MECHANICAL PROPERTIES OF SOLIDS

Deforming Forces

If a force is applied on a body which is neither free to move nor free to rotate, the molecules of the body are forced to undergo a change in their relative positions.

A force which changes the size or shape of a body is called a deforming force.

Elasticity

If a body regains its original size and shape after the removal of deforming force, it is said to be elastic body and this property is called elasticity.

Perfectly Elastic Body

If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body. The nearest approach to a perfectly elastic body is quartz and phosphor bronze.

Plasticity

If a body does not regain its original size and shape even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity. For example, if we stretch a piece of chewing–gum and release it, it will not regain its original size and shape.

Perfectly Plastic Body

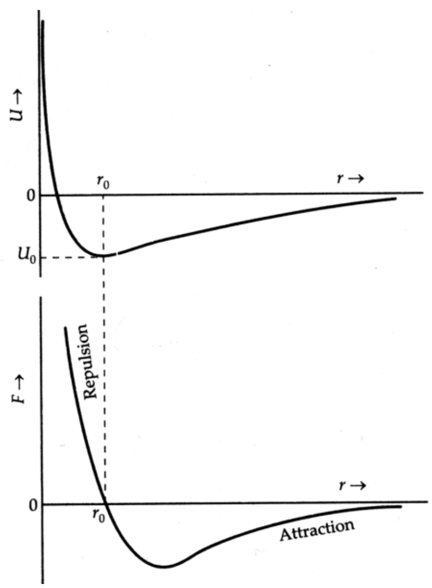
If a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be a perfectly plastic body. Putty and paraffin wax are nearly perfectly plastic bodies.

NOTE

- No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.

Elastic Behaviour in Terms of Interatomic forces

When the interatomic separation r is large, the potential energy of the atoms is negative and the interatomic force is *attractive.* At some particular separation r_0, the potential energy becomes minimum and the interatomic force becomes zero. The separation r_0 is called **normal or equilibrium separation.**

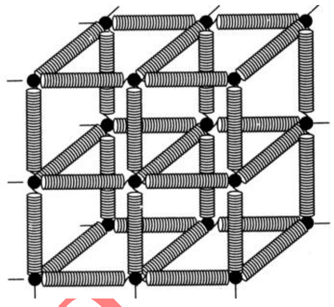


When separation reduces below r_0, the potential energy increases steeply and the interatomic force becomes *repulsive.*

Normally, the atoms occupy the positions ($r = r_0$) of minimum potential energy called the positions of stable equilibrium. When a tensile or compressive force is applied on a body, its atoms are pulled apart or pushed closer together to a distance r, greater than or smaller than r_0. When the deforming force is removed, the interatomic forces of attraction/ repulsion restore the atoms to their equilibrium positions. The body regains its original size and shape. The stronger the interatomic forces, the smaller will be the displacements of atoms from the equilibrium positions and hence greater is the elasticity (or modulus of elasticity) of the material.

Elastic behaviour on the basis of spring–ball model of a solid

The atoms in a solid may be regarded as mass points or small balls connected in three dimensional space through springs. The springs represent the interatomic forces. This is called spring ball model of a solid, as shown in figure.



Normally, the balls occupy the positions of minimum potential energy or zero interatomic force. When any ball is displaced from its equilibrium position, the various springs connected to it exert a resultant force on this ball. This force tends to bring the ball to its equilibrium position. This explains the elastic behaviour of solid in terms of microscopic nature of the solid.

Stress

If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state. *The internal restoring force set up per unit area of cross–section of the deformed body is called stress.* As the restoring force is equal and opposite to the external deforming force, therefore

$$Stress = \frac{Applied\ \ force}{Area} = \frac{F}{A}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. The dimensional formula of stress is $[ML^{-1}T^{-2}]$

It is a tensor.

Types of Stress

(i) **Normal Stress:** It is the restoring force set up perpendicular to cross sectional area. It changes the size of body. It is of two types:

- (a) **Tensile Stress:** It is the restoring force set up per until cross–sectional area of a body when the length of the body increases in the direction of the deforming force. It is also known as *longitudinal stress.*
- (b) **Compressional Stress:** It is the restoring force set up per unit cross–sectional area of a body when its lengths decreases under a deforming force.

(ii) **Hydrostatic Stress:** If a body is subjected to a uniform force from all sides, then the corresponding stress is called *hydrostatic stress.* Actually it is normal stress.

(iii) **Tangential or Shearing Stress:** When a deforming force acts tangentially to the surface of a body. The tangential force applied per unit area is equal to the tangential stress. It changes the shape of body.

Strain

When a deforming force acts on a body, the body undergoes a change in its shape and size. *The ratio of the change in any dimension produced in the body to the original dimension is called strain.*

$$Strain = \frac{Change\ in\ dimension}{Original\ dimension}$$

As strain is the ratio of two like quantities, it has no units and dimensions.

Types of Strain

(i) **Longitudinal Strain:** *It is defined as the increase in length per unit original length,* when the body is deformed by external forces.

$$Longitudinal\ strain = \frac{Change\ in\ length}{Original\ length} = \frac{\Delta l}{l}$$

(ii) **Volumetric Strain :** *It is defined as the change in volume per unit original volume,* when the body is deformed by external forces.

$$Longitudinal\ strain\ = \frac{Change\ in\ volume}{Original\ volume} = \frac{\Delta V}{V}$$

(iii) **Shear Strain:** *It is defined as the angle θ (in radian), through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.*

Shear strain = θ = tan θ

$$= \frac{Relative\ displacement\ between\ 2\ parallel\ planes}{Distance\ between\ parallel\ planes}$$

Elastic Limit

If a sufficiently large force is suspended from the wire, it is found that the wire does not regain its original length after the load is removed. *The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.* If the deforming force exceeds the elastic limit, the body acquires a *permanent set or deformation and is said to be overstrained.*

Hook's Law and Modulus of Elasticity

Hooke's law states that the extension produced in a wire is directly proportional to the load applied. Modified Hooke's law to the more general form as follows:

Within the elastic limit, the stress is directly proportional to strain. Thus within the elastic limit,

Stress ∝ Strain

or Stress = constant × strain or $\frac{Stress}{Strain} = constant$

The constant of proportionality is called modulus of elasticity or coefficient of elasticity of the material. Its value depends on the nature of the material of the body and the manner in which it is deformed.

NOTE:

- Like Boyle's law, Hooke's law is one of the earliest quantitative relationship in science.
- Hooke's law is valid only in the linear position of the stress–strain curve. The law is not valid for large values of strains.
- Stress is not a vector quantity since, unlike a force, the stress cannot be assigned a specific direction.
- When a wire, suspended from a ceiling, is stretched by a weight (F) suspended from its lower end, the ceiling exerts a force on the wire equal and opposite to the weight F. But the tension at any cross–section A of the wire is just F and not 2F. Hence the tensile stress which is equal to the tension per unit area is equal to F/A.

Modulus of Elasticity

The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain, within the elastic limit.

$$Modulus\ of\ \ elasticity,\ E = \frac{Stress}{Strain}$$

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$

Different types of moduli of elasticity

Corresponding to the three types of strain, we have three important moduli of elasticity:

(i) *Young's modulus (Y), i.e., the modulus of elasticity of length.*

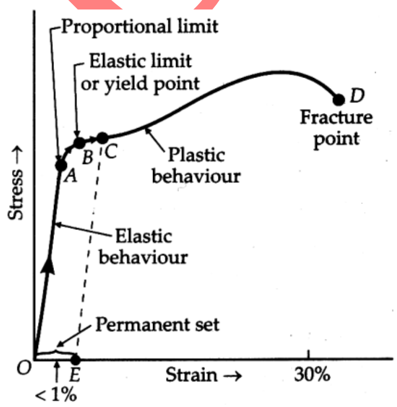
(ii) *Bulk modulus (k), i.e., the modulus of elasticity of volume.*

(iii) *Modulus of rigidity or shear modulus (η), i.e., modulus of elasticity of shape.*

Stress–Strain Curve for a Metallic Wire

Shows a stress–strain curve for a metal wire which is gradually being loaded.

(i) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the ***proportional limit***. In this region, the wire is perfectly elastic.



(ii) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called ***elastic region*** and the point B is called ***elastic limit or yield point.*** The stress corresponding to the yield point is called ***yield strength (S_y)***.

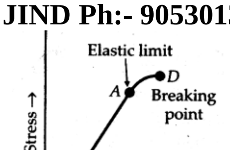
Upto point B, the elastic forces of the material are ***conservative*** i.e., when the material returns to its original size, the work done in producing the deformation is completely recovered.

(iii) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CE. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a ***permanent set.*** The fact that the stress–strain curve is not retraced on reversing the strain is called ***elastic hysteresis.***

(iv) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the ***fracture point.*** In the region between B and D, the length of wire goes on increasing even without any addition of load. This region is called *plastic region and the material* is said to undergo ***plastic flow or plastic deformation.*** The stress corresponding to the breaking point is called ***ultimate strength*** or ***tensile strength*** of the material.

Classification of Materials on the Basis of Stress–Strain Curve

(i) **Ductile Materials:** *The materials which have large plastic range of extension are called ductile materials.* Their fracture point is widely separated from the elastic limit. Such materials undergo

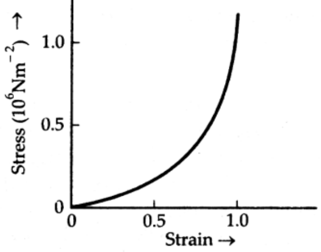


an irreversible increase in length before snapping. So they can be drawn into thin wires. For example, copper, silver, iron, aluminium, etc.

(ii) **Brittle materials:** *The materials which have very small range of plastic extension are called brittle materials.* Such materials break as soon as the stress is increased beyond the elastic limit. Their breaking point lies just close to their elastic limit, as shown in figure. For example, cast iron, glass, ceramics, etc.

Elastomers

The materials which can be elasticity stretched to large value of strain are called elastomers. For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic region in such cases is very large, but the materials does not obey Hooke's law. In our body, the elastic tissue is aorta.



Subjective Assignment – I

Q.1 The length of a suspended wire increases by 10^{-4} of its original length when a stress of 10^7 Nm^{-2} is applied on it. Calculate the Young's modulus of the material of the wire.

Q.2 A uniform wire of steel of length 2.5 m and density 8.0 gcm^{-3} weighs 50 g. When stretched by a force of 10 kgf, the length increases by 2 mm. Calculate Young's modulus of steel.

Q.3 A structural steel row has a radius of 10mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus, Y, of the structural steel is 2.0×10^{11} Nm^{-2}.

Q.4 What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.

Q.5 The breaking stress for a metal is 7.8×10^9 Nm^{-2}. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of the metal = 7.8×10^3 kg m^{-3}. Take g = 10 N kg^{-1}.

Q.6 A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg m^{-3} and Young's modulus for the rubber is 5×10^6 Nm^{-2}. Take g = 10 N kg^{-1}.

Q.7 A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is 50×10^6 Nm^{-2}. Estimate the largest mass that can be hung from it without breaking it. Take g = 10 N kg^{-1}.

Q.8 A composite wire of uniform diameter 3.0 mm consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, given that the Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.

Q.9 The maximum stress that can be applied to the material of a wire used to suspend an elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, what is the minimum diameter of the wire?

Q.10 A mass of 100 gram is attached to the end of a rubber string 49 cm long and having an area of cross–section 20 mm^2. The string is whirled round, horizontally at a constant speed of 40 rps in a circle of radius 51 cm. Find Young's modulus of rubber.

Q.11 A uniform heavy rod of weight W, cross–sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.

Q.12 A steel wire of uniform cross–section of 1 mm^2 is heated to 70^o C and stretched by typing its two ends rigidly. Calculate the change in the tension of the wire when the temperature falls from 70^o C to 35^oC. Coefficient of linear expansion of steel is 1.1×10^{-5} $^oC^{-1}$ and the Young's modulus is 2.0×10^{11} Nm^{-2}.

Q.13 A steel wire of length 5.0 m and cross–section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.0 m and cross–section 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?

Q.14 A wire increases by 10^{-3} of its length when a stress of 1×10^8 Nm^{-2} is applied to it. What is the Young's modulus of the material of the wire?

Q.15 What force is required to stretch a steel wire 1 cm^2 in cross–section to double its length? Given $Y = 2 \times 10^{11}$ Nm^{-2}.

Q.16 Find the stress to be applied to a steel wire to stretch it by 0.025% of its original length. Y for steel is
9×10^{-10} Nm^{-2}.

Q.17 A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg–wt. Find the increase in its length, if the Young's modulus of steel wire is $2. \times 10^{-12}$ dyne cm^{-2}.

Q.18 A wire elongates by 9 mm when a load of 10 kg is suspended from it. What is the elongation when its radius is doubled, if all other quantities are same as before?

Q.19. The breaking stress of aluminium is 7.5×10^7 Nm^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7×10^3 kg m^{-3}.

Q.20 A stress of 1kg mm^{-2} is applied to a wire of which Young's modulus is 10^{11} nm^{-2}. Find the percentage increase in length.

Q.21 Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is
1 cm, find by how much is each wire elongated? Given Y for steel = 20×10^{11} dyne cm^{-2} and Y for copper = 12×10^{11} dyne cm^{-2}.

Q.22 Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4 : 5 and their radii are in the ratio 4 : 3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B.

Q.23 Two wires of equal cross–section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires is found to be equal. Given Young's moduli of steel and copper are 2.0×10^{11} Nm^{-2} and 1.1×10^{11} Nm^{-2}. Find the ratio between the lengths of steel and copper wires.

Q.24 A lift is tied with thick iron wires and its mass is 1000 kg. If the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is 1.4×10^8 Nm^{-2}, find the minimum diameter of the wire.

Q.25 The length of a metal wire is l_1 when the tension in it is T_1 and l_2 when the tension in it is T_2. Find the original length of the wire.

Q.26 A metal bar of length l and area of cross–section A is rigidly clamped between two walls. The Young's modulus of the material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature is increased by ΔT. Find the force exerted at the ends of the bar.

Q.27 Two wires made of the same material are subjected to forces in the ratio of 1 : 4. Their lengths are in the ratio 8 : 1 and diameter in the ratio 2 : 1. Find the ratio of their extensions.

Answers

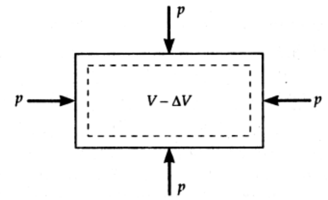
1.	10^{11} Nm^{-2}	2.	49×10^{11} dyne cm^{-2}		
3.	(i) 3.18×10^{8} Nm^{-2}, (ii) 1.59 mm, (iii) 0.16%			4.	0.16%
5.	10^{5} m	6.	0.15 m	7.	392.5 kg
8.	176.8 N	9.	1.0284×10^{-2} m	10.	3.95×10^{9} Nm^{-2}
11.	$\frac{Wl}{2Ay}$	12.	77.0 N	13.	2.22
14.	10^{11} Nm^{-2}	15.	2×10^{7} N	16.	2.25×10^{7} Nm^{-2}
17.	0.0041 cm	18.	2.25 mm	19.	2.83×10^{3} kg m^{-3}
20.	0.0098 %	21.	0.375 cm and 0.625 cm	22.	2.22 mm
23.	20 : 11	24.	0.01 m	25.	$\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$
26.	YA $\alpha\Delta T$	27.	1 : 2		

Bulk Modulus of Elasticity

Within the elastic limit, the ratio of normal stress to volumetric strain is called bulk modulus of elasticity.

Consider a body of volume V and surface area A. Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by ΔV as shown in figure. Then bulk modulus of elasticity is given by

$$B = \frac{Normal\ stress}{Volumetric\ strain} = \frac{F/A}{\Delta V/V}$$



or $$B = -\frac{F}{A}.\frac{V}{\Delta V} = -\frac{pV}{\Delta V}$$

where p(= F/A) is the normal pressure. Negative sign shows that the volume decreases with increase in stress.

Units and dimensions of B. The SI unit of bulk modulus is Nm^{-2} or Pascal (Pa) and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$

Compressibility. *The reciprocal of the bulk modulus of a material is called its compressibility.*

Compressibility = $\frac{1}{B}$

SI unit of compressibility = $N^{-1}m^{2}$

CGS unit of compressibility = $dyne^{-1}$ cm^{2}

The dimensional formula of compressibility is $[M^{-1}LT^{2}]$

Subjective Assignment – II

Q.1 The pressure of a medium is changed from 1.01×10^{5} Pa to 1.165×10^{5} Pa and change in volume is 10%, keeping temperature constant. Find the bulk modulus of the medium.

Q.2 The average depth of Indian ocean is about 3000 m. Calculate the fractional compression $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^{9} Nm^{-2}.

Q.3 A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk modulus of the material of the sphere. Density of sea water may be taken as 1.0×10^{3} kg m^{-3}.

Q.4 If the normal density of sea water is 1.00 g cm^{-3}, what will be its density at a depth of 3 km? Given compressibility of water = 0.0005 per atmosphere, 1 atomsphere pressure = 10^6 dyne cm^{-2}, g = 980 cm s^{-2}.

Q.5 A solid cube is subjected to a pressure of 5×10^5 Nm^{-2}. Each side of the cube is shortened by 1%. Find volumetric strain and bulk modulus of elasticity of the cube.

Q.6 Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 50^o to 70^oC. Coefficient of linear expansion of copper = 8.0×10^{-6} $^oC^{-1}$ and bulk modulus of elasticity = 3.6×10^{11} Nm^{-2}.

Q.7 A solid sphere of radius 10 cm is subjected to a uniform pressure = 5×10^8 Nm^{-2}. Determine the consequent change in volume. Bulk modulus of the material of sphere is equal to 3.14×10^{11} Nm^{-2}.

Q.8 Find the change in volume which 1 m^3 of water will undergo when taken from the surface to the bottom of a lake 100 m deep. Given volume elasticity of water is 22,000 atmosphere.

Q.9 A solid ball 300 cm in diameter is submerged in a lake at such a depth that the pressure exerted by water is 1.00 kgf cm^{-2}. Find the change in volume of the ball at this depth. B for material of the ball = 1.00×10^{13} dyne cm^{-2}.

Q.10 A spherical ball contracts in volume by 0.0098% when subjected to a pressure of 100 atm. Calculate its bulk modulus. Given 1 atm = 1.01×10^5 Nm^{-2}

Q.11 What increase in pressure will be needed to decrease the volume of 1.0 m^3 of water by 10 c.c.? The bulk modulus of water is 0.21×10^{10} Nm^{-2}.

Q.12 Determine the fractional change in volume as the pressure of the atmosphere (1.0×10^5 Pa) around a metal block is reduced to zero by placing the block in vacuum. The bulk modulus for the block is
1.25×10^{11} Nm^{-2}.

Q.13 Find the density of the metal under a pressure of 20,000 N cm^{-2}. Given density of the metal = 11 g cm^{-3}, bulk modulus of the metal = 8×10^9 Nm^{-2}.

Q.14 The compressibility of water is 4×10^{-5} per unit atmospheric pressure. What will be the decrease in volume of 100 cm^3 of water under pressure of 100 atmosphere?

Q.15 On taking a solid ball of rubber from the surface to the bottom of a lake of 180 m depth, the reduction of the volume of the ball is 0.1%. The density of water of the lake is 1.0×10^3 kg m^{-3}. Determine the value of the bulk modulus of elasticity of rubber. Take g = 10 ms^{-2}.

Q.16 A uniform pressure P is exerted on all sides of a solid cube at temperature t^oC. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied, if the bulk modulus and coefficient of volume expansion of the material are B and γ respectively?

Q.17 A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere.

Answers

1.	1.55×10^5 Pa	2.	3×10^7 Nm^{-2}, 1.36%	3.	9.8×10^{10} Nm^{-2}
4.	1.0149 g cm^{-3}	5.	0.03, 1.67×10^7 Nm^{-2}	6.	1.728×10^8 Nm^{-2}
7.	6.67×10^{-6} m^3	8.	4.4×10^{-4} m^3	9.	1.385 cm^3
10.	1.033×10^{11} Nm^{-2}	11.	2.1×10^4 Nm^{-2}	12.	8×10^{-7}
13.	11.28 g cm^{-3}	14.	0.4 cm^3	15.	1.8×10^9 Nm^{-2}

16. $\frac{p}{\gamma B}$ 17. $\frac{\Delta R}{R}=\frac{Mg}{3AB}$

Modulus of Rigidity or Shear Modulus

Within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.

Consider a rectangular block whose lower face is fixed and a tangential force F is applied over its upper face of area A. An equal and opposite force F comes into play on its lower fixed face. The two equal and opposite forces form a couple which exerts a torque. As the lower face of the block is fixed, the coupled shears the block into a parallelopied by displacing its upper face through distance AA' = Δl. Let AB = DC = l and $\angle ABA' = \theta$.

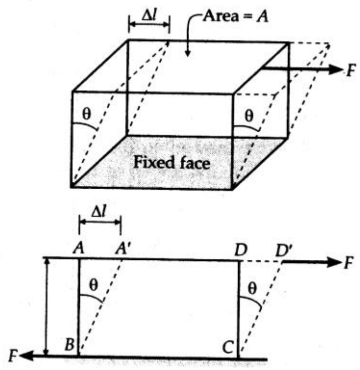
Tangential stress = $\frac{F}{A}$

Shear strain = $\theta \approx \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}$

The modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

Units and dimensions of η : The SI unit of modulus of rigidity is Nm^{-2} and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$



NOTE

- Elastic deformations in all bodies become plastic deformations with time.
- As only solids have length and shape, Young's modulus and shear modulus are relevant only for solids.
- As solids, liquids and gases all have volume elasticity, bulk modulus is relevant for all three sates of matter.
- Metals have large values of Young's modulus than alloys and elastomers. A material with large Y requires a large force to produce small changes in lengths.
- Elastic has a different meaning in physics than that in daily life. In daily life, a material which stretches more is said to be more elastic, but it is a misnomer. In physics, a material which stretches to a lesser extent for a given load is considered to be more elastic.

Subjective Assignment – III

Q.1 A cube of aluminium of each side 4 cm is subjected to a tangential (shearing) force. The top face of the cube is sheared through 0.012 cm with respect to the bottom face. Find (i) shearing strain (ii) shearing stress and shearing force. Given $\eta = 2.08 \times 10^{11}$ dyne cm^{-2}.

Q.2 A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced, if the shear modulus of lead is 5.6×10^9 Pa?

Q.3 A rubber block 1 cm × 3 cm × 10 cm is clamped at one end with its 10 cm side vertical. A horizontal force of 30 N is applied to the free surface. What is the horizontal displacement of the top face? Modulus of rigidity of rubber = 1.4×10^5 Nm^{-2}.

Q.4 A 60 kg motor rests on four cylindrical rubber blocks. Each cylinder has a height of 3 cm and a cross–sectional area of 15 cm^2. The shear modulus for this rubber is 2×10^6 Nm^{-2}. If a sideways force of 300 N is applied to the motor, how far will it move sideways?

Q.5 A metallic cube whose each side is 10 cm is subjected to a shearing force of 100 kg f. The top face is displaced through 0.25 cm with respect to the bottom. Calculate the shearing stress, strain and shear modulus.

Q.6 An Indian rubber cube of side 7 cm has one side fixed, while a tangential force equal to the weight of 200 kilogram is applied to the opposite face. Find the shearing strain produced and distance through which the strained side moves. Modulus of rigidity for rubber is 2×10^7 dyne cm^{-2}.

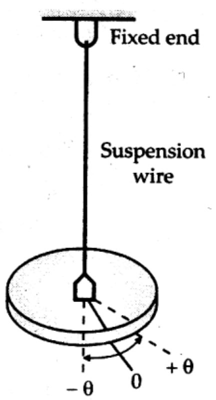
Q.7 A metal cube of side 10 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm with respect to its bottom.

Q.8 Two parallel and opposite forces, each 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block 25 cm on a side. Find the angle of shear and the displacement of the upper surface relative to the lower surface. The shear modulus for the metal is 8×10^{10} Nm^{-2}.

Answers

1. (i) 0.003 rad, (ii) 6.24×10^8 dyne cm^{-2}, 9.984×10^9 dyne
2. 1.6×10^{-4} m
3. 7.14 cm
4. 0.075 cm
5. 9.8×10^4 Nm^{-2}, 0.025 rad, 3.92×10^6 Nm^{-2}
6. 0.2 radian, 1.4 cm
7. 2×10^6 Nm^{-2}
8. 8.0×10^{-7} rad, 2.0×10^{-7} m

Some Other Elastic Effects



Elastic After Effect : The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. *The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.*

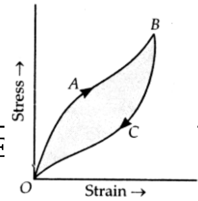
Elastic fatigue : If we set the wire into torsional vibrations, it will continue vibrating for a long times before its vibrations die out. If it is again made to vibrate, its vibrations will die out in a lesser time. Due to continuous alternating strains, the wire is said to have been *tired or fatigued.*

Elastic fatigue is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

A hard wire can be broken by bending it repeatedly in opposite directions, as it loses strength due to elastic fatigue. For the same reason, the railway bridges are declared unsafe after a reasonably good period to avoid the risk of a mishap.

Elastic Hysteresis

Figure shows the stress–strain curve for a rubber sample when loaded and then unloaded. For increasing load, the stress–strain curve is OAB and for



decreasing load, the curve is BCO. *The fact that the stress–strain curve is not retraced on reversing the strain is known as elastic hysteresis.*

The area under the curve OAB represents the work done per unit volume in stretching the rubber. The area under BCO represents the energy given up by rubber on unloading. So the shaded area of the hysteresis loop represents the energy lost as heat during the loading unloading cycle.

Applications of elastic hysteresis

(i) Car tyres are made with synthetic rubbers having small–area hysteresis loops because a car tyre of such a rubber will not get excessively heated during the journey.

(ii) A padding of vulcanized rubber having large area hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibrations, it dissipates a large amount of vibration energy.

Applications of Elasticity

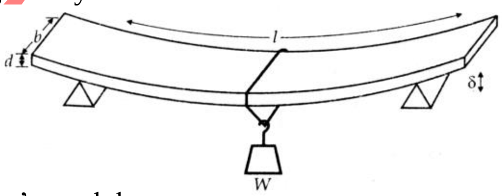
Any metallic part of a machinery is never subject to a stress beyond the elastic limit. This is because a stress beyond elastic limit will permanently deform that metallic part.

The thickness of metallic ropes used in cranes to lift heavy loads is decided from the knowledge of the elastic limit of the material and the factor of safety.

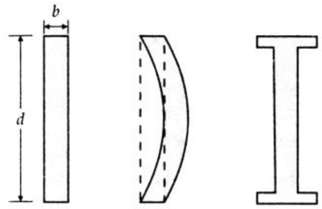
A single wire of this much radius would be a rigid rod. For the ease in manufacture and to impart flexibility and strength to the rope, it is always made of a large number of thin wires braided together.

The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a rectangular bar of length *l*, breadth b and thickness d supported at both ends, as shown in figure. When a load W is suspended at its middle, the bar gets depressed by an amount given by

$$\delta = \frac{Wl^3}{4Y\,bd^3}$$



Bending can be reduced by using a material with a large Young's modulus Y. As δ is proportional to d^{-3} and only to b^{-1}, *so depression more effectively reduced by increasing the depth d rather than the breadth b.* But a deep bar has a tendency to bend under the weight of a moving traffic, as shown in figure. This bending is called ***buckling.*** Hence a better choice is to have a bar of I–shaped cross–section, as shown in figure. This section provides a large load bearing surface and enough depth to prevent bending. Also, this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.



The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rock on the top should be less than the critical shear stress at which the rock begins to flow. Suppose the height of the mountain is h and the density of the mountain ρ. Hence stress exerted by mountain at the base = hρg. The material at the base experiences this force per unit area in the vertical direction, but sides of the mountain are free. Hence there is a tangential shear of the order of hρg. The elastic limit for a typical rock is about 3×10^8 Nm^{-2} and its density is 3×10^3 kg m^{-3}. Hence

$$h_{max}\,\rho g = 3 \times 10^8 \quad \text{or} \quad h_{max} = \frac{3\times10^8}{\rho g} = \frac{3\times10^8}{3\times10^3\times9.8} \simeq 10,0090 \text{ m} = 10 \text{ km}$$

This is nearly the height of the Mount Everest. A height greater than this will not be able to withstand the shearing stress due to the weight of the mountain.

A hollow shaft is stronger than a solid shaft made of equal quantity of same material : The torque required to produce until twist in a solid shaft of radius r, length *l* and made of material of modulus of rigidity η is given by

$$\tau = \frac{\pi\eta r^4}{2l}$$

The torque required to produce a unit twist in a hollow shaft of internal & external radii r_1 and r_2 is given by

$$\tau' = \frac{\pi\eta(r_2^4 - r_1^4)}{2l}$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4} \quad \text{.... (i)}$$

If both are made up of same mass and same material.

$\therefore$ m = m' or $\pi r^2 l\rho = \pi(r_2^2 - r_1^2)l\rho$ or $r^2 = r_2^2 - r_1^2$ .... (ii)

$\therefore$ $r^2 < r_2^2 + r_1^2$ .... (iii)

Using (ii) and (iii) in (i), we get $\tau' > \tau$

Thus torque required to twist hollow cylinder through a certain angle is greater than the torque necessary to twist a solid cylinder of same mass, length and material through the same angle. Hence a *hollow shaft is stronger than a solid shaft*. For this reason, elastic poles are given hollow structures.

Elastic Potential Energy of a Stretched Wire

When a wire is stretched, interatomic forces come into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

Expression for Elastic Potential Energy

Suppose a force F applied on a wire of length *l* increases its length by Δ*l*. Initially, the internal restoring force in the wire is zero. When the length is increased by Δ*l*, the internal force increases from zero to F (= applied force).

$\therefore$ Average internal force for an increase in length Δ*l* of wire

$$= \frac{0+F}{2} = \frac{F}{2}$$

Work done on the wire is

$$W = \text{Average force} \times \text{increase in length} = \frac{F}{2} \times \Delta l$$

This work done is stored as elastic potential energy U in the wire.

$$\therefore \quad U = \frac{1}{2} F \times \Delta l \quad = \frac{1}{2} \text{ stretching force} \times \text{increase in length}$$

Let A be the area of cross–section of the wire. Then

$$U = \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{l} \times Al \quad = \frac{1}{2} \text{ Stress} \times \text{Strain} \times \text{Volume of Wire}$$

Elastic potential energy per unit volume of the wire or elastic energy density is

$$u = \frac{U}{Volume} \quad \text{or} \quad u = \frac{1}{2}\, stress \times strain$$

But stress = Young's modulus × strain

$\therefore \quad u = \frac{1}{2}$ Young's modulus × strain^2

Subjective Assignment – IV

Q.1 A steel wire of 4.0 m is stretched through 2.0 mm. The cross–sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} Nm^{-2}, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.

Q.2 Calculate the increase in energy of a brass bar of length 0.2 m and cross–sectional area 1 cm^2 when compressed with a load of 5 kg weight along its length. Young's modulus of brass = 1.0×10^{11} Nm^{-2} and g = 9.8 ms^{-2}.

Q.3 When the load on a wire is increased from 3 kg wt to 5 kg wt, the elongation increases from 0.61 mm to 1.02 mm. How much work is done during the extension of the wire?

Q.4 A 40 kg boy whose leg bones are 4 cm^2 in area and 50 cm long falls through a height of 50 without breaking his leg bones. If the bones can stand a stress of 0.9×10^8 Nm^{-2}, calculate the Young's modulus for the material of the bone. Take g = 10 ms^{-2}.

Q.5 A steel wire of length 2.0 m is stretched through 2.0 mm. The cross–sectional area of the wire is 4.0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is 2.0×10^{11} Nm^{-2}.

Q.6 If the Young's modulus of steel is 2×10^{11} Nm^{-2}, calculate the work done in stretching a steel wire 100 cm in length and of cross–sectional area 0.03 cm^2 when a load of 20 kg is slowly applied without the elastic limit being reached.

Q.7 The limiting stress of a typical human bone is 0.9×10^8 Nm^{-2}, while Young's molecules is 1.4×10^{10} Nm^{-2}. How much energy can be absorbed by two legs (without breaking) if each has a typical length of 50 cm and an average cross–sectional area of 5 cm^2?

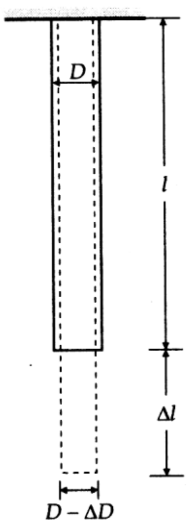
Answers

1.	(i) 2.5×10^4 Jm^{-3}, (ii) 0.2 J	2.	2.4×10^{-5} J	3.	16.023×10^{-3} J
4.	2.025×10^9 Nm^{-2}	5.	0.8 J	6.	0.032 J
7.	144.7 J				

Poisson's Ratio

When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced in the perpendicular direction is called lateral strain.

Within the elastic limit, the ratio of laterial strain to the longitudinal strain is called Poisson's ratio.



Suppose length of the loaded wire increases from l to $l + \Delta l$ and its diameter decreases from D to D – ΔD.

Longitudinal strain = $\frac{\Delta l}{l}$

Lateral strain = $-\frac{\Delta D}{D}$

Poisson's ratio is

$$\sigma = \frac{Lateral\ strain}{Longitudinal\ strain} = \frac{-\Delta D/D}{\Delta l/l} \quad \text{or} \quad \sigma = -\frac{l}{D}\cdot\frac{\Delta D}{\Delta l}$$

The negative sign indicates that longitudinal and lateral strains are in opposite sense. As the Poisson's ratio is the ratio of two strains, it has no units and dimensions.

Subjective Assignment – V

Q.1 Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.

Q.2 One end of a nylon of length 4.5 m and diameter 6 mm is fixed to a free limb. A monkey weighing 100 N jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in diameter. Given Young's modulus of nylon = 4.8×10^{11} Nm^{-2} and Poisson's ratio of nylon = 0.2.

Q.3 A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2 × 10–3, what is the percentage increase in volume?

Q.4 Calculate the Poisson's ratio for silver. Given its Young's modulus = 7.25×10^{10} Nm^{-2} and bulk modulus = 11×10^{10} Nm^{-2}.

Q.5 A material has Poisson's ratio 0.2. If a uniform rod of it suffers longitudinal strain 4.0×10^{-3}, calculate the percentage change in its volume.

Answers

1.	0.5	2.	3.32×10^{-5} m	3.	0
4.	0.39	5.	0.24%		

Miscellaneous Subjective Assignment

Q.1 A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, what will be the elongation of the wire in mm?

Q.2 A wire is cut to half its original length, (a) How would it affect the elongation under a given load? (b) How does it affect the maximum load it can support without exceeding the elastic limit?

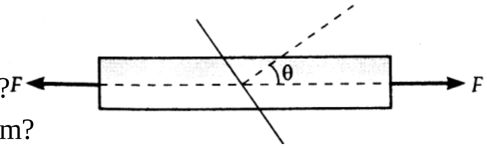
Q.3 A bar of cross–section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.

(a) What is the tensile stress on this plane?

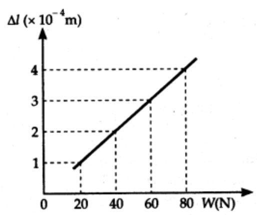
(b) What is the shearing stress on this plane?

(c) For what value of θ is the tensile stress maximum?

(d) For what value of θ is the shearing stress maximum?



Q.4 The graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to the other end. If the cross–sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



Q.5 A metallic wire is stretched by suspending weight from it. If α is the longitudinal strain and Y is the Young's modulus, show that elastic potential energy per unit volume is given by ½ $Y\alpha^2$.

Q.6 A copper wire of negligible mass, 1 m length and cross–sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad s^{-1}. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus. If on increasing the angular velocity to 100 rad s^{-1}, the wire breaks down, obtain the breaking stress.

Q.7 A load of 31.4 kg is suspended from a wire of radius 10^{-3} m and density 9×10^3 kg m^{-3}. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat of the material of the wire are 9.8×10^{10} Nm^{-2} and 490 J kg^{-1} K^{-1} respectively.

Q.8 A light rod of length 2 m is suspended horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross–section $A_1 = 0.1$ cm^2 and the other is of brass and is of cross–section $A_2 = 0.2$ cm^2. Find out the position along the rod at which a weight must be suspended to produce (i) equal stresses in both wires, (ii) equal strains in both wires. For steel, $Y = 20 \times 10^{10}$ Nm^{-2} and for brass $Y = 10 \times 10^{10}$ Nm^{-2}.

Q.9 A thin rod of negligible mass and area of cross–section 4×10^{-6} m^2, suspended vertically from one end has a length of 0.5 m at 100°C. The rod is cooled at 0°C, but prevented from contracting by attaching a mass at the lower end. Find (i) this mass and (ii) the energy stored in the rod. Given for this rod, $Y = 10^{11}$ Nm^{-2}, coefficient of linear expansion = 10^{-5} K^{-1} and g = 10 ms^{-2}.

Q.10 A wire of cross–sectional area A is stretched horizontally between two clamps located at a distance $2l$ metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the mid–point of the wire moves down be $x << l$, then find (i) the strain produced in the wire. (ii) the stress is the wire. (iii) If Y is the Young's modulus of wire, then find the value of x.

Q.11 A stone of 0.5 kg mass is attached to one end of a 0.8 m long aluminium wire of 0.7 mm diameter and suspended vertically. The stone is now rotated in a horizontal plane at a rate such that the wire makes an angle of 85° with the vertical. Find the increase in the length of the wire. The Young's modulus of aluminium = 7×10^{10} Nm^{-2}, sin 85° = 0.9962, cos 85° = 0.0872

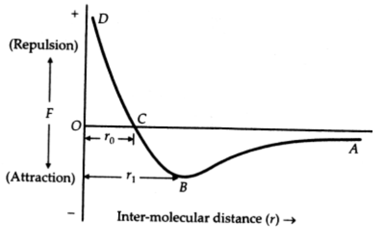
Q.12 Two rods of different materials but of equal cross–sections and lengths (1.0 m each) are joined to make a rod of length 2.0 m. The metal of one rod has coefficient of linear thermal expansion 10^{-5} $°C^{-1}$ and Young's modulus 3×10^{10} Nm^{-2}. The other metal has the values 2×10^{-5} $°C^{-1}$ and 10^{10} Nm^{-2} respectively. How much pressure must be applied to the ends of the composite rod to prevent its expansion when the temperature is raised by 100°C?

Answers

1.	l mm	2.	(a) halved, (b) no effect		
3.	$\frac{F}{A}\cos^2\theta$, (b) $\frac{F}{2A}\sin^2\theta$, (c) $\theta = 0^o$, (d) $\theta = 45^o$			4.	2×10^{11} Nm^{-2}
6.	10^{10} Nm^{-2}	7.	1/120 K	8.	(i) 1.33 m, (ii) 1 m
9.	(i) 40 kg, (ii) 0.1 J	10.	(i) $\frac{x^2}{2l^2}$, (ii) $\frac{wl}{Ax}$, (iii) $l\left[\frac{w}{YA}\right]^{1/3}$		
11.	1.67 mm	12.	5×10^7 Nm^{-2}		

Conceptual Problems

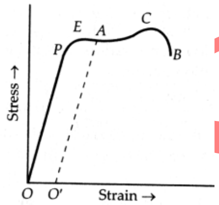
Q.1 In the diagram a graph between the inter–molecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.



Q.2 Crystalline solids have sharp melting points. Amorphous solids do not melt at a sharp temperature; rather these have a softening range of temperature. Explain.

Q.3 Which is more elastic–rubber or steel?

Q.4 The stress–strain graph for a metal wire is shown in figure. Up to the point E, the wire returns to its original state O along the curve EPO when it is gradually unloaded. Point B corresponds to the fracture of the wire.



(a) Up to which point on the curve is Hooke's law obeyed? This point is sometimes called "Proportionality limit"

(b) Which point on the curve corresponds to elastic limit and yield point of the wire?

(c) Indicate the elastic and plastic regions of the stress–strain graph.

(d) Describe what happens when the wire is loaded up to a stress corresponding to the point A on the graph, and the unloaded gradually. In particular, explain the dotted curve.

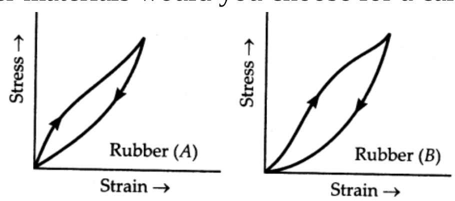
(e) What is peculiar about the portion of the stress–strain graph from C to B? Up to what stress can the wire be subjected without causing fracture?

Q.5 Two different types of rubber are found to have the stress–strain curves as shown in figure.

(a) In which significant ways do these curves differ from the stress–strain curve of a metal wire shown in figure.

(b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose? Why?

(c) Which of the two rubber materials would you choose for a car tyre?



Q.6 Read each of the statements below carefully and state, with reasons, if it is true or false

(a) When a material is under tensile stress, the restoring forces are caused by interatomic attraction while under compressional stress, the restoring forces are due to inter–atomic repulsion.

(b) A piece of rubber under an ordinary stress can display 1000% strain: yet when unloaded returns to its original length. This shows that the elastic restoring forces in a rubber piece are strictly conservative.

(c) Elastic restoring forces are strictly conservative only when Hooke's law is obeyed.

Q.7 Two wires of different materials are suspended from a rigid support. They have the same length and diameter and carry the same load at their free ends. (a) Will the stress and strain in each wire be the same? (b) Will the extension in both wires be the same?

Q.8 A cable is replaced by another of the same length and material but of twice the diameter. (a) How does this affect its elongation under a given load? (b) How many times will be the maximum load it can now support without exceeding the elastic limit?

Q.9 Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different?

Q.10 A uniform plank of Young's modulus Y is moved over a smooth horizontal surface by a constant horizontal force F. The area of transverse section of the plank is A. Find the compressive strain on the plank in the direction of the force.

Q.11 Why the bridges are declared unsafe alter long use?

Q.12 Two identical solid balls, one of ivory and the other of wet–clay, are dropped from the same height on the floor. Which will rise to a greater height after striking the floor and why?

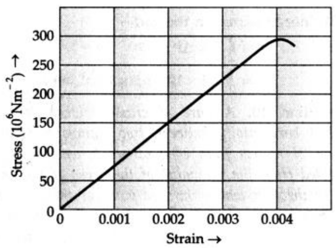
Q.13 The breaking force for a wire is F. What will be the breaking force for (a) two parallel wires of the same size (b) for a single wire of double the thickness?

Q.14 Why does modulus of elasticity of most of the materials decrease with the increase of temperature?

NCERT Exercise

Q.1 A steel wire of length 4.7 m and cross–section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross–section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Q.2 Figure shows the stress–strain curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



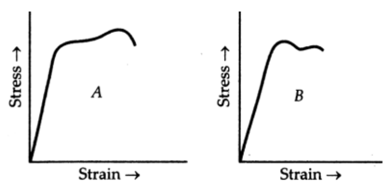
Q.3 The stress–strain graphs for materials A and B are shown in figure. The graphs are drawn to the same scale.

(a) which of the material has greater Young's modulus?

(b) Which material is more ductile?

(c) Which is more brittle?

(d) Which of the two is stronger material?



Q.4 Read each of the statements below carefully and state, with reasons, if it is true or false.

(a) The modulus of elasticity of rubber is greater than that of steel

(b) the stretching of a coil is determined by its shear modulus

Q.5 Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 Nm^{-2})

Q.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 G Pa. What is the vertical deflection of this face? (1 Pa = 1 Nm^{-2}).

Q.7 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50, 000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.

Q.8 A piece of copper having a rectangular cross–section of 15.2 mm × 19.1 mm is pulled in tension with 44, 500 N force, producing only elastic deformation. Calculate the resulting strain.

Q.9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?

Q.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Q.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross–sectional area of the wire is 0.005 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Q.12 Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, pressure increase = 100.0 atm, final volume = 100.5 litre (1 atm = 1.013×10^5 Pa)

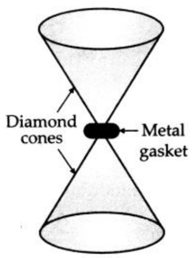
Q.13 What is the density of ocean water at a depth, where the pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kgm^{-3} ? Compressibility of water = 45.8×10^{-11} Pa^{-1}.

Q.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

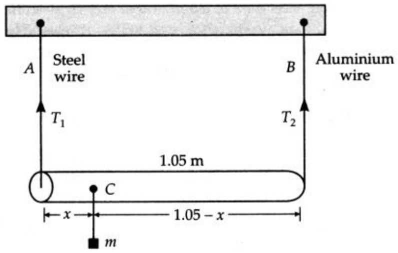
Q.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Q.16 How much should the pressure on a litre of water be changed to compress it by 0.10% ?

Q.17 Anvils made of single crystals of diamond, with the shape as shown in figure, are used to investigate behaviour of materials under very high pressure. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Q.18 A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross–sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2 respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?



Q.19 A mild steel wire of length 1.0 and cross–sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid–point of wire. Calculate the depression at the mid–point.

Q.20 Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3×10^9 Pa? Assume that each rivet is to carry one quarter of the load.

Q.21 The Marina trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

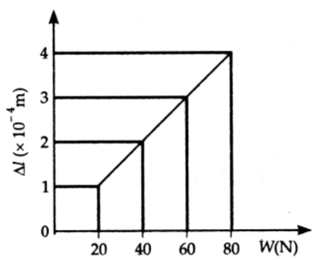
Answers

1.	1.79	2.	(a) 7.5×10^{10} Nm^{-2}, (b) 3×10^8 Nm^{-2}		
3.	(a) A, (b) A, (c) B, (d) A	4.	(a) false, (b) true		
5.	1.5×10^{-4} m, 1.3×10^{-4} m	6.	4×10^{-7} m		
7.	2.8×10^{-6}	8.	0.001277	9.	7.07×10^4 N
10.	1.3	11.	1.87×10^{-3} m	12.	2.026×10^9 Pa
13.	1.034×10^3 kgm^{-3}	14.	2.74×10^{-5}	15.	0.05 cm^3
16.	2.2×10^6 Nm^{-2} 0.7m,(b)0.43m	17.	2.55×10^{11} Nm^{-2}	18.	(a)
19.	1.07×10^{-2} m	20.	260 kN	21.	2.02×10^{-4} m^3

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

Q.1 The adjacent graph shows the extension (Δl) of a wire of length 1m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross–sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2
(c) 3×10^{-12} N/m^2 (d) 2×10^{-13} N/m^2

Q.2 The following four wires are made of the same material. Which of these will have the largest extension, when the same tension is applied?
(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm

Q.3 A wire of length L, and cross–sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is
(a) $YAx^2/ 2L$ (b) YAx^2/ L (c) $YAx/ 2L$ (d) $YAx^2 L$

Q.4 The pressure of a medium is changed from 1.01×10^5 Pa to 1.65×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is
(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa (c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

Q.5 A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is
(a) 2 P/3 (b) P (c) 3P/2 (d) 2 P

Answers

1.	a	2.	a	3.	a	4.	d	5.	b

AIEEE

Q.1 A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(a) $l / 2$ (b) l (c) $2\,l$ (d) zero

Q.2 A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is
(a) $F/ 2l$ (b) Fl (c) $2\,Fl$ (d) $Fl/2$

Q.3 A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J

Q.4 If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) 2 Y/S (b) S/2 Y (c) $2\,S^2Y$ (d) $S^2/ 2Y$

Q.5 Two wires are made of the same material and have the same volume. However wire 1 has cross–sectional area A and wire 2 has cross–sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?
(a) F (b) 4 F (c) 6 F (d) 9 F

Answers

1.	b	2.	d	3.	d	4.	d	5.	d

AIIMS Entrance Exam

Q.1 According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain
(a) increases (b) decreases (c) becomes zero (d) remains constant

Q.2 A thick copper rope of density 1.5×10^3 kgm^{-3} and Young's modulus 5×10^6 Nm^{-2}, 8 m in length, when hung from the ceiling of a room, the increase in its length due to its own weight is

(a) 9.6×10^{-5} m (b) 19.2×10^{-7} m (c) 9.6×10^{-2} m (d) 9.6 m

Q.3 If in a wire of Young's modulus Y, longitudinal strain X is produced, then the value of potential energy stored in its unit volume will be

(a) YX^2 (b) $2\ YX^2$ (c) $0.5\ Y^2\ X$ (d) $0.5\ YX^2$

Q.4 A metal ring of initial radius r and cross–sectional area A is fitted onto a wooden disc of radius R > r. If Young's modulus of the metal is Y, then the tension in the ring is

(a) $\frac{AYR}{r}$ (b) $\frac{Yr}{AR}$ (c) $\frac{AY(R-r)}{r}$ (d) $\frac{Y(R-r)}{Ar}$

Q.5 For a constant hydraulic stress on an object, the fractional change in the object's volume (ΔV/V) and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$ (c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto \frac{1}{B^2}$

Q.6 The compressibility of water is 4×10^{-5} per unit atmosphere pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(a) 0.4 cm^3 (b) 4×10^{-5} cm^3 (c) 0.025 cm^3 (d) 0.04 cm^3

Q.7 A stretched rubber has

(a) increased kinetic energy (b) increased potential energy
(c) decreased kinetic energy (d) decreased potential energy

Q.8 The breaking stress of a wire depends upon

(a) length of the wire (b) radius of the wire
(c) material of the wire (d) shape of the cross–section

Q.9 Which of the following affects the elasticity of a substance?

(a) hammering and annealing (b) change in temperature
(c) impurity in substance (d) all of these

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion if true, but reason is false.
(d) If both assertion and reason are false

Q.10 **Assertion:** Lead is more elastic than rubber.
Reason: If same load is loaded on the lead and rubber wire of same cross–sectional area, the strain of lead is very much less than that of rubber.

Q.11 **Assertion:** Stress is the internal force per unit area of a body.
Reason: Rubber is more elastic than steel.

Answers

1.	d	2.	c	3.	d	4.	c	5.	b
6.	a	7.	b	8.	c	9.	d	10.	a
11.	c								

DPMT Entrance Exam

Q.1 Which of the following has no dimensions?

(a) strain (b) angular velocity
(c) momentum (d) angular momentum

Q.2 The diameter of brass rod is 4 mm. Young's modulus of brass is 9×10^9 N/m^2. The force required to stretch 0.1% of its length is

(a) 360 πN (b) 36 N (c) $36\pi \times 10^5$ N (d) $144\ \pi \times 10^3$ N

Q.3 When a body of mass M is hung from a spring, the spring extends by 1 cm. If the body of mass 2 M be hung from the same spring, the extension of spring will be

(a) 1 cm (b) 2 cm (c) 0.5 cm (d) 4 cm

Q.4 A wire whose cross–sectional area is 2 mm^2 is stretched by 0.1 mm by a certain load, and if a similar wire of triple the area of cross section is stretched by the same load, then the elongation of the second wire would be

(a) 3.3 mm (b) 0.033 mm (c) 0.33 mm (d) 0.0033 mm

Q.5 A substance breaks down by a stress of 10^6 N/m^2. If the density of the wire is 3×10^3 kg/m^3, then the length of the wire of the substance which will break under its own weight when suspended vertically will be

(a) 66.6 m (b) 60.0 m (c) 33.3 mm (d) 30.3 mm

Q.6 With what minimum acceleration can a fireman slide down a rope whose breaking strength is two third of his weight?

(a) $\frac{g}{3}$ (b) $\frac{2}{3}g$ (c) $\frac{3}{2}g$ (d) $\frac{g}{2}$

Q.7 A wire of length L and cross–sectional area A is made of a material of Young's modulus Y. If the wire is stretched by the amount x, the work done is

(a) $\frac{YAx^2}{2L}$ (b) $\frac{YAx^2}{L}$ (c) Yax^2L (d) $\frac{YAx}{2L}$

Q.8 When a sphere is taken to bottom of sea 1 km deep, it contracts by 0.01%. The bulk modulus of elasticity of the material of sphere is (Given : Density of water = 1 g/cm^3)

(a) 9.8×10^{10} N/m^2 (b) 10.2×10^{10} N/m^2 (c) 0.98×10^{10} N/m^2 (d) 8.4×10^{10} N/m^2

Answers

1.	a	2.	a	3.	b	4.	b	5.	c
6.	a	7.	c	8.	a				

MECHANICAL PROPERTIES OF FLUIDS

Fluids

A fluid is a substance that can flow. It ultimately assumes the shape of the containing vessel because it cannot withstand shearing stress. Thus, both liquids and gases are fluids.

Fluid Statics

The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics. Its study includes hydrostatic pressure, Pascal's law.

Fluid dynamics

The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.

Thrust of a Liquid

Thrust

The total force exerted by a liquid on any surface in contact with it is called thrust. It is because of this thrust that a liquid flows out through the holes of the containing vessel. Thrust is a force. Its SI unit is Newton (N) & dimensional formula = $[MLT^{-2}]$

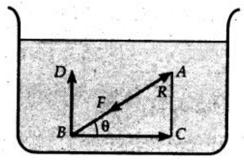
Liquid in Equilibrium

Consider a liquid contained in a vessel in the equilibrium state of rest. As shown in figure, suppose the liquid exerts a force F on the bottom surface in an inclined direction AB. The surface exerts an equal reaction R to water along BA.

The reaction R along BA has two rectangular components:

(i) Tangential component $BC = R\cos\theta$

(ii) Normal component, $BD = R\sin\theta$



Since a liquid cannot resist any tangential force, so the liquid near B should begin to flow along BC. Since the liquid is at rest, the force along BC should be zero.

$\therefore \quad R\cos\theta = 0$, as $R \neq 0$, so $\cos\theta = 0$ or $\theta = 90^\circ$

Hence a *liquid exerts force perpendicular to the surface of the container at every point.*

Pressure

The pressure at a point on a surface is the thrust acting normally per unit area around that point. If a total force F acts normally over a flat area A, then the pressure is $P = \frac{F}{A}$

- Pressure is a scalar quantity.
- SI units of pressure = Nm^{-2} or Pascal (Pa)
- CGS unit of pressure = dyne cm^{-2}
- Dimensional formula of pressure is $[ML^{-1}T^{-2}]$

Practical Application of Pressure

(i) A sharp knife cuts better than a blunt one

The area of a sharp edge is much less than the area of a blunt edge.

(ii) Railway tracks are laid on wooden sleepers

This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably.

(iii) **It is difficult for a man to walk on sand while a camel walks easily on sand** inspite of the fact that a camel is much heavier than a man. This is because camel's feet have a larger area than the feet of man.

(iv) **Pins and nails are made to have pointed ends:** Their pointed ends have very small area

Subjective Assignment – I

Q.1 The two thigh bones (femurs), each of cross–sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by femurs. Take g = 10 ms^{-2}.

Q.2 How much pressure will a man of weight 80 kgf exert on the ground when (i) he is lying and (ii) he is standing on his feet? Given that the area of the body of the man is 0.6 m^2 and that of a foot is 80 cm^2.

Q.3 A cylindrical vessel containing liquid is closed by a smooth piston of mass m. The area of cross–section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the piston.

Answers

1. 2×10^5 Nm^{-2} 2. (i) 1.307×10^3 Nm^{-2}, (ii) 4.9×10^4 Nm^{-2} 3. P_0 + mg/A

Density

The density of any material is defined as its mass per unit volume. If a body of mass M occupies volume V, then its density is

$$\rho = \frac{M}{V} \quad i.e., \quad Density = \frac{Mass}{Volume}$$

Density is a positive scalar quantity.

Units and dimensions of density

SI unit of density = kg m^{-3}

CGS unit of density = g cm^{-3}

Dimensional formula of density is $[ML^{-3}]$

Specific gravity or relative density

The specific gravity or relative density of a substance is defined as the ratio of the density of the substance to the density of water at 4°C. The density of water at 4°C is 1.0×10^3 kg m^{-3}.

$$Specific\ gravity = \frac{Density\ of\ substance}{Density\ of\ water\ at\ 4^oC}$$

Specific gravity is a dimensionaless positive scalar quantity.

Pascal's Law

This law tells as how pressure can be transmitted in a fluid. It can be stated in a number of equivalent ways as follows:

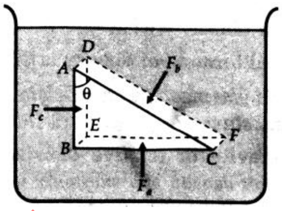
(i) *The pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.*

(ii) *A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.*

(iii) *The pressure in a fluid at rest is same at all points if we ignore gravity.*

Proof of Pascal's law

Consider a small element ABC – DEF in the form of a right angled prism in the interior of a fluid at rest. The element is so small that all its parts can be assumed to be at same depth from the liquid surface and, therefore, the effect of gravity is same for all of its points.



By Newton's law, the fluid force should balance in various directions.

Along horizontal direction, $F_b \sin\theta = F_c$

Along vertical direction, $F_b \cos\theta = F_a$

From the geometry of the figure, we get

and $A_b \sin\theta = A_c$ & $A_b \cos\theta = A_a$

From the above equations, we get

$$\frac{F_b \sin\theta}{A_b \sin\theta} = \frac{F_c}{A_c} \quad \text{and} \quad \frac{F_b \cos\theta}{A_b \cos\theta} = \frac{F_a}{A_a}$$

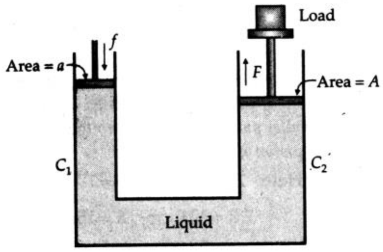
$$\therefore \quad \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c} \quad \text{or} \quad P_a = P_b = P_c$$

Hence, pressure exerted is same in all directions in a fluid at rest. This proves Pascal's law of transmission of fluid pressure.

Applications of Pascal's Law

Hydraulic lift

Hydraulic lift is an application of Pascal's law. It is used to lift heavy objects. It is a force multiplier. It consists of two cylinders C_1 and C_2 connected to each other by a pipe. The cylinders are fitted with water–tight frictionless pistons of different cross–sectional areas. The cylinders and the pipe contain a liquid. Suppose a force f is applied on the smaller piston of cross–sectional area a. Then



Pressure exerted on the liquid, $P = \frac{f}{a}$

According to Pascal's law, same pressure P is also transmitted to the larger piston of cross–sectional area A.

$$\therefore \quad F = P \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

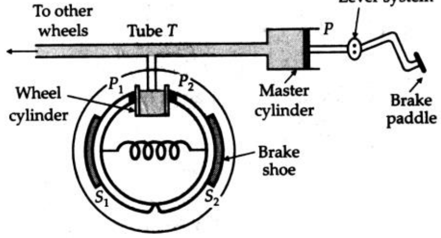
As A > a, therefore, F > f

Hence by making the ratio A/a large, very heavy loads (like cars and trucks) can be lifted by the application of a small force. However, there is no gain of work. The work done by force f is equal to the work done by F. The piston P_1 has to be moved down by a larger distance compared to the distance moved up by piston P_2.

Hydraulic Brakes

The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.

Construction



As shown in figure, a hydraulic brake consists of a tube T containing brake oil. One end of this tube is connected to a master cylinder fitted with piston P. The piston P is attached to the brake pedal through a lever system. The other end of the tube is connected to the wheel cylinder having two pistons P_1 and P_2. The pistons P_1 and P_2 are connected to the brake shoes S_1 and S_2

respectively. The area of cross–section of the wheel cylinder is larger than that of master cylinder.

Working

When the pedal is pressed, its lever system pushes the piston P into the master cylinder. The pressure is transmitted through the oil to the pistons P_1 and P_2 in the wheel cylinder, in accordance with Pascal's law. The pistons P_1 and P_2 are pushed outwards. The brake shoes get pressed against the inner rim of the wheel, retarding the motion of the wheel. As the cross–sectional area of wheel cylinder is larger than that of master cylinder, a small force applied to the pedal produces a large retarding force.

When the paddle is released, a spring pulls the brake shoes away from the rim. The pistons in both cylinders move towards their normal positions and the oil is forced back into the master cylinder.

Subjective Assignment – II

Q.1 In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1? What is the pressure necessary to accomplish that task?

Q.2 Two syringes of different cross–sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Q.3 Two pistons of hydraulic press have diameters of 30.0 cm and 2.5 cm. What is force exerted by larger piston, when 50.0 kg wt. is placed on the smaller piston? If the stroke of the smaller piston is 4.0 cm, through what distance will the larger piston move after 10 strokes?

Q.4 The average mass that must be lifted by a hydraulic press is 80 kg. If the radius of the larger piston is five times that of the smaller piston, what is the minimum force that must be applied?

Q.5 An automobile back is lifted by a hydraulic jack that consists of two pistons. The large piston is 1 m in diameter and the small piston is 10 cm in diameter. If W be weight of the car, how much smaller a force is needed on the small piston to lift the car?

Answers

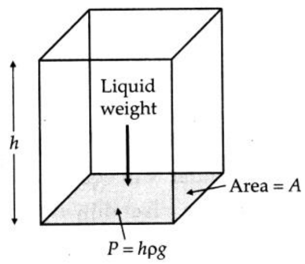
1.	1.5×10^3 N, 1.9×10^5 Pa	2.	(a) 90 N, (b) 0.67 cm	3.	7200 kg wt, 0.28 cm
4.	31.4 N	5.	1% of the weight of the car		

Pressure Exerted by a Liquid Column

Consider a vessel of height h and cross–sectional area A filled with a liquid of density ρ. The weight of the liquid column exerts a downward thrust on the bottom of the vessel and the liquid exerts pressure.

Weight of liquid column,

$$W = \text{Mass of liquid} \times g$$
$$= \text{Volume} \times \text{density} \times g$$
$$= Ah \times \rho \times g = Ah\ \rho g$$



Pressure exerted by the liquid column on the bottom of the vessel is

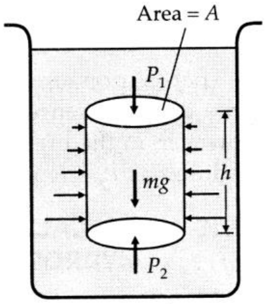
$$P = \frac{Thrust}{Area} = \frac{W}{A} = \frac{Ah\rho g}{A}$$

or $P = h\,\rho\,g$

Thus the pressure exerted by a liquid column at rest is proportional to (i) height of the liquid column and (ii) density of the liquid.

Effect of Gravity on Fluid Pressure

As the liquid cylinder is at rest, the resultant horizontal force should be zero. Various force acting on it in the vertical direction are:



1. Downward force on the top of the cylinder, $F_1 = P_1A$
2. Upward force on the bottom of the cylinder, $F_2 = P_2A$
3. Weight of the liquid cylinder acting downwards,

$W = \text{Mass} \times g = \text{Volume} \times \text{density} \times g = Ah\rho g$

where ρ is the density of the liquid.

As the liquid cylinder is in equilibrium,

Net upward force = Net downward force

or $F_1 + W = F_2$ or $F_2 - F_1 = W$

or $P_2A - P_1A = Ah\rho g$ or $P_2 - P_1 = h\rho g$

If we shift point 1 to the liquid surface, which is open to the atmosphere, then we can replace P_1 by atmospheric pressure P_a and P_2 by P in the above equitation and we get

$P - P_a = h\rho g$ or $P = P_a + h\rho g$

We can note the following points:

(i) The liquid pressure is the same at all points at the same horizontal level or at same depth.

(ii) Pressure at any point inside the fluid depends on the depth h.

(iii) The absolute (actual) pressure P, at a depth h below the liquid surface open to the atmosphere is greater than the atmospheric pressure by an amount $h\rho g$. The excess pressure $P - P_a$, at depth h is called a ***gauge pressure*** at the point.

(iv) Pressure does not depend on the cross–section or base–area or the shape of the vessel.

Effect of gravity on Pascal's law

If we neglect the effect of gravity, then

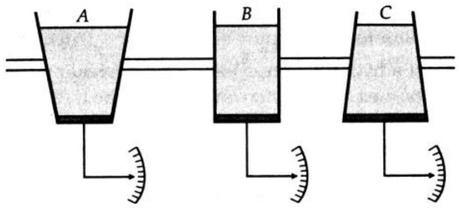
$P_2 - P_1 = h\rho g = 0$ or $P_2 = P_1$

That is, pressure at all points inside the liquid is same in the absence of gravity. This is Pascal's law. However, in the presence of gravity, Pascal's law gets modified as $P_2 - P_1 = h\rho g$.

Pascal's Vases : Hydrostatic Paradox

Pascal demonstrated experimentally that *the pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.*

When the there vessels are filled with the same liquid upto the same height, all the three meters records the same pressure.

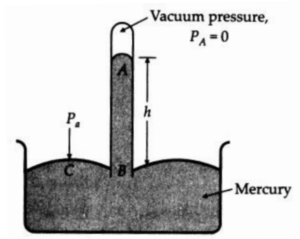


This appears anomalous because the three vessels have different shapes and contain different amounts of liquid. This apparently unexpected result is known as **hydrostatic paradox.**

Atmospheric Pressure

The gaseous envelope surrounding the earth is called the atmosphere. *The pressure exerted by the atmosphere is called atmospheric pressure.* The force exerted by air column of air on a unit area of the earth's surface is equal to the atmospheric pressure. The atmospheric pressure at sea level is 1.013×10^5 Nm^{-2} or Pa.

Torricelli's experiment of measuring atmospheric pressure



A 1 m long glass tube closed at one end is filled with clean and dry mercury. After closing the end of the tube with the thumb, the tube is inverted into a dish of mercury. As the thumb is removed, the mercury level in the tube falls down a little and comes to rest at a vertical height of 76 cm above the mercury level in the dish.

The space above mercury in the tube is almost a perfect vacuum and is called Torricellian vacuum. Therefore, pressure $P_A = 0$. Consider a point C on the mercury surface in the dish and point B in the tube at the same horizontal level. Then

$$P_B = P_C = \text{Atmospheric pressure, } P_a$$

If h is the height of mercury column and ρ is the density of mercury, then

$$P_B = P_A + h\rho g$$

or $P_a = 0 + h\rho g$ or $P_a = h\rho g$

For a mercury barometer, h = 76 cm = 0.76 m, $\rho = 13.6 \times 10^3$ kg m^{-3}, g = 9.8 ms^{-2}

$\therefore$ $P_a = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5$ Pa

Absolute Pressure and Gauge Pressure

The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure,

i.e., $P_g = P - P_a = h\rho g$

The gauge pressure is proportional to h. Many pressure measuring devices directly measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).

Various Units for Pressure:

(i) SI unit of pressure = Nm^{-2} or Pascal (Pa) (ii) CGS unit of pressure = dyne cm^{-2}

(iii) **Atmosphere** (atm). It is the pressure exerted by 76 cm of Hg column (at 0^oC, 95^o latitude and mean sea level).

1 atm = 1.013×10^5 Pa = 1.013×10^6 dyne cm^{-2}

(iv) In meteorology, the atmospheric pressure is measured in bar and millibar.

1 bar = 10^5 Pa = 10^6 dyne cm^{-2} 1 millibar = 10^{-3} bar = 100 Pa

(v) Atmospheric pressure is also measured in **torr,** a unit named after Torricelli.

1 torr = 1 mm of Hg 1 atm = 1.013 bar = 760 torr

Units for Blood Pressure

The blood pressure is measured in mm of Hg. When the heart is contracted to its smallest size, the pumping is hardest and the pressure of blood flowing in major arteries is nearly 120 mm of Hg. This is known as **systolic pressure.** When the heart is expanded to its largest size, the blood pressure is nearly 80 mm of Hg. This is known as ***diastolic pressure.***

NOTE

- While describing a fluid, we are concerned with properties that vary from point to point and not with properties associated with a specific piece of matter. So the role of force in a solid is replaced in a fluid by pressure and that of mass by density.
- A fluid exerts pressure not only on a solid piece immersed in fluid or on the walls of container, fluid pressure exists at all points in a fluid. A volume element (of fluid) inside a fluid is in a equilibrium because the pressures exerted on its various faces get balanced.

- Pressure at a point in a liquid acts equally in all directions.
- Pressure in a liquid is the same for all points at the same horizontal level.
- Pressure in a liquid increases with depth h according to the relation $P = P_a + h \rho g$
  This expression is valid only for incompressible fluids i.e., liquids.
- Liquid pressure is independent of the area and the shape of the containing vessel.
- The mean pressure on the walls of a vessel containing liquid upto height h is h ρ g/2
- Most of the pressure measuring devices measure the pressure difference between the true pressure and the atmospheric pressure. This difference is called *gauge pressure* and the pressure is called *absolute pressure.*
  Absolute pressure = Gauge pressure + Atmospheric pressure i.e., $P = P_g + P_a$
- The gauge pressure may be positive or negative depending on $P > P_a$ or $P < P_a$. In inflated tyres or the human circulatory system, the absolute pressure is greater than atmospheric pressure, so gauge pressure is positive, called the *overpressure*. However, when we suck a fluid through a straw, the absolute pressure in our lungs is less than atmospheric pressure and so the gauge pressure is negative.
- A diver in water at a depth of 10 m is under twice the atmospheric pressure.
- At a depth of 1 km in a sea, the increase in pressure is 100 atm. Submarines are designed to withstand such high pressures.
- The pressure at the centre of the earth is estimated to be 3 million atmospheres.
- The atmospheric pressure is nearly 100 kPa. The tyres of a car are usually inflated to a pressure of about 200 kPa.
- It is because of the blood pressure from inside that we do not feel such a high atmospheric pressure.
- A drop in the atmospheric pressure by 10 mm of Hg or more is a sign of an approaching storm.

Subjective Assignment – III

Q.1 What will be the length of mercury column in a barometer tube, when the atmospheric pressure is 75 cm of mercury and the tube is inclined at an angle of 60^o with the horizontal direction?

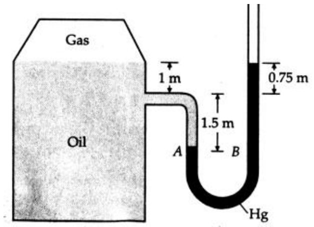
Q.2 The density of the atmosphere at sea level is $1.29 kg\ m^{-3}$. Assume that it does not change with altitude. Then how high would the atmosphere extend? Take $g = 9.81\ ms^{-2}$.

Q.3 A rectangular tank is 10 m long, 10 m broad and 3 m high. It is filled to the rim with water of density 10^3 kg m^{-3}. Calculate the thrust at the bottom and walls of the tank due to hydrostatic pressure.

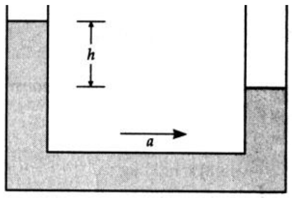
Q.4 The manual of a car instructs the owner to inflate the tyres to a pressure of 200 kPa. (a) What is the recommended gauge pressure? (b) What is the recommended absolute pressure? (c) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read?

Q.5 At a depth of 1000 m in an ocean (a) What is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea–level atmospheric pressure. (The density of sea water is 1.03×10^3 kg m^{-3}, $g = 10\ ms^{-2}$)

Q.6 What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in figure? Density of oil = 820 kg m^{-3}, density of mercury = 13.6×10^3 kg m^{-3}. Given 1 atmosphere pressure = 1.01×10^5 Pa.

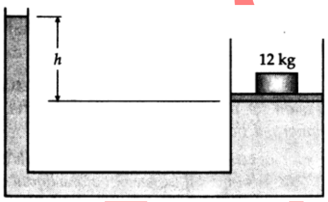


Q.7 A liquid stands at the same level in the U–tube when at rest. If A is the area of cross–section and g the acceleration due to gravity, what will be the difference in height h of the liquid in the two limbs of U–tube, when the system is given an acceleration 'a' towards right, as shown in figure if L is length of base.



Q.8 A vertical U–tube of uniform inner cross–section contains mercury in both of its arms. A glycerine (density 1.3g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8g cm^{-3} is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of the oil column.

Q.9 The area of cross–section of the wider tube shown in figure is 800 cm^2. If a mass of 12 kg is placed on the massless piston, what is the difference h in the level of water in the two tubes?



Q.10 A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. If the elevator is accelerating upwards at 4.9 ms^{-2}, what will be the air pressure in the elevator?

Answers

1.	86.6 cm	2.	8 km
3.	2.793×10^6 N	4.	(a) 200 kPa, (b) 301 kPa, 211 kPa
5.	(a) 104 atm, (b) 103 atm, (c) 4.12×10^5 N	6.	3.81×10^5 Pa, 2.8×10^5 pa
7.	$\frac{La}{g}$	8.	9.6 cm
9.	15.0 cm	10.	114 cm of Hg

Buoyancy

When body is immersed in a fluid, the fluid exerts pressure on all faces of the body. The upward thrust at the bottom is more than the downward thrust on the top because the bottom is at the greater depth than the top. Hence a resultant upward force acts on the body. *The upward force acting on a body immersed in a fluid is called up thrust or buoyant force and the phenomenon is called buoyancy.*

The force of buoyancy acts through the centre of gravity of the displaced fluid which is called centre of buoyancy.

Archimedes' principle

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the displaced fluid.

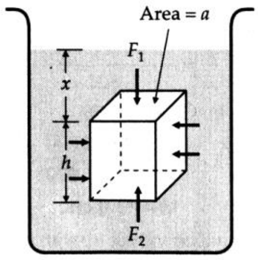
Proof: As shown in figure, consider a body of height h lying inside a liquid of density ρ, at a depth x below the free surface of the liquid. Area of cross–section of the body is a. The forces on the sides of the body cancel out.

Pressure at the upper face of the body, $P_1 = x\rho g$

Pressure at the lower face of the body, $P_2\ (x + h)\ \rho g$

Thrust acting on the upper face of the body is $F_1 = P_1\ a = x\rho ga$, acting vertically downwards.

Thrust acting on the lower face of the body is $F_2 = P_2\ a = (x + h)\ \rho ga$, acting vertically upwards.



The resultant force $(F_2 - F_1)$ is acting on the body in the upward direction and is called upthrust (U).

∴ $U = F_2 - F_1 = (x + h)\rho ga - x\rho ga = ah\rho g.$

But ah = V, the volume of body = volume of liquid displaced.

∴ $U = V\rho g = Mg$ [∵ $M = V\rho$ = mass of liquid displaced]

i.e., Upthrust or buoyant force = Weight of liquid displaced

This proves the Archimedes' principle.

Apparent weight of immersed body : The actual weight W of the immersed body acts downwards and the upthrust U acts upwards.

∴ Apparent weight = Actual weight – Buoyant force

$$W_{app} = W - U = V\sigma g - V\rho g = V\sigma g\left(1 - \frac{\rho}{\sigma}\right) \quad \text{or} \quad W_{app} = W\left(1 - \frac{\rho}{\sigma}\right)$$

Where $W = V\sigma g$ is the true weight of the body and σ is its density.

Law of Floatation

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.

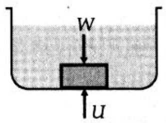
Explanation : When a body is immersed fully or partly in a liquid, following two vertical forces act on it:

(i) Its true weight W which acts vertically downward through its centre of gravity.

(ii) Force of buoyancy or upthrust U which acts vertically upwards through the centre of buoyancy.

Three cases are possible:

(a) When W > U: The downward pull of the weight of the body is higher than the upthrust. The net force (W – U) acts in the downward direction and hence the body sinks.

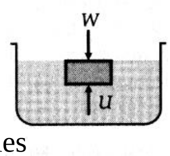


(a) W > U

$$W > U \Rightarrow \quad V\sigma g > V\rho g \quad \text{or} \quad \sigma > \rho$$

Thus a *body sinks in a liquid if its density greater than the density of the liquid.* That is why an iron piece or a stone sinks in water.

(b) When W = U : The weight of the body is just balanced by the upthrust. No net force acts on the body. The body floats fully immersed.

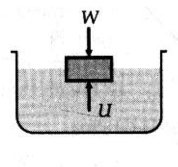


(b) W = U

$$W = U \Rightarrow \quad V\sigma g = V\rho g \quad \text{or} \quad \sigma = \rho$$

Thus a drop of olive oil stands at rest anywhere in a mixture of equal quantities of water and alcohol because the density of olive oil is equal to that of mixture.

(c) **When W < U :** The gravitational force W is less than the upward force U. The body floats partly immersed. This is because the body sinks only to the extent that W = U.



(c) W < U

Here σ < ρ. The density of the floating body is less than that of liquid. That is why a piece of cork floats on water.

If V is the total volume of the body and V' is the submerged volume, then at equilibrium,

Weight of the body = Weight of liquid displaced

or $V\sigma g = V'\rho g$ or $\frac{V'}{V} = \frac{\sigma}{\rho}$ or $\frac{Volume\ of\ submerged\ part}{Total\ volume\ of\ the\ body} = \frac{Density\ of\ body}{Density\ of\ liquid}$

Examples of Floating Bodies:

(i) The ship is made of steel (8 times denser than water) but its interior is made hollow by giving it a concave shape. It can displace much more water than its own weight. So the ship floats and can carry a lot of cargo.

(ii) Ice floats on water because the density of ice is less than that of water.

(iii) Human body is slightly more denser than water. An inflated rubber tube has low weight and large volume and increases the upthrust. It helps a person to float.

(iv) A person can swim in sea water more easily than in river water. The density of sea water is more than that of river water and so it exerts a greater upthrust.

(v) The average density of a fish is slightly greater than water. By means of an anatomical attachment called swim bladder whose size it can adjust, the fish is able to swim with case.

Equilibrium of Floating Bodies

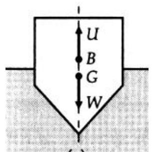
Conditions for the equilibrium of a Floating Body

(i) Weight of the liquid displaced must be equal to the weight of the body.

(ii) The centre of gravity of the body and the centre of buoyancy must lie on the same vertical line.

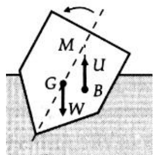
Stability of a floating body

When the centre of gravity of the body and the centre of buoyancy do not lie on the same vertical line, the two forces; the weight (W) of the body and the upthrust (U) form a couple which produces rotation.



(a)

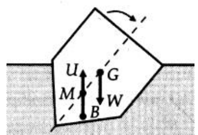
As the floating body is slightly displaced from the equilibrium position, the centre of buoyancy shifts to a new position. *The point at which the vertical line passing through the new centre of buoyancy meets the initial vertical line is called* **metacentre** (M).



(b)

(i) If the metacentre M lies above the centre of gravity G, the couple tends to bring the body back to its original position, as shown in figure. The floating body is in **stable equilibrium.**

(ii) If the metacentre M lies below the centre of gravity G, the couple tends to rotate the body away from the original position, as shown in figure. The floating body is in **unstable equilibrium.** The couple topples the floating body.



(c)

Subjective Assignment – IV

Q.1 The density of ice is 917 kg m^{-3}. What fraction of ice lies below water? The density of sea water is 1024 kg m^{-3}. What fraction of the ice berg do we see assuming that it has the same density as ordinary ice (917 kg m^{-3})?

Q.2 The density of ice is 0.918 g cm^{-3} and that of water is 1.03g cm^{-3}. An iceberg floats with a portion of 224 m^3 outside the surface of water. Find the total volume of the iceberg.

Q.3 A body of mass 6 kg is floating in a liquid with 2/3 of its volume inside the liquid. Find (i) buoyant force acting on the body, and (ii) ratio between the density of body and density of liquid.

Q.4 A piece of pure gold (ρ = 19.3 g cm^{-3}) is suspected to be hollow from inside. It weights 38.250g in air and 33.865 g in water. Calculate the volume of the hollow portion in gold, if any.

Q.5 A spring balance reads 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when

(i) an ice cube of mass 1.5 kg is put into the bucket

(ii) an iron piece of mass 7.8 kg suspended by another spring is immersed with half its volume inside the water in the bucket? Relative density of iron = 7.8

Q.6 A jeweller claims that he sells ornaments made of pure gold that has the relative density of 19.3. He sells a necklace weighing 25.250 gf to a person. The clever customer weights the necklace when immersed in pure water and finds that its weights 23.075gf in water. Is the ornament made of pure gold?

Q.7 A body of density ρ floats with a volume V_1 of its total volume V immersed in one liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2, where $\rho_1 > \rho_2$. Find the relative volumes immersed in two liquids.

Q.8 A sample of milk diluted with water has a density of 1032 kgm^{-3}. If pure milk has a density of 1080 kgm^{-3}, find the percentage of water by volume in milk.

Q.9 A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by one cm, when a man gets on it. What is the mass of the man?

Q.10 A piece of brass (alloy of zinc and copper) weights 12.9 g in air. When completely immersed in water it weights 11.3 g. What is the mass of copper contained in the alloy? Specific gravity of zinc and copper are 7.1 and 8.9 respectively.

Q.11 A metal cube of 5 cm side and relative density 9 is suspended by a thread so as to be completely immersed in a liquid of density 1.2×10^3 kg m^{-3}. Find the tension in the thread.

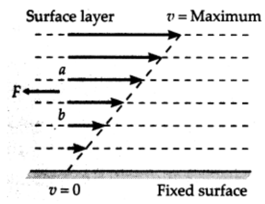
Answers

1.	0.917, 0.105	2.	2060 m^3	3.	60 N, 2/3
4.	2.403 cm^3	5.	(i) 11.5 kg f, (ii) 10.5 kg f	6.	11.61, No
7.	$V_1 = \left(\frac{\rho-\rho_2}{\rho_1-\rho_2}\right)V,\ V_2 = \left(\frac{\rho_1-\rho}{\rho_1-\rho_2}\right)V$			8.	60%
9.	60 kg	10.	7.61 g	11.	9.56 N

Viscosity

Viscosity is the property of fluid by virtue of which an internal force of friction comes into play when a fluid is in motion and which opposes the relative motion between its different layers. The backward dragging force, called viscous drag or viscous force, acts tangentially on the layers of the fluid in motion and tends to destroy its motion.

Cause of Viscosity: Consider a liquid moving slowly and steadily over a fixed horizontal surface. Each layer moves parallel to the fixed surface. The layer in contact with the fixed surface is at rest and the velocity of the very other layer increases uniformly upwards, as shown by arrows of increasing lengths in figure.



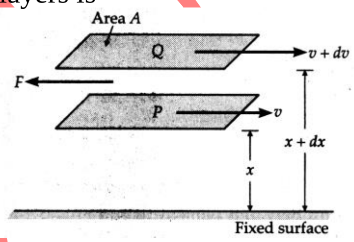
Coefficient of Viscosity

Suppose a liquid is flowing steadily in the form of parallel layers on a fixed horizontal surface. Consider two layers P and Q at distances x and x + dx from the solid surface and moving with velocities v and v + dv respectively. Then $\frac{dv}{dx}$ is the rate of change of velocity with distance in the direction of increasing distance and is called **velocity gradient.**

According to Newton, a force of viscosity F acting tangentially between two layers is

(i) Proportional to the area A of the layers in contact. $F \propto A$

(ii) Proportional to velocity gradient $\frac{dv}{dx}$ between the two layers. $F \propto \frac{dv}{dx}$



$$\therefore \quad F \propto A\frac{dv}{dx} \quad \text{or} \quad F = -\eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid.

It depends on the nature of the liquid and gives a measure of viscosity. Negative sign shows that the viscous force acts in a direction opposite to the direction of motion of the liquid.

If A = 1 and $\frac{dv}{dx} = 1$ then F = η (numerically)

Hence *coefficient of viscosity of a liquid may be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers each of unit area.*

Dimensions of η: $\eta = \frac{F}{A}.\frac{dx}{dv} \quad \therefore \quad [\eta] = \frac{MLT^{-2}.L}{L^2.LT^{-1}} = [ML^{-1}T^{-1}]$

Units of coefficient of viscosity

(i) The CGS unit of η is dyne s cm^{-2} or g cm^{-1} s^{-1} and is called **poise.**

$$1\,poise = \frac{1\,dyne}{1\,cm^2}.\frac{1cm}{1cm\,s^{-1}} = 1\,dyne\,s\,cm^{-2}$$

The coefficient of viscosity a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

(ii) The SI unit of η is N s m^{-2} or kg m^{-1} s^{-1} and is called ***decapoise or poiseuille.***

$$1\ poisecuille = \frac{1N}{1m^2}.\frac{1m}{1m\,s^{-1}} = 1N\,s\,m^{-2}$$

The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

Relation between poiseuille and poise

1 poiseuille or 1 decapoise = 1 Ns m^{-2}

$= (10^5 \text{ dyne}) \times s \times (10^2 \text{ cm})^{-2}$

$= 10 \text{ dyne s cm}^{-2} = 10 \text{ poise}$

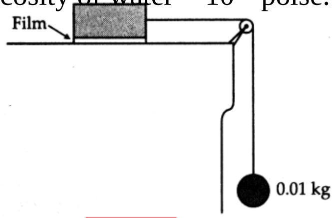
Subjective Assignment – V

Q.1 A metal plate 5 cm × 5 cm rests on a layer of castor oil 1 mm thick whose coefficient of viscosity is 1.55 Nsm^{-2}. Find the horizontal force required to move the plate with a speed of 2 cms^{-1}.

Q.2 A square metal plate of 10 cm side moves parallel to another plate with a velocity of 10 cms^{-1}, both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their distance apart?

Q.3 The velocity of water in a river is 180 kmh^{-1} near the surface. If the river is 5 m deep, find the shearing stress between horizontal layers of water. Coefficient of viscosity of water = 10^{-2} poise.

Q.4 A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as shown in figure. A liquid with a film thickness of 0.3 mm is placed between the plate and the table. When released the plate moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.



Q.5 A metal plate of area 0.02 m^2 is lying on a liquid layer of thickness 10^{-3} m and coefficient of viscosity 120 poise. Calculate the horizontal force required to move the plate with a speed of 0.025 ms^{-1}.

Answers

1.	0.0775 N	2.	0.05 cm	3.	10^{-3} Nm^{-2}
4.	3.45×10^{-3} Pa s	5.	6 N		

Comparison between Viscous Force and Solid Friction

Points of Similarly

(i) Both viscous force and solid friction come into play whenever there is relative motion.

(ii) Both oppose the motion.

(iii) Both are due to molecular attractions.

Points of Differences:

Sr. No.	*Viscous Force*	*Solid Friction*
1.	Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
2.	It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces
3.	It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

Effect of Temperature on Viscosity

(i) When a liquid is heated, the kinetic energy of its molecules increases and the intermolecular attractions become weaker. Hence *the viscosity of a liquid decreases with the increase in its temperature*. The coefficient of viscosity at any temperature, t $\eta_t = \frac{\eta_0}{1+\alpha t+\beta t^2}$

where η_t and η_0 are the coefficients of viscosity at t°C and 0°C respectively, and α and β are temperature coefficient of viscosity.

(ii) Viscosity of gases is due to the diffusion of molecules from one moving layer to another. But the rate of diffusion of a gas is directly proportional to the square root of its absolute temperature, so viscosity of a gas increases with temperature as

$$\eta \propto \sqrt{T}$$

Effect of pressure

(i) Except water the viscosity of liquids increases with the increase in pressure. In case of water, viscosity decreases with the increase in pressure for first few hundred atmospheres of pressure.

(ii) The viscosity of gases is independent of pressure.

Practical Applications of the Knowledge of Viscosity

(i) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given machine in different seasons.

(ii) Liquids of high viscosity are used as buffers at railway stations.

(iii) The knowledge of viscosity is used in determining the shape and molecular weight of some organic liquids like proteins, cellulose, etc.

(iv) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.

(v) Millikan used the knowledge of viscosity in determining the charge on an electron.

Poiseuille's Formula

The volume of a liquid flowing out per second through a horizontal capillary tube of length *l*, radius r, under a pressure difference p applied across its ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}$$

This formula is called *Poiseulle's formula.*

Derivation of Poiseuille's formula on the basis of dimensional analysis

The volume Q of liquid flowing out per second through a capillary tube depends on

(i) coefficient of viscosity η of the liquid, (ii) radius r of the tube,

(iii) pressure gradient (p/*l*) set up along the capillary tube.

Let $Q \propto \eta^a r^b \left(\frac{p}{l}\right)^c$ or $Q = k\eta^a r^b \left(\frac{p}{l}\right)^c$ ... (i)

where k is a dimensionless constant. The dimensions of various quantities are

$[Q] = \frac{Volume}{Time} \frac{[L^3]}{[T]} = [L^3T^{-1}]$ $\left[\frac{p}{l}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$ $[\eta] = [ML^{-1}T^{-1}]$, $[r] = [L]$

Substituting these dimensions in equation (1), we get

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c \quad \text{or} \quad [M^0L^3T^{-1}] = [M^{a+c} L^{-a+b-2c}T^{-a-2c}]$$

Equating the powers of M, L and T on both sides, we get

$a + c = 0$

$-a + b - 2c = 3$

$-a - 2c = -1$

On solving, we get a = – 1, b = 4, and c = 1

$$\therefore \qquad Q = k\eta^{-1}r^4\left[\frac{p}{l}\right]^1 = \frac{kpr^4}{\eta l}$$

Experimentally k is found to be π/8

$$\therefore \qquad Q = \frac{\pi pr^4}{8\eta l}$$

This is Poiseuille's formula for the flow of a liquid through a capillary tube.

Subjective Assignment – VI

Q.1 A capillary tube 1 mm in diameter and 20 cm in length is fitted horizontally to a vessel kept full of alcohol. The depth of the centre of capillary tube below the surface of alcohol is 30 cm. If the viscosity and density of alcohol are 0.012 cgs unit and 0.8g cm^{-3} respectively, find the amount of the alcohol that will flow out in 5 minutes. Given that g = 980 cms^{-2}.

Q.2 In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3 m above needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3}.

Q.3 Two tubes A and B of lengths 100 cm and 50 cm have radii 0.1 mm and 0.2 mm respectively. If a liquid passing through the two tubes is entering A at a presence of 80 cm of mercury and leaving B at a pressure of 76 cm of mercury, determine the pressure at the junction of A and B.

Q.4 Two capillary tubes AB and BC are joined end to end at B. AB is 16 cm long and of diameter 4 mm whereas BC is 4 cm long and of diameter 2 mm. The composite tube is held horizontally with A connected to a vessel of water giving a constant head of 3 cm and C is open to the air. Calculate the pressure difference between B and C.

Q.5 The level of liquid in a cylindrical vessel is kept constant at 30 cm. It has three identical horizontal tubes of length 39 cm, each coming out at heights 0, 4, 8 cm respectively. Calculate the length of a single overflow tube of the same radius as that of identical tubes which can replace the three when placed horizontally at bottom of the cylinder.

Q.6 Three capillary tubes of the same radius r but of lengths l_1, l_2 and l_3 are fitted horizontally to the bottom of a tall vessel containing a liquid at constant head and flowing through these tubes. Calculate the length of a single outflow tube of the same radius r which can replace the three capillaries.

Q.7 Water at 20^o is escaping from a cistern by way of a horizontal capillary tube 10 cm long and 0.4 mm in diameter, at a distance of 50 cm below the free surface of water in the cistern. Calculate the rate at which the water is escaping. Coefficient of viscosity of water is 20 decapoise.

Q.8 Alcohol flows through two capillary tubes under a constant pressure head. The diameters\of the two tubes are in the ratio of 4 : 1 and the lengths are in the ratio 4 : 1. Compare the rates of flow of alcohols through the two tubes.

Answers

1. 38.4 g 2. 0.238 poise 3. 76.12 cm of Hg

4.	2.4 cm	5.	15 cm	6.	$\frac{l_1 l_2 l_3}{l_2 l_3 + l_1 l_3 + l_1 l_2}$
7.	3.08×10^{-8} m^3 s^{-1}	8.	1024 : 1		

Stokes' Law

According to Stokes' law, the backward viscous force acting on a small spherical body of radius r moving with uniform velocity v through fluid of viscosity η is given by

$$F = 6\pi\eta rv$$

Derivation of Stokes' law : The viscous force F acting on a sphere moving through a fluid may depend on

(i) coefficient of viscosity η of the fluid (ii) radius r of the spherical body

(iii) velocity v of the body

Let $F = k\eta^a r^b v^c$ .... (i)

where k is dimensionless constant. The dimensions of various quantities are

$[F] = [MLT^{-2}]$, $[\eta] = [ML^{-1}T^{-1}]$ $[r] = [L]$, $[v] = [LT^{-1}]$

Substituting these dimensions in equation (1), we get

$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$

$= [M^a L^{-a+b+c} T^{-a-c}]$

Equating the powers M, L and T on both sides, we get

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

On solving, $a = b = c = 1$

$\therefore$ $F = k\eta r v$

For a small sphere, k is found to be equal to 6π.

Hence $F = 6\pi\eta rv$ *This proves Stokes' law.*

Conditions under which Stokes' law is valid:

(i) The fluid through which the body moves has infinite extension.

(ii) The body is perfectly rigid and smooth.

(iii) There is no slip between the body and fluid.

(iv) The motion of the body does not give rise to turbulent motion and eddies. Hence motion is streamlined.

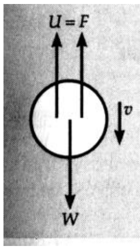
(v) The size of the body is small but it is larger than the distance between the molecules of the liquid. Thus the medium is homogeneous and continuous for such a body.

Terminal Velocity

When a body falls through a viscous fluid, it produces relative motion between its different layers. As a result, the body experiences a viscous force which tends to retard its motion. As the velocity of the body increase, the viscous force ($F = 6\pi \eta rv$) also increases. A stage is reached, when the weight of the body becomes just equal to the sum of the up thrust and viscous force. Then no net force acts on the body and it begins to move with a constant velocity. *The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity.*

Expression for terminal velocity

Consider a spherical body of radius r falling through a viscous liquid of density ρ and coefficient of viscosity η. Let σ be the density of the body. As the body falls, the various forces acting on the body are as shown in figure. These are



(i) Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \sigma g$$

(ii) Upward thrust equal to the weight of the liquid displaced.

$$U=\frac{4}{3}\pi r^3 \rho g$$

(iii) Force of viscosity F acting in the upward direction. According to Stokes' law, F = 6πηrv

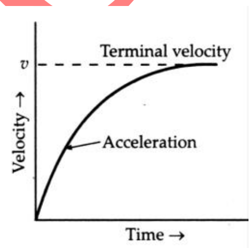
Clearly, the force of viscosity increases as the velocity of the body increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then the body begins to fall with a constant maximum velocity, called *terminal velocity*.

When the body attains terminal velocity v,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \rho g+6\pi\eta rv=\frac{4}{3}\pi r^3 \sigma g$$

or $$6\pi\eta rv=\frac{4}{3}\pi r^3 (\sigma-\rho) g$$ or $$v=\frac{2}{9}.\frac{r^2 (\sigma-\rho) g}{\eta}$$



This is the expression for terminal velocity.

Discussion of the result:

(i) Figure shows how the velocity of a small sphere dropped from rest into a viscous medium varies with time. Initially the body is accelerated and after some time, it acquires terminal velocity v.

(ii) *The terminal velocity is directly proportional to the radius of the body.* That is why bigger rain drops fall with a larger velocity compared to the smaller rain drops.

(iii) The terminal velocity is directly proportional the difference of the densities of the body and the fluid, i.e. (σ – ρ)

(a) If σ – ρ, the body will attain terminal velocity in the downward direction.

(b) If σ – ρ, the terminal velocity will be negative i.e., the body will rise through the fluid. That is why, air bubble in a liquid and clouds in a sky are seen to move in the upward direction.

(c) If σ = ρ, the body remains suspended in the fluid.

(iv) The *terminal velocity is inversely proportional to the coefficient of viscosity of the fluid.* The more viscous the fluid, the smaller the terminal velocity attained by a body.

(v) The *terminal velocity is independent of the height through which a body is dropped.*

(vi) Knowing the values of p, σ, r and v, we can determine the coefficient of viscosity η as follows:

$$\eta=\frac{2}{9}\frac{r^2 (\sigma -\rho) g}{v}$$

Subjective Assignment – VII

Q.1 An iron ball of radius 0.3 cm falls through a column of oil of density 0.94 g cm^{-3}. It is found to attain a terminal velocity of 0.5 cms^{-1}. Determine the viscosity of oil. Given that density of iron is 7.8 g cm^{-3}.

Q.2 Eight rain drops of radius 1 mm each falling down with terminal velocity of 5 cms^{-1} coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.

Q.3 Show that if n equal rain droplets falling through air with equal steady velocity of 10 cms^{-1} coalesce, the resultant drop attains a new terminal velocity of 10 $n^{2/3}$ cms^{-1}.

Q.4 A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of the initial acceleration, show that the time taken to attain the terminal velocity is independent of the fluid density.

Q.5 A gas bubble of diameter 2 cm rises steadily at the rate of 25 mms^{-1} through a solution of density 2.25g cm^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of the gas.

Q.6 The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cms^{-1}. Compute the viscosity of the oil at 20°C. Density of oil = 1.5×10^3 kg m^{-3}, density of copper = 8.9×10^3 kg m^{-3}.

Q.7 A spherical glass ball of mass 1.34×10^{-4} kg and diameter 4.4×10^{-3} m takes 6.4 s to fall steadily through a height of 0.381 m inside a large volume of oil of specific gravity 0.943. Calculate the viscosity of oil.

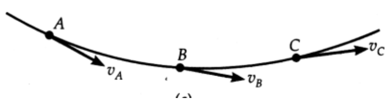
Answers

1.	268.9 poise	2.	20 cms^{-1}	3.	10 $n^{2/3}$ cms^{-1}
4.	$\frac{4}{9}.\frac{r^2\rho}{\eta}$	5.	1960 poise	6.	0.992 decapoise
7.	0.8025 Nsm^{-2}				

Streamline and Turbulent Flows

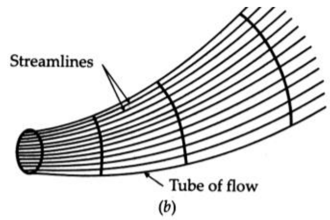
Streamline flow

When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow. A streamline may be defined as the path, the tangent to which at any point gives the direction of the flow of liquid at the point.



Tube of flow

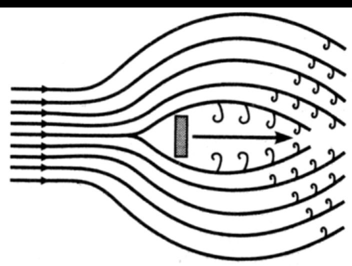
A bundle of streamlines forming a tubular region is called a tube of flow. The boundary of such a tube is always parallel to the velocity of fluid particles. No fluid can cross the boundaries of a tube of flow, and the flow behaves somewhat like a tube. In a steady flow, the shape of the flow tube does not change with time.



(*b*)

Turbulent flow

When the liquid velocity exceeds a certain limiting value, called *critical velocity*, the liquid flow becomes zig–zag. The path and the velocity of a liquid particle changes continuously, haphazardly. This flows is called *turbulent flow*. It is accompanied by random, irregular, local circular currents called vortices.



Properties of Streamlines

(i) In a steady flow, *no two streamlines can cross each other*. If they do so, the fluid particle at the point of intersection will have two different directions of flow. This will destroy the steady nature of the fluid flow.

(ii) The tangent at any point on the streamline gives the direction of velocity of fluid particle at that point.

(iii) Greater the number of streamlines passing normally through a section of the fluid, larger is the fluid velocity at the section.

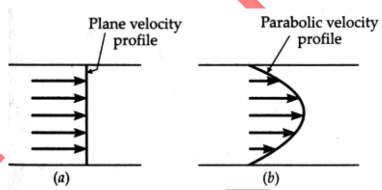
(iv) Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.

Laminar Flow

When the velocity of the flow of a liquid is less than its critical velocity, the liquid flows steadily. Each layer of the liquid slides over the other layer. It behaves as if different lamina are sliding over one another. Such a flow is called laminar flow. The surface obtained by joining the heads of the velocity vectors for the particles in a section of a flowing liquid is called a *velocity profile.*

(i) Velocity profile for a non–viscous liquid

In case of a non–viscous liquid, the velocity of all the particles at any section of a pipe is same, so the velocity profile is plane as shown in figure.



(ii) Velocity profile of a viscous liquid

When a viscous liquid flows through a pipe, the velocity of layer at the axis is maximum, the velocity decreases as we go towards the wall of the pipe and becomes zero for the layer in contact with the pipe. Hence the velocity profile for a viscous liquid is parabolic, as shown in figure.

Critical Velocity

The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent. The critical velocity v_c of a liquid flowing through a tube depends on

(i) coefficient of viscosity of the liquid (η) (ii) density of the liquid (ρ)

(iii) diameter of the tube (D)

Let $v_c = k\, \eta^a\, \rho^b\, D^c$

where k is a dimensionless constant. Writing the above equation in dimensional form, we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^a\, [ML^{-3}]^b\, [L]^c$$

$$[M^0LT^{-1}] = [M^{a+b}L^{-a-3b+c}T^{-a}]$$

Equating powers of M, L and T, we get

$a + b = 0$ $\qquad -a - 3b + c = 1$ $\qquad -a = -1$

On solving, we get a = 1, b = – 1, c = – 1

$\therefore \qquad v_c = k\eta\rho^{-1}D^{-1} = \dfrac{k\eta}{\rho D}$

Clearly, the critical velocity v_c will be large if η is large, and ρ and D are small. So we can conclude that

(i) The flow of liquids of higher viscosity and lower density through narrow pipes tends to be streamlined.

(ii) The flow of liquids of lower viscosity and higher density through broad pipes tends to become turbulent, because in that case the critical velocity will be very small.

Reynold's Number

It is dimensionless parameter whose value decides the nature of flow of a liquid through a pipe It is given by

$$R_e = \frac{\rho v D}{\eta}$$

where ρ = density of the liquid; v = velocity of the liquid

η = coefficient of viscosity of the liquid; D = diameter of the pipe.

Importance of Reynold's Number

If R_e lies between 0 and 2000, the liquid flow is streamlined or laminar. If $R_c > 3000$, the liquid flow is turbulent. If R_e lies between 2000 and 3000, the flow of liquid is unstable, it may change from laminar to turbulent and *vice–versa*. The exact value at which turbulence sets in a fluid is called *critical Reynold's number*.

Physical significance of Reynold's number

Consider a narrow tube having a cross–sectional area A. Suppose a fluid flows through it with a velocity v for a time interval Δt. Length of the fluid = Velocity × time = v Δt

Volume of the fluid flowing through the tube in time Δt = Av Δt

Mass of the fluid,

Δm = Volume × density = Av Δt × ρ

Inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A} = \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av\,\Delta t\,\rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta\frac{v}{D} \qquad \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta v/D} = \frac{\rho v D}{\eta} = R_e$$

Thus Reynold's number represents the ratio of the inertial force per unit area to viscous force per unit area.

Subjective Assignment – VIII

Q.1 The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa s. After some time the flow rate is increased to 3 L/min. Characteristic the flow for both the flow rates.

Q.2 What should be the maximum average velocity of water in a tube of diameter 0.5 cm so that the flow is laminar? The viscosity of water is 0.00125 Ns m^{-2}.

Q.3 Water flows at a speed of 6 cms^{-1} through a pipe of tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.01 poise. What is the nature of flow?

Q.4 Find the critical velocity for air flowing through a tube of 2 cm diameter. For air, $\rho = 1.3 \times 10^{-3}$ g cm^{-3} and $\eta = 181 \times 10^{-6}$ poise.

Answers

1.	steady to turbulent	2.	$0.5\ ms^{-1}$
3.	$R_e = 1200 < 2000$, so flow is laminar	4.	$140\ cms^{-1}$

Ideal Fluid

An ideal fluid is one which is non–viscous, incompressible, and its flow is steady and irrotational. Thus an ideal fluid has the following features connected with its flow:

(i) **Steady flow:** In a steady flow, the fluid velocity at each point does not change with time, either in magnitude or direction.

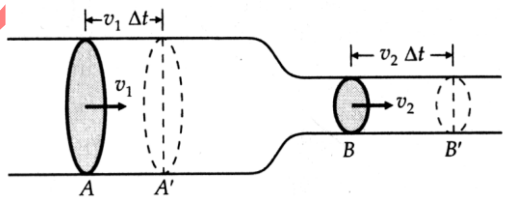
(ii) **Incompressible flow:** The density of the fluid remains constant during its flow.

(iii) **Non–viscous flow:** The fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

(iv) **Irrotational flow:** This means that there is no angular momentum of the fluid about any point. A very small wheel placed at any point inside such a fluid does not rotate about its cent re of mass.

Equation of Continuity

Consider a non–viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross–section. Let a_1 be the area of cross–section, v_1 fluid velocity, ρ_1 fluid density at section A; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.



An m = Volume × density

= Area of cross– section × length × density

∴ Mass of fluid that flows through section A in time Δt,

$$m_1 = a_1 v_1 \Delta t\ \rho_1$$

Mass of fluid that flows through section B in time Δt,

$$m_2 = a_2\ v_2\ \Delta t\ \rho_2$$

By conservation of mass, $m_1 = m_2$ or $a_1\ v_1\ \Delta t\ \rho_1 = a_2\ v_2\ \Delta t\ \rho_2$

As the fluid is incompressible, so $\rho_1 = \rho_2$ and hence $a_1\ v_1 = a_2\ v_2$ or av = constant

This *is the **equation of continuity**. It states that during the streamlined flow of the non–viscous and incompressible fluid through a pipe of varying cross–section, the product of area of cross–section and the normal fluid velocity* (av) *remains constant throughout the flow.*

NOTE

- The equation of continuity is a special case of the law of conservation of mass.
- The equation of continuity shows that $v \propto 1/a$, i.e., *the liquid velocity at any section of the pipe is inversely proportional to the area of cross–section of the pipe at that section.* This explains why the speed of water emerging from a PVC pipe increases when we press its outlet with our fingers and hence decrease its area of cross–section.

Deep water runs slowly

As the depth of water in a river or a steam increases, the area of cross–section available to the flowing water increases. Consequently, velocity decreases in accordance with the equation of continuity. Thus deep water runs slowly.

Energy of a Fluid in a Steady Flow

A liquid in a steady flow can have three kinds of energy (i) kinetic energy (ii) potential energy and (iii) pressure energy.

(i) ***Kinetic energy :*** *The energy possessed by a liquid by virtue of its motion is called its kinetic energy.*

$$K.E.=\frac{1}{2}mv^2$$

where m is the mass of the liquid and v is the velocity of the liquid.

K.E. per unit mass of the liquid = $\frac{1}{2}v^2$

The kinetic energy per unit weight of the liquid is known as *the velocity head.*

$\therefore$ Velocity head = $\frac{v^2}{2g}$ K.E. per unit volume = $\frac{1}{2}\frac{mv^2}{V}=\frac{1}{2}\rho v^2$

(ii) ***Potential energy:*** *The energy possessed by a liquid by virtue of its position above the earth's surface is called its potential energy.*

P.E. = mgh

where h is the average height of the liquid from the ground level.

P.E. per unit mass of the liquid = gh

The potential energy per unit weight of the liquid is known as the *potential head.*

$\therefore$ Potential head = $\frac{mgh}{mg}=h$ P.E. per unit volume = $\frac{mgh}{V}=\rho gh$

(iii) ***Pressure energy:*** *The energy possessed by a liquid by virtue of its pressure is called its pressure energy.* A liquid under pressure can do work and so possesses energy.

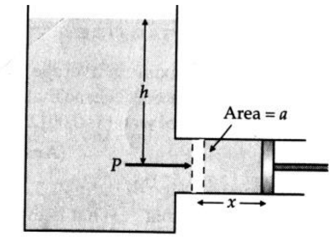
Let P be the pressure exerted by the liquid on a frictionless piston of area a. Suppose the piston moves through distance x under the pressure P.

The work done is

W = Force × distance = Pressure × area × distance = Pax = PV

where V = ax = volume swept by the piston

This work done is stored as the pressure energy of liquid of volume V.



$\therefore$ Pressure energy of volume V = PV

Pressure energy per unit volume

$= \frac{PV}{V}=P$ = Excess pressure

Pressure energy per unit mass = $\frac{PV}{m}=\frac{P}{\rho}$

Pressure energy per unit weight of the liquid is called *pressure head.*

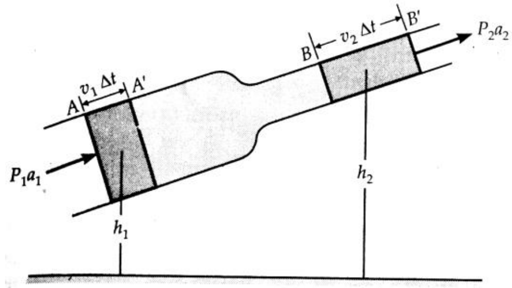
Pressure head = $\dfrac{P}{\rho g}$

Bernoulli's Principle

Bernoulli's principle states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non–viscous fluid in a streamlined irrotational flow remains constant along a streamline.

Mathematically, it can be expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$



Proof: Consider a non–viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross–section. Let a_1 be the area of cross–section at A, v_1 the fluid velocity, p_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2, P_2 and h_2 be the values of the corresponding quantities at B.

Let ρ be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume × density = Area of cross–section × length × density

or $\quad m = a_1 v_1 \Delta t\, \rho = a_2 v_2 \Delta t\, \rho \quad$ .... (1)

or $\quad a_1 v_1 = a_2 v_2 \quad$ ..... (2)

∴ Change in K.E. of the fluid = K.E. at B – K.E. at A

$$= \frac{1}{2} m (v_2^2 - v_1^2) = \frac{1}{2} a_1 v_1 \Delta t\, \rho (v_2^2 - v_1^2)$$

Change in P.E. of the fluid = P.E. at B – P.E. at A = mg $(h_2 - h_1)$ = $a_1 v_1 \Delta t \rho g (h_2 - h_1)$

Net work done on the fluid = Work done on the fluid at A – work done by the fluid at B

$= P_1 a_1 \times v_1 \Delta t - P_2 a_2 \times v_2 \Delta t \quad = P_1 a_1 v_1 \Delta t - P_2 a_1 v_1 \Delta t \quad = a_1 v_1 \Delta t (P_1 - P_2)$

By conservation of energy,

Net work done on the fluid = Change in K.E. of the fluid + Change in P.E. of the fluid

$$\therefore \quad a_1 v_1 \Delta t (P_1 - P_2) = \frac{1}{2} a_1 v_1 \Delta t\, \rho (v_2^2 - v_1^2) + a_1 v_1\, \Delta t \rho g (h_2 - h_1)$$

Dividing both sides by $a_1 v_1 \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

or $\quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad$ or $\quad P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad$ ... (3)

This proves Bernoulli's principle according to which the total energy per unit volume remains constant. Equation (3) can also be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

This is another form of *Bernoulli's principle according to which the sum of pressure head, velocity head and gravitational head remains constant in the streamline flow of an ideal fluid.*

Limitations of Bernoulli's equation

1. Bernoulli's equation ideally applies to fluids with zero viscosity or non–viscous fluids. In case of viscous fluids, we need to take into account the work done against viscous drag.
2. Bernoulli's equation has been derived on the assumption that there is no loss of energy due to friction. But in practice, when fluids flow, some of their kinetic energy gets converted into heat due to the work done against the internal forces of friction or viscous forces.
3. Bernoulli's equation is applicable only to incompressible fluids because it does not take into account the elastic energy of the fluids.
4. Bernoulli's equation is applicable only to streamline flow of a fluid and not when the flow is turbulent.
5. Bernoulli's equation does not take into consideration the angular momentum of the fluid. So it cannot be applied when the fluid flows along a curved path.

NOTE

- Bernoulli's principle is a fundamental principle of fluid dynamics based on the law of conservation of energy.
- In Bernoulli's equation : $P + \rho gh + \frac{1}{2}\rho v^2$ = constant, the term (P + ρgh) is called *static pressure*, because it is the pressure of the fluid even if it is at rest, and the term $\frac{1}{2}\rho v^2$ is the *dynamic pressure* of fluid which is the pressure by virtue of its velocity *v*. So Bernoulli's equation can be written as
  Static pressure + Dynamic pressure = Constant
- If a liquid is flowing through a horizontal tube, h remains constant and we can write

$$P + \frac{1}{2}\rho v^2 = constant$$

This shows that if *v* increases, P decreases and vice versa. Thus for the *streamline flow of an ideal liquid flowing horizontally, the pressure decreases where velocity increases and vice versa.* This is an important aspect of Bernoulli's principle which finds many applications.

Torricelli's Law of Efflux

Consider a tank containing a liquid of density ρ with a small hole on its side at a height y_1 from the bottom. Let y_2 be the height of the liquid surface from the liquid surface from the bottom and P be the air pressure above the liquid surface.

It A_1 and A_2 are the cross–sectional areas of the side hole and the tank respectively, and v_1 and v_2 are the liquid velocities at points 1 and 2, then from the equation of continuity, we get

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1}{A_2} v_1$$

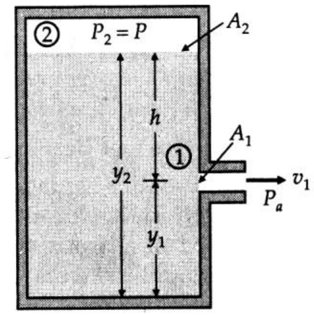
As $A_2 >> A_1$, so the liquid may be taken at rest at the top, i.e., $v_2 \simeq 0$. Applying Bernoulli's equation at points 1 and 2, we get

$$P_a + \frac{1}{2}\rho v_2^2 + \rho g y_1 = P + \rho g y_2$$

or

$$\frac{1}{2}\rho v_1^2 = \rho g (y_2 - y_1) + (P - P_a)$$

If we take $y_2 - y_1 = h$, then



$$\frac{1}{2}\rho v_1^2 = \rho gh + (P + P_a)$$

or $$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

Special cases (i) when P > > P_a, the term 2 gh may be ignored.

$$v_1 = \sqrt{\frac{2(P - P_a)}{\rho}}$$

Thus the speed of efflux is determined by container pressure P. Such a situation exists in rocket propulsion.

(ii) *When the tank is open to the atmosphere,* $P = P_a$ and $v_1 = \sqrt{2\,gh}$

Thus, *the velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called* ***Torricelli's law.***

The Venturimeter

It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle. It is also called **flow meter** or **venture tube.**

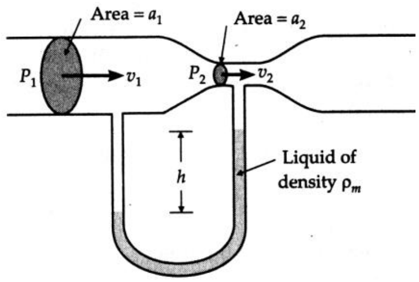
Construction

It consists of a horizontal tube having wider opening of cross–section a_1 and a narrow neck of cross–section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.

Working

Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity, $a_1 v_1 = a_2 v_2$ or $\frac{a_1}{a_2} = \frac{v_2}{v_1}$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) \quad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right] \quad = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

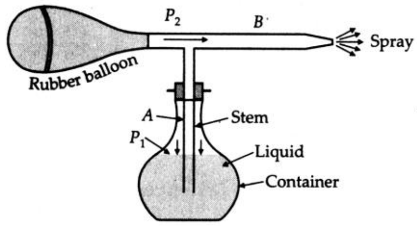
$P_1 - P_2 = h\rho_m g$

$$\therefore \quad h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right) \qquad \therefore \quad v_1 = \sqrt{\frac{2h\rho_m g}{\rho} \times \frac{a_2^2}{(a_1^2 - a_2^2)}}$$

Volume of the liquid flowing out per second, $V = a_1 v_1 = a_1 a_2\sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$

Atomizer or Sprayer

The working of an atomizer which is used to spray liquids is based on Bernoulli's principle. Figure shows the essential parts of an atomizer. When the rubber balloon is pressed, the air rushes out of the horizontal tube B decreasing the pressure to P_2 which is less than the atmospheric pressure P_1 in the container. As a result, the liquid rises up in the vertical tube A. When it collides with the high speed air in tube B, it breaks up into a fine spray.



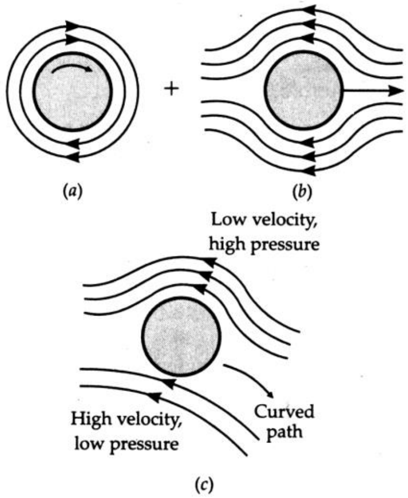
Dynamic Lift

Dynamic lift is the force that acts on a body, such as aeroplane wing, a hydrofoil or a spinning ball, *by virtue of its motion through a fluid.* It is responsible for the curved path of a spinning ball and the lift of an aircraft wing.

Curved path of a spinning ball : Magnus effect

When a ball is thrown horizontally with a large velocity and at the same time given a twisting motion to cause a spin, it deviates from its usual parabolic trajectory of spin free motion. This deviation can be explained on the basis of bernoulli's principle.

When the ball spins about an axis perpendicular to its horizontal motion, it carries with itself an air of layer due to viscous drag. The streamlines around it are in the form of concentric circles, as shown in figure. When the ball moves forward with velocity v, the air ahead of the ball rushes backward with velocity v to fill the space left empty by the ball. Thus the streamlines in air due to translatory motion of the ball are of the form shown in figure. The layer above the ball moves in a direction opposite to that of the spinning ball, so the resultant velocity decreases and hence pressure increases in accordance with Bernoulli's principle.

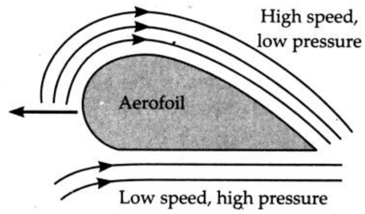


The layer below the ball moves in the direction of spin, the resultant velocity increases and hence pressure decreases. Due to the difference of pressure on the two sides of the ball, the ball curves downwards in the direction of spin.

The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect.

Aerofoil: Lift of an aircraft wing

Aerofoil is the name given to a solid object shaped to provide an upward vertical force as it moves horizontally through air. This upward force (dynamic lift) makes aeroplanes fly. The cross–section of the wing of an aeroplane looks like an aerofoil. The wing is so designed that its upper surface is more curved (and hence longer) than the lower surface and the front edge is broader than the rear edge.



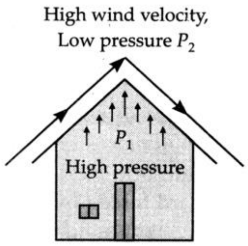
As the aircraft moves, the air moves faster over the upper surface of the wing than on the bottom. According to Bernoulli's principle, the air pressure above the upper surface decreases below the atmospheric pressure and that on the lower surface increases above the atmospheric pressure. The difference in pressure provides an upward lift, called *dynamic lift,* to the aircraft.

Blood flow and heart attack

In persons suffering with advanced heart condition, the artery gets constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction, a greater demand is placed on the activity of the heart. The speed of blood flow increases in this region. From Bernoulli's principle, the inside pressure drops and the artery may collapse due to external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops leading to a repeat collapse. This phenomenon is called vascular flutter which can be heard on a stethoscope. This may result in a heart attack.

Blowing off the roof during wind storm

During certain wind storm of cyclone, the roofs of some houses are blown off without damaging the other parts of the house. The high wind blowing over the roof creates a low pressure P_2 in accordance with Bernoulli's principle. The pressure P_1 below the roof is equal to the atmospheric pressure which is larger than P_2. The difference of pressure $(P_1 - P_2)$ causes an upward thrust and the roof is lifted up. Once the roof is lifted up, it is blown off with the wind.



Assignment – IX

Q.1 Water flows through a horizontal pipe of varying area of cross–section at the rate of 10 cubic metre per minute. Determine the velocity of water at a point where radius of pipe is 10 cm.

Q.2 Water flows through a horizontal pipe whose internal diameter is 2.0 cm at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1}?

Q.3 At what speed will the velocity of a stream of water be equal to 20 cm of mercury column?

Q.4 Calculate the total energy possessed by one kg of water at a point where the pressure is 20 gf/ mm^2, velocity is 0.1 ms^{-1} and the height is 50 cm above the ground level.

Q.5 The reading of pressure meter attached with a closed pipe is 3.5×10^5 Nm^{-2}. On opening the valve of the pipe, the reading of the pressure meter is reduced to 3.0×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

Q.6 A fully loaded Boeing aircraft 747 has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings. (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is $\rho = 1.2$ kg m^{-3} and g = 9.81 ms^{-2}.

Q.7 Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 ms^{-1} and the pressure is 2.0×10^4 Nm^{-2}. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively.

Q.8 The cross–sectional area of water pipe entering the basement is 4×10^{-4} m^2. The pressure at this point is 3×10^5 Nm^{-2} and the speed of water is 2 ms^{-1}. This pipe tapers to a cross–sectional area of 2×10^{-4} m^2 when it reaches second floor 8m above. Calculate the speed and pressure at the second floor.

Q.9 The pressure difference between two points along a horizontal pipe, through which water is flowing, is 1.4 cm of mercury. If, due to non–uniform cross–section, the speed of flow of water at the point of greater cross–section is 60 cm s^{-1}, calculate the speed at the other point.

Q.10 A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air? (sp. gr. of alcohol = 0.8 and density of air = 1 kg m^{-3}).

Q.11 A pitot tube is fixed in a main pipe of diameter 20 cm and difference of pressure indicated by the gauge is 5 cm of water column. Find the volume of water passing through the main pipe in one minute.

Q.12 A cylinder of height 20 m is completely filled with water. Find the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom. Given g = 10 ms^{-2}.

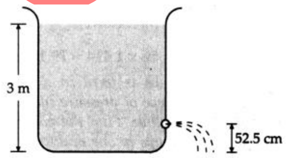
Q.13 A boat strikes an under water rock which punctures a hole 5 cm in diameter in its hull which is 1.5m below the water line. At what rate in litre per second does water enter?

Q.14 A drum of 30 cm radius has a capacity of 220 dm^3 of water. It contains 198 dm^3 of water and is placed on a solid block of exactly the same size as of drum. If a small hole is made at lower end of drum perpendicular to its length, find the horizontal range of water on the ground in the beginning.

Q.15 **Blood Velocity :** The flow of blood in a large artery of an anesthetized dog is diverted through a Venturimeter. The wider part of the meter has a cross–sectional area equal to that of the artery, A = 8 mm^2. The narrower part has an area a = 4 mm^2. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?

Q.16 A horizontal tube has different cross–sectional areas at points A and B. The diameter of A is 4 cm and that of B is 2 cm. Two manometer limbs are attached at A and B. When a liquid of density 8.0 g cm^{-3} flows through the tube, the pressure difference between the limbs of the manometer is 8 cm. Calculate the rate of flow of the liquid in the tube.

Q.17 Water is filled in a cylindrical container to a height of 3 m, as shown in figure. The ratio of the cross–sectional area of the orifice and the beaker is 0.1. Find the speed of the liquid coming out from the orifice.



Q.18 In a normal adult, the average speed of the blood through the aorta (which has a radius of 0.9 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius
0.5 cm. Calculate the speed of blood through the arteries.

Q.19 Water flows into a horizontal pipe whose one end is closed with a valve and the reading of a pressure gauge attached to the pipe is 3×10^5 Nm^{-2}. This reading of the pressure gauge falls to 1×10^5 Nm^{-2} when the valve is opened. Calculate the speed of water flowing into the pipe.

Q.20 Water flows at the rate of 4 litres per second through an orifice at the bottom of tank which contains water 720 cm deep. find the rate of escape of water if additional pressure of 16 kg f/cm^2 is applied at the surface of water.

Answers

1.	5.303 ms^{-1}	2.	1.0 cm	3.	7.3756 ms^{-1}
4.	200.905 J %	5.	10 ms^{-1}	6.	(a) 6.5×10^3 Nm^{-2}, (b) 8
7.	1 m/s, 2.75×10^4 Nm^{-2}	8.	4 ms^{-1}, 2.156×10^5 Nm^{-2}	9.	2 ms^{-1}
10.	79.18 ms^{-1}	11.	1.866 m^3	12.	20 ms^{-1}
13.	10.65 litres s^{-1}	14.	147.6 cm	15.	0.125 ms^{-1}
16.	406 cm^3 s^{-1}	17.	7.07 ms^{-1}	18.	0.036 ms^{-1}
19.	20 ms^{-1}	21.	19.28 litre s^{-1}		

Cohesive and Adhesive Forces

(i) **Cohesive force:** *It is the force of attraction between the molecules of the same substance.*
Example: Solids have definite shape and size due to strong forces of cohesion amongst their molecules.

(ii) **Adhesive force:** *It is the force of attraction between the molecules of two different substances.*
Example: It is due to force of adhesion that ink sticks to paper while writing.

Water wets the walls of its glass container because the force of adhesion between water and glass is greater than the force of cohesion between the water molecules. On the contrary, mercury does not wet glass because the force of cohesion between the mercury molecules is much greater than the force of adhesion between mercury and glass.

Molecular Range

It is the maximum distance upto which a molecule can exert some appreciable force of attraction on other molecules. It is of the order of 10^{-9} m in solids and liquids.

Sphere of Influence: *A sphere drawn around a molecule as centre and with a radius equal to the molecular range is called the sphere of influence of the molecule.* The molecule at the centre attracts all the molecules lying in its sphere of influence.

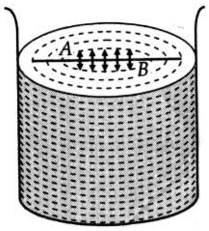
Surface Film: *A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.*

Surface Tension

A steel needle may be made to float on water though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called surface tension.

Surface tension *is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.*

Imagine a line AB on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side. The force acting on this line is proportional to the length of this line. If l is the length of the imaginary line and F the total force on either side of the line, then



$F \propto l$ or $F = \sigma l$

or $$\sigma = \frac{F}{l}$$

or $$\text{Surface tension} = \frac{Force}{Length}$$

Surface tension is measured as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.

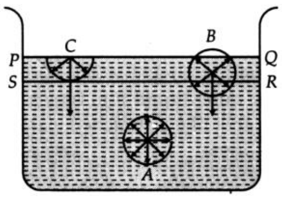
Units and dimensions of surface tension:

SI unit of surface tension = Nm^{-1} CGS unit of surface tension = dyne cm^{-1}

Dimensions of surface tension $= \frac{[Force]}{[Length]} = \frac{MLT^{-2}}{L} = [MT^{-2}]$

Molecular Theory of Surface Tension

In figure, PQRS is the surface film of a liquid. Consider the molecule A well inside the liquid. It is attracted equally in all directions by the molecules lying in its sphere of influence. Net force on such a molecule is zero.

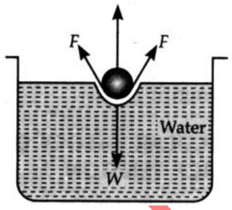


Now consider molecule B lying inside the surface film. Its sphere of influence lies partly outside. This molecule experiences less force upward and more force downward by the molecules in its sphere of influence. For molecule C, half its sphere of influence lies above the surface. The resultant downward force on such a molecule is maximum.

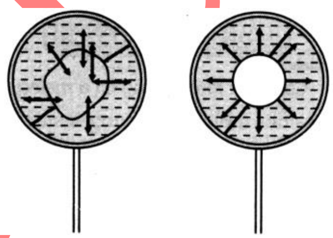
Due to this downward force, the potential energy of the molecules of the surface film is higher than those lying well inside the liquid. For a system to be stable, potential energy must be minimum. For the surface film to have minimum energy, the number of molecules in it must be minimum. Thus the surface film tends to have minimum surface area. As a result, the free surface of a liquid at rest behaves like an elastic stretched membrane.

Some Phenomena Based on Surface Tension

(i) Needle supported on water surface: Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating. The floating needle causes a little depression. The forces F, F due to surface tension of the curved surface are inclined as shown in figure. The vertical components of these two forces support the weight of the needle.



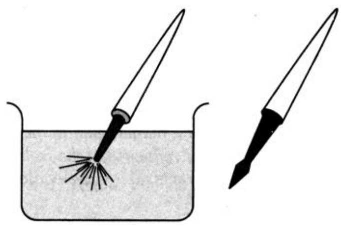
(ii) Endless wet thread on a soap film: If we take a circular frame of a stiff wire and dip it into a soap solution, a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape. But when the film is pricked at the centre, the loop is stretched outward and takes a symmetrical circular shape.



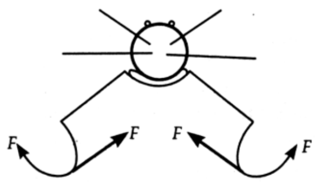
(iii) Rain drops are generally spherical in shape: Due to surface tension, the rain drops tend to minimize their surface area and the surface area of a sphere is minimum for a given volume.

(iv) Small mercury droplets are spherical and larger ones tend to flattened: Small mercury droplets are spherical because the forces of surface tension tend to reduce their area to a minimum value and a sphere has minimum surface area for a given volume. Larger drops of mercury are flattened due to the large gravitational force acting on them.

(v) The hair of a painting brush cling together when taken out of water: This is because the water films formed on them tend to contract to minimum area.



(vi) A bug floats on water due to surface tension: As shown in figure, a bug bends its legs on the surface of water such that the deformed surface gives rise to forces of surface tension which act tangential to the deformed surfaces. The weight of the bug is balanced by the upward components of these forces of surface tension.



(vii) Oil spreads on cold water but remains as a drop on hot water: This is because the surface tension of oil is less than that of the cold water but it is greater than that of the hot water.

Surface Energy

The free surfaced of a liquid possesses minimum area due to surface tension. To increase the surface area, molecules have to be brought from interior to the surface. Work has to be done against the forces of at traction. This work is stored as the potential energy of the molecules on the surface. So the molecules at the surface have extra energy compared to the molecules in the interior.

The extra energy possessed by the molecules of surface film of unit area compared to the molecules in the interior is called surface energy. It is equal to the work done in increasing the area of the surface film by unit amount.

$$\text{Surface energy} = \frac{\textit{Work done}}{\textit{Increase in surface area}}$$

The SI unit of surface energy is Jm^{-2}.

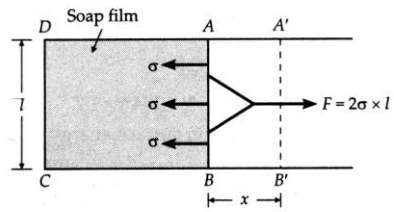
The relation between surface energy and surface tension

Consider a rectangular frame ABCD in which the wire AB is movable. Dip the frame in soap solution. A film is formed which pulls the wire AB inward due to surface tension with a force,

$$F = 2\,\sigma \times l$$

Here the factor 2 is taken because the soap film has two free surfaces.

Suppose AB is moved out through distance x to the position A'B'. Then



Work done = Force × distance

$= 2\sigma \times l \times x$

Increase in surface area of film = $2\,lx$

$$\therefore \quad \text{Surface energy} = \frac{\textit{Work done}}{\textit{Increase in surface area}} = \frac{2\sigma\, lx}{2\,lx} = \sigma$$

Thus surface energy of liquid is numerically equal to its surface tension.

Assignment – X

Q.1 A wire ring of 3 cm radius is rested on the surface of a liquid and then raised. The pull required is 3.03 g more before the film breaks than it is afterwards. Find the surface tension of the liquid.

Q.2 A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.

Q.3 A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended. Surface tension of mercury = 32×10^{-2} Nm^{-1}.

Q.4 A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.

Q.5 Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?

Q.6 If 500 erg of work is done in blowing a soap bubble to a radius r, what additional work is required to be done to blow it to a radius equal to 3r?

Q.7 A glass plate of length 10 cm, breadth 4 cm and thickness 0.4 cm, weighs 20g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight? Surface tension of water = 70 dyne cm^{-1}.

Q.8 If a number of little droplets of water of surface tension σ, all of the same radius r combine to form a single drop of radius R and the energy released is converted into kinetic energy, find the velocity acquired by the bigger drop.

Q.9 A soap film is formed on a rectangular frame of length 7 cm dipping in soap solution. The frame hangs from the arm of a balance. An extra weight of 0.38 g is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of soap solution. Given g = 980 cms^{-2}.

Q.10 A thin wire is bent in the form of a ring of diameter 3.0 cm. The ring is placed horizontally on the surface of soap solution and then raised up slowly. How much upward force is necessary to break the vertical film formed between the ring and the solution? Surface tension of a soap solution = 3.0×10^{-4} Ncm^{-1}.

Q.11 The length of a needle floating on water is 2.5 cm. How much minimum force, in addition to the weight of the needle, will be needed to lift the needle above the surface of water? Surface tension of
water = 7.2×10^{-4} N cm^{-1}.

Q.12 A rectangular plate of dimensions 6 cm × 4 cm and thickness 2 mm is placed with its largest face flat on the surface of water.

(i) What is the downward force on the plate due to surface tension? Surface tension of water = 7.0×10^{-2} N m^{-1}

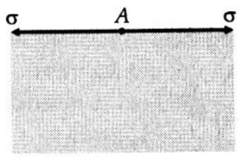
(ii) If the plate is placed vertical so that the longest side just touches the water surface, find the downward force on the plate.

Answers

1.	78.84 dyne cm^{-1}	2.	$2\pi D^2\sigma$	3.	3.98×10^{-2} J
4.	3168×10^{-8} J	5.	5 cm	6.	4000 erg
7.	13.4857 g f	8.	$\sqrt{\frac{6\sigma(R-r)}{rR}}$	9.	26.6 dyne cm^{-1}
10.	5.65×10^{-3} N	11.	3.6×10^{-3} N	12.	(i) 1.4×10^{-2} N, 8.68×10^{-3} N

Pressure Difference Across a Curved Liquid Surface

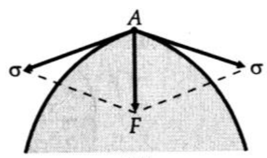
When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface. We consider the three possible liquid surfaces:



(a)

(i) As shown in figure, if the surface is plane, the molecule A on the surface is attracted equally in all directions. The resultant force due to surface tension is zero. Pressure on both sides of the surface is same i.e.,

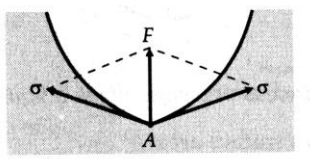
$$P_L = P_V$$



(b)

(ii) As shown in figure, if the surface is convex, there is a resultant downward force F on molecule A. For the surface to remain in equilibrium, the pressure on the liquid side must be greater than the pressure on the vapour side i.e.,

$$P_L > P_V$$



(c)

(iii) As shown in figure, if the surface is concave, there is a resultant upward force F due to surface tension on the molecule A. For the surface to remain in equilibrium, the pressure on the vapour side must be greater than the pressure on the liquid side i.e.,

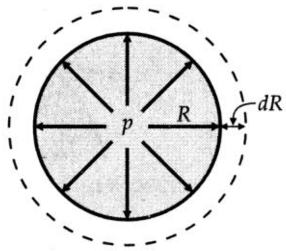
$$P_V > P_L$$

Thus we find that *whenever a liquid surface is curved, the pressure on its concave side is greater than the pressure on the convex side.*

Excess Pressure inside a Liquid Drop

Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to R + dR under the excess pressure p.

Initial surface area = $4\pi R^2$



Final surface area = $4\pi (R + dR)^2 = 4\pi (R^2 + 2R\,dR + dR^2) = 4\pi R^2 + 8\pi R\,dR$

dR^2 is neglected as it is small.

Increase in surface area = $4\pi R^2 + 8\pi R\,dR - 4\pi R^2 = 8\pi R\,dR$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area × surface tension = $8\pi R\,dR\,\sigma$

But work done = Force × Distance = Pressure × Area × Distance

= $p \times 4\pi R^2 \times dR$

Hence, $p \times 4\pi R^2 \times dR = 8\pi R\,dR\,\sigma$ $\quad\therefore\quad$ Excess pressure, $p = \frac{2\sigma}{R}$

Excess Pressure inside a Soap Bubble

Proceeding as in the case of a liquid drop in the above question, we obtain

Increase in surface area = $8\pi R\,dR$

But a soap bubble has air both inside and outside, so it has two free surfaces.

$\therefore$ Effective increase in surface area = $2 \times 8\pi R\,dR = 16\pi R\,dR$

Work done in enlarging the soap bubble = Increase in surface energy

= Increase in surface area × surface tension = $16\pi R\,dR\,\sigma$

But, Work done = Force × Distance = $p \times 4\pi R^2\,dR$

Hence $p \times 4\pi R^2 \times dR = 16\pi R\,dR\,\sigma$ or $p = \frac{4\sigma}{R}$

Excess pressure inside an air bubble inside a liquid

An air bubble inside a liquid is similar to a liquid drop in air. It has only one free spherical surface. Hence excess pressure is given by $p = \frac{2\sigma}{R}$

NOTE

- The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside the tiny fog droplets that they are rigid enough to behave like solids and resists fairly large deforming forces.

- When an ice skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball–bearings and help the skaters to run along smoothly.
- When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension σ, the excess pressure inside the bubble will be $p = \frac{2\sigma}{R} + h\rho g$

Assignment – XI

Q.1 The excess pressure inside a soap bubble of radius 6 mm is balanced by 2 mm column of oil of specific gravity 0.8. Find the surface tension of soap solution.

Q.2 Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. Also compare the works done in blowing these bubbles.

Q.3 A small hollow sphere having a small hole in it is immersed into water to a depth of 20 cm before any water penetrates into it. If the surface tension of water is 73 dyne cm^{-1}, find the radius of the hole.

Q.4 A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end? Given density of mercury = 13600 kg m^{-3} and surface tension of mercury 35×10^{-3} Nm^{-1}.

Q.5 The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at the temperature of the experiment is 7.30×10^{-2} Nm^{-1}.
1 atmospheric pressure = 1.01×10^{5} Pa, density of water = 1000 kgm^{-3}, g = 9.80 ms^{-2}. Also calculate the excess pressure.

Q.6 Calculate the total pressure inside a spherical bubble of radius 0.2 mm formed inside water at a depth of 10 cm. Surface tension of water at depth of 30 cm is 70 dyne cm^{-1}, barometric pressure is 76 cm, density of mercury is 13.6 g cm^{-3} and g = 980 cms^{-2}.

Q.7 Find the difference in excess pressure on the inside and outside of a rain drop if its diameter changes from 0.03 cm to 0.0002 cm by evaporation. Surface tension of water is 72 dyne cm^{-1}.

Q.8 There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 10^{3} kgm^{-3}. The bubble is at a depth of 10.0 cm below the free surface of a liquid. By what amount is the pressure inside the bubble greater than the atmospheric pressure?

Q.9 An ancient building has a dome of 5 m radius and uniform but small thickness. The surface tension of its masionary structure is about 500 Nm^{-1}. Treated as hemisphere, find max. load that dome can support.

Answers

1.	2.35×10^{-2} Nm^{-1}	2.	3 : 2, 4 : 9	3.	0.007449 cm
4.	2805.6 Nm^{-2}	5.	146 Pa, 1.02×10^{5} Pa	6.	1029728 dyne cm^{-2}
7.	1430400 dyne cm^{-2}	8.	1130 Nm^{-2}	9.	31420 N

Angle of Contact

The liquid surface is usually curved when it is in contact with a solid. The particular shape that it takes depends on the relative strengths of cohesive and adhesive forces. If

Adhesive force > cohesive force: Liquid wets the solid surface and has concave meniscus

Adhesive force < cohesive force: Liquid does not wet the solid surface and has a convex meniscus

Adhesive force = cohesive force: Liquid surface is plane

Angle of Contact *is defined as the angle θ between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.*

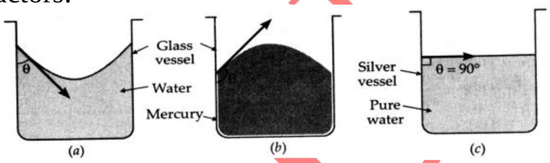
The value of angle of contact depends on the following factors:

(i) Nature of the solid and the liquid in contact

(ii) Cleanliness of the surface in contact

(iii) Medium above the free surface of the liquid

(iv) Temperature of the liquid



For those liquids which wet the walls of the vessel, the angle of contact is acute. For the liquids which do not wet the walls of the vessel, the angle of contact is obtuse. The angle of contact for water and glass is about 8^o, for mercury and glass it is 138^o and for pure water and silver, angle of contact is 90^o.

Shape of Liquid Meniscus in a Narrow Tube

Shape of liquid meniscus in a narrow tube: Consider a molecule O on the surface of the liquid in contact with the solid wall of the vessel. The various forces acting at the boundary of the three surfaces are as follows:

(i) Surface tension σ_{LV} of the liquid–vapour surface acting tangentially to the liquid surface.

(ii) Surface tension σ_{SV} of the solid–vapour surface acting parallel to the walls of the vessel.

(iii) Surface tension σ_{SL} of solid–liquid surface acting parallel to wall of the vessel directed into the liquid.

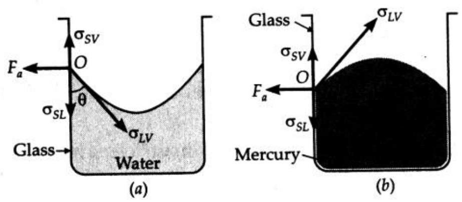
(iv) Adhesive force F_a between molecules of the vessel and liquid acting normal to the wall of the container.

For equilibrium, no forces should act on molecule O in any direction. Let θ be the angle of contact. Then the components of σ_{LV} parallel and perpendicular to water surface area $\sigma_{LV}\sin\theta$ and $\sigma_{LV}\cos\theta$ respectively. For equilibrium, we must have

$$Fa = \sigma_{LV}\cos\theta$$

and $$\sigma_{SV} = \sigma_{SL} + \sigma_{LV}\cos\theta$$

or $$\cos\theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$



The following there cases are possible:

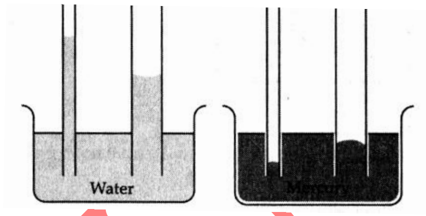
(i) If $\sigma_{SV} > \sigma_{SL}$, cos θ is positive and $\theta < 90^o$ i.e., angle of contact is acute. The liquid meniscus is concave upwards. This happens in the case of water taken in a glass vessel (figure a)

(ii) If $\sigma_{SV} < \sigma_{SL}$, cos θ is negative and $\theta > 90^o$, i.e., angle of contact is obtuse. The liquid meniscus is convex, upwards. This happens in the case of mercury taken in a glass vessel (figure b)

(iii) When $\sigma_{SV} = \sigma_{SL}$, cos θ = 0 and $\theta = 90^o$. The liquid meniscus is plane. This happens in the case of pure water taken in a silver vessel.

Capillarity

A tube of very fine (hair–like) bore is called a capillary tube. When a capillary tube of glass open at both ends is dipped in liquid which wets its walls (e.g., water, alcohol), the liquid rises in the tube. But when the capillary tube is dipped in a liquid which does not wet its walls (e.g., mercury), the level of liquid is depressed in the tube.



The phenomenon of rise or fall of a liquid in a capillary, tube in comparison to the surrounding is called capillarity.

Some examples of capillarity from daily life:

(i) A blotting paper soaks ink by capillary action. The pores of blotting paper act as capillaries.

(ii) Oil rises in the long narrow spaces between threads of a wick, the narrow spaces act as capillary tubes.

(iii) We use towels made of a coarse cloth for drying our skin after taking bath.

(iv) Sap rises from the roots of a plant to its leaves and branches due to capillarity action.

(v) The tip of the nib of a pen is split to provide capillary action for the ink to rise.

Rise of Liquid in a Capillary Tube : Ascent Formula

Consider a capillary tube of radius r dipped in a liquid of surface tension σ and density ρ. Suppose the liquid wets the sides of the tube. Then its meniscus will be concave. The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.

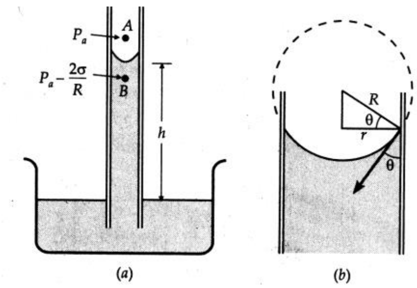
As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is $p = \frac{2\sigma}{R}$

where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in figure, we have

$$\frac{r}{R} = \cos\theta$$

or $$R = \frac{r}{\cos\theta}$$

∴ $$p = \frac{2\sigma\cos\theta}{r}$$



Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = h\rho g$$

or $$\frac{2\sigma\cos\theta}{r} = h\rho g$$ or $$h = \frac{2\sigma\cos\theta}{r\rho g}$$

This is the ***ascent formula*** for the rise liquid in a capillary tube. If we take into account the volume of the liquid contained in the meniscus, then the above formula gets modified as

$$h=\frac{2\sigma\cos\theta}{r\rho g}-\frac{r}{3}$$

However, the factor r/3 can be neglected for a narrow tube. The ascent formula shows that the height h to which a liquid rises in the capillary tube is

(i) *inversely proportional to the radius of the tube.*

(ii) *inversely proportional to the density of the liquid*

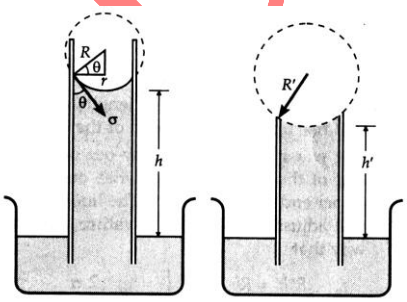
(iii) *directly proportional to the surface tension of the liquid.*

Hence a liquid rises more in a narrow tube than in wider tube.

Rise of liquid in a Capillary Tube of Insufficient Height

The height to which a liquid rises in a capillary tube is given by

$$h=\frac{2\sigma\cos\theta}{r\rho g}$$



The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by r = R cos θ. Therefore,

$$h=\frac{2\sigma\cos\theta}{R\cos\theta\,\rho g}=\frac{2\sigma}{R\rho g}$$

As σ, ρ and g are constants, so

$$hR=\frac{2\sigma}{\rho g}=constant \qquad \therefore \qquad hR = h'R'$$

where R' is the radius of curvature of the new meniscus at a height h'   As h' < h, so R' > R

Hence in a capillary tube of insufficient height, the liquid rises to the top and spreads out to a new radius of curvature R' given by $R'=\frac{hR}{h'}$ But the liquid will not overflow.

Assignment – XII

Q.1 Calculate the height to which water will rise in capillary tube of 1.5 mm diameter. Surface tension of water is 7.4×10^{-3} Nm^{-1}.

Q.2 Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water–glass and mercury–glass as 0^o and 135^o respectively. Determine the ratio of surface tensions of mercury and water. Take cos 135^o = – 0.71.

Q.3 Water rises in a capillary tube to a height of 2.0 cm. In another capillary whose radius is one–third of it, how much the water will rise? If the first capillary is inclined at an angle of 60^o with the vertical, then what will be the position of water in the tube?

Q.4 If a 5 cm long capillary tube with 0.1 mm internal diameter opens at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary (ii) water will rise up to the upper end of capillary (iii) water will overflow out of the upper end of capillary. Explain your answer.

Q.5 A glass U–tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is

the difference between the heights to which water rises in the two limbs? Surface tension of water is
$0.07\ Nm^{-1}$. Assume that the angle of contact between water and glass is 0^o.

Q.6 A capillary tube of inner diameter 0.5 mm is dipped in a liquid of specific gravity 13.6, surface tension 545 dyne cm^{-1} and angle of contact 130^o. Find the depression or elevation in the tube.

Q.7 The tube of mercury barometer is 5 mm in diameter. How much error does the surface tension cause in the reading? S.T. of mercury = $540 \times 10^{-3}\ Nm^{-1}$. Angle of contact = 135^o.

Q.8 Water rises to a height of 9 cm in a certain capillary tube. If in the same tube, level of Hg is depressed by 3 cm, compare the surface tension of water and mercury. Specific gravity of Hg is 13.6, the angle of contact for water is zero and that for Hg is 135^o.

Q.9 A capillary tube whose inside radius is 0.5 mm is dipped in water of surface tension 75 dyne cm^{-1}. To what height is the water raised by the capillary action above the normal level? What is the weight of water raised?

Q.10 A U–tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne cm^{-1} and zero contact angle. Calculate the density of the liquid, if the difference in the levels of the meniscus is 1.25 cm. Take g = 980 cms^{-2}.

Answers

1.	0.002014 m	2.	7.2 : 1	3.	6.0 cm, 4.0 cm
4.	30.58 cm	5.	4.76 mm	6.	– 2.1 cm
7.	-0.2293×10^{-2} m	8.	0.152		
9.	3.061 cm, 23.55 dyne or 0.024 g wt			10.	0.8 g cm^{-3}

Factors Affecting Surface

(i) **Effect of contamination:** If water surface has dust, grease or oil, the surface tension of water reduces. A small piece of camphor put in clear water dances vigorously due to decrease of surface tension of water.

(ii) **Effect of solute:** (a) A highly soluble substance like sodium chloride increases the surface tension of water. (b) A sparingly soluble substance like phenol or soap reduces the surface tension of water.

(iii) **Effect of temperature:** The surface tension of liquids decreases with increase of temperature. The surface tension of a liquid becomes zero at a particular temperature, called *critical temperature* of that liquid. For small temperature differences, surface tension decrease almost linearly as $\sigma_t = \sigma_0 (1 - \alpha t)$

where σ_t and σ_0 are the surface tensions at t^oC and 0^oC respectively, and α is the temperature coefficient of surface tension.

Detergents and Surface Tension

Cleaning action of detergents: Oil stains and grease on dirty clothes cannot be removed by simply washing the clothes with water because water does not wet them. By adding detergent or soap to water, the greasy dirt can be easily removed. The cleansing action of detergents can be explained as follows:

(i) Soap or detergent molecules have the shape of a hairpin.

(ii) When detergent is dissolved in water, the heads of its hairpin shape molecules get attracted to water surface.

(iii) When clothes with greasy strains are dipped in water containing detergent, the pointed ends of detergent molecules get attached to the molecules of grease. So a water–grease interface is formed. Thus surface tension is greatly reduced. The greasy dirt is held suspended.

(iv) When the clothes are rinsed in water, the greasy dirt is washed away by running water.

So when detergent is added to water, the surface tension of water is reduced, its area of contact with grease is increased and hence its cleaning ability is increased.

Conceptual Problems

Q.1 (i) A balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds). (i i) The force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air.

Q.2 A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_o. After some time, the ball falls with a const ant velocity. Calculate the viscous force on the ball.

Q.3 A tank filled with fresh water has hole in its bottom and water is flowing out of it. If the size of the hole is increased what will be the change in:

(a) Volume of water flowing out per second? (b) Velocity of the out coming water?

(c) If in the above tank, the fresh water is replaced by sea water, will the velocity of out coming water change?

Q.4 In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why?

Q.5 Two soap bubbles of different diameters are in contact with a certain portion common to both the bubbles. What will be the shape of the common boundary as seen from inside the smaller bubble? Support your answer with a neat diagram. Give reason for your answer.

Q.6 In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why?

Q.7 A big size balloon of mass M is held stationary in air with the help of a small block of mass M/2 tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations.

Q.8 A tarnado consists of rapidly whirling air vortex. Why is the pressure always much lower in the centre than at the outside? How does this condition account for the destructive power of tarnado?

Problems of Higher order Thinking Skills

Q.1 A piece of ice with a stone frozen in it floats on water taken in a beaker. Will the level of water increase or decrease or remain the same when ice melts completely?

Q.2 An ice block with a cork piece embedded inside floats in water. What will happen to the level of water when ice melts?

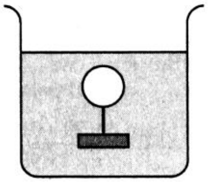
Q.3 A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level?

Q.4 To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force, with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel?

Q.5 A block of wood is floating on water at 0^oC with a certain volume V above the water level. The temperature of water is slowly raised from 0^oC to 20^oC. How will the volume V change with the rise in temperature?

Q.6 A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon?

Q.7 A balloon filled with air is weighed so that it barely floats in water, as shown in figure. Explain why it sinks to the bottom when it is submerged more by a small distance.

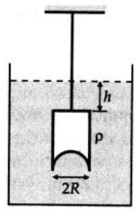


Q.8 A beaker containing water is placed on a spring balance. A stone of weight W is hung and lowered into the water without touching the sides and bottom of the beaker. Explain how the reading will change.

Q.9 A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density = 13.6 g cm^{-3}). A homogenous sphere floats with half its volume immersed in mercury and the other half in oil. What is the density of material of sphere?

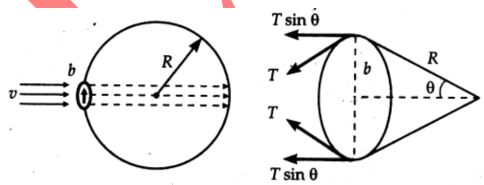
Q.10 An iceberg weighs 400 tonnes. The specific gravity of iceberg is 0.92 and the specific gravity of water is 1.02. What percentage of iceberg is below the water surface?

Q.11 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. How much is the force on the bottom of the cylinder by the liquid?



Q.12 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?

Q.13 A bubble having surface tension T and radius R is formed on a ring of radius b (b < < R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring (fig).



Q.14 A cylindrical vessel of radius 3 cm has at the bottom a horizontal capillary tube of length 20 cm and internal radius 0.4 mm. If the vessel is filled with water, find the time taken by it to empty one half of its contents. Given that the viscosity of water is 0.01 poise.

Q.15 A soap bubble of radius 4 cm and surface tension 30 dyne cm^{-1} is blown at the end of a tube of length 10 cm and internal radius 0.20 cm. If the viscosity of air is 1.89×10^{-4} poise, find the time taken by the bubble to the reduced to a radius of 2 cm.

Q.16 A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^{4} kg m^{-3} enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given coefficient of viscosity of water = 1.0×10^{-3} N sm^{-2},
g = 10 ms^{-2} and density of water = 1.0×10^{3} kg m^{-3}.

Q.17 Water stands at a height H in a tank whose side walls are vertical. A hole is made in one of the walls at a depth h below the water surface. (i) Find at what distance from the foot of the wall does the emerging stream of water strike the floor? (ii) For what value of h, this range is maximum? (iii) can a hole be made at another depth so that the second stream has the same range?

Q.18 A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross–sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. What is the pressure at another point where the cross–sectional area is 5 cm^2?

Q.19 A liquid is kept in a cylindrical vessel which is being rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rps, find the difference in the heights of the liquid at the centre of the vessel and at its sides.

Q.20 Calculate the rate of flow of glycerine of density 1.25×10^3 kgm^{-3} through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and pressure–drop across its length is 10 Nm^{-2}.

Q.21 Water from a tap emerges vertically downward with an initial speed of 1.0 ms^{-1}. The cross sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross–sectional area of the stream 0.15m below the tap?

NCERT Exercise

Q.1 Explain why:

(a) The blood pressure in humans is greater at the feet than at the brain.

(b) Atmospheric pressure at a height of about 6 km decrease to nearly half its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector.

Q.2 Explain why:

(i) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(ii) Water on a clean glass surface tends to spread out mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not).

(iii) Surface tension of a liquid is independent of the area of the surface.

(iv) Detergents should have small angles of contact.

(v) A drop of liquid under no external forces is always spherical in shape.

Q.3 Fill in the blanks using the word(s) from the list appended with each statement:

(i) Surface tension of liquids generally ______________ with temperatures (increases/decreases)

(ii) Viscosity of gases ____________ with temperature, whereas viscosity of liquids __________ with temperature (increase/ decreases)

(iii) For solids with elastic modulus of rigidity, the shearing force is proportional to ___________ while for fluids it is proportional to _____________ (shear strain/rate of shear strain)

(iv) For a fluid is steady flow, the increase in flow speed at a constriction follows from ______ while the decrease of pressure there follows from __________ (conservation of mass/ Bernoulli’s principle)

(v) For the model of a plane in a wind tunnel, turbulence occurs at a ________ speed than the critical speed for turbulence for an actual plane (greater/ smaller)

3. (i) decreases (ii) increases, decreases, (c) shear strain, rate of shear strain, (d) conservation of mass, Bernoulli’s principle, (v) greater

Q.4 Explain why:

(i) To keep a piece of paper horizontal, you should blow over, not under, it.

(ii) When we try to close a water tap with our fingers, fast jets of water gush though the openings between our fingers.

(iii) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(iv) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(v) A spinning cricket ball in air does not follow a parabolic trajectory.

Q.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Q.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984. Determine the height of the wine column for normal atmospheric pressure.

Q.7 A vertical off–shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in Bombay High? Take the depth of the sea to be roughly 3 km, and ignore ocean currents.

Q.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross–section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Q.9 A U–tube contains water and methylated sprit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

Q.10 In previous exercise, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in two arms? Specific gravity of mercury = 13.6.

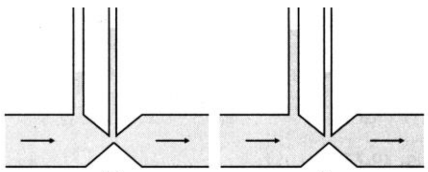
Q.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

Q.12 Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation? Explain.

Q.13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kgs^{-1}, what is the pressure difference between the two ends of the tube? Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Nsm^{-2}.

Q.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is lift of the wing if its area is 2.5 m^2? Density of air = 1.3 kg m^{-3}.

Q.15 Figure (a) and (b) refer to the steady flow of a (non–viscous) liquid. Which of the two figures is incorrect? Why?

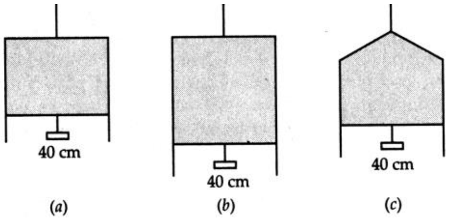


(a) (b)

Q.16 The cylindrical tube of a spray pump has a cross section of 8.0 cm^2, one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the spread of ejection of the liquid through the holes?

Q.17 A U–shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N(which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Q.18 Figure (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in figure(b) and (c)? Explain your answer physically.

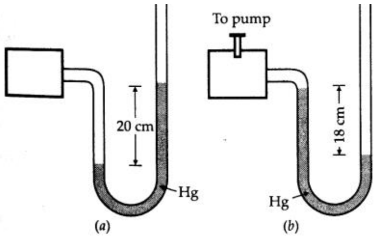


Q.19 What is the pressure inside a drop of mercury of radius 3.00 mm of room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} Nm^{-1}. The atmospheric pressure is 1.01 $\times 10^5$ Pa. Also give the excess pressure inside the drop.

Q.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm? Given that the surface tension of soap solution at the temperature (20°C) is 2.50×10^{-2} Nm^{-1}. If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atm = 1.01 $\times$ 10^5 Pa).

Q.21 A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door closed.

Q.22 A manometer reads the pressure of a gas in a enclosure as shown in figure (a). When some of the gas is removed by a pump, the manometer reads as in figure (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.



(i) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.

(ii) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in volume of the gas)

Q.23 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height.

(i) Is the force exerted by the water on the base of the vessel the same in the two cases?

(ii) If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

Q.24 During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? The density of whole blood = 1.06×10^3 kg m^{-3}.

Q.25 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present? (b) Do the dissipative forces becomes more important as the fluid velocity increases? Discuss qualitatively.

Q.26 (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar? (b) What is the corresponding flows rate? Take viscosity of blood to be 2.084×10^{-3} Pa s and density of blood = 1.06×10^{3} kg m^{-3}.

Q.27 A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. Take air density to be 1 kg m^{-3} and g = 9.81 ms^{-2}.

Q.28 In Millikan's oil drop experiment, what is the terminal speed of a drop of radius 2.0×10^{-5} m and density 1.2×10^{3} kg m^{-3}? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Nsm^{-2}. How much is viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Q.29 Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of thin glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6×10^{3} kgm^{-3}.

Q.30 The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U–shaped tube open at both ends. If the U–tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to the zero, and density of water to be 1.0×10^{3} kg m^{-3}. Take g = 9.8 ms^{-2}.

Q.31 (a) If is known that density ρ of air decreases with height y as $\rho = \rho_0 e^{-y/y_0}$

where ρ_0 = 1.25 kg m^{-3} is the density at sea level and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m^3 is used to lift a pay load of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise? Take y_0 = 8000 m and ρ_{He} = 0.18 kg m^{-3}

Answers

5.	6.2×10^{6} Pa	6.	10.5 m	7.	yes
8.	6.92×10^{5} Nm^{-2}	9.	0.8	10.	0.221 cm
11.	No	12.	No	13.	9.8×10^{12} Pa
14.	1512.9 N	15.	figure a is incorrect	16.	0.637 ms^{-1}
17.	2.5×10^{-2} Nm^{-1}	18.	4.5×10^{-2} N	19.	1.013×10^{5} Pa, 310 Pa
20.	20 Pa, 105714 Pa	21.	54.88 N		

22. (i) (a) P_a = 96 cm of Hg, P_g = 20 cm of Hg, (b) P_a = 58 cm of Hg, P_g = – 18 cm of Hg, (ii) 19 cm

23.	(i) same	24.	0.1925 m	25.	(a) decreases, (b) yes
26.	(a) 0.98 ms^{-1}, (b) 1.23×10^{-5} m^{-3} s^{-1}			27.	4396 kg
28.	3.9×10^{-10} N	29.	– 5.34 mm	30.	5.290 mm
31.	(b) 8 km				

IIT–JEE Objective Assignment

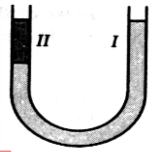
Multiple Choice Questions with One Correct Answer

Q.1 A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

(a) same everywhere (b) lower in the front side

(c) lower in the rear side (d) lower in the upper side

Q.2 A U–tube of uniform cross–section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be



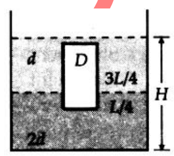
(a) 1.12 (b) 1.1 (c) 1.05 (d) 1.0

Q.3 A man is sitting in a boat, which is floating on a pond. If the man drinks some water from the pond, the level of water in the pond

(a) decreases

(b) increases (c) remains unchanged

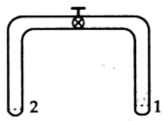
(d) may increase or decrease depending on the weight of the man



Q.4 A homogenous solid cylinder of length L ($L < H/2$), cross–sectional area A/5 is immersed such that it floats with its axis vertical at the liquid–liquid interface with length L/4 in the denser liquid as shown in the figure. The lower density liquid is D open to atmosphere having pressure P_0. Then density of solid is given by

(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$ (c) 4d (d) $\frac{d}{5}$

Q.5 A glass tube of uniform internal radius (r) has a value separating the two identical ends. Initially, the value is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub–hemi–spherical soap bubble as shown in figure. Just after opening the valve,



(a) air from end 1 flows towards end 2. No change in volume of the soap bubbles.

(b) air from end 1 flows towards end 2. Volume of soap bubble at end 1 decreases

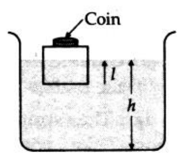
(c) no changes occur

(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Q.6 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is

(a) Mg (b) $Mg - V\rho g$ (c) $Mg + \pi R^2 h\rho g$ (d) $\rho g (V + \pi R^2 h)$

Q.7 A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance *l* and h are shown here. After some time the coin falls into the water. Then



(a) *l* decreases and h increases

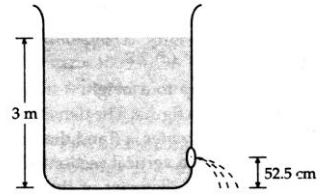
(b) *l* increases and h decreases

(c) both *l* and h increase (d) both *l* and h decrease

Q.8 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, quantities of water flowing out per second from both holes are the same. Then R is equal to

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (d) L (d) $L/2\pi$

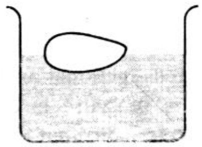
Q.9 Water is filled in a container upto height 3 m. A small hole of area a is punched in the wall of the container at a height 52.5 cm from the bottom. The cross–sectional area of the container is A. If a/A = 0.1, then v^2 (where v is the velocity of water coming out of the hole) is



(a) 50 m^2s^{-2} (b) 51 m^2s^{-2} (c) 48 $m^2\ s^{-2}$ (d) 51.5 m^2s^{-2}

Multiple Choice Questions with One or More than One Correct Answer

Q.10 A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is



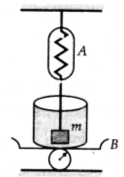
(a) zero

(b) equal to the weight of the liquid displaced

(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body

Q.11 The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation



(a) the balance A will read more than 2 kg

(b) the balance B will read more than 5 kg

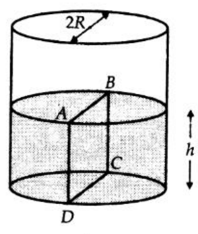
(c) the balance A will read less than 2 kg and B will read more than 5 kg

(d) the balances A and B will read 2 kg and 5 kg respectively

Q.12 A vessel contains oil (density = 0.8 g cm^{-3}) over mercury (density = 13.6 cm^{-3}). A homogenous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material (in g cm^{-3}) is

(a) 3.3 (b) 6.4 (c) 7.2 (d) 2.8

Q.13 Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



(a) $|2P_0Rh + \pi R^2\rho gh - 2RT|$ (b) $|2P_0Rh + R\rho gh^2 - 2RT|$

(c) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$ (d) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$

Q.14 Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross–sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross–sectional area of the stream 0.15 m below the tap is

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2 (c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

Reasoning Type

Q.15 **Statement – 1:** The steam of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement–2: In any steady flow of an incompressible fluid, volume flow rate of fluid remains constant.

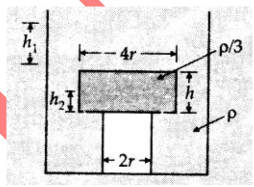
(a) Statement – 1 is true, Statement – 2 is true.
Statement – 2 is a correct explanation for Statement – 1.

(b) Statement – 1 is true, statement – 2 is true.
Statement – 2 is not a correct explanation for Statement – 1.

(c) Statement – 1 is true, Statement – 2 is false. (d) Statement – 1 is false, statement – 2 is true.

Comprehension Based Questions ***[Question N o. 16 to 18]***

A cylinder tank has a hole of diameter 2r in its bottom. The hole is covered with a wooden cylindrical block of diameter 4r, height h and density ρ/3.



Situation 1: Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

situation 2: The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. Height h_2 is smaller than h (height of the block) and thus block is exposed to atmosphere.

Read the passage given above and answer the following questions

Q.16 Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up.

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$ (c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

Q.17 Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force.

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$ (c) $\frac{2h}{3}$ (d) h

Q.18 In situation 2, if h_2 is further decreased, then

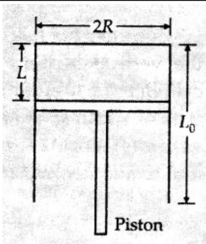
(a) cylinder will not move up and remains at its original position

(b) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \frac{h}{4}$, cylinder again starts moving up (d) for $h_2 = \frac{h}{5}$, cylinder again starts moving up

Paragraph Type ***[Q. No. 19 to 21]***

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



Read the passage given above and answer the following questions

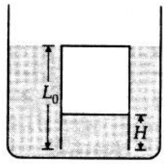
Q.19 The piston is now pulled out slowly and held at a distance 2 L from the top. The pressure in the cylinder between its top and the piston will then be

(a) P_0 (b) $P_0/2$ (c) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

Q.20 While the piston is at a distance 2 L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0+Mg}\right)(2L)$ (b) $\left(\frac{P_0\pi R^2-Mg}{\pi R^2 P_0}\right)(2L)$ (c) $\left(\frac{P_0\pi R^2+Mg}{\pi R^2 P_0}\right)(2L)$ (d) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0-Mg}\right)(2L)$

Q.21 The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same levels the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies.



(a) $\rho g(L_0-H)^2+P_0(L_0-H)+L_0P_0=0$ (b) $\rho g(L_0-H)^2-P_0(L_0-H)-L_0P_0=0$

(c) $\rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$ (d) $\rho g(L_0-H)^2-P_0(L_0-H)+L_0P_0=0$

Integer Answer Type

Q.22 A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmosphere pressure =m 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension]

Q.23 Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap–water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity].

Answers

1	b	2	b	3	c	4	a	5	b
6	d	7	d	8	a	9	b	10	a
11	b, c	12	c	13	b	14	c	15	a
16	c	17	b	18	a	19	a	20	d
21	c	22	6 mm	23	6				

AIEEE – Objective Assignment – II

Q.1 A jar is filled with two non–mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?

(a) $\rho_1<\rho_3<\rho_2$ (b) $\rho_3<\rho_1<\rho_2$ (c) $\rho_1>\rho_3>\rho_2$ (d) $\rho_1<\rho_2<\rho_3$

Q.2 Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is

(a) directly proportional to R but inversely proportional to v

(b) directly proportional to both radius R and velocity v

(c) inversely proportional to both radius R and velocity v

(d) inversely proportional to R but directly proportional to velocity.

Q.3 If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid.

(a) 0.2 m/s (b) 0.4 m/s (c) 0.133 m/s (d) 0.1 m/s

Q.4 A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is

(a) $\frac{Vg(\rho_1-\rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$ (c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$

Q.5 A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is

(a) 10 (b) 20 (c) 25.5 (d) 5

Q.6 If two soap bubbles of different radii are connected by a tube,

(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(b) air flows from bigger bubble to the smaller bubble till the size are interchanged

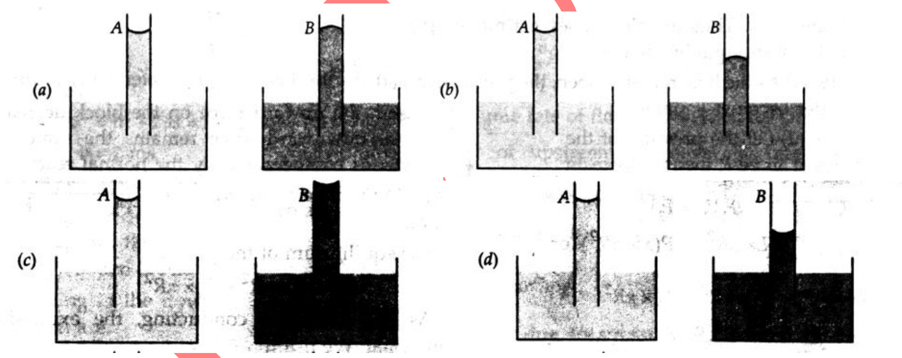
(c) air flows from the smaller bubble to the bigger

(d) there is no flow of air

Q.7 A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in capillary tube will be

(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm

Q.8 A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap–water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

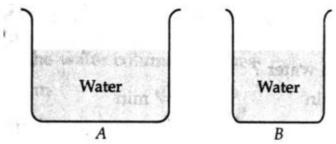


Answers

1	a	2	b	3	d	4	b	5	b
6	c	7	b	8	d				

DCE & DPMT – Objective Assignment – III

Q.1 From the following figures, the correct observation is



(a) the pressure on bottom of tank A is greater than that at bottom of B

(b) the pressure on bottom of the tank A is smaller than at bottom of B

(c) the pressure depends on the shape of the container

(d) the pressure on the bottoms of A and B is the same

Q.2 A wooden block is taken to bottom of a deep, calm lake of water and then released. It rises up with a

(a) constant acceleration (b) decreasing acceleration

(c) constant velocity (d) decreasing velocity

Q.3 If there were no gravity, which of the following will not be there for a fluid?
(a) Viscosity (b) Surface tension
(c) Pressure (d) Archimedes' upward thrust

Q.4 A bubble is at the bottom of the lake of depth h. As the bubble comes to sea level, its radius increases three times. If atmospheric pressure is equal to *l* metre of water column, then h is equal to
(a) 26 *l* (b) *l* (c) 25 *l* (d) 30 *l*

Q.5 Radius of one arm of hydraulic lift is four times of radius of other arm. What force should be applied on narrow arm to lift 100 kg?
(a) 26.5 N (b) 62.5 N (c) 6.25 N (d) 8.3 N

Q.6 A liquid X of density 3.36 g/cm^3 is poured in the right arm of a U–tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same. Density of Y?
(a) 0.8 g/cm^3 (b) 1.2 g/cm^3 (c) 1.4 g/cm^3 (d) 1.6 g/cm^3

Q.7 The unit of coefficient of viscosity is
(a) Nm/s (b) Nm^2/s (c) $N/(m^2s^{-1})$ (d) Nms^2

Q.8 An object is moving through the liquid. The viscous damping force acting on it is proportional to the velocity. Then dimensions of constant of proportionality are
(a) $[ML^{-1}T^{-1}]$ (b) $[MLT^{-1}]$ (c) $[M^0LT^{-1}]$ (d) $[ML^0T^{-1}]$

Q.9 The rate of flow of liquids in a tube of radius r, length *l*, whose ends are maintained at a pressure difference P is $V = \frac{\pi Q p r^4}{\eta l}$, where η is coefficient of viscosity and Q is
(a) 8 (b) 1/8 (c) 16 (d) 1/16

Q.10 Motion of a liquid in a tube is best described by
(a) Bernoulli's theorem (b) Poiseuille's equation (c) Stokes' law (d) Archimedes' principle

Q.11 which one is not a dimensional number?
(a) Acceleration due to gravity (b) Surface tension of water
(c) Velocity of light (d) Reynold's number

Q.12 Critical velocity of the liquid
(a) decreases when radius decreases (b) increases when radius increases
(c) decreases when density increases (d) increases when density increases

Q.13 A steel ball is dropped in oil, then
(a) the ball attains constant velocity after some time (b) the ball stops
(c) the speed of ball will keep on increasing (d) none of the above

Q.14 A sphere of mass m and radius r is falling in the column of a viscous fluid. Terminal velocity attained by falling object is proportional to
(a) r^2 (b) 1/r (c) r (d) $-1/r^2$

Q.15 The ratio of the terminal velocities of two drops of radii R and R/2 is
(a) 2 (b) 1 (c)1/2 (d) 4

Q.16 The radii of two drops are in the ratio of 3 : 2, their terminal velocities are in the ratio
(a) 9 : 4 (b) 2 : 3 (c) 3 : 2 (d) 2 : 9

Q.17 Bernoulli's equation is an example of conservation of
(a) energy (b) momentum (c) energy momentum (d) mass

Q.18 An aeroplane gets its upward lift due to a phenomenon described by the
(a) Archimedes' principle (b) Bernoulli's principle
(c) Buoyancy principle (d) Pascal law

Q.19 The rate of flow of liquid through an orifice of a tank does not depend upon
(a) the size of orifice (b) density of liquid

(c) the height of fluid column (d) acceleration due to gravity

Q.20 The velocity of efflux of a liquid through an orifice in bottom of the tank does not depend upon
(a) size of orifice (b) height of liquid
(c) acceleration due to gravity (d) none of the above

Q.21 A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?
(a) 9 min (b) 7 min (c) 5 min (d) 3 min

Q.22 The SI unit of surface tension is
(a) dyne/cm (b) N/m^2 (c) N/m (d) Nm

Q.23 The water droplets in free fall are spherical due to
(a) gravity (b) viscosity
(c) surface tension (d) intermolecular attraction

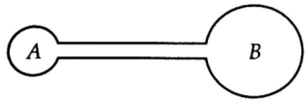
Q.24 One large soap bubble of diameter D breakes into 27 bubbles having surface tension T. The change in surface energy is
(a) $2\pi TD^2$ (b) $4\pi TD^2$ (c) πTD^2 (d) $8\pi TD^2$

Q.25 Two drops of equal radius coalesce to form a bigger drop. What is ratio of surface energy of bigger drop to smaller one?
(a) $2^{1/2} : 1$ (b) 1 : 1 (c) $2^{2/3} : 1$ (d) none of these

Q.26 8 mercury drops coalesce to from 1 mercury drop, the energy change by a factor of
(a) 1 (b) 2 (c) 4 (d) 6

Q.27 If a mercury drop is divided into 8 equal parts, its total energy
(a) remains same (b) becomes twice (c) becomes half (d) becomes 4 times

Q.28 There is a small bubble at one end and bigger bubble at other end of a rod. What will happen?
(a) smaller will grow until they collapse
(b) bigger will grow until they collapse
(c) remain in equilibrium (d) none of the above



Q.29 If a liquid does not wet glass, its angle of contact is
(a) zero (b) acute (c) obtuse (d) right angle

Q.30 In a capillary tube experiment, a vertical, 30 cm long capillary tube is dipped in water. The water rises upto a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes
(a) 10 cm (b) 20 cm (c) 30 cm (d) zero

Q.31 Two capillaries of lengths L and 2L and of radii R and 2R respectively are connected in series. The net rate of flow of fluid through them will (Given, rate of the flow through single capillary, $X = \pi PR^4/ 8\eta L$) be
(a) $\frac{8}{9}X$ (b) $\frac{9}{8}X$ (c) $\frac{5}{7}X$ (d) $\frac{7}{5}X$

Q.32 The rate of flow of water in a capillary tube of length l and radius r is V. The rate of flow in another capillary tube of length $2l$ and radius 2r for same pressure difference would be
(a) 16 V (b) 9 V (c) 8 V (d) 2 V

Q.33 The water flows from a tap of diameter 1.25 cm with a rate of $5 \times 10^{-5}\ m^3\ s^{-1}$. The density and coefficient of viscosity of water are $10^3\ kg\ m^{-3}$ and 10^{-3} Pas, respectively. The flow of water is
(a) steady with Reynold's number 5100 (b) turbulent with Reynold's number 5100
(c) steady with Reynold's number 3900 (d) turbulent with Reynold's number 3900

Answers

1	d	2	a	3	d	4	a	5	b
6	a	7	c	8	d	9	b	10	b
11	d	12	c	13	a	14	a	15	d
16	a	17	a	18	b	19	d	20	a
21	b	22	c	23	c	24	b	25	d

26	b	27	b	28	b	29	b	30	c
31	a	32	c	33	b				

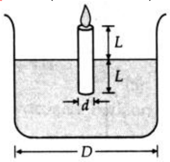
AIIMS – Objective Assignment – IV

Q.1 A body is just floating in a liquid (their densities are equal). If the body is slightly pressed down and released, it will
(a) start oscillating
(b) sink to the bottom
(c) come back to the same position immediately
(d) come back to the same position slowly

Q.2 When a large bubble rises from the bottom of a lake to the surface, its radius doubles. The atmospheric pressure is equal to that of a column of water of height H. The depth of the lake is
(a) H (b) 2 H (c) 7 H (d) 8 H

Q.3 By sucking through a straw, a student can reduce the pressure in his lungs to 750 mm of mercury (density = 13.6 g cm^{-3}). Using straw, he can drink water from a glass upto a maximum depth of
(a) 10 cm (b) 75 cm (c) 13.6 cm (d) 1.36 cm

Q.4 A candle of diameter d is floating on a liquid in a cylindrical container of diameter D (D > > d) as shown in figure. If it is burning at the rate of 2 cm h^{-1}, then the top of candle will



(a) remain at the same height
(b) fall at the rate of 1 cm h^{-1}
(c) fall at the rate of 2 cm h^{-1}
(d) go up at the rate of 1 cm h^{-1}

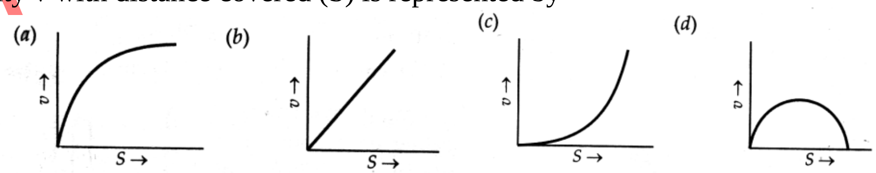
Q.5 A small ball of density ρ is dropped from a height h into a liquid of density σ (σ > ρ). Neglecting damping forces, the maximum depth to which the body sinks is
(a) $\frac{h\sigma}{\rho-\sigma}$ (b) $\frac{h\rho}{\rho-\sigma}$ (c) $\frac{h(\sigma-\rho)}{\rho}$ (d) $\frac{h(\sigma-\rho)}{\sigma}$

Q.6 A vertical U–tube contains mercury in both its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured into the other arm until the upper surfaces of oil and glycerine are at the same level. The length of the oil column is (density of mercury = 1.3 g cm^{-3})
(a) 8.5 cm (b) 9.6 cm (c) 10.7 cm (d) 11. 8 cm

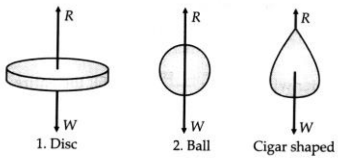
Q.7 Under a constant pressure head, the rate of flow of orderly volume flow of liquid through a capillary tube is V. If the length of the capillary is doubled and the diameter of the bore is halved, the rate of flow would become
(a) V/4 (b) 16 V (c) V/8 (d) V/32

Q.8 A sphere of mass M and radius R is falling in a viscous fluid. The terminal velocity attained by the falling object will be proportional to
(a) R^2 (b) R (c) 1/ R (d) $1/R^2$

Q.9 A lead shot of 1 mm diameter falls through a long column of glycerine. The variation of its velocity v with distance covered (S) is represented by



Q.10 When a body falls in air, the resistance of air depends on a greater extent on the shape of the body. Three different shapes are given. Identify the combination of air resistance, which truly represents the physical situation. (The cross sectional areas are the same).



(a) 1 < 2 < 3 (b) 2 < 3 < 1 (c) 3 < 2 < 1 (d) 3 < 1 < 2

Q.11 Scent sprayer is based on

(a) Charles' law (b) Boyle's law (c) Archimedes' principle (d) Bernouli's theorem

Q.12 Bernouli's principle is based on the law of conservation of

(a) energy (b) mass (c) linear momentum (d) angular momentum

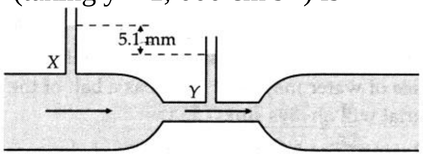
Q.13 In old age arteries carrying blood in the human body become narrow resulting in an increase in the blood pressure. This follows from

(a) Pascal's law (b) Stoke's law (c) Bernoulli's principle (d) Pascal's law

Q.14 In incompressible fluid flows steadily through a cylindrical pipe which has radius 2 R at point A and R at a point B further along the flow direction. If the velocity at A is v, then that at b is

(a) v/2 (b) v (c) 2 v (d) 4v

Q.15 Figure shows a venturimeter, through which water is flowing. The speed of water at X is 2 cm s^{-1}. The speed of water at Y (taking y = 1, 000 cm s^{-2}) is



(a) 23 cm s^{-1} (b) 32 cm s^{-1} (c) 101 cm s^{-1} (d) 1,024 cm s^{-1}

Q.16 The property utilized in the manufacture of lead shots is

(a) specific weight of liquid lead (b) specific gravity of liquid lead

(c) compressibility of liquid lead (d) surface tension of liquid lead

Q.17 The rain drops are in spherical shape due to

(a) viscosity (b) surface tension (c) thrust on drop (d) residual pressure

Q.18 Work of 3.0×10^{-4} J is required to be done in increasing the size of a soap film from 10cm × 6cm to 10 cm × 11cm. The surface tension of the soap film is

(a) 5×10^{-2} Nm^{-1} (b) 3×10^{-2} Nm^{-1} (c) 1.5×10^{-2} Nm^{-1} (d) 1.2×10^{-2} Nm^{-1}

Q.19 Two small drops of mercury, each of radius R coalesce to form a single large drop. The ratio of the total surface energies before and after the change is

(a) $1 : 2^{1/3}$ (b) $2^{1/3} : 1$ (c) 2 : 1 (d) 1 : 2

Q.20 The potential energy possessed by a soap bubble, having surface tension equal to 0.04 Nm^{-1} of diameter 1 cm, is

(a) $2\pi \times 10^{-6}$ J (b) $4\pi \times 10^{-6}$ J (c) $6\pi \times 10^{-6}$ J (d) $8\pi \times 10^{-6}$ J

Q.21 The radius of a soap bubble is r and the surface tension of soap solution is T. Keeping the temperature constant, the extra energy needed to double radius of the soap bubble by blowing is

(a) $32\pi r^2T$ (b) $24\pi r^2T$ (c) $16\pi r^2T$ (d) $8\pi r^2T$

Q.22 Extra pressure inside a soap bubble of radius (r) is proportional to

(a) r (b) 1/r (c) r^2 (d) $1/r^2$

Q.23 The surface tension of soap solution is 25×10^{-3} Nm^{-1}. The excess pressure inside a soap bubble of diameter 1 cm is

(a) 10 Pa (b) 20 Pa (c) 5 Pa (d) none of these

Q.24 A spherical drop of water has 1 mm radius. If the surface tension of water is 70×10^{-3} Nm^{-1}, then difference of pressure between inside and outside of the spherical drop is

(a) 35 Nm^{-2} (b) 70 Nm^{-2} (c) 140 Nm^{-2} (d) zero

Q.25 At critical temperature, the surface tension of a liquid is

(a) zero (b) infinity

(c) same as that any other temperature (d) cannot be determined

Q.26 The surface tension of liquid decreases with a rise in

(a) temperature of the liquid (b) viscosity of the liquid

(c) diameter of container (d) thickness of container

Assertions and Reasons

Mechanical Properties Of Solids

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false (d) If both assertion and reason are false

Q.27 **Assertion:** The size of a hydrogen balloon increases as it rises in air.
Reason: The material of the balloon can be easily stretched.

Q.28 **Assertion:** A hydrogen filled balloon stops rising after it has attained a certain height in the sky.
Reason: The atmospheric pressure decreases with height and becomes zero when maximum height is attained.

Q.29 **Assertion:** In taking into account the fact that any object, which floats must have an average density less than that of water, during World War I, a number of cargo vessels were made of concrete.
Reason: Concrete cargo vessels were filled with air.

Q.30 **Assertion:** The machine parts are jammed in winter.
Reason: The viscosity of the lubricants used in the machines increase at low temperature.

Q.31 **Assertion:** For Reynold's number $R_e > 2000$, the flow of fluid is turbulent.
Reason: Inertial forces are dominant compared to the viscous forces at such high Reynold's numbers.

Q.32 **Assertion:** The shape of an automobile is so designed that its front resembles the streamline pattern of the fluid through which it moves.
Reason: The resistance offered by the fluid is maximum.

Q.33 **Assertion:** A thin stainless steel needle can lay floating on a still water surface.
Reason: Any object floats, when the buoyancy force balances the weight of the object.

Q.34 **Assertion:** A needle placed carefully on the surface of water may float, whereas a ball of the same material will always sink.
Reason: The buoyancy of an object depends both on the material and shape of the object.

Q.35 **Assertion:** Smaller drops of liquid resist deforming forces better than the larger drops.
Reason: Excess pressure inside a drop is directly proportional to its surface area.

Q.36 **Assertion:** Bubble of soap is larger than that of water.
Reason: Surface tension of soap bubble is less than that of water.

Answers

1	b	2	c	3	c	4	b	5	b
6	b	7	d	8	a	9	a	10	c
11	c	12	a	13	c	14	d	15	b
16	d	17	b	18	b	19	b	20	d
21	b	22	b	23	b	24	c	25	a
26	a	27	b	28	c	29	a	30	a
31	a	32	d	33	c	34	c		
35	c	36	a						

THERMAL PROPERTIES OF MATTER

Heat

Heat is a form of energy which produces in us the sensation of hotness or coldness. For example, if we touch a piece of ice, heat flows from our body towards ice and we feel cold. Similarly, when we stand near a fire, heat from the fire flows towards our body and we feel hot.

(a) **Caloric theory of heat:** According to this theory, *heat is an invisible, weightless and odourless fluid called* **caloric.**

(b) **Dynamic theory of heat:** According to this theory, all substances (solids, liquids and gases) are made of molecules. These molecules are in a state of continuous random motion.

Depending on temperature and nature of the substance, the molecules may possess three types of motion:

(i) *Translatory motion:* That is, the motion in a straight line which is common in gases.

(ii) *Vibratory motion:* That is, the to and fro motion of the molecules about their mean positions. This occurs usually at high temperature.

(iii) *Rotatory motion:* That is, the rotation of the molecules about their axes. This is common in liquids and gases.

When a body is heated, all these molecular motions become fast. The kinetic energy of a molecule due to each type of motion increases. So we can regard *heat as an energy of molecular motion which is equal to the sum of total kinetic energy possessed by the molecules of a body by virtue of their translational, vibrational and rotational motions.*

CGS unit of heat

The CGS unit of heat is **calorie** (cal). *One calorie is defined as the heat energy required to raise the temperature of one gram of water through 1^oC (from 14.5 to 15.5^oC)*

SI unit of heat: Like all other forms of energy, the SI unit of heat is joule (J)

Joule's Mechanical Equivalent of Heat

From experiments, Joule established a relation between the work done and heat produced. He showed that whenever a given amount of work (W) is converted into heat, always the same amount of heat (Q) is produced, thus $W \propto Q$ or $W = JQ$ or $J = \frac{W}{Q}$

If $Q = 1$, then $J = W$

The proportionality constant J is called *Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.*

$J = 4.186 \text{ cal}^{-1} = 4.186 \times 10^7 \text{ erg cal}^{-1}$

NOTE: J is not a physical quantity. It just a conversion factor.

Temperature

Temperature is the degree of hotness or coldness of a body. Temperature may be defined as the thermal state of a body which decides the direction of flow of heat energy from one body to another when they are placed in thermal contact each other. The temperature of a body is the measure of the average kinetic energy of its molecules. When a body is heated, its molecules move faster. Their average K.E. increases. This increases the temperature of the body.

In thermodynamics the concept of temperature follows from the zeroth law of thermodynamics.

Thermometry: *The branch of physics that deal with the measurement of temperature is called thermometry.*

Thermometer: *Any device used to measure the temperature of a body is called a thermometer.*

Principle of a thermometer: *A thermometer makes use of some measurable property (called thermometric property)* of a substance *which changes linearly with temperature.*

The thermometric properties of different substances and the corresponding thermometers are as follows:

(i) Length of a liquid column in a capillary (Mercury thermometer).

(ii) Pressure of a gas at constant volume (Constant volume gas thermometer)

(iii) Volume of a gas at constant pressure (Constant pressure gas thermometer)

(iv) Electrical resistance of a metal wire (Platinum resistance thermometer).

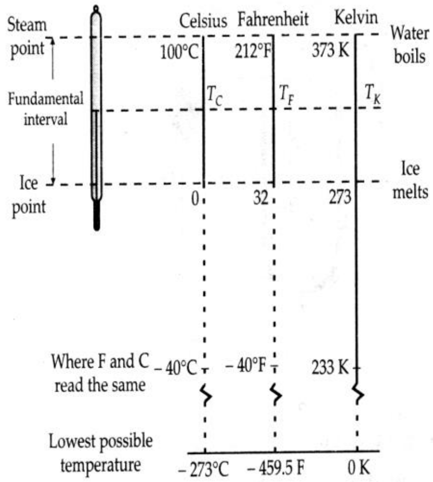
(v) Thermoelectrical e.m.f. (Thermoelectric or thermocouple thermometer)

(vi) Radiated power (Pyrometers).

Fixed points on a temperature scale: To construct a temperature scale, two fixed points (two well–defined thermodynamic states) are chosen and are assigned two arbitrary numbers for their temperature. One number fixes the origin of the scale and the other fixes the size of the unit of the scale. The temperature at which pure ice melts at standard pressure (ice–liquid water equilibrium state) is usually chosen as the *lower fixed point.* The temperature at which pure water boils at atmospheric pressure (liquid water–vapour) equilibrium state) is chosen as the *upper fixed point.*

Thermometric Scales

The commonly used temperature scales are as follows:



(i) ***The Celsius Scale:*** On this scale, lower fixed point (ice point) is taken as 0°C and the upper fixed point (steam point) as 100°C. The interval between the two fixed points is divided into hundred equal parts (hence the name centigrade) and each part is called 1°C.

(ii) ***The Fahrenheit scale:*** On this scale, the lower fixed point is taken as 32°F and the upper fixed point as 212°F. The interval between them is divided into 180 equal parts and each part represents 1°F.

(iii) ***The Reaumer Scale:*** On this scale, the lower fixed point is taken as 0°R and the upper fixed point as 80°R. The interval between them is divided into 80 equal parts and each part represents 1°R.

(iii) ***The Kelvin Scale:*** On this scale, the lower fixed point is taken as 273.15 K and the upper fixed point as 373.15 K. The interval between the two fixed points is divided into 100 equal parts. The SI unit of temperature is Kelvin (K).

Conversion of temperature from one scale to another

To convert the temperature from one scale to another, the following relation is used:

$$\frac{Temperature\ on\ one\ scale - Lower\ \ fixed\ \ point}{Upper\ fixed\ \ point - Lower\ fixed\ \ point}$$

$$= \frac{Temperature\ on\ other\ scale - Lower\ \ fixed\ \ point}{Upper\ fixed\ \ point - Lower\ fixed\ \ point}$$

If the temperature of body is measured as T_C, T_F, T_R and T_K on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100-0} = \frac{T_F - 32}{212-32} = \frac{T_R - 0}{80-0} = \frac{T_K - 273.15}{373.15-273.15}$$

or

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273.15}{100}$$

Triple point of water

The triple point of water is the state at which the three phase of water namely ice, liquid water and water vapour are equally stable and co–exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column. Thus for water,

P_{tr} = 0.46 cm of Hg, T_{tr} = 273.16 K or 0.01°C

Subjective Assignment – I

Q.1 A faulty thermometer has its fixed points marked as 5° and 95°. Temperature of a body as measured by the faulty thermometer is 59°. Find the correct temperature of the body on Celsius scale.

Q.2 A thermometer has wrong calibration. It reads the melting point of ice – 10°C. It reads 60°C in place of 50°. Calculate the temperature of boiling point of water on this scale.

Q.3 At what temperature, do the readings of Celcius and Fahrenheit scales coincide?

Q.4 A constant volume gas thermometer using helium records a pressure of 20.0 kPa at triple–point of water, and pressure of 14.3 kPa at temp. of 'dry ice' (solid CO_2). What is temperature of 'dry ice'?

Q.5 A constant volume thermometer using helium gas records a pressure of 1.75×10^4 Pa at normal freezing point of water, and a pressure of 2.39×10^4 Pa at normal boiling point of water. Obtain from these observations the temperature of absolute zero on the Celsius scale.

Q.6 A plantinum wire has resistance of 10Ω at 0°C & 20Ω at 273°C. Find value of coefficient of resistance.

Q.7 A faulty thermometer reads 5°C in melting ice and 99°C in steam. Find the correct temperature in °F when the faulty thermometer reads 52°C.

Q.8 When a thermometer is taken from the melting ice to warm liquid, the mercury level rises to $2/5^{th}$ of distance between the lower and the upper fixed points. Find the temperature of liquid in °C and K.

Q.9 At what temperature is the Fahrenheit scale reading equal to twice of Celsium scale reading?

Q.10 A constant volume gas thermometer using sulphur records a pressure of 2×10^4 Pa at the triple point of water and 2.87×10^4 Pa at temperature of melting sulphur. Calculate the melting point of sulphur.

Q.11 The resistance of a resistance thermometer at 19°C is 3.50Ω and at 99°C is 3.66Ω. At what temperature will its resistance be 4.30Ω?

Answers

1.	60°C	2.	130°C	3.	– 40°	4.	195.30 K
5.	– 273	6.	1/273 $°C^{-1}$	7.	122°F		
8.	40°C, 313.15 K	9.	160°C or 320°F	10.	190.4 K	11.	419°c

Thermal Expansion

Almost all solids, liquids and gases expand on heating. T*he increase in size of a body when it is heated is called thermal expansion.*

Different types of thermal expansion

(i) ***Linear expansion:*** *It is the increase in the length of a metal rod on heating.*

(ii) ***Superficial expansion:*** *It is the increase in the surface area of a metal sheet on heating.*

(iii) ***Cubical expansion:*** *It is the increase in the volume of block on heating.*

Cause of thermal expansion: All solids consist of atoms and molecules. At any given temperature, these atoms and molecules are held at equilibrium distance by forces of attraction. When a solid is heated, the amplitude of vibration of its atoms and molecules increases. The average interatomic separation increases. This results in the thermal expansion of the solid.

Coefficient of Linear Expansion: Suppose a solid rod of length l is heated through a temperature ΔT and its final (increased) length is l'. It is found from experiments that

(i) Increase in length ∝ rise in temperature i.e., $l' - l \propto \Delta T$

(ii) Increase in length ∝ original length i.e., $l' - l \propto l$

Combining the above two factors, we get $l' - l \propto l\,\Delta T$ *or* $l' - l = \alpha\, l\, \Delta T$

The proportionality constant α is called ***coefficient of linear expansion.*** Its value depends on the nature of the solid. Clearly, $l' - l \propto \alpha\, \Delta T$ and $\alpha = \dfrac{l' - l}{l\,\Delta T} = \dfrac{\Delta l}{l\,\Delta T}$ or

$$\alpha = \frac{\textit{Increase in length}}{\textit{Original length} \times \textit{Rise in temperature}}$$

Hence *the coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit original length per degree rise in its temperature.*

The unit of α is $^{o}C^{-1}$ or K^{-1}

Coefficient of Superficial Expansion

Suppose a metal sheet of initial surface area S is heated through temperature ΔT and its final surface area becomes S'.

Then $S' - S \propto \Delta T$ and $S - S \propto S$

∴ $S' - S \propto S\,\Delta T$ or $S' - S = \beta\, S\, \Delta T$

The proportionality constant β is called *coefficient of superficial expansion* and its value depends on the nature of the material. Clearly,

$S' = S[1 + \beta\,\Delta T]$ and $\beta = \dfrac{S' - S}{S\,\Delta T} = \dfrac{\Delta S}{S\,\Delta T}$

or $$\beta = \frac{\textit{Increase in surface area}}{\textit{Original surface area} \times \textit{Rise in temperaure}}$$

Hence the *coefficient of superficial expansion of a metal sheet is defined as the increase in its surface area per unit original surface area per degree rise in its temperature.*

The unit of β is $^{o}C^{-1}$ or K^{-1}.

Coefficient of cubical expansion

Suppose a solid block of initial volume V is heated through a temperature ΔT and its final volume is V'.

Then $V' - V \propto \Delta T$

and $V' - V \propto V$ or $V' - V = \gamma\, V\, \Delta T$

The proportionality constant γ is called the **coefficient of cubical expansion** which depends on the nature of the material of the solid. Clearly,

$V' = V[1 + \gamma\,\Delta T]$ and $\gamma = \dfrac{V' - V}{V\,\Delta T} = \dfrac{\Delta V}{V\,\Delta T}$

or $$\gamma = \frac{\textit{Increase in volume}}{\textit{Original volume} \times \textit{Rise in temperature}}$$

Hence *the coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.*

The unit of γ is $^oC^{-1}$ or K^{-1}.

Coefficient of cubical expansion of an ideal gas at constant pressure: For an ideal gas,

$PV = nRT$ ... (i)

At constant pressure,

$P \Delta V = nR \Delta T$ .... (ii) [$\because$ n and R are constants]

Dividing (ii) by (i), we get

$$\frac{\Delta V}{V}=\frac{\Delta T}{T} \text{ or } \frac{\Delta V}{V\ \Delta T}=\frac{1}{T} \text{ or } \gamma=\frac{1}{T}$$

Hence for ideal gas, the coefficient of volume expansion decreases with the increase in temperature.

Relation between α, β and γ

Consider a cube of side l. Its original volume is $V = l^3$

Suppose the cube is heated so that its temperature increases by ΔT. Its each side will become

$l' = l + \Delta l = l + \alpha l \Delta T = l(1 + \alpha \Delta T)$

The new volume of the cube will be $V' = l'^3 = l^3 (1 + \alpha \Delta T)^3$

$= V(1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$

As α is small, so the terms containing α^2 and α^3 can be neglected. Then

$V' = V(1 + 3\alpha \Delta T)$

By the definition of the coefficient of cubical expansion,

$$\gamma=\frac{\Delta V}{V \Delta T}=\frac{V'-V}{V \Delta T}=\frac{V(1+3\alpha\Delta T)-V}{V\ \Delta T}=3\alpha$$

Similarly, it can be proved that $\beta = 2\alpha$ Hence $\frac{\alpha}{1}=\frac{\beta}{2}=\frac{\gamma}{3}$.

NOTE:

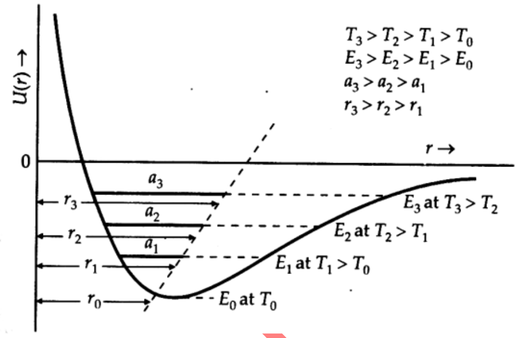
- The three coefficients of expansion α, β and γ are not constant for a given solid. Their values depend on the temperature range.
- For most of the solids, the value of α lies between 10^{-6} to 10^{-5} K^{-1} in the temperature range 0 to 100°C. The value of α is more for ionic solids than that for non–ionic solids.
- The coefficient of linear expansion of a solid rod is independent of the geometrical shape of its cross–section.
- The coefficient of volume expansion of solids and liquids is rather small, particularly very small for pyrex glass (1×10^{-5} K^{-1}) and invar (Fe–Ni alloy with $\gamma = 2 \times 10^{-6}$ K^{-1})
- For an ideal gas γ varies inversely with temp. i.e., $\gamma = 1/T$. At 0°C or 273 K, $\gamma = 1/273 = 3.7 \times 10^{-3}$ K^{-1}, which is much larger than that for solids and liquids.
- Water contracts on heating between 0°C and 4°C. This is called ***anomalous expansion of water.*** It has the minimum volume and hence the maximum density (1000 kg m^{-3}) at 4°C. Silver iodide also contracts on heating between 80°C to 140°C.

Molecular Explanation of Thermal Expansion

As shown in figure, the graph between the potential energy U (r) of two neighbouring atoms in a crystalline solid and their interatomic separation r is an *asymmetric parabola.* The potential energy curve is asymmetric about its minimum because the attractive part of the potential energy rises slowly compared to the repulsive part.

At the temperature $T_0 = 0$ K, the atoms remain at the equilibrium separation r_0 and their oscillation energy E_0 is minimum. As the temperature increases, the energy of the atoms increases and they start vibrating about their equilibrium positions with the interatomic separation oscillating between its minimum and maximum values: r_{min} and r_{max}. The average interatomic separation becomes

$$r = \frac{r_{min} + r_{max}}{2}$$



Clearly, as the temperature increases, the amplitude of vibration of the atoms increases. Due to the asymmetry of the P.E. curve, the equilibrium position shifts to the right on the curve (as shown by the dashed inclined line), *i.e., the average interatomic separation increases. It is thus in consequence of this increase in the average interatomic separation with temperature that a solid expands when heated.*

Practical Applications of Thermal Expansion

(i) *A small gap is left between the iron rails of railway tracks.*

(ii) *Space is left between the girders used for supporting bridges.*

(iii) *Iron ring to be put on rim of a cart wheel is always of slightly smaller diameter than that of wheel.*

(iv) *Clock pendulums are made of invar.*

(v) *A glass stopper jammed in the neck of a glass bottle can be removed by warming neck of the bottle.*

(vi) *Only platinum wire is used for fusing into glass.*

Expansion of a Liquid

When a liquid is heated, it s container also expands. The observed expansion of the liquid is called ***apparent expansion*** which is different from the ***real expansion*** of the liquid.

Coefficient of apparent expansion: *It is defined as the apparent increase in volume per unit original volume for l°C rise in temperature.* The coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Coefficient of real expansion: *It is defined as the real increase in volume per unit original volume for 1°C rise in temperature.* The coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

It can be proved that $\gamma_r = \gamma_a + \gamma_g$

where γ_g is coefficient of cubical expansion of glass (material) of the container.

Variation of Density with Temperature

When a given mass of a solid or a liquid is heated, its volume increases and hence density decreases. If V and V' are the volumes and ρ and ρ' are the densities of a given mass M at temperatures T and T + ΔT respectively, then

$$V' = V(1 + \gamma\,\Delta T)$$

or $$\frac{M}{\rho'} = \frac{M}{\rho}(1+\gamma\Delta T) \quad \text{or} \quad \rho' = \rho\,(1+\gamma\,\Delta T)^{-1}$$

Expanding by Binomial theorem and neglecting the terms containing higher powers of $\gamma\ \Delta T$, we get

$\rho' = \rho\ (1 - \gamma\ \Delta T)$

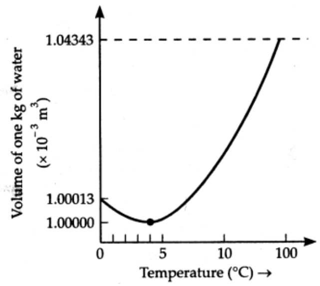
Clearly, the density of a solid or a liquid decreases with the increase in temperature.

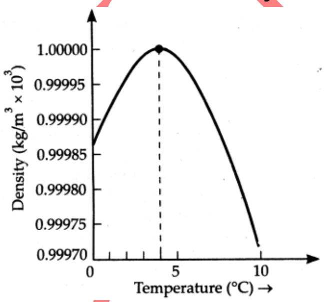
Anomalous Expansion of Water

When water at 0^oC is heated, its volume decreases and, therefore, its density increases, until its temperature reaches 4^oC. Above 4^oC, the volume increases, and therefore the density decreases. ***Thus water at 4^oC has the maximum density.***

Practical Importance of anomalous expansion of water

The anomalous expansion of water has a favourable effect on aquatic life. Since the density of water is maximum at 4^oC, water at the bottom of lakes remains at 4^oC, water at the bottom of lakes remains at 4^oC even if it freezes at top surface. This allows marine animals to remain alive and move freely near bottom.





Subjective Assignment – II

Q.1 Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66 m long be 3.63 cm at 10^oC, then at what temperature will the lines just touch? Coefficient of linear expansion of
steel = $11 \times 10^{-6}\ {}^oC^{-1}$

Q.2 A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27^oC. To what temperature should the ring be heated so as to fit the rim of the wheel? Coefficient of linear expansion = $12 \times 10^{-6}\ {}^oC^{-1}$

Q.3 A clock with an iron pendulum keeps correct time at 20^oC. How much will it lose or gain if temperature changes to 40^oC? Coefficient of cubical expansion of iron = $36 \times 10^{-6}\ {}^oC^{-1}$.

Q.4 A metal ball 0.1 min radius is heated from 273 to 348 K. Calculate the increase in surface area of the ball. Given coefficient of superficial expansion = $0.000034\ K^{-1}$.

Q.5 On heating a glass block of 10, 000 cm^3, from 25^oC to 40^oC, its volume increases by 4 cm^3. Calculate coefficient of linear expansion of glass.

Q.6 If the volume of a block of metal changes by 0.12% when it is heated through 20^oC, what is the coefficient of linear expansion of metal?

Q.7 How much the temperature of a brass rod should be increased so as to increase its length by 1%? Given that α for brass = 0.00002^oC^{-1}.

Q.8 A steel scale measures the length of a copper rod as 80 cm when both are at 20^oC, the calibration temperature of the scale. What would the scale read for the length of the rod when both are at 40^oC? Given α for steel = $1.1 \times 10^{-5}\ {}^oC^{-1}$ and α for copper $1.7 \times 10^{-5}\ {}^oC^{-1}$.

Q.9 A steel metre scale is to be ruled so that millimeter intervals are accurate to within about 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling? Given α for steel = $1.1 \times 10^{-5}\ {}^oC^{-1}$.

Q.10 A cylinder of diameter 1.0 cm at 30^oC is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30^oC. To what temperature must the plate be heated? For steel, $\alpha = 1.1 \times 10^{-5}\ {}^oC^{-1}$.

Q.11 What should be lengths of steel and copper rods at 0^oC that the length of steel rod is 5 cm longer than copper at all temperatures? Given α for copper = $1.7\times10^{-5}\ ^oC^{-1}$ and α for steel = $1.1\times 10^{-5}\ ^oC^{-1}$.

Q.12 A clock having a brass pendulum beats seconds at 30^oC. How many seconds will it lose or gain per day when temperature falls to 10^oC? Given α for brass = $1.9 \times 10^{-5}\ ^oC^{-1}$.

Q.13 A steel wire 2 mm in diameter is stretched between two clamps, when its temperature is 40^oC. Calculate the tension in the wire when its temperature falls to 30^oC. Given α for steel = $1.1 \times 10^{-6}\ ^oC^{-1}$ and y for steel = 21×10^{11} dyne cm^{-2}.

Q.14 Calculate the force required to prevent a steel wire of 1 mm^2 cross–section from contracting when it cools from 60^oC to 15^oC, if young's modulus for steel is 2×10^{11} Nm^{-2} and its coefficient of linear expansion is 0.000011^oC^{-1}.

Q.15 The volume of a metal sphere is increased by 1% of its original volume when it is heated from 320 K to 522 K. Calculate the coefficients of linear, superficial and cubical expansion of the metal.

Q.16 Density of mercury is 13.6g cm^{-3} at 0^oC and its coefficient of cubical expansion is $1.82 \times 10^{-4}\ ^oC^{-1}$. Calculate the density of mercury at 50^oC.

Q.17 Suppose that one early morning when the temperature is 10^oC, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 litre of gasoline, which is also at 10^oC. During the day, the temperature rises to 30^oC. How much gasoline will overflow? Given α for steel = $1.2 \times 10^{-5}\ ^oC^{-1}$ and γ for gasoline = $9.5 \times 10^{-4}\ ^oC^{-1}$.

Q.18 A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in this flask? Given α for glass = $9 \times 10^{-6}\ ^oC^{-1}$ and γ for mercury = $1.8 \times 10^{-4}\ ^oC^{-1}$.

Answers

1.	60^oC	2.	218^oC	3.	10.368 s		
4.	$3.206 \times 10^{-4}\ m^2$	5.	$8.89 \times 10^{-6}\ ^oC^{-1}$	6.	$2.0 \times 10^{-5}\ ^oC^{-1}$		
7.	500^oC	8.	80.0096	9.	4.5^oC	10.	57.3^oC
11.	9.17 cm,14.17 cm	12.	gain of 16.42 s	13.	7.26×10^6 dyne	14.	99 N
15.	$1.67 \times 10^{-5}\ ^oC^{-1}$, $3.34 \times 10^{-5}\ ^oC^{-1}$, $5 \times 10^{-5}\ ^oC^{-1}$			16.	13.48 g cm^{-3}		
17.	1.37 litre	18.	150 cm^3				

Specific Heat

The specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is needed to raise the temperature of M mass of a substance through ΔT, then specific heat is given by

$$c=\frac{\Delta Q}{M \times \Delta T}$$

The CGS unit of specific heat is cal $g^{-1}\ ^oC^{-1}$ and the SI unit is J $kg^{-1}\ K^{-1}$.

$\therefore$ The amount of heat required to raise the temperature of M mass of a substance through ΔT is

ΔQ = Mc QT.

Molar specific heat

The molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is required to raise the temperature of n moles of a substance through ΔT, then molar specific heat is given by

$$C = \frac{\Delta Q}{n\,\Delta T}$$

The CGS unit of molar specific heat is cal mol^{-1} $^{o}C^{-1}$ and SI units is J mol^{-1} K^{-1}.

Therefore, the amount of heat required to raise the temperature of n moles of a substance through ΔT is

$$\Delta Q = nC\ \Delta T$$

Heat capacity or thermal capacity

The heat capacity of a body is defined as the amount of heat required to raise its temperature through one degree. By definition, the amount of heat required to raise the temperature of unit mass of a body is equal to specific heat c. So heat required for m mass is m × c.

∴ Heat capacity = Mass × Specific heat or S = mc

The CGS unit of heat capacity is cal $^{o}C^{-1}$ and the SI unit is JK^{-1}.

Water equivalent

The water equivalent of a body is defined as the mass of water which requires the same amount heat as is required by the given body for the same rise of temperature.

Water equivalent = Mass × Specific Heat or w = mc

The CGS unit of water equivalent is g and the SI unit is kg.

Calorimetry

Principle of calorimetry or the law of mixtures

Whenever two bodies at different temperature are placed in contact with one another, heat flows from the body at higher temperature to the body at lower temperature till the two bodies acquire the same temperature. *The principle of calorimetry states that the heat gained by the cold body must be equal to the heat lost by the hot body, provided there is no exchange of heat with the surroundings.*

Heat gained = Heat lost

This principle is a consequence of the law of conservation of energy and useful for solving problems relating to calorimetry.

Effect of Pressure on Melting Point

The increase in pressure will help in its contraction. So we expect a decrease in the melting point of ice as the pressure on it is increased. *The melting point of those substances which expand on melting (e.g., paraffin wax, phosphorus, sulphur, etc.) increases with the increase in pressure while the melting point of those substances which contract on melting (e.g., ice, cast iron, bismuth etc.) decreases with increase in pressure.*

Effect of pressure on freezing point of ice: Regelation

When two pieces of ice are pressed against one another for few seconds and then released, they get frozen at the surface of contact. As the pressure is increased, the melting point of ice is lowered and ice melts. When pressure is released, the water so formed (at a temperature < $0^{o}C$) immediately freezes again. This phenomenon of refreezing is called regelation.

The phenomemon in which ice melts when pressure is increased and again freezes when pressure is removed is called ***regelation*** *(re = again; gelare = freeze)*

Practical application of regelation

1. By pressing snow in our hand, we can transform it into a snow–ball. When snow is pressed, its crystals melt. As the pressure is released, water refreezes forming as snow–ball.
2. When the wheels of cart pass over snow, ice melts due to increase in pressure exerted by the wheels. When pressure is released, water so formed refreezes on the wheels. That is why wheels are covered with ice.
3. Skating is possible due to the formation of water layer below the skates. Water is formed due to the increase of pressure and it acts as lubricant.
4. The ice of a glacier, pressed against the sides of its valley melts, and in this way adopts itself to the shape of the valley.

Effect of pressure on the boiling point of a liquid

The boiling point of a liquid increases with the increase in pressure

Practical Application

1. Cooking is difficult at mountains. The atmospheric pressure at mountains is much lower than that at plains, so water starts boiling at a temperature much lower than 100°C. Hence cooking is difficult.
2. The pressure inside a pressure cooker is increased much above the atmospheric pressure by not allowing the steam to escape. This increases the boiling point. Hence the vegetables are cooked inside a pressure cooker in a shorter time.

Latent Heat

The amount of heat required to change the state of unit mass of a substance at constant temperature and pressure is called latent heat of the substance.

If m mass of a substance undergoes a change from one state to another, then the amount of heat required for the process is Q = mL

where L is the latent heat of the substance and is a characteristic of the substance. Its value also depends on the pressure. Clearly, $L = \frac{Q}{m}$

∴ SI unit of latent heat = J kg^{-1} or CGS unit of latent heat = cal g^{-1}

Latent heat of fusion: *The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion or latent heat of melting. It is denoted by L_f.*

Latent heat of vaporisation: *The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point. It is denoted by L_v.*

Subjective Assignment – III

Q.1 A thermally isolated vessel contains 100g of water at 0°C when air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed, if no water is left in the vessel. Latent heat of vaporization of water at 0°C = 2.10×10^6 J kg^{-1} and latent heat of fusion of ice = 3.36×10^5 J kg^{-1}.

Q.2 When 0.15 kg of ice of 0°C is mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice (c_{water} = 4186 J kg^{-1} K^{-1})

Q.3 Calculate the heat required to convert 3 kg of ice at – 12°C kept in a calorimeter to steam at 100°C at atmospheric pressure. Given:
specific heat capacity of ice = 2100 J kg K^{-1}, specific heat capacity of water = 4186 J kg^{-1} K^{-1}
latent heat of fusion of ice = 3.35×10^5 J Kg^{-1} and latent heat of steam = 2.256×10^6 J kg^{-1}

Q.4 When 10 g of coal is burnt, it raises the temperature of 2 litres of water from 20°C to 55°C. Calculate the heat of combustion of fuel.

Q.5 A normal diet furnishes 2000 kcal to a 60 kg person in a day. If this energy was used to heat the person with no losses to the surroundings, how much would the person's temperature increases? The specific heat of the human body = 0.83 cal g^{-1} $°C^{-1}$.

Q.6 0.75 gram of petroleum was burnt in a bomb calorimeter which contains 2kg of water and has a water equivalent 500 gram. The rise in temperature was 3^oC. Determine the calorific value of petroleum.

Q.7 The heat of combustion of ethane gas is 373 kcal per mole. Assuming that 60% of the heat is useful, how many litres of ethane measured at S.T.P. must be burnt to convert 50 kg of water at 10^oC to steam at 100^oC? One mole of a gas occupies 22.4 litre at S.T.P.

Q.8 A refrigerator converts 50 gram of water at 15^oC into ice at 0^oC in one hour. Calculate the quantity of heat removed per minute. Take specific heat of water = 1 cal g^{-1} $^oC^{-1}$ and latent heat of ice = 80 cal g^{-1}.

Q.9 How many grams of ice at – 14^oC are needed to cool 200 gram of water from 25^oC to 10^oC? Take specific heat of ice = 0.5 cal $g^{-1o}C^{-1}$ and latent heat of ice = 80 cal g^{-1}.

Q.10 An electric heater of power 100 W raises the temperature of 5 kg of a liquid from 25^oC to 31^oC in 2 minutes. Calculate the specific heat of the liquid.

Q.11 A piece of iron of mass 100g is kept inside a furnace from a long time and then put in a calorimeter of water equivalent 10g containing 240g of water at 20^oC. The mixture attains an equilibrium temperature of 60^oC. Find the temperature of ice. Given specific heat of iron = 470 J kg^{-1} $^oC^{-1}$.

Q.12 When 0.45 kg of ice of 0^oC mixed with 0.9 kg of water at 55^oC in a container, the resulting temperature is 10^oC. Calculate the heat of fusion of ice. (c_{water} = 4186 J kg^{-1} K^{-1})

Q.13 Calculate the heat required to convert 0.6 kg of ice at – 20^oC, kept in a calorimeter to steam at 100^oC at atmospheric pressure. Given the specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of water is 4186 J kg^{-1} K^{-1}, latent heat of ice = 3.35×10^5 J kg^{-1}, latent heat of seam = 2.256×10^6 J kg^{-1}.

Answers

1.	86.2 g	2.	3.34×10^5 J kg^{-1}	3.	9.1×10^6 J
4.	7000 cal g^{-1}	5.	40.16^oC	6.	10^4 cal g^{-1}
7.	3131.5 litres	8.	79.2 cal min^{-1}	9.	31 g
10.	400 J kg^{-1} $^oC^{-1}$	11.	953.6^oC	12.	334400 J kg^{-1}
13.	1.8×10^8 J				

Modes of Transfer of Heat : Introduction

Heat can be transferred from one place to another by three different methods. These are (i) conduction, (ii) convection and (iii) radiation.

Thermal Conduction

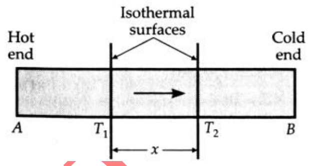
It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.

When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

Steady State and Temperature Gradient

In the process of conduction, each cross–section of the rod receives heat from the adjacent cross–section of the hotter side. A part of this heat is absorbed by the cross–section itself whose temperature increases, another part is lost into the atmosphere by convection and radiation from the sides of cross–section and the rest is conducted to the next cross–section. *In this state the temperature of every cross–section of the rod goes on increasing with time.* The rod is said to be in the **variable state** of heat conduction.

Suppose the sides of the rod are covered with some insulating material so that no heat is lost from the sides to the surroundings. After some time, a steady state is reached when the temperature of every cross–section of the rod becomes constant. In this state, no heat is absorbed by the rod. *This state of the rod when temperature of every cross–section of the rod becomes constant and there is no further adsorption of heat in any part is called* ***steady state.*** During steady state,



(i) the temperatures of two different parts of rod are different, but the temperature of each part remains constant.

(ii) every transverse section of the rod is an *isothermal surface.*

(iii) the temperature decreases as we move away from the hot end

(iv) the quantity of heat flowing per second through every cross–section is constant.

The rate of change of temperature with distance in the direction of flow of heat is called ***temperature gradient.*** If T_1 and T_2 are the temperatures of two isothermal surfaces separated by distance x, then

$$\text{Temperature gradient} = \frac{T_1 - T_2}{x}$$

Thermal Conductivity

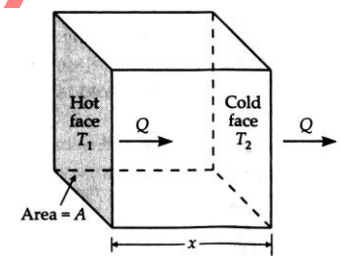
As shown in figure, consider a block of a material of cross–sectional area A and thickness x. Suppose its opposite faces are at temperatures T_1 and T_2, with $T_1 > T_2$. It is found that the amount of heat Q that flows from hot to cold face during the steady state.

(i) is directly proportional to the cross–sectional area A,

(ii) is directly proportional to the temperature difference ($T_1 - T_2$) between the opposite faces,

(iii) is inversely proportional to thickness x of the block, and

(iv) depends on the nature of the materials of the block.



$$\therefore \quad Q \propto \frac{A(T_1 - T_2)t}{x} \quad \text{or} \quad Q = \frac{KA(T_1 - T_2)t}{x}$$

The proportionality constant K is called *coefficient of thermal conductivity* of the given materials. Its value depends on the nature of the material. If A = 1, $T_1 - T_2 = 1$, t = 1, x = 1, then Q = K

Hence, *the coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.*

If A be the area of the cross–section at a place dx be a small thickness along the direction of heat flow and dT be the temperature difference across this thickness dx, then rate of flow of heat or heat current H will be

$$H = \frac{dQ}{dt} = -KA\frac{dT}{dx}$$

The quantity dT/dx is called the temperature gradient. The negative sign indicates that dT/dx is negative in the direction of flow of heat i.e., temperature decreases along the positive x–direction. Thus the negative sign in the above equation ensures that K is positive.

Units and dimensions of K

The numerical value of K is $K = \frac{Q.x}{A(T_1 - T_2)t}$

$$\therefore \quad \text{SI unit of K} = \frac{J.m}{m^2.K.s} = \text{J s}^{-1}\,\text{m}^{-1}\,\text{K}^{-1} \text{ or } \text{W m}^{-1}\,\text{K}^{-1} \quad \text{or} \quad \text{CGS unit of K} = \text{cal s}^{-1}\,\text{cm}^{-1}\,{}^{o}\text{C}^{-1}$$

Dimensions of K = $\frac{[ML^2T^{-2}].[L]}{[L^2].[K][T]} = [MLT^{-3}K^{-1}]$

Heat Current and Thermal Resistance

Heat flows in a conductor due to temperature difference between its two points. *The flow of heat unit time is called heat current*, denoted by H. Thus $H = \frac{Q}{t}$

Its SI unit is Js^{-1} or watt (W)

From Ohm's law, electric resistance is given by $R = \frac{V}{I}$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, *the ratio of the temperature difference between the ends of a conductors to the heat current through it is called thermal resistance*, denoted by R_H. Thus

$$R_H = \frac{\Delta T}{H} \qquad \text{As} \qquad Q = KA\frac{\Delta T}{\Delta x}t$$

$$\therefore \qquad H = \frac{Q}{t} = KA\frac{\Delta T}{\Delta x} \qquad \text{and} \qquad R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$$

Hence *greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material.*

Unit and dimensions of R_H

As $R_H = \Delta T/H$, so

SI unit of $R_H = \frac{K}{J\,s^{-1}} = \frac{K}{W} = KW^{-1}$

Dimensions of R_H = $\frac{[K]}{[ML^2T^{-1}].[T^{-1}]} = [M^{-1}L^{-2}T^3K]$

Application of Conductivity in Daily Life

(i) In winter, a metallic handle appears colder than the wooden door.

(ii) Cooking utensils are provided with wooden handles: Wood is a bad conductor of heat. A wooden handle does not allow heat to be conducted from the hot utensil to the hand.

(iii) A new quilt is warmer than an old quilt: A new quilt contains more air in its pores as compared to the old quilt. As air is bad conductor of heat, it does not allow heat to be conducted away from our body to the surroundings and we feel warmer in it.

(iv) Birds swell their feathers in winter: By doing so the birds enclose air between their feathers and the body.

(v) Ice is packed in saw dust: Saw dust and air trapped inside it are poor conductors of heat.

(vi) Eskimos make double wall houses of the blocks of ice: Air trapped between the two walls of ice does not allow the heat from the surrounds to the ice which may otherwise melt the ice.

(vii) When a wire gauge is placed over the burning Bunsen's burner, the flame does not go beyond the gauge: Copper is a very good conduct or of heat. The copper gauge absorbs most of the heat.

(viii) A refrigerator is provided with insulated walls: Generally, fibre glass is used as an insulating material. This is done to minimize the changes of heat flowing into the refrigerator from outside.

(ix)

Subjective Assignment – IV

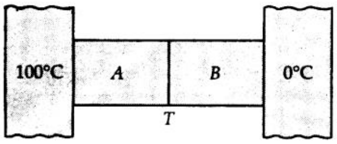
Q.1 Calculate the rate of loss of heat through a glass window of area 1000 cm^2 and thickness 0.4 cm when temperature inside is 37°C and outside is –5°C.

Q.2 Steam at 100°C is passed into a copper cylinder 10 mm thick and a 200 cm^2 area. Water at 100°C collects at the rate of 150g min^{-1}. Find the temperature of the outer surface, if the conductivity of copper is 0.8 cal s^{-1} cm^{-1} $°C^{-1}$ and the latent heat of steam is 540 cal g^{-1}.

Q.3 A metal rod of length 20 cm and diameter 2 cm is covered with non–conducting substance. One of its ends is maintained at 100°C, while the other end is put in ice at 0°C. It is found that 25g of ice melts in 5 minutes. Calculate the coefficient of thermal conductivity of the metal. Given latent heat of ice = 80 cal g^{-1}.

Q.4 A layer of ice 2 cm thick is formed on a pond. The temperature of air is – 20°C. Calculate how long it will take for the thickness of ice to increase by 1 mm. Density of ice = 1g cm^{-3}. Latent heat of ice = 80 cal g^{-1}. Conductivity of ice = 0.008 cal s^{-1} cm^{-1} $°C^{-1}$.

Q.5 Two metal cubes A and B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The co–efficient of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C, respectively.

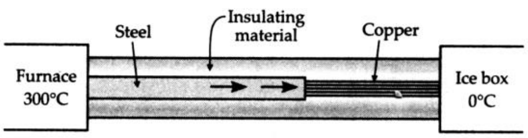


After steady state is reached, what will be the temperature T of the interface?

Q.6 Three bars of equal lengths and equal area of cross–section are connected in series. Their thermal conductivities are in the ratio of 2 : 4 : 3. If the open ends of the first and the last bars are at temperatures 200°C and 18°C respectively in the steady state, calculate the temperatures of both the junctions.

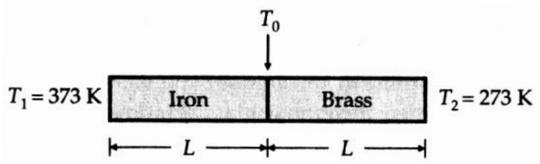
Q.7 One end of a copper rod of uniform cross–section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melted by equal to that of the steam produced in the same interval of time? Assume that the whole system is insulated from the surrounding. Latent heat of fusion of ice = 80 cal g^{-1}. Latent heat of vaporization of water = 540 cal g^{-1}.

Q.8 What is the temperature of the steel–copper junction in the steady state of the system shown in figure? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace =300°C, temperature of the other end = 0°C. The area of cross–section of the steel rod is twice that of the copper rod. Thermal conductivity of steel = 50.2 Js^{-1} m^{-2} $°C^{-1}$ and of copper = 385 $Js^{-1}m^{-1}$ $°C^{-1}$.



Q.9 An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of + 20°C inside it, when the outside temperature is – 10°C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 Watt/m/°C respectively.

Q.10 An iron bar (L_1 = 0.1 m, A_1 = 0.02 m^2, K_1 = 79 W m^{-1} K^{-1}) and a brass bar (L_2 = 0.1 m, A_2 = 0.02 m^2, K_2 = 109 W m^{-1} K^{-1}) are soldered end to end as shown in figure. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar



Q.11 Heat is flowing through a rod of length 25.0 cm having cross–sectional area 8.80 cm^2. The coefficient of thermal conductivity for the material of the rod is K = 9.2×10^{-2} kcal s^{-1} m^{-1} ^{o}C^{-1}. The temperature of the ends of the rod are 125^{o}C and 0^{o}C in the steady state. Calculate (i) temperature gradient in the rod (ii) temperature of a point at a distance of 10.0 cm from the hot end and (iii) rate of flow of heat.

Q.12 An iron boiler is 1 cm thick and has a heating area of 2 m^2. The two surfaces of the boiler are at 234^{o}C and 100^{o}C respectively. If the latent heat of steam is 536 kcal kg^{-1} and thermal conductivity of iron is 1.6×10^{-2} kcal s^{-2} kcal s^{-1} m^{-1} ^{o}C^{-1}, then how much water will be evaporated into steam per minute?

Q.13 One end of a 0.25 m long metal bar is in steam and the other is in contact with ice. If 12g of ice melts per minute, what is the thermal conductivity of the metal? Given cross–section of the bar = 5×10^{-4} m^2 and latent heat of ice is 80 cal g^{-1}.

Q.14 A layer of ice 0.15 m thick has formed on the surface of a deep pond. If the temperature of upper surface of ice is constant and equal to that of the air which is – 12^{o}C, determine the time it will take to increase the thickness of ice layer by 0.2 mm. Take latent heat of ice = 80 cal g^{-1}, density of ice = 0.91g cm^{-3} and thermal conductivity of ice = 0.5 cal s^{-1} m^{-1} K^{-1}.

Q.15 Water is boiled in a rectangular steel tank of thickness 2 cm by a constant temperature furnace. Due to vaporization, water level falls at a steady rate of 1 cm in 9 minutes. Calculate the temperature of the furnace. Given K for steel = 0.2 cal s^{-1} m^{-1} ^{o}C^{-1}.

Q.16 Estimate the rate at which ice would melt in a wooden box 2.0 cm thick and of inside measurements

200 cm × 120 cm × 120 cm assuming that the external temperature is 30^{o}C and coefficient of thermal conductivity of wood is 0.0004 cal s^{-1} cm^{-1o}C.

Q.17 Steam at 373 K is passed through a tube of radius 50 cm and length 3 m. If thickness of the tube be 2mm and conductivity of its material be 2×10^4 cal s^{-1} cm^{-1} K^{-1}, calculate the rate of loss of heat in Js^{-1}. The outside temperature is 282 K.

Q.18 Thermal conductivity of copper is four times that of brass. Two rods of copper and brass of same length and cross–section are joined end to end. The free end of copper rod is at 0^{o}C and that of brass rod at 100^{o}C. Calculate the temperature of junction at equilibrium. Neglect radiation losses.

Q.19 The temperature difference between the two ends of a bar 1.0m long is 50^{o}C and that for the other bar 1.25m long 75^{o}C. Both the bars have same area of cross–section. If the rates of conduction of heat in the two bars are the same, find the ratio of the coefficients of thermal conductivity of materials of the two bars.

Q.20 The ratio of the areas of cross–section of two rods of different materials is 1 : 2, and the ratio of the thermal conductivities of their materials is 4 : 3.On keeping equal temperature–difference between ends of these rods, rates of conduction of heat are equal. Determine ratio of the lengths of the rods.

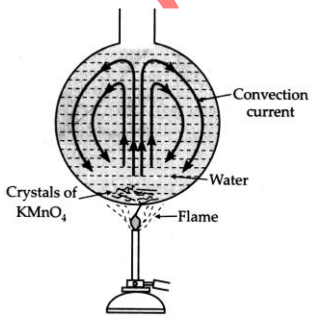
Q.21 A room at 20^{o}C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal s^{-1} m^{-1} ^{o}C^{-1} and J = 4.2 J cal^{-1}.

Answers

1.	231 cal s^{-1}	2.	91.56°C	3.	0.424 cal s^{-1} cm^{-1} $°C^{-1}$		
4.	102.5 s	5.	60°C	6.	116°C and 74°C		
7.	1.396 m	8.	44.4°C	9.	9000 W		
10.	(i) 315 K, (ii) 91.6m^{-1} K^{-1}, (iii) 916.1 W						
11.	(i) – 5°C cm^{-1}, (ii) 75°C, (iii) 4.048 × 10^{-2} $kcals^{-1}$			12.	48 kg		
13.	80 cal s^{-1} m^{-1} $°C^{-1}$	14.	364.2 s	15.	110°C	16.	9.36 gs^{-1}
17.	36026 Js^{-1}	18.	20°C	19.	6 : 5		
20.	2 : 3	21.	15.2°C				

Convection

It is the process by which heat flows from the region of higher temperature to the region of lower temperature by the actual movement of the material particles. Fluids (liquids and gases) are heated mainly by the process of convection in which buoyancy and gravity play an important role. As shown in figure, when a fluid is heated from below, the hot portion at the bottom expands and becomes less dense. Because of buoyancy, this lighter portion rises up. The denser colder fluid takes its place by moving downwards. *Thus convection current* is set up in the fluid. The actual movement of a liquid can be seen by colouring the liquid with potassium permanganate crystals placed of the bottom of the vessel.



Natural Convection

If the material moves due to difference in density, the process of heat transfer is called natural or free convection. Natural convection arises due to unequal heating (of fluid) and gravity. Natural convection is responsible for the origin of different types of winds in the atmosphere.

Forced Convection

If the heated material is forced to move by an agency like a pump or a blower, the process of heat transfer is called forced convection. Air–conditioning, central heating systems and heating a liquid by brisk stirring are examples of forced convection.

Phenomena Based on Thermal Convection

(i) In regulating the temperature of human body

In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection.

(ii) In maintaining comfortable room temperature in cold countries

In cold countries during winter, the outside temperature is much below 0°C while the room temperature is comfortably maintained around 20°C. However, the inside air close to the glass window is cooler than 20°C while the outside close to the window is warmer than the chilling temperature of the atmosphere. Thus heat is continuously transferred from the room to the outside by convection of air inside the room.

(iii) In the formation of trade winds

Natural convection plays an important role in the formation of trade winds. The surface of the earth and hence the air above it near the equator gets strongly heated by the sun. The heated air expands and rises upwards. The colder air from polar region rushes in towards the equator. This produces northward wind in northern hemisphere and southward in southern hemisphere. Due to rotation of the earth about its axis from west to east, the air close to the equator has an eastward speed of 1600 kmh^{-1}, while it is zero close to the poles. As a result, the actual direction of the wind in the northern hemisphere is north east and in the southern hemisphere, south west. These winds are called *trade winds.*

(iv) Land and Sea Breezes

These are local convection currents. Specific heat of water is higher than that of soil. So land and hence air above it is heated faster in summer during day time than air above the sea. The air above land expands and rises and its place is taken up by the colder air from sea to land and is called sea breeze. At night the land gets cooled faster than water. So colder air flows from land to sea and is called land breeze.

(v) Monsoons

Water has much more specific heat than soil or rock. In summer, the land mass of the Indian subcontinent gets much hotter than the Indian Ocean. This sets up convention current with hot air from the land rising and moving towards the Indian Ocean, while the moisture–laden air from the Ocean moves towards the land. When obstructed by height and gets cooled. The moisture condenses and causes wide–spread rains in India. In winter, the landmass is cooler than the ocean. Winds blow from the land to ocean. These winds take up moisture as they pass the Bay of Bengal and cause rainfall in Tamilnadu and Srilanka.

Radiation

It is the process by which heat is transmitted from one place to another without heating the intervening medium. The heat from the sun reaches the earth by the process of radiation, convering millions of kilometers of the empty space or vacuum.

Prevost's Theory of Heat Exchange

The salient features of this theory are:

(i) *All bodies at temperature above 0 K emit heat to the surroundings and gain heat from the surroundings at all times.*

(ii) *The amount of heat radiated per second depends on the nature of the emitting surface, its surface area and its temperature.*

(iii) *The rate of heat radiated by a body increases with the increase of its temperature and is unaffected by the presence of surrounding bodies.*

(iv) *There is a continuous exchange of heat between a body and its surroundings.*

(v) *The rise or fall in temperature of a body is the net result of the exchange of heat between the body and the surroundings.*

(vi) *The exchange of heat between a body and its surrounding continues till a dynamic thermal equilibrium is established between them and their temperatures become equal.*

When the temperature of a body is equal to that of its surroundings, it radiates heat to the surroundings at the same rate at which it absorbs. The body is then in the state of *dynamic equilibrium.* In this state, if a body absorbs a large fraction of total heat falling upon it, surroundings, otherwise its temperature will change. This shows that a body which is a good absorber is also a good radiator of heat & vice–versa.

Electromagnetic Waves

These are the waves constituted by oscillating electric and magnetic fields. The oscillations of the two fields are mutually perpendicular to each other as well as to the direction of propagation of the waves. Every body at any temperature emits electromagnetic waves. These waves can have different wavelengths. The atoms or molecules of a substance can be excited to higher energy states by thermal collisions or by some other means. When such atoms or molecules de–excite to lower energy states, electromagnetic radiations are emitted.

Thermal Radiation

The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy. All bodies having temperature above 0 K emit thermal radiation continuously. For example, the radiation emitted by red–hot iron or light from a filament lamp is thermal radiation.

Properties of thermal radiation

(i) These are electromagnetic waves having wavelengths range from 1 μm to 100 μm. These are also called infrared waves.

(ii) Like light, thermal radiations travel in straight lines.

(iii) These radiations obey the laws of reflection and refraction like light does.

(iv) They show the phenomena of interference, diffraction and polarization.

(v) Thermal radiations produce heat when they are absorbed by a body.

Newton's Law of Cooling

The rate at which a body loses heat by radiation depends on (i) the temperature of the body, (ii) the temperature of the surrounding medium, and (iii) the nature and extent of the exposed surface.

***Newton's law of cooling** states that the rate of cooling (or rate of loss of heat) of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.* This is in accordance with Newton's law of cooling that a hot water bucket cools fast initially until it gets lukewarm after which it stays so for a longer time.

Mathematical expression for Newton's law of cooling

Consider a hot body at temperature T. Let T_0 be the temperature of its surroundings. According to Newton's law of cooling,

Rate of loss of hear ∝ Temperature difference between the body and its surroundings

or $$-\frac{dQ}{dt} \propto (T-T_0) \qquad \text{or} \qquad -\frac{dQ}{dt} = k(T-T_0) \qquad \text{... (1)}$$

where k is a proportionality constant depending upon the area and nature of the surface of the body. Let m be the mass and c the specific heat of the body at temperature T. If the temperature of the body falls by small amount dT in time dt, then the amount of heat lost is

$$dQ = mc\, dT$$

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$

Combining the above equations, we get

$$-mc\frac{dT}{dt} = k(T-T_0)$$

or $$\frac{dT}{dt} = -\frac{k}{mc}(T-T_0) = -K(T-T_0) \qquad \text{... (2)}$$

where K = k/ mc is another constant. The negative sign indicates that as the time passes, the temperature of the body decreases. The above equation can be written as

$$\frac{dT}{T-T_0} = -K\, dt$$

On integrating both sides, we get

$$\int \frac{1}{T-T_0}\, dT = -K\int dt$$

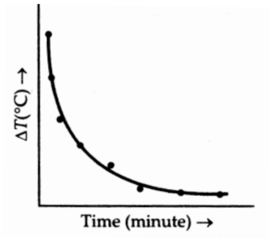
or $$\log_e (T - T_0) = -Kt + c \qquad \text{... (3)}$$

or $$T - T_0 = e^{-Kt + c}$$

or $T = T_0 + e^c \ e^{-Kt}$

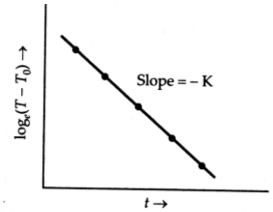
or $T = T_0 + Ce^{-Kt}$ ... (4)

where c is a constant of integration and $C = e^c$.

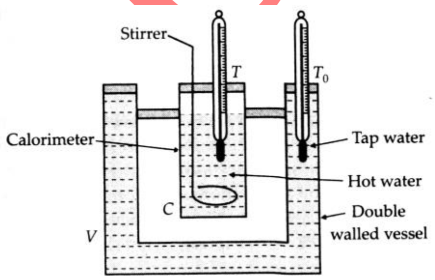


If we plot a graph by taking different values of temperature difference $\Delta T = T - T_0$ along y–axis and the corresponding values of t along x–axis, we get a curve of the form shown in figure. The rate of cooling is higher initially and then decrease as the temperature of the body falls.

If we plot a graph, by taking $\log_e (T - T_0)$ along y–axis and time t along x–axis, we must get a straight line, as shown in figure.



Experimental Verification of Newton's Law of Cooling



The experimental set–up used for verifying Newton's law of cooling is shown in figure. The set–up consists of a double walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T of hot water in calorimeter and T_0 of water in between the double walls respectively. Temperature of hot water in the calorimeter is noted after fixed intervals of time, say after every one minute of stirring the water gently with a stirrer.

Continue noting its temperature till it attains a temperature about 5^oC above that that of surroundings. Plot a graph between $\log_e (T - T_0)$ and time (t). The nature of the graph is observed to be a straight line, having a negative slope, as shown in figure. This verifies newton's law of cooling.

Absorptive Power

The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ. It is denoted by a_λ. A perfect black body absorbs all the heat radiations incident upon it. So its absorptive power is unity. If the radiant energy dQ in wavelength range λ and $\lambda + d\lambda$ is incident on a body of absorptive power a_λ, then amount of radiant energy absorbed by the body = a_λ dQ.

Emissive Power

The amount of heat energy radiated by a body per second depends upon (i) the area of its surface, (ii) the temperature of its surface and (iii) the nature of its surface. The strength of emission is measured by a quantity called emissive power. The *emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ.*

If a heat radiation of wavelength range λ to $\lambda + d\lambda$ is incident on the surface of a body of emissive power e_λ, then the amount of radiant energy emitted per second per unit area = $e_\lambda \, d\lambda$. The SI unit of emissive power
is $Js^{-1} m^{-2}$ or Wm^{-2}.

Emissivity

The emissivity of a body is defined as the ratio of the heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the *same temperature i.e.*, it is the ratio of the emissive power (e) of a body to the emissive power (E) of a black body at the same temperature. It is denoted by ε.

Thus $\varepsilon = \frac{e}{E}$

It is dimensionless quantity. Its value lies between 0 and 1. The emissivity of a perfect black body is 1.

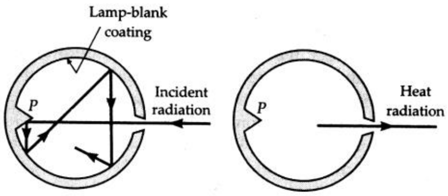
Black Body

A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it. The absorptive power of a perfect black body is unity. When a black body is heated to a high temperature, it emits radiations of all possible wavelengths within a certain wavelength range. The radiations emitted by a block body are called *full or black body radiations.*

In practice, a surface coated with lamp black or platinum black absorbs 95 to 97% of the incident radiation. But on heating, it does not emit full radiation spectrum. So it acts as a black body only for absorption of heat radiation. It is observed that if a hollow cavity is heated, the radiation coming out from its inner surface through a small opening is a full radiation spectrum. Such a radiation is called **cavity radiation.** Hence the small opening of a heated hollow cavity acts as a perfect black body both for absorption and emission of heat radiation.

Fery's Black Body

Fery's black body consists of a hollow double walled metal sphere coated inside with lamp black and nickel polished from outside. Heat radiations entering the sphere through the small opening are completely absorbed due to multiple reflections. The conical projection opposite the opening prevents direct reflection. To use it as a source of heat radiation, the enclosure is heated in a suitable bath to maintain its temperature constant.



(*a*) A black body absorber. (*b*) A black body emitter.

The radiations coming out from the small hole are black body radiations. The *wavelength range of emitted radiation is independent of the material of the body and depends only on the temperature of the black body.*

Kirchhoff's Law

Kirchhoff's law of heat radiation states that at any given temperature, the ratio of the emissive power to the absorptive power corresponding to the certain wavelength is constant for all bodies and this constant is equal to emissive power of the perfect black body at the same temperature and corresponding to the same wavelength. If e_λ and a_λ are emissive and the absorptive powers of a body corresponding to wavelength λ,

then $$\frac{e_\lambda}{a_\lambda} = E_\lambda \ (constant) \quad .... (i)$$

where E_λ is the emissive power of perfect black body at the same temperature and corresponding to the same wavelength. Thus, if a_λ is large, then e_λ will also be large i.e., if a body absorbs a radiation of certain wavelength strongly, then it will also strongly emit the radiation of that wavelength.

As the emissivity ε of a body is defined as the ratio of its emissive power to that of the emissive power of a black body at the same temperature, so

$$\frac{e_\lambda}{E_\lambda} = \varepsilon \quad .... (ii)$$

From equation (i) and (ii), we get

$$a_\lambda = \varepsilon$$

Thus *the absorptive power of a body is equal to its emissivity.* This is another form of ***Kirchhoff's law.***

Hence *a good absorber is a good emitter.* Since a good absorber is a poor reflector, so the ability of a body to emit radiation is related oppositely to its ability to reflect. That is, *a good emitter is a poor reflector.*

Applications of Kirchhoff's Law

(i) Take a piece of china having some dark paintings engraved on it. Heat it in a furnace to about 1000°C and then examine in a dark room immediately. The dark paintings will appear much brighter than white china. This is because the dark paintings are better absorbers and, therefore, also better emitters.

(ii) A green glass heated in a furnace when taken out in dark glows with red light. Green glass (when cold) is a good absorber of red light and a good reflector of green light. When heated, it becomes a good emitter of red light in accordance with Kirchhoff's law.

(iii) If a polished metal ball with a spot of platinum black on it is heated in a furnace to about 1200 K and then taken out into a dark room, the black spot appears brighter than the polished surface. This is because the black spot is a better absorber and hence, by Kirchhoff's law, a better emitter of radiation.

(iv) A Dewar flask or thermos bottle consists of a double–walled glass vessel with its inner and outer walls coated with silver. Radiation from the inner walls is reflected back into the contents of the bottle. Similarly, the outer wall reflects back any incoming radiation. The space between the walls is evacuated to reduce losses due to conduction and convection. The device helps in keeping hot contents hot and cold contents cold for a long time.

Fraunhoffer Lines

When light from the sun is seen through a spectrometer, it is found to be crossed by several dark lines which are called Fraunhoffer lines. The sun has a hot central core which emits continuous spectrum. The central hot core is surrounded by various elements in the vaporized state and comparatively cooler than the core. When white light from the central core passes through the elements in the vapour state, they absorb their characteristic wavelengths, thus producing dark lines in the sun's spectrum.

Importance

By comparing the wavelengths of Fraunhoffer's lines with those emitted by elements on the earth, we have identified various elements like H_2, He, N_2, O_2, Na, Fe, Cu, etc. in the atmosphere of the sun.

Stefan–Boltzmann Law

This law states that the total heat energy emitted by a perfect black body per second per unit area is directly proportional to the fourth power of the absolute temperature of its surface. Thus

$$E \propto T^4 \qquad or \qquad E = \sigma T^4$$

where σ is universal constant and known as Stefan Boltzmann constant.

In SI units, $\sigma = 5.67 \times 10^{-8}\ J\ s^{-1}\ m^{-2}\ K^{-4} = 5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$

In CGS units, $\sigma = 5.67 \times 10^{-5}\ erg\ s^{-1}\ cm^{-2}\ K^{-4}$

If a black body is in an enclosure at temperature T_0, then the rate at which the black body absorbs radiation from the enclosure is σT_0^4. Therefore, the net loss of energy by the black body per unit time per unit area is

$$E = \sigma(T^4 - T_0^4)$$

If the body is not a perfect black body and has emissivity ε, then above relations get modified as follows:

$$E = \varepsilon \sigma T^4 \qquad or \qquad E = \varepsilon \sigma (T^4 - T_0^4).$$

Subjective Assignment – V

Mechanical Properties Of Solids

Q.1 A body cools in 7 minutes from 60^oC to 40^oC. What will be its temperature after the next 7 minutes? The temperature of the surroundings is 10^oC. Assume that Newton's law of cooling holds good throughout the process.

Q.2 A pan filled with hot food cools from 94^oC to 86^oC in 2 minutes when the room temperature is at 20^oC. How long will it take to cool from 71^oC to 69^oC?

Q.3 Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of $5.67\ W\ cm^{-2}$. Given $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.

Q.4 Luminosity of Rigel star in Orion constellation is 17,000 times that of our sun. If the surface temperature of the sun is 6000 K, calculate the temperature of the star.

Q.5 Due to the change in mains voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed?

Q.6 Consider the sun to be a perfect sphere of radius 6.8×10^8 m. Calculate energy radiated by the sun in one minute. Surface temperature of sun = 6200 K. Stefan's constant = $5.67 \times 10^{-8}\ Jm^{-2}\ s^{-1}\ K^{-4}$.

Q.7 At what temperature will the filament of 100 W lamp operate if it is supposed to be perfectly black body of area 1 cm^2?

Q.8 A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600^oC, and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250? Neglect heat loss due to convection. (Stefan Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$)

Q.9 A spherical body with radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated?

Q.10 Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227^oC, when placed in an enclosure at 27^oC. Given Stefan's constant = $5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.

Q.11 Two bodies A and B are kept in evacuated vessels maintained at a temperature of 27^oC. The temperature of A is 527^oC and that of B is 127^oC. Compare the rates at which heat is lost from A and B.

Q.12 A small hole is made in a hollow sphere whose walls are at 273^oC. Find the total energy radiated per second per cm^2. Find the total energy radiated per second per cm^2. Given Stefan's constant = 5.7×10^{-5} erg $cm^{-2}\ s^{-1}\ K^{-4}$.

Q.13 To what temperature must a black body is raised in order to double total radiation, if original temperature is 727^oC?

Q.14 A black body initially at 27^oC is heated to 327^oC. How many times is total heat emitted at the higher temperature than that emitted at lower temperature? What is the wavelength of the maximum energy radiation at the higher temperature? Wien's constant = 2.898×10^{-3} mK.

Q.15 An electric bulb with tungsten filament having an area of 0.25 cm^2 is raised to a temperature of 3000 K, when a current passes through it. Calculate the electrical energy consumed in watt, if the emissivity of the filament is 0.35. Stefan's constant, $\sigma = 5.67 \times 10^{-5}$ erg $s^{-1}\ K^{-4}$. If due to fall in main voltage the filament temperature falls to 2500 K, what will be wattage of the bulb?

Q.16 An iron ball having a surface area of 200 cm^2 and at a temperature of 527^oC is placed in an enclosure at 27^oC. If the surface emissivity of iron be 0.4, at what rate is heat being lost by radiation by the ball?

Answers

1.	28^oC	2.	42 s	3.	1000 K
4.	68520 K	5.	216	6.	2.92×10^{28} J
7.	2049 K	8.	296 W	9.	1800 W

10.	45.48 cal s^{-1}	11.	23			12.	5.61×10^7 erg
13.	916°C	14.	48.300Å	15.	49.19 W, 19.38 W	16.	45.59 cal s^{-1}

Wien's Displacement Law

The total energy radiated by a black body is not uniformly distributed over all the wavelengths but is maximum for a particular wavelength λ_m. The value of λ_m decreases with the increase of temperature. *Wien's displacement law states that the wavelength (λ_m) corresponding to which the energy emitted by a black body is maximum is inversely proportional to its absolute temperature (T).* Mathematically,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

where b is Wien's constant. Its value is 2.9×10^{-3} mK.

Importance

Wien's law can be used to estimate the surface temperatures of the moon, sun and other starts. Light from the moon shows a maximum of intensity at λ_m = 14 μm. By applying Wien's law, the temperature of the surface of the moon turns out to be 200 K. Similarly, solar radiation shows a maximum at λ_m = 4753Å. This corresponds to a surface temperature of 6060 K.

Subjective Assignment – VI

Q.1 Wavelength corresponding to E_{max} for the moon is 14 microns. Estimate the surface temperature of the moon, if b = 2.284×10^{-3} mK.

Q.2 The surface temperature of a hot body is 1227°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant = 0.2898 cm K.

Q.3 The spectral energy distribution of the sun has a maximum at 4753 Å. If the temperature of the sun is 6050 K, What is the temperature of a star for which this maximum is at 9506 Å?

Q.4 An indirectly heated filament is radiating maximum energy of wavelength 2.16×10^{-5} cm. Find the net amount of heat energy lost per second per unit area, the temperature of surrounding air is 13°C. Given b = 0.288 cm K, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.

Q.5 The sun radiates maximum energy at wavelength 4753Å. Estimate the surface temperature of the sun, if b = 2.888×10^{-3} mK.

Q.6 The temperature of an ordinary electric bulb is around 3000 K. At what wavelength will it radiate maximum energy? Will this wavelength be within visible region? Given b = 0.288 cm K.

Q.7 A furnace is at a temperature of 2000 K. At what wavelength will it radiate maximum intensity? Is it in the visible region?

Answers

1.	206 K	2.	19320 Å	3.	3025 K
4.	1.824×10^{12} erg s^{-1} cm^{-2}	5.	6076 K	6.	9600 Å, No
7.	14400 Å, No				

Phases and Phase Diagrams

A graph drawn between the pressure and volume of a system at constant temperature is called an isotherm. AB represents the vapour phase (steam) which is compressible. This means when pressure is increased from A to B, the volume is decreased. Here steam is at 350°C and this is possible only at high pressure of about 160 atmosphere. From B to C, the pressure remains constant. At B the substance is in vapour state and at C it is in liquid state. Thus along BC liquid and vapour coexist in equilibrium.

Let V_l and V_g be the molar volumes of water in liquid and gaseous phases respectively. If V is the total volume of the system, then the fractions of the volume in liquid and gaseous phases will be

$$x_l = \frac{V_g - V}{V_g - V_l} \quad \text{and} \quad x_g = 1 - x_l$$

When pressure becomes more than 163 atm, the substance is in the liquid state (water). Along CD the pressure is increased. There is almost no change in volume. This shows that the liquids are incompressible. As the temperature is increased, the volume difference $V_l - V_g$ decreases and at a temperature 374.1°C and pressure

216 atm, $V_l - V_g = 0$. For this isotherm there is no horizontal portion. This temperature is called the critical temperature (T_C) of water. This means that water can exist as liquid till 374.1°C only, there is only one phase i.e. vapour. This means if a gas is above critical temperature whatever pressure is applied we cannot liquefy it.

Critical Constants

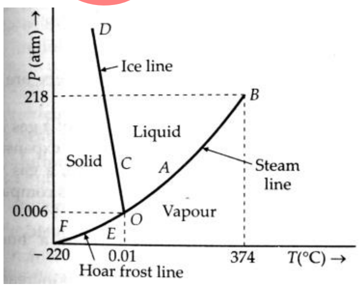
The critical temperature, T_c is the temperature below which a gas can be liquefied by the application of pressure. The pressure required is called critical pressure P_c and the volume occupied by unit mass of the gas at critical temperature and critical pressure is called critical volume V_c.

Pressure–temperature phase diagram for water

Figure shows the P–T phase diagram for water. It consists of the following *three* curves:

(i) **Vaporisation curve (Steam Line AB)**

It is a graph between pressure and the boiling point of the substance in the liquid state. Each point on this curve fixes a set of pressure and temperature at which the liquid and the gaseous phases can co–exist. If the pressure is increased, the vapour will at once condense into liquid but if the pressure is deceased, the liquid will evaporate. So, all points above the vaporisation curve correspond to liquid phase and below it to vapour phase.



(ii) **Fusion Curve (Ice Line CD)**

It is a graph between the pressure and the melting point of the substance in the solid state. Each point on this curve gives the value of the pressure and the temperature at which the solid and liquid phases can co–exist. If pressure is increased, the solid would melt into liquid but if the pressure is decreased liquid will turn into solid. So all the points above the fusion curve correspond to liquid phase and those below it to solid phase.

(iii) **Sublimation curve (Hoar frost line EF)**

It is a graph between pressure and temperature at which a solid directly changes to vapour state. Each point on this curve gives the values of pressure and temperature at which the solid and vapour phases can co–exist. If pressure is increased, the vapour changes to solid phase and if the pressure is decreased, the solid changes to vapour state. So all the points above this curve correspond to solid phase while those below it correspond to vapour state.

Conclusions

(i) In the space above the steam line and on the right of ice–line, water exists in liquid phase as water.

(ii) In the space below the steam line and on the right of hoar frost line, water exists in gaseous phase as steam.

(iii) In the space above the hoar–frost line and on the left of ice–line, water exists in solid phase as ice.

Triple Point

It is a unique point on P–T diagram at which all the three phases of a substance can co–exist in equilibrium with each other. The three curves AB, CD and EF on being extended meet at point O which represents the triple point. The values of pressure and temperature corresponding to this point for water are 0.46 cm of Hg and 273.16 K.

The negative slope of ice line for water indicates that melting point of ice decreases with the increase in pressure. The triple point of such substances is above its melting point at normal pressure.

Conceptual Problems

Q.1 Two thermometers are constructed in the same way except that one has a spherical bulb and the other and elongated cylindrical bulb. Which of the two will respond quickly to temperature changes?

Q.2 Why should a thermometer bulb have a small heat capacity?

Q.3 Why are gas thermometers are more sensitive than mercury thermometers?

Q.4 Why is a constant volume gas thermometer preferred as a standard thermometer than a constant pressure gas thermometer?

Q.5 Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle at the mean temperature $(T_1 + T_2)/2$. Why?

Q.6 Two hollow glass balls are connected by a tube, which has a pellet of mercury in the middle. Can the temperature of the surrounding air be determined from the position of the drop?

Q.7 The difference between lengths of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible?

Q.8 Why are loops provided in long metal pipes used for carrying oil and any other liquid over long distances?

Q.9 A long cylindrical vessel having linear coefficient of expansion α is filled with a liquid up to a certain level. On heating, it is observed that the length of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid?

Q.10 A metal ball is heated through a certain temperature. Out of mass, radius, surface area and volume, which will undergo largest percentage increase and which one the least?

Q.11 If an electric fan be switched in a closed room, will the air of the room be cooled? If not, why do we feel cold?

Q.12 The coolant used in a nuclear reactor should have high specific heat. Why?

Q.13 Why are two thin blankets are warmer than a single blanket of double the thickness?

Q.14 Place a safety pin on a sheet of paper. Hold the sheet over a burninig candle, until the paper becomes yellow and charr. On removing the pin, its white trace is observed on the paper. Why?

Q.15 If a drop of water falls on a very hot iron, it does not evaporate for a long time. Give reason.

Q.16 Why rooms are provided with the ventilators near the roof?

Q.17 Why snow is a better heat insulator than ice?

Q.18 Can we boil water inside an earth satellite?

Q.19 Suppose you want to cool your drink. Should you keep ice cubes floating on the top or should you arrange to keep the ice cubes at the bottom?

Q.20 Why are clear nights colder than cloudy nights?

Q.21 How does the boiling point of water change with pressure?

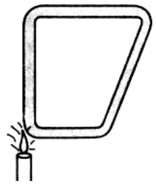
Q.22 Suggest suitable methods for measuring the temperature of
(i) surface of the sun, (ii) surface of the earth, (iii) an insect and (iv) liquid helium

Q.23 There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if (i) they are heated to the same temperature (ii) same amount of heat is given to each of them?

Q.24 Two vessels of different materials are identical in size and wall–thickness. They are filled with equal quantities of ice at 0^oC. If the ice melts completely in 10 and 25 minutes respectively, compare the coefficients of thermal conductivity of the materials of the vessels.

Q.25 Two vessels A and B of different materials but having identical shape, size and wall thickness are filled with ice and kept at the same place. Ice melts at the rate of 100 g min^{-1} and 150g min^{-1} in A and B respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.

Q.26 Water in a closed tube is heated with one arm placed vertically above an arc lamp. Water will begin to circulate along the tube in a counterclockwise direction. Is this true or false?



Q.27 A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 200°C. Which of these objects will cool fastest and which one slowest when left in air at room temperature? Give reasons.

Q.28 There are two rods of the same metal, same length, same area of cross–section, but one of square cross–section and the other of circular cross section. One end of each is kept immersed in steam. After the steady state is reached, the other ends of the rods are touched. Which one will be hotter? Given reason.

Q.29 A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster?

Q.30 On a hot day, a car is left in sunlight with all the windows closed. After some time, it is found that the inside of the car is considerably warmer than the air outside. Explain, why.

Q.31 A blackened platinum wire, when gradually heated, first appears dull red, then blue and finally white. Explain why?

Q.32 In a coal fire, the pockets formed by coals appear brighter than the coals themselves. Is the temperature of such a pocket higher than the surface temperature of a glowing coal?

Q.33 Answer the following questions:

(a) A vessel with a movable piston maintained at a constant temperature by a thermostat contain a certain amount of liquid in equilibrium with its vapour. Does this vapour obey Boyle's law? In other words, what happens when the volume of vapour is decreased? Does the vapour pressure increase?

(b) What is meant by 'superheated water' and 'supercooled vapour' Do these states of water lie on its P–V–T surface? Give some practical applications of these states of water.

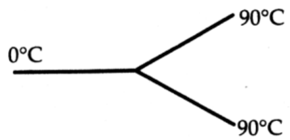
Q.34 A fat man is used to consuming about 3000 kcal worth of food every day. His food contains 50g of butter plus a plate of sweets everyday, besides items which provide him with other nutrients (proteins, vitamins, minerals, etc.) in addition to fats and carbohydrates. The caloric value of 10g of butter is 60 kcal and that of a plate of sweets is of average 700 kcal. What dietary strategy should he adopt to cut down his calories to about 2100 kcal per day? Assume the man cannot resist eating the full plate of sweets once it is offered to him !

Problems on Higher Order Thinking Skills

Q.1 2kg of ice at – 20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/ kg/ °C, and 0.5 kcal/ kg/ °C, while the latent heat of fusion of ice is 80 kcal/ kg.

Q.2 Two rods, one of aluminum and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increase by the same amount when their temperatures are raised by t°C, then find the ratio $l_1/(l_1 + l_2)$

Q.3 Three rods made of the same material and having the same cross–section have been joined as shown in figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. What will be the temperature of the junction of the three rods?



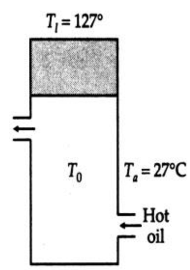
Q.4 When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then find the ratio k_1/k_2.

Q.5 An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal/ kg – K and B = 2×10^{-2} cal/ kg – K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal/kg. Specific heat of water = 10^3 cal/ kg – K)

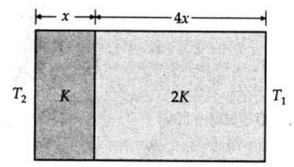
Q.6 Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/ (m – °C–sec), thickness t = 5 mm, emmissivity = 0.6. Temperature of the top of lid is maintained at T_l = 127°C. If the ambient temperature T_a = 27°C, calculate

(a) rate of heat loss per unit area due to radiation from the lid

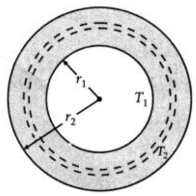
(b) temperature of the oil (Given $\sigma = \frac{17}{3} \times 10^{-8}$)



Q.7 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2 K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). What is the rate of flow of heat through the slab in a steady state?



Q.8 Figure shows a system of two concentric spherical shells of radii r_1 and r_2 and kept at temperatures T_1 and T_2. Find the radial rate of flow of heat through a substance of thermal conductivity K filled in the space between the two shells.



Q.9 A 5m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire : Young's modulus = 2.1×10^{11} Pa; Density = 7860 kg/ m^3; specific heat = 420 J / kg – K)

Q.10 A sphere of diameter 7 cm and mass 266.5g floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 g cm^{-3}, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

Answers

1.	6 kg	2.	$\frac{\alpha_s}{\alpha_a+\alpha_s}$	3.	60^oC
4.	$\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$	5.	0.5 kg	6.	(a) 595 Wm^{-2}, (b) 420 K
7.	$\frac{KA(T_2-T_1)}{3x}$	8.	$\frac{4\pi Kr_1r_2(T_1-T_2)}{(r_2-r_1)}$	9.	0.00457 K
10.	0.000084^oC^{-1}				

NCERT Exercise

Q.1 A brass wire 1.8m long at 27^oC is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of – 39^oC, what is the tension developed in the wire, if its diameter is 2.0 mm? Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ ^oC^{-1}$, Young's modulus of brass = 0.91×10^{11} Pa.

Q.2 A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250^oC, if the original lengths are at 40.0^oC? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand. Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ ^oC^{-1}$ and that of steel = $1.2 \times 10^{-5}\ ^oC^{-1}$.

Q.3 A child running a temperature of 101^oF is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98^oF in 20 min, what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

Q.4 A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6h. The outside temperature is 45^oC and coefficient of thermal conductivity of thermocole = 0.01 $Js^{-1}\ m^{-1}\ ^oC^{-1}$. Heat of fusion of water = 335×10^3 J kg^{-1}.

Q.5 The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Q.6 Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is relation between T_A and T_B?

Q.7 The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: $R = R_0\,[1 + 5 \times 10^{-3}\,(T - T_0)]$

The resistance of 101.6Ω at the triple point of water, and 165.5 Ω at the normal melting point of lead (600.5 K). What is t he temperature when the resistance is 123.4 Ω?

Q.8 Answer the following:

(a) The triple–point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting points of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0^oC and 100^oC respectively. On the absolute scale, one of the fixed points is the triple–point of water, which on the Kelvin absolute scale is assigned the number
273.16 K. What is the other fixed point on this (Kelvin) scale?

(c) The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T – 273.15$. Why do we have 273.15 in this relation, and not 273.16?

(d) What is the temperature of the triple–pint of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Q.9 Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	**Pressure thermometer A**	**Pressure thermometer B**
Triple–point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

(b) What do you think is the reason for slightly different answers from A and B?

Q.10 A steel tape 1m long is correctly calibrated for a temperature of 27.0°C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C? Coefficient of linear expansion of steel = 14.20×10^{-5} °C^{-1}?

Q.11 A large steel wheel is to be fitted on to a shaft of the same material. At 27°C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice' (solid carbon dioxide). At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range.

Q.12 A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0°C. What is the change in the diameter of the hole when the sheet is heated to 227°C? Coefficient of linear expansion of copper = 1.70×10^{-5} °C^{-1}.

Q.13 The coefficient of volume expansion of glycerine is 49×10^{-5} °C^{-1}. What is the fractional change in its density for a 30°C rise in temperature?

Q.14 A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = 0.91 J g^{-1} °C^{-1}.

Q.15 A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is maximum amount of ice that can melt? (Specific heat of copper = 0.39 Jg^{-1} °C^{-1}, and latent heat of fusion of water = 335 Jg^{-1}).

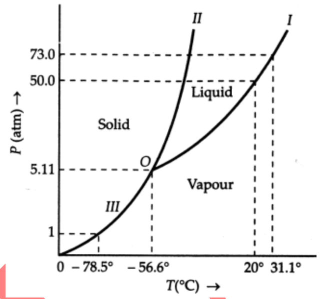
Q.16 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.

Q.17 Given below are observations on molar specific heats at room temperature of some common gases.

Gas	**Molar Specific Heat (C_V) (cal mol^{-1} K^{-1})**
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. [Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K]. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Q.18 Answer the following questions based on the P – T phase diagram of carbon dioxide as shown in figure.



(i) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co–exist in equilibrium?

(ii) What is the effect of decrease of pressure on the fusion and boiling point of CO_2

(iii) What are the critical temperature and pressure for CO_2? What is their significance?

(iv) Is CO_2 solid, liquid or gas at (a) – 70°C under 1 atm., (b) – 60°C under 10 atm., (c) 15°C under 56 atm?

Q.19 Answer the following questions based on the P – T phase diagram of CO_2:

(i) CO_2 at 1 atm pressure and temperature – 60°C is compressed isothermally. Does it go through the liquid phase?

(ii) What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?

(iii) Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm. pressure and temperature – 65°C as it is heated up to room temperature at constant pressure.

(iv) CO_2 is heated to a temperature 70°C and compressed isothermally. What changes in its properties do you expect to observe?

Q.20 A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg min^{-1}, when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 Js^{-1} m^{-1} $°C^{-1}$ and heat of vaporisation of water = 2256 Jg^{-1}.

Q.21 Explain why:

(a) a body with large reflectivity is a poor emitter.

(b) a brass tumbler feels much colder than a wooden tray on a chilly day.

(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.

(d) the earth without its atmosphere would be inhospitably cold.

(e) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Q.22 A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C, the temperature of the surrounding is 20°C.

Answers

1. 3.77×10^2 N   2. 0.34 cm , No   3. 4.31 g min^{-1} 4. 3.687 kg

5. Neon, T_C = – 248.58°C, T_F = – 415.44°F, CO_2 T_C = – 56.6°C, T_F = – 69.88°F   6. $T_A = \frac{4}{7}T_B$

7. 384.8 K   9. (a) 392.46 K, (b) 391.75 K   10. 63.0136 cm, 63 cm

11. – 68.8°C   12. 1.44×10^{-2} cm   13. 0.0147

14. 103.02 °C   15. 1.455 kg   16. 0.1 cal g^{-1} $°C^{-1}$

18. (i) 56.6°C, 5.11 atm, (ii) decrease, (iii) P_C = 73.0 atm, T_C = 31.1°C, (iv) (a) vapour, (b) solid (c) liquid

19. (i) No, (ii) directly into solid, (iii) go to liquid phase and then to the vapour phase, (iv) will deviated more and more from ideal gas behaviour   20. 238°C   22. 9 min

Objective Assignment – I (CBSE PMT)

Q.1 According to kinetic theory of gases, at absolute zero of temperature
(a) water freezes (b) liquid helium freezes
(c) molecular motion stops (d) liquid hydrogen freezes

Q.2 Mercury thermometer can be used to measure temperature upto
(a) 260°C (b) 100°C (c) 360°C (d) 500°C

Q.3 For measuring temperatures in the range of 2,000 to 2,500°C, we should employ
(a) barometer (b) pyrometer (c) gas thermometer
(d) platinum–rhodium thermometer

Q.4 A Centigrade and Fahrenheit thermometers are dipped in boiling water. The water temperature is lowered, until the Fahrenheit thermometer registers 140°. What is the fall in temperature registered by the centrigrade thermometer?
(a) 80° (b) 60° (c) 360° (d) 30°

Q.5 The coefficient of linear expansion of brass and steel are α_1 and α_2 respectively. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, the difference in their lengths (l_1 and l_2) will remain the same at all temperatures, if
(a) $\alpha_1 l_1 = \alpha_2 l_2$ (b) $\alpha_1 l_2 = \alpha_2 l_1$ (c) $\alpha^2_1 l_1 = \alpha^2_2 l_2$ (d) $\alpha_1 l^2_2 = \alpha^2_2 l_1$

Q.6 The thermal capacity of 40g of aluminium (specific heat = 0.2 cal°g^{-1}C^{-1}) is
(a) 40 cal °C^{-1} (b) 160 cal °C^{-1} (c) 200 cal °C^{-1} (d) 8 cal °C^{-1}

Q.7 80g of water at 30°C are poured on a large block of ice at 0°C. The mass of ice that melts is
(a) 30 g (b) 80 g (c) 150g (d) 1,600 g

Q.8 If 1g of steam is mixed with 1g of ice, the resultant temperature of the mixture is
(a) 270° (b) 230° (c) 100° (d) 50°

Q.9 10g of ice cubes at 0°C are released in a tumbler (water equivalent 55 g) at 40°C. Assuming that negligible heat is taken from the surroundings, the temperature of water in the tumbler becomes nearly (L = 80 cal/g)
(a) 31°C (b) 22°C (c) 19°C (d) 15°C

Q.10 On a new scale of temperature (which is linear) called the W scale, freezing and boiling points of water are respectively 39°W and 239°W. What will be the temperature on the new scale corresponding to a temperature of 39°C on the celcius scale?
(a) 200°W (b) 139°W (c) 78°W (d) 117°W

Q.11 The two ends of a rod of length L and a uniform cross–sectional area A are kept at two temperatures
T_1 and T_2($T_1 > T_2$). The rate of heat transfer, dQ/dt through the rod in a steady state is given by
(a) $\frac{dQ}{dt} = \frac{K(T_1 - T_2)}{LA}$ (b) $\frac{dQ}{dt} = KLA(T_1 - T_2)$
(c) $\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$ (d) $\frac{dQ}{dt} = \frac{KL(T_1 - T_2)}{A}$

Q.12 The presence of gravitational field is required for the heat transfer by
(a) stirring liquids (b) conduction (c) natural convection (d) radiation

Q.13 Consider two rods of same length and different specific heats (c_1, c_2), thermal conductivities (K_1, K_2) and area of cross–sections (A_1, A_2) and both having temperature (T_1, T_2) at their ends. If their rate of loss of heat due to conduction is equal, then

(a) $K_1A_1 = K_2A_2$ (b) $K_1A_1/c_1 = K_2A_2/c_2$ (c) $K_2A_1 = K_1A_2$ (d) $K_2A_1/c_2 = K_1A_2/c_1$

Q.14 Which of the following circular rods (radius r and length l) each made to the same material and whose ends are maintained at the same temperature will conduct most heat?

(a) $r = 2r_0$, $l = 2l_0$ (b) $r = 2r_0$, $l = l_0$ (c) $r = r_0$, $l = 2l_0$ (d) $r = r_0$, $l = l_0$

Q.15 A cylindrical rod having temperatures T_1 and T_2 at its end. The rate of flow of heat Q_1 cal s^{-1}. If all the dimensions (length and radius) are doubled keeping temperature constant, then the rate of flow of heat Q_2 will be

(a) $Q_2 = 2Q_1$ (b) $Q_2 = Q_1/2$ (c) $Q_2 = Q_1/4$ (d) $Q_2 = 4Q_1$

Q.16 Heat is flowing through the cylindrical rods of the same material. The diameters of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of the amount of heat conducted through them per unit time will be

(a) 1 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 8

Q.17 Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and 2K respectively. The equivalent thermal conductivity of the slab is

(a) 2K/3 (b) 4K/3 (c) $\sqrt{2}$ K (d) 3 K

Q.18 A beaker full of hot water is kept in a room. If it cools from 80°C to 75°C in t_1 minutes, from 75°C to 70°C in t_2 minutes and from 70°C to 65°C in t_3 minutes, then

(a) $t_1 = t_2 = t_3$ (b) $t_1 < t_2 = t_3$ (c) $t_1 < t_2 < t_3$ (d) $t_1 > t_2 > t_3$

Q.19 Which of the following is best close to an ideal black body?

(a) platinum black (b) black lamp

(c) cavity maintained at constant temperature

(d) a lump of charcoal heated to high temperature

Q.20 Unit of Stefan's constant is

(a) $W\ m^2\ K^4$ (b) $W\ m^2\ K^{-4}$ (c) $W\ m^{-2}\ K^{-1}$ (d) $W\ m^{-2}\ K^{-4}$

Q.21 A black body is at a temperature of 500 K. It emits energy at a rate, which is proportional to

(a) 500 (b) $(500)^2$ (c) $(500)^3$ (d) $(500)^4$

Q.22 A black body is at 727°C. It emits energy at a rate, which is proportional to

(a) $(727)^2$ (b) $(1000)^2$ (c) $(727)^4$ (d) $(1000)^4$

Q.23 A black body at 227°C radiates heat at the rate of 7 cal $cm^{-2}\ s^{-1}$. At a temperature of 727°C, the rate of heat radiated in the same units will be

(a) 50 (b) 112 (c) 80 (d) 60

Q.24 For a black body at temperature of 727°C, its radiating power is 60 W and temperature of surroundings is 227°C. If temperature of black body is changed to 1,227°C, then its radiating power will be

(a) 120 W (b) 240 W (c) 304 W (d) 320 W

Q.25 The radiant energy from the sun, incident normally at the surface of earth, is 20 kcal $m^{-2}\ min^{-1}$. What would have been the radiant energy incident normally on the earth, if the sun had a temperature twice of the present one?

(a) 40 kcal $m^{-2}\ min^{-1}$ (b) 80 kcal $m^{-2}\ min^{-1}$ (c) 160 kcal $m^{-2}\ min^{-1}$ (d) 320 kcal $m^{-2} min^{-1}$

Q.26 If the temperature of the sun is doubled, the rate of energy received on earth will be increased by a factor of

(a) 2 (b) 4 (c) 8 (d) 16

Q.27 Assuming the sun to have a spherical outer surface of radius R, radiating like a black body at temperature t°C, the power received by a unit surface (normal to the incident rays) at a distance r from the centre of the sun is

(a) $\frac{4\pi R^2 \sigma t^4}{r^2}$ (b) $\frac{R^2 \sigma (t+273)^4}{4\pi r^2}$ (c) $\frac{16\pi R^2 \sigma t^4}{r^2}$ (d) $\frac{R^2 \sigma (t+273)^4}{r^2}$

Q.28 Which of the following statement is true about the radiation emitted by human body?
(a) the radiation emitted lies in the ultraviolet region and hence is not visible
(b) the radiation is emitted during the summers and absorbed during the winters
(c) the radiation is emitted only during the day (d) the radiation emitted is in the infrared region.

Q.29 If λ_m denotes the wavelength at which the radiative emission from a black body at a temperature T K is maximum, then
(a) λ_m is independent of temperature (b) $\lambda_m \propto T^4$
(c) $\lambda_m \propto T$ (d) $\lambda_m \propto 1/T$

Q.30 The Wien's displacement law expresses relation between
(a) wavelength corresponding to maximum energy and absolute temperature
(b) radiated energy and wavelength (c) temperature and emissive power
(d) colour of light and temperature

Q.31 A black body emits radiation of maximum intensity of wavelength λ at 2,000 K. Its corresponding wavelength at 3,000 K will be
(a) $16\lambda/81$ (b) $81\lambda/16$ (c) $2\lambda/3$ (d) $4\lambda/3$

Q.32 A black body at 1,227°C emits radiations with maximum intensity at a wavelength of 5000Å. If the temperature of the body is increased by 1,000°C, the maximum intensity will be observed at
(a) 3000 Å (b) 4000Å (c) 5000Å (d) 6000Å

Q.33 Two closed containers A and B are party filled with water. The volume of A is twice that of B and it contains half the amount of water in B. If both are at the same temperature, the water vapour in the containers will have pressure in the ratio of
(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 4 : 1

Answers

1.	c	2.	c	3.	b	4.	c	5.	a
6.	d	7.	a	8.	c	9.	b	10.	d
11.	c	12.	c	13.	a	14.	b	15.	a
16.	d	17.	b	18.	c	19.	c	20.	d
21.	d	22.	d	23.	b	24.	d	25.	d
26.	d	27.	d	28.	d	29.	d	30.	a
31.	c	32.	a	33.	b				

Objective Assignment – II (AIIMS)

Q.1 We plot a graph, having temperature in °C on x–axis and in °F on y–axis. If the graph is straight line, then it
(a) passes through origin (b) intercepts the positive x–axis
(c) intercepts the positive y–axis (d) intercepts the negative axis of both x–and y–axis

Q.2 At a common temperature, a block of wood and a block of metal feel equally cold or hot. The temperatures of block and wood are
(a) equal to the temperature of the body (b) less than the temperature of the body
(c) greater than temperature of the body (d) either (b) or (c)

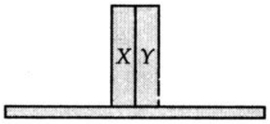
Q.3 A quantity of heat required to change the unit mass of a solid substance, from solid state to liquid state, while the temperature remains constant, is known as
(a) latent heat (b) sublimation (c) hoar frost (d) latent heat of fusion

Q.4 When a solid is converted into a gas, directly by heating, then this process is known as
(a) boiling (b) sublimation (c) vaporization (d) condensation

Q.5 A constant pressure air thermometer gave a reading of 47.5 units of volume, when immersed in ice–cold water, and 67 units in a boiling liquid. The boiling point of the liquid is

(a) 100^oC (b) 112^oC (c) 125^oC (d) 135^oC

Q.6 A bimetallic strip consists of metals X and Y. It is mounted rigidly at the base as shown below: The metal X has a higher coefficient of expansion compared to that for metal Y. When the bimetallic strip is placed in a cold bath,



(a) it will bend towards the right (b) it will bend towards the left

(c) it will not bend but shrink (d) it will neither bend nor shrink

Q.7 The density of a substance at 0^oC is 10g cm^{-3} and at 100^oC, its density is 9.7 g cm^{-3}.The coefficient of linear expansion of the substance is

(a) 10^{-4} (b) 10^{-2} (c) 10^{-3} (d) 10^{2}

Q.8 Calorimeters are made of which of the following?

(a) glass (b) metal (c) wood (d) either (a) or (c)

Q.9 Hailstone of 0^oC falls from a height of 1 km on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt? (g = 10 ms^{-2})

(a) 1/33 (b) 1/8 (c) $(1/33) \times 10^{-4}$ (d) all of it

Q.10 The bulb of one thermometer is spherical, while that of other is cylindrical. If both of them have equal amount of mercury, which one will respond quickly to the temperature?

(a) spherical (b) cylindrical (c) elliptical (d) both (a) and (c)

Q.11 On a cold morning, a metal surface will feel colder to touch than a wooden surface, because

(a) metal has high specific heat (b) metal has high thermal conductivity

(c) metal has low specific heat (d) metal has low thermal conductivity

Q.12 Woolen clothes keep the body warm, because wool

(a) is a bad conductor (b) increases the temperature of body

(c) decreases the temperature (d) all of these

Q.13 Heat travels through vacuum by

(a) conduction (b) convection (c) radiation (d) both (a) and (b)

Q.14 Ratio of the amount of heat radiation, transmitted through the body to the amount of heat radiation incident on it, is known as

(a) conductance (b) inductance (c) transmittance (d) absorbance

Q.15 Three objects coloured black, gray and white can withstand hostile conditions upto $2,800^oC$. These objects are thrown into a furnace, where each of them attains a temperature of $2,000^oC$. Which object will glow brightest?

(a) the white object (b) the black object

(c) all glow with equal brightness (d) gray object

Q.16 If amount of heat energy received per unit area from sun is measured on earth, mars and Jupiter, it will be

(a) the same for all (b) in decreasing order jupiter, mars, earth

(c) in increasing order Jupiter, mars, earth (d) in decreasing order mars, earth, Jupiter

Q.17 A black body is at a temperature 300 K. It emits energy at a rate, which is proportional to

(a) 300 (b) 300^2 (c) 300^3 (d) 300^4

Q.18 If temperature of a black body increases from 7^oC to 287^oC, then rate of energy radiation increases by

(a) $(287/7)^4$ (b) 16 (c) 4 (d) 2

Q.19 A black body is heated from 27^oC to 127^oC. The ratio of their energies of radiations emitted will be

(a) 3 : 4 (b) 9 : 16 (c) 27 : 64 (d) 81 : 256

Q.20 A black body, at a temperature of 227°C, radiates heat at a rate of 20 cal $m^{-2}s^{-1}$. When its temperature is raised to 727°C, heat radiated by it (in cal m^{-2} s^{-1}) will be closest to

(a) 40 (b) 160 (c) 320 (d) 640

Q.21 A metal rod at a temperature of 150°C, radiates energy at a rate of 20 W. If its temperature is increased to 300°C, then it will radiate at the rate of

(a) 17.5 W (b) 37.2 W (c) 40.8 W (d) 68.3 W

Q.22 Surface temperature of stars A and B are 727°C and 327°C respectively. What is the ratio $H_A : H_B$ for the heat radiated per second by the two stars?

(a) 5 : 3 (b) 25 : 9 (c) 625 : 81 (d) 125 : 27

Q.23 A black body at a high temperature T K radiates energy at the rate of E Wm^{-2}. When the temperature falls to T/2 K, the radiated energy will be

(a) E/4 (b) E/2 (c) 2 E (d) E/16

Q.24 Suppose the sun expands so that its radius and its surface temperature becomes half of its present value. The total energy emitted by it then will increase by a factor of

(a) 10^4 (b) 625 (c) 256 (d) 16

Q.25 The sun radiates energy in all directions. The average radiation received on the earth's surface from the sun per second is 1.4 kW m^{-2}. The average earth–sun distance is 1.5×10^{11}m. The mass lost by the sun per day (1 day = 86,400s) is

(a) 4.4×10^9 kg (b) 7.6×10^{14} kg (c) 3.8×10^{12} kg (d) 3.8×10^{14} kg

Q.26 Energy from the sun is received on earth at the rate of 2 cal cm^{-2} min^{-1}. If average wavelength of solar light be taken at 5,500 Å, then how many photons are received on the earth per cm^2 per min? ($h = 6.6 \times 10^{-34}$ Js, 1 cal = 4.2 J)

(a) 1.5×10^{13} (b) 2.9×10^{13} (c) 2.3×10^{13} (d) 1.75×10^{19}

Q.27 According to Wien's displacement law

(a) λT = constant (b) $\lambda \propto 1/T$ (c) λ/T = constant (d) both (a) and (b)

Q.28 For an enclosure maintained at 1,000 K, the maximum radiation occurs at wavelength λ_m. If the temperature is raised to 2,000 K, the peak will shift to

(a) $\lambda_m/2$ (b) $3\lambda_m/2$ (c) $5\lambda_m/2$ (d) $7\lambda_m/2$

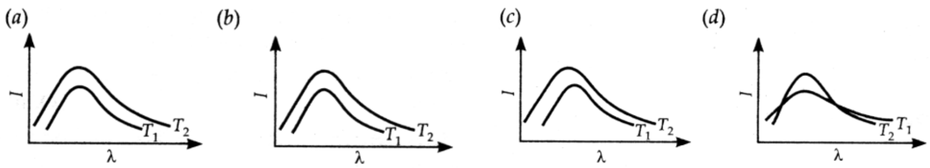
Q.29 The sun emits a light with maximum wavelength 510 nm, while another star X emits a light with maximum wavelength of 350 nm. What is the ratio of surface temperature of the sun and the star X?

(a) 1.45 (b) 0.68 (c) 0.46 (d) 2.1

Q.30 On increasing the temperature of a substance gradually, its colour becomes

(a) red (b) green (c) yellow (d) white

Q.31 Shown below are the black body radiation curves at temperature T_1 and T_2 (($T_2 > T_1$). Which of the following plots is correct?



Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion

(c) If assertion is true, but reason is false (d) If both assertion and reason are false

Q.32 **Assertion:** Good conductors of heat are also good conductors of electricity and vice–versa.
Reason: Mainly electrons are responsible for these conductors.

Q.33 **Assertion:** At room temperature water does not sublimate from ice to steam.
Reason: The critical point of water is much above the room temperature.

Q.34 **Assertion:** Water kept in an open vessel will quickly evaporate on the surface of the moon.
Reason: The temperature at the surface of the moon is much higher than the boiling point of water.

Q.35 **Assertion:** In a pressure cooker the water is brought to boil. The cooker is then removed from the stove. Now on removing the lid of the pressure cooker, the water starts boiling again.
Reason: The impurities in water bring down its boiling point.

Q.36 **Assertion:** It is hotter over the top of a fire than at the same distance of the sides.
Reason: Air surrounding the fire conducts more heat upwards

Q.37 **Assertion:** Air at some distance above the fire is hotter than the same distance below it.
Reason: Air surrounding the fire carries heat upwards.

Q.38 **Assertion:** Woollen clothes keep the body warm in winter.
Reason: Air is a bad conductor of heat

Q.39 **Assertion:** The earth without its atmosphere would be inhospitably cold.
Reason: All heat would escape in the absence of atmosphere.

Q.40 **Assertion:** While measuring the thermal conductivity of a liquid experimentally, the upper layer is kept hot and lower layer is kept cold.
Reason: This avoids heating of the liquid by convection.

Q.41 **Assertion:** A body that is a good radiator is also a good absorber of radiation at a given wavelength.
Reason: According to Kirchhof's law the absorptivity of a body is equal to its emissivity at a given wavelength.

Q.42 **Assertion:** Temperature near the sea coast are moderate.
Reason: Water has a high thermal conductivity.

Q.43 **Assertion:** Perspiration from human body helps in cooling the body.
Reason: A thin layer of water on the skin enhances its emissivity.

Q.44 **Assertion:** A hallow metallic closed container maintained at a uniform temperature can act as a source of black body radiation.
Reason: All metals act as black bodies.

Q.45 **Assertion:** Blue star is at higher temperature than red star.
Reason: Wien's displacement law states that $T \propto 1/\lambda_m$.

Q.46 **Assertion:** For higher temperatures, the peak emission wavelength of a black body shifts to lower wavelengths.
Reason: Peak emission wavelength of a black body is proportional to the fourth–power of temperature.

Q.47 **Assertion:** The radiation from the sun's surface varies as the fourth power of its absolute temperature.

Reason: Sun is not a black body.

Answers

1.	c	2.	a	3.	d	4.	b	5.	b
6.	b	7.	a	8.	b	9.	a	10.	b
11.	b	12.	a	13.	c	14.	c	15.	b
16.	c	17.	d	18.	b	19.	d	20.	c
21.	d	22.	c	23.	d	24.	b	25.	d
26.	c	27.	d	28.	a	29.	b	30.	c
31.	c	32.	a	33.	a	34.	c	35.	c
36.	c	37.	a	38.	a	39.	a	40.	a
41.	a	42.	b	43.	c	44.	c	45.	a
46.	c	47.	c						

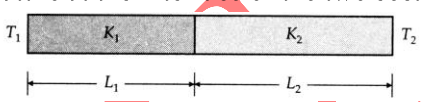
Objective Assignment – III (AIEEE)

Q.1 Heat give to a body, which raises its temperature by $1^{o}C$ is

(a) water equivalent (b) temperature gradient

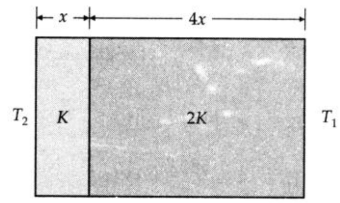
(c) thermal capacity (d) specific heat

Q.2 One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed to two sections of lengths L_1 and L_2 and the coefficients of thermal conductivity K_1 and K_2 respectively. The temperature at the interface of the two sections is



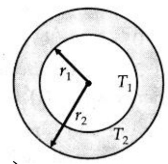
(a) $\frac{K_2L_2T_1+K_1L_1T_2}{K_1L_1+K_2L_2}$ (b) $\frac{K_2L_1T_1+K_1L_2T_2}{K_2L_1+K_1L_2}$ (c) $\frac{K_1L_2T_1+K_2L_1T_2}{K_1L_2+K_2L_1}$ (d) $\frac{K_1L_1T_1+K_2L_2T_2}{K_1L_1+K_2L_2}$

Q.3 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in a steady state is $\left(\frac{A(T_2-T_1)K}{x}\right)f$, where f equals to



(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

Q.4 The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(a) $(r_2 - r_1)/ r_1r_2$ (b) $\log_e (r_2/r_1)$ (c) $r_1r_2 (r_2 - r_1)$ (d) $(r_2 - r_1)$

Q.5 According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings and n is equal to

(a) two (b) three (c) four (d) one

Q.6 A pressure cooker reduces cooking time for food, because

(a) heat is more evenly distributed in the cooking space

(b) the higher pressure inside the cooker crushes the food material

(c) cooking involves chemical changes helped by a rise in temperature

(d) boiling point of water involved in cooking is increased

Q.7 Which of the following is more close to a black body?

(a) blackboard paint (b) green leaves (c) black holes (d) red roses

Q.8 If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously, will be
(a) 4 (b) 16 (c) 32 (d) 64

Q.9 Two spheres of the same material have radii 1m and 4m and temperatures 4,000 K and 2,000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 1 : 9 (c) 4 : 1 (d) 16 : 1

Q.10 Assuming the sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant power, incident on earth, at a distance r from the sun.
(a) $\frac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$ (b) $\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (c) $\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$ (d) $\frac{R^2 \sigma T^4}{r^2}$
Here, r_0 is the radius of the earth and σ is Stefan's constant.

Q.11 The earth radiates in the infra–red region of the spectrum. The spectrum is correctly given by
(a) Planck's law of radiation (b) Stefan's law of radiation
(c) Rayleigh Jeans law (d) Wien's law

Q.12 A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(a) E/c (b) 2E/c (c) Ec (d) E/c^2

Q.13 On a hilly region, water boils at 95°C. The temperature expressed in Fahrenheit is
(a) 100°F (b) 20.3°F (c) 150°F (d) 203°F

Q.14 If boiling point of water is 95°F, what will be reading at Celsius scale?
(a) 7°C (b) 65°C (c) 63°C (d) 35°C

Q.15 50g of ice at 0°C is mixed with 50g of water at 80°C, final temperature of mixture of will be
(a) 0°C (b) 40°C (c) 60°C (d) 4°C

Q.16 Which one of the following processes depends on gravity?
(a) conduction (b) convection (c) radiation (d) none of these

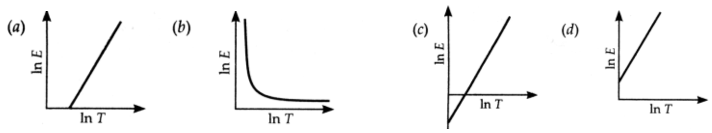
Q.17 A composite rod made of copper ($\alpha = 1.8 \times 10^{-5} K^{-1}$) and steel ($\alpha = 1.2 \times 10^{-5} K^{-1}$) is heated. Then
(a) it bends with steel on concave side (b) it bends with copper on concave side
(c) it does not expand (d) data is insufficient

Q.18 Heat current is maximum in which of the following (rods are of identical dimensions)?

Q.19 The sprinkling of water reduces slightly the temperature of a closed room because
(a) temperature of water is less than that of the room (b) specific heat of water is high
(c) water has large latent heat of vaporization (d) water is a bad conductor of heat

Q.20 Two rods of lengths d_1 and d_2 and coefficients of thermal conductivities K_1 and K_2 are kept in contact with each other end to end. The equivalent thermal conductivity is
(a) $K_1d_1 + K_2d_2$ (b) $K_1 + K_2$ (c) $\frac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (d) $\frac{d_1 + d_2}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$

Q.21 Which of the following graphs correctly represents the relation between E and T, where E is the amount of radiation emitted per unit time from unit area of body and T is the absolute temperature?



Q.22 A back body at a temperature of 227°C radiates heat at the rate of 20 cal $m^{-2}s^{-1}$. When its temperatures rises to 727°C, the rate of heat radiated will be

(a) 40 cal m^{-2} s^{-1} (b) 160 cal m^{-2} s^{-1} (c) 320 cal m^{-2} s^{-1} (d) 640 cal $m^{-2}s^{-1}$

Q.23 Temperature of a black body increases from 327°C to 927°C, the initial energy possessed is 2 kJ. What is its final energy?

(a) 32 kJ (b) 320 kJ (c) 1200 kJ (d) none of these

Q.24 In determining the temperature of a distant star, one makes use of

(a) Kirchhoff's law (b) Stefan's law (c) Wien's displacement law (d) none of the above

Q.25 The wavelength of radiation emitted by a body depends upon

(a) the nature of the surface (b) the area of the surface

(c) the temperature of the surface (d) all of the above factors

Q.26 Temperatures of two stars are in ratio 3 : 2. If wavelength of maximum intensity of first body is 4000Å, what is corresponding wavelength of second body?

(a) 9000 Å (b) 6000 Å (c) 2000 Å (d) 8000 Å

Q.27 A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then

(a) the blue piece will look blue and red will look as usual

(b) red looks brighter red and blue looks ordinary blue

(c) blue shines like brighter red compared to the red piece (d) both the pieces will look equally red.

Q.28 The unit of Stefan's constant is

(a) $Wm^{-2}K^{-1}$ (b) WmK^{-4} (c) Wm^{-2} K^{-4} (d) $Nm^{-2}K^{-4}$

Answers

1.	c	2.	c	3.	d	4.	c	5.	d
6.	d	7.	a	8.	d	9.	a	10.	c
11.	d	12.	b	13.	d	14.	d	15.	a
16.	b	17.	a	18.	a	19.	c	20.	d
21.	c	22.	c	23.	a	24.	c	25.	c
26.	b	27.	c	28.	c				

Objective Assignment – IV (IIT – JEE)

Multiple Choice Questions with One Correct Answer

Q.1 A constant volume air thermometer works on

(a) Archemedes' principle (b) Boyle's law

(c) Pascal's law (d) Gay Lussac's law

Q.2 The temperature coefficient of resistance of a wire is 0.00125°C^{-1}. At 300 K, its resistance is 1Ω. The resistance of the wire will be 2Ω at

(a) 1,154 K (b) 1,100 K (c) 1,400 K (d) 1,127 K

Q.3 Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_A and α_S respectively. If the length of each rod increases by the same amount, when their temperatures are raised by t°C, then find the ratio $l_1/(l_1 + l_2)$

(a) α_S/ α_A (b) α_A/ α_S (c) $\alpha_S/ (\alpha_A + \alpha_S)$ (d) $\alpha_A /(\alpha_A + \alpha_S)$

Q.4 A metal ball immersed in alcohol weighs W_1 at 0^oC and W_2 at 50^oC. The co–efficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

(a) $W_1 > W_2$ (b) $W_1 = W_2$ (c) $W_1 < W_2$ (d) all of these

Q.5 When a block of iron floats in mercury at 0^oC, fraction k_1 of its volume is submerged, while at the temperature 60^oC, a fraction k_2 is seen to be submerged. If the co–efficient of volume expansion of iron γ_{Fe} and that of mercury of γ_{Hg}, then the ratio k_1/k_2 can be expressed as

(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

Q.6 Calorie is defined as the amount of heat required to raise temperature of 1g of water by 1^oC and it is defined under which of the following conditions?

(a) From 14.5^oC to 15.5^oC at 760 mm of Hg (b) From 98.5^oC to 99.5^oC at 760 mm of Hg

(c) From 13.5^oC to 14.5^oC at 76mm of Hg (d) From 3.5^oC to 4.5^oC at 76mm of Hg

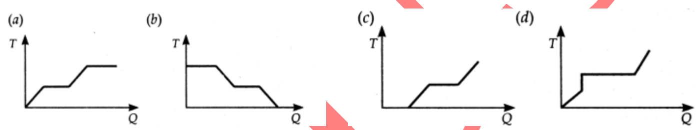
Q.7 Compared to burn due to air at 100^oC, a burn due to steam at 100^oC is

(a) more dangerous (b) less dangerous (c) equally dangerous (d) none of the above

Q.8 540g of ice at 0^oC mixed with 540g of water at 80^oC. The final temperature of mixture is

(a) 0^oC (b) 40^oC (c) 80^oC (d) less than 0^oC

Q.9 A block of ice at -10^oC is slowly heated and converted to steam at 100^oC. Which of the following curves represents the phenomenon qualitatively?



Q.10 2kg of ice at -20^oC is mixed with 5 kg of water at 20^oC in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal $kg^{-1o}C^{-1}$ and 0.5 kcal kg^{-1} $^oC^{-1}$, white the latent heat of fusion of ice is 80 kcal kg^{-1}

(a) 7 kg (b) 6 kg (c) 4 kg (d) 2 kg

Q.11 Water of volume 2 litre in a container is heated with a coil of 1 kW at 27^oC. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. In how much time, temperature will rise from 27^oC to 77^oC? Given that the specific heat of water is 4.2 kJ kg^{-1}.

(a) 8 min 20 s (b) 6 min 2s (c) 7 min (d) 14 min

Q.12 In which of the following processes, convection does not take place primarily?

(a) sea and land breeze (b) boiling of water (c) warming of glass of the bulb due to filament

(d) heating air around a furnace

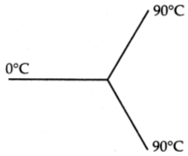
Q.13 A wall has two layers A and B, each made of a different material. Both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36^oC. The temperature difference across the layer A

(a) 6^oC (b) 12^oC (c) 18^oC (d) 24^oC

Q.14 Three rods of identical cross–sectional area and made from the same metal form the sides of an isosceles triangle ABC, right–angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$ respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, T_C/T is

(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$ (c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$

Q.15 Three rods made of same material and having the same cross–section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0^oC and 90^oC respectively. The temperature of the junction of the three rods will be



(a) 45^oC (b) 60^oC (c) 30^oC (d) 20^oC

Q.16 Two identical rods are connected between two containers. One of them is at 100^oC and another is at 0^oC. If rods are connected in parallel then the rate of melting of ice is q_1g/sec. If they are connected in series then the rate is q_2. The ratio q_2/q_1 is

(a) 2 (b) 4 (c) 1/2 (d) 1/4

Q.17 Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2.Both spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) 1/3 (b) $1/\sqrt{3}$ (c) $\sqrt{3}/1$ (d) $(1/3)^{1/3}$

Q.18 A spherical body of area A and emissivity e = 0.6 is kept inside a perfectly black body. Energy radiated per second by the body at temperature T is

(a) $0.4\ \sigma AT^4$ (b) $0.8\ \sigma AT^4$ (c) $0.6\ \sigma AT^4$ (d) $1.0\ \sigma AT^4$

Q.19 An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) initially, it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

(c) it cannot be distinguished at all times

(d) initially, it is the darkest body and at later times it cannot be distinguished

Q.20 A spherical black body with a radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 (b) 450 (c) 900 (d) 1800

Q.21 The earth receives at its surface radiation from the sun at the rate of 1,400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. Treating sun as a black body, it follows from the above data that its surface temperature is

(a) 5,801 K (b) 10^6 K (c) 50.1 K (d) $5{,}801^oC$

Q.22 Two spheres of same material have radii 1m and 4m and temperature 4,000 K and 2,000K respectively. The energy radiated per second by the first sphere is

(a) greater than that by the second (b) less that that by the second

(c) equal in both cases (d) the information is incomplete to draw any conclusion

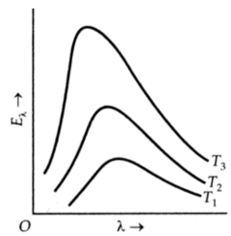
Q.23 Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following options is the correct match?

(a) Sun – T_3, tungsten filament – T_1, welding arc – T_2

(b) Sun – T_2, tungsten filament – T_1, welding arc – T_3

(c) Sun – T_3, tungsten filament – T_2, welding arc – T_1

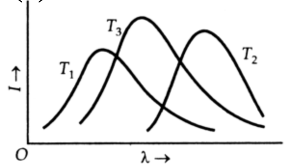
(d) Sun – T_1, tungsten filament – T_2, welding arc – T_3



Q.24 The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperatures of the sun and the North Star is

(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

Q.25 The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown: Their temperatures are such that

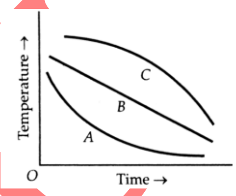


(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

Q.26 A sphere, a cube and a thin circular plate, all made of the same material and having the same mass, are initially heated to a temperature of 3,000°C. Which of these will cool fastest?

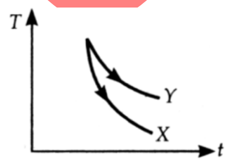
(a) sphere (b) cube (c) plate (d) none of these

Q.27 A block of steel heated to 100°C is left in a room to cool. Which of the curves shown in the figure, represents the correct behaviour?



(a) A (b) B
(c) C (d) none

Q.28 The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies X and Y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies.



(a) $e_X > e_Y$ and $a_X < a_Y$ (b) $e_X < e_Y$ and $a_X > a_Y$ (c) $e_X > e_Y$ and $a_X > a_Y$ (d) $e_X < e_Y$ and $a_X < a_Y$

Q.29 Three discs A, B and C having radii 2m, 4m and 6m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The powers radiated by them are Q_A, Q_B and Q_C respectively.

(a) Q_A is maximum (b) Q_B is maximum (c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

Multiple Choice Questions with One or More than One Correct Answer

Q.30 A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then, R is

(a) proportional to ΔT (b) inversely proportional to ΔT
(c) proportional to $|\alpha_B - \alpha_C|$ (d) inversely proportional to $|\alpha_B - \alpha_C|$

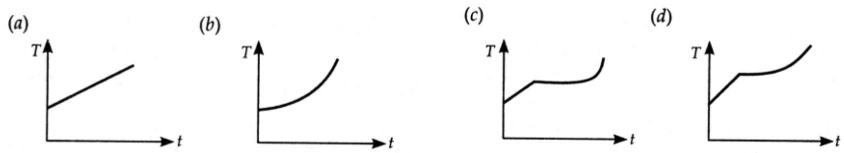
Q.31 Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (c) 4 : 9

Q.32 Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C, till the temperature of the calorimeter and its contents rises to 80°C. The mass of steam condensed (in kg) is

(a) 0.130 (b) 0.065 (c) 0.260 (d) 0.195

Q.33 If liquefied oxygen at 1 atmospheric pressure is heated from 50 K to 300 K by supplying heat at constant rate, the graph of temperature vs time will be



Q.34 A cylinder of radius R, made of a material of thermal conductivity K_1 is surrounded by a cylindrical sheet of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $K_1 + K_2$ (b) $\frac{K_1 + 3K_2}{4}$ (c) $\frac{K_1K_2}{K_1 + K_2}$ (d) $\frac{3K_1 + K_2}{4}$

Q.35 A black body is at a temperature of 2,880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1,499 nm and 1,500 nm is U_3. The Wien's constant, $b = 2.88 \times 10^6$ nm K. Then,

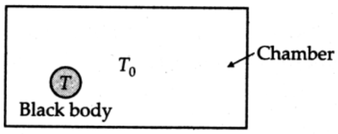
(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

Q.36 Two bodies A and B have thermal emissivities 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5,802 K, then

(a) the temperature of B is 1934 K (b) the temperature of B is 11604 K

(c) the temperature of B is 2901 K (d) $\lambda_B = 1.5$ μm

Q.37 Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remain constant. Which of the following statements is/are true?



(a) the rate of emission of energy from the black body remains the same

(b) the rate of emission of energy from the black body increases

(c) the rate of absorption of energy by the black body increases

(d) the energy radiated by the black body equals the energy absorbed by it.

Answers

1.	d	2.	d	3.	c	4.	a	5.	a
6.	a	7.	a	8.	a	9.	a	10.	b
11.	a	12.	c	13.	b	14.	b	15.	b
16.	d	17.	d	18.	c	19.	a	20.	d
21.	a	22.	c	23.	a	24.	b	25.	b
26.	c	27.	a	28.	c	29.	b	30.	b, d
31.	c	32.	a	33.	c	34.	b	35.	d
36.	d	37.	a, d						