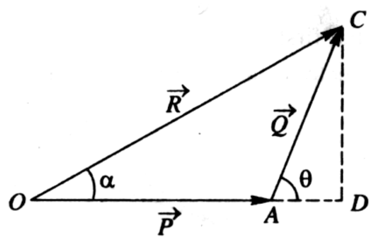
Working a sling. A sling consists of a Y–shaped wooden or metallic frame, to which a rubber band is attached, as shown in figure. When a stone held at the point O on the rubber band is pulled, the tensions T_1 and T_2 are produced along OA and OB in the two segments of the rubber band. According to the parallelogram law of forces, the resultant T of the tensions T_1 and T_2 acts on the stone along OC. As the stone is released, it moves under the action of the resultant tension T in forward direction with a high speed.



Analytical Method of Vector Addition

1. Triangle law of vector addition

Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the sides $\overrightarrow{OA}$ and $\overrightarrow{AC}$ of a triangle OAC taken in the same order. Then according to triangle law of vector addition, the third or closing side of the triangle taken in the opposite order represents the resultant $\vec{R}\ (=\overrightarrow{OC})$ in magnitude and direction. Let $\vec{R}$ make an angle α with P i.e., (figure) ∠AOC = α.



(i) Magnitude of Resultant: From C, draw CD perpendicular to OA produced. Suppose ∠CAD = θ. θ is the angle between the two vectors.

In the right angled triangle ODC, $(OC)^2 = (OD)^2 + (CD)^2 = (OA + AD)^2 + (CD)^2$

As in right angled triangle ACD, AD = Q cosθ and CD = Q sinθ

$\therefore \quad R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$

$= P^2 + Q^2 + 2PQ\cos\theta \qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots \text{(i)}$

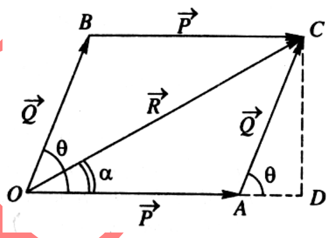
(ii) **Direction of resultant.** From right angled triangle ODC,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\therefore \quad \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} \qquad \ldots \text{(ii)}$$

Thus, the magnitude of the resultant is given by equation (i) and the direction of $\vec{R}$ with $\vec{P}$ is given by equation (ii).

2. Parallelogram law of Vector Addition



Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ of the parallelogram OACB (see figure). Suppose the angle between the vectors is θ i.e., ∠AOB = θ. According to parallelogram law of vector addition, the diagonal $\overrightarrow{OC}$ represents the resultant $\vec{R} = (=\overrightarrow{OC})$ in magnitude and direction. Suppose $\vec{R}$ makes angle α with P i.e., ∠AOC = α.

(i) **Magnitude of resultant:** From C, draw CD perpendicular on OA produced. From geometry, ∠DAC=θ. In right angled triangle ODC,

$$(OC)^2 = (OD)^2 + (CD)^2 = (OA + AD)^2 + (CD)^2$$

$$\therefore \quad R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \qquad [\because AD = Q\cos\theta;\ CD = Q\sin\theta]$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots \text{(iii)}$$

(ii) **Direction of resultant.** From right angled triangle ODC,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\therefore \quad \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} \qquad \ldots \text{(iv)}$$

Thus, the magnitude of the resultant is given by equation (iii) and the direction of $\vec{R}$ with $\vec{P}$ is given by equation (iv).

Different cases:

(a) When the vectors act along the same direction i.e., θ = 0°

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ} = \sqrt{(P+Q)^2} \quad \therefore \quad R = P + Q$$

$$\tan\alpha = \frac{Q\sin 0^\circ}{P+Q\cos 0^\circ} = \frac{Q(0)}{P+Q(1)} = 0 \quad \therefore \quad \alpha = 0^\circ$$

Therefore, the magnitude of the resultant of two vectors in the same direction is equal to the sum of the magnitudes of two vectors and the direction of the resultant is along the direction of $\vec{P}$ and $\vec{Q}$.

(b) When the vectors are at right angles i.e., θ = 90°

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ} = \sqrt{P^2 + Q^2}$$

$$\tan\alpha = \frac{Q\sin 90^\circ}{P+Q\cos 90^\circ} = \frac{Q(1)}{P+Q(0)} = \frac{Q}{P} \quad \therefore \quad \tan\alpha = \frac{Q}{P}$$

(c) When the vectors act along opposite directions i.e., $\theta = 180^o$

$$R=\sqrt{P^2+Q^2+2PQ\cos180^o}=\sqrt{P^2-Q^2}$$

$\therefore$ $R = P - Q$

$$\text{tam } \alpha = \frac{Q \sin 180^o}{P+Q\cos180^o} = \frac{Q(0)}{P+Q(-1)}=0$$

$\therefore$ $\alpha = 0^o \;\; 180^o$

Thus the magnitude of the resultant of two vectors acting in the opposite directions is equal to the difference in the magnitudes of the vectors. The resultant acts in the direction of the larger vector.

Subjective Assignment – I

Q.1 The angle between two vectors of equal magnitude is 120^o. Prove that the magnitude of their resultant is equal to either of them.

Q.2 Two forces of 30 N and 40 N are inclined to each other at an angle of 60^o. Find their resultant. What will be the angle if the forces are inclined at right angles to each other?

Q.3 The resultant of two equal forces acting at right angles to each other is 1414 N. Find the magnitude of each force.

Q.4 The magnitude of two vectors are equal and the angle between them is θ. Show that their resultant divides angle θ equally.

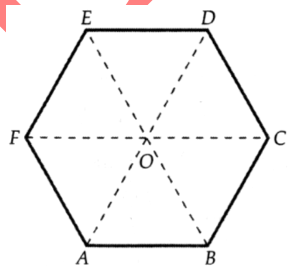
Q.5 The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of the resultant is 12 N. If the resultant is at 90^o with the force of smaller magnitude, what are the magnitudes of forces?

Q.6 A particle has a displacement of 12 m toward east and 5 m toward north and then 6 m vertically upward. Find the magnitude of the resultant displacement.

Q.7 A person moves 30 m north, then 20 m east and finally $30\sqrt{2}$ m south–west. What is the displacement form the original position?

Q.8 ABCDE is a pentagon. Prove that $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EA}=\vec{0}$.

Q.9 In figure, ABCDEF is a regular hexagon. Prove that $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$.



Q.10 A boy travels 10 m due north and then 7m due east. Find the displacement of the boy.

Q.11 Find the resultant of two forces, one 6N due east and other 8 N due north.

Q.12 Calculate the angle between a 2N force and a 3N force so that their resultant is 4N.

Q.13 The resultant vector of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. On reversing the direction of $\vec{Q}$, the resultant vector becomes $\vec{S}$. Show that: $R^2 + S^2 = 2 (P^2 + Q^2)$

Q.14 Two equal forces have the square of their resultant equal to three times their product. Find the angle between them.

Q.15 At what angle do the two forces (P + Q) and (P – Q) act so that the resultant is $\sqrt{3P^2+Q^2}$.

Q.16 A particle is acted upon by four forces simultaneously:

(a) 30 N due east (b) 20 N north (c) 50 N due west and (d) 40 N due south.

Find the resultant force on the particle

Q.17 Two boys raising a load pull at an angle to each other. If they exert forces of 30 N and 60 N respectively and their effective pull is at right angles to the direction of the pull of the first boy, what is the angle between their arms? What is the effective pull?

Q.18 Find the angle between two vectors $\vec{P}$ and $\vec{Q}$ if resultant of the vectors is given by $R^2 = P^2 + Q^2$.

Answers

2.	60.83 N, 53.06°	3.	1000 N	5.	Q = 13 N and P = 5 N	6.	14.32 m
7.	10 m west	10.	12.21 m, of N	11.	10N, 53° with 6 N force		
12.	75°31'	14.	60°	15.	60°	16.	$20\sqrt{2}$N, 45° south of west
17.	120°, $30\sqrt{3}$ N	18.	90°				

Some Properties of Vectors

(i) Multiplication of a vector by a real number: If we multiply a vector $\vec{A}$ by a real positive number n, we get the vector n $\vec{A}$ which has the same direction as $\vec{A}$ and magnitude nA i.e.,

$$n\vec{A} = n(\vec{A})$$

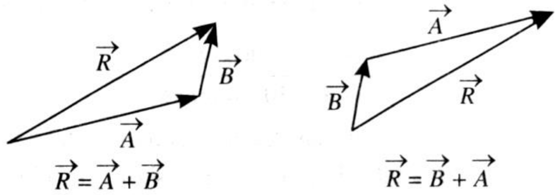
Thus the magnitude of the vector becomes n times while its direction remains unchanged. If a vector is multiplied by a negative real number (i.e., – n), the magnitude of the vector becomes nA but direction is opposite to that of $\vec{A}$ i.e.,

$$-n(\vec{A}) = -n\vec{A}$$

(ii) Multiplication of a vector by a scalar: If we multiply a vector $\vec{A}$ by a scalar S, the result is another vector with direction of $\vec{A}$ but magnitude SA. If S is negative, S $\vec{A}$ has a direction opposite to that of $\vec{A}$.

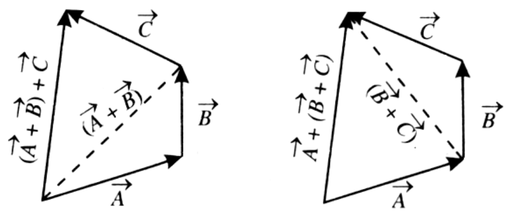
(iii) Vector addition obeys commutative law: According to this law, the resultant of the vectors remains the same in whatever order they may be added.

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$



(iv) Vector addition obeys associative law: According to this law, the resultant of the vectors remains the same in whatever grouping they may be added.

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

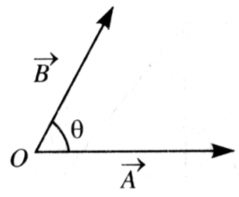


Subtraction of Vectors

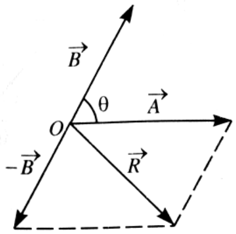
In order to subtract vector $\vec{B}$ from vector $\vec{A}$, first reverse the direction of $\vec{B}$, thus producing $-\vec{B}$. Then add $\vec{A}$ and $(-\vec{B})$ by parallelogram law of vector addition.

$\therefore \quad \vec{R}=\vec{A}+(-\vec{B})=\vec{A}-\vec{B}$

Consider two vectors $\vec{A}$ and $\vec{B}$ of the same kind and inclined to each other at an angle θ as shown in figure.



(i)

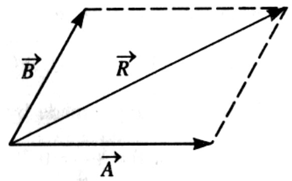


(ii)

(i) In order to find $\vec{A}-\vec{B}$, reverse the direction of $\vec{B}$, thus producing $-\vec{B}$ as shown in figure. (ii) Then find the sum of $\vec{A}$ and $-\vec{B}$ by parallelogram law of vector addition. The required difference is $\vec{R}$ and is shown in figure (ii)

$\therefore \quad \vec{R}=\vec{A}+(-\vec{B})=\vec{A}-\vec{B}$

Resolution of a Vector in a Plane



We can replace a single vector by any two (or more) vectors whose sum gives us back the original vector. This is called resolution of a vector. *This process of splitting a single vector into two or more vectors in different directions in a plane such that their sum gives back the original vector is called* ***resolution of a vector.***

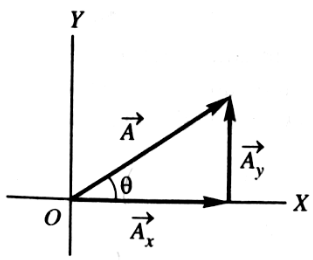
The vectors into which the given vector $\vec{R}$ is resolved (or splitted) are called the **vector components** or $\vec{R}$. Figure shows the vector $\vec{R}$ resolved into two non–parallel vector $\vec{A}$ and $\vec{B}$ such that:

$$\vec{R}=\vec{A}+\vec{B}$$

Therefore, $\vec{A}$ and $\vec{B}$ are the vector components of $\vec{R}$.

Rectangular Components of a Vector in a Plane

When a vector in a plane is resolved (i.e., splitted) into two components at right angles to each other, the component vectors are called ***rectangular components*** *of the vector.*



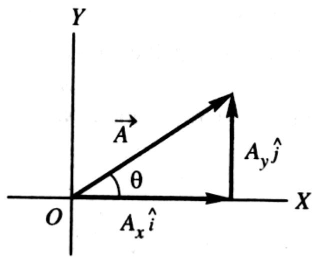
(i)

Figure (i) shows a vector $\vec{A}$ in the X–Y plane. It is resolved into two rectangular components $\vec{A}_x$ and $\vec{A}_y$ (along X–axis and Y–axis)

$\therefore \quad \vec{A}=\vec{A}_x+\vec{A}_y$

The vector components $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of vector $\vec{A}$. Therefore, we can replace vector $\vec{A}$ by $\vec{A}_x$ and $\vec{A}_y$. From the geometry of figure (i), we see that magnitudes of components $\vec{A}_x$ and $\vec{A}_y$ are related to the magnitude of $\vec{A}$ by;

$A_x = A\cos\theta$; $A_y = A\sin\theta$



(ii)

These are x–component and y–component respectively of vector $\vec{A}$.

Direction of $\vec{A}$ with X–axis, $\tan\theta = \dfrac{A_y}{A_x}$

We can express the vectors $\vec{A}_x$ and $\vec{A}_y$ in terms of unit vectors $\hat{i}$ and $\hat{j}$.

$$\vec{A}_x = \vec{A}_x\,\hat{i}\,;\ \vec{A}_y = \vec{A}_y\,\hat{j}$$

$$\therefore \qquad \vec{A} = \vec{A}_x\,\hat{i} + \vec{A}_y\,\hat{j}$$

Note that A_x(=A cosθ) is not a vector; $A_x\,\hat{i}$ is a vector. Similarly, A_y (=A sinθ) is not a vector; $A_y\,\hat{j}$ is a vector

Ex.1 Find the resultant of the following forces acting simultaneously at a point.

(i) a force of 50 N acting along OX axis

(ii) a force of 40 N acting at an angle of 60^o to OX axis

(iii) a force of 60 N acting 330^o with OX axis.

Ans. 122 N, 2.2^o

Rectangular Components in Three Dimensions

Consider a vector $\vec{A}$ in space. It has three rectangular components $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ as shown in figure.

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

Magnitude of $\vec{A}$, $\quad A = \sqrt{A_x^2 + A_y^2 + A_x^2}$

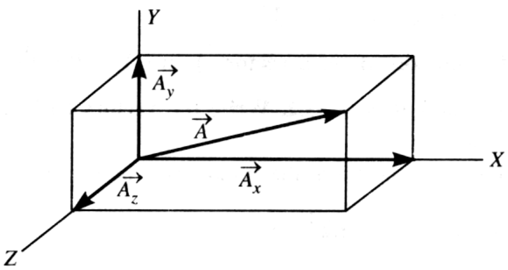
Note that $\vec{A}_x$ is the x–component of $\vec{A}$; $\vec{A}_y$ is the y–component of $\vec{A}$ and $\vec{A}_z$ is the z–component of $\vec{A}$.

An algebraic sum is one in which the sign of quantity is to be taken into account. The component along OX axis is positive while in opposite direction, it is negative. Similarly, component along OY is positive and that in the opposite direction is negative.

Note that A_x and A_y are perpendicular to each other and their resultant is

$R' = \sqrt{A_x^2 + A_y^2}$. Now A_z is $\perp$ R'. Therefore, magnitude of

$A = \sqrt{R'^2 + A_z^2} = \sqrt{A_x^2 + A_y^2 + A_z^2}$.



We can also express vector $\vec{A}$ in terms of unit vectors as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Note that A_x is not a vector; $A_x\hat{i}$ is a vector. Same is true for other two components.

Addition of vector in three dimensions

Suppose that there are three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ in space and their sum is $\vec{R}$.

$$\therefore \qquad \vec{R} = \vec{A} + \vec{B} + \vec{C}$$

The three corresponding algebraic equations are:

$$R_x = A_x + B_x + C_x;\ R_y = A_y + B_y + C_y;\ R_z = A_z + B_z + C_z$$

Direction cosines of vector: If α, β and γ are the angles which vector $\vec{A}$ makes with X, Y and Z axes respectively, then,

$$\cos\alpha = \frac{A_x}{A} \quad \text{or} \quad A_x = A\cos\alpha$$

$$\cos\beta = \frac{A_y}{A} \quad \text{or} \quad A_y = A\cos\beta$$

$$\cos\gamma = \frac{A_z}{A} \quad \text{or} \quad A_z = A\cos\gamma$$

Here cos α, cos β and cos γ are called **direction cosines** of the vector $\vec{A}$.

Now $\quad A^2 = A_x^2 + A_y^2 + A_z^2$

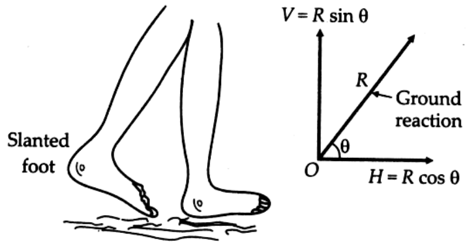
or $\quad A^2 = A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma$

or $\quad 1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

Therefore, the sum of the squares of the direction cosines of a vector is always unity.

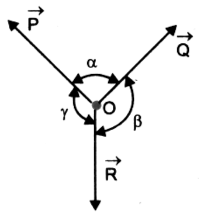
Ex.1 *Can the walk of a man be an example of resolution of vectors? If yes, how?*

Walking of a man is an example of resolution of forces. While walking, a person presses the ground with his feet slightly slanted in the backward direction, as shown in figure. The ground exerts upon him an equal and opposite reaction R. Its horizontal component H = R cos θ enables the person to move forward while the vertical component V = R sin θ balances his weight.



LAMI'S Theorem

According to this theorem, *if a particle under the simultaneous action of three forces is in equilibrium, then each force has a constant ratio with the sine of the angle between the other two forces.*



Let three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ are acting on a particle O (figure) such that the particle in equilibrium. These forces are represented by the three sides of a triangle taken in the same order as shown in figure.

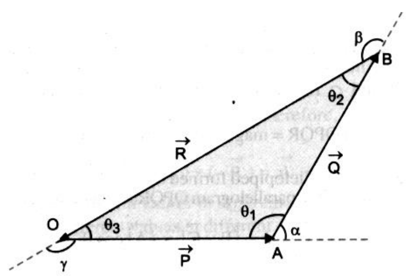
Then $\quad \dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha} = c$

Proof: Since $\vec{P}, \vec{Q}$ and $\vec{R}$ are represented by the sides OA, AB and BO of a ΔOAB respectively, then

$$\frac{P}{OA} = \frac{Q}{AB} = \frac{R}{BO} \qquad \text{..... (1)}$$

Using sine formula, we get

$$\frac{OA}{\sin\theta_2} = \frac{Q}{\sin\theta_3} = \frac{BO}{\sin\theta_1} \qquad \text{.... (2)}$$

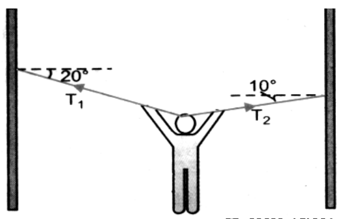


Using equation (2) in equation (1), we get

$$\frac{P}{\sin\theta_2} = \frac{Q}{\sin\theta_3} = \frac{R}{\sin\theta_1} \quad \text{or}$$

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

Since sin (180 – θ) = sin θ



$\therefore \quad \dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha}$ which is **Lami's theorem.**

Numerical Illustration: A girl of weight 100 N hangs from the rope extending between two poles show in figure. Calculate the tensions in the two parts of the rope. (take cos 10^o = 0.9848 and cos 20^o = 0.9397)

Ans: T_1 = 196.96 N, T_2 = 187.94 N

Subjective Assignment – II

Q.1 A force is represented by;

$$\vec{F} = (2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ newton.}$$

What is the magnitude of force?

Q.2 Two vectors are given as $\vec{A} = 3\hat{i} + 9\hat{j} - 6\hat{k}$ and $\vec{B} = 8\hat{i} - 4\hat{j} + 8\hat{k}$. Find $|\vec{A} + \vec{B}|$.

Q.3 The components of a vector $\vec{A}$ are $A_x = 0$; $A_y = 22$. The components of vector $\vec{B}$ are $B_x = 33.2$ and $B_y = -33.2$. Write the vector (i) in unit vector notation and (ii) perform the addition.

Q.4 Determine the vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives unit vector along y–direction.

Q.5 The x– and y – components of $\vec{A}$ are 4 and 6 and those of $\vec{A} + \vec{B}$ are 10 and 9. Find the components, magnitude and direction of $\vec{B}$.

Q.6 Given $\vec{A} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{B} = (3\hat{i} - 5\hat{j} + \hat{k})$. Find the angle between $\vec{A}$ and $\vec{B}$.

Q.7 If $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$, find the magnitude of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

Q.8 Find the unit vector parallel to the resultant of the vectors $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.

Q.9 Determine the vector which when added to the resultant of $\vec{A} = 2\hat{i} - 4\hat{j} - 6\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} + 3\hat{k}$ gives the unit vector along z–axis.

Q.10 Find the value of λ in the unit vector $0.4\hat{i} + 0.8\hat{j} + \lambda\hat{k}$.

Q.11 Given three coplanar vectors $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -3\hat{j}$. Find the magnitude of the sum of the three vectors.

Q.12 A force is inclined at 30^o to the horizontal. If its rectangular component in the horizontal direction is 50 N, find the magnitude of the force and its vertical component.

Q.13 A velocity of 10 ms^{-1} has its y–component $5\sqrt{2}$ ms^{-1}. Calculate its X–component.

Q.14 An aeroplane takes off at an angle of 30^o to the horizontal. If the component of its velocity along the horizontal is 200 km h^{-1}, what is its actual velocity? Also find the vertical component of its velocity.

Q.15 A child pulls a rope attached to a stone with a force of 60 N. The rope makes an angle of 40^o to the ground. (i) Calculate the effective value of the pull tending to move the stone along the ground. (ii) Calculate the force tending to lift the stone.

Answers

1. 7 N
2. $\sqrt{150}$
3. (i) $\vec{A} = 0\hat{i} + 22\hat{j}$; $\vec{B} = 33.2\hat{i} - 33.2\hat{j}$, (ii) $33.2\hat{i} - 11.2\hat{j}$
4. $-5\hat{i} + 2\hat{j} - 4\hat{k}$
5. 26.56^o with x–axis
6. 90^o
7. 5, $\sqrt{29}$
8. $1/\sqrt{61}(6\hat{i} - 3\hat{j} - 4\hat{k})$
9. $-6\hat{i} + \hat{j} + 4\hat{k}$

10.	$\sqrt{0.2}$	11.	$\sqrt{5}$	12.	57.74 N, 28.87 N
13.	$5\sqrt{2}\ \text{ms}^{-1}$	14.	230.94 km h^{-1}, 115.47 km h^{-1}	15.	(i) 45.96 N, (ii) 38.57 N

Multiplication of Vectors

The following two kinds of multiplication operations for vectors:

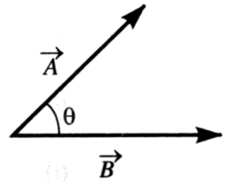
(i) Multiplication of one vector by a second vector so as to produce a scalar. It is called *scalar product or dot product of two vectors.*

(ii) Multiplication of one vector by a second vector so as to produce a vector. It is called *vector product or cross product of two vectors.*

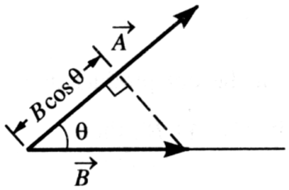
Scalar or Dot Product

Consider two vectors $\vec{A}$ and $\vec{B}$ with angle θ between them as shown in figure. The scalar product of vectors $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A}.\vec{B} = AB\cos\theta$$

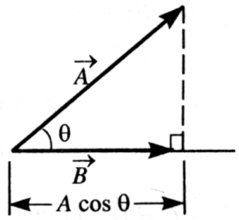


where A and B are the magnitudes of the vectors and θ is the angle between them when their tails touch. *The* ***scalar product*** *of two vectors* $\vec{A}$ *and* $\vec{B}$ *is defined as the product of magnitude of one vector (say A) and the scalar component of the other vector (B cos θ) along the direction of the first vector* ($\vec{A}$). $\vec{B}$ has a scalar component B cos θ along the direction of $\vec{A}$.



(*i*)

$$\therefore \quad \vec{A}.\vec{B} = (A) \times \text{scalar component of } \vec{B} \text{ along the direction of } \vec{A}.$$
$$= (A)\,(B\cos\theta)$$
$$\text{or} \quad \vec{A}.\vec{B} = AB\cos\theta$$



(*ii*)

Properties of Scalar (or Dot) Product

The following properties of scalar product (or dot product) are worth noting:

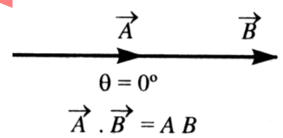
(i) For given vectors $\vec{A}$ and $\vec{B}$, the value of the scalar product depends upon the angle θ between them

For θ = O° ; $\vec{A}.\vec{B} = AB\cos 0^\circ = AB$

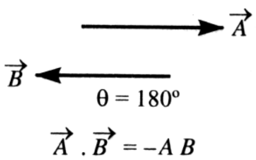
For θ = 180° ; $\vec{A}.\vec{B} = AB\cos 180^\circ = -AB$

For θ = 90° ; $\vec{A}.\vec{B} = AB\cos 90^\circ = 0$

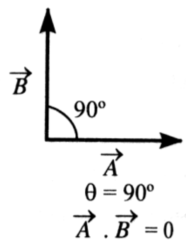
Thus the dot product of two mutually perpendicular vectors is zero.



(*i*)



(*ii*)

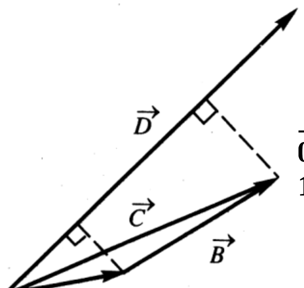


(*iii*)

(ii) *The dot product of two vectors obeys commutative law.*

$$\vec{A}.\vec{B} = A(B\cos\theta) = AB\cos\theta$$

$$\vec{B}.\vec{A} = B(A\cos\theta) = AB\cos\theta$$



$\therefore \quad \vec{A}.\vec{B} = \vec{B}.\vec{A}$

This simply means that order of vectors in the dot product does not matter.

(iii) *The dot product obeys the distributive law.* If $\vec{C} = \vec{A} + \vec{B}$, then,

$$\vec{D}.\vec{C} = \vec{D}.(\vec{A}+\vec{B})$$

$$= \vec{D}.\vec{A} + \vec{D}.\vec{B}$$

or $\quad \vec{D}.\vec{C} = \vec{D}.\vec{A} + \vec{D}.\vec{B}$

(iv) *The dot product of a vector with itself gives square of its magnitude.*

$$\vec{A}.\vec{A} = (A)(A)\cos 0^o \ (\theta = 0^o) \quad \therefore \quad \vec{A}.\vec{A} = A^2$$

Examples of dot product of two vectors

(i) Work done, W = FS cos θ = $\vec{F}.\vec{S}$

Thus work done by a constant force is the dot product of force $(\vec{F})$ and displacement $(\vec{S})$.

(ii) Instantaneous power, P = Fv cosθ = $\vec{F}.\vec{v}$

Thus instantaneous power is the dot product of force $(\vec{F})$ and velocity $(\vec{v})$.

Unit Vectors and the Dot Product

(i) Since $\hat{i}$ is parallel to $\hat{i}$ (i.e., θ = 0°) and each has a unit magnitude,

$\therefore \quad \hat{i}.\hat{i} = (1)(1)\cos 0^o = 1$. Similarly, $\hat{j}.\hat{j} = 1$ and $\hat{k}.\hat{k} = 1$.

$\therefore \quad \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ii) Since $\hat{i}$ and $\hat{j}$ are perpendicular and each has a unit magnitude,

$\therefore \quad \hat{i}.\hat{j} = (1)(1)\cos 90^o = 0$. Similarly, $\hat{i}.\hat{k} = 0$ and $\hat{j}.\hat{k} = 0$

$\therefore \quad \hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$

Consider two three–dimensional vectors $\vec{A}$ and $\vec{B}$. These can be expressed in the rectangular form as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\ ;\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\therefore \quad \vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

With the distributive law, it will yield,

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

This is a very useful relation.

Also $\quad \vec{A}.\vec{B} = AB\cos\theta \quad \therefore \quad \cos\theta = \dfrac{\vec{A}.\vec{B}}{AB} = \dfrac{A_xB_x + A_yB_y + A_zB_z}{AB}$

Thus we can find the angle θ between the vectors $\vec{A}$ and $\vec{B}$.

Subjective Assignment – III

Q.1 If $\vec{R} = \vec{A} - \vec{B}$, show that $R^2 = A^2 + B^2 - 2AB\cos\theta$ where θ is the angle between $\vec{A}$ and $\vec{B}$.

Q.2 Find the angle between the vectors $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$..

Q.3 The sum and difference of two vectors are perpendicular to each other. Prove that the vectors are equal in magnitude.

Q.4 The sum and difference of two vectors are equal in magnitude i.e. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. Prove that vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Q.5 If the magnitudes of two vectors are 3 and 4 and the magnitude of their scalar product is 6, find the angle between the vectors.

Q.6 Prove that vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}$ are perpendicular to each other.

Q.7 The resultant vector of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. On reversing the direction of $\vec{Q}$, the resultant vector becomes $\vec{S}$. Show that: $R^2 + S^2 = 2(P^2 + Q^2)$

Q.8 If unit vectors $\hat{A}$ and $\hat{B}$ are inclined at an angle θ, then prove that: $|\hat{A}-\hat{B}|=2\sin\frac{\theta}{2}$

Q.9 Find the magnitude and direction of vector $\hat{i} + \hat{j}$.

Q.10 Find the unit vector parallel to the resultant of vectors $\vec{A} = 2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$.

Q.11 Find the angle between the vectors $\vec{A}=2\hat{i}-4\hat{j}+6\hat{k}$ and $\vec{B}=3\hat{i}+\hat{j}+2\hat{k}$.

Q.12 Find the value of m so that the vector $3\hat{i}-2\hat{j}+\hat{k}$ is perpendicular to the vector $2\hat{i}+6\hat{j}+m\hat{k}$.

Q.13 Under a force of $10\hat{i}-3\hat{j}+6\hat{k}$ Newton, a body of mass 5kg is displaced from the position $6\hat{i}+5\hat{j}-3\hat{k}$ to the position $10\hat{i}-2\hat{j}+7\hat{k}$. Calculate the work done.

Q.14 The sum and difference of two vectors $\vec{A}$ and $\vec{B}$ are $\vec{A}+\vec{B}=2\hat{i}+6\hat{j}+\hat{k}$ and $\vec{A}-\vec{B}=4\hat{i}+2\hat{j}-11\hat{k}$. Find the magnitude of each vector and their scalar product $\vec{A}.\vec{B}$.

Q.15 A force $\vec{F}=5\hat{i}+4\hat{j}$ Newton displaces a body through $\vec{S}=3\hat{i}+4\hat{k}$ metre in 3s. Find the power.

Q.16 If the resultant of the vectors $3\hat{i}+4\hat{j}+5\hat{k}$ and $5\hat{i}+3\hat{j}+4\hat{k}$ makes an angle θ with x–axis, then find cos θ.

Q.17 If vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ have magnitudes 8, 15 and 17 units and $\vec{A}+\vec{B}=\vec{C}$, find the angle between $\vec{A}$ and $\vec{B}$.

Q.18 Prove that: $(\vec{A}+2\vec{B}).(2\vec{A}-3\vec{B})=2A^2+AB\cos\theta-6B^2$

Answers

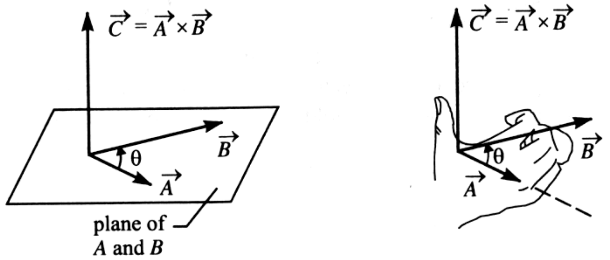
2.	69.5^o	5.	60^o	9.	$\sqrt{2},\ 45^o$
10.	$\frac{6}{\sqrt{61}}\hat{i}+\frac{3}{\sqrt{61}}\hat{j}-\frac{4}{\sqrt{61}}\hat{k}$	11.	60^o	12.	6
13.	121 J	14.	$\sqrt{50},\ \sqrt{41},\ -25$	15.	5 W
16.	0.5744	17.	90^o		

Vector Product or Cross Product

Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure, where the angle θ is the smaller of the angles between the two vectors. The vector or cross product of vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$ given by:

$$\vec{C}=\vec{A}\times\vec{B}$$

It is called vector product of vectors $\vec{A}$ and $\vec{B}$ because $\vec{C}$ is itself a vector.



(i) (ii)

The vector $\vec{C}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and its direction is given by **right–hand rule.**

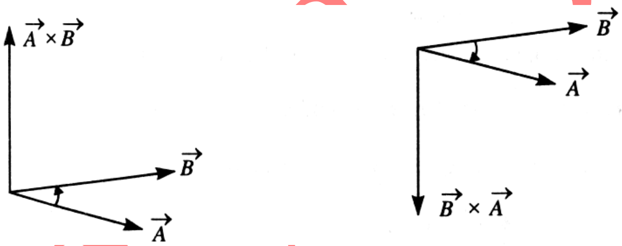
Properties of Vector Product

The following are the important properties of vector product or cross product:

(i) For given vectors $\vec{A}$ and $\vec{B}$, the value of the cross product for angle 90^o between them is equal to the product of magnitudes of the two vectors i.e.,

For $\theta = 90^o$, $\vec{A} \times \vec{B} = AB \sin 90^o = AB$

(ii) The cross product of two vectors does not obey commutative law i.e., $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$. Thus referring to figure, the direction of vector $\vec{A} \times \vec{B}$ is opposite to that of vector $\vec{B} \times \vec{A}$. Since the magnitude in each case is AB sin θ, it follows, therefore, that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$



(iii) The cross product of a vector with itself is zero i.e., $\vec{A} \times \vec{A} = 0$

(iv) Suppose two vectors $\vec{A}$ and $\vec{B}$ are parallel or antiparallel. The angle θ between them is either 0^o or 180^o. Then $\vec{A} \times \vec{B} = 0$. The cross product of parallel (or antiparallel vectors) is zero.

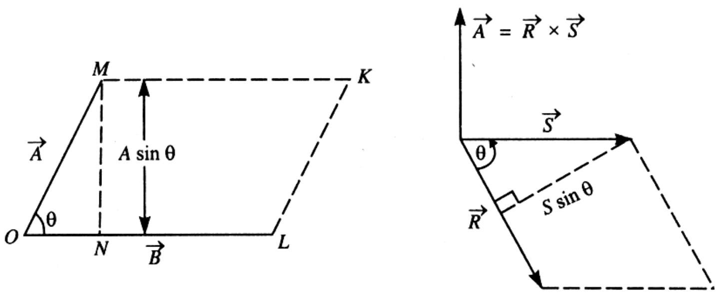
(v) The cross product obeys distributive law i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(vi) The magnitude of the cross product of two vectors is equal to the area of parallelogram formed by them.

Suppose two vectors $\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by the two adjacent sides $\overrightarrow{OM}$ and $\overrightarrow{OL}$ of the parallelogram OLKM (figure). The area of the parallelogram is given by:

$= OL \times MN = B(A \sin \theta) = AB \sin \theta$



But AB sin θ is the magnitude of the cross product $\vec{A}\times\vec{B}$. Therefore, the magnitude of the cross product of two vectors is equal to the area of the parallelogram formed by them.

Examples of cross product of two vectors

(i) Linear velocity, $\vec{v}=\vec{\omega}\times\vec{r}$

Thus linear velocity of a particle in rotational motion is equal to the cross product of its angular velocity $(\vec{\omega})$ and displacement vector $(\vec{r})$

(ii) Centripetal acceleration, $\vec{a}_c=\vec{\omega}\times\vec{v}$

Thus the centripetal acceleration of a particle in rotational motion is equal to the cross product of its angular velocity $(\vec{\omega})$ and its linear velocity $(\vec{v})$.

Unit Vectors and the Cross Product

(i) Let us evaluate $\hat{i}\times\hat{i}$. The result is zero. It is because the two vectors are parallel (sin θ = 0) and each has a unit magnitude.

∴ $\hat{i}\times\hat{i}=(1)\,(1)\sin 0^{o}=0$

Similarly, $\hat{j}\times\hat{j}=0$ and $\hat{k}\times\hat{k}=0$

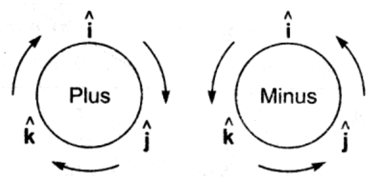
∴ $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

(ii) To evaluate $\hat{i}\times\hat{j}$, refer to figure which shows the unit vectors on XYZ – coordinate system. The magnitude of each unit vector is 1 and so the magnitude of $\hat{i}\times\hat{j}$ is 1 i.e., $|\hat{i}\times\hat{j}|=(1)\,(1)\sin 90^{o}=1$. The direction of $\hat{i}\times\hat{j}$ is given by the right–hand rule and from figure, it is along the positive Z–axis. But this is just the unit vector $\hat{k}$. Therefore, we have $\hat{i}\times\hat{j}=\hat{k}$. Simply reversing the order of the unit vectors gives $\hat{j}\times\hat{i}=-\hat{k}$.

$\hat{i}\times\hat{j}=-(\hat{j}\times\hat{i})=\hat{k}$

Similarly $\hat{j}\times\hat{k}=-(\hat{k}\times\hat{j})=\hat{i}$

and $\hat{k}\times\hat{i}=-(\hat{i}\times\hat{k})=\hat{j}$

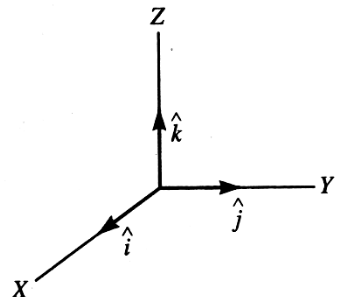


Consider three dimensional vectors $\vec{A}$ and $\vec{B}$.

These vectors can be expressed in terms of rectangular vectors as:

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k};\ \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$



Another way to express the result is

$$\vec{A}\times\vec{B}=\vec{C}=C_x\hat{i}+C_y\hat{j}+C_z\hat{k}$$

$$C_x=A_yB_z-A_zB_y:\ C_y=A_zB_x-A_xB_z;\ C_z=A_xB_y-A_yB_x$$

Subjective Assignment – IV

Q.1 Find the cross product of vectors $\vec{A}=2\hat{i}$ and $\vec{B}=-2\hat{i}+4\hat{j}$

Q.2 Determine the area of the parallelogram whose adjacent sides are $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\hat{j}$.

Q.3 Show that vectors $\vec{A}=\hat{i}-5\hat{j}$ and $\vec{B}=2\hat{i}-10\hat{j}$ are parallel to each other.

Q.4 Find a unit vector perpendicular to both the vectors $\vec{A} = 3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4\hat{k}$.

Q.5 If $\vec{A}=\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{B}=3\hat{i}+\hat{j}+2\hat{k}$, then find the vector product $\vec{A}\times\vec{B}$.

Q.6 Prove that the vectors $\vec{A}= 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B}=12\hat{i} + 9\hat{j} + \hat{k}$ are parallel to each other.

Q.7 If $\vec{A} = 2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B} = 3\hat{i}+2\hat{j}+4\hat{k}$, then find the value of $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

Q.8 Find the value of a for which the vectors $3\hat{i}+3\hat{j}+9\hat{k}$ and $\hat{i}+a\hat{j}+3\hat{k}$ are parallel.

Answers

1.	$8\hat{k}$	2.	$3\sqrt{3}$ sq. units	4.	$\dfrac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$
5.	$4\hat{i}+4\hat{j}-8\hat{k}$	7.	$-20\hat{i}+10\hat{j}+10\hat{k}$	8.	a = 1

Conceptual Questions

Q.1 $\vec{A}$ and $\vec{B}$ are two vectors, can $\vec{A}+\vec{B}$ be zero?

Q.2 Can three vectors (i) lying in a plane (ii) not lying in a plane give zero resultant?

Q.3 What is the condition for zero resultant vector for more than three vectors acting simultaneously on a particle?

Q.4 If $\vec{A}\times\vec{B} = \vec{C}\times\vec{B}$, show that $\vec{C}$ need not be equal to $\vec{A}$. When will $\vec{A}$ be equal to $\vec{C}$.

Q.5 Does a scalar quantity depend on the frame of reference chosen?

Q.6 Does it make a sense to call a physical quantity a vector, when its magnitude is zero?

Q.7 Can a vector be multiplied by both dimensional and non–dimensional scalars?

Q.8 Can magnitude of the rectangular component of a vector be greater than the magnitude of that vector?

Q.9 What is the vector sum of n coplanar forces, each of magnitude F, if each force makes an angle of $2\pi/n$ with the preceding force?

Q.10 What is the condition for two vectors to be collinear?

NCERT Questions

Q.1 Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Q.2 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) Adding any two scalars.
(b) Adding a scalar to a vector of the same dimensions.
(c) Multiplying any vector by any scalar.
(d) Multiplying any two scalars.
(e) Adding any two vectors.
(f) Adding a component of a vector to the same vector.

Q.3 Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar.

(b) Each component of a vector is always a scalar.

(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to magnitude of average velocity of particle over same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector.

Q.4 Establish the following vector inequalities geometrically or otherwise:

(a) $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$ (b) $|\vec{a}+\vec{b}|\ge|\vec{a}|-|\vec{b}|$ (c) $|\vec{a}-\vec{b}|\le|\vec{a}|+|\vec{b}|$ (d) $|\vec{a}-\vec{b}|\ge|\vec{a}|-|\vec{b}|$

When does the equality sign above apply?

Q.5 Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, which of the following statements are correct:

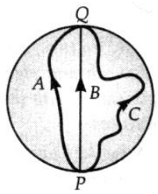
(a) $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ must each be a null vector,

(b) The magnitude of $(\vec{a}+\vec{c})$ equals the magnitude of $(\vec{b}+\vec{d})$,

(c) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}, \vec{c}$ and $\vec{d}$,

(d) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear?

Q.6 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skated?



Q.7 On an open ground, a motorist follows a track that turns to his left by an angle of 60^{o} after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with total path length covered by the motorist in each case.

Q.8 (a) If $\hat{i}$ and $\hat{j}$ are unit vectors along X– and Y–axis respectively, then what is the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

(b) Find the components of $\vec{a} = 2\hat{i} + 3\hat{j}$ along the directions of vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$

Q.9 Read each statement below carefully and state, with reasons and examples, if it is true or false: A scalar quantity is one that (a) is conserved in a process, (b) can never take negative values, (c) must be dimensionless, (d) does not vary from one point to another in space, (e) has the same value for observers with different orientations of axes.

Q.10 A vector has magnitude and direction. Does it have a location in space? Can it vary with time? Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Q.11 A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

Q.12 Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

Q.13 State for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Q.14 Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.

Answers

1. Impulse 2. (a) No, (b) No, (c) Yes, (d) Yes, (e) No, (f) No

3. (a) True, (b) False, (c) False, (d) True, (e) True

5. (a) incorrect, (b) correct, (c) correct (d) correct

6. Displacement of each girls = $\overrightarrow{PQ}$ = 400m, For girl B the magnitude of displacement vector = actual length of path

7. (i) displacement = 1 km, Total path length = 1.5 km (ii) displacement = zero, Total path length = 3 km
(iii) displacement = 866 m, Total path length = 4 km

8. (a) $\sqrt{2}, \alpha = 45^{o}$; $\sqrt{2}$, $\beta = -45^{o}$ (b) $\frac{5}{2}(\hat{i}+\hat{j})$, $-\frac{1}{2}(\hat{i}-\hat{j})$

9. (a) False, (b) False, (c) False, (d) False, (e) True

11. No, anything that has both magnitude and direction is not necessarily a vector. It must obey the laws of vector addition. An infinitesimally small rotation is considered a vector.

12. Only a plane area can be associated with a vector.

13. **Scalar:** Volume, mass, speed, density, number of moles and angular frequency.
Vectors: Acceleration, velocity, displacement and angular velocity.

14. Work and current

Objective Assignment

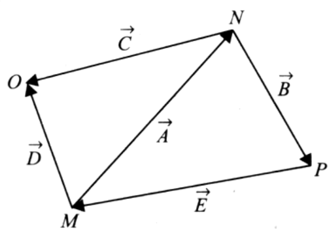
Single Choice Type Questions

Q.1 The sum and difference of two perpendicular vectors of equal lengths are
(a) also perpendicular and of equal length
(b) also perpendicular and of different lengths
(c) of equal length and have an obtuse angle between them
(d) of equal length and have an acute angle between them

Q.2 The minimum number of vectors having different planes which can be added to give zero resultant is
(a) 2 (b) 3 (c) 4 (d) 5

Q.3 A vector perpendicular to $\hat{i} + \hat{j} + \hat{k}$ is
(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} - \hat{k}$ (c) $-\hat{i} - \hat{j} - \hat{k}$ (d) $3\hat{i} + 2\hat{j} - 5\hat{k}$

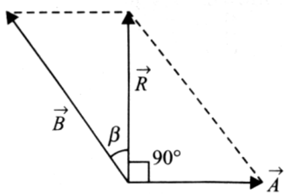
Q.4 From figure, the correct relation is



(a) $\vec{A}+\vec{B}+\vec{E}=\vec{0}$ (b) $\vec{C}-\vec{D}=-\vec{A}$ (c) $\vec{B}+\vec{E}-\vec{C}=-\vec{D}$ (d) All of the above

Q.5 Out of the following set of forces, the resultant of which cannot be zero
(a) 10, 10, 10 (b) 10, 10, 20 (c) 10, 20, 20 (d) 10, 20, 40

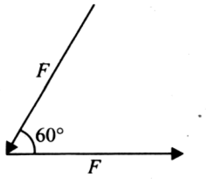
Q.6 The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $\vec{A}$ and its magnitude is equal to half of the magnitude of vector $\vec{B}$. The angle between $\vec{A}$ and $\vec{B}$ is



(a) 120^o (b) 150^o (c) 135^o (d) None of these

Q.7 The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{a}$ and $\vec{b}$ is 3 : 1. Now, $|\vec{a}|$ is equal to
(a) $|\vec{b}|$ (b) $2|\vec{b}|$ (c) $3|\vec{b}|$ (d) $4|\vec{b}|$

Q.8 Two forces, each equal to F, act as shown in figure. Their resultant is



(a) F/2 (b) F (c) $\sqrt{3}$ F (d) $\sqrt{5}$ F

Q.9 Vector $\vec{A}$ is 2 cm long and is 60^{o} above the x–axis in the first quadrant. Vector $\vec{B}$ is 2 cm long and is 60^{o} below the x–axis in the fourth quadrant. The sum $\vec{A}+\vec{B}$ is a vector of magnitude

(a) 2cm along + y–axis (b) 2 cm along + x–axis
(c) 2 cm along – u–axis (d) 2 cm along –x–axis

Q.10 What is the angle between two vector forces of equal magnitude such that the resultant is one–third as such as either of the original forces?

(a) $\cos^{-1}\left(-\frac{17}{18}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$ (c) 45^{o} (d) 120^{o}

Q.11 The angle between $\vec{A}+\vec{B}$ and $\vec{A}\times\vec{B}$ is

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) π

Q.12 The projection of a vector $\vec{r}=3\hat{i}+\hat{j}+2\hat{k}$ on the x–y plane has magnitude

(a) 3 (b) 4 (c) $\sqrt{14}$ (d) $\sqrt{10}$

Q.13 If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

(a) 120^{o} (b) 60^{o} (c) 90^{o} (d) 0^{o}

Q.14 If vectors $\vec{A}=\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{B}=5\hat{i}$ represent the two sides of a triangle, then the third side of the triangle can have length equal to

(a) 6 (b) $\sqrt{56}$
(c) both of the above (d) none of the above

Q.15 Given $|\vec{A}_1|=2, |\vec{A}_2|=3$ and $|\vec{A}_1+\vec{A}_2|=3$. Find the value of $\left(\vec{A}_1+2\vec{A}_2\right).\left(3\vec{A}_1-4\vec{A}_2\right)$

(a) – 64 (b) 60 (c) – 62 (d) 61

Q.16 Three vectors $\vec{A}, \vec{B}, \vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ and $\vec{A}.\vec{C}=0$. The vector $\vec{A}$ is parallel to

(a) $\vec{B}$ (b) $\vec{C}$ (c) $\vec{B}.\vec{C}$ (d) $\vec{B}\times\vec{C}$

Q.17 If $\vec{A}=\vec{B}+\vec{C}$, and the magnitude of $\vec{A}, \vec{B}, \vec{C}$ are 5, 4 and 3 units, then angle between $\vec{A}$ and $\vec{C}$ is

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\cos^{-1}\left(\frac{4}{5}\right)$ (c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\pi/2$

Q.18 Given: $\vec{A}=A\cos\theta\hat{i}+A\sin\theta\hat{j}$. A vector $\vec{B}$ which is perpendicular to $\vec{A}$ is given by

(a) $B\cos\theta\hat{i}-B\sin\theta\hat{j}$ (b) $B\sin\theta\hat{i}-B\cos\theta\hat{j}$

(c) $B\cos\theta\hat{i}+B\sin\theta\hat{j}$ (d) $B\sin\theta\hat{i}+B\cos\theta\hat{j}$

Q.19 The angle which the vector $\vec{A}=2\hat{i}+3\hat{j}$ makes with y–axis, where $\hat{i}$ and $\hat{j}$ are unit vectors along x– and y– axes, respectively, is

(a) $\cos^{-1}(3/5)$ (b) $\cos^{-1}(2/3)$ (c) $\tan^{-1}(2/3)$ (d) $\sin^{-1}(2/3)$

Q.20 In going from one city to another, a car travels 75 km north, 60 km north–west and 20 km east. The magnitude of displacement between the two cities is (Take $\frac{1}{\sqrt{2}}=0.7$)

(a) 170 km (b) 137 km (c) 119 km (d) 140 km

Q.21 What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ are the adjacent sides of a parallelogram drawn from a common point and the area of the parallelogram is AB/2?

(a) 15^o (b) 30^o (c) 45^o (d) 60^o

Q.22 Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$?

(a) 0^o (b) 90^o (c) 60^o (d) 180^o

Q.23 Given: $\vec{A}=4\hat{i}+6\hat{j}$ and $\vec{B}=2\hat{i}+3\hat{j}$. Which of the following is correct?

(a) $\vec{A}\times\vec{B}=\vec{0}$ (b) $\vec{A}.\vec{B}=24$

(c) $\frac{|\vec{A}|}{|\vec{B}|}=\frac{1}{2}$ (d) $\vec{A}$ and $\vec{B}$ are antiparallel

Q.24 Given: $\vec{A}=2\hat{i}+p\hat{j}+q\hat{k}$ and $\vec{B}=5\hat{i}+7\hat{j}+3\hat{k}$ If $\vec{A}||\vec{B}$, then the values of p and q are, respectively,

(a) $\frac{14}{5}$ and $\frac{6}{5}$ (b) $\frac{14}{3}$ and $\frac{6}{5}$ (c) $\frac{6}{5}$ and $\frac{1}{3}$ (d) $\frac{3}{4}$ and $\frac{1}{4}$

Q.25 If the angle between vectors $\vec{a}$ and $\vec{b}$ is an acute angle, then the difference $\vec{a}-\vec{b}$ is

(a) the major diagonal of the parallelogram (b) the minor diagonal of the parallelogram
(c) any of the above (d) none of the above

Q.26 Given that $\vec{A}+\vec{B}=\vec{C}$. if $|\vec{A}|=4$, $|\vec{B}|=5$ and $|\vec{C}|=\sqrt{61}$. The angle between $\vec{A}$ and $\vec{B}$ is

(a) 30^o (b) 60^o (c) 90^o (d) 120^o

Q.27 Given vector $\vec{A}=2\hat{i}+3\hat{j}$, the angle between $\vec{A}$ and y–axis is

(a) $\tan^{-1}(3/2)$ (b) $\tan^{-1}(2/3)$ (c) $\sin^{-1}(2/3)$ (d) $\cos^{-1}(2/3)$

Q.28 If $\vec{b}=3\hat{i}+4\hat{j}$ and $\vec{a}=\hat{i}-\hat{j}$, the vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$ is

(a) $\frac{5}{\sqrt{2}}(\hat{i}-\hat{j})$ (b) $\frac{5}{\sqrt{2}}(\hat{i}+\hat{j})$ (c) $5(\hat{i}-\hat{j})$ (d) $5(\hat{i}+\hat{j})$

Q.29 Choose the wrong statement

(a) Three vectors of different magnitudes may be combined to give zero resultant.
(b) Two vectors of different magnitudes can be combined to give a zero resultant.
(c) The product of a scalar and a vector is a vector quantity.
(d) All of the above are wrong statements.

Q.30 What displacement at an angle 60^o to the x–axis has an x–component of 5 m? $\hat{i}$ and $\hat{j}$ are unit vectors in x and y directions, respectively.

(a) $5\hat{i}$ (b) $5\hat{i}+5\hat{j}$

(c) $5\hat{i}+5\sqrt{3}\,\hat{j}$ (d) All of the above

Q.31 Mark the correct statement

(a) $|\vec{a}+\vec{b}|\geq|\vec{a}|+|\vec{b}|$ (b) $|\vec{a}+\vec{b}|\leq|\vec{a}|+|\vec{b}|$ (c) $|\vec{a}-\vec{b}|\geq|\vec{a}|+|\vec{b}|$ (d) all of the above

Q.32 Out of the following forces, the resultant of which cannot be 10 N?

(a) 15 N and 20 N (b) 10 N and 10 N (c) 5 N and 12 N (d) 12 N and 1 N

Q.33 In an equilateral triangle ABC, AL, BM and CN are medians. Forces along BC and BA represented by them will have a resultant represented by

(a) 2AL (b) 2BM (c) 2CN (d) AC

Q34 If a parallelogram is formed with two sides represented by vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}+\vec{b}$ represents the

(a) major diagonal when the angle between vectors is acute

(b) minor diagonal when the angle between vectors is obtuse

(c) both of the above (d) none of the above

Q.35 Two forces of $\vec{F}_1 = 500$ N due east and $\vec{F}_2 = 250$ N due north have their common initial point. $\vec{F}_2 - \vec{F}_1$ is

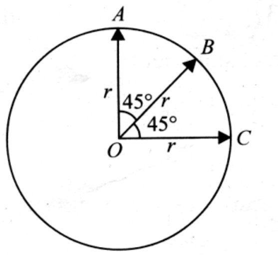
(a) $250\sqrt{5}$N, $\tan^{-1}(2)$ W of N (b) 250 N, $\tan^{-1}(2)$ W of N

(c) zero (d) 750 N, $\tan^{-1}(3/4)$ N of W

Q.36 The resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ shown in figure is

(a) r (b) 2r

(c) $r(1+\sqrt{2})$ (d) $r(\sqrt{2}-1)$



Q.37 Two vectors $\vec{a}$ and $\vec{b}$ are at an angle of 60^o with each other. Their resultant makes an angle of 45^o with $\vec{a}$. If $|\vec{b}|=2$ units, then $|\vec{a}|$ is

(a) $\sqrt{3}$ (b) $\sqrt{3}-1$ (c) $\sqrt{3}+1$ (d) $\sqrt{\frac{3}{2}}$

Q.38 The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant vector becomes perpendicular to $\vec{P}$. Then, the magnitude of $\vec{R}$ is equal to

(a) P + Q (b) P (c) P – Q (d) Q

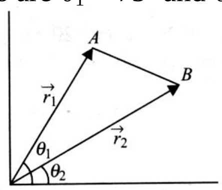
Q.39 A vector $\vec{A}$ when added to the vector $\vec{B} = 3\hat{i}+4\hat{j}$ yields a resultant vector that is in the positive y–direction and has a magnitude equal to that of $\vec{B}$. Find the magnitude of $\vec{A}$.

(a) $\sqrt{10}$ (b) 10 (c) 5 (d) $\sqrt{15}$

Q.40 ABCDEF is a regular hexagon with point O as centre. The value of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AE}$ is

(a) $2\,\overrightarrow{AO}$ (b) $4\,\overrightarrow{AO}$ (c) $6\overrightarrow{AO}$ (d) 0

Q.41 In a two dimensional motion of a particle, the particle moves from point A, position vector $\vec{r}_1$, to point B, position vector $\vec{r}_2$. If the magnitudes of these vectors are, respectively, $r_1 = 3$ and $r_2 = 4$ and the angles they make with the x–axis are $\theta_1 = 75^o$ and $\theta_2 = 15^o$, respectively, then find the magnitude of the displacement vector.



(a) 15 (b) $\sqrt{13}$ (c) 17 (d) $\sqrt{15}$

Q.42 The angle between two vectors $\vec{A}$ and $\vec{B}$ is θ. Resultant of these vectors $\vec{R}$ makes an angle $\theta/2$ with $\vec{A}$. which of the following is true?

(a) A = 2B (b) A = B/ 2 (c) A = B (d) AB = 1

Q.43 The resultant of three vectors 1, 2 and 3 units whose directions are those of the sides of an equilateral triangle is at an angle of

(a) 30^o with the first vector (b) 15^o with the first vector

(c) 100^o with the first vector (d) 150^o with the first vector

Q.44 A particle moves in the xy plane with only an x–component of acceleration of 2 ms^{-2}. The particle starts from the origin at t = 0 with an initial velocity having an x–component of 8 ms^{-1} and y–component of – 15 ms^{-1}. The total velocity vector at any time t is

(a) $[(8 + 2t)\hat{i} - 15\hat{j}]ms^{-1}$ (b) zero

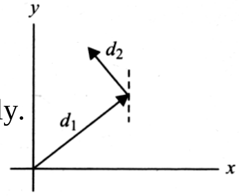
(c) $2t\hat{i} + 15\hat{j}$ (d) directed along z–axis

Q.45 What is the resultant of three coplanar forces: 300 N at 0^o, 400 N at 30^o, and 400 N at 150^o?

(a) 500 N (b) 700 N (c) 1, 100 N (d) 300 N

Multiple Choice Questions

Q.46 Which of the following statement is/are correct (se figure)?



(a) The signs of x–component of $\vec{d}_1$ is positive and that of $\vec{d}_2$ is negative.

(b) The signs of the y–component of $\vec{d}_1$ and $\vec{d}_2$ are positive & negative, respectively.

(c) The signs of x–and y–components of $\vec{d}_1 + \vec{d}_2$ are positive.

(d) None of these

Q.47 Given two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$. θ is the angle between $\vec{A}$ and $\vec{B}$. Which of the following statements is/are correct?

(a) $|\vec{A}|\cos\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.

(b) $|\vec{A}|\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

(c) $|\vec{A}|\cos\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.

(d) $|\vec{A}|\sin\theta\left(\frac{\hat{i}-\hat{j}}{2}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

Q.48 If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are two vectors, then the unit vector

(a) perpendicular to $\vec{A}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$ (b) parallel to $\vec{A}$ is $\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$

(c) perpendicular to $\vec{B}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$ (d) parallel to $\vec{A}$ is $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

Q.49 If $(\vec{v}_1 + \vec{v}_2)$ is perpendicular to $(\vec{v}_1 - \vec{v}_2)$, then

(a) $\vec{v}_1$ is perpendicular to $\vec{v}_2$ (b) $|\vec{v}_1| = |\vec{v}_2|$

(c) $\vec{v}_1$ is null vector (d) the angle between $\vec{v}_1$ and $\vec{v}_2$ can have any value

Q.50 Two vectors $\vec{A}$ and $\vec{B}$ lie in one plane. Vector $\vec{C}$ lies in a different plane. Then, $\vec{A}+\vec{B}+\vec{C}$

(a) cannot be zero (b) can be zero

(c) lies in the plane of $\vec{A}$ or $\vec{B}$ (d) lies in a plane different from that of any of three vectors

Assertion–Reason (Reasoning type)

Direction: Each of these questions contains two statements: Statement I (assertion) and Statement II (reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below:

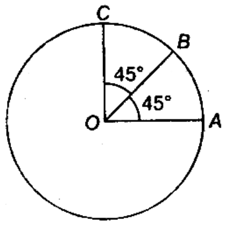
(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true, Statement II is true; statement II is not a correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

Q.51 **Statement – I:** The resultant of three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ as shown in the figure is R $(1+\sqrt{2})$. R is the radius of the circle



Statement – II: $\overrightarrow{OA}+\overrightarrow{OC}$ is acting along $\overrightarrow{OB}$ and $(\overrightarrow{OA}+\overrightarrow{OC})+\overrightarrow{OB}$ is acting along $\overrightarrow{OB}$.

Q.52 **Statement – I:** Two forces acting at a point, will have the resultant force directed outwards from the point at which the forces are acting.

Statement – II: The resultant of two forces, acts along the diagonal formed by the parallelogram with the sides being the given forces.

Q.53 **Statement – I:** If $\vec{A}$ is parallel to $\vec{B}$, *then* $\vec{A}\times\vec{B}$ is a null vector.

Statement – II: The cross product of two vectors is given by $\vec{A}\times\vec{B}=$ AB sin θ.

Q.54 **Statement – I:** Scalars can be added algebraically.

Statement – II: Vectors cannot be added algebraically.

Q.55 **Statement – I:** Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is 45^o.

Statement – II: $\hat{i}+\hat{j}$ is equally include to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90^o.

Entrance Questions

Q.56 A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10s is

(a) 10 units (b) $7\sqrt{2}$ units (c) 7 units (d) 8.5 units [AIEEE 2009]

Q.57 If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between A and B is

(a) π (b) π/3 (c) π/2 (d) π/4 [AIEEE 2004]

Q.58 A force $\vec{F}=(5\hat{i}+3\hat{j}+2\hat{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r}=(2\hat{i}-\hat{j})m$. The work done on the particle in joule is [AIEEE 2004]

(a) – 7 (b) + 7 (c) + 10 (d) + 13

Q.59 If the order of two vectors A and B is reversed, then in cross product of two vectors, the resultant vector

(a) changes only in direction (b) changes in magnitude [BITSAT 2006]

(c) changes both in magnitude and direction (d) does not change both in magnitude and direction

Q.60 The component of vector $A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ along the direction of $\hat{i} - \hat{j}$ *is* [BITSAT 2008]

(a) $a_x - a_y + a_z$ (b) $a_x - a_y$ (c) $\frac{(a_x - a_y)}{\sqrt{2}}$ (d) $a_x + a_y + a_z$

Answers

1	a	2	c	3	d	4	d	5	d
6	b	7	b	8	b	9	b	10	a
11	c	12	d	13	a	14	c	15	a
16	d	17	a	18	b	19	c	20	c
21	b	22	b	23	a	24	a	25	b
26	b	27	b	28	a	29	b	30	c
31	b	32	d	33	b	34	c	35	a
36	c	37	b	38	d	39	a	40	c
41	b	42	c	43	a	44	a	45	a
46	a, c	47	a, b	48	a, b, c	49	a, d	50	a, d
51	a	52	d	53	a	54	b	55	a
56	b	57	a	58	b	59	a	60	c

MOTION IN A STRAIGHT LINE

Mechanics: Mechanics is the branch of physics that *deals with the conditions of rest or motion of the material objects.*

Sub–branches of mechanics:

(i) **Statics.** It is the branch of mechanics that *deals with the study of objects at rest or in equilibrium,* even when they are under the action of several forces. The measurement of time is not essential in statics.

(ii) **Kinematics:** It is the branch of mechanics that *deals with the study of motion of objects without considering the cause of motion.*

(iii) **Dynamics:** It is the branch of mechanics that *deals with the study of motion of objects taking into consideration the cause of their motion.*

Rest: *An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time e.g.,* a book lying on a table.

Motion: *An object is said to be in motion if its position changes w.r.t. its surroundings with the passage of time e.g.,* a train moving on rails.

Rest and motion are relative terms. A passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to objects outside the train. Hence rest and motion are relative terms.

Absolute rest and motion are unknown

Types of Motion of a Body

Mainly the motion of a body can be of following three types:

(1) Rectilinear or translatory motion

Rectilinear motion *is that motion in which a particle or point mass body is moving along a straight line.*

Translatoy motion *is that motion in which a body, which is not a point mass body mass body is moving such that all its constituent particles move simultaneously along parallel straight lines and shift through equal distance in a given interval of time. e.g.,* A body slipping along the inclined plane has traslatory motion. Rectilinear or translatory motion can be uniform or non–uniform.

(2) Circular or Rotatory Motion

A circular motion *is that motion in which a particle or a point mass body is moving on a circle.*

A rotatory motion *is that motion in which a body, which is not appoint mass body, is moving such that all its constituent particles move simultaneously along concentric circles, whose centres lie on a line, called axis of rotation and shift through equal angle in a given time.*

Circular or rotatory motion can be two dimensional or three dimensional motion and can be uniform or non–uniform motion. If the circular or rotatory motion is uniform, it is periodic also.

(3) Oscillatory or Vibratory Motion

Oscillatory motion *is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position) in a definite interval of time.* e.g., the motion of the pendulum of wall clock is oscillatory motion.

If in the oscillatory motion, the amplitude is very small i.e. microscopic, the motion of body is said to be **vibratory motion.**

Frame of Reference

A rectangular coordinate system, consisting of three mutually perpendicular axes, labeled X–,Y– and Z–axis. The point of intersection O of these three axes is called origin which serves as a **reference point or the position of the observer.**

The position of the object at a given instant of time can be described in terms of position coordinates (x, y, z), i.e., the distances of the given position of object along the X–, Y– and Z–axis. *This coordinate system alongwith a clock constitutes a* **frame or reference.**

Thus the frame of reference is a system of coordinate axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc. of a moving object.

Frames of reference can be of two types:

(a) Inertial frame of reference (b) Non–inertial frame of reference

(a) Inertial frame of reference is one in which Newton's first law* of motion holds goods.

(b) Non–inertial frame of reference is one in which Newton's first law of motion does not hold good.

Point Object

If the position of an object changes by distances much greater than its own size in a reasonable duration of time, then the object may be regarded as a point object.

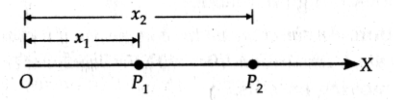
Example:

(i) Earth can be regarded as a point object for studying its motion around the sun.

(ii) A train under a journey of several hundred kilometers can be regarded as a point object.

One dimensional motion

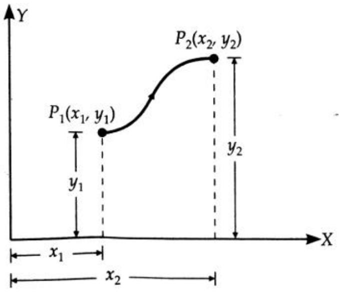
The motion of an object is said to be one dimensional if only one of the three coordinates specifying the position of the object changes with time. Hence the object moves along a straight line. This motion is also called **rectilinear or linear motion.**



Examples of one dimensional motion:

(i) Motion of a train along a straight track. (ii) Motion of a freely falling body.

Two dimensional motion: *The motion of an object is said to be two dimensional if only two of the three coordinates specifying its position change with time.*

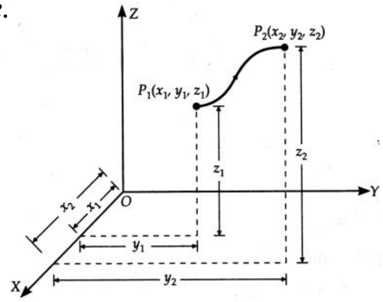


Examples of two dimensional motion:

(i) Motion of planets around the sun.

(ii) (ii) A car moving along a zig–zag path on a level road.

Three dimensional motion: *The motion of an object is said to be three dimensional if all the three coordinates specifying its position change with time.*

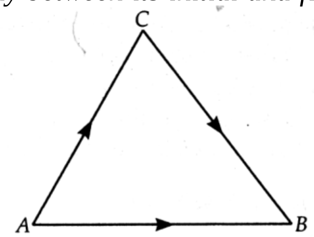


Examples of three dimensional motion:

(i) A kite flying on a windy day.

(ii) Motion of an aeroplane in space.

Distance or path length: *It is the length of the actual path traversed by a body between its initial and final positions*



Distance covered = AC + CB

Distance is a *scalar quantity* because it has only magnitude and no direction.

Distance covered is always positive or zero.

The SI unit of distance is metre (m).

The CGS unit of distance is centimetre (cm).

Displacement

The displacement of an object is the change in the position of an object in a fixed direction. It is the shortest (or the straight line) path measured in the direction from initial point to the final point. Displacement has both magnitude and direction, so it is a vector quantity.

Displacement = $\overrightarrow{AB}$

Displacement may be positive, negative or zero.

The SI unit of displacement is metre (m).

The CGS unit of displacement is centrimetre (cm).

Characteristics of Displacement

(i) Displacement has the units of length.

(ii) *The displacement of an object can be positive, negative or zero.*

(iii) *Displacement is not dependent on the choice of the origin O of the position coordinates.*

(iv) *The actual distance traveled by an object in a given time interval is greater than or equal to the magnitude of the displacement.*

(v) *The displacement of an object between two points is the unique path that takes the body from its initial to final position.*

(vi) *The displacement of an object between two positions does not give any information regarding the shape of the actual path followed by the object between these two positions.*

(vii) *The magnitude of the displacement of an object between two positions gives the shortest distance between these positions.*

(viii) *Displacement is a vector quantity. Displacement of an object between two given positions is independent of the actual path followed by the object in moving from one position to another.*

Speed

The rate of change of position of an object with time in any direction is called its speed. It is equal to the distance traveled by the object per unit time.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed has only magnitude and no direction, so it is *a scalar quantity.*

The SI unit of speed is ms^{-1}. The CGS unit of speed is cms^{-1}.

The dimensional formula of speed is $[M^0L^1T^{-1}]$

Different Types of Speed

(i) ***Uniform speed:*** An object is said to be moving with uniform speed, if it covers equal distances in equal intervals of time, however small these time intervals may be.

(ii) ***Variable speed:*** An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.

(iii) ***Average speed:*** For an object moving with variable speed, the average speed is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(iv) ***Instantaneous speed:*** *The speed of an object at any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.*

Instantaneous speed,

$$\upsilon = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Here $\frac{dx}{dt}$ is the first order derivative of distance x with respect to time t.

The **speedometer** of an automobile indicates its instantaneous speed at any instant.

Average Speed in Different Situations

A body covering different distances with different speeds. Suppose a body covers distances $s_1, s_2, s_3, \ldots$, with speed $v_1, v_2, v_3, \ldots$. Respectively, then its average speed will be

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{s}{t}$$

$$= \frac{s_1+s_2+s_3+.....}{t_1+t_2+t_3+......} \quad \text{or} \quad v_{av} = \frac{s_1+s_2+s_3+.....}{\left(\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}\right)}$$

Special case: If $s_1 = s_2 = s$ i.e., the body covers equal distances with different speeds, then

$$v_{av} = \frac{2s}{s\left(\frac{1}{v_1}+\frac{1}{v_2}\right)} = \frac{2\,v_1 v_2}{v_1+v_2}$$

Clearly, *the average speed is the harmonic mean of the individual speeds.*

A body moving with different speeds in different time intervals. Suppose a body travels with speeds v_1, v_2, v_3, …. In time intervals t_1, t_2, t_3, …. respectively, then

Total distance traveled = $v_1t_1 + v_2t_2 + v_3t_3 + ….$

Total time taken = $t_1 + t_2 + t_3 + …..$

$$\therefore \qquad v_{av} = \frac{v_1t_1+v_2t_2+v_3t_3+....}{t_1+t_2+t_3+....}$$

Special case: It $t_1 = t_2 = t_3 = …. t_n = t$ (say), then

$$v_{av} = \frac{(v_1+v_2+v_3+...+v_n)t}{nt} = \frac{v_1+v_2+v_3+.......+v_n}{n}$$

Clearly, *average speed is the arithmetic mean of the individual speeds.*

Velocity

The rate of change of position of an object with time in a given direction is called its velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

The SI unit of velocity is ms^{-1}

The CGS unit of velocity is cms^{-1}

The dimensional formula for the velocity is $[M^0L^1T^{-1}]$

Different Types of Velocities:

(i) ***Uniform velocity:*** *A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these time intervals may be.*

(ii) ***Variable velocity:*** *A body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.*

(iii) ***Average velocity:*** *For an object moving with variable velocity, average velocity is defined as the ratio of its total displacement to the total time interval in which that displacement occurs.*

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Total time taken}}$$

$$\vec{v}_{av} = \frac{\vec{x}_2-\vec{x}_1}{t_2-t_1} = \frac{\Delta\vec{x}}{\Delta t}$$

(iv) ***Instantaneous velocity:*** *The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.*

$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Thus instantaneous velocity of an object is equal to the first order derivative of its displacement with respect to time.

Uniform motion: *An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however, small these time intervals may be, in the same fixed direction.*

Some Important Features of Uniform Motion

(i) The velocity in uniform motion does not depend on the choice of origin.

(ii) The velocity in uniform motion does not depend on the choice of the time interval $(t_2 - t_1)$

(iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

(iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(v) For an object in uniform motion, no force is required to maintain its motion.

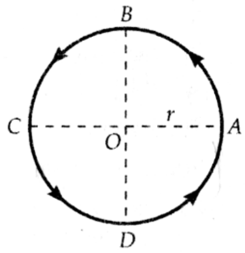
(vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant or at each point of the path.

Non–uniform motion

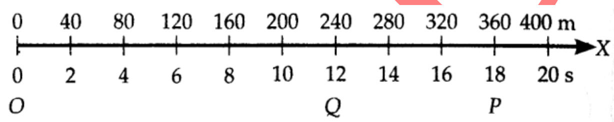
A body is said to be in non–uniform motion if its velocity changes with time. Here either the speed of the body or its direction of motion or both change with time.

Subjective Assignment – I

Q. 1 In figure, a particle moves along a circular path of radius r. It starts from point A and moves anticlockwise. Find the distance travelled by the particle as it (i) moves from A to B (ii) moves from A to C (iii) moves A to D (iv) completes one revolution. Also find the magnitude of displacement in each case.



Q.2 A car is moving along X–axis. As shown in figure, it moves from 0 to P in 18s and returns from P to Q in 6s. What are the average velocity and average speed of the car in going from (i) from O to P and (ii) from O to P and back to Q?



Q.3 A body travels from A to B at 40 ms^{-1} and from B to A at 60 ms^{-1}. Calculate the average speed and average velocity?

Q.4 On a 60 km track, a train travels the first 30 km with a uniform speed of 30 km h^{-1}. How fast must the train travel the next 30 km so as to average 40 km h^{-1} for the entire trip?

Q.5 A body covers one–third of its journey with speed 'u', next one–third with speed 'v' and the last one–third with speed 'w'. Calculate the average sped of the body during the entire journey.

Q.6 A body travelling along a straight line traversed one–half of the total distance with a velocity v_0. The remaining part of the distance was covered with a velocity v_1, for half the time and with velocity v_2 for the other half of time. Find the mean velocity averaged over the whole time of motion.

Q.7 A body travels the first half of the total distance with velocity v_1 and the second half with velocity v_2. Calculate the average velocity.

Q.8 A car covers the first half of the distance between two places at a speed of 40 kmh^{-1} and the second half at 60 kmh^{-1}. What is the average speed of the car?

Q.9 A train moves with a speed of 30 kmh^{-1} in the first 15 minutes, with another speed of 40 kmh^{-1} the next 15 minutes, and then with a speed of 60 kmh^{-1} in the last 30 minutes. Calculate the average sped of the train for this journey.

Q.10 A body travels a distance s_1 with velocity v_1 and distance s_2 with velocity v_2 in the same direction. Calculate the average velocity of the body.

Q.11 A car travels along a straight line for the first half time with speed 50 kmh^{-1} and the second half time with speed 60 kmh^{-1}. Find the average speed of the car.

Answers

1. (i) $S=\frac{1}{2}\pi r, \vec{S}=\sqrt{2}$ (ii) $S=\pi r, \vec{S}=2r$ (iii) $S=\frac{3\pi r}{2}, \vec{S}=\sqrt{2}\,r$ (iv) $S=2\pi r, \vec{S}=0$

2. (i) $v = 20\ ms^{-1}$, $\vec{v}= 20\ ms^{-1}$ (ii) $\vec{v}= 10\ ms^{-1}$, $v = 20\ ms^{-1}$

3. $48\ ms^{-1}$, 0

4. $60\ km\ h^{-1}$

5. $\frac{3uvw}{uv+uw+vw}$

6. $\frac{2v_0(v_1+v_2)}{v_1+v_2+2v_0}$

7. $\frac{2v_1 v_2}{v_1+v_2}$

8. $48\ kmh^{-1}$

9. $47.5\ kmh^{-1}$

10. $\frac{(s_1+s_2)v_1 v_2}{s_1 v_2+s_2 v_1}$

11. $55\ kmh^{-1}$

Acceleration

The rate of change of velocity of an object with time is called its acceleration. It tells how fast the velocity of an object changes with time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. It has the same direction as that of the change in velocity.
The SI unit of acceleration is ms^{-2}.
The CGS unit of acceleration is cm^{-2}.
The dimensional formula of acceleration is $[M^0L^1T^{-2}]$

Different Types of Acceleration

(i) ***Uniform acceleration:*** *The acceleration of an object is said to be uniform acceleration if its velocity changes by equal amounts in equal intervals of time, however small these time intervals may be.*

(ii) ***Variable acceleration:*** *The acceleration of an object is said to be variable acceleration if its velocity changes by unequal amounts in equal intervals of time.*

(iii) ***Average acceleration:*** For an object moving with variable velocity, the average *acceleration is defined as the ratio of the total change in velocity of the object to the total time interval taken.*

$$\vec{a}_v=\frac{\vec{v}_2-\vec{v}_1}{t_2-t_1}=\frac{\Delta\vec{v}}{\Delta t}$$

(iv) ***Instantaneous acceleration:*** *The acceleration of an object at a given instant of time or at a given point of its motion, is called its instantaneous acceleration.*

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t}=\frac{d\vec{v}}{dt} \qquad \text{As} \qquad \vec{v}=\frac{d\vec{x}}{dt}$$

$$\therefore \quad \vec{a}=\frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right)=\frac{d^2\vec{x}}{dt^2}$$

Thus, acceleration is the first order derivative of velocity and second order derivative of position with respect to time.

Subjective Assignment – II

Q.1 The position of an object moving along x–axis is given by $x = a + bt^2$, where a = 8.5m, b = 2.5 ms^{-2} and t is measured in seconds. What is its velocity at t = 0s and t = 2s? What is the average velocity between
t = 2s and t = 4s?

Q.2 The displacement (in metre) of a particle moving along x–axis is given by $x = 18t + 5t^2$. Calculate:
(i) the instantaneous velocity at t = 2 s,
(ii) average velocity between t = 2s and t = 3s,

(iii) instantaneous acceleration

Q.3 The displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$. Find the position, velocity and acceleration of the particle at t = 0. When will the velocity of the particle become zero? Can we call the motion of the particle as one with uniform acceleration?

Q.4 The velocity of a particle is given by the equation, $v = 2t^2 + 5\ cms^{-1}$. Find (i) the change in velocity of the particle during the time interval between $t_1 = 2s$ and $t_2 = 4s$ (ii) the average acceleration during the same interval and (iii) the instantaneous acceleration at $t_2 = 4s$.

Q.5 The distance x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation, $t = \sqrt{x} + 3$, where x is in metres and t in seconds. Find the displacement of the particle when its velocity is zero.

Q.6 The acceleration of a particle in ms^{-2} is given by $a = 3t^2 + 2t + 2$, where time t is in second. If the particle starts with a velocity $v = 2\ ms^{-1}$ at t = 0, then find the velocity at the end of 2s.

Q.7 The displacement x of a particle at time t along a straight line is given by $x = \alpha - \beta t + \gamma t^2$. Find the acceleration of the particle.

Q.8 A particle moves along X–axis in such a way that its x–coordinate varies with time t as $x = 2 - 5t + 6t^2$. Find the initial velocity of the particle.

Q.9 The displacement x of a particle along X–axis is given by $x = 3 + 8t + 7t^2$. Obtain its velocity and acceleration at t = 2s.

Q.10 The distance traversed by a particle moving along a straight line is given by $x = 180t + 50\ t^2$ metre. Find:

(i) the initial velocity of the particle (ii) the velocity at the end of 4s and

(iii) the acceleration of the particle

Answers

1. 0, 10 ms^{-1}, 15.0 ms^{-1}
2. (i) 38 ms^{-1}, (ii) 43 ms^{-1}, (iii) 10 ms^{-2}
3. 25m, – 15 ms^{-1}, 8 ms^{-2}, 1.875 s
4. 24 cms^{-1}, 12 cms^{-2}, 16 cms^{-2}
5. 0
6. 18 ms^{-1}
7. 2γ
8. – 5 units
9. 36 ms^{-1}, 14 ms^{-2}
10. (i) 180 ms^{-1}, (ii) 580 ms^{-1}, (iii) 100 ms^{-2}

Equations of Motion by Calculus Method

First equation of motion: Acceleration is defined as

$$a = \frac{dv}{dt}$$

or $$dv = a dt \quad \dots (1)$$

When time = 0, velocity = u(say)

When time = t, velocity = v (say)

Integrating equation (1) within the above limits of time and velocity, we get

$$\int_u^v dv = \int_0^t a\ dt$$

or $$[v]_u^v = a\int_0^t dt = a[t]_0^t$$ or $$v - u = a(t - 0)$$

or $$v = u + at \quad \dots (2)$$

Second equation of motion: Velocity is defined as

$$v = \frac{ds}{dt}$$

or $\quad ds = vdt = (u + at)dt \quad \ldots (3)$

When time = 0, distance travelled = 0

When time = t, distance travelled = s (say)

Integrating equation (3) within the above limits of time and distance, we get

$$\int_0^s ds = \int_0^t (u + at)dt = u\int_0^t dt + a\int_0^t t\ dt$$

or $\quad [s]_0^s = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$

or $\quad s - 0 = u(t - 0) + a\left[\frac{t^2}{2} - 0\right] \quad$ or $\quad s = ut + \frac{1}{2}at^2 \quad \ldots (4)$

Third equation of motion: By the definitions of acceleration and velocity,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

or $\quad ads = vdv \quad \ldots (5)$

When time = 0, velocity = u, distance travelled = 0

When time = t, velocity = v, distance travelled = s

Integrating equation (5) within the above limits of velocity and distance, we get

$$\int_0^s a\ ds = \int_u^v v\ dv \quad \text{or} \quad a\int_0^s ds = \int_u^v v\ dv \quad \text{or} \quad a[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

or $\quad a[s - 0] = \frac{v^2}{2} - \frac{u^2}{2}$

or $\quad 2\ as = v^2 - u^2$

or $\quad v^2 - u^2 = 2\ as \quad \ldots (6)$

Fourth equation of motion. By definition of velocity,

$$v = \frac{ds}{dt}$$

or $\quad ds = vdt = (u + at)dt \quad \ldots (7)$

When time = (n – 1) second, distance travelled = s_{n-1} (say)

When time = n second, distance travelled = s_n (say)

Integrating equation (7) within the above limits of time and distance, we get

$$\int_{s_{n-1}}^{s_n} ds = \int_{n-1}^{n} (u + at)dt$$

or $\quad [s]_{s_{n-1}}^{s_n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt \quad$ or $\quad s_n - s_{n-1} = u[t]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$

$$= u\,[n - (n - 1)] + \frac{a}{2}[n^2 - (n-1)^2]$$

$$= u + \frac{a}{2}[n^2 - (n^2 - 2n + 1)] \qquad \text{or} \qquad s_{nth} = u + \frac{a}{2}(2n - 1)$$

where $s_{nth} = s_n - s_{n-1}$ = distance travelled in nth second.

Subjective Assignment – III

Q.1 A jet plane starts from rest with an acceleration of 3 ms^{-2} and makes a run for 35s before taking off. What is the minimum length of the runway and what is the velocity of the jet at take off?

Q.2 An electron travelling with a speed of 5×10^3 ms^{-1} passes through an electric field with an acceleration of 10^{12} ms^{-2}. (i) How long will it take for the electron to double its speed? (ii) What will be the distance covered by the electron in this time?

Q.3 A driver takes 0.20s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving car at a speed of 54 kmh^{-1} and the brakes cause a declaration of 6.0 ms^{-2}, find the distance travelled by the car after the sees the need to put the brakes.

Q.4 On a foggy day two drivers spot each other when they are just 80 metres apart. They are travelling at 72 km h^{-1} and 60 km h^{-1}, respectively. Both of them applied brakes retarding their cars at the rate of 5 ms^{-2}. Determine whether they avert collision or not.

Q.5 A hundred metre sprinter increases her speed from rest uniformly at the rate of 1 ms^{-2} upto three quarters of the total run and covers the last quater with uniform speed. How much time does she take to cover the first half and the second half of the run?

Q.6 A motor car starts from rest and accelerates uniformly for 10s to a velocity of 20 ms^{-1}. It then runs at a constant speed and is finally brought to rest in 40 m with a constant acceleration. Total distance covered is 640 m. Find the value of acceleration, retardation and total time taken.

Q.7 An athletic runs a distance of 1500 m in the following manner. (i) Starting from rest, he accelerates himself uniformly at 2 ms^{-2} till he covers a distance of 900 m. (ii) He, then runs the remaining distance of 600 m at the uniform speed developed. Calculate the time taken by the athlete to cover the two parts of the distance covered. Also find the time, when he is at the centre of the track.

Q.8 A man is s = 9m behind the door of a train when it starts moving with acceleration a = 2 ms^{-2}. The man runs at full speed. How far does he have to run and after what time does he get into the train? What is his full speed?

Q.9 A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t second, then calculate:

(i) the maximum velocity attained by the car, and

(ii) the total distance travelled by the car in terms of α, β and t.

Q.10 A body covers 12m in 2^{nd} second and 20m in 4^{th} second. How much distance will it cover in 4 seconds after the 5^{th} second?

Q.11 Two buses A and B are at positions 50m and 100m from the origin at time t = 0. They start moving in the same direction simultaneously with uniform velocity of 10 ms^{-1} and 5 ms^{-1}. Determine the time and position at which A overtakes B.

Q.12 An object is moving along +ve x–axis with a uniform acceleration of 4 ms^{-2}. At time t = 0, x = 5 m and v = 3 ms^{-1}.

(a) What will be the velocity and position of the object at time t = 2s?

(b) What will be the position of the object when it has a velocity of 5 ms^{-1}?

Q.13 A race car accelerates on a straight road from rest to a speed of 180 kmh^{-1} in 25s. Assuming uniform acceleration of the car throughout, find the distance covered in this time

Q.14 A bullet travelling with a velocity of 16 ms^{-1} penetrates a tree trunk and comes to rest in 0.4m. Find the time taken during the retardation.

Q.15 A car moving along a straight highway with a speed of 72 kmh^{-1} is brought to a stop within a distance of 100 m. What is the retardation of the car and how long does it take for the car to stop?

Q.16 On turning a corner a car driver driving at 36 kmh^{-1}, finds a child on the road 55 m ahead. He immediately applies brakes, so as to stop within 5m of the child. Calculate the retardation produced and the time taken by the car to stop.

Q.17 The reaction time for an automobile driver is 0.6s. If the automobile can be decelerated at 5 ms^{-2}, calculate the total distance travelled in coming to stop from an initial velocity of 30 kmh^{-1}, after a signal is observed.

Q.18 A car starts from rest and accelerates uniformly for 10s to a velocity of 8 ms^{-1}. It then runs at a constant velocity and is finally brought to rest in 64m with a constant retardation. The total distance covered by the car is 584 m. Find the value of acceleration, retardation and total time taken.

Q.19 Two trains – one travelling at 72 kmh^{-1} and other at 90 kmh^{-1} are heading towards one another along a straight level track. When they are 1.0 km apart, both the drivers simultaneously see the other's train and apply brakes which retard each train at the rate of 1.0 ms^{-2}. Determine whether the trains would collide or not.

Q.20 A burglar's car had a start with an acceleration of 2ms^{-2}. A police vigilant party came after 5 seconds and continued to chase the burglar's car within a uniform velocity of 20 ms^{-1}. Find the time in which the police van overtakes the burglar's car.

Q.21 A ball rolls down in inclined track 2m long in 4s. Find (i) acceleration (ii) time taken to cover the second metre of the track and (iii) speed of the ball at the bottom of the track.

Q.22 A bus starts from rest with a constant acceleration of 5 ms^{-2}. At the same time a car travelling with a constant velocity of 50 ms^{-1} overtakes and passes the bus. (i) Find at what distance will the bus overtake the car? (ii) How fast will be the bus be travelling then?

Q.23 A body starting from rest accelerates uniformly at the rate of 10 cms^{-2} and retards uniformly at the rate of 20 cms^{-2}. Find the least time in which it can complete the journey of 5 km if the maximum velocity attained by the body is 72 kmh^{-1}.

Q.24 A body covers a distance of 20m in the 7^{th} second and 24 m in the 9^{th} second. How much shall it cover in 15^{th} s?

Q.25 A body covers a distance of 4m is 3^{rd} second and 12m in 5^{th} second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds?

Q.26 An object is moving with uniform acceleration. Its velocity after 5 seconds is 25 ms^{-1} and after 8 seconds, it is 34 ms^{-1}. Find the distance travelled by the object in 12^{th} second.

Answers

1.	105 ms^{-1}, 1837.5m	2.	(i) 5×10^{-9} s, (ii) 3.75×10^{-5} m
3.	21.75 m	4.	Will be averted
5.	10s, 4.24 s	6.	2 ms^{-2}, 5 ms^{-2}, 39 s
7.	30s, 10s, 27.4 s	8.	3s, 18m, 6 ms^{-1}
9.	$v=\frac{\alpha\beta t}{\alpha+\beta}, S=\frac{\alpha\beta t^2}{2(\alpha+\beta)}$	10.	136 m
11.	10s, 150 m	12.	(i) 11 ms^{-1}, 19m, (ii) 7m
13.	625 m	14.	0.05 s
15.	2 ms^{-2}, 10s	16.	1 ms^{-2}, 10s
17.	11.94 m 18. 0.8 ms^{-2}, 0.5 ms^{-2}, 26 s	19.	No 20. 5 s
21.	(i) 0.25 ms^{-2}, (ii) 1.17 s, (iii) 1 ms^{-2}	22.	(i) 1000 m, (ii) 100 ms^{-1}
23.	400 s 24. 36 m	25.	60m 26. 44.5 m

Motion Under Gravity

Free fall: Motion of body falling towards the earth from a small height is called *free fall*. The acceleration with which a body falls is called acceleration due to gravity and is denoted by g.

Equations of motion for a freely falling body: For a freely falling body, the following equations of motion hold good:

(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$ (iii) $v^2 - u^2 = 2gs$

When a body falls freely under the action of gravity, it velocity increases and the value of g is taken positive. When a body is thrown vertically upward, its velocity decreases and the value of g is taken negative.

Subjective Assignment – IV

Q.1 A ball thrown vertically upwards with a speed of 19.6 ms^{-1} from the top of a tower returns to the earth in 6s. Find the height of the tower.

Q.2 A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistoreyed building. The height of the point from where the ball is thrown is 25 m from the ground. (i) How high will the ball rise? (ii) How long will it be before the ball hits the ground? (iii) Trace the trajectory of this ball.

Q.3 A ball thrown up is caught by the thrower after 4s. How high did it go and with what velocity was it thrown? How far was it below the highest point 3 s after it was thrown?

Q.4 A balloon is ascending at the rate of 9.8 ms^{-1} at a height of 39.2 m above the ground when a food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground? Take g = 9.8 ms^{-2}.

Q.5 A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. After two seconds (i) What is the velocity of the packet? (ii) How far is it below the helicopter? Take g = 9.8 ms^{-2}.

Q.6 A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long is he in the air? At what height did he bail out from the plane?

Q.7 Two balls are thrown simultaneously, A vertically upwards with a speed of 20 $ms^{-1}$from the ground, and B vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what points do the two balls collide? Take g = 9.8 ms^{-2}.

Q.8 A tennis ball is dropped on to the floor from a height of 4m. It rebounds to a height of 3m. If the ball was in contact with the floor for 0.01 s, what was its average acceleration during contact?

Q.9 A stone falls from a cliff and travels 24.5 m in the last second before it reaches the ground at the foot of the cliff. Find the height of the cliff.

Q.10 A stone is thrown vertically upwards with a velocity of 4.9 ms^{-1}. Calculate (i) the maximum height reached (ii) the time taken to reach the maximum height (iii) the velocity with which it returns to the ground and (iv) the time taken to reach the ground.

Q.11 A stone thrown upwards from the top of a tower 85m high, reaches the ground in 5 s. Find (i) the greatest height above the ground (ii) the velocity with which it reaches the ground and (iii) the time taken to reach the maximum height. Take g = 10 ms^{-2}.

Q.12 From the top of a multi–storeyed building, 39.2m tall, a boy projects a stone vertically upwards with an initial velocity of 9.8 ms^{-1} such that it finally drops to the ground. (i) When will the stone reach the ground? (ii) When will it pass through the point of projection? (iii) What will be its velocity before striking the ground? Take g = 10 ms^{-2}.

Q.13 A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 ms^{-2}. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached?

Q.14 A balloon is ascending at the rate of 14 ms^{-1} at a height of 98m above the ground when the food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground? Take g = 9.8 ms^{-2}.

Q.15 A stone is dropped from a balloon rising upwards with a velocity of 16 ms^{-1}. The stone reaches the ground in 4s. Calculate the height of the balloon when the stone was dropped.

Q.16 From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with velocity of 25 ms^{-1}. Find when and where the two balls will meet. Take g = 9.8 ms^{-2}.

Q.17 A body is dropped from rest at a height of 150 m, and simultaneously, another body is dropped from rest from a point 100m above the ground. What is their difference in height after they have fallen (i) 2s, (ii) 3s? How does the difference in height vary with time? Take g = 10 ms^{-2}.

Q.18 A body falling freely under gravity passes two points 30m apart in 1s. Find from what point above the upper point it began to fall? Take g = 9.8 ms^{-2}.

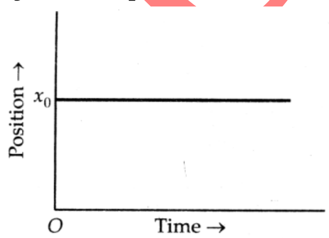
Q.19 Four balls are dropped from the top of a tower at intervals of one–one second. The first ball reaches the ground after 4s of dropping. What are the distances between first and second, second and third, third and fourth balls at this instant?

Answers

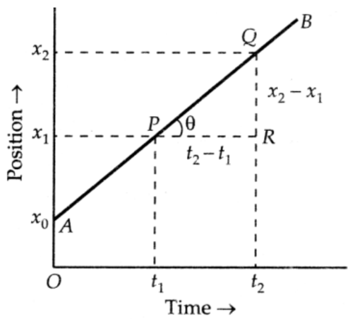
1.	58.8 m.	2.	+ 20m, 5 s, vertical line
3.	19.6 ms^{-1}, 19.6 m, 4.9 m	4.	4s, 29.4 ms^{-1}
5.	(i) + 19.6 ms^{-1}, (ii) 19.6 m	6.	2.86s, 15.86s, 235 m
7.	15.1 m	8.	1652 ms^{-2}
9.	44.1 m		
10.	(i) 1.225m, (ii) 0.5s, (iii) 4.9 ms^{-1}, (iv) 1s	11.	(i) 88.2 m, (ii) 42 ms^{-1}, (iii) 0.8 s
12.	4s, 2s, 29.4 ms^{-1}	13.	36.4 km
14.	6.12s, 45.98ms^{-1}	15.	14.4 m
16.	78.4 from top, 4s		
17.	50m, difference in height remains constant at 50 m		
18.	32.1 m	19.	34.3 m, 24.5m, 14.7 m

Position–Time Graphs

Position–time graph for a stationary object. The position of a stationary object does not change with time.



Position–time graph for uniform motion: The position–time graph for an object in uniform motion along a straight line path is a straight line inclined to the time–axis.



Slope of position–time graph AB

$$= \tan\theta = \frac{QR}{PR} = \frac{x_2 - x_1}{t_2 - t_1}$$

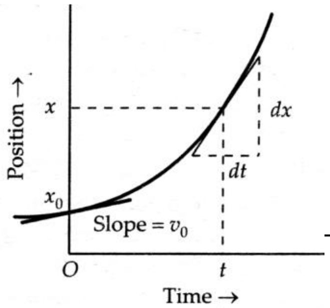
$$= \frac{\text{Displacement}}{\text{Time}} = \text{Velocity}(v).$$

Hence the slope of the position–time graph gives velocity of the object.

Position–time graph for uniformly accelerated motion. The position–time relation for uniformly accelerated motion along a straight line is

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Clearly, $x \propto t^2$ i.e., x is a quadratic function of t. So the *position–time graph for uniformly accelerated motion is a parabola.*



Slope of position–time graph

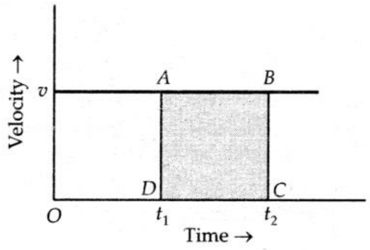
$$= \frac{\text{Small change in vertical coordiante}}{\text{Small change in horizontal coordiante}}$$

$$= \frac{dx}{dt} = \text{velocity at instant t.}$$

Thus the slope of the position–time graph gives the instantaneous velocity of the object.

Velocity–time graph for uniform motion. When an object has uniform motion, it moves with uniform velocity v in the same fixed direction. So *the velocity–time graph for uniform motion is a straight line parallel to the time–axis.*

Area under the velocity–time graph between times t_1 and t_2



= Area of rectangle ABCD

= AD × DC = $v(t_2 - t_1)$

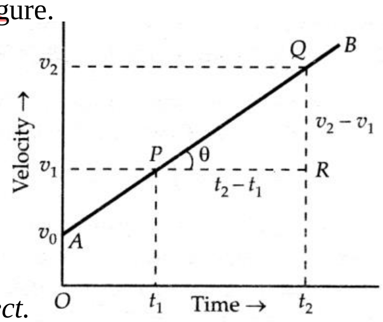
= Velocity × time

= Displacement

Hence *the area under the velocity–time graph gives the displacement of the object in the given time interval.*

Velocity–time graph for uniformly accelerated motion. The *velocity–time graph for a uniformly accelerated motion is a straight line inclined to the time axis,* as shown in figure.

Slope of velocity–time graph AB



$$= \tan\theta = \frac{QR}{PR} = \frac{v_2 - v_1}{t_2 - t_1}$$

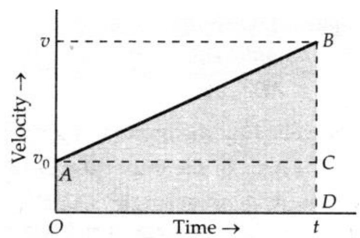
$$= \frac{\text{Change in velocity}}{\text{Time interval}}$$

= Acceleration (a)

Hence *the slope of the velocity–time graph gives the acceleration of the object.*

Distance covered as area under the velocity–time graph. The straight line AB is the velocity–time graph of an object moving along a straight line path with uniform acceleration a. Let its velocities be v_0 and v at times 0 and t respectively.

Area under the velocity–time graph AB



= Area of trapezium OABD

$$= \frac{1}{2}(OA + BD) \times OD = \frac{1}{2}(v_0 + v) \times (t - 0)$$

= Average velocity × time interval

= Distance travelled in time t

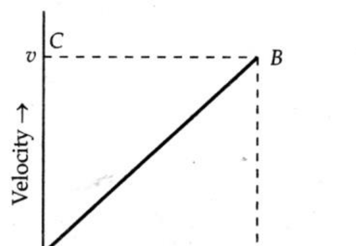
Hence *the area under the velocity–time graph gives the distance travelled by the object in the given time interval.*

Derivation of Equations of Motion By Graphical Method

Equation of motion by graphical method: Consider an object moving along a straight line path with initial velocity u and uniform acceleration a. Suppose it travels distance s in time t. Its velocity–time graph is straight line. Here OA = ED = u, OC = EB = v and OE = t = AD.

(i) We know that,

Acceleration = Slope of velocity–time graph AB



or $\quad a = \dfrac{DB}{AD} = \dfrac{DB}{OE} = \dfrac{EB - ED}{OE} = \dfrac{v - u}{t}$

or $\quad v - u = at \qquad$ or $\quad v = u + at$

This proves the *first equation of motion.*

(ii) From part (i), we have

$$a = \frac{DB}{AD} = \frac{DB}{t} \quad \text{or} \quad DB = at$$

Distance travelled by the object in time t is

s = Area of the trapezium OABE

= Area of rectangle OADE + Area of triangle ADB

$$= OA \times OE + \frac{1}{2} DB \times AD = ut + \frac{1}{2} at \times t \quad \text{or} \quad s = ut + \frac{1}{2} at^2$$

This proves the *second equation of motion.*

(iii) Distance travelled by object in time t is s = Area of trapezium OABE.

$$= \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + EB) \times OE$$

Acceleration,

a = Slope of velocity–time graph AB

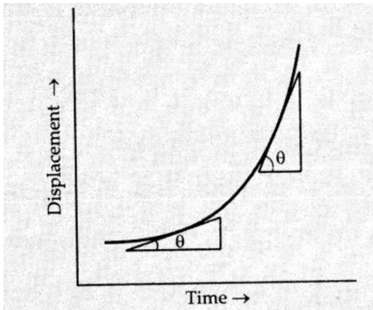
or $\quad a = \dfrac{DB}{AD} = \dfrac{EB - ED}{OE} \quad$ or $\quad OE = \dfrac{EB - ED}{a}$

$\therefore \quad s = \dfrac{1}{2}(EB + ED) \times \dfrac{(EB - ED)}{a}$

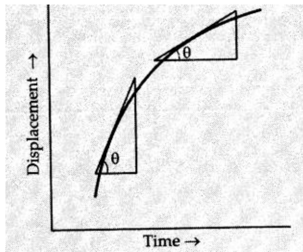
$$= \frac{1}{2a}(EB^2 - ED^2) = \frac{1}{2a}(v^2 - u^2)$$

or $\quad v^2 - u^2 = 2as$

This proves the *third equation of motion.*



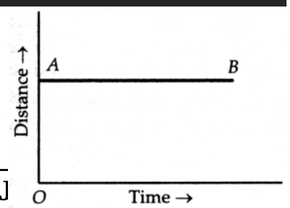
- A straight line graph has a *single slope.* So if the displacement–time graph is a straight line, it represents a constant velocity. If the velocity–time graph is a straight line, it represents a constant acceleration.
- A curved graph has *multiple slopes.* In figure, as the displacement–time graph bends upwards with the passage of time, the value of θ increases, slope (= tan θ) of the curve increases, consequently the velocity increases and hence the motion is accelerated.



- In the figure, as the displacement–time graph bends downwards with the passage of time, the value of θ decreases, the slope of the curve decreases, consequently the velocity decreases and hence the motion is decelerated.
-

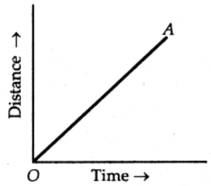
Analysing Nature of Motion From Various Graphs

Nature of motion from different types of distance–time graphs:

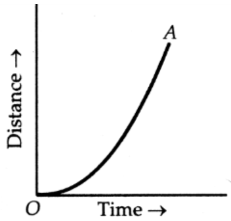


(i) For a body at rest, the distance–time graph is a straight line AB, as shown in figure. As the slope of AB is zero, so speed of the body is zero.

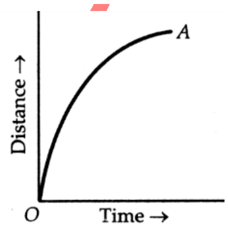
(ii) For a body moving with uniform speed, the distance–time graph is a straight line inclined to the time–axis, as shown in figure. As the graph passes through O, so distance travelled at t = 0 is also zero.



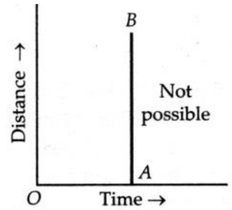
(iii) The distance–time graph in figure, represents accelerated (speeding up) motion, because the slope of the graph is increasing with time.



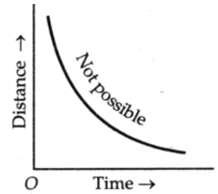
(iv) The distance–time graph in figure, represents decelerated (speeding down) motion, because slope of the graph is decreasing with time.



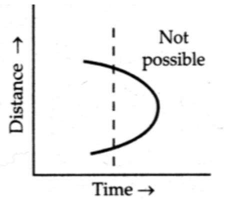
(v) In figure, distance–time graph is a straight line parallel to distance–axis. It represents infinite speed which is not possible.



(vi) The distance covered by a body cannot decrease with the increase of time. So the distance–time graph of the type shown in figure is not possible.

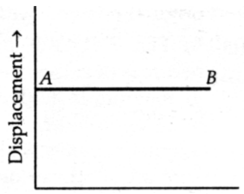


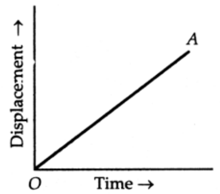
(vii) The distance–time graph shown in figure, is not possible because it represents two different positions of the body at the same instant which is not possible.



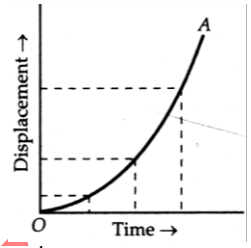
The nature of motion from the different types of displacement–time graphs

(i) For a stationary body, the displacement–time graph is a stationary body, the displacement–time graph is a straight line AB parallel to time–axis. The zero slope of line AB indicates zero velocity.

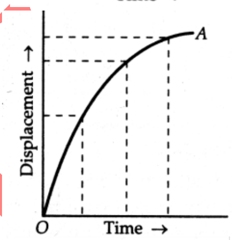




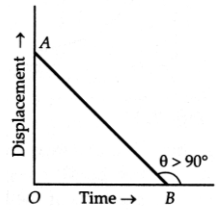
(ii) In figure, the displacement–time graph is a straight line OA inclined to time–axis. It has a single slope. So it represents a constant velocity and hence zero acceleration.



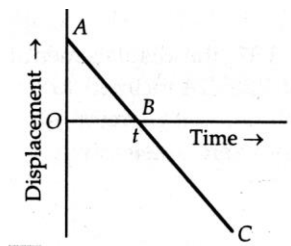
(iii) In figure, greater displacements are taking place in equal intervals of time. So the displacement–time curve OA represents an increasing velocity or an accelerated motion. For constant acceleration, the displacement–time graph is a parabola bending upwards.



(iv) In figure, decreasing displacements are taking place in equal intervals or time. So displacement–time curve OA represents a decreasing velocity or deceleration. For uniform deceleration, the displacement–time graph is a parabola bending downwards.



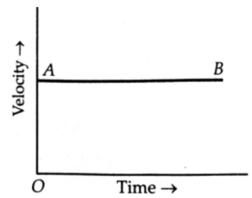
(v) In figure, displacement is decreasing uniformly with time and becomes zero after a certain time. Displacement–time graph has a negative slope ($\theta > 90^o$ and $\tan\theta < 0$) and represents a uniform negative velocity. It indicates that the body is returning back to its original position with a uniform velocity.



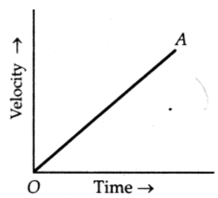
(vi) In figure, the displacement of the body becomes negative after time t and then increases in magnitude with time. It indicates that the original position B and then moves towards C with a uniform velocity.

The nature of the motion from different types of velocity–time graphs

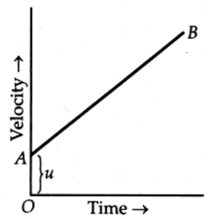
(i) For a body moving with a uniform velocity, the v–t graph is a straight line parallel to the time–axis as shown in figure. The zero slope of line AB indicates zero acceleration.

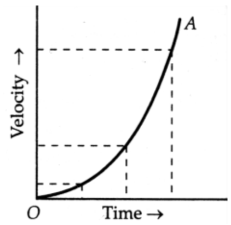


(ii) When a body starts from rest and moves with uniform acceleration, its v–t graph is straight line OA inclined to the time–axis and passing through the origin O, as shown in figure. Greater is the slope of the v–t graph, greater will be the acceleration.

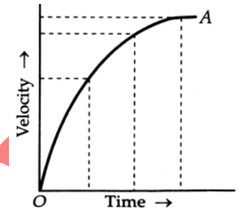


(iii) In figure, the straight line v–t graph does not pass through origin O. The body has an initial velocity u(=OA) and then it moves with a uniform acceleration.

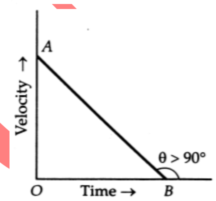




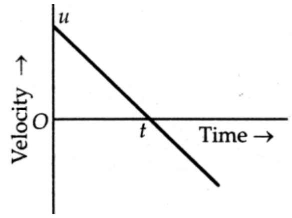
(iv) In figure, greater changes in velocity are taking place in equal intervals of time. So the v–t graph bending upwards represents an increasing acceleration.



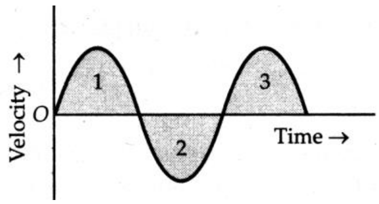
(v) In figure, smaller changes in velocity are taking place in equal intervals of time. So the v–t graph bending downwards represents a decreasing acceleration.



(vi) In figure, the body starts with an initial velocity u. The velocity decreases uniformly with time, becoming zero after some time. As $\theta > 90^o$, the graph has a negative slope. The v–t graph represents uniform negative acceleration.



(vii) In figure, the v–t graph represents a body projected upwards with an initial velocity u. The velocity decreases with time (negative uniform acceleration), becoming zero after certain time t. Then the velocity becomes negative and increases in magnitude, showing body is returning to original position with positive uniform acceleration.

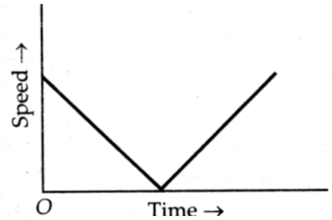


(viii) The area between the velocity–time graph and the time–axis gives the displacement. In figure, the v–t graph represents variable acceleration.

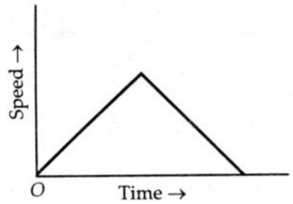
Displacement covered = Area 1 – Area 2 + Area 3

Distance covered = Area 1 + Area 2 + Area 3.

The nature of motion from the different type of sped – time graphs

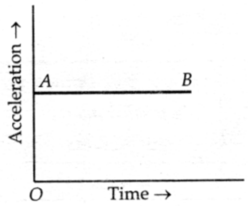


(i) For a body projected upwards, the speed–time graph is of the type shown in figure. When the body moves, its speed decreases uniformly, becoming zero at the highest point. As the body moves down, its speed increases uniformly. It returns with the same speed with which it was thrown up.

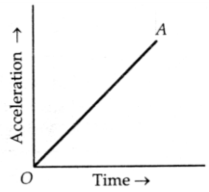


(ii) For a ball dropped on the ground from a certain height, the speed–time graph is of the type shown in figure. As the ball falls, its speed increases. As the ball bounces back, its speed decreases uniformly and becomes zero at the highest point.

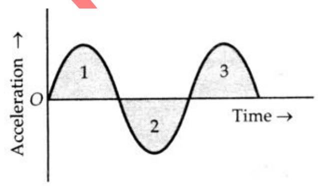
Discuss the nature of motion from the different types of acceleration – time graphs



(i) For a body moving with constant acceleration, the acceleration–time graph is a straight line AB parallel to the time–axis, as shown in figure.



(ii) When the acceleration of a body increases uniformly with time, its a–t graph is a straight line OA inclined to the time–axis as shown in figure.



(iii) For a body moving with variable acceleration, the a–t graph is a curve. The area between the a–t graph and the time–axis gives the change in the velocity, as shown in figure.

Change in velocity = Area 1 – Area 2 + Area 3

Subjective Assignment – V

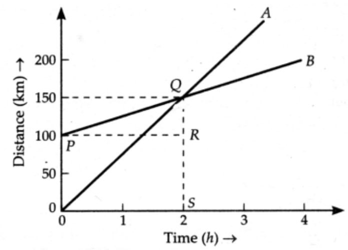
Q.1 Figure shows the distance–time graphs of two trains, which start moving simultaneously in the same direction. From the graphs, find:

(i) How much ahead of A is B when the motion starts?

(ii) What is the speed of B?

(iii) When and where will A catch B?

(iv) What is the difference between the speed of A and B?



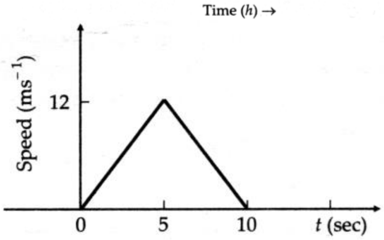
Q.2 The speed–time graph of a particle moving along a fixed direction is shown in figure. Find:

(i) distance travelled by the particle between 0 sec to 10 sec

(ii) average speed between this interval

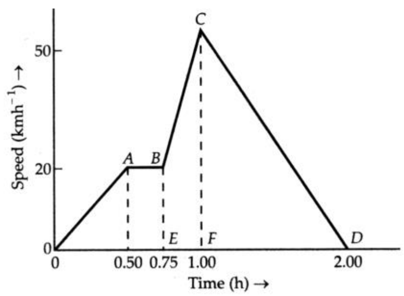
(iii) the time when the speed was minimum

(iv) the time when speed was maximum



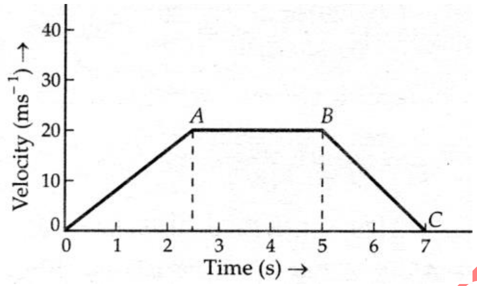
Q.3 A body starting from rest accerates uniformly along a straight line at the rate of 10 ms^{-2} for 5 seconds. It moves for 2 second with uniform velocity of 50 ms^{-1}. Then it retards uniformly and comes to rest in 3s. Draw velocity–time graph of the body and find the total distance travelled by the body.

Q.4 A train moves from one station to another in two hour's time. Its speed–time graph during the motion is shown in figure. (i) Determine the maximum acceleration during the journey (ii) Also calculate the distance covered during the time interval from 0.75 hour to 1 hour.



Q.5 A ball is thrown upward within an initial velocity of 100 ms^{-1}. After how much time will it return? Draw velocity–time graph for the ball and find from the graph (i) the maximum height attained by the ball and (ii) height of the ball after 15s. Take g = 10 ms^{-2}

Q.6 The velocity–time graph for a vehicle is shown in figure. Draw acceleration–time graph from it.



Q.7 The velocity of a train increases at a constant rate α from 0 to v and then remains constant for some time interval and then finally decreases to zero at a constant rate β. If the total distance covered by the particle be x, then show that the total time taken will be

$$t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right]$$

Q.8 Figure shows the position–time graphs of three cars A, B and C. On basis of the graphs, answer the following questions:

(i) Which car has the highest speed and which the lowest?

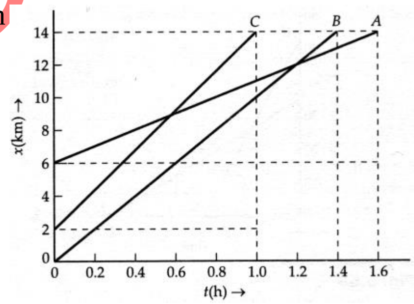
(ii) Are the three cars ever at the same point on the road?

(iii) When A passes C, where is B?

(iv) How far did car A travel between time it passed cars B and C?

(v) What is the relative velocity of car C with respect to car A?

(vi) What is the relative velocity of car B with respect to car C?

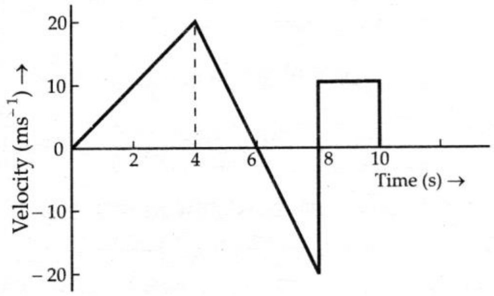


Q.9 An insect crawling up a wall crawls 5 cm upwards in the first minute but then slides 3 cm downwards in the next minute. It again crawls up 5 cm upwards in the third minute but again slides 3 cm downwards in the fourth minute. How long will the insect take to reach a crevice in the wall at a height of 24 cm from its starting point? How does the position–time graph of the insect look like?

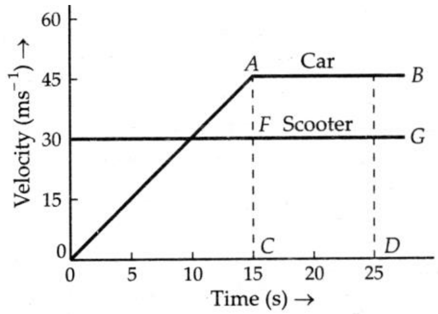
Q.10 A driver of a car travelling at 52 km h^{-1} applies the brakes and decelerates uniformly. The car stops in
5 seconds. Another driver going at 34 km h^{-1} applies his brakes slower and stops after 10 seconds. On the same graph paper, plot the speed versus time graph for two cars. Which of the two cars travelled farther after the brakes were applied?

Q.11 A motor car, starting from rest, moves with uniform acceleration and attains a velocity of 8 ms^{-1} in 8s. It then moves with uniform velocity and finally brought to rest in 32m under uniform retardation. The total distance covered by the car is 464 m. Find (i) the acceleration (ii) the retardation and (iii) the total time taken

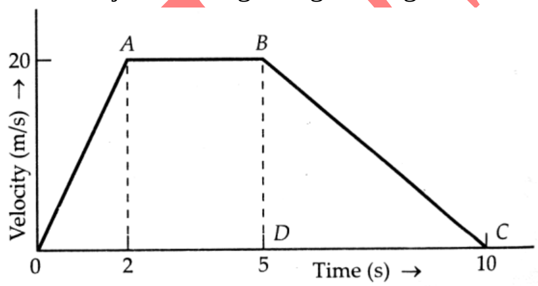
Q.12 The velocity–time graph of an object moving along a straight line is as shown in figure. Find the net distance covered by the object in time interval between t = 0 to t = 10s. Also find the displacement in time 0 to 10s.



Q.13 As soon as a car just starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity–time graphs are shown in figure. Calculate (i) the difference between the distance travelled by the car and the scooter in 15s (ii) the time when the car will catch up the scooter and (iii) the distance of car and scooter from the starting point at that instant.



Q.14 The velocity–time graph of an object moving along a straight line is as shown below:



Calculate the distance covered by object between:

(i) t = 0 to t = 5 s (ii) t = 0 to t = 10 s

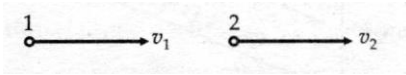
Answers

1. (i) 100 km, (ii) 25 kmh^{-1}, (iv) 50 kmh^{-1}
2. (i) 60m, (ii) 6 ms^{-1}, (iii) 0 & 10s, (iv) 5s
3. 300 m
4. (i) 120 kmh^{-2}, (ii) 8.75 km
5. (i) 500m, (ii) 375 m
8. (i) C has the highest speed and A has the lowest speed, (ii) No, (iii) 6 km from the origin (iv) 6 km, (v) 7 kmh^{-1}, (vi) – 2 kmh^{-1}
9. 21 min
10. Second car travelled farther
11. (i) 1 ms^{-2}, (ii) 1 ms^{-2}, (iii) 66s
12. 100m, 60m
13. (i) 112.5 m, (ii) 22.5 s, (iii) 675 m
14. (i) 80m, (ii) 130m

Relative Velocity

Relative velocity: *The relative velocity of an object 2 with respect to object 1, when both are in motion is the time rate of change of position of object 2 with respect to that of object 1.*

Expression for relative velocity: As shown in figure, consider the objects 1 and 2 moving along the same direction with constant velocities $\vec{v}_1$ and $\vec{v}_2$ (relative to the earth) respectively.



Suppose the position coordinates of the two objects are x_1 (0) and $x_2(0)$ at time t = 0. At time t = t, their position coordinates will be

$$x_1(t) = x_1(0) + v_1 t \quad \ldots. (1)$$

$$x_2(t) = x_2(0) + v_2 t \quad \ldots. (2)$$

Substracting (1) from (2), we find that

$$x_2(t) - x_1(t) = x_2(0) - x_1(0) + (v_2 - v_1)\, t$$

or $[x_2(t) - x_2(0)] - [x_1 (t) - x_1 (0)] = (v_2 - v_1)\, t$

or Displacement of object 2 in time t – Displacement of object 1 in time t = $(v_2 - v_1)$t

or Relative displacement of object 2 w.r.t. object 1 in time t = $(v_2 - v_1)$ t

or $$\frac{\text{Relative displacement of object 2 w.r.t. object 1}}{\text{Time t}} = v_2 - v_1$$

or Relative velocity of object 2 w.r.t. object 1,

$$v_{21} = v_2 - v_1$$

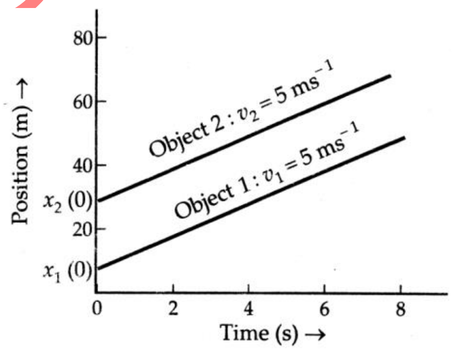
Similarly, relative velocity of object 1 w.r.t. object 2,

$$v_{12} = v_1 - v_2$$

Relative Velocity in Terms of Position–Time Graphs

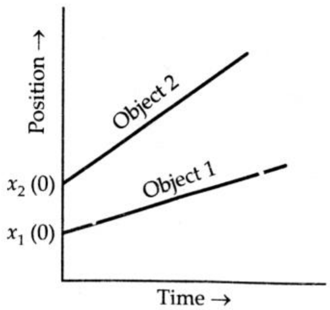
Case 1: ***When the two objects move with same velocity in the same direction.*** That is $v_1 = v_2$ and relative velocity, $v_2 - v_1 = 0$

Then, $x_2(t) - x_1(t) = x_2(0) - x_1(0)$

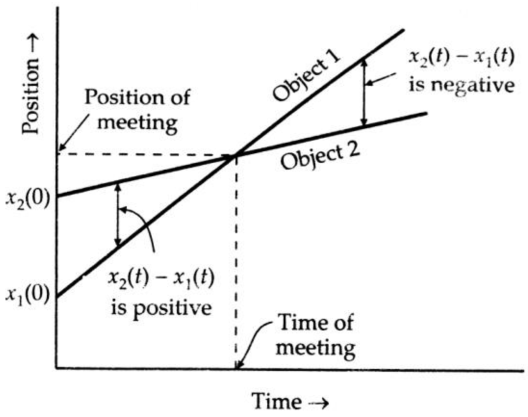


Thus the two objects remain a constant distance apart throughout their motion. Their position–time graphs are parallel straight lines as shown in figure. Here the initial positions of the objects are $x_1(0)$ = 10m and $x_2(0)$ = 30 m. Velocities are $v_1 = v_2 = 5$ ms^{-1}. The two objects remain 20 m apart at all instants.

Case 2: ***When $v_2 > v_1$ or relative velocity $(v_2 - v_1)$ is positive.*** The relative separation $x_2(t) - x_1(t)$ increases by the amount $(v_2 - v_1)$ after every second. So the position–time graphs gradually move apart, as shown in figure.



Case 3: ***When $v_2 < v_1$ or relative velocity $(v_2 - v_1)$ is negative.*** Initially the object 2 is ahead of object 1 and $x_2(t) - x_1(t)$ is positive. The relative separation $x_2(t) - x_1(t)$ first decreases till the two objects meet at the position $x_1(t) = x_2(t)$. Then the separation $x_2(t) - x_1(t)$ becomes negative. The object 1 overtakes the object 2 and the relative separation between them again begins to increase.



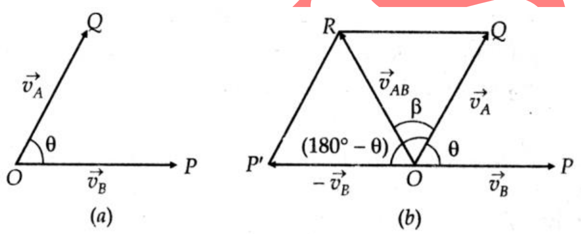
Determination of Relative Velocity

Rule to determine relative velocity. The relative velocity of a body A with respect to another body B when both are in motion can be obtained by adding to the velocity of A, a velocity equal and opposite to that of B. Thus

$$v_{AB} = v_A + (-v_B)$$

This law can be applied even to the bodies moving in directions inclined to each other.

Consider two bodies A and B moving with velocities v_A and v_B respectively, making an angle θ with each other as shown in figure.



To find the relative velocity v_{AB} of the body A with respect to B, draw OP' = – v_B as shown in figure (b). Now we add v_A and $(-v_B)$ which make an angle $(180^o - \theta)$ with each other. The relative velocity of A with respect to B is given by the diagonal OR of the parallelogram OQRP'.

The magnitude of the relative velocity v_{AB} is

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A\, v_B\, \cos(180^o - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A\, v_B\, \cos\theta}$$

Suppose the relative velocity v_{AB} makes angle β with v_A. Then

$$\tan\beta = \frac{v_B\, \sin(180^o - \theta)}{v_A + v_B\, \cos(180^o - \theta)}$$

$$= \frac{v_B\, \sin\theta}{v_A - v_B\, \cos\theta} \qquad \text{or} \qquad \beta = \tan^{-1}\left(\frac{v_B\, \sin\theta}{v_A - v_B\, \cos\theta}\right)$$

This gives the direction of the relative velocity v_{AB}.

(i) When both the bodies are moving along parallel straight lines in the same direction. We have $\theta = 0^o$.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A\, v_B \cos 0^o}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B}$$

$$= \sqrt{(v_A - v_B)^2} = v_A - v_B$$

Thus *the relative velocity of A with respect to B is equal to the difference between the magnitudes of their velocities.*

(ii) When the two bodies are moving along parallel straight lines in opposite directions. We have θ = 180^o.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 180^o}$$

$$= \sqrt{v_A^2 + v_B^2 + 2v_A v_B}$$

$$= \sqrt{(v_A + v_B)^2} = v_A + v_B$$

Thus *the relative velocity of body* A w.r.t. *body B is* equal to the sum of the magnitudes of their velocities. That is why when two fast trains cross each other in opposite directions, each appears to go very fast relative to the other.

Subjective Assignment – VI

Q.1 A car A moving at 10 ms^{-1} on a straight road, is ahead for B moving in the same direction at 6 ms^{-1}. Find the velocity of A relative to B and vice versa.

Q.2 Two parallel rail tracks run north south. Train A moves north with a speed of 54 kmh^{-1} and train B moves south with a sped of 90 kmh^{-1}. What is the

(i) relative velocity of B with respect to A?

(ii) relative velocity of ground with respect to B?

(iii) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 kmh^{-1} with respect to the train A) as observed by a man standing on the ground?

Q.3 Two trains 120m and 80m in length are running in opposite directions with velocities 42 kmh^{-1} and 30 kmh^{-1}. In what time they will completely cross each other?

Q.4 The speed of a motor launch with respect to still water = 7 ms^{-1} and the speed of stream is u = 3 ms^{-1}. When the launch began travelling upstream, a float was dropped from it. The launch travelled 4.2 km upstream, turned about and caught up with the float. How long is it before the launch reaches the float.

Q.5 A jet airplane travelling at the speed of 450 kmh^{-1} ejects the burnt gases at the speed of 1200 kmh^{-1} relative to the jet airplane. Find the speed of the burnt gases w.r.t. a stationary observer on earth.

Q.6 Two cars A and B are moving with velocities of 60 kmh^{-1} and 45 kmh^{-1} respectively. Calculate the relative velocity of A w.r.t. B, if (i) both cars are travelling eastwards and (ii) car A is travelling eastwards and car B is travelling westwards.

Q.7 An open car is moving on a road with a speed of 100 kmh^{-1}. A man sitting in the car fires a bullet from the gun in the opposite direction. If the speed of the bullet is 250 kmh^{-1} relative to the car, then find its (bullet's) speed with respect to an observer on the ground.

Q.8 A car A is moving with a speed of 60 kmh^{-1} and car B is moving with a speed of 75 kmh^{-1}, along parallel straight paths, starting from the same point. What is the position of car A w.r.t. B after 20 minutes?

Q.9 Two buses start simultaneously towards each other from towns A and B which are 480 km apart. The first bus takes 8 hours to travel from A to B while the second bus takes 12 hours to travel from B to A. Determine when and where the buses will meet.

Q.10 Two trains A and B, each of length 100m, are running on parallel tracks. One overtakes the other in 20s and one crosses the other in 10s. Calculate the velocities of each train.

Q.11 A man swims in a river with and against water at the rate of 15 kmh^{-1} and 5 kmh^{-1}. Find the man's speed in still water and the speed of the river.

Q.12 A motorboat covers the distance between the two spots on the river in 8h and 12h downstream and upstream respectively. Find the time required by the boat to cover this distance in still water.

Q.13 A car is travelling on a straight level road with a sped of 60 kmh^{-1}. It is followed by another car B which is moving with a sped of 70 kmh^{-1}. When the distance between them is 2.5 km, the car B is given a deceleration of 20 kmh^{-2}. After what distance and time will the car B catch up with car A?

Answers

1.	4 ms^{-1}, – 4 ms^{-1}	2 .	(i) – 40 ms^{-1}, (ii) 25 ms^{-1}, (iii) 10 ms^{-1}		
3.	10 s	4.	35 min		
5.	750 kmh^{-1}	6.	(i) 15 kmh^{-1} eastwards, (ii) 105 kmh^{-1} eastwards		
7.	150 kmh^{-1}	8.	5 km behind		
9.	4.8 h, 288 km from A	10.	15 ms^{-1}, 5 ms^{-1}		
11.	10 kmh^{-1}, 5 kmh^{-1}	12.	9.6 h	13.	32.5 km, 0.5h

Conceptual Questions

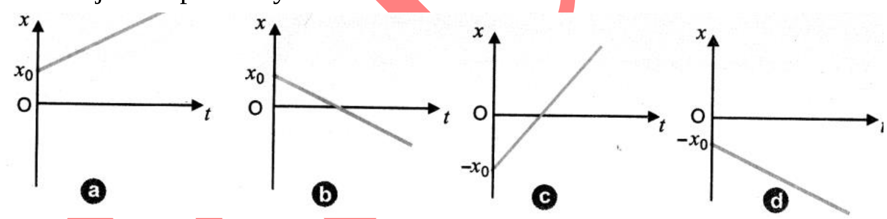
Q.1 State in the following case, whether the motion is one, two or three dimensional motion:

(a) a kite flying on a windy day (b) a speeding car on a long straight high way

(c) a carom coin rebounding from the side of the board

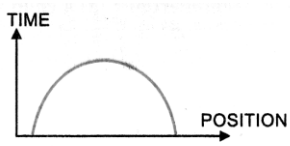
(d) a planet revolving around its star

Q.2 An object is in uniform motion along a straight line. What will be positive–time graph for the motion of the object if (a) x_0 = + ve, v = + ve (b) x_0 = + ve, v = – ve, (c) x_0 = – ve, v = + ve and (d) both x_0 and v are negative? The letters x_0 and v represent position of the object at time t = 0 and uniform velocity of the object respectively?

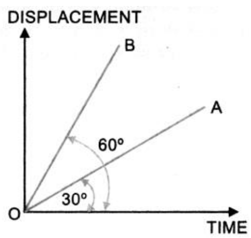


Q.3 A drunkard walking in a narrowlane takes 5 steps forward and 3 steps backward each step of 1m long, per second and so on. Determine how long the drunkard takes to fall in a pit 15m away from the start.

Q.4 Is the time variation of position, shown in figure observed in nature?



Q.5 Two straight lines drawn on the same displacement – time graph make angles 30° and 60° with time–axis respectively figure. Which line represents greater velocity? What is the ratio of two velocities?



Q.6 Wind is blowing west to east along two parallel tracks. Two trains moving with the same speed in opposite directions on these tracks have the steam tracks. If one steam track is double than the other, what is the speed of each train?

Q.7 For ordinary terrestrial experiments, which of the observers below are inertial and which are non–inertial? (a) a child revolving in a giant wheel. (b) a driver in a sports car moving with a constant high speed of 200 km/h on a straight road. (c) the pilot of an aeroplane which is taking off. (d) a cyclist negotiating a sharp turn. (e) the guard of a train which is slowing down to stop at a station.

Q.8 A body covered a distance of l metre along a semicircular path. Calculate the magnitude of displacement of the body and the ratio of distance to displacement.

Q.9 Mention the condition when an object in motion (a) can be considered point object (b) can not be considered point object.

Q.10 Under what condition will the distance and displacement of a moving object will have the same magnitude?

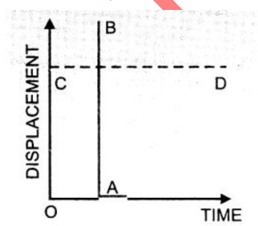
Q.11 Can the speed of a body be negative?

Q.12 Can an object have constant speed but variable velocity?

Q.13 What does the tangent at a point to the position–time graph for an object in non–uniform motion along a straight line represent?

Q.14 What will be nature of velocity – time graph for a uniform motion?

Q.15 If the displacement – time graph of a particle is parallel to



(a) displacement axis (b) the time axis, what will be the velocity of the particle?

Q.16 Can position – time graph have negative slope?

Q.17 Can a body have a constant velocity but a varying speed?

Q.18 A cyclist moving on a circular track of radius 100m completes one revolution in 4 minutes. What is his (i) average speed (ii) average velocity in one full revolution?

Q.19 The displacement x of the body in motion is given by $x = A \sin(\omega t + \theta)$; Determine the time at which the displacement is maximum.

Q.20 When two bodies move uniformly towards each other, the distance between them decreases by 6 metres/second. If both the bodies move in the same direction with their same speeds, the distance between them increases by 4 metres/second. What are the speeds of the two bodies.

Q.21 Define relative velocity of an object w.r.t. another. Draw position – time graphs of two objects moving along a straight line, when their relative velocity is (i) zero and (ii) non–zero.

Q.22 What do you understand by non uniform motion? Explain variable velocity and instantaneous velocity of an object in one dimensional motion.

NCERT Questions

Q.1 In which of the following examples of motion can the body be considered approximately a point object:

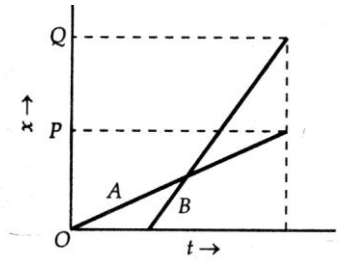
(i) a railway carriage moving without jerks between two stations.

(ii) a monkey sitting on the top of a man cycling smoothly on a circular track.

(iii) a spinning cricket ball that turns sharply on hitting the ground, and

(iv) tumbling beaker that has slipped off the edge of a table?

Q.2 The position–time (x–t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in figure. Choose the correct entries in the brackets below:



(a) A/B lives closer to the school than B/A

(b) A/B starts from the school earlier than B/A.

(c) A/B walks faster than B/A

(d) A and B reach home at the (same/different) time.

(e) A/B overtakes B/A on the road (once/twice)

Q.3 A women starts from her home at 9.00 A.M. walks with a speed of 5 kmh^{-1} on a straight road upto her office 2.5 km away, stays at office upto 5 P.M. and returns home by an auto with a speed of 25 kmh^{-1}. Choose suitable scales and plot the x–t graph of her motion.

Q.4 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, following again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Plot the x–t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13m away from the start.

Q.5 A jet airplane travelling at the speed of 500 kmh^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of latter with respect to an observer on ground?

Q.6 Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50s, the guard of B just brushes past the driver of A, what was the original distance between them?

Q.7 On a two–lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Q.8 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every
T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Q.9 A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.

(i) What is the direction of acceleration during the upward motion of the ball?

(ii) What are the velocity and acceleration of the ball at the highest point of its motion?

(iii) Chose the x = 0 and t = 0 to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of X–axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.

(iv) To what height does the ball rise and after how long does the ball return to the player's hands?

Q.10 Read each statement below carefully and state with reasons and examples, if it is true or false. A particle in one–dimensional motion

(a) will zero speed at an instant may have non–zero acceleration at that instant

(b) with zero speed may have non–zero velocity,

(c) with constant speed must have zero acceleration,

(d) with positive value of acceleration must be speeding up.

Q.11 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one–tenth of its speed. Plot the speed–time graph of its motion between t = 0 to 12 s.

Q.12 Explain clearly, with examples, the distinction between:

(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval;

(b) magnitude of average velocity over an interval of time, and the average sped over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]

(c) Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity, consider one–dimensional motion only].

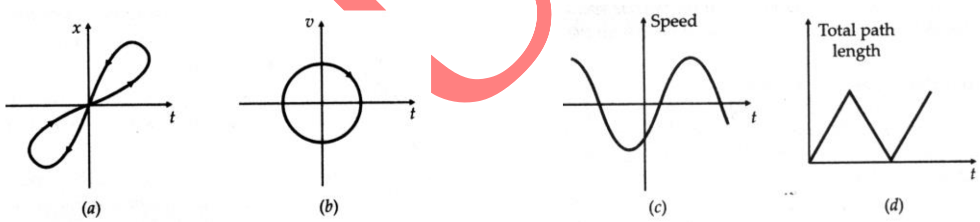
Q.13 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the

(a) magnitude of average velocity, and

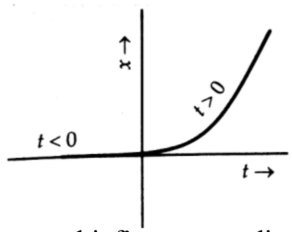
(b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min?

Q.14 The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Q.15 Look at the graphs (a) to (d) [figure] carefully and state, with reason, which of these cannot possibly represent one–dimensional motion of a particle.

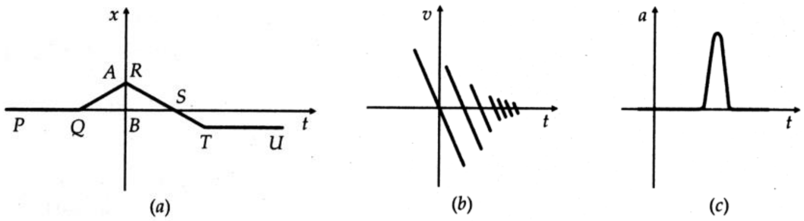


Q.16 Figure, shows the x–t plot of one–dimensional motion of a particle. It is correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t > 0? If not, suggest an suitable physical context for this graph.

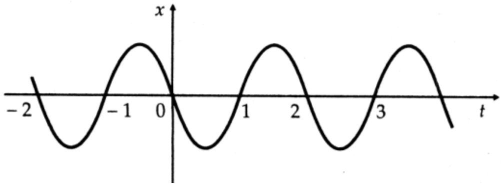


Q.17 A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?

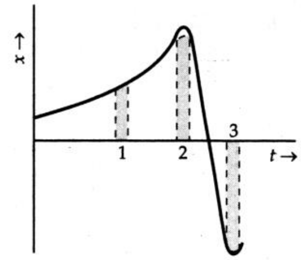
Q.18 Suggest a suitable physical situation for each of the following graphs.



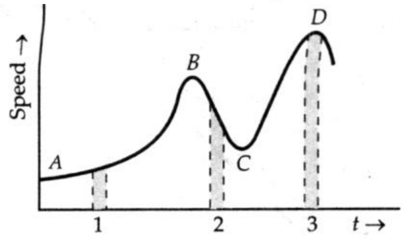
Q.19 Figure gives the x–t plot of a particle executing one–dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at t = 0.3s, 1.2s, –1.2s.



Q.20 Figure gives the x–t plot of a particle in one–dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



Q.21 Figure gives a speed–time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. (a) In which interval is the average acceleration greatest in magnitude? (b) In which interval is the average speed greatest? (c) Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. (d) What are the accelerations at the points A, B, C and D?



Q.22 A three–wheeler starts from rest, accelerates uniformly with 1 ms^{-2} on a straight road for 10s, and then moves with uniform velocity. Plot distance covered by the vehicle during the nth second (n = 1, 2, 3…) versus n. What do you except this plot to be during accelerated motion: a straight line or a parabola?

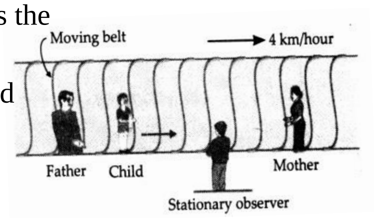
Q.23 On a long horizonally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a sped of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the

(i) speed of the child running in the direction of motion of the belt,

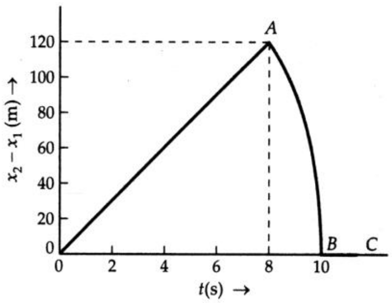
(ii) speed of child running opposite to direction of motion of belt, and

(iii) time taken by the child in (i) and (ii)?

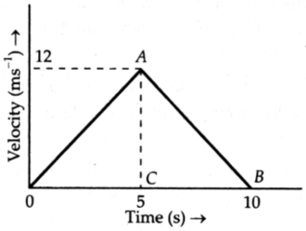
Which of the answers alter if motion is viewed by one of the parents?



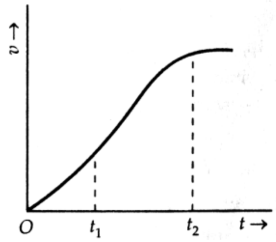
Q.24 Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the following graph correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stone do not rebound after hitting the ground. Take g = 10 ms^{-2}. Give equations for the linear and curved parts of the plot.



Q.25 The speed-time graph of a particle moving along a fixed direction is shown in figure. Obtain the distance travelled by the particle between (i) t = 0 to 10 s (ii) t = 2 to 6 s. What is the average speed of the particle in intervals in (i) and (ii) ?



Q.26 The velocity–time graph of particle in one–dimensional motion is shown in figure.



Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2:

(a) $x(t_2) = x(t_1) + v(t_1) \times (t_2 - t_1) + \frac{1}{2} a (t_2 - t_1)^2$

(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(c) $v_{av} = \{x(t_2) - x(t_1)\}/(t_2 - t_1)$

(d) $a_{av} = \{v(t_2) - v(t_1)\}/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \left(\frac{1}{2}\right) a_{av}(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the v–t curve bounded by the t–axis and the dotted line shown.

Answers

1. (i) and (ii) as point object
2. (a) A lives closer than B, (b) A start earlier than B, (c) B walks faster than A, (d) Same time, (e) B overtakes A once
4. 37 sec. 5. 1000 km/h 6. 1250 m 7. $1\ ms^{-2}$
8. T = 9 minute, v = 40 km/h
9. (i) vertically downward, (ii) v = 0, a = $9.8\ ms^{-2}$ vertical downward,
   (iii) upward motion → position = +ve, v = – ve, a = +ve
   downward motion → position = +ve, v = + ve, a = +ve
   (iv) 44.1m, 6 sec
10. (a) true, (b) false, (c) true, (d) false
13. (i) $\bar{v}_{av}$ = 5 km/h, v_{av} = 5 km/h (ii) $\bar{v}_{av}$ = 0 km/h, v_{av} = 6 km/h (iii) $\bar{v}_{av}$ = 1.875 km/h, v_{av} = 5.625 km/h
15. all are impossible 16. No 17. $105\ ms^{-1}$
19. (i) x < 0, v < 0, a > 0, (ii) x > 0, v > 0, a < 0, (iii) x < 0, v > 0, a > 0
20. v_{av} is greatest in interval 3, v_{av} is least in interval 1, v_{av} is +ve in interval 1 & 2, v_{av} is –ve in interval 3
21. (a) a_{av} is greatest in interval 2, (b) v_{av} is greatest in interval 3,
    (c) v is +ve in all 3 intervals, a is +ve in 1 & 3 interval, a is –ve in 2 interval
    (d) a = 0 at points A, B, C & D
22. Straight Line 23. (a) (i) 13 km/h, (ii) 5 km/h, (iii) 20 sec
    (b) only answer of (i) and (ii) are altered
24. (i) $x_2 - x_1 = 15t$, (ii) $x_2 = 200 + 30t - 5t^2$
25. (i) s = 20m, v_{av} = 6 m/s (ii) s = 36 m, v_{av} = 9 m/s
26. only relation c, d and f are correct

Objective Assignment – I

Q.1 A particle is constrained to move on a straight line path. It returns to the starting point after 10 s. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is true?

(a) displacement of the particle is zero (b) displacement of the particle is 30 m
(c) average speed of the particle is 3 m/s (d) both (a) and (c)

Q.2 A car travels first half of the distance between two places with a speed of 30 km/hr and remaining half with a speed of 50 km/hr. The average speed of the car is

(a) 37.5 km/hr (b) 42 km/hr (c) 40 km/h (d) 49 km/h

Q.3 If the first one–third of a journey is traveled at 20 km/h, next one–third at 40 km/h and the last one–third at 60 km/h, the average speed of whole journey will be

(a) 32.7 km/h (b) 35 km/h (c) 40 km/h (d) 45 km/h

Q.4 Given a = 2t + 5. Calculate the velocity of the body after five sec if it starts from rest.

(a) 50 m/s (b) 25 m/s (c) 100 m/s (d) 75 m/s

Q.5 The displacement of a particle moving in straight line is given by $x = 2t^2 + t + 5$, where x is expressed in metres and t in seconds. The acceleration at t = 2 sec is

(a) 4 m/s^2 (b) 10 m/s^2 (c) 8 m/s^2 (d) 15 m/s^2

Q.6 The velocity of a particle at an instant is 10 m/s. After 3s its velocity will become 16 m/s. The velocity at 2 s, before the given instant will be

(a) 6 m/s (b) 4 m/s (c) 2 m/s (d) 1 m/s

Q.7 A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 s is

(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$ (c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(n+1)}{n}$

Q.8 If a ball is thrown vertically upwards with 40 m/s, its velocity after two sec will be

(a) 10 m/s (b) 30 m/s (c) 20 m/s (d) 40 m/s

Q.9 A stone released with zero velocity from top of the tower reaches the ground in 4 sec. The height of the tower is about

(a) 20 m (b) 80 m (c) 40 m (d) 160 m

Q.10 A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds. It remained in air for

(a) 12 sec (b) 13 sec (c) 25 sec (d) 26 sec

Q.11 A body sliding on a smooth inclined plane requires 4 seconds to reach the bottom, starting from rest at the top. How much time does it take to cover one–fourth the distance starting from rest at the top?

(a) 1 sec (b) 4 sec (c) 2 sec (d) 16 sec

Q.12 A body falling from rest describes distances s_1, s_2 and s_3 in the first, second and third seconds of its fall, then the ratio $s_1 : s_2 : s_3$ is

(a) 1 : 1 : 1 (b) 1 : 3 : 5 (c) 1 : 2 : 3 (d) 1 : 4 : 9

Q.13 When a ball is thrown vertically upwards, at the maximum height

(a) the velocity is zero and therefore there is no acceleration acting on the particle
(b) the acceleration is present and therefore velocity is not zero
(c) the acceleration depends on the velocity as a = dv/dt
(d) the acceleration is independent of the velocity

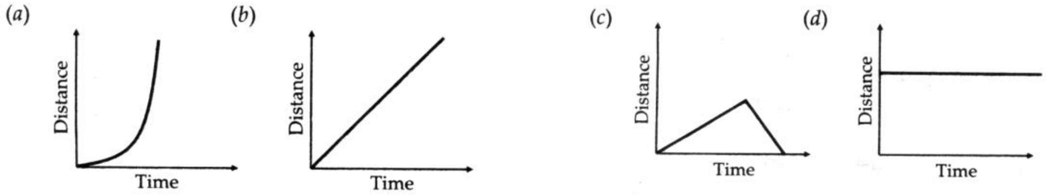
Q.14 A police jeep is chasing with velocity 45 km/h. A thief in another jeep is moving with 155 km/h. Police fires a bullet with a muzzle velocity 180 m/s. The bullet strikes the jeep of the thief with a velocity

(a) 27 ms^{-1} (b) 150 ms^{-1} (c) 250 ms^{-1} (d) 450 ms^{-1}

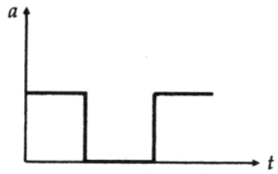
Q.15 Velocity–time curve for a body projected vertically upwards is

(a) ellipse (b) hyperbola (c) parabola (d) straight line

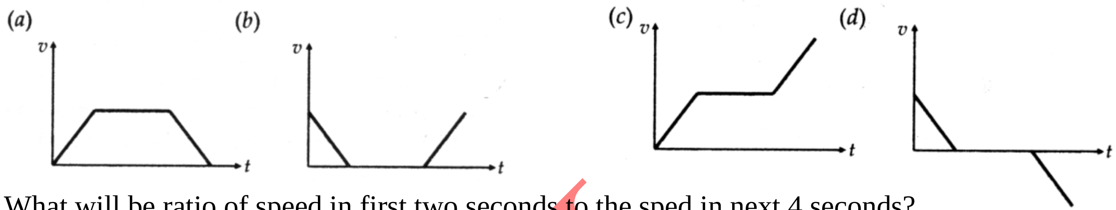
Q.16 Which of the following distance–time graph shows accelerated motion?



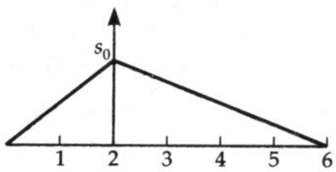
Q.17 Acceleration–time graph of a body is shown.



The corresponding velocity–time graph of the same body is:



Q.18 What will be ratio of speed in first two seconds to the sped in next 4 seconds?



(a) $\sqrt{2}:1$ (b) 3 : 1 (c) 2 : 1 (d) 1 : 2

Q.19 You drive a car at a speed of 70 km/h in a straight road for 8.4 km, and then the car runs out of petrol. You walk for 30 min to reach a petrol pump at a distance of 2 km. The average velocity from the beginning of your drive till you reach the petrol pump is

(a) 16.8 km/h (b) 35 km/h (c) 64 km/h (d) 18.6 km/h

Answers

1.	d	2.	a	3.	a	4.	a	5.	a
6.	a	7.	a	8.	c	9.	b	10.	b
11.	c	12.	d	13.	d	14.	b	15.	d
16.	a	17.	c	18.	c	19.	a		

Objective Assignment – II

Q.1 A car covers the first half of the distance between two places at 40 km/h and another half at 60 km/h. The average sped of the car is

(a) 40 km/h (b) 48 km/h (c) 50 km/h (d) 60 km/h

Q.2 A bus traveling the first one–third distance at a speed of 10 km/h, the next one–third at 20 km/h and the last one–third at 60 km/h. The average speed of bus is

(a) 9 km/h (b) 16 km/h (c) 18 km/h (d) 12 km/h

Q.3 A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?

(a) 16 m (b) 24 m (c) 40 m (d) 56 m

Q.4 The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration will be zero at time t equal to

(a) a/3b (b) zero (c) 2a/ 3b (d) a/b

Q.5 Motion of a particle is given by equation $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleration of the particle at t = 1sec is

(a) 10 m/s^2 (b) 32 m/s^2 (c) 23 m/s^2 (d) 16 m/s^2

Q.6 A particle moves along a straight line such that its displacement at any time t is given by $s = (t^3 - 6t^2 + 3t + 4)$ metres. The velocity when the acceleration is zero is

(a) 3 m/s (b) 42 m/s (c) – 9 m/s (d) – 15 m/s

Q.7 The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) be independent of β (b) drop to zero where α = β

(c) go on decreasing with time (d) go on increasing with time

Q.8 The position x of a particle with respect to time t along x–axis is given by $x = 9t^2 - t^3$, where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the +x direction?

(a) 54 m (b) 81 m (c) 24m (d) 32m

Q.9 The acceleration experienced by a moving motor boat, after its engine is cut off, is given by $dv/dt = -kv^3$, where k is constant. If v_0 is the magnitude of velocity at cut–off, the magnitude of the velocity at a time t after the cut off is

(a) $v_0/2$ (b) v_0 (c) $\frac{v_0}{\sqrt{2v_0^2 kt+1}}$ (d) v_0e^{-kt}

Q.10 A particle moving along x–axis has acceleration f at time t, given by $f = f_0\left(1-\frac{t}{T}\right)$, where f_0 and T are constant. The particle at t = 0 has zero velocity. In the time interval between t = 0 and the instant when

f = 0, the particle's velocity v_x is

(a) $\frac{1}{2}f_0T^2$ (b) f_0T^2 (c) $\frac{1}{2}f_0T$ (d) f_0T

Q.11 The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance traveled by the particle in time t will be

(a) $v_0t+\frac{1}{3}bt^2$ (b) $v_0t+\frac{1}{2}bt^2$ (c) $v_0t+\frac{1}{6}bt^3$ (d) $v_0t+\frac{1}{3}bt^3$

Q.12 The velocity of train increases uniformly from 20 km/h to 60 km/h in 4 hours. The distance traveled by the train during this period is

(a) 160 km (b) 180 km (c) 100 km (d) 120 km

Q.13 A car moving with a speed of 40 km/h can be stopped by applying brakes after atleast 2 m. If the same car is moving with a speed of 80 km/h, what is the minimum stopping distance?

(a) 4 m (b) 6 m (c) 8 m (d) 2 m

Q.14 A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is

(a) 33.3 km/h (b) $20\sqrt{2}$ km/h (c) $25\sqrt{2}$ km/h (d) 35 km/h

Q.15 A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If total time elapsed is t, then maximum velocity acquired by car will be

(a) $\frac{(\alpha^2-\beta^2)t}{\alpha\beta}$ (b) $\frac{(\alpha^2+\beta^2)t}{\alpha\beta}$ (c) $\frac{(\alpha+\beta)t}{\alpha\beta}$ (d) $\frac{\alpha\beta t}{\alpha+\beta}$

Q.16 If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(a) ut (b) $1/2\ gt^2$ (c) $ut - 1/2\ gt^2$ (d) (u + gt)t

Q.17 What will be the ratio of the distances moved by a freely falling body from rest in 4^{th} and 5^{th} seconds of journey?

(a) 4 : 5 (b) 7 : 9 (c) 16 : 25 (d) 1 : 1

Q.18 Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16m and 25m, respectively. The ratio of the time taken by them to reach the ground is

(a) 4/5 (b) 5/4 (c) 12/5 (d) 5/12

Q.19 A ball is thrown vertically upward. It has a speed of 10 m/sec when it has reached one half of its maximum height. How high does the ball rise? Take g = 10 m/s^2.

(a) 10 m (b) 5 m (c) 15 m (d) 20 m

Q.20 A body dropped from a height h with initial velocity zero, strikes the ground with a velocity 3 m/s. Another body of same mass dropped from the same height h with an initial velocity of 4 m/s. The final velocity of second mass, with which it strikes the ground is

(a) 5 m/s (b) 12 m/s (c) 3 m/s (d) 4 m/s

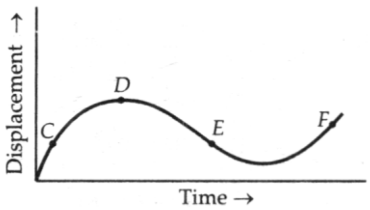
Q.21 The water drop falls at regular intervals from a tap 5m above the ground. The third drop is leaving the tap at instant the first drop touches the ground. How far above the ground is the second drop at that instant?

(a) 3.75 m (b) 4.00 m (c) 1.25 m (d) 2.50 m

Q.22 A rubber ball is dropped from a height of 5m on a plane. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) 3/5 (b) 2/5 (c) 16/25 (d) 9/25

Q.23 The displacement–time graph of a moving particle is shown ahead. The instantaneous velocity of the particle is negative at the point



(a) E (b) F (c) C (d) D

Answers

1.	b	2.	c	3.	d	4.	a	5.	b
6.	c	7.	d	8.	a	9.	c	10.	c
11.	c	12.	a	13.	c	14.	c	15.	d
16.	b	17.	b	18.	a	19.	a	20.	a
21.	a	22.	b	23.	a				

Objective Assignment – III

Q.1 A body dropped from top of a tower falls through 40 m during the last two seconds of its fall. The height of tower (g = 10 ms^{-2}) is

(a) 60 m (b) 45 m (c) 80 m (d) 50 m

Q.2 A train of 150 metre length is going towards north direction at a speed of 10 m/s. A parrot flies at the speed of 5 m/s towards south direction parallel to the railways track. The time taken by the parrot to cross the train is

(a) 12 sec (b) 8 sec (c) 15 sec (d) 10 sec

Q.3 A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then

(a) $S_2 = S_1$ (b) $S_2 = 2\,S_1$ (c) $S_2 = 3S_1$ (d) $S_2 = 4S_1$

Q.4 A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus?

(a) 10 ms^{-1} (b) 20 ms^{-1} (c) 40 ms^{-1} (d) 25 ms^{-1}

Q.5 The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is

(a) increasing with time (b) decreasing with time (c) constant but not zero (d) zero

Q.6 A particle starts from rest and has an acceleration of 2 ms^{-2} for 10s. After that, the particle travels for 30s with constant speed and then undergoes a retardation of 4 ms^{-2} and comes back to rest. The total distance covered by the particle is

(a) 650 m (b) 700 m (c) 750 m (d) 800 m

Q.7 The body A starts from rest with an acceleration a_1.After 2s, another body B starts from rest with an acceleration a_2. If they travel equal distances in the 5^{th} second after the start of A, then the ratio $a_1 : a_2$ is equal to

(a) 5 : 9 (b) 5 : 7 (c) 9 : 5 (d) 9 : 7

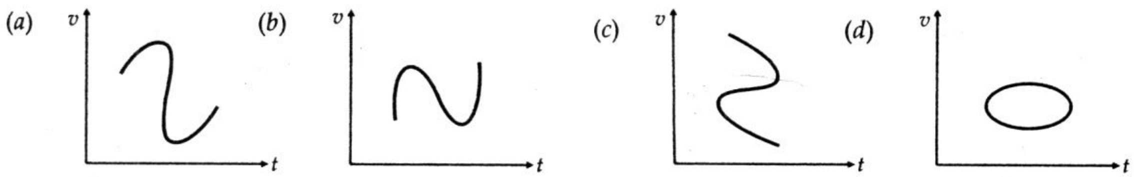
Q.8 Three different objects of masses m_1, m_2 and m_3 are allowed to fall from the same point O along three different frictionless paths. The speeds of three objects, on reaching the ground, will be in the ratio of

(a) $m_1 : m_2 : m_3$ (b) $m_1 : 2m_2 : 3m_3$ (c) $1/m_1 : 1/m_2 : 1/m_3$ (d) 1 : 1 : 1

Q.9 When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity

(a) $\sqrt{3}\,v_0$ (b) $3v_0$ (c) $9v_0$ (d) $3v_0/2$

Q.10 Which of the following velocity–time graph shows a realistic situation for a body in motion?



Q.11 A train started from rest from a station and accelerated at 2 ms^{-2} for10s. Then, it ran at constant speed for 30s and thereafter it decelerated at 4 ms^{-2} until it stopped at the next station. The distance between two station is

(a) 650 m (b) 700 m (c) 750 m (d) 800 m

Q.12 A ball falls from 20 m height on floor and rebounds to 5m. Time of contact is 0.02 sec. Find acceleration during impact.

(a) 1200 m/s^2 (b) 1000 m/s^2 (c) 2000 m/s^2 (d) 1500 m/s^2

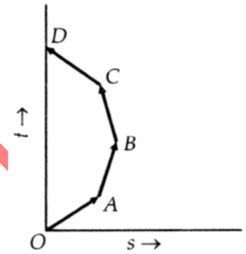
Q.13 A ball is dropped from top of a tower of 100 m height. Simultaneously another ball was thrown upward from bottom of the tower with a speed of 50 m/s. They will cross each other after (g = 10 m/s^2):

(a) 1 sec (b) 2 sec (c) 3 sec (d) 4 sec

Q.14 A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 second. If this ball is thrown downward from the top of the same tower with speed v, it reaches the ground in t_2 seconds. In what time will the ball reach the ground if it is allowed to fall freely under gravity from the top of the tower?

(a) $\frac{t_1+t_2}{2}$ (b) $\frac{t_1-t_2}{2}$ (c) $\sqrt{t_1t_2}$ (d) $t_1 + t_2$

Q.15 Which of the following options is correct for the object having a straight line motion represented by the following graph?



(a) object moves with constantly increasing velocity from O to A and then it moves with constant velocity.

(b) velocity of the object increases uniformly

(c) average velocity is zero

(d) the graph shown is impossible.

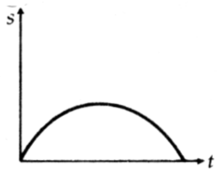
Q.16 The driver of a train traveling at 115 kmh^{-1} sees on the same track, 100 m in front of him, a slow train traveling in the same direction at 25 kmh^{-1}. The least retardation that must be applied to faster train to avoid collision is

(a) 25 ms^{-2} (b) 50 ms^{-2} (c) 75 ms^{-2} (d) 3.125 ms^{-2}

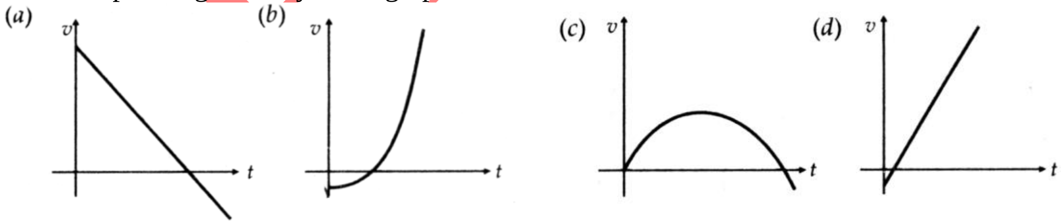
Q.17 The acceleration 'a' of a particle staring from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time 't' will be

(a) $\frac{\alpha t^2}{2}+\beta$ (b) $\frac{\alpha t^2}{2}+\beta t$ (c) $\alpha t^2+\frac{1}{2}\beta t$ (d) $\frac{\alpha t^2+\beta}{2}$

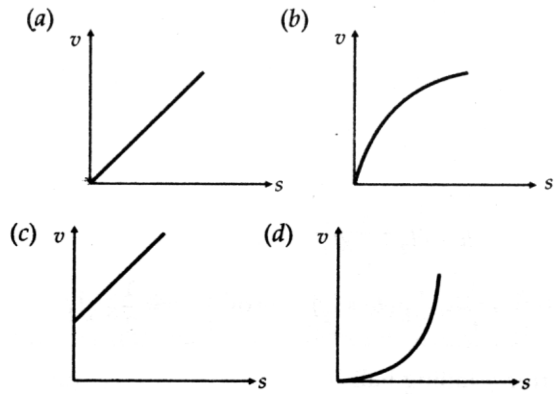
Q.18 The graph of displacement vs. time is



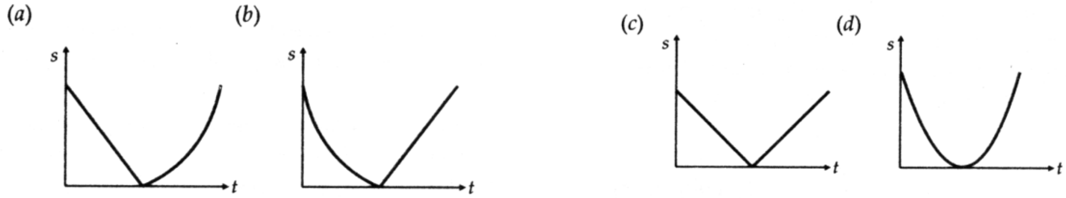
The corresponding velocity–time graph will be



Q.19 A body starting from rest moves along a straight line with constant acceleration. The variation of speed v with distance s is



Q.20 A ball is thrown vertically upwards. Which of the following plots represents the speed–time graph of ball during its flight, if the air resistance is not ignored?

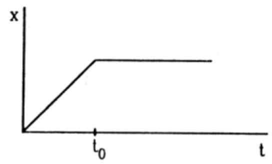


Answers

1.	b	2.	d	3.	d	4.	b	5.	a
6.	c	7.	a	8.	d	9.	a	10.	b
11.	c	12.	d	13.	b	14.	c	15.	c
16.	d	17.	b	18.	a	19.	b	20.	a

IIT JEE Assignment

Q.1 Figure shows the displacement–time graph of a particle moving on the X–axis



(a) the particle is continuously going in positive x direction
(b) the particle is at rest
(c) the velocity increases up to a time t_0, and then becomes constant
(d) the particle moves at a constant velocity up to a time t_0, and then stops

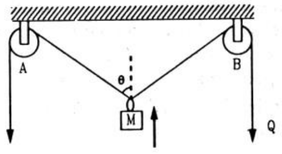
Q.2 A particle has a velocity u towards east at t = 0. Its acceleration is towards west and is constant. Let x_A and x_B be the magnitude of displacements in the first 10 seconds and the next 10 seconds
(a) $x_A < x_B$ (b) $x_A = x_B$ (c) $x_A > x_B$
(d) the information is insufficient to decide the relation of x_A with x_B

Q.3 A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by
(a) $v=\frac{v_1+v_2}{2}$ (b) $v=\sqrt{v_1v_2}$ (c) $\frac{2}{v}=\frac{1}{v_1}+\frac{1}{v_2}$ (d) $\frac{1}{v}=\frac{1}{v_1}+\frac{1}{v_2}$

Q.4 A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next equal distance. The average velocity v is given by
(a) $v=\frac{v_1+v_2}{2}$ (b) $v=\sqrt{v_1v_2}$ (c) $\frac{2}{v}=\frac{1}{v_1}+\frac{1}{v_2}$ (d) $\frac{1}{v}=\frac{1}{v_1}+\frac{1}{v_2}$

Q.5 A stone is released from an elevator going up with an acceleration a. The acceleration of the stone after the release is
(a) a upward (b) (g – a) upward (c) (g – a) downward (d) g downward

Q.6 In the arrangement shown in figure, the ends P and Q of an inextensible string move downwards with uniform speed u. Pulleys A and B are fixed. The mass M moves upwards with a speed



(a) $2u \cos \theta$ (b) $u/\cos \theta$

(c) $2u/ \cos \theta$ (d) $u\cos\theta$

Q.7 Consider the motion of the tip of the minute hand of a clock. In one hour

(a) the displacement is zero (b) the distance covered is zero

(c) the average speed is zero (d) the average velocity is zero

Q.8 A particle moves along the X–axis as $x = u(t - 2s) + a(t - 2s)^2$

(a) the initial velocity of the particle is u (b) the acceleration of the particle is a

(c) the acceleration of the particle is 2a (d) at t = 2 s particle is at the origin

Q.9 Pick the correct statements:

(a) Average speed of a particle in a given time is never less than the magnitude of the average velocity

(b) It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d}{dt}|\vec{v}|=0$

(c) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval

(d) Average velocity of a particle moving on a straight line is zero in a time interval. It is possible that instantaneous velocity is never zero in the interval. (Infinite accelerations are not allowed)

Q.10 An object may have

(a) varying speed without having varying velocity

(b) varying velocity without having varying speed

(c) non zero acceleration without having varying velocity

(d) nonzero acceleration without varying speed

Q.11 Mark the correct statements for a particle going on a straight line:

(a) If the velocity and acceleration have opposite sign, the object is slowing down.

(b) If the position and velocity have opposite sign, the particle is moving towards the origin

(c) If the velocity is zero at an instant, the acceleration should also be zero at that instant

(d) If the velocity is zero for a time interval, the acceleration is zero at any instant within time interval

Q.12 The velocity of a particle is zero at t = 0

(a) The acceleration at t = 0 must be zero (b) The acceleration at t = 0 may be zero

(c) If the acceleration is zero from t = 0 to t = 10 s, the speed is also zero in this interval

(d) If the speed is zero from t = 0 to t = 10 s the acceleration is also zero in this interval

Q.13 Mark the correct statements:

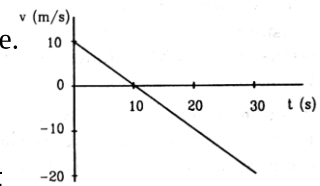
(a) The magnitude of the velocity of a particle is equal to its speed

(b) The magnitude of average velocity in an interval is equal to its average speed in that interval

(c) It is possible to have a situation in which speed of a particle is always zero but average speed is not zero

(d) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero

Q.14 Velocity time plot for a particle moving on a straight line is shown in figure.



(a) The particle has a constant acceleration

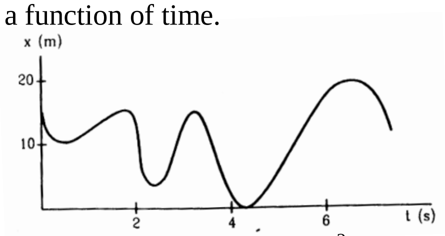
(b) The particle has never turned around

(c) The particle has zero displacement

(d) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10s to 20s.

Q.15 Figure shows the position of a particle moving on the X–axis as a function of time.

(a) The particle has come to rest 6 times
(b) The maximum speed is at t = 6s
(c) The velocity remains positive for t = 0 to t = 6 s
(d) The average velocity for the total period shown is negative



Q.16 The acceleration of a particle as seen from two frames S_1 and S_2 have equal magnitude 4 m/s^2.

(a) The frames must be at rest with respect to each other
(b) Frames may be moving w.r.t. each other but neither should be accelerated with respect to other
(c) The acceleration of S_2 with respect to S_1 may either be zero or 8 m/s^2
(d) The acceleration of S_2 with respect to S_1 may be anything between zero and 8 m/s^2

Comprehension Type Questions

Two cars A and B travel in straight line. The distance of A from the starting point is given as a function of time by x_A (t) = pt + qt^2, with p = 2.60 ms^{-1} and q = 1.20 ms^{-2}. The distance of B from straight point is x_B (t) = rt^2 – st^3 where r = 2.80 ms^{-2} and s = 0.20 ms^{-3}. Answer the following questions:

Q.17 Which car is ahead just after they have the starting point?

(a) Car A moves ahead (b) Car B moves ahead
(c) Cars A and B move simultaneously (d) Data is insufficient to decide

Q.18 At what time (s) are the cars at the same point?

(a) 2.60 (b) 2.27 s (c) 5.73 s (d) both 2.27 and 5.73 s

Q.19 At what time (s) do the cars A and B have the same acceleration?

(a) 2.67 s (b) 6.27 s (c) 4.33 s (d) both 6.27 and 4.33 s

Matrix–Match Type Questions

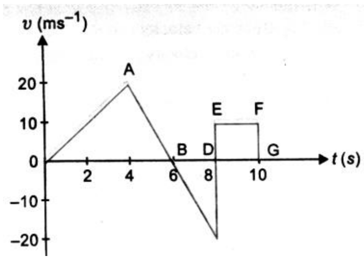
Directions: In each of the following questions, match column I and column II and select the correct match out of the four given choices.

Q.20 A ball thrown up is caught by the thrower after 4 seconds. Use g = 9.8 ms^{-2}

Column – I	Column – II
(A) The height of ball after 2 seconds	(p) 14.7 m
(B) The height of ball after 3 seconds	(q) 19.6 m
(C) The speed of the ball after 3 seconds	(r) 9.8 ms^{-1}
(D) The speed of the ball after 4 seconds	(s) 19.6 ms^{-1}

Q.21 The velocity (v) – time (t) graph of a body moving in a straight line is shown in figure. Match the quantities of column I and column II.

Column – I	Column – II
(A) distance travelled in 8 seconds	(p) 40 m
(B) distance travelled in 10 seconds	(q) 60 m
(C) Displacement in 8 seconds	(r) 80 m
(D) Displacement in 10 seconds	(s) 100 m



Q.22 Column – I	Column – II
(A) Linear Motion	(p) Motion of a bullet fired directly on a target from a distance
(B) Translatory motion	(q) Motion of a particle on a straight line confined in a well defined limits
(C) Oscillatory Motion	(r) A body sliding down on an inclined plane

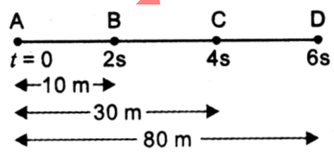
	Column – I	Column – II
	(D) Projectile motion	(s) Motion of a bus on a straight path

Q.23

	Column – I	Column – II
	(A) One dimensional motion	(p) The motion of earth around the sun in an orbit
	(B) Two dimensional motion	(q) Motion of the gas molecules in the given region
	(C) Three dimensional motion	(r) Sommer sault by a swimmer from a height before entering the water
	(D) Rotational Motion	(s) Oscillations of a mass suspended from a vertical spring

Q.24 A point mass body starts from rest and moves from A to D on a straight path ABCD as shown in figure.

Column – I	Column – II
(A) Constant velocity	(p) Motion from A to B
(B) Variable velocity	(q) Motion from B to C
(C) Uniform acceleration	(r) Motion from A to D
(D) Variable acceleration	(s) Motion from A to C



Q.25 A bird flies for 4s with a velocity of $|t - 2|$ m/s in a straight line, where t = time in seconds. It covers a distance of

(a) 2 m (b) 4 m (c) 6 m (d) 8 m

Q.26 A particle has an initial velocity of 9 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in the fifth second of its motion is

(a) 0 (b) 0.5 m (c) 2 m (d) none of these

Q.27 For a particle moving along a straight line, displacement x depends on time t as $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$. The ratio of its initial acceleration to its initial velocity depends

(a) only on α and β (b) only on β and γ (c) only on α and γ (d) only on α

Q.28 Water drops fall at regular intervals from a roof. At an instant when a drop is about to leave the roof, the separations between 3 successive drops below the roof are in the ratio

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 1 : 3 : 5 (d) 1 : 5 : 13

Q.29 A balloon starts rising from the ground with an acceleration of 1.25 m/s^2. After 8 s, a stone is released from the balloon. The stone will

(a) cover a distance of 40 m (b) have a displacement of 50 m

(c) reach the ground in 4s (d) begin to move down after being released

Q.30 An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects

(a) have the same speed (b) have the same velocity

(c) move in the same direction (d) move in opposite directions

Q.31 Which of the following statements are true for a moving body?

(a) If its speed changes, it velocity must change and it must have some acceleration

(b) If its velocity changes, its speed must change and it must have some acceleration

(c) If its velocity changes, its speed may or may not change, and it must have some acceleration

(d) If its speed changes but direction of motion does not change, its velocity may remain constant

Q.32 Let *v* and a denote the velocity and acceleration respectively of a body

(a) a can be non zero when v = 0 (b) a must be zero when v = 0

(c) a may be zero when $v \neq 0$

(d) The direction of a must have some correlation with the direction of v

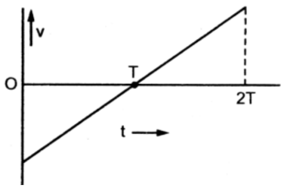
Q.33 Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one–dimensional motion.

(a) $|\vec{v}|$ must decrease when $\vec{a} < 0$ (b) Speed must increase when $\vec{a} > 0$

(c) Speed will increase when both $\vec{v}$ and $\vec{a}$ $are < 0$

(d) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$

Q.34 The figure shows the velocity (v) of a particle plotted against time (t).



(a) The particle changes its direction of motion at some point.

(b) The acceleration of the particle remains constant

(c) The displacement of the particle is zero

(d) The initial and final speeds of the particle are the same

Q.35 A particle moves along the x–axis as follows: it starts from rest at t = 0 from a point x = 0 and comes to rest at t = 1 at a point x = 1. No other information is available about its motion for the intermediate time (0 < t < 1). If α denotes the instantaneous acceleration of the particle then

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$

(b) $|\alpha|$ cannot exceed 2 at any point in its path

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path

(d) α must change sign during the motion, but no other assertion can be made with information given

Q.36 The displacement (x) of a particle depends on time (t) as $x = \alpha t^2 - \beta t^3$

(a) The particle will return to its starting point after time α/β

(b) The particle will come to rest after time $2\alpha/3\beta$

(c) The initial velocity of the particle was zero but its initial acceleration was not zero

(d) No net force will act on the particle at $t = \alpha/3\beta$

Q.37 A particle moves with an initial velocity v_0 and retardation αv, where v is its velocity at any time t.

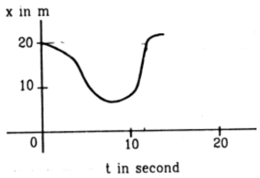
(a) The particle will cover a total distance v_0/α (b) The particle will come to rest after a time $1/\alpha$

(c) The particle will continue to move for a very long time

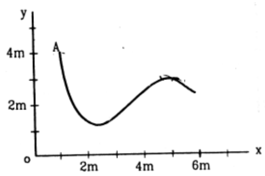
(d) The velocity of the particle will become $v_0/2$ after a time $1/\alpha$

Subjective Questions

Q.38 Figure shows x–t graph of a particle. Find the time t such that the average velocity of the particle during the period 0 to t is zero.



Q.39 A particle starts from a point A and travels along the solid curve shown in figure. Find approximately the position B of the particle such that the average velocity between the positions A and B has the same direction as the instantaneous velocity at B.



Q.40 A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h, crossing the turning ten seconds later than bike. Assuming that they travel at constant speeds, how far from turning will the jeep catch up with bike?

Q.41 A car travelling at 60 km/h overtakes another car travelling at 42 km/h. Assuming each car to be 5.0m long, find the time taken during the overtake and the total road distance used for the overtake.

Answers

1	d	2	d	3	a	4	c	5	d
6	b	7	a, d	8	c, d	9	a, b, c	10	b, d
11	a, b, d	12	b, c, d	13	a	14	a, d	15	a
16	d	17.	a	18.	d	19.	a		
20.	A – q; B – p; C – r; D – s	21.	A – r; B – s, C – p, D – q						
22.	A – q, r, s, B – q, r, s, C – q, D – p	23.	A – s, B – p, r, C – q, D –r						
24.	A – q; B – p, r, s,; C – p; D – r, s	25	d	26	b				
27	b	28	c	29	c	30	a, b, c		
31	a, c	32	a, c	33	c, d	34	a, b, c, d		
35	a, c	36	a, b, c, d	37	a, c	38.	12 s		
39.	x = 5m, y = 3m	40.	1.0 km	41.	2 s, 38 m				