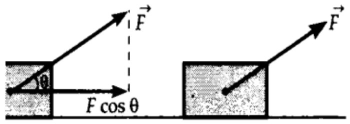
Work Power and Energy

Work

Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force. Thus work is done on a body only if the following two conditions are satisfied.



(i) A force acts on the body

(ii) The point of application of the force moves in the direction of the force.

Work done = Component of force in the direction of displacement × magnitude of displacement

$$W = F\cos\theta \times s \qquad \text{or} \qquad W = Fs\cos\theta \qquad \text{or} \qquad W = \vec{F}.\vec{S}$$

Thus work done is the dot product of force and displacement vectors. Hence work is a scalar quantity.

Dimensional formula of work $\left[ML^2T^{-2}\right]$

S.I. unit = Joule , cgs unit = erg , $1J = 10^7$ ergs

Work done may be positive, negative or zero.

Work done in terms of rectangular components : In terms of rectangular components, the force and displacement vectors can be written as

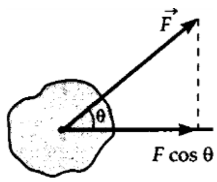
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \qquad \text{and} \qquad \vec{s} = s_x\hat{i} + s_y\hat{j} + s_z\hat{k}$$

$$\therefore \qquad W = \vec{F}.\vec{s} = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\ (s_x\hat{i} + s_y\hat{j} + s_z\hat{k})$$

$$\text{or} \qquad w = F_x\ s_x + F_y\ s_y + F_z\ s_z$$

NATURE OF WORK DONE IN DIFFERENT SITUATIONS :

Positive work : If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive. As shown in fig. When θ is acute, $\cos\theta$ is positive .



$$\therefore \qquad W = Fs\cos\theta = \text{positive value}$$

Positive Work : $(\theta < 90^0)$

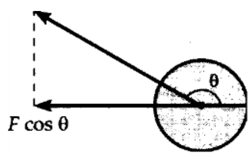
Example : (i) When a body falls freely under gravity $(\theta = 0^0)$, the work done by the gravity is positive.

(ii) When a horse pulls a cart, the applied force and displacement are in the same direction, the work done by the horse is positive.

(iii) When a gas filled in a cylinder fitted with a movable piston is allowed to expand, the work done by the gas is positive, because the force due to gas pressure and displacement act in the same direction.

(iv) When a spring is stretched, both the stretching force and the displacement act in the same direction. So, work done is positive.

Negative Work : If a force acting on the body has a component in the opposite direction of displacement, the work done is negative. As shown in Fig. When θ is obtuse, $\cos\theta$ is negative.



$$\therefore \qquad W = Fs\cos\theta = \text{a negative value}$$

Example : (i) When a body slides against a rough horizontal surface, its displacement is opposite to the force of friction. The work done by the friction is negative.

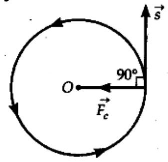
(ii) When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.

(iii) When a body is lifted, the work done by the gravitational force acts vertically downwards while the displacement is in the vertically upward direction.

(iv) When a positive charge is moved towards another positive charge, the work done by the force of repulsion (between them) is negative because displacement $\vec{s}$ is in the opposition direction of repulsive force $\vec{F}$ or $\vec{s}$ or both are zero.

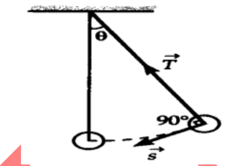
Zero Work : If no component of force act in the direction of displacement, the work done by the force is zero.

When $\theta = 90^o$, $\cos 90^o = 0$, $\therefore\ W = 0$



Example : (i) For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other, as shown in fig. So the work done by the centripetal force is zero.

(ii) When a coolie walks on a horizontal platform with a load on his head, the applies a force on it in the upward direction equal to its weight. The displacement of the load is along the horizontal direction. Thus the angle between $\vec{F}$ and $\vec{S}$ is 90^0. So $W = Fs\cos 90^0 = 0$ i.e., the work done by the collie on the load in zero.



(iii) As shown in fig. the tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.

Subjective Assignment – I

1. A gardener pushes a lawn roller through a distance of 20*m*. If he applies a force of 20kg wt. in a direction inclined at 60^0 to the ground, find the work done by him. Take $g = 9.8$ ms^{-2}.
2. A person is holding a bucket by applying a force of 10*N*. He moves a horizontal distance of 5m and then climbs up a vertical distance of 10m. Find the total work done by him.
3. A cyclist comes to a skidding stop in 10m. During this process, the force on the cycle due to the road is 200 N and is directly opposite to the motion.
   (a) How much work does the road do on the cycle?
   (b) How much work does the cycle do on the road.
4. A force $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ acts on a particle and displaces it through $\vec{s} = 6\hat{i} + 9\hat{k}$. Calculate the work done if the force is in Newton and displacement in metre.
5. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$, where *K* is a positive constant, acts on a particle moving in the *XY*–plane. Starting from the origin, the particle is taken along the positive *X*–axis to a point (*a*, 0) and then parallel to the *y*–axis to the point (*a*, *a*). Calculate the total work done by the force on the particle.
6. A uniform chain of length 2m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? Take g = 10 ms^{-2}.
7. Calculate the work done in raising a stone of mass 5 kg specific gravity 3, lying at the bed of a lake through a height of 5 m.
8. A cluster of clouds at a height of 1000 m above the earth burst and enough rain fell to cover an area of 10^6 m^2 with a depth of 2 cm. How much work would have been done in raising water to the height of clouds? Take g = 9.8 ms^{-2} and density of water = 10^3 kgm^{-3}
9. A particle is acted upon by constant forces $\vec{F_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{F_2} = -\hat{i} + 2\hat{j} - 3\hat{k}$, is displaced from the point A (2, 1, 0) to the point B (–3, –4, 2). Find the total work done by these forces.
10. A man weighting 50 kg f supports a body of 25 kgf on his head. What is the work done when he moves a distance of 20 m up an incline of 1 in 10? Take g = 9.8 ms^{-2}.

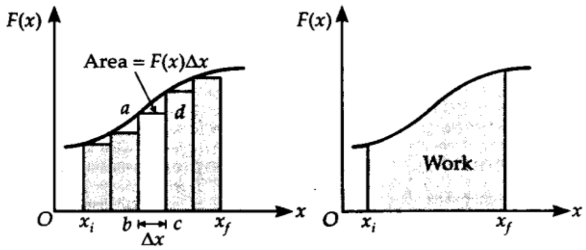
Answers

1.	1960	2.	100 J	3.	(a) – 2000 J, (b) 0	4.	69 J
5.	– Ka^2	6.	3.6 J	7.	163.3 J	8.	1.96×10^{11} J
9.	2 units	10.	1470 J				

Work Done by a variable Force :

Suppose a variable force F acts on a body along the fixed direction, say x–axis. The magnitude of the force F depends on x, as shown by force–displacement graph. The displacement can be divided into a large number of small equal displacements Δx. During a small displacement Δx, the force F can be assumed to be constant. Then the work done is

$W = F\Delta x =$ Area of rectangle *abcd.*



Adding areas of all the rectangles in fig. (a), we get the total work done as

$$W = \sum_{x_i}^{x_f} F\,\Delta x = \text{Sum of areas of all rectangles erected over all the small displacement.}$$

Thus the total work done is $$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F\;\Delta x = \int_{x_i}^{x_f} F\;dx$$

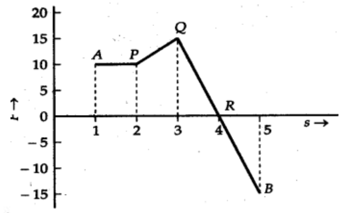
= Area under the force over the given displacement.

Note : When the force varies both in magnitude and direction, the work done is given by

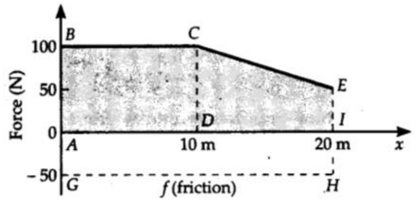
$$W = \int_{S_1}^{S_2} F\;dx\cos\theta \;=\; \int_{S_1}^{S_2} \vec{F}\;.\;d\,\vec{s}$$

Subjective Assignment – II

1 A body moves from point A to B under the action of a force, varying in magnitude as shown in fig. Obtain the work done. Force is expressed in newton and displacement in metre.



2 A woman pushes a trunk on railway platform which has a rough surface. She supplies a force of 100N over a distance of 10m. Thereafter she gets progressively tired and her applied force reduces linearly with distance to 50N. The total distance by which the trunk has been moved is 20m. Plot the force applied by the woman and the frictional force, which is 50N. Calculate the work done by the two forces over 20m.

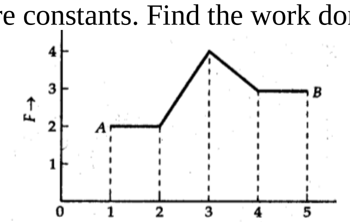


3 A particle moves along the X–axis from x = 0 to x = 5m under the influence of a force given by $F = 7 - 2x + 3x^2$. Find the work done in the process.

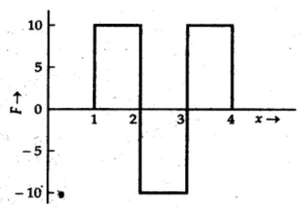
4 A force $F = (15 + 0.50x)$ acts on a particle in the X–direction, where F is in Newton and x in metre. Find the work done by this force during a displacement from $x = 0$ to $x = 2.0$ m.

5 A force $F = a + bx$ acts on a particle in the X–direction, where a and b are constants. Find the work done by this force during a displacement from $x = 0$ to $x = d$.

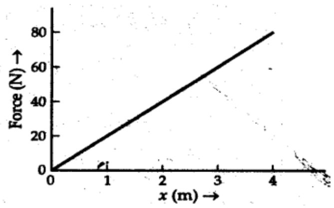
6 A body moves from a point A to B under the action of a force shown in fig. Force F is in Newton and distance x in metre. What is the amount of work done?



7 Fig. shows the F–x graph. Here the force F is in newton and distance x in metre. What is the work done?



8 Calculate work done in moving the object from $x = 2$ m to $x = 3$m from the following graph :



9 The relation between the displacement x and the time t for a body of mass 2 kg moving under the action of a force is given by $x = t^3/3$, where x is in metre and t in second, calculate the work done by the body in first 2 seconds.

Answers

1.	22.5 J	2.	1750 J, – 1000 J	3.	135 J
4.	31 J	5.	$\left(a+\frac{bd}{2}\right)d$	6.	11.5 J
7.	10 J	8.	50J	9	16 J

Energy

Energy of a body is defined as its capacity or ability to do work. The energy of a body is measured by the amount of work the body can perform, therefore

(i) Like work, energy is a scalar quantity.

(ii) The dimensional formula of energy is $\left[ML^2T^{-2}\right]$ i.e., same as that of work.

(iii) Energy is measured in the same units as the work. The *SI* unit of energy is joule and the *CGS* unit is erg.

Energy has several forms : Mechanical energy, sound energy, heat energy, light energy, chemical energy, atomic energy, nuclear energy, electric energy, magnetic energy, solar energy, etc.

Mechanical Energy : The energy produced by mechanical means is called mechanical energy. It has two forms:

(i) Kinetic Energy (ii) Potential Energy

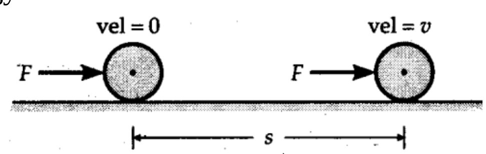
Kinetic Energy

The energy possessed by virtue of its motion is called its kinetic energy. A moving object can do work. The amount of work that a moving object can do before coming to rest is equal to its kinetic energy.

Examples : (i) A moving hammer drives a nail into the wood. Being in motion, it has kinetic energy or ability to do work.

(ii) A fast moving stone can break a window pane. The stone has kinetic energy due to its motion and so it can do work.

The kinetic energy of a body can be determined by calculating the amount of work required to bring the body into motion from its state of rest, as shown in fig.



Consider a body of mass *m* initially at rest. A force $\vec{F}$ applied on the body produces a displacement $\vec{ds}$ in its own direction $(\theta = 0^0)$.

The small work done is

$$dW = \vec{F} \,.\, \vec{ds} = Fds\cos 0^0 = F\,ds$$

According to Newton's second law of motion, $F = ma = m\frac{dv}{dt}$

$$\therefore \quad dW = Fds = m\frac{dv}{dt}\,.\,ds = mv\,dv \qquad \left[\because \frac{ds}{dt} = v\right]$$

The total work done to increase its velocity from 0 to v is given by

$$W = \int dW = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy (K) of the body.

$\therefore$ $$K = \frac{1}{2}mv^2$$

Hence, the kinetic energy of a body is equal to one-half the product of mass of body and square of its velocity.

Relation between K.E. and linear momentum

As linear momentum, $p = mv$

Therefore $K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{1}{2m}(mv)^2$ or $K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$

Work-energy theorem

It states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.

Proof of W-E theorem for a constant force : Suppose a constant force F acting on a body of mass m produces acceleration a in it. After covering distance s, suppose the velocity of the body changes from u to v. We use the equation of motion, $v^2 - u^2 = 2as$

Multiplying both sides by $\frac{1}{2}m$, we get $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$

By Newton's second law, $ma = F$, the applied force.

Therefore, $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W$ or $K_f - K_i = W$

Change in K.E. of the body = Work done on the body by the net force. This proves the work energy theorem for a constant force.

Proof of W–E theorem for a variable force : Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\vec{ds}$ in its own direction ($\theta = 0^\circ$).

The small work done is $dW = \vec{F}\cdot\vec{ds} = F\,ds\cos 0^\circ = F\,ds$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$\therefore$ $$dW = m\frac{dv}{dt}.ds = mv\,dv \qquad \left[\because \frac{ds}{dt} = v\right]$$

If the applied force increases the velocity from u to v, then the total work done on the body will be

$$W = \int dW = \int_u^v mv\,dv = m\int_u^v v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

or $$W = k_f - k_i$$

= Change in K.E. of the body

This proves the work energy theorem for a variable force.

Note :

- The work-energy theorem is not independent of Newton's second law. It may be viewed as scalar form of second law.
- The W.E. theorem holds in all inertial frames. It can be extended to non–inertial frames provided we include the pseudo force in the calculation of the net force acting on the body under consideration.
- When force and displacement are in same direction, the kinetic energy of the body increases. The increase in K.E. is equal to the work done on the body.
- When force and displacement are oppositely directed, the kinetic energy of the body decreases. The decrease in K.E. is equal to the work done by the body against the retarding force.
- When a body moves along a circular path with uniform speed, there is no change in its kinetic energy. By W-E theorem, the work done by the centripetal force is zero.
- When K.E. increases, the work done is positive and when K.E. decreases, the work done is negative.
- In deriving the W–E theorem, it has been assumed that the work done by the force is effective only in changing the K.E. of the body. However, the work done on a body may also be stored as the P.E. of the body.

Subjective Assignment – III

1. A body of mass 4kg initially at rest is subject to a force 16 N. What is the kinetic energy acquired by the body at the end of 10s?
2. A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg which it burns out in 3s. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket? What is the energy content per unit mass of the fuel? (Ignore the small mass variation of the rocket during fuel burning).
3. A bullet weighing 10 g is fired with a velocity of 800 ms^{-1}. After passing through a mud wall and 1m thick its velocity decreases to 100ms^{-1}. Find the average resistance offered by the mud wall.
4. A shot travelling at the rate of 100 ms^{-1} is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 cm thick?
5. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?
6. It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force?
7. A block of mass m = 1 kg, moving on a horizontal surface with speed $v_i = 2\text{ms}^{-1}$ enters a rough patch ranging from $x = 0.10m$ to $m = 2.01m$. The retarding force F_r, on the block in this range in inversely proportional to x over this range. $F_r = \frac{-k}{x}$ $\quad 0.1 < x < 2.01$ m

   $= 0$ for x < 0.1 m and x > 2.01 m

   where k = 0.5 J. What is the final kinetic energy and speed v_f of the block as it crosses this patch
8. Two identical 5kg blocks are moving with same speed of 2ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (ii) internal forces
9. If the linear momentum of a body increases by 20% what will be the % increase in the kinetic energy of the body.
10. If the kinetic energy of a body increases by 300% by what % will the linear momentum of the body increase?
11. A body of mass 0.3 kg is taken up an inclined plane to length 10m and height 5m and then allowed to slide down to the bottom again. The coefficient of friction between the body & plane is 0.15. What is

(i) work done by the gravitational force over the round trip

(ii) work done by the applied force over the upward journey,

(iii) work done by frictional force over the round trip,

(iv) kinetic energy of the body at the end of the trip?

How is the answer to (iv) related to the first three answers?

12. The momentum of a body of mass 5 kg is 500 kg ms^{-1}. Find its K.E.

13. A bullet of mass 20g is found to pass two points 30m apart in a time interval of 4*s*. Calculate the kinetic energy of the bullet if it moves with constant speed.

14. A body of mass 2 kg is resting on a rough horizontal surface. A force of 20N is now applied to it for 10*s*, friction between the surfaces in contact is 0.2, calculate :

(a) Work done by the applied force in 10s.

(b) Change in kinetic energy of the object in 10s. Take g = 10 ms^{-2}

15. An electron and a proton are detected in a cosmic ray experiment, the electron with K.E. of 5 keV and the proton with K.E. of 50 keV. Find the ratio of their speeds. Given $m_e = 9.11 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg.

16. While catching a cricket ball of mass 200 g moving with a velocity of 20 ms^{-1}, the player draws his hands backwards through 20 cm. Find the work done in catching the ball and the average force exerted by the ball on the hand.

17. A body of mass 1 kg is allowed to fall freely under gravity. Find the momentum and kinetic energy of the body 5 second after it start falling. Take $g = 10$ ms^{-2}.

18. Two bodies of masses 1g and 16g are moving with equal kinetic energies. Find the ratio of the magnitudes of their linear momenta.

19. If the momentum of a body is increased by 50%, then what will be the percentage increase in the kinetic energy of the body?

20. The kinetic energy of a body decreases by 19%. What is the percentage decrease in its linear momentum?

Answers

1.	3200 J	2.	1000 JK g^{-1}, 2/3 N	3.	3150N
4.	150 ms^{-1}	5.	63.2 ms^{-1}	6.	– 8.75 J
7.	0.5 J, 1 ms^{-1}	8.	(i) –20 J, (ii) zero	9.	44%
10.	100%	11.	(i) 0 (ii) 18.5 J		(iii) – 7.6 J (iv) 10.9 J
12.	2.5×10^4 J	13.	0.5625 J	14.	8000 J, 6400 J
15.	$\left(\frac{v_e}{v_p} = 4.28\right)$	16.	40 J, 200 N	17.	50 kg ms^{-1}, 1250 J
18.	1 : 4	19.	125%	20.	10%

Potential Energy

Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or by its configuration. Potential energy is also called mutual energy or energy of configuration. It is measured by the amount of work that a body or system can do in passing from its present position or configuration to some standard position or configuration, called zero position or zero configuration.

Examples of potential energy due to position :

(i) A body lying on the roof a building has some potential energy. When allowed to fall down, it can do work.

(ii) The potential energy of water stored to great heights in dams is used to run turbines for generating hydroelectricity.

Examples of potential energy due to configuration :

(i) In a toy car, the wound spring has potential energy. As the spring is released, its potential energy changes into kinetic energy which moves the toy car.

(ii) A stretched bow possesses potential energy. As soon as it is released, it shoots the arrow in the forward direction with a large velocity. The potential energy of the stretched bow gets converted into the kinetic energy.

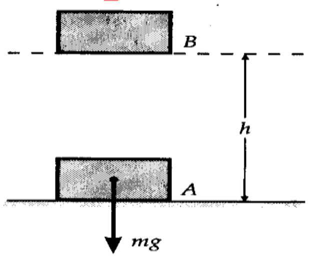
(iii) Due to the potential energy of the compressed spring in a loaded gun, the bullet is fired with a large velocity on firing the gun.

Three common types of potential energies are as follows :

(i) **Gravitational potential energy :** It is the potential energy associated with the state of separation of two bodies, which attract one another through the gravitational force.

(ii) **Elastic potential energy :** It is the potential energy associated with the state of compression or extension of an elastic (spring like) object.

(iii) **Electrostatic potential energy :** The energy due to the interaction between two electric charges is electrostatic potential energy.



Gravitational Potential energy : The gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

Expression for gravitational potential energy : Consider a body of mass m lying on the surface of the earth, as shown in fig. Let g be the acceleration due to gravity at this place. For heights much smaller than the radius of the earth $(h << R_E)$ the value g can be taken constant. Force needed to lift the body up with zero acceleration,

$$F = \text{Weight of the body} = mg$$

Work–done on the body in raising it through height h,

$$W = F \, . \, h = mg \, . \, h$$

This work done against gravity is stored as the gravitational potential energy (U) of the body.

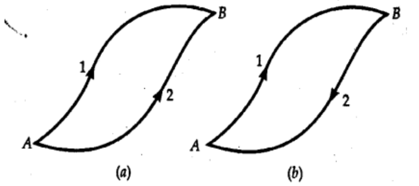
$$\therefore \quad U = mg\,h$$

At the surface of the earth, $h = 0$

$\therefore$ Gravitational P.E. at the earth's surface = zero.

Conservative and non–conservative forces

Conservative force : A force is conservative if the work done by the force in displacing a particle from one point to anther is independent of the path followed by the particle and depends only on the end points.



Suppose a particle moves from point A to point B along either path 1 or path 2, as shown in fig. If a conservative force F acts on the particle, then the work done on the particle is same along the two paths.

Mathematically, we can write

$$W_{AB} \text{ (along path 1)} = W_{AB} \text{ (along path 2)} \qquad \text{....(1)}$$

Now suppose the particle moves in a round trip, from point A to point B along path 1 and then back to point A along path 2, as shown in fig. (b). For a conservative force.

Work done on the particle along the path 2 from A to B.

= – work done on the particle along the path 2 from B to A.

$$W_{AB} \text{(along path 2)} = -\, W_{BA} \text{ (along path 2)}$$

From (1) and (2), we have W_{AB} (along path 1) = – W_{BA} (along path 2)

or $\quad W_{AB}$ (along path 1) + W_{BA} (along path 2) = 0 $\quad$ or $\quad W_{\text{closed path}} = 0$

Hence a force is conservative if the work done by the force in moving a particle around any closed path is zero.

Examples : Gravitational force, electrostatic force and elastic force of a spring are all conservative forces.

Non–conservative force : If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non–conservative force. The work done in moving an object against a non-conservative force along a closed path is not zero.

Example : Forces of friction and viscosity are non-conservative forces.

Conservative nature of gravitational force

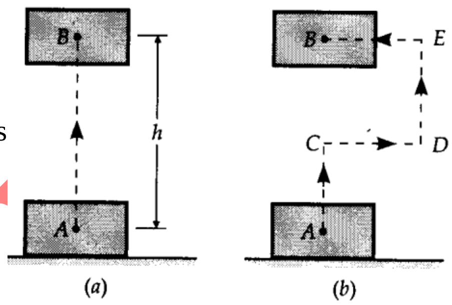
(i) As shown in fig. (a), suppose a body of mass *m* is raised to a height *h* vertically upwards from position *A* to *B*. The work done against gravity is

$$W = mg \times AB = mg\ h$$

As shown in fig. (b), now suppose the body is taken from position *A* to *B* along the path *ACDEB*. During the horizontal path *CD* and *EB*, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done only along vertical paths *AC* and *DE*. The total work done is

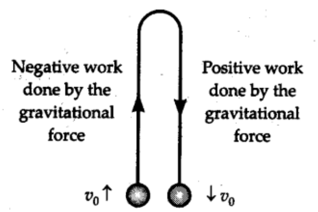
$$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$$
$$= mg \times AC + 0 + mg + DE + 0$$
$$= mg(AC + DE) = mg \times h$$

or $$W = mg\ h$$



Thus the work done in moving a body against gravity is independent of the path taken and depends only on the initial and final positions of the body. Hence gravitational force is a conservative force.

(ii) Suppose a ball is thrown vertically upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the ball descends, the gravitational force does positive work on it, increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and K.E. with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This again shows that the gravitational force is a conservative force.



Potential energy in relation to conservative force

The potential energy is the energy associated with the configuration of a system in which a conservative force acts. When the conservative force $F(x)$ (for simplicity, in one dimension) does work *W* on a particle within the system, the change in potential energy ΔU of the system is equal to the negative of the work done by conservative force, i.e., $\Delta U = -W$

But $$W = \int_{x_i}^{x_f} F(x)\,dx$$

$\therefore$ $$\Delta U = -\int_{x_i}^{x_f} F(x)\ dx$$

Differentiating the above equation, we get $\frac{dU(x)}{dx} = -F(x)$ or $F(x) = -\frac{dU(x)}{dx}$

Hence potential energy *U* may be defined as a function whose negative gradient gives the force. Conversely, we may define conservative force as a force which is equal to the negative gradient of the potential energy *U*.

Properties of the conservative force :

(i) A force *F* is conservative if it can be defined from the scalar potential energy function $U(x)$ by the relation,

$$F(x) = -\frac{d\ U(x)}{dx}$$

(ii) The work done by a conservative force on an object is path independent and depends only on the end points.

$$W = \int_{x_i}^{x_f} F(x)\ dx = K_f - K_i = U_i - U_f$$

(iii) The work done by the conservative force is zero if the object moving around any closed path returns to its initial position.

$$W_{\text{closed path}} = \oint F(x)\,dx = 0$$

(iv) If only the conservative forces are acting on body, then its total mechanical energy is conserved.

Principle of conservation of mechanical energy

This principle states that if only the conservative forces are doing work on a body, then its total mechanical energy (K.E. + P.E.) remains constant.

Proof : Suppose that a body undergoes displacement Δx under the action of a conservative force $F(x)$. Then from work-energy theorem, the change in K.E. is $\Delta K = F(x)\,\Delta x$

As the force is conservative the change in potential energy is given by

ΔU = Negative of the work done = $-F(x)\,\Delta x$

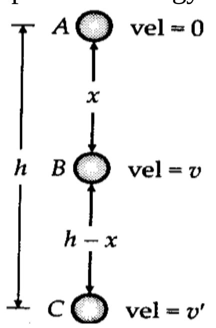
Combining the above two equations, we get $\Delta K = -\Delta U$

or $\Delta K + \Delta U = 0$ or $\Delta(K+U) = 0$ or $K + U$ = constant

or $K_i + U_i = K_f + U_f$

Although, individually the kinetic energy K and potential energy U may change from one state of the system to another, but their sum or the total mechanical energy of the system remains constant under the conservative force.

Conservation of mechanical energy in case of a freely falling body : Consider a body of mass m lying at position A at a height h above the ground. As the body falls, it kinetic energy increases at the expense of potential energy.



At point *A* : The body is at rest. K.E. of the body, $K_A = 0$

P.E. of the body , $U_A = mgh$

Total mechanical energy, $E_A = K_A + U_A = mg\,h$

At point *B* : Suppose the body falls freely through height x and reaches the point B with velocity v. Then

$v^2 - 0^2 = 2gx$ [Using $v^2 - u^2 = 2as$]

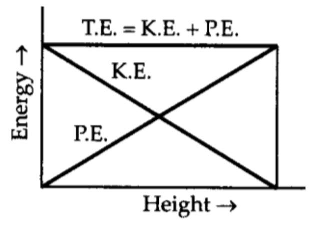
or $v^2 = 2gx$ so $K_B = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx = mgx$

$U_B = mg(h-x)$, $E_B = K_B + U_B = mgx + mg(h-x) = mg\,h$

At point *C* : Suppose the body finally reaches a point C on the ground with velocity v'. Then considering motion from A to C, $v'^2 - 0^2 = 2gh$ or $v'^2 = 2gh$

$\therefore$ $K_C = \frac{1}{2}mv'^2 = \frac{1}{2}m \times 2gh = mg\,h$, $U_C = mg \times 0 = 0$

$E_C = K_C + U_C = mg\,h$



Clearly, as the body falls, its P.E. decreases and K.E. increases by an equal amount. However, its total mechanical energy remains constant (= mgh) at all points. Thus total mechanical energy is conserved during free fall of a body.

Subjective Assignment – IV

1. A vehicle of mass 15 quintal climbs up a hill 200 m high. It then moves on a level road with speed of 30 ms^{-1}. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.

2. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Take g = 9.8 ms^{-2}.

3. A girl of mass 40 kg sits in a swing formed by a rope of 6m length. A person pulls the swing to a side so that the rope makes an angle of 60^0 with the vertical. What is the gain in potential energy of the girl?

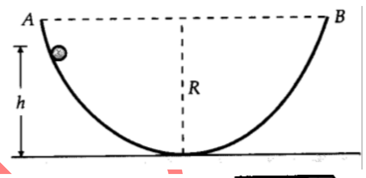
4. How high must a body be lifted to gain an amount of potential energy equal to the kinetic energy it has when moving at speed 20 ms^{-1}? The value of acceleration due to gravity at a place is $g = 9.8\ ms^{-2}$.

5. The string of a pendulum is 2.0 m long. The bob is pulled sideways so that the string becomes horizontal and then the bob is released. What is the speed with which the bob arrives at the lowest point? Assume that 10% of the initial energy is dissipated against air resistance, $g = 10\ ms^{-2}$.

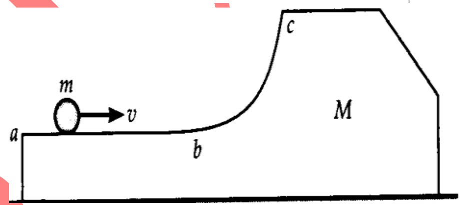
6. A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce? Where does this energy go?

7. A ball at rest is dropped from a height of 12m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy?

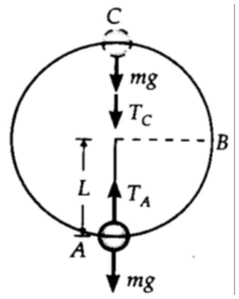
8. Fig. shows a frictionless hemispherical bowl of radius *R*. A ball of mass *m* is pushed down the wall from a point *A*. It just rises up to the edge of the bowl. calculate the speed with which the ball is pushed down along the wall.



9. A body of mass $M = 9.8$ kg with a small disc of mass $m = 0.2$ kg placed on its horizontal surface *ab*, rests on a smooth horizontal plane, as shown in fig. The disc can move freely along the smooth groove *abc* of mass *M*. To what height (relative to the initial position) will the disc rise after separating from the body of mass *M* when initial velocity $v = 5\ ms^{-1}$ is given to it in the horizontal direction?



10. A bob of mass *m* is suspended by a light string of length *L*. It is imparted a horizontal velocity v_0 at the lowest point *A* such that it completes a circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, *C*. This is shown in fig. Obtain an expression for
(i) v_0 ;
(ii) the speeds at points *B* and *C* ;
(iii) The ratio of the kinetic energies (K_B / K_C) at *B* and *C*. Comment on the nature of the trajectory of the bob after it reaches the point *C*.



11. A ball falls under gravity from a height of 10m with an initial downwards velocity *u*. It collides with the ground, loses 50% of its energy in collision and then rises back to the same height. Find the initial velocity *u*.

12. A simple pendulum of length 1m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of $2 \times 10^2\ ms^{-1}$. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back. Take $g = 10\ ms^{-2}$.

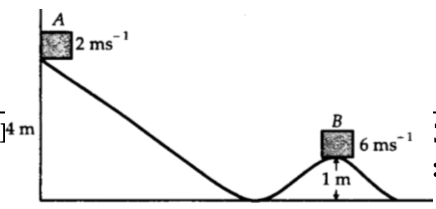
13. A ball is thrown vertically up with a velocity of 20 ms^{-1}. At what height, will its K.E. be half its original value?

14. 230 joules were spent in lifting a 10 kg weight to a height of 2m. Calculate the acceleration with which it was raised. Take $g = 10\ ms^{-2}$.

15. Calculate the work done in lifting a 300 N weight to a height of 10m with an acceleration 0.5 ms^{-2}. Take $g = 10\ ms^{-2}$.

16. A bullet of mass 10 g travels horizontally with speed of 100 ms^{-1} and is absorbed by a wooden block of mass 990 g suspended by a string. Find the vertical height through which the block rises. Take $g = 10\ ms^{-2}$.

17. A 3.0 kg block as shown in fig. has a speed of 2ms^{-1} at *A* and 6 ms^{-1} at *B*. If the distance from *A* to *B* along the curve is 12m, how large a



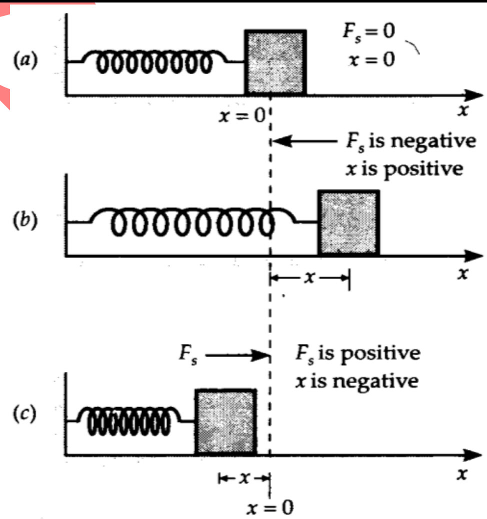
frictional force acts on it? Assuming the same friction, how far from *B* will it stop?

Answers

1.	2.94×10^6 ; 3.615×10^6 J	2.	1.4 ms^{-1}	3.	1176 J
4.	20.2 m	5.	6 ms^{-1}	6.	0.20
7.	9 m	8.	$v = \sqrt{2g(R-h)}$	9.	1.25 m
10.	$(i)\sqrt{5gL}$, $(ii)\sqrt{3gL}$, $\sqrt{gL}$, (iii) 3 : 1	11.	14 ms^{-1}	12.	0.2m
13.	10.20 m	14.	1.5 ms^{-2}	15.	3150 J
16.	5 cm	17.	3.35 N, 24.5 m		

Potential energy of a spring

Consider an elastic spring of negligibly small mass with its one end attached to a rigid support. Its other end is attached to a block of mass *m* which can slide over a smooth horizontal surface. The position $x = 0$ is the equilibrium position, as shown in fig. When the spring is stretched [fig.(b)] or compressed [Fig. (c)] by pulling or pushing the block, a spring force F_s begins to act in the spring towards the equilibrium position.



According to Hooke's law, the spring force F_s is proportional to the displacement of the block from the equilibrium position, i.e.,

$F_s \propto x$ or $F_s = -kx$

The proportionality constant *k* is called spring constant. Its *SI* unit is Nm^{-1}. The spring it stiff if *k* is large and soft if *k* is small. The negative sign shown F_s acts in the opposite direction of *x*. The work done by the spring force for the small extension *dx* is

$$dW_s = F_s\, dx = -kx\, dx$$

If the block is moved from an initial displacement x_i to the final displacement x_f, the work done by the spring force is

$$S_s = \int dW_s = -\int_{x_i}^{x_f} kx\, dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} \quad \text{or} \quad W_s = \frac{1}{2}Kx_i^2 - \frac{1}{2}kx_f^2$$

If the block is pulled from x_i and allowed to return to x_i, then

$$W_s = -\int_{x_i}^{x_f} kx\, dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_i^2 = 0$$

The above discussion shows that

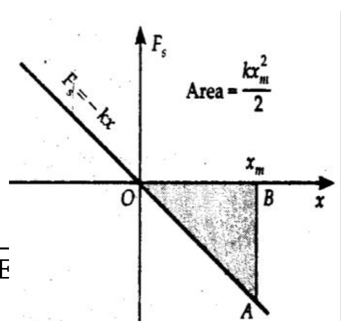
(i) The spring force is position dependent as is clear in Hooke's law : $F_s = -kx$.

(ii) The work done by the spring force depends on initial and final positions and

(iii) The work done by the spring force in a cyclic process is zero.

Thus the spring force is a conservative force.

In order to pull the block outwards with a slow constant speed (quasi–static motion), an external force *F* equal and opposite to F_s has to be applied. The work done by the



external force will be equal to the increase in P.E. of the spring and is given by

$$\Delta U = W = \frac{1}{2}k\, x_f^2 - \frac{1}{2}kx_i^2$$

If we take the potential energy $U(x)$ of the spring to be zero when the block is in equilibrium position, the P.E. of the spring for an extension x will be $U(x) - 0 = \frac{1}{2}kx^2 - 0$ or $U(x) = \frac{1}{2}kx^2$

Conservation of energy in an elastic spring : If we stretch a spring to a distance x_m its P.E. is $\frac{1}{2}kx_m^2$. When it is released, it begins to move under the spring force till it reaches the equilibrium position $x = 0$, where it has maximum velocity. All the P.E. is converted into K.E. Due to inertia of motion, the body overshoots the $x = 0$ position. It velocity decreases unit it momentarily stops at position $x = -x_m$, where all the K.E. is converted into P.E. The spring force again pulls the body towards the position $x = 0$. Thus the body keeps on oscillating. The total mechanical energy remains constant.

At the extreme positions. Here $x = \pm\, x_m$ and velocity $v = 0$.

$$K = \frac{1}{2}mv^2 = 0 \;, \qquad U = \frac{1}{2}kx_m^2 = \text{a maximum value}$$

At any intermediate position x. For x between $-\,x_m$ to $+\,x_m$, the energy is partly kinetic and partly potential.

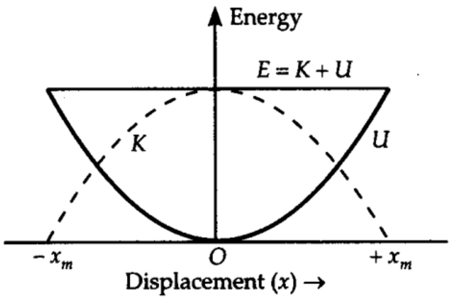
Total energy = K.E. + P.E.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\therefore \qquad K = \frac{1}{2}mv^2 = \frac{1}{2}k(x_m^2 - x^2)$$

Velocity ,

$$v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}$$



At the equilibrium position. Here $x = 0$.

$$\therefore \qquad U = \frac{1}{2}k(0)^2 = 0 \,, \qquad K = \frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

Maximum speed,

$$v_m = \sqrt{\frac{k}{m}}\, x_m$$

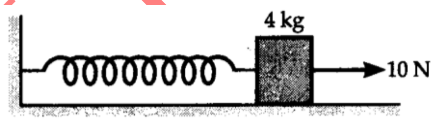
The variations of K.E. , P.E. and total energy with displacement x are shown in fig. As both K.E. and P.E. depend on x^2, their graphs are parabolic. Total mechanical energy $E = K + U$ remains constant, so its graph is a straight line parallel to displacement axis.

Notes :

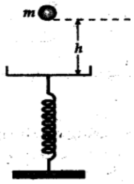
- The notion of potential energy applies to only those forces where the work done against the force gets stored up an energy by virtue of position or configuration of the body When external constraints are removed, this energy appears as kinetic energy.
- The potential energy of a body subjected to a conservative force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice.
- For the gravitational P.E. the zero of potential energy is chosen to be the ground.
- For the spring potential energy $\frac{1}{2}kx^2$, the zero of the potential energy is the equilibrium position of the oscillating mass.
- Every mechanical force is not associated with a potential energy. The work done by friction over a closed path is not zero because no potential energy can be associated with friction.

Subjective Assignment – V

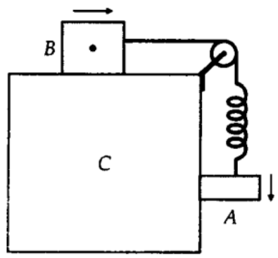
1. Two springs have force constant k_1 and $k_2 (k_1 > k_2)$. On which spring is more work done, if (i) they are stretched by the same force and (ii) they are stretched by the same amount.
2. The length of a steel wire increases by 0.5 cm when it is loaded with a weight of 5 kg. Calculate (i) force constant of the wire and (ii) Work done is stretching the wire
3. The potential energy of a spring when stretched through a distance *x* is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance *x*?
4. To simulate car accidents, auto manufactures study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant 6.25×10^3 Nm^{-1}. What is the maximum compression of the spring?
5. Consider example 4 taking the coefficient of friction, μ to be 0.5 and calculate the maximum compression of the spring.
6. The spring shown in fig. has a force constant of 24 Nm^{-1}. The mass of the block attached to the spring is 4kg. Initially the block is at rest and spring is unstretched. The horizontal surface is frictionless. If a constant horizontal force of 10N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5 m?



7. A ball of mass *m* is dropped from a height *h* on a platform fixed at the top of a vertical spring, as shown in fig. The platform is depressed by a distance *x*. What is the spring constant *k*?



8. A massless platform is kept on a light elastic spring. When a sand particle of mass 0.1 kg is dropped on the pan from a height of 0.24m, the particle strikes the pan, and the spring compresses by 0.01 m. From what height should the particle be dropped to cause a compression of 0.04 m?
9. Two blocks *A* and *B* are connected to each other as shown in fig. The string and spring is massless and pulley frictionless. Block *B* slides over the horizontal top surface of stationary block *C* and the block *A* slides along the vertical side of *C* both with same uniform speed. The coefficient of friction between the blocks is 0.2 and the spring constant of spring is 1960 Nm^{-1}. If mass of block *A* is 2kg, calculate (i) the mass of block *B* and (ii) Energy stored in spring.



10. A spring gun has a spring constant of 18 N cm^{-1}. The spring is compressed 12 cm by a ball of mass 15 g. How much is the potential energy of the spring? If the trigger is pulled, what will the velocity of the ball be?
11. A solid of mass 2 kg moving with a velocity of 10 ms^{-1} strikes an ideal weightless spring and produces a compression of 25cm in it. Calculate the force constant of the spring.
12. A 16 kg block moving on a frictionless horizontal surface with a velocity of 5 ms^{-1} compresses an ideal spring and comes to rest. If the force constant of the spring be 100 Nm^{-1}, then how much is the spring compressed?
13. A block of mass 2 kg is dropped from a height of 40 cm on a spring whose force–constant is 1960 Nm^{-1}. What will be the maximum distance *x* through which the spring is compressed?
14. An object is attached to a vertical spring and slowly lowered to its equilibrium position. This stretches the spring by a distance *d*. If the same object is attached to the same vertical spring but permitted to fall freely, through what distance does it stretch the spring?

Answers

1. (i) $W_2 > W_1$ (ii) $W_1 > W_2$ 2. (i) 1.0×10^4 Nm^{-1} (ii) 0.125 J
3. 30 J 4. 2.0 m 5. 1.354 m

6.	1 ms^{-1}	7.	$\dfrac{2mg(h+x)}{x^2}$	8.	3.96 m
9.	(i) 10 kg (ii) 0.098 J	10.	12.96 J , 41.6 ms^{-1}	11.	3200 Nm^{-1}
12.	2.0 m	13.	10 cm	14.	2d

Einstein's mass-energy equivalence

In 1905, Albert Einstein discovered that mass can be converted into energy and vice versa. He showed that mass and energy are equivalent and related by the relation.

$$E = mc^2$$

where c, the speed of light in vacuum is approximately 3×10^8 ms^{-1}. According t Einstein's mass–energy relation, it mass m disappears, an energy $E(=mc^2)$ appears in some form. Conversely, when energy E disappears, a mass $m(=E/c^2)$ appears.

Applications of mass-energy equivalence :

(i) Annihilation of matter : When an electron $({}_{-1}^{0}e)$ and a positron $\left({}_{1}^{0}e\right)$ come close to each other, they annihilate (destroy) each other forming two γ–rays (electromagnetic radiation) of total energy given by Einstein's mass-energy relation.

(ii) Pair production : When a γ–ray photon of energy 1.02 MeV passes closed to a massive nucleus, it materializes into a pair of particles-an electron and a positron. Thus energy gets converted into matter.

(iii) Energy generation in the sum and stars : The energy generated in the sun and stars is due to the conversion of mass into energy.

Principle of conservation of energy : Energy is being transformed from one form to another at every stage, yet its total amount remains the same. This is the principle of conservation of energy which can be stated in a number of ways :

(i) Energy can neither be created, nor destroyed. It may be transformed from one form to another.

(ii) The total energy of an isolated system remains constant.

(iii) As the entire universe may be regarded as an isolated system, the total energy of the universe is constant. If one part of the universe loses energy, another part must gain an equal amount of energy.

Notes :

- ♦ In the principle of conservation of energy, we include mass into total energy, because mass can be converted into energy.
- ♦ The principle of conservation of energy cannot be proved mathematically, but is an empirical principle. The deductions made on the basis of this principle are found to be true.

Subjective Assignment – VI

1. Express : (i) The energy required to break one bond (10^{-20} J) in *DNA* in eV.
   (ii) The kinetic energy of an air molecule (10^{-21} J) in eV.
   (iii) The daily intake of a human adult (10^{7} J) in kilocalories.
2. How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW?
3. If 1000 kg of water is heated from 0^oC to 100^oC, calculate the increase in the mass of water.
4. Calculate the energy in MeV equivalent to the rest mass of an electron. Given that the rest mass of an electron, $m_0 = 9.1 \times 10^{-31}$ kg, $1\text{MeV} = 1.6 \times 10^{-13}$J and speed of light $c = 3 \times 10^8$ ms^{-1}.
5. Estimate the amount of energy released in the following nuclear fusion reaction : ${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + {}_{0}^{1}n$ Given mass of ${}_{1}^{2}H = 2.0141$ amu, mass of ${}_{2}^{3}He = 3.0160$ amu, mass of ${}_{0}^{1}n = 1.0087$ amu and 1 amu $= 1.661 \times 10^{-27}$ kg. Express your answer in units of MeV.
6. When slow neutrons are incident on a target containing ${}_{92}^{235}U$, a possible fission reaction is ${}_{92}^{235}U + {}_{0}^{1}n \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3{}_{0}^{1}n$ Estimate the amount of energy released using the following data : Given , mass of ${}_{92}^{235}U = 235.04$ amu, mass of ${}_{0}^{1}n = 1.0087$ amu, mass of ${}_{56}^{141}Ba = 140.91$ amu, mass of ${}_{36}^{92}Kr = 91.926$ amu and energy equivalent to 1 amu = 931 MeV.

7. About 4×10^9 kg of matter is converted into energy in the sun each second. What is the power output of the sun.

8. Show that energy equivalent to atomic mass unit equals nearly 933 MeV of energy. Given 1 atomic mass unit = 1.66×10^{-27} kg.

9. 500 kg of water is heated from 20^0 to 100^oC. Calculate the increase in the mass of water. Given specific heat of water = 4.2×10^3 J kg^{-1} $^0C^{-1}$.

10. 1 mg of uranium is completely destroyed in an atomic bomb. How much energy is liberated.

11. An electron–positron pair annihilates at rest to produce γ–rays. Calculate the energy produced in MeV if the rest mass of electron is 9.1×10^{-31} kg.

Answers

1.	(i) 0.06 eV	(ii) 0.062 eV	(iii) 2400 kcal		
2.	9.6 g	3.	0.466×10^{-8} kg	4.	0.512 MeV
5.	3.27 MeV	6.	173.725 MeV	7.	3.6×10^{27} W
9.	1.87×10^{-9} kg	10.	9×10^{10} J	11.	1.02 MeV

Power

Power is defined as the rate of doing work. If an agent does work W in time t, then its average power is given by

$$P_{av} = \frac{W}{t}$$

The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.

Power is a scalar quantity, because it is the ratio of two scalar quantities work (W) and time (t).

Dimension of power : $$[P] = \frac{[W]}{[t]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2\,T^{-3}]$$

Units of power : The *SI* unit of power is watt (W). The power of an agent is one watt if it does work at the rate of 1 joule per second.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} \quad \text{or} \quad 1\text{ W} = 1\text{ Js}^{-1}$$

The bigger units of power are kilowatt (kW) and horse power (hp)

1 kilowatt = 1000 watt or $1 \text{ kW} = 10^3$ W

1 horse power = 746 watt or 1 hp = 746 W

Instantaneous power : The instantaneous power is defined as the limiting value of the average power as the time interval approaches zero. If ΔW work is done in a small time interval Δt, then the instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power as dot product : The work done by a force $\vec{F}$ for a small displacement $\vec{dr}$ is given by $dW = \vec{F} \,.\, \vec{dr}$

So, the instantaneous power can be expressed as $P = \frac{dW}{dt} = \vec{F} \,.\, \frac{\vec{dr}}{dt}$

But $\frac{\vec{dr}}{dt} = \vec{v}$, the instantaneous velocity

$\therefore$ $$P = \vec{F} \,.\, \vec{v}$$

Thus the power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

Kilowatt hour (kWh) : Kilowatt hour (kWh) or Board of Trade (B.O.T) unit is the commercial unit of electrical energy. One kilowatt hour is the electrical energy consumed by an appliance of 1000 watt in 1 hour.

Relation between kWh and joule

1KwH = 1 Kw × 1H = 1000 W × 1 h

$= 100\ Js^{-1} \times 3600\ s$

$1kWh = 3.6 \times 10^6\ J$

Subjective Assignment – VII

1. A man weighing 60 kg climbs up a staircase carrying a load of 20 kg on his head. The stair case has 20 steps each of height 0.2m. If he takes 10s to climb, find his power.
2. A car of mass 2000 kg is lifted up a distance of 30m by a crane in 1min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel? What is the power supplied by each crane? Neglect power dissipation against friction.
3. The human heart discharges 75 ml of blood at each beat against a pressure of 0.1m of *Hg*. Calculate the power of heart assuming that pulse frequency is 80 beats per minute. Density of $Hg = 13.6 \times 10^3\ kgm^{-3}$
4. An electric motor is used to lift an elevator and its load (total mass = 1500 kg) to a height of 20m. The time taken for the job is 20s. What is the work done? What is the rate at which work is done? If the efficiency of the motor is 75%, at which rate is the energy supplied to the motor.
5. Calculate the horse power of a man who can chew ice at the rate of 30g per minute. Given 1*hp* = 746 W and J = 4.2 J cal^{-1}.
6. A machine gun fires 60 bullets per minute with a velocity of 700 ms^{-1}. If each bullet has a mass of 50 g, find the power developed by the gun.
7. An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watt as well as in horse power.
8. A well 20m deep and 3m in diameter contains water to a depth of 14 metre. How long will a 5 hp engine take to empty it.
9. The turbine pits at the Niagra falls are 50m deep. The average horse power developed is 500. If the efficiency of the generator is 85%, how much water passes through the turbines per minute? Take $g = 10\ ms^{-2}$.
10. A man cycles up a hill, whose slope is 1 in 20 with a velocity of 6.4 kmh^{-1} along the hill. The weight of the man and the cycle is 98kg. What work per minute is he doing? What is his horse power.
11. A lift is designed to carry a load of 4000 kg through 10 floors of a building averaging 6m per floor in 10 seconds. Calculate the horse power of the lift.
12. A machine can take out 1000 kg of mud per hour from a depth of 100 m. If efficiency of the machine is 0.9, calculate its power.
13. One coolie takes 1 min to raise a box through a height 2m. Another takes 30s for the same job and does the same amount of work. Which one of these two has a greater power and which one uses greater energy.
14. An engine of 4.9 kW power is used to pump water from a well 50m deep. Calculate the quantity of water in kilolitres which it can pump out in one hour.
15. Water is pumped out of a well 10m deep by means of a pump rated at 10 kW. Find the efficiency of the motor if 4200 kg of water is pumped out every minute. Take $g = 10\ ms^{-2}$.
16. A 30 m deep well is having water upto 15m. An engine evacuates it in one hour. Calculate the power of the engine if the diameter of the well is 4m.
17. The human heart force 4000 cm^3 of blood per minute through the arteries under pressure of 130mm. The density of blood is 1.03 g cm^{-3}. What is the horse power of the heart?
18. A car of mass 1000 kg accelerates uniformly from rest to a velocity of 534 kh h^{-1} in 5 seconds. Calculate (i) its acceleration (ii) its gain in K.E. (iii) average power of the engine during this period, neglect friction.

Answers

1.	313.6 W	2.	same amount, 980W, 4900W	3.	1.33 W
4.	2.94×10^5 J, 1.47×10^4 W, 1.96×10^4 W			5.	0.225 hp
6.	12250 W	7.	59 hp, 44000 W	8.	3381.6 s
9.	5.39×10^5 kg	10.	0.144 hp , 5122.1 J	11.	315.28 hp
12.	302.47 W				

13. Second coolie has double power than first, both spend same amount of energy

14.	36.0 kilo litre	16.	70%		16.	11.55 kW
17.	1.17×10^{-4} hp	18.	(i) $3 ms^{-2}$	(ii) 1.125×10^{5} J	(iii) 22500 W	

Collisions

A collision is said to occur between two bodies, either if they physically collide against each other or if the path of one is affected by the force exerted by the other. For a collision to take place, the actual physical contact is not necessary.

The collisions between particles are of following types :

1. Elastic collision : If there is no loss of kinetic energy during a collision, it is called an elastic collision.

Characteristics of elastic collisions :

(i) The momentum is conserved.
(ii) Total energy is conserved
(iii) The kinetic energy is conserved.
(iv) forces involved during the collision are conservative.
(v) The mechanical energy is not converted into heat, light, sound, etc.

Examples : Collision between subatomic particles, collision between glass balls, etc.

2. Inelastic collision : If there is a loss of kinetic energy during a collision, it is called an inelastic collision.

Characteristics of inelastic collisions :

(i) The momentum is conserved.
(ii) Total energy is conserved.
(iii) The kinetic energy is not conserved.
(iv) Some or all of the forces involved are non–conservative.
(v) A part of the mechanical energy is converted into heat, light, sound , etc.

Examples : Collision between two vehicles, collision between a ball and floor .

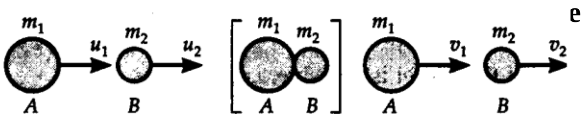
3. **Perfectly inelastic collision :** If two bodies stick together after the collision and move as a single body with a common velocity, then the collision is said to be perfectly inelastic collision. In such collisions, momentum is conserved, but the loss of kinetic energy is maximum.
   Examples : Mud thrown on a wall and sticking to it, a man jumping into a moving trolley, a bullet fired into a wooden block and remaining embedded in it, etc.
4. **Superelastic or explosive collision :** In such a collision, there is an increase in kinetic energy. This occurs if there is a release of potential energy on an impact.
   Examples : Bursting of a cracker when it hits the floor forcefully, the collision of a trolley with another may release a compressed spring and thereby releasing the energy stored in the spring.
5. **Head on or one–dimensional collision :** It is the collision in which the colliding bodies move along the same straight line path before and after the collision.
   Example : Collision between two railway compartments.
6. **Oblique or two–dimensional collision :** If two bodies do not move along the same straight line path but lie in the same plane before and after the collision, the collision is said to be oblique or two–dimensional collision.
   Example : Collision between two carom coins.

Notes :

- Total linear momentum is conserved at each instant of every collision.
- Total energy is conserved in all collisions.
- The total kinetic energy may or may not be conserved during a collision.
- Even for an elastic collision, the kinetic energy conservation holds after the collision is over and does not hold at every instant of the collision.
- When two bodies collide; they get deformed and may be momentarily at rest with respect to each other.
- The impact and deformation during a collision may convert part of the initial kinetic energy into heat and sound.

Elastic collision in one dimension

As shown in fig. consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 > u_2$. After some time, the two bodies collide head–on and



continue moving in the same direction with velocities v_1 and v_2 respectively. The two bodies will separates after the collision if $v_2 > v_1$.

As linear momentum is conserved in any collision, so ,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad ...(1)$$

or $$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

or $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad ...(2)$$

Since K.E. is also conserved in an elastic collision so

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

or $$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \qquad ...(3)$$

Dividing (3) by (2), we get $$u_1 + v_1 = v_2 + u_2$$

or $$u_1 - u_2 = v_2 - v_1 \qquad ...(4)$$

or Relative velocity of A w.r.t. B before collision = Relative velocity of B w.r.t. A after collision.

or Relative velocity of approach = Relative velocity of separation

Thus, in an elastic one–dimensional collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.

Velocities of the bodies after the collision : From equation (4), we get $v_2 = u_1 - u_2 + v_1$

Putting this value of v_2 in equation (1), we get $\mathrm{m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)}$

$$= m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$

or $$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

or $$\mathrm{v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2} \qquad ...(5)$$

Interchanging the subscripts 1 and 2 in the in the above equation, we get

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \qquad ...(6)$$

Equations (5) and (6) give the final velocities of the colliding bodies in terms of their initial velocities.

Special cases : (i) When two bodies of equal masses collide . Let $m_1 = m_2 = m$ (say)

From equation (5), $v_1 = \frac{2mu_2}{2m} = u_2$ = velocity of body of mass m_2 before collision

From equation (6), $v_2 = \frac{2mu_1}{2m} = u_1$ = velocity of body of mass m_1 before collision.

Hence when two bodies of equal masses suffer one dimensional elastic collision, their velocities get exchanged after the collision.

(ii) When a body collides against a stationary body of equal mass : Here $\mathrm{m_1 = m_2 = m}$ (say) and $u_2 = 0$

From equation (5), $v_1 = 0$

From equation (6), $v_2 = u_1$

hence when as elastic body collides against another elastic body of equal mass, initially at rest, after the collision the first body comes to rest while second body moves with the initial velocity of the first.

(iii) When a light body collides against a massive stationary body : Here $\mathrm{m_1 << m_2}$ and $\mathrm{u_2 = 0}$. Neglecting m_1 in equation (5), we get $v_1 = -\frac{m_2u_1}{m_2} = -u_1$.

From (6), $v_2 = 0$

Hence when a light body collides against a massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically remains at rest. A light ball on striking a wall rebounds almost with the same speed and the wall remains at rest.

(iv) When a massive body collides against a light stationary body : Here $m_1 >> m_2$ and $u_2 = 0$.

Neglecting m_2 in equation (5), we get $v_1 = \frac{m_1 u_1}{m_1} = u_1$ and $v_2 = \frac{2m_1 u_1}{m_1} = 2u_1$

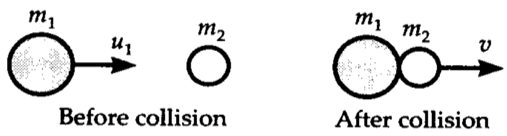
Hence when a massive body collides against a light body at rest, the velocity of the massive body remains almost unchanged while the light body starts moving with twice the velocity of the massive body.

Perfectly inelastic collision in one dimension

When the two colliding bodies stick together and move as a single body with a common velocity after the collision, the collision is perfectly inelastic.

As shown in fig. a body of mass m_1 moving with velocity u_1 collides head–on with another body of mass m_2 at rest. After the collision the two bodies move together with a common velocity v.

As the linear momentum is conserved, so



$$m_1 u_1 + m_2 \times 0 = (m_1 + m_2)v$$

or $$v = \frac{m_1}{m_1 + m_2} u_2$$

The loss in kinetic energy on collision is

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} u_1\right]^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} u_1^2 = \frac{1}{2} m_1 u_1^2 \left[1 - \frac{m_1}{m_1 + m_2}\right]$$

or $$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} u_1^2$$

This is a positive quantity. The kinetic energy is lost mainly in the form of heat and sound.

Moreover , $$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2} m_1 u_1^2} = \frac{m_1 + m_2}{m_1} \frac{v^2}{u_1^2} = \frac{m_1 + m_2}{m_1}\left(\frac{m_1}{m_1 + m_2}\right)^2$$

or $$\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2}$$

Which is <1. This again shows that the kinetic energy after the collision is less than the kinetic energy before the collision.

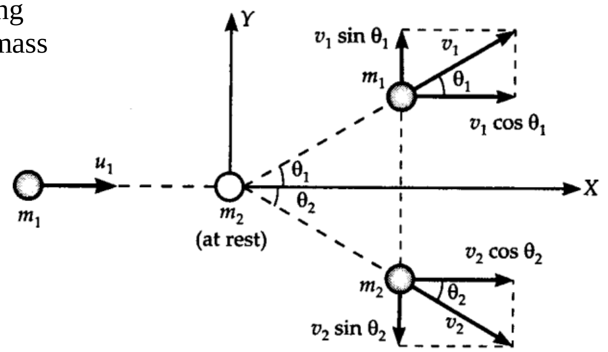
If the target is massive, i.e., $m_2 >> m_1$, then $\frac{K_f}{K_i} = 0$ i.e., $K_f = 0$.

Hence, when a light moving body collides against any massive body at rest and sticks to it, practically all of its kinetic energy is lost.

Elastic collision in two dimensions

As shown in fig. . Suppose a particle of mass m_1 moving along X–axis with velocity u_1 collides with another particle of mass m_2 at rest. After the collision, let the two particles move with velocities v_1 and v_2, making angles θ_1 and θ_2 with X–axis.

After the collision, the rectangular components of the momentum of m_1 are



(i) $m_1 v_1 \cos\theta_1$, along +ve *X*–axis

(ii) $m_1 v_1 \sin\theta_1$, long +ve *Y*–axis

After the collision, the rectangular components of the momentum of m_2 are

(i) $m_2 v_2 \cos\theta_2$, along +ve *X*–axis **(ii)** $m_2 v_2 \sin\theta_2$, along –ve *Y*–axis

Applying the principle of conservation of momentum along *X*–axis.

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \qquad \ldots(1)$$

The initial momentum of m_1 or m_2 along *Y*–axis is zero. Applying the principle of conservation of momentum along *Y*–axis. ,

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \qquad \ldots(2)$$

As the K.E. is conserved in as elastic collision, so $\frac{1}{2}m_1 u_1^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$ $\qquad \ldots(3)$

The four unknown quantities v_1, v_2, θ_1 and θ_2 cannot be calculated using the three equations (1), (2) and (3). By measuring one of the four unknowns, say θ_1, experimentally ; the values of other three unknowns can be solved.

Special Cases :

(i) Glancing collision : For such collisions $\theta_1 = 0^0$ and $\theta_2 = 90^0$. From equations (1) and (2), we get

$$u_1 = v_1 \quad \text{and} \quad v_2 = 0$$

K.E. of the target particle = $\frac{1}{2}m_2 v_2^2 = 0$

Hence in a glancing collision, the incident particle does not lose any kinetic energy and is scattered almost undeflected.

(ii) Head–on collision : In such a collision, the target particle moves in the direction of the incident particle, i.e., $\theta_2 = 0^0$. Then equation (1) and (2) take forms :

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \quad \text{and} \quad 0 = m_1 v_1 \sin\theta_1$$

Equation (3) for the kinetic energy remains unchanged.

(iii) Elastic collision of two identical particles : As the two particles are identical, so $m_1 = m_2 = m$ (say). By conservation of K.E. for elastic collision.

$$\frac{1}{2}\mathrm{mu_1}^2 = \frac{1}{2}\mathrm{mv_1}^2 + \frac{1}{2}\mathrm{mv_2}^2 \quad \text{or} \quad \mathrm{u_1^2 = v_1^2 + v_2^2}$$

By conservation of linear momentum , $m\vec{u}_1 = m\vec{v}_1 + m\vec{v}_2$ or $\vec{u}_1 = \vec{v}_1 + \vec{v}_2$

$\therefore$ $\vec{u}_1 \cdot \vec{u}_1 = (\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 + \vec{v}_2) = \vec{v}_1 \cdot \vec{v}_1 + \vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_1 + \vec{v}_2 \cdot \vec{v}_2$

or $u_1^2 = v_1^2 + v_2^2 + 2\vec{v}_1 \cdot \vec{v}_2$

or $u_1^2 = u_1^2 + 2\vec{v}_1 \cdot \vec{v}_2$ $\qquad [\because v_1^2 + v_2^2 = u_1^2]$

or $\vec{v}_1 \cdot \vec{v}_2 = 0$

This shows that the angle between $\vec{v}_1$ and $\vec{v}_2$ is 90^0. Hence two identical particles move at right angles to each other after elastic collision in two dimensions.

Coefficient of Restitution : The coefficient of restitution gives a measure of the degree of restitution of a collision and is defined as the ratio of the magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision. It is given by

$$e = \frac{|v_1 - v_2|}{|u_1 - u_2|} = -\frac{v_1 - v_2}{u_1 - u_2}$$

The value of *e* depends on the materials of the colliding bodies. For two glass balls, *e* = 0.95 and for the lead balls, *e* = 0.
20

The coefficient of restitution can be used to distinguish between the different types of collisions as follows

(i) For a perfectly elastic collision, *e* = 1, i.e., relative velocity of separation is equal to the relative velocity of approach.

(ii) For an inelastic collision, $0 < e < 1$, i.e., relative velocity of separation is less than the relative velocity of approach.

(iii) For a perfectly inelastic collision, $e = 0$, the relative velocity of separation is zero. The two bodies move together with a common velocity.

(iv) For a superelastic collision. $e > 1$ i.e., the kinetic energy increases.

Table : Different type of collisions

Collision	Kinetic energy	Coefficient of restitution	Main domain
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Not conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Max. loss of K.E.	$e = 0$	During shooting
Super elastic	K.E. increases	$e > 1$	In explosions

Notes :

- At each instant of the collision, the total energy and total linear momentum are both conserved in elastic as well as inelastic collisions.
- In an elastic collision, the kinetic energy conservation holds only after the collision is over. It does not hold during the short duration of actual collision.
- At the time of collision, the two colliding objects are deformed and may be momentarily at rest with respect to each other.
- When two equal massess suffer a glancing collision with one of them at rest, after the collision, the two masses move at right angles to each other.

Subjective Assignment – VIII

1. Two bodies of masses 5kg and 3kg moving in the same direction along the same straight line with velocities 5 ms^{-1} and 3 ms^{-1} respectively suffer one-dimensional elastic collision. Find their velocities after the collision.

2. *A* 10 kg ball and 20 kg ball approach each other with velocities 20 ms^{-1} and 10 ms^{-1} respectively. What are their velocities after collision if the collision is perfectly elastic?
3. Two ball bearings of mass *m* each moving in opposite directions with equal speeds *v* collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.
4. A railway carriage of mass 9000 kg moving with a speed of 36 kmh^{-1} collides with a stationary carriage of the same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? What type of collision is this?
5. What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (i) 9 times in mass (ii) equal in mass and (iii) 1/19th of its mass?

6. In a nuclear reactor a neutron of high speed (typically 10^7 ms^{-1}) must be slowed to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope $^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus unlike deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite is called a moderator. **or**
   A body of mass *M* at rest is struck by a moving body of mass *m*. Prove that fraction of the initial K.E. of the mass *m* transferred to the struck body is $4m\,M/(m + M)^2$ in an elastic collision.
7. A ball is dropped to the ground from a height of 2m. The coefficient of restitution is 0.6. To what height will the ball rebound?
8. A ball is dropped vertically from a height of 3.6 m. It rebounds from a horizontal surface to a height of 1.6 m. Find the coefficient of restitution of the material of the ball.

9. A ball is dropped from a height *h*. It rebounds from the ground a number of times. Given that the coefficient of restitution is *e*, to what height does it go after the n^{th} rebounding?

10. A sphere of mass *m* moving with a velocity *u* hits another stationary sphere of same mass. If *e* is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision?

11. A ball moving with a speed of 9 ms^{-1} strikes an identical ball such that after the collision the direction of each ball makes an angle 30^0 with the original line of motion. Find the speed of the two balls after the collision. Is the kinetic energy conserved in the collision process?

12. A ball is dropped from a height of 3m. What is the height upto which the ball will rebound? The coefficient of restitution is 0.5

13. A ball is dropped from a height *h* on to a floor. If the coefficient of restitution is *e*, calculate the height to which the ball first rebounds?

14. A nucleus of radium $\left({}^{226}_{88}Ra\right)$ decays to ${}^{222}_{86}R_n$ by the emission of α–particle $\left({}^{4}_{2}He\right)$ of energy 4.8 MeV. If mass of ${}^{222}_{86}Rn = 222.0$ amu and mass of ${}^{4}_{2}He = 4.003$ amu, then calculate the recoil energy of the daughter nucleus ${}^{222}_{86}Rn$.

15. The nucleus Fe^{57} emits a γ–ray of energy 14.4 keV. If the mass of the nucleus is 56.935 amu, calculate the recoil energy of the nucleus . Take 1 amu = 1.66×10^{-27} kg.

16. A vehicle of mass 30 quintals moving with a speed of 18 km h^{-1} collides with another vehicle of mass 90 quintals moving with a speed of 14.4 km h^{-1} in the opposite direction. What will be the velocity of each after the collision?

17. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1 kg ball rebounds at one–third of its original speed, what is the mass of the other ball?

18. Two particles of masses 0.5 kg and 0.25 kg moving with velocities 4.0 ms^{-1} and – 3.0 ms^{-1} collide head on in a perfectly inelastic collision. Find (i) the velocity of the composite particle after the collision and (ii) the kinetic energy lost in the collision.

19. Two particles of masses 0.5 kg and 0.25 kg moving with velocities 4.0 ms^{-1} and – 3.0 ms^{-1} collide head on in a perfectly inelastic collision. Find (i) the velocity of the composite particle after the collision and (ii) the kinetic energy lost in the collision.

20. What percentage of the K.E. of a moving particle is transferred to a stationary particle when it strikes the stationary particle of four times its mass?

21. A neutron moving with a speed of 10^6 ms^{-1}- suffers a head–on collision with a nucleus of mass number 80. What is the fraction of energy retained by the nucleus?

Answers

1.	3.5 ms^{-1}, 5.5 ms^{-1}	2.	– 20ms^{-1}, 10 ms^{-1}				
3.	rebound with equal speeds	4.	5 ms^{-1}, in elastic				
5.	(i) 36% (ii) 100% (iii) 19%	7.	0.72 m	8.	0.667		
9.	he^{2n}	10.	$v_2/v_1 = 1+e/1-e$	11.	27 m		
12.	0.75 m	13.	$h\,e^2$	14.	0.0866 MeV		
15.	1.95×10^{-6} KeV	16.	30.6 km h^{-1} , 1.8 km h^{-1}	17.	0.2 kg		
18.	(i) 1.7 ms^{-1} (ii) 4.1 J	19.	(i) 1.7 ms^{-1} (ii) 4.1 J	20.	64%	21.	79/81

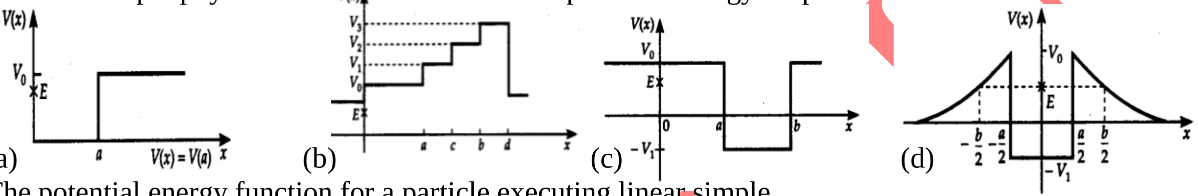
NCERT Exercise

Q.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.

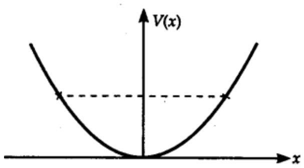
(i) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(ii) work done by gravitational force in the above case.
(iii) Work done by friction on a body sliding down an inclined plane.
(iv) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.
(v) Work done by the resistive forced of air on a vibrating pendulum in bringing it to rest.

Q.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1 . Compute the
(i) Work done by the applied force in 10s (ii) work done by the friction in 10s
(iii) Work done by the net force on the body in 10s, and
(iv) Change in kinetic energy of the body in 10s. Interpret your result.

Q.3 Given below (fig.) are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the region, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical context for which these potential energy shapes are relevant.



Q.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\ N\ m^{-1}$, the graph of V(x) versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential must "turn back" when it reaches $x = \pm 2$ m.

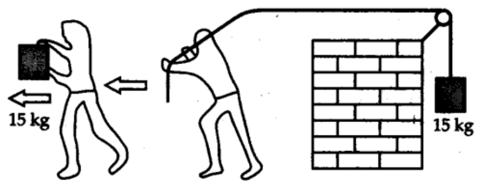


Q.5 Answer the following

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun is highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. yet the work done by the gravitational force every complete orbit of the comet is zero. Why?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?



(d) In figure, (i) the man walks 2m carrying a mass of 15 kg on his hands. In figure (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?

Q.6 Underline the correct alterative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/ decreases/ remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/ potential energy.

(c) The rate of change of total momentum of a many particle system is proportional to the external force/sum of the internal forces on the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/ total linear momentum/ total energy of the system of two bodies.

Q.7 State if each of the following statements is true of false. Give reason for your answer.

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Q.8 Answer carefully, with reasons:

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact)?

(b) Is total linear momentum conserved during the short time of an elastic collision of two balls?

(c) What are the answer to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Q.9 A body is initially at rest. It undergoes one–dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Q.10 A body is moving unidirectional under the influence of a source of constant power. Its displacement in time t is proportional to

(a) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Q.11 A body constrained to move along the Z–axis of a co–ordinate system is subject to a constant force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}, N,$ *where* $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the X–, Y–, and Z–axis of the system respectively. What is the work done by this force in moving the body a distance of 4m along the Z–axis?

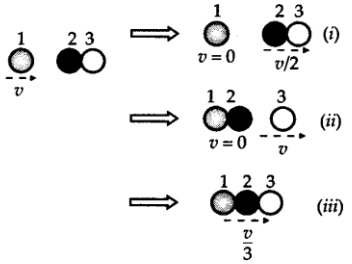
Q.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds.

Q.13 A rain drop of radius 2mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1}.

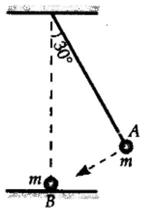
Q.14 A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30^o with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Q.15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

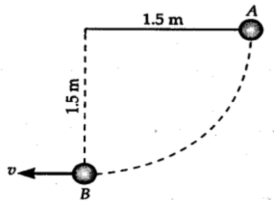
Q.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head–on by another ball bearing of the same mass moving initially with a speed v. If the collision is elastic, which of the situations shown in figure, is a possible result after collision?



Q.17 The bob A of a pendulum released from 30^o to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



Q.18 The bob of a pendulum is released from a horizontal position A as shown. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5% of its initial energy against air resistance?



Q.19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty?

Q.20 A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5m^{-1/2}$. What is the work done by the net force during its displacement from x = 0 to x = 2m.

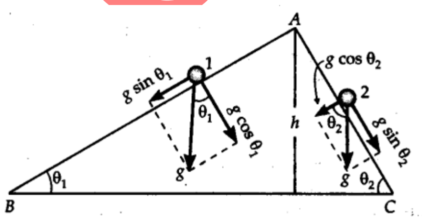
Q.21 The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, v = 36 km/h and the density of air is $1.2\ kg\ m^{-3}$.What is the electric power of produced.

Q.22 A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

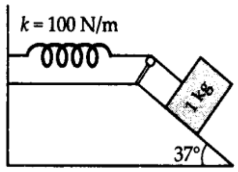
Q.23 A large family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15m with a permission to cover upto 70%.

Q.24 A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block.

Q.25 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (figure). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30^o$, $\theta_2 = 60^o$ and h = 10 m, what are the speeds and times taken by the two stones?



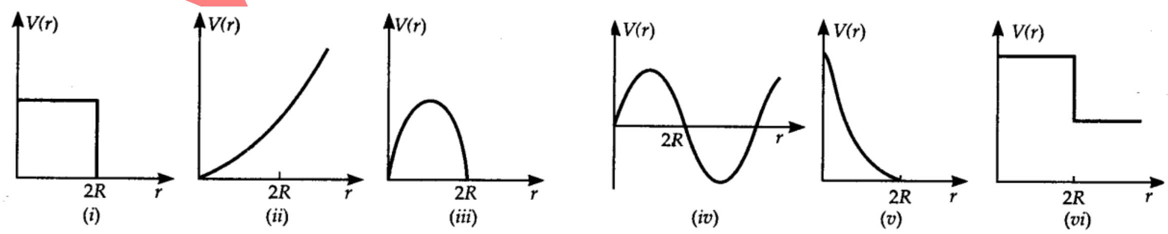
Q.26 A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 Nm^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that spring has negligible mass and the pulley is frictionless.



Q.27 A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does into rebound. What is the heat produced by the impact? Would your answer be different, if the elevator were stationary?

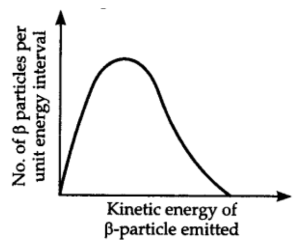
Q.28 A trolley to mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Q.29 Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billard balls? Here r is the distance between centres of the balls.



Q.30 Consider the decay of a free neutron at rest:

$$n \rightarrow p + e^-$$



Show that the two–body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β–decay of neutron or a nucleus (figure).

Answers

1. (i) + ve, (ii) –ve, (iii) –ve, (iv) + ve, (v) –ve
2. (i) 882 J, (ii) – 246.9 J, (iii) 635 J, (iv) 635 J
6. (a) decrease, (b) kinetic energy (c) external force, (d) total linear momentum , total energy
7. (a) false, (b) false, (c) false, (d) true
9. (ii)
10. (iii)
11. 12 J
12. $\frac{V_e}{V_p} = 13.53$
13. 0.082 J, – 0.1623 J
14. Elastic
15. 43.6 kW
16. (ii)
18. 5.3 ms^{-1}
19. No change
20. 50 J
21. (a) ρAvt, (b) $\frac{1}{2}\rho A v^3 t$, (c) 4.5 kW
22. (a) 49, 000 J, (b) 6.45×10^{-3} kg
23. (a) 200 m^2, (b) 20 : 21
24. 28.54 J, 0.212 m
25. same speed, $2\sqrt{2}$ sec, 10.14 ms^{-1}
26. 0.126
27. 8.82 J, same
28. 10.36 ms^{-1}, 25.9 m
29. except (v)

DPMT *objective assignment*

Q.1 When a body moves with constant speed in a circular path, then

(a) work done will be zero (b) acceleration will be zero

(c) no force acts on the body (d) its velocity remains constant

Q.2 A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $\vec{s} = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6J, then value of c is

(a) 0 (b) 6 (c) 1 (d) 12

Q.3 A body of mass 2 kg is placed on rough horizontal plane. The coefficient of friction between body and plane is 0.2.Then

(a) body will move in forward direction if F = 5 N

(b) body will move in backward direction with acceleration 0.5 m/s^2, if force F = 3N

(c) If F = 3N, then body will be in rest condition (d) both (a) and (c) are correct

Q.4 The kinetic energy of body of mass 2 kg and momentum of 2 Ns is

(a) 1 J (b) 3 J (c) 2 J (d) 4 J

Q.5 If momentum is increased by 20%, then kinetic energy increases by

(a) 48% (b) 40% (c) 44% (d) 35%

Q.6 Kinetic energy of particles of mass 10g and 40g is same, the ratio of their linear momentum is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{1}$

Q.7 If the potential energy of two molecules is given by $U = \frac{A}{r^6} - \frac{B}{r^{12}}$, then at equilibrium position, its potential energy is equal to

(a) $\frac{A^2}{4B}$ (b) $\frac{B^2}{4A}$ (c) $\frac{2B}{A}$ (d) $-\frac{B^2}{4A}$

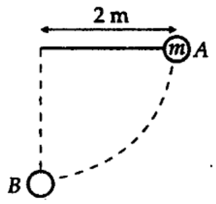
Q.8 The potential energy of a particle of mass 5 kg moving in the x–y plane is given by U = (–7x + 24y)J, x and y being in metre. If the particle starts from rest from origin, then speed of particle at t = 2s is

(a) 5 m/s (b) 14 m/s (c) 17.5 m/s (d) 10 m/s

Q.9 A simple pendulum hanging freely and at rest is vertical because in that position

(a) kinetic energy is zero (b) potential energy is zero

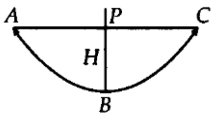
(c) kinetic energy is minimum (d) potential energy is minimum



Q.10 A body m of mass 1 kg is dropped from position A. If dropped from position A. If 10% of its energy is lost at heat, what will be its velocity at B?

(a) 6 ms^{-1} (b) 5.5 ms^{-1} (c) 6.32 ms^{-1} (d) 5.6 ms^{-1}

Q.11 A simple pendulum, with a bob of mass m, oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(a) zero (b) mgH (c) 2 gH (d) $\sqrt{2gH}$

Q.12 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Taking g = 10 m/s2, the work done against friction is

(a) 200 J (b) 100 J (c) zero (d) 1000 J

Q.13 Two springs of spring constant 1500 N/m and 3000 N/m respectively are stretched with a same force. Their potential energies will be in the ratio of

(a) 4 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 2

Q.14 A block of mass m falls through a height h on a spring of spring constant k. The maximum displacement in the spring is

(a) $\frac{2mgh}{k}$ (b) mg/ k (c) $\sqrt{\frac{2mgh}{k}}$ (d) none of these

Q.15 A 15g ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression ($g = 10\ m/s^2$) is

(a) 6.0 m (b) 10.0 m (c) 12.0 m (d) 8.0 m

Q.16 An electric motor exerts a force of 40 N on a cable and pulls it by distance of 30 m in one minute. The power supplied by the motor (in watt) is

(a) 10 (b) 2 (c) 200 (d) 20

Q.17 A particle moves with a velocity $\left(5\hat{i}-3\hat{j}+6\hat{k}\right)$ m/s under the influence of a constant force $\vec{F}=\left(10\hat{i}+10\hat{j}+20\hat{k}\right)$ N. The instantaneous power applied to the particle is

(a) 200 Js^{-1} (b) 40 J s^{-1} (c) 140 Js^{-1} (d) 170 J s^{-1}

Q.18 A body moves a distance of 10 m along a straight line under the action of a 5 N force. If the work done is 25J, then angle between the force and direction of motion of the body is

(a) 60^o (b) 75^o (c) 30^o (d) 45^o

Q.19 When a body moves with a constant speed along a circle

(a) no work is done on it (b) no acceleration is produced in it

(c) its velocity remains constant (d) no force acts on it

Q.20 A body, constrained to move in y–direction, is subjected to a force given by $\vec{F} = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ N. The work done by this force in moving the body through a distance of $10\hat{j}$ m along y–axis, is

(a) 150 J (b) 20 J (c) 190 J (d) 160 J

Q.21 If $\vec{F} = (60\hat{i} + 15\hat{j} - 3\hat{k})$N and $\vec{v} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ m/s, then instantaneous power is

(a) 195 watt (b) 45 watt (c) 75 watt (d) 100 watt

Q.22 A position dependent force, F = $(7 - 2x + 3x^2)$ N acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5m. The work done in joule is

(a) 135 (b) 270 (c) 35 (d) 70

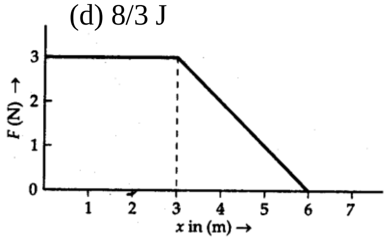
Q.23 A force acts on a 3g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is

(a) 490 mJ (b) 450 mJ (c) 576 mJ (d) 530 mJ

Q.24 A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $s = \frac{1}{3}t^2$, where t is in seconds. Work done by the force in 2 seconds is

(a) 19/5 J (b) 5/19 J (c) 3/8 J (d) 8/3 J

Q.25 A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work done by the force in moving the object from x = 0 to x = 6 m is



(a) 18.0 J (b) 13.5 J

(c) 9.0 J (d) 4.5 J

Answers

1.	A	**2.**	B	**3.**	D	**4.**	A	**5.**	C	**6.**	B	**7.**	A	**8.**	D
9.	D	**10.**	A	**11.**	D	**12.**	B	**13.**	B	**14.**	D	**15.**	B	**16.**	D
17.	C	**18.**	A	**19.**	A	**20.**	A	**21.**	B	**22.**	A	**23.**	C	**24.**	D
25.	B														

CBSE ***objective assignment***

Q.1 A bullet is fired and gets embedded in a block kept on table. If table is frictionless, then

(a) kinetic energy gets conserved (b) potential energy gets conserved

(c) momentum gets conserved (d) both (a) and (c)

Q.2 A body of mass 5 kg, moving with velocity 10 m/sec collides with another body of the mass 20 kg at rest and comes to rest. The velocity of the second body due to collision is

(a) 2.5 m/sec (b) 7.5 m/sec (c) 5 m/sec (d) 10 m/sec

Q.3 A 10 kg ball moving with velocity 2 m/s collides with a 20 kg mass initially at rest. If both of them coalesce, the final velocity of combined mass is

(a) 3/4 m/s (b) 1/3 m/s (c) 3/2 m/s (d) 2/3 m/s

Q.4 A body P moving with a velocity of 20 m/sec collides with another body Q of same mass at rest. If after collision P comes to rest, then what is the velocity of body Q?

(a) 10 m/sec (b) 30 m/sec (c) 20 m/sec (d) 40 m/sec

Q.5 A body of mass 4m at rest explodes into three pieces. Two of the pieces each of mass m move with a speed v each in mutually perpendicular directions. The total kinetic energy released is

(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{3}{2}mv^2$ (d) $\frac{5}{2}mv^2$

Q.6 A particle of mass m having velocity v moving towards north collides with similar particle moving with same velocity towards east. The two particles stick together and move towards north east with a velocity

(a) $\sqrt{2}\,v$ (b) $v/\sqrt{2}$ (c) v/ 2 (d) 2 v

Q.7 A ball is dropped from height h on the ground where coefficient of restitution is e. After one bounce the maximum height is

(a) e^2h (b) $e\sqrt{h}$ (c) eh (d) $\sqrt{eh}$

Q.8 A body of mass 10 kg initially at rest acquires velocity of 10 ms^{-1}. What is the work done?

(a) – 500 J (b) 500 J (c) 50 J (d) – 50 J

Q.9 A ball moves in a frictionless inclined table without slipping. The work done by the table surface on the ball is

(a) positive (b) negative (c) zero (d) none of these

Q.10 A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is

(a) 324 J (b) 486 J (c) 256 J (d) 524 J

Q.11 A mass of 1 kg is thrown up with a velocity of 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400 g comes down with a velocity 25 m/s. The velocity of other part is

(a) 40 m/s ↑ (b) 40 m/s ↓ (c) 100 m/s ↑ (d) 60 m/s ↑

Q.12 A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of

(a) $\sqrt{2}:1$ (b) 1 : 4 (c) 1 : 2 (d) $1:\sqrt{2}$

Q.13 A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is

(a) m_2/m_1 (b) m_1/m_2 (c) 1 (d) m_1v_2/m_2v_1

Q.14 A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$, then

(a) $E_1 < E_2$ (b) $\frac{E_1}{E_2}=\frac{m_1}{m_2}$ (c) $E_1 > E_2$ (d) $E_1 = E_2$

Q.15 Two bodies of masses m and 4m are moving with equal kinetic energies. Ratio of their linear momenta

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 1 : 1

Q.16 Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is

(a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 4

Q.17 The kinetic energy acquired by a mass m in travelling distance d, starting from rest, under the action of a constant force is directly proportional to

(a) m (b) m^0 (c) $\sqrt{m}$ (d) $1/\sqrt{m}$

Q.18 If kinetic energy of a body is increased by 300%, then percentage change in momentum will be

(a) 100% (b) 150% (c) 265% (d) 73.2%

Q.19 A particle is projected making an angle of 45^o with horizontal having kinetic energy K. The kinetic energy at highest point will be

(a) $K/\sqrt{2}$ (b) K/ 2 (c) 2 K (d) K

Q.20 A particle of mass M is moving in a horizontal circle of radius R with uniform speed v. When it moves from one point to a diametrically opposite point, its

(a) kinetic energy changes by $mv^2/4$ (b) momentum does not change

(c) momentum changes by 2 Mv (d) kinetic energy changes by Mv^2

Q.21 A bullet of mass 10g leaves a rifle at an initial velocity of 1000 m/s and strikes the earth at the same level with a velocity of 5000 m/s. The work done in joule overcoming the resistance of air will be

(a) 375 (b) 3750 (c) 5000 (d) 500

Q.22 A child is sitting on a swing. Its minimum and maximum heights from the ground 0.75m and 2m respectively, its maximum speed will be

(a) 10 m/s (b) 5 m/s (c) 8 m/s (d) 15 m/s

Q.23 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Work done against friction (take $g = 10\ m/s^2$) is

(a) 1000 J (b) 200 J (c) 100 J (d) zero

Q.24 The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm, the potential energy stored in it is

(a) U/ 4 (b) 4U (c) 8 U (d) 16 U

Q.25 Two springs A and B having spring constants k_A and k_B ($k_A = 2\ k_B$) are stretched by applying force of equal magnitude. If energy stored in spring A is E_A, then energy stored in B will be

(a) $2E_A$ (b) $E_A/4$ (c) $E_A/2$ (d) $4\ E_A$

Q.26 A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be

(a) 0.15 m (b) 0.12 m (c) 1.5 m (d) 0.5 m

Q.27 A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is

(a) $mg(h+d)-\frac{1}{2}kd^2$ (b) $mg(h-d)-\frac{1}{2}kd^2$ (c) $mg(h-d)+\frac{1}{2}kd^2$ (d) $mg(h+d)+\frac{1}{2}kd^2$

Q.28 The potential energy between two atoms in a molecule, is given by U(x) = $\frac{a}{x^{12}}-\frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium, when

(a) $x=\left(\frac{2a}{b}\right)^{1/6}$ (b) $x=\left(\frac{11a}{5b}\right)^{1/6}$ (c) x = 0 (d) $x=\left(\frac{a}{2b}\right)^{1/6}$

Q.29 Water falls from a height of 60 m at the rate 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine? Take g = 10 m/s^2.

(a) 8.1 kW (b) 10.2 kW (c) 12.3 kW (d) 7.0 kW

Q.30 How much water a pump of 2 kW can raise in one minute to a height of 10 m? (take g = 10 m/s^2)

(a) 1000 litres (b) 1200 litres (c) 10 litres (d) 2000 litres

Q.31 250 N force is required to raise 75 kg mass from a pulley. If rope is pulled 12 m, then the load is lifted to 3m, the efficiency of pulley system will be

(a) 25% (b) 33.3% (c) 75% (d) 90%

Q.32 A shell, in flight, explodes into four unequal parts. Which of the following is conserved?

(a) Potential energy (b) Momentum (c) Kinetic energy (d) Both (a) and (c)

Q.33 A moving body of mass m and velocity 3 km/h collides with a rest body of mass 2m and sticks to it. Now the combined mass starts to move. What will be the combined velocity?

(a) 3 km/ hour (b) 4 km/ hour (c) 1 km/ hour (d) 2 km/ hour

Q.34 A metal ball of mass 2 kg moving with speed of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move as a single mass, then the loss in K.E. due to collision

(a) 100 J (b) 140 J (c) 40 J (d) 60 J

Q.35 Two identical balls A and B collide head on elastically. If velocities of A and B, before the collision are + 0.5 m/s and – 0.3 m/s respectively, then their velocities, after the collision, are respectively

(a) –0.5 m/s and +0.3 m/s (b) + 0.5 m/s and + 0.3 m/s

(c) +0.3 m/s and –0.5 m/s (d) – 0.3 m/s and + 0.5 m/s

Q.36 The coefficient of restitution e for a perfectly elastic collision is

(a) 1 (b) 0 (c) ∞ (d) –1

Q.37 A body of mass 1 kg is thrown upwards with a velocity of 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? [Take g = 10 m/s^2].

(a) 30 J (b) 40 J (c) 10 J (d) 20 J

Q.38 An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and mass m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water?

(a) $\frac{1}{2}m^2v^2$ (b) $\frac{1}{2}mv^3$ (c) mv^3 (d) $\frac{1}{2}mv^2$

Q.39 A block of mass M is attached to the lower end of a vertical spring. The spring is hung from the ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spring will be

(a) 2 Mg/ k (b) 4 Mg/ k (c) Mg/ 2k (d) Mg/ k

Answers

1.	C	**2.**	A	**3.**	D	**4.**	C	**5.**	C	**6.**	B	**7.**	A	**8.**	B
9.	C	**10.**	B	**11.**	C	**12.**	C	**13.**	A	**14.**	A	**15.**	A	**16.**	D
17.	B	**18.**	A	**19.**	B	**20.**	C	**21.**	B	**22.**	B	**23.**	C	**24.**	D
25.	A	**26.**	A	**27.**	A	**28.**	A	**29.**	A	**30.**	B	**31.**	C	**32.**	B

33.	C	**34.**	D	**35.**	D	**36.**	A	**37.**	D	**38.**	B	**39.**	A

AIIMS *objective assignment*

Q.1 A boy carrying a box on his head is walking on a level load from one place to another on a straight road is doing no work. This statement is

(a) correct (b) incorrect (c) partly correct (d) insufficient data

Q.2 Kinetic energy, with any reference, must be

(a) zero (b) positive (c) negative (d) both (b) and (c)

Q.3 If the kinetic energy of a body becomes four times of its initial value, then new momentum will

(a) become twice its initial value (b) becomes thrice its initial value

(c) become four times its initial value (d) remain constant

Q.4 A body of mass 5 kg has momentum of 10 kg ms^{-1}. When a force of 0.2 N is applied on it for 10s, what is the change in kinetic energy?

(a) 1.1 J (b) 2.2J (c) 3.3 J (d) 4.4 J

Q.5 A block of mass 10 kg is moving in x–direction with a constant speed of 10 ms^{-1}, it is subjected to a retarding force, F = – 0.1x Jm^{-1} during its travel from x = 20 to x = 30 m. Its final kinetic energy will be

(a) 475 J (b) 450 J (c) 275 J (d) 250 J

Q.6 The decrease in the potential energy of a ball of mass 20 kg, which falls from a height 50 cm is

(a) 968 J (b) 98 J (c) 1, 980 J (d) none of these

Q.7 If the water falls from a dam into a turbine wheel 19.6 m below, then the velocity of water at the turbine (Take g = 9.8 ms^{-1}) is

(a) 9.8 ms^{-1} (b) 19.6 ms^{-1} (c) 39.2 ms^{-1} (d) 98.0 ms^{-1}

Q.8 A gun fires a bullet of mass 50g with a velocity of 30 ms^{-1}. Because of this, the gun is pushed back with a velocity of 1 ms^{-1}. The mass of the gun is

(a) 5.5 kg (b) 3.5 kg (c) 1.5 kg (d) 0.5 kg

Q.9 A body of mass 5 kg is raised vertically to a height of 10 m by a force of 170 N. The velocity of the body at this height will be

(a) 37 ms^{-1} (b) 22 ms^{-1} (c) 15 ms^{-1} (d) 9.8 ms^{-1}

Q.10 A bomb of mass 3.0 kg explodes in air into two pieces of masses 2.0 kg and 1.0 kg. The smaller mass goes at a speed of 80 ms^{-1}. The total energy imparted to the two fragments is

(a) 1.07 kJ (b) 2.14 kJ (c) 2.4 kJ (d) 4.8 kJ

Q.11 If a spring extends by x on loading, then the energy stored by the spring is (if T is tension in the spring and k is spring constant)

(a) $T^2/ 2x$ (b) $T^2/ 2k$ (c) $2k/ T^2$ (d) $2T^2/ k$

Q.12 A spring 40 mm long is stretched by the application of a force. If 10 N force is required to stretch the spring through 1 mm, then work done in stretching the spring through 40 mm is

(a) 23 J (b) 68 J (c) 84 J (d) 8 J

Q.13 Which of the following is true?

(a) Momentum is conserved in all collisions, but kinetic energy is conserved only in inelastic collisions.

(b) Neither momentum nor kinetic energy is conserved in inelastic collisions.

(c) Momentum is conserved in all collisions but not kinetic energy.

(d) Both momentum and kinetic energy are conserved in all collision.

Q.14 Two particles are seen to collide and move jointly together after the collision. During such a collision, for the total system,

(a) linear momentum is conserved, but not the mechanical energy

(b) mechanical energy is conserved, but not the linear momentum

(c) both the mechanical energy and the linear momentum are conserved.

(d) neither the mechanical energy nor the linear momentum is conserved

Q.15 A metal ball of mass 2 kg moving with speed of 36 km h^{-1} has a head on collision with a stationary ball of mass 3 kg. If after collision, both the balls move together, then the loss in kinetic energy due to collision is

(a) 40 J (b) 60 J (c) 100 J (d) 140 J

Q.16 A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, the speed of the combined particle is

(a) v/2 (b) 2v (c) v/3 (d) 3v

Q.17 In elastic collision, 100% energy transfer takes place when

(a) $m_1 = m_2$ (b) $m_1 > m_2$ (c) $m_1 < m_2$ (d) $m_1 = 2m_2$

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false (d) If both assertion and reason are false

Q.18 **Assertion:** When a body moves along a circular path, no work is done by the centripetal force.

Reason: The centripetal force is used in moving the body along the circular path and hence no work is done.

Q.19 **Assertion:** Mass and energy are not conserved separately, but are conserved as a single entity called mass–energy.

Reason: Mass and energy are inter–convertible in accordance with Einstein's relation, $E = mc^2$

Q.20 **Assertion:** In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (i.e., when they are in contact).

Reason: Energy spent against friction does not follow the law of conservation of energy.

Answers

1.	A	**2.**	B	**3.**	A	**4.**	D	**5.**	A	**6.**	B	**7.**	B	**8.**	C
9.	B	**10.**	D	**11.**	B	**12.**	D	**13.**	C	**14.**	A	**15.**	B	**16.**	C
17.	A	**18.**	C	**19.**	A	**20.**	D								

DCE *objective assignment*

Q.1 A body moves a distance of 10m under the action of force F = 10 N. If the work done is 25J, the angle which the force makes with the direction of motion is

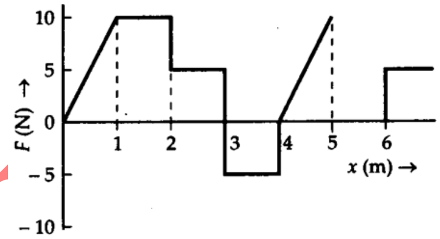
(a) 0^o (b) 30^o (c) 60^o (d) none of these

Q.2 What is $\vec{F}.\,\vec{ds}$?

(a) Torque (b) Impulse (c) Momentum (d) Work

Q.3 A particle moves under a force F = CX from X = 0 to X = X_1. The work done is

(a) CX_1^2 (b) $CX_1^2/2$ (c) zero (d) CX_1^3



Q.4 The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body from x = 1 m to x = 5 m will be

(a) 30 J (b) 15 J (c) 25 J (d) 20 J

Q.5 Two bodies of mass m and 4m have equal kinetic energy. What is the ratio of their momentum?

(a) 1 : 4 (b) 1 : 2 (c) 1 : 1 (d) 2 : 1

Q.6 If momentum decreases by 20%, K.E. will decrease by

(a) 40% (b) 36% (c) 18% (d) 8%

Q.7 A hockey player receives a corner shot at a speed of 15 m/s at an angle of 30^o with the y–axis and then shoots the ball of mass 100g along the positive x–axis with a speed of 30 m/s. If it remains in contact with the hockey stick for 0.01 s, the force imparted to the ball in the x–direction is

(a) 281.25 N (b) 187.5 N (c) 562.5 N (d) 375 N

Q.8 Which is odd out?

(a) Displacement (b) Momentum (c) Potential energy (d) Torque

Q.9 The dimension of k in the equation $W = \frac{1}{2}kx^2$ is

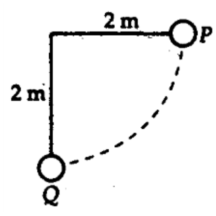
(a) $[M^1L^0T^{-2}]$ (b) $[M^0L^1T^{-1}]$ (c) $[M^1L^1T^{-2}]$ (d) $[M^1L^0T^{-1}]$

Q.10 The work done in stretching a spring of force constant k from length l_1 to l_2 is

(a) $k(l_2^2 - l_1^2)$ (b) $\frac{1}{2}k(l_2^2 - l_1^2)$ (c) $k(l_2 - l_1)$ (d) $\frac{k}{2}(l_2 + l_1)$

Q.11 When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by

(a) 100 J (b) 200 J (c) 300 J (d) 400 J



Q.12 The bob of a pendulum of length 2m lies at P. When it reaches Q, it loses 10% of its total energy due to air resistance. The velocity at Q is

(a) 6 m/s (b) 1 m/s (c) 2 m/s (d) 8 m/s

Q.13 Unit of power is

(a) kilowatt hour (b) kilowatt/ hour (c) watt (d) erg

Q.14 Power can be expressed as

(a) $\vec{F}.\vec{v}$ (b) $\frac{1}{2}\vec{F}.v^2$ (c) $\vec{F}.t$ (d) $\vec{F}\times\vec{v}$

Q.15 Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between t = 2s and t = 4s.

(a) 32 J (b) 46 J (c) 61 J (d) 102 J

Q.16 A block is kept on an inclined plane of inclination θ and length l. The velocity of particle at the bottom of incline (the coefficient of friction is μ) is

(a) $\sqrt{2gl(\mu\cos\theta - \sin\theta}$ (b) $\sqrt{2gl(\sin\theta - \mu\cos\theta}$ (c) $\sqrt{2gl(\sin\theta + \mu\cos\theta)}$ (d) $\sqrt{2gl(\cos\theta + \mu\sin\theta}$

Q.17 In an elastic collision

(a) both momentum and K.E. are conserved (b) both momentum and K.E. are not conserved

(c) only energy is conserved (d) only momentum is conserved

Q.18 In an inelastic collision, what is conserved?

(a) Kinetic energy (b) Momentum (c) Both (a) and (b) (d) Neither (a) nor (b)

Q.19 A body moving with a velocity v, breaks up into two equal parts. One of the parts retraces back with velocity v. Then the velocity of the other part is

(a) v in forward direction (b) 3v in forward direction

(c) v in backward direction (d) 3v in backward direction

Q.20 A bomb is kept stationary at a point. It suddenly explodes into two fragments of masses 1g and 3g. The total K.E. of the fragments is 6.4×10^4 J. What is the K.E. of the smaller fragment?

(a) 2.5×10^4 J (b) 3.5×10^4 J (c) 4.8×10^4 J (d) 5.2×10^4 J

Q.21 A particle of mass m_1 moving with velocity v collides with a mass m_2 at rest, then they get embedded. At the instant of collision , velocity of the system

(a) increases (b) decreases (c) remains constant (d) becomes zero

Q.22 A body of mass M_1 collides elastically with another mass M_2 at rest. There is maximum transfer of energy when

(a) $M_1 > M_2$ (b) $M_1 < M_2$

(c) $M_1 = M_2$ (d) same of all values of M_1 and M_2

Q.23 A spacecraft of mass M and moving with velocity v suddenly breaks in two piece of same mass m. After the explosion one of the masses m becomes stationary. What is the velocity of the other part of craft?

(a) $\frac{Mv}{M-m}$ (b) v (c) $\frac{Mv}{m}$ (d) $\frac{M-m}{m}v$

Q.24 If a neutron collides with a stationary α–particle with velocity v, what is resultant velocity of neutron?

(a) $\frac{1}{5}v$ (b) $\frac{2}{5}v$ (c) $\frac{3}{5}v$ (d) $\frac{4}{5}v$

Q.25 A bullet hits and gets embedded in a solid block resting on a frictionless surface. In this process which one of the following is correct?

(a) only momentum is conserved (b) only kinetic energy is conserved

(c) neither momentum nor kinetic energy is conserved

(d) both momentum and energy are conserved

Answers

1.	D	**2.**	D	**3.**	B	**4.**	B	**5.**	B	**6.**	B	**7.**	D	**8.**	C
9.	A	**10.**	B	**11.**	C	**12.**	A	**13.**	A	**14.**	A	**15.**	B	**16.**	B
17.	A	**18.**	B	**19.**	D	**20.**	C	**21.**	C	**22.**	C	**23.**	C	**24.**	C
25.	A														

AIEEE *objective assignment*

Q.1 A force $\vec{F}=(5\hat{i}+3\hat{j}+2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r}=(2\hat{i}-\hat{j})$ m. The work done on the particle (in joule) is

(a) – 7 (b) + 7 (c) + 10 (d) + 13

Q.2 An athlete in the Olympic Games covers a distance of 100 m in 10s. His kinetic energy can be estimated in the range

(a) 200J – 500 J (b) 2×10^5J–3×10^5 J (c) 20,000J – 50,000J (d) 2000J – 5000J

Q.3 A particle of mass 100g is thrown vertically upwards with a speed of 5 ms^{-1}. The work done by the force of gravity during the time the particle goes up is

(a) 1.25 J (b) 0.5 J (c) – 0.5 J (d) – 1.25 J

Q.4 A ball whose kinetic energy is E, is projected at an angle of 45^o to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(a) E (b) $E/\sqrt{2}$ (c) E/2 (d) zero

Q.5 A particle is projected at an angle of 60^o to the horizontal with a kinetic energy E. The kinetic energy at the highest point is

(a) E (b) E/ 4 (c) E/2 (d) zero

Q.6 A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x (c) x (d) $\log_e x$

Q.7 The potential energy of a 1 kg particle free to move along the x–axis is given by

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} \text{ (in joule)}$$

The total mechanical energy of the particle is 2J. Then the maximum speed (in ms^{-1}) is

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{2}$

Q.8 Consider the following two statements:

(A) Linear momentum of a system of particle is zero (B) Kinetic energy of a system of particles is zero

(a) A does not imply B and B does not imply A (b) A implies B but B does not imply A

(c) A does not imply B but B implies A (d) A implies B and B implies A

Q.9 A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20m above the ground. The velocity attained by the ball is

(a) $10\ ms^{-1}$ (b) $10\sqrt{30}\ ms^{-1}$ (c) $40\ ms^{-1}$ (d) $20\ ms^{-1}$

Q.10 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J (c) 120 J (d) 1,200 J

Q.11 A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it, until the string makes an angle 45^o with the initial vertical direction, is

(a) $\frac{Mg}{\sqrt{2}}$ (b) $(\sqrt{2}-1)Mg$ (c) $(\sqrt{2}+1)Mg$ (d) $\sqrt{2}\ Mg$

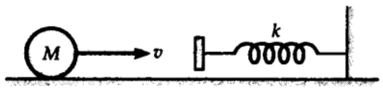
Q.12 A spring of spring constant $5 \times 10^3\ Nm^{-1}$ is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 12.50 Nm (b) 18.75Nm (c) 25.00 Nm (d) 6.25 Nm

Q.13 A spring of force constant $800\ Nm^{-1}$ has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J (c) 32 J (d) 24 J

Q.14 The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is



(a) $\frac{ML^2}{k}$ (b) $\frac{kL^2}{2M}$ (c) $\sqrt{Mk}\ L$ (d) zero

Q.15 A 2kg block slides on a horizontal floor with a speed of $4\ ms^{-1}$. It strikes an uncompressed spring and compresses it, till the block is motionless. The force of kinetic friction is 15 N and spring constant is $10{,}000\ ms^{-1}$. The spring compresses by

(a) 5.5 cm (b) 2.5 cm (c) 11.0 cm (d) 8.5 cm

Q.16 A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$ (c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$

Q.17 A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

Q.18 If mass–energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

(a) increase (b) remain unchanged (c) decrease (d)first increase then decrease

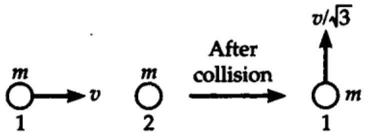
Q.19 A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\ ms^{-1}$. The kinetic energy of the other mass is

(a) 192 J (b) 96 J (c) 144 J (d) 288 J

Q.20 A block of mass 0.50 kg is moving with a speed of $2.00\ ms^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

Q.21 A mass m moves with a velocity v and collides elastically with another identical mass at rest. After collision the first mass moves at rest. After collision the first mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.



(a) v (b) $\sqrt{3}$ v (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{\sqrt{3}}$

Answers

1.	B	**2.**	D	**3.**	D	**4.**	C	**5.**	B	**6.**	A	**7.**	C	**8.**	C
9.	C	**10.**	B	**11.**	B	**12.**	B	**13.**	B	**14.**	C	**15.**	A	**16.**	B
17.	B	**18.**	A	**19.**	D	**20.**	C	**21.**	C						

IIT–JEE *objective assignment*

Q.1 Two masses of 1g and 4g are moving with equal kinetic energy. The ratio of the magnitudes of their momenta is

(a) 4 : 1 (b) $\sqrt{2}$:1 (c) 1 : 2 (d) 1 : 16

Q.2 If a machine is lubricated with oil

(a) the mechanical advantage of the machine increases

(b) the mechanical efficiency of the machine increases

(c) both its mechanical advantage and efficiency increase

(d) its efficiency increases, but its mechanical advantage decreases

Q.3 A spring of force–constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force–constant of

(a) (2/3) k (b) (3/2) k (c) 3k (d) 6 k

Q.4 An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

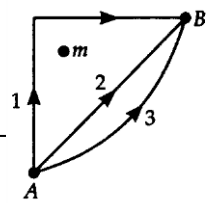
(a) $\frac{4\ Mg}{k}$ (b) $\frac{2\ Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

Q.5 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t (c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

Q.6 A wind–powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

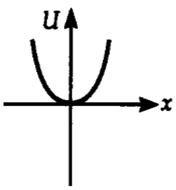
(a) v (b) v^2 (c) v^3 (d) v^4

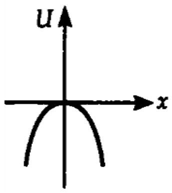


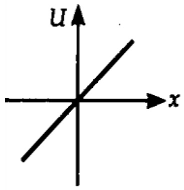
Q.7 If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

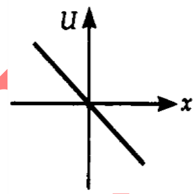
(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$ (c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

Q.8 A particle is acted by a force F = kx, where k is a +ve constant. Its potential energy at x = 0 is zero. Which curve correctly represents the variation of potential energy of the block with respect to x?

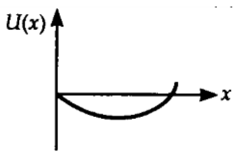
(a) 

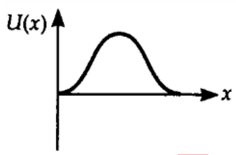
(b) 

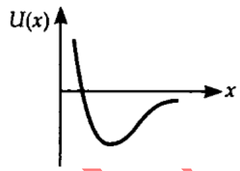
(c) 

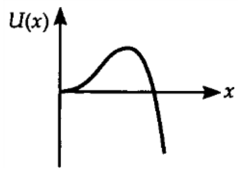
(d) 

Q.9 A particle, which is constrained to move along the x–axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy U(x) the particle is

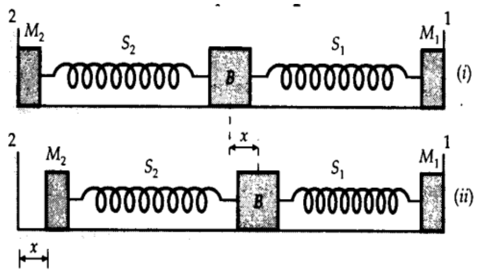
(a) 

(b) 

(c) 

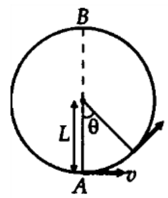
(d) 

Q.10 A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4k, respectively (see figure). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (see figure) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to equilibrium position of the block B. The ratio y/x is



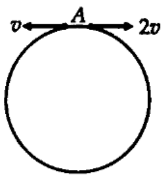
(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Q.11 A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reached the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



(a) $\theta = \pi/4$ (b) $\pi/4 < \theta < \pi/2$ (c) $\pi/2 < \theta < 3\pi/4$ (d) $3\pi/4 < \theta < \pi$

Q.12 Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(a) 4 (b) 3 (c) 2 (d) 1

Multiple Choice Questions with One or More than One Correct Answer

Q.13 A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant (b) its acceleration is constant

(c) its kinetic energy is constant (d) it moves in a circular path

Q.14 A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy–plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0), and then parallel to the y–axis to the point (a, a). The total work done by the force F on the particle is

(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

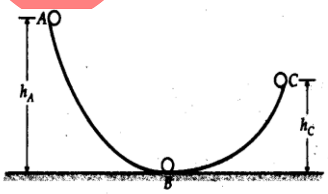
Q.15 A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

Q.16 A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(a) MgL (b) MgL/3 (c) MgL/9 (d) MgL/18

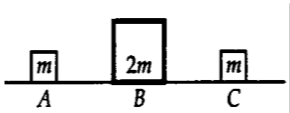
Q.17 A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively. Then



(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$ (c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$

Integer Answer Type

Q.18 Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Therefore, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.



Answers

1.	C	**2.**	B	**3.**	B	**4.**	B	**5.**	B	**6.**	C	**7.**	B	**8.**	B
9.	D	**10.**	C	**11.**	D	**12.**	C	**13.**	C,D	**14.**	C	**15.**	C	**16.**	D
17.	A,C	**18.**	$V_B = 6$ m/s ; $V = 4$ m/s												