THERMODYNAMICS

Thermodynamics

Thermodynamics is the branch of science that deals with the concepts of heat and temperature and the inter–conversion of heat and other forms of energy. It mainly deals with the transformation of heat into mechanical work and vice versa.

(i) **Thermodynamics System:** *An assembly of a very large number of particles having a certain value of pressure, volume and temperature is called a thermodynamic system.*

(ii) **Surrounding:** *Everything outside the system which can have a direct effect on the system is called its surroundings.*

(iii) **Thermodynamic Variables:** The quantities like pressure (P), volume (V), and temperature (T) which help us to study the behaviour of a thermodynamic system are called thermodynamic variables.

(iv) **Equation of State:** *The mathematical relation between the pressure, volume and temperature of a thermodynamic system is called its equation of state.* For example, the equation of state for n moles of an ideal gas can be written as

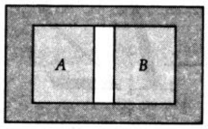
$$PV = nRT$$

Thermal Equilibrium

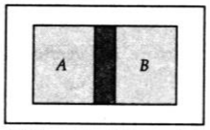
Two systems are said to be in thermal equilibrium with each other if they have the same temperature. Consider two gases A and B contained in two different vessels. Let the pressure and volume of the gases by (P_A, V_B) and (P_B, V_B) respectively.

As shown in figure (a), if the two vessels are separated by an *adiabatic wall* (an insulating wall that does not allow the flow of heat), then any possible pair of values (P_A, V_A) will be in equilibrium with any possible pair of values (P_B, V_B).

As shown in (b), if A and B are now separated by a *diathermic wall* (a conducting wall that allows heat to flow through it), then the pressure and volume variables of the two gases change to (P_A', V_A') and (P_B', V_B') such that the new states of A and B are in equilibrium with each other. There is no more flow of heat. The two systems attain equal temperature and we say they are in equilibrium with each other.



(*a*) Gases *A* and *B* separated by an adiabatic wall.



(*b*) Gases *A* and *B* separated by a diathermic wall.

Thermodynamic Equilibrium

A system is said to be in the state of thermodynamic equilibrium if the macroscopic variables describing the thermodynamic state of the system do not change with time. Consider a gas inside a closed rigid container completely insulated from the surroundings. If the pressure, volume, temperature, mass and composition of the gas do not change with time, then it is in a state of thermodynamic equilibrium.

A system in the state of thermodynamic equilibrium possesses the following equilibria simultaneously:

(i) **Mechanical Equilibrium:** There is no unbalanced force in its interior or between the system and the surroundings.

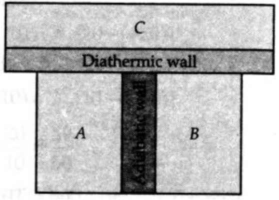
(ii) **Thermal Equilibrium:** All parts of the system and the surroundings are at the same temperature.

(iii) **Chemical Equilibrium:** The system does not undergo any spontaneous change in its internal structure due to chemical reaction, diffusion, etc.

Zeroth Law of Thermodynamics

It states that if two systems A and B are separately in thermal equilibrium with a third system C, then A and B are also in thermal equilibrium with each other.

Concept of Temperature: Zeroth law of thermodynamics implies that *temperature is a physical quantity which has the same value for all systems*



which are in thermal equilibrium with each other. Temperature of a system determines whether it is in thermal equilibrium or not with another system.

There exists a scalar quantity called temperature which is a property of all thermodynamic systems such that temperature equality is a necessary and sufficient condition for thermal equilibrium.

NOTE:

- The temperature which was first defined as the *degree of hotness* and later on the *condition determining the flow of heat,* is now regarded as *one of the seven fundamental quantities* like mass, length, time, etc.
- The zeroth law of thermodynamics was formulated by *R.H. Fowler* in 1931 long after the First and Second laws of thermodynamics were stated. But as this law leads to the concept of the fundamental quantity temperature, so this law was called the zeroth law.

Internal Energy

The internal energy of a system is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest. If the volume of the gas increases, work is done by the gas against intermolecular attraction and so its potential energy increases. Thus *intermolecular potential energy of a real gas is a function of its volume.*

As the temperature increases, the average kinetic energy of the gas molecules also increases. Thus *the internal kinetic energy of a gas is a function of its temperature.* The internal energy does not include the over–all kinetic energy of the system as a whole. It includes only the (disordered) energy associated with the random motion of the molecules of the system. We denote it by U.

Internal energy of a system is a thermodynamic state variable. That is its value depends only on the state of existence of the system and not on the path along which that state has been brought about.

Internal energy of an ideal gas is purely kinetic in nature. In an ideal gas, there are no molecular forces of attraction. So the gas does not possess intermolecular potential energy. Its internal energy is just the sum of kinetic energies associated with various random (translational, rotational and vibrational) motions of its molecules. Thus the internal energy of an ideal gas is wholly kinetic in nature and depends only on its temperature.

Difference between Heat and Work

(i) *Heat is a mode of energy transfer due to temperature difference between the system and the surroundings. Work is the mode of energy transfer brought about by means that do not involve temperature difference* such as moving the piston of a cylinder containing the gas, by raising or lowering the weight connected to it.

(ii) When heat is supplied to a gas, its molecules move faster in all directions at random. So *heat is a mode of energy transfer that produces random motion,* when a piston compresses a gas to do work on it, it forces the molecules to move in the direction of piston's motion. So *work may be regarded as the mode of energy transfer that produces organized motion.*

Sign Conventions Used:

(i) *Heat absorbed by a system is **positive.** Heat given out by a system is **negative.***

(ii) *Work done by a system is **positive.** Work done on a system is **negative.***

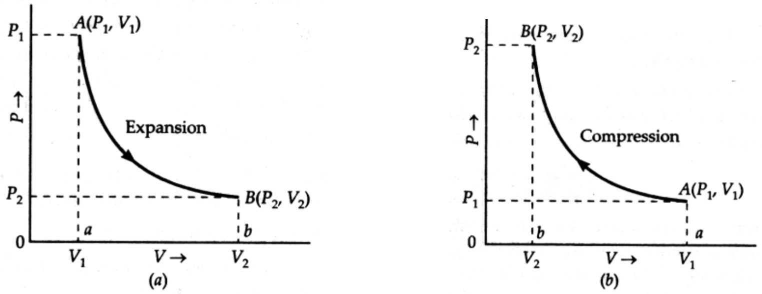
(iii) *The increase in internal energy of a system is **positive.** The decrease in internal energy of a system is **negative.***

NOTE:

- The thermodynamic state of a system is characterized by its internal energy, not heat. That is, internal energy is a state variable. For this reason, the statement like 'a gas in given state has a certain amount of energy' is meaningful.
- In thermodynamics, ***heat and work are not state variables.*** These are the modes of energy transfer to a system resulting in the change in its internal energy. Thus the statement like 'a gas in a given state has certain amount of heat of work' is meaningless, on the other hand, the statement like 'a certain amount of heat is supplied to the system or a certain amount of work is done by the system' is meaningful.

Indicator Diagram

A graphical representation of the state of system with the help of two thermodynamical variables is called an indicator diagram. A graph drawn between the pressure and volume of a gas under thermodynamic operation is called P–V diagram. Such diagrams are drawn with the help of a device called *indicator* which records the changes in volume and pressure accompanying the movement of the piston in the cylinder.



(*a*) (*b*)

Importance of P–V diagram

The area under the P–V diagram is numerically equal to the work done by a system or on the system.

Work Done During Expansion

Consider a gas contained a cylinder of cross–sectional area A and provided with a fractionless movable piston. Let P be the pressure of the gas. Force exerted by the gas on the piston, F = P × A

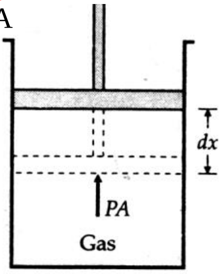
Suppose gas expands a little and pushes out the piston through a small distance dx.

The work done by the gas is dW = Fdx = PAdx = P dV

where dV = Adx, is the change in volume of the gas.

The total work done by the gas when its volume increases from V_1 to V_2 will be

$$W = \int dW = \int_{V_1}^{V_2} PdV.$$



Non–Cyclic Process

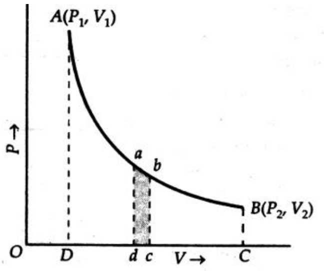
A non–cyclic process is one in which the system does not return to its initial state.

Indicator diagram method for the work done during expansion

In figure, the points A and B represent the *initial state* (P_1, V_1) and final state (P_2, V_2) respectively of a system on a P–V diagram. At any point a, let P and V be the pressure and volume respectively. Suppose that the volume increases from V to V + dV corresponding to a point b on the indicator diagram such that the pressure remains constant. Then ad = bc = P and cd = dV

The small work done when the system changes from state a to state b,

dW = PdV = ac × cd = area of shaded strip *abcd*



The total work done by the gas during the expansion from the initial state A (P_1, V_1) to the final state B (P_2, V_2) can be obtained by adding the areas of all such strips formed between AD and BC under the P–V diagram. Clearly, then the total work done will be

W = area ABCDA

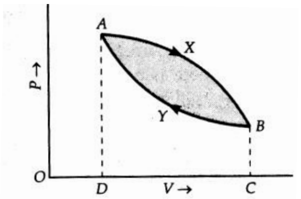
or $$W = \int_{V_1}^{V_2} P\, dV = \text{area under P – V diagram}$$

Hence *the work done by a system is numerically equal to the area enclosed between the P – V diagram and the volume axis.*

Work Done During a Cyclic Process

Any process in which the system returns to its initial state after undergoing a series of changes is known as a cyclic process. Suppose gas expands from the initial state A to the final state B after undergoing a series of changes of pressure and volume, along the path AXB. Work done by the gas during the expansion is

$W_1 = +$ area A X BCDA



Now the gas is subjected to a different series of changes of pressure and volume so that it returns to its initial state A via the path BYA.

Work done on the gas during the compression is

$W_2 = -$ area BYADCB

During compression, work is done on the gas which is taken negative. The net work done during the cyclic process is

$W = W_1 + W_2$ = area AXBCDA – area BYADCB = + area AXBYA

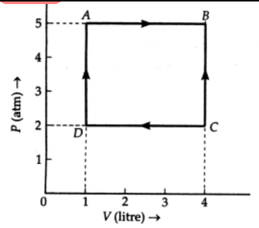
Conclusion: For a cyclic process:

(i) Work done per cycle is numerically equal to the area of the loop representing the cycle.

(ii) If the closed loop is traced in the *clockwise direction*, the expansion curve lies above the compression curves ($W_1 > W_2$), the area of the loop is *positive*, indicating that the *net work is done by the system.*

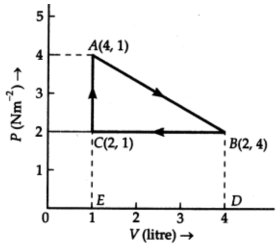
(iii) If the closed loop is traced in the anticlockwise direction, the expansion curve lies below the compression curve, ($W_1 < W_2$), the area of the loop is *negative*, indicating that *the net work is done on the system.*

Subjective Assignment – I



Q.1 One mole of an ideal gas undergoes a cyclic change ABCD. From the given diagram (figure), calculate the net work done in the process.

Q.2 One mole of an ideal gas undergoes a cyclic change ABCD where the (P, V) co–ordinates are A(5, 1), B (5, 3), C (2, 3), and D(2, 1). P is in atmosphere and V is in litre. Calculate work done along AB, BC, CD and DA and also net work done in the process. Given 1 atmosphere = 1.01×10^5 Nm^{-2}.



Q.3 The P – V diagram (figure), for a cyclic process is a triangle ABC drawn in order. The coordinates are in the A, B, C are (4, 1), (2, 4) and (2, 1). The coordinates are in the order P, V. Pressure is in Nm^{-2} and volume in litre. Calculate the work done during the process from A to B, B to C and C to A. Also calculate the work done in the complete cycle.

Q.4 An ideal monoatomic gas is taken round the cycle ABCDA, where co–ordinates of A, B, C, D on P – V diagram are A (p, V), B (2p, V), C (2p, 2V) and D (p, 2V). Calculate work done during the cycle.

Q.5 Calculate net work done by the gas whose thermodynamical behaviour is represented by right angled triangle ABC on P – V diagram. The P – V co–ordinates are A(20, 6), B (10, 12) and C (10, 6), where P is in Nm^{-2} and V is in m^3.

Answers

1.	9×10^9 erg	2.	1010 J, 0 J, – 404 J, 0 J, 606 J		
3.	9×10^{-3} J, -6×10^{-3} J, 0 J, 3×10^{-3} J	4.	pV	5.	30 J

First Law of Thermodynamics

According to the first law of thermodynamics, *if some heat is supplied to a system which is capable of doing work, then the quantity of heat absorbed by the system will be equal to the sum of the increase in its internal energy and the external work done by the system on the surroundings.* Let

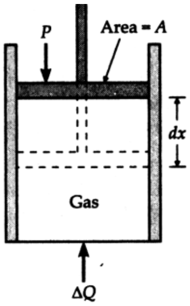
ΔQ = Heat supplied to the system by the surroundings

ΔW = Work done by the system on the surroundings

ΔU = Change in internal energy of the system

Then according to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$



As shown in figure, suppose the system is a gas contained in a cylinder provided with a movable piston. Then the gas does work in moving the piston. The work done by the system against a constant pressure P is

ΔW = Force × Distance = Pressure × Area × Distance = PA dx

or $\Delta W = P\Delta V$

where $\Delta V = Adx$ = the change in the volume of the gas.

So the first law of thermodynamics takes the form,

$$\Delta Q = \Delta U + P\,\Delta V$$

Subjective Assignment – II

Q.1 1 g of water at 100^oC is converted into steam at the same temperature. If the volume of steam is $1671 cm^3$, find the change in the internal energy of the system. Latent heat of steam = $2256\ Jg^{-1}$. Given 1 atmospheric pressure = $1.013 \times 10^5\ Nm^{-2}$.

Q.2 $1.0\ m^3$ of water is converted into $1671\ m^3$ of steam at atmospheric pressure and 100^oC temperature. The latent heat of vaporisation of water is $2.3 \times 10^6\ J\ kg^{-1}$. If 2.0 kg of water be converted into steam at atmospheric pressure and 100^oC temperature, then how much will be the increase in its internal energy? Density of water $1.0 \times 10^3\ kg\ m^{-3}$, atmospheric pressure = $1.01 \times 10^5\ Nm^{-2}$.

Q.3 At 0^oC and normal atmospheric pressure, the volume of 1 g of water increases from $1\ cm^3$ to $1.091\ cm^3$ on freezing. What will be the change in its internal energy? Normal atmospheric pressure is $1.013 \times 10^5\ Nm^{-2}$ and latent heat of melting of ice = $80\ cal\ g^{-1}$.

Q.4 5 moles of oxygen are heated at constant volume from 10^oC to 20^oC. What will be the change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure, $C_p = 8\ cal\ mole^{-1}\ {}^oC^{-1}$ and $R = 8.36\ J\ mole^{-1}\ {}^oC^{-1}$.

Q.5 A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20^oC is given a heat of 20000 J. Find (i) change in temperature, (ii) work done and (iii) change in internal energy. Given

specific heat, $c = 400\ Jkg^{-1}\ {}^oC^{-1}$, coefficient of cubical expansion, $\gamma = 9 \times 10^{-5}\ {}^oC^{-1}$

density, $\rho = 9000\ kg\ m^{-3}$, atmospheric pressure, $P = 10^5\ Nm^{-2}$

Q.6 Calculate the change in internal energy of a block of copper of mass 200g when it is heated from 25^oC to 75^oC. Take specific heat of copper = $0.1\ cal\ g^{-1}\ {}^oC^{-1}$, and assume the change in volume as negligible.

Q.7 Calculate the change in internal energy when 5g of air is heated from 0^oC to 4^oC at constant volume. The specific heat of air at constant volume is $0.172\ cal\ g^{-1}\ {}^oC^{-1}$.

Q.8 The internal energy of a monoatomic gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross–section $8.5\ cm^2$. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2^oC, find the distance moved by the piston. Atmosphere pressure = 100 kPa.

Answers

1.	2086.829 J	2.	4.263×10^6 J	3.	– 80.0022 cal
4.	300 cal	5.	(i) 50^oC, (ii) 0.05 J, (iii) 19999.95 J		
6.	4200 J	7.	14.4 J	8.	20 cm

Specific Heat of a Gas

When a gas is heated, its volume and pressure change with the increase in temperature. So the amount of heat required to raise the temperature of 1 gram of gas through 1^oC is not fixed. A gas can have any value of specific heat depending on the conditions under which it is heated. This can be clearly seen in the following examples.

Limits of specific heat of a gas

Consider m mass of a gas enclosed in a cylinder fitted with an air–tight and frictionless piston.

(i) Suppose the gas is *suddenly compressed.* No heat is supplied to the gas i.e., ΔQ = 0. But the temperature of the gas rises due to compression.

$$\therefore \qquad c = \frac{\Delta Q}{m\Delta T} = \frac{0}{m\Delta T} = 0$$

i.e., *specific heat of the gas is* ***zero.***

(ii) Now the gas is heated and allowed to expand such that the rise in temperature of the gas due to the heat supplied is equal to the fall in temperature of the gas due to the expansion of the gas itself. Then the net rise in temperature is zero i.e., ΔT = 0.

$$\therefore \qquad c = \frac{\Delta Q}{m\Delta T} = \frac{\Delta Q}{m \times 0} = \infty$$

i.e., the specific heat of the gas is ***infinite.***

(iii) Again, the gas is heated and a allowed to expand at such a rate that the fall in temperature due to expansion is less than the rise in temperature due to heat supplied. The temperature of the gas will rise i.e., ΔT is positive.

$$\therefore \qquad c = \frac{\Delta Q}{m\Delta T} = a \ \ positive \ \ value$$

i.e., *specific heat of the gas is* ***positive.***

(iv) Again, the gas is heated and allowed to expand at such a rate that the fall in temperature due to expansion is more than the rise in temperature due to heat supplied. The temperature of gas will decrease i.e. ΔT is negative.

$$\therefore \qquad c = \frac{\Delta Q}{m\Delta T} = a \ \ negative \ \ value$$

i.e., the specific heat of the gas is ***negative.***

Thus *the specific heat of a gas may have any positive or negative value ranging from zero to infinity.* The exact value depends on the conditions of pressure and volume when the gas is being heated.

Two Principal Specific Heats of a Gas

One when the gas is heated at *constant volume* and another when the gas is heated at *constant pressure.* These are known as the two *principle specific heats* of the gas and may be defined as follows:

(i) **Molar specific heat at constant volume:** *It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1^{o}C at constant volume.* It is denoted by C_V.

(ii) **Molar specific heat at constant pressure:** It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1^{o}C at constant pressure. It is denoted by C_p.

C_P is greater than C_V:

When a gas is heated at constant volume, all the heat supplied is used to increase the temperature or internal energy of the gas.

When a gas is heated at constant pressure, the gas expands. It does work against the external pressure. The heat supplied is used in two ways: (i) partly to increase the internal energy and (ii) partly to do work against external pressure. So at constant pressure, an additional amount of heat equivalent to work done is utilized.

Hence more amount of heat is required to increase the temperature of 1 mole of a gas at constant pressure than that at constant volume i.e., $C_P > C_V$. *The difference between the two specific heats is equal to the amount of heat equivalent to the work performed by the gas during expansion at constant pressure.*

Specific Heat Relation: Mayer's Formula

Consider one mole of an ideal gas. Heat the gas to raise its temperature by ΔT. According to the first law of thermodynamics, the heat supplied ΔQ is used partly to increase the internal energy and partly in doing the work of expansion. That is,

$$\Delta Q = \Delta U + P\,\Delta V$$

If the heat ΔQ is absorbed at constant volume, then ΔV = 0 and we have

$$C_V = \left(\frac{\Delta Q}{\Delta T}\right)_V = \left(\frac{\Delta U}{\Delta T}\right)_V = \left(\frac{\Delta U}{\Delta T}\right) + 0 \quad \text{or}$$

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_V \qquad \therefore \qquad C_V = \left(\frac{\Delta U}{\Delta T}\right)$$

We have dropped the subscript V because the internal energy U of an ideal gas depends only on its temperature T. If now the heat ΔQ is absorbed at constant pressure, then

$$C_P = \left(\frac{\Delta Q}{\Delta T}\right)_P = \left(\frac{\Delta U}{\Delta T}\right)_P + P\left(\frac{\Delta V}{\Delta T}\right)_P \quad \text{or} \quad C_P = \left(\frac{\Delta U}{\Delta T}\right) + P\left(\frac{\Delta V}{\Delta T}\right)_P$$

Again, we have dropped the subscript P from the first term because U of an ideal gas depends only on T. Clearly

$$C_P - C_V = P\left(\frac{\Delta V}{\Delta T}\right)_P$$

But for one mole of an ideal gas, PV = RT

Differentiating both sides w.r.t. T for constant pressure P,

$$\frac{\Delta(PV)}{\Delta T} = \frac{\Delta(RT)}{\Delta T} \quad \text{or} \quad P\left(\frac{\Delta V}{\Delta T}\right)_P = R \quad (\because\ \Delta P = 0)$$

Hence $\qquad C_P - C_V = R$

This is the required relation between C_P and C_V. It is also known as *Mayer's formula.*

NOTE:

- For one mole of a gas:
  - $C_P - C_V = R$. (When C_P, C_V are in units of work)
  - $C_P - C_V = \frac{R}{J}$ (When C_P, C_V are in units of heat)

    where R is universal gas constant for one mole of a gas.
- The above relations will remain same even if we consider any number of moles of a gas because in that case both sides will get multiplied by the same number.
- As R is always positive, it follows that $C_P > C_V$
- For one gram of a gas:
  - $C_P - C_V = r$ (When C_P, C_V are in units of work)

    where $r = \frac{R}{M}$ = gas constant for 1 g of a gas.
  - $C_P - C_V = \frac{r}{j}$ (when C_P and C_V are in units of heat)
- Heat lost or gained by n moles of a gas,
  - $Q = nC_p\,\Delta T$ (At constant pressure)
  - $Q = nC_V\,\Delta T$ (At constant volume)
- The ratio of the two principal specific heats is represented by γ. $\therefore \quad \gamma = \frac{C_P}{C_V}$
- The value of γ depends on the atomicity of the gas.

Q.1 Calculate the specific heat at constant volume for a gas. Given specific heat at constant pressure is 6.85 cal mol^{-1} K^{-1}, R = 8.31 J mol^{-1} K^{-1} and J = 4.18 J cal^{-1}.

Q.2 Calculate the difference between two principal specific heats of 1 g of helium gas at S.T.P. Given atomic weight of helium = 4 and J = 4.186 J cal^{-1} and R = 8.31 J mol^{-1} K^{-1}.

Q.3 The difference between two specific heats of a gas is 5000 J kg^{-1} K^{-1} and the ratio of specific heats is 1.6. Find the two specific heats.

Q.4 Specific heat of argon at constant pressure is 0.125 cal g^{-1} K^{-1} and at constant volume is 0.075 cal g^{-1} K^{-1}. Calculate the density of argon at STP. Given J = 4.18 × 10^7 erg cal^{-1} and normal pressure = 1.01 × 10^6 dyne cm^{-2}.

Q.5 Calculate the value of c_V for air, given that c_p = 0.23 calorie g^{-1} K^{-1}. Density of the air at S.T.P. is 1.293 g $litre^{-1}$and J = 4.2 × 10^7 erg $calorie^{-1}$.

Q.6 For air, specific heat at constant pressure is 0.237 cal g^{-1} $^oC^{-1}$ and specific heat at constant volume is 0.169 cal g^{-1} $^oC^{-1}$, density of air = 0.001293 g cm^{-3} at S.T.P. Calculate the value of J.

Q.7 An ideal gas has a specific heat at a constant pressure, C_p = (5/2) R. The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 K and a pressure of 1.6 × 10^6 Nm^{-2}. An amount of
2.49 × 10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

Q.8 Calculate the ratio of specific heats for nitrogen, given that specific heat at constant pressure is 0.236 cal g^{-1} K^{-1} and density at S.T.P. is 0.001234 g cm^{-3}.

Q.9 The specific heat of argon at constant pressure is 0.127 and ratio of specific heats is 1.667. Calculate the value of J. One litre of argon weighs 1.786 g at N.T.P.

Q.10 One mole of oxygen is heated at constant pressure from 0^oC. What must be the quantity of heat that should be supplied to the gas for the volume to be doubled? The specific heat of oxygen under these conditions is 0.218 cal g^{-1} K^{-1}.

Answers

1.	4.862 cal mol^{-1} K^{-1}	2.	0.496 cal g^{-1} K^{-1}
3.	C_V = 8333.33 J kg^{-1} K^{-1}, C_p = 13333.3 J kg^{-1} K^{-1}	4.	1.77 × 10^{-3} g cm^{-3}
5.	0.162 cal g^{-1} K^{-1}	6.	4.219 × 10^7 erg cal^{-1}
7	3.6 × 10^6 Nm^{-2}, 675 K	8.	1.434
9.	4.07 × 10^7 erg cal^{-1}	10.	1904 cal

Thermodynamic Process

A thermodynamic process is said to occur if the thermodynamic variables of a system undergo a change with time. Different types of thermodynamic processes are as follows:

(i) **Isothermal Process:** *It is a thermodynamic process which occurs at a constant temperature.*

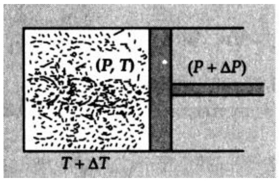
(ii) **Isobaric Process:** *It is a thermodynamics process which occurs at a constant pressure.*

(iii) **Isochoric Process:** *It is a thermodynamic process which occurs at a constant volume.*

(iv) **Adiabatic Process:** *It is a thermodynamic process in which there is no exchange of heat energy between system and surroundings.*

Quasi–Static Process

A quasi–static process is an infinitely slow process such that the system remains in thermal and chemical equilibrium with the surroundings. In a quasi–static (meaning nearly static) process, at every stage, the temperature and pressure of the surroundings differ only infinitesimally from those of the system.

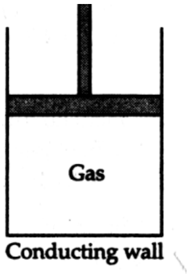


Practically, the processes that are sufficiently slow and do not involve accelerated motion of the piston, large temperature gradients, etc, are nearly quasi–static processes.

Isothermal Process

An isothermal process is one in which the pressure and volume of the system change but temperature remains constant.

As shown in figure, consider an ideal gas enclosed in a cylinder provided with a piston and having conducting walls. If the gas is slowly compressed, the heat produced due to the work done on the gas is transferred to the surroundings so that temperature of the gas remains constant.



Essential Conditions for an Isothermal Process to Take Place:

(i) The walls of the container must be perfectly conducting to allow free exchange of heat between the system and the surroundings.

(ii) The process of compression or expansion should be very slow, so as to provide sufficient time for the exchange of heat.

Equation of isothermal process

The ideal gas equation for n moles of a gas is PV = nRT

For a fixed mass (n fixed) of a gas undergoing an isothermal process (T fixed), the above equation gives

$$PV = \text{constant}$$

This equation is the **equation of state** of an isothermal process.

First Law of Thermodynamics Applied to Isothermal Process

The internal energy of an ideal gas depends only on its temperature. As temperature remains constant in an isothermal process, there is no change in internal energy of the gas i.e., $\Delta U = 0$. Applying first law of thermodynamics to an isothermal process,

$$\Delta Q = \Delta U + P\,\Delta V = 0 + P\,\Delta V \quad \text{or} \quad \Delta Q = P\,\Delta V$$

(i) **When a gas expands isothermally,** ΔV and hence $P\,\Delta V$ is positive and so ΔQ will also be positive. Therefore, when a gas expands isothermally, *an amount of heat equivalent to the work done by the gas has to be supplied from an external source.*

(ii) **When a gas is compressed isothermally,** ΔV and hence $P\,\Delta V$ is negative and ΔQ will also be negative. Therefore, *when a gas is compressed isothermally, an amount of heat equivalent to the work done on the gas has to be removed from the gas.*

(iii) In an isothermal expansion or compression, the internal energy of the gas remains unchanged.

Work Done in an Isothermal Process

Consider n moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in figure. Let P be the pressure of the gas. Work done by the gas when the piston moves up through a small distance dx is given by

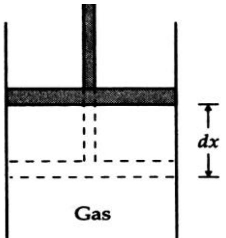
$$dW = PA\,dx = PdV$$

where A is the cross–sectional area of the piston and dV = Adx, is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state (P_1, V_1) to the final state (P_2, V_2). The total amount of work done will be

$$W_{iso} = \int_{V_1}^{V_2} P\,dV$$

For n moles of a gas, PV = nRT or $P = \dfrac{nRT}{V}$

$$\therefore \quad W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT\,[\ln V]_{V_1}^{V_2}$$



$$= nRT\,[\ln V_2 - \ln V_1] = nRT\,\ln\frac{V_2}{V_1}$$

$$W_{iso} = 2.303\,nRT\,\log\frac{V_2}{V_1} = 2.303\,nRT\,\log\frac{P_1}{P_2}$$

This is the expression for the work done during the isothermal expansion of n moles of an ideal gas.

Adiabatic Process

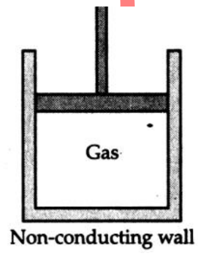
An adiabatic process is one in which the pressure, volume and temperature of the system change but there is no exchange of heat between the system and surroundings. Consider a gas enclosed in a cylinder having perfectly insulated walls. Suppose the gas is allowed to expand *very quickly*. Work is done by the gas during its expansion, so its internal energy decreases. As the heat cannot enter the system from the surroundings, so the temperature of the gas falls.

Essential Conditions for an adiabatic process to take place:

(i) The walls of the container must be perfectly insulated so that there cannot be any exchange.

(ii) The process of compression or expansion should be sudden, so that heat does not get time to get exchanged with the surroundings.

(i) Adiabatic Relation Between P and V



According to the first law of thermodynamics, dQ = dU + dW

For one mole of a gas, dU = C_V dT and dW = P dV

Also, for an adiabatic process, dQ = 0

$\therefore$ $C_V\,dT + P\,dV = 0$ .... (i)

According to the ideal gas equation, PV = RT

Differentiating above equation both sides, we get

P dV + V dP = R dT

or $dT = \dfrac{PdV + VdP}{R}$ .... (ii)

Using equation (ii) in equation (i), we get

$\therefore$ $C_V\cdot\dfrac{PdV + V\,dP}{R} + PdV = 0$ or $C_V PdV + C_V VdP + RP\,dV = 0$

or $C_V VdP + (C_V + R)\,P\,dV = 0$ or $C_V V\,dP + C_P P\,dV = 0$ $[\because\ C_P = C_V + R]$

Dividing both sides by C_V PV, we get

$\dfrac{dP}{P} + \dfrac{C_P}{C_V}\dfrac{dV}{V} = 0$ or $\dfrac{dP}{P} + \gamma\dfrac{dV}{V} = 0$ $\left[\because \gamma = \dfrac{C_P}{C_V}\right]$

Integrating both sides, we get

$\displaystyle\int\frac{dP}{P} + \gamma\int\frac{dV}{V} = C$ where C is constant of integration

$\therefore$ $\log_e P + \gamma\log_e V = C$

or $\log_e PV^{\gamma} = C$ or $PV^{\gamma} = e^{C}$ or $PV^{\gamma} = K$

where K is another constant. This is the adiabatic relation between pressure P and volume V of ideal gas.

(ii) Adiabatic Relation Between P and T : For one mole of a gas PV = RT, therefore

$$V = \frac{RT}{P}$$

Putting in PV^{γ} = K, we get $P\left(\dfrac{RT}{P}\right)^{\gamma} = K$

or $$P^{1-\gamma}\,T^{\gamma} = \frac{K}{R^{\gamma}} = \text{another constant}$$

i.e., $$P^{1-\gamma}\,T^{\gamma} = \text{constant}$$

This is the adiabatic relation between pressure P and temperature T of an ideal gas.

(iii) **Adiabatic relation between V and T:** Again, for one mole of a gas PV = RT, therefore

$$P = \frac{RT}{V}$$

Putting in $PV^{\gamma} = K$, we get

$$\frac{RT}{V}.V^{\gamma} = K \quad \text{or} \quad TV^{\gamma-1} = \frac{K}{R} = \text{ another constant i.e., } \quad TV^{\gamma-1} = \text{constant}$$

This is the adiabatic relation between volume V and temperature T of an ideal gas.

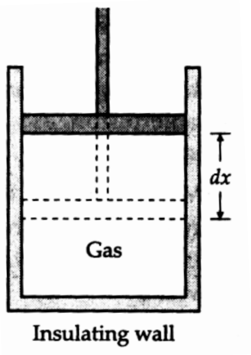
Work Done in an Adiabatic Process

Consider n moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let P be the pressure of the gas. When the piston moves up through a small distance dx, the work done by the gas will be dW = PA dx = P dV

where A is the cross–sectional area of the piston and dV = A dx is the increase in the volume of the gas.

Suppose the gas expands adiabatically and changes from the initial state (P_1, V_1, T_1) to the final state (P_2, V_2, T_2). The total work done by the gas will be

$$W_{adia} = \int_{V_1}^{V_2} P\, dV$$



For an adiabatic change $PV^{\gamma} = K$ or $P = KV^{-\gamma}$

$$\therefore \quad W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma}\, dV = K\int_{V_1}^{V_2} V^{-\gamma}\, dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma-1}[KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

But $K = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\therefore \quad W_{adia} = \frac{1}{\gamma-1}[P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}] \quad \text{or} \quad W_{adia} = \frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$$

Also, $P_1 V_1 = nRT_1$ and $P_2V_2 = nRT_2$

$$\therefore \quad W_{adia} = \frac{1}{\gamma-1}[nRT_1 - nRT_2] \quad \text{or} \quad W_{adia} = \frac{nR}{\gamma-1}[T_1 - T_2]$$

This equation gives the work done during the adiabatic expansion of n moles of an ideal gas.

First Law of Thermodynamics Applied to Isochoric Process

In an isochoric process, the volume V of the system remains constant. As there is no change in volume ($\Delta V = 0$), no work is done on or by the system ($W = P\ \Delta V = 0$). According to the first law of thermodynamics,

$$Q = \Delta U + W = \Delta U + 0 = \Delta U$$

Hence in an isochoric process, the entire heat given to or taken from the system goes to change its internal energy and temperature of the system. The change in temperature can be determined from the equation

$$Q = nC_V\ \Delta T$$

First Law of Thermodynamics Applied to a Cyclic Process

In a cyclic process, the system returns to its initial state after undergoing a series of changes. As internal energy is a state function, so $\Delta U = 0$ for a cyclic process. From first law of thermodynamics, $Q = \Delta U + W = 0 + W = W$

Hence *in a cyclic process, the total heat absorbed by a system is equal to the work done by the system.*

First Law of Thermodynamic Applied to Isobaric Process

A thermodynamic process which occurs at a constant pressure is called an isobaric process. For example, freezing of water, formation of steam, etc. Suppose the pressure P of a gas remains constant and its volume changes from V_1 and V_2, then the work done by the gas is

$$W = \int_{V_1}^{V_2} P\, dV = P \int_{V_1}^{V_2} dV = P(V_2 - V_1) = nR\,(T_2 - T_1)$$

As the temperature of the gas changes, so its internal energy also changes. Hence in *an isobaric process, the absorbed heat goes partly to increase internal energy and partly to do work.*

Boiling Process

Suppose m mass of a liquid is heated at the temperature of its boiling point so that it changes into vapour at a pressure. Let V_i be the volume of the liquid and V_f that of vapour. The work done by the liquid during its expansion at temperature T is given by

$$\Delta W = P\,\Delta V = P\,(V_f - V_i)$$

Let L be the heat of vaporization of the liquid. It represents the heat needed per unit mass to change from liquid to vapour phase at constant temperature and pressure. Then the amount of heat required for vaporisation of m mass of liquid will be

$$\Delta Q = mL$$

Let U_i and U_f be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \text{or} \quad mL = U_f - U_i + P\,(V_f - V_i) \quad \text{or} \quad U_f - U_i = mL - P\,(V_f - V_i)$$

Melting Process

Suppose m mass of a solid is heated at its melting point. When the solid melts, the change in its volume ΔV is negligibly small. So work done by the solid is $\Delta W = P\,\Delta V = P \times 0 = 0$

Let L be the latent heat of fusion. It represents the head needed per unit mass to change from solid to liquid phase at constant temperature and pressure. Then the amount of heat required for fusion of m mass of a solid will be

$$\Delta Q = mL$$

Let U_i and U_f be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \text{or} \quad mL = U_f - U_i + 0 \quad \therefore \quad U_f - U_i = mL$$

Hence the internal energy of a system increases by mL during the melting process.

Subjective Assignment – IV

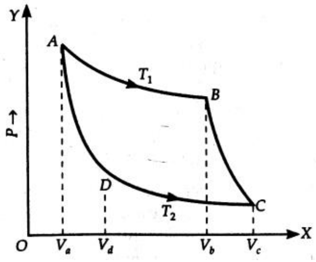
Q.1 The compression ratio of a certain diesel engine is 15. This means that air in the cylinder is compressed to 1/15 of its initial volume. If initial pressure is 1.0×10^5 Pa and the initial temperature is 300 K, find final pressure and temperature after compression. Air is mostly a mixture of oxygen and nitrogen and $\gamma = 1.4$.

Q.2 If, at 50°C and 75 cm of mercury pressure, a definite mass of a gas is compressed (i) slowly, (ii) suddenly, then what will be the final pressure and temperature of the gas in each case if the final volume is one–fourth of the initial volume? ($\gamma = 1.5$).

Q.3 A tyre pumped to a pressure of 3.375 atmosphere and at 27°C suddenly bursts. Calculate the temperature of escaping air. Given $\gamma = 1.5$.

Q.4 Calculate the fall in temperature of helium initially at 15°C, when it is suddenly expanded to 8 times its volume. Given $\gamma = 5/3$.

Q.5 Two different adiabatic parts for the same gas intersect two isothermals at T_1 and T_2 as shown in P–V diagram (figure). How does the ratio (V_a/V_d) compare with the ratio (V_b/V_c)?



Q.6 Two samples of a gas initially at same temperature and pressure are compressed from a volume V to V/2. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Q.7 Three moles of an ideal gas kept at a constant temperature of 300 K are compressed from a volume of
4 litre to 1 litre. Calculate the work done in the process. Given R = 8.31 J mol^{-1} K^{-1}.

Q.8 A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27°C to 97°C. Calculate the work done and heat produced in the gas. Given $\gamma = 1.5$.

Q.9 One gram molecule of an ideal gas at S.T.P. is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy in the process. Given $\gamma = 1.4$.

Q.10 A sample of gas $(\gamma = 1.5)$ is compressed adiabatically from a volume of 1600 cm^3 to 400 cm^3. If the initial pressure is 150 kPa, what is the final pressure and how much work is done on the gas in the process?

Q.11 A gas is suddenly compressed to one–fourth of its original volume. Calculate the rise in temperature, the original temperature being 27°C and$\gamma = 1.5$.

Q.12 A quantity of air at normal temperature is compressed (i) slowly (ii) suddenly to one third of its volume. Find the rise in temperature, if any in each case, $\gamma = 1.4$.

Q.13 Find the final volume of a gram molecule of a gas after an isothermal expansion at 127°C, if the original volume is 400 cm^3. Given the amount of work done by a gram molecular of a gas during expansion is 2302.6 joule, R = 8.3 joule $mole^{-1}$ K^{-1}.

Q.14 Calculate work done to compress isothermally 1g of hydrogen gas at N.T.P. to half its initial volume. Find the amount of heat evolved and change in internal energy. Given R = 8.31 J mol^{-1} K^{-1}.

Q.15 A sample of hydrogen of mass 6g is allowed to expand isothermally at 27°C, till its volume is doubled.
(a) how many moles of H_2 do we have?
(b) what is the final temperature of the H_2?
(c) calculate work done during expansion? (Given, R = 8.31 J mol^{-1} K^{-1}.

Q.16 50g of oxygen at N.T.P. is compressed adiabatically to a pressure of 5 atmosphere. Calculate the work done on the gas, if $\gamma = 1.4$ and R = 8.31 J mol^{-1} K^{-1}.

Answers

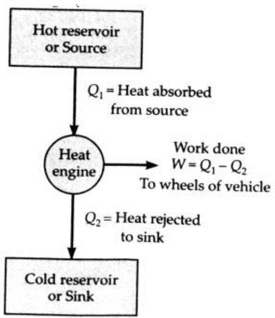
1.	44.3×10^5 Pa, 886 K	2.	(i) 300 cm of Hg, 50°C, (ii) 600 cm of Hg, 373°C		
3.	– 73°C	4.	216 K or 216°C	5.	equal
6.	adiabatic process	7.	-1.037×10^4 J	8.	276.67 cal, – 1162 J
9.	1369.5 J	10.	1200 kPa, – 480 J	11.	300°C
12.	(i) zero (ii) 150.6°C	13.	800 cm^3		
14.	–786.2J, 187.2 cal., zero	15.	(i) 3 moles (ii) 27°C (iii) 5184 J	16.	– 5167 J

Heat Engine

It is a device which converts continuously heat energy into mechanical energy in a cyclic process. As shown in figure, a heat engine has the following essential parts:

(i) **Source:**

It is a heat reservoir at higher temperature T_1. It is supposed to have infinite thermal capacity so that any amount of heat can be drawn from it without changing its temperature.



(ii) **Sink**

It is a heat reservoir at a lower temperature T_2. It has also infinite thermal capacity so that any amount of heat can be added to it without changing its temperature.

(iii) **Working Substance**

Working substance is any material (sold, liquid or gas) which performs mechanical work when heat is supplied to it.

Efficiency of heat Engine

The efficiency of a heat engine is defined as the ratio of the net work done by the engine in one cycle to the amount of heat absorbed by the working substance from the source. As the working substance returns to its initial state after completing one cycle, there is no change in its internal energy. Hence by first law of thermodynamics,

Net heat absorbed in a cycle = Work done i.e., $Q_1 - Q_2 = W$

The efficiency of heat engine is given by

$$\eta = \frac{\text{work done by engine (output)}}{\text{Heat absorbed from the source (input)}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \quad \text{or} \quad \eta = 1 - \frac{Q_2}{Q_1}$$

Efficiency of a heat engine is always less than unity.

Types of heat engines: The heat engines are of two types:

(i) **External combustion engine:** In such a heat engine, the heat needed for the working substance is produced by burning the fuel outside the cylinder and piston arrangement of the engine. A steam engine is an external combustion engine.

(ii) **Internal Combustion Engine:** In such a heat engine, the heat needed for the engine is produced by burning the fuel inside the main cylinder. The petrol and diesel engines are internal combustion engines.

Limitations of the First Law of Thermodynamics

(i) ***It does not indicate the direction of transfer of heat***

Heat always flows from a hot body to a cold one. First law does not give any reason as to why heat cannot flow from a cold body to a hot one.

(ii) ***It does not tell anything about conditions under which heat can be converted into mechanical work***

First law explains the stopping of a revolving wheel due to conversion of its kinetic energy into heat due to friction. But it fails to explain as to why the heat energy cannot be converted into kinetic energy of rotation of the wheel and put it back into rotation.

(iii) ***It does not indicate the extent to which heat energy can be converted into mechanical work continuously***

No heat engine can convert all the heat extracted from the source into mechanical work continuously without rejecting a part of it to the surrounding. First law has no explanation for this fact.

Second Law of thermodynamics

It can be stated in a number of ways as follows:

(i) ***Kelvin–Planck Statement:*** *It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.*

(ii) ***Clausius Statement:*** *It is impossible for a self–acting machine, unaided by any external agency, to transfer heat from a body to another at higher temperature.*

Significance of Second Law

The second law of thermodynamics puts a fundamental limit to the efficiency of a heat engine and the coefficient of performance of a refrigerator.

(i) According to second law, *the efficiency of a heat engine can never be unity*. This in turn, implies that the heat released to the cold reservoir can never be made zero.

(ii) According to second law, *the coefficient of performance of a refrigerator can never be infinite*. This implies that the external work (W) can never be zero.

Limitations of the second law of thermodynamics

(i) The second law of thermodynamics cannot be provided directly. But its validity has not been contradicted by any machine designed so far.

(ii) It is applicable to a cyclic process in which the system returns to its original state after a complete cycle of changes.

(iii) It makes no predictions as to what will happen under certain conditions but simply sates what will happen under given a set of conditions.

Reversible and Irreversible Processes

(a) ***Reversible Process:*** *Any process which can be made to proceed in the reverse direction by variation in its conditions such that any change occurring in any part of the direct process is exactly reversed in the corresponding part of reverse process is called a reversible process.* Thus if some work is done by the system in the direct process, an equal amount of work must be done on the system in the reverse process. If some heat is absorbed by the system in the direction process, it must release an equal amount of heat to the surroundings in the reverse process. At the end of the reversible process, both the system and surroundings must return to their initial states.

Necessary conditions for a reversible process

(i) The process must be *quasi–static*. For this, the process must be carried out infinitesimally slowly so that the system remains in thermal and mechanical equilibrium with the surroundings throughout.

(ii) The dissipative forces such as viscosity, friction, inelasticity, etc. should be absent.

Examples

(i) An infinitesimally slow compression and expansion of an ideal gas at constant temperature

(ii) The process of gradual compression and extension of an elastic spring is approximately reversible.

(iii) A working substance taken along the complete Carnot's cycle.

(iv) The process of electrolysis is reversible if the resistance offered by the elecrolyte is negligibly small.

A complete reversible process is an idealized concept as it can never be realized because dissipative forces cannot be completely eliminated.

(b) ***Irreversible Process:*** *Any process which cannot be retraced in the reverse direction exactly is called an irreversible process.* Most of the processes occurring in the nature are irreversible processes.

Examples

(i) Diffusion of gases. (ii) Rusting of iron (iii) Sudden expansion or contraction of a gas

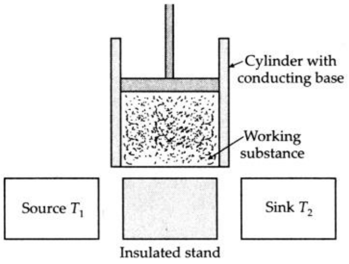
Carnot Engine

It is an ideal reversible heat engine that operates between two temperatures T_1 (source) and T_2 (sink). It is a theoretical heat engine with which the efficiency of practical engines is compared.

Construction: As shown in figure, a Carnot engine has the following main parts:

(i) ***Cylinder:*** This main part of the engine has conducting base and insulating walls. It is fitted with an insulating and frictionless piston.

(ii) ***Source:*** It is a heat reservoir at a higher temperature T_1 from which the engine draws heat. It is supposed that the source has an infinite thermal capacity and so any amount of heat can be drawn from it without changing its temperature.



(iii) ***Sink:*** It is heat reservoir at a lower temperature T_2 to which any amount of heat can be rejected by the engine. It has also infinite thermal capacity and so any amount of heat can be added to it without changing its temperature.

(iv) ***Working substance:*** The working substance is an ideal gas contained in the cylinder.

(v) ***Insulating stand:*** When the base of the cylinder is attached to the insulating stand, the working substance gets isolated from the surroundings.

Carnot cycle: The working substance is carried through a reversible cycle of the following *four* steps:

Step 1 Isothermal Expansion

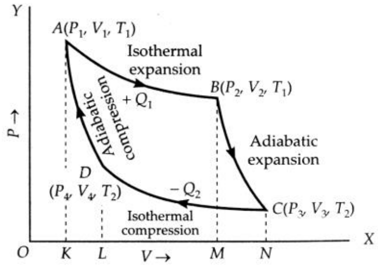
Place the cylinder on the source so that the gas acquires the temperature T_1 of the source. The gas is allowed to expand by slow outward motion of the piston. The temperature of the gas falls. As the gas absorbs the required amount of heat from the source, it expands isothermally. It Q_1 heat is absorbed from the source and W_1 work is done by the gas in isothermal expansion which takes its state from (P_1, V_1, T_1) to (P_2, V_2, T_1), then

$$W_1 = Q_1 = nRT_1 \left(\frac{V_2}{V_1}\right) = \text{area ABMKA}$$

Step 2 Adiabatic Expansion

The gas is now placed on the insulating stand and allowed to expand slowly till its temperature falls to T_2. It W_2 work is done by the gas in the adiabatic expansion which takes its state from (P_2, V_2, T_1) to (P_3, V_3, T_2), then

$$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1} = area\ BCNMB$$



Step 3 Isothermal Compression

The gas is now placed in thermal contact with the sink at temperature T_2. The gas is slowly compressed so that as heat is produced, it easily flows to the sink. The temperature of the gas remains constant at T_2. If Q_2 heat is released by the gas to the sink and W_3 work is done on the gas by the surroundings in the isothermal compression which takes its state from (P_3, V_3, T_2) to (P_4, V_4, T_2), then

$$W_2 = Q_2 = nRT_2\, ln \left(\frac{V_3}{V_4}\right) = area\ CNLDC$$

Step 4 Adiabatic Compression

The cylinder is again placed on the insulating stand. The gas is further compressed slowly till it returns to its initial state (P_1, V_1, T_1). If W_4 is the work done in the adiabatic compression from (P_4, V_4, T_2) to (P_1, V_1, T_1), then

$$W_4 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \ area\ DAKLD$$

Network done by the gas per cycle

Total work done by the gas = $W_1 + W_2$ (in step 1 and 2)

Total work done on the gas = $W_3 + W_4$ (in step 3 and 4)

$\therefore$ Network done by the gas in one complete cycle, $W = W_1 + W_2 - (W_3 + W_4)$

But $W_2 = W_4$

$\therefore$ $W = W_1 - W_3 = Q_1 - Q_2$

Also, W = area ABMKA + area BCNMB – area CNLDC – area DAKLD or W = area ABCDA

Hence *in a Carnot engine, the mechanical work done by the gas per cycle is numerically equal to the area of the Carnot cycle.*

Efficiency of Carnot Engine

It is defined as the ratio of the net work done per cycle by the engine to the amount of heat absorbed per cycle by the working substance from the source.

$$\therefore \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \qquad \text{or} \qquad \eta = 1 - \frac{nRT_2 \ln (V_3/V_4)}{nRT_1 \ln (V_2/V_1)}$$

Now step 2 is an adiabatic process, therefore

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \qquad \text{.... (i)}$$

Similarly, step 4 is an adiabatic compression, therefore

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \qquad \text{.... (ii)}$$

On dividing (i) by (ii), we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \qquad \text{or} \qquad \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{Hence} \quad \eta = 1 - \frac{T_2}{T_1}$$

NOTE

- The efficiency of a Carnot engine
  - depends upon the temperatures of the source and the sink.
  - is independent of the nature of the working substance.
  - is the same for all reversible engines working between the same two temperature
  - is directly proportional to the temperature difference $(T_1 - T_2)$
  - is always less than 100% because $Q_2 < Q_1$
- The efficiency of a Carnot engine will be unity or 100% if $T_1 = \infty$ or $T_2 = 0$ K. As 0 K or infinite temperature cannot be realized, hence a Carnot engine working on reversible cycle cannot have 100% efficiency.
- If $T_1 = T_2$, then $\eta = 0$. This means that the conversion of heat into mechanical work is impossible without having the source and sink at difference temperatures.
- As $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ $\therefore$ If $T_2 = 0$ K, then $Q_2 = 0$.

  Since $T_2 = 0$ cannot be realized, so $Q_2 = 0$ is also not possible. This means that it is not possible to convert whole of the heat energy absorbed from the source into mechanical work continuously, without rejecting a part of it to the sink.

Non–practicability of Carnot engine

Carnot engine is an ideal engine. It cannot be realized in practice due to the following reasons:

(i) It is difficult to realize source and sink of infinite thermal capacity.

(ii) The working substance should be an ideal gas. But no real gas fulfills the ideal gas behaviour.

(iii) The cylinder cannot be provided perfect frictionless piston.

(iv) It is difficult to attain the conditions of reversibility because the processes of expansion and compression have to be carried out very slowly.

Carnot Theorem

It states that (i) no engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same two temperatures and (ii) the efficient of the Carnot engine in independent of the nature of the working substance.

Subjective Assignment – V

Q.1 What is the efficiency of a Carnot engine working between ice point and steam point?

Q.2 One of the most efficient engines ever developed operated between 2100 K and 700 K. Its actual efficiency is 40%. What percentage of its maximum possible efficiency is this?

Q.3 The source temperature of a Carnot engine is 127°C. It takes 500 cal of heat from the source and rejects 400 cal to the sink during each cycle. What is the temperature of the sink?

Q.4 A carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. Find the work done by the engine.

Q.5 The efficiency of a Carnot cycle is 1/6. If on reducing the temperature of the sink by 65°C, the efficiency becomes 1/3, find the initial and final temperatures between which the cycle is working.

Q.6 A reversible engine converts one fifth of heat which it absorbs from source into work. When the temperature of the sink is reduced by 70°, its efficiency is doubled. Calculate the temperature of the source and the sink.

Q.7 A Carnot engine has the same efficiency (i) between 100 K and 500 K and (ii) between T K and 900 K. Calculate the temperature T K of the sink.

Q.8 A Carnot engine whose heat sink is at 27°C has an efficiency of 40%. By how many degrees should the temperature of the source be changed to increase the efficiency by 10% of the original efficiency?

Q.9 An ideal engine operates by taking in steam from a boiler at a temperature of 327°C and rejecting heat to the sink at a temperature of 27°C. The engine runs at 500 rpm and the heat taken is 600 kcal in each revolution. Calculate (i) the Carnot efficiency of the engine (ii) the work done in each cycle (iii) the heat rejected in each revolution and (iv) the power output of this engine.

Q.10 Five moles of an ideal gas are taken in a Carnot engine working between 100°C and 30°C. The useful work done in one cycle is 420 joule. Calculate the ratio of the volume of the gas at the end and beginning of the isothermal expansion. $R = 8.4$ J mole^{-1} K^{-1}.

Q.11 A Carnot cycle is performed by air initially at 327°C. Each stage represents a compression or expansion in the ratio 1 : 6. Calculate (i) the lowest temperature and (ii) efficiency of the cycle. Given $\gamma = 1.4$.

Q.12 A Carnot engine takes in heat from a reservoir of heat at 427°C and gives out heat to the sink at 77°C. How many calories per second must it take from the reservoir in order to produce useful mechanical work at the rate of 357 W?

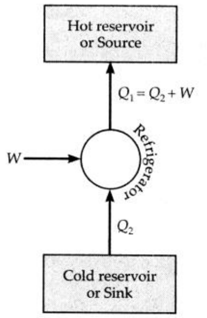
Q.13 Two Carnot engines A and B are operated in series. This first one A receives heat at 900 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 400 K. Calculate the temperature T for the situation when (i) the efficiencies of the two engines are equal (ii) the work outputs of the two engines are equal.

Answers

1.	26.8%	2.	60%	3.	47°C
4.	8.4×10^6 J	5.	117°C, 52°C	6.	350 K, 280 K
7.	180 K	8.	35.7 K or 35.7°C		
9.	50%, 1.26×10^6 J, 300 kcal, 1.05×10^4 kW			10.	1.153
11.	20°C, 51.2%	12.	170 cal s^{-1}	13.	(ii) 600 K, (ii) 650 K

Refrigerator or Heat Pump

A refrigerator is a Carnot's heat engine working in the reverse direction. In a refrigerator, the working substance absorbs an amount of heat Q_2 from the cold reservoir at temperature T_2. An amount of work W is done on it by some external agency (a compressor pump driven by an electric motor) and rejects a larger quantity of heat Q_1 to the source at temperature T_1. In domestic refrigerators, food and ice constitute the cold reservoir and the surrounding act as hot reservoir. Work is done by an electric motor and Freon (CCl_2F_2) is used as a working substance. The working substance is carried through a cycle of the following four steps:



(i) The gas is allowed to expand suddenly (adiabatically) from high to low pressure. This cools it and converts it into a vapour–liquid mixture.

(ii) The cold fluid is allowed to absorb heat Q_2 isothermally from the cold reservoir. This converts the mixture into vapour.

(iii) Then the vapour is adiabatically compressed till it heats up to the temperature of the surrounding.

(iv) Finally the vapour is compressed isothermally in contact with the surroundings. The vapour releases heat Q_1 (= Q_2 + W) to the surroundings and returns to the initial state. Here W is the work done on the gas per cycle.

Coefficient of Performance

It may be defined as the ratio of the amount of heat removed (Q_2) per cycle to the mechanical work (W) required to be done on it.

$$\therefore \qquad \beta = \frac{Q_2}{W}$$

By first law of thermodynamics, the heat released to the hot reservoir is $Q_1 = Q_2 + W$

$$\therefore \qquad W = Q_1 - Q_2 \quad \text{and} \quad \beta = \frac{Q_2}{Q_1 - Q_2}$$

$$\text{or} \qquad \beta = \frac{1}{\left(\frac{Q_1}{Q_2} - 1\right)} = \frac{1}{\frac{T_1}{T_2} - 1} \qquad \left[\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right] \qquad \therefore \qquad \beta = \frac{T_2}{T_1 - T_2}$$

NOTE

- The coefficient of performance represents the efficiency of a refrigerator. Higher the value of β, more efficient is the refrigerator.
- In a heat engine, heat cannot be fully converted into work. Likewise a refrigerator cannot work without some external work done on the system. Hence *the coefficient of performance cannot be infinite*. Practical refrigerators have a coefficient of performance close to ten.
- Lesser the temperature difference ($T_1 - T_2$) between the atmosphere and the cooling chamber, higher is the coefficient of performance of the refrigerator.
- **Why is defrosting necessary?** As the refrigerator works, T_2 decreases due to formation of too much ice and T_1 remains almost constant. The temperature difference ($T_1 - T_2$) increases. This decreases the value of β. Defrosting increases T_2 and hence improves the coefficient of performance.

Subjective Assignment – VI

Q.1 In a refrigerator, heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat shall be delivered to the room for each joule of electrical energy consumed ideally?

Q.2 (i) Calculate the least amount of work that must be done to freeze one gram of water at 0^oC by means of a refrigeration machine. The temperature of the surrounding is 27^oC. (ii) How much heat is passed on to the surroundings in the process?

Q.3 How much energy in watt hour may be required to convert 2 kg of water into ice at 0^oC, assuming that the refrigerator is ideal? Given temperature of freezer is -15^oC, room temperature is 25^oC and initial temperature of water is 25^oC.

Q.4 Ice in a cold storage melts at the rate of 2 kg per hour when the external temperature is 20°C. Find the minimum power output of the motor used to drive the refrigerator which just prevents the ice from melting. The latent heat of ice = 80 cal g^{-1} and J = 4.2 J cal^{-1}.

Q.5 A Carnot engine having a perfect gas as the working substance is driven backward and is used for freezing water already at 0°C. If the engine is driven by a 500W electric motor having an efficiency of 60%, how long will it take to freeze 15 kg of water. Take 15°C and 0°C as the working temperatures of engine and assume there are no heat losses in refrigerating system. Latent heat of ice = 333×10^3 J kg^{-1}.

Q.6 Refrigerator A works between – 10°C and 27°C, while refrigerator B works between – 27°C and 17°C, both removing heat equal to 2000 J from the freezer. Which of the two is the better refrigerator?

Q.7 A refrigerator whose coefficient of performance is 5 extracts heat from the cooling compartment at the rate of 250 J/cycle. How much electric energy is spent per cycle? How much heat per cycle is discharged to the room?

Q.8 A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in a time interval of 20 minutes. Assume that the room temperature is 20°C.Calculcate the minimum power needed to accomplish it.

Answers

1.	13 J	2.	7.91 cal, 87.91 cal	3.	36.96 Wh
4.	13.67 W	5.	914.8 s	6.	Refrigerator A
7.	50J, 300 J	8.	102.5 W		

Conceptual Problems

Q.1 How does the interval energy of an ideal gas differ from that of real gas?

Q.2 Out of a solid, liquid and gas of the same mass and at the same temperature, which one has the greatest internal energy? Which one least? Justify.

Q.3 Cooling is produced when a gas at high pressure suddenly expands. Why?

Q.4 Whose molecules: liquids water at 0°C or ice at 0°C have greater potential energy? Give reason.

Q.5 Can two isothermal curves intersect?

Q.6 Why a gas has two principal specific heat capacities? OR

Gases have two specific heats, but the solids and liquids possess only one specific heat. Why?

Q.7 A liquid is being converted into steam at its boiling point. What will be the specific heat of the liquid at this time?

Q.8 Why can a ship not use the internal energy of sea water to operate its engine?

Q.9 If you are aksed to increase the efficiency of a Carnot engine by increasing the temperature of the source or by decreasing the temperature of the sink by 10 K, which method would you prefer and why?

Q.10 The temperature of the surface of t eh sun is approximately 6000 K. If we take a big lens and focus the sun rays, can we produce a temperature of 8000 K?

Q.11 Is the efficiency of a heat engine more in hilly areas than in the plains?

Q.12 Can a kitchen be cooled by leaving the door of an electric refrigerator open? OR

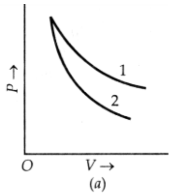
If a door of a working refrigerator is kept open for a long time in a closed room, will it make the room warm or cool?

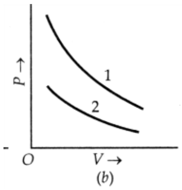
Q.13 A thermos flask contains coffee. It is vigorously shaken. Consider the coffee as the system. (i) Has any heat been added to it? (ii) Has any work been done on it? (iii) Does its temperature rise? (iv) Has its internal energy changed?

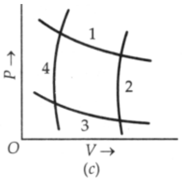
Q.14 When you whistle out air on to your palm held close to your mouth, the air feels cold; but when you blow out air from your mouth, keeping it wide open, the air feels hot. Why?

Q.15 A gas expands in such a manner that its pressure and volume comply with the condition PV^2 = constant. Will the gas cool or get heated on expansion?

Q.16 In figures (a), (b) and (c) given below, identify the isothermal and adiabatic process in each diagram.







NCERT Exercise

Q.1 A geyser heats water flowing at the rate of 3.0 litres per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g?

Q.2 What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen at room temperature to raise its temperature by 45°C at constant pressure?

Q.3 Explain why

(a) Two bodies at different temperatures T_1 and T_2 if brought in thermal contact do not necessarily settle to mean temperature $(T_1 + T_2)/2$.

(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat.

(c) Air pressure in a car tyre increases during driving.

(d) The climate of a harbour town is more temperate than that of a town in a desert at same latitude.

Q.4 A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?

Q.5 In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 223 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take 1 cal = 4.19 J)

Q.6 Two cylinders A and B of equal capacity are connected to each other via a stopcock. The cylinder A contains a gas at standard temperature and pressure, while the cylinder B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following:

(i) What is the final pressure of the gas in A and B?

(ii) What is the change in internal energy of the gas?

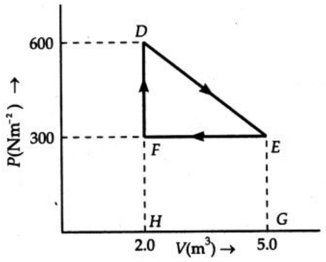
(iii) What is the change in temperature of the gas?

(iv) Do the intermediate states of the system (before settling to final equilibrium state) lie on its P – V – T surface?

Q.7 A steam engine delivers 5.4×10^8 J of work per minute and services 3.6×10^9 J of heat per minute from its boiler. What is the efficiency of the engine? How much heat is wasted per minute?

Q.8 An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate 75 joules per second, at what rate is the internal energy increasing?

Q.9 A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.



Q.10 A refrigerator is to maintain eatables kept inside at 9°C. If room temperature is 36°C, calculate the coefficient of performance.

Answers

1. 15.75 g min^{-1} 2. 933.75 J 4. 2.64
5. + 16.9 J 6. (i) 0.5 atm, (ii) no change, (iii) remain same, (iv) no
7. 3.1×10^9 J 8. 25 W 9. 450 J 10. 10.4

Problems of Higher Order Thinking Skills

Q.1 A certain amount of gas occupies volume V_0 at pressure P_0 and temperature T_0. It is allowed to expand (i) isobarically, (ii) adiabatically and (iii) isothermally, such that final volume is same in each case. In which case the work done is maximum and in which case it is minimum? Explain.

Q.2 Two samples of a gas initially at the same temperature and pressure are compressed from a volume V to V/2, one isothermally, the other adiabatically. In which sample is the final pressure greater?

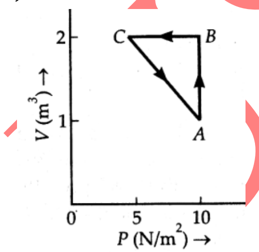
Q.3 Two gases have the same initial pressure P_0, volume V_0 and temperature T_0. They expand to the same volume, one adiabatically and the other isothermally.

(i) In which case is the final pressure greater?
(ii) In which case is the work done greater?
(iii) In which case is the final temperature greater?

Q.4 The work of 146 J is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. Identify the atomicity of the gas.

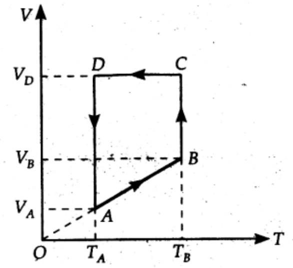
Q.5 In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. What happens to the temperature of the gas?

Q.6 An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in figure. If the net heat supplied to the gas in the cycle is 5 J, what is the work done by the gas in the process $C \rightarrow A$?



Q.7 A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively then, what is the ratio T_1/T_2?

Q.8 A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in figure. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C, calculate,



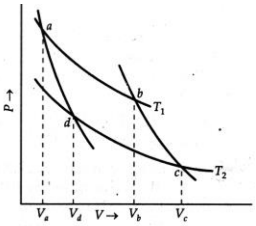
(a) the temperature of the gas at point B
(b) heat absorbed or released by the gas in each process
(c) the total work done by the gas during the complete cycle
Express your answer in terms of the gas constant R.

Q.9 An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding works involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively. Find (i) W_4 and (ii) efficiency of the cycle.

Q.10 Let the temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine be 1500°C and 500°C respectively. Which of these, increasing T_1 by 100°C or decreasing T_2 BY 100°C, would result in a greater improvement in the efficiency of the engine?

Q.11 An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to T/2. (i) How many degrees of freedom do the gas molecules have? (ii) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.

Q.12 Two different adiabatic paths for the same gas intersect two isothermals at T_1 and T_2 as shown in P–V diagram of figure. How does the ratio V_a/V_d compare with the ratio V_b/V_c?

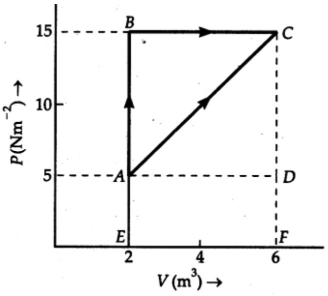


Q.13 In figure, an ideal gas changes its state from state A to C by two paths ABC and AC.

(i) Find the path along which work done is the least.

(ii) The internal energy of gas at A is 10 J and amount of heat supplied to change its state to C through the path AC is 200 J. Calculate the internal energy at C.

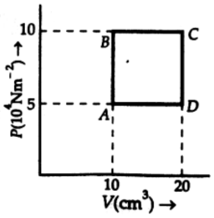
(iii) The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.



Q.14 A sample of 2 kg of monoatomic helium (assumed ideal) is taken the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (figure).

(i) What is the temperature of Helium in each of the state A, B, C and D?

(ii) How much is the heat involved in each of the process ABC and ADC?



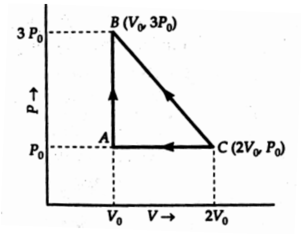
Q.15 One mole of an ideal monoatomic gas is taken round the cyclic process ABCA as shown in figure. Calculate

(i) the work done by the gas,

(ii) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB

(iii) the net heat absorbed by the gas in the path BC, and

(iv) the maximum temperature attained by gas during the cycle.



Answers

4. diatomic   6. – 5J   7. $\left(\frac{T_1}{T_2}\right)=\left(\frac{L_2}{L_1}\right)^{2/3}$

8. (a) 327°C, (b) 1500 R, 831.6 R, – 900 R, – 831.6 R, (c) 600 R   9. (i) 765 J, (ii) 10.83%

10. $\eta_2 > \eta_1$ 12. $\frac{V_a}{V_d}=\frac{V_b}{V_c}$ 13. (i) AC, (ii) 170 J, (iii) 10 J

14. (i) T_A = 120.5 K, T_B = 241 K, T_C = 482 K, T_D = 241 K, (ii) Q_{ABC} = 3.25 MJ, Q_{ADC} = 2.75 MJ

15. (i) P_oV_o, (ii) $Q_{CA} = -\frac{5}{2}P_oV_o,\ Q_{AB}=3P_oV_o$, (iii) $\frac{P_oV_o}{2}$, (iv) $\frac{25}{8}\frac{P_oV_o}{R}$

Objective Assignment – I (C.B.S.E.)

Q.1 The first law of thermodynamics confirms the law of

(a) conservation of momentum of molecules   (b) conservation of energy

(c) flow of heat in a particular direction

(d) conservation of heat energy and mechanical energy

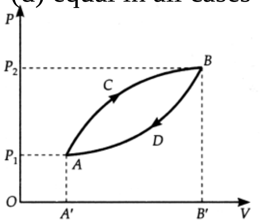
Q.2 A gas expands under constant pressure P from volume V_1 to V_2. The work done by the gas is

(a) $P(V_2 - V_1)$ (b) $P(V_1 - V_2)$ (c) $P\left(V_1^{\gamma} - V_2^{\gamma}\right)$ (d) $P\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

Q.3 A sample of gas expands from volume V_1 to V_2. The amount of work done by the gas is greatest, when the expansion is

(a) isothermal (b) isobaric (c) adiabatic (d) equal in all cases

Q.4 A thermodynamic system is taken from state A to B along ACB and is brought back to A long BDA as shown in the given P–V diagram. The net work done during the complete cycle is given by the area



(a) $P_1ACBP_2P_1$ (b) ACBB' A'A

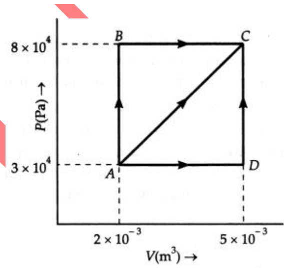
(c) ACBDA (d) ADBB' A'A

Q.5 A thermodynamic process is shown in the figure. The pressures and volumes corresponding to some points in the figure are

$$P_A = 3 \times 10^4 \text{ Pa}, V_A = 2 \times 10^{-3} \text{ m}^3$$

$$P_B = 8 \times 10^4 \text{ Pa}, V_D = 5 \times 10^{-3} \text{ m}^3$$

In process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be



(a) 560 J (b) 800 J (c) 600 J (d) 640 J

Q.6 We consider a thermodynamic system. If ΔU represents the increase in its internal energy and W, the work done by the system; which of the following statements is true?

(a) ΔU = – W in an isothermal process (b) ΔU = W in an isothermal process

(c) ΔU = – W in an adiabatic process (d) ΔU = W in an adiabatic process

Q.7 The internal energy change in a system that has absorbed 2 kcal of heat and done 500 J of work is

(a) 6400 J (b) 5400 J (c) 7900 J (d) 8900 J

Q.8 In thermodynamic processes which of the following statements is not true?

(a) in an isochoric process pressure remains constant

(b) in an isochoric process the temperature remains constant

(c) in an adiabatic process PV^{γ} = constant

(d) in an adiabatic process the system is insulated from the surroundings

Q.9 Which of the following is not thermo–dynamical function?

(a) Enthalpy (b) Work done (c) Gibb's energy (d) Internal energy

Q.10 An ideal gas A and a real gas B have their volumes increased from V to 2V under isothermal conditions. The increase in initial energy

(a) will be same in both A and B (b) will be zero in both the gases

(c) of B will be more than that of A (d) of A will be more than that of B.

Q.11 110 J of heat is added to a gaseous system, whose internal energy increases by 40 J. Then the amount of external work done is

(a) 150 J (b) 70 J (c) 110 J (d) 40 J

Q.12 In an isothermal change of an ideal gas, ΔU = 0. The change in heat energy ΔQ is equal to

(a) 0.5 ΔW (b) ΔW (c) 1.5 ΔW (d) 2 ΔW

Q.13 For hydrogen gas $C_P - C_V = a$ and for oxygen gas $C_P - C_V = b$, so relation between a and b is given by

(a) a = 16b (b) 16a = b (c) a = 4b (d) a = b

Q.14 The molar specific heat at constant pressure of an ideal gas is 7R/2. The ratio of specific heat at constant pressure to that at constant volume is

(a) 9/7 (b) 8/7 (c) 7/5 (d) 5/7

Q.15 For a certain gas, the ratio of specific heats is given to be γ = 1.5. For this gas

(a) $C_v = 3R/J$ (b) $C_p = 3R/J$ (c) $C_v = 5R/J$ (d) $C_p = 5R/J$

Q.16 If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, the change in internal energy of a mass of gas when the volume changes from V to 2V at constant pressure P, is

(a) $\frac{PV}{(\gamma-1)}$ (b) PV (c) $\frac{R}{(\gamma-1)}$ (d) $\frac{\gamma PV}{(\gamma-1)}$

Q.17 One mole of an ideal gas requires 207J heat to raise the temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required is

(a) 198.7 J (b) 29 J (c) 215.3 J (d) 124 J

Q.18 An ideal gas at 27°C is compressed adiabatically to 8/27 of its initial volume. If $\gamma = 5/3$, then the rise in temperature is

(a) 275 K (b) 375 K (c) 475K (d) 175 K

Q.19 A diatomic gas initially at 18°C is compressed adiabatically to 1/8th of its original volume. The temperature after compression will be

(a) 18°C (b) 887.4°C (c) 395.5°C (d) 144°C

Q.20 If in adiabatic change, the pressure P and temperature T of a monoatomic gas are related by the relation $P \propto T^c$, where c is equal to

(a) 5/2 (b) 5/3 (c) 2/5 (d) 3/5

Q.21 One mole of an ideal gas at an initial temperature of T K does 6R joules of work adiabatically. If the ratio of specific heats of this gas at constant pressure and at constant volume is 5/3, the final temperature of gas will be

(a) (T + 2.4) K (b) (T – 2.4) K (c) (T + 4)K (d) (T – 4)K

Q.22 Which of the following processes is reversible?

(a) transfer of heat by conduction (b) transfer of heat by radiation

(c) isothermal compression (d) electrical heating of a nichrome wire

Q.23 The efficiency of a Carnot engine operating with reservoir temperatures kept at 100°C and –23°C will be

(a) $\frac{100+23}{100}$ (b) $\frac{100-23}{100}$ (c) $\frac{373+250}{373}$ (d) $\frac{373-250}{373}$

Q.24 A scientist says that the efficiency of his heat engine, which works at source temperature 127°C and sink temperature 27°C, is 26%. Then

(a) it is impossible (b) it is possible but less probable

(c) it is quite probable (d) data are incomplete

Q.25 An ideal gas heat engine operates in Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cal of heat at higher temperature. The amount of heat converted into work is

(a) 1.2×10^4 cal (b) 2.4×10^4 cal (c) 4.8×10^4 cal (d) 6×10^4 cal

Q.26 An ideal Carnot engine, whose efficiency is 40% receives heat at 500 K. If its efficiency is 50%, then the intake temperature for the same exhaust temperature is

(a) 600 K (b) 700 K (c) 800 K (d) 900 K

Q.27 A reversible engine converts one–sixth of the heat input into work. When the temperature of the sink is reduced by 62°C; the efficiency of the engine is doubled. The temperatures of the source and sink are

(a) 80°C, 37°C (b) 90°C, 37°C (c) 95°C, 37°C (d) 99°C, 37°C

Q.28 A engine has an efficiency of 1/6. When the temperature of sink is reduced by 62°C, its efficiency is doubled. The temperature of source is

(a) 124°C (b) 37°C (c) 62°C (d) 99°C

Q.29 The efficiency of Carnot engine is 50% and temperature of sink is 500 K. If temperature of source is kept constant and its efficiency is raised to 60%, then the required temperature of the sink will be

(a) 100 K (b) 400 K (c) 500 K (d) 600 K

Answers

1.	b	2.	a	3.	b	4.	c	5.	a
6.	c	7.	c	8.	a	9.	b	10.	b
11.	b	12.	b	13.	d	14.	c	15.	b
16.	a	17.	d	18.	b	19.	c	20.	a
21.	d	22.	c	23.	d	24.	c	25.	a
26.	a	27.	d	28.	d	29.	b		

Objective Assignment – II (A.I.I.M.S.)

Q.1 Which of the following is not a thermodynamics co–ordinate?
(a) P (b) T (c) V (d) R

Q.2 Which of the following is path dependent?
(a) U (b) P dV (c) P (d) V

Q.3 The volume of a gas expands by 0.25 m^3 at a constant pressure of 10^3 Nm^{-1}. The work done is equal to
(a) 2.5 erg (b) 250 J (c) 250 W (d) 250 N

Q.4 The latent heat of vaporization of water is 2,240 J. If the work done in the process of vaporization of 1g is 168 J, then increase in internal energy is
(a) 2,408 J (b) 2,240 J (c) 2,072 J (d) 1,904 J

Q.5 If the amount of heat given to a system is 35 J and the amount of work done by the system is – 15 J, then the change in the internal energy of the system is
(a) – 50 J (b) 20 J (c) 30 J (d) 50 J

Q.6 If C_P and C_V are the specific heats for a gas at constant pressure and at constant volume respectively, then the relation $C_P - C_V = R$ is exact for
(a) ideal and real gases at all pressures
(b) ideal gas at all pressures and real gas at moderate pressure
(c) ideal gas and nearly true for real gases at high pressure
(d) ideal gas and nearly true for real gases at moderate pressure

Q.7 In an adiabatic process, the quantity which remains constant is
(a) volume (b) pressure (c) temperature (d) total heat of the system

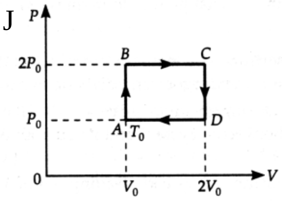
Q.8 The gas law $\left(\frac{PV}{T} = constant\right)$ is true for
(a) isothermal change only (b) adiabatic change only
(c) both isothermal and adiabatic changes (d) neither isothermal nor adiabatic changes

Q.9 A perfect gas is contained in a cylinder kept in vacuum. If the cylinder suddenly bursts, then the temperature of the gas
(a) becomes zero K (b) is decreased (c) is increased (d) remains unchanged

Q.10 The increase in internal energy of a system is equal to the work done on the system. Which process does the system undergo?
(a) isochoric (b) adiabatic (c) isobaric (d) isothermal

Q.11 A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from source per cycle is
(a) 1,200 J (b) 1,600 J (c) 2,400 J (d) 3,200 J

Q.12 N moles of a monoatomic gas it carried round the reversible rectangular cycle ABCDA as shown in the diagram. The temperature at A is T_0. The thermodynamic efficiency of the cycle is



(a) 15% (b) 20% (c) 25% (d) 50%

Q.13 Heat capacity of a substance is infinite. It means

(a) heat is given out (b) heat is taken out

(c) no change in temperature, whether heat is taken in or given out (d) all of these

Q.14 If the temperature of the source is increased, the efficiency of a Carnot engine

(a) increases (b) decreases

(c) remains constant (d) first increases and then remains constant

Assertions and Reasons

Directions – In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion

(b) If both assertion and reason are true but reason is not correct explanation of the assertion

(c) If assertion is true, but reason is false

(d) If both assertion and reason are false

Q.15 **Assertion:** The melting of solid causes an increase in the internal energy.

Reason: latent heat is the heat required to melt a unit mass of the solid.

Q.16 **Assertion:** The isothermal curves intersect each other at a certain point.

Reason: The isothermal changes take place slowly, so the isothermal curves have very little slope.

Q.17 **Assertion:** Heat energy is completely transformed into work during the isothermal expansion of a gas.

Reason: During an isothermal process, the change in internal energy of a gas due to decrease in pressure is nullified by the change due to increase in volume.

Q.18 **Assertion:** The temperature of a gas does not change, when it undergoes an adiabatic expansion.

Reason: During an adiabatic process, heat energy is exchanged between a system and its surroundings.

Q.19 **Assertion:** Air quickly leaking out of a balloon becomes cooler.

Reason: The leaking air undergoes adiabatic expansion.

Q.20 **Assertion:** In adiabatic compression, the internal energy and temperature of the system get decreased.

Reason: The adiabatic compression is a slow process.

Q.21 **Assertion:** When a bottle of cold carbonated drink is opened, a slight fog forms around the opening.

Reason: Adiabatic expansion of gas causes lowering of temperature and condensation of water vapours.

Q.22 **Assertion:** Reversible systems are difficult to find in real world.

Reason: Most processes are dissipative nature.

Q.23 **Assertion:** Thermodynamic processes in nature are irreversible.

Reason: Dissipative effects cannot be eliminated.

Q.24 **Assertion:** It is not possible for a system, unaided by an external agency to transfer heat from a body at a lower temperature to another at a higher temperature.

Reason: It is not possible to violate the second law of thermodynamics.

Q.25 **Assertion:** The temperature of the surface of the sun is approximately 6,000 K. If we take a big lens and focus the sun rays, we can produce a temperature of 8,000 K.

Reason: This higher temperature can be produced according to second law of thermodynamics.

Q.26 **Assertion:** The Carnot cycle is useful in understanding the performance of heat engines.

Reason: The Carnot cycle provides a way of determining the maximum possible efficiency achievable with reservoirs of given temperatures.

Q.27 **Assertion:** When a glass of hot milk placed in a room is allowed to cool, its entropy decreases.

Reason: Allowing hot object to cool does not violate the II law of thermodynamics

Q.28 **Assertion:** In an isolated system, the entropy increases.

Reason: The process in an isolated system is adiabatic.

Answers

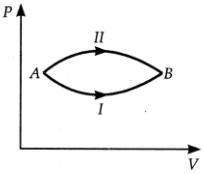
1.	d	2.	b	3.	b	4.	c	5.	d
6.	d	7.	d	8.	c	9.	d	10.	b
11.	b	12.	d	13.	c	14.	a	15.	a
16.	d	17.	b	18.	d	19.	a	20.	d
21.	a	22.	a	23.	a	24.	a	25.	d
26.	a	27.	b	28.	b				

Objective Assignment – III (A.I.E.E.E. & D.C.E.)

Q.1 Which of the following parameters does not characterize the thermodynamic state of matter?

(a) Temperature (b) Pressure (c) Work (d) Volume

Q.2 A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_1 = \Delta U_2$ (b) $\Delta U_1 > \Delta U_2$ (c) $\Delta U_1 < \Delta U_2$

(d) relation between ΔU_1 and ΔU_2 cannot be determined

Q.3 The internal energy change, when a system goes from state A and B is 40 kJ mole^{-1}. If the system goes from A to B by a reversible path and returns to state A by an irreversible path, what would be the net change in internal energy?

(a) 40 kJ (b) > 40 kJ (c) < 40 kJ (d) zero

Q.4 Which of the following is incorrect regarding the first law of thermodynamics?

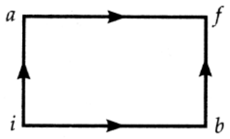
(a) It introduces the concept of the internal energy

(b) It introduces the concept of the entropy (c) It is not applicable to any cyclic process

(d) It is a restatement of the principle of conservation of energy

Q.5 When a system is taken from initial state i to final state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. If along the path ibf, Q = 36 cal, then W along the path ibf is

(a) 6 cal (b) 16 cal (c) 66 cal (d) 14 cal



Q.6 Which of the following statements is correct for any thermodynamic system?

(a) The internal energy changes in all processes

(b) Internal energy and entropy are state functions (c) The change in entropy can never be zero

(d) The work done in an adiabatic process is always zero

Q.7 If c_p and c_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $c_p - c_v = R/28$ (b) $c_p - c_v = R/14$ (c) $c_p - c_v = R$ (d) $c_p - c_v = 28\,R$

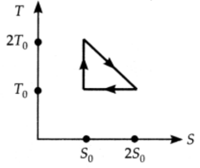
Q.8 The temperature entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is

(a) 1/3

(b) 1/2

(c) 2/3

(d) 1/4



Q.9 "Heat cannot itself flow from a body at lower temperature to a body at higher temperature" is a statement of consequence of

(a) second law of thermodynamics (b) conservation of momentum

(c) conservation of mass (d) first law of thermodynamics

Q.10 Even Carnot engine cannot give 100% efficiency, because we cannot

(a) prevent radiation (b) find ideal sources

(c) reach absolute zero temperature (d) eliminate friction

Q.11 Which statement is incorrect?

(a) all reversible cycles have same efficiency.

(b) reversible cycle has more efficiency than an irreversible one

(c) Carnot cycle is a reversible one

(d) Carnot cycle has the maximum efficiency of all the cycles

Q.12 A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is

(a) zero (b) 8.4×10^6 J (c) 4.2×10^6 J (d) 16.8×10^6 J

Q.13 A Carnot engine, having an efficiency of 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount energy absorbed from the reservoir at lower temperature is

(a) 1 J (b) 90 J (c) 99 J (d) 100 J

Q.14 First law of thermodynamics corresponds to

(a) conservation of energy (b) heat flow from hotter to cooler body

(c) law of conservation of angular momentum (d) Newton's law of cooling

Q.15 Which one of the following is not a state function?

(a) temperature (b) entropy (c) pressure (d) work

Q.16 Which is an intensive property?

(a) volume (b) mass (c) refractive index (d) weight

Q.17 Which one is correct?

(a) In an isobaric process, $\Delta P = 0$ (b) In an isochoric process, $\Delta W = 0$

(c) In an isotheraml process, $\Delta T = 0$ (d) In an isothermal process, $\Delta Q = 0$

Q.18 In an adiabatic system which is true?

(a) $P^{\gamma}T^{\gamma-1}$ = constant (b) $P^{\gamma}T^{1-\gamma}$ = constant (c) PT^{γ} = constant (d) $P^{1-\gamma}T^{\gamma}$ = constant

Q.19 In an adiabatic process, the state of a gas is changed from P_1, V_1, T_1 to P_2, V_2, T_2. Which of the following relations is correct?

(a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ (b) $P_1V_1^{\gamma-1} = P_2V_2^{\gamma-1}$ (c) $T_1P_1^{\gamma} = T_2P_2^{\gamma}$ (d) $T_1V_1^{\gamma} = T_2V_2^{\gamma}$

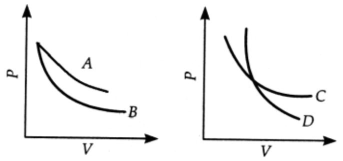
Q.20 In the following figure, four curves A, B, C and D are shown. The curves are

(a) isothermal for A and D while adiabatic for B and C

(b) adiabatic for A and C while isothermal for B and D

(c) isothermal for A and B while adiabatic for C and D

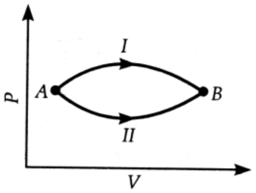
(d) isothermal for A and C while adiabatic for B and D



Q.21 A gas at state A changes to state B through path I and II shown in figure. The changes in internal energy are ΔU_1 and ΔU_2 respectively. Then

(a) $\Delta U_1 > \Delta U_2$

(b) $\Delta U_1 < \Delta U_2$ (c) $\Delta U_1 = \Delta U_2$ (d) $\Delta U_1 = \Delta U_2 = 0$



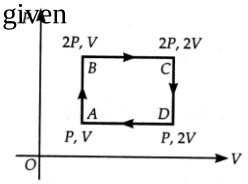
Q.22 An ideal monoatomic gas is taken around the cycle ABCDA as shown in the PV diagram. The work done during the cycle is given by

(a) 1/2 PV

(b) PV

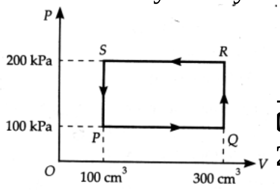
(c) 2 PV

(d) 4 PV



Q.23 A thermodynamic system is taken through cycle PQRSP process. The net work done by the system is

(a) 20 J



(b) – 20 J
(c) 400 J (d) – 374 J

Q.24 Universal gas constant is

(a) $\frac{C_P}{C_V}$ (b) $C_P - C_V$ (c) $C_P + C_V$ (d) $\frac{C_V}{C_P}$

Q.25 1g of water at atmospheric pressure has volume of 1 cm^3 and when boiled it becomes 1681 cm^3 of steam. The heat of vaporization of water is 540 cal/g. Then the change in its internal energy in this process is

(a) 540 cal (b) 500 cal (c) 1681 cal (d) none of these

Q.26 Calculate the work done if temperature is changed from 0^oC to 200^oC at one atmosphere (R = 2 cal K^{-1})

(a) 100 cal (b) 200 cal (c) 400 cal (d) 800 cal

Q.27 One mole of an ideal gas at an initial temperature of T K does 6R joules of work adiabatically. If the ratio of specific heats of this gas at constant pressure and at constant volume is 5/3, the final temperature of gas will be

(a) (T + 2.4) K (b) (T – 2.4)K (c) (T + 4)K (d) (T – 4)K

Q.28 A container having 1 mole of a gas at a temperature 27^oC has a movable piston which maintains a constant pressure of 1 atm in the container. The gas is compressed until temperature becomes 127^oC. The work done (C_P for gas is 7.03 J/mol–K) is

(a) 703 J (b) 814 J (c) 121 J (d) 2035 J

Q.29 A gas is suddenly compressed to 1/4th of its original volume at normal temperature. The increase in its temperature ($\gamma = 1.5$) is

(a) 273 K (b) 573 K (c) 373 K (d) 473 K

Q.30 In an adiabatic process pressure is increased by 2/3%. If $C_P/C_V = 3/2$, then the volume decreases by about

(a) 4/9% (b) 2/3% (c) 4% (d) 9/4%

Q.31 The efficiency of a Carnot engine when source temperature is T_1 and sink temperature is T_2 will be

(a) $\frac{T_1 - T_2}{T_1}$ (b) $\frac{T_2 - T_1}{T_2}$ (c) $\frac{T_1 - T_2}{T_2}$ (d) T_1/T_2

Q.32 A Carnot engine takes heat from a reservoir at 627^oC and rejects heat to a sink at 27^oC. Its efficiency will be

(a) 3/5 (b) 1/3 (c) 2/3 (d) 200/209

Q.33 The temperature of reservoir of Carnot's engine operating with an efficiency of 70% is 1000 K. The temperature of source is

(a) 300 K (b) 400 K (c) 500 K (d) 700 K

Q.34 A Carnot engine has efficiency 1/5. Efficiency becomes 1/3 when temperature of sink is decreased by 50 K. What is the temperature of sink?

(a) 325 K (b) 375 K (c) 300 K (d) 350 K

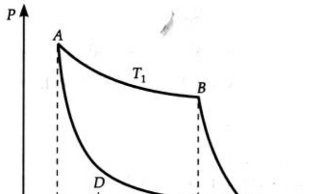
Q.35 The efficiency of Carnot engine is 0.6. It rejects 20 J of heat to sink. The work done by the engine is

(a) 20 J (b) 30 J (c) 33.3 J (d) 50 J

Q.36 In a Carnot engine, when $T_2 = 0^oC$ and $T_1 = 200^oC$, its efficiency is η_1 and when $T_1 = 0^oC$ and $T_2 = -200^oC$, its efficiency is η_2, then what is η_1/η_2?

(a) 0.577 (b) 0.733 (c) 0.638 (d) cannot be calculated

Q.37 The P–V diagram shows that two adiabatic parts of the same gas intersect two isothermals at T_1 and T_2. How the ratios (V_a/V_d) and (V_b/V_c) are related to each other?



(a) $\left(\frac{V_a}{V_d}\right)=2\left(\frac{V_b}{V_c}\right)$ (b) $\left(\frac{V_a}{V_b}\right)=\left(\frac{V_b}{V_c}\right)$

(c) $\left(\frac{V_a}{V_d}\right)=\left(\frac{V_b}{V_d}\right)$ (d) $\left(\frac{V_a}{V_d}\right)=\left(\frac{V_b}{V_c}\right)^2$

Q.38 The freezer in a refrigerator is located at the top section so that
(a) the entire chamber of the refrigerator is cooled quickly due to convection
(b) the motor is not heated (c) the heat gained from the environment is high
(d) the heat gained from the environment is low

Answers

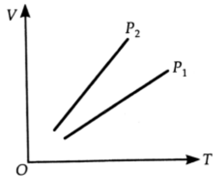
1.	c	2.	a	3.	d	4.	b	5.	a
6.	b	7.	a	8.	a	9.	a	10.	a
11.	a	12.	b	13.	b	14.	a	15.	d
16.	c	17.	d	18.	d	19.	a	20.	d
21.	c	22.	b	23.	b	24.	b	25.	b
26.	c	27.	d	28.	a	29.	a	30.	a
31.	a	32.	c	33.	a	34.	b	35.	b
36.	b	37.	b	38.	a				

Objective Assignment –IV (IIT – JEE)

Multiple Choice Questions with One Correct Answer

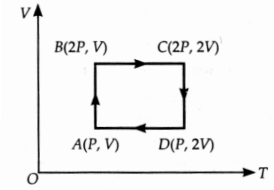
Q.1 In a given process of an ideal gas, dW= 0 and dQ < 0. Then for the gas
(a) the temperature will decrease (b) the volume will increase
(b) the pressure will remain constant (d) the temperature will increase

Q.2 The volume (V) versus temperature (T) graphs for a certain amount of a perfect gas at two pressures P_1 and P_2 are shown in the figure. It follows from the graphs that



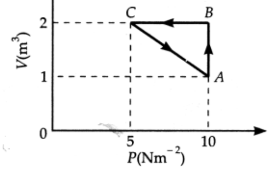
(a) $P_1 > P_2$ (b) $P_1 < P_2$
(c) $P_1 = P_2$ (d) information is insufficient to draw any conclusion

Q.3 An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P–V diagram. The work done during the cycle is



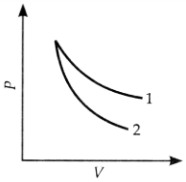
(a) PV
(b) 2 PV
(c) PV/2
(d) zero

Q.4 An ideal gas is taken through the cycle A → B → C → A as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process C → A is



(a) – 5 J
(b) – 10 J
(c) – 15 J
(d) – 20 J

Q.5 P–V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



(a) He and O_2 (b) O_2 and He (c) He and Ar (d) O_2 and N_2

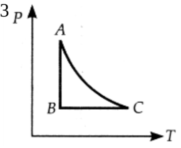
Q.6 An ideal gas initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process, which of the following is true?

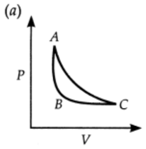
(a) $W > 0;\ P_3 > P_1$ (b) $W < 0;\ P_3 > P_1$ (c) $W > 0;\ P_3 < P_1$ (d) $W < 0;\ P_3 < P_1$

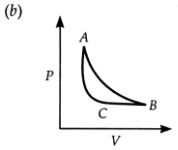
Q.7 A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. L_1 and L_2 are lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

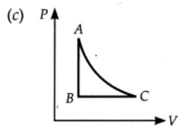
(a) $(L_1/L_2)^{2/3}$ (b) L_1/L_2 (c) L_2/L_1 (d) $(L_2/L_1)^{2/3}$

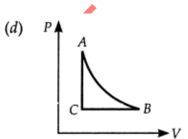
Q.8 The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process. Find the corresponding PV diagram.



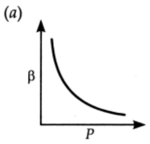


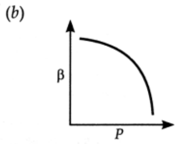


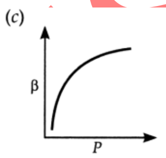


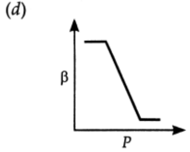


Q.9 Which of the following graphs correctly represents the variation of $\beta = -\dfrac{dV/dP}{V}$ with P for an ideal gas at constant temperature?

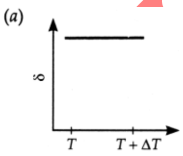


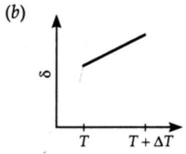


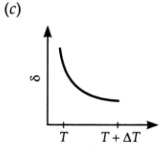


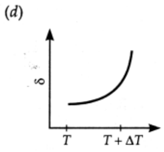


Q.10 An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/V\Delta T$ varies with temperature as









Q.11 One mole of a monoatomic gas is heated at a constant pressure of 1 atmosphere from 0 K to 100 K. If the gas constant R = 8.32 J/ mol K, the change in internal energy of the gas is approximately

(a) 2.3 J (b) 46 J (c) 8.67×10^3 J (d) 1.25×10^3 J

Q.12 An ideal gas heat engine is operating between 227°C and 127°C. It absorbs 10^4 J of heat at the higher temperature. The amount of heat converted into work is

(a) 2000 J (b) 4000 J (c) 8000 J (d) 5600 J

Multiple Choice Questions with One or More than One Correct Answer

Q.13 An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure P/2, volume 2V) along a straight line path in the P–V diagram. Select the correct statement(s) from the following:

(a) the work done by the gas in the process A to B exceeds the work that would be done by it, if the system were taken from A to B along an isotherm.

(b) in the T–V diagram, the path AB becomes a part of a parabola

(c) in the P–T diagram, the path AB becomes a part of a hyperbola

(d) in going from A to B, the temperature T of gas first increases to a maximum value & then decreases

Q.14 During the melting of a slab of ice at 273 K at atmospheric pressure,

(a) positive work is done by the ice–water system on the atmosphere

(b) positive work is done on the ice–water system by the atmosphere

(c) the internal energy of the ice–water system increases

(d) the internal energy of the ice–water system decreases

Q.15 70 calories are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30° to 35°C) at constant volume is

(a) 30 (b) 50 (c) 70 (d) 90

Q.16 For an ideal gas

(a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V (T_2 - T_1)$, where C_V is the molar specific heat at constant volume and n, the number of moles of the gas

(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process

(c) no heat is added or removed in an adiabatic process

(d) the internal energy does not change in an isothermal process

Q.17 Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If rise in temperature of gas in A is 30 K, then the rise in temperature of the gas in B is

(a) 18 K (b) 30 K (c) 50 K (d) 42 K

Q.18 Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that is B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be P and 1.5 P respectively. Then

(a) $2\,m_A = 3m_B$ (b) $3m_A = 2m_B$ (c) $4m_A = 9m_B$ (d) $9m_A = 4m_B$

Answers

1.	a	2.	a	3.	a	4.	a	5.	b
6.	b	7.	d	8.	b	9.	a	10.	c
11.	d	12.	a	13.	a, b, d	14.	b, c	15.	b
16.	a, b, c, d	17.	d	18.	b				

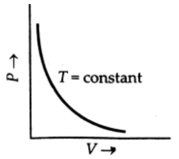
KINETIC THEORY OF GASES

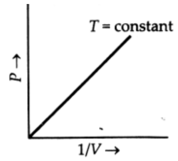
Boyle's Law

It states that the volume of a given mass of a gas is inversely proportional to its pressure, provided the temperature remains constant. Mathematically

$$V \propto \frac{1}{P} \quad \text{or} \quad V = \frac{K}{P} \quad \text{or} \quad PV = K$$

where K is a constant. Its value depends on (i) mass of the gas, (ii) its temperature and (iii) the units in which P and V are measured. If P_1 and V_1 are the initial values of pressure and volume and P_2 and V_2 are their final values, then according to Boyle's law, $P_1V_1 = P_2V_2$





Charles' Law

It states that if the pressure remains constant, then the volume of a given mass of a gas increases by $\frac{1}{273.15}$ *of its volume at 0°C rise or fall of temperature.*

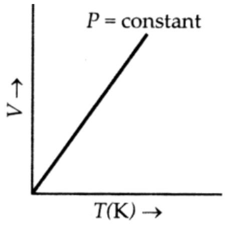
Let V_0 be the volume of the given mass of a gas at 1°C. According to Charles' law, its volume at 1°C is

$$V_1 = V_0 + \frac{V_0}{273.15} = V_0\left(1 + \frac{1}{273.15}\right)$$

Volume of the gas at 2°C, $V_2 = V_0 = \left(1 + \frac{2}{273.15}\right)$

∴ Volume of the gas at t°C,

$$V_t = V_0\left(1 + \frac{t}{273.15}\right) = V_0\left(\frac{273.15 + t}{273.15}\right)$$



If T_0 and T are temperatures on Kelvin scale corresponding to 0°C and t°C, then

$T_0 = 273.15 + 0 = 273.15$ and $T = 273.15 + t$

$$\therefore \quad V_t = V_0\frac{T}{T_0} \quad \text{or} \quad \frac{V_t}{T} = \frac{V_0}{T_0} \quad \text{or} \quad \frac{V}{T} = \text{constant} \quad \text{i.e., } V \propto T$$

So Charles' law can be stated in another way. *Pressure remaining constant, the volume of a given mass of a gas is directly proportional to its absolute temperature.*

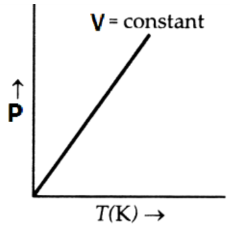
Gay Lussac's Law

It states that if the volume remains constant, the pressure of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0°C for each 1°C rise or fall of temperature. If P_0 and P_t are the pressures of a given mass of gas at 0°C and t°C respectively, then according to Gay Lussac's law,

$$P_t = P_0\left(1 + \frac{t}{273.15}\right) = P_0\left(\frac{273.15 + t}{273.15}\right) \quad \text{or} \quad P_t = P_0\frac{T}{T_0}$$

where T_0 (K) = 273.15 and T (K) = 273.15 + t

$$\therefore \quad \frac{P_t}{P_0} = \frac{T}{T_0} \quad \text{or} \quad \frac{P}{T}\ \text{constant} \quad \text{or} \quad P \propto T$$



So Gay Lussac's law can be stated in another way. *Volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.*

Perfect Gas Equation

Ideal/perfect gas equation: This equation gives the relation between pressure P, volume V and absolute temperature T of a gas. The equation is PV = nRT

where n is the number of moles of the gas and R is the universal gas constant.

Derivation: According to Boyle's law, for a given mass of a gas at constant temperature, $V \propto \frac{1}{P}$

According to Charles' law, for a given mass of a gas at constant pressure, $V \propto T$

Combining the above two laws,

$$V \propto \frac{T}{P} \quad \text{or} \quad V = \text{constant} \times \frac{T}{P} \quad \text{or} \quad \frac{PV}{T} = \text{constant}$$

For one mole of a gas, the constant has same value for all gases and is called *universal gas constant*, denoted by R. So the above equation becomes PV = RT

For n moles of a gas, PV = nRT

This is *perfect or ideal gas equation.*

If v is the volume of 1 gram mass of the gas and M_0 is the molecular mass, then the number of moles is

$$n = \frac{Mass\ of\ \ gas\ (in\ g)}{Molecular\ mass} = \frac{1}{M_0}$$

$$\therefore \qquad Pv = \frac{1}{M_0} RT \qquad \text{or} \qquad Pv = rT$$

This is perfect gas equation for 1 gram of the gas. Here $r = R/M_0$ is the gas constant for one gram of the gas.

Boltzmann's constant (k_B) is the gas constant per molecule.

$$\therefore \qquad k_B = \frac{R}{N_A} \qquad \text{or} \qquad R = k_B N_A$$

As number of moles, $n = \frac{No.\ of\ \ molecules}{Avogadro's\, number} = \frac{N}{N_A}$

$$\therefore \qquad PV = nRT = \frac{N}{N_A}.\ k_B\, N_A.T \qquad \text{or} \qquad PV = k_B NT$$

Units of Universal Gas Constant

From ideal gas equation, $R = \frac{PV}{nT} = \frac{pressure \times volume}{number\ of\ \ moles \times temperature} = \frac{work\ done}{number\, of\ moles \times temperature}$

SI unit of R = $\frac{J}{mole \times K}$ = J mole^{-1} K^{-1}

CGS unit of R = cal mole^{-1} ^{o}C^{-1}

Numerical Value of R

Consider one mole of a gas at S.T.P. Then $R = \frac{P_0 V_0}{T_0}$

Standard pressure, P_0 = 0.76 m of Hg column = 0.76 × 13.6 × 10^3 × 9.8 Nm^{-2}

Standard temperature, T_0 = 273.15 K

Volume of one mole of gas at S.T.P. is V_0 = 22.4 litre = 22.4 × 10^{-3} m^3

$$\therefore \qquad R = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273.15} \qquad \text{or} \qquad R = 8.31\ \text{J mol}^{-1}\ \text{K}^{-1}$$

In the CGS system, $\quad R = \frac{8.31}{4.2}\ cal\ \ mole^{-1\,o}C^{-1} = 1.98\ cal\ \ mole^{-1\,o}C^{-1}$

Numerical value of k_B $\quad k_B = \frac{R}{N_A} = \frac{8.31\, J\ \ mole^{-1}\, K^{-1}}{6.02 \times 10^{23}\ \ mole^{-1}} = 1.38 \times 10^{-23}\ \text{JK}^{-1}$

Ideal Gases

A gas which obeys the ideal gas equation: PV = nRT, at all temperatures and pressures is called an ideal gas or perfect gas. While deriving the ideal gas equation, the following two assumptions are used:

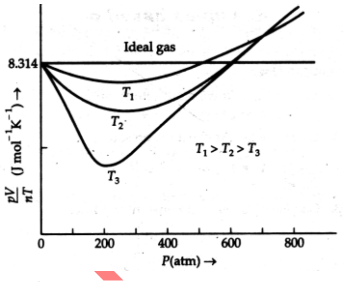
(i) The size of the gas molecules is negligibly small.

(ii) There is no force of attraction amongst the molecules of the gas.

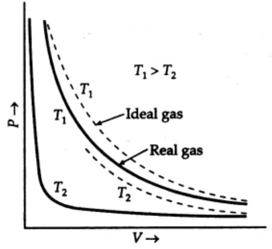
At low pressure and high temperatures, the above assumptions are valid and some real gases like hydrogen, oxygen, nitrogen, helium, etc., almost behave like an ideal gas.

Deviations from ideal behaviour

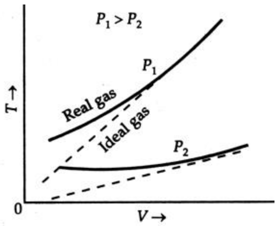
(i) Figure shows the graph of PV/nT against pressure P for three different temperatures. For an ideal gas PV/nT = R = 8.314 J mol^{-1} K^{-1}. Clearly, departures from ideal gas behaviour become less at low pressures and high temperatures.



(ii) Figure shows the comparison between the experimental P–V curves and the theoretical curves predicted by Boyle's law.



(iii) Figure shows the comparison between experimental T–V curves and the theoretical curves predicted by Charles's law.



Subjective Assignment – I

Q.1 A narrow uniform glass tube 80 cm long and open at both ends is half immersed in mercury. Then, the top of the tube is closed and it is taken out of mercury. A column of mercury 22 cm long then remains in the tube. What is the atmospheric pressure?

Q.2 A gas at 27^oC in a cylinder has a volume of 4 litre and pressure 100 Nm^{-2}. (i) Gas is first compressed at constant temperature so that the pressure is 150 Nm^{-2}. Calculate the change in volume (ii) It is then heated at constant volume so that temperature becomes 127^oC. Calculate the new pressure.

Q.3 As an air bubble rises from the bottom of a lake to the surface, its volume is doubled. Find the depth of the lake. Take atmospheric pressure as to be 76 cm of Hg.

Q.4 Using the ideal gas equation, determine the value of R. Given that one gram molecule of a gas at S.T.P. occupies 22.4 litres.

Q.5 A balloon partially filled with Helium has a volume of 30 m^3, at the earth's surface, where pressure is 76 cm of Hg and temperature is 27^oC, what will be the increase in volume of gas if balloon rises to a height, where pressure is 7.6 cm of Hg and temperature is (– 54^oC)?

Q.6 A vessel contains two non–reacting gases: neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressures is 3 : 2. Estimate the ratio of (i) number of molecules and (ii) mass density of neon and oxygen in the vessel. Atomic number of Ne = 20.2, molecular mass of O_2 = 32.0

Q.7 A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at 27^oC temperature and 1.0×10^5 Nm^{-2} pressure. If the gram–molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal.
(R = 8.314 J $mole^{-1}$ K^{-1}). Total mass of the mixture = 28 g.

Q.8 Air is filled in a bottle and it is corked at 35°C. If the cork can come out at 3 atmospheric pressure, then upto what temperature should the bottle be heated in order to remove the cork?

Q.9 A narrow uniform glass tube contains air enclosed by 15 cm long thread of mercury. When the tube is vertical with the open end upper most, the air column is 30 cm long. When the tube is inverted, the length of the air column becomes 45 cm. Calculate the atmospheric pressure.

Q.10 An open glass tube is immersed in mercury so that a length of 8 cm of the tube projects above the mercury. The tube is then closed and raised through 44 cm. What length of the tube will be occupied by the air after it has been raised? Given 1 atm = 76 cm of Hg.

Q.12 A vessel of volume of 2000 cm^3 contains 0.1 mole of O_2 and 0.2 mole of CO_2. If temperature of the mixture is 300 K, find the pressure exerted by it.

Q.13 A vessel of volume, V = 5.0 litres contains 1.4 g of nitrogen at temperature, T = 1800 K. Find the pressure of the gas if 30% of its molecules are dissociated into atoms at this temperature.

Answers

1.	70.9 cm	2.	(i) 1.333 litres, (ii) 200 Nm^{-2}	3.	10.34 m
4.	8.31 J mol^{-1} K^{-1}	5.	189 m^3	6.	1.5, 0.947
7.	m_{Ar} = 23.93 g, m_{Ne} = 4.07 g			8.	651°C
9.	75 cm of Hg	10.	15.4 cm	12.	3.74×10^5 Pa
13.	1.94×10^5 Nm^{-2}				

Kinetic Theory of an Ideal Gas

All matter is made of molecules. The molecules of a gas are in state of rapid and continuous motion. Their velocity depends on temperature.

Assumptions

1. *All gases consist of molecules. The molecules are rigid, elastic spheres identical in all respects for a given gas and different for different gases.*
2. *The size of a molecule is negligible compared with the average distance between the molecules.*
3. *The molecules are in a state of continuous random motion*, moving in all directions with all possible velocities.
4. *During the random motion, the molecules collide with one another and the walls of the vessel.* During collision, their velocities are changed in magnitude and direction.
5. *The collisions are perfectly elastic and there are no forces of attraction or repulsion between the molecules.* Thus all internal energy of the gas is kinetic.
6. *Between two collisions a molecules moves in a straight path with a uniform velocity.* The average distance covered by a molecule between two successive collisions is called *mean free path.*
7. *The collisions are almost instantaneous i.e.*, the time during which a collision lasts is negligible compared to the time of the free path between the molecules.
8. Inspite of the molecular collisions, *the density remains uniform throughout the gas.*

Expression for Pressure Exerted by a Gas

Consider an ideal gas enclosed in a cubical vessel. Suppose the sides of the cube are parallel to the co–ordinate axes, s shown in figure. Let n be the number of gas molecules per unit volume and m be the mass of each molecule. A molecule moving with velocity (v_x, v_y, v_z) hits the planar wall (perpendicular to x–axis) of area A. As the collision is elastic, the molecule rebounds with the same velocity. The y–and z–components of velocity do not change while the x–components reverses sign. So the velocity after the collisions is ($-v_x$, v_y, v_z)

The change in momentum of the molecule

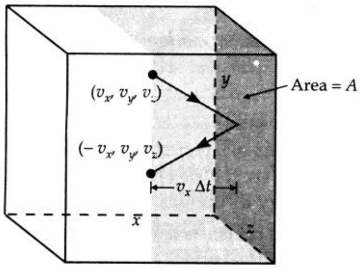
$= -mv_x - mv_x = -2\,mv_x$

By the conservation of momentum, the momentum imparted to the wall in each collision = 2 mv_x.

In small time interval Δt, all those molecules which lie within distance $v_x\,\Delta t$ from the wall of area A will hit this wall. That is, the molecules which lie in the volume $Av_x\,\Delta t$ only will hit the wall in time Δt. On the average, half of such molecules are moving towards the wall and other half away from the wall.

∴ Number of molecules hitting wall of area A in time Δt

$$= \frac{1}{2} A v_x \Delta t \times \text{number of molecules per unit volume} = \frac{1}{2} A v_x \Delta t\, n$$



Total momentum transferred to the wall in time Δt is

$$\Delta p = 2\, m v_x \times \frac{1}{2} A v_x \Delta t\, n = n m v_x^2\, A\, \Delta t$$

$$\text{Force exerted on the wall of area A} = \frac{\Delta p}{\Delta t} = n m v_x^2 A$$

$$\text{Pressure on the wall} = \frac{Force}{Area} = \frac{n m v_x^2 A}{A} \qquad \text{or} \qquad P = n m v_x^2$$

As the molecules move with different velocities, so we replace v_x^2 by its average value v_x^2 in the above equation.

$$\therefore \qquad P = n m \overline{v_x^2}$$

Again, the gas isotropic. So the molecular velocities are almost equally distributed in different directions. By symmetry,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}) = \frac{1}{3}\overline{v^2}$$

where $\overline{v^2}$ is the mean square velocity of the molecules.

$$\text{Hence} \qquad P = \frac{1}{3} n m\, \overline{v^2} \qquad \text{.... (i)}$$

$$\text{Density of gas,} \quad \rho = \frac{Mass}{Volume} = \frac{nm}{1} = nm$$

$$\therefore \qquad P = \frac{1}{3} \rho\, \overline{v^2}$$

According to Pascal's law, a gas transmits pressure equally in all directions.

Relation between pressure and K.E. per unit volume

According to kinetic theory of gases, the pressure exerted by a gas of density ρ and r.m.s. velocity v is given by

$$P = \frac{1}{3} \rho\, v^2$$

Mass per unit volume of the gas = Volume × density

$$= 1 \times \rho = \rho$$

Average K.E. of translation per unit volume of the gas,

$$E' = \frac{1}{2} \rho v^2 \qquad \therefore \qquad \frac{P}{E'} = \frac{\frac{1}{3}\rho v^2}{\frac{1}{2}\rho v^2} = \frac{2}{3} \quad \text{or} \quad P = \frac{2}{3} E' = \frac{2}{3} \times \text{ Average K.E. per unit volume}$$

Hence *the pressure exerted by a gas is equal to two–thirds of average kinetic energy of translation per unit volume of the gas.*

Kinetic Interpretation of Temperature

Consider one mole of a gas. Let P, V, T and M be the pressure, volume, temperature and molecular mass of the gas respectively.

Density, $\rho = \dfrac{M}{V}$

According to kinetic theory, the pressure exerted by the gas is

$$P = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}\frac{M}{V}\overline{v^2} \quad \text{or} \quad PV = \frac{1}{3}M\overline{v^2} = \frac{2}{3}\cdot\frac{1}{2}M\overline{v^2}$$

But $\frac{1}{2}M\overline{v^2}$ is the average kinetic energy E of one mole of the gas. $\therefore \quad PV = \frac{2}{3}E$

The ideal gas equation for one mole of a gas is PV = RT $\therefore \quad \frac{2}{3}E = RT$ or $E = \frac{3}{2}RT$

The above equation gives the *mean kinetic energy of one mole of the gas.* If N is the Avogadro’s number, then the mean kinetic energy per molecule is given by

$$\bar{E} = \frac{E}{N} = \frac{3}{2}\frac{R}{N}T \quad \text{or} \quad \bar{E} = \frac{3}{2}k_B T$$

where k_B = R/N is the gas constant per molecule and is called *Boltzmann’s constant.* Thus *the mean kinetic energy per molecule is proportional to the absolute temperature of the gas. It is independent of the pressure, volume and the nature of the ideal gas.* Clearly,

$$E = \frac{1}{2}M\overline{v^2} = \frac{3}{2}RT \quad \text{or} \quad \overline{v^2} = \frac{3RT}{M}$$

The square root of $\overline{v^2}$ is known as root mean square velocity and is given by

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3\,RT}{M}} \qquad i.e., \quad v_{rms} \propto \sqrt{T}$$

Thus *faster the motion of the molecules of a gas, higher will be their kinetic energy and hence higher will be the temperature of the gas. Hence the temperature of a gas is the measure of the average kinetic energy of its molecules.* This is what we mean by the kinetic interpretation of temperature. Also, at T = 0, $v_{rms} = 0$

So we can define **absolute zero** *as that temperature at which all molecular motion stops.*

Subjective Assignment – II

Q.1 Calculate the r.m.s. velocity of air molecules at S.T.P. Given density of air at S.T.P. is 1.296 kg m^{-3}.

Q.2 A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

Q.3 Calculate the kinetic energy of one mole of argon at 127°C. Given, Boltzmann’s constant, $k_B = 1.381 \times 10^{-23}$ J $molecule^{-1}$ K^{-1}. Avogadro’s number, $N = 6.02 \times 10^{23}$ mol^{-1}.

Q.4 Calculate the kinetic energy per molecule and also r.m.s. velocity of a gas at 127°C. Given $k_B = 1.38 \times 10^{-23}$ J $molecule^{-1}$ K^{-1} and mass per molecule of the gas = 6.4×10^{-27} kg.

Q.5 Calculate the number of molecules in 2×10^{-6} m^3 of a perfect gas at 27°C and at a pressure of 0.01 m of mercury. Mean kinetic–energy of a molecule at 27°C = 4×10^{-11} J and g = 9.8 ms^{-2}.

Q.6 (a) Calculate (i) the root mean square speed and (ii) the mean kinetic energy of one gram molecule of hydrogen at S.T.P. Given that density of hydrogen at S.T.P. is 0.09 kg m^{-3} and R = 8.31 J $mole^{-1}$ K^{-1}.

(b) Given that mass of a molecule of hydrogen is 3.34×10^{-27} kg, calculate Avogadro’s number.

(c) Calculate Boltzmann’s constant

Q.7 At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth? Escape velocity of earth is 11.0 kms^{-1} and mass of one molecule of oxygen is 5.34×10^{-26} kg. Boltzmann constant = 1.38×10^{-23} J $molecule^{-1}$ K^{-1}.

Q.8 A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare (i) average kinetic energies of hydrogen and oxygen molecules (ii) root mean square speed of the molecules and (iii) pressure of gases in A and B.

Q.9 A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of

(i) average kinetic energy per molecule, and

(ii) root mean square speed (v_{rms}) of the molecules of the two gases. Atomic mass of argon = 39.9 u; molecular mass of chlorine = 70.9 u.

Q.10 Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are m_1 and m_2 and the number of molecules in the gases are n_1 and n_2 respectively.

Answers

1.	482.24 ms^{-1}	2.	114 cm of Hg	3.	4988.2 J
4.	(i) 8.28×10^{-21} J, (ii) 1.608×10^3 ms^{-1}			5.	10^8
6.	(a) (i) 1837.5 ms^{-1}, (ii) 3.4×10^3 J, (b) 6.03×10^{23}, (c) 1.38×10^{-23} JK^{-1}				
7.	1.56×10^5 K	8.	(i) 1 : 1, (ii) 4 : 1, (iii) 32 : 1	9.	(i) 1 : 1, (ii) 1.333
10.	$T=\frac{n_1T_1+n_2T_2}{n_1+n_2}$				

Derivation of the Gas Laws

Boyle's Law: *It states that the volume (V) of a given mass of a gas is inversely proportional to its pressure (P) provided the temperature (T) remains unchanged, i.e.,*

$V \propto 1/P$ or PV = constant (At constant T)

Derivation

According to kinetic theory, pressure exerted by a gas is $P=\frac{1}{3}\rho\overline{v^2}=\frac{1}{3}\frac{M}{V}\overline{v^2}$ $\therefore$ $PV=\frac{1}{3}M\overline{v^2}$

But at constant temperature, total kinetic energy of gas $\frac{1}{2}M\overline{v^2}$ or $\overline{v^2}$ will be constant.

$\therefore$ At a constant temperature, PV = constant

This proves the Boyle's law.

Charles' Law: *It states that at constant pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature, i.e.,*

$V \propto T$ (At constant P)

Derivation: According to kinetic theory of gases,

Pressure of a gas, $P=\frac{1}{3}\rho\overline{v^2}=\frac{1}{3}\frac{M}{V}\overline{v^2}$ or $V=\frac{1}{3}\cdot\frac{M}{P}\overline{v^2}$

For a given mass of a gas and at constant pressure P, we have $V \propto \overline{v^2}$

According to kinetic theory, $\overline{v^2} \propto T$ $\therefore$ $V \propto T$

This proves the Charles' law.

Gay Lussac's Law: *It states that at constant volume, the pressure exerted by a given mass of a gas is directly proportional to its absolute temperature, i.e.,*

$P \propto T$ (At constant V)

Derivation: According to kinetic theory of gases, $P=\frac{1}{3}\frac{M}{V}\overline{v^2}$

For a given mass and at constant volume V, we have $P \propto \overline{v^2}$

But $v^2 \propto T$ $\therefore$ $P \propto T$

This proves Gay Lussac's law, also called Regnault's law.

Derivation of perfect gas equation

The pressure exerted by a gas is given by $P=\frac{1}{3}\frac{M}{V}\overline{v^2}$ $\quad\therefore\quad$ $PV=\frac{1}{3}M\,\overline{v^2}$

But $\quad \overline{v^2}\propto T \quad\therefore\quad PV\propto T \quad$ or $\quad PV = RT$

where R is called the gas constant for one mole of gas. The above equation is called the perfect gas equation.

Avogadro's Law

It states that equal volumes of all gases under similar conditions of temperature and pressure, contain equal number of molecules.

Derivation

Consider equal volume (say V each) of two gases A and B at the same temperature T and pressure P. According to kinetic theory, pressure exerted by a gas is

$$P=\frac{1}{3}\frac{M}{V}\overline{v^2}=\frac{1}{3}\frac{mn}{V}\overline{v^2}$$

As pressures exerted by two gases are equal i.e., $P_1 = P_2$

$$\therefore \quad \frac{1}{3}\frac{m_1 n_1 \overline{v_1^2}}{V}=\frac{1}{3}\frac{m_2 n_2 \overline{v_2^2}}{V} \quad \text{or} \quad m_1 n_1 \overline{v_1^2}=m_2 n_2 \overline{v_2^2} \qquad \text{.... (1)}$$

Again, at a given temperature the kinetic energy per molecule of each and every gas is constant and is independent of the nature of gas or the mass of gas molecule. Therefore, for the given gases, we have

$$\frac{1}{2}m_1\overline{v_1^2}=\frac{1}{2}m_2\overline{v_2^2} \quad \text{or} \quad m_1\overline{v_1^2}=m_2\overline{v_2^2} \qquad \text{.... (2)}$$

Dividing (1) by (2), we get : $n_1 = n_2$

$\therefore$ Number of molecules in gas A = Number of molecules in gas B

This proves Avogadro's law.

Graham's Law of Diffusion

It states that rate of diffusion of a gas is inversely proportional to the square root of its density.

Derivation:

Let us consider two gases A and B diffusing into one another. Let ρ_1 and ρ_2 be their densities and v_1 and v_2 be their respective r.m.s. velocities.

Pressure exerted by gas A, $P_1=\frac{1}{3}\rho_1 v_1^2$ and Pressure exert ed by gas B, $P_2=\frac{1}{3}\rho_2 v_2^2$

When steady state of diffusion is reached, $P_1 = P_2$

$$\therefore \quad \frac{1}{3}\rho_1 v_1^2=\frac{1}{3}\rho_2 v_2^2 \quad \text{or} \quad \frac{v_1^2}{v_2^2}=\frac{\rho_2}{\rho_1} \quad \text{or} \quad \frac{v_1}{v_2}=\sqrt{\frac{\rho_2}{\rho_1}}$$

As diffusion is a direct consequence of molecular motion and rate of diffusion of a gas is directly proportional to its r.m.s. velocity. Thus if r_1 and r_2 be the rates of diffusion of gases A and B respectively, then

$$\frac{r_1}{r_2}=\frac{v_1}{v_2}=\sqrt{\frac{\rho_2}{\rho_1}}$$

Dalton's Law of Partial Pressure

It states that the total pressure exerted by a mixture of non–reacting gases occupying a given volume is equal to the sum of the partial pressures which each gas would exert if it alone occupied the same volume at the given temperature.

Derivation

Consider a mixture of gases occupying a volume V. Let m_1, m_2, m_3, ... be the molecular masses of the gases; n_1, n_2, n_3, ... the number of their molecules; P_1, P_2, P_3, ... the pressures exerted by individual gases and v_1, v_2, v_3, ... be the r.m.s. velocities of the molecules of various gases. According to kinetic theory

$$P_1=\frac{1}{3}\frac{m_1n_1}{V}v_1^2,\ P_2=\frac{1}{3}\frac{m_2n_2}{V}v_2^2,\ P_3=\frac{1}{3}\frac{m_3n_3}{V}v_3^2,........$$

Adding, we get

$$P_1+P_2+P_3+.......=\frac{1}{3}\frac{m_1n_1}{V}v_1^2+\frac{1}{3}\frac{m_2n_2}{V}v_2^2+\frac{1}{3}\frac{m_3n_3}{V}v_3^2+.......$$

As the temperature of all the gases in the mixture is the same, therefore

$$\frac{1}{2}m_1v_1^2=\frac{1}{2}m_2v_2^2=m_3v_3^2=....=\frac{1}{2}mv^2\ (say) \qquad \text{or} \qquad m_1v_1^2=m_2v_2^2=m_3v_3^2=......=mv^2$$

$$\therefore \qquad P_1+P_2+P_3+.........=\frac{1}{3V}(n_1+n_2+n_3+......)mv^2=\frac{1}{3}\frac{mn}{V}v^2$$

where n = $n_1 + n_2 + n_3 + ...$, is the total number of molecules in the mixture.

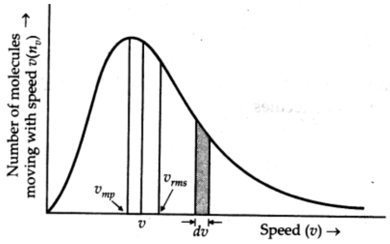
But $\frac{1}{3}\frac{mv}{V}v^2=P$, the total pressure exerted by the mixture

$\therefore$ $P = P_1 + P_2 + P_3 + ...$

This proves the Dalton's law of partial pressures.

Maxwell's Speed Distribution

In any gas, the molecules randomly collide against each other. So the velocity of any individual gas molecule changes continuously. At any instant, the speeds of the molecules vary over a wide range. However, the velocities distribution remains fixed in a steady state. *James Clerk Maxwell* was the first to derive a mathematical relation for the most probable distribution of speeds among the molecules of a gas.



Maxwell's Law of Speed Distribution in a gas at temperature T is

$$dN_v = 4\pi Na^3\ e^{-bv^2}\ v^2\ dv=n_v\ dv$$

where $a=\sqrt{\frac{m}{2\pi k_BT}}$, $b=\frac{m}{2k_BT}$

N = the total number of gas molecules

dN_v = the number of molecules having speeds between v and $v + dv$

The graph of n_v versus v is known as maxwellian speed distribution and is shown in figure.

The important features of the speed distribution curve as follows:

(i) At any temperature, the speed of molecules varies from zero to infinity.

(ii) The area of shaded region gives the number of molecules whose velocities lie in between v and $v + dv$.

(iii) *The speed possessed by the largest fraction of molecules at a given temperature is called most probable speed.* (v_{mp}). It corresponds to the maximum of the curve.

(iv) *The distribution is not symmetric about the most probable speed. Instead, it is skewed.* The area under the curve to the right of the maximum is greater than that to the left. This is because the lowest speed is zero whereas there is no limit to the upper speed that a molecule can have.

(v) The total area under the speed distribution curve gives the total number of molecules in the given sample of the gas.

Average, Root Mean Square and Most Probable Speeds

Average Speed

It is defined as the arithmetic mean of the speeds of the molecules of a gas at a given temperature. If v_1, v_2, v_3, ..., v_n are the speeds of the n gas molecules, then the average speed $\bar{v}$ is given by

$$\bar{v}=\frac{v_1+v_2+v_3+......v_n}{n}$$

By using Maxwell speed distribution law, it can be shown that

$$\bar{v} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8\,RT}{\pi M}} = \sqrt{\frac{8PV}{\pi M}} \qquad [k_B = R/N,\ mN = M,\ RT = PV]$$

where m is the mass of a single molecule and M is the molecular mass of the gas.

Root Mean Square Speed

It is defined as the square root of the mean of the squares of the speeds of the individual molecules of a gas. If $v_1, v_2, v_3, \ldots\ldots, v_n$ are the speeds of the n gas molecules, then the root mean square speed for the gas is given by

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots\ldots + v_n^2}{n}}$$

From Maxwell's speed distribution law, it can be seen that $v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$

$$\therefore \qquad v_{rms} \propto \sqrt{T}$$

Thus, *the root mean square speed of the gas molecules is directly proportional to the square root of the temperature of the gas.*

At a given temperature, $v_{rms} \propto \frac{1}{\sqrt{M}}$

Most Probable Speed

It is defined as the speed possessed by the maximum number of molecules in a gas sample at a given temperature. From Maxwell's speed distribution law, it can be seen that

$$v_{mp} = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

Relations between $\bar{v}, v_{rms}$ and v_{mp}. As the Maxwell's speed distribution curve is not symmetric, so $\bar{v}$, v_{rms} and v_{mp} are not same. Clearly,

$$\bar{v} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8}{3\pi}}\,v_{rms} = 0.92\,v_{rms} \qquad v_{mp} = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{2}{3}}\,v_{rms} = 0.816\,v_{rms}$$

Also, $v_{rms} = \sqrt{3}\sqrt{\frac{k_BT}{m}} = 1.73\sqrt{\frac{k_BT}{m}}$ $\qquad \bar{v} = \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_BT}{m}} = 1.60\sqrt{\frac{k_BT}{m}}$

$$v_{mp} = \sqrt{2}\sqrt{\frac{k_BT}{m}} = 1.41\sqrt{\frac{k_BT}{m}}$$

Ratio $v_{rms} : \bar{v} : v_{mp} = 1.73 : 1.60 : 1.41$ $\qquad$ Hence, $v_{rms} > \bar{v} > v_{mp}$

Subjective Assignment – III

Q.1 Four molecules of a gas have speeds 2, 4, 6 and 8 kms^{-1} respectively. Calculate their average speed and root mean square speed.

Q.2 If three gas molecules have velocities of 0.5, 1 and 2 kms^{-1} respectively, calculate the ratio of their root mean square speed and the average speed.

Q.3 Calculate the r.m.s. velocity of oxygen molecules at S.T.P. The molecular weight of oxygen is 32.

Q.4 The r.m.s. velocity of hydrogen at S.T.P. is u ms^{-1}. If the gas is heated at constant pressure till its volume is three fold, what will be its final temperature and the r.m.s. velocity?

Q.5 The r.m.s. speed of oxygen molecules at a certain temperature T is v. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, what is the changed r.m.s. speed?

Q.6 At what temperature is the r.m.s. velocity of hydrogen molecule equal to that of an oxygen molecule at 47^{o}C?

Q.7 Calculate the temperature at which r.m.s. velocity of gas molecules is double its value at 27°C, pressure of the gas remaining the same.

Q.8 Uranium has two isotopes of masses 235 and 238 units. If both are present in Uranium hexafluoride gas, which would have the larger average speed? If atomic mass of fluorine is 19 units, estimate the percentage difference in speeds at any temperature.

Q.9 The velocities of ten molecules of any gas are given v, 0, 2 v, 4v, 3v, 2v, v, 3v, 5v, v. Calculate their root mean square velocity.

Answers

1.	v_{av} = 5 kms^{-1}, v_{rms} = 5.48 kms^{-1}	2.	1.13	3.	461.23 ms^{-1}
4.	819 K, $\sqrt{3}\,u\ ms^{-1}$	5.	2 v	6.	20 K
7.	927°C	8.	0.44%	9.	2.64 v

Degrees of Freedom

The degrees of freedom of a dynamical system are defined as the total number of co–ordaintes or independent quantities required to describe completely the position and configuration (arrangement of constituent atoms in space) of the system.

The degrees of freedom of a system may also be defined as the total number of independent ways in which the particles of the system can absorb energy.

The number of degrees of freedom of a system is equal to the total number of coordinates required to specify the positions of the constituent particles of the system minus the number of independent relations existing between the particles.

If N = number of particles in the system, k = number of independent relations between the particles, then the number of degrees of freedom of the system is f = 3N – k

(a) Degrees of Freedom of a Monoatomic Gas

The molecule of a monoatomic gas like He, Ne, Ar, etc. consists of a single atom (a point–mass). It is capable of translatory motion only. So it has there degrees of freedom.

(b) Degrees of Freedom of a Diatomic Gas

The molecule of a diatomic gas like N_2, O_2, H_2, CO, etc. consists of two atoms A and B, a fixed distance apart. Corresponding to the translatory motion, the molecule has 3 degrees of freedom. The molecule has two additional degrees of freedom due to rotational motion, about two mutually perpendicular axes passing through its centre of mass. As the atoms are point–masses, so rotation is not possible about the line AB. The rotatory motion contributes 2 degrees of freedom, so that the total degrees of freedom is 5.

At high temperature ($\simeq$ 5000 K), a diatomic molecule has two additional degrees of freedom due to vibrational motion. Each vibrational motion has both kinetic and potential energies.

Law of Equipartition of Energy

It states that in any dynamical system in thermal equilibrium, the energy is equally distributed amongst its various degrees of freedom and the energy associated with each degree of freedom per molecule is ½ $k_B T$, where k_B is Boltzmann's constant and T is the absolute temperature of the system.

Proof: Consider one mole of a monoatomic gas is thermal equilibrium at temperature T. A monoatomic gas molecule can be taken as a point mass. So each such molecule has 3 degrees of freedom due to translatory motion. According to the kinetic theory of a gases, the average translational kinetic energy of a gas molecule is given by $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$

where $\overline{v^2}$ is the mean square velocity of a gas molecule of mass m.

If $\overline{v_x^2}$, $\overline{v_y^2}$ and $\overline{v_z^2}$ are the components of mean square velocity of the gas molecules along the three coordinate axes, then

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

∴ $$\frac{1}{2} m\overline{v_x^2} + \frac{1}{2} m\overline{v_y^2} + \frac{1}{2} \overline{v_z^2} = \frac{3}{2} k_B T$$

As the molecular motion is random, there is no preferred direction of motion. So the average kinetic energy of each molecule along each of the three axes is the same.

∴ $$\frac{1}{2} m\overline{v_x^2} = \frac{1}{2} m\overline{v_y^2} = \frac{1}{2} m\overline{v_z^2}$$

Combining the above two equations, we get $\frac{1}{2} m\overline{v_x^2} = \frac{1}{2} m\overline{v_y^2} = \frac{1}{2} m\overline{v_z^2} = \frac{1}{2} k_B T$

Thus *the average kinetic energy per molecule per degree of freedom is* $\frac{1}{2} k_B T$. This result was first deduced by Boltzmann and is called the *law of equipartition of energy*.

NOTE:

- The law of equipartition of energy holds good for all degrees of freedom whether translational, rotational or vibrational.
- Each square term in the total energy expression of a molecule contributes towards one degree of freedom.
- A monoatomic gas molecule has only translational kinetic energy, $\varepsilon_t = \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2$

  So a monoatomic gas molecule has only three (translational) degrees of freedom.
- In addition of translational kinetic energy, a diatomic molecule has two rotational kinetic energies.

  $$\varepsilon_t + \varepsilon_r = \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2 + \frac{1}{2} I_y\, \omega_y^2 + \frac{1}{2} I_z\, \omega_z^2$$

  Here the line joining the two atoms has been taken as X–axis about which there is no rotation. So the degree of freedom of a diatomic molecule is 5, it does not vibrate.
- Diatomic molecule like CO has a mode of vibration even at moderate temperatures. Its atoms vibrate along the interatomic axis and contribute a vibrational energy term ε_v to the total energy.

  $$\varepsilon = \varepsilon_t + \varepsilon_r + \varepsilon_v$$

  $$= \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2 + \frac{1}{2} m\eta^2 + \frac{1}{2} k\eta^2$$

  where k is the force constant of the oscillator, η the vibrational coordinate and $\eta = \frac{dx}{dt}$.

  So a diatomic molecule has 7 degree of freedom if it vibrate.
- Each translational and rotational degree of freedom corresponds to one mode of absorption of energy and has energy $\frac{1}{2} k_B T$. Each vibrational frequency has two modes of energy (kinetic and potential) with corresponding energy equal to $2 \times \frac{1}{2} k_B T = k_B T$

Specific Heats of Monoatomic, Diatomic and Polyatomic Gases

(i) **Specific Heats of Monoatomic Gas:** In case of a monoatomic gases, like He, Ar, etc., a molecule has three translational degree of freedom. According to the law of equipartition of energy, average energy associated with each degree of freedom per molecule = $\frac{1}{2} k_B T$.

∴ Average energy associated with three degrees of freedom per molecules = $\frac{3}{2} k_B T$

The total internal energy of one mole of a monoatomic gas,

$$U = \frac{3}{2} k_B T \times N_A = \frac{3}{2} RT \qquad [\because k_B\, N_A = R]$$

The molar specific heat at constant volume will be $C_V\,(monoatomic) = \dfrac{dU}{dT} = \dfrac{d}{dT}\left(\dfrac{3}{2}RT\right) = \dfrac{3}{2}R$

The molar specific heat at constant pressure, $C_P\,(monoatomic) = C_V + R = \dfrac{3}{2}R + R = \dfrac{5}{2}R$

Specific heat ratio, $\gamma = \dfrac{C_P}{C_V} = \dfrac{(5/2)\,R}{(3/2)\,R} = \dfrac{5}{3} = \mathbf{1.67}$

(ii) **Specific Heats of Diatomic Gas**

(a) Diatomic molecules such as N_2, O_2, etc., behave as *rigid rotator* at moderate temperatures. Such molecules have 5 degrees of freedom : 3 translational and 2 rotational. According to the law of equipartition of energy, the total energy of a mole of such a gas is

$$U = \frac{5}{2}k_BT \times N_A = \frac{5}{2}RT$$

$$\therefore \quad C_V\,(rigid\ \ diatomic) = \frac{dU}{dT} = \frac{5}{2}R \qquad \text{and}$$

$$C_P\,(rigid\ diatomic) = C_V + R = \frac{7}{2}R$$

$$\gamma\,(rigid\ diatomic) = \frac{(7/2)\ R}{(5/2)\ R} = \frac{7}{5} = \mathbf{1.4}$$

(b) If the diatomic molecule is not rigid but has also a vibrational mode, then each molecule has an additional energy equal to $2 \times \left(\dfrac{1}{2}\right)k_BT = k_BT$, because a vibrational frequency has both kinetic and potential energy modes.

$$\therefore \quad U = \left(\frac{5}{2}k_BT + k_BT\right)N_A = \frac{7}{2}k_B\ N_AT = \frac{7}{2}RT$$

$$C_V\ \text{(diatomic with vibratinoal mode)} = \frac{dU}{dT} = \frac{7}{2}R$$

$$C_P\ \text{(diatomic with vibratinoal mode)} = C_V + R = \frac{9}{2}R$$

$$\gamma\ \text{(diatomic with vibrational mode)} = \frac{(9/2)\,R}{(7/2)\,R} = \frac{9}{7} = \mathbf{1.28}$$

(iii) **Specific Heats of Triatomic Gas**

(a) A non–linear triatomic gas molecule has six degrees of freedom.

$$\therefore \quad U = \frac{6}{2}k_BT \times N_A = 3RT$$

$$C_V = \frac{dU}{dT} = 3R \qquad \text{and} \qquad C_P = C_V + R = 4R$$

$$\gamma = \frac{C_P}{C_V} = \frac{4}{3} = \mathbf{1.33}$$

(b) A linear triatomic molecule has seven degrees of freedom.

$$\therefore \quad U = \frac{7}{2}k_BT \times N_A = \frac{7}{2}\ RT$$

$$C_V = \frac{dU}{dT} = \frac{7}{2}R \qquad \text{and} \qquad C_P = C_V + R = \frac{9}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{(9/2)\,R}{(7/2)\,R} = \frac{9}{7} = \mathbf{1.28}$$

Specific Heats of a Polyatomic Gas

Consider one mole of a perfect polyatomic gas at absolute temperature T. Suppose the total degrees of freedom of each molecule be f. According to the law of equipartition of energy,

average energy of each molecule = $\frac{f}{2}k_BT$

∴ Internal energy of one mole of the gas, $U = \frac{f}{2}k_BT \times N_A = \frac{f}{2}RT$

$$C_V = \frac{dU}{dT} = \frac{f}{2}RT \qquad \text{and} \qquad C_P = C_V + R$$

$$= \frac{f}{2}R + R\left(\frac{f}{2}+1\right)R \qquad \gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2}+1\right)R}{\frac{f}{2}R} \quad \text{or} \quad \gamma = 1 + \frac{2}{f}$$

NOTE:

♦ Generally, a polyatomic atomic gas has 3 translational, 3 rotational degrees of freedom and a certain number (f ') of vibrational modes. Therefore, the internal energy of one mole of such a gas is

$$U = \left(\frac{3}{2}k_BT + \frac{3}{2}k_BT + f'\,k_BT\right)N_A = (3 + f')\,k_B N_A T = (3 + f')\,RT$$

$$\therefore \quad C_V = \frac{dU}{dT} = (3 + f')\,R$$

$$C_P = C_V + R = (4 + f')\,R \quad \text{and} \quad \gamma = \frac{C_P}{C_V} = \frac{4 + f'}{3 + f'}$$

Subjective Assignment – IV

Q.1 Calculate the total number of degree of freedom possessed by the molecules in 1 cm^3 of H_2 gas at N.T.P.

Q.2 Calculate the internal energy of 1 g of oxygen at N.T.P.

Q.3 Hydrogen is heated in a vessel to a temperature of 10,000 K. Let each molecule possess an average energy E_1. A few molecules escape into the atmosphere at 300 K. Due to collisions, their energy changes to E_2. Calculate ratio E_1/E_2.

Q.4 Calculate the molecular K.E. of 1 gram of Helium (Molecular weight 4) at 127°C.

Q.5 How many degrees of freedom are associated with 2 g of He at N.T.P.? Calculate the amount of heat energy required to raise the temperature of this amount from 27°C to 127°C. Given Boltzmann's constant = 1.38×10^{-16} erg $molecule^{-1}$ K^{-1} and Avogadro's number = 6.02×10^{23}.

Q.6 Calculate the limiting ratio of the internal energy possessed by helium and hydrogen gases at 10,000 K.

Q.7 A cylinder of fixed capacity 44.8 litres contains helium gas at standard pressure and temperature. What is the amount of heat needed to raise the temperature of the gas by 15.0°C? R = 8.31 J mol^{-1} K^{-1}.

Q.8 One mole of a monoatomic gas is mixed with three moles of a diatomic gas. What is the molecular specific heat of the mixture at constant volume? Take R = 8.31 J mol^{-1} K^{-1}.

Q.9 One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? Here γ denotes the ratio of specific heat at constant pressure to that at constant volume.

Q.10 A gaseous mixture consists of 16g of helium and 16g of oxygen. Find the ratio C_P/C_V of the mixture.

Q.11 A gaseous mixture enclosed in a vessel contains 1 gram mole of a gas A (with $\gamma = 5/3$) and another gas B (with $\gamma = 7/5$) at a temperature T. The gas A and B do not react with each other and assumed to be ideal. Find the number of gram moles of B if γ for the gaseous mixture is 19/13.

Answers

1.	1.34375×10^{20}	2.	177.2 J	3.	140/3
4.	12.465 J	5.	623.1 J	6.	3 : 7
7.	373.95 J	8.	18.7 J mol^{-1} K^{-1}	9.	1.5

10. 1.62   11. 2

Specific Heat of Solids

Dulong and Petit's Law

Near the room temperature the molar specific heat of most of the solids at constant volume is equal to 3R or 6 cal mol^{-1} *K*$^{-1}$ *or 25 J mol*$^{-1}$ *K*$^{-1}$*. This statement is known as Dulong and Petit's Law.*

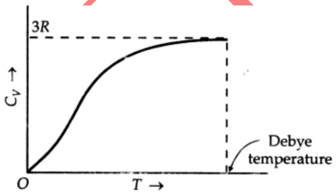
Proof: In a solid, the atoms vibrate about their mean positions. During vibration, the kinetic energy (E_k) of an atom changes continuously into potential energy (E_p) and vice–versa. So the average values of E_k and E_p are equal in a solid. Since an atom can vibrate along three mutually perpendicular directions, it has three degrees of freedom. Applying the law of equipartition of energy, we get

$$E_k = 3 \times \frac{1}{2} k_B T = \frac{3}{2} k_B T \quad \text{and} \quad E_p = 3 \times \frac{1}{2} k_B T = \frac{3}{2} k_B T$$

∴ Average vibrational energy per atom = $E_k + E_p = 3\, k_B T$

The total vibrational energy or the internal energy of one mole of atoms of the solid is given by

$$U = N_A \times 3k_B T = 3RT \qquad [\because R = k_B\ N_A]$$



Since for a solid ΔV is negligible, so $\Delta Q = \Delta U + P\,\Delta V \simeq \Delta U$

Therefore, $C_V = \dfrac{\Delta Q}{\Delta T} = \dfrac{\Delta U}{\Delta T} = 3R$

This proves the Dulong and Petit's law.

Variation of Specific Heat of a Solid with Temperature

Figure shows the variation of molar specific heat (C_V) of a solid as a function of temperature. Clearly, (C_V) of all solids is closed to the Dulong and Petit's value 3R. At lower temperatures, the molar specific heat decreases rapidly with temperature, tending to become zero at 0 K. Physically, this is related to the fact that the number of degrees of freedom of a molecule decreases as we go to low temperatures. In fact, some of the modes of motion get frozen. *The temperature at which the molar specific heat of a solid at constant volume becomes equal to 3R is called* ***Debye temperature.***

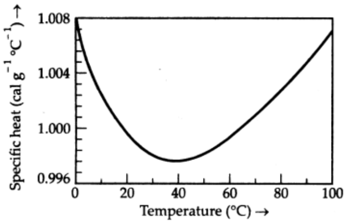
Specific Heat of Water

Water may be treats like a solid. By the law of equipartition of energy, the average vibrational energy per atom is $3\, k_B T$. Now a water molecule has three atoms : two hydrogen and one oxygen.

∴ Average vibrational energy per water molecule = $3 \times 3k_B T = 9\, k_B T$

The total vibrational or the internal energy of one mole of water molecules.

$$U = N_A \times 9\, k_B T = 9\, RT \qquad [\because R = k_B\ N_A]$$



Neglecting ΔV, like for a solid, we get

$$\Delta Q = \Delta U + P\Delta V = \Delta U$$

$$\therefore \quad C_V = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 9R$$

This predicted value is found to be in good agreement with the observed value. The specific heat of water is nearly 75 J mol^{-1} K^{-1} ≈ 9 R.

Variation of Specific Heat of Water with Temperature

Figure shows the variation of specific heat of water with temperature in the temperature range 0°C to 100°C. Water shows perculiar behaviour, its specific heat first decreases and then increases with temperature. For this reason, we have to specify the unit temperature interval for defining calorie.

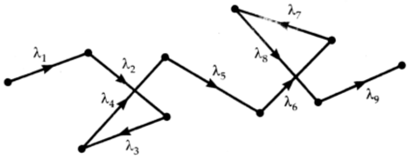
One calorie is defined as the amount of heat required to raise the temperature of 1g of water from 14.5°C to 15.5°C. Hence 1 calorie = 4.186 J

Mean Free Path

The molecules of a gas are in state of continuous, rapid and random motion. As these molecules have a finite though small size, so they collide against one another frequently. *Between two successive collisions, a molecule moves along a straight line path with uniform velocity*. This path is called **free path.** But after every collision, velocity of each molecule changes both in magnitude and direction. Hence each molecule follows a series of straight line zig–zag paths, as shown in figure.

The ***mean free path*** *of a gas molecule may be defined as the average distance travelled by the molecule between two successive collisions.* As shown in figure, if a molecule covers free paths λ_1, λ_2, λ_3, ..., after successive collisions, then its mean free path is given by

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ...}{Total\ number\ of\ collisions}$$



Expression for Mean Free Path

In order to derive the expression for the mean free path, we make use of the following assumptions:

(i) Each molecule of the gas is a sphere of diameter d.

(ii) All molecules of the gas except the molecule A under consideration are at rest.

As shown in figure, suppose the molecule A has average speed $\bar{v}$. It will collide with all those molecules whose centres lie within a distance d from its path. In time Δt, it will obviously collide with all those molecules in the cylinder of volume $\pi d^2 \bar{v}\ \Delta t$. Let n be the number of molecules per unit volume. Number of collisions suffered by the molecule A in time Δt

= Volume of the cylinder swept by molecule A in time Δt × number of molecules per unit volume

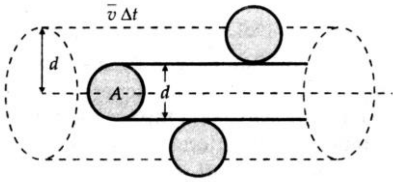
$$= \pi d^2 \bar{v}\ \Delta t \times n$$

Mean free path of a gas molecule,

$$\bar{\lambda} = \frac{Distance\ covered\ in\ time\ \Delta t}{Number\ of\ collisions\ suffered\ in\ time\ \Delta t} = \frac{\bar{v}\ \Delta t}{\pi d^2 \bar{v}\ \Delta t\ n} = \frac{1}{n\pi\ d^2}$$

In the above derivation, we have assumed the other molecules to be at rest. Taking into consideration the motion of all the gas molecules, the mean free path comes out to be $\bar{\lambda} = \frac{1}{\sqrt{2}\ n\pi\ d^2}$

If m is the mass of each gas molecule, then the density of gas is



$$\rho = mn \qquad \text{or} \qquad n = \frac{\rho}{m}$$

$$\therefore \qquad \bar{\lambda} = \frac{m}{\sqrt{2}\,\pi d^2 \rho}$$

For one mole of gas,

$$PV = RT \quad \text{or} \quad P = \frac{RT}{V} = \frac{N}{V} \times \frac{R}{N} \times T = nk_B T \qquad \text{or} \qquad n = \frac{P}{k_B T} \qquad \therefore \quad \bar{\lambda} = \frac{k_B T}{\sqrt{2}\,\pi\ d^2\ P}$$

Factors on which the mean free path depends: It is obvious from the above expressions for $\bar{\lambda}$ that

(i) $\bar{\lambda} \propto m$, i.e., the mean free path is directly proportional to the mass of the gas molecule.

(ii) $\bar{\lambda} \propto \frac{1}{\rho}$, i.e., the mean free path is inversely proportional to the density of the gas.

(iii) $\bar{\lambda} \propto \frac{1}{d^2}$, the mean free path is inversely proportional to the square of the molecular diameter.

(iv) $\bar{\lambda} \propto T$, i.e., the mean free path is directly proportional to the absolute temperature of the gas.

(v) $\bar{\lambda} \propto \frac{1}{P}$, i.e., the mean free path is inversely proportional to the pressure of the gas.

Avogadro's Number

It is the number of atoms present in one gram atom of an element or the number of molecules present in one gram molecule of the substance. In general, *it is the number of particles present in one mole of the substance.* Its most accepted value is

$$N_A = 6.0225 \times 10^{23}\ \text{mole}^{-1}$$

Importance of Avogadro's Number

(i) **To calculate the actual weight of one atom of an element**. Weight of one atom of an element

$$= \frac{Atomic\ weight\ (in\ gram)}{Avogadro's\ number}$$

(ii) **To calculate the actual weight of one molecule of a substance.** Weight of one molecule of a substance

$$= \frac{Molecular\ weight\ (in\ gram)}{Avogadro's\ number}$$

(iii) **To calculate the number of atoms present in given amount of an element.** Number of atoms present in m gram of an element = $\frac{Avogadro's\ number}{Atomic\ weight} \times m$

(iv) **To calculate the number of molecules present in given amount of a substance.** Number of molecules present in m gram of a substance = $\frac{Avogadro's\ number}{Molecular\ \ weight} \times m$

(v) **To calculate the number of molecules present in given volume of the gas**. At S.T.P., the 22.4 litres of every gas contain an Avogadro's number of molecules.

$\therefore$ Number of molecules present in V litres of a gas at S.T.P. = $\frac{Avogadro's\ Number}{22.4} \times V$

Subjective Assignment – V

Q.1 The density of water is 1000 kg m^{-3}. The density of water vapour at 100°C and 1 atm pressure is 0.6 kg m^{-3}. The volume of a molecule multiplied by the total number gives, what is called molecular volume. Estimate the ratio (or fraction) of the molecular volume to the total volume occupied by the water vapour under the above conditions of temperature and pressure.

Q.2 Estimate the volume of a water molecule. Given density of water is 1000 kg m^{-3} and Avogadro's number = 6×10^{23} mole^{-1}.

Q.3 What is the average distance between atoms (interatomic distance) in water? Use the data given in questions 1 and 2.

Answers

1. 6×10^{-4} 2. 3×10^{-29} m^3 3. 40 Å

Brownian Motion

Any object suspended in a fluid is continuously bombarded by the fluid molecules from all directions. If the object is sufficiently small but still visible under a microscope (such as pollen grains which are about 10^{-5} m in diameter), the impact of molecules from all sides gives rise to an unbalanced force in a certain direction. As soon as the particle moves a little, the magnitude and direction of the unbalanced force change making the particle move in a new direction. The suspended object thus moves about in a zig–zag manner and tumbles about randomly. This is said to be Brownian motion.

Factors affecting the Brownian Motion: The Brownian motion increases

(i) with the decrease in size of the suspended particles.
(ii) with the increase in temperature of the fluid.
(iii) with the decrease in density of the fluid.
(iv) with the decrease in viscosity of the fluid.

Conceptual Problems

Q.1 A gas is filled in a cylinder fitted with a piston at a constant temperature. Explain on the basis of kinetic theory:
  (i) The pressure of the gas increases by raising the temperature.
  (ii) On pulling the piston out, the pressure of the gas decreases.

Q.2 There are N molecules of a gas in a container. If the number of molecules is increased to 2N, what will be (i) pressure of the gas, (ii) total energy of the gas and (iii) r.m.s. speed of the gas?

Q.3 Although the velocity of air molecules is nearly 0.5 km s^{-1}, yet the smell of scent spreads at a much slower rate. Why?

Q.4 When do the real gases obey more correctly the gas equation : PV = nRT?

Q.5 Explain qualitatively how the extent of Brownian motion is affected by

(a) size of the Brownian particle, (b) density of the medium,

(c) temperature of the medium, (d) viscosity of the medium?

Q.6 For Brownian motion of particles of suspensions in liquids, what should be the typical size of suspended particles? Why should not the size of the particles to too small (say of atomic dimensions 10^{-10} m) or too large (say of the order of 1 m)?

Q.7 What is the simplest evidence in nature that you can think of to suggest that atoms are not point particles?

Q.8 In an experiment, the specific heats of some inert gases (at ordinary temperatures) are measured to be as follows:

Substance	**Atomic Mass (u)**	**Density (10^3 kg m^{-3})**
Helium	4.00	0.748
Neon	20.18	0.147
Argon	39.94	0.0760
Krypton	83.80	0.0358
Xenon	131.3	0.0226

Try to discover a regularity in the data and explain it on the basis of kinetic energy.

Q.9 What is meant by molar specific heat of a gas? The molar specific heat of hydrogen H_2 is about $\frac{5}{2}$R in the temperature range of about 250 K to 750 K. At lower temperatures, molar specific heat of hydrogen decreases to the value typical of monoatomic gases: $\frac{3}{2}$R. At higher temperatures, it tends to the value $\frac{7}{2}$R. What do you think is happening?

Q.10 (a) When a molecule (or an elastic ball) hits a (massive) wall, it rebounds with the same speed. When a ball hits a massive bat held firmly, the same thing happens. However, when the bat is moving towards the ball, the ball rebounds with a different speed. Does the ball move faster or slower?

(b) When gas in a cylinder is compressed by pushing in a piston, its temperature rises. Guess at an explanation of this in terms of kinetic theory using (a) above.

(c) What happens when a compressed gas pushes a piston out and expands? What would you observe?

(d) Sachin Tendulkar uses a heavy cricket bat while playing. Does it help him in any way?

Answers

2. (i) double, (ii) total energy would double but average energy per molecule remain same, (iii) same

3. because of Brownian motion. 4. At low pressure and high temperature

5. (a) increase, (b) increase, (c) increase, (d) decrease 8. specific heat is nearly 3 cal mol^{-1} k^{-1}

10. At lower temperature only translatory motion. At moderate temperature translatory and rotational motion. At high temperature vibrational motion also comes into play.

NCERT Exercise

Q.1 Estimate the fraction of the molecular volume to the actual volume occupied by oxygen gas at STP. Take the radius of an oxygen molecule to roughly 3Å.

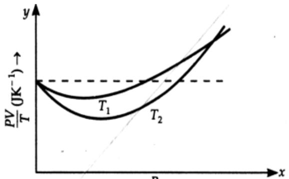
Q.2 Molar volume is the volume occupied by 1 mole of any (ideal) gas at standard temperature and pressure (0°C, 1 atmosphere pressure). Show that it is 22.4 litres.

Q.3 Figure shows plot of PV/T versus P for 1.00 × 10^{-3} kg of oxygen gas at two different temperatures.

(a) What does the dotted plot signify?

(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y–axis?



(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y–axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)?
(Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}).

Q.4 An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drop to 17°C. Estimate the mass of oxygen taken out of the cylinder, R = 8.3 J mol^{-1} K^{-1}, molecule weight of oxygen = 32.

Q.5 An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow, when it reaches the surface, which is at temperature of 35°C?

Q.6 Estimate the total number of molecules inclusive of oxygen, nitrogen, water vapour and other constituents in a room of capacity 25.0 m^3 at a temperature of 27°C and 1 atmospheric pressure.

Q.7 Estimate the average energy of a helium atom at (i) room temperature (27°C) (ii) the temperature on the surface of the sum (6000 K) and (iii) the temperature of 10^7 K.

Q.8 Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexaflouroride (polyatomic). (i) Do the vessels contain equal number of respective molecules? (iii) Is the root mean squared speed of molecules same in the three cases? If not, in which case is v_{rms} the largest?

Q.9 At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at – 20°C?

Q.10 Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule movies freely between two successive collisions

Q.11 A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Q.12 From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 cm^3 s^{-1}. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 cm^3 s^{-1}. Identify the gas.

Q.13 A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so–called law of atmospheres,

$$n_2 = n_1 \exp\left[-mg(h_2 - h_1)/k_BT\right]$$

where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column.

$$n_2 = n_1 \exp\left[-mg N_A (\rho - \rho')(h_2 - h_1)/\rho RT\right]$$

where ρ is the density of the suspended particle, and ρ' that of surrounding medium [N_A is Avogadro's number, and R the universal gas constant]

Q.14 Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms:

Substance	**Atomic Mass (u)**	**Density (10^3 kg m^{-3})**
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

Answers

1. 3×10^{-3} 2. 22.4

3. (a) ideal behaviour, (b) $T_1 > T_2$, (c) 0.26 JK^{-1}, (d) 6.3×10^{-5} kg, No

4. 0.141 kg 5. 5.275×10^{-6} m^3 6. 6.117×10^{26}

7. (i) 6.21×10^{-21} J, (ii) 1.242×10^{-19} J, (iii) 2.07×10^{-16} J 8. (i) yes, (ii) maximum for neon

9. 2523.7 K 10. 500, 1.1×10^{-7} m, 4.6×10^{9} Hz

11. decrease in length by 23.8 cm 12. oxygen gas 14. 1.29Å, 1.59Å, 1.73Å, 1.88Å

Objective Assignment – I (CBSE and AIIMS)

Q.1 At constant volume temperature is increased, then
(a) collisions on walls will be less
(b) number of collisions per unit time will increase
(c) collisions will be in straight lines
(d) collisions will not change

Q.2 Three containers of the same volume contain three different gases. The masses of the molecules are m_1, m_2 and m_3 and the number of molecules in their respective containers are N_1, N_2 and N_3. The gas pressures in the containers are P_1, P_2 and P_3 respectively. All the gases are now mixed and put in one of these containers. The pressure P of the mixture will be
(a) $P < (P_1 + P_2 + P_3)$
(b) $P = \frac{P_1 + P_2 + P_3}{3}$
(c) $P = P_1 + P_2 + P_3$
(d) $P > (P_1 + P_2 + P_3)$

Q.3 The relation PV = RT can describe behaviour of a real gas at
(a) high temperature and high density
(b) high temperature and low density
(c) low temperature and low density
(d) low temperature and high density

Q.4 The equation of a state for 5g of oxygen at a pressure P and temperature T, when occupying a volume V, will be
(a) PV = 5RT/32
(b) PV = 5RT/16
(c) PV = 5RT/2
(d) PV = 5RT

Q.5 Relation between pressure P and average kinetic energy E per unit volume of a gas is
(a) P = 2E/3
(b) P = E/3
(c) P = 3E/2
(d) P = 3E

Q.6 At 0K, which of the following properties of a gas will be zero?
(a) kinetic energy
(b) potential energy
(c) vibrational energy
(d) density

Q.7 The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to
(a) m^0
(b) m
(c) $\sqrt{m}$
(d) $m^{-1/2}$

Q.8 The temperature of a gas is raised from 27°C to 927°C. The r.m.s. molecular speed
(a) gets halved
(b) gets doubled
(c) gets $\sqrt{927/27}$ times the earlier value
(d) remains unchanged

Q.9 An ant is walking on the horizontal surface. The number of degrees of freedom of ant will be
(a) 1
(b) 2
(c) 3
(d) 6

Q.10 The number of degrees of freedom for a diatomic gas molecule is
(a) 2
(b) 3
(c) 5
(d) 8

Q.11 The degree of freedom of a triatomic gas is
(a) 1
(b) 2
(c) 6
(d) 8

Q.12 A polyatomic gas with n degrees of freedom has a mean energy per molecule given by
(a) nkT/N
(b) nkT/2N
(c) nkT/2
(d) 3kT/2

Q.13 Temperature of oxygen kept in a vessel is raised by 1°C at constant volume. Heat supplied to the gas may be taken partly as translational and partly rotational kinetic energies. Their respective shares are
(a) 60%, 40%
(b) 50%, 50%
(c) 100%, zero
(d) 40%, 60%

Q.14 If for a gas $R/C_V = 0.67$, this gas is made up of molecules, which are
(a) diatomic
(b) mixture of diatomic and polyatomic
(c) monoatomic
(d) polyatomic

Q.15 If γ is the ratio of specific heats of a perfect gas, then the number of degrees of freedom of a molecule of the gas is

(a) $\frac{25(\gamma-1)}{2}$ (b) $\frac{9(\gamma-1)}{2}$ (c) $\frac{3\gamma-1}{2\gamma-1}$ (d) $\frac{2}{\gamma-1}$

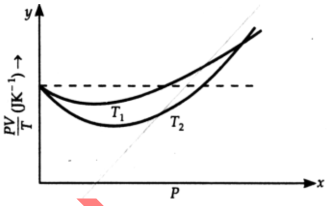
Q.16 The value of critical temperature in terms of van der Wall's constants a and b is given by

(a) $T_c = \frac{a}{2Rb}$ (b) $T_c = \frac{a}{27bR}$ (c) $T_c = \frac{8a}{27Rb}$ (d) $T_C = \frac{27a}{8Rb}$

Q.17 For Boyle's law to hold good, the gas should be

(a) perfect and of constant mass and temperature (b) real and of constant mass and temperature
(c) perfect and at constant temperature but variable mass
(d) real and at constant temperature but variable mass

Q.18 The figure shows the plot of PV/nT versus P for oxygen gas at two different temperatures. Read the following statements concerning the above curves:



(i) The dotted line corresponds to ideal gas behaviour (ii) $T_1 > T_2$

(iii) The value of PV/nT at the point, where the curves meet on the Y–axis is the same for all gases.
Which of the above statements is true?

(a) (i) only (b) (i) and (ii) only (c) all the above (d) none of the above

Q.19 A gas behaves as an ideal gas at

(a) low pressure and high temperature (b) low pressure and low temperature
(c) high pressure and low temperature (d) high pressure and high temperature

Q.20 A gas in a container A is in thermal equilibrium with another gas of the same mass of container B. If we denote the corresponding pressures and volumes by the suffixes A and B, then which of the following statements is most likely to be true:

(a) $P_A = P_B, V_A \neq V_B$ (b) $P_A \neq P_B, V_A = V_B$ (c) $P_A/V_A = P_B/V_B$ (d) $P_AV_A = P_BV_B$

Q.21 An ideal gas is heated from 27°C to 627°C at constant pressure. If initial volume was 4m^3, then the final volume of the gas will be

(a) 2m^3 (b) 4 m^3 (c) 6 m^3 (d) 12 m^3

Q.22 In kinetic theory of gas a molecule of mass m of an ideal gas collides with a wall of vessel with velocity v. The change in the linear momentum of the molecule is

(a) 2 mv (b) mv (c) – mv (d) zero

Q.23 The temperature of a gas is held constant, while its volume is decreased. The pressure exerted by the gas on the walls of the container increases, because its molecules

(a) strike the walls with higher velocities (b) strike the walls with large force
(c) strike the walls more frequently (d) are in contact with the walls for a shorter time

Q.24 In a vessel, the gas is at a pressure P. If the mass of all the molecules is halved and their speed is doubled, then the resultant pressure will be

(a) 4 P (b) 2 P (c) P (d) P/2

Q.25 We have a jar filled with gas characterised by parameters P, V and T and another jar B filled with a gas with parameters 2P, V/4, 2T; where the symbols have their usual meanings. The ratio of the number of molecules of jar A to those of jar B is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

Q.26 The absolute zero is the temperature, at which

(a) all substances exist in solid state (b) water freezes
(c) molecular motion ceases (d) none of these

Q.27 The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J. The average kinetic energy of a gas molecule at 227°C will be

(a) 52.2×10^{-21} J (b) 5.22×10^{-21} J (c) 10.35×10^{-21} J (d) 11.35×10^{-21} J

Q.28 When we heat a gas–sample from 27°C to 327°C, then the initial average kinetic energy of the molecules was E. What will be the average kinetic energy?

(a) 327 E (b) 300 E (c) 2 E (d) $\sqrt{2}$ E

Q.29 v_{rms}, v_{av} and v_{mp} are root mean square, average and most probable speeds of molecules of a gas obeying Mexwellian velocity distribution. Which of the following statements is correct?

(a) $v_{rms} < v_{av} < v_{mp}$ (b) $v_{rms} > v_{av} > v_{mp}$ (c) $v_{mp} < v_{rms} < v_{av}$ (d) $v_{mp} > v_{rms} > v_{av}$

Q.30 A bulb contains one mole of hydrogen mixed with one mole of oxygen at temperature T. The ratio of r.m.s. values of velocity of hydrogen molecules to that of oxygen molecules is

(a) 1 : 16 (b) 1 : 4 (c) 4 : 1 (d) 16 : 1

Q.31 In an adiabatic change, the pressure and temperature of a monoatomic gas are related as $P \propto T^c$, where c equals

(a) 2/5 (b) 5/2 (c) 3/5 (d) 5/3

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reasons is not correct explanation of the assertion.
(c) If assertion is true, but reason is false
(d) If both assertion and reason are false

Q.32 **Assertion:** Air pressure in a car tyre increases during driving.
Reason: Absolute zero temperature is not zero energy temperature.

Q.33 **Assertion:** For an ideal gas at constant temp., the product of the pressure and volume is a constant.
Reason: The mean square velocity of the molecules is inversely proportional to mass.

Q.34 **Assertion:** The root mean square and most probable speeds of the molecules in a gas are the same.
Reason: The Maxwell distribution for the speed to molecules in a gas is symmetrical.

Q.35 **Assertion:** The ratio C_p/C_v is more for helium gas than that for hydrogen gas.
Reason: Atomic mass of helium is more than that of hydrogen.

Q.36 **Assertion:** The ration C_p/C_v for diatomic gas is more than that for a monoatomic gas.
Reason: Molecules of a monoatomic gas have more degrees of freedom than those of a diatomic gas.

Q.37 **Assertion:** The melting point of ice decreases with increase of pressure.
Reason: Ice contracts on melting.

Q.38 **Assertion:** In pressure temperature (P–T) phase diagram of water, the slope of the melting curve is found to be negative.
Reason: Ice contracts on melting to water.

Answers

1.	b	2.	c	3.	b	4.	a	5.	a
6.	a	7.	d	8.	b	9.	b	10.	c
11.	c	12.	c	13.	a	14.	c	15.	d
16.	c	17.	a	18.	c	19.	a	20.	d
21.	d	22.	a	23.	c	24.	c	25.	d
26.	c	27.	c	28.	c	29.	b	30.	c
31.	b	32.	b	33.	b	34.	d	35.	a
36.	a	37.	a	38.	b				

Objective Assignment – II (AIEEE and IIT – JEE)

Q.1 Two thermally insulated vessels 1 and 2 are filled with air at temperatures, (T_1, T_2), volumes (V_1, V_2) and pressures (P_1, P_2) respectively. If the value joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $\frac{T_1+T_2}{2}$ (c) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

Q.2 An insulated container of gas has two chambers separated by an insulated partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2}$ (b) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (c) $\frac{P_1V_1T_2+P_2V_2T_2}{P_1V_1+P_2V_2}$ (d) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$

Q.3 Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will

(a) increase (b) decrease (c) remain the same
(d) decreases for some, while increases for others

Q.4 At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(a) – 73 K (b) 3 K (c) 20 K (d) 80 K

Q.5 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v for the gas is

(a) 4/3 (b) 2 (c) 5/3 (d) 3/2

Q.6 One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure to that at constant volume.

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3

Q.7 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of gas with $\gamma = 5/3$, then value of γ of the resulting mixture is

(a) 7/5 (b) 2/5 (c) 3/2 (d) 12/7

Q.8 A gaseous mixture consists of 16g of helium and 16g of oxygen. The ratio of the two specific heats of the mixture is

(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62

Q.9 The work of 146 kJ is performed in order to compress one kilomole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is

(a) a mixture of monoatomic and diatomic (b) monoatomic
(c) diatomic (d) triatomic

Q.10 One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J (c) 6×10^4 J (d) 7×10^4 J

Q.11 Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature $(7/3)T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$ (c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$

Q.12 A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (relative molar mass 4) at a temperature 2T has a pressure of

(a) P/8 (b) P (c) 2P (d) 8P

Q.13 The average translational kinetic energy of O_2 molecules (relative molar mass 32) at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 molecules (relative molar mass 28) in eV at the same temperature is

(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768

Q.14 The average translational energy and the r.m.s. speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21}J and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(a) 12.42×10^{-21} J, 968 ms^{-1} (b) 8.78×10^{-21}J, 684 ms^{-1}
(c) 6.21×10^{-21} J, 968 ms^{-1} (d) 12.42×10^{-21}J, 684 ms^{-1}

Q.15 A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(a) 4RT (b) 9RT (c) 11RT (d) 15RT

Q.16 The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(a) $\sqrt{2/7}$ (b) $\sqrt{1/7}$ (c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

Q.17 From the following statements, concerning ideal gas at any given temperature T, select the correct one(s):

(a) The co–efficient of volume expansion at constant pressure is the same for all ideal gases

(b) Average translational kinetic energy per molecule of oxygen gas is 3kT, k being Boltzmann constant

(c) The mean–free path of molecules increases with increases in the pressure

(d) In a gaseous mixture, average translational kinetic energy of molecules of each component is different.

Q.18 An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(a) 1/T (b) 2/T (c) 3/T (d) 4/T

Multiple Choice Questions with One or More than One Correct Answer

Q.19 At room temperature, the r.m.s. speed of the molecules of a certain diatomic gas is found to be 1,920 ms^{-1}. The gas is

(a) H_2 (b) F_2 (c) O_2 (d) Cl_2

Q.20 The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the r.m.s. velocity of the gas molecules is v; at 480 K, it becomes

(a) 4v (b) 2v (c) v/2 (d) v/4

Q.21 Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square sped and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of the molecule is m. Then

(a) no molecule can have a speed greater than $\sqrt{2}v_{rms}$ (b) no molecule can have a speed less than $v_p/\sqrt{2}$

(c) $v_p < \bar{v} < v_{rms}$ (d) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$.

Q.22 A vessel contains a mixture of 1 mole of oxygen and 2 moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) depends on the moment of inertia of the two molecules

Q.23 When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied, which increases the internal energy of the gas is

(a) 2/5 (b) 3/5 (c) 3/7 (d) 5/7

Q.24 If one mole of monoatomic gas (γ = 5/3) is mixed with one mole of a diatomic gas (γ = 7/5), the value of adiabatic exponent γ for mixture is

(a) 1.35 (b) 1.40 (c) 1.50 (d) 1.75

Q.25 A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(a) 2/3 P (b) P (c) 3/2 P (d) 2 P

Q.26 Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessels A contains only O_2, B and N_2 and C a mixture of equal quantities O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1 that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is

(a) $\frac{v_1 + v_2}{2}$ (b) v_1 (c) $(v_1 . v_2)^{1/2}$ (d) $\sqrt{\frac{3kT}{M}}$

where M is the mass of an oxygen molecule.

Q.27 C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(c) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas
(d) $C_p . C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Answers

1.	a	2.	b	3.	c	4.	c	5.	d
6.	a	7.	c	8.	d	9.	c	10.	b
11.	d	12.	c	13.	c	14.	d	15.	c
16.	c	17.	a, b, c, d	18.	c	19.	a	20.	b
21.	c, d	22.	a	23.	d	24.	c	25.	a
26.	b	27.	b, d						

THERMAL PROPERTIES OF MATTER

Heat

Heat is a form of energy which produces in us the sensation of hotness or coldness. For example, if we touch a piece of ice, heat flows from our body towards ice and we feel cold. Similarly, when we stand near a fire, heat from the fire flows towards our body and we feel hot.

(a) **Caloric theory of heat:** According to this theory, *heat is an invisible, weightless and odourless fluid called* **caloric.**

(b) **Dynamic theory of heat:** According to this theory, all substances (solids, liquids and gases) are made of molecules. These molecules are in a state of continuous random motion.

Depending on temperature and nature of the substance, the molecules may possess three types of motion:

(i) *Translatory motion:* That is, the motion in a straight line which is common in gases.

(ii) *Vibratory motion:* That is, the to and fro motion of the molecules about their mean positions. This occurs usually at high temperature.

(iii) *Rotatory motion:* That is, the rotation of the molecules about their axes. This is common in liquids and gases.

When a body is heated, all these molecular motions become fast. The kinetic energy of a molecule due to each type of motion increases. So we can regard *heat as an energy of molecular motion which is equal to the sum of total kinetic energy possessed by the molecules of a body by virtue of their translational, vibrational and rotational motions.*

CGS unit of heat

The CGS unit of heat is **calorie** (cal). *One calorie is defined as the heat energy required to raise the temperature of one gram of water through $1^{\circ}C$ (from 14.5 to $15.5^{\circ}C$)*

SI unit of heat: Like all other forms of energy, the SI unit of heat is joule (J)

Joule's Mechanical Equivalent of Heat

From experiments, Joule established a relation between the work done and heat produced. He showed that whenever a given amount of work (W) is converted into heat, always the same amount of heat (Q) is produced, thus $W \propto Q$ or $W = JQ$ or $J = \frac{W}{Q}$

If $Q = 1$, then $J = W$

The proportionality constant J is called *Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.*

$$J = 4.186\ \text{cal}^{-1} = 4.186 \times 10^{7}\ \text{erg cal}^{-1}$$

NOTE: J is not a physical quantity. It just a conversion factor.

Temperature

Temperature is the degree of hotness or coldness of a body. Temperature may be defined as the thermal state of a body which decides the direction of flow of heat energy from one body to another when they are placed in thermal contact each other. The temperature of a body is the measure of the average kinetic energy of its molecules. When a body is heated, its molecules move faster. Their average K.E. increases. This increases the temperature of the body.

In thermodynamics the concept of temperature follows from the zeroth law of thermodynamics.

Thermometry: *The branch of physics that deal with the measurement of temperature is called thermometry.*

Thermometer: *Any device used to measure the temperature of a body is called a thermometer.*

Principle of a thermometer: *A thermometer makes use of some measurable property (called thermometric property)* of a substance *which changes linearly with temperature.*

The thermometric properties of different substances and the corresponding thermometers are as follows:

(i) Length of a liquid column in a capillary (Mercury thermometer).

(ii) Pressure of a gas at constant volume (Constant volume gas thermometer)

(iii) Volume of a gas at constant pressure (Constant pressure gas thermometer)

(iv) Electrical resistance of a metal wire (Platinum resistance thermometer).

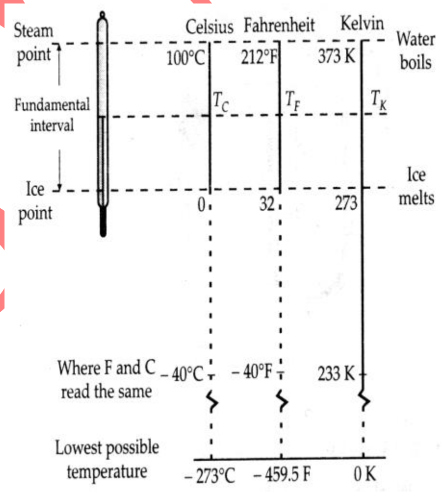
(v) Thermoelectrical e.m.f. (Thermoelectric or thermocouple thermometer)

(vi) Radiated power (Pyrometers).

Fixed points on a temperature scale: To construct a temperature scale, two fixed points (two well–defined thermodynamic states) are chosen and are assigned two arbitrary numbers for their temperature. One number fixes the origin of the scale and the other fixes the size of the unit of the scale. The temperature at which pure ice melts at standard pressure (ice–liquid water equilibrium state) is usually chosen as the *lower fixed point.* The temperature at which pure water boils at atmospheric pressure (liquid water–vapour) equilibrium state) is chosen as the *upper fixed point.*

Thermometric Scales

The commonly used temperature scales are as follows:



(i) ***The Celsius Scale:*** On this scale, lower fixed point (ice point) is taken as 0°C and the upper fixed point (steam point) as 100°C. The interval between the two fixed points is divided into hundred equal parts (hence the name centigrade) and each part is called 1°C.

(ii) ***The Fahrenheit scale:*** On this scale, the lower fixed point is taken as 32°F and the upper fixed point as 212°F. The interval between them is divided into 180 equal parts and each part represents 1°F.

(iii) ***The Reaumer Scale:*** On this scale, the lower fixed point is taken as 0°R and the upper fixed point as 80°R. The interval between them is divided into 80 equal parts and each part represents 1°R.

(iii) ***The Kelvin Scale:*** On this scale, the lower fixed point is taken as 273.15 K and the upper fixed point as 373.15 K. The interval between the two fixed points is divided into 100 equal parts. The SI unit of temperature is Kelvin (K).

Conversion of temperature from one scale to another

To convert the temperature from one scale to another, the following relation is used:

$$\frac{Temperature\ on\ one\ scale - Lower\ \ fixed\ \ point}{Upper\ \ fixed\ \ point - Lower\ fixed\ \ point}$$

$$= \frac{Temperature\ on\ other\ scale - Lower\ \ fixed\ \ point}{Upper\ \ fixed\ \ point - Lower\ \ fixed\ \ point}$$

If the temperature of body is measured as T_C, T_F, T_R and T_K on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T_K - 273.15}{373.15 - 273.15}$$

or

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273.15}{100}$$

Triple point of water

The triple point of water is the state at which the three phase of water namely ice, liquid water and water vapour are equally stable and co–exist in equilibrium. It is unique because it occurs at a specific *temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.* Thus for water,

P_{tr} = 0.46 cm of Hg, T_{tr} = 273.16 K or 0.01°C

Subjective Assignment – I

Q.1 A faulty thermometer has its fixed points marked as 5° and 95°. Temperature of a body as measured by the faulty thermometer is 59°. Find the correct temperature of the body on Celsius scale.

Q.2 A thermometer has wrong calibration. It reads the melting point of ice – 10°C. It reads 60°C in place of 50°. Calculate the temperature of boiling point of water on this scale.

Q.3 At what temperature, do the readings of Celcius and Fahrenheit scales coincide?

Q.4 A constant volume gas thermometer using helium records a pressure of 20.0 kPa at triple–point of water, and pressure of 14.3 kPa at temp. of 'dry ice' (solid CO_2). What is temperature of 'dry ice'?

Q.5 A constant volume thermometer using helium gas records a pressure of 1.75×10^4 Pa at normal freezing point of water, and a pressure of 2.39×10^4 Pa at normal boiling point of water. Obtain from these observations the temperature of absolute zero on the Celsius scale.

Q.6 A plantinum wire has resistance of 10Ω at 0°C & 20Ω at 273°C. Find value of coefficient of resistance.

Q.7 A faulty thermometer reads 5°C in melting ice and 99°C in steam. Find the correct temperature in °F when the faulty thermometer reads 52°C.

Q.8 When a thermometer is taken from the melting ice to warm liquid, the mercury level rises to 2/5th of distance between the lower and the upper fixed points. Find the temperature of liquid in °C and K.

Q.9 At what temperature is the Fahrenheit scale reading equal to twice of Celsium scale reading?

Q.10 A constant volume gas thermometer using sulphur records a pressure of 2×10^4 Pa at the triple point of water and 2.87×10^4 Pa at temperature of melting sulphur. Calculate the melting point of sulphur.

Q.11 The resistance of a resistance thermometer at 19°C is 3.50Ω and at 99°C is 3.66Ω. At what temperature will its resistance be 4.30Ω?

Answers

1.	60°C	2.	130°C	3.	– 40°	4.	195.30 K
5.	– 273	6.	1/273 $°C^{-1}$	7.	122°F		
8.	40°C, 313.15 K	9.	160°C or 320°F	10.	190.4 K	11.	419°c

Thermal Expansion

Almost all solids, liquids and gases expand on heating. T*he increase in size of a body when it is heated is called thermal expansion.*

Different types of thermal expansion

(i) ***Linear expansion:*** *It is the increase in the length of a metal rod on heating.*

(ii) ***Superficial expansion:*** *It is the increase in the surface area of a metal sheet on heating.*

(iii) ***Cubical expansion:*** *It is the increase in the volume of block on heating.*

Cause of thermal expansion: All solids consist of atoms and molecules. At any given temperature, these atoms and molecules are held at equilibrium distance by forces of attraction. When a solid is heated, the amplitude of vibration of its atoms and molecules increases. The average interatomic separation increases. This results in the thermal expansion of the solid.

Coefficient of Linear Expansion: Suppose a solid rod of length *l* is heated through a temperature ΔT and its final (increased) length is *l'*. It is found from experiments that

(i) Increase in length ∝ rise in temperature i.e., $l' - l \propto \Delta T$

(ii) Increase in length ∝ original length i.e., $l' - l \propto l$

Combining the above two factors, we get $l' - l \propto l\,\Delta T$ or $l' - l = \alpha\, l\, \Delta T$

The proportionality constant α is called ***coefficient of linear expansion.*** Its value depends on the nature of the solid. Clearly, $l' - l \propto \alpha\, \Delta T$ and $\alpha = \frac{l' - l}{l\,\Delta T} = \frac{\Delta l}{l\,\Delta T}$ or

$$\alpha = \frac{\textit{Increase in length}}{\textit{Original length} \times \textit{Rise in temperature}}$$

Hence *the coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit original length per degree rise in its temperature.*

The unit of α is $^{o}C^{-1}$ or K^{-1}

Coefficient of Superficial Expansion

Suppose a metal sheet of initial surface area S is heated through temperature ΔT and its final surface area becomes S'.

Then $S' - S \propto \Delta T$ and $S - S \propto S$

∴ $S' - S \propto S\,\Delta T$ or $S' - S = \beta\, S\, \Delta T$

The proportionality constant β is called *coefficient of superficial expansion* and its value depends on the nature of the material. Clearly,

$S' = S[1 + \beta\,\Delta T]$ and $\beta = \frac{S' - S}{S\,\Delta T} = \frac{\Delta S}{S\,\Delta T}$

or $$\beta = \frac{\textit{Increase in surface area}}{\textit{Original surface area} \times \textit{Rise in temperaure}}$$

Hence the *coefficient of superficial expansion of a metal sheet is defined as the increase in its surface area per unit original surface area per degree rise in its temperature.*

The unit of β is $^{o}C^{-1}$ or K^{-1}.

Coefficient of cubical expansion

Suppose a solid block of initial volume V is heated through a temperature ΔT and its final volume is V'.

Then $V' - V \propto \Delta T$

and $V' - V \propto V$ or $V' - V = \gamma\, V\, \Delta T$

The proportionality constant γ is called the **coefficient of cubical expansion** which depends on the nature of the material of the solid. Clearly,

$V' = V[1 + \gamma\,\Delta T]$ and $\gamma = \frac{V' - V}{V\,\Delta T} = \frac{\Delta V}{V\,\Delta T}$

or $$\gamma = \frac{\textit{Increase in volume}}{\textit{Original volume} \times \textit{Rise in temperature}}$$

Hence *the coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.*

The unit of γ is $^{o}C^{-1}$ or K^{-1}.

Coefficient of cubical expansion of an ideal gas at constant pressure: For an ideal gas,

$$PV = nRT \qquad \text{... (i)}$$

At constant pressure,

$$P\,\Delta V = nR\,\Delta T \qquad \text{.... (ii)} \qquad [\because \text{ n and R are constants}]$$

Dividing (ii) by (i), we get

$$\frac{\Delta V}{V}=\frac{\Delta T}{T} \text{ or } \frac{\Delta V}{V\,\Delta T}=\frac{1}{T} \text{ or } \gamma=\frac{1}{T}$$

Hence for ideal gas, the coefficient of volume expansion decreases with the increase in temperature.

Relation between α, β and γ

Consider a cube of side *l*. Its original volume is $V = l^3$

Suppose the cube is heated so that its temperature increases by ΔT. Its each side will become

$$l' = l + \Delta l = l + \alpha\, l\, \Delta T = l\,(1 + \alpha\, \Delta T)$$

The new volume of the cube will be $V' = l'^3 = l^3\,(1 + \alpha\,\Delta T)^3$

$$= V\,(1 + 3\alpha\,\Delta T + 3\alpha^2\,\Delta T^2 + \alpha^3\,\Delta T^3)$$

As α is small, so the terms containing α^2 and α^3 can be neglected. Then

$$V' = V\,(1 + 3\alpha\,\Delta T)$$

By the definition of the coefficient of cubical expansion,

$$\gamma=\frac{\Delta V}{V\,\Delta T} = \frac{V'-V}{V\,\Delta T}=\frac{V\,(1+3\alpha\,\Delta T)-V}{V\,\Delta T} = 3\alpha$$

Similarly, it can be proved that $\beta = 2\alpha$ Hence $\frac{\alpha}{1}=\frac{\beta}{2}=\frac{\gamma}{3}$.

—

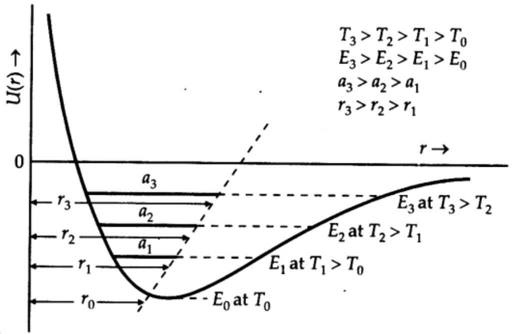
NOTE:

- The three coefficients of expansion α, β and γ are not constant for a given solid. Their values depend on the temperature range.
- For most of the solids, the value of α lies between 10^{-6} to 10^{-5} K^{-1} in the temperature range 0 to 100°C. The value of α is more for ionic solids than that for non–ionic solids.
- The coefficient of linear expansion of a solid rod is independent of the geometrical shape of its cross–section.
- The coefficient of volume expansion of solids and liquids is rather small, particularly very small for pyrex glass (1×10^{-5} K^{-1}) and invar (Fe–Ni alloy with $\gamma = 2 \times 10^{-6}$ K^{-1})
- For an ideal gas γ varies inversely with temp. i.e., γ = 1/T. At 0°C or 273 K, $\gamma = 1/273 = 3.7 \times 10^{-3}$ K^{-1}, which is much larger than that for solids and liquids.
- Water contracts on heating between 0°C and 4°C. This is called ***anomalous expansion of water.*** It has the minimum volume and hence the maximum density (1000 kg m^{-3}) at 4°C. Silver iodide also contracts on heating between 80°C to 140°C.

Molecular Explanation of Thermal Expansion

As shown in figure, the graph between the potential energy U (r) of two neighbouring atoms in a crystalline solid and their interatomic separation r is an *asymmetric parabola*. The potential energy curve is asymmetric about its minimum because the attractive part of the potential energy rises slowly compared to the repulsive part.

At the temperature T_0 = 0 K, the atoms remain at the equilibrium separation r_0 and their oscillation energy E_0 is minimum. As the temperature increases, the energy of the atoms increases and they start vibrating about their equilibrium positions with the interatomic separation oscillating between its minimum and maximum values: r_{min} and r_{max}. The average interatomic separation becomes



$$r = \frac{r_{\min} + r_{\max}}{2}$$

Clearly, as the temperature increases, the amplitude of vibration of the atoms increases. Due to the asymmetry of the P.E. curve, the equilibrium position shifts to the right on the curve (as shown by the dashed inclined line), *i.e., the average interatomic separation increases. It is thus in consequence of this increase in the average interatomic separation with temperature that a solid expands when heated.*

Practical Applications of Thermal Expansion

(i) *A small gap is left between the iron rails of railway tracks.*
(ii) *Space is left between the girders used for supporting bridges.*
(iii) *Iron ring to be put on rim of a cart wheel is always of slightly smaller diameter than that of wheel.*
(iv) *Clock pendulums are made of invar.*
(v) *A glass stopper jammed in the neck of a glass bottle can be removed by warming neck of the bottle.*
(vi) *Only platinum wire is used for fusing into glass.*

Expansion of a Liquid

When a liquid is heated, it s container also expands. The observed expansion of the liquid is called ***apparent expansion*** which is different from the ***real expansion*** of the liquid.

Coefficient of apparent expansion: *It is defined as the apparent increase in volume per unit original volume for* l^oC *rise in temperature*. The coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Coefficient of real expansion: *It is defined as the real increase in volume per unit original volume for* 1^oC *rise in temperature*. The coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

It can be proved that $\gamma_r = \gamma_a + \gamma_g$

where γ_g is coefficient of cubical expansion of glass (material) of the container.

Variation of Density with Temperature

When a given mass of a solid or a liquid is heated, its volume increases and hence density decreases. If V and V' are the volumes and ρ and ρ' are the densities of a given mass M at temperatures T and T + ΔT respectively, then

$$V' = V\,(1 + \gamma\,\Delta T)$$

or $$\frac{M}{\rho'} = \frac{M}{\rho}(1+\gamma\Delta T) \quad \text{or} \quad \rho' = \rho\,(1 + \gamma\,\Delta T)^{-1}$$

Expanding by Binomial theorem and neglecting the terms containing higher powers of $\gamma\,\Delta T$, we get

$$\rho' = \rho\,(1 - \gamma\,\Delta T)$$

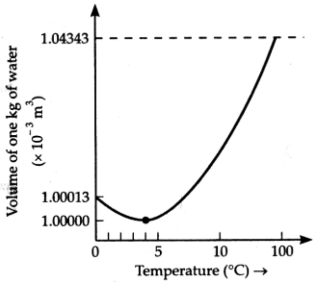
Clearly, the density of a solid or a liquid decreases with the increase in temperature.

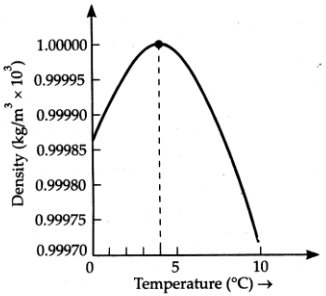
Anomalous Expansion of Water

When water at 0^oC is heated, its volume decreases and, therefore, its density increases, until its temperature reaches 4^oC. Above 4^oC, the volume increases, and therefore the density decreases. ***Thus water at*** 4^oC ***has the maximum density.***

Practical Importance of anomalous expansion of water

The anomalous expansion of water has a favourable effect on aquatic life. Since the density of water is maximum at 4°C, water at the bottom of lakes remains at 4°C, water at the bottom of lakes remains at 4°C even if it freezes at top surface. This allows marine animals to remain alive and move freely near bottom.





Subjective Assignment – II

Q.1 Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66 m long be 3.63 cm at 10°C, then at what temperature will the lines just touch? Coefficient of linear expansion of steel = 11×10^{-6} °C^{-1}

Q.2 A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27°C. To what temperature should the ring be heated so as to fit the rim of the wheel? Coefficient of linear expansion = 12×10^{-6} °C^{-1}

Q.3 A clock with an iron pendulum keeps correct time at 20°C. How much will it lose or gain if temperature changes to 40°C? Coefficient of cubical expansion of iron = 36×10^{-6} °C^{-1}.

Q.4 A metal ball 0.1 min radius is heated from 273 to 348 K. Calculate the increase in surface area of the ball. Given coefficient of superficial expansion = 0.000034 K^{-1}.

Q.5 On heating a glass block of 10, 000 cm^3, from 25°C to 40°C, its volume increases by 4 cm^3. Calculate coefficient of linear expansion of glass.

Q.6 If the volume of a block of metal changes by 0.12% when it is heated through 20°C, what is the coefficient of linear expansion of metal?

Q.7 How much the temperature of a brass rod should be increased so as to increase its length by 1%? Given that α for brass = 0.00002°C^{-1}.

Q.8 A steel scale measures the length of a copper rod as 80 cm when both are at 20°C, the calibration temperature of the scale. What would the scale read for the length of the rod when both are at 40°C? Given α for steel = 1.1×10^{-5} °C^{-1} and α for copper 1.7×10^{-5} °C^{-1}.

Q.9 A steel metre scale is to be ruled so that millimeter intervals are accurate to within about 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling? Given α for steel = 1.1×10^{-5} °C^{-1}.

Q.10 A cylinder of diameter 1.0 cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30°C. To what temperature must the plate be heated? For steel, $\alpha = 1.1 \times 10^{-5}$ °C^{-1}.

Q.11 What should be lengths of steel and copper rods at 0°C that the length of steel rod is 5 cm longer than copper at all temperatures? Given α for copper = 1.7×10^{-5} °C^{-1} and α for steel = 1.1×10^{-5} °C^{-1}.

Q.12 A clock having a brass pendulum beats seconds at 30°C. How many seconds will it lose or gain per day when temperature falls to 10°C? Given α for brass = 1.9×10^{-5} °C^{-1}.

Q.13 A steel wire 2 mm in diameter is stretched between two clamps, when its temperature is 40°C. Calculate the tension in the wire when its temperature falls to 30°C. Given α for steel = 1.1×10^{-6} °C^{-1} and y for steel = 21×10^{11} dyne cm^{-2}.

Q.14 Calculate the force required to prevent a steel wire of 1 mm^2 cross–section from contracting when it cools from 60°C to 15°C, if young's modulus for steel is 2×10^{11} Nm^{-2} and its coefficient of linear expansion is 0.000011°C^{-1}.

Q.15 The volume of a metal sphere is increased by 1% of its original volume when it is heated from 320 K to 522 K. Calculate the coefficients of linear, superficial and cubical expansion of the metal.

Q.16 Density of mercury is 13.6g cm^{-3} at $0^\circ C$ and its coefficient of cubical expansion is $1.82 \times 10^{-4}\,{}^\circ C^{-1}$. Calculate the density of mercury at $50^\circ C$.

Q.17 Suppose that one early morning when the temperature is $10^\circ C$, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 litre of gasoline, which is also at $10^\circ C$. During the day, the temperature rises to $30^\circ C$. How much gasoline will overflow? Given α for steel = $1.2 \times 10^{-5}\,{}^\circ C^{-1}$ and γ for gasoline = $9.5 \times 10^{-4}\,{}^\circ C^{-1}$.

Q.18 A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in this flask? Given α for glass = $9 \times 10^{-6}\,{}^\circ C^{-1}$ and γ for mercury = $1.8 \times 10^{-4}\,{}^\circ C^{-1}$.

Answers

1.	$60^\circ C$	2.	$218^\circ C$	3.	10.368 s		
4.	3.206×10^{-4} m^2	5.	$8.89 \times 10^{-6}\,{}^\circ C^{-1}$	6.	$2.0 \times 10^{-5}\,{}^\circ C^{-1}$		
7.	$500^\circ C$	8.	80.0096	9.	$4.5^\circ C$	10.	$57.3^\circ C$
11.	9.17 cm,14.17 cm	12.	gain of 16.42 s	13.	7.26×10^6 dyne	14.	99 N
15.	$1.67 \times 10^{-5}\,{}^\circ C^{-1}$, $3.34 \times 10^{-5}\,{}^\circ C^{-1}$, $5 \times 10^{-5}\,{}^\circ C^{-1}$			16.	13.48 g cm^{-3}		
17.	1.37 litre	18.	150 cm^3				

Specific Heat

The specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is needed to raise the temperature of M mass of a substance through ΔT, then specific heat is given by

$$c = \frac{\Delta Q}{M \times \Delta T}$$

The CGS unit of specific heat is cal $g^{-1}\,{}^\circ C^{-1}$ and the SI unit is J kg^{-1} K^{-1}.

∴ The amount of heat required to raise the temperature of M mass of a substance through ΔT is

$\Delta Q = Mc\ QT.$

Molar specific heat

The molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is required to raise the temperature of n moles of a substance through ΔT, then molar specific heat is given by

$$C = \frac{\Delta Q}{n\ \Delta T}$$

The CGS unit of molar specific heat is cal $mol^{-1}\,{}^\circ C^{-1}$ and SI units is J mol^{-1} K^{-1}.

Therefore, the amount of heat required to raise the temperature of n moles of a substance through ΔT is

$\Delta Q = nC\ \ \Delta T$

Heat capacity or thermal capacity

The heat capacity of a body is defined as the amount of heat required to raise its temperature through one degree. By definition, the amount of heat required to raise the temperature of unit mass of a body is equal to specific heat c. So heat required for m mass is m × c.

∴ Heat capacity = Mass × Specific heat or S = mc

The CGS unit of heat capacity is cal $^{o}C^{-1}$ and the SI unit is JK^{-1}.

Water equivalent

The water equivalent of a body is defined as the mass of water which requires the same amount heat as is required by the given body for the same rise of temperature.

Water equivalent = Mass × Specific Heat or $w = mc$

The CGS unit of water equivalent is g and the SI unit is kg.

Calorimetry

Principle of calorimetry or the law of mixtures

Whenever two bodies at different temperature are placed in contact with one another, heat flows from the body at higher temperature to the body at lower temperature till the two bodies acquire the same temperature. *The principle of calorimetry states that the heat gained by the cold body must be equal to the heat lost by the hot body, provided there is no exchange of heat with the surroundings.*

Heat gained = Heat lost

This principle is a consequence of the law of conservation of energy and useful for solving problems relating to calorimetry.

Effect of Pressure on Melting Point

The increase in pressure will help in its contraction. So we expect a decrease in the melting point of ice as the pressure on it is increased. *The melting point of those substances which expand on melting (e.g., paraffin wax, phosphorus, sulphur, etc.) increases with the increase in pressure while the melting point of those substances which contract on melting (e.g., ice, cast iron, bismuth etc.) decreases with increase in pressure.*

Effect of pressure on freezing point of ice: Regelation

When two pieces of ice are pressed against one another for few seconds and then released, they get frozen at the surface of contact. As the pressure is increased, the melting point of ice is lowered and ice melts. When pressure is released, the water so formed (at a temperature < 0°C) immediately freezes again. This phenomenon of refreezing is called regelation.

The phenomemon in which ice melts when pressure is increased and again freezes when pressure is removed is called ***regelation*** *(re = again; gelare = freeze)*

Practical application of regelation

1. By pressing snow in our hand, we can transform it into a snow–ball. When snow is pressed, its crystals melt. As the pressure is released, water refreezes forming as snow–ball.
2. When the wheels of cart pass over snow, ice melts due to increase in pressure exerted by the wheels. When pressure is released, water so formed refreezes on the wheels. That is why wheels are covered with ice.
3. Skating is possible due to the formation of water layer below the skates. Water is formed due to the increase of pressure and it acts as lubricant.
4. The ice of a glacier, pressed against the sides of its valley melts, and in this way adopts itself to the shape of the valley.

Effect of pressure on the boiling point of a liquid

The boiling point of a liquid increases with the increase in pressure

Practical Application

1. Cooking is difficult at mountains. The atmospheric pressure at mountains is much lower than that at plains, so water starts boiling at a temperature much lower than 100°C. Hence cooking is difficult.
2. The pressure inside a pressure cooker is increased much above the atmospheric pressure by not allowing the steam to escape. This increases the boiling point. Hence the vegetables are cooked inside a pressure cooker in a shorter time.

Latent Heat

The amount of heat required to change the state of unit mass of a substance at constant temperature and pressure is called latent heat of the substance.

If m mass of a substance undergoes a change from one state to another, then the amount of heat required for the process is Q = mL

where L is the latent heat of the substance and is a characteristic of the substance. Its value also depends on the pressure. Clearly, $L = \frac{Q}{m}$

∴ SI unit of latent heat = J kg^{-1} or CGS unit of latent heat = cal g^{-1}

Latent heat of fusion: *The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion or latent heat of melting. It is denoted by* L_f.

Latent heat of vaporisation: *The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point. It is denoted by* L_v.

Subjective Assignment – III

Q.1 A thermally isolated vessel contains 100g of water at 0°C when air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed, if no water is left in the vessel. Latent heat of vaporization of water at 0°C = 2.10 × 10^6 J kg^{-1} and latent heat of fusion of ice = 3.36 × 10^5 J kg^{-1}.

Q.2 When 0.15 kg of ice of 0°C is mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice (c_{water} = 4186 J kg^{-1} K^{-1})

Q.3 Calculate the heat required to convert 3 kg of ice at – 12°C kept in a calorimeter to steam at 100°C at atmospheric pressure. Given:
specific heat capacity of ice = 2100 J kg K^{-1}, specific heat capacity of water = 4186 J kg^{-1} K^{-1}
latent heat of fusion of ice = 3.35 × 10^5 J Kg^{-1} and latent heat of steam = 2.256 × 10^6 J kg^{-1}

Q.4 When 10 g of coal is burnt, it raises the temperature of 2 litres of water from 20°C to 55°C. Calculate the heat of combustion of fuel.

Q.5 A normal diet furnishes 2000 kcal to a 60 kg person in a day. If this energy was used to heat the person with no losses to the surroundings, how much would the person's temperature increases? The specific heat of the human body = 0.83 cal g^{-1} °C^{-1}.

Q.6 0.75 gram of petroleum was burnt in a bomb calorimeter which contains 2kg of water and has a water equivalent 500 gram. The rise in temperature was 3°C. Determine the calorific value of petroleum.

Q.7 The heat of combustion of ethane gas is 373 kcal per mole. Assuming that 60% of the heat is useful, how many litres of ethane measured at S.T.P. must be burnt to convert 50 kg of water at 10°C to steam at 100°C? One mole of a gas occupies 22.4 litre at S.T.P.

Q.8 A refrigerator converts 50 gram of water at 15°C into ice at 0°C in one hour. Calculate the quantity of heat removed per minute. Take specific heat of water = 1 cal g^{-1} °C^{-1} and latent heat of ice = 80 cal g^{-1}.

Q.9 How many grams of ice at – 14°C are needed to cool 200 gram of water from 25°C to 10°C? Take specific heat of ice = 0.5 cal g^{-1}°C^{-1} and latent heat of ice = 80 cal g^{-1}.

Q.10 An electric heater of power 100 W raises the temperature of 5 kg of a liquid from 25°C to 31°C in 2 minutes. Calculate the specific heat of the liquid.

Q.11 A piece of iron of mass 100g is kept inside a furnace from a long time and then put in a calorimeter of water equivalent 10g containing 240g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of ice. Given specific heat of iron = 470 J kg^{-1} °C^{-1}.

Q.12 When 0.45 kg of ice of 0°C mixed with 0.9 kg of water at 55°C in a container, the resulting temperature is 10°C. Calculate the heat of fusion of ice. (c_{water} = 4186 J kg^{-1} K^{-1})

Q.13 Calculate the heat required to convert 0.6 kg of ice at – 20°C, kept in a calorimeter to steam at 100°C at atmospheric pressure. Given the specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of water is 4186 J kg^{-1} K^{-1}, latent heat of ice = 3.35 × 10^5 J kg^{-1}, latent heat of seam = 2.256 × 10^6 J kg^{-1}.

Answers

1.	86.2 g	2.	3.34×10^5 J kg^{-1}	3.	9.1×10^6 J
4.	7000 cal g^{-1}	5.	40.16°C	6.	10^4 cal g^{-1}
7.	3131.5 litres	8.	79.2 cal min^{-1}	9.	31 g
10.	400 J kg^{-1} °C^{-1}	11.	953.6°C	12.	334400 J kg^{-1}
13.	1.8×10^8 J				

Modes of Transfer of Heat : Introduction

Heat can be transferred from one place to another by three different methods. These are (i) conduction, (ii) convection and (iii) radiation.

Thermal Conduction

It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.

When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

Steady State and Temperature Gradient

In the process of conduction, each cross–section of the rod receives heat from the adjacent cross–section of the hotter side. A part of this heat is absorbed by the cross–section itself whose temperature increases, another part is lost into the atmosphere by convection and radiation from the sides of cross–section and the rest is conducted to the next cross–section. *In this state the temperature of every cross–section of the rod goes on increasing with time.* The rod is said to be in the **variable state** of heat conduction.

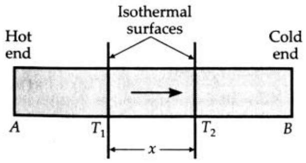
Suppose the sides of the rod are covered with some insulating material so that no heat is lost from the sides to the surroundings. After some time, a steady state is reached when the temperature of every cross–section of the rod becomes constant. In this state, no heat is absorbed by the rod. *This state of the rod when temperature of every cross–section of the rod becomes constant and there is no further adsorption of heat in any part is called* ***steady state.*** During steady state,

(i) the temperatures of two different parts of rod are different, but the temperature of each part remains constant.

(ii) every transverse section of the rod is an *isothermal surface.*

(iii) the temperature decreases as we move away from the hot end

(iv) the quantity of heat flowing per second through every cross–section is constant.



The rate of change of temperature with distance in the direction of flow of heat is called ***temperature gradient.*** If T_1 and T_2 are the temperatures of two isothermal surfaces separated by distance x, then

$$\text{Temperature gradient} = \frac{T_1 - T_2}{x}$$

Thermal Conductivity

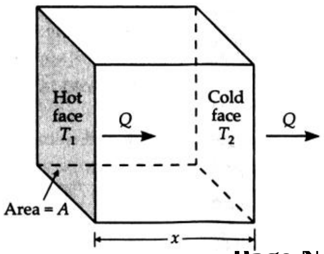
As shown in figure, consider a block of a material of cross–sectional area A and thickness x. Suppose its opposite faces are at temperatures T_1 and T_2, with $T_1 > T_2$. It is found that the amount of heat Q that flows from hot to cold face during the steady state.

(i) is directly proportional to the cross–sectional area A,

(ii) is directly proportional to the temperature difference ($T_1 - T_2$) between the opposite faces,

(iii) is inversely proportional to thickness x of the block, and

(iv) depends on the nature of the materials of the block.



$$\therefore \quad Q \propto \frac{A(T_1-T_2)t}{x} \quad \text{or} \quad Q=\frac{KA(T_1-T_2)t}{x}$$

The proportionality constant K is called *coefficient of thermal conductivity* of the given materials. Its value depends on the nature of the material. If A = 1, $T_1 - T_2 = 1$, t = 1, x = 1, then Q = K

Hence, *the coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.*

If A be the area of the cross–section at a place dx be a small thickness along the direction of heat flow and dT be the temperature difference across this thickness dx, then rate of flow of heat or heat current H will be

$$H=\frac{dQ}{dt}=-KA\frac{dT}{dx}$$

The quantity dT/dx is called the temperature gradient. The negative sign indicates that dT/dx is negative in the direction of flow of heat i.e., temperature decreases along the positive x–direction. Thus the negative sign in the above equation ensures that K is positive.

Units and dimensions of K

The numerical value of K is $K=\dfrac{Q.x}{A(T_1-T_2)t}$

$\therefore$ SI unit of K $=\dfrac{J.m}{m^2.K.s}$ = J s^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1} or CGS unit of K = cal s^{-1} cm^{-1} $^{o}C^{-1}$

Dimensions of K = $\dfrac{[ML^2T^{-2}].[L]}{[L^2].[K][T]}=[MLT^{-3}K^{-1}]$

Heat Current and Thermal Resistance

Heat flows in a conductor due to temperature difference between its two points. *The flow of heat unit time is called heat current,* denoted by H. Thus $H=\dfrac{Q}{t}$

Its SI unit is Js^{-1} or watt (W)

From Ohm's law, electric resistance is given by $R=\dfrac{V}{I}$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, *the ratio of the temperature difference between the ends of a conductors to the heat current through it is called thermal resistance,* denoted by R_H. Thus

$$R_H=\frac{\Delta T}{H} \quad \text{As} \quad Q=KA\frac{\Delta T}{\Delta x}t$$

$$\therefore \quad H=\frac{Q}{t}=KA\frac{\Delta T}{\Delta x} \quad \text{and} \quad R_H=\frac{\Delta T}{H}=\frac{\Delta x}{KA}$$

Hence *greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material.*

Unit and dimensions of R_H

As $R_H = \Delta T/H$, so

SI unit of $R_H=\dfrac{K}{J\,s^{-1}}=\dfrac{K}{W}=KW^{-1}$

Dimensions of $R_H = \frac{[K]}{[ML^2T^{-1}].[T^{-1}]} = [M^{-1}L^{-2}T^3K]$

Application of Conductivity in Daily Life

(i) In winter, a metallic handle appears colder than the wooden door.

(ii) Cooking utensils are provided with wooden handles: Wood is a bad conductor of heat. A wooden handle does not allow heat to be conducted from the hot utensil to the hand.

(iii) A new quilt is warmer than an old quilt: A new quilt contains more air in its pores as compared to the old quilt. As air is bad conductor of heat, it does not allow heat to be conducted away from our body to the surroundings and we feel warmer in it.

(iv) Birds swell their feathers in winter: By doing so the birds enclose air between their feathers and the body.

(v) Ice is packed in saw dust: Saw dust and air trapped inside it are poor conductors of heat.

(vi) Eskimos make double wall houses of the blocks of ice: Air trapped between the two walls of ice does not allow the heat from the surrounds to the ice which may otherwise melt the ice.

(vii) When a wire gauge is placed over the burning Bunsen's burner, the flame does not go beyond the gauge: Copper is a very good conduct or of heat. The copper gauge absorbs most of the heat.

(viii) A refrigerator is provided with insulated walls: Generally, fibre glass is used as an insulating material. This is done to minimize the changes of heat flowing into the refrigerator from outside.

(ix)

Subjective Assignment – IV

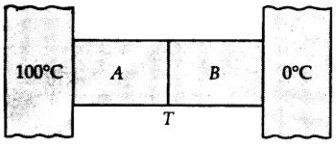
Q.1 Calculate the rate of loss of heat through a glass window of area 1000 cm^2 and thickness 0.4 cm when temperature inside is 37^{o}C and outside is –5^{o}C.

Q.2 Steam at 100^{o}C is passed into a copper cylinder 10 mm thick and a 200 cm^2 area. Water at 100^{o}C collects at the rate of 150g min^{-1}. Find the temperature of the outer surface, if the conductivity of copper is 0.8 cal s^{-1} cm^{-1} ^{o}C^{-1} and the latent heat of steam is 540 cal g^{-1}.

Q.3 A metal rod of length 20 cm and diameter 2 cm is covered with non–conducting substance. One of its ends is maintained at 100^{o}C, while the other end is put in ice at 0^{o}C. It is found that 25g of ice melts in 5 minutes. Calculate the coefficient of thermal conductivity of the metal. Given latent heat of ice = 80 cal g^{-1}.

Q.4 A layer of ice 2 cm thick is formed on a pond. The temperature of air is – 20^{o}C. Calculate how long it will take for the thickness of ice to increase by 1 mm. Density of ice = 1g cm^{-3}. Latent heat of ice = 80 cal g^{-1}. Conductivity of ice = 0.008 cal s^{-1} cm^{-1} ^{o}C^{-1}.

Q.5 Two metal cubes A and B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The co–efficient of thermal conductivity of A and B are 300 W/m^{o}C and 200 W/m^{o}C, respectively.

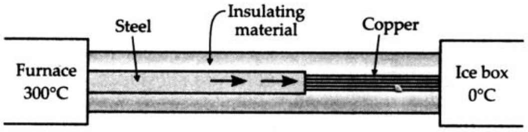


After steady state is reached, what will be the temperature T of the interface?

Q.6 Three bars of equal lengths and equal area of cross–section are connected in series. Their thermal conductivities are in the ratio of 2 : 4 : 3. If the open ends of the first and the last bars are at temperatures 200^{o}C and 18^{o}C respectively in the steady state, calculate the temperatures of both the junctions.

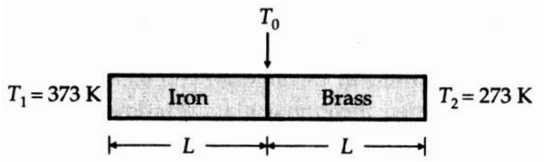
Q.7 One end of a copper rod of uniform cross–section and of length 1.5 m is kept in contact with ice and the other end with water at 100^{o}C. At what point along its length should a temperature of 200^{o}C be maintained so that in steady state, the mass of ice melted by equal to that of the steam produced in the same interval of time? Assume that the whole system is insulated from the surrounding. Latent heat of fusion of ice = 80 cal g^{-1}. Latent heat of vaporization of water = 540 cal g^{-1}.

Q.8 What is the temperature of the steel–copper junction in the steady state of the system shown in figure? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace =300°C, temperature of the other end = 0°C. The area of cross–section of the steel rod is twice that of the copper rod. Thermal conductivity of steel = 50.2 Js^{-1} m^{-2} $^oC^{-1}$ and of copper = 385 $Js^{-1}m^{-1}$ $^oC^{-1}$.



Q.9 An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of + 20°C inside it, when the outside temperature is – 10°C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 Watt/m/°C respectively.

Q.10 An iron bar (L_1 = 0.1 m, A_1 = 0.02 m^2, K_1 = 79 W m^{-1} K^{-1}) and a brass bar (L_2 = 0.1 m, A_2 = 0.02 m^2, K_2 = 109 W m^{-1} K^{-1}) are soldered end to end as shown in figure. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar



Q.11 Heat is flowing through a rod of length 25.0 cm having cross–sectional area 8.80 cm^2. The coefficient of thermal conductivity for the material of the rod is K = 9.2 × 10^{-2} kcal s^{-1} m^{-1} $^oC^{-1}$. The temperature of the ends of the rod are 125°C and 0°C in the steady state. Calculate (i) temperature gradient in the rod (ii) temperature of a point at a distance of 10.0 cm from the hot end and (iii) rate of flow of heat.

Q.12 An iron boiler is 1 cm thick and has a heating area of 2 m^2. The two surfaces of the boiler are at 234°C and 100°C respectively. If the latent heat of steam is 536 kcal kg^{-1} and thermal conductivity of iron is 1.6×10^{-2} kcal s^{-2} kcal s^{-1} m^{-1} $^oC^{-1}$, then how much water will be evaporated into steam per minute?

Q.13 One end of a 0.25 m long metal bar is in steam and the other is in contact with ice. If 12g of ice melts per minute, what is the thermal conductivity of the metal? Given cross–section of the bar = 5 × 10^{-4} m^2 and latent heat of ice is 80 cal g^{-1}.

Q.14 A layer of ice 0.15 m thick has formed on the surface of a deep pond. If the temperature of upper surface of ice is constant and equal to that of the air which is – 12°C, determine the time it will take to increase the thickness of ice layer by 0.2 mm. Take latent heat of ice = 80 cal g^{-1}, density of ice = 0.91g cm^{-3} and thermal conductivity of ice = 0.5 cal s^{-1} m^{-1} K^{-1}.

Q.15 Water is boiled in a rectangular steel tank of thickness 2 cm by a constant temperature furnace. Due to vaporization, water level falls at a steady rate of 1 cm in 9 minutes. Calculate the temperature of the furnace. Given K for steel = 0.2 cal s^{-1} m^{-1} $^oC^{-1}$.

Q.16 Estimate the rate at which ice would melt in a wooden box 2.0 cm thick and of inside measurements 200 cm × 120 cm × 120 cm assuming that the external temperature is 30°C and coefficient of thermal conductivity of wood is 0.0004 cal s^{-1} $cm^{-1\,o}C$.

Q.17 Steam at 373 K is passed through a tube of radius 50 cm and length 3 m. If thickness of the tube be 2mm and conductivity of its material be 2 × 10^4 cal s^{-1} cm^{-1} K^{-1}, calculate the rate of loss of heat in Js^{-1}. The outside temperature is 282 K.

Q.18 Thermal conductivity of copper is four times that of brass. Two rods of copper and brass of same length and cross–section are joined end to end. The free end of copper rod is at 0°C and that of brass rod at 100°C. Calculate the temperature of junction at equilibrium. Neglect radiation losses.

Q.19 The temperature difference between the two ends of a bar 1.0m long is 50°C and that for the other bar 1.25m long 75°C. Both the bars have same area of cross–section. If the rates of conduction of heat in the two bars are the same, find the ratio of the coefficients of thermal conductivity of materials of the two bars.

Q.20 The ratio of the areas of cross–section of two rods of different materials is 1 : 2, and the ratio of the thermal conductivities of their materials is 4 : 3.On keeping equal temperature–difference between ends of these rods, rates of conduction of heat are equal. Determine ratio of the lengths of the rods.

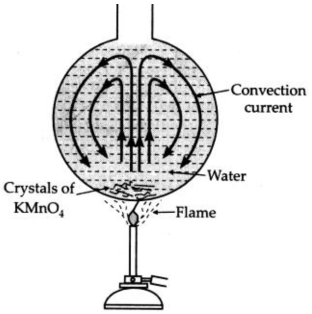
Q.21 A room at 20°C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal s^{-1} m^{-1} $^oC^{-1}$ and J = 4.2 J cal^{-1}.

Answers

1.	231 cal s^{-1}	2.	91.56°C	3.	0.424 cal s^{-1} cm^{-1} $^oC^{-1}$		
4.	102.5 s	5.	60°C	6.	116°C and 74°C		
7.	1.396 m	8.	44.4°C	9.	9000 W		
10.	(i) 315 K, (ii) 91.6m^{-1} K^{-1}, (iii) 916.1 W						
11.	(i) – 5°C cm^{-1}, (ii) 75°C, (iii) 4.048 × 10^{-2} $kcals^{-1}$			12.	48 kg		
13.	80 cal s^{-1} m^{-1} $^oC^{-1}$	14.	364.2 s	15.	110°C	16.	9.36 gs^{-1}
17.	36026 Js^{-1}	18.	20°C	19.	6 : 5		
20.	2 : 3	21.	15.2°C				

Convection

It is the process by which heat flows from the region of higher temperature to the region of lower temperature by the actual movement of the material particles. Fluids (liquids and gases) are heated mainly by the process of convection in which buoyancy and gravity play an important role. As shown in figure, when a fluid is heated from below, the hot portion at the bottom expands and becomes less dense. Because of buoyancy, this lighter portion rises up. The denser colder fluid takes its place by moving downwards. *Thus convection current* is set up in the fluid. The actual movement of a liquid can be seen by colouring the liquid with potassium permanganate crystals placed of the bottom of the vessel.



Natural Convection

If the material moves due to difference in density, the process of heat transfer is called natural or free convection. Natural convection arises due to unequal heating (of fluid) and gravity. Natural convection is responsible for the origin of different types of winds in the atmosphere.

Forced Convection

If the heated material is forced to move by an agency like a pump or a blower, the process of heat transfer is called forced convection. Air–conditioning, central heating systems and heating a liquid by brisk stirring are examples of forced convection.

Phenomena Based on Thermal Convection

(i) In regulating the temperature of human body

In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection.

(ii) In maintaining comfortable room temperature in cold countries

In cold countries during winter, the outside temperature is much below $0^{o}C$ while the room temperature is comfortably maintained around $20^{o}C$. However, the inside air close to the glass window is cooler than $20^{o}C$ while the outside close to the window is warmer than the chilling temperature of the atmosphere. Thus heat is continuously transferred from the room to the outside by convection of air inside the room.

(iii) In the formation of trade winds

Natural convection plays an important role in the formation of trade winds. The surface of the earth and hence the air above it near the equator gets strongly heated by the sun. The heated air expands and rises upwards. The colder air from polar region rushes in towards the equator. This produces northward wind in northern hemisphere and southward in southern hemisphere. Due to rotation of the earth about its axis from west to east, the air close to the equator has an eastward speed of 1600 kmh^{-1}, while it is zero close to the poles. As a result, the actual direction of the wind in the northern hemisphere is north east and in the southern hemisphere, south west. These winds are called *trade winds.*

(iv) Land and Sea Breezes

These are local convection currents. Specific heat of water is higher than that of soil. So land and hence air above it is heated faster in summer during day time than air above the sea. The air above land expands and rises and its place is taken up by the colder air from sea to land and is called sea breeze. At night the land gets cooled faster than water. So colder air flows from land to sea and is called land breeze.

(v) Monsoons

Water has much more specific heat than soil or rock. In summer, the land mass of the Indian subcontinent gets much hotter than the Indian Ocean. This sets up convention current with hot air from the land rising and moving towards the Indian Ocean, while the moisture–laden air from the Ocean moves towards the land. When obstructed by height and gets cooled. The moisture condenses and causes wide–spread rains in India. In winter, the landmass is cooler than the ocean. Winds blow from the land to ocean. These winds take up moisture as they pass the Bay of Bengal and cause rainfall in Tamilnadu and Srilanka.

Radiation

It is the process by which heat is transmitted from one place to another without heating the intervening medium. The heat from the sun reaches the earth by the process of radiation, convering millions of kilometers of the empty space or vacuum.

Prevost's Theory of Heat Exchange

The salient features of this theory are:

(i) *All bodies at temperature above 0 K emit heat to the surroundings and gain heat from the surroundings at all times.*

(ii) *The amount of heat radiated per second depends on the nature of the emitting surface, its surface area and its temperature.*

(iii) *The rate of heat radiated by a body increases with the increase of its temperature and is unaffected by the presence of surrounding bodies.*

(iv) *There is a continuous exchange of heat between a body and its surroundings.*

(v) *The rise or fall in temperature of a body is the net result of the exchange of heat between the body and the surroundings.*

(vi) *The exchange of heat between a body and its surrounding continues till a dynamic thermal equilibrium is established between them and their temperatures become equal.*

When the temperature of a body is equal to that of its surroundings, it radiates heat to the surroundings at the same rate at which it absorbs. The body is then in the state of *dynamic equilibrium.* In this state, if a body absorbs a large fraction of total heat falling upon it, surroundings, otherwise its temperature will change. This shows that a body which is a good absorber is also a good radiator of heat & vice–versa.

Electromagnetic Waves

These are the waves constituted by oscillating electric and magnetic fields. The oscillations of the two fields are mutually perpendicular to each other as well as to the direction of propagation of the waves. Every body at any temperature emits electromagnetic waves. These waves can have different wavelengths. The atoms or molecules of a substance can be excited to higher energy states by thermal collisions or by some other means. When such atoms or molecules de–excite to lower energy states, electromagnetic radiations are emitted.

Thermal Radiation

The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy. All bodies having temperature above 0 K emit thermal radiation continuously. For example, the radiation emitted by red–hot iron or light from a filament lamp is thermal radiation.

Properties of thermal radiation

(i) These are electromagnetic waves having wavelengths range from 1 µm to 100 µm. These are also called infrared waves.

(ii) Like light, thermal radiations travel in straight lines.

(iii) These radiations obey the laws of reflection and refraction like light does.

(iv) They show the phenomena of interference, diffraction and polarization.

(v) Thermal radiations produce heat when they are absorbed by a body.

Newton's Law of Cooling

The rate at which a body loses heat by radiation depends on (i) the temperature of the body, (ii) the temperature of the surrounding medium, and (iii) the nature and extent of the exposed surface.

Newton's law of cooling *states that the rate of cooling (or rate of loss of heat) of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.* This is in accordance with Newton's law of cooling that a hot water bucket cools fast initially until it gets lukewarm after which it stays so for a longer time.

Mathematical expression for Newton's law of cooling

Consider a hot body at temperature T. Let T_0 be the temperature of its surroundings. According to Newton's law of cooling,

Rate of loss of hear $\propto$ Temperature difference between the body and its surroundings

or $-\frac{dQ}{dt} \propto (T - T_0)$ or $-\frac{dQ}{dt} = k(T - T_0)$ ... (1)

where k is a proportionality constant depending upon the area and nature of the surface of the body. Let m be the mass and c the specific heat of the body at temperature T. If the temperature of the body falls by small amount dT in time dt, then the amount of heat lost is

dQ = mc dT

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$

Combining the above equations, we get

$$-mc\frac{dT}{dt} = k(T - T_0)$$

or $\frac{dT}{dt} = -\frac{k}{mc}(T - T_0) = -K(T - T_0)$ ... (2)

where K = k/ mc is another constant. The negative sign indicates that as the time passes, the temperature of the body decreases. The above equation can be written as

$$\frac{dT}{T-T_0}=-K\,dt$$

On integrating both sides, we get

$$\int \frac{1}{T-T_0}\,dT=-K\int dt$$

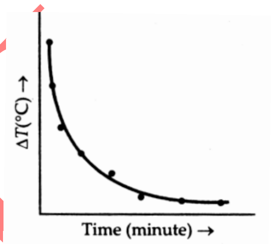
or $\log_e (T - T_0) = - Kt + c$ ... (3)

or $T - T_0 = e^{-Kt + c}$

or $T = T_0 + e^c\ e^{- Kt}$

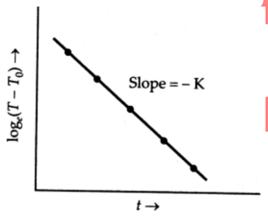
or $T = T_0 + Ce^{- Kt}$ ... (4)

where c is a constant of integration and $C = e^c$.

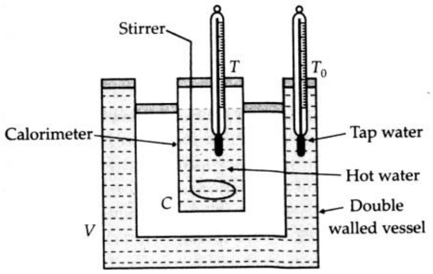


If we plot a graph by taking different values of temperature difference $\Delta T = T - T_0$ along y–axis and the corresponding values of t along x–axis, we get a curve of the form shown in figure. The rate of cooling is higher initially and then decrease as the temperature of the body falls.

If we plot a graph, by taking $\log_e (T - T_0)$ along y–axis and time t along x–axis, we must get a straight line, as shown in figure.



Experimental Verification of Newton's Law of Cooling



The experimental set–up used for verifying Newton's law of cooling is shown in figure. The set–up consists of a double walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T of hot water in calorimeter and T_0 of water in between the double walls respectively. Temperature of hot water in the calorimeter is noted after fixed intervals of time, say after every one minute of stirring the water gently with a stirrer.

Continue noting its temperature till it attains a temperature about 5°C above that that of surroundings. Plot a graph between $\log_e (T - T_0)$ and time (t). The nature of the graph is observed to be a straight line, having a negative slope, as shown in figure. This verifies newton's law of cooling.

Absorptive Power

The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ. It is denoted by a_λ. A perfect black body absorbs all the heat radiations incident upon it. So its absorptive power is unity. If the radiant energy dQ in wavelength range λ and $\lambda + d\lambda$ is incident on a body of absorptive power a_λ, then amount of radiant energy absorbed by the body = $a_\lambda\, dQ$.

Emissive Power

The amount of heat energy radiated by a body per second depends upon (i) the area of its surface, (ii) the temperature of its surface and (iii) the nature of its surface. The strength of emission is measured by a quantity called emissive power. The *emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ.*

If a heat radiation of wavelength range λ to $\lambda + d\lambda$ is incident on the surface of a body of emissive power e_λ, then the amount of radiant energy emitted per second per unit area = $e_\lambda \, d\lambda$. The SI unit of emissive power is $Js^{-1} m^{-2}$ or Wm^{-2}.

Emissivity

The emissivity of a body is defined as the ratio of the heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the same temperature i.e., it is the ratio of the emissive power (e) of a body to the emissive power (E) of a black body at the same temperature. It is denoted by ε.

Thus $\varepsilon = \dfrac{e}{E}$

It is dimensionless quantity. Its value lies between 0 and 1. The emissivity of a perfect black body is 1.

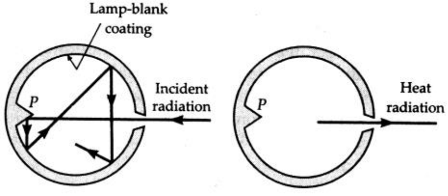
Black Body

A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it. The absorptive power of a perfect black body is unity. When a black body is heated to a high temperature, it emits radiations of all possible wavelengths within a certain wavelength range. The radiations emitted by a block body are called *full or black body radiations.*

In practice, a surface coated with lamp black or platinum black absorbs 95 to 97% of the incident radiation. But on heating, it does not emit full radiation spectrum. So it acts as a black body only for absorption of heat radiation. It is observed that if a hollow cavity is heated, the radiation coming out from its inner surface through a small opening is a full radiation spectrum. Such a radiation is called **cavity radiation.** Hence the small opening of a heated hollow cavity acts as a perfect black body both for absorption and emission of heat radiation.

Fery's Black Body

Fery's black body consists of a hollow double walled metal sphere coated inside with lamp black and nickel polished from outside. Heat radiations entering the sphere through the small opening are completely absorbed due to multiple reflections. The conical projection opposite the opening prevents direct reflection. To use it as a source of heat radiation, the enclosure is heated in a suitable bath to maintain its temperature constant.



(a) A black body absorber. (b) A black body emitter.

The radiations coming out from the small hole are black body radiations. The *wavelength range of emitted radiation is independent of the material of the body and depends only on the temperature of the black body.*

Kirchhoff's Law

Kirchhoff's law of heat radiation states that at any given temperature, the ratio of the emissive power to the absorptive power corresponding to the certain wavelength is constant for all bodies and this constant is equal to emissive power of the perfect black body at the same temperature and corresponding to the same wavelength. If e_λ and a_λ are emissive and the absorptive powers of a body corresponding to wavelength λ,

then $$\frac{e_\lambda}{a_\lambda} = E_\lambda \ (constant) \qquad \text{.... (i)}$$

where E_λ is the emissive power of perfect black body at the same temperature and corresponding to the same wavelength. Thus, if a_λ is large, then e_λ will also be large i.e., if a body absorbs a radiation of certain wavelength strongly, then it will also strongly emit the radiation of that wavelength.

As the emissivity ε of a body is defined as the ratio of its emissive power to that of the emissive power of a black body at the same temperature, so

$$\frac{e_\lambda}{E_\lambda} = \varepsilon \qquad \text{.... (ii)}$$

From equation (i) and (ii), we get

$$a_\lambda = \varepsilon$$

Thus *the absorptive power of a body is equal to its emissivity*. This is another form of ***Kirchhoff's law.***

Hence *a good absorber is a good emitter*. Since a good absorber is a poor reflector, so the ability of a body to emit radiation is related oppositely to its ability to reflect. That is, *a good emitter is a poor reflector*.

Applications of Kirchhoff's Law

(i) Take a piece of china having some dark paintings engraved on it. Heat it in a furnace to about 1000°C and then examine in a dark room immediately. The dark paintings will appear much brighter than white china. This is because the dark paintings are better absorbers and, therefore, also better emitters.

(ii) A green glass heated in a furnace when taken out in dark glows with red light. Green glass (when cold) is a good absorber of red light and a good reflector of green light. When heated, it becomes a good emitter of red light in accordance with Kirchhoff's law.

(iii) If a polished metal ball with a spot of platinum black on it is heated in a furnace to about 1200 K and then taken out into a dark room, the black spot appears brighter than the polished surface. This is because the black spot is a better absorber and hence, by Kirchhoff's law, a better emitter of radiation.

(iv) A Dewar flask or thermos bottle consists of a double–walled glass vessel with its inner and outer walls coated with silver. Radiation from the inner walls is reflected back into the contents of the bottle. Similarly, the outer wall reflects back any incoming radiation. The space between the walls is evacuated to reduce losses due to conduction and convection. The device helps in keeping hot contents hot and cold contents cold for a long time.

Fraunhoffer Lines

When light from the sun is seen through a spectrometer, it is found to be crossed by several dark lines which are called Fraunhoffer lines. The sun has a hot central core which emits continuous spectrum. The central hot core is surrounded by various elements in the vaporized state and comparatively cooler than the core. When white light from the central core passes through the elements in the vapour state, they absorb their characteristic wavelengths, thus producing dark lines in the sun's spectrum.

Importance

By comparing the wavelengths of Fraunhoffer's lines with those emitted by elements on the earth, we have identified various elements like H_2, He, N_2, O_2, Na, Fe, Cu, etc. in the atmosphere of the sun.

Stefan–Boltzmann Law

This law states that the total heat energy emitted by a perfect black body per second per unit area is directly proportional to the fourth power of the absolute temperature of its surface. Thus

$$E \propto T^4 \qquad \text{or} \qquad E = \sigma T^4$$

where σ is universal constant and known as Stefan Boltzmann constant.

In SI units, $\sigma = 5.67 \times 10^{-8}\ \text{J s}^{-1}\ \text{m}^{-2}\ \text{K}^{-4} = 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$

In CGS units, $\sigma = 5.67 \times 10^{-5}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{K}^{-4}$

If a black body is in an enclosure at temperature T_0, then the rate at which the black body absorbs radiation from the enclosure is σT_0^4. Therefore, the net loss of energy by the black body per unit time per unit area is

$$E = \sigma(T^4 - T_0^4)$$

If the body is not a perfect black body and has emissivity ε, then above relations get modified as follows:

$$E = \varepsilon \sigma T^4 \qquad \text{or} \qquad E = \varepsilon \sigma (T^4 - T_0^4).$$

Subjective Assignment – V

Q.1 A body cools in 7 minutes from 60^{o}C to 40^{o}C. What will be its temperature after the next 7 minutes? The temperature of the surroundings is 10^{o}C. Assume that Newton's law of cooling holds good throughout the process.

Q.2 A pan filled with hot food cools from 94^{o}C to 86^{o}C in 2 minutes when the room temperature is at 20^{o}C. How long will it take to cool from 71^{o}C to 69^{o}C?

Q.3 Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of 5.67 W cm^{-2}. Given $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}.

Q.4 Luminosity of Rigel star in Orion constellation is 17,000 times that of our sun. If the surface temperature of the sun is 6000 K, calculate the temperature of the star.

Q.5 Due to the change in mains voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed?

Q.6 Consider the sun to be a perfect sphere of radius 6.8×10^8 m. Calculate energy radiated by the sun in one minute. Surface temperature of sun = 6200 K. Stefan's constant = 5.67×10^{-8} Jm^{-2} s^{-1} K^{-4}.

Q.7 At what temperature will the filament of 100 W lamp operate if it is supposed to be perfectly black body of area 1 cm^2?

Q.8 A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600^{o}C, and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250? Neglect heat loss due to convection. (Stefan Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4})

Q.9 A spherical body with radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated?

Q.10 Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227^{o}C, when placed in an enclosure at 27^{o}C. Given Stefan's constant = 5.7×10^{-8} Wm^{-2} K^{-4}.

Q.11 Two bodies A and B are kept in evacuated vessels maintained at a temperature of 27^{o}C. The temperature of A is 527^{o}C and that of B is 127^{o}C. Compare the rates at which heat is lost from A and B.

Q.12 A small hole is made in a hollow sphere whose walls are at 273^{o}C. Find the total energy radiated per second per cm^2. Find the total energy radiated per second per cm^2. Given Stefan's constant = 5.7×10^{-5} erg cm^{-2} s^{-1} K^{-4}.

Q.13 To what temperature must a black body is raised in order to double total radiation, if original temperature is 727^{o}C?

Q.14 A black body initially at 27^{o}C is heated to 327^{o}C. How many times is total heat emitted at the higher temperature than that emitted at lower temperature? What is the wavelength of the maximum energy radiation at the higher temperature? Wien's constant = 2.898×10^{-3} mK.

Q.15 An electric bulb with tungsten filament having an area of 0.25 cm^2 is raised to a temperature of 3000 K, when a current passes through it. Calculate the electrical energy consumed in watt, if the emissivity of the filament is 0.35. Stefan's constant, $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} K^{-4}. If due to fall in main voltage the filament temperature falls to 2500 K, what will be wattage of the bulb?

Q.16 An iron ball having a surface area of 200 cm^2 and at a temperature of 527^{o}C is placed in an enclosure at 27^{o}C. If the surface emissivity of iron be 0.4, at what rate is heat being lost by radiation by the ball?

Answers

1.	28^{o}C	2.	42 s	3.	1000 K
4.	68520 K	5.	216	6.	2.92×10^{28} J
7.	2049 K	8.	296 W	9.	1800 W
10.	45.48 cal s^{-1}	11.	23	12.	5.61×10^7 erg

13.	916°C	14.	48.300Å	15.	49.19 W, 19.38 W	16.	45.59 cal s^{-1}

Wien's Displacement Law

The total energy radiated by a black body is not uniformly distributed over all the wavelengths but is maximum for a particular wavelength λ_m. The value of λ_m decreases with the increase of temperature. *Wien's displacement law states that the wavelength (λ_m) corresponding to which the energy emitted by a black body is maximum is inversely proportional to its absolute temperature (T).* Mathematically,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

where b is Wien's constant. Its value is 2.9×10^{-3} mK.

Importance

Wien's law can be used to estimate the surface temperatures of the moon, sun and other starts. Light from the moon shows a maximum of intensity at $\lambda_m = 14$ μm. By applying Wien's law, the temperature of the surface of the moon turns out to be 200 K. Similarly, solar radiation shows a maximum at $\lambda_m = 4753$Å. This corresponds to a surface temperature of 6060 K.

Subjective Assignment – VI

Q.1 Wavelength corresponding to E_{max} for the moon is 14 microns. Estimate the surface temperature of the moon, if $b = 2.284 \times 10^{-3}$ mK.

Q.2 The surface temperature of a hot body is 1227°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant = 0.2898 cm K.

Q.3 The spectral energy distribution of the sun has a maximum at 4753 Å. If the temperature of the sun is 6050 K, What is the temperature of a star for which this maximum is at 9506 Å?

Q.4 An indirectly heated filament is radiating maximum energy of wavelength 2.16×10^{-5} cm. Find the net amount of heat energy lost per second per unit area, the temperature of surrounding air is 13°C. Given b = 0.288 cm K, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.

Q.5 The sun radiates maximum energy at wavelength 4753Å. Estimate the surface temperature of the sun, if $b = 2.888 \times 10^{-3}$ mK.

Q.6 The temperature of an ordinary electric bulb is around 3000 K. At what wavelength will it radiate maximum energy? Will this wavelength be within visible region? Given b = 0.288 cm K.

Q.7 A furnace is at a temperature of 2000 K. At what wavelength will it radiate maximum intensity? Is it in the visible region?

Answers

1.	206 K	2.	19320 Å	3.	3025 K
4.	1.824×10^{12} erg s^{-1} cm^{-2}	5.	6076 K	6.	9600 Å, No
7.	14400 Å, No				

Phases and Phase Diagrams

A graph drawn between the pressure and volume of a system at constant temperature is called an isotherm. AB represents the vapour phase (steam) which is compressible. This means when pressure is increased from A to B, the volume is decreased. Here steam is at 350°C and this is possible only at high pressure of about 160 atmosphere. From B to C, the pressure remains constant. At B the substance is in vapour state and at C it is in liquid state. Thus along BC liquid and vapour coexist in equilibrium.

Let V_l and V_g be the molar volumes of water in liquid and gaseous phases respectively. If V is the total volume of the system, then the fractions of the volume in liquid and gaseous phases will be

$$x_l = \frac{V_g - V}{V_g - V_l} \quad \text{and} \quad x_g = 1 - x_l$$

When pressure becomes more than 163 atm, the substance is in the liquid state (water). Along CD the pressure is increased. There is almost no change in volume. This shows that the liquids are incompressible.

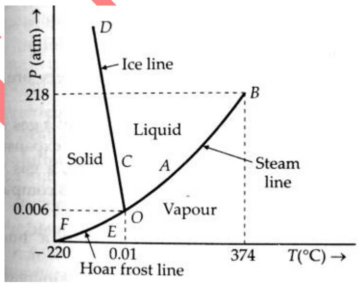
As the temperature is increased, the volume difference $V_l - V_g$ decreases and at a temperature 374.1°C and pressure
216 atm, $V_l - V_g = 0$. For this isotherm there is no horizontal portion. This temperature is called the critical temperature (T_C) of water. This means that water can exist as liquid till 374.1°C only, there is only one phase i.e. vapour. This means if a gas is above critical temperature whatever pressure is applied we cannot liquefy it.

Critical Constants

The critical temperature, T_c is the temperature below which a gas can be liquefied by the application of pressure. The pressure required is called critical pressure P_c and the volume occupied by unit mass of the gas at critical temperature and critical pressure is called critical volume V_c.

Pressure–temperature phase diagram for water

Figure shows the P–T phase diagram for water. It consists of the following *three* curves:



(i) **Vaporisation curve (Steam Line AB)**

It is a graph between pressure and the boiling point of the substance in the liquid state. Each point on this curve fixes a set of pressure and temperature at which the liquid and the gaseous phases can co–exist. If the pressure is increased, the vapour will at once condense into liquid but if the pressure is deceased, the liquid will evaporate. So, all points above the vaporisation curve correspond to liquid phase and below it to vapour phase.

(ii) **Fusion Curve (Ice Line CD)**

It is a graph between the pressure and the melting point of the substance in the solid state. Each point on this curve gives the value of the pressure and the temperature at which the solid and liquid phases can co–exist. If pressure is increased, the solid would melt into liquid but if the pressure is decreased liquid will turn into solid. So all the points above the fusion curve correspond to liquid phase and those below it to solid phase.

(iii) **Sublimation curve (Hoar frost line EF)**

It is a graph between pressure and temperature at which a solid directly changes to vapour state. Each point on this curve gives the values of pressure and temperature at which the solid and vapour phases can co–exist. If pressure is increased, the vapour changes to solid phase and if the pressure is decreased, the solid changes to vapour state. So all the points above this curve correspond to solid phase while those below it correspond to vapour state.

Conclusions

(i) In the space above the steam line and on the right of ice–line, water exists in liquid phase as water.

(ii) In the space below the steam line and on the right of hoar frost line, water exists in gaseous phase as steam.

(iii) In the space above the hoar–frost line and on the left of ice–line, water exists in solid phase as ice.

Triple Point

It is a unique point on P–T diagram at which all the three phases of a substance can co–exist in equilibrium with each other. The three curves AB, CD and EF on being extended meet at point O which represents the triple point. The values of pressure and temperature corresponding to this point for water are 0.46 cm of Hg and 273.16 K.

The negative slope of ice line for water indicates that melting point of ice decreases with the increase in pressure. The triple point of such substances is above its melting point at normal pressure.

Conceptual Problems

Q.1 Two thermometers are constructed in the same way except that one has a spherical bulb and the other and elongated cylindrical bulb. Which of the two will respond quickly to temperature changes?

Q.2 Why should a thermometer bulb have a small heat capacity?

Q.3 Why are gas thermometers are more sensitive than mercury thermometers?

Q.4 Why is a constant volume gas thermometer preferred as a standard thermometer than a constant pressure gas thermometer?

Q.5 Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle at the mean temperature $(T_1 + T_2)/2$. Why?

Q.6 Two hollow glass balls are connected by a tube, which has a pellet of mercury in the middle. Can the temperature of the surrounding air be determined from the position of the drop?

Q.7 The difference between lengths of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible?

Q.8 Why are loops provided in long metal pipes used for carrying oil and any other liquid over long distances?

Q.9 A long cylindrical vessel having linear coefficient of expansion α is filled with a liquid up to a certain level. On heating, it is observed that the length of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid?

Q.10 A metal ball is heated through a certain temperature. Out of mass, radius, surface area and volume, which will undergo largest percentage increase and which one the least?

Q.11 If an electric fan be switched in a closed room, will the air of the room be cooled? If not, why do we feel cold?

Q.12 The coolant used in a nuclear reactor should have high specific heat. Why?

Q.13 Why are two thin blankets are warmer than a single blanket of double the thickness?

Q.14 Place a safety pin on a sheet of paper. Hold the sheet over a burninig candle, until the paper becomes yellow and charr. On removing the pin, its white trace is observed on the paper. Why?

Q.15 If a drop of water falls on a very hot iron, it does not evaporate for a long time. Give reason.

Q.16 Why rooms are provided with the ventilators near the roof?

Q.17 Why snow is a better heat insulator than ice?

Q.18 Can we boil water inside an earth satellite?

Q.19 Suppose you want to cool your drink. Should you keep ice cubes floating on the top or should you arrange to keep the ice cubes at the bottom?

Q.20 Why are clear nights colder than cloudy nights?

Q.21 How does the boiling point of water change with pressure?

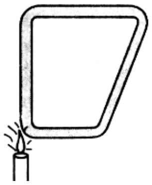
Q.22 Suggest suitable methods for measuring the temperature of
(i) surface of the sun, (ii) surface of the earth, (iii) an insect and (iv) liquid helium

Q.23 There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if (i) they are heated to the same temperature (ii) same amount of heat is given to each of them?

Q.24 Two vessels of different materials are identical in size and wall–thickness. They are filled with equal quantities of ice at 0^{o}C. If the ice melts completely in 10 and 25 minutes respectively, compare the coefficients of thermal conductivity of the materials of the vessels.

Q.25 Two vessels A and B of different materials but having identical shape, size and wall thickness are filled with ice and kept at the same place. Ice melts at the rate of 100 g min^{-1} and 150g min^{-1} in A and B respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.

Q.26 Water in a closed tube is heated with one arm placed vertically above an arc lamp. Water will begin to circulate along the tube in a counterclockwise direction. Is this true or false?



Q.27 A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 200°C. Which of these objects will cool fastest and which one slowest when left in air at room temperature? Give reasons.

Q.28 There are two rods of the same metal, same length, same area of cross–section, but one of square cross–section and the other of circular cross section. One end of each is kept immersed in steam. After the steady state is reached, the other ends of the rods are touched. Which one will be hotter? Given reason.

Q.29 A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster?

Q.30 On a hot day, a car is left in sunlight with all the windows closed. After some time, it is found that the inside of the car is considerably warmer than the air outside. Explain, why.

Q.31 A blackened platinum wire, when gradually heated, first appears dull red, then blue and finally white. Explain why?

Q.32 In a coal fire, the pockets formed by coals appear brighter than the coals themselves. Is the temperature of such a pocket higher than the surface temperature of a glowing coal?

Q.33 Answer the following questions:

(a) A vessel with a movable piston maintained at a constant temperature by a thermostat contain a certain amount of liquid in equilibrium with its vapour. Does this vapour obey Boyle's law? In other words, what happens when the volume of vapour is decreased? Does the vapour pressure increase?

(b) What is meant by 'superheated water' and 'supercooled vapour' Do these states of water lie on its P–V–T surface? Give some practical applications of these states of water.

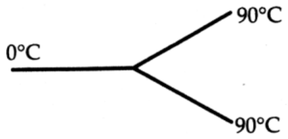
Q.34 A fat man is used to consuming about 3000 kcal worth of food every day. His food contains 50g of butter plus a plate of sweets everyday, besides items which provide him with other nutrients (proteins, vitamins, minerals, etc.) in addition to fats and carbohydrates. The caloric value of 10g of butter is 60 kcal and that of a plate of sweets is of average 700 kcal. What dietary strategy should he adopt to cut down his calories to about 2100 kcal per day? Assume the man cannot resist eating the full plate of sweets once it is offered to him !

Problems on Higher Order Thinking Skills

Q.1 2kg of ice at – 20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/ kg/ °C, and 0.5 kcal/ kg/ °C, while the latent heat of fusion of ice is 80 kcal/ kg.

Q.2 Two rods, one of aluminum and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increase by the same amount when their temperatures are raised by t°C, then find the ratio $l_1/(l_1 + l_2)$

Q.3 Three rods made of the same material and having the same cross–section have been joined as shown in figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. What will be the temperature of the junction of the three rods?



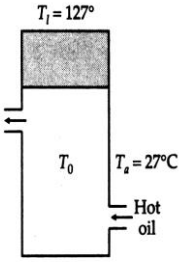
Q.4 When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is $\gamma_{Hg'}$ then find the ratio k_1/ k_2.

Q.5 An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal/ kg – K and B = 2×10^{-2} cal/ kg – K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8 × 10^4 cal/kg. Specific heat of water = 10^3 cal/ kg – K)

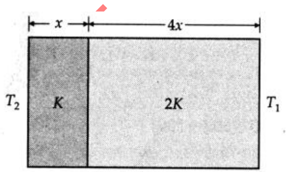
Q.6 Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/ (m – °C–sec), thickness t = 5 mm, emmissivity = 0.6. Temperature of the top of lid is maintained at T_l = 127°C. If the ambient temperature T_a = 27°C, calculate

(a) rate of heat loss per unit area due to radiation from the lid

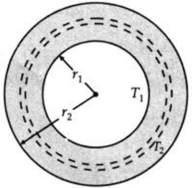
(b) temperature of the oil (Given $\sigma = \frac{17}{3} \times 10^{-8}$)



Q.7 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2 K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). What is the rate of flow of heat through the slab in a steady state?



Q.8 Figure shows a system of two concentric spherical shells of radii r_1 and r_2 and kept at temperatures T_1 and T_2. Find the radial rate of flow of heat through a substance of thermal conductivity K filled in the space between the two shells.



Q.9 A 5m long cylindrical steel wire with radius 2 × 10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire : Young's modulus = 2.1 × 10^{11} Pa; Density = 7860 kg/ m^3; specific heat = 420 J / kg – K)

Q.10 A sphere of diameter 7 cm and mass 266.5g floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 g cm^{-3}, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

Answers

1. 6 kg
2. $\frac{\alpha_s}{\alpha_a + \alpha_s}$
3. 60°C
4. $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$
5. 0.5 kg
6. (a) 595 Wm^{-2}, (b) 420 K
7. $\frac{KA(T_2 - T_1)}{3x}$
8. $\frac{4\pi K r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$
9. 0.00457 K
10. 0.000084°C^{-1}

NCERT Exercise

Q.1 A brass wire 1.8m long at 27°C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of – 39°C, what is the tension developed in the wire, if its diameter is 2.0 mm? Coefficient of linear expansion of brass = 2.0 × 10^{-5} °C^{-1}, Young's modulus of brass = 0.91 × 10^{11} Pa.

Q.2 A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are at

40.0°C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand. Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ ^{o}C^{-1}$ and that of steel = $1.2 \times 10^{-5}\ ^{o}C^{-1}$.

Q.3 A child running a temperature of 101°F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min, what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

Q.4 A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6h. The outside temperature is 45°C and coefficient of thermal conductivity of thermocole = 0.01 $Js^{-1}\ m^{-1}\ ^{o}C^{-1}$. Heat of fusion of water = 335×10^{3} J kg^{-1}.

Q.5 The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Q.6 Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is relation between T_A and T_B?

Q.7 The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: $R = R_0\,[1 + 5 \times 10^{-3}\,(T - T_0)]$

The resistance of 101.6Ω at the triple point of water, and 165.5 Ω at the normal melting point of lead (600.5 K). What is t he temperature when the resistance is 123.4 Ω?

Q.8 Answer the following:

(a) The triple–point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting points of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0°C and 100°C respectively. On the absolute scale, one of the fixed points is the triple–point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale?

(c) The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation, and not 273.16?

(d) What is the temperature of the triple–pint of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Q.9 Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	**Pressure thermometer A**	**Pressure thermometer B**
Triple–point of water	1.250×10^{5} Pa	0.200×10^{5} Pa
Normal melting point of sulphur	1.797×10^{5} Pa	0.287×10^{5} Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

(b) What do you think is the reason for slightly different answers from A and B?

Q.10 A steel tape 1m long is correctly calibrated for a temperature of 27.0°C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C? Coefficient of linear expansion of steel = $14.20 \times 10^{-5}\ ^{o}C^{-1}$?

Q.11 A large steel wheel is to be fitted on to a shaft of the same material. At 27°C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice' (solid carbon dioxide). At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range.

Q.12 A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0°C. What is the change in the diameter of the hole when the sheet is heated to 227°C? Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\,^{\circ}C^{-1}$.

Q.13 The coefficient of volume expansion of glycerine is $49 \times 10^{-5}\,^{\circ}C^{-1}$. What is the fractional change in its density for a 30°C rise in temperature?

Q.14 A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = $0.91\ J\ g^{-1}\,^{\circ}C^{-1}$.

Q.15 A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is maximum amount of ice that can melt? (Specific heat of copper = $0.39\ Jg^{-1}\,^{\circ}C^{-1}$, and latent heat of fusion of water = $335\ Jg^{-1}$).

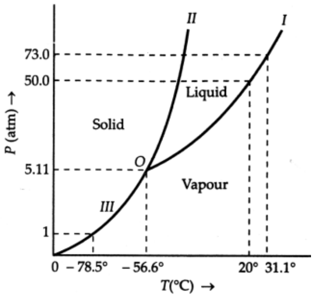
Q.16 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.

Q.17 Given below are observations on molar specific heats at room temperature of some common gases.

Gas	**Molar Specific Heat (C_V) (cal mol^{-1} K^{-1})**
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. [Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K]. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Q.18 Answer the following questions based on the P – T phase diagram of carbon dioxide as shown in figure.



(i) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co–exist in equilibrium?

(ii) What is the effect of decrease of pressure on the fusion and boiling point of CO_2

(iii) What are the critical temperature and pressure for CO_2? What is their significance?

(iv) Is CO_2 solid, liquid or gas at (a) – 70°C under 1 atm., (b) – 60°C under 10 atm., (c) 15°C under 56 atm?

Q.19 Answer the following questions based on the P – T phase diagram of CO_2:

(i) CO_2 at 1 atm pressure and temperature – 60°C is compressed isothermally. Does it go through the liquid phase?

(ii) What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?

(iii) Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm. pressure and temperature – 65°C as it is heated up to room temperature at constant pressure.

(iv) CO_2 is heated to a temperature 70°C and compressed isothermally. What changes in its properties do you expect to observe?

Q.20 A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg min^{-1}, when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 Js^{-1} m^{-1} $^oC^{-1}$ and heat of vaporisation of water = 2256 Jg^{-1}.

Q.21 Explain why:

(a) a body with large reflectivity is a poor emitter.

(b) a brass tumbler feels much colder than a wooden tray on a chilly day.

(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.

(d) the earth without its atmosphere would be inhospitably cold.

(e) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Q.22 A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C, the temperature of the surrounding is 20°C.

Answers

1.	3.77×10^2 N	2.	0.34 cm , No	3.	4.31 g min^{-1}	4.	3.687 kg
5.	Neon, $T_C = -248.58^oC$, $T_F = -415.44^oF$, CO_2 $T_C = -56.6^oC$, $T_F = -69.88^oF$					6.	$T_A = \frac{4}{7}T_B$
7.	384.8 K	9.	(a) 392.46 K, (b) 391.75 K	10.	63.0136 cm, 63 cm		
11.	– 68.8°C	12.	1.44×10^{-2} cm	13.	0.0147		
14.	103.02 °C	15.	1.455 kg	16.	0.1 cal g^{-1} $^oC^{-1}$		
18.	(i) 56.6°C, 5.11 atm, (ii) decrease, (iii) P_C = 73.0 atm, T_C = 31.1°C, (iv) (a) vapour, (b) solid (c) liquid						
19.	(i) No, (ii) directly into solid, (iii) go to liquid phase and then to the vapour phase, (iv) will deviated more and more from ideal gas behaviour			20.	238°C	22.	9 min

Objective Assignment – I (CBSE PMT)

Q.1 According to kinetic theory of gases, at absolute zero of temperature

(a) water freezes (b) liquid helium freezes

(c) molecular motion stops (d) liquid hydrogen freezes

Q.2 Mercury thermometer can be used to measure temperature upto

(a) 260°C (b) 100°C (c) 360°C (d) 500°C

Q.3 For measuring temperatures in the range of 2,000 to 2,500°C, we should employ

(a) barometer (b) pyrometer (c) gas thermometer

(d) platinum–rhodium thermometer

Q.4 A Centigrade and Fahrenheit thermometers are dipped in boiling water. The water temperature is lowered, until the Fahrenheit thermometer registers 140°. What is the fall in temperature registered by the centrigrade thermometer?

(a) 80° (b) 60° (c) 360° (d) 30°

Q.5 The coefficient of linear expansion of brass and steel are α_1 and α_2 respectively. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, the difference in their lengths (l_1 and l_2) will remain the same at all temperatures, if

(a) $\alpha_1 l_1 = \alpha_2 l_2$ (b) $\alpha_1 l_2 = \alpha_2 l_1$ (c) $\alpha^2{}_1 l_1 = \alpha^2{}_2 l_2$ (d) $\alpha_1 l^2{}_2 = \alpha^2{}_2 l_1$

Q.6 The thermal capacity of 40g of aluminium (specific heat = 0.2 $cal^og^{-1}C^{-1}$) is

(a) 40 cal $^oC^{-1}$ (b) 160 cal $^oC^{-1}$ (c) 200 cal $^oC^{-1}$ (d) 8 cal $^oC^{-1}$

Q.7 80g of water at 30°C are poured on a large block of ice at 0°C. The mass of ice that melts is

(a) 30 g (b) 80 g (c) 150g (d) 1,600 g

Q.8 If 1g of steam is mixed with 1g of ice, the resultant temperature of the mixture is

(a) 270^o (b) 230^o (c) 100^o (d) 50^o

Q.9 10g of ice cubes at 0^oC are released in a tumbler (water equivalent 55 g) at 40^oC. Assuming that negligible heat is taken from the surroundings, the temperature of water in the tumbler becomes nearly (L = 80 cal/g)

(a) 31^oC (b) 22^oC (c) 19^oC (d) 15^oC

Q.10 On a new scale of temperature (which is linear) called the W scale, freezing and boiling points of water are respectively 39^oW and 239^oW. What will be the temperature on the new scale corresponding to a temperature of 39^oC on the celcius scale?

(a) 200^oW (b) 139^oW (c) 78^oW (d) 117^oW

Q.11 The two ends of a rod of length L and a uniform cross–sectional area A are kept at two temperatures T_1 and $T_2 (T_1 > T_2)$. The rate of heat transfer, dQ/dt through the rod in a steady state is given by

(a) $\frac{dQ}{dt} = \frac{K(T_1 - T_2)}{LA}$ (b) $\frac{dQ}{dt} = KLA(T_1 - T_2)$

(c) $\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$ (d) $\frac{dQ}{dt} = \frac{KL(T_1 - T_2)}{A}$

Q.12 The presence of gravitational field is required for the heat transfer by

(a) stirring liquids (b) conduction (c) natural convection (d) radiation

Q.13 Consider two rods of same length and different specific heats (c_1, c_2), thermal conductivities (K_1, K_2) and area of cross–sections (A_1, A_2) and both having temperature (T_1, T_2) at their ends. If their rate of loss of heat due to conduction is equal, then

(a) $K_1A_1 = K_2A_2$ (b) $K_1A_1/c_1 = K_2A_2/c_2$ (c) $K_2A_1 = K_1A_2$ (d) $K_2A_1/c_2 = K_1A_2/c_1$

Q.14 Which of the following circular rods (radius r and length l) each made to the same material and whose ends are maintained at the same temperature will conduct most heat?

(a) $r = 2r_0$, $l = 2l_0$ (b) $r = 2r_0$, $l = l_0$ (c) $r = r_0$, $l = 2l_0$ (d) $r = r_0$, $l = l_0$

Q.15 A cylindrical rod having temperatures T_1 and T_2 at its end. The rate of flow of heat Q_1 cal s^{-1}. If all the dimensions (length and radius) are doubled keeping temperature constant, then the rate of flow of heat Q_2 will be

(a) $Q_2 = 2Q_1$ (b) $Q_2 = Q_1/2$ (c) $Q_2 = Q_1/4$ (d) $Q_2 = 4Q_1$

Q.16 Heat is flowing through the cylindrical rods of the same material. The diameters of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of the amount of heat conducted through them per unit time will be

(a) 1 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 8

Q.17 Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and 2K respectively. The equivalent thermal conductivity of the slab is

(a) 2K/3 (b) 4K/3 (c) $\sqrt{2}$ K (d) 3 K

Q.18 A beaker full of hot water is kept in a room. If it cools from 80^oC to 75^oC in t_1 minutes, from 75^oC to 70^oC in t_2 minutes and from 70^oC to 65^oC in t_3 minutes, then

(a) $t_1 = t_2 = t_3$ (b) $t_1 < t_2 = t_3$ (c) $t_1 < t_2 < t_3$ (d) $t_1 > t_2 > t_3$

Q.19 Which of the following is best close to an ideal black body?

(a) platinum black (b) black lamp

(c) cavity maintained at constant temperature

(d) a lump of charcoal heated to high temperature

Q.20 Unit of Stefan's constant is

(a) $W\ m^2\ K^4$ (b) $W\ m^2\ K^{-4}$ (c) $W\ m^{-2}\ K^{-1}$ (d) $W\ m^{-2}\ K^{-4}$

Q.21 A black body is at a temperature of 500 K. It emits energy at a rate, which is proportional to

(a) 500 (b) $(500)^2$ (c) $(500)^3$ (d) $(500)^4$

Q.22 A black body is at 727°C. It emits energy at a rate, which is proportional to
(a) $(727)^2$ (b) $(1000)^2$ (c) $(727)^4$ (d) $(1000)^4$

Q.23 A black body at 227°C radiates heat at the rate of 7 cal cm^{-2} s^{-1}. At a temperature of 727°C, the rate of heat radiated in the same units will be
(a) 50 (b) 112 (c) 80 (d) 60

Q.24 For a black body at temperature of 727°C, its radiating power is 60 W and temperature of surroundings is 227°C. If temperature of black body is changed to 1,227°C, then its radiating power will be
(a) 120 W (b) 240 W (c) 304 W (d) 320 W

Q.25 The radiant energy from the sun, incident normally at the surface of earth, is 20 kcal m^{-2} min^{-1}. What would have been the radiant energy incident normally on the earth, if the sun had a temperature twice of the present one?
(a) 40 kcal m^{-2} min^{-1} (b) 80 kcal m^{-2} min^{-1} (c) 160 kcal m^{-2} min^{-1} (d) 320 kcal $m^{-2}min^{-1}$

Q.26 If the temperature of the sun is doubled, the rate of energy received on earth will be increased by a factor of
(a) 2 (b) 4 (c) 8 (d) 16

Q.27 Assuming the sun to have a spherical outer surface of radius R, radiating like a black body at temperature t°C, the power received by a unit surface (normal to the incident rays) at a distance r from the centre of the sun is
(a) $\frac{4\pi R^2\sigma t^4}{r^2}$ (b) $\frac{R^2\sigma(t+273)^4}{4\pi r^2}$ (c) $\frac{16\pi R^2\sigma t^4}{r^2}$ (d) $\frac{R^2\sigma(t+273)^4}{r^2}$

Q.28 Which of the following statement is true about the radiation emitted by human body?
(a) the radiation emitted lies in the ultraviolet region and hence is not visible
(b) the radiation is emitted during the summers and absorbed during the winters
(c) the radiation is emitted only during the day (d) the radiation emitted is in the infrared region.

Q.29 If λ_m denotes the wavelength at which the radiative emission from a black body at a temperature T K is maximum, then
(a) λ_m is independent of temperature (b) $\lambda_m \propto T^4$
(c) $\lambda_m \propto T$ (d) $\lambda_m \propto 1/T$

Q.30 The Wien's displacement law expresses relation between
(a) wavelength corresponding to maximum energy and absolute temperature
(b) radiated energy and wavelength (c) temperature and emissive power
(d) colour of light and temperature

Q.31 A black body emits radiation of maximum intensity of wavelength λ at 2,000 K. Its corresponding wavelength at 3,000 K will be
(a) 16λ/81 (b) 81λ/16 (c) 2λ/3 (d) 4λ/3

Q.32 A black body at 1,227°C emits radiations with maximum intensity at a wavelength of 5000Å. If the temperature of the body is increased by 1,000°C, the maximum intensity will be observed at
(a) 3000 Å (b) 4000Å (c) 5000Å (d) 6000Å

Q.33 Two closed containers A and B are party filled with water. The volume of A is twice that of B and it contains half the amount of water in B. If both are at the same temperature, the water vapour in the containers will have pressure in the ratio of
(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 4 : 1

Answers

1.	c	2.	c	3.	b	4.	c	5.	a
6.	d	7.	a	8.	c	9.	b	10.	d
11.	c	12.	c	13.	a	14.	b	15.	a
16.	d	17.	b	18.	c	19.	c	20.	d
21.	d	22.	d	23.	b	24.	d	25.	d
26.	d	27.	d	28.	d	29.	d	30.	a
31.	c	32.	a	33.	b				

Objective Assignment – II (AIIMS)

Q.1 We plot a graph, having temperature in °C on x–axis and in °F on y–axis. If the graph is straight line, then it

(a) passes through origin
(b) intercepts the positive x–axis
(c) intercepts the positive y–axis
(d) intercepts the negative axis of both x–and y–axis

Q.2 At a common temperature, a block of wood and a block of metal feel equally cold or hot. The temperatures of block and wood are

(a) equal to the temperature of the body
(b) less than the temperature of the body
(c) greater than temperature of the body
(d) either (b) or (c)

Q.3 A quantity of heat required to change the unit mass of a solid substance, from solid state to liquid state, while the temperature remains constant, is known as

(a) latent heat (b) sublimation (c) hoar frost (d) latent heat of fusion

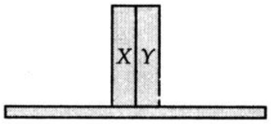
Q.4 When a solid is converted into a gas, directly by heating, then this process is known as

(a) boiling (b) sublimation (c) vaporization (d) condensation

Q.5 A constant pressure air thermometer gave a reading of 47.5 units of volume, when immersed in ice–cold water, and 67 units in a boiling liquid. The boiling point of the liquid is

(a) 100°C (b) 112°C (c) 125°C (d) 135°C

Q.6 A bimetallic strip consists of metals X and Y. It is mounted rigidly at the base as shown below: The metal X has a higher coefficient of expansion compared to that for metal Y. When the bimetallic strip is placed in a cold bath,



(a) it will bend towards the right
(b) it will bend towards the left
(c) it will not bend but shrink
(d) it will neither bend nor shrink

Q.7 The density of a substance at 0°C is 10g cm^{-3} and at 100°C, its density is 9.7 g cm^{-3}.The coefficient of linear expansion of the substance is

(a) 10^{-4} (b) 10^{-2} (c) 10^{-3} (d) 10^{2}

Q.8 Calorimeters are made of which of the following?

(a) glass (b) metal (c) wood (d) either (a) or (c)

Q.9 Hailstone of 0°C falls from a height of 1 km on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt? (g = 10 ms^{-2})

(a) 1/33 (b) 1/8 (c) $(1/33) \times 10^{-4}$ (d) all of it

Q.10 The bulb of one thermometer is spherical, while that of other is cylindrical. If both of them have equal amount of mercury, which one will respond quickly to the temperature?

(a) spherical (b) cylindrical (c) elliptical (d) both (a) and (c)

Q.11 On a cold morning, a metal surface will feel colder to touch than a wooden surface, because

(a) metal has high specific heat
(b) metal has high thermal conductivity
(c) metal has low specific heat
(d) metal has low thermal conductivity

Q.12 Woolen clothes keep the body warm, because wool

(a) is a bad conductor
(b) increases the temperature of body
(c) decreases the temperature
(d) all of these

Q.13 Heat travels through vacuum by

(a) conduction (b) convection (c) radiation (d) both (a) and (b)

Q.14 Ratio of the amount of heat radiation, transmitted through the body to the amount of heat radiation incident on it, is known as

(a) conductance (b) inductance (c) transmittance (d) absorbance

Q.15 Three objects coloured black, gray and white can withstand hostile conditions upto 2,800°C. These objects are thrown into a furnace, where each of them attains a temperature of 2,000°C. Which object will glow brightest?

(a) the white object (b) the black object
(c) all glow with equal brightness (d) gray object

Q.16 If amount of heat energy received per unit area from sun is measured on earth, mars and Jupiter, it will be
(a) the same for all (b) in decreasing order jupiter, mars, earth
(c) in increasing order Jupiter, mars, earth (d) in decreasing order mars, earth, Jupiter

Q.17 A black body is at a temperature 300 K. It emits energy at a rate, which is proportional to
(a) 300 (b) 300^2 (c) 300^3 (d) 300^4

Q.18 If temperature of a black body increases from 7°C to 287°C, then rate of energy radiation increases by
(a) $(287/7)^4$ (b) 16 (c) 4 (d) 2

Q.19 A black body is heated from 27°C to 127°C. The ratio of their energies of radiations emitted will be
(a) 3 : 4 (b) 9 : 16 (c) 27 : 64 (d) 81 : 256

Q.20 A black body, at a temperature of 227°C, radiates heat at a rate of 20 cal $m^{-2}s^{-1}$. When its temperature is raised to 727°C, heat radiated by it (in cal m^{-2} s^{-1}) will be closest to
(a) 40 (b) 160 (c) 320 (d) 640

Q.21 A metal rod at a temperature of 150°C, radiates energy at a rate of 20 W. If its temperature is increased to 300°C, then it will radiate at the rate of
(a) 17.5 W (b) 37.2 W (c) 40.8 W (d) 68.3 W

Q.22 Surface temperature of stars A and B are 727°C and 327°C respectively. What is the ratio $H_A : H_B$ for the heat radiated per second by the two stars?
(a) 5 : 3 (b) 25 : 9 (c) 625 : 81 (d) 125 : 27

Q.23 A black body at a high temperature T K radiates energy at the rate of E Wm^{-2}. When the temperature falls to T/2 K, the radiated energy will be
(a) E/4 (b) E/2 (c) 2 E (d) E/16

Q.24 Suppose the sun expands so that its radius and its surface temperature becomes half of its present value. The total energy emitted by it then will increase by a factor of
(a) 10^4 (b) 625 (c) 256 (d) 16

Q.25 The sun radiates energy in all directions. The average radiation received on the earth's surface from the sun per second is 1.4 kW m^{-2}. The average earth–sun distance is 1.5×10^{11}m. The mass lost by the sun per day (1 day = 86,400s) is
(a) 4.4×10^9 kg (b) 7.6×10^{14} kg (c) 3.8×10^{12} kg (d) 3.8×10^{14} kg

Q.26 Energy from the sun is received on earth at the rate of 2 cal cm^{-2} min^{-1}. If average wavelength of solar light be taken at 5,500 Å, then how many photons are received on the earth per cm^2 per min?
($h = 6.6 \times 10^{-34}$ Js, 1 cal = 4.2 J)
(a) 1.5×10^{13} (b) 2.9×10^{13} (c) 2.3×10^{13} (d) 1.75×10^{19}

Q.27 According to Wien's displacement law
(a) λT = constant (b) $\lambda \propto 1/T$ (c) λ/T = constant (d) both (a) and (b)

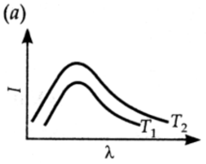
Q.28 For an enclosure maintained at 1,000 K, the maximum radiation occurs at wavelength λ_m. If the temperature is raised to 2,000 K, the peak will shift to
(a) $\lambda_m/2$ (b) $3\lambda_m/2$ (c) $5\lambda_m/2$ (d) $7\lambda_m/2$

Q.29 The sun emits a light with maximum wavelength 510 nm, while another star X emits a light with maximum wavelength of 350 nm. What is the ratio of surface temperature of the sun and the star X?
(a) 1.45 (b) 0.68 (c) 0.46 (d) 2.1

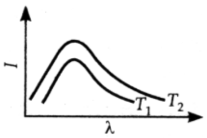
Q.30 On increasing the temperature of a substance gradually, its colour becomes
(a) red (b) green (c) yellow (d) white

Q.31 Shown below are the black body radiation curves at temperature T_1 and T_2 (($T_2 > T_1$). Which of the following plots is correct?

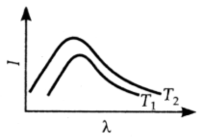
(a)



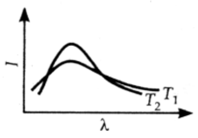
(b)



(c)



(d)



Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion
(c) If assertion is true, but reason is false (d) If both assertion and reason are false

Q.32 **Assertion:** Good conductors of heat are also good conductors of electricity and vice–versa.
Reason: Mainly electrons are responsible for these conductors.

Q.33 **Assertion:** At room temperature water does not sublimate from ice to steam.
Reason: The critical point of water is much above the room temperature.

Q.34 **Assertion:** Water kept in an open vessel will quickly evaporate on the surface of the moon.
Reason: The temperature at the surface of the moon is much higher than the boiling point of water.

Q.35 **Assertion:** In a pressure cooker the water is brought to boil. The cooker is then removed from the stove. Now on removing the lid of the pressure cooker, the water starts boiling again.
Reason: The impurities in water bring down its boiling point.

Q.36 **Assertion:** It is hotter over the top of a fire than at the same distance of the sides.
Reason: Air surrounding the fire conducts more heat upwards

Q.37 **Assertion:** Air at some distance above the fire is hotter than the same distance below it.
Reason: Air surrounding the fire carries heat upwards.

Q.38 **Assertion:** Woollen clothes keep the body warm in winter.
Reason: Air is a bad conductor of heat

Q.39 **Assertion:** The earth without its atmosphere would be inhospitably cold.
Reason: All heat would escape in the absence of atmosphere.

Q.40 **Assertion:** While measuring the thermal conductivity of a liquid experimentally, the upper layer is kept hot and lower layer is kept cold.
Reason: This avoids heating of the liquid by convection.

Q.41 **Assertion:** A body that is a good radiator is also a good absorber of radiation at a given wavelength.
Reason: According to Kirchhof's law the absorptivity of a body is equal to its emissivity at a given wavelength.

Q.42 **Assertion:** Temperature near the sea coast are moderate.
Reason: Water has a high thermal conductivity.

Q.43 **Assertion:** Perspiration from human body helps in cooling the body.
Reason: A thin layer of water on the skin enhances its emissivity.

Q.44 **Assertion:** A hallow metallic closed container maintained at a uniform temperature can act as a source of black body radiation.
Reason: All metals act as black bodies.

Q.45 **Assertion:** Blue star is at higher temperature than red star.
Reason: Wien's displacement law states that $T \propto 1/\lambda_m$.

Q.46 **Assertion:** For higher temperatures, the peak emission wavelength of a black body shifts to lower wavelengths.

Reason: Peak emission wavelength of a black body is proportional to the fourth–power of temperature.

Q.47 **Assertion:** The radiation from the sun's surface varies as the fourth power of its absolute temperature.

Reason: Sun is not a black body.

Answers

1.	c	2.	a	3.	d	4.	b	5.	b
6.	b	7.	a	8.	b	9.	a	10.	b
11.	b	12.	a	13.	c	14.	c	15.	b
16.	c	17.	d	18.	b	19.	d	20.	c
21.	d	22.	c	23.	d	24.	b	25.	d
26.	c	27.	d	28.	a	29.	b	30.	c
31.	c	32.	a	33.	a	34.	c	35.	c
36.	c	37.	a	38.	a	39.	a	40.	a
41.	a	42.	b	43.	c	44.	c	45.	a
46.	c	47.	c						

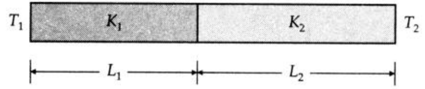
Objective Assignment – III (AIEEE)

Q.1 Heat give to a body, which raises its temperature by $1^{o}C$ is

(a) water equivalent (b) temperature gradient

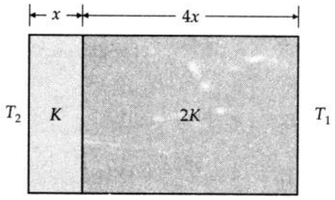
(c) thermal capacity (d) specific heat

Q.2 One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed to two sections of lengths L_1 and L_2 and the coefficients of thermal conductivity K_1 and K_2 respectively. The temperature at the interface of the two sections is



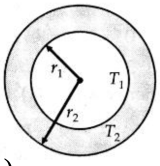
(a) $\frac{K_2L_2T_1+K_1L_1T_2}{K_1L_1+K_2L_2}$ (b) $\frac{K_2L_1T_1+K_1L_2T_2}{K_2L_1+K_1L_2}$ (c) $\frac{K_1L_2T_1+K_2L_1T_2}{K_1L_2+K_2L_1}$ (d) $\frac{K_1L_1T_1+K_2L_2T_2}{K_1L_1+K_2L_2}$

Q.3 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in a steady state is $\left(\frac{A(T_2-T_1)K}{x}\right)f$, where f equals to



(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

Q.4 The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(a) $(r_2 - r_1)/ r_1r_2$ (b) $\log_e (r_2/r_1)$ (c) $r_1r_2 (r_2 - r_1)$ (d) $(r_2 - r_1)$

Q.5 According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings and n is equal to

(a) two (b) three (c) four (d) one

Q.6 A pressure cooker reduces cooking time for food, because

(a) heat is more evenly distributed in the cooking space

(b) the higher pressure inside the cooker crushes the food material

(c) cooking involves chemical changes helped by a rise in temperature
(d) boiling point of water involved in cooking is increased

Q.7 Which of the following is more close to a black body?
(a) blackboard paint (b) green leaves (c) black holes (d) red roses

Q.8 If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously, will be
(a) 4 (b) 16 (c) 32 (d) 64

Q.9 Two spheres of the same material have radii 1m and 4m and temperatures 4,000 K and 2,000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 1 : 9 (c) 4 : 1 (d) 16 : 1

Q.10 Assuming the sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant power, incident on earth, at a distance r from the sun.
(a) $\frac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$ (b) $\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (c) $\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$ (d) $\frac{R^2 \sigma T^4}{r^2}$
Here, r_0 is the radius of the earth and σ is Stefan's constant.

Q.11 The earth radiates in the infra–red region of the spectrum. The spectrum is correctly given by
(a) Planck's law of radiation (b) Stefan's law of radiation
(c) Rayleigh Jeans law (d) Wien's law

Q.12 A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(a) E/c (b) 2E/c (c) Ec (d) E/c^2

Q.13 On a hilly region, water boils at 95°C. The temperature expressed in Fahrenheit is
(a) 100°F (b) 20.3°F (c) 150°F (d) 203°F

Q.14 If boiling point of water is 95°F, what will be reading at Celsius scale?
(a) 7°C (b) 65°C (c) 63°C (d) 35°C

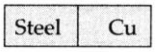
Q.15 50g of ice at 0°C is mixed with 50g of water at 80°C, final temperature of mixture of will be
(a) 0°C (b) 40°C (c) 60°C (d) 4°C

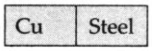
Q.16 Which one of the following processes depends on gravity?
(a) conduction (b) convection (c) radiation (d) none of these

Q.17 A composite rod made of copper ($\alpha = 1.8 \times 10^{-5} K^{-1}$) and steel ($\alpha = 1.2 \times 10^{-5} K^{-1}$) is heated. Then
(a) it bends with steel on concave side (b) it bends with copper on concave side
(c) it does not expand (d) data is insufficient

Q.18 Heat current is maximum in which of the following (rods are of identical dimensions)?

(a)


(b)


(c)


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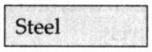
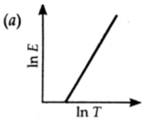
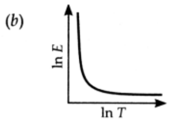


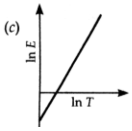
Q.19 The sprinkling of water reduces slightly the temperature of a closed room because
(a) temperature of water is less than that of the room (b) specific heat of water is high
(c) water has large latent heat of vaporization (d) water is a bad conductor of heat

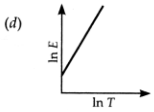
Q.20 Two rods of lengths d_1 and d_2 and coefficients of thermal conductivities K_1 and K_2 are kept in contact with each other end to end. The equivalent thermal conductivity is
(a) $K_1d_1 + K_2d_2$ (b) $K_1 + K_2$ (c) $\frac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (d) $\frac{d_1 + d_2}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$

Q.21 Which of the following graphs correctly represents the relation between E and T, where E is the amount of radiation emitted per unit time from unit area of body and T is the absolute temperature?









Q.22 A back body at a temperature of 227°C radiates heat at the rate of 20 cal $m^{-2}s^{-1}$. When its temperatures rises to 727°C, the rate of heat radiated will be

(a) 40 cal m^{-2} s^{-1} (b) 160 cal m^{-2} s^{-1} (c) 320 cal m^{-2} s^{-1} (d) 640 cal $m^{-2}s^{-1}$

Q.23 Temperature of a black body increases from 327°C to 927°C, the initial energy possessed is 2 kJ. What is its final energy?

(a) 32 kJ (b) 320 kJ (c) 1200 kJ (d) none of these

Q.24 In determining the temperature of a distant star, one makes use of

(a) Kirchhoff's law (b) Stefan's law (c) Wien's displacement law (d) none of the above

Q.25 The wavelength of radiation emitted by a body depends upon

(a) the nature of the surface (b) the area of the surface

(c) the temperature of the surface (d) all of the above factors

Q.26 Temperatures of two stars are in ratio 3 : 2. If wavelength of maximum intensity of first body is 4000Å, what is corresponding wavelength of second body?

(a) 9000 Å (b) 6000 Å (c) 2000 Å (d) 8000 Å

Q.27 A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then

(a) the blue piece will look blue and red will look as usual

(b) red looks brighter red and blue looks ordinary blue

(c) blue shines like brighter red compared to the red piece (d) both the pieces will look equally red.

Q.28 The unit of Stefan's constant is

(a) $Wm^{-2}K^{-1}$ (b) WmK^{-4} (c) Wm^{-2} K^{-4} (d) $Nm^{-2}K^{-4}$

Answers

1.	c	2.	c	3.	d	4.	c	5.	d
6.	d	7.	a	8.	d	9.	a	10.	c
11.	d	12.	b	13.	d	14.	d	15.	a
16.	b	17.	a	18.	a	19.	c	20.	d
21.	c	22.	c	23.	a	24.	c	25.	c
26.	b	27.	c	28.	c				

Objective Assignment – IV (IIT – JEE)

Multiple Choice Questions with One Correct Answer

Q.1 A constant volume air thermometer works on

(a) Archemedes' principle (b) Boyle's law

(c) Pascal's law (d) Gay Lussac's law

Q.2 The temperature coefficient of resistance of a wire is 0.00125°C^{-1}. At 300 K, its resistance is 1Ω. The resistance of the wire will be 2Ω at

(a) 1,154 K (b) 1,100 K (c) 1,400 K (d) 1,127 K

Q.3 Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_A and α_S respectively. If the length of each rod increases by the same amount, when their temperatures are raised by t°C, then find the ratio $l_1/(l_1 + l_2)$

(a) α_S/ α_A (b) α_A/ α_S (c) $\alpha_S/ (\alpha_A + \alpha_S)$ (d) $\alpha_A /(\alpha_A + \alpha_S)$

Q.4 A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C. The co–efficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

(a) $W_1 > W_2$ (b) $W_1 = W_2$ (c) $W_1 < W_2$ (d) all of these

Q.5 When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the co–efficient of volume expansion of iron γ_{Fe} and that of mercury of γ_{Hg}, then the ratio k_1/k_2 can be expressed as

(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

Q.6 Calorie is defined as the amount of heat required to raise temperature of 1g of water by 1°C and it is defined under which of the following conditions?

(a) From 14.5°C to 15.5°C at 760 mm of Hg (b) From 98.5°C to 99.5°C at 760 mm of Hg

(c) From 13.5°C to 14.5°C at 76mm of Hg (d) From 3.5°C to 4.5°C at 76mm of Hg

Q.7 Compared to burn due to air at 100°C, a burn due to steam at 100°C is

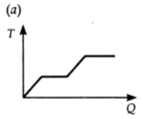
(a) more dangerous (b) less dangerous (c) equally dangerous (d) none of the above

Q.8 540g of ice at 0°C mixed with 540g of water at 80°C. The final temperature of mixture is

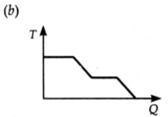
(a) 0°C (b) 40°C (c) 80°C (d) less than 0°C

Q.9 A block of ice at – 10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively?

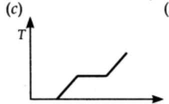
(a)



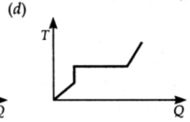
(b)



(c)



(d)



Q.10 2kg of ice at – 20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal $\text{kg}^{-1}{}^{\circ}\text{C}^{-1}$ and 0.5 kcal kg^{-1} $^{\circ}\text{C}^{-1}$, white the latent heat of fusion of ice is 80 kcal kg^{-1}

(a) 7 kg (b) 6 kg (c) 4 kg (d) 2 kg

Q.11 Water of volume 2 litre in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. In how much time, temperature will rise from 27°C to 77°C? Given that the specific heat of water is 4.2 kJ kg^{-1}.

(a) 8 min 20 s (b) 6 min 2s (c) 7 min (d) 14 min

Q.12 In which of the following processes, convection does not take place primarily?

(a) sea and land breeze (b) boiling of water (c) warming of glass of the bulb due to filament

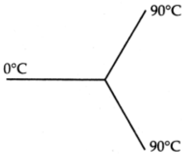
(d) heating air around a furnace

Q.13 A wall has two layers A and B, each made of a different material. Both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer A

(a) 6°C (b) 12°C (c) 18°C (d) 24°C

Q.14 Three rods of identical cross–sectional area and made from the same metal form the sides of an isosceles triangle ABC, right–angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$ respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, T_C/T is

(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$ (c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$



Q.15 Three rods made of same material and having the same cross–section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be

(a) 45°C (b) 60°C (c) 30°C (d) 20°C

Q.16 Two identical rods are connected between two containers. One of them is at 100°C and another is at 0°C. If rods are connected in parallel then the rate of melting of ice is q_1g/sec. If they are connected in series then the rate is q_2. The ratio q_2/q_1 is

(a) 2 (b) 4 (c) 1/2 (d) 1/4

Q.17 Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2.Both spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) 1/3 (b) $1/\sqrt{3}$ (c) $\sqrt{3}/1$ (d) $(1/3)^{1/3}$

Q.18 A spherical body of area A and emissivity e = 0.6 is kept inside a perfectly black body. Energy radiated per second by the body at temperature T is

(a) $0.4\ \sigma AT^4$ (b) $0.8\ \sigma AT^4$ (c) $0.6\ \sigma AT^4$ (d) $1.0\ \sigma AT^4$

Q.19 An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) initially, it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

(c) it cannot be distinguished at all times

(d) initially, it is the darkest body and at later times it cannot be distinguished

Q.20 A spherical black body with a radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 (b) 450 (c) 900 (d) 1800

Q.21 The earth receives at its surface radiation from the sun at the rate of 1,400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. Treating sun as a black body, it follows from the above data that its surface temperature is

(a) 5,801 K (b) 10^6 K (c) 50.1 K (d) 5,801°C

Q.22 Two spheres of same material have radii 1m and 4m and temperature 4,000 K and 2,000K respectively. The energy radiated per second by the first sphere is

(a) greater than that by the second (b) less that that by the second

(c) equal in both cases (d) the information is incomplete to draw any conclusion

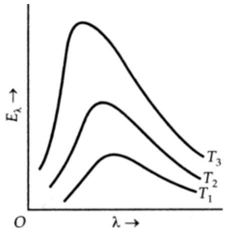
Q.23 Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following options is the correct match?

(a) Sun – T_3, tungsten filament – T_1, welding arc – T_2

(b) Sun – T_2, tungsten filament – T_1, welding arc – T_3

(c) Sun – T_3, tungsten filament – T_2, welding arc – T_1

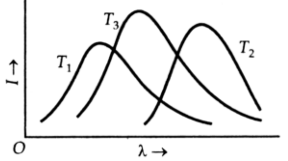
(d) Sun – T_1, tungsten filament – T_2, welding arc – T_3



Q.24 The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperatures of the sun and the North Star is

(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

Q.25 The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown: Their temperatures are such that

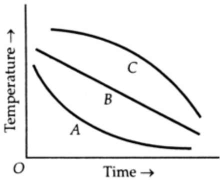


(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

Q.26 A sphere, a cube and a thin circular plate, all made of the same material and having the same mass, are initially heated to a temperature of 3,000°C. Which of these will cool fastest?

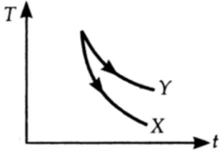
(a) sphere (b) cube (c) plate (d) none of these

Q.27 A block of steel heated to 100°C is left in a room to cool. Which of the curves shown in the figure, represents the correct behaviour?



(a) A (b) B
(c) C (d) none

Q.28 The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies X and Y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies.



(a) $e_X > e_Y$ and $a_X < a_Y$ (b) $e_X < e_Y$ and $a_X > a_Y$ (c) $e_X > e_Y$ and $a_X > a_Y$ (d) $e_X < e_Y$ and $a_X < a_Y$

Q.29 Three discs A, B and C having radii 2m, 4m and 6m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The powers radiated by them are Q_A, Q_B and Q_C respectively.

(a) Q_A is maximum (b) Q_B is maximum (c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

Multiple Choice Questions with One or More than One Correct Answer

Q.30 A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then, R is

(a) proportional to ΔT (b) inversely proportional to ΔT
(c) proportional to $|\alpha_B - \alpha_C|$ (d) inversely proportional to $|\alpha_B - \alpha_C|$

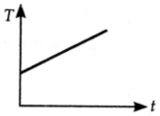
Q.31 Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

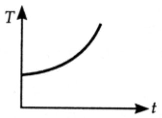
(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (c) 4 : 9

Q.32 Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C, till the temperature of the calorimeter and its contents rises to 80°C. The mass of steam condensed (in kg) is

(a) 0.130 (b) 0.065 (c) 0.260 (d) 0.195

Q.33 If liquefied oxygen at 1 atmospheric pressure is heated from 50 K to 300 K by supplying heat at constant rate, the graph of temperature vs time will be

(a)


(b)


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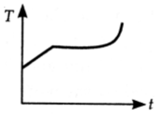

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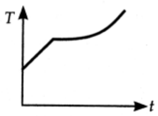


Q.34 A cylinder of radius R, made of a material of thermal conductivity K_1 is surrounded by a cylindrical sheet of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $K_1 + K_2$ (b) $\frac{K_1 + 3K_2}{4}$ (c) $\frac{K_1 K_2}{K_1 + K_2}$ (d) $\frac{3K_1 + K_2}{4}$

Q.35 A black body is at a temperature of 2,880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1,499 nm and 1,500 nm is U_3. The Wien's constant, $b = 2.88 \times 10^6$ nm K. Then,

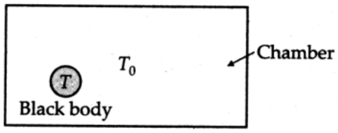
(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

Q.36 Two bodies A and B have thermal emissivities 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5,802 K, then

(a) the temperature of B is 1934 K (b) the temperature of B is 11604 K

(c) the temperature of B is 2901 K (d) $\lambda_B = 1.5$ μm

Q.37 Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remain constant. Which of the following statements is/are true?



(a) the rate of emission of energy from the black body remains the same

(b) the rate of emission of energy from the black body increases

(c) the rate of absorption of energy by the black body increases

(d) the energy radiated by the black body equals the energy absorbed by it.

Answers

1.	d	2.	d	3.	c	4.	a	5.	a
6.	a	7.	a	8.	a	9.	a	10.	b
11.	a	12.	c	13.	b	14.	b	15.	b
16.	d	17.	d	18.	c	19.	a	20.	d
21.	a	22.	c	23.	a	24.	b	25.	b
26.	c	27.	a	28.	c	29.	b	30.	b, d
31.	c	32.	a	33.	c	34.	b	35.	d
36.	d	37.	a, d						