GRAVITATION

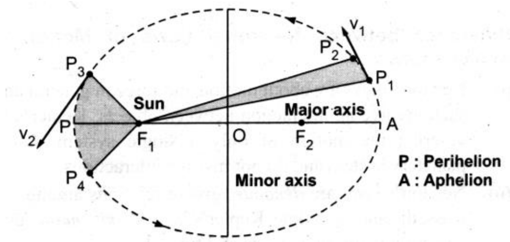
Kepler's Laws of Planetary Motion

First Law: The Law of Orbits

All planets move in elliptical orbits with the Sun at one focus.

This means that one of the two foci (F_1, F_2) of each of the elliptical orbits in which the planets move, coincides with the position of the Sun, (figure).

The closest point P is called the ***perihelion*** *and the farthest point A is called the* ***aphelion.***



Second Law: The Law of Areas

The line joining the centres of a planet and the Sun sweeps equal areas in equal intervals of time, i.e., the areal velocity of the planet around the Sun is constant.

For instance, two equal areas ($P_1F_1P_2$ and $P_3F_1P_4$) have been swept by the line joining the planet with the Sun in the same time. Since, the distances P_1P_2 and P_3P_4 are not equal, the linear velocity of the planet is not constant. Obviously, when the planet is approaching the Sun, its velocity is increasing (i.e, $v_2 > v_1$).

Third Law: The Harmonic Law or The Law of Periods

The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semi–major axis of its elliptical orbit.

That is, $T^2 \propto r^3$

where T = time period of revolution of the planet and

$r = PO = AO = \dfrac{PA}{2} =$ semi–major axis of the ellipse

= mean distance of the planet from the Sun.

NOTES

1. Kepler's laws are applicable not only to the solar system but to the moons going around the planets as well as to the artificial satellites.
2. Kepler's laws are valid whenever inverse–square law is involved.
3. Kepler's laws, which are empirical laws (i.e., laws based on observation, not theory), sum up neatly how planets of the solar system behave without indicating why they do so.

Derivation of Kepler's Laws

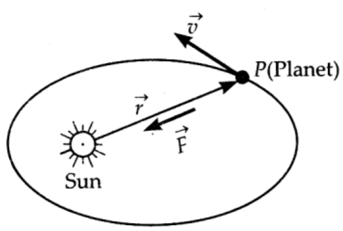
Proof of Kepler's first law of planetary motion. The gravitational force exerted by the sum,

$$\vec{F} = -\frac{GM_sM_p}{r^3}\vec{r}$$

where M_s is the mass of the sun. This force is radial and central. Negative sign indicates that $\vec{F}$ is oppositely directed to $\vec{r}$.

The torque exerted on the planet P about the sun is

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{GM_sm_p}{r^3}\right)\vec{r} = \vec{0} \qquad [\because \vec{r} \times \vec{r} = \vec{0}]$$



But $\vec{\tau}$ = rate of change of a angular momentum

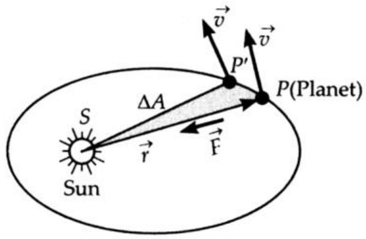
$$= \frac{d\vec{L}}{dt}$$

$$\therefore \quad \frac{d\vec{L}}{dt} = 0 \qquad \text{or} \qquad \vec{L} = \text{constant}$$

This shows that the angular momentum of the planet about the sun remains constant both in magnitude and direction. Since the direction of $\vec{L}(=\vec{r}\times\vec{p})$ is fixed, $\vec{r}$ and $\vec{v}$ lie in a plane normal to $\vec{L}$. The central force under the action of which the planet moves varies as the square of the distance between the planet and sun and this orbit is an ellipse.

Proof of Kepler's second law

Consider a planet moving in an elliptical orbit with the sun at focus S. Let $\vec{r}$ be the position vector of the planet w.r.t. the sun and $\vec{F}$ be the gravitational force on the planet due to the sun. Torque exerted on the planet by this force about the sun is



$$\vec{\tau}=\vec{r}\times\vec{F}=0$$

[$\because$ $\vec{r}$ and $\vec{F}$ are oppositely directed]

But $\quad \vec{\tau}=\dfrac{d\vec{L}}{dt}$

$\therefore \quad \dfrac{d\vec{L}}{dt}=0 \quad$ or $\quad L = \text{constant}$

Suppose the planet moves from position P to P' in time Δt. The area swept by the radius vector $\vec{r}$ is

$$\Delta\vec{A} = \text{Area of triangular region SPP'}$$

$$= \frac{1}{2}\vec{r}\times\vec{P}P'$$

But $\quad \vec{P}P'=\Delta\vec{r}=\vec{v}\,\Delta t=\dfrac{\vec{p}}{m}\Delta t$

$\therefore \quad \Delta\vec{A}=\dfrac{1}{2}\vec{r}\times\dfrac{\vec{p}}{m}\Delta t$

or $\quad \dfrac{\Delta\vec{A}}{\Delta t}=\dfrac{1}{2m}(\vec{r}\times\vec{p})=\dfrac{\vec{L}}{2m}$

or $\quad \dfrac{\Delta\vec{A}}{\Delta t}=\text{constant} \quad$ [$\because$ $\vec{L}$ and m are constant]

Thus the areal velocity of the planet remains constant i.e., *the radius vector joining planet to the sun sweeps out equal areas in equal intervals of time.* This proves Kepler's second law of planetary motion.

Proof of Kepler's Third Law

Suppose a planet of mass m moves around the sun in a circular orbit of radius r with orbital speed v. Let M be the mass of the sun. The force of gravitation between the sun and the planet provides the necessary centripetal force.

$\therefore \quad \dfrac{mv^2}{r}=\dfrac{GMm}{r^2} \quad$ or $\quad v^2=\dfrac{GM}{r}$

But orbital speed,

$$v=\frac{\text{Circumference}}{\text{Period of revolution}}=\frac{2\pi r}{T}$$

$\therefore \quad \dfrac{4\pi^2 r^2}{T^2}=\dfrac{GM}{r} \quad$ or $\quad T^2=\dfrac{4\pi^2}{GM}r^3=K_s r^3$

Thus $\quad T^2 \propto r^3$

This proves Kepler's third law.

Assignment

Q.1 Calculate the period of revolution of Neptune around the sun, given that diameter of its orbit is 30 times the diameter of earth's orbit, both orbits being assumed to be circular.

Q.2 In an imaginary planetary system, the central star has the same mass as our sun, but is brighter so that only a planet twice the distance between the earth and the sun can support life. Assuming biological evolution (including aging process etc.) on that planet similar to ours, what would be the average life span of a 'human' on that planet in terms of its natural year? The average life span of a human on the earth may be taken to be 70 years.

Q.3 The planet Mars has two moon, Phobos and Delmos.

(i) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of Mars.

(ii) Assume that Earth and mars move in circular orbits around the Sun, with the Martian orbit being 1.52 times the orbital radius of the Earth. What is the length of the Martian year in days?

Q.4 The distances of two planets from the sun are 10^{13} m and 10^{12} m respectively. Find the radio of time periods and speeds of the two planets.

Q.5 If the earth be one half its present distance from the sun, how many days will the present one year on the surface of earth change?

Q.6 The distance of planet Jupiter from the sun is 5.2 times that of the earth. Find the period of revolution of Jupiter around the sun.

Q.7 A geostationary satellite is orbiting the earth at a height 6R above the surface of earth, where R is the radius of the earth. Find the time period of another satellite at a height of 2.5 R from the surface of earth in hours.

Answers

1.	164.3 years	2.	25 planet years		
3.	(i) 6.48×10^{23} kg, (ii) 684 days	4.	$10\sqrt{10},\ 1/\sqrt{10}$		
5.	year decreases by 236 days	6.	11.86 years	7.	$6\sqrt{2}$ h

Difference between Newton's Laws of Motion and Kepler's Laws

(i) Newton's laws are about motion of force in general and as such involve an interaction between objects. Kepler's laws describe the motion of only a single system, i.e., the planetary system and do not involve interactions.

(ii) Newton's laws are *dynamic*, giving relations among force, mass, distance and time. Kepler's laws are *kinematic*, giving a relation between distance and time.

Sample Example – 1

The distance of the planet Jupiter from the Sun is 5.2 times that of the Earth. Find the period of Jupiter's revolution around the Sun.

Solution:

Since, the square of the time period (T) of revolution of a planet around the Sun is proportional to the cube of the mean distance (r) of the planet from the Sun, $T_j^2 \propto r_j^3$ and $T_e^2 \propto r_e^3$

where j and e stand for Jupiter and the Earth respectively.

Clearly, $\dfrac{T_j^2}{T_e^2} = \dfrac{r_j^3}{r_e^3}$ or $\left(\dfrac{T_j}{T_e}\right)^2 = \left(\dfrac{r_j}{r_e}\right)^3$ or $T_j = \left(\dfrac{r_j}{r_e}\right)^{3/2} \times T_e$

Since $\dfrac{r_j}{r_e}$ = 5.2 and T_e = 1 year,

$Tj = (5.2)^{3/2} \times 1\ year$ $\qquad T_j = (5.2)^{3/2} \times 1$ year = **11.86 year**

Newton's Universal Law of Gravitation

Newton arrived at the universal law of gravitation which is stated as follows:

Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Mathematically, the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by

$$F = G\,\frac{m_1 m_2}{r^2}$$

Here, G is a constant of proportionality and is called the **Universal Gravitational Constant.**

Let $\qquad m_1 = m_2 = 1$ unit, $r = 1$ unit

$\therefore \qquad F = G$ or $\qquad G = F$

Gravitational constant is *thus numerically equal to the force of attraction between two unit masses placed a unit distance apart, the distance being measured from their centres of gravity.*

The numerical value of G depends upon the units in which mass, distance and force are expressed.

Value of G = 6.67×10^{-11} Nm^2 Kg^{-2}

Dimensions of G = $[M^{-1} L^3 T^{-2}]$

Through the gravitational force of attraction between bodies of ordinary size is negligible, it becomes sufficiently large in case of heavenly bodies.

Experimental Evidence of Support of Newton's Law of Gravitation

(i) The rotation of Earth around the Sun and that of Moon around the Earth is explained on the basis of the law.

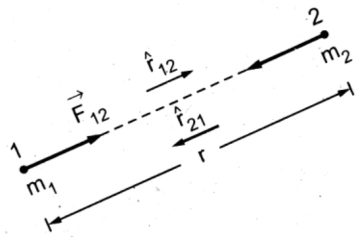
(ii) The formation of tides in oceans is due to the force of attraction between the Moon and the sea water.

(iii) The times of solar and lunar eclipses calculated in advanced on the basis of this law agree closely with the actual observations.

(iv) The orbits and time periods of the modern artificial satellites are predicted very accurately on the basis of the law.

Newton's Law of Gravitation in Vector Form

Let $\vec{F}_{12}$ represent the gravitational force exerted on particle 1 by another particle 2. Since gravitational force is *attractive*, it is directed from 1 towards 2. The customary way to show the direction of $\vec{F}_{12}$ is to use a unit vector r_{21} which is directed from 2 to 1 as shown in figure. Thus, we have



$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{21}$$

where m_1 and m_2 are the masses of the particles and r is the distance between them. Similarly, force exerted on particle 2 by particle 1 is given by

$$\vec{F}_{21} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12}$ is a unit vector directed from particle 1 to particle 2. Hence

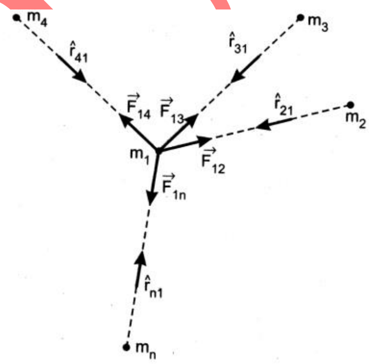
$$\vec{F}_{12} = -\vec{F}_{21} \qquad (as\ \hat{r}_{21} = -\hat{r}_{12}) \qquad \ldots (7)$$

Above equation implies that the gravitational force acting between the two particles forms an *action–reaction pair*.

Comments

1. Since $\vec{F}_{12}$ and $\vec{F}_{21}$ are directed towards the centre of mass of the two particles, the gravitational force is a *central force.*
2. Gravitational force is always attractive while electric and magnetic forces can be attractive or repulsive.
3. Gravitational force is independent of the medium between the particles whereas electric and magnetic forces depend on the nature of the intervening medium.
4. Gravitational force is a *conservative force* which means that work done by it is independent of path followed. This fact can also be stated by saying that work done in moving a particle round a closed path under the action of gravitational force is zero.
5. Newton's law of gravitation is valid for objects lying at huge distances (interplanetary distances) and also for very small distances (inter molecular distances), i.e., it holds over a wide range of distance.
6. Newton's law of gravitation is of universal application and *it holds irrespective of the state and the nature of the attracting bodies.*

Gravitation and Principle of Superposition



If instead of two particles as considered earlier, several particles are present, each pair will experience a mutual attraction. *The total gravitational force acting on a given particle is the vector sum of the gravitational forces exerted by other particles.*

Thus, the principle of superposition is valid. For *n* interacting particles, we can write the principle of superposition for gravitational force as

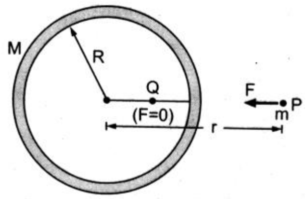
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + ... + \vec{F}_{1n}$$

where $\vec{F}_1$ is the net gravitational force acting on particle m_1 due to other particles, i.e., m_2, m_3, m_4, ... m_n, as shown in figure. Clearly,

$$F_1 = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{21} - G\frac{m_1 m_3}{r_{13}^2}\hat{r}_{31} .... - G\frac{m_1 m_2}{r_{1n}^2}\hat{r}_{n1} \quad \text{or} \quad \vec{F}_1 = -GM_1\left[\frac{m_2}{r_{12}^2}\hat{r}_{21} + \frac{m_3}{r_{13}^2}\hat{r}_{31} + ... \frac{m_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

Gravitational Force between an Extended Object and a Particle

(i) Spherical Shell



Case I: If a particle of mass m is located at P, a point outside a spherical shell of mass M, (figure), the shell attracts the particle as though the mass of the shell were concentrated at its centre,

i.e., $F = G\dfrac{Mm}{r^2}$

Case II: If the particle of mass m is located at Q, a point inside the shell, the force acting on it is zero, i.e., F = 0. We can express these two important results as follows:

$F = G\dfrac{Mm}{r^2}$ for $r \geq R$

$F = 0$ for $r < R$

(ii) Solid Sphere

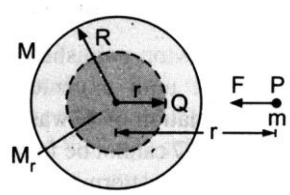
Case I: If a particle of mass m is located at P, (figure), a point *outside a homogeneous solid sphere* of mass M, the sphere attracts the particle as though the mass of the sphere were concentrated at its centre,

i.e., $F = G\dfrac{Mm}{r^2}$

[This result follows from case I (for a spherical shell), since a solid sphere can be considered to be a collection of concentric spherical shells].

Case II: If a particle of mass m is located at Q, a point *inside a homogeneous solid sphere* of mass M, (figure), the force on m due only due to the mass M_r contained within the sphere of radius r (represented by the dashed circle), i.e.,

$$F = G\frac{M_r m}{r^2} = \frac{GMm}{R^3}r$$



We can express these two important results as follows:

$$F = G\frac{Mm}{r^2} \qquad for \geq R$$

$$F = \frac{GMm}{R^3}r \qquad for < R$$

Deduction of Newton's Law of Gravitation from Kepler's Laws

We shall now show how Newton *might* have discovered the law of gravitation from the Kepler's laws.

(i) *Kepler's first law* states that the orbit of a planet is an ellipse. A particular case of an ellipse is the *circle* where the two foci coincide with the centre.

(ii) According to the *Kepler's second law,* the force of attraction acting on the planet due to the Sun is central points towards the centre of the circle. Such a force is called the *centripetal force.*

Consider a planet of mass m moving with a speed v in a circle of radius r.

Centripetal force acting on the planet, i.e.

$$F = \frac{4\pi r^2 mr}{T^2}$$

(iii) From *Kepler's third law,* $T^2 \propto r^3$

or $T^2 = kr^3$

From equation (1) and (2),

$$F = \frac{4\pi^2 mr}{kr^3} = \left(\frac{4\pi^2}{k}\right)\frac{m}{r^2}$$

Thus, the force exerted on the planet by the Sun is proportional to the mass (m) of the planet and inversely proportional to the square of the distance (r^2) from the Sun. By Newton's third law of motion, this force should be equal in magnitude to the force exerted on the Sun by the planet. Therefore, this force should also be proportional to mass (M) of the Sun.

$$F = \left(\frac{4\pi^2}{k}\right)\frac{Mn}{r^2} \text{ or } F = G\frac{Mn}{r^2} \qquad \ldots (4)$$

where $G(= 4\pi^2/k)$ is a constant independent of the mass of the Sun or the planet.

Comments

1. From equation $F \propto \frac{1}{r^2}$ ... (5)

This means that the force exerted on a planet by the Sun varies inversely as the square of the distance from the Sun, i.e., *gravitational force is inverse square force.* Though we have taken the help of all the three laws of Kepler to deduce Newton's law of gravitation, equation is a direct outcome of Kepler's third law. *Thus, Kepler's third law enables us to determine the way in which the gravitational force varies with the distance, i.e., it established the inverse square nature of gravitational force.*

2. Although we have not proved here, Kepler's *first law* is also a direct consequence of the fact that the gravitational force varies as $1/r^2$. It can be shown that under an inverse square force, the orbit of a planet is a conic section (i.e., circle, ellipse, parabola or hyperbola) with the Sun at one focus.

3. $\vec{L} = 2m\dfrac{\overrightarrow{dA}}{dl}$ or $\dfrac{\overrightarrow{dA}}{dl} = \dfrac{\vec{L}}{2m}$

According to Kepler's second law, $\dfrac{\overrightarrow{dA}}{dl}$ = constant or $\dfrac{\vec{L}}{2m}$ constant or $\vec{L}$ = constant

Hence, this implies that the angular momentum of the planet is constant, i.e., *Kepler's second law follows from conservation of angular momentum.*

As $\vec{L}$ *is* constant $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \vec{0}$

But as $\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{r} \times \vec{F} = \vec{0}$ or rF sinθ = 0 or θ = 0° or 180°

Thus, $\vec{r}$ and $\vec{F}$ must act along the same line. Such a force $\vec{F}$, which acts along $\vec{r}$, is called the *central force. Thus, Kepler's second law establishes that the gravitational force is central.* In fact, this law applies to any situation that involves central force whether inverse square or not.

4. We have derived *Newton's law of gravitation from Kepler's laws* on the assumption that Newton was guided by these laws while formulating the law of gravitation. By comparing the acceleration of Moon (a) with the acceleration due to gravity on the Earth's surface (g), he only checked the correctness of the inverse square nature of gravitational force on which his law was based.

There is another view point according to which it is believed that having discovered the $(1/r^2)$ nature of gravitational force by comparing (a) and (g), Newton formulated his universal law of gravitational. Later on, he was able to *derive Kepler's laws using his laws of motion and universal law of gravitation.*

We have already talked about the derivation of 1st law (comment 2) and 2nd law. The 3rd law can also be derived as discussed from there it follows that

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 = Kr^3$$

where K is a constant whose value depends on M.

In case of planets moving around the Sun, $M = M_S$ (mass of the Sun),

$$K = K_S = \left(\frac{4\pi^2}{GM_S}\right) = 2.97 \times 10^{-19}\ s^2/m^3$$

For Moon and other satellites around the Earth.

$$K = K_E = \left(\frac{4\pi^2}{GM_E}\right) = 10^{-13}\ s^2/m^2$$

5. The historical connection between Kepler's laws and Newton's law of gravitation can best be understood from the following two statements of Newton.

(i) *"If I have seen farther from others, it is because I stand on the shoulders of giants".*

(ii) *"From Kepler's third law, I deduced the inverse–square property of gravitational force and thereby compared the force requisite to keep the Moon in her orbit with the force of gravity at the surface of the Earth, and found them answer pretty nearly"* (Newton's nostalgic look back–a year before his death)

But all this is now a part of glorious history of physics.

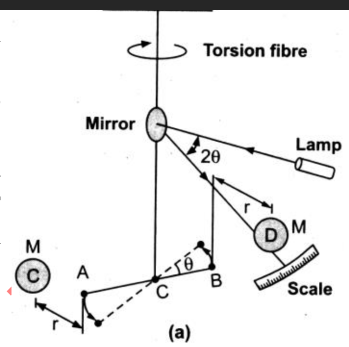
Sample Example – 3

The planet Mars has two Moons: Phobos and Deimos. Phobos has a period of 7 h 39 min and orbital radius or 9.4×10^3 km. Calculate the mass of the Mars.

Sol. 6.48×10^{23} kg

Determination of G: Cavendish Experiment

An English chemist, Henry Cavendish, was the first to perform such an experiment in 1798. He used a very sensitive type of balance, called the **torsion balance (figure)** to find the force of attraction between two small lead spheres A, B (each of mass m = 0.729 kg) attached to the ends of a suspended rod of length, l and two larger stationary lead spheres C and D (each of mass m = 158 kg). First of all, the spring (or torsion) constant (k) of the torsion fibre is determined. The deflection (θ) that is produced on bring the larger spheres near the smaller ones is determined (by using a lamp and scale arrangement)



Referring to figure, force on each small sphere,

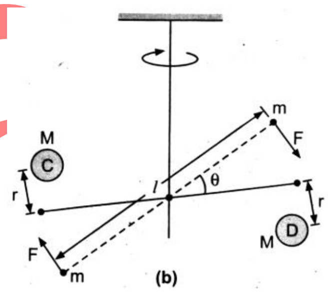
$$F = G\,\frac{Mn}{r^2}$$

where r is the separation between the large and the small sphere

Force (F, F) acting on A and b form a couple, l being the arm of the couple.

Moment of this couple,

$$\tau = F \times l = G\frac{Mn}{r^2}l \qquad \text{... (1)}$$



Further, τ, k and θ are related to each other as

$$\tau = k\theta \qquad \text{... (2)}$$

From equations (1) and (2),

$$G\frac{Mn}{r^2}l = k\theta \quad \text{or} \quad G = \frac{k\theta r^2}{Mml} \qquad \text{... (3)}$$

Since, k, θ, r, M, m and l are known, G can be found out.

Sample Example – 4

In Cavendish's experiment, let each small mass be 20g and each large mass be 5 kg. The rod connecting the small masses is 50 cm long, while the small and the large spheres are separated by 10.0 cm. The torsion constant is 4.8×10^{-8} kg m^2/s^2 and the resulting angular deflection is 0.4^o. Calculate the value of the universal gravitational constant G from this data.

Sol. 6.72×10^{-11} Nm^2/kg^2

Comments

1. The symbol G represents a universal constant and is referred to as *'big G'* whereas g which represents acceleration due to gravity near the Earth's surface is sometimes called *'small g'*.
2. 'g' is a vector quantity with $[LT^{-2}]$ as its dimensional formula. Its SI unit is m/s^2.
3. It is independent of the mass, shape and size of the bodies falling under gravity.

Subjective Assignment – I

Q.1 Calculate the force of attraction between two balls each of mass 1 kg each, when their centres are 10 cm apart. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Q.2 The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30} kg . The mean distance of the Jupiter from the sun is 7.8×10^{11} m. Calculate the gravitational force which the sun exerts on Jupiter. Assuming that Jupiter moves in a circular orbit around the sun, calculate the speed of the Jupiter.

Q.3 Two particles, each of mass m, go round a circle of a radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Q.4 A mass M is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum?

Q.5 Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC, as shown in figure.

(a) What is the force acting on a mass 2m placed at the centroid G of the triangle?

(b) What is the force if the mass at the vertex A is doubled? Take AG = BG = CG = 1 m

Q.6 The centres of two identical spheres are 1.0 m apart. If the gravitational force between the spheres be 1.0 N, then what is the mass of each sphere? ($G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$)

Q.7 Find the gravitational attraction between two H–atoms of a hydrogen molecules. Given $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$, mass of atom=$1.67\times10^{-27}$ kg and distance between two H–atoms = 1 Å.

Q.8 Calculate the force of gravitation between two bodies, each of mass 100 kg and 1 m apart on the surface of the earth. Will the force of attraction be different if the same bodies are taken on the moon, their separation remaining constant?

Q.9 An apple of mass 0.25 kg falls from a tree. What is the acceleration of the apple towards the earth? Also calculate the acceleration of the earth towards the apple. Mass of the earth = 5.983×10^{24} kg, radius of the earth = 6.378×10^6m and $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$.

Q.10 How far from earth must a body be along a line towards the sun so that the sun's gravitational pull on it balances that of the earth. Distance between sun and earth's centre is 1.5×10^{10} km. Mass of sun is 3.24×10^5 times mass of earth.

Answers

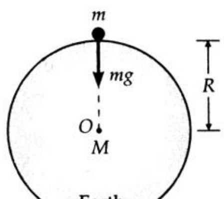
1. 6.67×10^{-9} N
2. 4.145×10^{23} N, $1.304 \times 10^4\ ms^{-1}$
3. $v = \sqrt{\frac{Gm}{4R}}$
4. M/2
5. (a) 0, (b) $2Gm^2\ \hat{j}$
6. 1.225×10^5 kg
7. 1.86×10^{-44} N
8. 6.67×10^{-7} N, No
9. $9.810\ ms^{-2}$, $4.099 \times 10^{-25}\ ms^{-2}$
10. 2.63×10^7 km

Relation between g and G

Consider the earth to be a sphere of mass M and radius R. Suppose a body of mass m is lying on its surface, as shown in figure. According to the law of gravitation, the force of attraction between the earth and the body is

$$F = \frac{GMm}{R^2}$$

Here we have used shell theorem according to which the gravitational force due to a sphere on a mass outside it acts as if the entire mass of the sphere is concentrated at its centre. The force of gravity F produces an acceleration g (called acceleration due to gravity) in the body of mass m. From Newton's second law of motion, we get



$$F = mg$$

From the above two equations, we have

$$mg = \frac{GMm}{R^2} \quad \text{or} \quad g = \frac{GM}{R^2}$$

This gives acceleration due to gravity on the surface of the earth. *The value of g is independent of the mass, size and shape of the body falling under gravity.*

Subjective Assignment – II

Q.1 **Weighing the Earth:** You are given the following data: $g = 9.81\ ms^{-2}$, $R_E = 6.37 \times 10^6$ m, the distance to the moon $r = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Q.2 If the earth were made of lead of relative density 11.3, what then would be the value of acceleration due to gravity on the surface of the earth? Radius of the earth = 6.4×10^6 m and $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$.

Q.3 The acceleration due to gravity at the moon's surface is $1.67\ ms^{-2}$. If the radius of the moon is 1.74×10^6 m, calculate the mass of the moon. Use the known value of G.

Q.4 Two lead spheres of 20 and 2 cm diameter respectively are placed with centes 100 cm apart. Calculate the attraction between them, given the radius of the earth as 6.37×10^8 cm and its mean density as $5.53 \times 10^3\ kg\ m^{-3}$. Specific gravity of lead = 11.5. If the lead spheres are replaced by brass spheres of same radii, would the force of attraction be same?

Q.5 Compare the gravitational acceleration of the earth due to attraction of the sun with that due to attraction of the moon. Given that mass of sun, $M_S = 1.99 \times 10^{30}$ kg, mass of moon, $M_m = 7.35 \times 10^{22}$ kg, distance of sun from earth, $r_{es} = 1.49 \times 10^{11}$ m and distance of moon from earth $r_{em} = 3.84 \times 10^8$ m.

Q.6 A body weighs 90 kg f on the surface of the earth. How much will it weigh on the surface of Mars whose mass is 1/9 and the radius is 1/2 of that of the earth?

Q.7 If the radius of the earth shrinks by 2.0%, mass remaining constant, then how would the value of acceleration due to gravity change?

Q.8 A man can jump 1.5m high on the earth. Calculate the approximate height he might be able to jump on a planet whose density is one–quarter that of the earth and whose radius is one–third of the earth's radius.

Q.9 An astronaut on the moon measures the acceleration due to gravity to be $1.7\ ms^{-2}$. He knows that the radius of the moon is about 0.27 times that of the earth. Find the ratio of the mass of the earth to that of the moon. If the value of g on the earth's surface is $9.8\ ms^{-2}$.

Q.10 The acceleration due to gravity on the surface of the earth is $10\ ms^{-2}$. The mass of the planet Mars as compared to earth is 1/10 and radius is 1/2. Determine the gravitational acceleration of a body on the surface of Mars.

Q.11 On a planet whose size is the same and mass 4 times as that of the earth, find the energy needed to lift a 2kg mass vertically upwards through 2 m distance in joule. The value of g on the surface of earth is $10\ ms^{-2}$.

Answers

1.	(i) 5.97×10^{24} kg, (ii) 6.02×10^{24} kg	2.	$22.21\ ms^{-2}$
3.	7.58×10^{22} kg	4.	15.4×10^{-11} N, No
5.	179.8 : 1	6.	40 kg f

7.	increasing by 4 %	8.	18 m		
9.	79	10.	4 ms^{-2}	11.	160 J

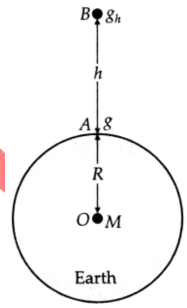
Variation of g with Altitude (Height)

Consider the earth to be a sphere of mass M, radius R and centre O. Then the acceleration due to gravity at a point A on the surface of the earth will be

$$g = \frac{GM}{R^2} \qquad \text{... (i)}$$

If g_h is the acceleration due to gravity at a point B at a height h from the earth's surface, then

$$g_h = \frac{GM}{(R+h)^2} \qquad \text{... (ii)}$$



Dividing equation (ii) by (i), we get

$$\frac{gh}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

or $$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

or $$\frac{g_h}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$ or $$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

Expanding R.H.S. by using binomial theorem, we get $\frac{g_h}{g} = 1 - \frac{2h}{R} +$ terms containing higher powers of h/R

If h < < R, then $\frac{h}{R} << 1$, so that higher powers of h/R can be neglected, we get

$$\frac{g_h}{g} = 1 - \frac{2h}{R}$$

or $$g_h = g\left(1 - \frac{2h}{R}\right) \qquad \text{... (iv)}$$

Hence the value of acceleration due to gravity decrease with the increase in height h, that is why the value of g is less at mountains than at plains. While solving numerical problems, equation this should be used when h is comparable to R and equation this should be used when h < < R.

NOTE

- The decrease in the value of g at height h is

  $$g - g_h = \frac{2gh}{R}$$

  Clearly $g - g_h \propto h$
- The percentage decrease in the value of g at height h is

  $$\frac{g-g_h}{g} \times 100 = \frac{2h}{R} \times 100\%$$
- The loss in weight of a body at a height h

  $$= mg - mg_h = \frac{2mgh}{R}$$

- At an altitude h = 320 km, $g_h = 0.69g$, i.e., the value of g decrease by 10%

Subjective Assignment – III

Q.1 At what height from the surface of the earth, will the value of g be reduced by 36% from the value at the surface? Radius of the earth = 6400 km.

Q.2 At what height above the earth's surface, the value of g is half of its value on earth's surface? Given its radius is 6400 km.

Q.3 Find the percentage decrease in the weight of a body when taken to a height of 32 km above the surface of the earth. Radius of the earth is 6400 km.

Q.4 A mass of 0.5 kg is weighed on a balance at the top of a tower 20 m high. The mass is then suspended from the pan of the balance by a fine wire 20 m long and is reweighed. Find the change in weight. Assume that the radius of the earth is 6400 km.

Q.5 A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch spring at a place 1600 km above the earth's surface? Radius of the earth = 6400 km.

Q.6 The radius of the earth is 6000 km. What will be the weight of a 120 kg body if it is taken to a height of 2000 km above the surface of the earth?

Q.7 At what height above the surface of the earth will the acceleration due to gravity be 25% of its value on the surface of the earth? Assume that the radius of the earth is 6400 km.

Q.8 How far away from the surface of earth does the acceleration due to gravity becomes 4% of its value on the surface of earth? Radius of earth = 6400 km.

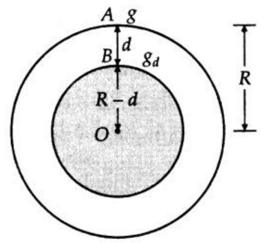
Answers

1.	1600 km	2.	2649.6 km	3.	1%
4.	3.125×10^{-6} kg f	5.	0.64 cm	6.	67.5 kg f
7.	6400 km	8.	25, 600 km		

Variation of g with Depth

Consider the earth to be a sphere of mass M, radius R and centre O. The acceleration due to gravity at any point A on the surface of the earth will be

$$g = \frac{GM}{R^2}$$



Assuming the earth to be a homogenous sphere of average density ρ, then its total mass will be

$$M = \text{Volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \quad g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} \quad \text{or} \quad g = \frac{4}{3}\pi G R \rho$$

Let g_d be the acceleration due to gravity at a point B at depth d below the surface of the earth. A body at B is situated at the surface of inner solid sphere and lies inside the spherical shell of thickness d. The gravitational force of attraction on a body inside a spherical shell is always zero. Therefore, a body at B experiences gravitational force due to inner shaded sphere of radius (R – d) and mass M', where

$$M' = \frac{4}{3}\pi (R-d)^3 \rho$$

$$\therefore \quad g_d = \frac{GM'}{(R-d)^2} = \frac{G}{(R-d)^2} \times \frac{4}{3}\pi (R-d)^3 \rho \quad \text{or} \quad g_d = \frac{4}{3}\pi g (R-d) \rho$$

$$\frac{g_d}{g}=\frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G\rho}=\frac{R-d}{R}=1-\frac{d}{R} \qquad \text{or} \qquad g_d=g=\left(1-\frac{d}{R}\right)$$

Clearly, *the acceleration due to gravity decreases with the increase in depth d.* That is why the acceleration due to gravity is less in mines than that on earth's surface.

Weight of a body at the centre of the earth: At the centre of the earth, d = R,

$$g_d=g\left(1-\frac{R}{R}\right)=0$$

Weight of a body of mass m at the centre of the earth,

$= mg_d = m \times 0 = 0$

Hence *the weight of a body at the centre of the earth is zero though its mass is not zero.*

NOTE

- The acceleration due to gravity decreases both with the increase in height and increase in depth. So *it is maximum at the surface of the earth and zero at the centre of the earth.*
- Decrease in the value of g at depth d is

$$g - g_d = \frac{d}{R}g$$

- Percentage decrease in the value of g at depth d is

$$\frac{g-g_d}{g}\times 100=\frac{d}{R}\times 100\%$$

Relation between height h and depth d for the some change in g. Acceleration due to gravity at a height h above the earth's surface,

$$g_h=g\left(1-\frac{2h}{R}\right)$$

Acceleration due to gravity at a depth d below the earth's surface,

$$g_d=d\left(1-\frac{d}{R}\right)$$

For the same change in g, we have $g_h = g_d$

$$\therefore \qquad I-\frac{2h}{R}=1-\frac{d}{R} \text{ or } \frac{2h}{R}=\frac{d}{R} \text{ or } d=2h$$

Hence *the acceleration due to gravity at a height h above the earth's surface will be same as that at depth d = 2h, below the earth's surface.* But this fact holds only when h < < R.

Subjective Assignment – IV

Q.1 Find the percentage decrease in weight of a body, when taken 16 km below the surface of the earth. Take radius of the earth as 6400 km.

Q.2 How much below the surface of the earth does the acceleration due to gravity become 1% of its value at the earth's surface? Radius of the earth = 6400 km.

Q.3 At what height above the earth's surface, the value of g is same as in a mine 80 km deep?

Q.4 Imagine a tunnel dug along a diameter of the earth. Show that a particle dropped from one end of tunnel execute simple Harmon motion. What is the period of this motion? Assume the earth to be a sphere of uniform mass density (equal to its known average density = 5520 kg m^{-3}). Calculate it's the period of oscillation.

Q.5 How much below the surface of the earth does the acceleration due to gravity (i) reduces to 36%, (ii) reduces by 36% of its value on the surface of the earth? Radius of the earth = 6400 km.

Q.6 Compare the weights of a body when it is (i) 100 km above the surface of the earth and (ii) 100 km below the surface of the earth. Radius of the earth is 6300 km.

Answers

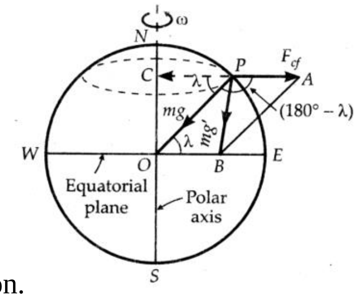
1.	0.25%	2.	6336 km	3.	40 km
4.	1.414 hr	5.	(i) 4096 km, (ii) 2304 km	6.	0.984

Variation of 'g' with Latitude (Or Rotation of the Earth)

The latitude of a place is defined as the angle which the line joining the place to the centre of the earth makes with the equatorial plane.

Consider the earth to be a perfect sphere of radius R, mass M and centre at O. It revolves with uniform angular velocity ω about polar axis NS. As the earth revolves, every particle lying on its surface also revolves along a horizontal circle with same angular velocity ω. The centre of circle lies on polar axis NS.

Consider a particle of mass m lying at point P, whose latitude is λ. The particle P describes a horizontal circle of radius r = PC.



In right angle ΔPCO,

$$\cos\lambda = \frac{PC}{PO} = \frac{r}{R}$$

or $r = R\cos\lambda$

The centrifugal force acting on the particle P is

$$F_{cf} = mr\omega^2$$

This force acts along PA, directed a way from the centre C of the circle of rotation.

Let g be the acceleration due to gravity in the absence of rotational motion of the earth. Then the gravitational pull acting on the particle P = mg, is directed along PO, towards the centre of the earth.

Let g' be the acceleration due to gravity in the presence of rotational motion of the earth. Then the apparent weight of the particle P = mg'. This must be the resultant of true weight mg and centrifugal force F_{cf}. It acts along the diagonal PB of the parallelogram OP AB. Clearly, ∠OPA = $180^o - \lambda$.

By the application of the parallelogram law of forces, we get

$$mg' = \sqrt{(mg)^2 + (mr\omega^2)^2 + 2mg \times mr\omega^2 \times \cos(180^o - \lambda)}$$

or $$g' = \sqrt{g^2 + r^2\omega^4 - 2gr\omega^2\cos\lambda}$$

$$= \sqrt{g^2 + R^2\cos^2\lambda^2\,\omega^4 - 2gR\cos\lambda\,\omega^2\cos\lambda}$$ [Using equation (i)]

$$= g\left[1 + \left(\frac{R\omega^2}{g}\right)^2\cos^2\lambda - \frac{2R\omega^2}{g}\cos^2\lambda\right]^{1/2}$$

As $R = 6.38 \times 10^6$m, $g = 9.8\ ms^{-2}$, $\omega = \frac{2\pi}{86400}\ rad\ s^{-1}$

$$\therefore \quad \frac{R\omega^2}{g} = \frac{6.38\times10^6}{9.8} \times \left(\frac{2\pi}{86400}\right)^2 = \frac{1}{291}$$

Now $\frac{R\omega^2}{g}$ is very small, so its square term may be neglected. Then $g' = g\left(1 - \frac{2R\omega^2}{g}\cos^2\lambda\right)^{1/2}$

Expanding by binomial theorem and neglecting higher terms, we get

$$g' = g\left[1 - \frac{\frac{1}{2}\times 2R\omega^2\cos^2\lambda}{g}\right] \text{ or } g' = g - R\,\omega^2\cos^2\lambda$$

As λ increases, cos λ decreases and g' increases. So as *we move from equator to pole, the acceleration due to gravity increases.*

Special Case

(i) At the equator. $\lambda = 0^o$, $\cos\lambda = 1$, hence

$$g_e = g - R\omega^2$$

(ii) At the poles. $\lambda = 90^o$, $\cos\lambda = 0$, hence

$$g_p = g - R\omega^2 \times 0 = g$$

Thus acceleration due to gravity is minimum at the equator and maximum at the poles. The difference in the two values is $g_p - g_e = g - g\ (g - R\omega^2) = R\omega^2$

NOTE

- Acceleration due to gravity decreases due to rotation of the earth (g' < g).
- Acceleration due to gravity increases with the increase in latitude of the place.
- The effect of rotation of the earth is maximum at the equator and minimum at the poles. In fact, rotational motion of the earth has no effect on the value of g at the poles.
- If the earth stops rotating, the weight of a body would increase due to the absence of the centrifugal force.
- Both the rotation of the earth and its equatorial bulg contribute additively to lower the value of g at the equator than at the poles.
- Even for the rotating earth, the direction of acceleration due to gravity is towards the centre of the earth both at the equator ($\lambda = 0^o$) and at the poles ($\lambda = \pm 90^o$). At intermediate latitudes, this direction slightly deviates from the centre of the earth. The maximum deviation is about 0.1^o.

Subjective Assignment – V

Q.1 Calculate that imaginary angular velocity of the earth for which effective acceleration due to gravity at the equator becomes zero. In this condition what will be the length (in hours) of the day? Given radius of the earth = 6400 km and g = 10 ms^{-2}.

Q.2 Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh $3/5^{th}$ as much as at present. Take the equatorial radius as 6400 km.

Q.3 If the earth were a perfect sphere of radius 6.37×10^6 m, rotating about its axis with a period of 1 day (= 8.64×10^4 s), how much would the acceleration due to gravity (g) differ from the poles to the equator.

Q.4 Calculate the value of acceleration due to gravity at a place of latitude 45^o. Radius of the earth = 6.38×10^3 km.

Q.5 If the earth stops rotating about its axis, then what will be the change in the value of g at a place in the equatorial plane? Radius of the earth = 6400 km.

Q.6 How many times faster than its present speed the earth should rotate so that the apparent weight of an object at equator becomes zero? Given radius of the earth = 6.37×10^6 m. What would be the duration of the day in that case?

Answers

1. 1.25×10^{-3} rad s^{-1}, 1.414 h
2. 7.8×10^{-4} rad s^{-1}
3. 3.4 cms^{-2}
4. 9.783 ms^{-2}

5. $3.4\ cms^{-2}$

6. 17 times faster, 1.412 h

Intensity of Gravitational Field

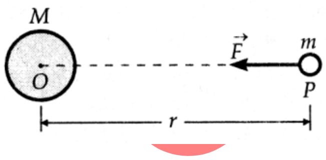
The gravitational field intensity at any point in the gravitational field due to a given mass is defined as the force experienced by a unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

The gravitational field intensity is a vector quantity, denoted by $\vec{E}$. It always acts towards the mass producing the gravitational field.

Intensity of gravitational field due to a body

Consider a body of mass M. To determine its gravitational field intensity at a point P at distance r from its centre O, place a test mass m (m << M) at the point P.

Let $\vec{F}$ be the force of gravitation experienced by test mass m. The gravitational field intensity at point P will be



$$\vec{E} = \frac{\vec{F}}{m}$$

The direction of $\vec{E}$ is same as that of $\vec{F}$.

According to Newton's law of gravitations,

$$F = \frac{GMm}{r^2}$$

$$\therefore \quad E = \frac{F}{m} = \frac{GM}{r^2} \quad \text{.... (i)}$$

At r = ∞, E = 0. Thus gravitational field intensity decreases as distance r increases and becomes zero at infinity. If the test mass m is free to move, it will move towards the mass M with acceleration a under the force F, so

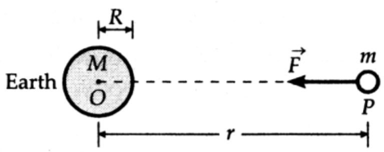
$$a = \frac{F}{m} \quad \text{.... (ii)}$$

From equations (i) and (ii), we get a = E

Thus the intensity of gravitational field at any point is equal to the free acceleration produced in the test mass when placed at that point.

Intensity of gravitational field due to earth

As shown in figure, let earth be a sphere of radius R and mass M. Suppose a test mass m be placed at a point P at distance r form its centre O. According to Newton's law of gravitation, the force of attraction on test mass m is



$$F = \frac{GMm}{r^2}$$

The gravitational field intensity at point P will be

$$E = \frac{F}{m} = \frac{GM}{r_0^2}$$

But GM/r^2 is equal to the acceleration due to gravity at the point P. *Hence the gravitational field intensity of the earth at any point is equal to acceleration produced in the freely falling body at that point.*

For any point on the surface of the earth, r = R, so

$$E_{surface} = \frac{GM}{R_2} = g$$

This is the acceleration due to gravity at the surface of the earth.

Units of E

As gravitational field intensity is force per unit mass, so its SI unit is Nkg^{-1} and cgs unit is dyn g^{-1}.

Dimensions of E

$$\text{As } E = \frac{F}{m}$$

$$\therefore \text{ Dimensions of } E = \frac{MLT^{-2}}{M} = [LT^{-2}]$$

Gravitational Potential Energy

When two bodies are placed close to one another, they interact through the gravitational force. Due to this, they possess mutual gravitational potential energy. When the distance between the two bodies is changed, work is done either by the gravitational force between the two bodies or against this force. In either case, the gravitational potential energy of the bodies changes.

The gravitational potential energy of a body is the energy associated with it due to its position in the gravitational field of another body and is measured by the amount of work done in bringing a body from infinity to a given point in the gravitational field of the other.

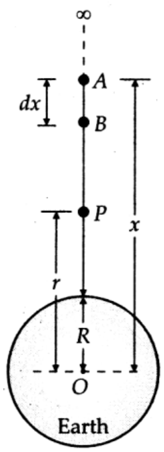
Expression for gravitational potential energy

As shown in figure, suppose the earth is a uniform sphere of mass M and radius R. We wish to calculate the potential energy of a body of mass m located at point P such that OP = r and r > R.

Suppose at any instant the body is at point A such that

$$OA = x$$

The gravitational force of attraction on the body at A is



$$F = \frac{GMm}{x^2}$$

The small work done in moving the body through small distance AB (= dx) is given by

$$dW = Fdx = \frac{GMm}{x^2}dx$$

The total work done in bringing the body from infinity ($x = \infty$) to the point P ($x = r$) will be

$$W = \int dW = \int_{\infty}^{r} \frac{GMm}{x^2}dx = GMm\int_{\infty}^{r} x^{-2}dx$$

$$= GMm\left[-\frac{1}{x}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right] = -\frac{GMm}{r}$$

By definition, this work done is the gravitational potential energy U of the body of mass m located at distance r from the centre of the earth.

$$\therefore \quad U = -\frac{GMm}{r} \qquad \text{... (i)}$$

Some Important Points:

1. The negative sign in equation (i) indicates that the *potential energy is due to the gravitational attraction between the earth and the body*. When the body is brought form infinity to a distance r, work is done by the gravitational force of attraction. As the mutual energy of the two bodies is expended, so their energy r educes by this amount.
2. As the distance r increases, the gravitational P.E. increases because it becomes zero i.e., maximum.

3. If a body of mass m is moved from a point at distance r_1 to a point at distance r_2, then the change in potential energy of the body will be

$$\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = GMm\left[-\frac{1}{x}\right]_{r_1}^{r_2}$$

$$= GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

If $r_1 > r_2$, then ΔU be negative. So *when a body is brought closer to the earth, its gravitation P.E. decreases.*

4. If a body is moved from the surface of the earth ($r_1 = R$) to a point at height h above the surface of the earth ($r_2 = R + h$), then the change in its gravitational P.E. will be

$$\Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMm}{R}\left[1 - \frac{R}{R+h}\right]$$

$$= \frac{GMm}{R}\left[1 - \frac{1}{\left(1+\frac{h}{R}\right)}\right] \qquad = \frac{GMm}{R}\left[1 - \left(1+\frac{h}{r}\right)^{-1}\right]$$

Applying binomial theorem, we get

$$\Delta U = \frac{GMm}{R}\left[1 - \left(1 - \frac{h}{R} + \text{terms containing higher powers of } \frac{h}{R}\right)\right]$$

If $h << R$, then higher powers of h / R can be neglected.

Hence $\Delta U = \frac{GMm}{R}\left[1 - \left(1 - \frac{h}{R}\right)\right] = \frac{GMmh}{R^2}$

But $\frac{GM}{R^2} = g =$ acceleration due to gravity on the earth's surface

$\therefore \quad \Delta U = mgh$

Gravitational Potential

The gravitational potential at a point is the potential energy associated with a unit mass due to its position in the gravitational field of another body. The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.

Gravitational potential,

$$V = \frac{\text{Work done}}{\text{Mass}} = \frac{W}{m}$$

The gravitational potential is a scalar quantity. Its SI units is J kg^{-1} and cgs units is erg g^{-1}. The dimensional formula of gravitational potential is $[M^0L^2T^{-2}]$

Gravitational potential at a point due to the earth

The work done in bringing a body of mass m from infinity to a point at distance r from the centre of the earth is

$$W = -\frac{GMm}{r}$$

Hence the gravitational potential due to the earth at distance r from its centre is

$$V = \frac{W}{m} = -\frac{GM}{r}$$

At the surface of the earth, r = R, therefore

$$V_{surface} = -\frac{GM}{R}$$

Relation between gravitational potential energy and gravitational potential

From the above equations, we find that

$$U = -\frac{GMm}{r} = \left(-\frac{GM}{r}\right) \times m$$

∴ Gravitational potential energy = Gravitational potential × mass

Assignment

Q.1 Find the intensity of gravitational field when a force of 100 N acts on a body of mass 10 kg in the gravitational field.

Q.2 Two bodies of masses 10 kg and 1000 kg are at a distance 1 m apart. At which point on the line joining them will the gravitational field intensity be zero?

Q.3 Two masses, 800 kg and 600 kg, are at a distance 0.25 m apart. Compute the magnitude of the intensity of the gravitational field at a point distant 0.20 m from the 800 kg mass and 0.15 m from the 600 kg mass.

Q.4 At a point above the surface of the earth, the gravitational potential is -5.12×10^7 J kg^{-1} and the acceleration due to gravity is 6.4 ms^{-2}. Assuming the mean radius of the earth to be 6400 km, calculate the height of this point above the earth's surface.

Q.5 Three mass points each of mass m are placed at the vertices of an equilateral triangle of side ℓ. What is the gravitational field and potential due to three masses at the centroid of the triangle?

Q.6 Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side ℓ. Also obtain the potential at the centre of the square.

Q.7 Two bodies of masses m_1 and m_2 are placed at a distance r apart. Show that at the position where the gravitational field due to them is zero, the potential is given by

$$V = -\frac{G}{r}[m_1 + m_2 + 2\sqrt{m_1 m_2}]$$

Q.8 A non–homogenous sphere of radius R has the following density variation:

$\rho = \rho_0$ for $r \le R/3$

$\rho = \rho_0/2$ for $R/2 < r \le 3R/4$

$\rho = \rho_0/8$ for $3R/4 < r \le R$

What is the gravitational field due to the sphere at r = R/4, R/2, 5R/6 and 2 R?

Q.9 Two bodies of masses 100 kg and 1000 kg are at a distance 1.00 metre apart. Calculate the gravitational field intensity and the potential at the middle–point of the line joining them.

Q.10 The mass of the earth is 6.0×10^{24} kg. Calculate (i) the potential energy of a body of mass 33.5 kg and (ii) the gravitational potential, at a distance of 3.35×10^{10} m from the centre of the earth.

Q.11 The radius of the earth is R and the acceleration due to gravity at its surfaced is g. Calculate the work required in raising a body of mass m to a height h from the surface of the earth.

Q.12 Find the work done to bring 4 particles each of mass 100 gram from large distances to the vertices of a square of side 20 cm.

Answers

1. $10\ N\ kg^{-1}$
2. 1/11 m from 10 kg
3. 2.22×10^{-6} N
4. 1600 km
5. $E = 0,\ V = -3\sqrt{3}\frac{Gm}{\ell}$
7. $U = -\frac{5.41Gm^2}{\ell}$, $V = -\frac{4\sqrt{2}Gm}{\ell}$
8. (i) $0.33\ \pi\ GR\rho_0$, (ii) $0.43\ \pi\ GR\rho_0$, (iii) $0.48\ G\ \pi\ R\rho_0$, (iv) $0.1\pi\ GR\rho_0$
9. $2.40 \times 10^{-7}\ Nkg^{-1}$, $-1.47 \times 10^{-7}\ J\ kg^{-1}$
10. (i) -4.02×10^{5} J (ii) $-12 \times 10^{3}\ J\ kg^{-1}$
11. $\frac{mgh}{1+\frac{h}{R}}$
12. -1.80×10^{-11} J

Escape Velocity

Escape velocity is the minimum velocity with which body must be projected vertically upwards in order that it may just escape the gravitational field of the earth.

Expression for escape velocity: Consider the earth to be a sphere of mass M and radius R with centre O. Suppose a body of mass m lies at point P at distance x from its center, as shown in figure. The gravitational force of attraction on the body at P is $F = \frac{GMm}{x^2}$

The small work done in moving the body through small distance PQ = dx against the gravitational force is given by

$$dW = Fdx = \frac{GMm}{x^2}dx$$

The total work done in moving the body from the surface of the earth (x = R) to a region beyond the gravitational field of the earth ($x = \infty$) will be

$$W = \int dW = \int_R^\infty \frac{GMm}{x^2}dx$$

$$= GMm\int_R^\infty x^{-2}dx = GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$= GMm\left[-\frac{1}{\infty}+\frac{1}{R}\right] = \frac{GMm}{R}$$

If v_e is the escape velocity of the body, then the kinetic energy $\frac{1}{2}mv_e^2$ imparted to the body at the surface of the earth will just sufficient to perform work W.

$\therefore \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R}$ or $v_e^2 = \frac{2GM}{R}$

Escape velocity $\quad v_e = \sqrt{\frac{2GM}{R}}$ ... (i)

As $\quad g = \frac{GM}{R^2}$ or $GM = gR^2$

$\therefore \quad v_e = \sqrt{\frac{2gR^2}{R}}$ or $v_e = \sqrt{2gR}$ ... (ii)

If ρ is the mean density of the earth, then

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \qquad v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi\rho G R^2}{3}} \qquad \text{... (iii)}$$

Equations (i), (ii) and (iii) give different expressions for the escape velocity of a body. Clearly, *the escape velocity does not depend on the mass of the body projected.*

NOTE

- For the earth, $g = 9.8\ ms^{-2}$ and $R = 6.4 \times 10^6$ m, so

$$v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= 11.2 \times 10^3\ ms^{-1} = 11.2\ kms^{-1}$$

- In deriving the expression for escape velocity, we have neglected the air resistance on the body. In actual practice, the value of escape velocity is slightly greater than the above calculated value.
- The escape velocity does not depend on angle of projection from the earth's surface. But as the earth rotates about its axis, so it becomes easier to attain escape velocity if the body is projected in the direction in which the launch site is moving.
- As the escape velocity depends on the mass and radius of the planet from the surface of which the body is projected, so value of escape velocity is different for different planets.
- A planet will have atmosphere if the root mean square velocity of its atmospheric molecules is less than the escape velocity for the given planet. That is why moon has no atmosphere ($v_e = 2.3\ kms^{-1}$) while Jupiter has a thick atmosphere ($v_e = 60\ kms^{-1}$). Even the lightest hydrogen cannot escape from its surface.

Assignment – II

Q.1 Find the velocity of escape at the earth given that its radius is 6.4×10^6 m and the value of g at its surface is $9.8\ ms^{-2}$.

Q.2 A black hole is a body from whose surface nothing may even escape. What is the condition for a uniform spherical body of mass M to be a black hole? What should be the radius of such a black hole if its mass is nine times the mass of the earth?

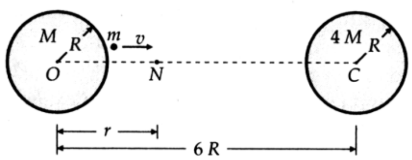
Q.3 Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface is $11.2\ km\ s^{-1}$.

Q.4 Show that the moon would depart for ever if its speed were increased by 42%.

Q.5 Calculate the escape velocity for an atmospheric particle 1600 km above the earth's surface, given that the radius of earth is 6400 km and acceleration due to gravity on surface of earth is $9.8\ ms^{-2}$.

Q.6 The radius of a planet is double that of the earth but their average densities are the same. If the escape velocities at the planet and at the earth and v_p and v_E respectively, then prove that $v_P = 2\ v_E$.

Q.7 Two uniform solid spheres of equal radii R, but mass M and 4 M have a center to centre separation 6 R, as shown in figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the center of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.



Q.8 Find the velocity of escape at the moon. Given that its radius is 1.7×10^6 m and the value of 'g' is $1.63\ ms^{-2}$.

Q.9 If earth has a mass 9 times and radius twice that of a planet Mars, calculate the minimum velocity required by a rocket to pull out of gravitational force of Mars. Take the escape velocity on the surface of earth to be 11.2 kms^{-1}.

Q.10 The escape velocity of a projectile on the surface of the earth is 11.2 kms^{-1}. A body is projected out with twice this speed. What is the speed of the body far away from the earth i.e. at infinity? Ignore the presence of the sun and other planets, etc.

Q.11 A body is a height equal to the radius of the earth from the surface of the earth. With what velocity be it thrown so that it goes out of the gravitational field of the earth? Given $M_e = 60 \times 10^{24}$ kg, $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$.

Q.12 A body of mass 100 kg falls on the earth from infinity. What will be its velocity on reaching the earth? What will be its K.E.? Radius of the earth is 6400 km and g = 9.8 ms^{-2}. Air friction is negligible.

Answers

1.	11.2 kms^{-1}	2.	nearly 8 cm	3.	59.7 kms^{-1}
5.	10.02 kms^{-1}	7.	$v=\sqrt{\frac{3GM}{5R}}$	8.	2.354 kms^{-1}
9.	5.28 kms^{-1}	10.	19.4 kms^{-1}	11.	7.9 kms^{-1}
12.	11.2 kms^{-1}, 6.27×10^9 J				

Natural and Artificial Satellites

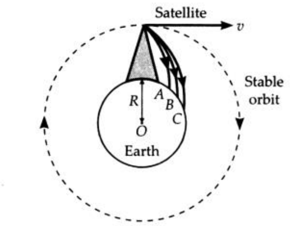
Satellite: *A satellite is a body which continuously revolves on its own around a much larger body in a stable orbit.*

Natural Satellite: *A satellite created by nature is called a natural satellite.* Moon is a natural satellite of the earth which, in turn, is a satellite of the sun. In fact, each planet is a satellite of the sun.

Artificial satellite: *A man made satellite is called an artificial satellite.* Russians were the first to put an artificial satellite, SPUTNIK–I, in an orbit around the earth on October 4, 1957.

Principle of a Launching a Satellite

Consider a higher tower with its top projecting outside the earth's atmosphere. Let us throw a body horizontally from the top of the tower with different velocities. When the velocity is low, the body describes a parabolic path under the effect of gravity and hits the earth's surface at A.



As we go on increasing the velocity of horizontal projection, the body will hit the ground at a point farther and farther from the foot of the tower. At a certain horizontal velocity, the body will not hit the earth, but will always be in a state of free fall under gravity and attempt to fall to the earth but missing it all the time. Then the body will follow a stable circular path around the earth and will become a satellite of the earth. This horizontal velocity is called orbital velocity.

Use of Multistage Rockets

Much higher velocities are required to take the satellite to a suitable height. Such high velocities can be imparted to the satellites by using multistage rockets. Generally 3–stage rockets are used. The satellite is placed on the third stage. At lift off, the exhaust gases build up a very large upthrust so that the rocket accelerates upwards. The rocket rises vertically through the denser atmosphere with a minimum time. When the fuel of the first stage gets exhausted, its casing is detached. Now the rocket is tilted gradually, the second stage comes into operation and its velocity increases further. The second stage gets detached. The final stage of the rocket turns the satellite in a horizontal direction and gives it a proper speed. With this speed, the satellite moves around the earth in a stable orbit.

Orbital velocity: *Orbital velocity is the velocity required to put the satellite into its orbit around the earth*

Let M = mass of the earth,

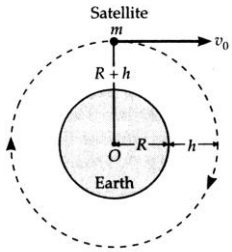
R = radius of the earth,

m = mass of the satellite

v_0 = orbital velocity of the satellite

h = height of the satellite above the earth's surface

R + h = orbital radius of the satellite



According to the law of gravitation, the force of gravity on the satellite is

$$F = \frac{GMm}{(R+h)^2}$$

The centripetal force required by the satellite to keep it in its orbit is

$$F = \frac{mv_0^2}{R+h}$$

In equilibrium, the centripetal force is just provided by the gravitational pull of the earth, so

$$\frac{mv_0^2}{R+h} = \frac{GMm}{(R+h)^2}$$

or $$v_0^2 = \frac{GM}{R+h}$$

∴ Orbital velocity, $$v_0 = \sqrt{\frac{GM}{R+h}} \quad \text{... (i)}$$

If g is the acceleration due to gravity on the earth's surface, then

$$g = \frac{GM}{R^2}$$

or GM = gR^2

Hence $$v_0 = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

When the satellite revolves close to the surface of the earth, h = 0 and the orbital velocity will become

$$v_0 = \sqrt{gR}$$

As g = 9.8 ms^{-1} and R = 6.4 × 10^6 m, so

$$v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3 \text{ ms}^{-1}$$

$$\mathbf{= 7.92\ kms^{-1}}$$

Some important points

From equation (i), it is clear that the orbital velocity of a satellite

(i) is independent of the mass of the satellite.

(ii) decreases with the increase in the radius of the orbit i.e. with increase in the height of the satellite.

(iii) depends on the mass and radius of the planet about which the satellite revolves.

The escape velocity of a body from the earth's surface is

$$v_e = \sqrt{2gR}$$

The orbital velocity of a satellite revolving close to the earth's surface is

$$v_0 = \sqrt{gR}$$

$$\therefore \quad \frac{v_e}{V_0} = \sqrt{\frac{2gR}{gR}} = \sqrt{2} \qquad \text{or} \qquad v_e = \sqrt{2}\, v_0$$

Hence the escape velocity of a body from the earth's surface is $\sqrt{2}$ *times its velocity in a circular orbit just above the earth's surface.*

Time period of a satellite

It is the time taken by a satellite to complete one revolution around the earth. It is given by

$$T = \frac{\text{Circumference of the orbit}}{\text{Orbital velocity}} = \frac{2\pi(R+h)}{v_0}$$

As orbital velocity,

$$v_0 = \sqrt{\frac{GM}{R+h}}$$

$$\therefore \quad T = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

But g = GM/ R^2 or GM = gR^2, therefore

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad \text{... (i)}$$

If the earth is a sphere of mean density ρ, then its mass would be

$$M = \text{Volume} \times \text{density } \frac{4}{3}\pi R^3\rho$$

$$\therefore \quad \therefore \quad T = 2\pi\sqrt{\frac{(R+h)^3}{G\times\frac{4}{3}\pi R^3\rho}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

When the satellite revolves closed to the earth, h = 0 and the time period will be

$$\therefore \quad T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

Putting g = 9.8 ms^{-2} and R = 6.4 × 10^6 m, we get

$$T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} = 5078 \text{ s} = \mathbf{84.6\ min.}$$

Height of a satellite above the earth's surface

Squaring both sides of equation (i), we get

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$\text{or} \quad (R+h)^3 = \frac{T^2R^2g}{4\pi^2} \quad \text{or} \quad R+h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3}$$

$\therefore$ Height of satellite, $h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$

Angular momentum:

The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius r (= R + h) is given by

$$L = mv_0r = m\sqrt{\frac{GM}{r}}\,r = \sqrt{GMm^2r}$$

Subjective Assignment

Q.1 An artificial satellite revolves around the earth at a height of 1000 km. The radius of the earth is 6.38×10^3 km. Mass of the earth is 6×10^{24} kg and G = 6.67×10^{-11} kg^{-2}. Find its orbital velocity and period of revolution.

Q.2 A remote sensing satellite of the earth revolves in a circular orbit at a height of 250 km above the earth's surface. What is the (i) orbital speed and (ii) period of revolution of the satellite? Radius of the earth, R = 6.38×10^6 m, and acceleration due to gravity on the surface of the earth, g = 9.8 ms^{-2}.

Q.3 An artificial satellite is going round the earth, close to its surface. What is the time taken by it to complete one round? Given radius of the earth = 6400 km.

Q.4 An earth's satellite make a circle around the earth in 90 minutes. Calculate the height of the satellite above the earth's surface. Given radius of the earth is 6400 km and g = 980 cms^{-2}.

Q.5 In a two–stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the second stage gives it the necessary critical speed to put it in a circular orbit around the Earth. Which stage requires more expenditure of fuel? (Neglect damping due to air resistance, especially in the first stage). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km, G = 6.67×10^{-11} Nm^2 kg^{-2}.

Q.6 An artificial satellite circled around the earth at a distance of 3400 km. Calculate its orbital velocity and period of revolution. Radius of earth = 6400 km and g = 9.8 ms^{-2}.

Q.7 An artificial satellite of mass 100 kg is in a circular orbit of 500 km above the earth's surface.
(i) find the acceleration due to gravity at any point along the satellite path
(ii) what is the centripetal acceleration of the satellite? Take g = 9.8 ms^{-2}

Q.8 A space–ship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the space–ship in the orbit to overcome the gravitational pull?

Answers

1.	6297 s, 7364 ms^{-1}	2.	7.76 kms^{-1}, 5370 s	3.	1.41 hour
4.	268 km	5.	second stage	6.	6400 ms^{-1}, 9621 s
7.	(i) 8.45 ms^{-2}, (ii) 8.45 ms^{-2}			10.	3.278 kms^{-1}

Geostationary Satellites

A satellite which revolves around the earth in its equatorial plane with the same angular speed and in the same direction as the earth rotates about its own axis is called a geostationary or synchronous satellite.

Height of a geostationary satellite: The height of a satellite above the earth's surface is given by

$$h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

But T = 24 h = 86400 s,
R = radius of the earth = 6400 km,

$g = 9.8\ ms^{-2} = 0.0098\ kms^{-2}$

$$\therefore \quad h = \left[\frac{(86400)^2 \times (6400)^2 \times 0.098}{4 \times 9.87}\right]^{1/3} - 6400$$

$= 42330 - 6400 =$ **35930 km**

Necessary conditions for a geostationary satellite

These are as follows:

1. It should revolve in an orbit concentric and coplanar with the equatorial plane of the earth
2. Its sense of rotation should be same as that of the earth i.e., from west to east
3. Its period of revolution around the earth should be exactly same as that of the earth about its own axis i.e., 24 hours
4. It should revolve at a height of nearly 36,000 km above the earth's surface

Uses of geostationary satellites

1. In communicating radio, T.V. and telephone signals across the world. Geostationary satellites act as reflectors of such signals.
2. In studying upper regions of the atmosphere.
3. In forecasting weather.
4. In determining the exact shape and dimensions of the earth.
5. In studying meteorites.
6. In studying solar radiations and cosmic rays SYNCOMS → Sumchronous Communication Satellites

Polar satellite

A satellite that revolves in a polar orbit is called a polar satellite. A polar orbit is one whose plane is perpendicular to the equatorial plane of the earth. A polar orbit passes over north and south poles of the earth and has a smaller radius of 500 – 800 km.

Uses of Polar Satellites:

(i) Polar satellites are used in weather and environment monitoring. They provide more reliable information than geostationary satellites because their orbits are closed to the earth.

(ii) They are used in spying work for military purposes.

(iii) British polar satellite first detected hole in the ozone layer.

(iv) They are used to study topography of Moon, Venus and Mars.

Total Energy and Binding Energy of a Satellite

Consider a satellite of mass m moving around the earth with velocity v_0 in an orbit of radius r. Because of gravitational pull of the earth, the satellite has *potential energy* which is given by

$$U = -\frac{GMm}{r}$$

The kinetic energy of a satellite due to its orbital motion is

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)\left[\because v_0 = \sqrt{\frac{GM}{r}}\right]$$

Total energy of the satellite is

$$E = U + K = -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r} \qquad \text{or} \qquad E = -\frac{GMm}{2r}$$

The total energy of the satellite is *negative*. It indicates that the *satellite is bound to the earth.*

Binding energy of a satellite

The energy required by a satellite to leave its orbit around the earth and escape to infinity is called its binding energy.

The total energy of a satellite is $-\frac{GMm}{2r}$. In order to escape to infinity, it must be supplied an extra energy equal to $+\frac{GMm}{2r}$ so that its total energy E becomes equal to zero. Hence

Binding energy of a satellite = $\frac{GMm}{2r}$.

NOTE:

- The total mechanical energy of an object (say satellite in orbit), is negative if it is bound. This implies that orbit may be an ellipse or circle. But it is *not always negative*. It can be positive in which case its trajectory is a hyperbola and the object is not bound to the central star or its equivalent. These statements are evidently true when the zero of potential energy is chosen at infinitely.

Assignment

Q.1 A 400 kg satellite is in a circular orbit of radius 2 R_E about the earth. How much energy is required to transfer it to a circular orbit of radius 4 R_E? What are the changes in the kinetic and potential energies?

Q.2 A satellite orbits the earth at a height of 500 km from its surface. Compute its (i) kinetic energy, (ii) potential energy, and (iii) total energy. Mass of the satellite = 300 kg, Mass of the earth = 6.0×10^{24} kg, radius of the earth = 6.4×10^6 m, G = 6.67×10^{-11} Nm^2 kg^{-2}. Will your answer alter if the earth were to shrink suddenly to half its size?

Q.3 A rocket is launched vertically from the surface of the earth with an initial velocity of 10 kms^{-1}. How far above the surface of earth would it go? Radius of the earth = 6400 km and g = 9.8 ms^{-2}.

Q.4 A body is to be projected vertically upwards from earth's surface to reach a height of 9R, where R is the radius of earth. What is the velocity required to do so? Given g = 10 ms^{-2} and radius of earth = 6.4×10^6 m.

Q.5 Show that the velocity of a body released at a distance r from the centre of the earth, when it strikes the surface of the earth is given by

$$v=\sqrt{2GM\left(\frac{1}{R}-\frac{1}{r}\right)}$$

where R and M are the radius and mass of the earth respectively. Also show that the velocity with which the meteorites strike the surface of the earth is equal to the escape velocity.

Q.6 Calculate the energy required to move an earth satellite of mass 10^3 kg from a circular orbit of radius 2R to that of radius 3R. Given mass of the earth, M = 5.98×10^{24} kg and radius of the earth, R = 6.37×10^6 m.

Answers

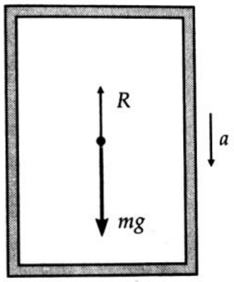
1. $\Delta U = 6.26 \times 10^9$ J, UK = -3.13×10^9 J, 3.13×10^9 J
2. (i) 8.7×10^9 J, (ii) -17.4×10^9 J, (iii) -8.7×10^9 J, No
3. 2.5×10^4 km 4. 1.073×10^4 ms^{-1} 6. 5.02×10^9 J

Weightlessness

When a body presses against a supporting surface, the supporting surface exerts a force of reaction on him. This force of reaction produces the feeling of weight in the body. If somehow the force of reaction becomes zero, the apparent weight of the body becomes zero.

A body is said to be in a state of weightlessness when the reaction of the supporting surface is zero or its apparent weight is zero.

A body can be in the state of weightlessness under the following circumstances:



(i) **In a freely falling lift:** Consider a person of true weight mg standing in a lift which is moving vertically downwards with acceleration a. If R is the reaction of the floor on the man, then

$$mg - R = ma$$

∴ Apparent weight,

$$R = m(g - a)$$

If the cable of the lift breaks, it begins to fall freely. Then a = g, and

$$R = m(g - g) = 0$$

Hence the men falling freely develops a feeling of weightlessness.

(ii) **Inside a spacecraft:** Consider a spacecraft revolving around the earth in an orbit of radius r. The acceleration of the satellite is GM/r^2, towards the centre of the earth. Here M is the mass of the earth. Suppose a body of mass m lies on an inside surface of the satellite. Forces acting on this body will be

(a) Gravitational pull of the earth $= \left(\frac{GM}{r^2}\right)$

(b) Reaction force R of the surface

By Newton's second law,

$$\frac{GMm}{r^2} - R = ma = m\left(\frac{GM}{r^2}\right)$$

∴ $R = 0$

Thus the surface does not exert any force on the body and hence its apparent weight is zero.

(iii) **At null points in space:** At certain points in space, called the null points, the gravitational forces due to various masses cancel out. As the value of g is zero at these points, so the effective weight of the body is zero.

(iv) **At the centre of the earth:** As the value of g is zero at the centre of the earth, so weight of a body is zero at the centre of the earth.

Problems of weightlessness

(i) Eating and drinking become difficult in the state of weightlessness. An astronaut cannot drink water from a glass because on tilting the glass, the water comes out in the form of floating drops. He takes food in the form of paste from tube squeezed into his mouth.

(ii) Space–flight for a long time adversely affects the human organism.

(iii) While walking in a space craft, an astronaut is pushed away from the floor and he may crash against the ceiling of the spacecraft.

(iv) It is not possible to perform experiments on simple pendulum in the state of weightlessness. As g = 0, so $T = 2\pi\sqrt{L/g} = \infty$

NOTE

- When a body is in a free fall, a gravitational pull mg does act on it. It is said to be weightless because it exerts no force on its support.
- An astronaut experiences weightlessness in space. This is not because the gravitational force is small on him. It is because the astronaut and the satellite both are in a state of free fall towards the earth.

- In the state of weightlessness, though the bodies have no weight, they have inertia on account of their mass. So bodies floating in a space craft may collide with each other and crash.

Inertial and Gravitational Mass

Inertial mass: *The mass of a body which measures its inertia is called its inertial mass. It is equal to the ratio of the external force applied on the body to the acceleration produced in it along a smooth horizontal surface.*

$$\text{Inertial mass} = \frac{\text{Applied force}}{\text{Acceleration produced}} \quad \text{or} \quad m_i = \frac{F}{a}$$

The inertial mass of a body is a measure of its ability to resist the production of acceleration by an external force. If some force is applied on two different bodies, then the inertial mass of that body will be more in which the acceleration produced is less and vice versa.

Properties of Inertial Mass:

1. Inertial mass of a body is directly proportional to the quantity of matter possessed by it.
2. It is independent of size, shape and state of the body.
3. It is conserved both in physical and chemical process.
4. It is not affected by the presence of other bodies.
5. When different bodies are put together, their inertial masses get added together irrespective of the nature of their materials.
6. The inertial mass of a body increases with its speed. When a body of rest mass m_0 moves with speed v, its inertial mass will be

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Gravitational Mass

The mass of a body which determines the gravitational pull due to earth acting upon it is called its gravitational mass. If M is the mass of the earth and R its radius, then according to Newton's law of gravitation, the gravitational pull acting on a body of mass m_g placed on the earth's surface is given by

$$F = \frac{GMm_g}{R^2}$$

∴ Gravitational mass, $m_g = \frac{FR^2}{GM}$

Greater the gravitational mass of a body, greater is the gravitational pull of the earth on it. So if two bodies lying at equal heights from the ground experience equal force of gravity, then their gravitational masses must be equal. This forms the principle of a pan balance.

Equivalence of inertial and gravitational masses

According to Newton's law of gravitation, the gravitational force acting on a body of gravitational mass m_g placed on the surface of the earth is given by

$$F = \frac{GMm_g}{R^2}$$

If a body of inertial mass m_i is allowed to fall freely, then from Newton's second law,

F = inertial mass × acceleration due to gravity = m_i g

From the above two equations, we get

$$m_i\, g = \frac{GMm_g}{R^2}$$

or $\frac{m_i}{m_g} = \frac{GM}{gR^2} = k \text{ (a constant)}$

or $m_i = km_g$ or $m_1 \propto m_g$

Thus the inertial mass of a body is proportional to its gravitational mass.

Comparison between inertial and gravitational masses :

Similarities :

1. Both represent the quantity of matter in a body
2. Both are equivalent in magnitude and have same units of measurement.
3. Both do not depend on the shape or state of matter.
4. Both are not affected by the presence of other bodies.
5. Both are scalar quantities.

Differences :

1. Inertial mass is the measure of difficulty of accelerating a body while gravitational mass measures the force of attraction between the body and the earth.
2. Inertial mass is determined from Newton's second law of motion while gravitational mass is determined from Newton's law of gravitation.
3. Inertial mass can be measured only under dynamic conditions i.e., when the body is in motion, which is neither convenient nor practical. Gravitational mass can be easily measured by using a par balance.

IIT Entrance Exam

Multiple Choice Questions with One Correct Answers

Q.1 If the radius of earth were to shrink by one percent (its mass remaining the same), then the acceleration due to gravity on the earth's surface

(a) would decreases (b) would remain unchanged

(c) would increase (d) cannot be predicted

Q.2 A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

Q.3 If the distance between the earth and the sun were half its present value, the number of days in year would have been

(a) 64.5 (b) 129 (c) 182.5 (d) 730

Q.4 A geo–stationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400 km) will approximately be

(a) (1/2)h (b) 1 h (c) 2 h (d) 4 h

Q.5 A binary star system consists of two starts A and B which have time periods T_A and T_B, radii R_A and R_B and masses M_A and M_B. Then

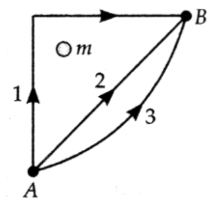
(a) if $T_A > T_B$, then $R_A > R_B$ (b) if $T_A > T_B$, then $M_A > M_B$

(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (d) $T_A = T_B$

Q.6 A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth, R being the radius of earth. The time period of another satellite at a height of 2.5 R from the surface of earth is

(a) $6\sqrt{2}$ hours (b) 6 hours (c) $6\sqrt{3}$ hours (d) 10 hours

Q.7 If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown in the figure) in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 and W_3.



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

Q.8 If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of earth to a height equal to the radius R of the earth, is

(a) 1/2 mgR (b) 2 mgR (c) mgR (d) 1/4 mgR

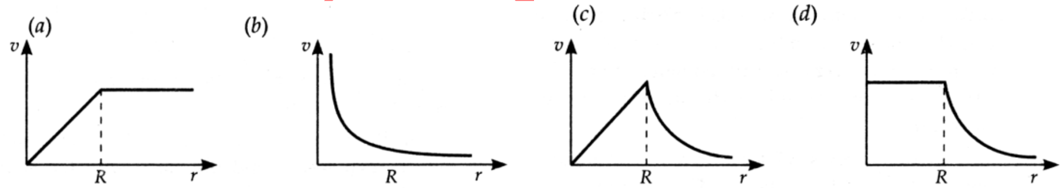
Q.9 An artificial satellite moving in a circular orbit around the earth has total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\ E_0$ (c) $2\ E_0$ (d) E_0

Q.10 A spherically symmetric system of particles has a mass density

$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

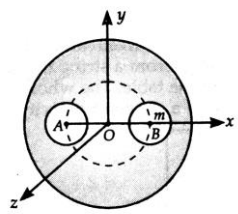


Multiple Choice questions with one or More than one Correct Answers

Q.11 Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then

(a) T^2 is proportional to R^3 (b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$ (d) T^2 is proportional to $R^{7/3}$

Q.12 A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then



(a) the gravitational force due to this object at the origin is zero
(b) the gravitational force at the point B (2, 0, 0) is zero
(c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$
(d) the gravitational potential is the same at all points of circle $y^2 + z^2 = 4$

Q.13 The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$, if $r_1 < R$ and $r_2 < R$ (b) $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$, if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$, if $r_1 > R$ and $r_2 > R$ (d) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$, if $r_1 < R$ and $r_2 < R$

Q.14 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth.

(a) the acceleration of S is always directed towards the centre of the earth

(b) the angular momentum of S bout the centre of the earth changes in direction, but its magnitude remains constant

(c) the total mechanical energy of S varies periodically with time

(d) the linear momentum of S remains constant in magnitude

Reasoning Type

Instructions: Each question contains statement – 1 (assertion) and statement – 2 (reason). Of these statements mark correct choice if

(a) Statement – 1 and 2 are true and statement –2 is a correct explanation for statement – 1

(b) Statement –1 and 2 are true and statement – 2 is not a correct explanation for statement – 1

(c) Statement – 1 is true, statement – 2 is false

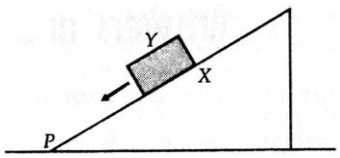
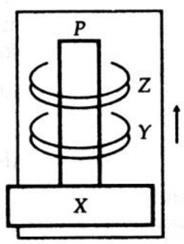
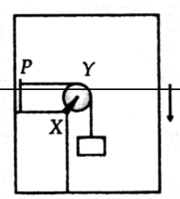
(d) Statement – 1 is false, statement – 2 is true

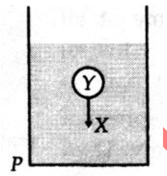
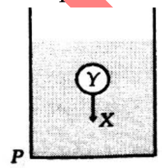
Q.15 Statement – 1: An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement – 2: An object moving around the earth under the influence of earth's gravitational field is in a state of free fall.

Match Matrix Type

Q.16 Column II shows in which two objects are labeled as X and Y. Also in each case a point P is shown. Column I gives some statements about X and/or Y. Match these statements to the appropriate system (s) from Column II.

	Column – I		**Column – II**
(a)	The force exerted by X on Y has a magnitude Mg	(p)	Block Y of mass left on a fixed inclined plane X, slides on it with a constant velocity 
(b)	The gravitational energy of X is continuously increasing	(q)	Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the top most point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 
(c)	Mechanical energy of the system X + Y is continuously decreasing	(r)	A pulley Y of mass m_0 is fixed to a table through a clamp X, a block of mass M hangs from a string that 

			goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going with a constant velocity in downward direction
(d)	The torque of the weight of Y about point P is zero	(s)	A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid. 
		(t)	A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container. 

Answers

1	c	2	d	3	b	4	c	5	d
6	a	7	b	8	a	9	c	10	c
11	b	12	a, c, d	13	a, b	14	a, c	15	a
16	a – p, t; b – q, s, t; c – p, r, t; d – q								

AIEEE

Q.1 If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will
(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit with the same velocity
(c) become stationary in its orbit
(d) move towards the earth.

Q.2 Average density of the earth
(a) does not depend on g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g

Q.3 The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct?
(a) $d = \frac{h}{2}$
(b) $d = \frac{3h}{2}$
(c) d = 2h
(d) d = h

Q.4 If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to radius R of the earth, is
(a) $\frac{1}{4}$mgr
(b) $\frac{1}{2}$mgr
(c) 2mgr
(d) mgr

Q.5 Energy required to move a body of mass m from an orbit of radius 2 R to 3 R is

(a) $GMm/12\ R^2$ (b) $GMm/3R^2$ (c) $GMm/8R$ (d) $GMm/6R$

Q.6 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J (c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

Q.7 The kinetic energy needed to project a body of mass m from earth's surface (radius R) to infinity is

(a) mgR/2 (b) 2mgR (c) mgR (d) mgR / 4

Q.8 The escape velocity of a body depends upon mass as

(a) m^0 (b) m (c) m^2 (d) m^3

Q.9 The escape velocity for a body projected vertically upwards from the surface of earth is 11 km s^{-1}. If the body is projected at an angle 45° with the vertical, the escape velocity will be

(a) $1/\sqrt{2}\ km\,s^{-1}$ (b) $11 km\,s^{-1}$ (c) $11\sqrt{2}\ km\ s^{-1}$ (d) 22 km s^{-1}

Q.10 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1} (c) 11 km s^{-1} (d) 110 km s^{-1}

Q.11 A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\frac{gR}{R-x}$ (c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

Q.12 The time period of an earth satellite in circular orbit is independent of

(a) the mass of the satellite (b) radius of its orbit

(c) both the mass and radius of the orbit

(d) neither the mass of the satellite nor the radius of its orbit.

Q.13 The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hours (b) 80 hours (c) 40 hours (d) 20 hours

Q.14 Suppose that the gravitational force varies inversely as the nth power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{(n+1)/2}$ (b) $R^{(n-1)/2}$ (c) R^n (d) $R^{(n-2)/2}$

Q.15 Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) 4.5 R (b) 7.5 R (c) 1.5 R (d) 2.5 R

Q.16 The height at which acceleration due to gravity becomes g/a (where g = the acceleration to gravity on the surface of the earth) in terms of R, the radius of the earth is

(a) 2 R (b) $\frac{R}{\sqrt{2}}$ (c) R/2 (d) $\sqrt{2}\,R$

Reasoning Type

Instructions: Each question contains statement – 1 (assertion) and statement – 2 (reason). Of these statements mark correct choice if

(a) Statement – 1 and 2 are true and statement –2 is a correct explanation for statement – 1

(b) Statement –1 and 2 are true and statement – 2 is not a correct explanation for statement – 1
(c) Statement – 1 is true, statement – 2 is false
(d) Statement – 1 is false, statement – 2 is true

Q.17 Statement – 1: For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement – 2: If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as I/r^2, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

Answers

1.	b	2.	c	3.	c	4.	b	5.	d
6.	d	7.	c	8.	a	9.	b	10.	d
11.	d	12.	a	13.	c	14.	a	15.	b
16.	a	17.	a						

AIIMS Entrance Exam

Q.1 The force of gravitation is
(a) repulsive (b) electrical (c) conservative (d) non-conservative

Q.2 Which of the following is an evidence to show that there must be a force acting on earth and directed toward the sun?
(a) deviation of the falling bodies towards east
(b) revolution of the earth round the sun
(c) phenomenon of day and night (d) apparent motion of sun round the earth.

Q.3 A man waves his arms, while walking. This is
(a) to keep constant velocity (b) to ease the tension
(c) to increase the velocity (d) to balance the effect of earth's gravity.

Q.4 All the known planets move in
(a) straight path (b) circular path (c) elliptical path (d) hyperbolic path

Q.5 Kepler's second law is based on
(a) Newton's first law (b) Newton's second law
(c) Special theory of relativity (d) conservation of angular momentum.

Q.6 The radius vector, drawn from the sun to a planet sweeps out equal areas in equal times. This is the statement of
(a) Kepler's first law (b) Kepler's second law
(c) Kepler's third law (d) Newton's third law

Q.7 The orbital speed of Jupiter is
(a) greater than the orbital speed of earth (b) less than the orbital speed of earth
(c) equal to the orbital speed of earth (d) proportional to distance from the earth.

Q.8 For a planet moving around the sun in an elliptical orbit of semi–major and semi–minor axes a and b respectively and period T,
(a) the torque acting on the planet about the sun is non-zero
(b) the angular momentum of the planet about the sun is constant
(c) the planet moves with a constant speed around the sun
(d) the areal velocity is $\pi ab/ T$

Q.9 If mass of a body is M on the earth surface, then the mass of the same body on the moon surface is
(a) M / 6 (b) zero (c) M (d) none of these

Q.10 Two spheres of same size, one of mass 2 kg and another of mass 4 kg are dropped simultaneously from the top of Qutab Minar (height = 72 km). When they are 1 m above the ground, the two spheres have the same

(a) momentum (b) kinetic energy (c) potential energy (d) acceleration

Q.11 Two planets of radii and are made from the same material. The ratio of the acceleration of gravity g_1 / g_2 at the surfaces of the planets is

(a) r_1/r_2 (b) r_2/r_1 (c) $(r_1 / r_2)^2$ (d) $(r_2 / r_1)^2$

Q.12 If the radius of earth shrinks by one percent and its mass remaining the same, then acceleration due to gravity on the earth's surface will

(a) decrease (b) increase (c) remain constant (d) either (a) or (c)

Q.13 At what depth below the surface of the earth, is the value of g same as that of a height of 5 km?

(a) 10 km (b) 7.5 km (c) 5 km (d) 2.5 km

Q.14 A body weighed 250 N on the surface. Assuming the earth to be a sphere of uniform mass density, how much would it weigh half way down to the centre of earth?

(a) 240 N (b) 210 N (c) 195 N (d) 125 N

Q.15 Knowing that mass of moon is M / 81 (where M is the mass of earth), find the distance of the point, where gravitational field due to earth and moon cancel each other. Given that the distance between the earth and moon is 60 R, where R is the radius of earth.

(a) 2 R (b) 4 R (c) 6 R (d) 8 R

Q.16 The velocity with which a projectile, must be fired so that it escapes earth's gravitation, does not depend on

(a) mass of the earth (b) mass of the projectile
(c) radius of the projectile's orbit (d) gravitational constant

Q.17 The angular velocity of rotation of a star (of mass M and radius R) at which the matter starts to escape from its equator, is

(a) $\sqrt{2GM^2/R}$ (b) $\sqrt{2GM/R^3}$ (c) $\sqrt{2GM/R}$ (d) $\sqrt{2GR/M}$

Q.18 In what manner, does the escape velocity of a particle depend upon its mass?

(a) m^2 (b) m (c) m^0 (d) m^{-1}

Q.19 Escape velocity of a body, when projected from the earth's surface is 11.2 km s^{-1}. If it is projected at an angle of 60^o with the horizontal, then escape velocity will be

(a) 11.2 km s^{-1} (b) 11.6 km s^{-1} (c) 12.8 km s^{-1} (d) 16.2 km s^{-1}

Q.20 The mass of moon is 1/81 of earth's mass and its radius 1/4 of that of earth. If the escape velocity from the earth's surface is 11.2 km s^{-1}, its value for the moon is

(a) 0.14 km s^{-1} (b) 0.76 km s^{-1} (c) 2.45 km s^{-1} (d) 5.28 km s^{-1}

Q.21 The escape velocity from the earth is 11.2 km s^{-1}. The escape velocity from a planet having twice the radius and the same mean density as the earth is

(a) 22.4 km s^{-1} (b) 11.2 km s^{-1} (c) 5.5 km s^{-1} (d) 15.5 km s^{-1}

Q.22 There is no atmosphere on the moon, because

(a) it is closer to the earth and also it has the inactive inert gases in it.
(b) it is too far from the sun and has very low pressure in its outer surface.
(c) escape velocity of gas molecules is greater than their root mean square velocity
(d) escape velocity of gas molecules is less than their root mean square velocity

Q.23 A missile is launched with a velocity less than escape velocity. The sum of its kinetic and potential energies is

(a) zero (b) negative (c) positive (d) first (a) then (b)

Q.24 A satellite of the earth is revolving in a circular orbit with a uniform speed v. If the gravitational force suddenly disappears, the satellite will

(a) continue to move with velocity v along the original orbit

(b) move with a velocity v tangentially to the original orbit

(c) fall down with increasing velocity

(d) ultimately come to rest, somewhere on the original orbit

Q.25 Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are going around the earth in orbits of radii r_1 and r_2 ($r_1 > r_2$). Which statement about their velocities is correct?

(a) $v_1 = v_2$ (b) $v_1/r_1 = v_2/r_2$ (c) $v_1 > v_2$ (d) $v_1 < v_2$

Q.26 If v be the orbital velocity of a satellite in a circular orbit close to the earth's surface and v_e is the escape velocity from the earth, then relation between the two is

(a) $v_e = v$ (b) $v_e = \sqrt{2}\,v$ (c) $v = \sqrt{3}\,v_e$ (d) $v_e = 2v$

Q.27 A satellite is in an orbit around the earth. If its kinetic energy is doubled, then

(a) it will maintain its path (b) it will fall on the earth

(c) it will rotate with a great speed (d) it will escape out of earth's gravitational field

Assertion and Reason Type

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice is

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason if false

(d) If both assertion and reason are false

Q.28 **Assertion:** The stars twinkle, while the planets do not.

Reason: The starts are much bigger in size than the planets.

Q.29 **Assertion:** A planet is a heavenly body revolving a round the sun.

Reason: Star is a self–luminous body made of gaseous material.

Q.30 **Assertion:** The comets do not obey Kepler's laws of planetary motion.

Reason: The comets do not have elliptical orbits.

Q.31 **Assertion:** The square of period of revolution of a planet is proportional to the cube of its distance from the sun.

Reason: Sun's gravitational field is inversely proportional to square of its distance from the planet.

Q.32 **Assertion:** The earth is slowing down and as a result, the moon is coming nearer to it.

Reason: The angular momentum of the earthmoon system is not conserved.

Q.33 **Assertion:** The length of the day is slowly increasing.

Reason: The dominant effect causing a slow down in the rotation of the earth is the gravitational pull of other planets in the solar system.

Q.34 **Assertion:** The earth without its atmosphere would be inhospitably cold.

Reason: All heat would escape in the absence of atmosphere.

Q.35 **Assertion:** The time–period of pendulum on a satellite orbiting the earth is infinity.

Reason: Time–period of a pendulum is inversely proportional to $\sqrt{g}$.

Q.36 **Assertion:** If a pendulum falls freely, then its time period is infinite.

Reason: Free falling body has acceleration equal to g.

Q.37 **Assertion:** An astronaut experiences weightlessness in a space satellite.

Reason: When a body falls freely, it does not experience gravity.

Answers

1.	c	2.	b	3.	d	4.	c	5.	d
6.	b	7.	b	8.	b	9.	c	10.	d
11.	a	12.	b	13.	a	14.	d	15.	c
16.	b	17.	b	18.	c	19.	a	20.	a
21.	a	22.	d	23.	c	24.	b	25.	d
26.	b	27.	d	28.	b	29.	b	30.	b
31.	b	32.	d	33.	d	34.	a	35.	a
36.	b	37.	b						

CBSE PMT

Q.1 Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) 3 F (b) F (c) F/3 (d) F/9

Q.2 The earth (mass = 6×10^{24} kg) revolves around the sun with an angular velocity of 2×10^{-7} rad/s in a circular orbit of radius 1.5×10^{8} km. The force exerted by the sun on the earth, in Newton, is

(a) 36×10^{21} (b) 27×10^{39} (c) zero (d) 18×10^{25}

Q.3 Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(a) $\frac{1}{2}\frac{\sqrt{GM}}{R}$ (b) $\sqrt{\frac{4Gm}{R}}$ (c) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (d) $\sqrt{\frac{Gm}{R}}$

Q.4 If the gravitational force between two objects were proportional to 1/R (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to

(a) R (b) R^0 (independent of R)

(c) $1/R^2$ (d) 1/ R

Q.5 Gravitational force is required for

(a) stirring of liquid (b) convection (c) conduction (d) radiation

Q.6 What will be the formula of mass of the earth in terms of g, R and G?

(a) $G\frac{R}{g}$ (b) $g\frac{R^2}{G}$ (c) $g^2\frac{R}{G}$ (d) $G\frac{g}{R}$

Q.7 The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations? (where G is the gravitational constant and R is the radius of the earth)

(a) $\rho = \frac{3g}{4\pi GR}$ (b) $\rho = \frac{3g}{4\pi GR^3}$ (c) $\rho = \frac{4\pi GR^2}{3G}$ (d) $\rho = \frac{4\pi GR^3}{3G}$

Q.8 The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be

(a) 2 R (b) 4 R (c) 1/4 R (d) 1/2 R

Q.9 Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

(a) g' = g/9 (b) g' = 27 g (c) g' = 9g (d) g' = 3g

Q.10 The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B?

(a) (2/9) m (b) 18 m (c) 6 m (d) (2/3) m

Q.11 A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be

(a) 36 N (b) 32 N (c) 144 N (d) 50 N

Q.12 The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times mass of mars. An object weighs 200 N on the surface of earth. Its weight on the surface of mars will be

(a) 20 N (b) 8 N (c) 80 N (d) 40 N

Q.13 A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball?

(a) it will fall down to the earth gradually (b) it will go very far in the space

(c) it will continue to move with the same speed along the original orbit of spacecraft

(d) it will move with the same speed, tangentially to the spacecraft.

Q.14 The largest and the shortest distances of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major–axis of the orbit drawn from the sun, is

(a) $\frac{r_1+r_2}{4}$ (b) $\frac{r_1+r_2}{r_1-r_2}$ (c) $\frac{2r_1r_1}{r_1+r_2}$ (d) $\frac{r_1+r_2}{3}$

Q.15 The period of revolution for planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?

(a) 4 (b) 5 (c) 2 (d) 3

Q.16 The distances of two planets from the sun are 10^3 and 10^{12} m respectively. The ratio of time periods of the planets is

(a) $\sqrt{10}$ (b) $10\sqrt{10}$ (c) 10 (d) $1/\sqrt{10}$

Q.17 A body of mass m is placed on earth surface which is taken from earth surface to a height of h = 3 R. Then change in gravitational potential energy is

(a) $\frac{mgR}{4}$ (b) $\frac{2}{3}mgR$ (c) $\frac{3}{4}mgR$ (d) $\frac{mgR}{2}$

Q.18 The escape velocity of a sphere of mass m is given by (G = universal gravitational constant; M_e = Mass of the earth and R_e = radius of the earth)

(a) $\sqrt{\frac{2GM_em}{R_e}}$ (b) $\sqrt{\frac{2GM_e}{R_e}}$ (c) $\sqrt{\frac{GM_e}{R_e}}$ (d) $\sqrt{\frac{2GM_e+R_e}{R_e}}$

Q.19 For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60^o with the vertical, then escape velocity will be

(a) 11 km/s (b) $11\sqrt{3}$ km/s (c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s

Q.20 The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45^o to the vertical, then the escape velocity is

(a) 11.2 × 2 km/s (b) 11.2 km/s (c) $11.2/\sqrt{2}$ km/s (d) $11.2\sqrt{2}$ km/s

Q.21 The escape velocity of body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(a) 22.4 km/s (b) 44.8 km/s (c) 5.6 km/s (d) 11.2 km/s

Q.22 For a planet having mass equal to mass of the earth, the radius is one fourth of radius of the earth. Then escape velocity for this planet will be

(a) 11.2 km/sec (b) 22.4 km/sec (c) 5.6 km/sec (d) 44.8 km/sec

Q.23 With what velocity should a particle be projected so that its height becomes equal to radius of earth?

(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{2}$ (c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$

Q.24 The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the earth. The value of f is

(a) 1/2 (b) $\sqrt{2}$ (c) $1/\sqrt{2}$ (d) 1/3

Q.25 A satellite A of mass m is at a distance of r from the surface of the earth. Another satellite B of mass 2m is at a distance of 2r from the earth's surface. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1 : 2\sqrt{2}$

Q.26 The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$ (c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$

Q.27 For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(a) 1/2 (b) $1/\sqrt{2}$ (c) 2 (d) $\sqrt{2}$

Q.28 The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy?

(a) $(3/4)\,mv^2$ (b) $(1/2)\,mv^2$ (c) mv^2 (d) $-(1/2)\,mv^2$

Q.29 Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

(a) the potential energies of earth and satellite in the two cases are equal

(b) S_1 and S_2 are moving with the same speed

(c) the kinetic energies of the two satellites are equal

(d) the time period of S_1 is four times that of S_2

Q.30 A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct?

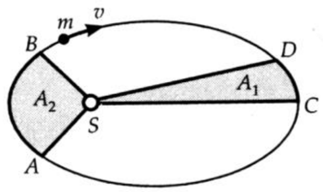
(a) T is conserved (b) V is always positive (c) E is always negative

(d) L is conserved but direction of vector L changes continuously

Q.31 A satellite in force free space sweeps stationery interplanetary dust at a rate of $dM/dt = \alpha v$, where M is mass and v is the speed of satellite and α is a constant. The acceleration of satellite is

(a) $\frac{-\alpha v^2}{2M}$ (b) $-\alpha v^2$ (c) $\frac{-2\alpha v^2}{M}$ (d) $\frac{-\alpha v^2}{M}$

Q.32 The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B, then



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$

(c) $t_1 = t_2$ (d) $t_1 > t_2$

Answers

1.	b	2.	a	3.	a	4.	b	5.	b
6.	b	7.	a	8.	d	9.	d	10.	b
11.	b	12.	c	13.	c	14.	c	15.	a
16.	b	17.	c	18.	b	19.	a	20.	b
21.	a	22.	b	23.	a	24.	c	25.	d
26.	a	27.	a	28.	d	29.	b	30.	c
31.	d								

Problems

Q.1 Draw graphs showing the variation of acceleration due to gravity with (i) height above the earth's surface and (ii) depth below the earth's surface.

Q.2 Are we living at the bottom of a gravitational well? Give reason.

Q.3 Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very great height. Why?

Q.4 A person sitting in a satellite feels weightlessness but a person standing on moon has weight though moon is also a satellite of the earth. Given reason.

Q.5 The sun attracts all bodies on the earth. At midnight, when the sun is directly below, it pulls on a body in the same direction as the pull of the earth on that body; at noon, when the sun is directly above, it pulls on a body in a direction opposite to the pull of the earth. Then will the weight of the body be greater at mid–night than at noon?

Q.6 Suppose the gravitational force varies inversely as the nth power of distance. Then, find the expression for the time period of a planet in a circular orbit of radius r around the sun.

Q.7 A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. What is the value of T_2/T_1?

Q.8 A geo–stationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, what will be the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400 km)?

Q.9 Find the period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α.

Q.10 Two bodies of masses m_1 and m_2 are initially at rest placed infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Show that their relative velocity of approach at separation r between them is

$$v=\sqrt{\frac{2G(m_1+m_2)}{r}}$$

Q.11 A spherical cavity is made inside a sphere of density ρ. If its centre lies at a distance ℓ from the centre of the sphere, calculate the gravitational field strength of the field inside the cavity.

Q.12 The distance between the centres of two stars is 10a. The masses of these stars are M and 16 M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

Q.13 An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. (i) Determine the height of the satellite above the earth's surface. (ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely on the earth, find the speed with which it hits the surface of the earth. Take g = 9.8 ms^{-2}, radius of the earth = 6400 km.

Q.14 A particle is projected upward from the surface of the earth (radius R) with a K.E. equal to half the minimum value needed for it to escape. To which height does it rise above the surface of the earth?

Answer

6.	$T \propto r^{\left(\frac{n+1}{2}\right)}$	7.	2	8.	T_S = 2hr
9.	$T = 2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$	11.	$\dfrac{4\pi}{3}G\ell\rho$	12.	$\dfrac{3}{2}\sqrt{\dfrac{5Gm}{a}}$
13.	(i) 6.4×10^6m, (ii) 7.92 km/s	14.	R		

NCERT Exercise

Q.1 Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means.

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the Earth due to the Sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of Sun. Why?

Q.2 Choose the correct alternative:

(i) Acceleration due to gravity increases/ decreases with increasing altitude.

(ii) Acceleration due to gravity increases/decreases with increasing depth (assume the Earth to be a sphere of uniform density)

(iii) The effect of rotation on the effective value of acceleration due to gravity is greatest at the equator/poles.

(iv) Acceleration due to gravity is independent of mass of the Earth/mass of the body.

(v) The formula – GMm $(1/r_2 - 1/r_1)$ is more/less accurate than the formula mg $(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the Earth.

Q.3 Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Q.4 To, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one thousandth that of the sun.

Q.5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to compete one revolution? Take the diameter of the milky way to be 10^5 ly.

Q.6 Choose the correct alternatives:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

Q.7 Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched? Explain your answer.

Q.8 A comet orbits the sun in a highly elliptical obit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the sun.

Q.9 Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientation problem.

Q.10 The gravitation intensity at the centre C of the drumhead defined by a hemispherical shell has the direction indicated by the arrow.

(i) a, (ii) b, (iii) c, (iv) zero

Q.11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) d.

Q.12 A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero? Mass of sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. Orbital radius = 1.5×10^{11} m.

Q.13 A Saturn year is 29.5 times the earth years. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun.

Q.14 A body weights 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Q.15 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Q.17 A rocket is fired vertically with a speed of 5 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of earth = 6.0×10^{24} kg, mean radius of the earth = 6.4×10^6 m, G = 6.67×10^{-11} $Nm^2\ kg^{-2}$.

Q.18 The escape velocity of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Q.19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence? Mass of the satellite =200 kg, mass of earth = 6.0×10^{24} kg, radius of earth =6.4×10^6 m, G = 6.67×10^{-11} $Nm^2\ kg^{-2}$.

Q.20 Two stars each of 1 solar mass (= 2×10^{30} kg) are approaching each other for a head–on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until collide.

Q.21 Two heavy spheres each of mass 100 kg and radius 0.1 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres?

Q.22 A geostationary satellite orbits the earth at a height of nearly 36, 000 km from the surface of the earth. What is the potential due to earth's gravity at the site of the satellite? Mass of the earth = 6×10^{24} kg and radius = 6400 km.

Q.23 A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev/s. Will an object placed on its equator remain stuck to its surface due to gravity? Mass of the sun = 2×10^{30} kg.

Q.24 A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system? Mass of the spaceship 1000 kg, mass of the sun = 2×10^{30} kg, mass of Mars = 6.4×10^{23} kg, radius of Mars 3395 km, radius of the orbit of Mars = 2.28×10^{8} KM?

Q.25 A rocket is fired vertically from the surface of Mars with a speed of 2 kms^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it? Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km

Answers

1. (a) No, (b) Yes, (c) $T_m/T_s = 61.5 \times 10^6$
2. (i) decreases, (ii) decreases, (iii) equator, (iv) mass of the body, (v) more
3. 0.63 AU   5. 3.54×10^8 years   6. (a) kinetic energy, (b) less
7. (a) independent, (b) yes, (c) no, (d) yes
8. (a) variable, (b) variable, (c) constant, (d) variable, (e) variable, (f) constant
9. (b), (c) and (d)   10. (iii)   11. (ii)
12. 2.59×10^8 m   13. 14.32×10^8 km   14. 28 N
15. 28 N   16. 125 N
17. 1.6×10^6 m   18. 31.68 kms^{-1}
19. 5.89×10^9 J   20. 2.6×10^6 ms^{-1}
21. $V = -2.668 \times 10^{-8}$ J kg^{-1}, E = 0   22. -9.43×10^6 J kg^{-1} 23. Yes
24. 2.9×10^{11} J   25. 1655 km

MECHANICAL PROPERTIES OF SOLIDS

Deforming Forces

If a force is applied on a body which is neither free to move nor free to rotate, the molecules of the body are forced to undergo a change in their relative positions.

A force which changes the size or shape of a body is called a deforming force.

Elasticity

If a body regains its original size and shape after the removal of deforming force, it is said to be elastic body and this property is called elasticity.

Perfectly Elastic Body

If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body. The nearest approach to a perfectly elastic body is quartz and phosphor bronze.

Plasticity

If a body does not regain its original size and shape even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity. For example, if we stretch a piece of chewing–gum and release it, it will not regain its original size and shape.

Perfectly Plastic Body

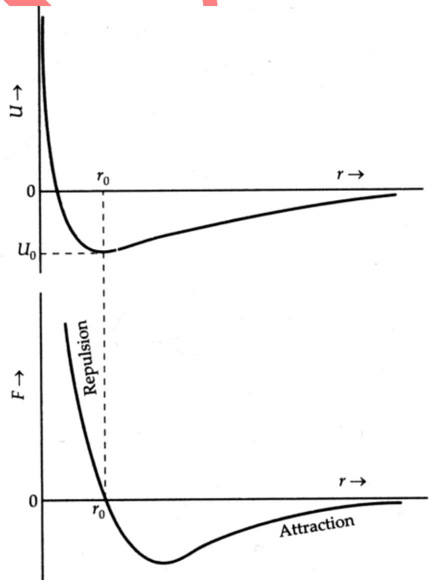
If a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be a perfectly plastic body. Putty and paraffin wax are nearly perfectly plastic bodies.

NOTE

- ♦ No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.

Elastic Behaviour in Terms of Interatomic forces

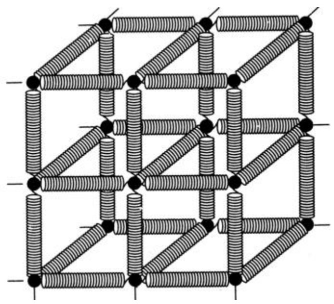
When the interatomic separation r is large, the potential energy of the atoms is negative and the interatomic force is *attractive.* At some particular separation r_0, the potential energy becomes minimum and the interatomic force becomes zero. The separation r_0 is called **normal or equilibrium separation.**



When separation reduces below r_0, the potential energy increases steeply and the interatomic force becomes *repulsive.*

Normally, the atoms occupy the positions ($r = r_0$) of minimum potential energy called the positions of stable equilibrium. When a tensile or compressive force is applied on a body, its atoms are pulled apart or pushed closer together to a distance r, greater than or smaller than r_0. When the deforming force is removed, the interatomic forces of attraction/ repulsion restore the atoms to their equilibrium positions. The body regains its original size and shape. The stronger the interatomic forces, the smaller will be the displacements of atoms from the equilibrium positions and hence greater is the elasticity (or modulus of elasticity) of the material.

Elastic behaviour on the basis of spring–ball model of a solid



The atoms in a solid may be regarded as mass points or small balls connected in three dimensional space through springs. The springs represent the interatomic forces. This is called spring ball model of a solid, as shown in figure.

Normally, the balls occupy the positions of minimum potential energy or zero interatomic force. When any ball is displaced from its equilibrium position, the various springs connected to it exert a resultant force on this ball. This force tends to bring the ball to its equilibrium position. This explains the elastic behaviour of solid in terms of microscopic nature of the solid.

Stress

If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state. *The internal restoring force set up per unit area of cross–section of the deformed body is called stress.* As the restoring force is equal and opposite to the external deforming force, therefore

$$Stress = \frac{Applied\ \ force}{Area} = \frac{F}{A}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. The dimensional formula of stress is $[ML^{-1}T^{-2}]$

It is a tensor.

Types of Stress

(i) **Normal Stress:** It is the restoring force set up perpendicular to cross sectional area. It changes the size of body. It is of two types:

(a) **Tensile Stress:** It is the restoring force set up per until cross–sectional area of a body when the length of the body increases in the direction of the deforming force. It is also known as *longitudinal stress.*

(b) **Compressional Stress:** It is the restoring force set up per unit cross–sectional area of a body when its lengths decreases under a deforming force.

(ii) **Hydrostatic Stress:** If a body is subjected to a uniform force from all sides, then the corresponding stress is called *hydrostatic stress.* Actually it is normal stress.

(iii) **Tangential or Shearing Stress:** When a deforming force acts tangentially to the surface of a body. The tangential force applied per unit area is equal to the tangential stress. It changes the shape of body.

Strain

When a deforming force acts on a body, the body undergoes a change in its shape and size. *The ratio of the change in any dimension produced in the body to the original dimension is called strain.*

$$Strain = \frac{Change\ in\ dimension}{Original\ dimension}$$

As strain is the ratio of two like quantities, it has no units and dimensions.

Types of Strain

(i) **Longitudinal Strain:** *It is defined as the increase in length per unit original length,* when the body is deformed by external forces.

$$Longitudinal\ \ strain = \frac{Change\ in\ length}{Original\ length} = \frac{\Delta l}{l}$$

(ii) **Volumetric Strain :** *It is defined as the change in volume per unit original volume,* when the body is deformed by external forces.

$$Longitudinal\ \ strain\ = \frac{Change\ in\ volume}{Original\ volume} = \frac{\Delta V}{V}$$

(iii) **Shear Strain:** *It is defined as the angle θ (in radian), through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.*

Shear strain = θ = tan θ

$$= \frac{Relative\ displacement\ between\ 2\ \ parallel\ \ planes}{Distance\ between\ \ parallel\ \ planes}$$

Elastic Limit

If a sufficiently large force is suspended from the wire, it is found that the wire does not regain its original length after the load is removed. *The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.* If the deforming force exceeds the elastic limit, the body acquires a *permanent set or deformation and is said to be overstrained.*

Hook's Law and Modulus of Elasticity

Hooke's law states that the extension produced in a wire is directly proportional to the load applied. Modified Hooke's law to the more general form as follows:

Within the elastic limit, the stress is directly proportional to strain. Thus within the elastic limit,

Stress $\propto$ Strain

or Stress = constant × strain or $\frac{Stress}{Strain} = constant$

The constant of proportionality is called modulus of elasticity or coefficient of elasticity of the material. Its value depends on the nature of the material of the body and the manner in which it is deformed.

NOTE:

- Like Boyle's law, Hooke's law is one of the earliest quantitative relationship in science.
- Hooke's law is valid only in the linear position of the stress–strain curve. The law is not valid for large values of strains.
- Stress is not a vector quantity since, unlike a force, the stress cannot be assigned a specific direction.
- When a wire, suspended from a ceiling, is stretched by a weight (F) suspended from its lower end, the ceiling exerts a force on the wire equal and opposite to the weight F. But the tension at any cross–section A of the wire is just F and not 2F. Hence the tensile stress which is equal to the tension per unit area is equal to F/A.

Modulus of Elasticity

The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain, within the elastic limit.

$$Modulus\ of\ elasticity,\ E = \frac{Stress}{Strain}$$

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$

Different types of moduli of elasticity

Corresponding to the three types of strain, we have three important moduli of elasticity:

(i) Young's modulus (Y), i.e., the modulus of elasticity of length.

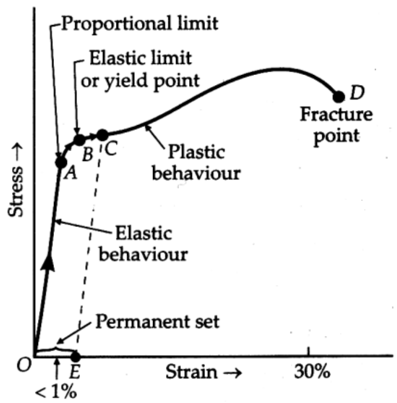
(ii) Bulk modulus (k), i.e., the modulus of elasticity of volume.

(iii) Modulus of rigidity or shear modulus (η), i.e., modulus of elasticity of shape.

Stress–Strain Curve for a Metallic Wire

Shows a stress–strain curve for a metal wire which is gradually being loaded.

(i) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the ***proportional limit***. In this region, the wire is perfectly elastic.



(ii) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called ***elastic region*** and the point B is called ***elastic limit or yield point.*** The stress corresponding to the yield point is called ***yield strength (S_y)***.

Upto point B, the elastic forces of the material are ***conservative*** i.e., when the material returns to its original size, the work done in producing the deformation is completely recovered.

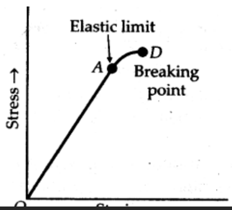
(iii) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CE. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a ***permanent set.*** The fact that the stress–strain curve is not retraced on reversing the strain is called ***elastic hysteresis.***

(iv) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the ***fracture point.*** In the region between B and D, the length of wire goes on increasing even without any addition of load. This region is called *plastic region and the material* is said to undergo ***plastic flow or plastic deformation.*** The stress corresponding to the breaking point is called ***ultimate strength*** or ***tensile strength*** of the material.

Classification of Materials on the Basis of Stress–Strain Curve

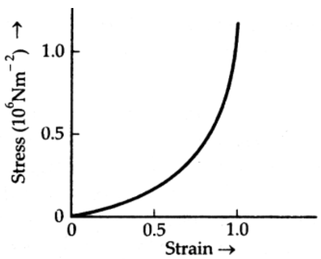
(i) **Ductile Materials:** *The materials which have large plastic range of extension are called ductile materials.* Their fracture point is widely separated from the elastic limit. Such materials undergo an irreversible increase in length before snapping. So they can be drawn into thin wires. For example, copper, silver, iron, aluminium, etc.

(ii) **Brittle materials:** *The materials which have very small range of plastic extension are called brittle materials.* Such materials break as soon as the stress is increased beyond the elastic limit. Their breaking point lies just close to their elastic limit, as shown in figure. For example, cast iron, glass, ceramics, etc.



Elastomers

The materials which can be elasticity stretched to large value of strain are called elastomers. For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic region in such cases is very large, but the materials does not obey Hooke's law. In our body, the elastic tissue is aorta.



Subjective Assignment – I

Q.1 The length of a suspended wire increases by 10^{-4} of its original length when a stress of 10^7 Nm^{-2} is applied on it. Calculate the Young's modulus of the material of the wire.

Q.2 A uniform wire of steel of length 2.5 m and density 8.0 gcm^{-3} weighs 50 g. When stretched by a force of 10 kgf, the length increases by 2 mm. Calculate Young's modulus of steel.

Q.3 A structural steel row has a radius of 10mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus, Y, of the structural steel is 2.0×10^{11} Nm^{-2}.

Q.4 What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.

Q.5 The breaking stress for a metal is 7.8×10^9 Nm^{-2}. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of the metal = 7.8×10^3 kg m^{-3}. Take g = 10 N kg^{-1}.

Q.6 A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg m^{-3} and Young's modulus for the rubber is 5×10^6 Nm^{-2}. Take g = 10 N kg^{-1}.

Q.7 A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is 50×10^6 Nm^{-2}. Estimate the largest mass that can be hung from it without breaking it. Take g = 10 N kg^{-1}.

Q.8 A composite wire of uniform diameter 3.0 mm consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, given that the Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.

Q.9 The maximum stress that can be applied to the material of a wire used to suspend an elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, what is the minimum diameter of the wire?

Q.10 A mass of 100 gram is attached to the end of a rubber string 49 cm long and having an area of cross–section 20 mm^2. The string is whirled round, horizontally at a constant speed of 40 rps in a circle of radius 51 cm. Find Young's modulus of rubber.

Q.11 A uniform heavy rod of weight W, cross–sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.

Q.12 A steel wire of uniform cross–section of 1 mm^2 is heated to 70^o C and stretched by typing its two ends rigidly. Calculate the change in the tension of the wire when the temperature falls from 70^o C to 35^{o}C. Coefficient of linear expansion of steel is 1.1×10^{-5} ^{o}C^{-1} and the Young's modulus is 2.0×10^{11} Nm^{-2}.

Q.13 A steel wire of length 5.0 m and cross–section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.0 m and cross–section 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?

Q.14 A wire increases by 10^{-3} of its length when a stress of 1×10^8 Nm^{-2} is applied to it. What is the Young's modulus of the material of the wire?

Q.15 What force is required to stretch a steel wire 1 cm^2 in cross–section to double its length? Given Y = 2×10^{11} Nm^{-2}.

Q.16 Find the stress to be applied to a steel wire to stretch it by 0.025% of its original length. Y for steel is
9×10^{-10} Nm^{-2}.

Q.17 A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg–wt. Find the increase in its length, if the Young's modulus of steel wire is $2. \times 10^{-12}$ dyne cm^{-2}.

Q.18 A wire elongates by 9 mm when a load of 10 kg is suspended from it. What is the elongation when its radius is doubled, if all other quantities are same as before?

Q.19. The breaking stress of aluminium is 7.5×10^7 Nm^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7×10^3 kg m^{-3}.

Q.20 A stress of 1kg mm^{-2} is applied to a wire of which Young's modulus is 10^{11} nm^{-2}. Find the percentage increase in length.

Q.21 Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is
1 cm, find by how much is each wire elongated? Given Y for steel = 20×10^{11} dyne cm^{-2} and Y for copper = 12×10^{11} dyne cm^{-2}.

Q.22 Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4 : 5 and their radii are in the ratio 4 : 3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B.

Q.23 Two wires of equal cross–section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires is found to be equal. Given Young's moduli of steel and copper are 2.0×10^{11} Nm^{-2} and 1.1×10^{11} Nm^{-2}. Find the ratio between the lengths of steel and copper wires.

Q.24 A lift is tied with thick iron wires and its mass is 1000 kg. If the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is 1.4×10^{8} Nm^{-2}, find the minimum diameter of the wire.

Q.25 The length of a metal wire is l_1 when the tension in it is T_1 and l_2 when the tension in it is T_2. Find the original length of the wire.

Q.26 A metal bar of length l and area of cross–section A is rigidly clamped between two walls. The Young's modulus of the material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature is increased by ΔT. Find the force exerted at the ends of the bar.

Q.27 Two wires made of the same material are subjected to forces in the ratio of 1 : 4. Their lengths are in the ratio 8 : 1 and diameter in the ratio 2 : 1. Find the ratio of their extensions.

Answers

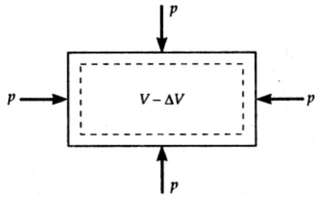
1.	10^{11} Nm^{-2}	2.	49×10^{11} dyne cm^{-2}		
3.	(i) 3.18×10^{8} Nm^{-2}, (ii) 1.59 mm, (iii) 0.16%			4.	0.16%
5.	10^{5} m	6.	0.15 m	7.	392.5 kg
8.	176.8 N	9.	1.0284×10^{-2} m	10.	3.95×10^{9} Nm^{-2}
11.	$\frac{Wl}{2Ay}$	12.	77.0 N	13.	2.22
14.	10^{11} Nm^{-2}	15.	2×10^{7} N	16.	2.25×10^{7} Nm^{-2}
17.	0.0041 cm	18.	2.25 mm	19.	2.83×10^{3} kg m^{-3}
20.	0.0098 %	21.	0.375 cm and 0.625 cm	22.	2.22 mm
23.	20 : 11	24.	0.01 m	25.	$\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$
26.	YA αΔT	27.	1 : 2		

Bulk Modulus of Elasticity

Within the elastic limit, the ratio of normal stress to volumetric strain is called bulk modulus of elasticity.

Consider a body of volume V and surface area A. Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by ΔV as shown in figure. Then bulk modulus of elasticity is given by

$$B = \frac{Normal\ stress}{Volumetric\ strain} = \frac{F/A}{\Delta V/V}$$



or $$B = -\frac{F}{A}.\frac{V}{\Delta V} = -\frac{pV}{\Delta V}$$

where p(= F/A) is the normal pressure. Negative sign shows that the volume decreases with increase in stress.

Units and dimensions of B. The SI unit of bulk modulus is Nm^{-2} or Pascal (Pa) and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$

Compressibility. *The reciprocal of the bulk modulus of a material is called its compressibility.*

Compressibility = $\frac{1}{B}$

SI unit of compressibility = $N^{-1}m^{2}$

CGS unit of compressibility = $dyne^{-1}$ cm^{2}

The dimensional formula of compressibility is $[M^{-1}LT^{2}]$

Subjective Assignment – II

Q.1 The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10%, keeping temperature constant. Find the bulk modulus of the medium.

Q.2 The average depth of Indian ocean is about 3000 m. Calculate the fractional compression $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 Nm^{-2}.

Q.3 A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk modulus of the material of the sphere. Density of sea water may be taken as 1.0×10^3 kg m^{-3}.

Q.4 If the normal density of sea water is 1.00 g cm^{-3}, what will be its density at a depth of 3 km? Given compressibility of water = 0.0005 per atmosphere, 1 atomsphere pressure = 10^6 dyne cm^{-2}, g = 980 cm s^{-2}.

Q.5 A solid cube is subjected to a pressure of 5×10^5 Nm^{-2}. Each side of the cube is shortened by 1%. Find volumetric strain and bulk modulus of elasticity of the cube.

Q.6 Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 50^o to 70^{o}C. Coefficient of linear expansion of copper = 8.0×10^{-6} ^{o}C^{-1} and bulk modulus of elasticity = 3.6×10^{11} Nm^{-2}.

Q.7 A solid sphere of radius 10 cm is subjected to a uniform pressure = 5×10^8 Nm^{-2}. Determine the consequent change in volume. Bulk modulus of the material of sphere is equal to 3.14×10^{11} Nm^{-2}.

Q.8 Find the change in volume which 1 m^3 of water will undergo when taken from the surface to the bottom of a lake 100 m deep. Given volume elasticity of water is 22,000 atmosphere.

Q.9 A solid ball 300 cm in diameter is submerged in a lake at such a depth that the pressure exerted by water is 1.00 kgf cm^{-2}. Find the change in volume of the ball at this depth. B for material of the ball = 1.00×10^{13} dyne cm^{-2}.

Q.10 A spherical ball contracts in volume by 0.0098% when subjected to a pressure of 100 atm. Calculate its bulk modulus. Given 1 atm = 1.01×10^5 Nm^{-2}

Q.11 What increase in pressure will be needed to decrease the volume of 1.0 m^3 of water by 10 c.c.? The bulk modulus of water is 0.21×10^{10} Nm^{-2}.

Q.12 Determine the fractional change in volume as the pressure of the atmosphere (1.0×10^5 Pa) around a metal block is reduced to zero by placing the block in vacuum. The bulk modulus for the block is
1.25×10^{11} Nm^{-2}.

Q.13 Find the density of the metal under a pressure of 20,000 N cm^{-2}. Given density of the metal = 11 g cm^{-3}, bulk modulus of the metal = 8×10^9 Nm^{-2}.

Q.14 The compressibility of water is 4×10^{-5} per unit atmospheric pressure. What will be the decrease in volume of 100 cm^3 of water under pressure of 100 atmosphere?

Q.15 On taking a solid ball of rubber from the surface to the bottom of a lake of 180 m depth, the reduction of the volume of the ball is 0.1%. The density of water of the lake is 1.0×10^3 kg m^{-3}. Determine the value of the bulk modulus of elasticity of rubber. Take g = 10 ms^{-2}.

Q.16 A uniform pressure P is exerted on all sides of a solid cube at temperature t^{o}C. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied, if the bulk modulus and coefficient of volume expansion of the material are B and γ respectively?

Q.17 A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere.

Answers

1.	1.55×10^5 Pa	2.	3×10^7 Nm^{-2}, 1.36%	3.	9.8×10^{10} Nm^{-2}
4.	1.0149 g cm^{-3}	5.	0.03, 1.67×10^7 Nm^{-2}	6.	1.728×10^8 Nm^{-2}
7.	6.67×10^{-6} m^3	8.	4.4×10^{-4} m^3	9.	1.385 cm^3
10.	1.033×10^{11} Nm^{-2}	11.	2.1×10^4 Nm^{-2}	12.	8×10^{-7}
13.	11.28 g cm^{-3}	14.	0.4 cm^3	15.	1.8×10^9 Nm^{-2}
16.	$\frac{p}{\gamma B}$	17.	$\frac{\Delta R}{R} = \frac{Mg}{3AB}$		

Modulus of Rigidity or Shear Modulus

Within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.

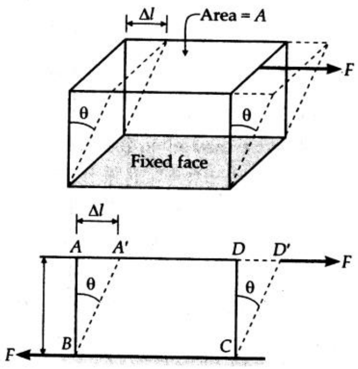
Consider a rectangular block whose lower face is fixed and a tangential force F is applied over its upper face of area A. An equal and opposite force F comes into play on its lower fixed face. The two equal and opposite forces form a couple which exerts a torque. As the lower face of the block is fixed, the coupled shears the block into a parallelopied by displacing its upper face through distance AA' = Δl. Let AB = DC = l and $\angle$ABA' = θ.

Tangential stress = $\frac{F}{A}$

Shear strain = $\theta \approx \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}$

The modulus of rigidity is given by

$$\eta = \frac{Tangential\ stress}{Shear\ strain} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$



Units and dimensions of η : The SI unit of modulus of rigidity is Nm^{-2} and its CGS unit is dyne cm^{-2}. Its dimensional formula is [ML^{-1} T^{-2}]

NOTE

- ♦ Elastic deformations in all bodies become plastic deformations with time.
- ♦ As only solids have length and shape, Young's modulus and shear modulus are relevant only for solids.
- ♦ As solids, liquids and gases all have volume elasticity, bulk modulus is relevant for all three sates of matter.
- ♦ Metals have large values of Young's modulus than alloys and elastomers. A material with large Y requires a large force to produce small changes in lengths.
- ♦ Elastic has a different meaning in physics than that in daily life. In daily life, a material which stretches more is said to be more elastic, but it is a misnomer. In physics, a material which stretches to a lesser extent for a given load is considered to be more elastic.

Subjective Assignment – III

Q.1 A cube of aluminium of each side 4 cm is subjected to a tangential (shearing) force. The top face of the cube is sheared through 0.012 cm with respect to the bottom face. Find (i) shearing strain (ii) shearing stress and shearing force. Given $\eta = 2.08 \times 10^{11}$ dyne cm^{-2}.

Q.2 A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced, if the shear modulus of lead is 5.6×10^9 Pa?

Q.3 A rubber block 1 cm × 3 cm × 10 cm is clamped at one end with its 10 cm side vertical. A horizontal force of 30 N is applied to the free surface. What is the horizontal displacement of the top face? Modulus of rigidity of rubber = 1.4×10^5 Nm^{-2}.

Q.4 A 60 kg motor rests on four cylindrical rubber blocks. Each cylinder has a height of 3 cm and a cross–sectional area of 15 cm^2. The shear modulus for this rubber is 2×10^6 Nm^{-2}. If a sideways force of 300 N is applied to the motor, how far will it move sideways?

Q.5 A metallic cube whose each side is 10 cm is subjected to a shearing force of 100 kg f. The top face is displaced through 0.25 cm with respect to the bottom. Calculate the shearing stress, strain and shear modulus.

Q.6 An Indian rubber cube of side 7 cm has one side fixed, while a tangential force equal to the weight of 200 kilogram is applied to the opposite face. Find the shearing strain produced and distance through which the strained side moves. Modulus of rigidity for rubber is 2×10^7 dyne cm^{-2}.

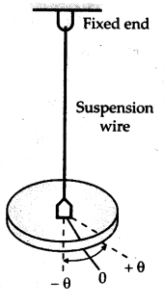
Q.7 A metal cube of side 10 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm with respect to its bottom.

Q.8 Two parallel and opposite forces, each 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block 25 cm on a side. Find the angle of shear and the displacement of the upper surface relative to the lower surface. The shear modulus for the metal is 8×10^{10} Nm^{-2}.

Answers

1. (i) 0.003 rad, (ii) 6.24×10^8 dyne cm^{-2}, 9.984×10^9 dyne
2. 1.6×10^{-4} m
3. 7.14 cm
4. 0.075 cm
5. 9.8×10^4 Nm^{-2}, 0.025 rad, 3.92×10^6 Nm^{-2}
6. 0.2 radian, 1.4 cm
7. 2×10^6 Nm^{-2}
8. 8.0×10^{-7} rad, 2.0×10^{-7} m

Some Other Elastic Effects



Elastic After Effect : The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. *The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.*

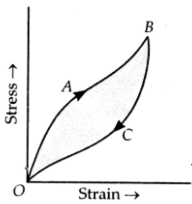
Elastic fatigue : If we set the wire into torsional vibrations, it will continue vibrating for a long times before its vibrations die out. If it is again made to vibrate, its vibrations will die out in a lesser time. Due to continuous alternating strains, the wire is said to have been *tired or fatigued.*

Elastic fatigue is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

A hard wire can be broken by bending it repeatedly in opposite directions, as it loses strength due to elastic fatigue. For the same reason, the railway bridges are declared unsafe after a reasonably good period to avoid the risk of a mishap.

Elastic Hysteresis

Figure shows the stress–strain curve for a rubber sample when loaded and then unloaded. For increasing load, the stress–strain curve is OAB and for



decreasing load, the curve is BCO. *The fact that the stress–strain curve is not retraced on reversing the strain is known as elastic hysteresis.*

The area under the curve OAB represents the work done per unit volume in stretching the rubber. The area under BCO represents the energy given up by rubber on unloading. So the shaded area of the hysteresis loop represents the energy lost as heat during the loading unloading cycle.

Applications of elastic hysteresis

(i) Car tyres are made with synthetic rubbers having small–area hysteresis loops because a car tyre of such a rubber will not get excessively heated during the journey.

(ii) A padding of vulcanized rubber having large area hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibrations, it dissipates a large amount of vibration energy.

Applications of Elasticity

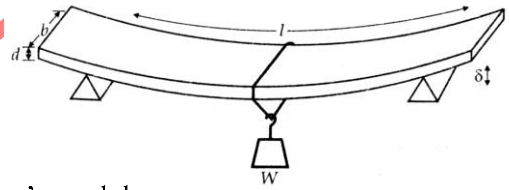
Any metallic part of a machinery is never subject to a stress beyond the elastic limit. This is because a stress beyond elastic limit will permanently deform that metallic part.

The thickness of metallic ropes used in cranes to lift heavy loads is decided from the knowledge of the elastic limit of the material and the factor of safety.

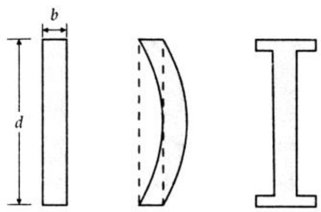
A single wire of this much radius would be a rigid rod. For the ease in manufacture and to impart flexibility and strength to the rope, it is always made of a large number of thin wires braided together.

The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a rectangular bar of length *l*, breadth b and thickness d supported at both ends, as shown in figure. When a load W is suspended at its middle, the bar gets depressed by an amount given by

$$\delta = \frac{Wl^3}{4Y\,bd^3}$$



Bending can be reduced by using a material with a large Young's modulus Y. As δ is proportional to d^{-3} and only to b^{-1}, *so depression more effectively reduced by increasing the depth d rather than the breadth b.* But a deep bar has a tendency to bend under the weight of a moving traffic, as shown in figure. This bending is called ***buckling.*** Hence a better choice is to have a bar of I–shaped cross–section, as shown in figure. This section provides a large load bearing surface and enough depth to prevent bending. Also, this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.



The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rock on the top should be less than the critical shear stress at which the rock begins to flow. Suppose the height of the mountain is h and the density of the mountain ρ. Hence stress exerted by mountain at the base = hρg. The material at the base experiences this force per unit area in the vertical direction, but sides of the mountain are free. Hence there is a tangential shear of the order of hρg. The elastic limit for a typical rock is about 3×10^8 Nm^{-2} and its density is 3×10^3 kg m^{-3}. Hence

$$h_{max}\,\rho g = 3 \times 10^8 \quad \text{or} \quad h_{max} = \frac{3\times10^8}{\rho g} = \frac{3\times10^8}{3\times10^3\times9.8} \simeq 10,0090 \text{ m} = 10 \text{ km}$$

This is nearly the height of the Mount Everest. A height greater than this will not be able to withstand the shearing stress due to the weight of the mountain.

A hollow shaft is stronger than a solid shaft made of equal quantity of same material : The torque required to produce until twist in a solid shaft of radius r, length l and made of material of modulus of rigidity η is given by

$$\tau = \frac{\pi \eta r^4}{2l}$$

The torque required to produce a unit twist in a hollow shaft of internal & external radii r_1 and r_2 is given by

$$\tau' = \frac{\pi\eta(r_2^4 - r_1^4)}{2l}$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4} \qquad \text{.... (i)}$$

If both are made up of same mass and same material.

$\therefore$ m = m' or $\pi r^2 l\rho = \pi(r_2^2 - r_1^2)l\rho$ or $r^2 = r_2^2 - r_1^2$ .... (ii)

$\therefore$ $r^2 < r_2^2 + r_1^2$ .... (iii)

Using (ii) and (iii) in (i), we get $\tau' > \tau$

Thus torque required to twist hollow cylinder through a certain angle is greater than the torque necessary to twist a solid cylinder of same mass, length and material through the same angle. Hence a *hollow shaft is stronger than a solid shaft*. For this reason, elastic poles are given hollow structures.

Elastic Potential Energy of a Stretched Wire

When a wire is stretched, interatomic forces come into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

Expression for Elastic Potential Energy

Suppose a force F applied on a wire of length l increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl, the internal force increases from zero to F (= applied force).

$\therefore$ Average internal force for an increase in length Δl of wire

$$= \frac{0+F}{2} = \frac{F}{2}$$

Work done on the wire is

W = Average force × increase in length = $\frac{F}{2} \times \Delta l$

This work done is stored as elastic potential energy U in the wire.

$\therefore$ $U = \frac{1}{2} F \times \Delta l$ $= \frac{1}{2}$ stretching force × increase in length

Let A be the area of cross–section of the wire. Then

$U = \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{l} \times Al$ $= \frac{1}{2}$ Stress × Strain × Volume of Wire

Elastic potential energy per unit volume of the wire or elastic energy density is

$u = \frac{U}{Volume}$ or $u = \frac{1}{2}\, stress \times strain$

But stress = Young's modulus × strain

$\therefore \quad u = \frac{1}{2}$ Young's modulus × strain^2

Subjective Assignment – IV

Q.1 A steel wire of 4.0 m is stretched through 2.0 mm. The cross–sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} Nm^{-2}, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.

Q.2 Calculate the increase in energy of a brass bar of length 0.2 m and cross–sectional area 1 cm^2 when compressed with a load of 5 kg weight along its length. Young's modulus of brass = 1.0×10^{11} Nm^{-2} and g = 9.8 ms^{-2}.

Q.3 When the load on a wire is increased from 3 kg wt to 5 kg wt, the elongation increases from 0.61 mm to 1.02 mm. How much work is done during the extension of the wire?

Q.4 A 40 kg boy whose leg bones are 4 cm^2 in area and 50 cm long falls through a height of 50 without breaking his leg bones. If the bones can stand a stress of 0.9×10^8 Nm^{-2}, calculate the Young's modulus for the material of the bone. Take g = 10 ms^{-2}.

Q.5 A steel wire of length 2.0 m is stretched through 2.0 mm. The cross–sectional area of the wire is 4.0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is 2.0×10^{11} Nm^{-2}.

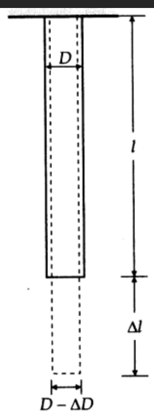
Q.6 If the Young's modulus of steel is 2×10^{11} Nm^{-2}, calculate the work done in stretching a steel wire 100 cm in length and of cross–sectional area 0.03 cm^2 when a load of 20 kg is slowly applied without the elastic limit being reached.

Q.7 The limiting stress of a typical human bone is 0.9×10^8 Nm^{-2}, while Young's molecules is 1.4×10^{10} Nm^{-2}. How much energy can be absorbed by two legs (without breaking) if each has a typical length of 50 cm and an average cross–sectional area of 5 cm^2?

Answers

1.	(i) 2.5×10^4 Jm^{-3}, (ii) 0.2 J	2.	2.4×10^{-5} J	3.	16.023×10^{-3} J
4.	2.025×10^9 Nm^{-2}	5.	0.8 J	6.	0.032 J
7.	144.7 J				

Poisson's Ratio



When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced in the perpendicular direction is called lateral strain.

Within the elastic limit, the ratio of laterial strain to the longitudinal strain is called Poisson's ratio.

Suppose length of the loaded wire increases from l to $l + \Delta l$ and its diameter decreases from D to D – ΔD.

Longitudinal strain = $\frac{\Delta l}{l}$

Lateral strain = $-\frac{\Delta D}{D}$

Poisson's ratio is

$$\sigma = \frac{Lateral\ strain}{Longitudinal\ strain} = \frac{-\Delta D / D}{\Delta l / l} \quad \text{or} \quad \sigma = -\frac{l}{D} \cdot \frac{\Delta D}{\Delta l}$$

The negative sign indicates that longitudinal and lateral strains are in opposite sense. As the Poisson's ratio is the ratio of two strains, it has no units and dimensions.

Subjective Assignment – V

Q.1 Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.

Q.2 One end of a nylon of length 4.5 m and diameter 6 mm is fixed to a free limb. A monkey weighing 100 N jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in diameter. Given Young's modulus of nylon = 4.8×10^{11} Nm^{-2} and Poisson's ratio of nylon = 0.2.

Q.3 A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2 × 10–3, what is the percentage increase in volume?

Q.4 Calculate the Poisson's ratio for silver. Given its Young's modulus = 7.25×10^{10} Nm^{-2} and bulk modulus = 11×10^{10} Nm^{-2}.

Q.5 A material has Poisson's ratio 0.2. If a uniform rod of it suffers longitudinal strain 4.0×10^{-3}, calculate the percentage change in its volume.

Answers

1.	0.5	2.	3.32×10^{-5} m	3.	0
4.	0.39	5.	0.24%		

Miscellaneous Subjective Assignment

Q.1 A wire elongates by *l* mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, what will be the elongation of the wire in mm?

Q.2 A wire is cut to half its original length, (a) How would it affect the elongation under a given load? (b) How does it affect the maximum load it can support without exceeding the elastic limit?

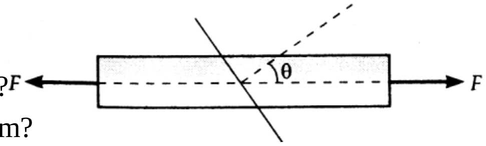
Q.3 A bar of cross–section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.

(a) What is the tensile stress on this plane?

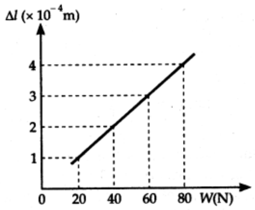
(b) What is the shearing stress on this plane?

(c) For what value of θ is the tensile stress maximum?

(d) For what value of θ is the shearing stress maximum?



Q.4 The graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to the other end. If the cross–sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



Q.5 A metallic wire is stretched by suspending weight from it. If α is the longitudinal strain and Y is the Young's modulus, show that elastic potential energy per unit volume is given by ½ $Y\alpha^2$.

Q.6 A copper wire of negligible mass, 1 m length and cross–sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad s^{-1}. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus. If on increasing the angular velocity to 100 rad s^{-1}, the wire breaks down, obtain the breaking stress.

Q.7 A load of 31.4 kg is suspended from a wire of radius 10^{-3} m and density 9×10^3 kg m^{-3}. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat of the material of the wire are 9.8×10^{10} Nm^{-2} and 490 J kg^{-1} K^{-1} respectively.

Q.8 A light rod of length 2 m is suspended horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross–section $A_1 = 0.1$ cm^2 and the other is of brass and is of cross–section $A_2 = 0.2$ cm^2. Find out the position along the rod at which a weight must be suspended to produce (i) equal stresses in both wires, (ii) equal strains in both wires. For steel, $Y = 20 \times 10^{10}$ Nm^{-2} and for brass $Y = 10 \times 10^{10}$ Nm^{-2}.

Q.9 A thin rod of negligible mass and area of cross–section 4×10^{-6} m^2, suspended vertically from one end has a length of 0.5 m at 100°C. The rod is cooled at 0°C, but prevented from contracting by attaching a mass at the lower end. Find (i) this mass and (ii) the energy stored in the rod. Given for this rod, $Y = 10^{11}$ Nm^{-2}, coefficient of linear expansion = 10^{-5} K^{-1} and g = 10 ms^{-2}.

Q.10 A wire of cross–sectional area A is stretched horizontally between two clamps located at a distance $2l$ metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the mid–point of the wire moves down be $x << l$, then find (i) the strain produced in the wire. (ii) the stress is the wire. (iii) If Y is the Young's modulus of wire, then find the value of x.

Q.11 A stone of 0.5 kg mass is attached to one end of a 0.8 m long aluminium wire of 0.7 mm diameter and suspended vertically. The stone is now rotated in a horizontal plane at a rate such that the wire makes an angle of 85° with the vertical. Find the increase in the length of the wire. The Young's modulus of aluminium = 7×10^{10} Nm^{-2}, sin 85° = 0.9962, cos 85° = 0.0872

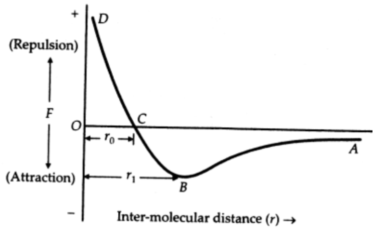
Q.12 Two rods of different materials but of equal cross–sections and lengths (1.0 m each) are joined to make a rod of length 2.0 m. The metal of one rod has coefficient of linear thermal expansion 10^{-5} °C^{-1} and Young's modulus 3×10^{10} Nm^{-2}. The other metal has the values 2×10^{-5} °C^{-1} and 10^{10} Nm^{-2} respectively. How much pressure must be applied to the ends of the composite rod to prevent its expansion when the temperature is raised by 100°C?

Answers

1.	l mm	2.	(a) halved, (b) no effect		
3.	$\frac{F}{A}\cos^2\theta$, (b) $\frac{F}{2A}\sin^2\theta$, (c) $\theta = 0^o$, (d) $\theta = 45^o$			4.	2×10^{11} Nm^{-2}
6.	10^{10} Nm^{-2}	7.	1/120 K	8.	(i) 1.33 m, (ii) 1 m
9.	(i) 40 kg, (ii) 0.1 J	10.	(i) $\frac{x^2}{2l^2}$, (ii) $\frac{wl}{Ax}$, (iii) $l\left[\frac{w}{YA}\right]^{1/3}$		
11.	1.67 mm	12.	5×10^7 Nm^{-2}		

Conceptual Problems

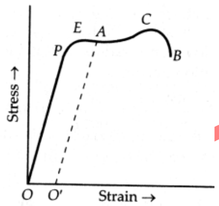
Q.1 In the diagram a graph between the inter–molecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.



Q.2 Crystalline solids have sharp melting points. Amorphous solids do not melt at a sharp temperature; rather these have a softening range of temperature. Explain.

Q.3 Which is more elastic–rubber or steel?

Q.4 The stress–strain graph for a metal wire is shown in figure. Up to the point E, the wire returns to its original state O along the curve EPO when it is gradually unloaded. Point B corresponds to the fracture of the wire.



(a) Up to which point on the curve is Hooke's law obeyed? This point is sometimes called "Proportionality limit"

(b) Which point on the curve corresponds to elastic limit and yield point of the wire?

(c) Indicate the elastic and plastic regions of the stress–strain graph.

(d) Describe what happens when the wire is loaded up to a stress corresponding to the point A on the graph, and the unloaded gradually. In particular, explain the dotted curve.

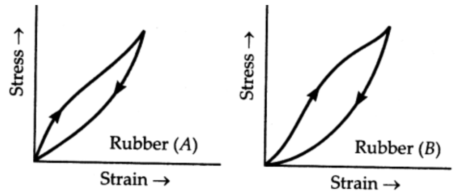
(e) What is peculiar about the portion of the stress–strain graph from C to B? Up to what stress can the wire be subjected without causing fracture?

Q.5 Two different types of rubber are found to have the stress–strain curves as shown in figure.

(a) In which significant ways do these curves differ from the stress–strain curve of a metal wire shown in figure.

(b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose? Why?

(c) Which of the two rubber materials would you choose for a car tyre?



Q.6 Read each of the statements below carefully and state, with reasons, if it is true or false

(a) When a material is under tensile stress, the restoring forces are caused by interatomic attraction while under compressional stress, the restoring forces are due to inter–atomic repulsion.

(b) A piece of rubber under an ordinary stress can display 1000% strain: yet when unloaded returns to its original length. This shows that the elastic restoring forces in a rubber piece are strictly conservative.

(c) Elastic restoring forces are strictly conservative only when Hooke's law is obeyed.

Q.7 Two wires of different materials are suspended from a rigid support. They have the same length and diameter and carry the same load at their free ends. (a) Will the stress and strain in each wire be the same? (b) Will the extension in both wires be the same?

Q.8 A cable is replaced by another of the same length and material but of twice the diameter. (a) How does this affect its elongation under a given load? (b) How many times will be the maximum load it can now support without exceeding the elastic limit?

Q.9 Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different?

Q.10 A uniform plank of Young's modulus Y is moved over a smooth horizontal surface by a constant horizontal force F. The area of transverse section of the plank is A. Find the compressive strain on the plank in the direction of the force.

Q.11 Why the bridges are declared unsafe alter long use?

Q.12 Two identical solid balls, one of ivory and the other of wet–clay, are dropped from the same height on the floor. Which will rise to a greater height after striking the floor and why?

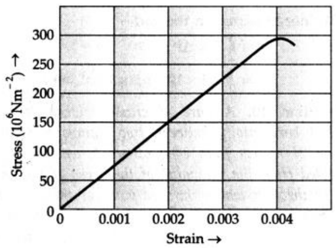
Q.13 The breaking force for a wire is F. What will be the breaking force for (a) two parallel wires of the same size (b) for a single wire of double the thickness?

Q.14 Why does modulus of elasticity of most of the materials decrease with the increase of temperature?

NCERT Exercise

Q.1 A steel wire of length 4.7 m and cross–section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross–section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Q.2 Figure shows the stress–strain curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



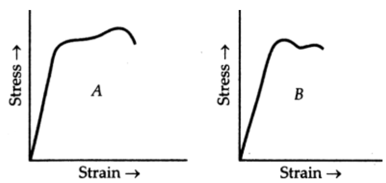
Q.3 The stress–strain graphs for materials A and B are shown in figure. The graphs are drawn to the same scale.

(a) which of the material has greater Young's modulus?

(b) Which material is more ductile?

(c) Which is more brittle?

(d) Which of the two is stronger material?



Q.4 Read each of the statements below carefully and state, with reasons, if it is true or false.

(a) The modulus of elasticity of rubber is greater than that of steel

(b) the stretching of a coil is determined by its shear modulus

Q.5 Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 Nm^{-2})

Q.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 G Pa. What is the vertical deflection of this face? (1 Pa = 1 Nm^{-2}).

Q.7 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50, 000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.

Q.8 A piece of copper having a rectangular cross–section of 15.2 mm × 19.1 mm is pulled in tension with 44, 500 N force, producing only elastic deformation. Calculate the resulting strain.

Q.9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?

Q.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Q.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross–sectional area of the wire is 0.005 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Q.12 Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, pressure increase = 100.0 atm, final volume = 100.5 litre (1 atm = 1.013×10^5 Pa)

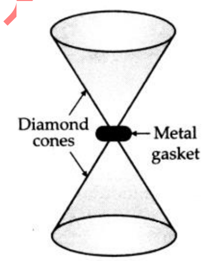
Q.13 What is the density of ocean water at a depth, where the pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kgm^{-3} ? Compressibility of water = 45.8×10^{-11} Pa^{-1}.

Q.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

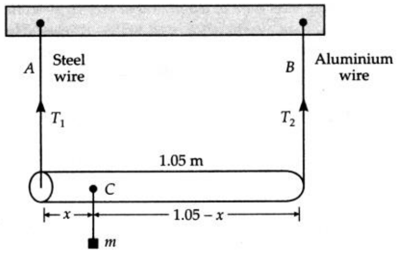
Q.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Q.16 How much should the pressure on a litre of water be changed to compress it by 0.10% ?

Q.17 Anvils made of single crystals of diamond, with the shape as shown in figure, are used to investigate behaviour of materials under very high pressure. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Q.18 A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross–sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2 respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?



Q.19 A mild steel wire of length 1.0 and cross–sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid–point of wire. Calculate the depression at the mid–point.

Q.20 Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3×10^9 Pa? Assume that each rivet is to carry one quarter of the load.

Q.21 The Marina trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

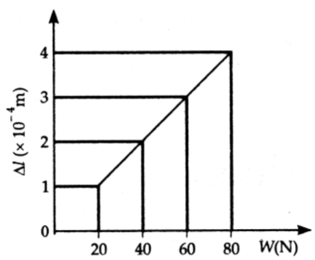
Answers

1.	1.79	2.	(a) 7.5×10^{10} Nm^{-2}, (b) 3×10^8 Nm^{-2}		
3.	(a) A, (b) A, (c) B, (d) A	4.	(a) false, (b) true		
5.	1.5×10^{-4} m, 1.3×10^{-4} m	6.	4×10^{-7} m		
7.	2.8×10^{-6}	8.	0.001277	9.	7.07×10^4 N
10.	1.3	11.	1.87×10^{-3} m	12.	2.026×10^9 Pa
13.	1.034×10^3 kgm^{-3}	14.	2.74×10^{-5}	15.	0.05 cm^3
16.	2.2×10^6 Nm^{-2}	17.	2.55×10^{11} Nm^{-2}	18.	(a)
	0.7m,(b)0.43m				
19.	1.07×10^{-2} m	20.	260 kN	21.	2.02×10^{-4} m^3

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

Q.1 The adjacent graph shows the extension (Δl) of a wire of length 1m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross–sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2

(c) 3×10^{-12} N/m^2 (d) 2×10^{-13} N/m^2

Q.2 The following four wires are made of the same material. Which of these will have the largest extension, when the same tension is applied?

(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm

Q.3 A wire of length L, and cross–sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is

(a) $YAx^2/ 2L$ (b) YAx^2/ L (c) $YAx/ 2L$ (d) $YAx^2 L$

Q.4 The pressure of a medium is changed from 1.01×10^5 Pa to 1.65×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa (c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

Q.5 A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(a) 2 P/3 (b) P (c) 3P/2 (d) 2 P

Answers

1.	a	2.	a	3.	a	4.	d	5.	b

AIEEE

Q.1 A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)

(a) $l / 2$ (b) l (c) $2\,l$ (d) zero

Q.2 A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is

(a) $F/ 2l$ (b) Fl (c) $2\,Fl$ (d) $Fl/2$

Q.3 A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is

(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J

Q.4 If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(a) 2 Y/S (b) S/2 Y (c) $2\,S^2Y$ (d) $S^2/ 2Y$

Q.5 Two wires are made of the same material and have the same volume. However wire 1 has cross–sectional area A and wire 2 has cross–sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) F (b) 4 F (c) 6 F (d) 9 F

Answers

1.	b	2.	d	3.	d	4.	d	5.	d

AIIMS Entrance Exam

Q.1 According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain

(a) increases (b) decreases (c) becomes zero (d) remains constant

Q.2 A thick copper rope of density 1.5×10^3 kgm^{-3} and Young's modulus 5×10^6 Nm^{-2}, 8 m in length, when hung from the ceiling of a room, the increase in its length due to its own weight is

(a) 9.6×10^{-5} m (b) 19.2×10^{-7} m (c) 9.6×10^{-2} m (d) 9.6 m

Q.3 If in a wire of Young's modulus Y, longitudinal strain X is produced, then the value of potential energy stored in its unit volume will be

(a) YX^2 (b) $2\ YX^2$ (c) $0.5\ Y^2\ X$ (d) $0.5\ YX^2$

Q.4 A metal ring of initial radius r and cross–sectional area A is fitted onto a wooden disc of radius R > r. If Young's modulus of the metal is Y, then the tension in the ring is

(a) $\frac{AYR}{r}$ (b) $\frac{Yr}{AR}$ (c) $\frac{AY(R-r)}{r}$ (d) $\frac{Y(R-r)}{Ar}$

Q.5 For a constant hydraulic stress on an object, the fractional change in the object's volume ($\Delta V/V$) and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$ (c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto \frac{1}{B^2}$

Q.6 The compressibility of water is 4×10^{-5} per unit atmosphere pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(a) 0.4 cm^3 (b) 4×10^{-5} cm^3 (c) 0.025 cm^3 (d) 0.04 cm^3

Q.7 A stretched rubber has

(a) increased kinetic energy (b) increased potential energy

(c) decreased kinetic energy (d) decreased potential energy

Q.8 The breaking stress of a wire depends upon

(a) length of the wire (b) radius of the wire

(c) material of the wire (d) shape of the cross–section

Q.9 Which of the following affects the elasticity of a substance?

(a) hammering and annealing (b) change in temperature

(c) impurity in substance (d) all of these

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion if true, but reason is false.

(d) If both assertion and reason are false

Q.10 **Assertion:** Lead is more elastic than rubber.

Reason: If same load is loaded on the lead and rubber wire of same cross–sectional area, the strain of lead is very much less than that of rubber.

Q.11 **Assertion:** Stress is the internal force per unit area of a body.

Reason: Rubber is more elastic than steel.

Answers

1.	d	2.	c	3.	d	4.	c	5.	b
6.	a	7.	b	8.	c	9.	d	10.	a
11.	c								

DPMT Entrance Exam

Q.1 Which of the following has no dimensions?

(a) strain (b) angular velocity

(c) momentum (d) angular momentum

Q.2 The diameter of brass rod is 4 mm. Young's modulus of brass is 9×10^9 N/m^2. The force required to stretch 0.1% of its length is

(a) 360 πN (b) 36 N (c) $36\pi \times 10^5$ N (d) $144\,\pi \times 10^3$ N

Q.3 When a body of mass M is hung from a spring, the spring extends by 1 cm. If the body of mass 2 M be hung from the same spring, the extension of spring will be

(a) 1 cm (b) 2 cm (c) 0.5 cm (d) 4 cm

Q.4 A wire whose cross–sectional area is 2 mm^2 is stretched by 0.1 mm by a certain load, and if a similar wire of triple the area of cross section is stretched by the same load, then the elongation of the second wire would be

(a) 3.3 mm (b) 0.033 mm (c) 0.33 mm (d) 0.0033 mm

Q.5 A substance breaks down by a stress of 10^6 N/m^2. If the density of the wire is 3×10^3 kg/m^3, then the length of the wire of the substance which will break under its own weight when suspended vertically will be

(a) 66.6 m (b) 60.0 m (c) 33.3 mm (d) 30.3 mm

Q.6 With what minimum acceleration can a fireman slide down a rope whose breaking strength is two third of his weight?

(a) $\frac{g}{3}$ (b) $\frac{2}{3}g$ (c) $\frac{3}{2}g$ (d) $\frac{g}{2}$

Q.7 A wire of length L and cross–sectional area A is made of a material of Young's modulus Y. If the wire is stretched by the amount x, the work done is

(a) $\frac{YAx^2}{2L}$ (b) $\frac{YAx^2}{L}$ (c) $Yax^2 L$ (d) $\frac{YAx}{2L}$

Q.8 When a sphere is taken to bottom of sea 1 km deep, it contracts by 0.01%. The bulk modulus of elasticity of the material of sphere is (Given : Density of water = 1 g/cm^3)

(a) 9.8×10^{10} N/m^2 (b) 10.2×10^{10} N/m^2 (c) 0.98×10^{10} N/m^2 (d) 8.4×10^{10} N/m^2

Answers

1.	a	2.	a	3.	b	4.	b	5.	c
6.	a	7.	c	8.	a				

MECHANICAL PROPERTIES OF FLUIDS

Fluids

A fluid is a substance that can flow. It ultimately assumes the shape of the containing vessel because it cannot withstand shearing stress. Thus, both liquids and gases are fluids.

Fluid Statics

The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics. Its study includes hydrostatic pressure, Pascal's law.

Fluid dynamics

The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.

Thrust of a Liquid

Thrust

The total force exerted by a liquid on any surface in contact with it is called thrust. It is because of this thrust that a liquid flows out through the holes of the containing vessel. Thrust is a force. Its SI unit is Newton (N) & dimensional formula = $[MLT^{-2}]$

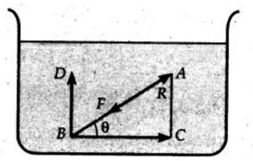
Liquid in Equilibrium

Consider a liquid contained in a vessel in the equilibrium state of rest. As shown in figure, suppose the liquid exerts a force F on the bottom surface in an inclined direction AB. The surface exerts an equal reaction R to water along BA.

The reaction R along BA has two rectangular components:

(i) Tangential component $BC = R\cos\theta$

(ii) Normal component, $BD = R\sin\theta$



Since a liquid cannot resist any tangential force, so the liquid near B should begin to flow along BC. Since the liquid is at rest, the force along BC should be zero.

∴ $R\cos\theta = 0$, as $R \neq 0$, so $\cos\theta = 0$ or $\theta = 90^o$

Hence a *liquid exerts force perpendicular to the surface of the container at every point.*

Pressure

The pressure at a point on a surface is the thrust acting normally per unit area around that point. If a total force F acts normally over a flat area A, then the pressure is $P = \frac{F}{A}$

Pressure is a scalar quantity.

SI units of pressure = Nm^{-2} or Pascal (Pa)

CGS unit of pressure = dyne cm^{-2}

Dimensional formula of pressure is $[ML^{-1}T^{-2}]$

Practical Application of Pressure

(i) A sharp knife cuts better than a blunt one

The area of a sharp edge is much less than the area of a blunt edge.

(ii) Railway tracks are laid on wooden sleepers

This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably.

(iii) **It is difficult for a man to walk on sand while a camel walks easily on sand** inspite of the fact that a camel is much heavier than a man. This is because camel's feet have a larger area than the feet of man.

(iv) **Pins and nails are made to have pointed ends:** Their pointed ends have very small area

Subjective Assignment – I

Q.1 The two thigh bones (femurs), each of cross–sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by femurs. Take g = 10 ms^{-2}.

Q.2 How much pressure will a man of weight 80 kgf exert on the ground when (i) he is lying and (ii) he is standing on his feet? Given that the area of the body of the man is 0.6 m^2 and that of a foot is 80 cm^2.

Q.3 A cylindrical vessel containing liquid is closed by a smooth piston of mass m. The area of cross–section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the piston.

Answers

1. 2×10^5 Nm^{-2} 2. (i) 1.307×10^3 Nm^{-2}, (ii) 4.9×10^4 Nm^{-2} 3. $P_0 + mg/A$

Density

The density of any material is defined as its mass per unit volume. If a body of mass M occupies volume V, then its density is

$$\rho = \frac{M}{V} \quad i.e., \quad Density = \frac{Mass}{Volume}$$

Density is a positive scalar quantity.

Units and dimensions of density

SI unit of density = kg m^{-3}

CGS unit of density = g cm^{-3}

Dimensional formula of density is $[ML^{-3}]$

Specific gravity or relative density

The specific gravity or relative density of a substance is defined as the ratio of the density of the substance to the density of water at 4°C. The density of water at 4°C is 1.0×10^3 kg m^{-3}.

$$Specific\ gravity = \frac{Density\ of\ substance}{Density\ of\ water\ at\ 4^oC}$$

Specific gravity is a dimensionaless positive scalar quantity.

Pascal's Law

This law tells as how pressure can be transmitted in a fluid. It can be stated in a number of equivalent ways as follows:

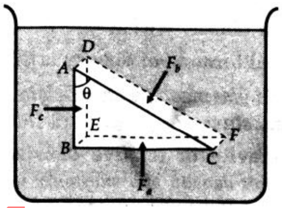
(i) *The pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.*

(ii) *A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.*

(iii) *The pressure in a fluid at rest is same at all points if we ignore gravity.*

Proof of Pascal's law

Consider a small element ABC – DEF in the form of a right angled prism in the interior of a fluid at rest. The element is so small that all its parts can be assumed to be at same depth from the liquid surface and, therefore, the effect of gravity is same for all of its points.



By Newton's law, the fluid force should balance in various directions.

Along horizontal direction, $F_b \sin\theta = F_c$

Along vertical direction, $F_b \cos\theta = F_a$

From the geometry of the figure, we get

and $A_b \sin\theta = A_c$ & $A_b\cos\theta = A_a$

From the above equations, we get

$$\frac{F_b \sin\theta}{A_b \sin\theta} = \frac{F_c}{A_c} \quad \text{and} \quad \frac{F_b \cos\theta}{A_b \cos\theta} = \frac{F_a}{A_a}$$

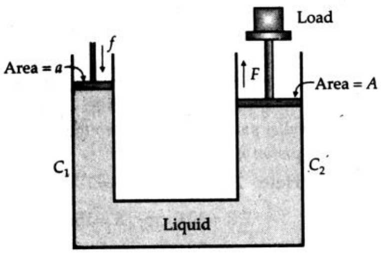
$$\therefore \quad \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c} \quad \text{or} \quad P_a = P_b = P_c$$

Hence, pressure exerted is same in all directions in a fluid at rest. This proves Pascal's law of transmission of fluid pressure.

Applications of Pascal's Law

Hydraulic lift

Hydraulic lift is an application of Pascal's law. It is used to lift heavy objects. It is a force multiplier. It consists of two cylinders C_1 and C_2 connected to each other by a pipe. The cylinders are fitted with water–tight frictionless pistons of different cross–sectional areas. The cylinders and the pipe contain a liquid. Suppose a force f is applied on the smaller piston of cross–sectional area a. Then



Pressure exerted on the liquid, $P = \frac{f}{a}$

According to Pascal's law, same pressure P is also transmitted to the larger piston of cross–sectional area A.

$$\therefore \quad F = P \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

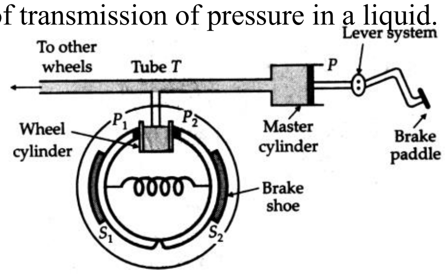
As A > a, therefore, F > f

Hence by making the ratio A/a large, very heavy loads (like cars and trucks) can be lifted by the application of a small force. However, there is no gain of work. The work done by force f is equal to the work done by F. The piston P_1 has to be moved down by a larger distance compared to the distance moved up by piston P_2.

Hydraulic Brakes

The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.

Construction



As shown in figure, a hydraulic brake consists of a tube T containing brake oil. One end of this tube is connected to a master cylinder fitted with piston P. The piston P is attached to the brake pedal through a lever system. The other end of the tube is connected to the wheel cylinder having two pistons P_1 and P_2. The pistons P_1 and P_2 are connected to the brake shoes S_1 and S_2

respectively. The area of cross–section of the wheel cylinder is larger than that of master cylinder.

Working

When the pedal is pressed, its lever system pushes the piston P into the master cylinder. The pressure is transmitted through the oil to the pistons P_1 and P_2 in the wheel cylinder, in accordance with Pascal's law. The pistons P_1 and P_2 are pushed outwards. The brake shoes get pressed against the inner rim of the wheel, retarding the motion of the wheel. As the cross–sectional area of wheel cylinder is larger than that of master cylinder, a small force applied to the pedal produces a large retarding force.

When the paddle is released, a spring pulls the brake shoes away from the rim. The pistons in both cylinders move towards their normal positions and the oil is forced back into the master cylinder.

Subjective Assignment – II

Q.1 In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1? What is the pressure necessary to accomplish that task?

Q.2 Two syringes of different cross–sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Q.3 Two pistons of hydraulic press have diameters of 30.0 cm and 2.5 cm. What is force exerted by larger piston, when 50.0 kg wt. is placed on the smaller piston? If the stroke of the smaller piston is 4.0 cm, through what distance will the larger piston move after 10 strokes?

Q.4 The average mass that must be lifted by a hydraulic press is 80 kg. If the radius of the larger piston is five times that of the smaller piston, what is the minimum force that must be applied?

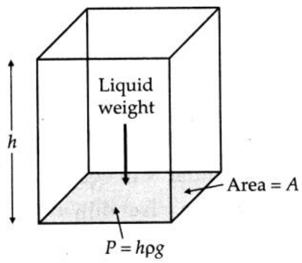
Q.5 An automobile back is lifted by a hydraulic jack that consists of two pistons. The large piston is 1 m in diameter and the small piston is 10 cm in diameter. If W be weight of the car, how much smaller a force is needed on the small piston to lift the car?

Answers

1.	1.5×10^3 N, 1.9×10^5 Pa	2.	(a) 90 N, (b) 0.67 cm	3.	7200 kg wt, 0.28 cm
4.	31.4 N	5.	1% of the weight of the car		

Pressure Exerted by a Liquid Column

Consider a vessel of height h and cross–sectional area A filled with a liquid of density ρ. The weight of the liquid column exerts a downward thrust on the bottom of the vessel and the liquid exerts pressure.



Weight of liquid column,

$$W = \text{Mass of liquid} \times g = \text{Volume} \times \text{density} \times g = Ah \times \rho \times g = Ah\rho g$$

Pressure exerted by the liquid column on the bottom of the vessel is

$$P = \frac{Thrust}{Area} = \frac{W}{A} = \frac{Ah\rho g}{A}$$

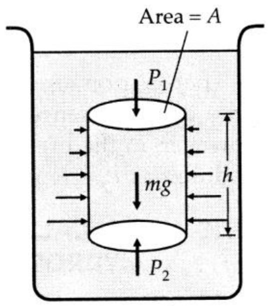
or $P = h\rho g$

Thus the pressure exerted by a liquid column at rest is proportional to (i) height of the liquid column and (ii) density of the liquid.

Effect of Gravity on Fluid Pressure

As the liquid cylinder is at rest, the resultant horizontal force should be zero. Various force acting on it in the vertical direction are:

1. Downward force on the top of the cylinder, $F_1 = P_1A$
2. Upward force on the bottom of the cylinder, $F_2 = P_2A$
3. Weight of the liquid cylinder acting downwards,



$$W = \text{Mass} \times g = \text{Volume} \times \text{density} \times g = Ah\rho g$$

where ρ is the density of the liquid.

As the liquid cylinder is in equilibrium,

Net upward force = Net downward force

or $F_1 + W = F_2$ or $F_2 - F_1 = W$

or $P_2A - P_1A = Ah\rho g$ or $P_2 - P_1 = h\rho g$

If we shift point 1 to the liquid surface, which is open to the atmosphere, then we can replace P_1 by atmospheric pressure P_a and P_2 by P in the above equitation and we get

$$P - P_a = h\rho g \quad \text{or} \quad P = P_a + h\rho g$$

We can note the following points:

(i) The liquid pressure is the same at all points at the same horizontal level or at same depth.

(ii) Pressure at any point inside the fluid depends on the depth h.

(iii) The absolute (actual) pressure P, at a depth h below the liquid surface open to the atmosphere is greater than the atmospheric pressure by an amount $h\rho g$. The excess pressure $P - P_a$, at depth h is called a ***gauge pressure*** at the point.

(iv) Pressure does not depend on the cross–section or base–area or the shape of the vessel.

Effect of gravity on Pascal's law

If we neglect the effect of gravity, then

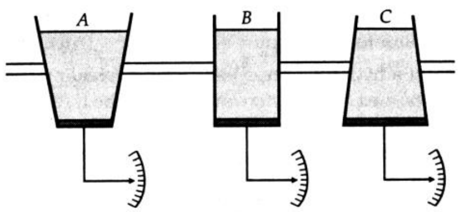
$$P_2 - P_1 = h\rho g = 0 \quad \text{or} \quad P_2 = P_1$$

That is, pressure at all points inside the liquid is same in the absence of gravity. This is Pascal's law. However, in the presence of gravity, Pascal's law gets modified as $P_2 - P_1 = h\rho g$.

Pascal's Vases : Hydrostatic Paradox

Pascal demonstrated experimentally that *the pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.*

When the there vessels are filled with the same liquid upto the same height, all the three meters records the same pressure.

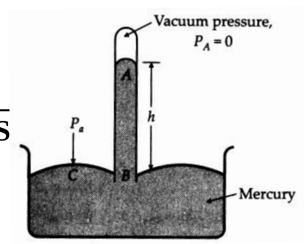


This appears anomalous because the three vessels have different shapes and contain different amounts of liquid. This apparently unexpected result is known as **hydrostatic paradox.**

Atmospheric Pressure

The gaseous envelope surrounding the earth is called the atmosphere. *The pressure exerted by the atmosphere is called atmospheric pressure.* The force exerted by air column of air on a unit area of the earth's surface is equal to the atmospheric pressure. The atmospheric pressure at sea level is 1.013×10^5 Nm^{-2} or Pa.

Torricelli's experiment of measuring atmospheric pressure



A 1 m long glass tube closed at one end is filled with clean and dry mercury. After closing the end of the tube with the thumb, the tube is inverted into a dish of mercury. As the thumb is removed, the mercury level in the tube falls down a little and comes to rest at a vertical height of 76 cm above the mercury level in the dish.

The space above mercury in the tube is almost a perfect vacuum and is called Torricellian vacuum. Therefore, pressure $P_A = 0$. Consider a point C on the mercury surface in the dish and point B in the tube at the same horizontal level. Then

$$P_B = P_C = \text{Atmospheric pressure, } P_a$$

If h is the height of mercury column and ρ is the density of mercury, then

$$P_B = P_A + h\rho g$$

or $P_a = 0 + h\rho g$ or $P_a = h\rho g$

For a mercury barometer, $h = 76$ cm $= 0.76$ m, $\rho = 13.6 \times 10^3$ kg m^{-3}, $g = 9.8$ ms^{-2}

$\therefore$ $P_a = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5$ Pa

Absolute Pressure and Gauge Pressure

The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure,

i.e., $P_g = P - P_a = h\rho g$

The gauge pressure is proportional to h. Many pressure measuring devices directly measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).

Various Units for Pressure:

(i) SI unit of pressure = Nm^{-2} or Pascal (Pa) (ii) CGS unit of pressure = dyne cm^{-2}

(iii) **Atmosphere** (atm). It is the pressure exerted by 76 cm of Hg column (at 0°C, 95° latitude and mean sea level).

$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \times 10^6 \text{ dyne cm}^{-2}$

(iv) In meteorology, the atmospheric pressure is measured in bar and millibar.

$1 \text{ bar} = 10^5 \text{ Pa} = 10^6 \text{ dyne cm}^{-2}$ $1 \text{ millibar} = 10^{-3} \text{ bar} = 100 \text{ Pa}$

(v) Atmospheric pressure is also measured in **torr,** a unit named after Torricelli.

1 torr = 1 mm of Hg 1 atm = 1.013 bar = 760 torr

Units for Blood Pressure

The blood pressure is measured in mm of Hg. When the heart is contracted to its smallest size, the pumping is hardest and the pressure of blood flowing in major arteries is nearly 120 mm of Hg. This is known as **systolic pressure.** When the heart is expanded to its largest size, the blood pressure is nearly 80 mm of Hg. This is known as ***diastolic pressure.***

NOTE

- ♦ While describing a fluid, we are concerned with properties that vary from point to point and not with properties associated with a specific piece of matter. So the role of force in a solid is replaced in a fluid by pressure and that of mass by density.
- ♦ A fluid exerts pressure not only on a solid piece immersed in fluid or on the walls of container, fluid pressure exists at all points in a fluid. A volume element (of fluid) inside a fluid is in a equilibrium because the pressures exerted on its various faces get balanced.
- ♦ Pressure at a point in a liquid acts equally in all directions.
- ♦ Pressure in a liquid is the same for all points at the same horizontal level.

- Pressure in a liquid increases with depth h according to the relation $P = P_a + h \rho g$
  This expression is valid only for incompressible fluids i.e., liquids.
- Liquid pressure is independent of the area and the shape of the containing vessel.
- The mean pressure on the walls of a vessel containing liquid upto height h is $h \rho g/2$
- Most of the pressure measuring devices measure the pressure difference between the true pressure and the atmospheric pressure. This difference is called *gauge pressure* and the pressure is called *absolute pressure.*
  Absolute pressure = Gauge pressure + Atmospheric pressure i.e., $P = P_g + P_a$
- The gauge pressure may be positive or negative depending on $P > P_a$ or $P < P_a$. In inflated tyres or the human circulatory system, the absolute pressure is greater than atmospheric pressure, so gauge pressure is positive, called the *overpressure*. However, when we suck a fluid through a straw, the absolute pressure in our lungs is less than atmospheric pressure and so the gauge pressure is negative.
- A diver in water at a depth of 10 m is under twice the atmospheric pressure.
- At a depth of 1 km in a sea, the increase in pressure is 100 atm. Submarines are designed to withstand such high pressures.
- The pressure at the centre of the earth is estimated to be 3 million atmospheres.
- The atmospheric pressure is nearly 100 kPa. The tyres of a car are usually inflated to a pressure of about 200 kPa.
- It is because of the blood pressure from inside that we do not feel such a high atmospheric pressure.
- A drop in the atmospheric pressure by 10 mm of Hg or more is a sign of an approaching storm.

Subjective Assignment – III

Q.1 What will be the length of mercury column in a barometer tube, when the atmospheric pressure is 75 cm of mercury and the tube is inclined at an angle of 60^o with the horizontal direction?

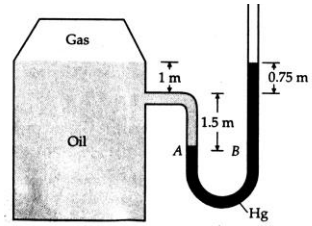
Q.2 The density of the atmosphere at sea level is $1.29 kg\ m^{-3}$. Assume that it does not change with altitude. Then how high would the atmosphere extend? Take $g = 9.81\ ms^{-2}$.

Q.3 A rectangular tank is 10 m long, 10 m broad and 3 m high. It is filled to the rim with water of density 10^3 kg m^{-3}. Calculate the thrust at the bottom and walls of the tank due to hydrostatic pressure.

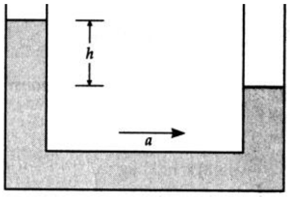
Q.4 The manual of a car instructs the owner to inflate the tyres to a pressure of 200 kPa. (a) What is the recommended gauge pressure? (b) What is the recommended absolute pressure? (c) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read?

Q.5 At a depth of 1000 m in an ocean (a) What is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea–level atmospheric pressure. (The density of sea water is 1.03×10^3 kg m^{-3}, $g = 10\ ms^{-2}$)

Q.6 What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in figure? Density of oil = 820 kg m^{-3}, density of mercury = 13.6×10^3 kg m^{-3}. Given 1 atmosphere pressure = 1.01×10^5 Pa.

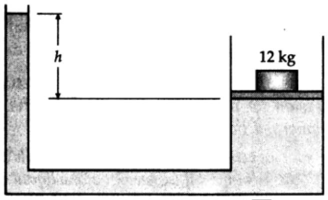


Q.7 A liquid stands at the same level in the U–tube when at rest. If A is the area of cross–section and g the acceleration due to gravity, what will be the difference in height h of the liquid in the two limbs of U–tube, when the system is given an acceleration ‘a’ towards right, as shown in figure if L is length of base.



Q.8 A vertical U–tube of uniform inner cross–section contains mercury in both of its arms. A glycerine (density 1.3g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8g cm^{-3} is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of the oil column.

Q.9 The area of cross–section of the wider tube shown in figure is 800 cm^2. If a mass of 12 kg is placed on the massless piston, what is the difference h in the level of water in the two tubes?



Q.10 A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. If the elevator is accelerating upwards at 4.9 ms^{-2}, what will be the air pressure in the elevator?

Answers

1.	86.6 cm	2.	8 km
3.	2.793×10^6 N	4.	(a) 200 kPa, (b) 301 kPa, 211 kPa
5.	(a) 104 atm, (b) 103 atm, (c) 4.12×10^5 N	6.	3.81×10^5 Pa, 2.8×10^5 pa
7.	$\frac{La}{g}$	8.	9.6 cm
9.	15.0 cm	10.	114 cm of Hg

Buoyancy

When body is immersed in a fluid, the fluid exerts pressure on all faces of the body. The upward thrust at the bottom is more than the downward thrust on the top because the bottom is at the greater depth than the top. Hence a resultant upward force acts on the body. *The upward force acting on a body immersed in a fluid is called up thrust or buoyant force and the phenomenon is called buoyancy.*

The force of buoyancy acts through the centre of gravity of the displaced fluid which is called centre of buoyancy.

Archimedes' principle

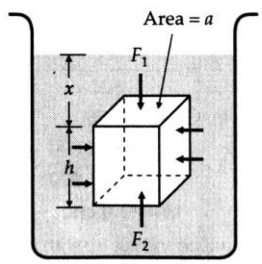
It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the displaced fluid.

Proof: As shown in figure, consider a body of height h lying inside a liquid of density ρ, at a depth x below the free surface of the liquid. Area of cross–section of the body is a. The forces on the sides of the body cancel out.

Pressure at the upper face of the body, $P_1 = x\rho g$

Pressure at the lower face of the body, P_2 $(x + h)\rho g$

Thrust acting on the upper face of the body is $F_1 = P_1 a = x\rho g a$, acting vertically downwards.



Thrust acting on the lower face of the body is $F_2 = P_2\, a = (x + h)\, \rho g a$, acting vertically upwards.

The resultant force $(F_2 - F_1)$ is acting on the body in the upward direction and is called upthrust (U).

∴ $U = F_2 - F_1 = (x + h)\rho g a - x\rho g a = a h \rho g$.

But ah = V, the volume of body = volume of liquid displaced.

∴ $U = V\rho g = Mg$ [∵ $M = V\rho$ = mass of liquid displaced]

i.e., Upthrust or buoyant force = Weight of liquid displaced

This proves the Archimedes' principle.

Apparent weight of immersed body : The actual weight W of the immersed body acts downwards and the upthrust U acts upwards.

∴ Apparent weight = Actual weight – Buoyant force

$$W_{app} = W - U = V\sigma g - V\rho g = V\sigma g\left(1 - \frac{\rho}{\sigma}\right) \quad \text{or} \quad W_{app} = W\left(1 - \frac{\rho}{\sigma}\right)$$

Where $W = V\sigma g$ is the true weight of the body and σ is its density.

Law of Floatation

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.

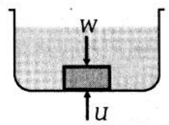
Explanation : When a body is immersed fully or partly in a liquid, following two vertical forces act on it:

(i) Its true weight W which acts vertically downward through its centre of gravity.

(ii) Force of buoyancy or upthrust U which acts vertically upwards through the centre of buoyancy.

Three cases are possible:

(a) When W > U: The downward pull of the weight of the body is higher than the upthrust. The net force (W – U) acts in the downward direction and hence the body sinks.

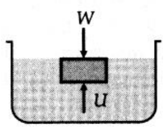


(a) W > U

$$W > U \Rightarrow \quad V\sigma g > V\rho g \quad \text{or} \quad \sigma > \rho$$

Thus a *body sinks in a liquid if its density greater than the density of the liquid.* That is why an iron piece or a stone sinks in water.

(b) When W = U : The weight of the body is just balanced by the upthrust. No net force acts on the body. The body floats fully immersed.

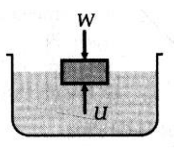


(b) W = U

$$W = U \Rightarrow \quad V\sigma g = V\rho g \quad \text{or} \quad \sigma = \rho$$

Thus a drop of olive oil stands at rest anywhere in a mixture of equal quantities of water and alcohol because the density of olive oil is equal to that of mixture.

(c) When W < U : The gravitational force W is less than the upward force U. The body floats partly immersed. This is because the body sinks only to the extent that W = U.



(c) W < U

Here σ < ρ. The density of the floating body is less than that of liquid. That is why a piece of cork floats on water.

If V is the total volume of the body and V' is the submerged volume, then at equilibrium,

Weight of the body = Weight of liquid displaced or $V\sigma g = V'\rho g$ or $\frac{V'}{V} = \frac{\sigma}{\rho}$ or $\frac{\text{Volume of submerged part}}{\text{Total volume of the body}} = \frac{\text{Density of body}}{\text{Density of liquid}}$

Examples of Floating Bodies:

(i) The ship is made of steel (8 times denser than water) but its interior is made hollow by giving it a concave shape. It can displace much more water than its own weight. So the ship floats and can carry a lot of cargo.

(ii) Ice floats on water because the density of ice is less than that of water.

(iii) Human body is slightly more denser than water. An inflated rubber tube has low weight and large volume and increases the upthrust. It helps a person to float.

(iv) A person can swim in sea water more easily than in river water. The density of sea water is more than that of river water and so it exerts a greater upthrust.

(v) The average density of a fish is slightly greater than water. By means of an anatomical attachment called swim bladder whose size it can adjust, the fish is able to swim with case.

Equilibrium of Floating Bodies

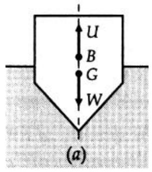
Conditions for the equilibrium of a Floating Body

(i) Weight of the liquid displaced must be equal to the weight of the body.

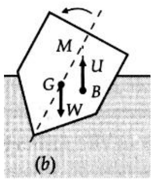
(ii) The centre of gravity of the body and the centre of buoyancy must lie on the same vertical line.

Stability of a floating body

When the centre of gravity of the body and the centre of buoyancy do not lie on the same vertical line, the two forces; the weight (W) of the body and the upthrust (U) form a couple which produces rotation.

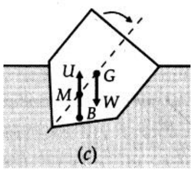


As the floating body is slightly displaced from the equilibrium position, the centre of buoyancy shifts to a new position. *The point at which the vertical line passing through the new centre of buoyancy meets the initial vertical line is called* **metacentre** (M).



(i) If the metacentre M lies above the centre of gravity G, the couple tends to bring the body back to its original position, as shown in figure. The floating body is in **stable equilibrium.**

(ii) If the metacentre M lies below the centre of gravity G, the couple tends to rotate the body away from the original position, as shown in figure. The floating body is in **unstable equilibrium.** The couple topples the floating body.



Subjective Assignment – IV

Q.1 The density of ice is 917 kg m^{-3}. What fraction of ice lies below water? The density of sea water is 1024 kg m^{-3}. What fraction of the ice berg do we see assuming that it has the same density as ordinary ice (917 kg m^{-3})?

Q.2 The density of ice is 0.918 g cm^{-3} and that of water is 1.03g cm^{-3}. An iceberg floats with a portion of 224 m^3 outside the surface of water. Find the total volume of the iceberg.

Q.3 A body of mass 6 kg is floating in a liquid with 2/3 of its volume inside the liquid. Find (i) buoyant force acting on the body, and (ii) ratio between the density of body and density of liquid.

Q.4 A piece of pure gold (ρ = 19.3 g cm^{-3}) is suspected to be hollow from inside. It weights 38.250g in air and 33.865 g in water. Calculate the volume of the hollow portion in gold, if any.

Q.5 A spring balance reads 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when

(i) an ice cube of mass 1.5 kg is put into the bucket

(ii) an iron piece of mass 7.8 kg suspended by another spring is immersed with half its volume inside the water in the bucket? Relative density of iron = 7.8

Q.6 A jeweller claims that he sells ornaments made of pure gold that has the relative density of 19.3. He sells a necklace weighing 25.250 gf to a person. The clever customer weights the necklace when immersed in pure water and finds that its weights 23.075gf in water. Is the ornament made of pure gold?

Q.7 A body of density ρ floats with a volume V_1 of its total volume V immersed in one liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2, where $\rho_1 > \rho_2$. Find the relative volumes immersed in two liquids.

Q.8 A sample of milk diluted with water has a density of 1032 kgm^{-3}. If pure milk has a density of 1080 kgm^{-3}, find the percentage of water by volume in milk.

Q.9 A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by one cm, when a man gets on it. What is the mass of the man?

Q.10 A piece of brass (alloy of zinc and copper) weights 12.9 g in air. When completely immersed in water it weights 11.3 g. What is the mass of copper contained in the alloy? Specific gravity of zinc and copper are 7.1 and 8.9 respectively.

Q.11 A metal cube of 5 cm side and relative density 9 is suspended by a thread so as to be completely immersed in a liquid of density 1.2×10^3 kg m^{-3}. Find the tension in the thread.

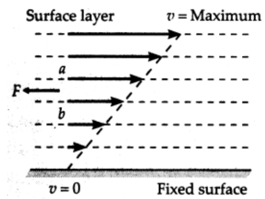
Answers

1.	0.917, 0.105	2.	2060 m^3	3.	60 N, 2/3
4.	2.403 cm^3	5.	(i) 11.5 kg f, (ii) 10.5 kg f	6.	11.61, No
7.	$V_1 = \left(\frac{\rho - \rho_2}{\rho_1 - \rho_2}\right)V, V_2 = \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right)V$			8.	60%
9.	60 kg	10.	7.61 g	11.	9.56 N

Viscosity

Viscosity is the property of fluid by virtue of which an internal force of friction comes into play when a fluid is in motion and which opposes the relative motion between its different layers. The backward dragging force, called viscous drag or viscous force, acts tangentially on the layers of the fluid in motion and tends to destroy its motion.

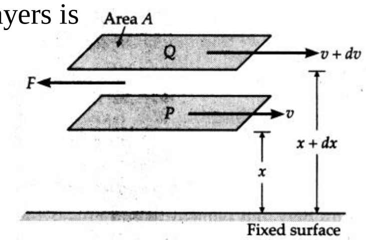
Cause of Viscosity: Consider a liquid moving slowly and steadily over a fixed horizontal surface. Each layer moves parallel to the fixed surface. The layer in contact with the fixed surface is at rest and the velocity of the very other layer increases uniformly upwards, as shown by arrows of increasing lengths in figure.



Coefficient of Viscosity

Suppose a liquid is flowing steadily in the form of parallel layers on a fixed horizontal surface. Consider two layers P and Q at distances x and x + dx from the solid surface and moving with velocities v and v + dv respectively. Then $\frac{dv}{dx}$ is the rate of change of velocity with distance in the direction of increasing distance and is called **velocity gradient.**

According to Newton, a force of viscosity F acting tangentially between two layers is



(i) Proportional to the area A of the layers in contact. $F \propto A$

(ii) Proportional to velocity gradient $\frac{dv}{dx}$ between the two layers. $F \propto \frac{dv}{dx}$

$$\therefore \quad F \propto A\frac{dv}{dx} \quad \text{or} \quad F = -\eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid.

It depends on the nature of the liquid and gives a measure of viscosity. Negative sign shows that the viscous force acts in a direction opposite to the direction of motion of the liquid.

If A = 1 and $\frac{dv}{dx} = 1$ then F = η (numerically)

Hence *coefficient of viscosity of a liquid may be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers each of unit area.*

Dimensions of η: $\eta = \frac{F}{A}.\frac{dx}{dv} \quad \therefore \quad [\eta] = \frac{MLT^{-2}.L}{L^2.LT^{-1}} = [ML^{-1}T^{-1}]$

Units of coefficient of viscosity

(i) The CGS unit of η is dyne s cm^{-2} or g cm^{-1} s^{-1} and is called **poise.**

$$1\,poise = \frac{1\,dyne}{1\,cm^2}.\frac{1cm}{1cm\,s^{-1}} = 1\,dyne\,s\,cm^{-2}$$

The coefficient of viscosity a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

(ii) The SI unit of η is N s m^{-2} or kg m^{-1} s^{-1} and is called ***decapoise or poiseuille.***

$$1\,poisecuille = \frac{1N}{1m^2}.\frac{1m}{1m\,s^{-1}} = 1N\,s\,m^{-2}$$

The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

Relation between poiseuille and poise

1 poiseuille or 1 decapoise = 1 Ns m^{-2}

$= (10^5 \text{ dyne}) \times s \times (10^2 \text{ cm})^{-2}$

$= 10$ dyne s cm^{-2} = 10 poise

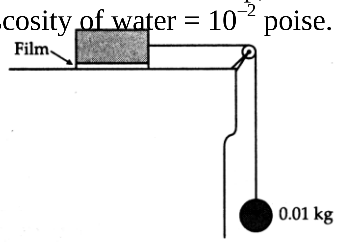
Subjective Assignment – V

Q.1 A metal plate 5 cm × 5 cm rests on a layer of castor oil 1 mm thick whose coefficient of viscosity is 1.55 Nsm^{-2}. Find the horizontal force required to move the plate with a speed of 2 cms^{-1}.

Q.2 A square metal plate of 10 cm side moves parallel to another plate with a velocity of 10 cms^{-1}, both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their distance apart?

Q.3 The velocity of water in a river is 180 kmh^{-1} near the surface. If the river is 5 m deep, find the shearing stress between horizontal layers of water. Coefficient of viscosity of water = 10^{-2} poise.

Q.4 A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as shown in figure. A liquid with a film thickness of 0.3 mm is placed between the plate and the table. When released the plate moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.



Q.5 A metal plate of area 0.02 m^2 is lying on a liquid layer of thickness 10^{-3} m and coefficient of viscosity 120 poise. Calculate the horizontal force required to move the plate with a speed of 0.025 ms^{-1}.

Answers

1.	0.0775 N	2.	0.05 cm	3.	10^{-3} Nm^{-2}
4.	3.45×10^{-3} Pa s	5.	6 N		

Comparison between Viscous Force and Solid Friction

Points of Similarly

(i) Both viscous force and solid friction come into play whenever there is relative motion.

(ii) Both oppose the motion.

(iii) Both are due to molecular attractions.

Points of Differences:

Sr. No.	***Viscous Force***	***Solid Friction***
1.	Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
2.	It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces
3.	It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

Effect of Temperature on Viscosity

(i) When a liquid is heated, the kinetic energy of its molecules increases and the intermolecular attractions become weaker. Hence *the viscosity of a liquid decreases with the increase in its temperature*. The coefficient of viscosity at any temperature, t

$$\eta_t = \frac{n_0}{1+\alpha t+\beta t^2}$$

where η_t and η_0 are the coefficients of viscosity at t^oC and 0^oC respectively, and α and β are temperature coefficient of viscosity.

(ii) Viscosity of gases is due to the diffusion of molecules from one moving layer to another. But the rate of diffusion of a gas is directly proportional to the square root of its absolute temperature, so viscosity of a gas increases with temperature as

$$\eta \propto \sqrt{T}$$

Effect of pressure

(i) Except water the viscosity of liquids increases with the increase in pressure. In case of water, viscosity decreases with the increase in pressure for first few hundred atmospheres of pressure.

(ii) The viscosity of gases is independent of pressure.

Practical Applications of the Knowledge of Viscosity

(i) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given machine in different seasons.

(ii) Liquids of high viscosity are used as buffers at railway stations.

(iii) The knowledge of viscosity is used in determining the shape and molecular weight of some organic liquids like proteins, cellulose, etc.

(iv) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.

(v) Millikan used the knowledge of viscosity in determining the charge on an electron.

Poiseuille's Formula

The volume of a liquid flowing out per second through a horizontal capillary tube of length *l*, radius r, under a pressure difference p applied across its ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}$$

This formula is called *Poiseulle's formula.*

Derivation of Poiseuille's formula on the basis of dimensional analysis

The volume Q of liquid flowing out per second through a capillary tube depends on

(i) coefficient of viscosity η of the liquid, (ii) radius r of the tube,

(iii) pressure gradient (p/*l*) set up along the capillary tube.

Let $Q \propto \eta^a r^b \left(\frac{p}{l}\right)^c$ or $Q = k\eta^a r^b \left(\frac{p}{l}\right)^c$ ... (i)

where k is a dimensionless constant. The dimensions of various quantities are

$[Q] = \frac{Volume}{Time} \frac{[L^3]}{[T]} = [L^3T^{-1}]$ $\left[\frac{p}{l}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$ $[\eta] = [ML^{-1}T^{-1}]$, $[r] = [L]$

Substituting these dimensions in equation (1), we get

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c \quad \text{or} \quad [M^0L^3T^{-1}] = [M^{a+c} L^{-a+b-2c} T^{-a-2c}]$$

Equating the powers of M, L and T on both sides, we get

$$a + c = 0$$
$$-a + b - 2c = 3$$
$$-a - 2c = -1$$

On solving, we get a = – 1, b = 4, and c = 1

$$\therefore \quad Q = k\eta^{-1} r^4 \left[\frac{p}{l}\right]^1 = \frac{kpr^4}{\eta l}$$

Experimentally k is found to be π/8

$$\therefore \quad Q = \frac{\pi pr^4}{8\eta l}$$

This is Poiseuille's formula for the flow of a liquid through a capillary tube.

Subjective Assignment – VI

Q.1 A capillary tube 1 mm in diameter and 20 cm in length is fitted horizontally to a vessel kept full of alcohol. The depth of the centre of capillary tube below the surface of alcohol is 30 cm. If the viscosity and density of alcohol are 0.012 cgs unit and 0.8g cm^{-3} respectively, find the amount of the alcohol that will flow out in 5 minutes. Given that g = 980 cms^{-2}.

Q.2 In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3 m above needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3}.

Q.3 Two tubes A and B of lengths 100 cm and 50 cm have radii 0.1 mm and 0.2 mm respectively. If a liquid passing through the two tubes is entering A at a presence of 80 cm of mercury and leaving B at a pressure of 76 cm of mercury, determine the pressure at the junction of A and B.

Q.4 Two capillary tubes AB and BC are joined end to end at B. AB is 16 cm long and of diameter 4 mm whereas BC is 4 cm long and of diameter 2 mm. The composite tube is held horizontally with A connected to a vessel of water giving a constant head of 3 cm and C is open to the air. Calculate the pressure difference between B and C.

Q.5 The level of liquid in a cylindrical vessel is kept constant at 30 cm. It has three identical horizontal tubes of length 39 cm, each coming out at heights 0, 4, 8 cm respectively. Calculate the length of a single overflow tube of the same radius as that of identical tubes which can replace the three when placed horizontally at bottom of the cylinder.

Q.6 Three capillary tubes of the same radius r but of lengths l_1, l_2 and l_3 are fitted horizontally to the bottom of a tall vessel containing a liquid at constant head and flowing through these tubes. Calculate the length of a single outflow tube of the same radius r which can replace the three capillaries.

Q.7 Water at 20° is escaping from a cistern by way of a horizontal capillary tube 10 cm long and 0.4 mm in diameter, at a distance of 50 cm below the free surface of water in the cistern. Calculate the rate at which the water is escaping. Coefficient of viscosity of water is 20 decapoise.

Q.8 Alcohol flows through two capillary tubes under a constant pressure head. The diameters\of the two tubes are in the ratio of 4 : 1 and the lengths are in the ratio 4 : 1. Compare the rates of flow of alcohols through the two tubes.

Answers

1.	38.4 g	2.	0.238 poise	3.	76.12 cm of Hg
4.	2.4 cm	5.	15 cm	6.	$\frac{l_1 l_2 l_3}{l_2 l_3 + l_1 l_3 + l_1 l_2}$
7.	3.08×10^{-8} m^3 s^{-1}	8.	1024 : 1		

Stokes' Law

According to Stokes' law, the backward viscous force acting on a small spherical body of radius r moving with uniform velocity v through fluid of viscosity η is given by

$$F = 6\pi\eta rv$$

Derivation of Stokes' law : The viscous force F acting on a sphere moving through a fluid may depend on

(i) coefficient of viscosity η of the fluid

(ii) radius r of the spherical body

(iii) velocity v of the body

Let $F = k\eta^a r^b v^c$ .... (i)

where k is dimensionless constant. The dimensions of various quantities are

$[F] = [MLT^{-2}]$, $[\eta] = [ML^{-1}T^{-1}]$ $[r] = [L]$, $[v] = [LT^{-1}]$

Substituting these dimensions in equation (1), we get

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$= [M^a L^{-a+b+c} T^{-a-c}]$$

Equating the powers M, L and T on both sides, we get

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

On solving, $a = b = c = 1$

$\therefore$ $F = k\eta r v$

For a small sphere, k is found to be equal to 6π.

Hence $F = 6\pi \eta rv$ *This proves Stokes' law.*

Conditions under which Stokes' law is valid:

(i) The fluid through which the body moves has infinite extension.

(ii) The body is perfectly rigid and smooth.

(iii) There is no slip between the body and fluid.

(iv) The motion of the body does not give rise to turbulent motion and eddies. Hence motion is streamlined.

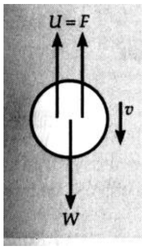
(v) The size of the body is small but it is larger than the distance between the molecules of the liquid. Thus the medium is homogeneous and continuous for such a body.

Terminal Velocity

When a body falls through a viscous fluid, it produces relative motion between its different layers. As a result, the body experiences a viscous force which tends to retard its motion. As the velocity of the body increase, the viscous force ($F = 6\pi \eta rv$) also increases. A stage is reached, when the weight of the body becomes just equal to the sum of the up thrust and viscous force. Then no net force acts on the body and it begins to move with a constant velocity. *The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity.*

Expression for terminal velocity

Consider a spherical body of radius r falling through a viscous liquid of density ρ and coefficient of viscosity η. Let σ be the density of the body. As the body falls, the various forces acting on the body are as shown in figure. These are



(i) Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \sigma g$$

(ii) Upward thrust equal to the weight of the liquid displaced.

$$U = \frac{4}{3}\pi r^3 \rho g$$

(iii) Force of viscosity F acting in the upward direction. According to Stokes' law, $F = 6\pi\eta rv$

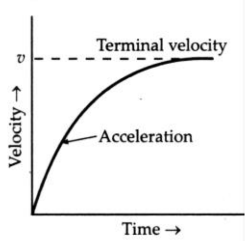
Clearly, the force of viscosity increases as the velocity of the body increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then the body begins to fall with a constant maximum velocity, called *terminal velocity*.

When the body attains terminal velocity v,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \sigma g$$

or $$6\pi\eta r v = \frac{4}{3}\pi r^3 (\sigma - \rho) g$$ or $$v = \frac{2}{9}.\frac{r^2 (\sigma - \rho) g}{\eta}$$



This is the expression for terminal velocity.

Discussion of the result:

(i) Figure shows how the velocity of a small sphere dropped from rest into a viscous medium varies with time. Initially the body is accelerated and after some time, it acquires terminal velocity v.

(ii) *The terminal velocity is directly proportional to the radius of the body*. That is why bigger rain drops fall with a larger velocity compared to the smaller rain drops.

(iii) The terminal velocity is directly proportional the difference of the densities of the body and the fluid, i.e. (σ – ρ)

(a) If σ – ρ, the body will attain terminal velocity in the downward direction.

(b) If σ – ρ, the terminal velocity will be negative i.e., the body will rise through the fluid. That is why, air bubble in a liquid and clouds in a sky are seen to move in the upward direction.

(c) If σ = ρ, the body remains suspended in the fluid.

(iv) The *terminal velocity is inversely proportional to the coefficient of viscosity of the fluid.* The more viscous the fluid, the smaller the terminal velocity attained by a body.

(v) The *terminal velocity is independent of the height through which a body is dropped.*

(vi) Knowing the values of ρ, σ, r and v, we can determine the coefficient of viscosity η as follows:

$$\eta = \frac{2}{9}\frac{r^2 (\sigma - \rho) g}{v}$$

Subjective Assignment – VII

Q.1 An iron ball of radius 0.3 cm falls through a column of oil of density 0.94 g cm^{-3}. It is found to attain a terminal velocity of 0.5 cms^{-1}. Determine the viscosity of oil. Given that density of iron is 7.8 g cm^{-3}.

Q.2 Eight rain drops of radius 1 mm each falling down with terminal velocity of 5 cms^{-1} coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.

Q.3 Show that if n equal rain droplets falling through air with equal steady velocity of 10 cms^{-1} coalesce, the resultant drop attains a new terminal velocity of 10 $n^{2/3}$ cms^{-1}.

Q.4 A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of the initial acceleration, show that the time taken to attain the terminal velocity is independent of the fluid density.

Q.5 A gas bubble of diameter 2 cm rises steadily at the rate of 25 mms^{-1} through a solution of density 2.25g cm^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of the gas.

Q.6 The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cms^{-1}. Compute the viscosity of the oil at 20°C. Density of oil = 1.5×10^3 kg m^{-3}, density of copper = 8.9×10^3 kg m^{-3}.

Q.7 A spherical glass ball of mass 1.34×10^{-4} kg and diameter 4.4×10^{-3} m takes 6.4 s to fall steadily through a height of 0.381 m inside a large volume of oil of specific gravity 0.943. Calculate the viscosity of oil.

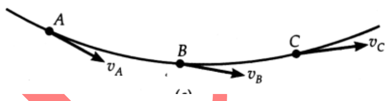
Answers

1.	268.9 poise	2.	20 cms^{-1}	3.	$10\, n^{2/3}\, cms^{-1}$
4.	$\frac{4}{9}.\frac{r^2\rho}{\eta}$	5.	1960 poise	6.	0.992 decapoise
7.	0.8025 Nsm^{-2}				

Streamline and Turbulent Flows

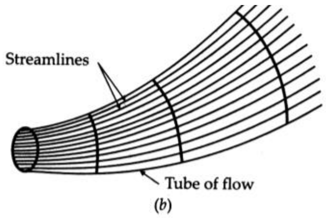
Streamline flow

When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow. A streamline may be defined as the path, the tangent to which at any point gives the direction of the flow of liquid at the point.



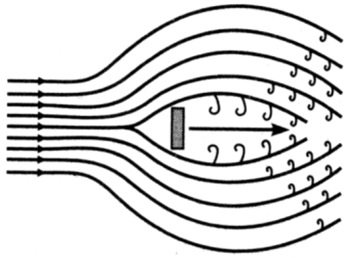
Tube of flow

A bundle of streamlines forming a tubular region is called a tube of flow. The boundary of such a tube is always parallel to the velocity of fluid particles. No fluid can cross the boundaries of a tube of flow, and the flow behaves somewhat like a tube. In a steady flow, the shape of the flow tube does not change with time.



Turbulent flow

When the liquid velocity exceeds a certain limiting value, called *critical velocity*, the liquid flow becomes zig–zag. The path and the velocity of a liquid particle changes continuously, haphazardly. This flows is called *turbulent flow*. It is accompanied by random, irregular, local circular currents called vortices.



Properties of Streamlines

(i) In a steady flow, *no two streamlines can cross each other*. If they do so, the fluid particle at the point of intersection will have two different directions of flow. This will destroy the steady nature of the fluid flow.

(ii) The tangent at any point on the streamline gives the direction of velocity of fluid particle at that point.

(iii) Greater the number of streamlines passing normally through a section of the fluid, larger is the fluid velocity at the section.

(iv) Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.

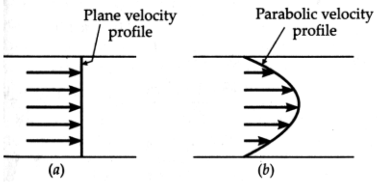
Laminar Flow

When the velocity of the flow of a liquid is less than its critical velocity, the liquid flows steadily. Each layer of the liquid slides over the other layer. It behaves as if different lamina are sliding over one another.

Such a flow is called laminar flow. The surface obtained by joining the heads of the velocity vectors for the particles in a section of a flowing liquid is called a *velocity profile.*

(i) Velocity profile for a non–viscous liquid

In case of a non–viscous liquid, the velocity of all the particles at any section of a pipe is same, so the velocity profile is plane as shown in figure.



(ii) Velocity profile of a viscous liquid

When a viscous liquid flows through a pipe, the velocity of layer at the axis is maximum, the velocity decreases as we go towards the wall of the pipe and becomes zero for the layer in contact with the pipe. Hence the velocity profile for a viscous liquid is parabolic, as shown in figure.

Critical Velocity

The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent. The critical velocity v_c of a liquid flowing through a tube depends on

(i) coefficient of viscosity of the liquid (η) (ii) density of the liquid (ρ)

(iii) diameter of the tube (D)

Let $v_c = k\,\eta^a \rho^b D^c$

where k is a dimensionless constant. Writing the above equation in dimensional form, we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^a\,[ML^{-3}]^b\,[L]^c$$

$$[M^0LT^{-1}] = [M^{a+b}L^{-a-3b+c}T^{-a}]$$

Equating powers of M, L and T, we get

$a + b = 0$ $-a - 3b + c = 1$ $-a = -1$

On solving, we get a = 1, b = – 1, c = – 1

$$\therefore \quad v_c = k\eta\rho^{-1}D^{-1} = \frac{k\eta}{\rho D}$$

Clearly, the critical velocity v_c will be large if η is large, and ρ and D are small. So we can conclude that

(i) The flow of liquids of higher viscosity and lower density through narrow pipes tends to be streamlined.

(ii) The flow of liquids of lower viscosity and higher density through broad pipes tends to become turbulent, because in that case the critical velocity will be very small.

Reynold's Number

It is dimensionless parameter whose value decides the nature of flow of a liquid through a pipe It is given by

$$R_e = \frac{\rho v D}{\eta}$$

where ρ = density of the liquid v = velocity of the liquid

η = coefficient of viscosity of the liquid D = diameter of the pipe.

Importance of Reynold's Number

If R_e lies between 0 and 2000, the liquid flow is streamlined or laminar. If $R_c > 3000$, the liquid flow is turbulent. If R_e lies between 2000 and 3000, the flow of liquid is unstable, it may change from laminar to turbulent and *vice–versa*. The exact value at which turbulence sets in a fluid is called *critical Reynold's number*.

Physical significance of Reynold's number

Consider a narrow tube having a cross–sectional area A. Suppose a fluid flows through it with a velocity v for a time interval Δt. Length of the fluid = Velocity × time = v Δt

Volume of the fluid flowing through the tube in time Δt = Av Δt

Mass of the fluid,

$$\Delta m = \text{Volume} \times \text{density} = Av\,\Delta t \times \rho$$

Inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A} = \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av\,\Delta t\,\rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta\frac{v}{D} \qquad \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta v/D} = \frac{\rho v D}{\eta} = R_e$$

Thus Reynold's number represents the ratio of the inertial force per unit area to viscous force per unit area.

Subjective Assignment – VIII

Q.1 The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa s. After some time the flow rate is increased to 3 L/min. Characteristic the flow for both the flow rates.

Q.2 What should be the maximum average velocity of water in a tube of diameter 0.5 cm so that the flow is laminar? The viscosity of water is 0.00125 Ns m^{-2}.

Q.3 Water flows at a speed of 6 cms^{-1} through a pipe of tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.01 poise. What is the nature of flow?

Q.4 Find the critical velocity for air flowing through a tube of 2 cm diameter. For air, $\rho = 1.3 \times 10^{-3}$g cm^{-3} and $\eta = 181 \times 10^{-6}$ poise.

Answers

1.	steady to turbulent	2.	0.5 ms^{-1}
3.	$R_e = 1200 < 2000$, so flow is laminar	4.	140 cms^{-1}

Ideal Fluid

An ideal fluid is one which is non–viscous, incompressible, and its flow is steady and irrotational. Thus an ideal fluid has the following features connected with its flow:

(i) **Steady flow:** In a steady flow, the fluid velocity at each point does not change with time, either in magnitude or direction.

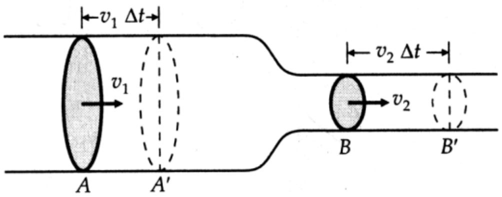
(ii) **Incompressible flow:** The density of the fluid remains constant during its flow.

(iii) **Non–viscous flow:** The fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

(iv) **Irrotational flow:** This means that there is no angular momentum of the fluid about any point. A very small wheel placed at any point inside such a fluid does not rotate about its cent re of mass.

Equation of Continuity

Consider a non–viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross–section. Let a_1 be the area of cross–section, v_1 fluid velocity, ρ_1 fluid density at section A; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.



An m = Volume × density

= Area of cross– section × length × density

∴ Mass of fluid that flows through section A in time Δt,

$m_1 = a_1 v_1 \Delta t \, \rho_1$

Mass of fluid that flows through section B in time Δt,

$m_2 = a_2 v_2 \Delta t \, \rho_2$

By conservation of mass, $m_1 = m_2$ or $a_1 v_1 \Delta t \rho_1 = a_2 v_2 \Delta t \rho_2$

As the fluid is incompressible, so $\rho_1 = \rho_2$ and hence $a_1 v_1 = a_2 v_2$ or av = constant

This *is the* ***equation of continuity.*** *It states that during the streamlined flow of the non–viscous and incompressible fluid through a pipe of varying cross–section, the product of area of cross–section and the normal fluid velocity* (av) *remains constant throughout the flow.*

NOTE

- The equation of continuity is a special case of the law of conservation of mass.
- The equation of continuity shows that $v \propto 1/a$, i.e., *the liquid velocity at any section of the pipe is inversely proportional to the area of cross–section of the pipe at that section.* This explains why the speed of water emerging from a PVC pipe increases when we press its outlet with our fingers and hence decrease its area of cross–section.

Deep water runs slowly

As the depth of water in a river or a steam increases, the area of cross–section available to the flowing water increases. Consequently, velocity decreases in accordance with the equation of continuity. Thus deep water runs slowly.

Energy of a Fluid in a Steady Flow

A liquid in a steady flow can have three kinds of energy (i) kinetic energy (ii) potential energy and (iii) pressure energy.

(i) ***Kinetic energy :*** *The energy possessed by a liquid by virtue of its motion is called its kinetic energy.*

$$K.E. = \frac{1}{2} m v^2$$

where m is the mass of the liquid and *v* is the velocity of the liquid.

K.E. per unit mass of the liquid = $\frac{1}{2} v^2$

The kinetic energy per unit weight of the liquid is known as *the velocity head.*

∴ Velocity head = $\frac{v^2}{2g}$ K.E. per unit volume = $\frac{1}{2}\frac{mv^2}{V} = \frac{1}{2}\rho v^2$

(ii) ***Potential energy:*** *The energy possessed by a liquid by virtue of its position above the earth's surface is called its potential energy.*

P.E. = mgh

where h is the average height of the liquid from the ground level.

P.E. per unit mass of the liquid = gh

The potential energy per unit weight of the liquid is known as the *potential head.*

$\therefore$ Potential head = $\frac{mgh}{mg} = h$ P.E. per unit volume = $\frac{mgh}{V} = \rho gh$

(iii) ***Pressure energy:*** *The energy possessed by a liquid by virtue of its pressure is called its pressure energy.* A liquid under pressure can do work and so possesses energy.

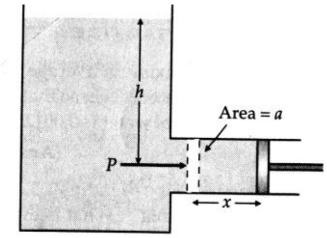
Let P be the pressure exerted by the liquid on a frictionless piston of area a. Suppose the piston moves through distance x under the pressure P.

The work done is

W = Force × distance = Pressure × area × distance = Pax = PV

where V = ax = volume swept by the piston

This work done is stored as the pressure energy of liquid of volume V.



$\therefore$ Pressure energy of volume V = PV

Pressure energy per unit volume

$$= \frac{PV}{V} = P = \text{Excess pressure}$$

Pressure energy per unit mass = $\frac{PV}{m} = \frac{P}{\rho}$

Pressure energy per unit weight of the liquid is called *pressure head.*

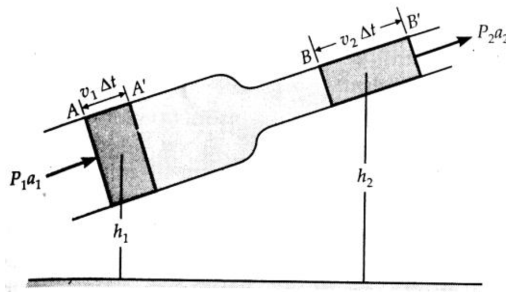
Pressure head = $\frac{P}{\rho g}$

Bernoulli's Principle

Bernoulli's principle states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non–viscous fluid in a streamlined irrotational flow remains constant along a streamline.

Mathematically, it can be expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gh = constant$$



Proof: Consider a non–viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross–section. Let a_1 be the area of cross–section at A, v_1 the fluid velocity, p_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2, P_2 and h_2 be the values of the corresponding quantities at B.

Let ρ be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume × density = Area of cross–section × length × density

or $m = a_1 v_1 \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$ .... (1)

or $a_1 v_1 = a_2 v_2$ ..... (2)

∴ Change in K.E. of the fluid = K.E. at B – K.E. at A

$$= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}a_1 v_1 \Delta t\, \rho (v_2^2 - v_1^2)$$

Change in P.E. of the fluid = P.E. at B – P.E. at A = mg $(h_2 - h_1) = a_1 v_1 \Delta t\, \rho\, g\, (h_2 - h_1)$

Net work done on the fluid = Work done on the fluid at A – work done by the fluid at B

$= P_1 a_1 \times v_1 \Delta t - P_2 a_2 \times v_2 \Delta t \quad = P_1 a_1 v_1 \Delta t - P_2 a_1 v_1 \Delta t \quad = a_1 v_1 \Delta t (P_1 - P_2)$

By conservation of energy,

Net work done on the fluid = Change in K.E. of the fluid + Change in P.E. of the fluid

$$\therefore \quad a_1 v_1 \Delta t (P_1 - P_2) = \frac{1}{2}a_1 v_1 \Delta t\, \rho (v_2^2 - v_1^2) + a_1 v_1\, \Delta t \rho g (h_2 - h_1)$$

Dividing both sides by $a_1 v_1 \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{or} \quad P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad \text{... (3)}$$

This proves Bernoulli's principle according to which the total energy per unit volume remains constant. Equation (3) can also be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

This is another form of *Bernoulli's principle according to which the sum of pressure head, velocity head and gravitational head remains constant in the streamline flow of an ideal fluid.*

Limitations of Bernoulli's equation

1. Bernoulli's equation ideally applies to fluids with zero viscosity or non–viscous fluids. In case of viscous fluids, we need to take into account the work done against viscous drag.
2. Bernoulli's equation has been derived on the assumption that there is no loss of energy due to friction. But in practice, when fluids flow, some of their kinetic energy gets converted into heat due to the work done against the internal forces of friction or viscous forces.
3. Bernoulli's equation is applicable only to incompressible fluids because it does not take into account the elastic energy of the fluids.
4. Bernoulli's equation is applicable only to streamline flow of a fluid and not when the flow is turbulent.
5. Bernoulli's equation does not take into consideration the angular momentum of the fluid. So it cannot be applied when the fluid flows along a curved path.

NOTE

♦ Bernoulli's principle is a fundamental principle of fluid dynamics based on the law of conservation of energy.

♦ In Bernoulli's equation : $P + \rho g h + \frac{1}{2}\rho v^2$ = constant, the term $(P + \rho g h)$ is called *static pressure*, because it is the pressure of the fluid even if it is at rest, and the term $\frac{1}{2}\rho v^2$ is the *dynamic pressure* of fluid which is the pressure by virtue of its velocity *v*. So Bernoulli's equation can be written as
Static pressure + Dynamic pressure = Constant

- If a liquid is flowing through a horizontal tube, h remains constant and we can write

  $$P+\frac{1}{2}\rho v^2=constant$$

  This shows that if v increases, P decreases and vice versa. Thus for the *streamline flow of an ideal liquid flowing horizontally, the pressure decreases where velocity increases and vice versa.* This is an important aspect of Bernoulli's principle which finds many applications.

Torricelli's Law of Efflux

Consider a tank containing a liquid of density ρ with a small hole on its side at a height y_1 from the bottom. Let y_2 be the height of the liquid surface from the liquid surface from the bottom and P be the air pressure above the liquid surface.

It A_1 and A_2 are the cross–sectional areas of the side hole and the tank respectively, and v_1 and v_2 are the liquid velocities at points 1 and 2, then from the equation of continuity, we get

$$A_1v_1=A_2v_2 \quad \text{or} \quad v_2=\frac{A_1}{A_2}v_1$$

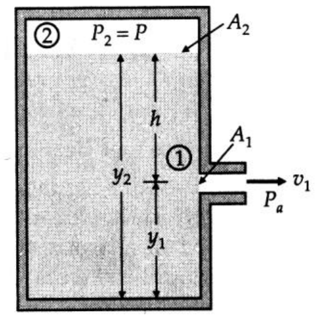
As $A_2 >> A_1$, so the liquid may be taken at rest at the top, i.e., $v_2 \simeq 0$. Applying Bernoulli's equation at points 1 and 2, we get

$$P_a+\frac{1}{2}\rho v_2^2+\rho g y_1=P+\rho g y_2$$

or $$\frac{1}{2}\rho v_1^2=\rho g(y_2-y_1)+(P-P_a)$$

If we take $y_2 - y_1 = h$, then

$$\frac{1}{2}\rho v_1^2=\rho gh+(P+P_a)$$



or $$v_1=\sqrt{2gh+\frac{2(P-P_a)}{\rho}}$$

Special cases (i) when $P >> P_a$, the term 2 gh may be ignored.

$$v_1=\sqrt{\frac{2(P-P_a)}{\rho}}$$

Thus the speed of efflux is determined by container pressure P. Such a situation exists in rocket propulsion.

(ii) *When the tank is open to the atmosphere,* $P=P_a$ and $v_1=\sqrt{2gh}$

Thus, *the velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called **Torricelli's law.***

The Venturimeter

It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle. It is also called **flow meter** or **venture tube.**

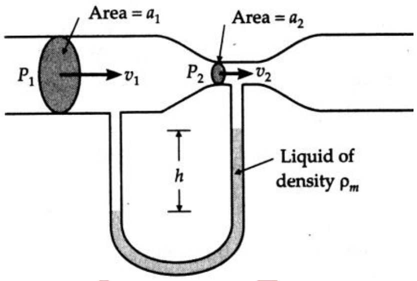
Construction

It consists of a horizontal tube having wider opening of cross–section a_1 and a narrow neck of cross–section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.

Working

Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity, $a_1 v_1 = a_2 v_2$ or $\frac{a_1}{a_2} = \frac{v_2}{v_1}$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) \qquad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right] \qquad = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

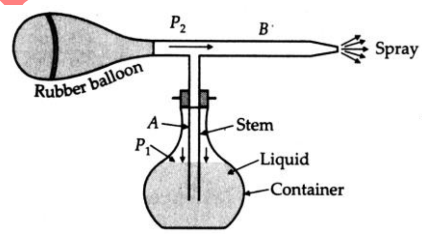
$$P_1 - P_2 = h\rho_m g$$

$$\therefore \quad h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right) \qquad \therefore \quad v_1 = \sqrt{\frac{2h\rho_m g}{\rho} \times \frac{a_2^2}{(a_1^2 - a_2^2)}}$$

Volume of the liquid flowing out per second, $V = a_1 v_1 = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$

Atomizer or Sprayer

The working of an atomizer which is used to spray liquids is based on Bernoulli's principle. Figure shows the essential parts of an atomizer. When the rubber balloon is pressed, the air rushes out of the horizontal tube B decreasing the pressure to P_2 which is less than the atmospheric pressure P_1 in the container. As a result, the liquid rises up in the vertical tube A. When it collides with the high speed air in tube B, it breaks up into a fine spray.

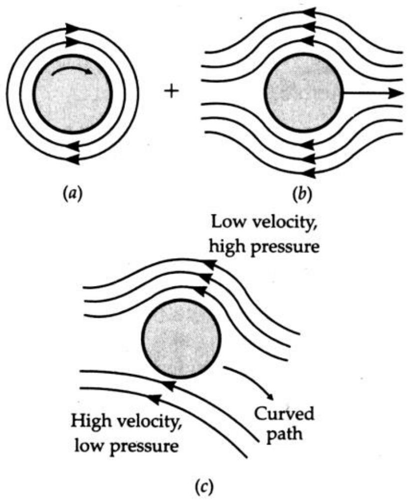


Dynamic Lift

Dynamic lift is the force that acts on a body, such as aeroplane wing, a hydrofoil or a spinning ball, *by virtue of its motion through a fluid*. It is responsible for the curved path of a spinning ball and the lift of an aircraft wing.

Curved path of a spinning ball : Magnus effect

When a ball is thrown horizontally with a large velocity and at the same time given a twisting motion to cause a spin, it deviates from its usual parabolic trajectory of spin free motion. This deviation can be explained on the basis of bernoulli's principle.

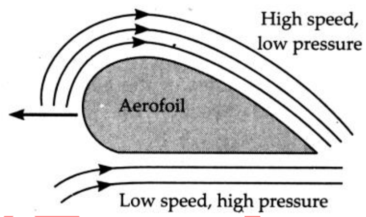


When the ball spins about an axis perpendicular to its horizontal motion, it carries with itself an air of layer due to viscous drag. The streamlines around it are in the form of concentric circles, as shown in figure. When the ball moves forward with velocity v, the air ahead of the ball rushes backward with velocity v to fill the space left empty by the ball. Thus the streamlines in air due to translatory motion of the ball are of the form shown in figure. The layer above the ball moves in a direction opposite to that of the spinning ball, so the resultant velocity decreases and hence pressure increases in accordance with Bernoulli's principle.

The layer below the ball moves in the direction of spin, the resultant velocity increases and hence pressure decreases. Due to the difference of pressure on the two sides of the ball, the ball curves downwards in the direction of spin.

The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect.

Aerofoil: Lift of an aircraft wing



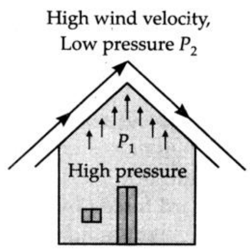
Aerofoil is the name given to a solid object shaped to provide an upward vertical force as it moves horizontally through air. This upward force (dynamic lift) makes aeroplanes fly. The cross–section of the wing of an aeroplane looks like an aerofoil. The wing is so designed that its upper surface is more curved (and hence longer) than the lower surface and the front edge is broader than the rear edge.

As the aircraft moves, the air moves faster over the upper surface of the wing than on the bottom. According to Bernoulli's principle, the air pressure above the upper surface decreases below the atmospheric pressure and that on the lower surface increases above the atmospheric pressure. The difference in pressure provides an upward lift, called *dynamic lift*, to the aircraft.

Blood flow and heart attack

In persons suffering with advanced heart condition, the artery gets constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction, a greater demand is placed on the activity of the heart. The speed of blood flow increases in this region. From Bernoulli's principle, the inside pressure drops and the artery may collapse due to external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops leading to a repeat collapse. This phenomenon is called vascular flutter which can be heard on a stethoscope. This may result in a heart attack.

Blowing off the roof during wind storm



During certain wind storm of cyclone, the roofs of some houses are blown off without damaging the other parts of the house. The high wind blowing over the roof creates a low pressure P_2 in accordance with Bernoulli's principle. The pressure P_1 below the roof is equal to the atmospheric pressure which is larger than P_2. The difference of pressure $(P_1 - P_2)$ causes an upward thrust and the roof is lifted up. Once the roof is lifted up, it is blown off with the wind.

Assignment – IX

Q.1 Water flows through a horizontal pipe of varying area of cross–section at the rate of 10 cubic metre per minute. Determine the velocity of water at a point where radius of pipe is 10 cm.

Q.2 Water flows through a horizontal pipe whose internal diameter is 2.0 cm at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1}?

Q.3 At what speed will the velocity of a stream of water be equal to 20 cm of mercury column?

Q.4 Calculate the total energy possessed by one kg of water at a point where the pressure is 20 gf/ mm^2, velocity is 0.1 ms^{-1} and the height is 50 cm above the ground level.

Q.5 The reading of pressure meter attached with a closed pipe is 3.5×10^5 Nm^{-2}. On opening the valve of the pipe, the reading of the pressure meter is reduced to 3.0×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

Q.6 A fully loaded Boeing aircraft 747 has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings. (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is $\rho = 1.2$ kg m^{-3} and g = 9.81 ms^{-2}.

Q.7 Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 ms^{-1} and the pressure is 2.0×10^4 Nm^{-2}. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively.

Q.8 The cross–sectional area of water pipe entering the basement is 4×10^{-4} m^2. The pressure at this point is 3×10^5 Nm^{-2} and the speed of water is 2 ms^{-1}. This pipe tapers to a cross–sectional area of 2×10^{-4} m^2 when it reaches second floor 8m above. Calculate the speed and pressure at the second floor.

Q.9 The pressure difference between two points along a horizontal pipe, through which water is flowing, is 1.4 cm of mercury. If, due to non–uniform cross–section, the speed of flow of water at the point of greater cross–section is 60 cm s^{-1}, calculate the speed at the other point.

Q.10 A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air? (sp. gr. of alcohol = 0.8 and density of air = 1 kg m^{-3}).

Q.11 A pitot tube is fixed in a main pipe of diameter 20 cm and difference of pressure indicated by the gauge is 5 cm of water column. Find the volume of water passing through the main pipe in one minute.

Q.12 A cylinder of height 20 m is completely filled with water. Find the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom. Given g = 10 ms^{-2}.

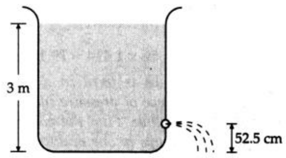
Q.13 A boat strikes an under water rock which punctures a hole 5 cm in diameter in its hull which is 1.5m below the water line. At what rate in litre per second does water enter?

Q.14 A drum of 30 cm radius has a capacity of 220 dm^3 of water. It contains 198 dm^3 of water and is placed on a solid block of exactly the same size as of drum. If a small hole is made at lower end of drum perpendicular to its length, find the horizontal range of water on the ground in the beginning.

Q.15 **Blood Velocity :** The flow of blood in a large artery of an anesthetized dog is diverted through a Venturimeter. The wider part of the meter has a cross–sectional area equal to that of the artery, A = 8 mm^2. The narrower part has an area a = 4 mm^2. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?

Q.16 A horizontal tube has different cross–sectional areas at points A and B. The diameter of A is 4 cm and that of B is 2 cm. Two manometer limbs are attached at A and B. When a liquid of density 8.0 g cm^{-3} flows through the tube, the pressure difference between the limbs of the manometer is 8 cm. Calculate the rate of flow of the liquid in the tube.

Q.17 Water is filled in a cylindrical container to a height of 3 m, as shown in figure. The ratio of the cross–sectional area of the orifice and the beaker is 0.1. Find the speed of the liquid coming out from the orifice.



Q.18 In a normal adult, the average speed of the blood through the aorta (which has a radius of 0.9 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius
0.5 cm. Calculate the speed of blood through the arteries.

Q.19 Water flows into a horizontal pipe whose one end is closed with a valve and the reading of a pressure gauge attached to the pipe is 3×10^5 Nm^{-2}. This reading of the pressure gauge falls to 1×10^5 Nm^{-2} when the valve is opened. Calculate the speed of water flowing into the pipe.

Q.20 Water flows at the rate of 4 litres per second through an orifice at the bottom of tank which contains water 720 cm deep. find the rate of escape of water if additional pressure of 16 kg f/cm^2 is applied at the surface of water.

Answers

1.	5.303 ms^{-1}	2.	1.0 cm	3.	7.3756 ms^{-1}
4.	200.905 J	5.	10 ms^{-1}	6.	(a) 6.5×10^3 Nm^{-2}, (b) 8 %
7.	1 m/s, 2.75×10^4 Nm^{-2}	8.	4 ms^{-1}, 2.156×10^5 Nm^{-2}	9.	2 ms^{-1}
10.	79.18 ms^{-1}	11.	1.866 m^3	12.	20 ms^{-1}
13.	10.65 litres s^{-1}	14.	147.6 cm	15.	0.125 ms^{-1}
16.	406 cm^3 s^{-1}	17.	7.07 ms^{-1}	18.	0.036 ms^{-1}
19.	20 ms^{-1}	21.	19.28 litre s^{-1}		

Cohesive and Adhesive Forces

(i) **Cohesive force:** *It is the force of attraction between the molecules of the same substance.*

Example: Solids have definite shape and size due to strong forces of cohesion amongst their molecules.

(ii) **Adhesive force:** *It is the force of attraction between the molecules of two different substances.*

Example: It is due to force of adhesion that ink sticks to paper while writing.

Water wets the walls of its glass container because the force of adhesion between water and glass is greater than the force of cohesion between the water molecules. On the contrary, mercury does not wet glass because the force of cohesion between the mercury molecules is much greater than the force of adhesion between mercury and glass.

Molecular Range

It is the maximum distance upto which a molecule can exert some appreciable force of attraction on other molecules. It is of the order of 10^{-9} m in solids and liquids.

Sphere of Influence: *A sphere drawn around a molecule as centre and with a radius equal to the molecular range is called the sphere of influence of the molecule.* The molecule at the centre attracts all the molecules lying in its sphere of influence.

Surface Film: *A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.*

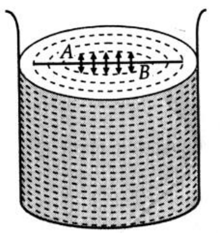
Surface Tension

A steel needle may be made to float on water though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called surface tension.

Surface tension *is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.*

Imagine a line AB on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side. The force acting on this line is proportional to the length of this line. If l is the length of the imaginary line and F the total force on either side of the line, then

$F \propto l$ or $F = \sigma l$



or $\sigma = \frac{F}{l}$

or Surface tension = $\frac{Force}{Length}$

Surface tension is measured as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.

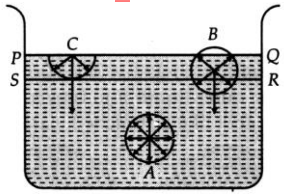
Units and dimensions of surface tension:

SI unit of surface tension = Nm^{-1} CGS unit of surface tension = dyne cm^{-1}

Dimensions of surface tension $= \frac{[Force]}{[Length]} = \frac{MLT^{-2}}{L} = [MT^{-2}]$

Molecular Theory of Surface Tension

In figure, PQRS is the surface film of a liquid. Consider the molecule A well inside the liquid. It is attracted equally in all directions by the molecules lying in its sphere of influence. Net force on such a molecule is zero.

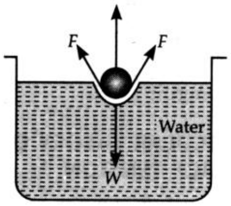


Now consider molecule B lying inside the surface film. Its sphere of influence lies partly outside. This molecule experiences less force upward and more force downward by the molecules in its sphere of influence. For molecule C, half its sphere of influence lies above the surface. The resultant downward force on such a molecule is maximum.

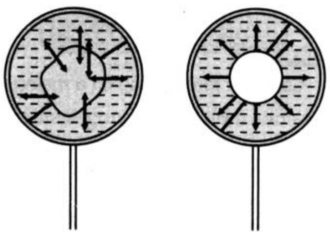
Due to this downward force, the potential energy of the molecules of the surface film is higher than those lying well inside the liquid. For a system to be stable, potential energy must be minimum. For the surface film to have minimum energy, the number of molecules in it must be minimum. Thus the surface film tends to have minimum surface area. As a result, the free surface of a liquid at rest behaves like an elastic stretched membrane.

Some Phenomena Based on Surface Tension

(i) **Needle supported on water surface:** Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating. The floating needle causes a little depression. The forces F, F due to surface tension of the curved surface are inclined as shown in figure. The vertical components of these two forces support the weight of the needle.



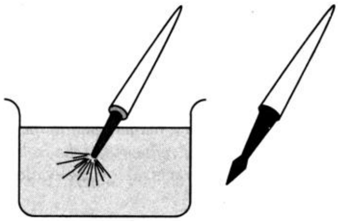
(ii) **Endless wet thread on a soap film:** If we take a circular frame of a stiff wire and dip it into a soap solution, a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape. But when the film is pricked at the centre, the loop is stretched outward and takes a symmetrical circular shape.



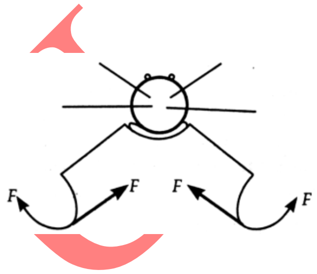
(iii) **Rain drops are generally spherical in shape:** Due to surface tension, the rain drops tend to minimize their surface area and the surface area of a sphere is minimum for a given volume.

(iv) **Small mercury droplets are spherical and larger ones tend to flattened:** Small mercury droplets are spherical because the forces of surface tension tend to reduce their area to a minimum value and a sphere has minimum surface area for a given volume. Larger drops of mercury are flattened due to the large gravitational force acting on them.

(v) **The hair of a painting brush cling together when taken out of water:** This is because the water films formed on them tend to contract to minimum area.



(vi) **A bug floats on water due to surface tension:** As shown in figure, a bug bends its legs on the surface of water such that the deformed surface gives rise to forces of surface tension which act tangential to the deformed surfaces. The weight of the bug is balanced by the upward components of these forces of surface tension.



(vii) **Oil spreads on cold water but remains as a drop on hot water:** This is because the surface tension of oil is less than that of the cold water but it is greater than that of the hot water.

Surface Energy

The free surfaced of a liquid possesses minimum area due to surface tension. To increase the surface area, molecules have to be brought from interior to the surface. Work has to be done against the forces of at traction. This work is stored as the potential energy of the molecules on the surface. So the molecules at the surface have extra energy compared to the molecules in the interior.

The extra energy possessed by the molecules of surface film of unit area compared to the molecules in the interior is called surface energy. It is equal to the work done in increasing the area of the surface film by unit amount.

$$\text{Surface energy} = \frac{\textit{Work done}}{\textit{Increase in surface area}}$$

The SI unit of surface energy is Jm^{-2}.

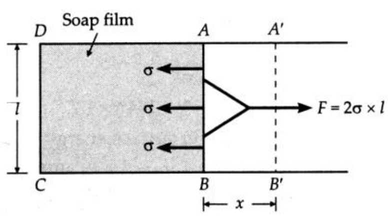
The relation between surface energy and surface tension

Consider a rectangular frame ABCD in which the wire AB is movable. Dip the frame in soap solution. A film is formed which pulls the wire AB inward due to surface tension with a force,

$$F = 2\,\sigma \times l$$

Here the factor 2 is taken because the soap film has two free surfaces.

Suppose AB is moved out through distance x to the position A'B'. Then



$$\text{Work done} = \text{Force} \times \text{distance}$$

$$= 2\sigma \times l \times x$$

$$\text{Increase in surface area of film} = 2\,lx$$

$$\therefore \quad \text{Surface energy} = \frac{\textit{Work done}}{\textit{Increase in surface area}} = \frac{2\sigma\, lx}{2lx} = \sigma$$

Thus surface energy of liquid is numerically equal to its surface tension.

Assignment – X

Q.1 A wire ring of 3 cm radius is rested on the surface of a liquid and then raised. The pull required is 3.03 g more before the film breaks than it is afterwards. Find the surface tension of the liquid.

Q.2 A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.

Q.3 A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended. Surface tension of mercury = 32×10^{-2} Nm^{-1}.

Q.4 A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.

Q.5 Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?

Q.6 If 500 erg of work is done in blowing a soap bubble to a radius r, what additional work is required to be done to blow it to a radius equal to 3r?

Q.7 A glass plate of length 10 cm, breadth 4 cm and thickness 0.4 cm, weighs 20g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight? Surface tension of water = 70 dyne cm^{-1}.

Q.8 If a number of little droplets of water of surface tension σ, all of the same radius r combine to form a single drop of radius R and the energy released is converted into kinetic energy, find the velocity acquired by the bigger drop.

Q.9 A soap film is formed on a rectangular frame of length 7 cm dipping in soap solution. The frame hangs from the arm of a balance. An extra weight of 0.38 g is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of soap solution. Given g = 980 cms^{-2}.

Q.10 A thin wire is bent in the form of a ring of diameter 3.0 cm. The ring is placed horizontally on the surface of soap solution and then raised up slowly. How much upward force is necessary to break the vertical film formed between the ring and the solution? Surface tension of a soap solution = 3.0×10^{-4} Ncm^{-1}.

Q.11 The length of a needle floating on water is 2.5 cm. How much minimum force, in addition to the weight of the needle, will be needed to lift the needle above the surface of water? Surface tension of
water = 7.2×10^{-4} N cm^{-1}.

Q.12 A rectangular plate of dimensions 6 cm × 4 cm and thickness 2 mm is placed with its largest face flat on the surface of water.

(i) What is the downward force on the plate due to surface tension? Surface tension of water = 7.0×10^{-2} N m^{-1}

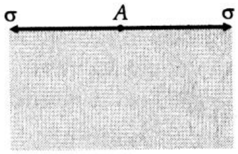
(ii) If the plate is placed vertical so that the longest side just touches the water surface, find the downward force on the plate.

Answers

1.	78.84 dyne cm^{-1}	2.	$2\pi D^2\sigma$	3.	3.98×10^{-2} J
4.	3168×10^{-8} J	5.	5 cm	6.	4000 erg
7.	13.4857 g f	8.	$\sqrt{\frac{6\sigma(R-r)}{rR}}$	9.	26.6 dyne cm^{-1}
10.	5.65×10^{-3} N	11.	3.6×10^{-3} N	12.	(i) 1.4×10^{-2} N, 8.68×10^{-3} N

Pressure Difference Across a Curved Liquid Surface

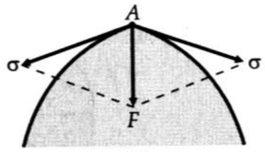
When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface. We consider the three possible liquid surfaces:



(a)

(i) As shown in figure, if the surface is plane, the molecule A on the surface is attracted equally in all directions. The resultant force due to surface tension is zero. Pressure on both sides of the surface is same i.e.,

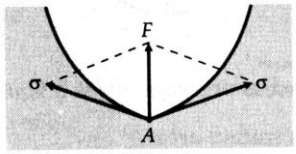
$$P_L = P_V$$



(b)

(ii) As shown in figure, if the surface is convex, there is a resultant downward force F on molecule A. For the surface to remain in equilibrium, the pressure on the liquid side must be greater than the pressure on the vapour side i.e.,

$$P_L > P_V$$



(c)

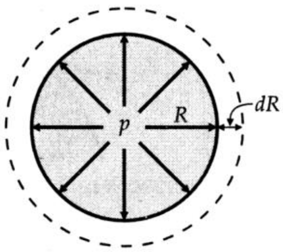
(iii) As shown in figure, if the surface is concave, there is a resultant upward force F due to surface tension on the molecule A. For the surface to remain in equilibrium, the pressure on the vapour side must be greater than the pressure on the liquid side i.e.,

$$P_V > P_L$$

Thus we find that *whenever a liquid surface is curved, the pressure on its concave side is greater than the pressure on the convex side.*

Excess Pressure inside a Liquid Drop

Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to R + dR under the excess pressure p.



Initial surface area = $4\pi R^2$

Final surface area = $4\pi (R + dR)^2 = 4\pi (R^2 + 2R\,dR + dR^2) = 4\pi R^2 + 8\pi R\,dR$

dR^2 is neglected as it is small.

Increase in surface area = $4\pi R^2 + 8\pi R\,dR - 4\pi R^2 = 8\pi R\,dR$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area × surface tension = $8\pi R\,dR\,\sigma$

But work done = Force × Distance = Pressure × Area × Distance

= $p \times 4\pi R^2 \times dR$

Hence, $p \times 4\pi R^2 \times dR = 8\pi R\,dR\,\sigma$ ∴ Excess pressure, $p = \frac{2\sigma}{R}$

Excess Pressure inside a Soap Bubble

Proceeding as in the case of a liquid drop in the above question, we obtain

Increase in surface area = $8\pi R\,dR$

But a soap bubble has air both inside and outside, so it has two free surfaces.

∴ Effective increase in surface area = $2 \times 8\pi R\,dR = 16\pi R\,dR$

Work done in enlarging the soap bubble = Increase in surface energy

= Increase in surface area × surface tension = $16\pi R\, dR\, \sigma$

But, Work done = Force × Distance $= p \times 4\pi R^2 dR$

Hence $p \times 4\pi R^2 \times dR = 16\pi R\, dR\, \sigma$ or $p = \frac{4\sigma}{R}$

Excess pressure inside an air bubble inside a liquid

An air bubble inside a liquid is similar to a liquid drop in air. It has only one free spherical surface. Hence excess pressure is given by $p = \frac{2\sigma}{R}$

NOTE

- The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside the tiny fog droplets that they are rigid enough to behave like solids and resists fairly large deforming forces.
- When an ice skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball–bearings and help the skaters to run along smoothly.
- When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension σ, the excess pressure inside the bubble will be $p = \frac{2\sigma}{R} + h\rho g$

Assignment – XI

Q.1 The excess pressure inside a soap bubble of radius 6 mm is balanced by 2 mm column of oil of specific gravity 0.8. Find the surface tension of soap solution.

Q.2 Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. Also compare the works done in blowing these bubbles.

Q.3 A small hollow sphere having a small hole in it is immersed into water to a depth of 20 cm before any water penetrates into it. If the surface tension of water is 73 dyne cm^{-1}, find the radius of the hole.

Q.4 A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end? Given density of mercury = 13600 kg m^{-3} and surface tension of mercury 35×10^{-3} Nm^{-1}.

Q.5 The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at the temperature of the experiment is 7.30×10^{-2} Nm^{-1}.
1 atmospheric pressure = 1.01×10^5 Pa, density of water = 1000 kgm^{-3}, g = 9.80 ms^{-2}. Also calculate the excess pressure.

Q.6 Calculate the total pressure inside a spherical bubble of radius 0.2 mm formed inside water at a depth of 10 cm. Surface tension of water at depth of 30 cm is 70 dyne cm^{-1}, barometric pressure is 76 cm, density of mercury is 13.6 g cm^{-3} and g = 980 cms^{-2}.

Q.7 Find the difference in excess pressure on the inside and outside of a rain drop if its diameter changes from 0.03 cm to 0.0002 cm by evaporation. Surface tension of water is 72 dyne cm^{-1}.

Q.8 There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 10^3 kgm^{-3}. The bubble is at a depth of 10.0 cm below the free surface of a liquid. By what amount is the pressure inside the bubble greater than the atmospheric pressure?

Q.9 An ancient building has a dome of 5 m radius and uniform but small thickness. The surface tension of its masionary structure is about 500 Nm^{-1}. Treated as hemisphere, find max. load that dome can support.

Answers

1.	2.35×10^{-2} Nm^{-1}	2.	3 : 2, 4 : 9	3.	0.007449 cm
4.	2805.6 Nm^{-2}	5.	146 Pa, 1.02×10^5 Pa	6.	1029728 dyne cm^{-2}
7.	1430400 dyne cm^{-2}	8.	1130 Nm^{-2}	9.	31420 N

Angle of Contact

The liquid surface is usually curved when it is in contact with a solid. The particular shape that it takes depends on the relative strengths of cohesive and adhesive forces. If

Adhesive force > cohesive force: Liquid wets the solid surface and has concave meniscus

Adhesive force < cohesive force: Liquid does not wet the solid surface and has a convex meniscus

Adhesive force = cohesive force: Liquid surface is plane

Angle of Contact *is defined as the angle θ between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.*

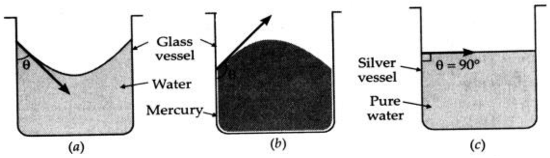
The value of angle of contact depends on the following factors:

(i) Nature of the solid and the liquid in contact

(ii) Cleanliness of the surface in contact

(iii) Medium above the free surface of the liquid

(iv) Temperature of the liquid



For those liquids which wet the walls of the vessel, the angle of contact is acute. For the liquids which do not wet the walls of the vessel, the angle of contact is obtuse. The angle of contact for water and glass is about 8^o, for mercury and glass it is 138^o and for pure water and silver, angle of contact is 90^o.

Shape of Liquid Meniscus in a Narrow Tube

Shape of liquid meniscus in a narrow tube: Consider a molecule O on the surface of the liquid in contact with the solid wall of the vessel. The various forces acting at the boundary of the three surfaces are as follows:

(i) Surface tension σ_{LV} of the liquid–vapour surface acting tangentially to the liquid surface.

(ii) Surface tension σ_{SV} of the solid–vapour surface acting parallel to the walls of the vessel.

(iii) Surface tension σ_{SL} of solid–liquid surface acting parallel to wall of the vessel directed into the liquid.

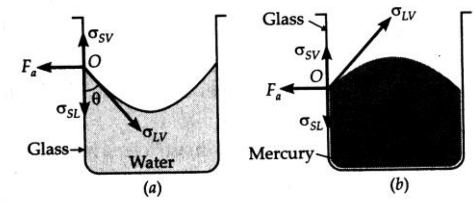
(iv) Adhesive force F_a between molecules of the vessel and liquid acting normal to the wall of the container.

For equilibrium, no forces should act on molecule O in any direction. Let θ be the angle of contact. Then the components of σ_{LV} parallel and perpendicular to water surface area σ_{LV} sin θ and σ_{LV} cos θ respectively. For equilibrium, we must have

$$Fa = \sigma_{LV} \cos\theta$$

and $$\sigma_{SV} = \sigma_{SL} + \sigma_{LV} \cos\theta$$

or $$\cos\theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$



The following there cases are possible:

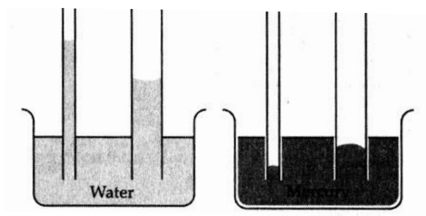
(i) If $\sigma_{SV} > \sigma_{SL}$, cos θ is positive and $\theta < 90^o$ i.e., angle of contact is acute. The liquid meniscus is concave upwards. This happens in the case of water taken in a glass vessel (figure a)

(ii) If $\sigma_{SV} < \sigma_{SL}$, cos θ is negative and $\theta > 90^o$, i.e., angle of contact is obtuse. The liquid meniscus is convex, upwards. This happens in the case of mercury taken in a glass vessel (figure b)

(iii) When $\sigma_{SV} = \sigma_{SL}$, cos θ = 0 and $\theta = 90^o$. The liquid meniscus is plane. This happens in the case of pure water taken in a silver vessel.

Capillarity

A tube of very fine (hair–like) bore is called a capillary tube. When a capillary tube of glass open at both ends is dipped in liquid which wets its walls (e.g., water, alcohol), the liquid rises in the tube. But when the capillary tube is dipped in a liquid which does not wet its walls (e.g., mercury), the level of liquid is depressed in the tube.



The phenomenon of rise or fall of a liquid in a capillary, tube in comparison to the surrounding is called capillarity.

Some examples of capillarity from daily life:

(i) A blotting paper soaks ink by capillary action. The pores of blotting paper act as capillaries.

(ii) Oil rises in the long narrow spaces between threads of a wick, the narrow spaces act as capillary tubes.

(iii) We use towels made of a coarse cloth for drying our skin after taking bath.

(iv) Sap rises from the roots of a plant to its leaves and branches due to capillarity action.

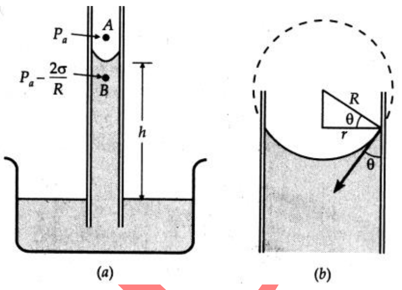
(v) The tip of the nib of a pen is split to provide capillary action for the ink to rise.

Rise of Liquid in a Capillary Tube : Ascent Formula

Consider a capillary tube of radius r dipped in a liquid of surface tension σ and density ρ. Suppose the liquid wets the sides of the tube. Then its meniscus will be concave. The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is $p=\frac{2\sigma}{R}$

where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in figure, we have



$$\frac{r}{R}=\cos\theta$$

or $$R=\frac{r}{\cos\theta}$$

∴ $$p=\frac{2\sigma\cos\theta}{r}$$

Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = h\rho g$$

or $$\frac{2\sigma\cos\theta}{r}=h\rho g$$ or $$h=\frac{2\sigma\cos\theta}{r\rho g}$$

This is the ***ascent formula*** for the rise liquid in a capillary tube. If we take into account the volume of the liquid contained in the meniscus, then the above formula gets modified as

$$h=\frac{2\sigma\cos\theta}{r\rho g}-\frac{r}{3}$$

However, the factor r/3 can be neglected for a narrow tube. The ascent formula shows that the height h to which a liquid rises in the capillary tube is

(i) inversely proportional to the radius of the tube.

(ii) inversely proportional to the density of the liquid

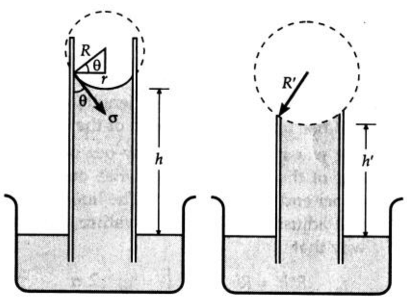
(iii) directly proportional to the surface tension of the liquid.

Hence a liquid rises more in a narrow tube than in wider tube.

Rise of liquid in a Capillary Tube of Insufficient Height

The height to which a liquid rises in a capillary tube is given by

$$h=\frac{2\sigma\cos\theta}{r\rho g}$$



The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by r = R cos θ. Therefore,

$$h=\frac{2\sigma\cos\theta}{R\cos\theta\,\rho g}=\frac{2\sigma}{R\rho g}$$

As σ, ρ and g are constants, so

$$hR=\frac{2\sigma}{\rho g}=\text{constant}$$ ∴ $$hR = h'R'$$

where R' is the radius of curvature of the new meniscus at a height h' As h' < h, so R' > R

Hence in a capillary tube of insufficient height, the liquid rises to the top and spreads out to a new radius of curvature R' given by $R' = \frac{hR}{h'}$ But the liquid will not overflow.

Assignment – XII

Q.1 Calculate the height to which water will rise in capillary tube of 1.5 mm diameter. Surface tension of water is 7.4×10^{-3} Nm^{-1}.

Q.2 Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water–glass and mercury–glass as 0^o and 135^o respectively. Determine the ratio of surface tensions of mercury and water. Take cos 135^o = – 0.71.

Q.3 Water rises in a capillary tube to a height of 2.0 cm. In another capillary whose radius is one–third of it, how much the water will rise? If the first capillary is inclined at an angle of 60^o with the vertical, then what will be the position of water in the tube?

Q.4 If a 5 cm long capillary tube with 0.1 mm internal diameter opens at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary (ii) water will rise up to the upper end of capillary (iii) water will overflow out of the upper end of capillary. Explain your answer.

Q.5 A glass U–tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is
0.07 Nm^{-1}. Assume that the angle of contact between water and glass is 0^o.

Q.6 A capillary tube of inner diameter 0.5 mm is dipped in a liquid of specific gravity 13.6, surface tension 545 dyne cm^{-1} and angle of contact 130^o. Find the depression or elevation in the tube.

Q.7 The tube of mercury barometer is 5 mm in diameter. How much error does the surface tension cause in the reading? S.T. of mercury = 540×10^{-3} Nm^{-1}. Angle of contact = 135^o.

Q.8 Water rises to a height of 9 cm in a certain capillary tube. If in the same tube, level of Hg is depressed by 3 cm, compare the surface tension of water and mercury. Specific gravity of Hg is 13.6, the angle of contact for water is zero and that for Hg is 135^o.

Q.9 A capillary tube whose inside radius is 0.5 mm is dipped in water of surface tension 75 dyne cm^{-1}. To what height is the water raised by the capillary action above the normal level? What is the weight of water raised?

Q.10 A U–tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne cm^{-1} and zero contact angle. Calculate the density of the liquid, if the difference in the levels of the meniscus is 1.25 cm. Take g = 980 cms^{-2}.

Answers

1.	0.002014 m	2.	7.2 : 1	3.	6.0 cm, 4.0 cm
4.	30.58 cm	5.	4.76 mm	6.	– 2.1 cm
7.	-0.2293×10^{-2} m	8.	0.152		
9.	3.061 cm, 23.55 dyne or 0.024 g wt			10.	0.8 g cm^{-3}

Factors Affecting Surface

(i) **Effect of contamination:** If water surface has dust, grease or oil, the surface tension of water reduces. A small piece of camphor put in clear water dances vigorously due to decrease of surface tension of water.

(ii) **Effect of solute:** (a) A highly soluble substance like sodium chloride increases the surface tension of water. (b) A sparingly soluble substance like phenol or soap reduces the surface tension of water.

(iii) **Effect of temperature:** The surface tension of liquids decreases with increase of temperature. The surface tension of a liquid becomes zero at a particular temperature, called *critical temperature* of that liquid. For small temperature differences, surface tension decrease almost linearly as $\sigma_t = \sigma_0 (1 - \alpha t)$

where σ_t and σ_0 are the surface tensions at t^oC and 0^oC respectively, and α is the temperature coefficient of surface tension.

Detergents and Surface Tension

Cleaning action of detergents: Oil stains and grease on dirty clothes cannot be removed by simply washing the clothes with water because water does not wet them. By adding detergent or soap to water, the greasy dirt can be easily removed. The cleansing action of detergents can be explained as follows:

(i) Soap or detergent molecules have the shape of a hairpin.

(ii) When detergent is dissolved in water, the heads of its hairpin shape molecules get attracted to water surface.

(iii) When clothes with greasy strains are dipped in water containing detergent, the pointed ends of detergent molecules get attached to the molecules of grease. So a water–grease interface is formed. Thus surface tension is greatly reduced. The greasy dirt is held suspended.

(iv) When the clothes are rinsed in water, the greasy dirt is washed away by running water.

So when detergent is added to water, the surface tension of water is reduced, its area of contact with grease is increased and hence its cleaning ability is increased.

Conceptual Problems

Q.1 (i) A balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds). (i i) The force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air.

Q.2 A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_o. After some time, the ball falls with a const ant velocity. Calculate the viscous force on the ball.

Q.3 A tank filled with fresh water has hole in its bottom and water is flowing out of it. If the size of the hole is increased what will be the change in:

(a) Volume of water flowing out per second? (b) Velocity of the out coming water?

(c) If in the above tank, the fresh water is replaced by sea water, will the velocity of out coming water change?

Q.4 In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why?

Q.5 Two soap bubbles of different diameters are in contact with a certain portion common to both the bubbles. What will be the shape of the common boundary as seen from inside the smaller bubble? Support your answer with a neat diagram. Give reason for your answer.

Q.6 In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why?

Q.7 A big size balloon of mass M is held stationary in air with the help of a small block of mass M/2 tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations.

Q.8 A tarnado consists of rapidly whirling air vortex. Why is the pressure always much lower in the centre than at the outside? How does this condition account for the destructive power of tarnado?

Problems of Higher order Thinking Skills

Q.1 A piece of ice with a stone frozen in it floats on water taken in a beaker. Will the level of water increase or decrease or remain the same when ice melts completely?

Q.2 An ice block with a cork piece embedded inside floats in water. What will happen to the level of water when ice melts?

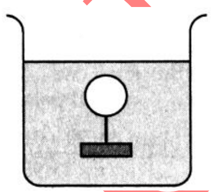
Q.3 A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level?

Q.4 To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force, with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel?

Q.5 A block of wood is floating on water at 0^oC with a certain volume V above the water level. The temperature of water is slowly raised from 0^oC to 20^oC. How will the volume V change with the rise in temperature?

Q.6 A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon?

Q.7 A balloon filled with air is weighed so that it barely floats in water, as shown in figure. Explain why it sinks to the bottom when it is submerged more by a small distance.

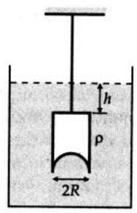


Q.8 A beaker containing water is placed on a spring balance. A stone of weight W is hung and lowered into the water without touching the sides and bottom of the beaker. Explain how the reading will change.

Q.9 A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density = 13.6 g cm^{-3}). A homogenous sphere floats with half its volume immersed in mercury and the other half in oil. What is the density of material of sphere?

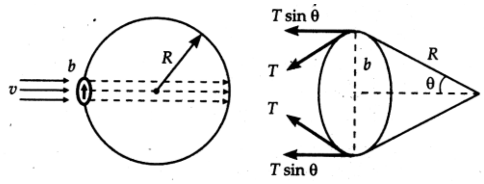
Q.10 An iceberg weighs 400 tonnes. The specific gravity of iceberg is 0.92 and the specific gravity of water is 1.02. What percentage of iceberg is below the water surface?

Q.11 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. How much is the force on the bottom of the cylinder by the liquid?



Q.12 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?

Q.13 A bubble having surface tension T and radius R is formed on a ring of radius b (b < < R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring (fig).



Q.14 A cylindrical vessel of radius 3 cm has at the bottom a horizontal capillary tube of length 20 cm and internal radius 0.4 mm. If the vessel is filled with water, find the time taken by it to empty one half of its contents. Given that the viscosity of water is 0.01 poise.

Q.15 A soap bubble of radius 4 cm and surface tension 30 dyne cm^{-1} is blown at the end of a tube of length 10 cm and internal radius 0.20 cm. If the viscosity of air is 1.89×10^{-4} poise, find the time taken by the bubble to the reduced to a radius of 2 cm.

Q.16 A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^{4} kg m^{-3} enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given coefficient of viscosity of water = 1.0×10^{-3} N sm^{-2},
g = 10 ms^{-2} and density of water = 1.0×10^{3} kg m^{-3}.

Q.17 Water stands at a height H in a tank whose side walls are vertical. A hole is made in one of the walls at a depth h below the water surface. (i) Find at what distance from the foot of the wall does the emerging stream of water strike the floor? (ii) For what value of h, this range is maximum? (iii) can a hole be made at another depth so that the second stream has the same range?

Q.18 A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross–sectional area is 10 cm^{2}, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. What is the pressure at another point where the cross–sectional area is 5 cm^{2}?

Q.19 A liquid is kept in a cylindrical vessel which is being rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rps, find the difference in the heights of the liquid at the centre of the vessel and at its sides.

Q.20 Calculate the rate of flow of glycerine of density 1.25×10^{3} kgm^{-3} through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and pressure–drop across its length is 10 Nm^{-2}.

Q.21 Water from a tap emerges vertically downward with an initial speed of 1.0 ms^{-1}. The cross sectional area of the tap is 10^{-4} m^{2}. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross–sectional area of the stream 0.15m below the tap?

NCERT Exercise

Q.1 Explain why:

(a) The blood pressure in humans is greater at the feet than at the brain.

(b) Atmospheric pressure at a height of about 6 km decrease to nearly half its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector.

Q.2 Explain why:

(i) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(ii) Water on a clean glass surface tends to spread out mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not).

(iii) Surface tension of a liquid is independent of the area of the surface.

(iv) Detergents should have small angles of contact.

(v) A drop of liquid under no external forces is always spherical in shape.

Q.3 Fill in the blanks using the word(s) from the list appended with each statement:

(i) Surface tension of liquids generally _____________ with temperatures (increases/decreases)

(ii) Viscosity of gases ___________ with temperature, whereas viscosity of liquids _________ with temperature (increase/ decreases)

(iii) For solids with elastic modulus of rigidity, the shearing force is proportional to __________ while for fluids it is proportional to ____________ (shear strain/rate of shear strain)

(iv) For a fluid is steady flow, the increase in flow speed at a constriction follows from ______ while the decrease of pressure there follows from _________ (conservation of mass/ Bernoulli's principle)

(v) For the model of a plane in a wind tunnel, turbulence occurs at a _______ speed than the critical speed for turbulence for an actual plane (greater/ smaller)

3. (i) decreases (ii) increases, decreases, (c) shear strain, rate of shear strain, (d) conservation of mass, Bernoulli's principle, (v) greater

Q.4 Explain why:

(i) To keep a piece of paper horizontal, you should blow over, not under, it.

(ii) When we try to close a water tap with our fingers, fast jets of water gush though the openings between our fingers.

(iii) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(iv) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(v) A spinning cricket ball in air does not follow a parabolic trajectory.

Q.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Q.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984. Determine the height of the wine column for normal atmospheric pressure.

Q.7 A vertical off–shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in Bombay High? Take the depth of the sea to be roughly 3 km, and ignore ocean currents.

Q.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross–section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Q.9 A U–tube contains water and methylated sprit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

Q.10 In previous exercise, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in two arms? Specific gravity of mercury = 13.6.

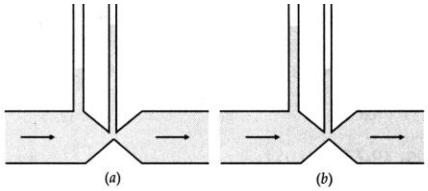
Q.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

Q.12 Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation? Explain.

Q.13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kgs^{-1}, what is the pressure difference between the two ends of the tube? Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Nsm^{-2}.

Q.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is lift of the wing if its area is 2.5 m^2? Density of air = 1.3 kg m^{-3}.

Q.15 Figure (a) and (b) refer to the steady flow of a (non–viscous) liquid. Which of the two figures is incorrect? Why?

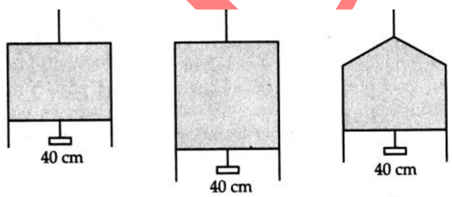


(a) (b)

Q.16 The cylindrical tube of a spray pump has a cross section of 8.0 cm^2, one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the spread of ejection of the liquid through the holes?

Q.17 A U–shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N(which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Q.18 Figure (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in figure(b) and (c)? Explain your answer physically.



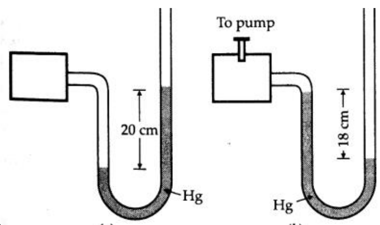
(a) (b) (c)

Q.19 What is the pressure inside a drop of mercury of radius 3.00 mm of room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} Nm^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.

Q.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm? Given that the surface tension of soap solution at the temperature (20°C) is 2.50×10^{-2} Nm^{-1}. If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atm = 1.01×10^5 Pa).

Q.21 A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door closed.

Q.22 A manometer reads the pressure of a gas in a enclosure as shown in figure (a). When some of the gas is removed by a pump, the manometer reads as in figure (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.



(a) (b)

(i) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.

(ii) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in volume of the gas)

Q.23 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height.

(i) Is the force exerted by the water on the base of the vessel the same in the two cases?

(ii) If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

Q.24 During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? The density of whole blood = 1.06×10^3 kg m^{-3}.

Q.25 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present? (b) Do the dissipative forces becomes more important as the fluid velocity increases? Discuss qualitatively.

Q.26 (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar? (b) What is the corresponding flows rate? Take viscosity of blood to be 2.084×10^{-3} Pa s and density of blood = 1.06×10^3 kg m^{-3}.

Q.27 A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. Take air density to be 1 kg m^{-3} and g = 9.81 ms^{-2}.

Q.28 In Millikan's oil drop experiment, what is the terminal speed of a drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3}? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Nsm^{-2}. How much is viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Q.29 Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of thin glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6×10^3 kgm^{-3}.

Q.30 The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U–shaped tube open at both ends. If the U–tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to the zero, and density of water to be 1.0×10^3 kg m^{-3}. Take g = 9.8 ms^{-2}.

Q.31 (a) If is known that density ρ of air decreases with height y as $\rho = \rho_0 e^{-y/y_0}$

where $\rho_0 = 1.25$ kg m^{-3} is the density at sea level and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m^3 is used to lift a pay load of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise? Take y_0 = 8000 m and ρ_{He} = 0.18 kg m^{-3}

Answers

5.	6.2×10^6 Pa	6.	10.5 m	7.	yes
8.	6.92×10^5 Nm^{-2}	9.	0.8	10.	0.221 cm
11.	No	12.	No	13.	9.8×10^{12} Pa
14.	1512.9 N	15.	figure a is incorrect	16.	0.637 ms^{-1}
17.	2.5×10^{-2} Nm^{-1}	18.	4.5×10^{-2} N	19.	1.013×10^5 Pa, 310 Pa
20.	20 Pa, 105714 Pa	21.	54.88 N		

22. (i) (a) $P_a = 96$ cm of Hg, $P_g = 20$ cm of Hg, (b) $P_a = 58$ cm of Hg, $P_g = -18$ cm of Hg, (ii) 19 cm

23.	(i) same	24.	0.1925 m	25.	(a) decreases, (b) yes
26.	(a) 0.98 ms^{-1}, (b) 1.23×10^{-5} m^{-3} s^{-1}			27.	4396 kg
28.	3.9×10^{-10} N	29.	– 5.34 mm	30.	5.290 mm
31.	(b) 8 km				

IIT–JEE Objective Assignment

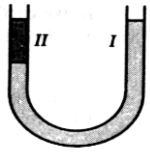
Multiple Choice Questions with One Correct Answer

Q.1 A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

(a) same everywhere (b) lower in the front side

(c) lower in the rear side (d) lower in the upper side

Q.2 A U–tube of uniform cross–section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be



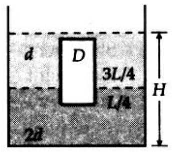
(a) 1.12 (b) 1.1 (c) 1.05 (d) 1.0

Q.3 A man is sitting in a boat, which is floating on a pond. If the man drinks some water from the pond, the level of water in the pond

(a) decreases

(b) increases (c) remains unchanged

(d) may increase or decrease depending on the weight of the man



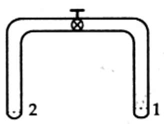
Q.4 A homogenous solid cylinder of length L (L < H/ 2), cross–sectional area A/5 is immersed such that it floats with its axis vertical at the liquid–liquid interface with length L/4 in the denser liquid as shown in the figure. The lower density liquid is D open to atmosphere having pressure P_0. Then density of solid is given by

(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$ (c) 4d (d) $\frac{d}{5}$

Q.5 A glass tube of uniform internal radius (r) has a value separating the two identical ends. Initially, the value is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub–hemi–spherical soap bubble as shown in figure. Just after opening the valve,

(a) air from end 1 flows towards end 2. No change in volume of the soap bubbles.

(b) air from end 1 flows towards end 2. Volume of soap bubble at end 1 decreases



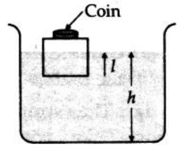
(c) no changes occur

(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Q.6 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is

(a) Mg (b) $Mg - V\rho g$ (c) $Mg + \pi R^2 h\rho g$ (d) $\rho g (V + \pi R^2 h)$

Q.7 A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance *l* and h are shown here. After some time the coin falls into the water. Then



(a) *l* decreases and h increases

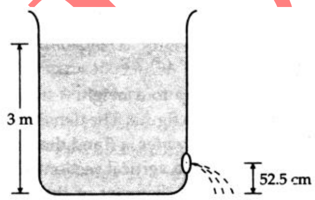
(b) *l* increases and h decreases

(c) both *l* and h increase (d) both *l* and h decrease

Q.8 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, quantities of water flowing out per second from both holes are the same. Then R is equal to

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (d) L (d) $L/2\,\pi$

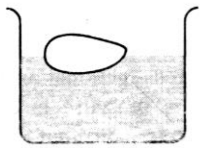
Q.9 Water is filled in a container upto height 3 m. A small hole of area a is punched in the wall of the container at a height 52.5 cm from the bottom. The cross–sectional area of the container is A. If a/A = 0.1, then v^2 (where v is the velocity of water coming out of the hole) is



(a) $50\ m^2s^{-2}$ (b) $51\ m^2s^{-2}$ (c) $48\ m^2\ s^{-2}$ (d) $51.5\ m^2s^{-2}$

Multiple Choice Questions with One or More than One Correct Answer

Q.10 A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is



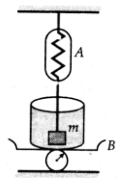
(a) zero

(b) equal to the weight of the liquid displaced

(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body

Q.11 The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation



(a) the balance A will read more than 2 kg

(b) the balance B will read more than 5 kg

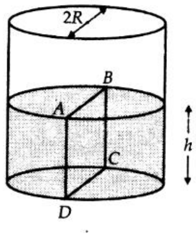
(c) the balance A will read less than 2 kg and B will read more than 5 kg

(d) the balances A and B will read 2 kg and 5 kg respectively

Q.12 A vessel contains oil (density = 0.8 g cm^{-3}) over mercury (density = 13.6 cm^{-3}). A homogenous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material (in g cm^{-3}) is

(a) 3.3 (b) 6.4 (c) 7.2 (d) 2.8

Q.13 Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



(a) $|2P_0Rh + \pi R^2\rho gh - 2RT|$ (b) $|2P_0Rh + R\rho gh^2 - 2RT|$

(c) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$ (d) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$

Q.14 Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross–sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross–sectional area of the stream 0.15 m below the tap is

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2 (c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

Reasoning Type

Q.15 **Statement – 1:** The steam of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement–2: In any steady flow of an incompressible fluid, volume flow rate of fluid remains constant.

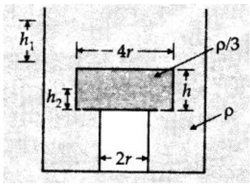
(a) Statement – 1 is true, Statement – 2 is true.
Statement – 2 is a correct explanation for Statement – 1.

(b) Statement – 1 is true, statement – 2 is true.
Statement – 2 is not a correct explanation for Statement – 1.

(c) Statement – 1 is true, Statement – 2 is false. (d) Statement – 1 is false, statement – 2 is true.

Comprehension Based Questions *[Question N o. 16 to 18]*

A cylinder tank has a hole of diameter 2r in its bottom. The hole is covered with a wooden cylindrical block of diameter 4r, height h and density ρ/3.



Situation 1: Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

situation 2: The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. Height h_2 is smaller than h (height of the block) and thus block is exposed to atmosphere.

Read the passage given above and answer the following questions

Q.16 Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up.

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$ (c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

Q.17 Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force.

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$ (c) $\frac{2h}{3}$ (d) h

Q.18 In situation 2, if h_2 is further decreased, then

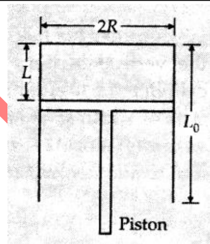
(a) cylinder will not move up and remains at its original position

(b) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \frac{h}{4}$, cylinder again starts moving up (d) for $h_2 = \frac{h}{5}$, cylinder again starts moving up

Paragraph Type *[Q. No. 19 to 21]*

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



Read the passage given above and answer the following questions

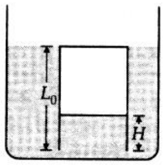
Q.19 The piston is now pulled out slowly and held at a distance 2 L from the top. The pressure in the cylinder between its top and the piston will then be

(a) P_0 (b) $P_0/2$ (c) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

Q.20 While the piston is at a distance 2 L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (b) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$ (c) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (d) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

Q.21 The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same levels the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies.



(a) $\rho g (L_0 - H)^2 + P_0 (L_0 - H) + L_0 P_0 = 0$ (b) $\rho g (L_0 - H)^2 - P_0 (L_0 - H) - L_0 P_0 = 0$

(c) $\rho g (L_0 - H)^2 + P_0 (L_0 - H) - L_0 P_0 = 0$ (d) $\rho g (L_0 - H)^2 - P_0 (L_0 - H) + L_0 P_0 = 0$

Integer Answer Type

Q.22 A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmosphere pressure =m 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension]

Q.23 Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap–water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity].

Answers

1	b	2	b	3	c	4	a	5	b
6	d	7	d	8	a	9	b	10	a

11	b, c	12	c	13	b	14	c	15	a
16	c	17	b	18	a	19	a	20	d
21	c	22	6 mm	23	6				

AIEEE – Objective Assignment – II

Q.1 A jar is filled with two non–mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?
(a) $\rho_1 < \rho_3 < \rho_2$ (b) $\rho_3 < \rho_1 < \rho_2$ (c) $\rho_1 > \rho_3 > \rho_2$ (d) $\rho_1 < \rho_2 < \rho_3$

Q.2 Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is
(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity.

Q.3 If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid.
(a) 0.2 m/s (b) 0.4 m/s (c) 0.133 m/s (d) 0.1 m/s

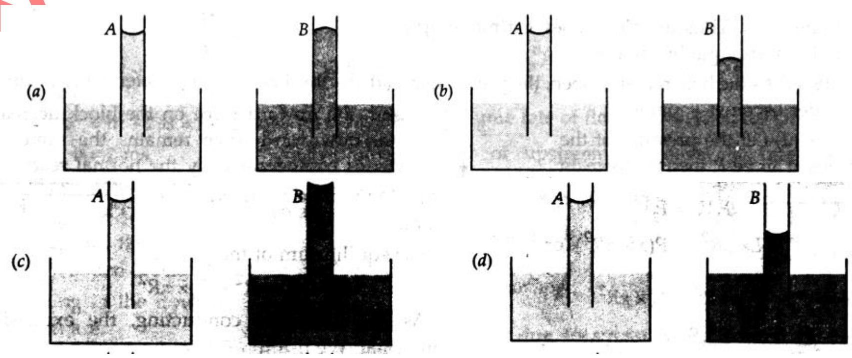
Q.4 A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is
(a) $\frac{Vg(\rho_1-\rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$ (c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$

Q.5 A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20 (c) 25.5 (d) 5

Q.6 If two soap bubbles of different radii are connected by a tube,
(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from bigger bubble to the smaller bubble till the size are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air

Q.7 A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in capillary tube will be
(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm

Q.8 A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap–water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

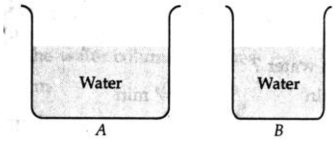


Answers

1	a	2	b	3	d	4	b	5	b
6	c	7	b	8	d				

DCE & DPMT – Objective Assignment – III

Q.1 From the following figures, the correct observation is



(a) the pressure on bottom of tank A is greater than that at bottom of B
(b) the pressure on bottom of the tank A is smaller than at bottom of B
(c) the pressure depends on the shape of the container
(d) the pressure on the bottoms of A and B is the same

Q.2 A wooden block is taken to bottom of a deep, calm lake of water and then released. It rises up with a
(a) constant acceleration (b) decreasing acceleration
(c) constant velocity (d) decreasing velocity

Q.3 If there were no gravity, which of the following will not be there for a fluid?
(a) Viscosity (b) Surface tension
(c) Pressure (d) Archimedes' upward thrust

Q.4 A bubble is at the bottom of the lake of depth h. As the bubble comes to sea level, its radius increases three times. If atmospheric pressure is equal to *l* metre of water column, then h is equal to
(a) 26 *l* (b) *l* (c) 25 *l* (d) 30 *l*

Q.5 Radius of one arm of hydraulic lift is four times of radius of other arm. What force should be applied on narrow arm to lift 100 kg?
(a) 26.5 N (b) 62.5 N (c) 6.25 N (d) 8.3 N

Q.6 A liquid X of density 3.36 g/cm^3 is poured in the right arm of a U–tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same. Density of Y?
(a) 0.8 g/cm^3 (b) 1.2 g/cm^3 (c) 1.4 g/cm^3 (d) 1.6 g/cm^3

Q.7 The unit of coefficient of viscosity is
(a) Nm/s (b) Nm^2/s (c) N/ (m^{2}s^{-1}) (d) Nms^2

Q.8 An object is moving through the liquid. The viscous damping force acting on it is proportional to the velocity. Then dimensions of constant of proportionality are
(a) [ML^{-1}T^{-1}] (b) [MLT^{-1}] (c) [M^{0}LT^{-1}] (d) [ML^{0}T^{-1}]

Q.9 The rate of flow of liquids in a tube of radius r, length *l*, whose ends are maintained at a pressure difference P is $V = \frac{\pi Q p r^4}{\eta l}$, where η is coefficient of viscosity and Q is
(a) 8 (b) 1/8 (c) 16 (d) 1/16

Q.10 Motion of a liquid in a tube is best described by
(a) Bernoulli’s theorem (b) Poiseuille’s equation (c) Stokes’ law (d) Archimedes’ principle

Q.11 which one is not a dimensional number?
(a) Acceleration due to gravity (b) Surface tension of water
(c) Velocity of light (d) Reynold’s number

Q.12 Critical velocity of the liquid
(a) decreases when radius decreases (b) increases when radius increases
(c) decreases when density increases (d) increases when density increases

Q.13 A steel ball is dropped in oil, then
(a) the ball attains constant velocity after some time (b) the ball stops
(c) the speed of ball will keep on increasing (d) none of the above

Q.14 A sphere of mass m and radius r is falling in the column of a viscous fluid. Terminal velocity attained by falling object is proportional to
(a) r^2 (b) 1/r (c) r (d) – 1/r^2

Q.15 The ratio of the terminal velocities of two drops of radii R and R/2 is
(a) 2 (b) 1 (c)1/2 (d) 4

Q.16 The radii of two drops are in the ratio of 3 : 2, their terminal velocities are in the ratio
(a) 9 : 4 (b) 2 : 3 (c) 3 : 2 (d) 2 : 9

Q.17 Bernoulli's equation is an example of conservation of
(a) energy (b) momentum (c) energy momentum (d) mass

Q.18 An aeroplane gets its upward lift due to a phenomenon described by the
(a) Archimedes' principle (b) Bernoulli's principle
(c) Buoyancy principle (d) Pascal law

Q.19 The rate of flow of liquid through an orifice of a tank does not depend upon
(a) the size of orifice (b) density of liquid
(c) the height of fluid column (d) acceleration due to gravity

Q.20 The velocity of efflux of a liquid through an orifice in bottom of the tank does not depend upon
(a) size of orifice (b) height of liquid
(c) acceleration due to gravity (d) none of the above

Q.21 A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?
(a) 9 min (b) 7 min (c) 5 min (d) 3 min

Q.22 The SI unit of surface tension is
(a) dyne/cm (b) N/m^2 (c) N/m (d) Nm

Q.23 The water droplets in free fall are spherical due to
(a) gravity (b) viscosity
(c) surface tension (d) intermolecular attraction

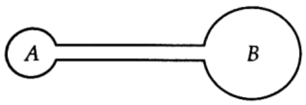
Q.24 One large soap bubble of diameter D breakes into 27 bubbles having surface tension T. The change in surface energy is
(a) $2\pi TD^2$ (b) $4\pi TD^2$ (c) πTD^2 (d) $8\pi TD^2$

Q.25 Two drops of equal radius coalesce to form a bigger drop. What is ratio of surface energy of bigger drop to smaller one?
(a) $2^{1/2} : 1$ (b) 1 : 1 (c) $2^{2/3} : 1$ (d) none of these

Q.26 8 mercury drops coalesce to from 1 mercury drop, the energy change by a factor of
(a) 1 (b) 2 (c) 4 (d) 6

Q.27 If a mercury drop is divided into 8 equal parts, its total energy
(a) remains same (b) becomes twice (c) becomes half (d) becomes 4 times

Q.28 There is a small bubble at one end and bigger bubble at other end of a rod. What will happen?
(a) smaller will grow until they collapse
(b) bigger will grow until they collapse
(c) remain in equilibrium (d) none of the above



Q.29 If a liquid does not wet glass, its angle of contact is
(a) zero (b) acute (c) obtuse (d) right angle

Q.30 In a capillary tube experiment, a vertical, 30 cm long capillary tube is dipped in water. The water rises upto a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes
(a) 10 cm (b) 20 cm (c) 30 cm (d) zero

Q.31 Two capillaries of lengths L and 2L and of radii R and 2R respectively are connected in series. The net rate of flow of fluid through them will (Given, rate of the flow through single capillary, $X = \pi PR^4 / 8\eta L$) be
(a) $\frac{8}{9}X$ (b) $\frac{9}{8}X$ (c) $\frac{5}{7}X$ (d) $\frac{7}{5}X$

Q.32 The rate of flow of water in a capillary tube of length l and radius r is V. The rate of flow in another capillary tube of length $2l$ and radius 2r for same pressure difference would be
(a) 16 V (b) 9 V (c) 8 V (d) 2 V

Q.33 The water flows from a tap of diameter 1.25 cm with a rate of 5×10^{-5} m^3 s^{-1}. The density and coefficient of viscosity of water are 10^3 kg m^{-3} and 10^{-3} Pas, respectively. The flow of water is
(a) steady with Reynold's number 5100 (b) turbulent with Reynold's number 5100
(c) steady with Reynold's number 3900 (d) turbulent with Reynold's number 3900

Answers

1	d	2	a	3	d	4	a	5	b
6	a	7	c	8	d	9	b	10	b
11	d	12	c	13	a	14	a	15	d
16	a	17	a	18	b	19	d	20	a
21	b	22	c	23	c	24	b	25	d
26	b	27	b	28	b	29	b	30	c
31	a	32	c	33	b				

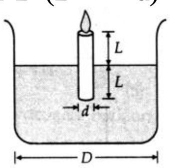
AIIMS – Objective Assignment – IV

Q.1 A body is just floating in a liquid (their densities are equal). If the body is slightly pressed down and released, it will
(a) start oscillating (b) sink to the bottom
(c) come back to the same position immediately (d) come back to the same position slowly

Q.2 When a large bubble rises from the bottom of a lake to the surface, its radius doubles. The atmospheric pressure is equal to that of a column of water of height H. The depth of the lake is
(a) H (b) 2 H (c) 7 H (d) 8 H

Q.3 By sucking through a straw, a student can reduce the pressure in his lungs to 750 mm of mercury (density = 13.6 g cm^{-3}). Using straw, he can drink water from a glass upto a maximum depth of
(a) 10 cm (b) 75 cm (c) 13.6 cm (d) 1.36 cm

Q.4 A candle of diameter d is floating on a liquid in a cylindrical container of diameter D (D > > d) as shown in figure. If it is burning at the rate of 2 cm h^{-1}, then the top of candle will



(a) remain at the same height
(b) fall at the rate of 1 cm h^{-1}
(c) fall at the rate of 2 cm h^{-1}
(d) go up at the rate of 1 cm h^{-1}

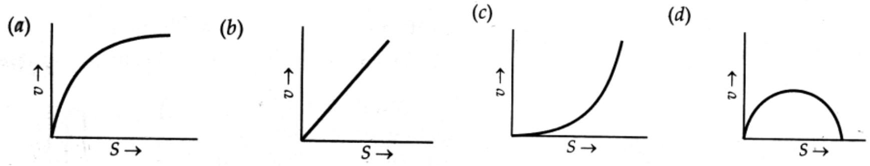
Q.5 A small ball of density ρ is dropped from a height h into a liquid of density σ ($\sigma > \rho$). Neglecting damping forces, the maximum depth to which the body sinks is
(a) $\frac{h\sigma}{\rho-\sigma}$ (b) $\frac{h\rho}{\rho-\sigma}$ (c) $\frac{h(\sigma-\rho)}{\rho}$ (d) $\frac{h(\sigma-\rho)}{\sigma}$

Q.6 A vertical U–tube contains mercury in both its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured into the other arm until the upper surfaces of oil and glycerine are at the same level. The length of the oil column is (density of mercury = 1.3 g cm^{-3})
(a) 8.5 cm (b) 9.6 cm (c) 10.7 cm (d) 11. 8 cm

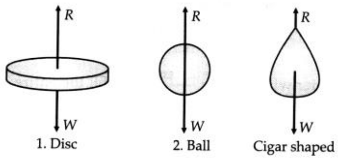
Q.7 Under a constant pressure head, the rate of flow of orderly volume flow of liquid through a capillary tube is V. If the length of the capillary is doubled and the diameter of the bore is halved, the rate of flow would become
(a) V/4 (b) 16 V (c) V/8 (d) V/32

Q.8 A sphere of mass M and radius R is falling in a viscous fluid. The terminal velocity attained by the falling object will be proportional to
(a) R^2 (b) R (c) 1/ R (d) $1/R^2$

Q.9 A lead shot of 1 mm diameter falls through a long column of glycerine. The variation of its velocity v with distance covered (S) is represented by



Q.10 When a body falls in air, the resistance of air depends on a greater extent on the shape of the body. Three different shapes are given. Identify the combination of air resistance, which truly represents the physical situation. (The cross sectional areas are the same).



(a) 1 < 2 < 3 (b) 2 < 3 < 1 (c) 3 < 2 < 1 (d) 3 < 1 < 2

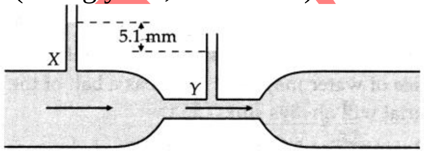
Q.11 Scent sprayer is based on
(a) Charles' law (b) Boyle's law (c) Archimedes' principle (d) Bernouli's theorem

Q.12 Bernouli's principle is based on the law of conservation of
(a) energy (b) mass (c) linear momentum (d) angular momentum

Q.13 In old age arteries carrying blood in the human body become narrow resulting in an increase in the blood pressure. This follows from
(a) Pascal's law (b) Stoke's law (c) Bernoulli's principle (d) Pascal's law

Q.14 In incompressible fluid flows steadily through a cylindrical pipe which has radius 2 R at point A and R at a point B further along the flow direction. If the velocity at A is v, then that at b is
(a) v/2 (b) v (c) 2 v (d) 4v

Q.15 Figure shows a venturimeter, through which water is flowing. The speed of water at X is 2 cm s^{-1}. The speed of water at Y (taking y = 1, 000 cm s^{-2}) is



(a) 23 cm s^{-1} (b) 32 cm s^{-1} (c) 101 cm s^{-1} (d) 1,024 cm s^{-1}

Q.16 The property utilized in the manufacture of lead shots is
(a) specific weight of liquid lead (b) specific gravity of liquid lead
(c) compressibility of liquid lead (d) surface tension of liquid lead

Q.17 The rain drops are in spherical shape due to
(a) viscosity (b) surface tension (c) thrust on drop (d) residual pressure

Q.18 Work of 3.0×10^{-4} J is required to be done in increasing the size of a soap film from 10cm × 6cm to 10 cm × 11cm. The surface tension of the soap film is
(a) 5×10^{-2} Nm^{-1} (b) 3×10^{-2} Nm^{-1} (c) 1.5×10^{-2} Nm^{-1} (d) 1.2×10^{-2} Nm^{-1}

Q.19 Two small drops of mercury, each of radius R coalesce to form a single large drop. The ratio of the total surface energies before and after the change is
(a) $1 : 2^{1/3}$ (b) $2^{1/3} : 1$ (c) 2 : 1 (d) 1 : 2

Q.20 The potential energy possessed by a soap bubble, having surface tension equal to 0.04 Nm^{-1} of diameter 1 cm, is
(a) $2\pi \times 10^{-6}$ J (b) $4\pi \times 10^{-6}$ J (c) $6\pi \times 10^{-6}$ J (d) $8\pi \times 10^{-6}$ J

Q.21 The radius of a soap bubble is r and the surface tension of soap solution is T. Keeping the temperature constant, the extra energy needed to double radius of the soap bubble by blowing is
(a) $32\pi r^2 T$ (b) $24\pi r^2 T$ (c) $16\pi r^2 T$ (d) $8\pi r^2 T$

Q.22 Extra pressure inside a soap bubble of radius (r) is proportional to
(a) r (b) 1/r (c) r^2 (d) $1/r^2$

Q.23 The surface tension of soap solution is 25×10^{-3} Nm^{-1}. The excess pressure inside a soap bubble of diameter 1 cm is

(a) 10 Pa (b) 20 Pa (c) 5 Pa (d) none of these

Q.24 A spherical drop of water has 1 mm radius. If the surface tension of water is 70×10^{-3} Nm^{-1}, then difference of pressure between inside and outside of the spherical drop is
(a) 35 Nm^{-2} (b) 70 Nm^{-2} (c) 140 Nm^{-2} (d) zero

Q.25 At critical temperature, the surface tension of a liquid is
(a) zero (b) infinity
(c) same as that any other temperature (d) cannot be determined

Q.26 The surface tension of liquid decreases with a rise in
(a) temperature of the liquid (b) viscosity of the liquid
(c) diameter of container (d) thickness of container

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false (d) If both assertion and reason are false

Q.27 **Assertion:** The size of a hydrogen balloon increases as it rises in air.
Reason: The material of the balloon can be easily stretched.

Q.28 **Assertion:** A hydrogen filled balloon stops rising after it has attained a certain height in the sky.
Reason: The atmospheric pressure decreases with height and becomes zero when maximum height is attained.

Q.29 **Assertion:** In taking into account the fact that any object, which floats must have an average density less than that of water, during World War I, a number of cargo vessels were made of concrete.
Reason: Concrete cargo vessels were filled with air.

Q.30 **Assertion:** The machine parts are jammed in winter.
Reason: The viscosity of the lubricants used in the machines increase at low temperature.

Q.31 **Assertion:** For Reynold's number $R_e > 2000$, the flow of fluid is turbulent.
Reason: Inertial forces are dominant compared to the viscous forces at such high Reynold's numbers.

Q.32 **Assertion:** The shape of an automobile is so designed that its front resembles the streamline pattern of the fluid through which it moves.
Reason: The resistance offered by the fluid is maximum.

Q.33 **Assertion:** A thin stainless steel needle can lay floating on a still water surface.
Reason: Any object floats, when the buoyancy force balances the weight of the object.

Q.34 **Assertion:** A needle placed carefully on the surface of water may float, whereas a ball of the same material will always sink.
Reason: The buoyancy of an object depends both on the material and shape of the object.

Q.35 **Assertion:** Smaller drops of liquid resist deforming forces better than the larger drops.
Reason: Excess pressure inside a drop is directly proportional to its surface area.

Q.36 **Assertion:** Bubble of soap is larger than that of water.
Reason: Surface tension of soap bubble is less than that of water.

Answers

1	b	2	c	3	c	4	b	5	b
6	b	7	d	8	a	9	a	10	c
11	c	12	a	13	c	14	d	15	b
16	d	17	b	18	b	19	b	20	d
21	b	22	b	23	b	24	c	25	a
26	a	27	b	28	c	29	a	30	a
31	a	32	d	33	c	34	c		
35	c	36	a						