SYSTEMS OF PARTICLES & ROTATIONAL MOTION

The Concept of a 'System', Internal Forces and External Forces

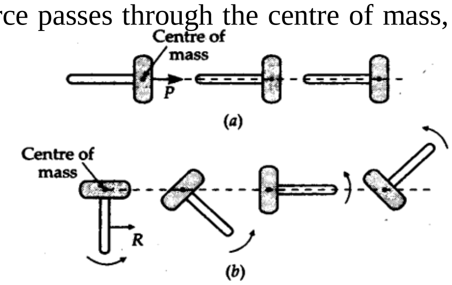
Every body of finite size is made up of a very large number of particles. It is almost impossible to describe the positions and velocities of all the particles individually. A collection of any number of particles interacting with one another is said to form a system. Note that 'particles interacting with one another' means that the particles are exerting forces on one another. These forces alone enable them to form a well defined system. Thus any object of finite size can be regarded as a system. All the forces exerted by various particles of the system on one another are called **internal forces**. Note that internal forces between any two particles of the system are mutual i.e., internal forces between a pair of particles are equal and opposite. Hence such forces cancel out in pairs.

To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system by the agencies outside the system is called an **external force**. The overall motion of a body is affected by external forces only.

Centre of Mass

Newton's laws of motion are applicable to point objects. The introduction of the concept of centre of mass enables us to apply them equally well to the motion of finite or extended objects. The centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translator by motion. The centre of mass of a system of particles is that single point which moves in the same way in which a single particle having the total mass of the system and acted upon by the same external force would move.

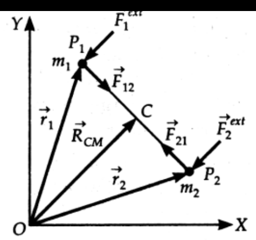
If a single force acts on a body and the line of action of the force passes through the centre of mass, the body will have only linear acceleration and no angular acceleration.



For example, consider a hammer resting on a plane surface. If a force P is applied on the hammer in such a way that its line of action passes through the centre of mass of the hammer, then the hammer moves along a straight line path, as shown in fig. (a). But when a force R is applied along a line not passing through its centre of mass, then the hammer rotates about its centre of mass, as shown in fig. (b).

Centre of mass vs. centre of gravity : The centre of mass of body is point where whole mass of the body may be assumed to be concentrated for describing its translatory motion. On the other hand, the centre of gravity is a point at which the resultant of the gravitational forces on all the particles of the body acts i.e., a point where whole weight may be assumed to act. In a uniform gravitational field such as that of the earth on a small body, the centre of gravity coincides with the centre of mass. But in the case of mount Everest, the centre of gravity lies a little below its centre of mass because the gravitational force decreases with altitude.

Centre of mass of a two particle system



Consider a system of two particles P_1 and P_2 of masses m_1 and m_2. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors at any instant t with respect to the origin O, as shown in fig.

The velocity and acceleration vectors of the two particles are

$$\left.\begin{array}{lll} \vec{v}_1 = \dfrac{d\vec{r}_1}{dt} & \text{and} & \vec{a}_1 = \dfrac{d\vec{v}_1}{dt} = \dfrac{d^2\vec{r}_1}{dt^2} \\ \vec{v}_2 = \dfrac{d\vec{r}_2}{dt} & \text{and} & \vec{a}_2 = \dfrac{d\vec{v}_2}{dt} = \dfrac{d^2\vec{r}_2}{dt^2} \end{array}\right\} \quad ....(1)$$

Total force $\vec{F}_1$ acting on particle P_1 is the sum of the internal force $\vec{F}_{12}$ due to P_2 and external force $\vec{F}_1^{ext}$ on.

Thus $\vec{F}_1 = \vec{F}_{12} + \vec{F}_1^{ext}$

$$m_1\vec{a}_1 = \vec{F}_{12} + \vec{F}_1^{ext} \quad ....(2)$$

Similarly , for m_2, $m_2\vec{a}_2 = \vec{F}_{21} + \vec{F}_2^{ext}$ ....(3)

On adding equations (2) and (3), we get

$$m_1\vec{a}_1 + m_2\vec{a}_2 = \vec{F}_{12} + \vec{F}_{21} + \vec{F}_1^{ext} + \vec{F}_2^{ext} \quad ....(4)$$

According to Newton's third law, the internal forces mutually exerted by the two particles are equal and opposite, i.e., $\vec{F}_{12} = -\vec{F}_{21}$ ....(5)

The total external force acting on the system,

$$\vec{F} = \vec{F}_1^{ext} + \vec{F}_2^{ext} \quad ....(6)$$

Using (5) and (6) in (4)

$$m_1\vec{a}_1 + m_2\vec{a}_2 = \vec{F} \quad ....(7)$$

Suppose the total mass of the two–particle system is M. Then $M = m_1 + m_2$.

Let us assume that the total external force $\vec{F}$ acting on the system of mass M produces acceleration $\vec{a}_{CM}$. Then according to Newton's second law,

$$M\vec{a}_{CM} = \vec{F}$$

$$M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 = m_1\frac{d^2\vec{r}_1}{dt^2} + m_2\frac{d^2\vec{r}_2}{dt^2} \qquad \text{[Using (1 and 7)]}$$

$$= \frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

or
$$\vec{a}_{CM} = \frac{1}{M}\frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

or
$$\frac{d^2\vec{R}_{CM}}{dt^2} = \frac{d^2}{dt^2}\,\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \qquad [\because M = m_1 + m_2]$$

The acceleration $\vec{a}_{CM}$ is called the acceleration vector of the centre of mass of the system and $\vec{R}_{CM}$ is called the position vector of the centre of mass

Clearly ,
$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \qquad ....(6)$$

This equation defines the position of the centre of mass of a system of two particles of masses m_1 and m_2 and having position vectors $\vec{r}_1$ and $\vec{r}_2$.

Note :

(i) The above equation shows that the position vector of a system of particles is the weighted average of the position vectors of the particles making the system, each particle making a contribution proportional to its mass.

(ii) We can write the above equation as $(m_1+m_2)\vec{R}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2$. Thus the product of the total mass of the system and the position vector of its centre of mass is equal to the sum of the products of individual masses and their respective position vectors.

(iii) If $m_1 = m_2 = m(\text{say})$, then $\vec{R}_{CM} = \frac{\vec{r}_1 + \vec{r}_2}{2}$. Thus the centre of mass of two equal masses lies exactly at the centre of the line joining the two masses.

(iv) If (x_1, y_1) and (x_2, y_2) are the coordinates of the locations of the two particles, the coordinates of their centre of mass are given by

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} \quad \text{and} \quad y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1+m_2}.$$

(v) If the centre of mass of two particles of the system were at the origin i.e., $\vec{r} = 0$, then from (8),

$m_1\vec{r}_1 + m_2\vec{r}_2 = 0$ or $m_2\vec{r}_2 = -m_1\vec{r}_1$.

$$\boxed{\vec{r}_2 = -\frac{m_1}{m_2}\vec{r}_1}$$

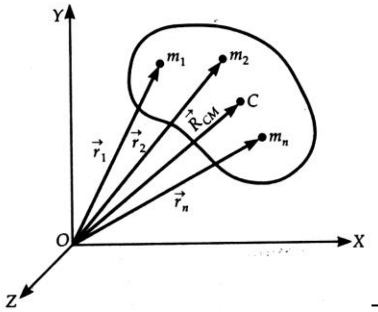
Show that if $\vec{r}_1$ is negative, $\vec{r}_2$ is positive. It means, if m_1 lies on the left of the origin (*c.m.*), then m_2 lies on the right of the origin (*c.m.*). Hence centre of mass of a system of two particles always on the straight line joining these particles. Further, if $m_1 > m_2$, then $\vec{r}_1 < \vec{r}_2$, i.e., centre of mass of two particle system lies closer to the heavier particle. In general, *the centre of mass divides internally, the line joining the two particles in the inverse ratio of masses.*

Centre of Mass of n–particle system

Consider a system of n particles having mass $m_1, m_2, m_3, ..., m_n$ and position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_2, ..., \vec{r}_n$ relative to the origin O, as shown in fig.

The total mass of the system is

$$m_1 + m_2 + m_3 + ... + m_n = M \quad \text{(say)}$$



The position vector $\vec{R}_{CM}$ of the centre of mass C can be obtained by adding the products $m_1\vec{r}_1, m_2\vec{r}_2, ..., m_n\vec{r}_n$ and dividing it by the total mass of the system.

Thus , $$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...+ m_n\vec{r}_n}{m_1 + m_2 + ...+ m_n} \quad \text{or} \quad \vec{R}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M} \quad ....(1)$$

Clearly, $\vec{R}_{CM}$ is the weighted average of the position vectors of all the particles of the system, the contribution of each particle being proportional to its mass.

Cartesian coordinates of the centre of mass : If x_{CM}, y_{CM} and z_{CM} are the Cartesian coordinates of the centre of mass of the n–particle system, then

$$\left.\begin{aligned} x_{CM} &= \frac{m_1x_1 + m_2x_2 + ...+ m_n x_n}{m_1 + m_2 + ...+ m_n} = \frac{\sum mx}{M} \\ y_{CM} &= \frac{m_1y_1 + m_2y_2 + ...+ m_n y_n}{m_1 + m_2 + ...+ m_n} = \frac{\sum my}{M} \\ z_{CM} &= \frac{m_1z_1 + m_2z_2 + ...+ m_n z_n}{m_1 + m_2 + ...+ m_n} = \frac{\sum mz}{M} \end{aligned}\right\} \quad ....(2)$$

Equations of motion for the centre of mass : Let $\vec{F}_1, \vec{F}_2, \vec{F}_3, ...\vec{F}_n$ be the external forces acting on the particles of masses $m_1, m_2, m_3, ..., m_n$ respectively. Let $\vec{F}_{tot}$ be the vector sum of all these external forces acting on the system. If $\vec{a}_{CM}$ is the acceleration of the centre of mass of the system, then the motion of the centre of mass is governed by the equation

$$M\vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n \quad \text{or} \quad M\vec{a}_{CM} = \vec{F}_{tot} \quad \text{where} \quad \vec{a}_{CM} = \frac{d^2\vec{R}_{CM}}{dt^2} \quad ....(3)$$

The equation (3) shows that the centre of mass of the system moves as if the entire mass of the system is concentrated at this point and this total external force acts on this point. The internal forces between various particles cancel out in pairs in accordance with Newton’s third law of motion. The definition of centre of mass given by equation (1) holds even though there may not be any actual matter present at the centre of mass.

Note : Inc case of a body with a continuous mass distribution, we can replace the summations in equations (2) by the following integrals :

$$\sum mx \to \int x\, dm, \quad \sum my \to \int y\, dm, \quad \sum Mz \to \int z\, dm$$

Then the coordinate of the centre of mass of a body of mass M will be

$$x_{CM} = \frac{1}{M}\int x\, dm, \quad y_{CM} = \frac{1}{2}\int y\, dm, \quad z_{CM} = \frac{1}{M}\int z\, dm$$

The equivalent vector representation for the centre of mass will be

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$

It we choose, the centre of mass as the origin of our coordinate system, then

$$\vec{R}_{CM} = (x, y, z) = 0 \qquad \text{or} \qquad \int \vec{r}\, dm = 0 \qquad \text{or} \qquad \int x\, dm = \int y\, dm = \int z\, dm = 0$$

Velocity of *CM* is constant in the absence of external force : Suppose an external force $\vec{F}_{tot}$ acts on a system of mass M and produces an acceleration $\vec{a}_{CM}$ in its centre of mass

$$\vec{F}_{tot} = M\vec{a}_{CM}$$

In the absence of any external force, $\vec{F}_{tot} = 0$, so $M\vec{a}_{CM} = 0$

$$\text{or} \qquad \vec{a}_{CM} = 0 \qquad \text{or} \qquad \frac{d\vec{v}_{CM}}{dt} = 0$$

As the derivative of a constant is zero, so $\vec{v}_{CM}$ = constant.

where $\vec{v}_{CM}$ is the velocity of the centre of mass. Hence in the absence of any external force, the centre of mass of system moves with a uniform velocity. This is Newton's first law of motion. The position vector of the centre of mass instant t is given by

$$\vec{R}_{CM}(t) = \vec{R}_{CM}(0) + \vec{v}_{CM}\, t$$

Momentum conservation and centre of mass motion

Consider a system of n particles of masses $m_1, m_2, m_3, ..., m_n$. Suppose the forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, ..., \vec{F}_n$ exerted on them produced accelerations $\vec{a}_1, \vec{a}_2, \vec{a}_3, ..., \vec{a}_n$ respectively. In the absence of any external force,

$$\vec{F}_{tot} = 0$$

$$\text{or} \qquad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$$

$$\text{or} \qquad m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ... + m_n\vec{a}_n = 0$$

$$\text{or} \qquad m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + ... + m_n\frac{d\vec{v}_n}{dt} = 0$$

$$\text{or} \qquad \frac{d}{dt}(m_1\vec{v}_1 + M_2\vec{v}_2 + M_3\vec{v}_3 + ... + m_n\vec{v}_n) = 0$$

$$\text{or} \qquad m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... + m_n\vec{v}_n = \text{constant}$$

$$\text{or} \qquad \vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ... + \vec{P}_n = \text{constant}$$

where $\vec{P}$ is the total linear momentum of the system.

Hence if no net external force acts on a system, the total linear momentum of the system is conserved. This is the law of conservation of linear momentum.

Now the position vector of the centre of mass of n–particle system is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...+ m_n\vec{r}_n}{m_1 + m_2 + m_3 + ...+ m_n} \quad \text{or} \quad \vec{R}_{CM} = \frac{1}{m}(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...+ m_n\vec{r}_n)$$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{R}_{CM}}{dt} = \frac{1}{M}\left(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + ...+ m_n\frac{d\vec{r}_n}{dt}\right)$$

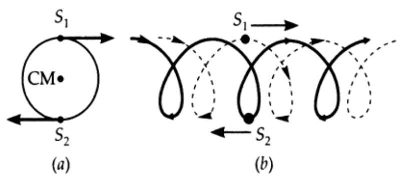
$$= \frac{1}{M}(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...+ m_n\vec{v}_n) = \frac{1}{M}(\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ...+ \vec{P}_n)$$

$$\text{or} \qquad \vec{v}_{CM} = \frac{1}{M}\vec{P} \quad \text{or} \quad \vec{P} = M\,\vec{V}_{CM}$$

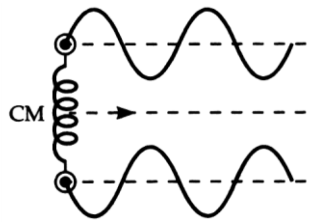
This equation shows that the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

Examples of binary systems in nature :

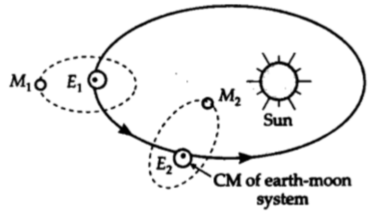
(i) **Binary stars :** Two stars bound to each other by the gravitational force and orbiting around their common centre of mass are called binary stars. Figure (a) shows binary stars S_1 and S_2 of equal mass moving in circular orbits around their common centre of mass, which is at rest. When no external force acts on the system, the centre of mass of the double star moves like a free particle. The orbits of the two stars are slightly complicated as shown in fig. (b).



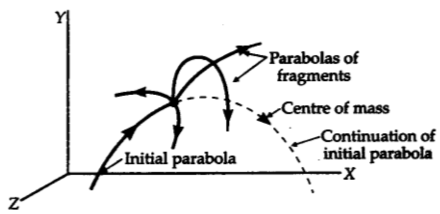
(ii) **Diatomic molecule :** A symmetric diatomic molecule like O_2 is also an example of binary system. The internal binding force between the two oxygen atoms is due to the chemical bond which can be regarded as a spring. When there is no external force (i.e., no collisions between the molecules themselves or with the walls of the vessel), the centre of mass of the line. As shown in figure.



(iii) **Earth–moon system :** The moon moves around the earth in a circular orbit and the earth moves around the sun in an elliptical orbit. It will be more correct to say that the centre of mass of the earth–moon system moves around the sun in an elliptical orbit, not the earth and moon themselves. As shown in figure.

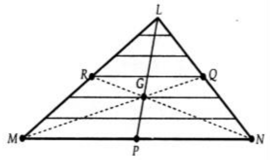


Motion of the CM of fire crackers exploding in air : Initially, a fire cracker moves along a parabolic path. It explodes in flight. Each fragment will follow its own parabolic path. Since the explosion is caused by internal forces only, the centre of mass of all the fragments will continue to move along the same parabolic path of the cracker as before explosion.



Subjective Assignment –I

1. Find the centre of mass of a triangular lamina.

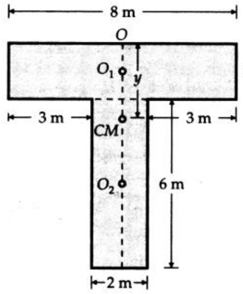


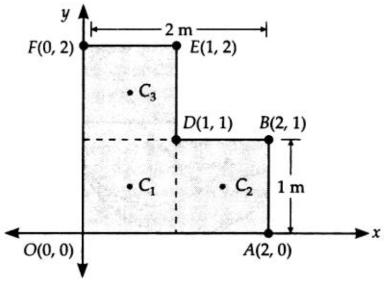
2. Three masses 3, 4 and 5kg are located at the corners of an equilateral triangle of side 1`m Locate the centre of mass of the system.

3. Two particles of masses 100g and 300g at a given time have positions $2\hat{i}+5\hat{j}+13\hat{k}$ and $-6\hat{i}+4\hat{j}-2\hat{k}$ m respectively and velocities $10\hat{i}-7\hat{j}-3\hat{k}$ and $7\hat{i}-9\hat{j}+6\hat{k}$ ms^{-1} respectively. Determine the instantaneous position and velocity of *CM*.

4. If three point masses m_1, m_2 and m_3 are situated at the vertices of an equilateral triangle of side a, then what will be the coordinates of the centre of mass of this system?

5. Find the position of the centre of mass of the *T* shaped plate from *O* in Fig.

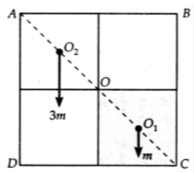




6. Find the centre of mass of a uniform L–shaped lamina (a thin flat plate) with dimensions as shown in fig. The mass of lamina is 3 kg.

7. A circular plate of uniform thickness has a diameter of 56cm. A circular portion of diameter 42 cm is removed from one edge of the plate. Find *CM* of the remaining portion.

8. A square of side 4 cm and uniform thickness is divided into four equal square as shown in figure. If one of the squares is cut off, find the position of the centre of mass of the remaining portion from *O*.



9. Show that centre of mass of a uniform rod of mass *M* and length *L* lies at middle point of the rod.

10. Determine the position of the centre of mass of a hemisphere of radius *R*.

11. Determine the coordinates of the centre of mass of a right circular solid cone of base radius *R* and height *h*.

12. Two bodies of mass 1 kg and 2kg are located at (1, 2) and (– 1, 3) respectively. Calculate the co-ordinates of the centre of mass.

13. The distance between the centres of carbon and oxygen atoms in the carbon monoxide gas molecule is 1.13 Å. Locate the centre of mass of the gas molecule relative to the carbon atom.

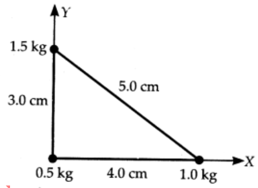
14. Three blocks of uniform thickness and masses m, m and $2m$ are placed at three corners of a triangle having co-ordinates $(2.5, 1.5), (3.5, 1.5)$ & $(3, 3)$ respectively. Find centre of mass of system.

15. Find the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100g, 150g and 200g respectively. Each side of the equilateral triangle is 0.5 m long.

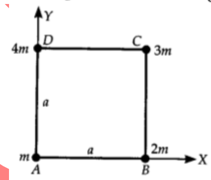
16. Three particles each of mass *m* are placed at three corners of an equilateral triangle of length *l*. Find the position of centre of mass in terms of coordinates.

17. Locate the centre of mass of a system of three particles 1.0 kg, 2.0 kg and 3.0 kg placed at the corners of an equilateral triangle of side 1m.

18. Two bodies of masses 10 kg and 2 kg are moving with velocities $2\hat{i}-7\hat{j}+3\hat{k}$ and $-10\hat{i}+35\hat{j}-3\hat{k}\ \text{ms}^{-1}$ respectively. Find the velocity of the centre of mass of the system.

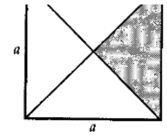


19. Three particles of masses 0.50 kg, 1.0 kg and 1.5 kg are placed at the corners of a right angle triangle, as shown in fig. Locate the centre of mass of the system.



20. Four particles of masses m, m, $2m$ and $2m$ are placed at the four corners of a square of side *a*. Find the centre of mass of the system.

21. Four particles of masses m, $2m$, $3m$ and $4m$ respectively are placed as the corners of a square of side *a*, as shown in fig. Locate the centre of mass.



22. From a square sheet of uniform density, a portion is removed shown shaded in Fig. Find the centre of mass of the remaining portion if the side of the square is *a*.

23. The centre of mass of three particles of masses 1 kg, 2kg and 3kg lies at the point (3m, 3m, 3m). Where should a fourth particle of mass 4kg be positioned so that the centre of mass of the four particle system lies at the point (1m, 1m, 1m)?

Answers

1. Centroid *G* of the triangle.

2. (0.54 , 0.36) m

3. $\vec{R}_{CM} = \dfrac{-16\hat{i}+17\hat{j}+7\hat{k}}{4}\ \text{m}$; $\vec{v}_{CM} = \dfrac{31\hat{i}-34\hat{j}+15\hat{k}}{4}\ \text{ms}^{-1}$

4. $x_{CM} = \dfrac{m_2 a + m_3(a/2)}{m_1+m_2+m_3}$; $y_{CM} = \dfrac{m_2\sqrt{3}\,a}{2(m_1+m_2+m_3)}$

5. $y = 2.71$ m

6. $x_{CM} = \dfrac{5}{6}$ m ; $y_{CM} = \dfrac{5}{6}$ m

7. 9 cm

8. $OO_2 = \dfrac{\sqrt{2}}{3}$ cm

9. $\dfrac{L}{2}$

10. $\left(\dfrac{3}{8}R, 0, 0\right)$

11. $\left(0, \dfrac{h}{4}, 0\right)$

12. $\left(-\dfrac{1}{3}, \dfrac{8}{3}\right)$

13. 0.6457 Å

14. (3, 2.25)

15. $\dfrac{5}{18}$ m, $\dfrac{1}{3\sqrt{3}}$ m

16. $\left(\dfrac{l}{2}, \dfrac{l}{2\sqrt{3}}\right)$

17. $\left(\dfrac{7}{12}\text{m}, \dfrac{\sqrt{3}}{4}\text{m}\right)$ with 1.0 kg mass at the origin.

18. $2\hat{k}\ \text{ms}^{-1}$

19. (1.33cm, 1.5 cm)

20. $\left(\dfrac{a}{2}, \dfrac{2}{3}a\right)$ with first mass *m* at the origin.

21. $\left(\dfrac{a}{2}, \dfrac{7a}{10}\right)$

22. $\left(\dfrac{7}{18}a, \dfrac{a}{2}\right)$

23. (– 2m, – 2m, – 2m)

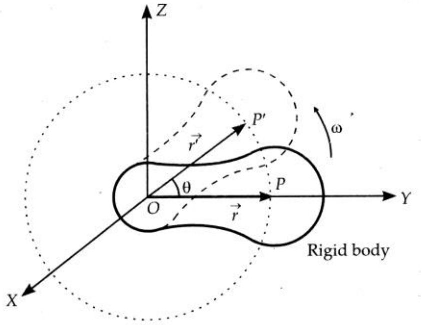
A body is said to be rigid if it does not undergo any change in its size and shape, however large the external force may be acting on it. More appropriately, a rigid body is one whose constituent particles retain their relative positions even when they move under the action of an external force. A rigid body cannot be deformed. If the body under-goes some displacement, every particle in it surfers the same displacement. If the body rotates through a certain angle, every particle of it rotates through the certain angle about the axis of rotation. Nobody can be perfectly rigid. In practice, solid bodies of steel, glass etc; can be regarded as rigid for moderate forces.

Centre of mass of a rigid body :

S.No.	Shape of body	Position of centre of mass
1.	Long thin rod	Middle point of the rod.
2.	Thin circular ring	Geometrical centre of the ring.
3.	Circular disc	Geometrical centre of the disc.
4.	Rectangular lamina	Point of intersection of diagonals.
5.	Rectangular cubical block	Point of intersection of diagonals.
6.	Cylinder	Middle point of the axis.
7.	Solid or hollow sphere	Geometrical centre of the sphere.
8.	Triangular lamina	Point of intersection of the medians.
9.	Right circular cone	A point on its axis at a distance of $\frac{h}{4}$ from its base, h = height of the cone.

Rotational motion of a Rigid body

A body is said to possess rotrational motion if all its particles move along circles in parallel planes. The centres of these circles lie on a fixed line perpendicular to the parallel planes and is called the **axis of rotation.** Fig shows a rigid body being rotated anticlockwise about Z–axis of an inertial frame of reference.

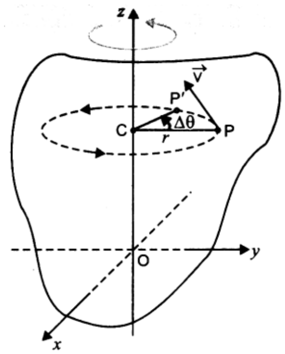


Let P be any particle of the body and $\vec{r}$ be its position vector. As the body rotates , the particle P moves along a circle of radius r whose centre lies on the axis of rotation. The radius vector $\vec{r}$ sweeps out an angle θ in certain time t. Similarly, all other particles of body move along circles with their centres on Z–axis and their radius vectors sweep the same angle θ in time t. This implies that all the particles have the same angular velocity $\omega(=\theta/t)$ which is also the angular velocity of the body.

Angular velocity : Angular velocity of a particle as the time rate of change of its angular displacement.

i.e., Angular velocity $= \dfrac{\text{angular displacement}}{\text{time taken}} = \dfrac{\Delta\theta}{\Delta t}$

At Δt tends to zero, the ratio $\dfrac{\Delta\theta}{\Delta t}$ approaches a limit which is the instantaneous angular velocity $\left(\dfrac{d\theta}{dt}\right)$ of the particle at P. We represent the instantaneous angular velocity by the Greek letter ω (omega).

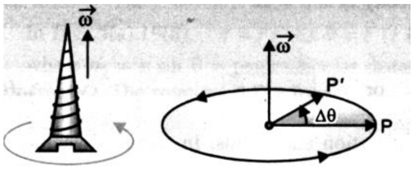


$$\boxed{\omega = \frac{d\theta}{dt}}$$

Angular velocity is measured in radian/sec and its dimensional formula is $[M^0 L^0 T^{-1}]$.

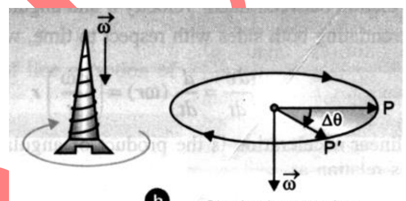
Angular velocity is a vector quantity whose direction is given by **right handed screw rule**.

For rotation about a fixed axis, the angular velocity vector lies along the axis of rotation and points out in the direction in which the tip of right handed screw would advance if the head of the screw is rotated with the body. For anticlockwise rotation, the direction of angular velocity $\vec{\omega}$ is along the axis of rotation and directed upwards. As shown in figure (a).



Anti clock wise rotation, $\vec{\omega}$ upwards

For clockwise rotation the direction of angular velocity $\vec{\omega}$ is along the axis of rotation and directed downwards. As shown in figure (b).



Clock wise rotation, $\vec{\omega}$ downwards

Linear velocity $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$

For particles on the axis, $r = 0$ and hence $v = \omega\, r = 0$. Thus, particles on the axis are stationary. Thus verifies that the axis is fixed. Further , as the body is rigid, all the particles of the body complete one revolution about the given axis together, in the same time.

Therefore, time period (T) of all the particles is the same. Therefore, $\omega = \frac{2\pi}{T}$ for all the particle is the same.

Thus just as in pure translation, all particles of the body have the same linear velocity at any instant of time, similarly, in pure rotation, all particles of the body have the same angular velocity at any instant of time.

Angular Acceleration

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

It is represented by Greek letter α(alpha).

i.e., angular acceleration = $\frac{\text{change in angular velocity}}{\text{time taken}}$.

$$\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt}}$$

SI unit of α is rad s^{-2} and its dimensional formula is $[M^\circ L^\circ T^{-2}]$. Angular acceleration is a vector quantity.

Linear acceleration $\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$.

Equations of rotational motion

Derivation of first equation of motion : Consider a rigid body rotating about a fixed axis with constant angular acceleration α. By definition,

$$\alpha = \frac{d\omega}{dt} \quad \text{or} \quad d\omega = \alpha\, dt \qquad \text{....(1)}$$

At $t = 0$, let $\omega = \omega_0$ At $t = t$, let $\omega = \omega$

Integrating equation (1) within the above limits of time and angular velocity, we get

$$\int_{\omega_0}^{\omega} d\omega \int_0^t \alpha\, dt = \alpha \int_0^t dt \qquad \text{or} \qquad [\,\omega\,]_{\omega_0}^{\omega} = \alpha [\,t\,]_0^t$$

$$\text{or} \qquad \omega - \omega_0 = \alpha(t-0) \qquad \text{or} \qquad \boxed{\omega = \omega_0 + \alpha t} \qquad \text{....(2)}$$

Derivation of second equation of motion : Let ω be the angular velocity of a rigid body at any instant *t*. By definition,

$$\omega = \frac{d\theta}{dt} \qquad \text{or} \qquad d\theta = \omega\, dt \qquad \text{....(3)}$$

At $t = 0$, let $\theta = 0$

At $t = t$, let $\theta = \theta$

Integrating equation (3) within the above limits of time and angular displacement, we get

$$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t (\omega_0 + \alpha t)\, dt \qquad \text{[Using (2)]}$$

$$= \omega_0 \int_0^t dt + \alpha \int_0^t t\, dt$$

$$[\theta]_0^{\theta} = \omega_0 [\,t\,]_0^t + \alpha \left[\frac{t^2}{2}\right]_0^t \qquad \text{or} \qquad \theta - 0 = \omega_0(t-0) + \frac{\alpha}{2}(t^2 - 0)$$

$$\text{or} \qquad \boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2} \qquad \text{....(4)}$$

Derivation of third equation of motion : The angular acceleration α may be expressed as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\,.\,\omega \qquad \text{or} \qquad \omega\, d\omega = \alpha\, d\theta \qquad \text{....(5)}$$

At $t = 0$, $\theta = 0$ and $\omega = \omega_0$ (initial angular velocity). At $t = t$, $\theta = \theta$ and $\omega = \omega$ (final angular velocity).

Integrating equation (5) within the above limits of θ and ω, we get

$$\int_{\omega_0}^{\omega} \omega\, d\omega = \int_0^{\theta} \alpha\, d\theta = \alpha \int_0^{\theta} d\theta \qquad \text{or} \qquad \left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \alpha [\,\theta\,]_0^{\theta}$$

$$\text{or} \qquad \frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \alpha(\theta - 0) \qquad \text{or} \qquad \boxed{\omega^2 - \omega_0^2 = 2\alpha\theta}$$

The angular displacement of body in *n*th second :

$$\theta_n^{\text{th}} = \omega_0 + \frac{\alpha}{2}(2n-1)$$

Subjective Assignment –II

1. On the application of a constant torque, a wheel is turned from rest through 400 radians in 10s.

(i) find angular acceleration.

(ii) If same torque continues to act , what will be angular velocity of the wheel after 20s from start?

2. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds.

(i) What is its angular acceleration, assuming the acceleration to be uniform?

(ii) How many revolutions does the wheel make during this time?

3. A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t second, show that the angular acceleration is given by $\alpha = \dfrac{4\pi n}{t^2}$ rad s^{-2}.

4. The radius of a wheel of a car is 0.4m. The car is accelerated from rest by an angular acceleration of 1.5 rad s^{-2} for 20s. How much distance the wheel covers in this time interval and what will be its linear velocity?

5. A grindstone has a constant acceleration of 4 rad s^{-2}. Starting from rest, calculate the angular speed of the grindstone 2.5 s later.

6. The speed of a motor increases from 600 rpm to 1200 rpm in 20s. What is its angular acceleration and how many revolutions does it make during this time?

7. On the application of a constant torque, a wheel is turned from rest through an angle of 200 rad in 8s.

(i) What is its angular acceleration? **(ii)** If the same torque continues to act, what will be the angular velocity of the wheel after 16s from the start?

8. The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15s after being switched off. Assuming constant angular deceleration, calculate the number of revolutions made by it before coming to rest.

9. A car is moving at a speed of 72 kmh^{-1}. The diameter of its wheels is 0.50m. If the wheels are stopped in 20 rotations by applying brakes, calculate the angular retardation produced by the brakes.

Answers

1.	(i) 8 rad s^{-2} (ii) 160 rad s^{-1}	2.	(i) 4π rad s^{-2} (ii) 576		
4.	$s = 120$ m ; $\omega = 30$ rad s^{-1} ; $v = 12$ ms^{-1}				
5.	10 rad s^{-1}	6.	π rad s^{-2}, 300	7.	(i) 6.25 rad s^{-2} (ii) 100 rad s^{-1}
8.	12.5	9.	-25.5 rad s^{-2}		

Moment of force or torque

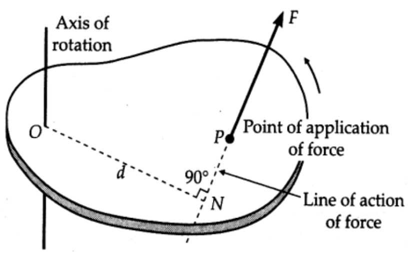
The turning effect of force is called moment of force or torque. It depends on two factors.

(i) The magnitude of the force

(ii) The perpendicular distance of the line of action of the force from the axis of rotation. It is called lever arm or moment arm.

Thus, greater the magnitude of the force, or greater the perpendicular distance between the line of action of the force and the axis of rotation, the greater is the moment of force, or greater is the turning effect.

The torque or moment of force is the turning effect of the force about the axis of rotation. It is measured as the product of the magnitude of the force and the perpendicular distance between the line of action of the force and the axis of rotation. Fig. shows a body free to rotate about a vertical axis through O. A horizontal force F applied on it at point P rotates it about this axis. If d is the perpendicular distance of the line of action of the force from the axis of rotation, then the torque or moment of force F about the axis of rotation is $\tau = F \times ON$



or $\tau = F \times d$ or Torque = Force × Lever arm

Dimensions of torque : As

Torque = Force × distance , so $\left[\tau\right]=\left[MLT^{-2}\right]\left[L\right]=\left[ML^{2}T^{-2}\right]$

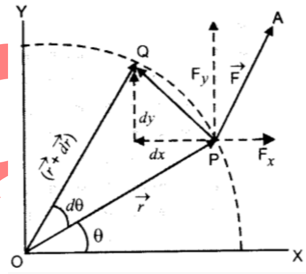
Units of torque : The *SI* unit of torque is netwon metre (*Nm*) and its *CGS* unit is dyne cm.

Sign convention : The moment of force is taken positive if the turning tendency of the force is anticlockwise and negative if it is clockwise.

Expression for torque in Cartesian co–ordinates from rotation of a particle in a plane : Physical meaning of torque.

Consider a particle of mass *m* rotating in plane *XY* about a fixed axis *OZ* (not shown). Let *P* be the position of the particle at any instant, where $\vec{OP}=\vec{r}$ and $\angle XOP=\theta$. Let the rotation occur under the action of a force $\vec{F}$ applied at *P*, along $\vec{PA}$, as shown in the figure.

In a small time *dt*, let the particle at *P* reach *Q* , where $\overrightarrow{OQ}=(\vec{r}+d\vec{r})$ and $\angle POQ=d\theta$.



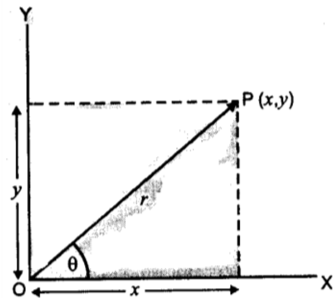
In vector triangle *OPQ* , $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=(\vec{r}+d\vec{r})-\vec{r}$

$\therefore \qquad \overrightarrow{PQ}=d\vec{r}$

Small amount of work done in rotating the particle from *P* to *Q* is

$$dW=\vec{F}\,.\,d\vec{r}$$

If F_x, F_y are rectangular components of force $\vec{F}$ and *dx, dy* are rectangular components of displacement $d\vec{r}$,



then $\left.\begin{aligned}\vec{F}&=(\hat{i}\,F_x+\hat{j}\,F_y)\\ d\vec{r}&=(\hat{i}\,dx+\hat{j}\,dy)\end{aligned}\right\}$ ....(1)

From (1), we get $\quad dW=(\hat{i}\,F_x+\hat{j}\,F_y)\,.\,(\hat{i}\,dx+\hat{j}\,dy)$

$$dW=F_x dx+F_y\,dy \qquad ....(2)$$

Let the co-ordinates of the point *P* be (x,y).

It is clear from fig. $\quad x=r\cos\theta \qquad ....(3)$

$\qquad y=r\sin\theta \qquad ....(4)$

Differentiating (2) w.r.t. θ, we get

$$\frac{dx}{d\theta}=\frac{d}{d\theta}(r\cos\theta)=r\frac{d}{d\theta}(\cos\theta)=r(-\sin\theta)$$

$\therefore \qquad dx=-y\;d\theta$

Again, differentiating (3), w.r.t. θ, we get

$$\frac{dy}{d\theta}=\frac{d}{d\theta}(r\sin\theta)=r\cos\theta=x$$

$\therefore \qquad dy=x\;d\theta$

Substituting in (2), we get $dW = F_x(-y\,d\theta) + F_y(x\,d\theta) = xF_y d\theta - yF_x\,d\theta$

$$dW = (xF_y - yF_x)\,d\theta$$

or $dW = \tau(d\theta)$ where $\boxed{\tau = (xF_y - yF_x)}$. ....(5)

τ is called torque. Equation (5) is the expression for torque in Cartesian co–ordinates.

Torque τ is a quantity in rotational motion, which when multiplied by a small angular displacement gives us work done in rotational motion. This quantity corresponds to force in linear motion, which when multiplied by a small linear displacement gives us work done in linear motion. This is the physical meaning of torque.

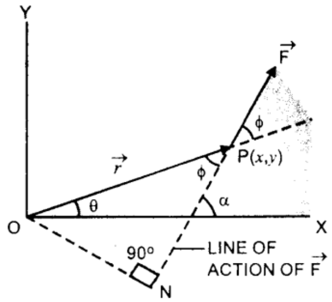
Expression for Torque in polar co–ordinates

Suppose the line of action of force $\vec{F}$ makes an angle α with X–axis, as shown in figure. Two rectangular components of F are :

$\therefore$ $F_x = F\cos\alpha$ ....(1)

$F_y = F\sin\alpha$ ....(2)

If x, y are the co–ordinates of the point P, where $\overrightarrow{OP} = \vec{r}$ and $\angle XOP = \theta$



then $x = r\cos\theta$ ....(3)

and $r\sin\theta$ ....(4)

Substituting these values in expression for Cartesian coordinates of torque,

$$\tau = xF_y - yF_x$$

$$\therefore \quad \tau = (r\cos\theta)F\sin\alpha - (r\sin\theta)(F\cos\alpha)$$

$$= rF[\sin\alpha\cos\theta - \cos\alpha\sin\theta]$$

$$\tau = rF\sin(\alpha - \theta) \quad \text{....(5)}$$

Let ϕ be the angle which the line of action of $\vec{F}$ makes with the position vector $\overrightarrow{OP} = \vec{r}$.

As is clear from figure. $\theta + \phi = \alpha$ or $\phi = \alpha - \theta$

Putting in (5), we get $\boxed{\tau = rF\sin\phi}$ ....(6)

Equation (6) is the expression for torque in polar co-ordinates.

This equation shows that ability of $\vec{F}$ to rotate the body depends not only on the magnitude of $\vec{F}$, but also on just how far from O, the force is applied and in what direction.

In figure, draw $ON \perp$ on line of action of $\vec{F}$.

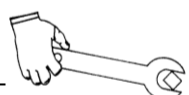
In ΔOPN, $\sin\phi = \dfrac{ON}{OP} = \dfrac{ON}{r}$, $\therefore$ $ON = r\sin\phi$

$$\tau = rF\sin\phi = F(r\sin\phi) = F(ON) \quad \text{....(7)}$$

Hence, torque due to a force is the product of force and perpendicular distance of line of action of force from the axis of rotation.

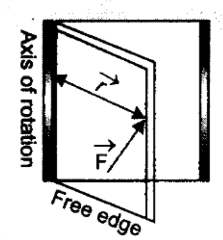
From equation (7), we find that

(i) Torque due to a force is maximum, when r is maximum. For example, we can open or close a door easily by applying force near the edge of the door (at maximum distance



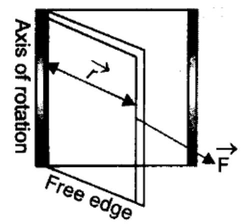
from the hinges). That is why a handle/knob is provided near the free edge of the plank of the door. Similarly, to unscrew a nut fitted tightly to a bolt, we need a **wrench with a long arm**, as shown in figure.

When the length of arm (r) is long, the force (F) required to produced a given turning effect $(\tau = rF\sin\phi)$ is smaller.



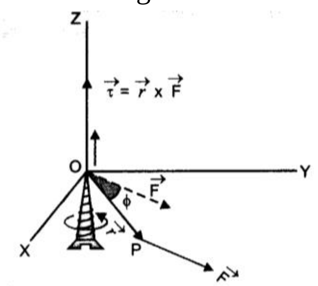
(ii) Torque will be maximum when $\sin\phi = \max. = +1$. Therefore, $\phi = 90°$, i.e., when force is applied in a direction perpendicular to $\vec{r}$. for example, it is easiest to open or close a door by applying force at the edge of the plank in a direction perpendicular to the plank of the door, as shown in fig

(iii) When $\phi = 0°$ or $180°$, $\sin\phi = \sin 0°$ or $\sin 180° = 0$. Therefore, $\tau = r\,F\sin\phi = 0$. i.e., torque of the force is zero. For example, the door cannot be rotated by applying force in a direction parallel to the plank of the door. as shown in the figure.



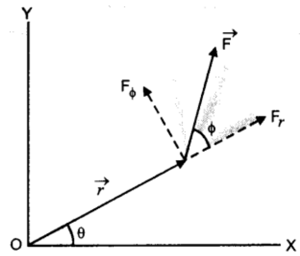
(iv) Equation (7) can be rewritten in vector from as $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$ ....(8)

Obviously, torque is a vector quantity whose direction is given by right handed screw rule or right hand thumb rule.



Note : Fig . shows relative orientation of $\vec{r}$ and $\vec{F}$. Force $\vec{F}$ acting actually at P has been shifted to origin O, in a direction parallel to itself. When $\vec{F}$ rotates the particle in XY plane in anticlockwise direction, the tip of the right handed screw moves along the positive Z direction, which is the direction of the torque τ.

In fig. we have shown two rectangular components of $\vec{F}$: $\vec{F_r} = F\cos\phi$, in the direction of increasing r. It is called "radial component of $\vec{F}$, and $F_\phi = F\sin\phi$ in the direction of increasing θ and perpendicular to F_r, this is called **transverse component** of $\vec{F}$.



From (6), $\tau = rF\sin\phi = r(F\sin\phi) = rF_\phi$

i.e., torque of a force is given by the product of transverse component of force and perpendicular distance from the axis of rotation. Thus torque is due to transverse component only. The radial component of force has no role to play in the torque.

Rectangular components of torque

For three dimensional motion, the position vector $\vec{r}$, and torque vector $\vec{r}$ can be written in terms of their rectangular components as follows :

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \qquad \text{and} \qquad \vec{\tau} = \tau_x\hat{i} + \tau_y\hat{j} + \tau_z\hat{k}$$

Now, $\vec{\tau} = \vec{r} \times \vec{F} \qquad = (\;+ y\hat{j} + z\hat{k}) \times (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

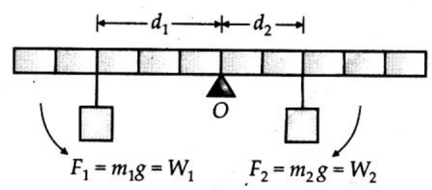
or $$\tau_x \hat{i} + \tau_y \hat{j} + \tau_z \hat{k} = \hat{i}\,(yF_z - zF_y) + \hat{j}\,(zF_x - xF_z) + \hat{k}\,(xF_y - yF_x)$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on the two sides of the above equation, we get the rectangular components of torque $\vec{\tau}$ as follows :

$$\tau_x = xF_z - zF_y \quad ; \quad \tau_y = zF_x - xF_z \quad ; \quad \tau_z = xF_y - yF_x$$

Rotational equilibrium and the principle of moments : Principle of moments :

When a body is in rotational equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point or the algebraic sum of moments about any point is zero. As shown in fig. consider a uniform rod free to rotate on a pivot O. Two weights W_1 and W_2 are hung from it at distances d_1 and d_2 from the pivot O.



Anticlockwise moment about $O = F_1 \times d_1 = W_1 \times d_1$

Clockwise moment about $O = F_2 \times d_2 = W_2 \times d_2$.

According to the principle of moments, the rod will be horizontal or in rotational equilibrium if

Anticlock wise moment = Clockwise moment

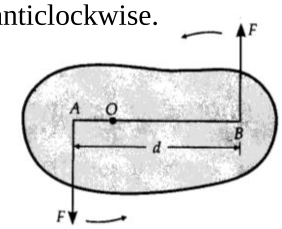
or $F_1 \times d_1 = F_2 \times d_2$ or $W_1 \times d_1 = W_2 \times d_2$

i.e., Load $\times$ load arm = Effort $\times$ effort arm

This is sometimes called the lever principle.

Couple : A pair of equal and opposite forces acting on a body along two different lines of action constitute a couple. A couple has a turning effect, but no resultant force acts on a body. So it cannot produce translational motion.

Moment of a couple : The moment of couple can be found by taking the moments of the two forces about any point and then adding them. In fig. , two opposite forces, each of magnitude F act at two points A and B of a rigid body, which can rotate about point O. The turning tendency of the two forces is anticlockwise.



Moment or torque of the couple about O is

$$\tau = F \times AO + F \times OB = F(AO + OB) = F \times AB$$

or $\tau = Fd$ $\quad [AB = d, \text{say}]$

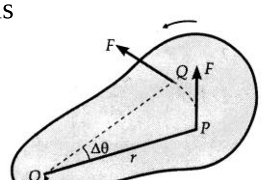
Moment of a couple = Force $\times$ perpendicular distance between two forces

Hence the moment of a couple is equal to the product of either of the forces and the perpendicular distance, called the arm of the couple, between their lines of action. Note that the torque exerted by couple about O does not depend on the position of O. Hence torque or moment of a couple is independent of the choice of the fulcrum or the point of rotation. Notably, a couple can only be balanced by an equal and opposite couple.

Work done by a torque

As shown in figure, suppose a body undergoes an angular displacement $\Delta\theta$ under the action of a tangential force F.

The work done in the rotational motion of the body or the work done by the torque is



$$\Delta W = F \times \text{ distance along the are } PQ$$

But $\quad \Delta\theta = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{\text{Arc } PQ}{r}$

$\therefore \quad \text{Arc } PQ = r\,\Delta\theta$

Hence $\quad \Delta W = Fr\,\Delta\theta \quad \text{or} \quad \Delta W = \tau\Delta\theta$

i.e., Work done by a torque = Torque × angular displacement

In case the torque applied is not constant, but variable, the total work done by the torque is given by

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

Power delivered by a torque : We know that $\Delta W = \tau\Delta\theta$

Dividing both sides by Δt, we get $\dfrac{\Delta W}{\Delta t} = \tau\dfrac{\Delta\theta}{\Delta t} \quad \text{or} \quad P = \tau\omega$

i.e., Power = Torque × Angular velocity.

Angular momentum

In linear motion, the linear momentum of a body gives a measure of its translatory motion. Analogous to it in rotational motion, the angular momentum gives a measure of the turning motion of the body. The angular momentum of a particle rotating about an axis is defined as the moment of the linear momentum of the particle about that axis. It is measured as the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

Angular momentum = Linear momentum × its perpendicular distance from the axis of rotation

$$L = pd$$

Dimensions of angular momentum

$$= L \times MLT^{-1} = \left[ML^2T^{-1}\right]$$

SI unit of angular momentum is kg m^2 s^{-1}.

CGS unit of angular momentum is *g* cm^2 s^{-1}.

The concept of angular momentum expression for angular momentum in Cartesian co–ordinates

If we can represent torque as the rate of change of some quantity, that quantity will be rotational analogue of linear momentum, and it can appropriately be called the angular momentum.

In order to express torque as the rate of change of some quantity, we rewrite expression for torque rotating a particle in *XY* plane,

$$\tau = xF_y - yF_x \qquad \text{....(1)}$$

If $p_x = mv_x$ and $p_y = mv_y$ are the x and y components of linear momentum of the body, then According to Newton's 2nd law of motion,

$$F_x = \frac{dp_x}{dt} = \frac{d}{dt}(mv_x) = m\frac{dv_x}{dt} \quad \text{and} \quad F_y = \frac{dp_y}{dt} = \frac{d}{dt}(mv_y) = m\frac{dv_y}{dt}$$

Substituting in (1), we get $\tau = xm\dfrac{dv_y}{dt} - ym\dfrac{dv_x}{dt}$ or $\tau = m\left[x\dfrac{dv_y}{dt} - y\dfrac{dv_x}{dt}\right]$ ....(2)

Using rule for differentiation of product, we may write,

$$\frac{d}{dt}(xv_y - yv_x) = x\frac{dv_y}{dt} + v_y\frac{dx}{dt} - y\frac{dv_x}{dt} - v_x\frac{dy}{dt}$$

$$= x\frac{dv_y}{dt} + v_y v_x - y\frac{dv_x}{dt} - v_x v_y \qquad \left[\because \frac{dx}{dt} = v_x \,;\, \frac{dy}{dt} = v_y\right]$$

$$\frac{d}{dt}(xv_y - yv_x) = x\frac{dv_y}{dt} - y\frac{dv_x}{dt} \qquad ....(3)$$

Substituting (3) in (2), we obtain $\tau = m\frac{d}{dt}(xv_y - yv_x)$

$$\tau = \frac{d}{dt}(xmv_y - ymv_x)$$

As $mv_y = p_y$ and $mv_x = p_x$ $\therefore$ $\tau = \frac{d}{dt}(xP_y - yP_x)$

But $\qquad \tau = \frac{dL}{dt}$

$$\therefore \qquad \boxed{(xp_y - yp_x) = L} \qquad ....(4)$$

Thus, we have obtained torque τ as the rate of change of a quantity L defined by equation (4). We call this quantity L as the angular momentum of the body.

An expression for angular momentum in Cartesian co–ordinates. Thus basically, angular momentum of a particle/body about a given axis is the moment of linear momentum of the particle/body about this axis.

Expression for angular momentum in Polar co–ordinates

Suppose $K(x, y)$ is position of a particle of mass m and linear momentum $\vec{p}$ rotating in XY plane about Z–axis.

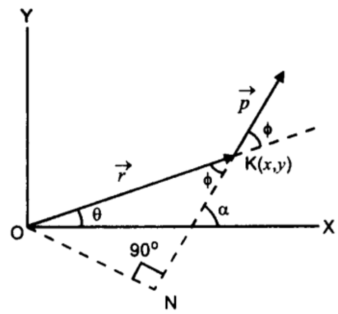
fig. Let $\overrightarrow{OK} = \vec{r}$ and $\angle XOK = \theta$

$$\therefore \qquad x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Let the line of action of linear momentum $\vec{p}$ make an angle α with OX and angle ϕ with $\vec{r}$.

$$\therefore \qquad p_x = p\cos\alpha \quad \text{and} \quad p_y = p\sin\alpha$$

Substituting these values in expression for Cartesian co-ordinates of angular momentum,



$$L = xp_y - yp_x$$

$$\therefore \qquad L = (r\cos\theta)(p\sin\alpha) - (r\sin\theta)(p\cos\alpha)$$

$$= pr[\sin\alpha\cos\theta - \cos\alpha\sin\theta]$$

$$L = pr\sin(\alpha - \theta)$$

From fig. , $\theta + \phi = \alpha$ $\therefore$ $\phi = \alpha - \theta$.

$$\therefore \qquad \boxed{L = pr\sin\phi} \qquad ....(1)$$

This is the expression for angular momentum of a particle in polar co-ordinates.

In the figure, draw ON perpendicular to the line of action of $\vec{P}$.

In $\quad \Delta OKN, \sin\phi = \frac{ON}{OK} = \frac{ON}{r}$

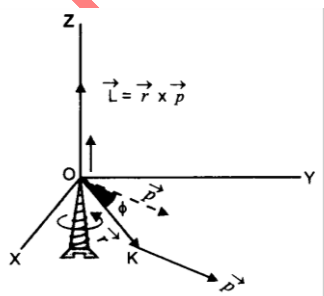
$ON = r\sin\phi$

Substituting in (1), we get $\boxed{L = p(ON)}$. ....(2)

Hence, angular momentum of a body abut a given axis is the product of linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation. This is the physical meaning of angular momentum.

In *SI*, the units of angular momentum are $(kg\,ms^{-1})(m) = kg\ m^2 s^{-1}$. This dimensional formula for angular momentum would be $\left[M^1 L^2 T^{-1}\right]$.

In equation (1), can be rewritten in vector from as $\boxed{\vec{L} = \vec{r} \times \vec{p}}$ ....(3)



This indicates that momentum is a vector quantity and its direction is given by right handed screw rule or right hand thumb rule. When the centre of mass rotates through smaller angle ϕ in *XY* plane, say from the direction of $\vec{r}$ to the direction of $\vec{p}$, the angular momentum vector $\vec{L}$ is along *OZ* i.e., the positive *Z*–direction, as illustrated in figure.

Note : 1. Proceeding as in the case of torque, we can show that radial component of linear momentum does not contribute to angular momentum of the particle. It is only the transverse component of linear momentum (perpendicular to position vector $\vec{r}$), which when multiplied by distance from the axis of rotation gives us angular momentum.

2. Again, proceeding as in case of torque, we may write three rectangular components of angular momentum as

$$\boxed{\begin{aligned} L_x &= yp_z - zp_y \\ L_y &= zp_x - xp_z \\ L_z &= xp_y - yp_x \end{aligned}}$$

Relation between torque and angular momentum

We know that , torque , $\quad \vec{\tau} = \vec{r} \times \vec{F}$

Angular momentum , $\quad \vec{L} = \vec{r} \times \vec{P}$

Differentiating both sides w.r.t. time *t*, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) \quad = \vec{r} \times \frac{d\vec{P}}{dt} + \vec{P} \times \frac{d\vec{r}}{dt}$$

$$= \vec{r} \times \vec{F} + \vec{P} \times \vec{v} \qquad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$$

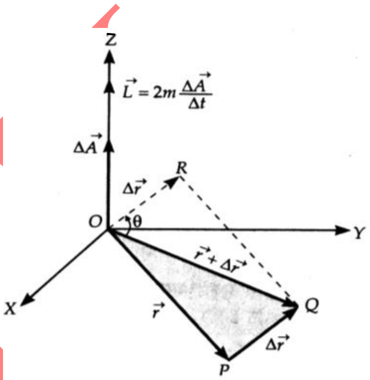
$$= \vec{\tau} + 0 \qquad \left[\because \vec{v} \times \vec{p} = \vec{v} \times m\vec{v} = 0\right]$$

$$\therefore \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Thus the torque acting on a particle is equal to its rate of change of angular momentum. This equation is the rotational analogue of Newton's second law of linear motion.

Geometrical meaning of angular momentum

Consider a particle of mass m rotating in the X–Y plane about the origin O. Let $\vec{r}$ and $(\vec{r} + \Delta\vec{r})$ be the position vectors of the particle at instants t and $(t + \Delta t)$ respectively, as shown in figure. The displacement of the particle in small time Δt is



$$\vec{PQ} = (\vec{r} + \Delta\vec{r}) - \vec{r} = \Delta\vec{r}$$

If $\vec{v}$ is the velocity of the particle at point P, then the small displacement covered in time Δt may be expressed as

$$\Delta\vec{r} = \vec{v}\,\Delta t$$

Complete the parallelogram $OPQR$. Then $\vec{OR} = \vec{PQ} = \Delta\vec{r}$.

Area of the parallelogram $OPQR = \vec{r} \times \Delta\vec{r}$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}(\vec{r} \times \Delta\vec{r})$.

The shaded area of ΔOPQ represents the area swept by the position vector in time Δt. By right hand rule, its direction is along Z–axis. If this area is represented by $\Delta\vec{A}$, then

$$\Delta\vec{A} = \frac{1}{2}(\vec{r} \times \Delta\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{v}\ \ \Delta t)$$

If $\vec{p}$ is the linear momentum of the particle, then

$$\vec{p} = m\vec{v} \qquad \text{or} \qquad \vec{v} = \frac{\vec{p}}{m}$$

$$\therefore \qquad \Delta\vec{A} = \frac{1}{2}\vec{r} \times \frac{\vec{p}}{m}\Delta t \qquad \text{or} \qquad \frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2m}(\vec{r} \times \vec{p})$$

But $\vec{r} \times \vec{p} = \vec{L}$, the angular momentum of the particle about Z–axis, so we have

$$\frac{\Delta\vec{A}}{\Delta t} = \frac{\vec{L}}{2m} = \frac{1}{2} \times \text{ Angular momentum per unit mass} \qquad \text{or} \qquad \vec{L} = 2m\frac{\Delta\vec{A}}{\Delta t}$$

The quantity $\Delta\vec{A}/\Delta t$ is the area swept out by the position vector per unit time and is called the areal velocity of the particle. Thus, Angular momentum = 2 × Mass × areal velocity.

This is the geometrical meaning of angular momentum. So geometrically, the angular momentum of a particle is equal to twice the product of its mass and areal velocity. Equivalently, we can say that the areal velocity of a particle is just half its angular momentum per unit mass.

Kepler's second law of planetary motion

A planet revolves around the sun under the influence of gravitational force which acts towards the sun i.e., the force is purely radial and tangential component F_t of the force is zero.

As torque, $\tau = rF_t$, therefore $\tau = 0$ or $\frac{dL}{dt} = 0$

or $L = \text{constant}$

or $2m\frac{\Delta A}{\Delta t} = \text{constant}$ $[\because L = 2m \times \text{areal velocity}]$

or $\frac{\Delta A}{\Delta t} = \text{constant}$ $[\because m \text{ of planet is constant}]$

This means that the areal velocity of a planet is constant. This is Kepler's second law of planetary motion which states that the line joining the planet to the sun sweeps out equal areas in equal intervals of time.

Torque and angular momentum for a system of particles

Consider a system of n particles. Let $\vec{L}_1, \vec{L}_2, \vec{L}_3, ...$ be the angular momenta of the various particles about the origin O of a reference frame. The total angular momentum of the system about the point O is given by the vector sum of angular momenta of all the individual particles. Thus

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + ... + \vec{L}_n$$

$$= \vec{r} \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + ... + \vec{r}_n \times \vec{p}_n$$

or
$$\vec{L} = \sum_{i=1}^{n} \vec{L}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

Similarly, the total torque acting on the system is equal to the vector sum of the torques of all the particles about the origin O. Thus

$$\vec{\tau}^{\,tot} = \vec{\tau}_1 + \vec{\tau}_2 + ... + \vec{\tau}_n$$

$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + ... \vec{r}_n \times \vec{F}_n$$

or
$$\vec{\tau}^{\,tot} = \sum_{i=1}^{n} \vec{\tau}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i$$

The total torque acting on the system is due to two sources :

(i) Torques exerted on the particles on the system by mutual internal forces between the particles.

(ii) Torques exerted on the individual particles of the system by the external forces.

According to Newton's third law, the internal torque on the system due to internal forces is zero because the forces between any two particles are equal and opposite and directed along the line joining the two particles. Hence the total torque is due to external forces only. So, we have

$$\vec{\tau}^{\,tot} = \vec{\tau}^{\,ext} = \sum_{i=1}^{n} \vec{\tau}_i^{\,tot}$$

This is in accordance with the common experience bodies do not start spinning on their own without external forces acting on them. Hence if the angular momentum of a system changes with time, this change can be due to the torques produced by external forces only. So, we can write $\vec{\tau}^{ext} = \dfrac{d\vec{L}}{dt}$.

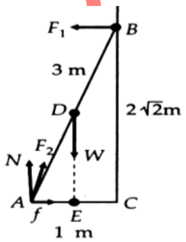
Thus the rate of change of total angular momentum of a system of particles about a fixed point is equal to the total external torque acting on the system about that point.

Subjective Assignment – III

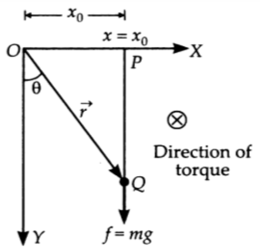
1. Find the torque of a force $7\hat{i} - 3\hat{j} - 5\hat{k}$ about the origin which acts on a particle whose position vector is $\hat{i} + \hat{j} - \hat{k}$.

2. A metal bar 70 cm along and 4.00 kg in mass is supported on two knife–edges placed 10 cm from each end. A 6.00 kg weight is suspended at 30 cm from one end. Find the reactions at the knife-edges. (Assume the bar to be of uniform cross section and homogenous). Take g = 9.8 ms^{-2}.

3. A 3m long ladder weighing 20 kg leans on a frictionless wall. Its feet rest on the floor 1m from the wall as shown in fig. Find the reaction forces of the wall and the floor.



4. A particle of mass *m* is released from point *P* at $x = x_0$ on the *X*–axis from origin *O* and falls vertically along the *Y*–axis ,as shown in fig.



(i) Find the torque τ acting on the particle at *a* time *t* when it is at point *Q* with respect to *O*.

(ii) Find the angular momentum *L* of the particle about *O* at this time *t*.

(iii) Show that $\tau = \dfrac{dL}{dt}$ in this example.

5. An electron of mass 9×10^{-31} kg revolves in a circle of radius 0.53 Å around the nucleus of hydrogen with a velocity of 2.2 × 10^6 ms^{-1}. Show that its angular momentum is equal to *h*/2where *h* is Planck's constant of value 6.6×10^{-34} Js.

6. Show that the angular momentum of a satellite of mass M_s is revolving around the earth having mass M_e in an orbit of radius *r* is $L = \sqrt{GM_e M_s^2 r}$.

7. Determine the angular momentum of a car of mass 1500 kg moving in a circular track of radius 50m with a speed of 40 ms^{-1}.

8. Mass of an electron is 9.0 × 10^{31} kg. It revolves around the nucleus of an atom in a circular orbit of radius 4.0 Å with a speed of 6.0 × 10^6 ms^{-1}. Calculate the angular momentum of the electron.

Answers

1.	$-8\hat{i} - 2\hat{j} - 10\hat{k}$	2.	55 N at K_1 and 43 *N* at K_2	3.	34.6 *u*, 199 *u*
4.	(i) mgx_0 (ii) $mgx_0\, t$	7.	3×10^6 $kgm^2 s^{-1}$	8.	2.16×10^{-33} $kgm^2 s^{-1}$

Equilibrium of rigid bodies

A rigid body is one for which the distances between different particles do not change, even though they move. Under the influence of an external force, a rigid body can execute two kinds of motion :

(i) Translational motion in which all particles move with the same velocity and **(ii)** rotational motion about an axis.

A rigid body is said to be in equilibrium if both the linear momentum and angular momentum of the rigid body remain constant with time. Hence for a body in equilibrium, the linear acceleration of its centre of mass would be zero and also the angular acceleration of the rigid body about any axis would be zero.

A body under the action of several forces will be in equilibrium, if it possesses the following two equilibrium simultaneously :

(i) Translational equilibrium : The resultant of all the external forces acting on the body must be zero, otherwise they would produce linear acceleration. Hence for translation equilibrium, $\sum \vec{F}_{ext} = 0$

or $$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum F_z = 0$$

Applying Newton's second law, $\sum \vec{F}_{ext} = M\vec{a}_{CM} = M\dfrac{d\vec{v}_{CM}}{dt} = 0$

or $$\frac{d\vec{v}_{CM}}{dt} = 0 \quad \text{or} \quad \vec{v}_{CM} = \text{constant}$$

This implies that a body in translational equilibrium will be either at rest ($v = 0$) or in uniform motion (v = constant). If the body is at rest, it is said to be in static equilibrium. It the body is in uniform motion along a straight path, it is in dynamic equilibrium.

(ii) Rotational equilibrium : The resultant of torques due to all the forces acting on the body about any point must be zero, otherwise they would produce angular acceleration. Hence for rotational equilibrium

$$\sum \vec{\tau}_{ext} = \sum \vec{r}_i \times \vec{F}_i^{ext} = 0$$

For rotational equilibrium, the choice of the point of rotation (or fixed point) is unimportant. If the total torque is zero about any point, then it will be zero about any other point when the body is in equilibrium.

Distinguish between stable, unstable and neutral equilibria of a body.

(i) Stable equilibrium : A body is said to be in stable equilibrium if tends to regain it equilibrium position after being slightly displaced and released. In stable equilibrium, a body has minimum potential energy and its centre of mass goes higher when it is slightly displaced.

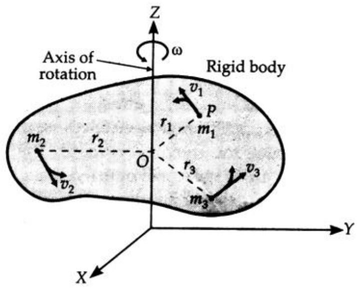
(ii) Unstable equilibrium : A body is said to be in unstable equilibrium if it gets further displaced from it equilibrium position after being slightly displaced and released. In unstable equilibrium, a body possesses maximum potential energy and its centre of mass goes lower on being slightly displaced.

(iii) Neutral equilibrium : If a body stays in equilibrium position even after being slightly displaced and released, it is said to be in neutral equilibrium. When a body is slightly displaced from its position of neutral equilibrium, its centre of mass is neither raised nor lowered and its potential energy remains constant.

Moment of inertia and its physical significance

A body rotating about a given axis tends to maintain its state of uniform rotation, unless an external torque is applied on it to change that state. This property of a body by virtue of which it opposes the torque tending to change its state of rest or of uniform rotation about an axis is called rotational inertia or moment of inertia. The moment of inertia of a rigid body about a fixed axis is defined as the sum of the products of the masses of the particles constituting the body and the squares of their respective distances from the axis of rotation.

Consider a rigid body rotating with uniform angular velocity ω about a vertical axis through *O*, as shown in figure. Suppose the body consists of *n* particles of masses $m_1, m_2, m_3, ..., m_n$ situated at distances $r_1, r_2, r_3, ..., r_n$ respectively from the axis of rotation. The moment of inertia of the body about the axis is given by



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ...+ m_n r_n^2$$

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The dimensional formula of moment of inertia is $\left[ML^2 T^0\right]$. The *SI* unit of moment of inertia is kg m^2 and its *CGS* units is g cm^2.

Physical significance of moment of inertia : The mass of a body resists a change in its state of linear motion, it is a measure of its inertia in linear motion. Similarly, the moment of inertia of a body about an axis of rotation resists a change in its rotational motion. The greater the moment of inertia of a body, the greater is the torque required to change its state of rotation. Thus moment of inertia of a body can be regarded as the measure of rotational inertia of the body. The moment of inertia of a body plays the same role in the rotational motion as the mass plays in linear motion. That is why moment of inertia is called the rotational analogue of mass in linear motion.

Factors on which the moment of inertia depends : The moment of inertia of a body is the measure of the manner in which its different parts are distributed at different distances from the axis of rotation. Unlike mass, it is not a fixed quantity as it depends on the position and orientation of the axis of rotation with respect to the body as a whole.

The moment of inertia of a body depends on

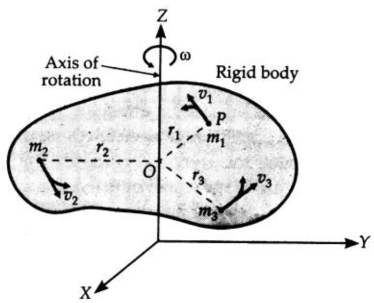
(i) Mass of the body **(ii)** Size and shape of the body

(iii) Distribution of mass about the axis of rotation.

(iv) Position and orientation of the axis of rotation w.r.t. the body.

Relation between rotational K.E. and moment of inertia

As shown in fig. consider a rigid body rotating about an axis *OZ* with uniform angular velocity ω. The body may be assumed to consist of *n* particles of masses $m_1, m_2, m_3, ..., m_n$; situated at distances $r_1, r_2, r_3, ..., r_n$ from the axis of rotation. As the angular velocity ω of all the *n* particles is same, so their linear velocities are



$$v_1 = r_1\omega,\ v_2 = r_2\omega,\ v_3 = r_3\,\omega,\ .....,\ v_n = r_n\,\omega$$

Hence the total kinetic energy of rotation of the body about the axis *OZ* is

Rotational K.E. $= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + ...+ \frac{1}{2}m_n v_n^2$

$$= \frac{1}{2}m_1 r_1^2\,\omega^2 + \frac{1}{2}m_2 r_2^2\,\omega^2 + \frac{1}{2}m_3 r_3^2\,\omega^2 + ...+ \frac{1}{2}m_n r_n^2\,\omega^2$$

$$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ... + m_n r_n^2)\omega^2$$

$$= \frac{1}{2}\left(\sum mr^2\right)\omega^2$$

But $\sum mr^2 = I$, the moment of inertia of the body about the axis of rotation.

$\therefore$ Rotation K.E. = $\frac{1}{2} I \omega^2$

When $\omega = 1$, rotational K.E. $= \frac{1}{2} I$. or $I = 2\times$ Rotational K.E.

Hence the moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the rotational kinetic energy of the body when it is rotating with unit angular velocity about that axis.

Radius of Gyration

For any body capable of rotation about a given axis, it is possible to find a radial distance from the axis where, if whole mass of the body is concentrated, its moment of inertia will remain unchanged. This radial distance is called radius of gyration and I denoted by *k*.

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which *f* the whole mass of the body were concentrated, its moment of inertia about the given axis would be the same as with the actual distribution of mass. The radius of gyration *k* is a geometrical property of the body and the axis of rotation. It gives a measure of the manner in which the mass of a rotating body is distributed with respect to the axis of rotation.

k has the dimensions of length Land is measured in metre or cm.

Expression for *k* : Suppose a rigid body consists of *n* particles of mass *m* each, situated at distances $r_1, r_2, r_3, ..., r_n$ from the axis of rotation *AB*.

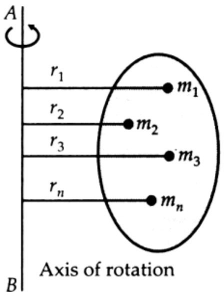
The moment of inertia of the body about the axis *AB* is

$$I = mr_1^2 + mr_2^2 + mr_3^2 + ... + mr_n^2$$

$$= m(r_1^2 + r_2^2 + r_3^2 + ... + r_n^2)$$

$$= m \times n \frac{(r_1^2 + r_2^2 + r_3^2 + ... + r_n^2)}{n}$$

or $$I = M \frac{(r_1^2 + r_2^2 + r_3^2 + ... + r_n^2)}{n}$$



where $M = m \times n$ = total mass of the body

If *k* is the radius of gyration about the axis *AB*, then $I = Mk^2$

$\therefore$ $$Mk^2 = M\left(\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}\right)$$

or $$k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}}$$ = Root mean square distance.

Hence the radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of its particles from the axis of rotation.

Factors on which radius of gyration of a body depends :

(i) Position and direction of the axis of rotation. **(ii)** Distribution of mass about the axis of rotation.

Theorem of perpendicular axis

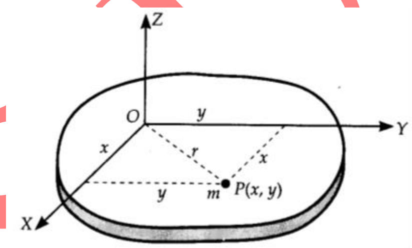
The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane and intersecting each other at the point where the perpendicular axis passes through the lamina.

Proof : Consider a plane lamina lying in the *XOY* plane. It can be assumed to be made up of large number of particles. Consider one such particle of mass *m* situated at point $P(x, y)$. Clearly, the distances of the particle from $X-, Y-$ and $Z-$axis are *y*, *x* and *r* respectively such that

$$r^2 = y^2 + x^2$$

Moment inertia of the particle about *X*–axes

$$= my^2$$



$\therefore$ Moment of inertia of whole lamina about *X*–axis is

$$I_x = \sum my^2$$

Moment of inertia of whole lamina about *Y*–axis is

$$I_y = \sum mx^2$$

Moment of inertia of whole lamina about *Z*–axis is

$$I_z = \sum mr^2 = \sum m(y^2 + x^2) = \sum my^2 + \sum mx^2$$

or $$I_z = I_x + I_y$$

This proves the theorem of perpendicular axes.

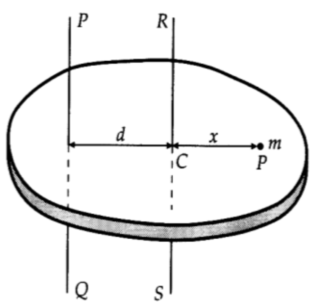
Theorem of parallel axes

The moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.

Proof : Let *I* be the moment of inertia of a body of mass *M* about an axis *PQ*. Let *RS* be a parallel axis passing through the centre of mass *C* of the body and at distance *d* from *PQ*. Let I_{CM} be the moment of inertia of the body about the axis *RS*. Consider a particle *P* of mass *m* at distance *x* from *RS* and so at distance (*x* + *d*) from *PQ*.

Moment of inertia of the particle about axis *PQ*

$$= m(x+d)^2$$



$\therefore$ Moment of inertia of the whole body about the axis *PQ* is

$$I = \sum m(x+d)^2 = \sum m(x^2 + d^2 + 2xd) = \sum mx^2 + \sum md^2 + \sum 2mxd$$

Now, $$\sum mx^2 = I_{CM}$$

$$\sum md^2 = (\sum m)d^2 = Md^2$$

$$\sum 2mxd = 2d \sum mx = 2d \times 0 = 0$$

This is because a body can balance itself about its centre of mass, so the algebraic sum of moments $(\sum mx)$ of mass of all its particles about the axis *RS* is zero.

Hence $$I = I_{CM} + Md^2$$

This proves the theorem of parallel axes.

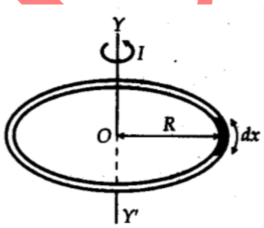
Moment of inertia of a thin circular ring

(a) M.I. of a ring about an axis through its centre and perpendicular to its plane :

Consider a thin uniform circular ring of radius *R* and mass *M*. As shown in fig. we wish to determine its moment of inertia *I* about an axis YY' passing through its centre *O* and perpendicular to it. The ring can be imagined to be made of a large number of small elements. Consider one such element of length *dx*.

Length of the ring = circumference = $2\pi R$.

Mass per unit length of ring $= \dfrac{M}{2\pi R}$



Mass of the small element $dM = \dfrac{M}{2\pi R}dx$

Moment of inertia of the small element about the axis YY'.

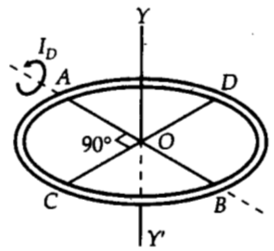
$$dI = \left(\frac{M}{2\pi R}dx\right)R^2 = \frac{MR}{2\pi}dx$$

The small elements lie along the entire circumference of the ring i.e., from $x = 0$ to $x = 2\pi R$. Hence the moment of inertia of the whole ring about the axis YY' will be

$$I = \int_0^{2\pi R}\frac{MR}{2\pi}dx = \frac{MR}{2\pi}\int_0^{2\pi R}dx = \frac{MR}{2\pi}[x]_0^{2\pi R} = \frac{MR}{2\pi}(2\pi R - 0)$$

or $$I = MR^2$$

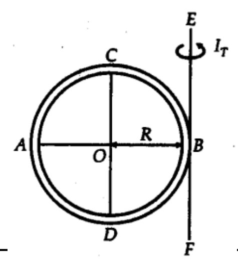
(b) M.I. of a ring about any diameter. According to the theorem of perpendicular axes, the moment of inertia about an axis YY' through *O* and perpendicular to the ring is equal to sum of its moments of inertia about two perpendicular diameters *AB* and *CD*, as shown in fig.



$$I_{AB} + I_{CD} = I_{YY'} \quad , \quad I_D + I_D = MR^2 \qquad \text{or} \qquad I_D = \frac{1}{2}MR^2$$

Here I_D is the M.I. of the ring about any diameter.

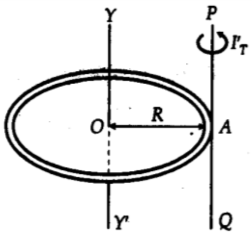
(c) M.I. of a ring about a tangent in its plane : As shown in fig. Let I_T be the moment of inertia of the ring about the tangent *EBF*. Applying the theorem of parallel axes, we get



$$I_T = M.I.\text{ about diameter } CD + MR^2 = \frac{1}{2}MR^2 + MR^2$$

or $\quad I_T = \dfrac{3}{2} MR^2$

(d) M.I. of a ring about a tangent perpendicular to its plane : Let I'_T be the moment of inertia of the ring about the axis *PAQ* tangent to the plane of the ring. Applying the theorem of parallel axes,



$$I_{PQ} = I_{YY'} + MR^2 = MR^2 + MR^2 \quad \text{or} \quad I'_T = 2MR^2$$

Note : We can determine the radius of gyration (k) of the ring about any axis by equating its M.I. about that axis to Mk^2. For example, the radius of gyration of a thin ring about any diameter is given by

$$I_D = \frac{1}{2} MR^2 = Mk^2 \quad \text{or} \quad k = R/\sqrt{2}$$

Moment of inertia of a uniform circular disc

(a) M.I. of a circular disc about an axis through its centre and perpendicular to its plane :

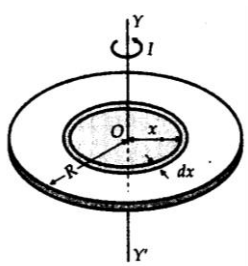
As shown in fig. consider a uniform disc of mass *M* and radius *R*. Suppose YY' is an axis passing through the centre *O* of the disc and perpendicular to its plane.

Area of the disc $= \pi R^2$

Mass per unit area of the disc $= \dfrac{M}{\pi R^2}$.

We can imagine the disc to be made up of a large number of concentric rings, whose radii vary from *O* to *R*. Let us consider one such concentric ring of radius *x* and width *dx*.

Area of the ring = Circumference × Width = $2\pi x \times dx$



Mass of the concentric ring, $m = \left(\dfrac{m}{\pi R^2}\right) 2\pi x\, dx = \dfrac{2Mx\, dx}{R^2}$

Moment of inertia of the concentric ring about the axis YY'

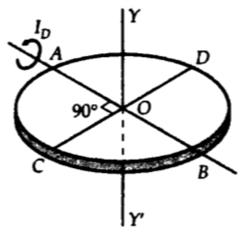
$$dI = mx^2 = \frac{2Mx\, dx}{R^2} \times x^2 = \frac{2Mx^3\, dx}{R^2}$$

The moment of inertia of the whole disc about the axis YY' can be obtained by integrating the above expression between the limits 0 to *R*.

$$I = \int_0^R \frac{2Mx^3\, dx}{R^2} = \frac{2M}{R^2} \int_0^R x^3\, dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R = \frac{2M}{4R^2}[R^4 - 0] = \frac{M}{2R^2} \times R^4$$

or $\quad I = \dfrac{1}{2} MR^2$

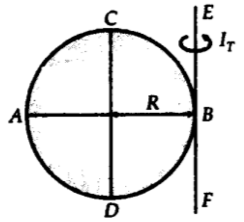
(b) M.I. of a disc about any diameter : In fig. *AB* and *CD* are two mutually perpendicular diameters in the plane of the disc. Applying the theorem of perpendicular axes, we get



$$I_{AB} + I_{CD} = I_{YY'}$$

or $\quad I_D + I_D = \dfrac{1}{2} MR^2 \quad$ or $\quad I_D = \dfrac{1}{4} MR^2$

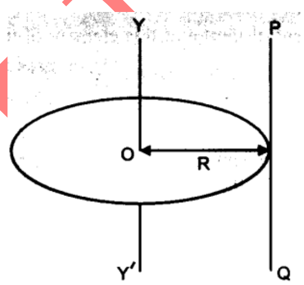
(c) M.I. of a disc about a tangent in its plane : Let I_T the moment of inertia of the disc about a tangent *EBF* in the plane of the disc. This tangent is parallel to the diameter *CD* of the disc. Applying the theorem of parallel axes, we get



$$L_T = \text{Moment of inertia of disc about } CD + MR^2$$

or $$I_T = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

(d) M.I. of a disc about a tangent perpendicular to its plane : In fig. let I'_T be the moment of inertia of disc about the tangent PQ perpendicular to the plane of disc. Applying the theorem of parallel axes, we get



$$I_{PQ} = I_{YY'} + MR^2 = \frac{1}{2}MR^2 + MR^2$$

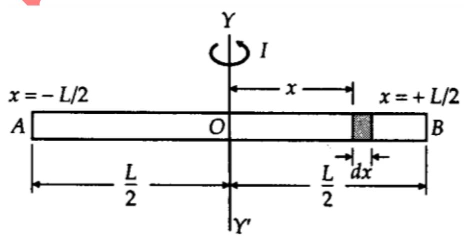
or $$I'_T = \frac{3}{2}MR^2$$

Moreover, the radius of gyration (k) in this case is given by

$$Mk^2 = \frac{3}{2}MR^2 \quad \text{or} \quad k = \sqrt{\frac{3}{2}}\,R$$

Moment of inertia of a thin uniform rod about a perpendicular axis through its centre

Consider a thin uniform rod *AB* of length *L* and mass *M*, free to rotate about an axis *YY′* through its centre *O* and perpendicular to its length.



$\therefore$ Mass per unit length of rod $= \dfrac{M}{L}$.

Consider a small mass element of length *dx* at a distance *x* from *O*.

Mass of the small element $= \dfrac{M}{L}\,dx$

Moment of inertia of the small element about *YY′*.

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L}\,dx \times x^2$$

The moment of inertia of the whole rod about the axis *YY′* can be obtained by integrating the above expression between the limits $x = -\dfrac{L}{2}$ and $x = +\dfrac{L}{2}$.

$$\therefore \quad I = \int dI = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{M}{L}x^2\,dx = \frac{M}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} x^2\,dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{+\frac{L}{2}} = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{M}{3L}\left[\frac{L^3}{8} + \frac{L^3}{8}\right] = \frac{M}{3L}\times\frac{L^3}{4}$$

or $$I = \frac{ML^2}{12}$$

Radius of gyration. Let *k* be the radius of gyration of the rod about the axis *YY′*. Then

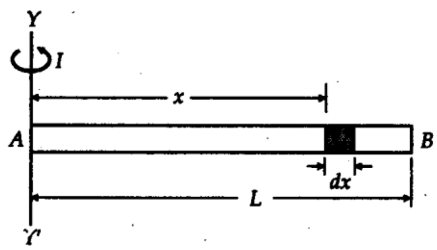
$$I = Mk^2$$

$$\therefore \quad Mk^2 = \frac{ML^2}{12} \quad \text{or} \quad k^2 = \frac{L^2}{12} \quad \text{or} \quad k = \frac{L}{2\sqrt{3}}$$

Thus, the radius of gyration of a uniform thin rod rotating about an axis passing through its centre and perpendicular to its length is $L/2\sqrt{3}$.

M.I. of a thin uniform rod about a perpendicular axis through its one end

Consider a thin uniform rod AB of length L and mass M, free to rotate about an axis YY' passing through its one end A and perpendicular to its length, as shown in fig.



Mass per unit length of the rod = $\dfrac{M}{L}$

Consider a small element of length dx of the rod at a distance x from the end A.

Mass of the small element = $\dfrac{M}{L}dx$

Moment of inertia of the small element about the axis YY'.

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L}dx \,.\, x^2$$

The moment of inertia of the whole rod about the axis YY' can be obtained by integrating the above expression between the limits $x = 0$ and $x = L$.

$$I = \int dI = \int_0^L \frac{M}{L}dx \,.\, x^2 = \frac{M}{L}\int_0^L x^2\, dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{M}{3L}\left[x^3\right]_0^L = \frac{M}{3L}\left[L^3 - 0\right] = \frac{ML^2}{3L}$$

or $$I = \frac{ML^2}{3}.$$

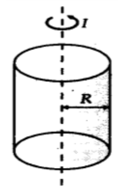
Radius of gyration : Let k be the radius of gyration of the rod about the axis YY'. Then

$$\frac{ML^2}{3} = Mk^2 \quad \text{or} \quad k^2 = \frac{L^2}{3} \quad \text{or} \quad k = \frac{L}{\sqrt{3}}$$

Thus radius of gyration of rod about an axis passing through its one end and perpendicular to its length is $L/\sqrt{3}$

M.I. of a hollow cylinder about its own axis

Consider a hollow cylinder of mass M and radius R. As shown in fig. every element of the cylinder is at the same perpendicular distance R from its axis. Hence the moment of inertia of the hollow cylinder about its own axis is



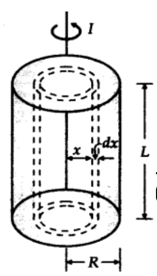
$$I = \int R^2\, dm = \int_0^M R^2\, dm = R^2\int_0^M dm = R^2 \times M \quad \text{or} \quad I = MR^2.$$

Moment of inertia of uniform solid cylinder about its own axis

Consider a solid cylinder of mass M, radius R and length L. We wish to determine its moment of inertia about its own axis YY'.

Volume of the cylinder $= \pi R^2 L$

Mass per unit volume of the cylinder, $\rho = \dfrac{M}{\pi R^2 L}$



We can imagine the cylinder to be made of a large number of coaxial cylindrical shells. Consider one such cylindrical shell of inner radius x and outer radius $x+dx$, as shown in fig.; The cross section of the shell is a ring of radius x and thickness dx.

$\therefore$ Cross-sectional area of the cylindrical shell

$$= \text{Circumference} \times \text{thickness} = 2\pi x\, dx$$

volume of the cylindrical shell = Cross-sectional area × length = $2\pi x\, dx \times L$

Mass of the cylindrical shell, $m = \text{Volume} \times \rho = 2\pi x L\, dx \times \dfrac{M}{\pi R^2 L} = \dfrac{2M}{R^2}\, x\, dx$

As the mass of the shell is distributed at the same distance x from its axis, so its moment of inertia about the axis YY' is

$$dI = mx^2 = \frac{2M}{R^2}\, x\, dx \times x^2 = \frac{2M}{R^2}\, x^3\, dx$$

The moment of inertia of the slid cylinder can be obtained by integrating the above expression between the limits $x = 0$ and $x = R$.

$$\therefore \qquad I = \int dI = \int_0^R \frac{2M}{R^2}\, x^3\, dx = \frac{2M}{R^2}\int_0^R x^3\, dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R = \frac{2M}{R^2}\left[R^4 - 0\right]$$

$$\text{or} \qquad I = \frac{1}{2} MR^2$$

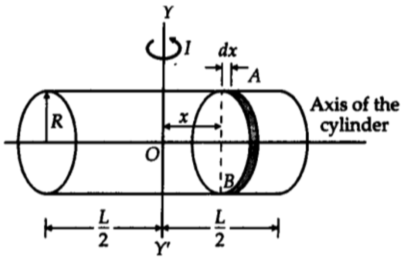
M.I. of a solid cylinder about an axis through its centre and perpendicular to its axis

Consider a uniform solid cylinder of mass M, radius R and length L, we wish to determine its moment of inertia about an axis YY' passing through its centre O and perpendicular to its length.

Mass per unit length = $\dfrac{M}{L}$

We can imagine the cylinder to be made up of a large number of thin circular discs placed perpendicular to the axis of the cylinder. As shown in fig. consider one such thin disc of thickness dx and at distance x from the centre O.

mass of the elementary disc = $\dfrac{M}{L}\, dx$



Radius of the elementary disc = R

Moment of inertia of the elementary disc about the diameter AB

$$= \frac{1}{4}\text{Mass} \times \text{radius}^2 = \frac{1}{4} \cdot \frac{M}{L}\, dx \times R^2 = \frac{MR^2}{4L}\, dx$$

Applying the theorem of parallel axes , the moment of inertia of the elementary disc about the axis YY',

$$dI = M.I. \text{ about the diameter } AB + \text{Mass} \times x^2$$

$$= \frac{MR^2}{4L}\, dx + \frac{M}{L}\, dx \times x^2 = \frac{M}{L}\left(\frac{R^2}{4} + x^2\right) dx$$

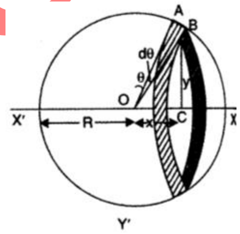
The moment of inertia of the cylinder about the axis YY' can be obtained by integrating the above expression between the limits $x = 0$ and $x = L/2$ and multiplying the result by 2 to cover both halves of cylinder. Thus,

$$I = 2\int dI = 2\int_0^{\frac{L}{2}} \frac{M}{L}\left(\frac{R^2}{4} + x^2\right) dx = \frac{2M}{L}\left[\frac{R^2}{4}\int_0^{\frac{L}{2}} dx + \int_0^{\frac{L}{2}} x^2\, dx\right]$$

$$= \frac{2M}{L}\left[\frac{R^2}{4}\left| x \right|_0^{L/2} + \left|\frac{x^3}{3}\right|_0^{\frac{L}{2}}\right] = \frac{2M}{L}\left[\frac{R^2}{4}\left(\frac{L}{2} - 0\right) + \left(\frac{(L/2)^3}{3} - 0\right)\right] = \frac{2M}{L}\left[\frac{R^2}{4}\cdot\frac{L}{2} + \frac{L^3}{24}\right]$$

or $\quad I = M\left[\dfrac{R^2}{4} + \dfrac{L^2}{12}\right]$

Moment of inertia of a hollow sphere about its diameter



Consider a solid sphere of mass M and radius R. XX' be its diameter about which the moment of inertia is to be calculated. As shown in the figure.

Surface area of the sphere $= 4\pi R^2$

$\therefore$ Mass per unit area of the hollow sphere = $\dfrac{M}{4\pi R^2}$ ....(1)

Now consider a circular ring of the hollow sphere such that the centre of the ring is at a distance x from the centre of the sphere. Let AC be the radius and AB be the width of the ring. $\angle AOB = d\theta$, $\angle YOA = \angle OAC = \theta$.

Let $\quad AC = y$

$\therefore \quad AB = R\,d\theta$ ....(2)

From right angled ΔOCA, $\quad AC = OA\cos\theta$

or $\quad y = R\cos\theta$ ....(3)

Also, $\quad OC = OA\sin\theta$ ....(4)

or $\quad x = R\sin\theta$

Differentiating equation (4), we get $dx = R\cos\theta\, d\theta$

Using equation (3), we get $\quad dx = y\, d\theta$ ....(5)

Surface area of the ring = circumference × Width

$$= 2\pi y \times AB \; = 2\pi y \times R\,d\theta \; = 2\pi R(y\, d\theta)$$

Using equation (5), we get

Surface area of the ring $= 2\pi R\,dx$

$$\text{Mass of the ring} = \frac{M}{4\pi R^2} \times 2\pi\, R\,dx = \frac{M}{2R}\,dx$$

XX' is an axis passing through the centre of the ring and perpendicular to its plane.

We know, moment of inertia of a ring about an axis passing through its centre and perpendicular to its plane is given by

$$dI = \text{Mass} \times (\text{Radius})^2 \qquad = \frac{M}{2R}\,dx \times y^2$$

Form $\Delta\, OAC$, $\quad R^2 = x^2 + y^2$

or $\quad y^2 = R^2 - x^2$

$$dI = \frac{M}{2R}\, dx \times (R^2 - x^2) = \frac{M}{2R}(R^2 - x^2)\, dx \qquad ....(6)$$

Moment of inertia of the hollow sphere about XX' axis can be calculated by integrating equation (6) between the limits $x = -R$ to $x = +R$.

$$\int dI = \int_{-R}^{R} \frac{M}{2R}(R^2 - x^2)\, dx$$

$$I = \frac{M}{2R}\left[R^2 \int_{-R}^{R} dx - \int_{-R}^{R} x^2\, dx\right] = \frac{M}{2R}\left[R^2 [x]_{-R}^{R} - \left[\frac{x^3}{3}\right]_{-R}^{R}\right]$$

$$= \frac{M}{2R}\left[R^2\{R - (-R)\} - \frac{1}{3}\{R^3 - (-R)^3\}\right] = \frac{M}{2R}\left[2R^3 - \frac{2R^3}{3}\right]$$

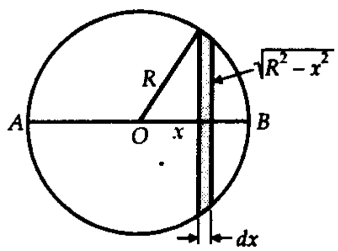
$$= \frac{M}{2R}\left[\frac{4R^3}{3}\right] = \frac{2}{3}MR^2 \quad \text{or} \quad I = \frac{2}{3}MR^2$$

Moment of inertia of a solid sphere about its diameter

Consider a uniform solid sphere of mass M and radius R. We wish to determine its moment of inertia about diameter AB

Volume of the sphere = $\frac{4}{3}\pi R^3$, $\qquad$ Mass per unit volume, $\rho = \frac{3M}{4\pi R^3}$

We can imagine the sphere to be made up of a large number of thin slices placed perpendicular to the diameter AB. Consider one such slice of thickness dx placed at distance x from the centre O.



Radius of the elementary slice $= \sqrt{R^2 - x^2}$

Volume of the elementary slice = Area × thickness

$$= \pi\left(\sqrt{R^2 - x^2}\right)^2 \times dx = \pi(R^2 - x^2)\, dx$$

Mass of the elementary slice $= \text{Volume} \times \rho = \pi(R^2 - x^2)\, dx \times \frac{3M}{4\pi R^3}$

$$= \frac{3M(R^2 - x^2)\, dx}{4R^3}$$

Moment of inertia of the thin slice about the axis AB passing through its centre and perpendicular to its plane,

$$dI = \frac{1}{2}\ \text{mass} \times (\text{radius})^2$$

$$= \frac{1}{2} \cdot \frac{3M(R^2 - x^2)\, dx}{4R^3} \cdot (R^2 - x^2) = \frac{3M(R^2 - x^2)^2\, dx}{8R^3}$$

The moment of inertia of the whole sphere about the diameter AB can be obtained by integrating the above expression between the limits $x = 0$ and $x = R$ and multiplying the result by 2 to include both halves of the sphere.

$$\therefore \qquad I = 2\int dI = 2\int_0^R \frac{3M(R^2-x^2)^2\,dx}{8R^3} = \frac{2\times 3M}{8R^3}\int_0^R (R^2-x^2)^2\,dx$$

$$= \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2x^2 + x^4)\,dx = \frac{3M}{4R^3}\left[R^4\int_0^R dx - 2R^2\int_0^R x^2\,dx + \int_0^{R} x^4\,dx\right]$$

$$= \frac{3M}{4R^3}\left[R^4\,|x|_0^R - 2R^2\left|\frac{x^3}{3}\right|_0^R + \left|\frac{x^5}{5}\right|_0^R\right] = \frac{3M}{4R^3}\left[R^4(R-0) - 2R^2\left(\frac{R^3}{3}-0\right) + \left(\frac{R^5}{5}-0\right)\right]$$

$$= \frac{3M}{4R^3}\left[R^5 - \frac{2}{3}R^5 + \frac{R^5}{5}\right] = \frac{3M}{4R^3}\times\frac{8}{15}R^5 \qquad \text{and} \qquad I = \frac{2}{5}MR^2$$

Moment of inertia of the solid sphere about a tangent.

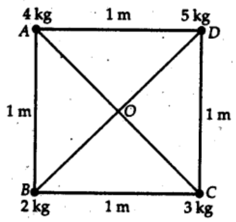
Applying the theorem of parallel axes, the moment of inertia of a solid sphere about a tangent is given by

$$I_T = M.I.\ \text{about a diameter} + \text{Mass}\times(\text{radius})^2$$

$$= \frac{2}{5}MR^2 + MR^2 \qquad \text{or} \qquad I_T = \frac{7}{5}MR^2$$

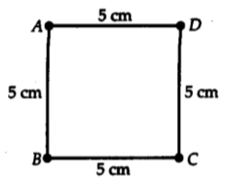
Subjective Assignment – IV

1. A wheel of mass 8kg and radius of gyration 25 cm is rotating at 300 rpm. What is its moment of inertia?
2. Three mass points m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of length a. What is the moment of inertia of system about an axis along the altitude of the triangle passing through m_1.
3. Three balls of masses 1, 2 and 3 kg respectively are arranged at the corners of an equilateral triangle of side 1m. What will be the moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle?
4. Four particles of masses 4kg, 2kg, 3kg and 5kg are respectively located at the four corners A, B, C and D of a square of side 1m as shown in fig. Calculate the moment of inertia of the system about
   (i) an axis passing through the point of intersection of the diagonals and perpendicular to the plane of the square.
   (ii) the side AB and (iii) the diagonal BD

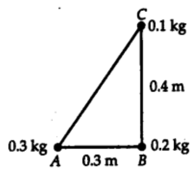


5. The moment of inertia of a uniform circular disc about its diameter is 100g cm^2. What is its moment of inertia (i) about its tangent (ii) about an axis perpendicular to its plane.
6. Calculate the moment of inertia of a cylinder of length 1.5 m, radius 0.05 m and density 8 × 10^3 kg m^{-3} about the axis of the cylinder.
7. Calculate the moment of inertia of the earth about its diameter, taking it to be a sphere of 10^{25} kg and diameter 12800 km.
8. Four spheres of diameter 2a and mass M each are placed with their centres on the four corners of a square of side b. Calculate moment of inertia of the system about one side of the square taken as its axis.
9. Two point masses of 2 kg and 10kg are connected by a weightless rod of length 1.2m. Calculate the M.I. of the system about an axis passing through the centre of mass and perpendicular to the system.
10. Find the moment of inertia of a rectangular bar magnet abut an axis passing through its centre and parallel to its thickness. Mass of the magnet is 100g, length is 12cm, breadth is 3cm and thickness is 2cm.
11. Calculate the ratio of radii of gyration of a circular ring and a disc of the same radius about the axis passing through their centres and perpendicular to their planes.

12. Find the radius of gyration of a rod of mass 100 g and length 100 cm about an axis passing through its centre and perpendicular to its length.
13. A wheel is rotating at a rate of 1000 rpm and its kinetic energy is 10^6J. Determine the moment of inertia of the wheel about its axis of rotation.
14. Calculate the kinetic energy of rotation of a circular disc of mass 1kg and radius 0.2 m rotating about an axis passing through its centre and perpendicular to its plane. The disc is making $30/\pi$ rotations per minute.
15. Energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. Find the moment of inertia of the wheel.
16. Calculate the rotational K.E. of the earth about its own axis. Mass of the earth = 6×10^{24} kg and radius of the earth = 6400 km.
17. A metre scale AB is held vertically with its one end A on the floor and is then allowed to fall. Find the speed of the other end B when it strikes the floor, assuming that the end on the floor does not slip.
18. A uniform circular disc of mass m is set rolling on a smooth horizontal table with a uniform linear velocity v. Find the total K.E. of the disc.
19. A solid sphere is rolling on a frictionless plane surface about its axis of symmetry. Find the rotational energy and the ratio of its rational energy to its total energy.
20. A wheel of mass 5kg and radius 0.40 m is rolling on a road without sliding with angular velocity 10 rad s^{-1}. The moment of inertia of the wheel about the axis of rotation is 0.65 kgm^2. What is the percentage of kinetic energy of rotation in the total kinetic energy of the wheel?
21. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1m. If the height of the inclined plane is 4m, what is its rotational K.E. when it reaches the foot of the plane?
22. A bucket of mass 8kg is supported by a light rope wound around a solid wooden cylinder of mass 12kg and radius 20 cm free to rotate about its axis. A man holding the free end of the rope, with the bucket and the cylinder at rest initially, lets go the bucket freely downwards in a well 50m deep. Neglecting friction, obtain the speed of the bucket and the angular speed of the cylinder just before the bucket enters water. Take g = 10 ms^{-2}.
23. A body of mass 50 g is revolving about an axis in a circular path. The distance of the centre of mass of the body from the axis of rotation is 50 cm. Find the moment of inertia of the body.
24. Find the moment of inertia of the hydrogen molecule about an axis passing through its centre of mass and perpendicular to the interatomic axis. Given mass of H-atom = 1.7×10^{-27} kg, interatomic distance = 4×10^{-10} m.
25. Three particles, each of 10 g are located at the corners of an equilateral triangle of side 5 cm. Determine the moment of inertia of this system about an axis passing through one corner of the triangle and perpendicular to the plane of the triangle.

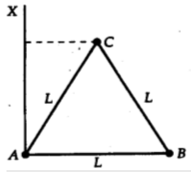


26. Four point masses of 20 g each are placed at the corners of a square *ABCD* of side 5 cm, as shown in Fig.. Find the moment of inertia of the system
    (i) about an axis coinciding with the side *BC* and
    (ii) about an axis through *A* and perpendicular to the plane of the square.



27. The point masses of 0.3 kg, 0.2 kg and 0.1 kg are placed at the corners of a right angled ΔABC, as shown in fig. Find the moment of inertia of the system (i) about an axis through *A* and perpendicular to the plane of the diagram and (ii) about an axis along *BC*.

28. Three particles, each of mass m, are situated at the vertices of an equilateral ΔABC of side L, as shown in fig. Find the moment of inertia of the system about the line AX perpendicular to AB and in the plane of $\Delta\, ABC$.



29. Four particles each of mass m are kept at the four corners of a square of edge a. Find the moment of inertia of the system about an axis perpendicular to the plane of the system and passing through the centre of the square.

30. What is the moment of inertia of a ring of mass 2 kg and radius 50 cm about an axis passing through its centre and perpendicular to its plane ? Also find the moment of inertia about a parallel axis through its edge.

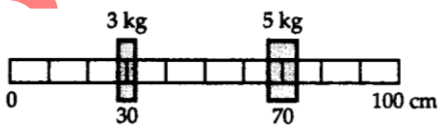
31. Calculate the moment of inertia of uniform circular disc of mass 500g, radius 10 cm about (i) diameter of the disc (ii) the axis tangent to the disc and parallel to its diameter and (iii) the axis through the centre of the disc and perpendicular to its plane.

32. Calculate moment of inertia of a circular disc of radius 10 cm, thickness 5 mm and uniform density $8 gcm^{-3}$, about a transverse axis through the centre of the disc.

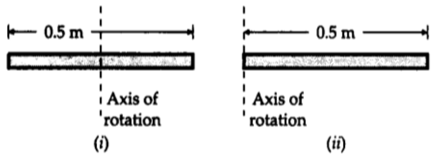
33. The radius of a sphere is 5 cm. Calculate the radius of gyration about (i) its diameter and (ii) about any tangent.

34. Calculate the radius of gyration of a cylindrical rod of mass M and length L about an axis of rotation perpendicular to its length and passing through its centre.

35. Two masses of 3 kg and 5 kg are placed at 30 cm and 70 cm marks respectively on a light wooden metre scale, as shown in Figure. What will the moment of inertia of the system about an axis through (i) 0 cm mark and (ii) 100 cm mark, and perpendicular to the metre scale?



36. Calculate the moment of inertia of a rod of mass 2kg and length 0.5m in each of the following cases as shown in fig.



37. A body of mass 2 kg is revolving in a horizontal circle of radius 2 m at the rate of 2 revolutions per second. Determine (i) moment of inertia of the body and (ii) the rotational kinetic energy of the body.

38. A flywheel of mass 500 kg and diameter 1 m makes 500 rpm. Assuming the mass to be concentrated along rim, calculate (i) angular velocity (ii) moment of inertia and (iii) rotational K.E. of the flywheel.

39. A rod revolving 60 times in a minute about an axis passing through an end at right angles to the length, has kinetic energy of 400J. Find the moment of inertia of the rod.

40. A thin metal hoop of radius 0.25 m and mass 2 kg starts from rest and rolls down an inclined plane. If its linear velocity on reaching the foot of the plane is 2 ms^{-1}, what is its rotation K.E. at that instant?

41. The earth has a mass of 6×10^{24} kg and a radius of 6.4×10^{6} m. Calculate the amount of work in joules that must be done if its rotation were to be slowed down so that the duration of the day becomes 30 hours instead of 24 hours. Moment of inertia of earth $= \frac{2}{5} MR^2$.

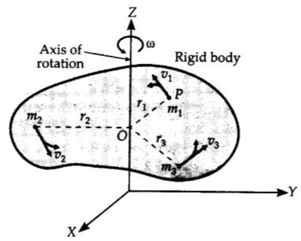
Answers

1.	0.5 kgm^2	2.	$\frac{a^2}{4}(m_2 + m_3)$	3.	2 kg m^2
4.	(i) 7 kg m^2 (ii) 8 kg m^2		(iii) 3.5 kg m^2	5.	(i) 500 g cm^2 (ii) 200 g cm^2

6.	0.1175 kg m^2	7.	1.64×10^{38} kg m^2	8.	$\frac{2}{5}M(4a^2+5b^2)$
9.	2.4 kg m^2	10.	1275 g cm^2	11.	$\sqrt{2}:1$
12.	0.289 m	13.	182.4 kg m^2	14.	0.01 J
15.	0.7 kg m^2	16.	2.6×10^{29} J	17.	5.4 ms^{-1}
18.	$\frac{3}{4}mv^2$	19.	$\frac{1}{5}mv^2$, 2 : 7	20.	44.8%
21.	26.13 J	22.	23.9 ms^{-1} ; 119.5 $rads^{-1}$	23.	0.0125 kg m^2
24.	13.6×10^{-47} kg m^2	25.	5×10^{-5} kg m^2	26.	(i) 1000g cm^2(ii) 2000gcm^2
27.	(i) 0.043 kg m^2 (ii) 0.027 kg m^2	28.	$\left(\frac{5mL^2}{4}\right)$	29.	$2ma^2$
30.	0.5 kgm^2, 1.0 kgm^2	31.	(i) 12500 g cm^2 (ii) 62500 g cm^2 (iii) 25000 g cm^2		
32.	6.28×10^4 gcm^2	33.	(i) 3.16 cm (ii) 5.915 cm	34.	$K = \frac{L}{\sqrt{12}}$
35.	(i) 2.72 kg m^2 (ii) 1.92 kg m^2	36.	(i) 0.042 kg m^2 (ii) 0.166 kg m^2		
37.	8 kgm^2 , 631.65 J	38.	(i) 52.38 rad s^{-1} (ii) 125 kg m^2 (iii) 1.715×10^5 J		
39.	20.26 kg m^2	40.	4 J	41.	9.36×10^{28} J

Relation between torque and moment of inertia

When a torque acts on a body capable of rotation about an axis, it produces an angular acceleration in the body. If the angular velocity of each particle is ω, then the angular acceleration, $\alpha = d\omega/dt$ will be same for all particles of the body. The linear acceleration will depend on their distances $r_1, r_2, ..., r_n$ from the axis of rotation. As shown in figure, consider a particle P of mass m_1 at a distance r_1 from the axis of rotation. Let its linear velocity be v_1.



Linear acceleration of the first particle, $a_1 = r_1 \alpha$

Force acting on the first particle , $F_1 = m_1 r_1 \alpha$

Moment of force F_1 about the axis rotation is $\tau_1 = F_1 r_1 = m_1 r_1^2 \alpha$

Total torque acting on the rigid body is

$$\tau = \tau_1 + \tau_2 + \tau_3 + ... + \tau_n$$

$$= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + ... + m_n r_n^2 \alpha$$

$$= (\Sigma mr^2)\alpha$$

But $\Sigma mr^2 = I$, moment of inertia of the body about the given axis.

$\therefore$ $$\tau = I\alpha$$

Torque = Moment of inertia × Angular acceleration

When α = 1 , τ = I.

Thus the moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce unit angular acceleration in the body about that axis.

Relation between angular momentum and moment of inertia

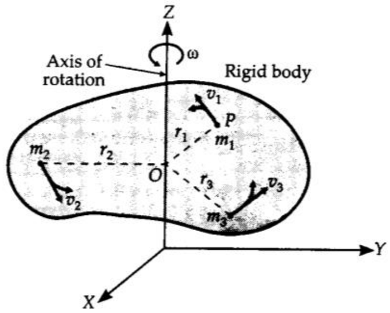
As shown in the figure consider a rigid body rotating about a fixed axis with uniform angular velocity ω. The body consists of *n* particles of masses $m_1, m_2, m_3, ..., m_n$; situated at distance $r_1, r_2, r_3, ..., r_n$ from the axis of rotation. The angular velocity ω of all the *n* particles will be same but their linear velocities will be different and are given by $v_1 = r_1\omega,\ v_2 = r_2\omega,\ v_3 = r_3\omega, ..., v_n = r_n\omega$.

Linear momentum of first particle, $p_1 = m_1 v_1 = m_1 r_1 \omega$

Moment of linear momentum of the first particle about the axis *YY′*, $\quad L_1 = p_1 r_1 = m_1 r_1^2 \omega$

The angular momentum of a rigid body about an axis is the sum of moments of linear momenta of all its particles about that axis. Thus

$$L = L_1 + L_2 + L_3 + ... + L_n$$
$$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + ... + m_n r_n^2 \omega$$
$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2)\omega$$
$$= (\Sigma\, mr^2)\omega$$



But $\Sigma mr^2 = I$, moment of inertia of the body about the given axis

$\therefore \quad\quad L = I\omega$

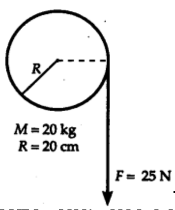
Angular momentum = M.I. × Angular velocity

when ω = 1, L = 1

Thus the moment of inertia of a body about an axis is numerically equal to the angular momentum of the rigid body when rotating with unit angular velocity about that axis.

Subjective Assignment – V

1. A torque of 2.0×10^{-4} Nm is applied to produce an angular acceleration of 4 rad s^{-2} in a rotating body. What is the moment of inertia of the body?
2. An automobile moves on a road with a speed of 54 kmh^{-1}. The radius of its wheels is 0.35m. What is the average negative torque transmitted by its brakes to a wheel if the vehicle is brought to rest in 15s? The moment of inertia of the wheel about the axis of rotation is 3 kgm^2.
3. A flywheel of mass 25 kg has a radius of 0.2m. What force should be applied tangentially to the rim of the flywheel so that it acquires an angular acceleration of 2 rad s^{-2}?
4. A torque of 10 Nm is applied to a flywheel of mass 10 kg and radius of gyration 50cm. What is the resulting angular acceleration.
5. A grindstone has a moment of inertia of 6 kg m^2. A constant torque is applied and the grindstone is found to have a speed of 150 rpm, 10 seconds after starting from rest, calculate the torque.
6. A flywheel of mass 25 kg has a radius of 0.2 m. It is making 240 r.p.m. What is the torque necessary to bring it to rest in 20s? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force?
7. A cord is wound around the circumference of a wheel of diameter 0.3m. The axis of the wheel is horizontal. A mass of 0.5 kg is attached at the end of the cord and it is allowed to fall from rest. If the weight falls 1.5m in 4s, what is the angular acceleration of the wheel? Also find out of moment of inertia of the wheel.
8. A cord of neglibible mass is wound round the rim of a fly wheel of mass 20g and radius 20cm. A steady pull of 25 N is applied on the cord as shown in figure. The flywheel is mounted on a horizontal axle with frictionless bearings.

   (a) Compute the angular acceleration of the wheel.

(b) Find the work done by the pull, when 2m of the cord is unwound.

(c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest

(d) Compare answer to part (b) and (c)

9. A body whose moment of inertia is 3 kgm^2, is at rest. It is rotated for 20s with a moment of force 6Nm. Find the angular displacement of the body. Also calculate the work done.

10. How much tangential force would be needed to stop the earth in one year, if it were rotating with angular velocity of 7.3×10^{-5} rad s^{-1}? Given the moment of inertia of the earth = 9.3×10^{37} kg m^2 and radius of the earth $= 6.4 \times 10^6\, m$.

11. The angular momentum of a body is 31.4 Js and its rate of revolution is 10 cycles per second. Calculate the moment of inertia of the body about the axis of rotation.

12. A 40 kg flywheel in the form of a uniform circular disc of 1m radius is making 120 rpm. Calculate the angular momentum.

13. A ring of diameter 0.4 m and of mass 10 kg is rotating about its axis at the rate of 2100 rpm. Find (i) moment of inertia (ii) angular momentum and (iii) rotational K.E. of the ring.

14. Calculate the angular momentum of earth rotating about its own axis. Mass of earth $= 5.98 \times 10^{27}$ kg, mean radius of the earth = 6.37×10^6 m, M.I. of the earth $= \frac{2}{5}MR^2$.

15. A cylinder of mass 5 kg and radius 30 cm, and free to rotate about its axis, receives an angular impulse of 3 kg m^2 s^{-1} initially followed by a similar impulse after every 4s. What is the angular speed of the cylinder 30s after the initial impulse? The cylinder is at rest initially.

16. The moment of inertia of a flywheel is 4kg m^2. What angular acceleration will be produced in it by applying a torque of 10 Nm on it?

17. The moment of inertia of a body is 2.5 kg m^2. Calculate the torque required to produce an angular acceleration of 18 rad s^{-2} in the body.

18. A cylinder of length 20cm and radius 10 cm is rotating about its central axis at an angular speed of 100 rad/s. What tangential force will stop the cylinder at a uniform rate in 10 seconds? The moment of inertia of the cylinder about its axis of rotation is 0.8 kg m^2.

19. A flywheel of moment of inertia 10^7 g cm^2 is rotating at a speed of 120 rotations per minute. Find the constant breaking torque required to stop the wheel in 5 rotations.

20. If a constant torque of 500 Nm turns a wheel of moment of inertia 100 kg m^2 about an axis through its centre, find the gain in angular velocity in 2s.

21. A sphere of mass 2kg and radius 5cm is rotating at the rate of 300 rpm. Calculate the torque required to stop it is 6.28 revolutions. Moment of inertia of the sphere about any diameter $= \frac{2}{5}MR^2$.

22. A body of mass 1.0 kg is rotating on a circular path of diameter 2.0 m at the rate of 10 rotations in 31.4s. Calculate (i) angular momentum of the body and (ii) rotational kinetic energy.

23. A circular ring of diameter 40 cm and mass 1 kg is rotating about an axis normal to its plane and passing through the centre with a frequency of 10 rotations per second. Calculate the angular momentum about its axis of rotation.

Answers

1.	0.5×10^{-4} kg m^2	2.	$-8.57\,\text{kgm}^2\,\text{s}^{-2}$	3.	5N
4.	4 rad s^{-2}	5.	3π Nm.	6.	π N.

7.	$\alpha = 1.25$ rad s^{-2} ; $I = 0.855$ kg m^2	8.	(a) 12.5 rad s^{-2} (b) 50 J (c) 50 J (d) Ans. same		
9.	400 rad , W = 2400 J	10.	3.363×10^{19} N	11.	0.5 kgm^2
12.	251.2 kg m^2s^{-1}	13.	(i) 0.4 kg m^2 (ii) 88 kg m^2 s^{-2} (iii) 9680 J		
14.	1.53×10^{34} kg m^2 s^{-1}	15.	106.67 rad s^{-1}	16.	2.5 rad s^{-2}
17.	45 Nm	18.	80 N		
19.	2.513×10^7 dyne cm	20.	10 rad s^{-1}		
21.	2.542×10^{-2} Nm	22.	(i) 2.0 kgm^2s^{-1} (ii) 2.0 J	23.	$0.8\,\pi$ kgm^2 s^{-1}.

Conservation of angular momentum :

Law of conservation of angular momentum : Suppose the external torque acting on a rigid body due to external forces is zero. Then $\tau = \frac{dL}{dt} = 0$

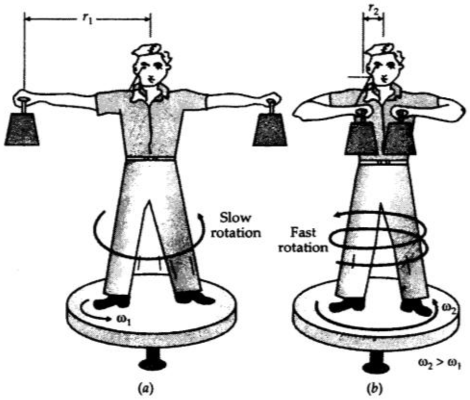
Hence, L = constant.

So, when the total external torque acting on a rigid body is zero, the total angular momentum of the body is conserved. This is the law of conservation of angular momentum.

Clearly, when $\tau = 0$, $L = I\ \omega$ = constant or $I_1\omega_1 = I_2\omega_2$

This means that when no external torque is acting, the angular velocity ω of the body can be increased or decreased by decreasing or increasing the moment of inertia of the body.

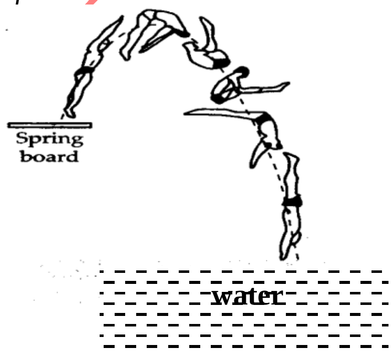
Illustrations of the law of conservation of angular momentum :



(i) **Planetary motion :** The angular velocity of a planet revolving in an elliptical orbit around the sun increases, when it comes closer to the sun because its moment of inertia about the axis through the sun decreases.

(ii) *A man carrying heavy weights in his hands and standing on a rotating turn–table can change the angular speed of the turn–table.*

(iii) *A diver jumping from a spring board exhibits somersaults in air before touching the water surface.*



(iv) *An ice–skater or a ballet dancer can increase her angular velocity by folding her arms and bringing the stretched leg close to the other leg.*

(v) *The speed of the inner layers of the whirlwind in a tarnado is alarmingly high.*

Subjective Assignment – VI

1. A small block is rotating in a horizontal circle at the end of a thread which passes down through a hole at the centre of table top. If the system is rotating at 2.5 rps in a circle of 30 cm radius, what will be the speed of rotation when the thread is pulled inwards to decrease the radius to 10cm? Neglect friction.
2. A star of mass twice the solar mass and radius 10^6 km rotates about its axis with an angular speed of 10^{-6} rad s^{-1}. What is the angular speed of the star when it collapses (due to inward gravitational force) to a radius of 10^4 km? Solar mass 1.99×10^{30} kg.
3. If the earth were to suddenly contract to half of its present radius (without any external torque on it), by what duration would the day be decreased? Assume earth to be a perfect solid sphere of moment of inertia $\frac{2}{5}MR^2$.
4. What will be the duration of the day, if earth suddenly shrinks to 1/64 of its original volume, mass remaining the same?
5. The maximum and minimum distances of a comet from the sun are 1.4×10^{12} m and 7×10^{10} m. If its velocity nearest to the sun is 6×10^4 ms^{-1}, what is the velocity in the farthest position? Assume that path of the comet in both the instantaneous positions is circular.
6. A horizontal disc rotating about a vertical axis passing through its centre makes 180 rpm. A small piece of wax of mass 10g falls vertically on the disc and adheres to it at a distance of 8 cm from its axis. If the frequency is thus reduced to 150 rpm, calculate the moment of inertia of the disc.
7. An ice shatter spins with arms outstretched at 1.9 rps. Her moment of inertia at this instant is 1.33kg m^2. She pulls in her arms to increase her rate of spin. If the moment of inertia is 0.48 kg m^2 after she pulls in her arms, what is her new rate of rotation?
8. A mass of 2 kg is rotating on a circular path of radius 0.8 m with angular velocity of 44 rad s^{-1}. If the radius of the path becomes 1.0m, what will be the value of angular velocity?
9. A ball tied to a string takes 4 s to complete revolution along a horizontal circle. If, by pulling the cord, the radius of the circle is reduced to half of the previous value, then how much time the ball will take in one revolution?
10. The sun rotates round itself once in 27 days. What will be period of revolution if the sun were to expand to twice its present radius ? Assume the sun to be a sphere of uniform density.
11. If the earth suddenly contracts by one-fourth of its present radius, by how much would the day be shortened?
12. Prove that for an earth satellite, the ratio of its velocity at apogee (when farthest from the earth) to its velocity at perigee (when nearest to the earth) is the inverse ratio of its distances from apogee and perigee.
13. A uniform disc rotating freely about a vertical axis makes 90 revolutions per minute. A small piece of wax of mass *m* gram falls vertically on the disc and sticks to it at a distance of r cm from the axis. If the number of rotations per minute reduces to 60, find the moment of inertia of the disc.

Answer

1.	22.5 rps	2.	0.01 rad s^{-1}	3.	18 h	4.	1.5 h
5.	3000 ms^{-1}	6.	3.2×10^{-8} kgm^2	7.	5.26 rps	8.	28.16 rad s^{-1}
9.	1s	10.	108 days	11.	10.5 h	13.	mr^2 g cm^2

Analogy between translational and rotational motions

Linear motion		**Rotational motion**	
Quantities :			
Displacement	s	Angular displacement	θ
Velocity	v	angular velocity	ω
acceleration	a	angular acceleration	α

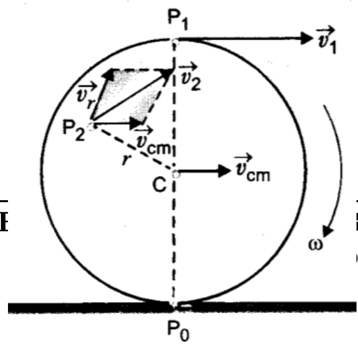
force	F	Torque	τ
mass	m	moment of inertia	I
Expressions :			
velocity	$v = \frac{ds}{dt}$	angular velocity	$\omega = \frac{d\theta}{dt}$
acceleration	$a = \frac{dv}{dt}$	angular acceleration	$\alpha = \frac{d\omega}{dt}$
force	$F = ma = \frac{d}{dt}(mv)$	Torque	$\tau = I\alpha = \frac{d}{dt}(I\omega)$
work done	$W = Fs$	work done	$W = \tau\,\theta$
linear K.E.	$E = \frac{1}{2}mv^2$	rotational K.E.	$E = \frac{1}{2}I\,\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Linear momentum	$p = mv$	angular momentum	$L = I\omega$
impulse	$F\,\Delta t = mv - mu$	angular impulse	$\tau\Delta t = I\,\omega_f - I\,\omega_i$
Equations of motion			
(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$		(i) $\omega = \omega_0 + \alpha t$ (ii) $\theta = \theta_0 t + \frac{1}{2}\alpha t^2$ (iii) $\omega^2 - \omega_0^2 = 2\alpha\theta$	
Dimensions :			
velocity	$[LT^{-1}]$	angular velocity	$[LT^{-1}]$
acceleration	$[LT^{-2}]$	angular acceleration	$[T^{-2}]$
mass	$[M]$	moment of inertia	$[ML^2]$
force	$[MLT^{-2}]$	torque $\tau = Fr$	$[ML^2T^{-2}]$
linear K.E.	$[ML^2T^{-2}]$	rotational K.E.	$[ML^2T^{-2}]$
momentum	$[MLT^{-1}]$	angular momentum	$[ML^2T^{-1}]$
power	$[ML^2T^{-3}]$	Power	$[ML^2T^{-3}]$

Rolling Motion

The rolling motion is one of the most common motions observed in daily life. All wheels, for example, used in transportation have rolling motion. We know that rolling motion is a combination of translation and rotation. Further, the translational motion of a system of particles is the motion of its centre of mass. Let us consider the rolling motion (without slipping) of a circular disc on a level surface. At any

instant, the point of contact P_0 of the disc with the surface is at rest (as there is no slipping). If $\vec{v}_{cm}$ is the velocity of centre of mass, which is at the geometric centre C of the disc, then the translational velocity of disc = $\vec{v}_{cm}$, which is parallel to the level surface.

As rotation of disc is about its symmetric axis, which passes through C, fig. therefore, velocity $\vec{v}_2$ of any point P_2 of disc is vector sum of $\vec{v}_{cm}$ and $\vec{v}_r$, which is the linear velocity on account of rotation.



The magnitude of $\vec{v}_r = r\,\omega$, where ω is angular velocity of rotation of the disc about the axis and r is the distance of the point from the axis (i.e., from C).

The velocity $\vec{v}_r$ is directed perpendicular to $\vec{r}$. In figure, we have shown velocity $\vec{v}_2$ of point P_2 as the resultant of $\vec{v}_{cm}$ and $\vec{v}_r$. At P_0 the linear velocity $\vec{v}_r$ at P_0 is $R\,\omega$, where R is radius of the disc. As P_0 is instantaneously at rest.

$$\boxed{\vec{v}_{cm} = R\,\omega}$$

Hence , for the disc to roll without slipping, the essential condition is $\vec{v}_{cm} = R\,\omega$. Obviously, velocity of point P_1 at the top of the disc , $v_1 = v_{cm} + R\omega = 2v_{cm}$ and is directed parallel to the level surface as shown in the figure.

Kinetic energy of Rolling motion

The general result for a system of particles in rolling motion is that total kinetic energy K of the system = K.E. of translational motion of centre of mass + K.E. of rotational motion (K_R) about the centre of mass.

i.e., $$K = K_T + K_R \qquad ....(1)$$

If m is mass of the body and v_{cm} is velocity of centre of mass of the body, then

K.E. of translation, $K_T = \frac{1}{2}mv_{cm}^2$

Since motion of rolling body about the centre of mass is rotation, therefore, K_R represents K.E. of rotation of the body

i.e., $$K_R = \frac{1}{2}I\,\omega^2$$

where I is moment of inertia of the body about the symmetry axis of the rolling body.

Putting in (1), we get

K.E. of rolling body, $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\,\omega^2$ ....(2)

From $v_{cm} = R\omega,\ \omega = \frac{v_{cm}}{R}$,

and $I = mk^2$, where k is corresponding radius of gyration of the body.

Putting in (2) , we get $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}mk^2\left(\frac{v_{cm}}{R}\right)^2$

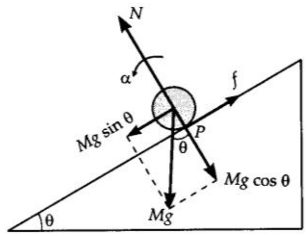
$$\boxed{K = \frac{1}{2}mv_{cm}^2\left[1 + \frac{k^2}{R^2}\right]}$$

This is a general equation, which applies to any rolling body, a disc, a ring, a cylinder and a sphere.

Note : When an object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping. See that friction is responsible for such a motion. But no work is done against friction because there is no relative motion between the body and the surface at the point of contact.

Solid cylinder rolling without slipping down an inclined plane

Consider a solid cylinder of mass M and radius R rolling down a plane inclined at an angle θ to the horizontal, as shown in fig. Suppose the cylinder roll down without slipping. The condition for rolling without slipping is that at each instant the line of contact of the cylinder with the surface at P is momentarily at rest and the cylinder rotates about this line as axis. The centre of mass of the cylinder moves in a straight line parallel to the inclined plane. Notably, it is the friction which prevent slipping.



The external forces acting on the cylinder are

(i) The weight Mg of the cylinder acting vertically downwards through the centre of mass of the cylinder.

(ii) The normal reaction N of the inclined plane acting perpendicular to the plane at P.

(iii) The frictional force f acting upwards and parallel to the inclined plane.

The weight Mg can be resolved into two rectangular components.

(i) $Mg\cos\theta$ perpendicular to the inclined plane.

(ii) $Mg\ \sin\theta$ acting down the inclined plane.

As there is no motion in a direction normal to the inclined plane, so

$$N = Mg\cos\theta$$

Applying Newton's second law to the linear motion of the centre of mass, the net force on the cylinder rolling down the inclined plane is

$$Ma = Mg\sin\theta - f$$

It is only the force of friction f which exerts torque τ on the cylinder and makes it rotate with angular acceleration α. It acts tangentially at the point of contact P and has lever arm equal to R.

$\therefore$ $\tau = \text{Force} \times \text{force arm} = f\ .\ R$

Also, $\tau = M.I. \times$ angular acceleration $= I\alpha$.

$\therefore$ $f\ R = I\alpha$ or $f = \dfrac{I\alpha}{R} = \dfrac{Ia}{R^2}$ $\quad \left[\because \alpha = \dfrac{a}{R}\right]$

Putting the value of f in equation (1), we get

$$Ma = Mg\sin\theta - \frac{Ia}{R^2}$$

$$a = g\sin\theta - \frac{Ia}{MR^2} \quad \text{or} \quad a + \frac{Ia}{MR^2} = g\sin\theta \quad \text{or} \quad a\left[1 + \frac{I}{MR^2}\right] = g\sin\theta$$

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}}$$

Moment of inertia of the solid cylinder about its axis $= \dfrac{1}{2}MR^2$.

$$\therefore \quad a = \frac{g\sin\theta}{1 + \dfrac{\frac{1}{2}MR^2}{MR^2}} \quad \text{or} \quad a = \frac{2}{3}g\ \sin\theta$$

Clearly, the linear acceleration a of solid cylinder rolling down an inclined plane is less than the acceleration due to gravity $g(a<g)$. The linear acceleration of the cylinder is constant for a given inclined plane (or given θ) and is independent of its mass *M* and radius *R*. However, for a hollow cylinder, $I = MR^2$, the value of *a* would decrease to $\frac{1}{2}g\sin\theta$.

From equation (1), the value of force of friction is

$$f = Mg\sin\theta - Ma = Mg\sin\theta - M\,.\frac{2}{3}g\,\sin\theta = \frac{1}{3}Mg\sin\theta$$

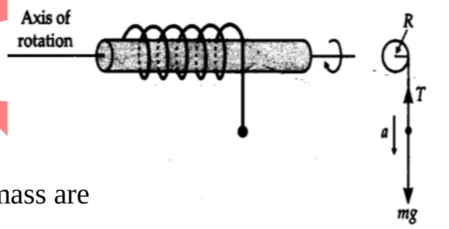
If μ_s is the coefficient of friction between the cylinder and the inclined plane, then

$$\mu_s = \frac{f}{N} = \frac{\frac{1}{3}Mg\sin\theta}{Mg\cos\theta} = \frac{1}{3}\tan\theta$$

To prevent slipping, the coefficient of static friction must be equal to or greater than the above value. That is

$$\mu_s \geq \frac{1}{3}\tan\theta \quad \text{or} \quad \tan\theta \leq 3\mu_s$$

Motion of a mass point attached to a string wound on a cylinder



As shown in the Fig. Consider a solid cylinder of mass m and radius R. It is mounted on a frictionless horizontal axle so that it can freely rotate about its axis. A light string is wound round the cylinder and mass *m* is suspended from it. When the mass m is released from rest, it moves down with acceleration *a*. Let *T* be the tension in the string.

(a) Linear acceleration of the point mass. The forces acting on the point mass are

(i) Its weight mg acting vertically downwards.

(ii) Tension T in the string acting upwards.

According to Newton's second law, the net down force on the point mass is

$$ma = mg - T \qquad \text{....(1)}$$

The tension *T* is the string acts tangentially on the cylinder and produces a torque τ given by

$$\tau = \text{Force} \times \text{lever arm} = T.\,R \qquad \text{....(2)}$$

If *I* is the moment of inertia of the cylinder and α, the angular acceleration produced in it, then

$$\tau = I\alpha \qquad \text{....(3)}$$

Form equation (2) and (3) , $TR = I\alpha$ or $T = \frac{I}{R}\alpha = \frac{Ia}{R^2}$ $\left[\because \alpha = \frac{a}{R}\right]$

Using above in equation (1), we get

$$ma = mg - \frac{Ia}{R^2} \quad \text{or} \quad ma + \frac{Ia}{R^2} = mg \quad \text{or} \quad ma\left(1+\frac{1}{mR^2}\right) = mg \quad \text{or} \quad a = \frac{g}{1+\frac{I}{mR^2}}$$

This gives the linear downward acceleration of the point mass.

(b) Angular acceleration of point mass : As *I*, *m* and *R* are positive quantities, so *a* is always less then '*g*'.

Angular acceleration, $\alpha = \frac{a}{R} = \frac{g/R}{1+\frac{I}{mR^2}} = \frac{Ig}{R^2\,.\frac{I}{mR^2}\left(\frac{mR^2}{I}+1\right)}$ or $T = \frac{mg}{1+\frac{mR^2}{I}}$

Clearly, T is less than the weight mg of the point mass.

Kinetic energy of rolling motion : The kinetic energy of a body rolling without slipping in the sum of kinetic energies of translation and rotation.

$$K = \text{K.E. f the translational motion of CM + K.E. of rotational motion of CM}$$

$$= \frac{1}{2} mv_{CM}^2 + \frac{1}{2} I\omega^2$$

where v_{CM} is the velocity of CM and I is the moment of inertia about the symmetry axis of the rolling body. If R is the radius and K the radius of gyration of the rolling body, then

$$v_{CM} = R\omega \text{ and } I = mk^2$$

$$\therefore \quad K = \frac{1}{2} mv_{CM}^2 + \frac{1}{2} mk^2 \left(\frac{v_{CM}}{R}\right)^2 \quad \text{or} \quad K = \frac{1}{2} mv_{CM}^2 \left(1 + \frac{k^2}{R^2}\right)$$

Subjective Assignment – VII

1. A cylinder of mass 5kg and radius 30 cm is rolling down an inclined plane at an angle of 45° with the horizontal. Calculate (i) force of friction, (ii) acceleration with which the cylinder rolls down and (iii) the minimum value of static friction so that cylinder does not slip while rolling down the plane.
2. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same in cylinder plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity.
3. A solid cylinder of radius 4cm and mass 250 g rolls down an inclined plane(1 in 10). Calculate the acceleration and the total energy of the cylinder after 5s.
4. A solid cylinder of mass 10kg is rolling perfectly on a plane of inclination 30°. Find the force of friction between the cylinder and the surface of the inclined plane.
5. A solid cylinder of mass 8 kg and radius 50cm is rolling down a plane inclined at an angle of 30° with the horizontal. Calculate (i) force of friction, (ii) acceleration with which the cylinder rolls down and (iii) the minimum value of coefficient of friction so that cylinder does not slip while rolling down the plane.
6. A body of mass 5 kg is attached to a weightless string wound round a cylinder of mass 8 kg and radius 0.3m. The body is allowed to fall. Calculate (i) tension in the string (ii) acceleration with which the body falls and (iii) the angular acceleration of the cylinder.

Answers

1. (i) 11.55 N (ii) 4.62 ms^{-2} (iii) $\frac{1}{3}$
2. $v_{sphere} > v_{cylinder} > v_{ring}$
3. 0.653 ms^{-2} , 2.0 J
4. 16.33 N
5. (i) 13.06 N (ii) 3.267 ms^{-2} (iii) 0.192
6. (i) 21.78 N (ii) 5.44 ms^{-2} (iii) 18.13 rad s^{-1}

Conceptual Problems

1. Is it correct to say that the centre of mass of a system of n particles is always given by the average position vectors of the constituent particles? If not, when it this statement true?
2. Why do we prefer to use a wrench of longer arm?

3. A man climbs a tall, old step ladder that has a tendency to sway. he feels much more unstable when standing near the top than when near the bottom why?
4. A faulty balance with unequal arms has its beam horizontal. Are the weights of the two pans equal?
5. A projectile acquires angular momentum about the point of projection during its flight. Does it violate the conservation of angular momentum?
6. Some heavy boxes are to be loaded along with some empty boxes on a cart. Which boxes should be put on the cart first and why?
7. Why we cannot rise from a chair without bending a little forward?
8. Does the moment of inertia of a rigid body change with the speed of rotation?
9. About which axis would a uniform cube have a minimum rotational inertia?
10. Two lenses of same mass and same radius are given. One is convex and other is concave. Which one will have greater moment of inertia, when rotating about an axis perpendicular to the plane and passing through the centre?
11. A disc is recast into a thin walled cylinder of same radius. Which will have large moment of inertia.
12. Why is it more difficult to revolve a stone tied to a large string than a stone tied to a smaller string?
13. Two satellites of equal masses, which can be considered as particles are orbiting the earth at different heights? Will their moments of inertia be same or different?
14. There is a stick half of which is wooden and half is of steel. It is pivoted at the wooden end and a force is applied at the steel end at right angles to its length. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why?
15. Why there are two propellers in a helicopter?
16. A thin wheel can stay up right on its rim for a considerable length of time when rolled with considerable velocity, while it falls from its upright position at the slightest disturbance when stationary. Give reason.
17. Many rivers flow towards the equator. What effect does the sediment they carry to the seas have on the rotation of the earth?
18. The moments of inertia of two rotating bodies *A* and *B* are I_A and $I_B (I_A > I_B)$ and their angular momenta are equal. Which one has a greater kinetic energy.
19. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be conserved?
20. If earth were to shrink suddenly, what would happen to the length of the day?
21 A body *A* of mass *M* while falling vertically downwards under gravity breaks into two parts; a body *B* of mass M/3 and a body *C* of mass 2M/3. How does the centre of mass of bodies *B* and *C* takes together shift compared to that of *A*?
22. A particle moves in a circular path with decreasing speed. What happens to its angular momentum?
23. If an external force can change the state of motion of *CM* of a body, how does the internal force of the brakes bring a car to rest?
24. Two men stand facing each other on two boats floating on still water at a distance apart. A rope is held at its ends by both. The two boats are found to meet always at the same point, whether each man pulls separately or both pull together, why? Will the time taken by them is different in the two cases? Neglect friction.
25. There are 100 passengers in a stationary railway compartment. A physical fight starts between the passengers over some difference of opinion. (i) will the position of *CM* of the compartment change? (ii) will the position of *CM* of system (compartment +100 passengers) change? Give reason.
26. Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion. Is there any external torque on the particle?

27. A rod of weight W is supported by two parallel edges A and B are is in equilibrium in horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. Find the normal reactions at the knife edges A and B.

28. How will you distinguish between a hard boiled egg and a raw egg by spinning it on a table top?

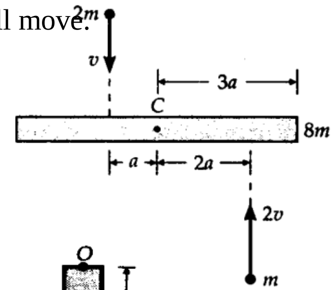
29. If two circular discs of the same mass and thickness are made from metals of different densities, which disc will have the larger moment of inertia about its central axis? Explain.

30. Two identical cylinders 'run a race' starting from rest at the top of an inclined plane, one slides without rolling and other rolls without slipping. Assuming that no mechanical energy is dissipated as heat, which one will win?

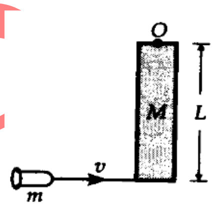
31. A uniform circular disc of radius R is rolling on a horizontal surface. Determine the tangential velocity (i) at the upper most point, (ii) at the centre of mass and (iii) at the point of contact.

32. A boat of 90 kg is floating in still water. A boy of mass 30 kg walks from the stern to the bow. The length of the boat is 3m. Calculate the distance through which the boat will move.

33. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point–masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v respectively strike the bar as shown in figure, and stick to the bar after collision. Determine (i) velocity of the centre of mass (ii) angular velocity about centre of mass and (ii) total kinetic energy just after collision.



34. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod with velocity v, as shown in fig. The bullet gets embedded in the rod. Find the angular velocity of the system just after the impact.



NCERT Exercise (short type questions)

1. Give the location of the centre of mass of mass of a

(i) sphere **(ii)** cylinder **(iii)** ring, and

(iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

2. In the HCl molecule, the separation between nuclei of the two atoms is about 1.27 Å (1Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that the chorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in all its nucleus.

3. A child sits stationary at one end of a long trolley moving uniformly with speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, then what is the effect of the speed of the centre of mass of the (trolley + child) system?

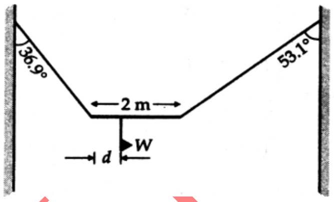
4. Show that the area of the triangle contained between vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.

5. Show that $\vec{a}\,.\,(\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelopiped formed on the three vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

6. Find the components along the x, y, z axes of the angular momentum $\vec{l}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x–y plane, the angular momentum has only a z–component.

7. Two particles each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken

8. A non–uniform bar of weight W is suspended at rest by two strings of negligible weights as shown in fig. The angles made by the strings with the vertical are 36.9^0 and 53.1^0 respectively. The bar is 2m long. Calculate the distance d of the centre of gravity of the bar from its left end.



9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

10. (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2\ MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

11. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

13. (i) A child stands at the centre of turntable with his two arms out stretched. The turntable is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/3 times the initial value? Assume that the turntable rotates without friction.

(ii) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

14. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

15. To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? Assume that the engine is 100% efficient.

16. From a uniform disc of radius R, a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of mass of the resulting flat body.

17. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick.

18. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination.

(a) Will it reach the bottom with the same speed in each case?

(b) Will it take longer to roll down one plane than the other?

(c) If so, which one and why?

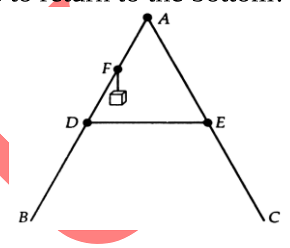
19. A hoop of radius 2m weights 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

20. The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the line joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

21. A solid cylinder rolls up an inclined plane of angle of inclination 30^{O}. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.

(a) How far will the cylinder go up the plane? **(b)** How long will it take to return to the bottom?

22. As shown in fig. the two sides of a step ladder *BA* and *CA* are 1.6 m long and hinged at *A*. A rope *DE*, 0.5 m is tied half way up. A weight 40 kg is suspended from a point *F*, 1.2m from *B* along the ladder *BA*. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forcers exerted by the floor on the ladder. (Take $g = 9.8$ m/s^2).



23. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.

(a) What is his new angular speed? (Neglect friction)

(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

24. A bullet of mass 10g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg . It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.

[Hint . The moment of inertia of the door about the vertical axis at one end is $ML^2/3$).

25. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

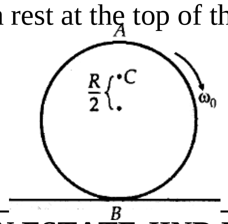
(i) What is the angular speed of the two–disc system?

(ii) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.

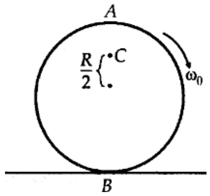
26. **(a)** Prove the theorem of perpendicular axes. **(b)** Prove the theorem of parallel axes.

27. Prove the result that the velocity *v* of translation of a rolling body(like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height *h* is given by $v^2 = \dfrac{2gh}{\left(1+\dfrac{k^2}{R^2}\right)}$ using dynamical consideration (i.e., by consideration of forces and torques). Note *k* is the radius of gyration of the body about its symmetry axis, and *R* is the radius of the body. The body starts from rest at the top of the plane.

28. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is *R*. What are the linear velocities of the points *A*, *B* and *C* on the disc shown in fig. Will the disc roll in the direction indicated?



29. Explain why friction is necessary to make the disc in fig. given in previous question roll in the direction indicated.



(i) Give the direction of frictional force at *B*, and the sense of frictional torque, before perfect rolling begins.

(ii) What is the force of friction after perfect rolling begins?

30. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10 π rad s^{-1}. Which of the two will start to roll earlier? The co–efficient of kinetic friction is $\mu_k = 0.2$.

31. A solid cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30^o. The coefficient of static friction, $\mu_s = 0.25$.

(i) Find the force of friction acting on the cylinder.

(ii) What is the work done against friction during rolling?

(iii) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?

32. Read each statement below carefully, and state, with reasons, if it is true or false :

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the *CM* of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

Answers

1. **(i)** Geometrical centre **(ii)** Centre of its axis of symmetry
(iii) Centre of the ring **(iv)** Point of intersection of the diagonals.
No it is not necessary that centre of mass of a body lies inside the body.

2. 1.235 Å from *H* nucleus.

3. No change in the speed of the centre of mass of the (trolley + child) system.

6. $\hat{k}\,(xp_y - yp_x)$ **8.** 72 cm

9. Front wheel 3675 N ; back wheel 5145 N

10. (a) $\frac{7}{5}MR^2$ **(b)** $\frac{3}{2}MR^2$ **11.** solid sphere **12.** 3125 J ; 62.5 kg m^2 s^{-1}

13. **(i)** 100 rpm **(ii)** 2.5 **14.** 25 rad s^{-2} ; 10 ms^{-2}

15. 36 kW **16.** *R*/6 from the centre of original disk **17.** 66.0 g

18. **(a)** Yes
(b) Yes the sphere will take longer time to roll down one plane than the other.
(c) The sphere will take larger time in case of the plane with smaller inclination because the acceleration $a \propto \sin\theta$.

19. 4 J **20.** 6.75×10^{12} rad s^{-1} **21.** **(a)** 3.8 m **(b)** 3.0 s

22. $N_B = 245N$; $N_C = 147N$; $T = 97N$ **23.** **(a)** 59 rpm **(b)** 1.97

24. 0.625 rad s^{-1} **25.** **(i)** $\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$ **(ii)** $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

28. $V_A = R\omega_0,\ \ V_B = R\omega_0,\ \ V = \frac{R}{2}\omega_0$; Not roll

29. **(i)** Opposite to direction of velocity **(ii)** zero

30. t_{disk} = 0.53s ; t_{ring} = 0.80 s **31.** **(i)** 16.33 N **(ii)** 0 J **(iii)** 37^0

32. **(a)** True **(b)** True **(c)** False **(d)** True **(e)** True

CBSE PMT Prelims Exam

Q.1 Three identical metal balls, each of the radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at

(a) line joining centres of any two balls (b) centre of one of the balls
(c) horizontal surface (d) point of intersection of the medians

Q.2 The centre of mass of a system of particles does not depend on

(a) position of the particles (b) relative distance between the particles
(c) masses of the particles (d) forces acting on the particles

Q.3 A solid sphere of radius R is placed on smooth horizontal surface. A horizontal force F is applied at height h from the lowest pint. For the maximum acceleration of centre of mass, which is correct?

(a) h = R (b) h = 2 R
(c) h = 0 (d) no relation between h and R

Q.4 Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the mass centre of particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position?

(a) $\frac{m_1}{m_1 + m_2}d$ (b) $\frac{m_1}{m_2}d$ (c) d (d) $\frac{m_2}{m_1}d$

Q.5 A rod has length 3 m and its mass acting per unit length is directly proportional to distance x from one of its end, then its centre of gravity from that end will be at

(a) 1.5 m (b) 2 m (c) 2.5 m (d) 3.0 m

Q.6 A disc is rolling, the velocity of its centre of mass is v_{cm}. Which one will be correct?

(a) the velocity of highest point is $2v_{cm}$ and point of contact is zero.
(b) the velocity of highest point is v_{cm} and point of contact is v_{cm}.
(c) the velocity of highest point is $2v_{cm}$ and point of contact is v_{cm}.
(d) the velocity of highest point is $2v_{cm}$ and point of contact is $2v_{cm}$.

Q.7 Identify the vector quantity among the following:

(a) distance (b) angular momentum (c) heat (d) energy

Q.8 Angular momentum is

(a) axial vector (b) polar vector (c) scalar (d) none of the above

Q.9 A couple produces

(a) linear and rotational motion (b) no motion
(c) purely linear motion (d) purely rotational motion

Q.10 A wheel has angular acceleration of 3.0 rad/sec^2 and an initial angular speed of 2.00 rad/sec. In a time of 2 sec, it has rotated through an angle (in radian) of

(a) 10 (b) 12 (c) 4 (d) 6

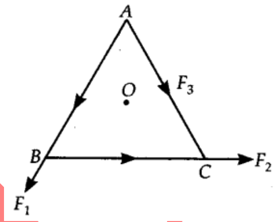
Q.11 Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

(a) $-21\hat{i} + 4\hat{j} + 4\hat{k}$ (b) $-14\hat{i} + 34\hat{j} - 16\hat{k}$ (c) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (d) $4\hat{i} + 4\hat{j} + 6\hat{k}$

Q.12 What is the torque of the force $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ N acting at the point $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ m about origin?

(a) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (b) $-17\hat{i} + 6\hat{j} + 13\hat{k}$ (c) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (d) $17\hat{i} - 6\hat{j} - 13\hat{k}$

Q.13 O is the centre of an equilateral triangle ABC. F_1, F_2 and F_3 are three forces acting along the sides AB, BC and AC as shown in the figure. What should be the magnitude of F_3, so that the total torque about O is zero?



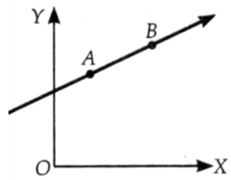
(a) $(F_1 + F_2)$ (b) $2(F_1 + F_2)$

(c) $(F_1 + F_2)/2$ (d) $(F_1 - F_2)$

Q.14 A particle of mass m = 5 is moving with a uniform speed $v = 3\sqrt{2}$ in the XOY plane along the line y = x + 4. The magnitude of the angular momentum of the particle about the origin is

(a) 60 units (b) $40\sqrt{2}$ *units* (c) zero (d) 7.5 units

Q.15 A particle of mass m movies in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then



(a) $L_A = L_B$

(b) the relationship between L_A and L_B depends upon the slope of the line AB

(c) $L_A < L_B$ (d) $L_A > L_B$

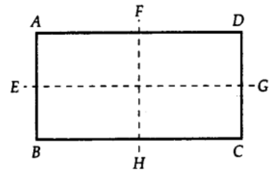
Q.16 A circular disc is to be made by using iron and aluminium so that it acquired maximum moment of inertia about geometrical axis. It is possible with

(a) aluminium at interior and iron surround to it (b) iron at interior and aluminium surround to it

(c) using iron and aluminium layers in alternate order

(d) sheet of iron is used at both external surfaces and aluminium sheet as internal layers.

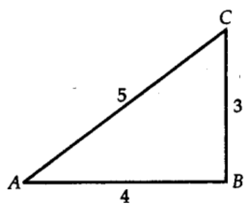
Q.17 In a rectangle ABCD (BC = 2AB). The moment of inertia is minimum along axis through



(a) BC (b) BD

(c) HF (d) EG

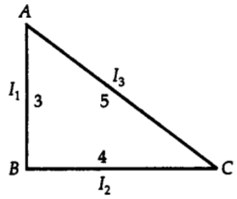
Q.18 ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plane about AB, BC and CA respectively. Which one of the following relations is correct?



(a) $I_{AB} + I_{BC} = I_{CA}$ (b) I_{CA} is maximum

(c) $I_{AB} > I_{BC}$ (d) $I_{BC} > I_{AB}$

Q.19 For the adjoining diagram, the correct relation between I_1, I_2 and I_3 is (I = moment of inertia)



a) $I_1 > I_2$ (b) $I_2 > I_1$

(c) $I_3 > I_1$ (d) $I_3 > I_2$

Q.20 The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at the diameter and normal to the disc is

(a) $\frac{1}{2}MR^2$ (b) MR^2 (c) $\frac{2}{5}MR^2$ (d) $\frac{3}{2}MR^2$

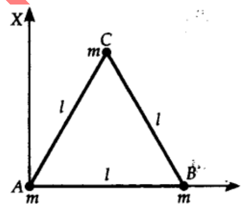
Q.21 Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be

(a) 5 I (b) 3 I (c) 6 I (d) 4 I

Q.22 The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter is

(a) $\frac{5}{4}MR^2$ (b) $\frac{2}{3}MR^2$ (c) $\frac{3}{2}MR^2$ (d) $\frac{4}{5}MR^2$

Q.23 Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in figure). The moment of inertia of the system about a line AX perpendicular to AB and in the pane of ABC, in gram–cm^2 units will be



(a) $\frac{3}{4}ml^2$ (b) 2ml^2 (c) $\frac{5}{4}ml^2$ (d) $\frac{3}{2}ml^2$

Q.24 A thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90^o. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is

(a) $\frac{ML^2}{24}$ (b) $\frac{ML^2}{12}$ (c) $\frac{ML^2}{6}$ (d) $\frac{\sqrt{2}\,ML^2}{24}$

Q.25 A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is

(a) $\frac{1}{2}mr^2\omega^2$ (b) $mr\omega^2$ (c) $mr^2\omega^2$ (d) $\frac{1}{2}mr\omega^2$

Q.26 A flywheel rotating about fixed axis has a kinetic energy of 360 joule when its angular speed is 30 radian/sec. The moment of inertia of the wheel about the axis of rotation is

(a) 0.6 kgm^2 (b) 0.15 kgm^2 (c) 0.8 kgm^2 (d) 0.75 kgm^2

Q.27 Two bodies have their moments of inertia 1 and 21 respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio

(a) 2 : 1 (b) 1 : 2 (c) $\sqrt{2}$: 1 (d) 1:$\sqrt{2}$

Q.28 The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

(a) 2 : 3 (b) 2 : 1 (c) $\sqrt{5}:\sqrt{6}$ (d) $\sqrt{2}:\sqrt{3}$

Q.29 The ratio of the radii of gyration of a circular disc to that of circular ring, each of same mass and same radius about their axes is

(a) $\sqrt{3}:\sqrt{2}$ (b) $1:\sqrt{2}$ (c) $\sqrt{2}:1$ (d) $\sqrt{2}:\sqrt{3}$

Q.30 A wheel having moment of inertia 2 kg m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

(a) $\frac{2\pi}{15}N\,m$ (b) $\frac{\pi}{12}N\,m$ (c) $\frac{\pi}{15}N\,m$ (d) $\frac{\pi}{18}N\,m$

Q.31 The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of

(a) 4s (b) 2s (c) 8 s (d) 10 s

Q.32 A disc is rotating with angular speed to ω. If a child sits on it, what is conserved

(a) linear momentum (b) angular momentum (c) kinetic energy (d) potential energy

Q.33 A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\frac{I_2\omega}{I_1+I_2}$ (b) ω (c) $\frac{I_1\omega}{I_1+I_2}$ (d) $\frac{(I_1+I_2)\omega}{I_1}$

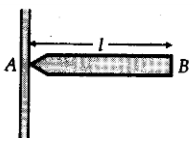
Q.34 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring will now rotate with an angular velocity

(a) $\frac{\omega(M+2m)}{M}$ (b) $\frac{\omega M}{M+2m}$ (c) $\frac{\omega(M-2m)}{M+2m}$ (d) $\frac{\omega M}{M+m}$

Q.35 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass *m*, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{4m}$ (b) $\frac{M\omega}{M+4m}$ (c) $\frac{(M+4m)\omega}{M}$ (d) $\frac{(M-4m)\omega}{M+4m}$

Q.36 A uniform rod *AB* of length *l* and mass m is free to rotate in a vertical plane about point A. The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about A is $ml^2/3$, the initial angular acceleration of the rod will be



(a) $gl/2$ (b) $\frac{3}{2}gl$ (c) $\frac{3g}{2l}$ (d) $\frac{2g}{3l}$

Q.37 If a sphere is rolling, the ratio of translational energy to total kinetic energy is given by

(a) 7: 10 (b) 2 : 5 (c) 10 : 7 (d) 5 : 7

Q.38 A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is

(a) 1/2 (b) 1/6 (c) 7/10 (d) 2/7

Q.39 The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is

(a) $\sqrt{\frac{10}{7}gh}$ (b) $\sqrt{gh}$ (c) $\sqrt{\frac{6}{5}gh}$ (d) $\sqrt{\frac{4}{3}gh}$

Q.40 For a hollow cylinder and a solid cylinder rolling without slipping on an inclined plane, then which of these reaches earlier

(a) solid cylinder (d) can't say anything. (c) both simultaneously (b) hollow cylinder

Q.41 A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on the inclined plane, then

(a) solid sphere reaches the bottom first
(b) solid sphere reaches the bottom last
(c) disc will reach the bottom first
(d) all reach the bottom at the same time

Q.42 A solid homogenous sphere of mass M and radius R is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of this sphere

(a) total kinetic energy is conserved

(b) the angular momentum of the sphere about the point of contact with the plane is conserved

(c) only the rotational kinetic energy about the centre of mass is conserved

(d) angular momentum about the centre of mass is conserved.

Q.43 A drum of radius R and mass M rolls down without slipping along an inclined plane of angle 0. The frictional force

(a) dissipates energy as heat
(b) decreases the rotational motion
(c) decreases the rotational and translational motion
(d) converts translational energy to rotational energy.

Q.44 A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom?

(a) $\sqrt{2gh}$
(b) $\sqrt{\frac{3}{4}gh}$
(c) $\sqrt{\frac{4}{3}gh}$
(d) $\sqrt{4gh}$

Q.45 A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is k. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{k^2+R^2}{R^2}$
(b) $\frac{k^2}{R^2}$
(c) $\frac{k^2}{k^2+R^2}$
(d) $\frac{R^2}{k^2+R^2}$

Q.46 If point P is the point of contact of a wheel on ground which rolls on ground without slipping, then value of displacement of point P when wheel completes half of rotation (if radius of wheel is 1 m) is

(a) 2m
(b) $\sqrt{\pi^2+4}\,m$
(c) π m
(d) $\sqrt{\pi^2+2}\,m$

Q.47 Two bodies of masses 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The center of mass of this system has a position vector

(a) $-2\hat{i}-2\hat{j}+\hat{k}$
(b) $2\hat{i}-\hat{j}-2\hat{k}$
(c) $-\hat{i}+\hat{j}+\hat{k}$
(d) $-2\hat{i}+2\hat{k}$

Q.48 Four thin rods of mass *M* and length *l* form a square frame. Moment of inertia perpendicular to its plane is axis through the centre and perpendicular to the plane of square

(a) $\frac{2}{3}Ml^2$
(b) $\frac{13}{3}Ml^2$
(c) $\frac{1}{3}Ml^2$
(d) $\frac{4}{3}Ml^2$

Answers

1	d	2	d	3	d	4	b	5	b
6	a	7	b	8	a	9	d	10	a
11	c	12	d	13	a	14	a	15	a
16	a	17	d	18	d	19	b	20	d
21	c	22	a	23	c	24	b	25	a
26	c	27	c	28	c	29	b	30	c
31	b	32	b	33	c	34	b	35	b
36	c	37	d	38	d	39	a	40	a
41	a	42	b	43	d	44	c	45	c

46	b	47	a	48	d

DPMT Entrance Exam

Q.1 Two bodies of mass 2kg and 4 kg are moving with velocities 2 m/s and 10 m/s respectively. What is velocity of their center of mass?

(a) 5.3 m/s (b) 7.3 m/s (c) 6.4 m/s (d) 8.1 m/s

Q.2 Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car will be

(a) 1 : 1 (b) $r_1 : r_2$ (c) $m_1 : m_2$ (d) $m_1m_2 : r_1r_2$

Q.3 Two wheels having radii in the ratio 1 : 3 are connected by a common belt. If the smaller wheel is accelerated from rest at a rate 1.5 rads^{-2} for 10 s, find the velocity of bigger wheel.

(a) 5 rads^{-1} (b) 15 rads^{-1} (c) 45 rads^{-1} (d) none of these

Q.4 Angular acceleration α of a body is given by the relation $\alpha = 4\,at^3 - 3bt^2$. If initial angular velocity of the body is ω_0, then its velocity at time t will be

(a) $\omega_0 + at^4 - bt^3$ (b) $\omega_0 + 4at^4 - 4bt^3$ (c) $\omega_0 + 12\,at^2 - 6bt$ (d) $\omega_0 - at^4 + bt^3$

Q.5 If $\vec{F}$ is force and $\vec{r}$ is the radius vector, then torque is given by

(a) $\vec{r} \times \vec{F}$ (b) $\vec{r} . \vec{F}$ (c) $|\vec{r}||\vec{F}|$ (d) none of these

Q.6 Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$.

(a) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (b) $4\hat{i} + 4\hat{j} + 6\hat{k}$ (c) $-14\hat{i} + 38\hat{j} - 16\hat{k}$ (d) $-21\hat{i} + 3\hat{j} + 6\hat{k}$

Q.7 Which one is a vector quantity?

(a) energy (b) torque (c) both of these (d) none of these

Q.8 A car is moving at a speed of 72 km/h. The diameter of its wheels is 0.5 m. If the wheels are stopped in 20 rotations by applying brakes, then angular retardation produced by the brakes is

(a) – 25.5 rad/s^2 (b) –33.5 rad/s^2 (c) –29.5 rad/s^2 (d) –45.5 rad/s^2

Q.9 An automobile engine develops 100 kW when rotating at a speed of 1800 rev/min. What torque does it deliver?

(a) 350 Nm (b) 531 Nm (c) 440 Nm (d) 628 Nm

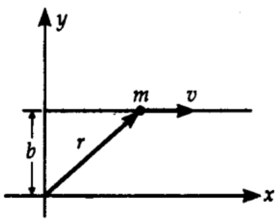
Q.10 Joule second is the unit of

(a) momentum (b) angular momentum (c) work (d) pressure

Q.11 Under a constant torque, the angular momentum of a body changes from A to 4A in 4 sec. The torque on the body will be

(a) $\frac{3}{4}A$ (b) $\frac{1}{4}A$ (c) $\frac{4}{3}A$ (d) 4 A

Q.12 A particle of mass m is moving with a constant velocity v parallel to x–axis in an x–y plane as shown in figure. Calculate angular momentum with respect to origin at any instant.



(a) $-mvb\,\hat{k}$ (b) zero

(c) $\frac{mvb}{2}\hat{k}$ (d) $mvb \cos\theta\,\hat{k}$

Q.13 Moment of inertia of a body depends upon

(a) axis of rotation (b) earth's gravitational constant

(c) relativistic effect of motion of earth around sun (d) none of these

Q.14 Moment of inertia of a thin circular disc of mass M and radius R about any diameter is

(a) $\frac{MR^2}{4}$ (b) MR^2 (c) $\frac{MR^2}{2}$ (d) 2 MR^2

Q.15 Moment of inertia of ring about its diameter is *I*. The moment of inertia of the same ring about the axis perpendicular to its plane and passing through centre is

(a) I/2 (b) 2 I (c) I/4 (d) 4 I

Q.16 The ratio of radii of gyration of a circular disc and a circular ring of the same radii and same mass about a tangential axis in the plane is

(a) 1 : 2 (b) $\sqrt{5}:\sqrt{6}$ (c) 2 : 3 (d) 2 : 1

Q.17 The radius of gyration of a disc of mass 100 g and radius 5 cm about an axis passing through its center of gravity and perpendicular to the plane is

(a) 1.52 cm (b) 3.54 cm (c) 2.51 cm (d) 6.54 cm

Q.18 A flywheel is attached to an engine to

(a) increase its speed (b) decrease its speed

(c) help in overcoming the dead point (d) decrease its energy

Q.19 The angular momentum of a system of particles is not conserved

(a) when a net external force acts upon the system

(b) when a net external torque is acting upon the system

(c) when a net external impulse is acting upon the system (d) none of these

Q.20 A diver in a swimming pool bends his head before diving. It

(a) increases his linear velocity (b) decreases his angular velocity

(c) increases his moment of inertia (d) decreases his moment of inertia

Q.21 A disc having mass *M* and radius *R* is rotating with angular velocity ω, another disc of means 2*M* and radius *R*/2 is placed coaxially on first disc gently. The angular velocity of system will now be

(a) $\frac{4\omega}{5}$ (b) $\frac{2\omega}{5}$ (c) $\frac{3\omega}{2}$ (d) $\frac{2\omega}{3}$

Q.22 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. If two objects of mass m are attached gently to opposite ends of a diameter of ring, ring will now rotate with an angular velocity given by

(a) $\frac{2\omega M}{(M-2m)}$ (b) $\frac{(M-2m)}{M}$ (c) $\frac{\omega M}{(M+2m)}$ (d) $\frac{\omega M}{(M-2m)}$

Q.23 A circular disc of mass M and radius R is rotating with an angular velocity ω about an axis passing through its centre and perpendicular to the plane of the disc. A small point like part of mass m detaches from the rim of the disc and continues to move with same angular speed. The angular velocity of remaining disc just after detaching will become

(a) $\left(\frac{M-2m}{M+m}\right)\omega$ (b) $\left(\frac{M+2m}{M+m}\right)\omega$ (c) $\left(\frac{M-2m}{M-m}\right)\omega$ (d) $\left(\frac{M+2m}{M-m}\right)\omega$

Q.24 Moment of inertia of a body is 1 kg m^2. If the body makes 2 revolutions per second, when its angular momentum is

(a) 2 π Js (b) 4 π J s (c) π/2 J s (d) π J s

Q.25 A constant torque of 31.4 N m is exerted on a pivoted wheel. If the angular acceleration of wheel is 4π rad/s^2, then the moment of inertia of the wheel is

(a) 2.5 kg m^2 (b) 4.5 kg m^2 (c) 3.5 kg m^2 (d) 5.5 kg m^2

Q.26 A particle performs uniform circular motion with an angular momentum L. If the frequency of particle motion is doubled and its KE is halved, the angular momentum becomes

(a) 2 L (b) 4 L (c) L/2 (d) L/4

Q.27 Angular momentum L of body with moment of inertia I and angular velocity ω rad/sec is equal to

(a) I/ω (b) $I\omega^2$ (c) Iω (d) none of these

Q.28 A circular disc of mass m and radius r is rolling forward on horizontal table with a velocity v. Its total kinetic energy is

(a) mv^2 (b) $\frac{3}{4}mv^2$ (c) $\frac{1}{4}mv^2$ (d) $\frac{1}{2}mv^2$

Q.29 When a sphere of moment of inertia I about an axis through centre of gravity and mass m rolls from rest down an inclined plane without slipping, its kinetic energy is

(a) $\frac{1}{2}I\omega^2$ (b) $\frac{1}{2}mv^2$ (c) $I\omega + mv$ (d) $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

Q.30 The acceleration of a solid cylinder rolling down an inclined plane of inclination 30° is

(a) g/2 (b) g (c) g/3 (d) g/4

Q.31 A cylinder is rolling down an inclined plane of inclination 60°. What is its acceleration?

(a) $g/\sqrt{3}$ (b) $g\sqrt{3}$ (c) $\sqrt{\frac{2}{3}}g$ (d) none of these

Q.32 Two identical co-centric rings each of mass m and radius R are placed perpendicularly. What is the moment of inertia about axis of one of the rings?

(a) $\frac{1}{2}MR^2$ (b) MR^2 (c) $\frac{3}{2}MR^2$ (d) $2MR^2$

Q.33 A uniform rod of Length L and mass 1.8 kg is made to rest on two measuring scales at its two ends. A uniform block of mass 2.7 kg is placed on the rod at a distance of L/ 4 from the left end. The force experienced by the measuring scale on the right end is

(a) 16 N (b) 27 N (c) 29 N (d) 45 N

Assertion and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both assertion and reason are false.

Q.34 **Assertion:** There are very small sporadic changes in the speed of rotation of the earth.

Reason: Shifting of large air masses in the earth's atmosphere produces a change in the moment of inertia of the earth causing its speed of rotation to change.

Q.35 **Assertion**: If the ice on the polar caps of the earth melts, then length of day will increase.
Reason: Moment of inertia of earth increases, as ice on polar caps melts

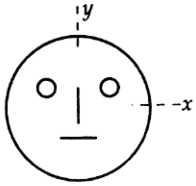
Answers

1	b	2	a	3	a	4	a	5	a
6	a	7	b	8	a	9	b	10	b
11	a	12	a	13	a	14	a	15	b
16	b	17	b	18	c	19	b	20	d
21	d	22	c	23	c	24	b	25	a
26	d	27	c	28	b	29	d	30	c
31	a	32	c	33	a	34	a	35	a

IIT Entrance Exam.

Only one correct Answers :

1. Look at a drawing given in the figure which has been drawn with ink of uniform line–thickness. The mass of ink used to draw each of the two inner circles, and each of the two lines segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle $(-a, a)$ right inner circle (a, a) vertical line (0, 0) and horizontal line $(0, -a)$. The y–coordinate of the centre of mass of the ink in this drawing is



(a) $\frac{a}{10}$ (b) $\frac{a}{8}$ (c) $\frac{a}{12}$ (d) $\frac{a}{3}$

2. Two particles A and B, initially at rest, move towards each other under mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of the centre of mass of the system is
(a) 3 v (b) v (c) 1.5 v (d) zero

3. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is
(a) 30 m/s (b) 20 m/s (c) 10 m/s (d) 5 m/s

4. An isolated particle of mass m is moving in horizontal plane (x–y) along the x–axis at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at
(a) $y = -5$ cm (b) $y = +20$ cm (c) $y = +5$ cm (d) $y = -20$ cm

5. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity u. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then
(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$ (c) $\omega_A = \omega$ (d) $\omega_B = \omega$

6. A mass m is moving with a constant velocity along a line parallel to the x–axis, away from the origin. Its angular momentum with respect to the origin.
(a) is zero (b) remains constant (c) goes on increasing (d) goes on decreasing

7. A particle of mass m is projected with a velocity v making an angle of 45^0 with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection, when the particle is at its maximum height h is

(a) zero (b) $\frac{mv^3}{4\sqrt{2}\,g}$ (c) $\frac{mv^3}{\sqrt{2}\,g}$ (d) $m\sqrt{2gh^3}$

8. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?

(a) Centre of the circle (b) on the circumference of the circle

(c) inside the circle (d) outside the circle

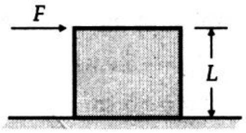
9. A particle is confined to rotate in a circular path with decreasing linear speed. Then which of the following is correct?

(a) $\vec{L}$ (angular momentum) is conserved about the centre

(b) only direction of angular momentum $\vec{L}$ is conserved.

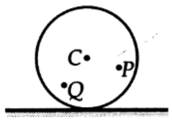
(c) it spirals towards the centre (d) its acceleration is towards the centre

10. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



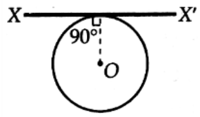
(a) infinitesimal (b) $\frac{mg}{4}$ (c) $\frac{mg}{2}$ (d) $mg(1-\mu)$

11. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is



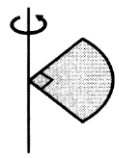
(a) $v_Q > v_C > v_P$ (b) $v_P > v_C > v_Q$ (c) $v_P = v_C,\ v_Q = v_C/2$ (d) $v_P < v_C > v_Q$.

12. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



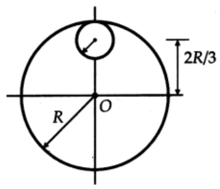
(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

13. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is



(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$ (c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}\,MR^2$

14. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10\,MR^2$ (d) $\frac{37}{9}MR^2$

15. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as

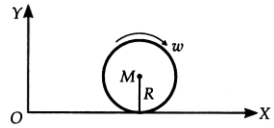
(a) $r = \sqrt{\frac{2}{15}}R$ (b) $r = \frac{2}{\sqrt{15}}R$ (c) $r = \frac{2}{15}R$ (d) $r = \frac{\sqrt{2}}{15}R$

16. Two points masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg (b) 0.70 m from mass of 0.7 kg

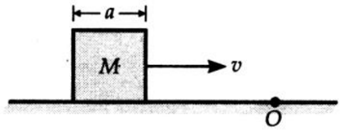
(c) 0.98 m from mass of 0.3 kg (d) 0.98 m from mass of 0.7 kg

17. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is



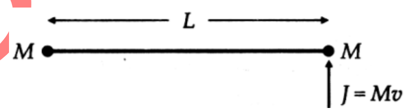
(a) $\left(\frac{1}{2}\right)MR^2\omega$ (b) $MR^2\omega$ (c) $\left(\frac{3}{2}\right)MR^2\omega$ (d) $2MR^2\omega$

18. A cubical black of side a is moving with velocity v on a horizontal smooth plane as shown in figure. It hits a ridge at point O. The angular speed of the block after it hits O is



(a) $\frac{3v}{4a}$ (b) $\frac{3v}{2a}$ (c) $\frac{3v}{\sqrt{2}a}$ (d) zero

19. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity?



(a) v/L (b) $2v/L$ (c) $v/3L$ (d) $v/4L$

20. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

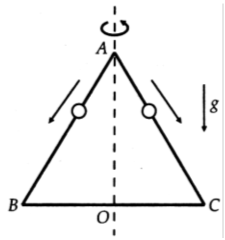
(a) up the incline, while ascending and down the incline, while descending.

(b) up the incline, while ascending as sell as descending

(c) down the incline, while ascending and up the incline, while descending

(d) down the incline, while ascending as well as descending

21. An equilateral triangle ABC formed from uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects , the quantities that are conserved as the beads slide down are



(a) Angular velocity and total energy (kinetic and potential)

(b) total angular momentum and total energy

(c) angular velocity and moment of inertia about the axis of rotation.

(d) total angular momentum and moment of inertia about the axis of rotation.

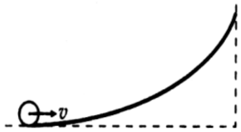
22. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be

(a) $2K$ (b) $K/2$ (c) K (d) $K/4$

23. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

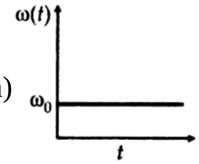
(a) 2 K (b) K/2 (c) K /4 (d) 4 K

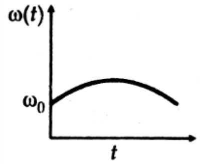
24. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

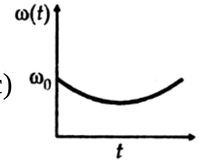


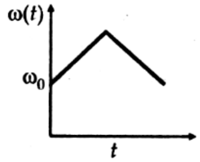
(a) ring (b) solid sphere (c) hollow sphere (d) disc

25. A circular platform is free to rotate in horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

(a) 

(b) 

(c) 

(d) 

26. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y–axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x–axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest w.r.t. the wire, from the y–axis is

(a) $\frac{a}{gk}$ (b) $\frac{a}{2gk}$ (c) $\frac{2a}{gk}$ (d) $\frac{a}{4gk}$

Multiple choice questions with one or more than one correct answer

27. A ball hits the floor and rebounds after an inelastic collision. In this case

(a) The momentum of the ball just after the collision is the same as that just before the collision

(b) The mechanical energy of the ball remains the same in the collision

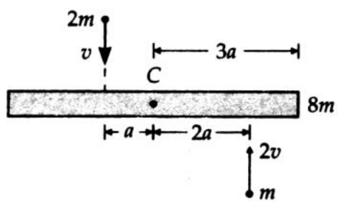
(c) The total momentum of the ball and the earth is conserved

(d) The total energy of the ball and the earth is conserved.

28. A shell is fired from a cannon with a velocity v(m/sec) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec) of the other piece immediately after the explosion is

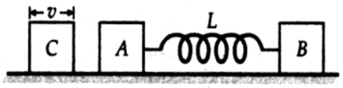
(a) $3v \cos\theta$ (b) $2v \cos\theta$ (c) $\frac{3}{2}v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

29. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v respectively, strike the bar [as shown in figure] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_C respectively, we have after collision



(a) $v_C = 0$ (b) $\omega = \frac{3v}{5a}$ (c) $\omega = \frac{v}{5a}$ (d) $E = \frac{3mv^2}{5}$

30. Two blocks A and B, each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in the figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then



(a) the kinetic energy of the A–B system, at maximum compression of the spring, is zero.

(b) The kinetic energy of the A–B system, at maximum compression of the spring is $mv^2/4$.

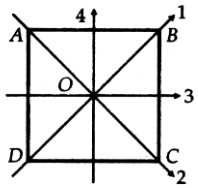
(c) The maximum compression of the spring is $v\sqrt{(m/k)}$.

(d) the maximum compression of the spring is $v\sqrt{(m/2k)}$.

31. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\dfrac{M\omega^2 L}{2}$ (b) $M\omega^2 L$ (c) $\dfrac{M\omega^2 L}{4}$ (d) $\dfrac{M\omega^2 L^2}{2}$

32. The moment of inertia of a thin square plate $ABCD$, as shown in the figure, of uniform thickness about an axis passing through the centre O and perpendicular to the plane of plate is



(a) $I_1 + I_2$ (b) $I_3 + I_4$ (c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

where I_1, I_2, I_3 and I_4 are respectively the moments of inertia about axis 1, 2, 3 and 4 which are in the plane of the plate.

33. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of plate about the axis CD then equal to

(a) I (b) $I\sin^2\theta$ (c) $I\cos^2\theta$ (d) $I\cos^2\theta/2$

34. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then

(a) the friction force is dissipative. (b) the friction force is necessarily changing

(c) the friction force will aid rotation but hinder translation

(d) the friction force is reduced if θ is reduced

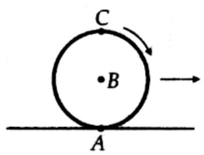
35. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame , one can surely say that

(a) Linear momentum of the system does not change in time

(b) kinetic energy of the system does not change in time

(c) angular momentum of the system does not change in time

(d) potential energy of the system does not change in time.



36. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then

(a) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(c) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (d) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

Reasoning types questions :

This question contains **A**(assertion) and **R** (reason).

(a) A and R are true and R is a correct explanation for A.

(b) A and R are true and R is not a correct explanation for A.

(c) A is true, R is false (d) A is false, R is true

37. **A :** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

R : The linear momentum of an isolated system remains constant.

38. **A :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

R : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

Comprehension based questions

Passage 1 : Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

Read the passage given above and answer the following questions :

39. The ratio $\frac{x_1}{x_2}$ is

(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

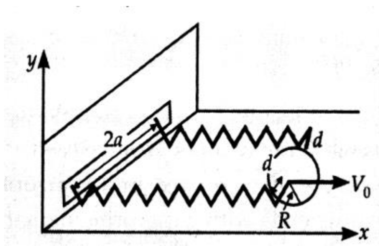
40. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on the disc by the other during this period is

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$ (c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

41. The loss of kinetic energy during the above process is

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$ (c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

Passage – 2 : A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the fig. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unscratched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ.



42. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$ (c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$

43. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$ (c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

44. The maximum value of V_0 for which the disk will roll without slipping is

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$ (c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

Answers

1.	A	**2.**	D	**3.**	C	**4.**	A	**5.**	C
6.	B	**7.**	B	**8.**	A	**9.**	B	**10.**	C
11.	B	**12.**	D	**13.**	A	**14.**	A	**15.**	B
16.	C	**17.**	C	**18.**	A	**19.**	A	**20.**	B
21.	B	**22.**	B	**23.**	B	**24.**	D	**25.**	B
26.	B	**27.**	C, D	**28.**	A	**29.**	A,C,D	**30.**	B,D
31.	A	**32.**	A,B,C	**33.**	A	**34.**	C,D	**35.**	A
36.	B,C	**37.**	D	**38.**	D	**39.**	C	**40.**	A
41.	B	**42.**	D	**43.**	D	**44.**	A		

AIEEE

Only one correct Answers :

1. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C (b) body B (c) depends on height of breaking (d) does not shift

2. Two identical particles move towards each other with velocities $2v$ and v respectively. The velocity of centre of mass is

(a) v (b) $v/3$ (c) $v/2$ (d) zero

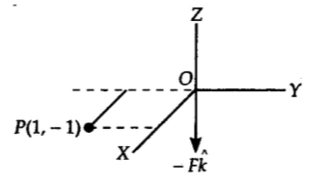
3. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) d (b) $\frac{m_2}{m_1}d$ (c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$

4. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumferences of the disc coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is

(a) 1/3 (b) 1/2 (c) 1/6 (d) 1/4

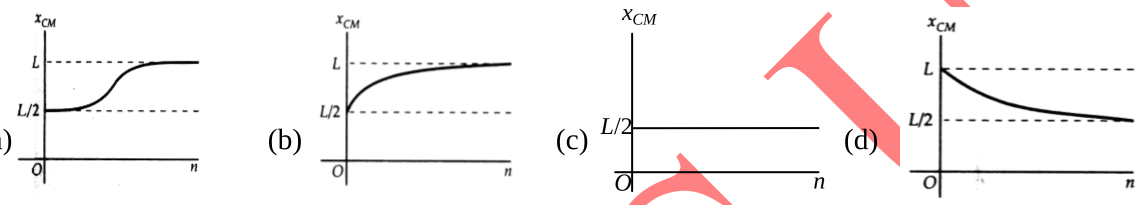
5. A force of $-F\hat{k}$ acts on *O*, the origin of the coordinate system. The torque about the point $(1, -1)$ is



(a) $F(\hat{i}+\hat{j})$ (b) $-F(\hat{i}-\hat{j})$

(c) $F(\hat{i}-\hat{j})$ (d) $-F(\hat{i}+\hat{j})$

6. A thin rod of length *L* is lying along the *x*–axis with its ends at *x* = 0 and *x* = *L*. Its linear density (mass/length) varies with *x* as $k(x/L)^n$, where *n* can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against *n*, which of the following graphs best approximates the dependence of x_{CM} on *n*?



7. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then

(a) $\vec{r}\cdot\vec{F}=0$ and $\vec{F}\cdot\vec{\tau}\neq 0$ (b) $\vec{r}\cdot\vec{\tau}\neq 0$ and $\vec{F}\cdot\vec{\tau}=0$

(c) $\vec{r}\cdot\vec{\tau}\neq 0$ and $\vec{F}\cdot\vec{\tau}\neq 0$ (d) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{\tau}=0$

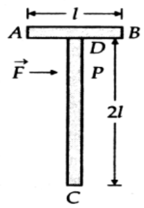
8. Angular momentum of the particle rotating with a central force is constant due to

(a) constant force (b) constant linear momentum (c) constant torque (d) zero torque

9. A particle performing uniform circular motion has angular momentum *L*. If its angular frequency is doubled and kinetic energy is halved, then the angular momentum becomes

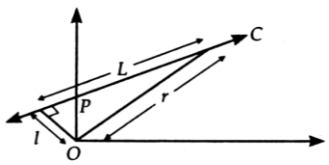
(a) $\frac{L}{4}$ (b) $2L$ (c) $4L$ (d) $\frac{L}{2}$

10. A T–shaped object with dimensions shown in the figure is lying on a smooth floor. A force $\vec{F}$ is applied at the point *P* parallel to *AB*, such that the object has only the translational motion without rotation. Find the location of *P* with respect to *C*



(a) $\frac{-2l}{3}$ (b) $\frac{3l}{2}$ (c) $\frac{4l}{3}$ (d) *l*

11. A particle of mass *m* moves along line *PC* with velocity *v* as shown. What is the angular momentum of the particle about *P* ?

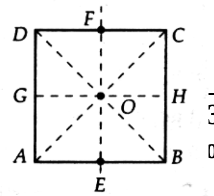


(a) *mvL* (b) *mvl* (c) *mvr* (d) zero

12. Four point masses, each of the value *m*, are placed at the corners of a square *ABCD* of side *l*. The moment of inertia of this system about an axis passing through *A* and parallel to *BD* is

(a) 3 ml^2 (b) ml^2 (c) 2 ml^2 (d) $\sqrt{3}\ ml^2$

13. For the given uniform square lamina *ABCD*, whose centre is *O*.



(a) $I_{AC} = \sqrt{2}\, I_{EF}$ (b) $\sqrt{2}\, I_{AC} = I_{EF}$

(c) $I_{AD} = 3I_{EF}$ (d) $I_{AC} = I_{EF}$

14. Moment of inertia of a circular wire of mass M and radius R about is its diameter is

(a) $\frac{1}{2} MR^2$ (b) $\frac{1}{4} MR^2$ (c) $2\, MR^2$ (d) MR^2

15. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = I_X$ (b) $I_Y = 16 I_X$ (c) $I_Y = 32 I_X$ (d) $I_Y = 64 I_X$

16. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{2}{3} ma^2$ (b) $\frac{5}{6} ma^2$ (c) $\frac{1}{12} ma^2$ (d) $\frac{7}{12} ma^2$

17. The moment of inertia of a uniform semi–circular disc of mass M and radius R about a line perpendicular to the plane of the disc through the centre is

(a) $\frac{1}{4} Mr^2$ (b) $\frac{2}{5} Mr^2$ (c) Mr^2 (d) $\frac{1}{2} Mr^2$

18. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moments of inertia about their diameters are respectively I_A and I_B, such that

(a) $I_A = I_B$ (b) $I_A > I_B$ (c) $I_A < I_B$ (d) $\frac{I_A}{I_B} = \frac{\rho_A}{\rho_B}$

Here ρ_A and ρ_B represent their densities.

19. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected?

(a) Moment of inertia (b) Angular momentum (c) Angular velocity (d) Rotational kinetic energy

20. Initial angular velocity of a circular disc of mass M is ω_1. Then, two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\frac{M+m}{M}\omega_1$ (b) $\frac{M}{M+m}\omega_1$ (c) $\frac{M}{M+4m}\omega_1$ (d) $\frac{M}{M+2m}\omega_1$

21. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω, which is equal to

(a) $\frac{m\omega}{m+M}$ (b) $\frac{m\omega}{m+2M}$ (c) $\frac{(m+2M)\omega}{m}$ (d) $\frac{(m-2M)\omega}{(m+2M)}$

22. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling).

(a) Solid sphere (b) hollow sphere (c) ring (d) all same

23. A uniform round body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then the acceleration is

(a) $\dfrac{g\sin\theta}{2+\dfrac{I}{MR^2}}$ (b) $\dfrac{g\sin\theta}{1+\dfrac{MR^2}{I}}$ (c) $\dfrac{g\sin\theta}{1-\dfrac{1}{MR^2}}$ (d) $\dfrac{g\sin\theta}{1-\dfrac{MR^2}{I}}$

24. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\dfrac{1}{3}\dfrac{l^2\omega^2}{g}$ (b) $\dfrac{1}{6}\dfrac{l\omega}{g}$ (c) $\dfrac{1}{2}\dfrac{l^2\omega^2}{g}$ (d) $\dfrac{1}{6}\dfrac{l^2\omega^2}{g}$

Answers

1.	D	**2.**	C	**3.**	D	**4.**	A	**5.**	A
6.	B	**7.**	D	**8.**	D	**9.**	A	**10.**	C
11.	D	**12.**	A	**13.**	D	**14.**	B	**15.**	D
16.	A	**17.**	D	**18.**	C	**19.**	B	**20.**	C
21.	B	**22.**	D	**23.**	A	**24.**	D		

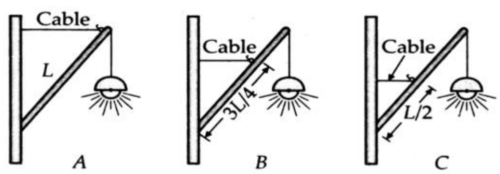
AIIMS

Q.1 Two spheres of masses M and 2M are initially at rest at a distance R apart. Due to mutual force of attraction, they approach each other. When they are at separation R/2, the acceleration of their centre of mass would be

(a) 0 (b) g (c) 3g (d) 12 g

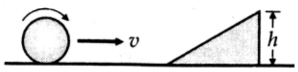
Q.2 A ladder is leaned against a smooth wall and it is allowed to slip on a frictionless floor. Which figure represents trace of its centre of mass?

Q.3 If a street light of mass M is suspended from the end of a uniform rod of length L in different possible patterns as shown in figure, then



(a) pattern A is more sturdy
(b) pattern B is more sturdy
(c) pattern C is more sturdy
(d) all will have same sturdiness

Q.4 A solid sphere is rolling on a frictionless surface with a translational velocity v (in ms^{-1}) as shown in the figure. If it is to climb the inclined surface, then v should be



(a) $\sqrt{10gh/7}$ (b) $\geq\sqrt{10gh/7}$

(c) $\sqrt{2gh}$ (d) $\geq\sqrt{2gh}$

Q.5 In an orbital motion, the angular momentum vectors is

(a) along the radius vector (b) parallel to the linear momentum

(c) in the orbital plane (d) perpendicular to the orbital plane

Q.6 The direction of angular velocity vector is along

(a) the tangent to the circular path (b) the inward radius

(c) the outward radius (d) the axis of rotation

Q.7 The motion of planets in the solar system is an example of the conservation of

(a) mass (b) linear momentum (c) angular momentum (d) energy

Q.8 The angular momentum of a moving body remains constant, if

(a) net external force is applied (b) net pressure is applied

(c) net external torque is applied (d) net external torque is not applied

Q.9 If there is change of angular momentum from J to 4 J in 4s, then the torque is

(a) (3/4) J (b) 1 J (c) (5/4) J (d) (4/3) J

Q.10 A body is projected from the ground with some angle to the horizontal. What happens to the angular momentum about the initial position in this motion?

(a) decreases (b) increases (c) remains same (d) first increases and then decreases

Q.11 Radius of gyration of a body depends upon

(a) axis of rotation (b) translational motion (c) shape of the body (d) area of the body

Q.12 A rod of length 1.4m and negligible mass has two masses of 0.3 kg and 0.7 kg tied to its two ends. Find the location of the point on this rod, where the rotational energy is minimum, when the rod is rotated about the point.

(a) 0.98 m from 0.3 kg (b) 0.98 m from 0.7 kg (c) 0.7 m from 0.3 kg (d) 0.7 m from 0.7 kg

Q.13 The moment of inertia of a rod about an axis through its centre and perpendicular to it is 1/12 ML^2 (where M is the mass and L, the length of the rod). The rod is bent in the middle, so that the two halves make an angle of 60^o. The moment of inertia of the bent rod about the same axis would be

(a) $\frac{1}{12}ML^2$ (b) $\frac{1}{8\sqrt{3}}ML^2$ (c) $\frac{1}{24}ML^2$ (d) $\frac{1}{48}ML^2$

Q.14 If the earth is treated as a sphere of radius R and mass M having period of rotation T, then its angular momentum about its axis of rotation is

(a) $\frac{4\pi MR^2}{5T}$ (b) $\frac{2\pi MR^2}{5T}$ (c) $\frac{MR^2T}{2\pi}$ (d) $\frac{MR^2T}{4\pi}$

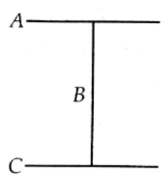
Q.15 A horizontal platform is rotating with uniform angular velocity ω around the vertical axis passing through its centre. At some instant of time, a viscous liquid of mass m is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period.

(a) decreases continuously (b) decreases initially and increases again

(c) remains unaltered (d) increases continuously

Q.16 If a solid sphere of mass 1 kg and radius 0.1 m rolls without slipping at a uniform velocity of 1 m/s along a straight line on a horizontal floor, the kinetic energy is

(a) $\frac{7}{5}J$ (b) $\frac{2}{5}J$ (c) $\frac{7}{10}J$ (d) 1 J



Q.17 In the diagram shown all three rods are of equal length and equal mass M. The system is rotated such that the rod B is the axis. What is the moment of inertia of the system?

(a) $\frac{ML^2}{6}$ (b) $\frac{4}{3}ML^2$ (c) $\frac{ML^2}{3}$ (d) $\frac{2}{3}ML^2$

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion

(c) If assertion is true, but reason is false

(d) If both assertion and reason are false

Q.18 **A :** A judo fighter in order to throw his opponent on to the mat tries to initially bend his opponent and then rotate him around his hip.

R : As the mass of the opponent is brought closer to the fighter's hip, the force required to throw the opponent is reduced.

Q.19 **A :** For a system of particles under central force field, the total angular momentum is conserved.

R : The torque acting on such a system is zero

Q.20 **A :** There are very small sporadic changes in the period of rotation of the earth.

R : Shifting on large air masses in the earth's atmosphere produces a change in the moment of inertia of the earth causing its period of rotation to change.

Q.21 **A :** The earth is slowing down and as a result the moon is coming nearer to it.

R : The angular momentum of earth man system is not conserved.

Q.22 **A :** If polar ice melts, days will be longer.

R : Moment of inertia decreases and thus angular velocity increases.

Q.23 **A :** If ice caps of the pole melt, the day length will shorten.

R : Ice flows towards the equator and decreases the moment of inertia of the earth and hence increases the frequency of rotation of the earth.

Answers

1	A	2	A	3	A	4	B	5	D
6	D	7	A	8	D	9	A	10	B
11	A	12	A	13	A	14	A	15	B
16	C	17	A	18	D	19	A	20	A
21	D	22	A	23	D				

Reasoning types questions :

The following questions consists of two statements **A**(assertion) and other **R** (reason). Answer these questions selecting an appropriate code given below :

(a) A and R are true and R is a correct explanation for A.

(b) A and R are true and R is not a correct explanation for A.

(c) A is true, R is false (d) A is false, R is true

1. **A :** Torque is a vector quantity and its direction is along the applied force.

   R : $\vec{\tau} = \vec{r} \times \vec{F}$

2. **A :** Direction of torque is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$.

   R : $\frac{d\vec{L}}{dt} = \vec{\tau}$.

3. **A :** Moment of inertia of a body is same whatever be the axis of rotation.

   R : Moment of inertia depends on the distribution of mass of the body.

4. **A :** Kinetic energy of rotation becomes half of its original value if the moment of inertia is doubled.

   R : K.E. = $\frac{1}{2} I \omega^2$.

5. **A :** A planet moves slower, when it is farthest from the sun in its orbit and vice–versa.

   R : Orbital velocity in an orbit of planet is constant.

6. **A :** If polar ice melts, days will be shortened.

   R : Moment of inertia decreases and then angular velocity increases.

7. **A :** A planet moves faster, when it is closer to the sun in its orbit.

   R : Moment of inertia decreases and hence angular velocity increases.

8. **A :** Angular velocity of a body decreases if the momentum of inertia of the body increases.

   R : $I\omega$ = constant.

9. **A :** Moment of inertia of two identical spheres, one solid and other hollow, are unequal.

   R : Moment of inertia of a body depends upon the distribution of the mass of the body about the axis of rotation.

10. **A :** Two discs of same mass and thickness but made of materials having different densities have unequal moment of inertia.

    R : Moment of inertia is inversely proportional to the density of the material.

11. **A :** For a system of particles under central force field, the total angular momentum is conserved.

    R : The torque acting on such a system is zero.

12. **A :** A judo fighter in order to throw his opponent onto the mat tries to initially bend his opponent and then rotate him around his hip.

    R : As the mass of the opponent is brought closer to the fighter's hip, the force required to throw the opponent is reduced.

13. **A :** The velocity of a body at the bottom of an inclined plane of a given height is more when it slides down the plane, compared to, when it is rolling down the same plane.

    R : In rolling down, a body acquires both kinetic energy of rotation and translation.

Answers

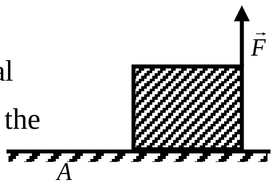
1.	D	**2.**	A	**3.**	D	**4.**	C	**5.**	C	**6.**	A	**7.**	A	**8.**	A
9.	A	**10.**	A	**11.**	A	**12.**	A	**13.**	B						

Matrix Mach type Questions

1. Match the items of column I with that of in column II and then correctly bubble the matrix given below :

	Column I		**Column II**
(a)	Centre of mass	(p)	Directly proportional to the time period about the given axis
(b)	Torque	(q)	Axis Vector
(c)	Moment of inertia of a body	(r)	does not change if no external force acts on the system.
(d)	Radius of gyration	(s)	dimensions of length.

2. A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force $\vec{F}$ is applied to its edge as shown in fig. Match the conditions given in column I with the situations given in column II.



	Column I		**Column II**
(a)	$\frac{mg}{4} > F < \frac{mg}{2}$	(i)	cube will move up
(b)	$F > mg/2$	(ii)	cube will not exhibit motion.
(c)	$F > mg$	(iii)	cube will begin to rotate and slip at A
(d)	$E = mg/4$	(iv)	Normal reaction effectively at $a/3$ from A, no motion

Answers

1.	(a) $\rightarrow$ (r)	;	(b) $\rightarrow$ (q)	;	(c) $\rightarrow$ (p)	;	(d) $\rightarrow$ (s)
2.	(a) $\rightarrow$ (ii)	;	(b) $\rightarrow$ (iii)	;	(c) $\rightarrow$ (i)	;	(d) $\rightarrow$ (iv)