MOTION IN A PLANE

Expression for position vector

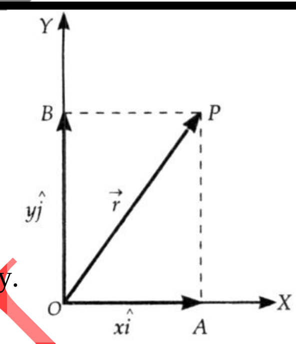


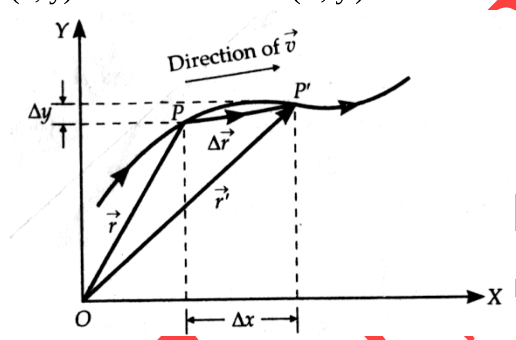
Figure shows the position vectors $\overrightarrow{OP}$ of a particle located at P with respect to the origin O. If (x, y) are the coordinates of point P, then

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$$

or $$\vec{r} = x\hat{i} + y\hat{j}$$

This equation expresses position vector $\vec{r}$ in terms of its rectangular components x and y.

Expression for Displacement Vector: Suppose a particle moves in the X–Y plane along the curved path shown in figure. The particle is at point P (x, y) at time t and at P' (x', y') at time t'.



Using triangle laws of vectors addition,

$$\overrightarrow{OP'} = \overrightarrow{OP} + \overrightarrow{PP'} \qquad \text{or,} \quad \vec{r}' = \vec{r} + \Delta\vec{r}$$

or $$\Delta\vec{r} = \vec{r}' - \vec{r}$$

This direction of $\Delta\vec{r}$ is form P to P'. In terms of rectangular components,

$$\Delta\vec{r} = (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$$

$$= (x' - x)\hat{i} + (y' - y)\hat{j} \quad \text{or} \quad \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

Average velocity in terms of its rectangular components

Average Velocity: Refer to above figure. Suppose the particle moves from point P to P' in time t to t'. *The average velocity of an object is ratio of the displacement and the corresponding time interval.* So it is given by

$$\bar{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \qquad [\text{Here } \Delta t = t' - t]$$

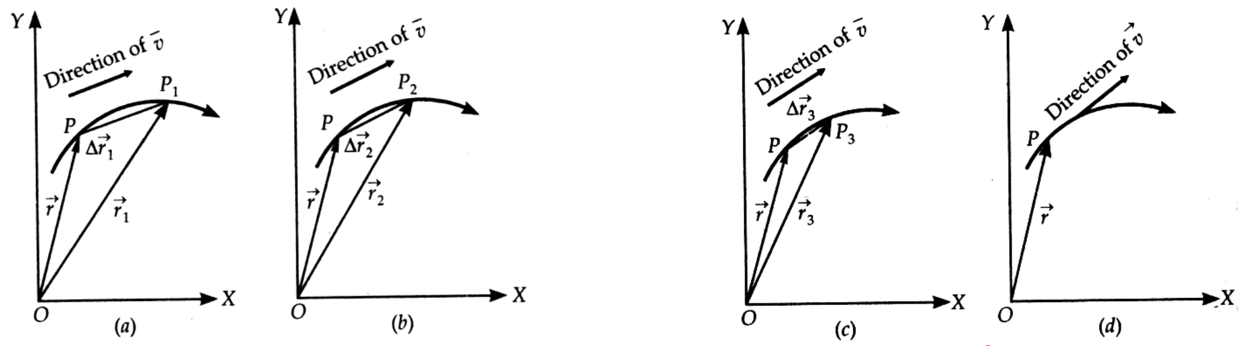
or $$\bar{v} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j}$$

This equation expresses average velocity $\bar{v}$ in terms of its rectangular components $\vec{v}_x$ and $\vec{v}_y$.

As $\bar{v} = \frac{\Delta\vec{r}}{\Delta t}$, so the direction of the average velocity is same as that of displacement vector $\Delta\vec{r}$, as shown in above figure.

Instantaneous velocity: *The instantaneous velocity of a particle is equal to the limiting value of its average velocity when the time interval approaches zero.* It is given by

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



As the average velocity approaches the velocity $\vec{v}$. The direction of $\vec{v}$ is parallel to the line tangent to the path. *The direction of (instantaneous) velocity at any point on the path of an object is tangent to the path at that point and is in direction of motion.*

Velocity in terms of rectangular components. The instantaneous velocity is given by

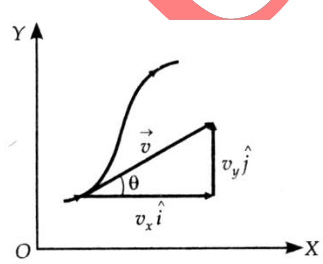
$$v = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x\,\hat{i} + \Delta y\,\hat{j}}{\Delta t}$$

$$= \left(\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right)\hat{i} + \left(\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\right)\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \quad \text{or} \qquad \vec{v}_x = v_x\hat{i} + v_y\hat{j}.$$

If the coordinate x and y are known as function of time t, then we can determine v_x and v_y. The magnitude of $\vec{v}$ will be $\quad v = \sqrt{v_x^2 + v_y^2}$

Figure, shows the rectangular compounds v_x and v_y of velocity $\vec{v}$. If $\vec{v}$ makes angle θ with X–axis, then

$$\tan\theta = \frac{v_y}{v_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



Average acceleration in terms of its rectangular components

Average Acceleration. *The average acceleration of an object is the ratio of the change in velocity and the corresponding time interval.* If the velocity of an object changes from $\vec{v}$ to $\vec{v}'$ in time Δt, then the average acceleration is given by

$$\vec{a} = \frac{\vec{v}' - \vec{v}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(v_x\,\hat{i} + v_y\,\hat{j})}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} \quad \text{or} \quad \vec{a} = a_x\hat{i} + a_y\hat{j}$$

This equation expresses average acceleration $\vec{a}$ in terms of its rectangular components $\vec{a}_x$ and $\vec{a}_y$. As $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, so the direction of average acceleration is same as that of the change in velocity $\Delta\vec{v}$.

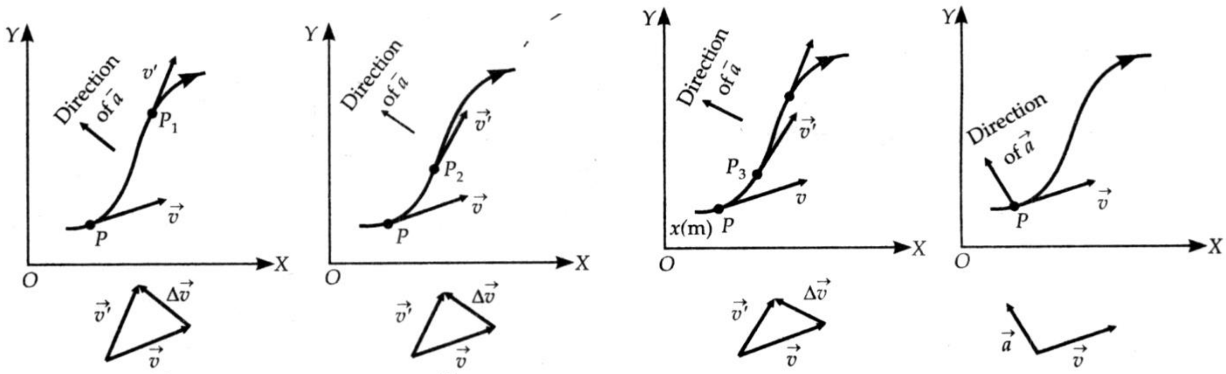
Instantaneous Acceleration

The instantaneous acceleration of an object is equal to the limiting value of its average acceleration when the time interval approaches zero. It is given by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Direction of Instantaneous Acceleration

In figure (a) to (d), the thick line curve represents the path of object's motion. In each case, the change in velocity $\Delta\vec{v}$ is obtained by using triangle law of vector addition. Again, in each case the direction of average acceleration $\vec{a}$ is shown parallel to $\Delta\vec{v}$.



(a) (b) (c) (d)

Note

For motion in one dimension, the velocity and acceleration are always along the same line either in same direction (for accelerated motion) or in opposite direction (for decelerated motion).

For motion in two or three dimensions, the angle between velocity and acceleration vectors may have any value between 0^o and 180^o.

Acceleration in terms of rectangular components

$$\vec{a}=\frac{d\vec{v}}{dt}=\lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t}=\lim_{\Delta t\to 0}\frac{\Delta(v_x\hat{i}+v_y\hat{j})}{\Delta t}=\left(\lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t}\right)\hat{i}+\left(\lim_{\Delta t\to 0}\frac{\Delta v_y}{\Delta t}\right)\hat{j}$$

$$=\frac{dv_x}{dt}\hat{i}+\frac{dv_y}{dt}\hat{j} \qquad \text{or} \qquad \vec{a}=a_x\hat{i}+a_y\hat{j}$$

This equation expresses acceleration $\vec{a}$ in terms of its rectangular component a_x and a_y. We can express a_x and a_y in terms of coordinates x and y as follows:

$$a_x=\frac{dv_x}{dt}=\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d^2x}{dt^2} \qquad \text{and} \qquad a_y=\frac{dv_y}{dt}=\frac{d}{dt}\left(\frac{dy}{dt}\right)=\frac{d^2y}{dt^2}$$

Motion in a Plane With Constant Acceleration

Velocity vector for uniform acceleration

If $\vec{v}_0$ and $\vec{v}$ be the velocity vectors at times t = 0 and t = t respectively, then the acceleration is given by

$$\vec{a}=\frac{\vec{v}-\vec{v}_0}{t-0}=\frac{\vec{v}-\vec{v}_0}{t}$$

or $\vec{v}=\vec{v}_0+\vec{a}\,t$

Writing the above equation in terms of rectangular components, we get

$$v_x\hat{i}+v_y\hat{j}=v_{0x}\hat{i}+v_{oy}\hat{j}+(a_x\hat{i}+a_y\hat{j})t$$

or $v_x\hat{i}+v_y\hat{j}=(v_{0x}+a_xt)\hat{i}+(v_{0y}+a_yt)\hat{j}$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ on both sides of the above equation, we get

$$v_x = v_{0x} + a_xt \quad \text{and} \quad v_y = v_{0y} + a_yt$$

The above two equations show that each rectangular component of velocity of an object moving with uniform acceleration in a plane depends upon time as if it were the velocity of one–dimensional uniformly accelerated motion.

Position vector for uniform acceleration

Consider a particle moving with uniform acceleration $\vec{a}$. Let $\overrightarrow{r_0}$ and $\vec{r}$ be its position vectors at times 0 and t and let the velocities at these instants be $\vec{v}_0$ and $\vec{v}$. Now

Displacement = Average velocity × time interval

or $$\vec{r}-\vec{r}_0=\frac{\vec{v}_0+\vec{v}}{2}\times t=\frac{v_0+(\vec{v}_0+\vec{a}\,t)}{2}\times t \quad =\vec{v}_0 t+\frac{1}{2}\vec{a}\,t_2$$

or $$\vec{r}=\vec{r}_0+\vec{v}_0\,t+\frac{1}{2}\vec{a}\,t^2$$

This equation gives position of a uniformly accelerated particle at time t. Writing the above equation in terms of rectangular components, we get

$$x\hat{i}+y\hat{j}=x_0\hat{i}+y_0\hat{j}+(v_{0x}\,\hat{i}+v_{0y}\,\hat{j})t+\frac{1}{2}(a_x\hat{i}+a_y\hat{j})t^2$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$x = x_0 + v_{0x}\,t + \frac{1}{2}a_x\,t^2$$

and $$y = y_0 + v_{0y}\,t + \frac{1}{2}a_y\,t^2$$

The above two equations show that the motions in x and y directions can be treated independently of each other. Thus, *the motion in a plane with uniform acceleration can be treated as the superposition of two separate simultaneous one–dimensional motions along two perpendicular directions.*

Note

In uniform acceleration, the position vector at time t,

$$\vec{r}=\vec{r}_o+\vec{v}_o\,t+\frac{1}{2}\vec{a}\,t^2$$

Similarly, position vector at time t' is

$$\vec{r}'=\overrightarrow{r_o}+\underset{\rightarrow}{v_o}t'+\frac{1}{2}\vec{a}\,t'^2$$

$$\therefore \quad \vec{r}'-\vec{r}=v_o(t'-t)+\frac{1}{2}\vec{a}(t'^2-t^2) \qquad \text{or} \qquad \vec{r}'=\vec{r}+v_o(t'-t)+\frac{1}{2}\vec{a}(t'^2-t^2)$$

Prove that $v^2-v_0^2=2\vec{a}.(\vec{r}_0-\vec{r}_0)$**.v**

From first equation of motion,

$$\vec{v}=\vec{v}_0+\vec{a}\,t \qquad \text{or} \qquad \vec{v}-\vec{v}_0=\vec{a}\,t \qquad \dots (1)$$

Now, displacement = Average velocity × time interval

or $$\vec{r}-\vec{r}_0=\frac{\vec{v}+\vec{v}_0}{2}\times t$$

or $$\vec{v}+\vec{v}_0=\frac{2(\vec{r}-\vec{r}_0)}{t} \qquad \dots (2)$$

Taking the dot products of the corresponding sides of the equation (1) and (2), we get

$$(\vec{v}-\vec{v}_0).(\vec{v}+\vec{v}_0)=\vec{a}\,t.2\frac{(\vec{r}-\vec{r}_0)}{t} \qquad \text{or} \qquad v^2-v_0^2=2a.(\vec{r}-\vec{r}_0)$$

Subjective Assignment – I

Q.1 A cyclist moves along a circular path of radius 70 m. If he completes one round in 11 s, calculate (i) total length of path, (ii) magnitude of the displacement, (iii) average speed, and (iv) magnitude of average velocity.

Q.2 A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds, the velocity changes to 5 ms^{-1} northwards. Find the average acceleration of the particle in this time interval.

Q.3 The position of a particle is given by $\vec{r} = 3.0t\,\hat{i} + 2.0t^2\,\hat{j} + 5.0\,\hat{k}$ where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find v(t) and a(t) of the particle. (b) Find the magnitude and direction of v(t) at t = 3.0 s.

Q.4 If the position vector of a particle is given by: $\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + (6t)\hat{k}$ m, calculate its acceleration at t = $\pi/4$.

Q.5 A body is moving with a uniform velocity of 10 ms^{-1} on a circular path of diameter 2.0 m. Calculate (i) the difference between the magnitude of the displacement of the body and the distance covered in half a round and (ii) the magnitude of the change in velocity of the body in half a round.

Q.6 A particle starts from origin at t = 0 with a velocity $5.0\,\hat{i}$ m/s and moves in x–y plane under action of a force which produces a constant acceleration of $(3.0\hat{i} + 2.0\hat{j})\ m/s^2$. (a) What is the y–coordinate of the particle at the instant its x–coordinate is 84 m? (b) What is the speed of the particle at this time?

Answers

1.	(i) 440 m, (ii) 0 (iii) 40 ms^{-1} (iv) 0	2.	$\frac{1}{\sqrt{2}} ms^{-2}$, along N–W direction
3.	$3.0\hat{i} + 4.0t\,\hat{j}$, $4.0\,\hat{j}$, 12.4 ms^{-1}, $\simeq 76^o$	4.	$-16\,\hat{j}\ ms^{-2}$
5.	1.14m, 20 ms^{-1}	6.	(a) 36.0 m, (b) 25.9 ms^{-1}

Relative Velocity in Two Dimensions

Relative velocity: *The relative velocity of an object A with respect to object B, when both are in motion, is the rate of change of position of object A with respect to object B.* Suppose two objects A and B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$, with respect to ground or the earth.

Then

Relative velocity of object A w.r.t object B, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Relative velocity of object B w.r.t. object A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

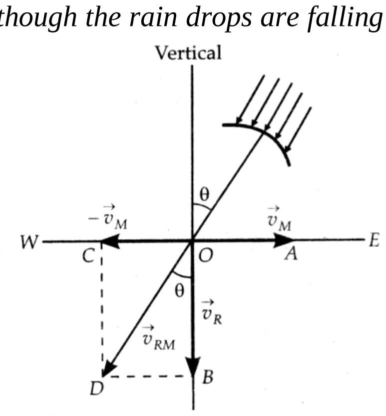
Clearly,

$$\vec{v}_{AB} = -\vec{v}_{BA} \text{ and } |\vec{v}_{AB}| = |\vec{v}_{BA}|$$

Now, relative velocity of object $A = \vec{v}_A + (-\vec{v}_B)$

Ex.1 *A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling velocity downwards.* Why?

Sol. **Rain and man:** The man experiences the velocity of rain relative to himself. To protect himself from the rain, the man should hold umbrella in the direction of relative velocity of rain w.r.t. the man.



As shown in figure, consider a man moving due east with velocity $\vec{v}_M$. Suppose the rain falls vertically with velocity $\vec{v}_R$. The relative velocity of rain w.r.t. the man is $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M = \vec{v}_R + (-\vec{v}_M) = \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OD}$

It $\overrightarrow{OD}$ makes angle θ with the vertical, then

$$\tan\theta = \frac{DB}{OB} = \frac{v_M}{v_R}$$

So the man can protect himself from rain by holding his umbrella at an angle θ with the vertical in the direction of his motion.

Subjective Assignment – II

Q.1 A boat is moving with a velocity $(3\hat{i}+4\hat{j})$ with respect to ground. The water in the river is moving with a velocity –3i – 4j with respect to ground. What is the relative velocity of boat with respect to river?

Q.2 A particle P is moving along a straight line with a velocity of 3 ms^{-1} and another particle Q has a velocity of 4 ms^{-1} at an angle of 30^o to the path of P. Find the speed of Q relative to P.

Q.3 To a driver east in a car with a velocity of 40 km h^{-1}, a bus appears to move towards north with a velocity of $40\sqrt{3}$ km h^{-1}. What is the actual velocity and direction of motion of the bus?

Q.4 A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u be the speed of man in still water and v that of stream, then show that:

$$t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}$$

Q.5 A train is moving with a velocity of 30 km h^{-1} due east and a car is moving with a velocity of 40 km h^{-1} due north. What the velocity of car is as appears to a passenger in the train?

Q.6 Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 ms^{-1} in east to west direction. What is the direction in which she should hold her umbrella?

Q.7 To a person moving eastwards with a velocity of 4.8 km h^{-1}, rain appears to fall vertically downwards with a speed of 6.4 km h^{-1}. Find the actual speed and direction of the rain.

Q.8 A ship is streaming towards east with a speed of 12 ms^{-1}. A woman runs across the deck at a speed of 5 ms^{-1} in the direction at right angles to the direction of motion of the ship i.e., towards north. What is the velocity of the woman relative to the sea?

Q.9 A plane is travelling eastwards at a speed of 500 km h^{-1}. But a 90 km h^{-1} wind is blowing southward. What is the direction and speed of the plane relative to the ground?

Q.10 A reckless drunk is playing with a gun in an airplane that is going directly east at 500 km h^{-1}. The drunk shoots the gun straight up at the ceiling of the plane. The bullet leaves the gun at a speed of 1000 km h^{-1}.Relative to an observer on earth, what angle does the bullet make with the vertical?

Answers

1.	$6\hat{i}+8\hat{j}$	2.	2.1 ms^{-1}
3.	80 km h^{-1}, β = 30^o, east of north	5.	50 km h^{-1}, $36^o52'$ west of north
6.	At an angle of 19^o with the vertical towards the west		
7.	8 kmh^{-1}, $53^o7'33''$ with the horizontal	8.	13 ms^{-1}, $22^o37'$ north of east
9.	10.2^o south of east, 508 km h^{-1}	10.	26.6^o

Projectile

A projectile is the name given to any body which once thrown into space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

Examples of projectile motion

(i) A javelin thrown by an athlete
(ii) An object dropped from an aeroplane
(iii) A bullet fired from a rifle.
(iv) A jet of water coming out from the side hole of vessel.
(v) A stone thrown horizontally from the top of a building

Projectile Given Horizontal Projection

As shown in figure, suppose a body is projected horizontally with velocity u from a point O at a certain height h above the ground level. The body is under the influence of two simultaneous independent motions:

(i) Uniform horizontal velocity u.

(ii) Vertically downward accelerated motion with constant acceleration g.

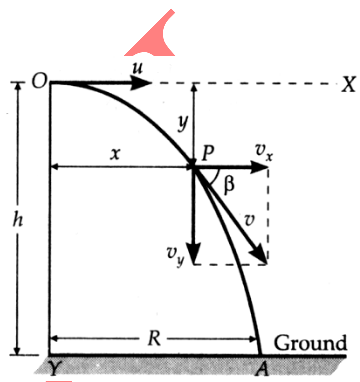
Under the combined effect of the above two motions, the body moves along the path OPA.

Trajectory of the projectile

After the time t, suppose the body reaches the point P (x, y).

The horizontal distance covered by the body in time t is

$$x = ut \quad \therefore \quad t = \frac{x}{u}$$



The vertical distance travelled by the body in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $$y = 0 \times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

[For vertical motion, u = 0]

or $$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2 \qquad \left[\because t = \frac{x}{u}\right]$$

or $$y = k\,x^2 \qquad \left[\text{here } k = \frac{g}{2x^2} = \text{a constant}\right]$$

As y is a quadratic function of x, so the trajectory of the projectile is a parabola.

Time of flight

It is the total time for which the projectile remains in its flight (from O to A). Let T be its time of flight. For the vertical downward motion of the body, we use

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad h = 0 \times T + \frac{1}{2}gT^2 \qquad \text{or} \qquad T = \sqrt{\frac{2h}{g}}.$$

Horizontal Range

It is the horizontal distance covered by the projectile during its time of flight. It is equal to OA = R. Thus

$$R = \text{Horizontal velocity} \times \text{time of flight} = u \times T \qquad \text{or} \qquad R = u\sqrt{\frac{2h}{g}}$$

Velocity of the projectile at any instant

At the instant t (when the body is at point P), let the velocity of the projectile be v. The velocity v has two rectangular components:

Horizontal component of velocity, $v_x = u$

Vertical component of velocity, $v_y = 0 + gt = gt$

∴ The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$$

If the velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u} \qquad \text{or} \qquad \beta = \tan^{-1}\left(\frac{gt}{u}\right).$$

Subjective Assignment – III

Q.1 A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to each the ground, and the speed with which it hits the ground. (Take g = 9.8 ms^{-2})

Q.2 A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

Q.3 A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45^o with the horizontal. Find the height of the tower and the speed with which the body was projected. Take g = 9.8 ms^{-2}.

Q.4 A bomb is dropped from an aeroplane when it is directly above a target at a height of 1000 m. The aeroplane is moving horizontally with a speed of 5000 kmh^{-1}. By how much distance will the bomb miss the target?

Q.5 A body is projected horizontally from the top of a cliff with a velocity of 9.8 ms^{-1}.What time elapses before horizontal and vertical velocities become equal? Take g = 9.8 ms^{-2}.

Q.6 A marksman wishes to hit a target just in the same level as the line of sight. How high from the target he should aim, if the distance of the target is 1600 m and the muzzle velocity of the gun is 800 ms^{-1}? Take g = 9.8 ms^{-2}.

Q.7 Two tall buildings face each other and are at a distance of 180m from each other. With what velocity must a ball be thrown horizontally from a window 55 m above the ground in one building, so that it enters a window 10.9 m above the ground in the second building?

Q.8 A plane is flying horizontally at a height of 1000m with a velocity of 100 ms^{-1} when a bomb is released from it. Find (i) the time taken by it to reach the ground (ii) the velocity with which the bomb hits the target and (iii) the distance of the target.

Q.9 From the top of a building 19.6m high, a ball is projected horizontal. After how long does it strike the ground? If the line joining the point of projection to the point where it hits the ground makes an angle of 45^o with the horizontal, what is the initial velocity of the ball?

Q.10 A body is thrown horizontally from the top of a tower and strikes the ground after two seconds at angle of 45^o with the horizontal. Find the height of the tower and the speed with which the body was thrown. Take g = 9.8 ms^{-2}

Q.11 Two tall buildings are situated 200 m apart. With what speed must a ball be thrown horizontally from the window 540 m above the ground in one building, so that it will enter a window 50 m above the ground in the other?

Q.12 A stone is dropped from the window of a bus moving at 60 kmh^{-1}. If the windows is 1.96 m high, find the distance along the track, which the stone moves before striking the ground.

Q.13 A mailbag is to be dropped into a post office from an aeroplane flying horizontally with a velocity of 270 kmh^{-1} at a height of 176.4 m above the ground. How far must the aeroplane be from the post office at the time of dropping the bag so that it directly falls into the post office?

Q.14 In between two hills of heights 100 m and 92m respectively, there is a valley of breadth 16m. If a vehicle jumps from the first hill to the second, what must be its minimum horizontal velocity so that it may not fall into the valley? Take g = 9 ms^{-2}.

Q.15 A ball is projected horizontally from a tower with a velocity of 4 ms^{-1}. Find the velocity of the ball after 0.7s. Take g = 10 ms^{-2}.

Answers

1.	10s, 99.1 ms^{-1},	2.	(i) 10s, (ii) 980 m, (iii) 138.59 ms^{-1}, $\beta = 45^o$		
3.	44.1 m, 29.4 ms^{-1}	4.	1984.13 m	5.	1 s
6.	19.6 m	7.	60 ms^{-1}		
8.	(i) 14.28s (ii) 172.1 ms^{-1}, $\beta = 54^o28'$, (iii) 1428.5 m			9.	2s, 9.8 ms^{-1}
10.	19.6 m, 19.6 ms^{-1}	11.	20 ms^{-1}	12.	10.54 m
13.	450 m	14.	12 ms^{-1}	15.	8.06 ms^{-1}, $60^o15'$

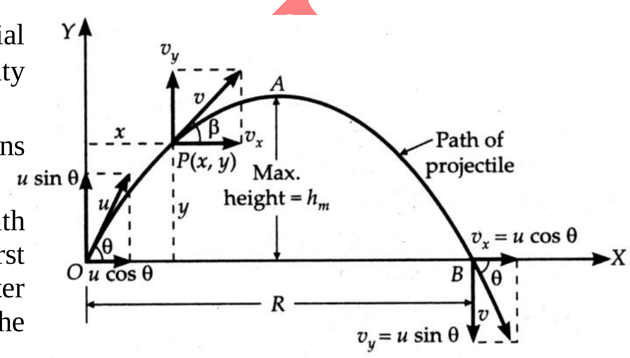
Projectile Given Angular Projection

Projectile fired at an angle θ with the horizontal

As shown in figure, suppose a body is projected with initial velocity u, making an angle θ with the horizontal. The velocity u has two rectangular components:

(i) The horizontal component u cos θ, which remains constant throughout the motion.

(ii) The vertical component u sin θ, which changes with time under the effect of gravity. This component first decreases, becomes zero at the highest point A, after which it again increases, till the projectile hits the ground.



Under the combined effect of the above two components, the body follows the parabolic path OAB as shown in the figure.

Equation of trajectory of a projectile

Suppose the body reaches the point P (x, y) after time t.

∴ The horizontal distance covered by the body in time t,

x = Horizontal velocity × time = u cos θ. t

or $$t = \frac{x}{u \cos\theta}$$

For vertical motion: u = u sin θ, a = – g, so the vertical distance covered in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $$y = u \sin\theta \,.\, \frac{x}{u\cos\theta} - \frac{1}{2} g \,.\, \frac{x^2}{u^2 \cos\theta}$$

or $$y = x \tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

or $$y = px - qx^2$$

where p and q are constants.

Thus y is a quadratic function of x. Hence the trajectory of a projectile is a parabola.

Time of maximum height

Let t_m be the time taken by the projectile to reach the maximum height h_m. At the highest point, vertical component of velocity = 0

As v = u + at ∴ $0 = u \sin\theta - g\, t_m$

or $$t_m = \frac{u \sin\theta}{g}$$

Time of flight

It is the time taken by the projectile from the instant it is projected till it reaches a point in the horizontal plane of its projection. The body reaches the point B after the time of flight T_f.

$\therefore$ Net vertical displacement covered during the time of flight = 0

As $s = ut + \frac{1}{2}at^2$

$\therefore$ $0 = u \sin\theta.\, T_f\ \frac{1}{2}\, gT_f^2$ or $T_f = \frac{2u \sin\theta}{g}$

Obviously, $T_f = 2t_m$. This is expected because the time of ascent is equal to the time of descent for the symmetrical parabolic path.

Maximum height of a projectile

It is the maximum vertical distance attained by the projectile above the horizontal plane of projection. It is denoted by h_m.

At the highest point A, vertical component of velocity = 0

As $v^2 - u^2 = 2as$

$\therefore$ $0^2 - (u \sin\theta)^2 = 2(-g)\, h_m$

or $h_m = \frac{u^2 \sin^2\theta}{2g}$

Horizontal Range (R)

It is the horizontal distance travelled by the projectile during its time of flight. So

Horizontal range = Horizontal velocity × time of flight

or $R = u\cos\theta \times \frac{2u \sin\theta}{g} = \frac{u^2}{g}.\, 2\sin\theta\cos\theta$

or $R = \frac{u^2 \sin 2\theta}{g}$ $[\because 2\sin\theta\cos\theta = \sin 2\theta]$

Condition for the maximum horizontal range. The horizontal range is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Clearly, R will be maximum when

$\sin 2\theta = 1 = \sin 90^o$

or $2\theta = 90^o$ or $\theta = 45^o$

Thus *the horizontal range of a projectile is maximum when it is projected an angle of 45^o with the horizontal.*

The maximum horizontal range is given by

$$R_m = \frac{u^2 \sin 90^o}{g} = \frac{u^2 \times 1}{g}$$

or $R_m = u^2/g$

Two angles of projection for the same horizontal range

The horizontal range of a projectile projected at an angle θ with the horizontal with velocity u is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Replacing θ by $(90^o - \theta)$, we get

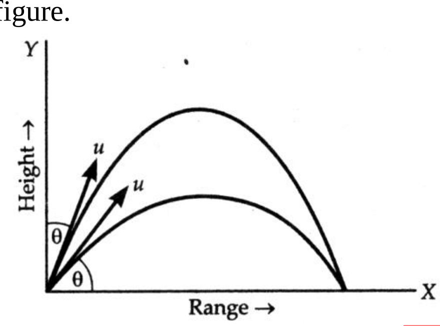
$$R' = \frac{u^2 \sin 2(90^o - \theta)}{g}$$

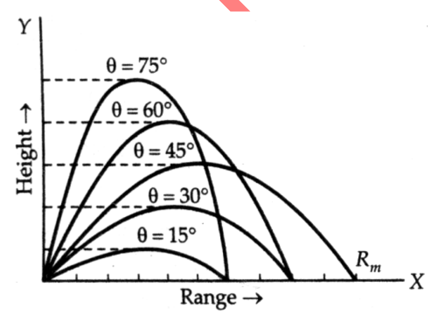
$$= \frac{u^2 \sin (180^o - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

i.e., $R' = R$

Hence for a given velocity of projection, a projectile has the same horizontal range for the angles of projection θ and $(90^o - \theta)$. As shown in figure, the horizontal range is maximum for 45^o. Clearly, R is same for $\theta = 15^o$ and 75^o but less than R_m. Again R is same for $\theta = 30^o$ and 60^o.

When the angle of projection is $(90^o - \theta)$ with the horizontal, the angle of projection with the vertical is θ. This indicates that the *horizontal range is same whether* θ is the angle of projection with the horizontal or with the vertical as shown in figure.





Velocity of projectile at any instant. As shown in figure, suppose the projectile has velocity v at the instant t when it is at point P(x, y). The velocity v has two rectangular components:

Horizontal component of velocity,

$$v_x = u \cos \theta$$

Vertical component of velocity,

$$v_y = u \sin \theta - gt \qquad \text{[Using } v = u + at]$$

The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(v\cos\theta)^2 + (u\sin\theta - gt)^2}$$

or $$v = \sqrt{v^2 + g^2t^2 - 2u\,gt\,\sin\theta}$$

It the velocity v makes an angle θ with the vertical, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Velocity of projectile at the end point. At the end of flight,

$$t = \text{total time of flight} = \frac{2u\sin\theta}{g}$$

So the resultant velocity is

$$v' = \sqrt{v^2 + g^2 . \frac{4u^2\sin^2\theta}{g^2} - 2ug.\frac{2u\sin\theta}{g}.\sin\theta}$$

$$= \sqrt{u^2} = u$$

$$\text{Also, } \tan\beta = \frac{u\sin\theta - g.\frac{2u\sin\theta}{g}}{u\cos\theta}$$

$$= -\frac{u\sin\theta}{u\cos\theta} = -\tan\theta = \tan(-\theta)$$

or $\beta = -\theta$

The negative sign shows that the projectile is moving downwards. Thus *in projectile motion, a body returns to the ground at the same angle and with the same speed at which it was projected.*

Note:

- A body is said to be projectile if it is projected into space with some initial velocity and then it continues to move in a vertical plane such that its horizontal acceleration is zero and vertical downward acceleration is equal to g.
- In projectile motion, the horizontal motion and the vertical motion are independent of each other i.e., neither motion affects the other.
- The horizontal range is maximum for $\theta = 45^o$ and $R_m = u^2/g$
- The horizontal range is same when the angle of projection is θ and $(90^o - \theta)$
- Again, the horizontal range is same for the angles of projection of $(45^o + \theta)$ and $(45^o - \theta)$
- At the highest point of the parabolic path, the velocity and acceleration of a projectile are perpendicular each other.
- The velocity at the end of flight of an oblique projectile is the same in magnitude as at the beginning but the angle that it makes with the horizontal is negative of the angle of projection.
- In projectile motion, the kinetic energy is maximum at the point of projection or point of reaching the ground and is minimum at the highest point.
- There are two values of time for which the projectile is at the same height. The sum of these two times is equal to the time of flight.
- The maximum horizontal range is four times the maximum height attained by the projectile, when fired at $\theta = 45^o$. Thus $h_m = R_m/4 = u^2/4g$.
- If a body is projected from a place above the surface of the earth, then for the maximum range the angle of projection should be slightly less than 45^o. For javelin throw and discus throw, the athlete throws the projectile at an angle slightly less than 45^o to the horizontal for achieving maximum range.
- The trajectory of a projectile is parabolic only when the acceleration of the projectile is constant and the direction of acceleration is different from the direction of projectile's initial velocity. The acceleration of a projectile thrown from the earth is equal to acceleration due to gravity (g) which remains constant if
  (i) the projectile does not go to a very large height.
  (ii) the range of the projectile is not very large.
  (iii) the initial velocity of the projectile is not large.
  Thus the trajectory of a bullet fired from a gun will be parabolic, but not so the trajectory of a missile.
- The shape of the trajectory of motion of an object is not determined by position alone but also depends on its initial position and initial velocity. Under the same acceleration due to gravity, the trajectory of an object can be a straight line or a parabola depending on the initial conditions.

Subjective Assignment – IV

Q.1 A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30^o above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, (c) the horizontal distance from the thrower to the point where the ball returns to the same level.

Q.2 A body is projected with a velocity of 30 ms^{-1} at an angle of 30^o with the vertical. Find the maximum height, time of flight and the horizontal range.

Q.3 A projectile has a range of 50 m and reaches a maximum height of 10 m .Calculate the angle at which the projectile is fired.

Q.4 A body stands at 39.2m from a building and throws a ball which just passes through a window 19.6 m (maximum height) above the ground. Calculate the velocity of projection of the ball.

Q.5 Find the angle of projection for which the horizontal range and the maximum height are equal.

Q.6 Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

Q.7 A ball is kicked at an angle of 30^o with the vertical. If the horizontal component of its velocity is 19.6 ms^{-1}, find the maximum height and horizontal range.

Q.8 Show that a given gun will shoot three times as high when elevated at an angle of 60^o as when fired at angle of 30^o but will carry the same distance on a horizontal plane.

Q.9 A ball is thrown at an angle θ and another ball is thrown at an angle $(90^o - \theta)$ with the horizontal direction from the same point with velocity 39.2 ms^{-1}. The second ball reaches 50 m higher than the first ball. Find their individual heights. Take $g = 9.8\ ms^{-2}$.

Q.10 Show that there are two angles of projection for which the horizontal range is the same. Also show that the sum of the maximum heights for these two angles is independent of the angle of projection.

Q.11 Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two time is equal to the time of flight.

Q.12 A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Will the bullet hit the monkey? Substantiate your answer with proper reasoning.

Q.13 At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 16 m high at a horizontal distance of 32 m? Take $g = 10\ ms^{-2}$.

Q.14 A target is fixed on the top of a pole 13 metre high. A person standing at a distance of 50 metre from the pole is capable of projecting a stone with a velocity $10\sqrt{g}\ ms^{-1}$. If he wants to strike the target in shortest possible time, at what angle should he project the stone?

Q.15 A particle is projected over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

Answers

1.	(a) 10.0 m, (b) 2.9 s, (c) 69.3 m	2.	34.44m, 5.3s, 79.53 m
3.	38.66^o	4.	27.72 ms^{-1}
5.	$75^o58'$	7.	58.8m, 135.8m
8.	3 : 1	9.	14.2m, 64.2 m
13.	$67^o54'$ or $48^o40'$	14.	$30^o58'$

Subjective Assignment – V

Q.1 A football player kicks ball at an angle of 37^o to the horizontal with an initial speed of 15 ms^{-1}. Assuming that the ball travels in a vertical plane, calculate (i) the time at which the ball reaches the highest point (ii) the maximum height reached (iii) the horizontal range of the projectile and (iv) the time for which the ball is in air.

Q.2 A body is projected with a velocity of 20 ms^{-1} in a direction making an angle of 60^o with the horizontal. Calculate its (i) position after 0.5s and (ii) velocity after 0.5 s.

Q.3 The maximum vertical height of a projectile is 10m. If the magnitude of the initial velocity is 28 ms^{-1}, what is the direction of the initial velocity? Take $g = 9.8\ ms^{-2}$.

Q.4 A bullet fired from a gun with a velocity of 140 ms^{-1} strikes the ground at the same level as the gun at a distance of 1 km. Find the angle of inclination with the horizontal at which the bullet is fired. Take $g = 9.8\ ms^{-2}$.

Q.5 A bullet is fired at an angle of 15^o with the horizontal and hits the ground 6 km away. Is it possible to hit a target 10 km away by adjusting the angle of projection assuming the initial speed to the same?

Q.6 A cricketer can throw a ball to maximum horizontal distance of 160 m. Calculate the maximum vertical height to which he can throw the ball. Given g = 10 ms^{-2}.

Q.7 A football is kicked 20 ms^{-1} at a projection angle of 45^o.A receiver on the goal line 25 metres away in the direction of the kick runs the same instant to meet the ball. What may be his speed, if he is to catch the ball before it hit the ground?

Q.8 A bullet fired at an angle of 60^o with the vertical hits the ground at a distance of 2 km. Calculate the distance at which the bullet will hit the ground when fired at an angle of 45^o, assuming the speed to be the same.

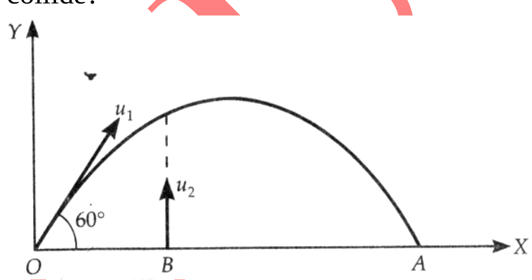
Q.9 A person observes a bird on a tree 39.6 m high and at a distance of 59.2 m. With what velocity the person should throw an arrow at an angle of 45^o so that it may hit the bird?

Q.10 A ball is thrown from the top of a tower with an initial velocity of 10 ms^{-1} an at angle of 30^o with the horizontal. If it hits the ground at a distance of 17.3m from the base of the tower, calculate the height of the tower. Given g = 10 ms^{-2}.

Q.11 A body is projected with velocity of 40 ms^{-1}. After 2s it crosses a vertical pole of height 20.4 m. Calculate the angle of projection and horizontal range.

Q.12 From the top of a tower 156.8m high, a projectile is thrown up with velocity of 39.2 ms^{-1} making an angle of 30^o with the horizontal direction. Find the distance from the foot of the tower where it strikes the ground and the time taken by it to do so.

Q.13 As shown in figure, a body is projected with velocity u_1 from the point A. At the same time another body is projected vertically upwards with the velocity u_2 from the point B. What should be the value of u_1/ u_2 for both bodies to collide?



Q.14 A body is projected such that its kinetic energy at the top is $3/4^{th}$ of its initial kinetic energy. What is the initial angle of projection of the projectile with the horizontal?

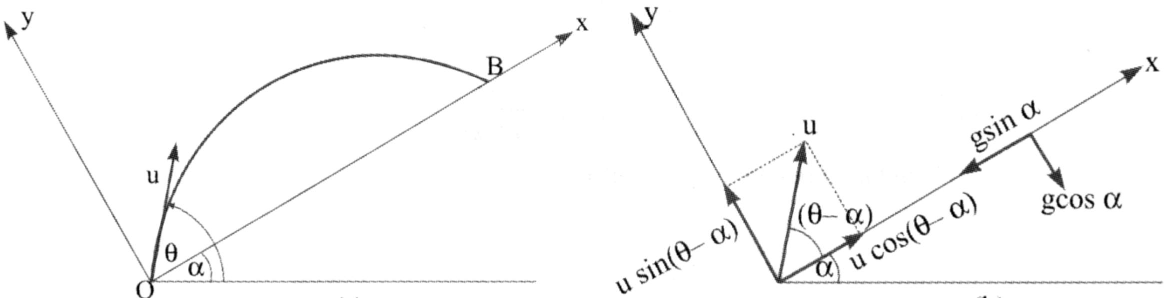
Answers

1. (i) 0.92s, (ii) 4.16m, (iii) 21.2m, (iv)1.84 s

2.	(i) x = 5m, y = 7.43m, (ii) 15.95 ms^{-1}, $\beta = 51.16^o$			3.	30^o	4.	15^o
5.	Yes	6.	80 m	7.	5.483 ms^{-1}	8.	2.31 km
9.	41.86 ms^{-1}	10.	10 m	11.	30^o, 141.39 m	12.	271.6 m, 8s
13.	$\frac{2}{\sqrt{3}}$	14.	30^o				

Projection up on an inclined plane

Let us consider a particle is projected with velocity u at an angle θ with the horizontal on an inclined plane of inclination α. In this case take x and y–axes along inclined plane and perpendicular to it.



We have, $u_x = u\cos(\theta - \alpha)$, $a_x = -g\sin\alpha$

and $u_y = u\sin(\theta - \alpha)$, $a_y = -g\cos\alpha$.

Time of flight (T):

The displacement along y–direction becomes zero to total time of flight T. Thus we have,

$$y = u_y T + \frac{1}{2} a_y T^2$$

or $$0 = u\sin(\theta - \alpha) - \frac{1}{2}(g\cos\alpha)T^2$$

which gives, $T = 0$ and $T = \dfrac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

T = 0 corresponds to O. Therefore time of flight

$$T = \frac{2u(\sin(\theta - \alpha)}{g\cos\alpha} \quad \ldots \text{(i)}$$

Range along inclined plane (R)

Using second equation of motion along x–axis, we have

$$R = u_x T + \frac{1}{2} a_x T^2$$

$$= u\cos(\theta - \alpha) \times \left[\frac{2u\sin(\theta - \alpha)}{g\cos\alpha}\right] - \frac{1}{2}(g\sin\alpha)\left[\frac{2u\sin(\theta - \alpha)}{g\cos\alpha}\right]^2$$

After simplifying, we get

$$R = \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

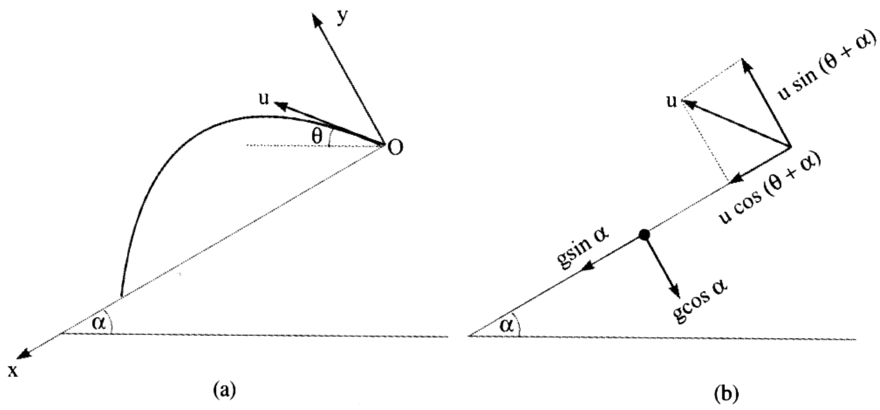
For maximum range, $\sin(2\theta - \alpha) = 1$

or $2\theta - \alpha = 90^\circ$ or $\theta = 45^\circ + \dfrac{\alpha}{2}$

$$\therefore \quad R_{max} = \frac{u^2(1 - \sin\alpha)}{g(1 - \sin^2\alpha)}$$

$$\therefore \quad R_{max} = \frac{u^2}{g(1 - \sin\alpha)}$$

Projection down the inclined plane



Here we have, $u_x = u\cos(\theta - \alpha)$, $a_x = g\sin\alpha$

$u_y = u\sin(\theta + \alpha)$, $a_y = -g\cos\alpha$

Time of flight

As displacement becomes zero along y–direction in time T,

$$\therefore \quad 0 = u_y T + \frac{1}{2} a_y T^2$$

or $$0 = u\sin(\theta + \alpha)T - \frac{1}{2}(g\cos\alpha)T^2$$

where gives T = 0 or $$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$

Range along inclined plane (R)

$$R = u_x T + \frac{1}{2} a_x T^2$$

$$= u\cos(\theta-\alpha)\left[\frac{2u\sin(\theta+\alpha)}{g\cos\alpha}\right] + \frac{1}{2} g\sin\alpha\left[\frac{2u\sin(\theta+\alpha)}{g\cos\alpha}\right]^2$$

After simplifying, we get

$$R = \frac{u^2}{g\cos^2\alpha}[\sin(2\theta+\alpha)+\sin\alpha] \qquad \dots \text{(ii)}$$

For maximum range, $\sin(2\theta + \alpha) = +1$

or $$(2\theta + \alpha) = 90^\circ \text{ or } \theta = 45^\circ - \frac{\alpha}{2}$$

$$\therefore \quad R_{max} = \frac{u^2(1+\sin\alpha)}{g\cos^2\alpha} = \frac{u^2(1+\sin\alpha)}{g(1-\sin^2\alpha)}$$

or $$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

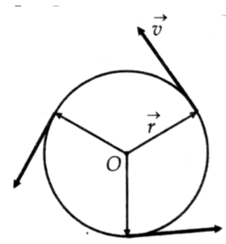
Uniform Circular Motion

If a particle moves along a circular path with a constant speed (i.e., it covers equal distances along the circumference of the circle in equal intervals of time), then its motion is said to be a uniform circular motion.

Examples:

(i) Motion of the tip of the second hand of a clock.

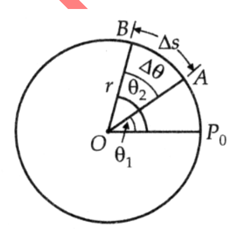
(ii) Motion of a point on the rim of a wheel rotation uniformly.



At each position, the velocity vector $\vec{v}$ is perpendicular to the radius vector $\vec{r}$. Thus the velocity of the body changes continuously due to the continuous change in the direction of motion of the body. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.

(i) Angular displacement

The angular displacement of a particle moving along a circular path is defined as the angle swept out by its radius vector in the given time interval.



As shown in figure, suppose a particles starts from the position P_0. Its angular position is θ_1 at time t_1 and θ_2 at time t_2. Let the particle cover a distance Δs in the time interval $t_2 - t_1$ $(= \Delta t)$. It revolves through angle $\theta_2 - \theta_1$ $(= \Delta\theta)$ in this interval. The angle of revolution $\Delta\theta$ is the angular displacement of the particle. If r is the radius of the circle, then

$$\Delta\theta = \frac{\Delta s}{r} \qquad \left[\because \text{Angle} = \frac{\text{Arc}}{\text{Radius}}\right]$$

The unit of angular displacement is radian. It is a dimensionless quantity.

(ii) Angular Velocity

The time rate of change of angular displacement of a particle is called its angular velocity. It is denoted by ω. It is measured in **radian per second** (rad s^{-1}) and its dimensional formula is $[M^0L^0T^{-1}]$

As shown in above figure, if $\Delta\theta$ is the angular displacement of a particle is time Δt, then its average angular velocity is

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

When the time interval $\Delta t \to 0$, the limiting values of the average velocity is called the instantaneous angular velocity, which is given by

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

(iii) Time Period

The time taken by a particle to complete one revolution along its circular path is called its period of revolution. It is denoted by T and is measured in **second.**

(iv) Frequency

The frequency of an object in circular motion is defined as the number of revolutions completed per unit time. It is denoted by ν (nu).

It v is the frequency of revolution of a particle, then time taken to complete v revolutions = 1 second, time taken to complete 1 revolution = $\frac{1}{\nu}$ second.

But time taken to complete 1 revolution is the time period T, so

$$T = \frac{1}{\nu} \quad \text{or} \quad \nu = \frac{1}{T}$$

Relations between angular velocity, frequency and time period

By definition of time period, a particle completes one revolution in time T i.e., it traverses an angle of 2π radian in time T.

$\therefore$ When time $t = T$

angular displacement $\theta = 2\pi$ radian

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{Time}}$$

or $$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu \qquad \left[\because \frac{1}{T} = \nu\right]$$

Relation between linear velocity and angular velocity

Consider a particle moving along a circular path of radius r. As shown in figure, suppose A to B in the Δt covering distance Δs along the arc AB. Hence the angular displacement of the particle is

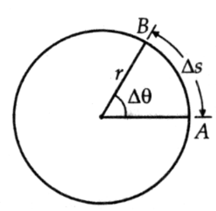
$$\Delta\theta = \frac{\Delta s}{r}$$

Dividing both sides by Δt, we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \to 0$ on both sides,

$$\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$



But $$\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega,$$ is the instantaneous angular velocity and

$$\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v,$$

is the instantaneous linear velocity.

$\therefore$ $$\omega = \frac{1}{r}.\, v$$

or $$v = \omega r$$

Linear velocity = Angular velocity × radius

In vector notation, we have the relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Angular acceleration

The time rate of change of angular velocity of a particle is called its angular acceleration. If $\Delta\omega$ is the change in angular velocity in time Δt, then the *average angular acceleration* is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

The *instantaneous acceleration* is equal to the limiting value of the average acceleration $\Delta\omega / \Delta t$ when Δt approaches zero. It is given by

$$\alpha = \lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The angular acceleration is measured in *radian per second^2 (rad s^{-2})* and has the dimensions $[M^oL^oT^{-2}]$.

The ration between linear velocity v and angular velocity ω is $v = r\omega$

Differentiating both sides w.r.t. time t, we get

$$\frac{dv}{dt}=\frac{d}{dt}(r\omega)$$

or $$\frac{dv}{dt}=r\left(\frac{d\omega}{dt}\right) \qquad [\because \text{ r is constant}]$$

or $$a = \alpha\, r$$

Linear acceleration = Angular acceleration × radius

In vector rotation, we have the relation

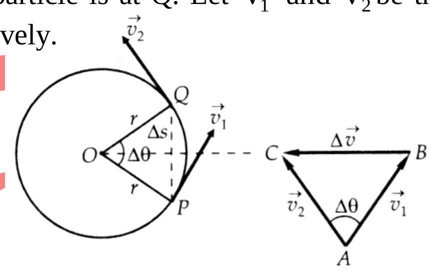
$$\vec{a} = \vec{\alpha}\times \vec{r}$$

Centripetal Acceleration

When a body is in uniform circular motion, its speed remains constant but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated. *A body undergoing uniform circular motion is acted upon by an accelerated which is directed along the radius towards the centre of the circular path. This acceleration is called centripetal (centre seeking) acceleration.*

Expression for centripetal acceleration

Consider a particle moving on a circular path of radius r and centre O, with a uniform speed v. As shown in figure (a), suppose at time t the particle is at P and at time t + Δt, the particle is at Q. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors at P and Q, directed along the tangents at P and Q respectively.



(a) (b)

Applying triangle law of vector addition in ΔBAC,

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$

$$\therefore \qquad \overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=\vec{v}_2-\vec{v}_1$$

Thus the change in velocity in time Δt is given by

$$\overrightarrow{BC}=\Delta\vec{v}$$

If Δt is small, the chord PQ becomes equal to arc PQ. Then OPQ can be considered as a triangle. ∠POQ = ∠BAC = Δθ. This is because the angle between the radii PO and QO is same as the angle between the tangents at P and Q.

Also OP = OQ = r, radius of the circle.

$$|\vec{v}_1|=|\vec{v}_2|=v \quad \text{i.e.,} \quad AB=AC=v$$

And ∠POQ = Δθ, ∠BAC = Δθ

Thus the two triangles POQ and BAC are similar. Hence

$$\frac{PQ}{OP}=\frac{BC}{AB}$$

or $$\frac{\Delta s}{r}=\frac{\Delta v}{v} \qquad \text{or} \qquad \Delta v = \frac{v}{r}\Delta s$$

Dividing both sides by Δt, we get

$$\frac{\Delta v}{\Delta t}=\frac{v}{r}\frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \to 0$ on both sides, we get

$$\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{v}{r}\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

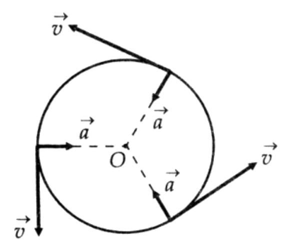
But $$\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}=v,$$ is the instantaneous velocity

$$\therefore \qquad a = \frac{v}{r}.v$$

$$\text{or} \qquad a = \frac{v^2}{r} = \omega^2 r \qquad [\because\ v = \omega r]$$

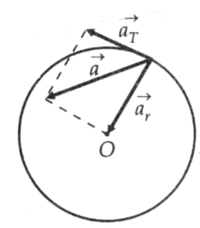
This gives the magnitude of the acceleration of a particle in uniform circular motion.

Direction of Acceleration



As Δt tends to zero, the angle $\Delta\theta$ also approaches zero. In this limit, as AB = AC, so $\angle ABC = \angle ACB = 90^o$. Thus the change in velocity $\Delta\vec{v}$ and hence the acceleration $\vec{a}$ is perpendicular to the velocity vector $\vec{v}_1$. But $\vec{v}_1$ is directed along tangent at point P, so acceleration $\vec{a}$ acts along the radius towards the centre of the circle. Such an acceleration is called *centripetal acceleration.* Its magnitude remains constant (= v^2/r) but its direction continuously changes and remains perpendicular to the velocity vector at all positions.

Circular Motion with Variable Speed



Consider a particle moving along a circular path of radius with a variable speed v. As the speed of the particle changes, so acceleration has a tangential component, $a_T = \frac{dv}{dt} = r\alpha$

As the direction of motion changes continuously, so the acceleration has a radial component.

$$a_r \qquad \text{or} \qquad a_c = \frac{v^2}{r}$$

The resultant acceleration of the particle will be

$$a = \sqrt{a_T^2 + a_r^2}$$

Note

- In uniform circular motion, the direction of velocity vector which acts along the tangent to the path, changes continuously but its magnitude always remains constant ($v = r\omega$). So circular motion is an accelerated motion.
- As v and r are constant, so that magnitude of the centripetal acceleration is a constant (= v^2/r). But the direction of a_c changes continuously, pointing always towards the centre. So centripetal acceleration is not a constant vector.
- The a resultant acceleration of an object in circular motion is towards the centre only if the speed is constant.
- For a body moving with a constant angular velocity, the angular acceleration is zero.
- In projectile motion, both the magnitude and direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes. Hence the equations of motion v = u + at, etc., are not applicable to circular motion. These equations hold only when both the magnitude and direction of acceleration are constant.

Subjective Assignment – VI

Q.1 Calculate the angular speed of (i) the hour hand of a watch and (ii) the earth about its own axis.

Q.2 Calculate the angular speed of flywheel making 420 revolutions per minute.

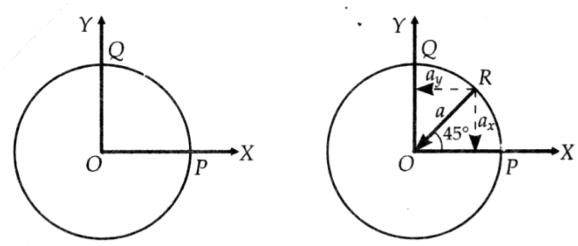
Q.3 A body of mass 10 kg revolves in a circle of diameter 0.40 m, making 1000 revolutions per minute. Calculate its linear velocity and centripetal acceleration.

Q.4 Find the magnitude of the centripetal acceleration of a particle on the tip of a fan blade, 0.30 metre in diameter, rotating at 1200 rev/minute.

Q.5 An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (i) What is the angular speed and the linear speed of the motion? (ii) Is the acceleration vector a constant vector? What is the magnitude? (iii) what is its linear displacement?

Q.6 The radius of the earth's orbit around the sun is 1.5×10^{11} m. Calculate the angular and linear velocity of the earth. Through how much angle does the earth revolve in 2 days?

Q.7 A particle moves in a circle of radius 4.0 cm clockwise at constant speed of 2 cms^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X–axis and Y–axis respectively (in cms^{-2}), find the acceleration of the particle at the instant half way between P and Q (figure).



(a) (b)

Q.8 What is the angular velocity of a second hand and minute hand of a clock?

Q.9 A threaded rod with 12 turns per cm and diameter 1.8 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at the rate of 216 rpm. How long will it take for the bar to move 1.50 cm along the rod?

Q.10 A circular wheel of 0.50 m radius is moving with a speed of 10 ms^{-1}. Find the angular speed.

Q.11 Assuming that the moon completes one revolution in a circular orbit around the earth in 27.3 days, calculate the acceleration of the moon towards the earth. The radius of the circular orbit can be taken as 3.85×10^5 km.

Q.12 The angular velocity of a particle moving along a circle of radius 50 cm is increased in 5 minutes from 100 revolutions per minute to 400 revolutions per minute. Find (i) angular acceleration and (ii) linear acceleration.

Q.13 Calculate the linear acceleration of a particle moving in a circle of radius 0.4m at the instant when its angular velocity is 2 rad s^{-1} and its angular acceleration is 5 rad s^{-2}.

Answers

1. $\dfrac{\pi}{21600}$ rad s^{-1}, $\dfrac{\pi}{43200}$ rad s^{-1}

2. 44 rad s^{-1}

3. $\dfrac{20\pi}{3}$ ms^{-1}, $\dfrac{2000\pi^2}{9}$ ms^{-2}

4. 4737.6 ms^{-2}

5. (i) 0.44 $rads^{-1}$, 5.28 cms^{-1}, (ii) 2.32 cms^{-2}

6. 1.99×10^{-7} rad s^{-1}, 2.99 ms^{-1}, 0.0344 rad

7. $-\dfrac{1}{\sqrt{2}}(\hat{x}+\hat{y})$

8. 0.1047 rad s^{-1}, 0.0017 rad s^{-1}

9. 5 s

10. 20 rad s^{-1}

11. 2.73×10^{-3} ms^{-2}

12. (i) $\dfrac{\pi}{30}$ rad s^{-2}, (ii) $\dfrac{5\pi}{3}$ cms^{-2}

13. 2.6 ms^{-2}, 38°40' with a_T

Conceptual Questions

Q.1 Can there be motion in two dimensions with an acceleration only in one dimension?

Q.2 A stone is thrown vertically upwards and then it returns to the thrower. Is it a projectile?

Q.3 Why does a projectile fired along the horizontal not follow a straight line path?

Q.4 What is the angle between the direction of velocity and acceleration at the highest point of a projectile path?

Q.5 A bullet is dropped from a certain height and at the same time, another bullet is fired horizontally from the same height. Which one will hit the ground earlier and why?

Q.6 A stone dropped from the window of a stationary bus takes 5 seconds to reach the ground. In what time the stone will reach the ground when the bus is moving with (a) constant velocity of 80 kmh^{-1} is (b) constant acceleration of 2 kmh^{-1}.

Q.7 A bomb thrown as projectile explodes in mid–air. What is the path traced by the centre of mass of the fragments assuming the friction to be negligible?

Q.8 A projectile is fired at an angle of 15^o to the horizontal with the speed v. If another projectile is projected with the same speed, then at what angle with the horizontal it must be projected so as to have the same range?

Q.9 Is the maximum height attained by projectile is largest when its horizontal range is maximum?

Q.10 What will be the effect on maximum height of a projectile when its angle of projection is changed from 30^o to 60^o, keeping the same initial velocity of projection?

Q.11 What is the angle between velocity vector and acceleration vector in uniform circular motion?

Q.12 For uniform circular motion, does the direction of centripetal acceleration depend upon the sense of rotation?

Q.13 A stone tied to the end of a string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why?

Q.14 A person sitting in a moving train throws a ball vertically upwards. How will the ball appear to move to an observer (i) sitting inside the train (ii) standing outside the train? Give reason?

Q.15 A bob hung from the celling of a room by a string is performing simple harmonic oscillations. What will be the trajectory of the bob, if the string is cut, when bob is (i) at one of its extreme positions, (ii) at its mean position?

Q.16 Two bombs of 5 kg and 10 kg are thrown from a cannon with the same velocity in the same direction. (i) Which bomb will reach the ground first? If the bombs are thrown in the same direction with different velocity, what would be the effect?

Q.17 A skilled gunman always keeps his gun slightly tilted above the line of sight while shooting, why?

Q.18 A railway carriage moves over a straight track with acceleration a. A passenger in the carriage drops a stone. What is the acceleration of the stone w.r.t. the carriage and the earth?,

NCERT Questions

Q.1 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (i) the average speed of the taxi and (ii) the magnitude of average velocity? Are the two equal?

Q.2 Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the relative velocity of rain with respect to the women? What is the direction in which she should hold her umbrella to protect herself from the rain?

Q.3 A man can swim with a speed of 4 kmh^{-1} in still water. How long does he take to cross the river 1 km wide, if the river flows steadily at 3 kmh^{-1} and he makes his strokes normal to the river current? How far from the river does he go, when he reaches the other bank?

Q.4 In a harbour, wind is blowing at the speed of 72 kmh^{-1} and the flag on the mast of a boat anchored in the harbour flutters along the N – E direction. If the boat starts moving at a speed of 51 kmh^{-1} to the north, what is the direction of the flag on the mast of the boat?

Q.5 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

Q.6 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball?

Q.7 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?

Q.8 An aircraft executes a horizontal loop of radius 1 km with a steady of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.

Q.9 Read each statement below carefully and state, with reasons, if it is true or false.

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Q.10 The position of a particle is given by $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\,m$ where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle. (b) What is the magnitude and direction of velocities of the particle at t = 2s?

Q.11 A particle starts from the origin at t = 0s with a velocity of $10.0\,\hat{j}$ m/s and moves in the x–y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ ms^{-2}. (a) At what time is the x–coordinate of the particle 16 m? What is the y–coordinate of the particle at that time? (b) What is the speed of the particle at that time?

Q.12 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10s apart is 30^o, what is the speed of the aircraft?

Q.13 A fighter plane flying horizontally at an altitude of 1.5 km with a speed 720 kmh^{-1} passes directly overhead an antiaircraft gun. At what angle from the vertical should the gun be fired for the shell muzzle speed 600 ms^{-1} to hit the plane? At what maximum altitude should the pilot fly the plane to avoid being hit? Take g = 10 ms^{-2}.

Q.14 A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate 0.5 ms^{-2}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Q.15 (a) Show that for a projectile angle between the velocity and the X–axis as a function of time is given by

$$\theta(t) = \tan^{-1}\frac{(v_{oy} - gt)}{v_{ox}}$$

(b) Show that the projection angle θ_0 for a projectile launched from the origin is given by:

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Answers

1.	49.3 kmh^{-1}, 21.43 kmh^{-1}			2.	31.6 ms^{-1}, 18°26'
3.	15 min, 0.75 km	4.	0.01° due east	5.	150.7 m
6.	50 m			7.	991.2 cm s^{-2}
8.	6.38			9.	(i) False, (ii) True, (iii) True
10.	$3.0\hat{i} - 4.0t\,\hat{j}$, $-4.0\,\hat{j}$, 8.54 ms^{-1}, 70° with x–axis			11.	(a)2 s, 24 m, (b) 21.26 ms^{-1}
12.	182.2 ms^{-1}	13.	19°30', 16 km	14.	0.86 ms^{-2}, 54°28'

Objective Assignment – I

Q.1 Given the speed of a boat in still water is u and the velocity of a river is v, the time taken by the boat to reach a place downstream at a distance d and back to the original place is

(a) $\frac{2vd}{v^2 - u^2}$ (b) $\frac{2ud}{u^2 - v^2}$ (c) $\frac{vd}{v^2 - u^2}$ (d) $\frac{ud}{u^2 - v^2}$

Q.2 Which of the following is true regarding projectile motion?

(a) horizontal velocity of projectile is constant (b) vertical velocity of projectile is constant

(c) acceleration is not constant (d) momentum is constant

Q.3 An aeroplane flying horizontally with a speed of 360 kmh^{-1} releases a bomb at a height of 490 m from the ground. If g = 9.8 ms^{-2}, it will strike the ground at

(a) 10 km (b) 100 km (c) 1 km (d) 16 km

Q.4 A bomber plane moves horizontally with a speed of 500 m/s and a bomb released from it, strikes the ground in 10 sec. Angle at which it strikes the ground (g = 10 m/s^2) will be

(a) $\tan^{-1}$ (1/5) (b) tan (1/5) (c) $\tan^{-1}$ (1) (d) $\tan^{-1}$ (5)

Q.5 If range and height of a projectile are equal, then angle of projection with the horizontal is

(a) 60° (b) $\tan^{-1}$ (4) (c) 30° (d) 45°

Q.6 The horizontal range of projectile is $4\sqrt{3}$ times of its maximum height. The angle of projection will be

(a) 40° (b) 90° (c) 30° (d) 45°

Q.7 Two bodies are projected with the same velocity. If one is projected at an angle of 30° and the other at 60° to the horizontal, then ratio of maximum heights reached is

(a) 3 : 1 (b) 1 : 2 (c) 1 : 3 (d) 2 : 1

Q.8 Two bodies are thrown up at angles of 45° and 60°, respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is

(a) $\sqrt{2/3}$ (b) $2/\sqrt{3}$ (c) $\sqrt{3/2}$ (d) $\sqrt{3}/2$

Q.9 A bullet is fired from a gun with a speed of 1000 m/s in order to hit a target 100 m away. At what height above the target should be gun be aimed? (The resistance of air is negligible and g = 10 m/s^2)

(a) 5 cm (b) 15 cm (c) 9 cm (d) 23 cm

Q.10 A cricketer hits a ball with a velocity 25 m/s at 60° above the horizontal. How far above the ground it passes over a fielder 50 m from the bat (assume the ball is struck very close to the ground)

(a) 8.2 m (b) 9.0 (c) 11.6 m (d) 12.7 m

Q.11 A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds from the instant of throwing will the ball reach the ground?

(a) 2 s (b) 5 s (c) 7 s (d) 9 s

Q.12 A boat crosses a river from port A to port B, which are just on the opposite side. The speed of the water is v_W and that of boat is v_B relative to water. Assume $v_B = 2v_W$. What is the time taken by the boat, if it has to cross the river directly on the AB line?

(a) $\frac{2D}{v_B\sqrt{3}}$ (b) $\frac{\sqrt{3}D}{2v_B}$ (c) $\frac{D}{v_B\sqrt{2}}$ (d) $\frac{D\sqrt{2}}{v_B}$

Q.13 A coastguard ship locates a pirate ship at a distance 560 m. It fires a cannon ball with an initial speed 82 m/s. At what angle from horizontal the ball must be fired so that it hits the pirate ship?

(a) 54^o (b) 125^o (c) 27^o (d) 18^o

Answers

1.	b	2.	a	3.	c	4.	a	5.	d
6.	c	7.	c	8.	c	9.	a	10.	a
11.	c	12.	a	13.	c				

Objective Assignment – II

Q.1 A boat is sent across a river with a velocity of 8 km h^{-1}. If the resultant velocity of boat is 10 km h^{-1}, then velocity of river is

(a) 12.8 km h^{-1} (b) 6 km h^{-1} (c) 8 km h^{-1} (d) 10 km h^{-1}

Q.2 The width of river is 1 km. The velocity of boat is 5 km/h. The boat covered the width of river in shortest time 15 min. Then the velocity of river stream is

(a) 3 km/hr (b) 4 km/hr (c) $\sqrt{29}$ km/hr (d) $\sqrt{41}$ km/hr

Q.3 A person aiming to reach to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at angle of 120^o with the direction of flow of water. The sped of water in the stream are

(a 0.25 m/s (b) 0.5 m/s (c) 1.0 m/s (d) 0.433

Q.4 A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x–axis an angle of

(a) 45^o (b) 60^o (c) 0^o (d) 30^o

Q.5 Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other in a time t, where t is

(a) $\frac{a}{\sqrt{v^2+v_1^2}}$ (b) $\frac{a}{v+v_1}$ (c) $\frac{a}{v-v_1}$ (d) $\sqrt{\frac{a^2}{v^2-v_1^2}}$

Q.6 Two particles A and B are connected by a rigid rod AB. The rod slides along perpendicular rails as shown here. The velocity of A to the left is 10 m/s. What is the velocity of B when $\alpha = 60^o$?

(a) 10 m/s (b) 9.8 m/s
(c) 5.8 m/s (d) 17.3 m/s

Q.7 The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$, the velocity of the particle is

(a) directed towards the origin (b) directed away from the origin
(c) parallel to the position vector (d) perpendicular to the position vector

Q.8 Two projectiles of same mass and with same velocity are thrown at an angle 60^o and 30^o with the horizontal, then which will remain same

(a) time of flight (b) range of projectile (c) maximum height acquired (d) all of them

Q.9 If a body A of mass M is thrown with velocity v at an angle 30^o to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60^o to the horizontal, the ratio of horizontal range of A to B will be

(a) 1 : 3 (b) 1 : 1 (c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

Q.10 For angles of projection of a projectile at angle $(45^o - \theta)$ and $(45^o + \theta)$, the horizontal range described by the projectile are in the ratio of

(a) 2 : 1 (b) 1 :1 (c) 2 : 3 (d) 1 : 2

Q.11 The maximum range of a gun of horizontal tarrain is 16 km. If g = 10 ms^{-2}, then muzzle velocity of a shell must be

(a) 160 ms^{-1} (b) $200\sqrt{2}\,ms^{-2}$ (c) 400 ms^{-1} (d) 800 ms^{-1}

Q.12 A cricket ball is hit at 45^o to the horizontal with a kinetic energy E. The kinetic energy at highest point is

(a) 0 (b) E/2 (c) $E/\sqrt{2}$ (d) E

Q.13 A particle A is dropped from a height and another particle B is projected in horizontal direction with sped of 5 m/sec from the same height, then correct statement is

(a) particle A will reach at ground first with respect to particle B

(b) particle B will reach at ground first with respect to particle A

(c) both particles will reach at ground simultaneously

(d) both particles will reach at ground with same speed

Q.14 A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time?

Given g = 9.8 m/s^2)

(a) more than 19.6 m/s (b) atleast 9.8 m/s

(c) any speed less than 149.6 m/s (d) only with speed 19.6 m/s

Q.15 A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the velocity of both is related as

(a) $v_B > v_m$ (b) $v_B < v_m$ (c) $v_B = v_m$ (d) v_B and v_m can't be related

Answers

1.	b	2.	a	3.	a	4.	b	5.	d
6.	d	7.	d	8.	b	9.	b	10.	b
11.	c	12.	b	13.	c	14.	a	15.	c

Objective Assignment – III

Q.1 Two projectiles are projected with the same velocity. If one is projected at an angle of 30^o and the other at 60^o to the horizontal, then the ratio of maximum heights reached is

(a) 3: 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

Q.2 An aeroplane moving horizontally with a speed of 720 km/h drops a food packet while flying at a height of 396.9m. The time taken by the food packet to reach the ground and its horizontal range are

(Take g = 9.8 m/s^2)

Q.3 A particle is projected making an angle an angle of 45^o with the horizontal having kinetic energy K. The kinetic energy at the highest point will be

(a) $\frac{K}{\sqrt{2}}$ (b) K/2 (c) 2K (d) K

Q.4 An object is projected vertically upward with a velocity $\sqrt{4gR/3}$ from earth. The velocity of the object at the maximum height reached by it will be

(a) $\sqrt{\frac{2gR}{3}}$ (b) $\sqrt{\frac{gR}{3}}$ (c) $\sqrt{2gR}$ (d) zero

Q.5 A body is projected at such an angle that the horizontal range is three times the greatest height attained. The angle of projection is

(a) $25^o\ 8'$ (b) $33^o\ 7'$ (c) $42^o\ 8'$ (d) 53.1^o

Q.6 Two particles of same mass and with same velocity are thrown at an angle of 60^o and 30^o with the horizontal. Which will remain the same?

(a) Time of flight (b) Range of projectile (c) Max. height acquired (d) All of them

Q.7 A bomber plane is moving horizontally with a speed of 500 m/s and bomb released from it strikes the ground in 10 sec. The angle at which it strikes the ground is (Take g = 10 m/s^2)

(a) $\tan^{-1} 5$ (b) $\tan^{-1} 1$ (c) $\tan^{-1} 1/5$ (d) $\sin^{-1} 1/5$

Q.8 Two particles are projected with the same initial velocity, one making angle θ with the horizontal and other making angle θ with the vertical. If their common range is R, then product of their times of flight is directly proportional to

(a) R (b) R^2 (c) 1/R (d) R^3

Q.9 A bomb is dropped from an aeroplane moving horizontally at a constant speed. When air resistance is taken into consideration, then the bomb

(a) flies with the aeroplane (b) falls on earth ahead of aeroplane

(c) falls on earth behind aeroplane (d) falls on earth exactly below aeroplane

Q.10 The maximum horizontal range of a projectile is 400 m. The maximum height attained by it will be:

(a) 100 m (b) 200 m (c) 400 m (d) 800 m

Q.11 For a given angle of projection, if the time of flight of a projectile is doubled, the horizontal range will increase to:

(a) four times (b) thrice (c) once (d) twice

Q.12 If retardation produced by air resistance of projectile is one–tenth of acceleration due to gravity, the time to reach maximum height:

(a) decreases to 11 percent (b) increases to 11 percent

(c) decreases to 90 percent (d) increases to 90 percent

Q.13 Two stones are projected with same velocity v at an angle θ and (90 – θ). If H and H_1 are the greatest heights in the two paths, what is the relation between R, H and H_1?

(a) $R = 4\sqrt{H\ H_1}$ (b) $R = \sqrt{HH_1}$ (c) $R = 4HH_1$ (d) None of the above

Q.14 A body is thrown with a velocity of 9.8ms^{-1} making an angle of 30^o with the horizontal. It will hit ground after a time.

(a) 3 s (b) 2s (c) 1.5 s (d) 1 s

Q.15 A person aims a gun at a bird from a point at a horizontal distance of 100 m. If the gun can impart a speed of 500 ms^{-1} to bullet, at what height above bird must he aim his gun is order to hit it? (Take g = 10 m/s^2)

(a) 10 cm (b) 20 cm (c) 50 cm (d) 100 cm

Q.16 For angle of projection of a projectile at angles $(45^o + \theta)$ and $(45^o - \theta)$, the horizontal ranges described by the projectile are in the ratio of (if $\theta \geq 45^o$)

(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 2 : 3

Q.17 An aeroplane moving horizontally with a speed of 180 km/hr drops a food packet while flying at a height of 490 m. The horizontal range is

(a) 500 m (b) 980 m (c) 670 m (d) 180 m

Q.18 The horizontal range is four times the maximum height attained by a projectile. The angle of projection is

(a) 90^o (b) 60^o (c) 45^o (d) 30^o

Q.19 At the top of the trajectory of a projectile, the acceleration is

(a) maximum (b) minimum (c) zero (d) g

Q.20 The range of projectile is R when the angle of projection is 30^o. Then the value of the other angle of projection for the same range is

(a) 45^o (b) 60^o (c) 50^o (d) 40^o

Q.21 A stone thrown at an angle θ to the horizontal, reaches a maximum height. The time of flight of the stone is

(a) $\frac{\sqrt{2h \sin\theta}}{g}$ (b) $\frac{2\sqrt{2h \sin\theta}}{g}$ (c) $2\sqrt{\frac{2h}{g}}$ (d) $\sqrt{\frac{2h}{g}}$

Q.22 A ball is projected horizontally with a velocity of 4 m/s. The velocity of ball after 0.7s ($g = 10\ m^{-2}$) is

(a) $11\ ms^{-1}$ (b) $10\ ms^{-1}$ (c) $8\ ms^{-1}$ (d) $3\ ms^{-1}$

Q.23 A stone released with zero velocity from the top of the tower reaches the ground in 4 seconds. The height of the tower is about

(a) 20 m (b) 40 m (c) 80 m (d) 160 m

Q.24 A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0\sqrt{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

(a) Yes, 60^o (b) Yes, 30^o (c) No (d) Yes, 45^o

Q.25 A boy playing on the roof of a 10m high building throws a ball with a speed of 10 m/s at an angle of 30^o with the horizontal. How far from the throwing point will the ball be at the height of 10m from the ground? [$g = 10\ m/s^2$, $\sin 30^o = 1/2$, $\cos 30^o = \sqrt{3}/2$]

(a) 5.20 m (b) 4.33 m (c) 2.60 m (d) 8.66 m

Answer

1.	b	2.	d	3.	b	4.	d
5.	d	6.	b	7.	c	8.	a
9.	c	10.	a	11.	a	12.	c
13.	a	14.	d	15.	b	16.	c
17.	a	18.	c	19.	d	20.	b
21.	c	22.	c	23.	c	24.	a
25.	d						