OSCILLATIONS

Periodic Motion

Any motion that repeats itself over and over again at regular intervals of time is called periodic or harmonic motion. The smallest interval of time after which the motion is repeated is called its time period. The time period is denoted by T and its SI unit is second.

Examples of Periodic Motion

(i) The motion of any planet around the sun in an elliptical orbit is periodic. The period of revolution of Mercury is 87.97 days.

(ii) The motion of the moon around the earth is periodic. Its time period is 27.3 days.

(iii) The motion of Halley's comet around the sun is periodic. It appears on the earth after every 76 years.

(iv) The motion of the hands of a clock is periodic.

(v) The heart beats of a human being are periodic. The periodic time is about 0.8 second for a normal person.

Oscillatory Motion

If a body moves back and forth repeatedly about its mean position, its motion is said to be oscillatory or vibratory or harmonic motion. Such a motion repeats itself over and over again about a mean position such that it remains confirmed within well defined limits (known as extreme positions) on either side of mean position.

Examples of Oscillatory Motion

(i) The swinging motion of the pendulum of a wall clock.

(ii) The oscillations of a mass suspended from a spring.

(iii) The motion of the piston of an automobile engine.

(iv) The vibrations of the string of a guitar.

(v) When a freely suspended bar magnet is displaced from its equilibrium position along north south line and released, it executes oscillatory motion.

Periodic Functions and Fourier Analysis

Periodic function

Any function that repeats itself at regular intervals of its argument is called a periodic function. Consider the function f(θ) satisfying the property,

$$f(\theta + T) = f(\theta)$$

This indicates that the value of the function f remains same when the argument is increased or decreased by an integral multiple of T for all values of θ. A function f satisfying this property is said to be periodic having a period T. For example, trigonometric functions like sin θ and cos θ are periodic with a period of 2π radians, because

$$\sin(\theta + 2\pi) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$$

If the independent variable θ stands for some dimensional quantity such as time t, then we can construct periodic functions with period T as follows:

$$f_1(t) = \sin\frac{2\pi t}{T} \quad \text{and} \quad g_1(t) = \cos\frac{2\pi t}{T}$$

We can check the periodicity by replacing t by t + T. Thus

$$f_1(t+T) = \sin\frac{2\pi}{T}(t+T) = \sin\left(\frac{2\pi t}{T} + 2\pi\right) = \sin\frac{2\pi t}{T} = f_1(t)$$

Similarly, $g_1(t + T) = g_1(t)$

It can be easily seen that functions with period T/n, where n = 1, 2, 3, ... also repeat their values after a time t. Hence it is possible to construct two infinite sets of periodic functions such as

$$f_n(t) = \sin\frac{2\pi nt}{T} \qquad n = 1, 2, 3, 4, ...$$

$$g_n(t) = \cos\frac{2\pi nt}{T} \qquad n = 0, 1, 2, 3, 4, ...$$

In the set of cosine functions we have included the constant function $g_0(t) = 1$. The constant function 1 is periodic for any value of T and hence does not alter the periodicity of $g_n(t)$.

Periodic, harmonic and Non–harmonic Functions

Any function that repeats itself at regular intervals of its argument is called a periodic function. The following sine and cosine functions are periodic with period T :

$$f(t) = \sin\omega t = \sin\frac{2\pi t}{T} \qquad \text{and} \qquad g(t) = \cos\omega t = \cos\frac{2\pi t}{T}$$

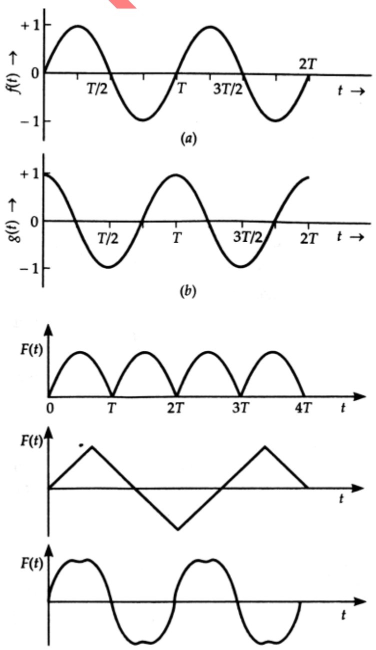

Figure shows how these functions vary with time t.

Obviously, these functions vary between a maximum value + 1 and minimum value – 1 passing through zero in between. *The periodic functions which can be represented by a sine or cosine curve are called **harmonic functions.** All harmonic functions are necessarily periodic but all periodic functions are not harmonic. The periodic functions which cannot be represented by single sine or cosine function are called non–harmonic functions.* Figure shows some periodic functions which repeat themselves in a period T but are not harmonic. Any non–harmonic periodic function can be constructed from two or more harmonic functions. One such function is: $F(t) = a_1 \sin\omega t + a_2 \sin 2\omega t$

It can be easily checked that the functions tan ωt and cot ωt are periodic with period $T = \pi/\omega$ while sec ωt and cosec ωt are periodic with period $T = 2\pi/\omega$. Thus

$$\tan\left\{\omega\left(t + \frac{\pi}{\omega}\right)\right\} = \tan(\omega t + \pi) = \tan\omega t$$

$$\sec\left\{\omega\left(t + \frac{2\pi}{\omega}\right)\right\} = \sec(\omega t + 2\pi) = \sec\omega t$$

But such functions take values between zero and infinity. So these functions cannot be used to represent displacement functions in periodic motions because displacement always takes a finite value in any physical situation.

Subjective Assignment – I

Q.1 On an average a human heart is found to beat 75 times in a minute. Calculate its beat frequency and period.

Q.2 Which of the following functions of time represent (a) periodic and (n) non–periodic motion? Give the period for each case of periodic motion. [ω is any positive constant].

(i) sin ωt + cos ωt (ii) sin ωt + cos 2ωt + sin 4ωt (iii) $e^{-\omega t}$ (iv) log ωt

Q.3 *Which of the following functions of time represent (a) simple harmonic motion, (b) periodic but not simple harmonic and (c) non–periodic motion? Find the period of each periodic motion. Here ω is a positive real constant.*

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(a) $\sin \omega t + \cos \omega t$. (b) $\sin \pi t + 2 \cos 2\pi t + 3 \sin 3\pi t$. (c) $\cos (2\omega t + \pi/3)$.

(d) $\sin 2 \omega t$ (e) $\cos \omega t + 2 \sin^2 \omega t$.

Answers

1. 0.8 s, 1.25 Hz
2. (i) periodic function with period $2\pi/\omega$, (ii) periodic function with period $2\pi/\omega$,
   (iii) non periodic, (iv) non periodic
3. (a) simple harmonic, (b) periodic but not simple harmonic, (c) Simple harmonic
   (d) Periodic but not simple harmonic (e) Periodic but not simple harmonic

Simple Harmonic Motion

A particle is said to execute simple harmonic motion if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from the mean position and is always directed towards the mean position. If the displacement of the oscillating body from the mean position is small, then

Restoring force $\propto$ Displacement $F \propto x$ or $F = -kx$

This equation defines S.H.M. Here k is a positive constant called ***force constant or spring factor*** and is *defined as the restoring force produced per unit displacement*. The SI unit of k is Nm^{-1}. The negative sign in the above equation shows that the restoring force F always acts in the opposite direction of the displacement x. Now, according to Newton's second law of motion, $F = ma$

$\therefore$ $ma = -kx$ or $a = -\frac{k}{m}x$ *i.e.,* $a \propto x$

Hence simple harmonic motion may also be defined as follows:

A particle is said to posses simple harmonic motion if it moves to and fro about a mean position under an acceleration which is directly proportional to its displacement from the mean position and is always directed towards that position.

Examples of Simple Harmonic Motion

(i) Oscillations of a loaded spring.
(ii) Vibrations of a tuning fork.
(iii) Vibrations of the balance wheel of a watch.
(iv) Oscillations of a freely suspended magnet in a uniform magnetic field.

Some Important Features of S.H.M.

(i) The motion of the particle is periodic.
(ii) It is the oscillatory motion of simplest kind in which the particle oscillates back and forth about its mean position with constant amplitude and fixed frequency.
(iii) Restoring force acting on the particle is proportional to its displacement from the mean position.
(iv) The force acting in the particle always opposes the increase in its displacement.
(v) A simple harmonic motion can always be expressed in terms of a single harmonic function of sine or cosine.

Differential Equation for S.H.M.

In S.H.M., the restoring force acting on the particle is proportional to its displacement. Thus

$$F = -kx$$

The negative sign shows that F and x are oppositely directed. Here k is spring factor or force constant.

By Newton's second law, $F = m\frac{d^2x}{dt^2}$

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where m is the mass of the particle and $\frac{d^2x}{dt^2}$ is its acceleration.

$$\therefore \qquad m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Put $\frac{k}{m} = \omega^2$, *then* $\frac{d^2x}{dt^2} = -\omega^2 x$ or $\frac{d^2x}{dt^2} + \omega^2 x = 0$ ... (1)

This is the *differential equation of S.H.M.* Here ω is the angular frequency. Clearly, x should be such a function whose second derivative is equal to the function itself multiplied with a negative constant. So a possible solution of equation (1) may be of the form

$$x = A\cos(\omega t + \phi_0)$$

Then $\frac{dx}{dt} = -\omega A\sin(\omega t + \phi_0)$ and $\frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi_0) = -\omega^2 x$

or $\frac{d^2x}{dt^2} + \omega^2 x = 0$

which is same as equation (1). Hence the solution of the equation (1) is

$$x = A\cos(\omega t + \phi_0) \qquad .... (2)$$

It gives displacement of the harmonic oscillator at any instant t. Here A is the *amplitude* of the oscillating particle.

$\phi = \omega t + \phi_0$, is the phase of the oscillating particle.

ϕ_0 is the initial *phase* (at t = 0) or epoch.

Time period of S.H.M.

If we replace t by $t + \frac{2\pi}{\omega}$ in equation (2), we get $x = A\cos\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi_0\right]$

i.e., the motion repeats after time interval $\frac{2\pi}{\omega}$. Hence $\frac{2\pi}{\omega}$ is the time period of S.H.M.

$$\therefore \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \qquad \left[\because \omega^2 = \frac{k}{m}\right] \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

In general, *m* is called inertia factor and *k* the spring factor.

Some Important Terms Connected with S.H.M.

(i) **Harmonic Oscillator**: *A particle executing simple harmonic motion is called harmonic oscillator.*

(ii) **Displacement:** *The distance of the oscillating particle from its mean position at any instant is called its displacement.* It is denoted by x.

(iii) **Amplitude:** *The maximum displacement of the oscillating particle on either side of its mean position is called its amplitude.* It is denoted by A. Thus $x_{max} = \pm A$.

(iv) **Oscillation or Cycle:** *One complete back and forth motion of a particle starting and ending at the same point is called a cycle or oscillation or vibration.*

(v) **Time period:** *The time taken by a particle to complete one oscillation is called its time period. Or, it is the smallest time interval after which the oscillatory motion repeats.* It is denoted by T.

(vi) **Frequency:** *It is defined as the number of oscillations completed per unit time by a particle.* It is denoted by *v*. Frequency is equal to the reciprocal of time period. That is,

$$v = \frac{1}{T}$$

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Unit of frequency is (second)$^{-1}$ or s^{-1}. It is also expressed as *cycles per second* (cps) or *hertz* (Hz).
SI unit of frequency = s^{-1} = cps = Hz

(vii) **Angular frequency:** *It is the quantity obtained by multiplying frequency v by a factor of 2π.* It is denoted by ω.

Thus, $$\omega = 2\pi v = \frac{2\pi}{T}$$

SI unit of angular frequency = rad s^{-1}

(viii) **Phase:** *The phase of a vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.* It is equal to the argument of sine or cosine function occurring in the displacement equation of the S.H.M. Suppose a simple harmonic equation is represented by $x = A\cos(\omega t + \phi_0)$

Then phase of the particle is : $\phi = \omega t + \phi_0$

Thus the phase ϕ gives an idea about the position and direction of motion of the oscillating particle.

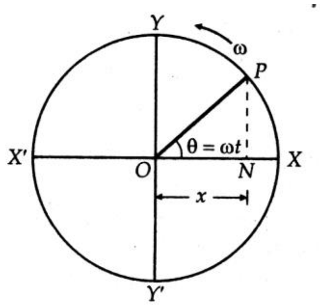
(xi) **Initial phase or epoch:** *The phase of a vibrating particle corresponding to time t = 0 is called initial phase or epoch.*

At t = 0, $\phi = \phi_0$

The constant ϕ_0 is called initial phase or epoch. It tells about initial state of motion of vibrating particle.

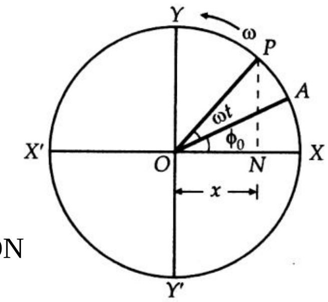
Uniform Circular Motion and S.H.M.

Consider a particle P moving along a circle of radius A with uniform angular velocity ω. Let N be the foot of the perpendicular drawn from the point P to the diameter XX'. Then N is called the *projection* of P on the diameter XX'. As P moves along the circle from X to Y, Y to X', X' to Y' and Y' to X; N moves from X to O, O to X', X' to O and O to X. Thus, as P revolves along the circumference of the circle, N moves to and fro about the point O along the diameter XX'. The motion of N about O is said to be simple harmonic. Hence *simple harmonic motion may be defined as the projection of uniform circular motion upon a diameter of a circle.* The particle P is called *reference particle or generating particle* and the circle along which the particle P revolves is called *circle of reference.*



Displacement in Simple Harmonic Motion

As shown in figure, consider a particle moving in anticlockwise direction with uniform angular velocity ω along a circle of radius A and centre O. Suppose at time t = 0, the reference particle is at point A such that ∠XOA = ϕ_0. At any time t, suppose the particle reaches the point P such that ∠AOP = ωt. Draw PN ⊥ XX'.



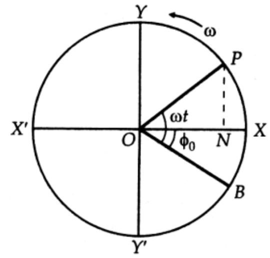
Clearly, displacement of projection N from centre O at any instant t is $x = ON$

In right–angled ΔONP,

$\angle PON = \omega t + \phi_0$

$$\therefore \quad \frac{ON}{OP} = \cos(\omega t + \phi_0) \quad \text{or} \quad \frac{x}{A} = \cos(\omega t + \phi_0) \quad \text{or} \quad x = A\cos(\omega t + \phi_0)$$

This equation gives displacement of a particle in S.H.M. at any instant t. The quantity $\omega t + \phi_0$ is called phase of the particle and ϕ_0 is called *initial phase or phase constant or epoch* of the particle. The quantity A is called *amplitude* of the



motion. It is a positive constant whose value depends on how the motion is initially started. Thus

$$\underset{\text{Displacement}}{x} = \underset{\text{Amplitude}}{A}\cos(\overbrace{\underset{\text{Angular frequency}}{\omega t} + \underset{\text{Initial phase}}{\phi_0}}^{\text{Phase}})$$

As shown in figure, if the reference particle starts motion from the point P such that $\angle BOX = \phi_0$ and $\angle BOP = \omega t$, then $x = \angle PON = \omega t - \phi_0$ $\therefore$ $x = A\cos(\omega t - \phi_0)$

Here $-\phi_0$ is the initial phase of the S.H.M.

Velocity in S.H.M.

Consider a particle P moving with uniform angular speed ω in a circle of radius A. Its velocity vector $\vec{V}$ is directed along the tangent and the magnitude of this velocity vector is

v = Angular velocity × radius = ω A

Draw PP' and QQ' perpendiculars to the diameter XX'. The motion of P' is simple harmonic. Clearly, the instantaneous velocity of a particle executing S.H.M. will be

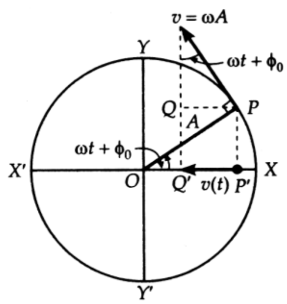
v(t) = Velocity of the particle P' at any instant t

= Projection of the velocity v of the reference particle P

= P' Q' = PQ = $-v\sin(\omega t + \phi_0)$ or $v(t) = -\omega A\sin(\omega t + \phi_0)$

The negative sign shows that the velocity of P' is directed towards left i.e., in the negative X–direction. Moreover,

$$v(t) = -\omega A\sqrt{1-\cos^2(\omega t+\phi_0)} = -\omega A\sqrt{1-\frac{x^2}{A^2}}$$



or $v(t) = -\omega\sqrt{A^2-x^2}$ $[\because x = A\cos(\omega t+\phi_0)]$

Special Cases

(i) When the particle is at the mean position, then x = 0, so

$$v(t) = -\omega\sqrt{A^2-0^2} = -\omega A$$

This is the maximum velocity which a particle in S.H.M. can execute and is called velocity amplitude, denoted by v_{max}

$$\therefore \quad v_{max} = \omega A = \frac{2\pi}{T}A$$

(ii) When the particle is at the extreme position, then x = ± A, so $v = -\omega\sqrt{A^2-A^2} = 0$

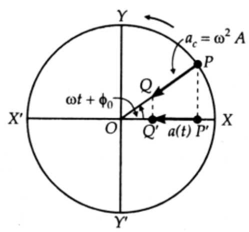
Thus the velocity of a particle in S.H.M. is zero at either of its extreme positions.

Acceleration of a Particle in S.H.M.

Consider a particle P moving with uniform angular speed ω in a circle of radius A. The particle has the centripetal acceleration $\vec{a}_c$ acting radially towards the centre O. The magnitude of this acceleration is $a_c = \omega^2 A$.

Draw PP' and QQ' perpendiculars to the diameter XX'. The motion of P' is simple harmonic. Clearly, the instantaneous acceleration of a particle executing S.H.M. will be

a (t) = Acceleration of particle P' at any instant t



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= Projection of the acceleration a_c of the reference particle P

= Projection of PQ on diameter XX'

= P' Q' = – a_c cos ($\omega t + \phi_0$)

or $\quad a(t) = -\omega^2 A \cos(\omega t + \phi_0) = -\omega^2 x$

This equation expresses the acceleration of a particle executing S.H.M. It shows that *the acceleration of a particle in S.H.M. is proportional to its displacement from the mean position and acts in the opposite direction of the displacement.*

Special Cases

(i) When the particle is at the mean position, then x = 0, so, acceleration = $-\omega^2$ (0) = 0. Hence *the acceleration of a particle in S.H.M. is zero at the mean position.*

(ii) When the particle is at the extreme position, then x = A, so acceleration = $-\omega^2 A$

This is the maximum value of acceleration which a particle in S.H.M. can possess and is called **acceleration** amplitude, denoted by a_{max}.

$$\therefore \quad a_{max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$$

Phase Relationship between Displacement, Velocity and Acceleration

If a particle executing S.H.M. passes through its positive extreme position (x = $\pm$ A) at time t = 0, then its displacement equation can be written as

$$x(t) = A \cos \omega t$$

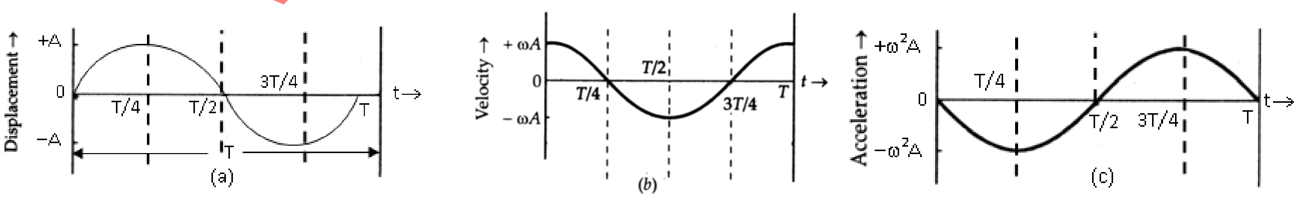
Velocity, $\quad v(t) = \dfrac{dx}{dt} = -\omega A \sin \omega t \quad = \omega A \cos\left(\omega t + \dfrac{\pi}{2}\right)$

Acceleration, $\quad a(t) = \dfrac{dv}{dt} = -\omega^2 A \cos \omega t \quad = \omega^2 A \cos(\omega t + \pi)$

Using the above relations, we determine the values of displacement, velocity and acceleration at various instant t for one complete cycle as illustrated below.

Time, t	**0**	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	**T**
Phase angle, $\omega t = \frac{2\pi}{T} t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement, $x(t)$	$+A$ max.	0 min.	$-A$ max.	0 min.	$+A$ max.
Velocity, $v(t)$	0 min.	$-\omega A$ max.	0 min.	$+\omega A$ max.	0 min.
Acceleration, $a(t)$	$-\omega^2 A$ max.	0 min.	$+\omega^2 A$ max.	0 min.	$-\omega^2 A$ max.

In figure, we have plotted separately x versus t, v versus t and a versus t curves for a simple harmonic motion.



Conclusions

From the above graphs, we can draw the following conclusions about simple harmonic motion:

(i) Displacement, velocity and acceleration, all vary harmonically with time.

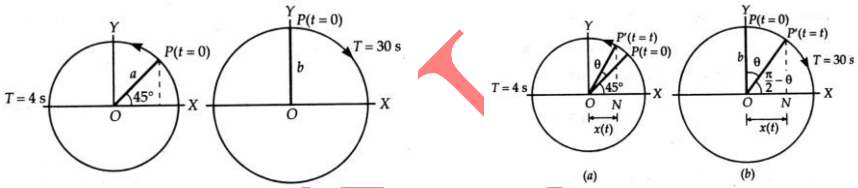
(ii) The velocity amplitude is ω times; and acceleration amplitude is ω^2 times displacement amplitude A.

(iii) Clearly, the velocity curve lies shifted to the left of the displacement curve by an interval of T/4. Thus the *particle velocity is ahead of its displacement by a phase angle of π/2 rad.* This means that whichever value displacement attains at any instant, velocity attains a similar value a T/4 time (a quarter of cycle) earlier. When the particle velocity is maximum, the displacement is minimum and vice versa.

(iv) Clearly, the acceleration curve lies shifted to the left of the displacement curve by an interval of T/2. *Thus the particle acceleration is ahead of its displacement by a phase angle of π rad.* Or, acceleration is ahead of velocity in phase by π/2 rad. When acceleration has maximum positive value, displacement has maximum negative value and vice versa. When displacement is zero, the acceleration is also zero.

Subjective Assignment – II

Q.1 The following figures depict two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figure. Obtain the simple harmonic motions of the x–projection of the radius vector of the rotating particle P in each case.



Q.2 A simple harmonic motion is represented by x = 10 sin (20t + 0.5)
Write down its amplitude, angular frequency, frequency, time period and initial phase, if displacement is measured in metres and time in seconds.

Q.3 A particle executes SHM with a time period of 2s and amplitude 5 cm. Find (i) displacement (ii) velocity and (iii) acceleration, after 1/3 second; starting from the mean position.

Q.4 A body oscillates with SHM according to the equation: $x(t) = 5 \cos (2\pi t + \pi/4)$,
where t is in sec. and x in metres. Calculate
(a) Displacement at t = 0 (b) Time period (c) Initial velocity

Q.5 A body oscillates with SHM according to the equation, $x = (5.0) \cos [(2\pi \text{ rad s}^{-1}) t + \pi/4]$
At t = 1.5 s, calculate (a) displacement, (b) speed and (c) acceleration of the body.

Q.6 The equation of a simple harmonic motion is given by $y = 6 \sin 10\pi t + 8 \cos 10\pi t$, where is in cm and t in sec. Determine the amplitude, period and initial phase.

Q.7 A particle executes S.H.M. of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position?

Q.8 The shortest distance traveled by a particle executing SHM from mean position in 2 s is equal to $(\sqrt{3}/2)$ times its amplitude. Determine its time period.

Q.9 The time–period of a simple pendulum is 2s and it can go to and fro from equilibrium position at a maximum distance of 5 cm. If at the start of the motion the pendulum is in the position of maximum displacement towards the right of the equilibrium position, then write the displacement equation of the pendulum.

Q.10 A particle executes S.H.M. of time period 10 seconds. The displacement of particle at any instant is given by : x = 10 sin ωt (in cm). Find (i) the velocity of body 2s after it passes through mean position (ii) the acceleration 2s after it passes the mean position.

Q.11 For a particle in SHM, displacement x of the particle as a function of time t is given as $x = A \sin(2\pi t)$
Here x is in cm and t is in seconds. Let the time taken by the particle to travel from x = 0 to x = A/2 be t_1 and the time taken to travel from x = A/2 to x = A be t_2. Find t_1/t_2

Q.12 In a HCl molecule, we may treat Cl to be of infinite mass and H alone oscillating. If the oscillation of HCl molecule shows frequency $9 \times 10^{13}\ s^{-1}$, deduce the force constant. The Avogadro number = 6×10^{26} per kg–mole.

Q.13 A particle is moving with SHM in a straight line. When the distance of the particle from the equilibrium position has values x_1 and x_2, the corresponding values of velocities u_1 and u_2. Show that the time period of oscillation is given by

$$T = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

Q.14 If the distance y of a point moving on a straight line measured from a fixed origin on it and velocity v are connected by the relation $4v^2 = 25 - y^2$, then show that the motion is simple harmonic and find its time period.

Q.15 A particle executing SHM along a straight line has a velocity of 4 ms^{-1} when at a distance 3 m from the mean position and 3 ms^{-1} when at a distance of 4 m from it. Find the time it takes to travel 2.5 m from the positive extremity of its oscillation.

Q.16 A particle executing linear SHM has a maximum velocity of 40 cm s^{-1} and a maximum acceleration of 50 cm s^{-2}. Find its amplitude and the period of oscillation.

Q.17 The vertical motion of a huge piston in a machine is approximately simple harmonic with a frequency of 0.50 s^{-1}. A block of 10 kg is placed on the piston. What is the maximum amplitude of the piston's SHM for the block and the piston to remain together?

Q.18 A block of mass one kg is fastened to a spring with a spring constant 50 Nm^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Write the expression for its x(t) and v(t).

Q.19 A person normally weighing 60 kg stands on a platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. If a machine on the platform gives the person's weight against time, deduce the maximum and minimum readings it will show. Take g = 10 ms^{-2}.

Q.20 A body of mass 0.1 kg is executing SHM according to the equation

$$y = 0.5 \cos\left(100t + \frac{3\pi}{4}\right) \textit{metre}$$

Find (i) the frequency of oscillation (ii) initial phase (iii) maximum velocity (iv) maximum acceleration and (v) total energy.

Answers

1. (a) $x = a\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$, (b) $x = b\cos\left(\frac{2\pi}{30}t - \frac{\pi}{2}\right)$
2. (i) 10 m, (ii) 20 rad s^{-1}, (iii) 3.18 Hz, (iv) 0.314 s, (v) 0.5 rad
3. (i) 4.33 cm, (ii) 7.85 cm s^{-1}, (iii) 42.77 cm s^{-2}
4. (a) $\frac{5}{\sqrt{2}}m$, (b) 1 s, (c) $-\frac{10\pi}{\sqrt{2}}m/s$
5. (a) – 3.535 m, (b) 22.22 m s^{-1}, (c) 139.56 m s^{-1}
6. 0.2 s, 10 cm, 53^o
7. 0.5 s
8. 12 s
9. 5 cos πt

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10.	(i) 1.94 cm s^{-1}, (ii) 3.75 cm s^{-2}			11.	$\frac{1}{2}$
12.	533.4 Nm^{-1}	14.	4π	15.	1.047 s
16.	32 cm, 5.03 s	17.	0.99 m		

18. v(t) = 0.707 cos 7.07 t ms^{-1}, x (t) = 0.1 sin 7.0 7t

19. 1074 N, 126 N   20. (i) $\frac{50}{\pi} Hz$, (ii) $\frac{3\pi}{4} rad$, (iii) 50 ms^{-1}, (iv) 5000 ms^{-2}, (v) 125 J

Subjective Assignment – III

Q.1 A simple harmonic oscillation is represented by the equation, y = 0.40 sin (440t + 0.61) here y and t are in m and s respectively. What are the values of (i) amplitude (ii) angular frequency (iii) frequency of oscillations (iv) time period of oscillations and (v) initial phase?

Q.2 The periodic time of a body executing SHM is 2s. After how much time interval from t = 0, will its displacement be half of its amplitude?

Q.3 A particle executes SHM represented by the equation: 10y = 0.1 sin 50 πt, where the displacement y in metre and time t in second. Find the amplitude and frequency of the particle.

Q.4 The displacement of a particle executing periodic motion is given by y = 4 $\cos^2$ (t/2) sin (1000 t). Find the independent constituent SHM's.

Q.5 A particle executing SHM completes 1200 oscillations per minute and passes through the mean position with a velocity of 31.4 ms^{-1}. Determine the maximum displacement of the particle from the mean position. Also obtain displacement equation of the particle if its displacement be zero at the instant t = 0.

Q.6 The acceleration of a particle performing SHM is 12 cm s^{-2} at a distance of 3 cm from the mean position. Calculate its time–period.

Q.7 In a pendulum, the amplitude is 0.05 m and a period of 2 s. Compute the maximum velocity.

Q.8 In what time after its motion begins, will a particle oscillating according to the equation, y = 7 sin 0.5 π t, move from the mean position to maximum displacement?

Q.9 A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the magnitude of its acceleration is equal to its velocity. Find the time period, maximum velocity and maximum acceleration of SHM.

Q.10 The velocity of a particle describing SHM is 16 cm s^{-1} at a distance of 8 cm from mean position and 8 cm s^{-1} at a distance of 12 cm from mean position. Calculate the amplitude of the motion.

Q.11 If a particle executes SHM of time period 4s and amplitude 2 cm, find its maximum velocity and that at half its full displacement. Also find the acceleration at the turning points and when the displacement is 0.75 cm.

Q.12 A block lying on a horizontal table executes SHM of period 1 second, horizontally. What is the maximum amplitude for which the block does not slide? Coefficient of friction between block and surface is 0.4, π^2 = 10.

Q.13 A horizontal platform moves up and down simple harmonically, the total vertical movement being 10 cm. What is the shortest period permissible, if objects resting on the platform are to remain in contact with it throughout the motion? Take g = 980 cm s^{-2}.

Q.14 In a gasoline engine, the motion of the piston is simple harmonic. The piston has a mass of 2 kg and stoke (twice the amplitude) of 10 cm. Find maximum acceleration and the maximum unbalanced force on the piston, if it is making 50 complete vibrations each minute.

Q.15 A man stands on a weighing machine placed on a horizontal platform. The machine reads 50 kg. By means of a suitable mechanism the platform is made to execute harmonic vibrations up and down with a frequency of 2 vibrations per second. What will be the effect on the reading of the weighing machine? The amplitude of vibration of the platform is 5 cm. Take g = 10 ms^{-2}.

Answers

1. (i) 0.40 m, (ii) 440 rad s^{-1}, (iii) 70 Hz, (iv) 0.0143 s, (v) 0.61 rad
2. 1/6 s
3. A = 0.01 m, v = 25 Hz
4. sin (1001 t), sin (1000 t), sin (999 t)
5. A = 0.025 m, y = 0.025 sin (40πt) metre
6. 3.142 s
7. 0.1571 ms^{-1}
8. 1 s
9. 3.63 s, 3.464 cms^{-1}, 6 cms^{-2}
10. 13.06 cm
12. 3.14 cm s^{-1}, 2.72 cm s^{-1}, 4.93 cm s^{-2}, 1.85 cm s^{-2}
13. 9.8 cm
14. 0.449 s
15. 1.371 ms^{-2}, 2.742 N
16. 895 N, 105 N

Energy in S.H.M.: Kinetic and Potential Energies

The energy of a harmonic oscillator is partly kinetic and partly potential. When a body is displaced from it equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back with a velocity, thus acquiring kinetic energy.

(i) **Kinetic energy:** At any instant, the displacement of a particle executing S.H. M. is given by

$$x = A\cos(\omega t + \phi_0)$$

∴ $$\text{Velocity, } v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$$

Hence kinetic energy of the particle at any displacement x is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi_0)$$

But $$A^2\sin^2(\omega t + \phi_0) = A^2[1 - \cos^2(\omega t + \phi_0)]$$

$$A^2 - A^2\cos^2(\omega t + \phi_0) = A^2 - x^2$$

∴ $$K = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi_0)$$ or $$K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

(ii) **Potential energy:** When the displacement of a particle from its equilibrium position is x, the restoring force acting on it is $F = -kx$

If we displace the particle further through a small distance dx, then work done against the restoring force is give by $dW = -F\,dx = +k\,x\,dx$

The total work done is moving the particle from mean position (x = 0) to displacement x is given by

$$W = \int dW = \int_0^x k\,x\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done against the restoring force is stored as the potential energy of the particle. Hence potential energy of a particle at displacement x is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi_0)$$

(iii) **Total energy:** At any displacement x, the total energy of a harmonic oscillatory is given by

$$E = K + U = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2$$

or $$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 mv^2 A^2$$

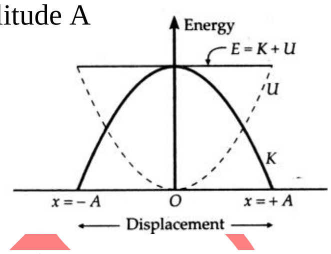
Thus the total mechanical energy of a harmonic oscillator is independent of time of displacement. Hence *in the absence of any frictional force, the total energy of a harmonic oscillator is conserved.*

Obviously, the total energy of particle in S.H.M. is

(i) directly proportional to the mass m of the particle

(ii) directly proportional to the square of its frequency v, and

(iii) directly proportional to the square of its vibrational amplitude A



Graphical representation: *At the mean position, x = 0*

Kinetic energy, $K=\frac{1}{2}k(A^2-0^2)=\frac{1}{2}kA^2$

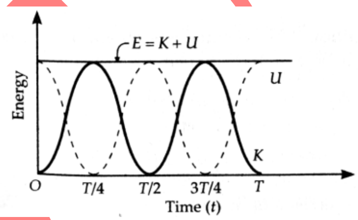
Potential energy, $U=\frac{1}{2}k(0^2)=0$

Hence at the mean position, the energy is all kinetic

At the extreme positions, $x = \pm A$

Kinetic energy, $K=\frac{1}{2}k(A^2-A^2)=0$

Potential energy, $U=\frac{1}{2}kA^2$



Hence at the tow extreme positions, the energy is all potential.

Figure shows the variations of kinetic energy K, potential energy U and total energy E with displacement x. The graphs for K and U are parabolic while that for E is a straight line parallel to the displacement axis. At x = 0, the energy is all kinetic and for $x = \pm A$, the energy is all potential.

Figure shows the vibrations of energies K, U and E of a harmonic oscillator with time t. Clearly, twice in each cycle, both kinetic and potential energies assume their peak values. Both of these energies are periodic functions of time, the time period of each being T/2.

A particle in linear SHM, the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Suppose a particle of mass m executes SHM of period T. The displacement of the particle at any instant t is given by $y = A\sin\omega t$

$\therefore$ Velocity, $v=\frac{dy}{dt}=\omega A\cos\omega t$

Kinetic energy, $E_k=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2A^2\cos^2\omega t$

Potential energy, $E_p=\frac{1}{2}m\omega^2y^2=\frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\therefore$ Average K.E. over a period of oscillation,

$$E_{k_{av}}=\frac{1}{T}\int_0^T E_k\,dt=\frac{1}{T}\int_0^T\frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$=\frac{1}{2T}m\omega^2A^2\int_0^T\frac{(1+\cos2\omega t)}{2}dt$$

$$=\frac{1}{4T}m\omega^2A^2\left[t+\frac{\sin2\omega t}{2\omega}\right]_0^T = \frac{1}{4T}m\omega^2A^2(T)=\frac{1}{4}m\omega^2A^2 \qquad ...(1)$$

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Average P.E. over a period of oscillation,

$$E_{p_{av}} = \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t \, dt$$

$$= \frac{1}{2T} m\omega^2 A^2 \int_0^T \frac{(1-\cos 2\omega t)}{2} dt$$

$$= \frac{1}{4T} m\omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T \quad = \frac{1}{4T} m\omega^2 A^2 (T) = \frac{1}{4} m\omega^2 A^2 \qquad \text{... (2)}$$

Clearly, from equation (1) and (2), $E_{k_{av}} = E_{p_{av}}$

Subjective Assignment – IV

Q.1 A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.

Q.2 A body executes SHM of time period 8 s. If its mass be 0.1 kg, its velocity 1 second after it passes through its mean position be 4 ms^{-1}, find its (i) kinetic energy (ii) potential energy and (iii) total energy.

Q.3 A spring of force constant 800 Nm^{-1} has an extension of 5 cm. What is the work done in increasing the extension from 5 cm to 15 cm?

Q.4 A particle of mass 10 g is describing SHM along a straight line with a period 2 s and amplitude of 10 cm. What is the kinetic energy when it is (i) 2 cm (ii) 5 cm, from its equilibrium position? How do you account for the difference between its two values?

Q.5 At a time when the displacement is half the amplitude, what fraction of the total energy is kinetic and what fraction is potential in S.H.M.?

Q.6 A particle is executing SHM of amplitude A. At what displacement from the mean position, is the energy half kinetic and half potential?

Q.7 A particle executes simple harmonic motion of amplitude A. (i) At what distance from the mean position is its kinetic energy equal to its potential energy? (ii) At what points is its speed half the maximum speed?

Q.8 A bob of simple pendulum of mass 1 g is oscillating with a frequency 5 vibrations per second and its amplitude is 3 cm. Find the kinetic energy of the bob in the lowest position.

Q.9 A body weighting 10g has a velocity of 6 cm s^{-1} after one second of its starting from mean position. If the time period is 6 seconds, find the kinetic energy, potential energy and the total energy.

Q.10 A particle executes SHM of period 8 seconds. After what time of its passing through the mean position will the energy be half kinetic and half potential?

Q.11 The total energy of a particle executing SHM of period 2π seconds is 1.024 × 10^{-3} J. The displacement of the particle at π/4 sec is $0.08\sqrt{2}m$. Calculate the amplitude of motion and mass of the particle.

Q.12 A particle which is attached to a spring oscillates horizontally with simple harmonic motion with a frequency of 1/ π Hz and total energy of 10 J. If the maximum speed of the particle is 0.4 ms^{-1}, what is the force constant of the spring? What will be the maximum potential energy of the spring during the motion?

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Q.13 The length of a weightless spring increases by 2 cm when a weight of 1.0 kg is suspended from it. The weight is pulled down by 10 cm and released. Determine the period of oscillation of the spring and its kinetic energy of oscillation. Take $g = 10\ ms^{-2}$.

Answers

1. $E_k = 0.1875$ J, $E_p = 0.0625$ J, E = 0.25 J
2. E = 1.6 J, $E_p = E_k = 0.8$ J
3. 8 J
4. 480 π^2 erg, 375 π^2 erg.
5. $E_p = \frac{1}{4},\ E_k = \frac{3}{4}$
6. $y = \pm\frac{A}{\sqrt{2}}$
7. (i) 0.71 times the amplitude on either side of mean position

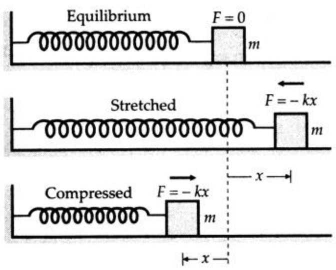
   (ii) 0.86 times the amplitude on either side of mean position
8. 4441.5 erg
9. $E_k = 180$ erg, $E_p = 540$ erg, E = 720 erg
10. 1 s
11. 0.16 m; 0.08 kg
12. $k = 500\ Nm^{-1}$, $u_{max} = 10$ J
13. 0.28 s, 2.5 J

Oscillations Due to a Spring

Horizontal oscillations of a body on a spring

Consider a massless spring lying on a frictionless horizontal table. Its one end is attached t a rigid support and the other end to a body of mass m. If the body is pulled towards right through a small distance x and released, it starts oscillating back and forth about its equilibrium position under the action of the restoring force of elasticity,

$$F = -kx$$



where k is the force constant (restoring force per unit compression of extension) of the spring. The negative sign indicates that the force is directed oppositely to x.

If d^2x/dt^2 is the acceleration of the body, then

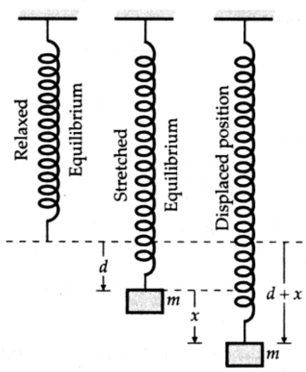
$$m\frac{d^2x}{dt^2} = -kx \qquad \text{or} \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

This shows that the acceleration is proportional to displacement x and acts opposite to it. Hence the body executes simple harmonic motion. Its time period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency of oscillation will be $v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Vertical oscillations of a body on a spring



If a spring is suspended vertically from a rigid support and a body of mass m is attached to its lower end, the spring gets stretched to a distance d due to the weight mg of the body. Because of the elasticity of the spring, a restoring force equal to kd begins to act in the upward direction. Here k is the spring factor of the spring. In the position of equilibrium,

$$mg = kd$$

If the body be pulled vertically downwards through a small distance x from its equilibrium position and then released, it begins to oscillate up and down about this position. The weight mg of the body acts vertically downwards while the restoring force k (d + x) due to elasticity acts vertically upwards. Therefore, the resultant force on the body is

$F = mg - k(d + x) = kd - kd - kx \quad [\because mg = kd] \quad$ or $\quad F = -kx$

If d^2x/dt^2 is the acceleration of the body, then $m\dfrac{d^2x}{dt^2} = -kx$ or $\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x = -\omega^2 x$

Thus acceleration is proportional to displacement x and is directed opposite to it. Hence the body executes S.H.M. and its time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{K}}$$

Obviously, *the force of gravity has no effect on force constant k and hence time period of the oscillating mass.*

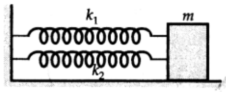
Springs Connected in Parallel

Figures show two springs connected in parallel. Let k_1 and k_2 be their spring constant. Let y be the extension produced in each spring. Restoring forces produced in the two springs will be

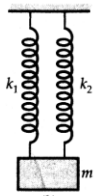
$F_1 = -k_1 y$ and $F_2 = -k_2 y$

The total restoring force is

$F = F_1 + F_2 = -(k_1 + k_2)y$ ... (1)



(a)



(b)

Let k_p be the force constant of the parallel combination. Then

$F = -k_p y$ ... (2)

From (1) and (2), $\quad k_p = k_1 + k_2$

Frequency of vibration of the parallel combination is

$$\nu_p = \frac{1}{2\pi}\sqrt{\frac{k_p}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

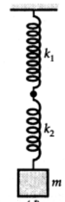
Springs Connected in Series

Figures represent two springs connected in series. Let x_1 and x_2 be the extensions produced in two springs. The restoring force F acting in the two springs is same.

$$\therefore \quad F = -k_1x_1 = -k_2x_2 \quad \text{or} \quad x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2}$$

Total extension, $\quad x = x_1 + x_2 = -\dfrac{F}{k_1} - \dfrac{F}{k_2} = -F\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right) = -\left(\dfrac{k_1 + k_2}{k_1 k_2}\right)$

or $\quad F = -\left(\dfrac{k_1 k_2}{k_1 + k_2}\right)x$ ... (1)

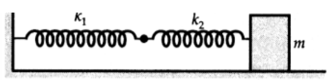


(d)

Let k_s be the force constant of the series combination. Then

$F = -k_s x$ ... (2)

From (1) and (2), $\quad k_s = \dfrac{k_1 k_2}{k_1 + k_2}$



(c)

Frequency of oscillation of the series combination is

$$\nu_s = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

Subjective Assignment – V

Q.1 The pan attached to a spring balance has a mass of 1 kg. A weight of 2 kg when placed on the pan stretches the spring by 10 cm. What is the frequency with which the empty pan will oscillate?

Q.2 A spring compressed by 0.2m develops a restoring force of 25 N. A body of mass 5 kg is placed over it. Deduce:

(i) force constant of the spring

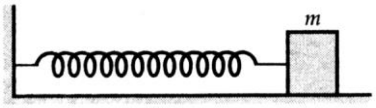
(ii) the depression of the spring under the weight of the body and

(iii) the period of oscillation, if the body is disturbed. Take $g = 10\ N\ kg^{-1}$.

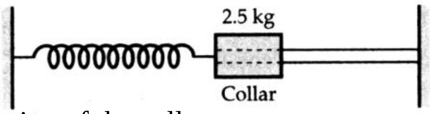
Q.3 A 0.2 kg of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7 cm more. If the 0.02 kg mass is removed, what will be the period of vibration of the system?

Q.4 A body of mass 12 kg is suspended by a coil spring of natural length 50 cm and force constant $2.0 \times 10^3\ Nm^{-1}$. What is the stretched length of the spring? If the body is pulled down further stretching the spring to a length of 59 cm and then released, what is the frequency of oscillation of the suspended mass? (Neglect the mass of the spring).

Q.5 An impulsive force gives an initial velocity of $-1.0\ ms^{-1}$ to the mass in the unstretched spring position (see figure). What is the amplitude of motion? Give x as a function of time t for the oscillating mass. Given m = 3 kg and $k = 1200\ Nm^{-1}$.



Q.6 A 5 kg collar is attached to a spring of force constant $500\ Nm^{-1}$. It slides without friction on a horizontal rod as shown in figure. The collar is displaced from its equilibrium position by 10.0 cm and released.



Calculate

(i) the period of oscillation,

(ii) the maximum speed, and (iii) the maximum acceleration of the collar.

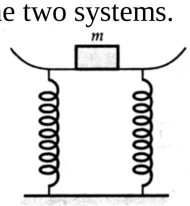
Q.7 A small trolley of mass 2.0 kg resting on a horizontal turn table is connected by a light spring to the centre of the table. When the turn table is set into rotation at a speed of 300 rpm, the length of the stretched spring is 40 cm. If the original length of the spring is 35 cm, determine the force constant of the spring.

Q.8 Two masses m_1 = 10 kg and m_2 = 0.5 kg are suspended together by a massless spring of force constant, $k = 12.5\ Nm^{-1}$. When they are in equilibrium position, m_1 is gently removed. Calculate the angular frequency and the amplitude of oscillation of m_2. Given $g = 10\ ms^{-2}$.

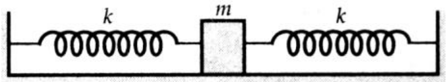
Q.9 Two identical springs, each of spring factor k, may be connected in the following ways. Deduce the spring factor of the oscillation of the body in each case.

Q.10 Two identical springs, each of force constant k are connected in (a) series (b) parallel, and they support a mass m. Calculate the ratio of the time periods of the mass in the two systems.

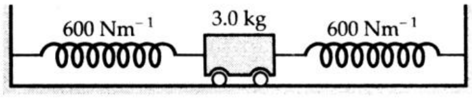
Q.11 A tray of mass 12 kg is supported by two identical springs as shown in figure. When the tray is pressed down slightly and released, it executes SHM with a time period of 1.5 s. What is the force constant of each spring? When a block of mass M is placed on the tray, the period of SHM change to 3.0 s. What is the mass of the block?



Q.12 The identical springs of spring constant k are attached to a block of mss m and to fixed supports as shown as below. Show that when mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.

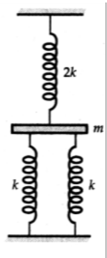


Q.13 A trolley of mass 3.0 kg is connected to two identical springs each of force constant $600\ Nm^{-1}$, as shown in figure. If the trolley is displaced from it equilibrium position by 5.0 cm and released, with is (i) the period of ensuing oscillations, (ii) the maximum speed of the trolley? (iii) How much is the total energy dissipated as heat by the time the trolley comes to rest due to damping forces?



Q.14 The periodic time of a mass suspended by a spring (force constant k) is T. If the spring is cut in three equal pieces, what will be the force constant of each part? If the same mass be suspended from one piece, what will be the periodic time?

Q.15 The time period of a body suspended by a spring be T. What will be the new period, if the spring is cut into two equal parts and when (i) the body is suspended from one part (ii) the body is suspended from both the parts connected in parallel.



Q.16 Three springs are connected to a mass m as shown in figure. When mass m oscillates, what is the effective spring constant and time period of vibration? Given k = 2 Nm^{-1} and m = 80 g.

Answers

1.	$\frac{7}{\pi} Hz$	2.	(i) 125 Nm^{-1}, (ii) 0.4 m, (iii) $\frac{2\pi}{5} s$		
3.	1.66 s	4.	55.88 cm, 2.06 Hz		
5.	5 cm, – 5 sin 20 t	6.	(i) 0.628 s, (ii) 1.0 m s^{-1}, (iii) 10 m s^{-2}		
7.	15795 Nm^{-1}	8.	5 rad s^{-1}, 0.8 m	9.	2K, $\frac{K}{2}$, 2K
10.	2	11.	105.17 Nm^{-1}, 36 kg	12.	$T=2\pi\sqrt{\frac{m}{2K}}$
13.	(i) 0.314 s, (ii) 1.0 ms^{-1}, (iii) 1.5 J	14.	$3k, T/\sqrt{3}$		
15.	(i) T/$\sqrt{2}$ *(ii) T/2*	16.	8 Nm^{-1}, 0.628 s		

VPLUS U

Simple Pendulum

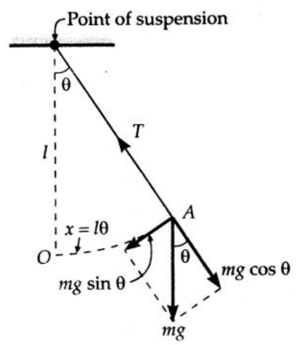
An ideal simple pendulum consists of a point mass suspended by a flexible, inelastic and weightless string from a rigid support of infinite mass. In practice, a simple pendulum is obtained by suspending a small metal bob by a long and fine cotton thread from a rigid support.

Expression for time period

In the equilibrium position, the bob of a simple pendulum lies vertically below the point of suspension. If the bob is slightly displaced on either side and released, it begins to oscillate about the mean position. Suppose at any instant during oscillation, the bob lies at position A when its displacement is OA = x and the thread makes angle θ with the vertical. The forces acting on the bob are

(i) Weight mg of the bob acting vertically downwards.

(ii) Tension T along the string



The force mg has two rectangular components (i) the component mg cos θ acting along the thread is balanced by the tension T in the thread and (ii) the tangential component mg sin θ is the net force acting on the bob and tends to bring it back to the mean position. Thus, the restoring force is

$$F = -mg \sin\theta = -mg\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - ....\right) = -mg\,\theta\left(1 - \frac{\theta^2}{6} + \frac{\theta^4}{120} - ......\right)$$

where θ is in radians. Clearly, oscillations are not simple harmonic because the restoring force F is not proportional to the angular displacement θ. However, if θ is so small that its higher powers can be neglected, then

$$F = -mg\,\theta$$

If *l* is the length of the simple pendulum, then

$$\theta\,(rad) = \frac{arc}{radius} = \frac{x}{l}$$

$$\therefore \quad F = -mg\frac{x}{l} \qquad \text{or} \qquad ma = -\frac{mg}{l}x \qquad \text{or} \qquad a = -\frac{g}{l}x = -\omega^2 x$$

Thus, the acceleration of the bob is proportional to its displacement x and is directed opposite to it. Hence for small oscillations, the motion of the bob is simple harmonic. Its time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/l}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Obviously, the time period of a simple pendulum depends on its length *l* and acceleration due to gravity g but is independent of the mass m of the bob.

Subjective Assignment – VI

Q.1 What is the length of a simple pendulum, which ticks seconds?

Q.2 A pendulum clock shows accurate time. If the length increases by 0.1%, deduce the error in time per day.

Q.3 Two pendulums of lengths 100 cm and 110.25 cm start oscillating in phase. After how many oscillations will they again be in same phase?

Q.4 A second's pendulum is taken in a carriage. Find the period of oscillation when the carriage moves with an acceleration of 4 ms^{-2} (i) vertically upwards (ii) vertically downwards, and (iii) in a horizontal direction.

Q.5 The bottom of a dip on a road has a radius of curvature R. A rickshaw of mass M left a little away from the bottom oscillates about the dip. Deduce an expression for the period of osciilation.

Q.6 The time taken by a simple pendulum to perform 100 vibrations is 8 minutes 9 seconds in Bombay and 8 minutes 20 seconds in Pune. Calculate the ratio of acceleration due to gravity in Bombay and Pune.

Q.7 If the length of a pendulum is decreased by 2%, find the gain or loss in time per day.

Q.8 What will be the time period of second's pendulum if its length is doubled?

Q.9 If the acceleration due to gravity on moon is one–sixth of that on the earth, what will be the length of a second pendulum there? Take g = 9.8 ms^{-2}.

Answers

1.	0.992 m			2.	lose 43.2 s
3.	larger complete 20 oscillation and smaller 21 oscillation				
4.	(i) 1.69 s, (ii) 2.59 s, (iii) 1.92 s	5.	$2\pi\sqrt{\frac{R}{g}}$	6.	1.0455
7.	Gain of 864 s	8.	2.828 s	9.	16.5 cm

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Oscillations of a liquid column in a U–tube

Initially, suppose the U–tube of cross–section A contains liquid of density ρ upto height h. Then mass of the liquid in the U–tube is

$$m = \text{volume} \times \text{density} = A \times 2h \times \rho$$

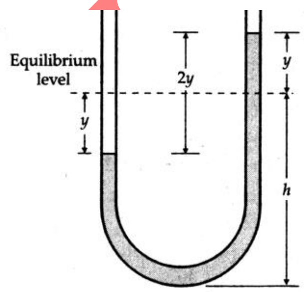
If the liquid in one arm is depressed by distance y, it rises by the same amount in the other arm. If left to itself, the liquid begins to oscillate under the restoring force,

F = Weight of liquid column of height 2 y

$$F = -A \times 2y \times \rho \times g = -2A\rho g y \qquad \text{i.e.,} \qquad F \propto y$$

Thus the force on the liquid is proportional to displacement and acts in its opposite direction. Hence the liquid in the U–tube executes SHM with force constant.

$$k = 2A\rho g$$



The time–period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A\times 2h\times\rho}{2A\rho g}} = 2\pi\sqrt{\frac{h}{g}}$$

If l is the length of the liquid column, then

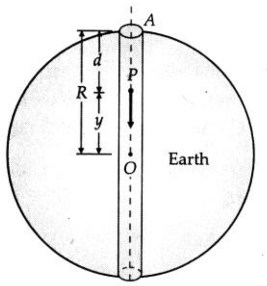
$$l = 2h \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{l}{2g}}$$

Oscillations of a body dropped in a tunnel along the diameter of the earth

As shown in figure, consider earth to be a sphere of radius R and centre O. A straight tunnel is dug along the diameter of the earth. Let g be the value of acceleration due to gravity at the surface of the earth.

Suppose a body of mass m is dropped into the tunnel and it is at point P i.e., at a depth d below the surface of the earth at any instant. If g' is acceleration due to gravity at P, then

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$



If y is distance of body from centre of the earth (displacement from mean position), then

$$R - d = y \qquad \therefore \qquad g' = g\frac{y}{R}$$

Force acting on the body at point P is

$$F = -mg' = -\frac{mg}{R}y \qquad \text{i.e.,} \qquad F \propto y$$

Negative sign shows that the force F acts in the opposite direction of displacement i.e., it acts towards the mean position O. Thus the body will execute SHM with force constant,

$$k = \frac{mg}{R}$$

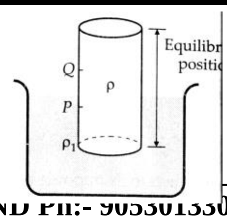
The period of oscillation of the body will be

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/R}} = 2\pi\sqrt{\frac{R}{g}}$$

Oscillations of a floating cylinder

In equilibrium, weight of the cork is balanced by the upthrust of the liquid.

Let the cork be slightly depressed through distance y from the equilibrium position and left to itself. It begins to oscillate under the restoring force,



F = Net upward force = Weight of liquid column of height y

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or $\quad F = -A\,y\,\rho_1 g = -A\,\rho_1\,g\,y \quad$ i.e., $\quad F \propto y$

Negative sign shows that F and y are in opposite directions. Hence the cork executes SHM with force constant,

$$k = A\,\rho_1\,g$$

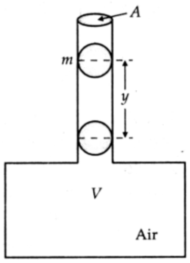
Also, mass of cork = A ρ h

$\therefore$ Period of oscillation of the cork is $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{A\rho h}{A\rho_1 g}} = 2\pi\sqrt{\dfrac{\rho h}{\rho_1 g}}$

Oscillations of a ball in the neck of an air chamber

Figure shows an air chamber of volume V, having a neck of area of cross–section A and a ball of mass m fitting smoothly in the neck. If the ball be pressed down a little and released, it starts oscillating up and down about the equilibrium position.

If the ball be depressed by distance y, then the decrease in volume of air in the chamber is ΔV = Ay.



$\therefore$ Volume strain = $\dfrac{\Delta V}{V} = \dfrac{Ay}{V}$

If pressure P is applied to the ball, then hydrostatic stress = P

$\therefore$ Bulk modulus of elasticity of air,

$$E = -\frac{P}{\Delta V/V} = -\frac{P}{Ay/V} \qquad \text{or} \qquad P = -\frac{EA}{V}y$$

Restoring force, $F = PA = -\dfrac{EAy}{V}A = -\dfrac{EA^2}{V}y$

Thus F is proportional to y and acts in its opposite direction. Hence the ball executes SHM with force constant,

$$k = \frac{EA^2}{V}$$

Period of oscillation of the ball is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{EA^2/V}} = 2\pi\sqrt{\frac{mV}{EA^2}}$$

(i) If the P–V variations are *isothermal*, then E = P,

$$\therefore \quad T = 2\pi\sqrt{\frac{mV}{PA^2}}$$

(ii) If the P – V variations are adiabatic, then E = γ P

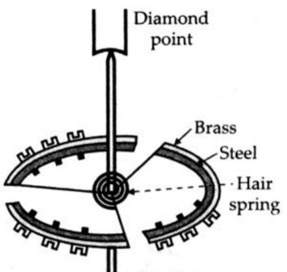
$$\therefore \quad T = 2\pi\sqrt{\frac{mV}{\gamma\,PA^2}}$$

Oscillations of the balance–wheel of a watch or Torsional pendulum

In a watch, a balance–wheel controls the movement of its hands. An axle passing through its centre is held between two diamond points. A hair–spring controls its oscillations. For an angular displacement θ, the hair–spring back the wheel into its equilibrium position. Here C is the restoring torque produced per unit angular displacement. Now

Torque = Moment of inertia × angular acceleration

$$\therefore \quad C\,\theta = -I \times \frac{d^2\theta}{dt^2} \qquad \text{or} \qquad \frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta = -\omega^2\,\theta$$



where I is the moment of inertia of the wheel about its axis of rotation. Clearly, angular acceleration $\frac{d^2\theta}{dt^2}$ is proportional to angular displacement θ and acts in its opposite direction. Hence oscillations of the balance wheel are simple harmonic.

Angular frequency, $\omega = \sqrt{\frac{C}{I}}$

Period of oscillation, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{C/I}} = 2\pi\sqrt{\frac{I}{C}}$

Subjective Assignment – VII

Q.1 A vertical U–tube of uniform cross–section contains water upto a height of 2.45 cm. If the water on one side is depressed and then released, its up and down motion in tube is simple harmonic. Calculate its time period. Given g = 980 cm s^{-2}.

Q.2 A test tube weighing 10 g and external diameter 2 cm is floated vertically in water by placing 10 g of mercury at its bottom. The tube is depressed in water a little and then released. Find the time of oscillation. Take g = 10 ms^{-2}.

Q.3 A cylindrical wooden block of cross–section 15.0 cm^2 and mass 230 g is floated over water with an extra weight of 50 g attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood is 0.30 and g = 9.8 ms^{-2}, deduce the frequency of the block.

Q.4 The balance wheel of a watch has a moment of inertia of 2×10^{-8} kg m^2 and the torsional constant of its hair spring is 9.8×10^{-6} Nm rad^{-1}. Calculate its frequency.

Q.5 A sphere is hung with a wire. 30^o rotation of the sphere about the wire generates a restoring torque of 4.6 Nm. If the moment of inertia of the sphere is 0.082 kg m^2, deduce the frequency of angular oscillations.

Q.6 If the earth were a homogeneous sphere and a straight hole was bored in it through its centre, show that a body dropped in the hole with execute SHM and calculate the time period of its vibration. Radius of earth is 6.4×10^6 m and g = 9.8 ms^{-2}

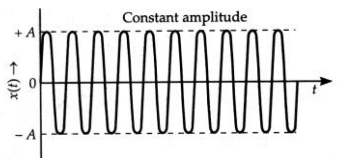
Answers

1.	0.314 s	2.	0.5 s	3.	14.7 Nm^{-1}, 1.15 Hz
4.	3.53 Hz	5.	1.65 Hz	6.	5077.6 s

Free, Damped and Maintained Oscillations

(i) **Free oscillations:** *If a body, capable of oscillation, is slightly displaced from its position of equilibrium and left to itself, it starts oscillating with a frequency of its own. Such oscillations are called free oscillations*. The frequency with which a body oscillates freely is called natural frequency and is given by

$$v_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



Some important features of free oscillations are

(i) In the absence of dissipative force, such a body vibrates with a constant amplitude and fixed frequency, as shown in figure. Such oscillations are also called **undamped oscillations.**

(ii) The amplitude of oscillation depends on the energy supplied initially to the oscillator.

(iii) The natural frequency of an oscillator depends on its mass, dimensions and restoring force i.e., on its inertial and elastic properties (m and k).

Examples

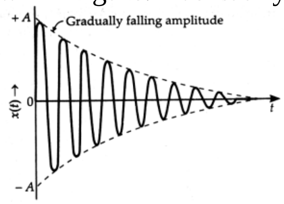
(i) The vibrations of the prongs of tuning fork struck against a rubber pad.

(ii) The vibrations of the string of a sitar when pulled aside and released.

(iii) The oscillations of the bob of a pendulum when displaced from its mean position and released.

(b) **Damped oscillations:** *The oscillations in which the amplitude decreases gradually with the passage of time are called damped oscillations.*

In actual practice, most of the oscillations occur in viscous media, such as air, water, etc. A part of the energy of the oscillating system is lost in the form of heat, in overcoming these resistive forces. As a result, the amplitude of such oscillations decreases exponentially with time, as shown in figure. Eventually, these oscillations die out.



In an oscillatory motion, friction produces three effects:

(i) It changes the simple harmonic motion into periodic motion.

(ii) It decreases the amplitude of oscillation.

(iii) It slightly reduces the frequency of oscillation.

Example

(i) As shown in figure, consider a block of mass m that oscillates vertically on a spring with spring constant k. The block is connected to a vane through a rod. The vane is submerged in a liquid. As the block oscillates up and down, the vane also oscillates in a similar manner inside the liquid. The liquid exerts an opposing force of viscosity on the vane. The energy of the oscillating system is lost in the liquid as heat. The amplitude of oscillation decreases continuously with time.

(ii) The oscillations of a swing in air.

(iii) The oscillations of the bob of a pendulum in a fluid.

DIFFERENTIAL EQUATION FOR DAMPED OSCILLATORS AND ITS SOLUTION

In a real oscillator, the damping force is proportional to the velocity v of the oscillator.

$F_d = -bv$

where b is damping constant which depends on the characteristics of the fluid and the body that oscillates in it. The negative sign indicates that the damping force opposes the motion.

$\therefore$ Total restoring force = – kx – bv or $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$ $\left[v = \frac{dx}{dt}\right]$

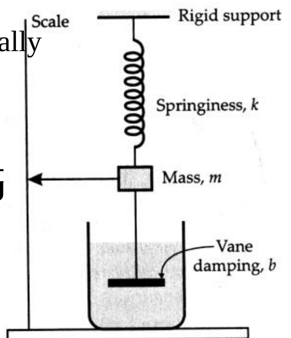
or $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

This is the differential equation for damped S.H.M. The solution of the equation is

$x(t) = A e^{-bt/2m} \cos(\omega' t + \phi)$

The amplitude of the damped S.H.M. is $A' = Ae^{-bt/2m}$

where A is amplitude of undamped S.H.M. Clearly, A' decreases exponentially with time. The angular frequency of the damped oscillator is



$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Time period, $T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$

Clearly, damping increases the time period (due to presence of the term $b^2/4\ m^2$ in the denominator). The mechanical energy of the damped oscillator at any instant t will be

$$E(t) = \frac{1}{2}kA'^2 = \frac{1}{2}kA^2 e^{-bt/m}$$

Obviously, the total energy decreases exponentially with time.

As damping constant, b = F/v

$\therefore$ SI unit of $b = \frac{N}{ms^{-1}} kg\ \frac{ms^{-2}}{ms^{-1}} = kg\ s^{-1}$ CGS unit of b = g s^{-1}

(c) **Maintained Oscillations:** *If to an oscillating system, energy is continuously supplied from outside at the same rate at which the energy is lost by it, then its amplitude can be maintained constant. Such oscillations are called maintained oscillations.* Here, the system oscillates with its own natural frequency.

Examples

(i) The oscillations of the balance wheel of a watch in which the main spring provides the required energy.

(ii) An electrically maintained tuning fork.

(iii) A child's swing in which energy is continuously fed to maintain constant amplitude.

Forced and Resonant Oscillations

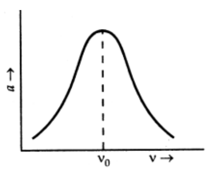
When a body oscillates under the influence of an external periodic force, not with its own natural frequency but with the frequency of the external periodic force, its oscillations are said to be forced oscillations. The external agent which exerts the periodic force is called the *drive* and the oscillating system under consideration is called the *driven body*.

Example

(i) When the stem of a vibrating tuning fork is pressed against a table, a loud sound is heard. This is because the particles of table are forced to vibrate with the frequency of the tuning fork.

(ii) When the free end of the string of a simple pendulum is held in hand and the pendulum is made to oscillate by giving jerks by the hand, the pendulum executes forced oscillations. Its frequency is same as that of the periodic force exerted by the hand.

(iii) The sound boards of all stringed musical instruments like sitar, violin, etc. execute forced oscillations and the frequency of oscillation is equal to the natural frequency of the vibrating string.



Suppose an external periodic force of frequency v is applied to an oscillator of natural frequency v_0. Initially, the body tries to vibrate with its own natural frequency, while the applied force tries to drive the body with its own frequency. But soon the free vibrations of the body die out and finally the body vibrate a with a constant amplitude and with the frequency of the driving force. In this steady state, the rate of loss of energy through friction equals the rate at which energy is fed to the oscillator by driver.

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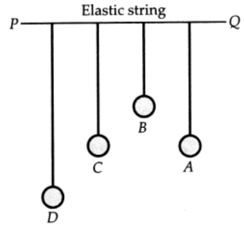
The variation of the amplitude of forced oscillations as the frequency of the driver varies from zero to a large value. Clearly, the amplitude of forced oscillations is very small *for* $v << v_0$ *and* $v >> v_0$. But when $v \simeq v_0$, the amplitude of the forced oscillations becomes very large. In this condition, the oscillator responds most favourably to the driving force and draws maximum energy from it. The case $v = v_0$ is called *resonance* and the oscillations are called *resonant oscillations.*

Resonant oscillations and resonance

It is a particular case of forced oscillations in which the frequency of the driving force is equal to the natural frequency of the oscillator itself and the amplitude of oscillations is very large. Such oscillations are called resonant oscillations and phenomenon is called resonance.



Example

(i) An aircraft passing near a building shatters its window panes, if the natural frequency of the window matches the frequency of the sound waves sent by the aircraft's engine.

(ii) The air–column in a resonance tube produces a loud sound when its frequency matches the frequency of the tuning fork.

(iii) A glass tumbler or a piece of china–ware on shelf is set into resonant vibrations when some note is sung or played.

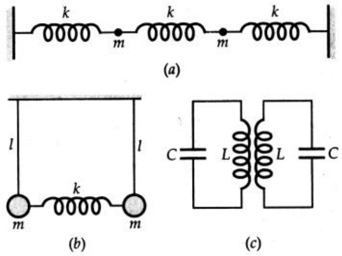
Principle of tuning of a radio receiver

Tuning of the ratio receiver is based on the principle of resonance. Waves from all stations are present around the antenna. When we tune our radio to a particular station, we produce a frequency of the radio circuit which matches with the frequency of that station. When this condition of resonance is achieved, the radio receives and responds selectively to the incoming waves from that station and thus gets tuned to that station.

Coupled Oscillations

A system of two or more oscillators linked together in such a way that there is mutual exchange of energy between them is called a coupled oscillator. The oscillations of such a system are called coupled oscillations.

Example:



(i) Two masses attached to each other by three springs between two rigid supports. The middle spring provides the coupling between the driver and the driven system.

(ii) Two LC–circuits placed close to each other. The circuits are linked by each other through the magnetic lines of force.

When two identical oscillators are coupled together, the general motion of such a system is complex. It is periodic but not simple harmonic.

NCERT Exercise

Q.1 Which of the following examples represent periodic motion?

(i) A swimmer completing one (return) trip from one bank of a river to the other and back.

(ii) A freely suspended bar magnet displaced from its N – S direction and released.

(iii) A hydrogen molecule rotating about its centre of mass.

(iv) An arrow released from a bow (v) Halley's comet

Q.2 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

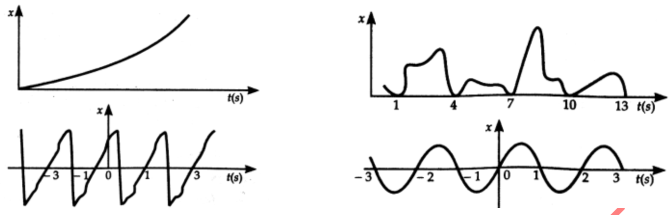
(i) The rotation of earth about its axis

(ii) Motion of an oscillating mercury column in a U–tube

(iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point

(iv) General vibrations of a polyatomic molecule about its equilibrium position.

Q.3 Figure depicts four x – t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?



Q.4 Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non–periodic motion? Give period for each case of periodic motion [ω is any positive constant]

(i) sin ωt – cos ωt (ii) $\sin^3 \omega t$ (iii) 3 cos (π/4 – 2ωt)

(iv) cos ωt + cos 3ωt + cos 5ωt (v) $\exp(-\omega^2 t^2)$ (vi) $1 + \omega t + \omega^2 t^2$

Q.5 A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A, (b) at the end B, (c) at the mid–point of AB going towards A,

(d) at 2 cm away from B going towards A (e) at 3 cm away from A going towards B, and

(f) at 4 cm away from A going towards A

Q.6 Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) a = 0.7 x, (b) $a = -200x^2$ (c) a = – 10 x (d) $a = 100\,x^3$

Q.7 (a) A particle in SHM is described by the displacement function, $x(t) = A\cos(\omega t + \phi),\ \omega = \dfrac{2\pi}{T}$

If the initial (t = 0), position of the particle is 1 cm and its initial velocity is π cm s^{-1}, what are its amplitude and initial phase angle? The angular frequency of the particle is π s^{-1}.

(b) A particle in SHM is described by the displacement function, $x(t) = B\sin(\omega t + \alpha),\ \omega = \dfrac{2\pi}{T}$

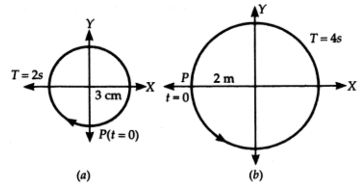
If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm s^{-1}, what are its amplitude and initial phase angle? The angular frequency of the particle is π s^{-1}

Q.8 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body?

Q.9 A spring of force constant 1200 Nm^{-1} is mounted horizontally on a horizontal table. A mass of 3.0 kg is attached to the free end of the spring, pulled sideways to a distance of 2.0 cm and released. (i) What is the frequency of oscillation of the mass? (ii) What is the maximum acceleration of the mass? (iii) What is the maximum speed of the mass?

Q.10 In above exercise, let us take the position of the mass, when the spring in unstretched, as x = 0, and the direction from left to right as the positive direction of X–axis. Give x as a function of time t for the oscillating mass, if at the moment we start the stop watch (t = 0), the mass is (i) at the mean position (ii) at the maximum stretched position (iii) at the maximum compressed position. *In what do these different functions of SHM differ? Frequency, amplitude or initial phase?*

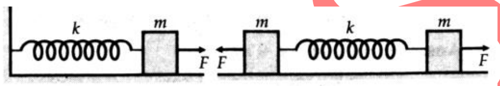
Q.11 Figure, corresponds to two circular motions. The radius of the circle, the period of revolution, the initial position, and sense of revolution (i.e., clockwise or anti–clockwise) are indicated on each figure. Obtain the corresponding simple harmonic motions of the x–projection of the radius rector of revolving particle P, in each case.



Q.12 Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case (x is in cm and t is in s).

(i) $x = -2\sin(3t + \pi/3)$ (ii) $x = \cos(\pi/6 - t)$ (iii) $x = 3\sin(2\pi t + \pi/4)$ (iv) $x = 2\cos \pi t$

Q.13 Figure shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. The spring is stretched by a force F at its free end. Figure, shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in figure, is stretched by the same force F.



(i) What is the maximum extension of the spring in the two cases?

(ii) If the mass in (a) and the two masses in (b) are released free, what is the period of oscillation in each case?

Q.14 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what is its maximum speed?

Q.15 The acceleration due to gravity on the surface of the moon is 1.7 ms^{-2}. What is the period of a simple pendulum on the moon if its time period on the earth is 3.5 s? Given g on earth = 9.8 ms^{-2}.

Q.16 Answer the following questions:

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle:

$T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. When then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For large angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free ball?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Q.17 A simple pendulum of length *l* and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Q.18 A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$, where ρ is the density of cork. (Ignore damping due to viscosity of liquid).

Q.19 One end of a U–tube containing mercury is connected to a suction pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in the U–tube executes SHM.

Q.20 An air chamber of volume V has a neck of area of cross–section A into which a ball of mass m can move without friction. Show that when the ball is pressed down through some distance and released, the ball executes SHM. Obtain the formula for the time period of this SHM, assuming pressure volume variations of the air to be (i) isothermal and (ii) adiabatic.

Q.21 You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values (a) the spring constant and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Q.22 Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equals the average pot entail energy over the same period.

Q.23 A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire.

Q.24 A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.

Q.25 A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.

Answers

1. (i) not periodic, (ii) periodic, (iii) periodic, (iv) not periodic, (v) periodic
2. (i) periodic but not simple harmonic, (ii) simple harmonic, (iii) simple harmonic, (iv) periodic but not simple harmonic
3. (i) not periodic, (ii) periodic, T = 2s, (iii) not periodic, (iv) periodic, T = 2s
4. (i) simple harmonic motion with $T = \frac{2\pi}{\omega}$, (ii) $T = \frac{2\pi}{\omega}$, periodic but not simple harmonic, (iii) simple harmonic motion with $T = \frac{\pi}{\omega}$, (iv) $T = \frac{2\pi}{\omega}$, periodic but not simple harmonic, (v) non periodic, (vi) non periodic
5.

Position	Velocity	Acceleration	Force
(a) At A	0 (at extreme position)	+ ve (acts from A to O)	+ ve (acts from A to O)
(b) At B	0 (at extreme position)	– ve (acts from B to O)	– ve (acts from B to O)
(c) At midpoint O, going towards A	– ve and maximum (acts from O to A)	0 (at mid-point)	0 (at mid-point)
(d) At C, going towards A	– ve (acts from C to O)	– ve (acts from C to O)	– ve (acts from C to O)
(e) At D, going towards B	+ ve (acts from D to O)	+ ve (acts from D to O)	+ ve (acts from D to O)
(f) At E, going towards A	– ve (acts from E to A)	+ ve (acts from E to O)	+ ve (acts from E to O)

6. c

7. (a) $\sqrt{2}$ cm, $\phi = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, (b) $\sqrt{2}$ cm, $\frac{\pi}{4}$ or $\frac{5\pi}{4}$

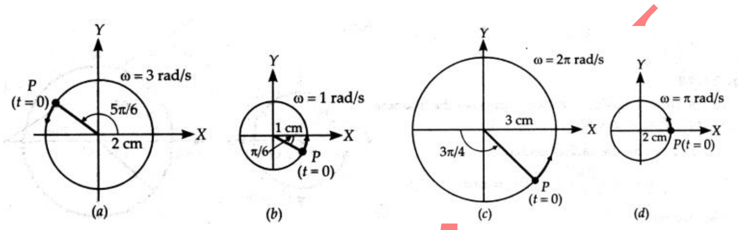
8. 219.13 N

9. (i) 3.2 s^{-1}, (ii) 8.0 ms^{-2}, (iii) 0.40 ms^{-1}

10. (i) 2 sin 20 t, (ii) 2 cos 20t, (iii) – 2 cos 20t, initial phase

11. (i) x (t) = =– 3 sin πt, (ii) x (t) = – 2 cos $\frac{\pi t}{2}$

12.



13. (i) F/k, (ii) (a) $2\pi\sqrt{\frac{m}{k}}$, (b) $2\pi\sqrt{\frac{m}{2k}}$

14. 100 m/ min

15. 8.4 s

17. $T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

21. (a) 5×10^4 N m^{-1}, (b) 1350.4 kg s^{-1}

23. 2.0 Nm rad^{-1}

24. (a) – $5\pi^2$ ms^{-2}, 0 (b) – $3\pi^2$ ms^{-2}, 0.4 π ms^{-1} (c) 0.50 π ms^{-1}, 0

25. $A = \sqrt{\frac{v_0^2}{\omega^2} + x_0^2}$

Miscellaneous Subjective Assignment – VIII

Q.1 Two simple harmonic motions are represented by the equations:

$$x_1 = 5 \sin (2\pi t + \pi/4),\ x_2 = 5\sqrt{2}\ (\sin 2\pi t + \cos 2\pi)$$

What is the ratio of their amplitudes?

Q.2 The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change if the water begins to drain out of the hollow sphere from a fine hole at its bottom.

OR

The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating body gets suddenly unplugged. How would the time period of oscillation of the pendulum change, till water is coming out?

Q.3 The period of vibration of a mass m suspended by a spring is T. The spring is cut into n equal parts and the body is again suspended by one of the pieces. Find the time period of oscillation of the mass.

Q.4 Two simple harmonic motions are represented by the equations:

$$y_1 = 0.1 \sin (100\ \pi t + \pi/3) \text{ and } y_2 = 0.1 \cos \pi t$$

What is the phase difference of the velocity of the particle 1 with respect to the velocity of particle 2?

Q.5 A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force, $f(t) \propto \cos \omega t \quad (\omega \neq \omega_0)$

is applied to the oscillator. How does the time displacement of oscillator vary?

Q.6 A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$ ($K = 1\ ms^{-2}$), where y is the vertical displacement. The time period now becomes T_2. What is the ratio T_1^2/T_2^2? Given $g = 10\ ms^{-2}$

Q.7 The bob of simple pendulum executes SHM in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\frac{4000}{3} kg\ m^{-3}$, find the relationship between t and t_0?

Q.8 A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then the mass is increased by m, the time period becomes 5T/3. What is the ratio m/M?

Q.9 Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically, such that their maximum velocities are equal, then find the ratio of the amplitude of M to that of N.

Q.10 A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. What is the period of oscillation when the two springs are connected in series?

Q.11 A particle executes simple harmonic motion between x = – A and x = + A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then how are T_1 and T_2 related?

Q.12 Two simple harmonic motions are represented by the equations:

$$y_1 = 10 \sin\frac{\pi}{4}(12t + 1),\ y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

Find the ratio of their amplitudes. What are time periods of the two motions?

Q.13 A point particle of mass 0.1 kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of the particle if the initial phase of oscillation is 45°.

Q.14 A simple harmonic motion has an amplitude A and time period T. What is the taken to travel from x = A to x = A/2?

Q.15 A block is resting on a piston which is moving vertically with simple harmonic motion of period 1.0 second. At what amplitude of motion will the block and piston separate? What is the maximum velocity of the piston at this amplitude?

Q.16 A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.

Q.17 Springs of spring constants k, 2k, 4k, 8k, ... are connected in series. A mass m kg is attached to the lower end of the last spring and the system is allowed to vibrate. What is the time period of oscillations?

Given m = 40g and k = 2.0 N cm^{-1}

Q.18 A uniform spring whose unstretched length is l has a force constant k. The springs is cut into two pieces of unstretched lengths l_1 and l_2, where $l_1 = nl_2$ and n is an integer. What are the corresponding force constants k_1 and k_2 in terms of n and k? What is the ratio k_1/k_2?

VPLUS

Q.19 A horizontal spring block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.

Q.20 The bob of pendulum of length l is pulled aside from its equilibrium position through an angle θ and then released. Find the speed v with which the bob passes through the equilibrium position.

Answers

1. 1: 2   3. $T' = \frac{T}{\sqrt{n}}$ 4. $-\pi/6$ 5. $y \propto \frac{1}{m(\omega_o^2 - \omega^2)}$

6. 6/5   7. $t = 2\,t_0$ 8. $\frac{m}{M} = \frac{16}{9}$ 9. $\sqrt{\frac{K_2}{K_1}}$

10. $T^2 = t_1^2 + t_2^2$ 11. $T_2 = 2T_1$ 12. $\frac{A_1}{A_2} = 1{:}1, T_1 = T_2 = \frac{2}{3}$ sec

13. $Y = 0.1 \sin\left(4t + \frac{\pi}{4}\right)$ 14. T/6   15. 0.248 m, 1.56 m s^{-1}

16. 0.02 m   17. 0.126 sec   18.

$$k_1 = \left(\frac{n+1}{n}\right)k,\quad k_2 = (n+1)k;\quad \frac{k_1}{k_2} = \frac{1}{n}$$

19. $A' = \sqrt{\frac{M}{m+M}}A;\ \nu' = \frac{1}{2\pi}\sqrt{\frac{K}{m+M}}$ 20. $v = \sqrt{2g\ell(1-\cos\theta)}$

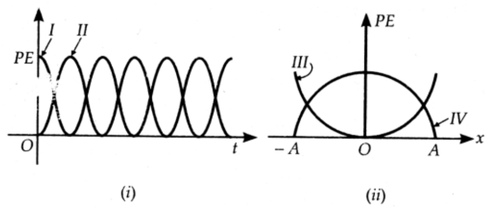
Objective Assignment – I [IIT Entrance Exam]

Multiple Choice Questions with One Correct Answer

Q.1 A particle executes simple harmonic motion between x = – A and x = + A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

Q.2 For a particle executing SHM the displacement x is given by x = A cos ωt. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x



(a) I, III (b) II, IV (c) II, III (d) I, IV

Q.3 A particle free to move along the x–axis has potential energy given by U(x) = k[1 – exp(– x)^2] for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then

(a) at points away from the origin, the particle is in unstable equilibrium

(b) for any finite non zero value of x, there is a force directed away from the origin

(c) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin

(d) for small displacements from x = 0, the motion is simple harmonic.

Q.4 A spring of force constant k is cut into two pieces, such that one piece is double the length of the other. Then, the long piece will have a force constant of

(a) 2/3 k (b) 3/2 k (c) 3 k (d) 6 k

Q.5 Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{k_1/k_2}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

Q.6 An object of mass 0.2 kg executes simple harmonic motion along the x–axis with a frequency of $(25/\pi)$ Hz. At the position x = 0.04, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is

(a) 6 cm (b) 4 cm (c) 8 cm (d) 2 cm

Q.7 A simple pendulum has a time period T_1, when on the earth's surface; and T_2, when taken to a height R above the earth's surface (R is the radius of the earth). The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

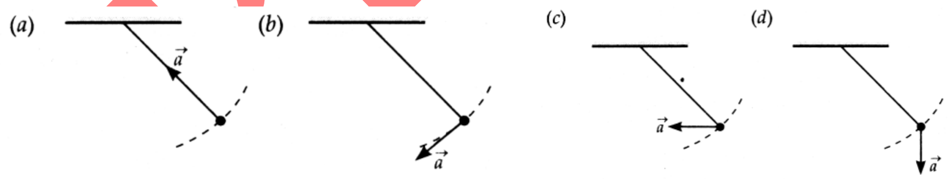
Q.8 The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$ (c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

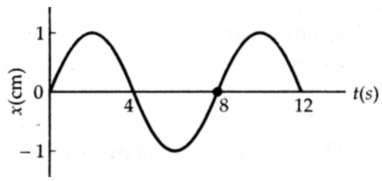
Q.9 A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $(K = 1 m/s^2)$ where y is the vertical displacement. The time period now becomes T_2.The ratio of $\frac{T_1^2}{T_2^2}$ $(g = 10 m/s^2)$ is

(a) 5/6 (b) 6/5 (c) 1 (d) 4/5

Q.10 A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector is $\vec{a}$ correctly shown in



Q.11 The x–t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 is

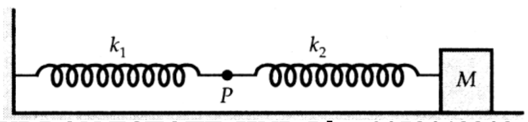


(a) $\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$ (b) $\frac{-\pi^2}{32}cm/s^2$

(c) $\frac{\pi^2}{32}cm/s^2$ (d) $-\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$

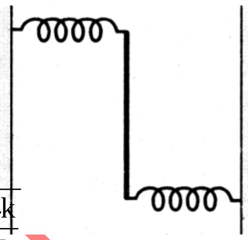
Q.12 The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$



(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

Q.13 A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Q.14 A uniform cylinder of length L and mass M having cross–sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half–submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

(a) $\frac{1}{2\pi}\left(\frac{k - A\rho g}{M}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$ (c) $\frac{1}{2\pi}\left(\frac{k + \rho g L}{M}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k - \rho g L}{M}\right)^{1/2}$

Q.15 One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross–section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(a) $2\pi\,(m/k)^{1/2}$ (b) $2\pi\sqrt{\frac{m(YA + kL)}{YAk}}$ (c) $2\pi(mYA/\,kL)^{1/2}$ (d) $2\pi(mL/YA)^{1/2}$

Q.16 A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Multiple Choice Questions with One or More than One Correct Answer

Q.17 The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents simple harmonic motion for which of the option (s)?

(a) for all values of A, B and C ($C \neq 0$) (b) A = B, C = 2B

(c) A = – B, C = 2B (d) A = B, C = 0

Q.18 Three simple harmonic motions in the same direction having the same amplitude a and same period are suspended. If each differs in phase from the next by 45^o, then

(a) the resultant amplitude is $(1+\sqrt{2})a$ (b) the phase of the resultant relative to the first is 90^o

(c) energy associated with resulting motion is $(3+2\sqrt{2})$ times energy associated with any single motion

(d) the resulting motion is not simple harmonic

Q.19 A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is

(a) f/2 (b) f (c) 2f (d) 4f

Q.20 A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(a) maximum potential energy is 100 J (b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J (d) maximum potential energy is zero

Q.21 A particle of mass m is executing oscillations about the origin on the x–axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If amplitude of oscillation is α, then its time period T is

(a) proportional to $1/\sqrt{a}$ (b) independent of a

(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

Q.22 A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and v respectively. The following relations hold good under the above conditions:

(a) $T \cos\theta = Mg$ (b) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(c) The magnitude of the tangenial acceleration of the bob $|a_T| = g\sin\theta$ (d) $T = Mg\cos\theta$

Match – Matrix Type

Q.23 Column – I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in column I with the characteristics in column II.

	Column – I		**Column – II**
(a)	The object moves on the x–axis under a conservative force in such a way that its speed and position satisfy $v=c_1\sqrt{c_2-x^2}$, where c_1 and c_2 are positive constants.	(p)	The object executes a simple harmonic motion
(b)	The object moves on the x–axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q)	The object does not change its direction
(c)	The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r)	The kinetic energy of the objects keeps on decreasing.
(d)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the	(s)	The object can change its direction only once.

	earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.		

Answer

1.	a	2.	a	3.	d	4.	b	5.	d
6.	a	7.	d	8.	a	9.	b	10.	c
11.	d	12.	d	13.	a	14.	b	15.	b
16.	d	17.	a, b, c	18.	a, c	19.	c	20.	b, c
21.	a	22.	b, c	23.	a – p; b – q, r; c – p, d – r, q				

Objective Assignment – II [AIEEE]

Q.1 The function $\sin^2 \omega t$ represents

(a) a periodic but not simple harmonic motion with a period $2\pi/\omega$

(b) a periodic, but not simple harmonic motion with a period π/ω

(c) a simple harmonic motion with a period $2\pi/\omega$

(d) a simple harmonic motion with a period π/ω

Q.2 Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin (100\ \pi t + \pi/3)$ and

$y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to velocity of particle 2 is

(a) $-\pi/6$ (b) $\pi/3$ (c) $-\pi/3$ (d) $\pi/6$

Q.3 The displacement of an object attached to a spring and executing S.H.M. is given by $x=2\times10^{-2} \cos \pi t$

(in m). The time at which the maximum speed first occurs is

(a) 0.5 s (b) 0.75 s (c) 0.125 s (d) 0.25 s

Q.4 A point mass oscillates along the X–axis according to the relation $x = x_0 \cos (\omega t - \pi/4)$. The acceleration of the particle is written as $a = a_0 \cos (\omega t + \delta)$, then

(a) $a_0 = x_0$; $\delta = -\pi/4$ (b) $a_0 = x_0\omega^2$; $\delta = \pi/4$ (c) $a_0 = x_0\omega^2$; $\delta = -\pi/4$ (d) $a_0 = x_0\omega^2$; $\delta = 3\pi/4$

Q.5 The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1}. The period of oscillation is

(a) 0.01 s (b) 0.1 s (c) 10 s (d) 100 s

Q.6 If a simple harmonic motion is represented by $\frac{d^2x}{dt^2}+\alpha x=0,$ its time period is

(a) $2\pi/\ \alpha$ (b) $2\pi/\ \sqrt{\alpha}$ (c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$

Q.7 If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following does not change with time?

(a) $a^2T^2 + 4\pi^2v^2$ (b) aT/x (c) $aT + 2\pi v$ (d) aT/v

VPLUS U

Q.8 A coin is placed on a horizontal plateform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) for an amplitude of g^2/ω^2 (b) for an amplitude of g/ω^2

(c) at the highest position of the platform (d) at the mean position of the platform

Q.9 If a spring has time period T and is cut into n equal parts, then the time period of each part will be

(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T

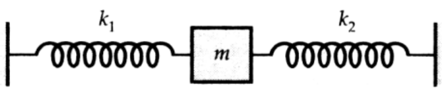
Q.10 A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period T. If the mass is increased by m, the time becomes 5T/3. Then the ratio of m/M is

(a) 3/5 (b) 25/9 (c) 16/9 (d) 5/3

Q.11 A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$ (c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

Q.12 Two springs of force constant k_1 and k_2 are connected to a mass m as shown in the figure. The frequency of oscillation of the mass is v. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $v/2$ (b) $v/4$ (c) $4v$ (d) $2v$

Q.13 Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically, such that their maximum velocities are equal, then the ratio of the amplitude of M to that of N is

(a) k_1/k_2 (b) $\sqrt{k_1/k_2}$ (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

Q.14 A child swinging on a swing in sitting position stands up. Then the time period of the swing will

(a) increase (b) decrease (c) remain the same

(d) increase, if the child is long and decrease, if the child is short.

Q.15 The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 50% (b) 21% (c) 30% (d) 10%

Q.16 The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) first increase and then decrease to the original value

(b) first decreases and then increase to the original value

(c) remain unchanged

(d) increase towards a saturation value

Q.17 The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is 4000/3 kg m^{-3}, what relationship between t and t_0 is true?

(a) $t = t_0$ (b) $t = t_0/2$ (c) $t = 2t_0$ (d) $t = 4t_0$

Q.18 The total energy of a particle executing simple harmonic motion is

(a) $\propto x$ (b) $\propto x^2$ (c) independent of x (d) $\propto x^{1/2}$

Q.19 In a simple harmonic oscillator, at the mean position

(a) kinetic energy is minimum, potential energy is maximum

(b) both kinetic and potential energies are maximum

(c) kinetic energy is maximum, potential energy is minimum

(d) both kinetic and potential energies are minimum

Q.20 A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statements is true?

(a) K.E. is maximum, when x = 0 (b) T.E. is zero, when x = 0

(c) K.E. is maximum, when x is maximum (d) P.E. is maximum, when x = 0

Q.21 A spring of force constant 800 N m^{-1} has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 8 J (b) 16 J (c) 24 J (d) 32 J

Q.22 A spring of spring constant 5×10^3 Nm^{-1} is stretched initially by 5cm from the unstretched position. Then the work done to stretch it further by another 5 cm is

(a) 6.25 Nm (b) 12.50 Nm (c) 18.75 Nm (d) 25.00 Nm

Q.23 A particle of mass m executes S.H.M. with amplitude a and frequency v. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $\pi^2 ma^2 v^2$ (b) $\frac{1}{4}\pi^2 ma^2 v^2$ (c) $4\pi^2 ma^2 v^2$ (d) $2\pi^2 ma^2 v^2$

Q.24 Starting from the origin, a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy by 75% of the total energy?

(a) 1/12 s (b) 1/6 s (c) 1/4 s (d) 1/3 s

Q.25 A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$ (c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

Q.26 In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. Then

(a) $\omega_1 = \omega_2$ (b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$, when damping is small and $\omega_1 > \omega_2$, when damping is large (d) $\omega_1 < \omega_2$

Answer

1.	b	2.	a	3.	a	4.	d	5.	a
6.	b	7.	b	8.	b	9.	b	10.	c
11.	b	12.	d	13.	d	14.	b	15.	d
16.	a	17.	c	18.	c	19.	c	20.	a
21.	a	22.	c	23.	a	24.	b	25.	b
26.	a								

Objective Assignment – III [AIIMS Entrance Exam]

Oscillations and Waves

Q.1 Which of the following functions represents a simple harmonic oscillation?

(a) $\sin \omega t - \cos \omega t$ (b) $\sin \omega t + \sin 2\omega t$

(c) $\sin \omega t - \sin 2\omega t$ (d) $\sin^2 \omega t$

Q.2 A particle executes simple harmonic motion with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a) T/12 (b) T/8 (c) T/4 (d) T/2

Q.3 The periodic time of a body executing S.H.M. is 4s. After how much interval from time t = 0, its displacement will be half of its amplitude?

(a) 1/2s (b) 1/3s (c) 1/4 s (d) 1/6 s

Q.4 The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

(a) $0.5\,\pi$ (b) π (c) $0.707\,\pi$ (d) 0.61 m

Q.5 A particle executes simple harmonic motion with an angular velocity of 3.5 rad s^{-1} and maximum velocity acceleration 7.5 m s^{-2} respectively. The amplitude of oscillations is

(a) 0.28 m (b) 0.36 m (c) 0.707 m (d) zero

Q.6 If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2s, then its maximum velocity is

(a) 0.10 ms^{-1} (b) 0.16 ms^{-1} (c) 0.24 ms^{-1} (d) 0.32 ms^{-1}

Q.7 For a particle executing simple harmonic motion, which of the following statements is not correct?

(a) total energy of the particle always remains the same

(b) restoring force is always directed towards a fixed point

(c) restoring force is maximum at the extreme positions

(d) acceleration of the particle is maximum at the equilibrium position

Q.8 A spring 40mm long is stretched by application of a force. If 10 N force is required to stretch the spring through 1 mm, then work done in stretching the spring through 40 mm is

(a) 84 J (b) 48 J (c) 24 J (d) 8 J

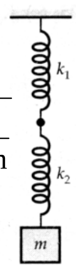
Q.9 If a spring of mass 30 kg has spring constant of 15 Nm^{-1}, then its time period is

(a) $2\,\pi$ s (b) $2\sqrt{2}\,\pi$s (c) $2\sqrt{2\pi}$ s (d) $2\sqrt{2}$ s

Q.10 If the period of oscillation of mass m suspended from a spring is 2s, then the period of mass 4m will be

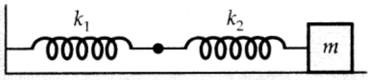
(a) 1s (b) 4s (c) 8s (d) 16 s

Q.11 The frequency of oscillation of the springs shown in the figure will be



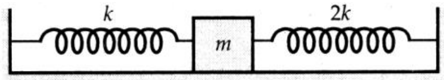
(a) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{(k_1+k_2)m}{k_1 k_2}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$

Q.12 Two springs are connected to a block of mass m placed on a frictionless surface as shown below: If both the springs have a spring constant k, the frequency of oscillation of the block is



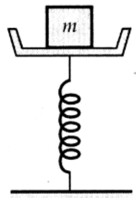
(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

Q.13 Two springs of force constants k and 2k are connected to a mass m as shown below: The frequency of oscillation of the mass is



(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3m}{k}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

Q.14 A mass of 2kg is put on a flat pan attached to a vertical spring fixed on the ground shown in the figure. The mass of pan and the spring is negligible. When pressed slightly and released, the mass executes S.H.M. The spring constant of the spring is 200 Nm^{-1}. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan? Take g = 10 ms^{-2}.



(a) 4 cm (b) 8 cm (c) 10 cm (d) any value less than 12 cm

Q.15 A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is 3.92×10^{-3} m. What must be the least period of these oscillations, so that the object is not detached from the platform?

(a) 0.1256 s (b) 0.1356 s (c) 0.1456 s (d) 0.1556 s

Q.16 If the metal bob of a simple pendulum is replaced by wooden bob, then its time period will

(a) increase (b) decrease (c) remain the same (d) first (a) then (b)

Q.17 The time period of a simple dendulum on a satellite, orbiting around the earth, is

(a) infinite (b) zero (c) 84.6 min (d) 24 hours

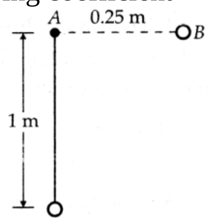
Q.18 A simple pendulum has a time period T. The pendulum is completely immersed in a non–viscous liquid, whose density is 1/10th of that of the material of the bob. The time period of the pendulum immersed in the liquid is

(a) T (b) T/10 (c) $\sqrt{\frac{9}{10}}T$ (d) $\sqrt{\frac{10}{9}}T$

Q.19 A lightly damped oscillator with a frequency v is set in motion by a harmonic driving force of frequency v'. When $v' < v$, then response of the oscillator is controlled by

(a) spring constant (b) inertia of the mass (c) oscillator frequency (d) damping coefficient

Q.20 A simple pendulum has a bob suspended by inextensible thread of length 1 metre from a point A of suspension. At the extreme position of oscillation, the thread is suddenly caught by the peg at a point B distant 0.25 m from A and the bob begins to oscillate in the new condition. The change in frequency of oscillation of the pendulum is approximately (g = 10 ms^{-2}) given by



(a) $\frac{\sqrt{10}}{2}$ Hz (b) $\frac{1}{4\sqrt{10}}$ Hz (c) $\frac{\sqrt{10}}{3}$ Hz (d) $\frac{1}{\sqrt{10}}$ Hz

Assertions and Reasons

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

V PLUS U

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false
(d) If both assertion and reason are false

Q.21 **Assertion:** In S.H.M., the motion is to and fro and periodic.
Reason: Velocity of the particle,
$$v=\omega\sqrt{a^2-x^2}$$ where x is displacement.

Q.22 **Assertion:** In simple harmonic motion, the velocity is maximum, when the acceleration is minimum.
Reason: Displacement and velocity in simple harmonic motion differ in phase by $\pi/2$

Q.23 **Assertion:** The amplitude of an oscillating pendulum decrease gradually with time.
Reason: The frequency of the pendulum decreases with time.

Q.24 **Assertion:** The time period of a pendulum on a satellite orbiting the earth is infinity.
Reason: The time period of a pendulum is inversely proportional to $\sqrt{g}$.

Q.25 **Assertion:** Water in a U–tube executes S.H.M. The time period for mercury filled upto the same height in the U–tube be greater than that in case of water.
Reason: The amplitude of an oscillating pendulum goes on increasing.

Q.26 **Assertion:** Resonance is a special case of forced vibration in which the nature and frequency of vibration of the body is same as impressed frequency and amplitude of forced vibration, is maximum.
Reason: The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

Answer

1.	a	2.	a	3.	b	4.	a	5.	d
6.	b	7.	d	8.	d	9.	b	10.	b
11.	d	12.	d	13.	a	14.	c	15.	a
16.	c	17.	a	18.	d	19.	a	20.	b
21.	b	22.	b	23.	c	24.	b	25.	d
26.	c								

Objective Assignment – IV [CBSE PMT Prelims Exam]

Q.1 The circular motion of a particle with constant speed is
(a) periodic but not simple harmonic (b) simple harmonic but not periodic
(c) periodic and simple harmonic (d) neither periodic not simple harmonic

Q.2 Which of the following is simple harmonic motion?
(a) Particle moving in a circle with uniform speed
(b) Wave moving through a string fixed at both ends
(c) Earth spinning about its axes (d) Ball bouncing between two rigid vertical walls

Q.3 A particle is executes S.H.M. along x–axis. The force acting on it is given by
(a) A cos (kx) (b) Ae^{-kx} (c) Akx (d) –Akx

Q.4 Which one of the following represents simple harmonic motion?
(a) Acceleration = kx (b) Acceleration = $k_0x + k_1x^2$
(c) Acceleration = – k(x + a) (d) Acceleration = k(x + a)
where k, k_0, k_1 are all positive.

Oscillations and Waves

Q.5 A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is

(a) 4 Hz (b) 3 Hz (c) 2 Hz (d) 1 Hz

Q.6 A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a)T/8 (b) T/12 (c) T/2 (d) T/4

Q.7 A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from x = A to x = A/2 is

(a) T/6 (b) T/4 (c) T/3 (d) T/2

Q.8 If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s^2 at any time, the angular frequency of the oscillator is equal to

(a) 10 rad/s (b) 0.1 rad/s (c) 100 rad/s (d) 1 rad/s

Q.9 The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

(a) π (b) 0.707 π (c) zero (d) 0.5 π

Q.10 Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion?

(a) when v is maximum, a is maximum (b) value of a is zero, whatever may be the value of v

(c) when v is zero, a is zero (d) when v is maximum, a is zero

Q.11 A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a and time period T. The speed of the pendulum at x = a/2 will be

(a) $\frac{\pi a\sqrt{3}}{T}$ (b) $\frac{\pi a\sqrt{3}}{2T}$ (c) $\frac{\pi a}{T}$ (d) $\frac{3\pi^2 a}{T}$

Q.12 A body is executing simple harmonic motion. When the displacements from the mean position are 4 cm and 5cm, the corresponding velocities of the body are 10cm/sec and 8 cm/sec. Then the time period of the body is

(a) 2 π sec (b) $\pi/2$ sec (c) π sec (d) $(3\pi/2)$ sec

Q.13 The total energy of particle performing SHM depends on

(a) k, a, m (b) k, a (c) k, a, x (d) k, x

Q.14 Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is

(a) $\pm$ a/2 (b) + a (c) $\pm$ a (d) – 1

Q.15 The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2\omega t$. The maximum values of the potential energy and the total energy are respectively

(a) $K_0/2$ and K_0 (b) K_0 and 2 K_0 (c) K_0 and K_0 (d) 0 and 2 K_0

Q.16 A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01m has a total mechanical energy of 160 J. Its

(a) P.E. is 160 J (b) P.E. is zero (c) P.E. is 100 J (d) P.E. is 120 J

Q.17 The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

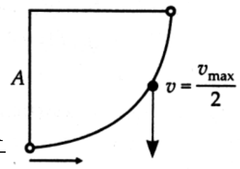
(a) 2/3 E (b) 1/8 E (c) 1/4 E (d) 1/2 E

Q.18 In a simple harmonic motion, when the displacement is one–half the amplitude, what fraction of the total energy is kinetic?

(a) 1/2 (b) 3/4 (c) zero (d) 1/4

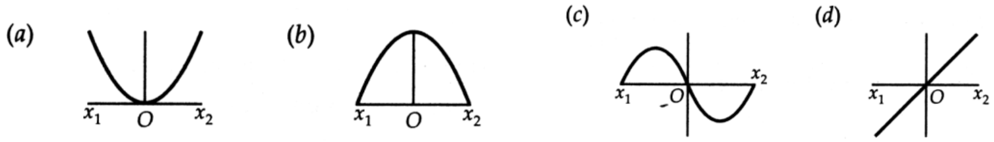
Q.19 A body executes simple harmonic motion with an amplitude A. At what displacement from the mean position is the potential energy of the body is one–fourth of its total energy?
(a) A/4 (b) A/2 (c) 3A/4 (d) some other fraction of A

Q.20 A particle starts with S.H.M. from the mean position as shown in the figure. Its amplitude is A and its time period is T. At one time, its speed is half that of the maximum speed. What is this displacement?



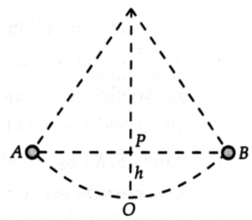
(a) $\frac{2A}{\sqrt{3}}$ (b) $\frac{3A}{\sqrt{2}}$ (c) $\frac{\sqrt{2}A}{3}$ (d) $\frac{\sqrt{3}A}{2}$

Q.21 A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equallibrium position being O. Its potential energy is plotted. It will be as given below in the graph



Q.22 The angular velocity and the amplitude of a simple pendulum are ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is
(a) $\frac{(a^2 - x^2\omega^2)}{x^2\omega^2}$ (b) $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$ (c) $\frac{(a^2 - \omega^2)}{x^2}$ (d) $\frac{x^2}{(a^2 - x^2)}$

Q.23 A simple pendulum with a bob of mass m oscillates from A to B and back to A such that OP is h. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(a) mgh (b) $\sqrt{2gh}$
(c) zero (d) 2 gh

Q.24 The bob of simple pendulum having length l, is displaced from mean position to an angular position θ with respect to vertical. If it is released, then velocity of bob at equilibrium position
(a) $\sqrt{2gl(1-\cos\pi)}$ (b) $\sqrt{2gl(1+\cos 0)}$ (c) $\sqrt{2gl\cos\theta}$ (d) $\sqrt{2\ gl}$

Q.25 A loaded vertical spring executes S.H.M. with a time period of 4 sec. The difference between the kinetic energy and potential energy of this system varies with a period of
(a) 2 sec (b) 1 sec (c) 8 sec (d) 4 sec

Q.26 A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system, if a mass 4m is suspended from the same spring?
(a) n/2 (b) 4n (c) n/4 (d) 2 n

Q.27 A body of mass 5 kg hangs from a spring and oscillates with a time period of 2 π seconds. If the body is removed, the length of the spring will decreases by
(a) g/k metres (b) k/g metres (c) 2 π metres (d) g metres

Q.28 The time period of mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be
(a) T/4 (b) T (c) T/2 (d) 2 T

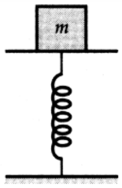
Q.29 Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by

(a) $\sqrt{k_1 k_2}$ (b) $(k_1 + k_2)/2$ (c) $k_1 + k_2$ (d) $k_1k_1/(k_1 + k_2)$

Q.30 A mass m is suspended from the two coupled springs connected in series. The force constants for springs are k_1 and k_2. The time period of the suspended mass will be

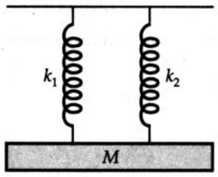
(a) $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$ (b) $T = 2\pi\sqrt{\frac{mk_1k_2}{k_1 + k_2}}$ (c) $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$ (d) $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$

Q.31 A mass of 2.0 kg is put on a flat pan attached on a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan (take $g = 10\ m/s^2$)?



(a) 10.0 cm (b) any value less than 12.0 cm (c) 4.0 cm (d) 8.0 cm

Q.32 A mass is suspended separately by two different springs in successive order, then time periods are t_1 and t_2 respectively. If it is connected by both springs as shown in figure, then time period is t_0, correct relation is



(a) $t_0^2 = t_1^2 + t_2^2$ (b) $t_0^{-2} = t_1^{-2} + t_2^{-2}$

(c) $t_0^{-1} = t_1^{-1} + t_2^{-1}$ (d) $t_0 = t_1 + t_2$

Q.33 Time period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its period becomes

(a) 8 sec (b) 12 sec (c) 16 sec (d) 4 sec

Q.34 If the length of a simple pendulum is increased by 2%, then the time period

(a) increases by 1% (b) decreases by 1% (c) increases by 2% (d) decreases by 2%

Q.35 Two masses M_A and M_B are hung from two strings of length l_A and l_B respectively. They are executing SHM with frequency relation $f_A = 2f_B$, then relation

(a) $l_A = \frac{l_B}{4}$, does not depend on mass (b) $I_A = 4l_B$, does not depend on mass

(c) $l_A = 2l_B$ and $M_A = 2M_B$ (d) $l_A = l_B/2$ and $M_A = M_B/2$

Q.36 A second's pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is

(a) moving down with uniform acceleration

(b) moving around the earth in geostationary orbit

(c) moving up with uniform velocity (d) moving up with uniform acceleration

Q.37 A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi$, $\sqrt{\left(\frac{\ell}{g}\right)}$, where g is equal to

(a) g (b) g – a (c) g + a (d) $\sqrt{(g^2 + a^2)}$

Q.38 A rectangular block of mass m and area of cross–section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T, then

(a) $T \propto \frac{1}{\sqrt{m}}$ (b) $T \propto \sqrt{\rho}$ (c) $T \propto \frac{1}{\sqrt{A}}$ (d) $T \propto \frac{1}{\rho}$

Q.39 In case of a forced vibration, the resonance peak becomes very sharp when the

(a) damping force is small (b) restoring force is small

(c) applied periodic force is small (d) quality factor is small

Q.40 A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force F sin ωt. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle maximum for $\omega = \omega_2$, then

(a) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$ (b) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$

(c) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$ (d) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

Q.41 When an oscillator completes 100 oscillations, its amplitude reduces to 1/3 of initial value. What will be its amplitude, when it completes 200 oscillations?

(a) 1/8 (b) 2/3 (c) 1/6 (d) 1/9

Q.42 Two simple pendulums of lengths 5m and 20m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed………… oscillations.

(a) 2 (b) 1 (c) 5 (d) 3

Q.43 Two simple pendulums of time periods 2.0s and 2.1s are made to vibrate simultaneously. They are in phase initially. After how many vibrations, they are in the same phase?

(a) 21 (b) 25 (c) 30 (d) 35

Q.44 Two SHM's with same amplitude and time period, when acting together in perpendicular directions with a phase difference of π/2, give rise to

(a) straight motion (b) elliptical motion (c) circular motion (d) none of these

Q.45 The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along

(a) circle (b) figure of eight (c) straight line (d) ellipse

Answer

1.	a	2.	b	3.	d	4.	c	5.	d
6.	b	7.	a	8.	a	9.	d	10.	d
11.	a	12.	c	13.	b	14.	c	15.	c
16.	c	17.	c	18.	b	19.	b	20.	d
21.	a	22.	c	23.	c	24.	a	25.	a
26.	a	27.	d	28.	c	29.	d	30.	d
31.	a	32.	b	33.	d	34.	a	35.	a
36.	a	37.	d	38.	c	39.	a	40.	a
41.	d	42.	b	43.	a	44.	c	45.	C

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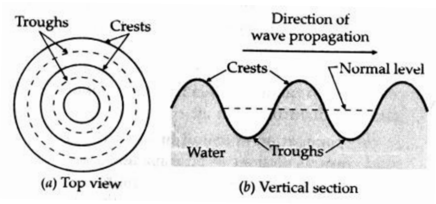
WAVES

Wave Motion

Wave motion is a kind of disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. In a wave, both information and energy propagate (in the form of signals) from one point to another but there is no motion of matter as a whole through a medium.



Characteristics of Wave Motion

(i) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(ii) Energy is transferred from one place to another without any actual transfer of particles of the medium.

(iii) Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.

(iv) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.

(v) The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about mean position. It is maximum at mean position and zero at extreme position.

(vi) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

Different types of Waves

(i) Mechanical Waves

The waves which require a material medium for their propagation are called mechanical waves. Such waves are also called elastic waves because their propagation depends on the elastic properties of the medium. These waves are governed by Newton's laws and can exist in a materials mediums, such as water, air, rock etc.

Examples: Water waves, sound waves, seismic waves (waves produced during earthquake), etc.

(ii) Electromagnetic Waves

The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation and are also called non–mechanical waves. Light from the sun and distant stars reaches us through inter–stellar space, which is almost vacuum. All electromagnetic waves travel through vacuum at the same speed c, given by

$$c = 3 \times 10^8\ ms^{-1} \text{ (speed of light)}$$

Examples: Visible and ultraviolet light, radiowaves, microwaves, X–rays, etc.

(iii) Matter Waves

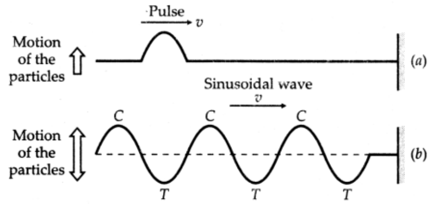
The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de–Broglie waves. These waves become important in the quantum mechanical description of matter.

Examples: Electron microscopes make use of the matter waves associated with fast moving electrons.

Transverse and Longitudinal Waves

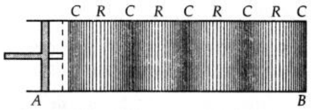
Transverse Waves:

These are the waves in which the individual particles of the medium oscillate perpendicular to the direction of wave propagation. Consider a horizontal string with its one end fixed to rigid support and other end held in the hand. If we continuously give up and down jerks to the free end of the string, a number of sinusoidal waves begin to travel along the string.



Each part of the string vibrates up and down while the wave travels along the string. So the waves in the string are transverse in nature. The point (C, C, ...) of maximum displacement in upward direction are called *crests.* The points (T, T, ...) of maximum displacement in the downward direction are called *troughs. One crest and one trough together form one wave.*

Longitudinal Waves



These are the waves in which the individual particles of the medium oscillate along the direction of wave propagation. Consider a long hollow cylinder AB closed at one end and having a movable piston at the other end.

If we continuously push and pull the piston in a simple harmonic manner, a sinusoidal sound wave travels along the cylinder in the form of alternate compressions and rarefactions, marked C, R, C, R etc. As the oscillations of an element of air are parallel to direction of wave propagation, the wave is a longitudinal wave. Hence sound waves produced in air are longitudinal waves.

Essential properties of a medium for the propagation of mechanical waves

Both transverse and longitudinal waves can propagate through those media which have the following properties:

(i) **Elasticity:** The medium must possess elasticity so that the particles can return to their mean positions after being disturbed.

(ii) **Inertia:** The medium must possess inertia or mass so that its particles can store kinetic energy.

(iii) **Minimum friction:** The frictional force amongst the particles of the medium should be negligibly small so that they continue oscillating for a sufficiently long time and the wave travels a sufficiently long distance through the medium.

Some Definitions in Connection with Wave Motion

(i) **Amplitude:** *It is the maximum displacement suffered by the particles of the medium about their mean positions.* It is denoted by A.

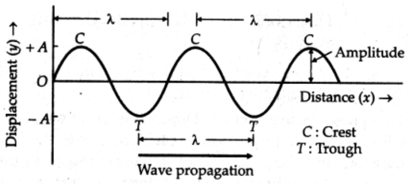
(ii) **Time period:** *It is the time in which a particle of medium completes one vibration to and fro about its mean position.* It is denoted by T.

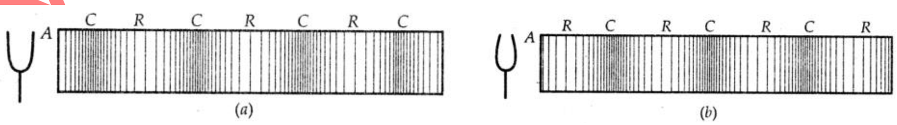
(iii) **Frequency:** *The frequency of a wave is the number of waves produced per unit time in the given medium.* It is equal to the number of oscillations completed per unit time by any particle of the medium. It is equal to the reciprocal of the medium. It is equal to the reciprocal of the time period T of the particle and is denoted by v. Thus $v = \frac{1}{T}$ SI unit of v is s^{-1} or hertz (Hz).

(iv) **Angular frequency:** *The rate of change of phase with time is called angular frequency of the wave.* It is clearly equal to $2\pi/T$, because the phase change in time T is 2π. It is denoted by ω.

Thus $\omega = \frac{2\pi}{T} = 2\pi v$ SI unit of ω = rad s^{-1}

(v) **Wavelength:** *It is the distance covered by a wave during the time in which a particle of the medium completes one vibration to and fro about its mean position. Or, it is the distance between two nearest particles of the medium which are vibrating in the same phase.* It is denoted by λ.





(a) (b)

(vi) **Wave number:** *The number of waves present in a unit distance of the medium is called wave number.* It is equal to the reciprocal of wavelength λ. Thus wave number, $\bar{v} = \frac{1}{\lambda}$

SI unit of wave number = m^{-1}

(vii) **Angular wave number or propagation constant:** *The quantity $2\pi/\lambda$ is called angular wave number or propagation constant of a wave.* It represents phase change per unit path difference. It is denoted by k. Thus $k = \frac{2\pi}{\lambda}$ The SI unit of k is radian per metre or rad m^{-1}

(viii) **Wave velocity or phase velocity:** *The distance covered by a wave per unit time in its direction of propagation is called its wave velocity or phase velocity.* It is denoted by v.

Relation between wave velocity, frequency and wavelength

We know that when a particle of medium completes one oscillation about its mean position in periodic time T, the wave travels a distance equal to its wavelength λ. Therefore,

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad v = \frac{\lambda}{T} \quad \text{or} \quad v = \nu\lambda \quad [\because \nu = 1/T]$$

i.e., Wave velocity = frequency × wavelength

Subjective Assignment – I

Q.1 How far does the sound travel in air when a tuning fork of frequency 256 Hz makes 64 vibrations? Velocity of sound in air = 320 ms^{-1}.

Q.2 A source of sound is placed at one end of an iron bar two kilometer long and two sounds are heard at the other end at an interval of 5.6 seconds. If the velocity of sound in air is 330 ms^{-1}, find the velocity of sound in iron.

Q.3 A radio station broadcasts its programme at 219.3 metre wavelength. Determine the frequency of radio waves if velocity of radio wave be 3×10^8 ms^{-1}.

Q.4 The audible frequency range of a human's ear is 20 Hz – 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperatures to be 340 ms^{-1}.

Q.5 The speed of a wave in a medium is 960 ms^{-1}. If 3600 waves are passing through a point in the medium in 1 minute, then calculate the wavelength.

Q.6 A stone is dropped into a well and its splash is heard at the mouth of the well after an interval of 1.45 s. Find the depth of the well. Given that velocity of sound in air at room temperature is equal to 332 ms^{-1}.

Q.7 A body sends waves 100 mm long through medium A and 0.25 m long in medium B. If the velocity of waves in medium A is 80 cm s^{-1}, calculate the velocity of waves in medium B.

Answers

1.	80 m	2.	4348 ms^{-1}	3.	1.368×10^6 Hz		
4.	0.017 m to 17m	5.	16 m	6.	9.9 m	7.	2 ms^{-1}

Speed or Transverse Waves

(a) **Speed of a transverse wave on a stretched string**

The wave velocity through a medium depends on its inertial and elastic properties. So the speed of transverse wave through a stretched string is determined by two factors:

(i) Tension T in the string is a measure of elasticity in the string. Without tension no disturbance can propagate in the string. Dimensions of T = [Force] = $[MLT^{-2}]$

(ii) Mass per unit length or *linear mass density m* of the string so that the string can store kinetic energy. Dimensions of $m = \frac{[\text{Mass}]}{[\text{Length}]} = [ML^{-1}]$

Now, dimensions of ratio $\frac{T}{m} = \frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}]$

VPLUS

As the speed v has the dimensions $[LT^{-1}]$, so we can express v in terms of T and m as $v = C\sqrt{\frac{T}{m}}$

The dimensionless constant C = 1. Hence the speed of transverse waves on a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

Clearly, *the speed of transverse wave along a stretched string depends only on the tension T and linear mass density m of the string. It does not depend on the frequency of the wave*. The frequency of a wave depends on the source generating that wave.

(b) Speed of transverse wave in a solid

The speed of transverse wave through a solid is determined by two factors: (i) Elasticity of shape or modulus of rigidity η of the solid. (ii) Mass per unit volume or density ρ determines its inertia. Now,

Dimensions of ratio $\frac{\eta}{\rho} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$

Dimensions of speed v = $[LT^{-1}]$

So we can express v in terms of η and ρ as $v = C\sqrt{\frac{\eta}{\rho}}$

The dimensionless constant C is found to be unity. Thus speed of traverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}$$

Subjective Assignment – II

Q.1 For aluminium the modulus of rigidity is 2.1×10^{10} Nm^{-2} and density is 2.7×10^3 kg m^{-3}. Find the speed of transverse waves in the medium.

Q.2 A steel wire 0.72 m long has a mass of 5.0×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire?

Q.3 In a sonometer experiment, the density of the material of the wire used is 7.5×10^3 kg m^{-3}. If the stress of the wire is 3.0×10^8 Nm^{-2}, find out the speed of the transverse wave in the wire.

Q.4 A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut with negligible tension. Find the speed of transverse waves in the wire at 10°C.

Q.5 A steel wire 70 cm long has a mass of 7 kg. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire?

Q.6 The speed of a transverse wave in a stretched string is 348 ms^{-1}, when the tension of the string is 3.6 kg wt. Calculate the speed of the transverse wave in the same string, if the tension in the string is changed to 4.9 kg wt.

Q.7 A wave–pulse is traveling on a string of linear mass density 10 g cm^{-1} under a tension of 1 kg wt. Calculate the time taken by the pulse to travel a distance of 50 cm on the string. Given g = 10 ms^{-2}.

Q.8 The diameter of an iron wire is 1.20 mm. If the speed of the transverse wave in the wire be 50.0 ms^{-1}, what is the tension in the wire? The density of iron is 7.7×10^3 kg m^{-3}.

Answers

1.	2.79×10^3 ms^{-1}	2.	93 ms^{-1}	3.	200 ms^{-1}	4.	72 ms^{-1}
5.	100 ms^{-1}	6.	406 ms^{-1}	7.	0.05 s	8.	21.78 N

Speed of a Longitudinal Wave

(a) Speed of a longitudinal wave in a liquid or gas: In a longitudinal wave, the particles of the medium oscillate forward and backward in the direction of propagation of the wave. They cause compressions and rarefactions of small volume elements of fluid. So the speed of a longitudinal wave through a fluid is determined by two factors:

(i) The volume elasticity or bulk modulus k of the fluid.

(ii) The density of the fluid which determines its inertia.

$\therefore$ Dimensions of the ratio $\frac{k}{\rho} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$

Dimensions of speed v = $[LT^{-1}]$

So the speed v can be expressed in terms of k and ρ as $v = C\sqrt{\frac{k}{\rho}}$

The dimensionless constant C is found to be unity. Hence the speed of a longitudinal wave in any fluid (liquid or gas) is given by

Clearly, *the speed of a longitudinal wave through a fluid depends only on its bulk modulus k and density ρ.*

(b) Speed of a longitudinal wave in a solid: The speed of a longitudinal wave through a solid of bulk modulus k, modulus of rigidity η and density ρ is given by $v = \sqrt{\frac{k + \frac{4}{3}\eta}{\rho}}$

(c) Speed of a longitudinal wave in a solid rod: When a long solid rod is given blows at one end, longitudinal waves travel through it in the form of compressions and rarefactions. As the sidewise expansion of the rod is negligible, we need to consider only longitudinal strain. In this case, the relevant modulus of elasticity is the Yung's modulus. Hence the speed of a longitudinal wave through a solid rod of Young's modulus Y and density ρ is given by $v = \sqrt{\frac{Y}{\rho}}$

Speed of Sound

Newton's formula for the speed of sound in a gas

Newton gave the first theoretical expression of the speed of sound in a gas. He assumed that sound waves travel through a gas under isothermal conditions. He argued that the small amount of heat produced in a compression is rapidly conducted to the surrounding rarefactions where slight cooling is produced. Thus the temperature of gas remains constant. If k_{iso} is the isothermal volume elasticity (bulk modulus of the gas at constant temperature), then the speed of sound in the gas will be

$$v = \sqrt{\frac{k_{iso}}{\rho}}$$

For an isothermal change, PV = constant (Boyle's law)

Differentiating both sides, we get

$$P\,dV + VdP = 0 \quad \text{or} \quad PdV = -V\,dP \quad \text{or} \quad P = -\frac{VdP}{dV} = -\frac{dP}{dV/V} = \frac{\text{Volume stress}}{\text{Volume strain}} = k_{iso}$$

Hence *the Newton's formula for the speed of sound in gas is* $v = \sqrt{\frac{P}{\rho}}$

At STP, $P = 0.76$ m of Hg $= 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5\ Nm^{-2}$

ρ = density of air = 1.293 kg m^{-3}

$\therefore$ Speed of sound in air STP, $v=\sqrt{\frac{1.013\times10^5}{1.293}}$ ≈ 280 ms^{-1}

This value is about 15% less than the experimental value (331 ms^{-1}) of the speed of sound in air at STP. Hence Newton's formula is not acceptable.

Laplace's correct

In 1816, the French scientist Laplace pointed out that sound travels through a gas under adiabatic conditions not under isothermal conditions (as suggested by Newton). This is because of the following reason:

(i) As sound travels through a gas, temperature rises in the regions of compressions and falls in the regions of rarefactions.

(ii) A gas is a poor conductor of heat.

(iii) The compressions and rarefactions are formed so rapidly that the heat generated in the regions of compressions does not get time to pass into the regions of rarefactions so as to equalize the temperature.

So when sound travels through a gas, the temperaure does not remain constant. The pressure volume variations are adiabatic. If k_{adia} is the adiabatic bulk modulus of the gas, then the formula for the speed of sound in the gas would be

$$v=\sqrt{\frac{k_{adia}}{\rho}}$$

For an adiabatic change, PV^{γ} = constant

Differentiating both sides, we get

$$P(\gamma V^{\gamma-1})\,dV + V^{\gamma}\,dP = 0 \quad \text{or} \quad \gamma\,PdV + VdP = 0 \qquad \gamma P = -\frac{dP}{dV/V} = k_{adia}$$

where $\gamma = C_p/C_v$, is the ratio of two specific heats.

Hence the Laplace formula for the speed of sound in a gas is $v=\sqrt{\frac{\gamma P}{\rho}}$

This modification of Newton's formula is known as Laplace correction.

For air $\gamma = 7/5$, so speed of sound in air at STP will be

$$v=\sqrt{\gamma}\sqrt{\frac{P}{\rho}}=\sqrt{\frac{7}{5}}\times 280 = 331.3\ ms^{-1}$$

This value is in close agreement with the experimental value. Hence the Laplace correction is justified.

Subjective Assignment – III

Q.1 For aluminium the bulk modulus and modulus of rigidity are $7.5 \times 10^{10}\ Nm^{-2}$ and $2.1 \times 10^{10}\ Nm^{-2}$. Find the velocity of longitudinal waves in the medium. Density of aluminium is 2.7×10^3 kg m^{-3}.

Q.2 For a steel rod, the Young's modulus of elasticity is $2.9 \times 10^{11}\ Nm^{-2}$ and density is 8×10^3 kg m^{-3}. Find the velocity of the longitudinal waves in the steel rod.

Q.3 At a pressure of $10^5\ Nm^{-2}$, the volume strain of water is 5×10^{-5}. Calculate the speed of sound in water. Density of water is 10^3 kg m^{-3}.

Q.4 Estimate the speed of sound in air at standard temperature and pressure by using (i) Newton's formula and (ii) Laplace formula. The mass of 1 mole of air = 29.0×10^{-3} kg. For air, $\gamma = 1.4$

Q.5 The speed of sound in a liquid is 1500 ms^{-1}. The density of the liquid is 1.0×10^3 kg m^{-3}. Determine the bulk modulus of elasticity of the liquid.

Q.6 The longitudinal waves starting from a ship return from the bottom of the sea to the ship after 2.64 s. If the bulk modulus of water be 220 kg mm^{-2} and the density $1.1. \times 10^3$ kg m^{-3}, calculate the depth of the sea. Take g = 9.8 N kg^{-1}.

Q.7 At normal temperature and pressure, 4g of helium occupies a volume of 22.4 litre. Determine the speed of sound in helium. For helium, $\gamma = 1.67$ and 1 atmospheric pressure = 10^5 Nm^{-2}.

Q.8 At 10^5 Nm^{-2} atmospheric pressure the density of air is 1.29 kg m^{-3}. If $\gamma = 1.41$ for air, calculate the speed of sound in air.

Answers

1.	6.18×10^3 ms^{-1}	2.	6.02×10^3 ms^{-1}	3.	1.414×10^3 ms^{-1}
4.	280 ms^{-1}, 331.5 ms^{-1}	5.	2.25×10^9 Nm^{-2}	6.	1848 m
7.	967 ms^{-1}	8.	330.6 ms^{-1}		

Factors Affecting Speed of Sound in a Gas

(i) Effect of pressure:

The speed of sound in a gas is given by the Laplace formula, $v = \sqrt{\frac{\gamma P}{\rho}}$

At constant temperature, PV = constant or $\frac{Pm}{\rho} = \text{constant}$ $\left[\because \rho = \frac{m}{V} \text{ or } V = \frac{m}{\rho}\right]$

Since m is a constant, so $\frac{P}{\rho}$ = constant i.e., when pressure changes, density also changes in the same ratio so that the factor P/ρ remains unchanged. Hence pressure has no effect on the speed of sound in a gas.

(ii) Effect of density: Suppose two gases have the same pressure P and same value of γ (both are either monoatomic, diatomic or triatomic). If ρ_1 and ρ_2 are the densities of the two gases, then the speeds of sound in them will be

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma P}{\rho_2}} \qquad \therefore \qquad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Hence *at constant pressure, the speed of sound in a gas is inversely proportional to the square root of its density*.

(iii) Effect of humidity

The speed o sound in air is given by $v = \sqrt{\frac{\gamma P}{\rho}}$ i.e., $v \propto \frac{1}{\sqrt{\rho}}$

As the density of water vapour (0.8 kgm^{-3} at STP) is less than that of dry air (1.293 kgm^{-3} at STP), so the presence of moisture in air decreases the density of air. Since, the speed of sound is inversely proportional to the square root of density, so sound travels faster in moist air than in dry air.

(iv) Effect of Temperature

For one mole of a gas, PV = RT. If M is the molecular mass of the gas and ρ its density, then

$$\rho = \frac{M}{V} \quad \text{or} \quad V = \frac{M}{\rho}$$

$$\therefore \quad \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M} \qquad \therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

VPLUSU

Clearly, $v \propto \sqrt{T}$

Hence *the speed of sound in a gas is directly proportional to the square root of its absolute temperature.*

Temperature coefficient for the speed of sound in air

It is defined as the increase in the velocity of sound for 1°C (or 1 K) rise in temperature of the gas.

As $v \propto \sqrt{T}$ and $T(K) = t^\circ C + 273$

$\therefore$ At $0^\circ C$, speed $v_0 \propto \sqrt{0+273}$ At $t^\circ C$, speed $v_t \propto \sqrt{t+273}$

Hence $\frac{v_t}{v_0} = \sqrt{\frac{t+273}{0+273}}$ or $v_t = v_0\left[1+\frac{t}{273}\right]^{1/2} \approx V_0\left[1+\frac{1}{2}.\frac{t}{273}\right]$ or $v_t = v_0 + \frac{v_0 \times t}{546}$

But speed of sound in air at $0^\circ C$,

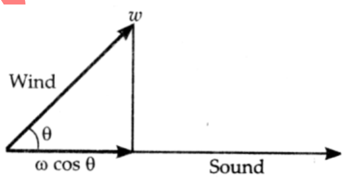
$$\therefore \quad v_t - v_0 = \frac{332 \times t}{546} = 0.61t$$

when $t = 1^\circ C$, $v_t - v_0 = 0.61\ ms^{-1} = 61\ cm\ s^{-1}$

Hence the *velocity of sound in air increase* by 61 cm s^{-1} for every 1°C rise of temperature. This is known as temperature coefficient for sound in air.

(v) Effect of Wind

As the sound is carried by air, so its velocity is affected by the wind velocity. Suppose the wind travels with velocity w at angle θ with direction of propagation of sound, as shown in figure. Clearly, the component of wind velocity y in the direction of sound is w cos θ.



$\therefore$ Resultant velocity of sound = v + w cos θ

When the wind blows in the direction of sound (θ = 0°), resultant velocity = v + w

(vi) Effect of frequency

The speed of sound in air is independent of its frequency. Sound waves of different frequencies travel with the same speed in air, though their wavelengths in air are different.

If the speed of sound were dependent on the frequency, we could not have enjoyed orchestra.

(vii) Effect of amplitude

To a large extent, the speed sound is independent of the amplitude of the sound wave. But if the amplitude is very large, the compressions and rarefactions may cause large temperature variations which may affect the speed of sound.

Subjective Assignment – IV

Q.1 Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000°C. Density of oxygen is 16 times that of hydrogen.

Q.2 Speed of sound in air is 332 ms^{-1} at S.T.P. What will be its value in hydrogen at S.T.P., if density of hydrogen at S.T.P. is $1/16^{th}$ that of air?

Q.3 At normal temperature and pressure the speed of sound in air is 332 ms^{-1}. What will be the speed of sound in hydrogen at 546°C and 3 atmospheric pressure? Air is 16 times heavier than hydrogen.

Q.4 Find the ratio of velocity of sound in hydrogen gas (γ = 7/5) to that in helium gas (γ = 5/3) at the same temperature. Given that molecular weights of hydrogen and helium are 2 and 4 respectively

Q.5 The ratio of densities of oxygen and nitrogen is 16 : 14. At what temperature, the speed of sound in oxygen will be equal to its speed in nitrogen at 14°C? .

Q.6 A gas is a mixture of two parts by volume of hydrogen and one part by volume of nitrogen. If the velocity of sound in hydrogen at 0^oC is 1300 ms^{-1}, find velocity of sound in gaseous mixture at 27^oC.

Q.7 Find the temperature at which the velocity of sound in air will be $1\frac{1}{2}$ times the velocity at 11^oC.

Q.8 At what temperature will the speed of sound be double its value at 0^oC?

Q.9 What is the ratio of the velocity of sound in hydrogen (γ = 7/5) to that in helium gas (γ = 5/3) at the same temperature?

Q.10 A sound wave propagating in air has a frequency of 4000 Hz. Calculate the percentage change in wavelength when the wavefront, initially in a region where T = 27^oC, enters a region where the temperature decreases to 10^oC.

Q.11 At what temperature will the velocity of sound in hydrogen be the same as in oxygen at 100^oC? Density of oxygen is 16 times the density of hydrogen.

Q.12 The speed of sound in dry air at S.T.P. is 332 ms^{-1}. Assuming air as composed of 4 parts of nitrogen and one part of oxygen, calculate velocity of sound in oxygen under similar conditions, when the densities of oxygen and nitrogen at S.T.P. are in the ratio of 16 : 14.

Q.13 At what temperature will the speed of sound be double its value at 273 K? OR Calculate the temperature at which the speed of sound will be two times its value at 0^oC.

Q.14 A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at S.T.P. Calculate the increase in wavelength, when temperature of air is 27^oC.

Answers

1.	– 193.44^oC	2.	1328 ms^{-1}	3.	2300 ms^{-1}	4.	1.68
5.	55^oC	6.	591 ms^{-1}	7.	3.66^oC	8.	819^oC
9.	$\sqrt{42}/5$	10.	3%	11.	– 249.9^oC		
12.	314.77 ms^{-1}	13.	1092 K	14.	0.07 m		

Displacement Relation for a Progressive Wave

Progressive Wave

A wave that travels from one point of the medium to another is called a progressive wave. A progressive wave may be transverse or longitudinal.

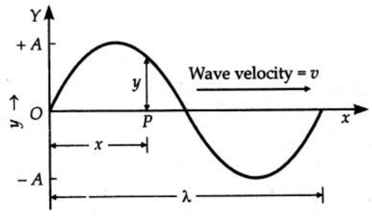
Plane progressive harmonic wave

If during the propagation of a wave through a medium, the particles of the medium vibrate simple harmonically about their mean positions, then the wave is said to be plane progressive harmonic wave. In a harmonic progressive wave of given frequency, all particles have same amplitude but the phase of oscillation changes from one particle to the next.

Displacement relation for a progressive harmonic wave

Suppose a simple harmonic wave starts from the origin O and travels along the positive direction of X–axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at origin O (x = 0) at any instant t is given by

$$y(0, t) = A \sin \omega t \quad \dots (1)$$



where T is the periodic time and A the amplitude of the wave.

Consider a particle P on the X–axis at a distance x from O. The disturbance starting from the origin O will reach P in x/v seconds. This means the particle P will start vibrating x/v seconds later than the particle at O. Therefore, Displacement of the particle at P any instant t

= displacement of the particle at O at a time x/ v seconds earlier

= displacement of the particle at O at time (t – x/v)

Thus the displacement of particle at P at any time t can be obtained by replacing t by (t – x/v) in equation (1).

$$\therefore \quad y(x,t) = A \sin \omega\left(t - \frac{x}{v}\right) = A \sin\left(\omega t - \frac{\omega}{v}x\right)$$

But $\quad \frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k \qquad (\because v = \nu\lambda)$

The quantity k = 2π/ λ is called *angular wave number or propagation constant.* Hence

$$y(x,t) = A \sin(\omega t - kx) \qquad \text{... (2)}$$

This equation represents a harmonic wave travelling along the positive direction of the X–axis. It can also be written in the following forms:

$$y(x,t) = A \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

or $$y(x,t) = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad \text{... (3)}$$

$$= A \sin \frac{2\pi}{T}\left(t - \frac{x}{\lambda}T\right) \qquad \text{But} \quad \frac{\lambda}{T} = v$$

$$\therefore \quad y(x,t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right)$$

Also $\quad y(x,t) = A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T}t - x\right) \quad$ or $\quad y(x,t) = A \sin \frac{2\pi}{\lambda}(vt - x) \qquad \text{... (4)}$

Equations (2), (3), (4) and (5) are the various forms of plane progressive wave. If the initial phase of the particle at O is ϕ_0, then the equation of wave motion will be

$$\overbrace{y(x,t)}^{\text{Displacement}} = \overbrace{A}^{\text{Amplitude}} \sin\overbrace{(\underbrace{\omega}_{\text{angular frequency}}\,\underbrace{t}_{\text{time}} - \underbrace{k}_{\text{Angular wave number}}\,\underbrace{x}_{\text{Position}} + \underbrace{\phi_0}_{\text{Initial phase}})}^{\text{Phase}}$$

A harmonic wave travelling along negative direction of X–axis can be written as

$$y(x,t) = A \sin(\omega t + kx + \phi_0)$$

Phase and Phase Difference

Phase of a wave

The phase of a harmonic wave is a quantity that gives complete information of the wave at any time and at any position. It is equal to the argument of the sine or cosine function representing the wave. Suppose a harmonic wave is given by

$$y(x,t) = A \sin(\omega t - kx + \phi_0) \qquad \text{... (1)}$$

Then the phase of the wave at position x and time t is given by $\phi = \omega t - kx + \phi_0 \qquad \text{... (2)}$

Clearly, the phase of a wave is periodic both in time and space.

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Phase Change with time

Taking x as constant, if we differentiate equation (2) w.r.t., time t, we get $\frac{\Delta\phi}{\Delta t} = \omega$

Thus the phase change at a given position (x = constant) in time Δt is given by $\Delta\phi = \omega\Delta t = \frac{2\pi}{T}\Delta t$

Hence *we can define the time period of a wave as the time in which the phase of the medium changes by 2π.*

Phase change with position

Taking t as constant, if we differentiate equation (2) w.r.t. position x, we get $\frac{\Delta\phi}{\Delta x} = -k$

Thus the phase difference, at any instant of time t, between two particles separated by distance Δx is given by

$$\Delta\phi = -k\Delta x = -\frac{2\pi}{\lambda}\Delta x$$

Hence *we can define the wavelength of a wave as the distance between two points (or particles) which have a phase difference of 2π at any given instant.*

Particle Velocity and Acceleration

Particle Velocity

The particle velocity V is different from the wave velocity v. It is *the velocity with which the particles of the medium vibrate about their mean positions.* The displacement relation for a harmonic wave travelling along positive X–direction is

$$y(x, t) = A \sin(\omega t - kx) \qquad \ldots (1)$$

Differentiating (1) w.r.t. time t, and taking x constant, we get the particle velocity

$$V = \frac{dy}{dt} = \omega A \cos(\omega t - kx) \qquad \ldots (2)$$

or $$V = \omega A \sin[(\omega t - kx) + \pi/2] \qquad \ldots (3)$$

It may be noted that

(i) While the wave velocity ($v = v\lambda$) remains constant, the particle velocity changes simple harmonically with time.

(ii) The particle velocity is ahead of displacement in phase by $\pi/2$ radian.

(iii) The maximum particle velocity or the *velocity amplitude* is

$$V_0 = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{T} \text{ times the displacement amplitude } A.$$

(iv) If we differentiate equation (1) w.r.t. position x, we get

$$\frac{dy}{dx} = -kA \cos(\omega t - kx) \qquad \ldots (4)$$

From equation (2) and (4), we get

$$\frac{V}{dy/dx} = \frac{\omega A \cos(\omega t - kx)}{-kA \cos(\omega t - kx)} = -\frac{\omega}{k} = -\frac{2\pi v}{2\pi/\lambda} = -v \text{ or } V = -v\frac{dy}{dx}$$

$\therefore$ Particle velocity at a point = – Wave velocity × slope of displacement curve at that point.

(b) Phase acceleration

If we differentiate equation (2) with respect to time t, we get the particle acceleration

$$a = \frac{dV}{dt} = -\omega^2 A \sin(\omega t - kx) = -\omega^2 y \quad \text{or} \quad a = \omega^2 A \sin[(\omega t - kx) + \pi]$$

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It may be noted that

(i) The maximum value of particle acceleration or the acceleration amplitude is

$$a_0 = \omega^2 \ A = \left(\frac{2\pi}{T}\right)^2 \ A \ = \left(\frac{2\pi}{T}\right)^2 \textit{times the displacement amplitude.}$$

(ii) The particle acceleration is ahead of the particle displacement in phase by π radian.

Subjective Assignment – V

Q.1 The displacement y of a particle in a medium can be expressed as $y = 10^{-6} \sin(100t + 20x + \pi/4)$ where t is in second and x in metre. What is the speed of the wave?

Q.2 A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3 ms^{-1} and 90 ms^{-2} respectively. Velocity of the wave is 20 ms^{-1}. Find the waveform.

Q.3 A wave traveling along a string is described by $y(x, t) = 0.005 \sin(80.0x - 3.0 t)$, in which the numerical constants are in SI units (0.005m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s.

Q.4 The equation of a plane progressive wave is $y = 10 \sin 2\pi (t - 0.005 x)$ where y and x are in cm and t in seconds. Calculate the amplitude, frequency, wavelength and velocity of the wave.

Q.5 A wave traveling along a string is described by equation $y(x, t) = 0.05 \sin(40x - 5t)$ in which the numerical constants are in SI units (0.05 m, 40 rad m^{-1} and 5 rad s^{-1}). Calculate the displacement at distance 35 cm and time 10 sec.

Q.6 A wave traveling along a string is given by $y(x, t) = 0.005 \sin(80x - 3t)$ where the numerical values are in SI units. Symbols have their usual meanings. Calculate:

(a) frequency of the wave (b) velocity of the wave (c) amplitude of particle velocity

Q.7 The speed of wave in a stretched string is 20 ms^{-1} and its frequency is 50 Hz. Calculate the phase difference in radian between two points situated at a distance of 10 cm on the string.

Q.8 Write the equation of a progressive wave propagating along the positive x – direction, whose amplitude is 5 cm, frequency 250 Hz and velocity 500 ms^{-1}.

Q.9 For the plane wave $y = 2.5 \times 10^{-0.02x} \cos(800t - 0.82x + \pi/2)$, write down

(i) the general expression for phase ϕ (ii) the phase at x = 0, t = 0

(iii) the phase difference between the points separated by 20 cm along x–axis

(iv) the change in phase at a given place 0.6 milli second and (v) the amplitude at x = 100 m

Take units of y, t, x as 10^{-5} cm, s and m respectively.

Q.10 A simple harmonic wave train of amplitude 1cm and frequency 100 vibrations is traveling in positive
x–direction with velocity 15 ms^{-1}. Calculate the displacement y, the particle velocity and particle acceleration at x = 180 cm from the origin at t = 5 s.

Q.11 A certain string has a linear mass density of 0.25 kg m^{-1} and is stretched with a tension of 25 N. One end is given a sinusoidal motion with frequency 5 Hz and amplitude 0.01 m. At time t = 0, the other end has zero displacement and is moving in the positive y–direction.

(i) Find the wave speed, amplitude, angular frequency, period, wavelength and wave number

(ii) Write a wave function representing the wave

(iii) Find the position of the point at x = 0.25m at time t = 0.1 s.

Q.12 The equation of transverse wave traveling in a rope is given by $y = 10 \sin \pi (0.01x - 2.00t)$ where y and x are in cm and t in seconds. Find the amplitude, frequency, velocity and wavelength of the wave.

Q.13 For a traveling harmonic wave, $y = 2.0 \cos(10t - 0.0080x + 0.18)$ where x and y are in cm and t is in seconds. What is the phase difference between two points separated by (i) a distance of 0.5 m and (ii) a time gap of 0.5s?

Q.14 Find the displacement of an air particle 3.5 m from the origin of disturbance at t = 0.05s, when a wave of amplitude 0.2 mm and frequency 500 Hz travels along it with a velocity 350 ms^{-1}.

Q.15 A simple harmonic wave–train is traveling in a gas in the positive direction of the X–axis. Its amplitude is 2 cm, velocity 45 ms^{-1} and frequency 75 s^{-1}. Write down the equation of the wave. Find the displacement of the particle of the medium at a distance of 135 cm from the origin in the direction of the wave at the instant t = 3s.

Q.16 The phase difference between the vibrations of two medium particles due to the transmission of a wave is $2\pi/3$. The distance between the particles is 15 cm. Determine the wavelength of the wave.

Q.17 The distance between two points on a stretched string is 20 cm. The frequency of the progressive wave is 400 Hz and velocity 100 ms^{-1}. Find the phase difference between these two points.

Q.18 A sound–source of frequency 500 Hz is producing longitudinal waves in a spring. The distance between two consecutive rarefractions is 24 cm. If the amplitude of vibration of a particle of the spring is 3.0 cm and the wave is traveling in the negative x–direction, then write the equation for the wave. Assume that the source is at x = 0 and at this point the displacement is zero at the time t = 0.

Answers

1. 20 ms^{-1} 2. $0.1 \sin(30t + 1.5x)$

3. (a) 0.005 m, (b) 7.85 cm, (c) 2.09 s, 0.48 Hz , zero 4. 10 cm, 1 Hz, 200 cm, 200 cm s^{-1}

5. $-0.055 \sin 36$ 6. (a) 0.48 Hz, (b) 7.5 cms^{-1}, (c) 0.015 ms^{-1}

7. $\pi/2$ rad 8. $0.05 \sin \pi(500t - x)$ metre

9. (i) $800t - 0.82x + \pi/2$, (ii) $\pi/2$ rad, (iii) -0.164 rad, (iv) 0.48 rad, (v) 0.025×10^{-5} cm

10. 0, 200π cm s^{-1}, 0

11. (i) 10 ms^{-1}, 31.4 rad s^{-1}, 0.2s, 2m, 3.14m^{-1}, (ii) $0.01 \sin(31.4t - 3.14x)$, (iii) 0.00707 m

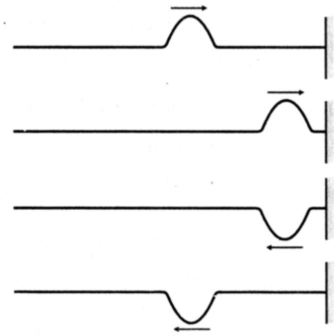
12. 10 cm, 1 Hz, 200 cm s^{-1}, 200 cm 13. (i) – 0.4 rad (ii) 5 rad 14. 0

15. $y = 2 \sin 2\pi \left(75t - \dfrac{x}{60}\right)$, –2cm 16. 45 cm 17. 1.6 π or 288^{o}

18. $y = 3.0 \sin 2\pi(25t + x/24)$

Boundary Effects

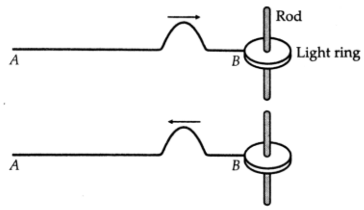
Reflection of wave from a rigid boundary



As shown in figure, consider a wave pulse travelling along a string (rarer medium) attached to a rigid support, such as a wall (denser medium). As the pulse reaches the wall, it exerts an upward force on the wall. By Newton's third law, the wall exerts an equal downward force on the string. This produces a reflected pulse in the downward direction, which travels in the reverse direction. Thus a crest is reflected as a trough.

Hence *when a travelling wave is reflected from a rigid boundary, it is reflected back with a phase reversal or phase difference of π radians.*

Reflection of a wave from an open boundary



As shown in figure, consider a wave pulse travelling along a string attached to a light ring, which slides without friction up and down a vertical rod. As the crest produced in string at A reaches the end B, it meets little or no opposition there. The ring rises above its equilibrium position. As the ring moves up, it stretches the string and produces a reflected crest which travels back towards A. There is no phase reversal and crest is reflected as a crest.

Hence *when a travelling wave is reflected from a free or open boundary, it suffers no phase change.*

Suppose an incident wave is represented by $y_i (x, t) = A \sin (\omega t - kx)$

For *reflection at a rigid boundary*, the reflected wave can be represented as

$$y_r (x, t) = A \sin (\omega t + kx + \pi) = - A \sin (\omega t + kx)$$

For reflection at an open boundary, the reflected wave can be represented as $y_r (x, t) = A \sin (\omega t + kx)$

NOTE

- Δ The incident and reflected waves obey the usual laws of reflection. The frequency, wavelength and velocity of the reflected wave are same as those of incident wave.
- Δ The wave transmitted into the second medium always goes without any change in phase.
- Δ The incident and refracted rays obey the Snell's law of refraction.
- Δ The wave velocity and wavelength of the refracted wave are different from those of the incident wave but their frequencies are equal. Hence $v = \frac{v_i}{\lambda_i} = \frac{v_r}{\lambda_r}$

Here the subscripts i and r stand for the incident and the refracted waves respectively.

Principle of superposition of waves

When one wave reaches a particle of the medium, the particle suffers one displacement. When two waves simultaneously cross this particle, it suffers two displacements, one due to each wave. *The resultant displacement of the particle is equal to the algebraic sum of the individual displacements given to it by the two waves. This is the principle of superposition of waves.*

The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves.

$$y = y_1 + y_2 + y_3 + \ldots\ldots + y_n = f_1 (vt - x) + f_2 (vt - x) + \ldots\ldots + fn (vt - x) = \sum_{i=1}^{n} f_i (vt - x)$$

The superposition of two waves may lead to following three different effects:

(i) When two waves of the same frequency moving with the same speed in the same direction in a medium superpose on each other, they give rise to effect called ***interference of waves.***

(ii) When two waves of same frequency moving with the same speed in the opposite directions in a medium superpose on each other, they produce ***stationary waves.***

(iii) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each other, they produce ***beats.***

NOTE

- ♦ The principle of superposition holds not only for the mechanical waves but also for electromagnetic waves.
- ♦ In case of mechanical waves, the superposition principle does not hold if the amplitude of disturbance is so large that the ordinary linear laws of mechanical action no longer hold good. For example, the superposition principle fails in case of shock waves generated by a violent explosion.

Stationary Waves

When two identical waves of same amplitude and frequency travelling in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called stationary or standing wave.

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The resultant wave keeps on repeating itself in the same fixed position. Some particles of the medium remain permanently at rest i.e., they have zero displacement. Their positions are called ***nodes.*** Some other particles always suffer maximum displacement. Their positions are called ***antinodes.*** The positions of nodes and antinodes do not change with time. In such waves, there is no transfer of energy along the medium in either direction.

Necessary condition for the formation of stationary waves

In actual practice a stationary wave is produced when a progressive wave and its reflected wave are superposed. *Hence a stationary wave can be produced only in a finite medium which has its boundaries.*

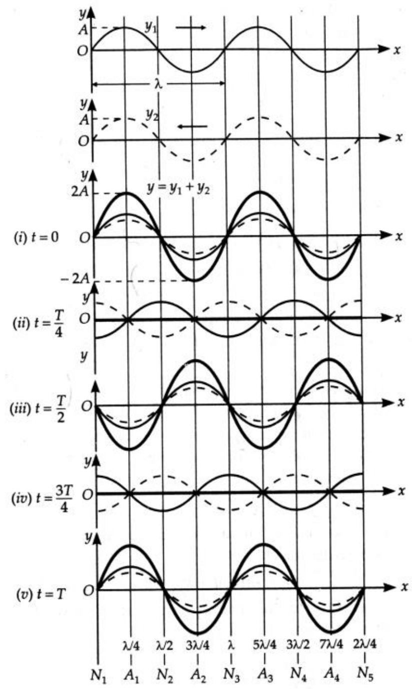
Two types of stationary waves

(i) ***Transverse stationary waves:*** *When two identical traverse waves travelling in opposite directions overlap, a transverse stationary wave is formed.* For example, transverse stationary waves are formed in a sonometer and Melde's experiment.

(ii) ***Longitudinal stationary waves:*** *When two identical longitudinal waves travelling in opposite directions overlap, a longitudinal stationary wave is formed.* For example, longitudinal stationary waves are formed in a resonance apparatus, organ pipes and Kundt's tube.

Graphical Treatment of Stationary Waves

The full line curve represents a harmonic wave of time period T and wavelength λ traveling from left to right while the dashed curve represents an identical wave travelling from right to left. The resultant wave is obtained by taking the algebraic sum of the displacements of the two waves at every point and is shown by the thick line curve.



(i) **At t = 0,** the two waves are in same phase i.e., the crests and troughs of the two waves coincide respectively with each other. The amplitude of the resultant wave is twice of that due to each individual wave. All particles are at their positions of maximum displacement (figure)

(ii) **At t = T/4,** each wave has advanced through a distance of $\lambda/4$ from the opposite direction. The two waves are in opposite phases. The resultant wave is the central straight line. All the particles of the medium are now passing through their mean positions (figure).

(iii) **At t = T/2,** each wave has advanced through a distance of $\lambda/2$ from the opposite direction. The two waves are again in same phase. The resultant wave is reciprocal of that at t = 0. All the particles are at their positions of maximum displacement but in the directions opposite to those at t = 0 (figure).

(iv) **At t = 3T/4,** each wave has advanced through a distance of $3\lambda/4$ from the opposite direction. The two waves are again in opposite phases. All the particles are again passing through their mean positions, but their directions of motion are opposite to those at t = T/4 (figure).

(v) **At t = T,** each wave has advanced through a distance λ from opposite direction. The two waves are again in same phase. The resultant wave is similar to that at t = 0. This completes one cycle (figure). The whole cycle continues to repeat again and again. The various segments of the string, on which stationary waves are formed, keep on vibrating up and down. The positions N_1, N_2, N_3, ... where the amplitude of oscillation is zero are called ***nodes.*** The positions A_1, A_2, A_3, ... where the amplitude of oscillation is maximum are called ***antinodes.***

Analytical Treatment of Stationary Waves

Consider two sinusoidal waves of equal amplitude and frequency travelling along a long string in opposite directions. Two wave travellign along position X–direction can be represented as $y_1 = A \sin(\omega t - kx)$

The wave travelling along negative X–direction can be represented as $y_2 = A \sin(\omega t + kx)$

According to the principle of superposition, the resultant wave is given by $y = y_1 + y_2$

$= A \sin(\omega t - kx) + A \sin(\omega t + kx)$

$= 2A \sin \omega t \cos kx \qquad [\because \sin(A+B) + \sin(A-B) = 2 \sin A \cos B]$

or $\quad y = (2A \cos kx) \sin \omega t$

This equation represents a stationary wave. It cannot represent a progressive wave because the argument of any of its trigonometric functions does not contain the combination ($\omega t \pm kx$). The stationary wave has the same angular frequency ω but has amplitude

$A' = 2A \cos kx$

Obviously in case of a stationary wave, the amplitude of oscillation is not same for all the particles. It varies harmonically with the location x of the particle.

Changes with position x

The amplitude will be zero at points, where $\cos kx = 0$

or $\quad kx = \left(\frac{2n+1}{2}\right)\pi, \quad$ where $n = 0, 1, 2, 3, \ldots\ldots$ or

$\frac{2\pi x}{\lambda} = \left(\frac{2n+1}{2}\right)\pi \qquad \left[\because k = \frac{2\pi}{\lambda}\right]$

or $\quad x = (2n+1)\frac{\lambda}{4} \qquad$ or $\quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots$

These positions of zero amplitude are called nodes. Clearly, separation between two consecutive nodes is $\lambda/2$. The amplitude will have a maximum value of 2A at points, where $\cos kx = \pm 1$

or $\quad kx = n\pi, \qquad$ where $n = 0, 1, 2, 3, \ldots\ldots$

or $\quad \frac{2\pi}{\lambda}x = n\lambda \qquad$ or $\quad x = n\frac{\lambda}{2} \qquad$ or $\quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots$

These positions of maximum amplitude are called ***antinodes.*** Clearly, the antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

Changes with time t

And the instant $t = 0, T/2, 3T/2, \ldots\ldots$, we have

$$\sin \omega t = \sin \frac{2\pi}{T} t = 0$$

Thus at these instants the displacement y becomes zero at all the points. That is, *all the particles of the medium pass through their mean positions simultaneously twice in each cycle.*

At the instants $\quad t = T/4, 3T/4, 5T/4, \ldots\ldots\ldots$,

we have $\quad \sin \omega t = \sin \frac{2\pi}{T} t = \pm 1.$

Thus at these instants the displacement y is maximum at all the points and becomes alternatively positive and negative. That is, *all the particles of the medium pass through their positions of maximum displacements twice in each cycle.*

Characteristics of Stationary Waves

(i) In a stationary wave, the disturbance does not advance forward. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite conditions after every half the time period.

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(ii) All particles of the medium, except those at nodes, execute simple harmonic motions with the same time period about their mean positions.

(iii) During the formation of a stationary wave, the medium is broken into loops or segments between equally spaced points called *nodes* which remain permanently at rest and midway between them are points called *antinodes* where the displacement amplitude is maximum.

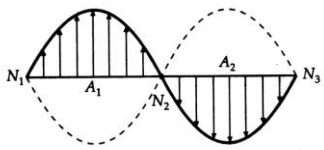
(iv) The distance between two successive nodes or antinodes is $\lambda/2$.

(v) The amplitudes of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

(vi) The maximum velocity is different at different points. Its value is zero at the nodes and progressively increases towards the antinode. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.

(vii) All the particles in a particular segment between two nodes vibrate in the same phase but the particles in two neighbouring segments vibrate in opposite phases, as shown in figure.

(viii) Twice in each cycle, the energy becomes alternately wholly potential and wholly kinetic. It is wholly kinetic when the particles are at their positions of maximum displacements and wholly kinetic when the particles pass through their mean positions.



(ix) There is no transference of energy across any section of the medium because no energy can flow past a nodal point which remains permanently at rest.

(x) A stationary wave has the same wavelength and time period as the two component waves.

Subjective Assignment – VI

Q.1 The constituent waves of a stationary wave have amplitude, frequency and velocity as 8 cm, 30 Hz and 180 cm s^{-1} respectively. Write down the equation of the stationary wave.

Q.2 Stationary waves are set up by the superposition of two waves given by $y_1 = 0.05 \sin (5\pi t - x)$ and $y_2 = 0.05 \sin (5\pi t + x)$ where x and y are in metres and t in seconds. Find the displacement of a particle situated at a distance x = 1m.

Q.3 The distance between two consecutive nodes in a stationary wave is 25 cm. If the speed of the wave is 300 ms^{-1}, calculate the frequency.

Q.4 The equation of a longitudinal stationary wave produced in a closed organ pipe is $y = 6\sin\frac{2\pi x}{6}\cos 160\pi t$ where x, y are in cm and t in seconds. Find (i) the frequency, amplitude and wavelength of the original progressive wave (ii) separation between two successive nodes and (iii) equation of the original progressive waves.

Q.5 (i) Write the equation of a wave identical to the wave represented by the equation: $y = 5 \sin \pi (4.0t - 0.02 x)$ but moving in opposite direction. (ii) Write the equation of stationary wave produced by the composition of the above two waves and determine the distance between two nearest nodes. All the distances in the equation are in mm.

Answers

1. $16\cos\frac{\pi x}{3}\sin 60\pi t$ 2. 0.054 m 3. 600 Hz

4. (i) v = 80 Hz, a = 3cm, λ = 6cm, (ii) 3cm (iii) $y = 3 \sin\left(\frac{\pi}{3}x - 160\pi t\right)$

5. (i) $y = 5 \sin \pi (4.0\, t + 0.02\, x)$, (ii) $y = 10 \cos 0.02\pi x \sin 4.0\, \pi t$, 50 mm

Analytical Treatment of Stationary Waves in a String fixed at both the ends

Consider a uniform string of length L stretched by a tension T along the x–axis, with its ends rigidly fixed at the end x = 0 and x = L. Suppose a transverse wave produced in the string travels along the string along positive
x–direction and gets reflected at the end x = L. The two waves can be represented as

$$y_1 = A \sin(\omega t - kx) \quad \text{and} \quad y_2 = -A \sin(\omega t + kx)$$

The negative sign before A is due to phase reversal of the reflected wave at the fixed end. By the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2 = -A[\sin(\omega t + kx) - \sin(\omega t - kx)] = -2A\cos\omega t \sin kx$$

$$[\sin(A+B) - \sin(A-B) = 2\cos A \sin B] \quad \text{or} \quad y = -2A \sin kx \cos \omega t \qquad \text{... (1)}$$

If stationary waves are formed, then the ends x = 0 and x = L must be nodes because they are kept fixed. So, we have the boundary conditions:

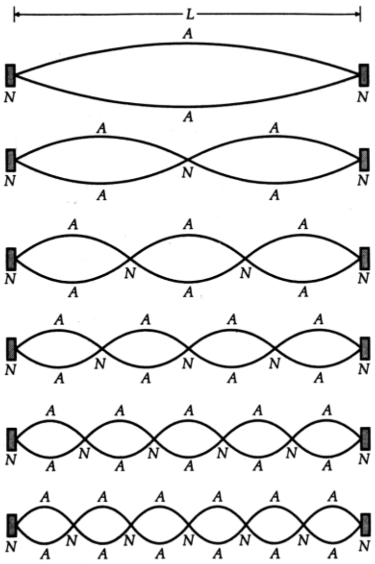
$$y = 0 \quad \text{at } x = 0 \quad \text{for all } t \quad \text{and} \quad y = 0 \quad \text{at } x = L \quad \text{for all } t$$

The first boundary condition (y = 0, x = 0) is satisfied automatically by equation (1). The second boundary condition (y = 0, x = L) will be satisfied if $y = -2 \sin kL \cos \omega t = 0$

This will be true for all values of t only if

$$\sin kL = 0 \quad \text{or} \quad kL = n\pi, \text{ where } n = 1, 2, 3, \ldots\ldots$$

or $$\frac{2\pi L}{\lambda} = n\pi$$



For each value of n, there is a corresponding value of λ, so can write

$$\frac{2\pi L}{\lambda_n} = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

The speed of transverse wave on a string of linear mass density m is given by $v = \sqrt{\frac{T}{m}}$

So the frequency of vibration of the string is $\nu_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{m}}$

For n = 1, 1st mode of vibration: If the string is plucked in the middle and released, it vibrates in one segment with nodes at its ends and an antinode in the middle. $\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu$ (say)

This is the lowest frequency with which the string can vibrate and is called *fundamental frequency or first harmonic*. For n = 2, 2nd mode of vibration: If the string is pressed in the middle and plucked at one–fourth length, then the string vibrates in two segments. $\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu$ *(first overtone or second harmonic)*

For n = 3, 3rd mode of vibration: If string is pressed at one–third of its length from one end and plucked at one–sixth length, it will vibrate in three segments. Then $\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu$ *(second overtone or third harmonic)*

Thus the various frequencies are in the ratio 1 : 2 : 3 : ...... and hence from a harmonic series. These frequencies are called ***harmonics*** with the fundamental itself as the *first harmonic*. The higher harmonic are called ***overtones.*** Thus second harmonic is first overtone, third harmonic is second overtone and so on. These are shown in figure.

VPLUS

Nodes: *These are the positions of zero amplitude.* In the n^{th} mode of vibration, there are (n + 1) nodes, which are located from one end at distances

$$x = 0, \frac{L}{n}, \frac{2L}{n}, ..., L$$

Antinodes: *These are positions of maximum amplitude.* In the n^{th} mode of vibration, there are n antinodes, which are located at distances

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, ..., \frac{(2n-1)L}{2n}$$

Laws of transverse vibration of a string

The fundamental frequency produced in a stretched string of length L under tension T and having mass per unit length m is given by

$$v = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

The above equation gives the following laws of vibrations of strings:

(i) ***Law of length***

The fundamental frequency of a vibrating string is inversely proportional to its length provided its tension and mass per unit length remain the same.

$$v \propto \frac{1}{L} \qquad \text{(T and m are constants)}$$

(ii) ***Law of tension***

The fundamental frequency of a vibrating string is proportional to the square root of its tension provided its length and the mass per unit length remain the same.

$$v \propto \sqrt{T} \qquad \text{(L and m are constants)}$$

(iii) ***Law of mass***

The fundamental frequency of a vibrating string is inversely proportional to the square root of its mass per unit length provided the length and tension remain the same.

$$v \propto \frac{1}{\sqrt{m}} \qquad \text{(L and T are constants)}$$

If ρ is the density of the string and D its diameter, then its mass per unit length will be

m = volume of unit length × density

$$= \pi\left(\frac{D^2}{4}\right)\rho$$

$$\therefore \quad v = \frac{1}{2L}\sqrt{\frac{T}{\pi D^2 \rho/4}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

Hence the law of mass may be expressed in the form of following two alternative laws:

(a) ***Law of diameter***

The fundamental frequency of a vibrating string is inversely proportional to its diameter provided its tension, length and density remain the same.

$$v \propto \frac{1}{D} \qquad \text{(T, L and ρ are constants)}$$

(b) ***Law of density***

The fundamental frequency of a vibrating string is inversely proportional to the square root of its density provided its tension, and diameter remain constant.

$$\nu \propto \frac{1}{\sqrt{\rho}}$$ (T, L and D are constants)

Subjective Assignment – VII

Q.1 A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg wt into between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance, when carrying an alternating current of frequency ν. Find the frequency of the alternating source.

Q.2 Calculate the fundamental frequency of a sonometer wire of length = 20 cm tension 25 N, cross – sectional area = 10^{-2} cm^2 and density of the material of wire 10^4 kg m^{-3}.

Q.3 The length of a sonometer wire is 0.75 m and density 9×10^3 kg m^{-3}. It can bear a stress of 8.1×10^8 Nm^{-2} without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire?

Q.4 A stretched wire emits a fundamental note of 256 Hz. Keeping the stretching force constant and reducing the length of wire by 10 cm, the frequency becomes 320 Hz. Calculate the original length of the wire.

Q.5 Find the fundamental note emitted by a string of length $10\sqrt{10}$ cm under tension of 31.4 kg. Radius of string is 0.55 mm and density = 9.8g cm^{-3}.

Q.6 A rope 5 m long has a total mass of 245g. It is stretched with a constant tension of 1 kg wt. If it is fixed at one end and shaken by hand at the other end, what frequency of shaking will make it break up into three vibrating segments? Take g = 980 cm s^{-2}.

Q.7 In an experiment it was found that the string vibrated in three loops when 8 g were placed on the scale pan. What mass must be placed on the pan to make the string vibrate in six loops? Neglect the mass of the string and the scale pan.

Q.8 The length of a wire between the two ends of a sonometer is 105 cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15?

Q.9 The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. How will the frequency be affected if its length is increased by 10%?

Q.10 A stone hangs in air from one end of a wire which is stretched over a sonometer. The wire is in unison with a certain tuning fork when the bridges of the sonometer are 45 cm apart. Now the stone hangs immersed in water at 4^oC and the distance between the bridges has to be altered by 9 cm to re–establish unison of the wire with the same fork. Calculate the density of the stone.

Q.11 A wire having a linear mass density of 5.0×10^{-3} kg m^{-1} is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.

Q.12 A sonometer wire is under a tension of 40 N and the length between the bridges is 50 cm. A meter long wire of the sonometer has a mass of 1.0g. Determine its fundamental frequency.

Q.13 A cord 80 cm long is stretched by a load of 8.0 kg f. The mass per unit length of the cord is 4.0×10^{-5} kg m^{-1}. Find (i) speed of the transverse wave in the cord and (ii) frequency of the fundamental and that of the second overtone.

Q.14 The length of a stretched wire is 1 m and its fundamental frequency is 300 Hz. What is the speed of the transverse wave in the wire?

Q.15 The mass of a 1m long steel wire is 20g. The wire is stretched under a tension of 800 N. What are the frequencies of its fundamental mode of vibration and the next three higher modes?

Q.16 If the tension in the string is increased by 5 kg wt, the frequency of the fundamental tone increases in the ratio 2 : 3. What was the initial tension in the string?

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Q.17 A sonometer wire has a length of 114 cm between its two fixed ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in ratio 1 : 3 : 4?

Q.18 Two wires of the same material are stretched with the same force. Their diameters are 1.2 mm and 1.6 mm, while their lengths are 90 cm and 60 cm respectively. If the frequency of vibrations of first is 256 Hz, find that of the other.

Q.19 A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz?

Q.20 The ratio of frequencies of two wires having same length and same tension and made of the same material is 2 : 3. If the diameter of one wire be 0.09 cm, then determine the diameter of the other.

Q.21 A 50 cm long wire is in unison with a tuning fork of frequency 256, when stretched by a load of density 9g cm^{-3} hanging vertically. The load is then immersed in water. By how much the length of the wire be reduced to bring it again in unison with the same tuning fork?

Q.22 A string vibrates with a frequency of 200 Hz. Its length is doubled and its tension is altered until it begins to vibrate with a frequency of 300 Hz. What is the ratio of new tension to the original tension?

Q.23 In Melde's experiment, a string vibrates in 3 loops when 8 grams were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops?

Answer

1.	50 Hz	2.	125 Hz	3.	200 Hz	4.	50 cm
5.	100 Hz	6.	4.24 Hz	7.	2 g		
8.	75 cm and 100 cm from one end			9.	45.45 Hz	10.	2.778 g cm^{-3}
11.	2.14 m	12.	200 Hz	13.	(i) 1400 ms^{-1}, (ii) 875 Hz, 2625 Hz		
14.	600 ms^{-1}	15.	100, 200, 300, 400 Hz	16.	4 kg wt		
17.	at a distance of 72 cm, 96 cm from one end			18.	288 Hz	19.	60 cm
20.	0.06 cm	21.	2.867 cm	22.	9 : 1	23.	2.88 g

Analytical treatment of stationary waves in an open organ pipe

Consider a cylindrical pipe of length L lying along the x–axis, with its open ends at x = 0 and x = L. The sound wave travelling along the pipe may be represented as $y_1 = A \sin(\omega t - kx)$

The wave reflected from right open end may be represented as $y_2 = A \sin(\omega t + kx)$

There is no phase reversal on reflection from the open end because it is a free or loose boundary. So the sign of A in the reflected wave is same as that in the incident wave.

$$y = y_1 + y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx)] = 2A \sin \omega t \cos kx = (2A \cos kx) \sin \omega t$$

For all values of t, the resultant displacement is maximum (+ve or –ve) or antinodes are formed at the open ends i.e., at x = 0 and x = L. This condition is satisfied if

$$\cos kL = \pm 1 \quad \text{or} \quad kL = n\pi,$$

where n = 1, 2, 3, ... or $\frac{2\pi}{\lambda} L = n\pi$ or $\lambda_n = \frac{2L}{n}$

The frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}}$$

For n = 1, $\nu_1 = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu\,(\text{say})$

This is the smallest frequency of the stationary waves produced in the open pipe. It is called *fundamental frequency or first harmonic.*

For n = 2, $\nu_2 = \frac{2v}{2L} = 2\nu$ (first overtone or second harmonic)

For n = 3, $\nu_3 = \frac{3v}{2L} = 3v$ (second overtone or third harmonic)

and so on. The various modes of vibration of an open pipe are shown in figure.

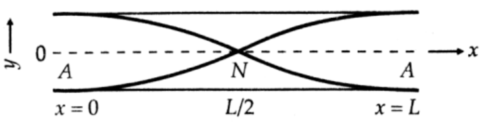
Normal modes of vibration of an open organ pipe

Both the ends of an open organ pipe are open. The waves are reflected from these ends with change of type. However, the particles continue to move in the same direction even after the reflection of the waves. Consequently, the particles have the maximum displacements at the open ends. Hence antinodes are formed at the open ends. The various nodes of vibration of an open organ pipe are shown in figure.

(i) **First mode of vibration:** In the simplest mode of vibration, there is node in the middle and two antinodes at the ends of the pipe.

Here length of the pipe,

$$L = 2.\frac{\lambda_1}{4} = \frac{\lambda_1}{2} \qquad \therefore \quad \lambda_1 = 2L$$



(*a*)

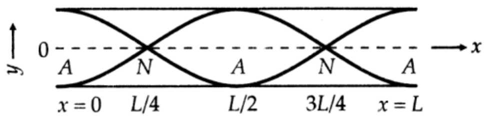
frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu\,(say)$$

This frequency is called *fundamental frequency or first harmonic.*

(ii) **Second mode of vibration:** Here antinodes at the open ends are separated by two nodes and one antinode.

$$L = 4.\frac{\lambda_2}{4} = \lambda_2$$



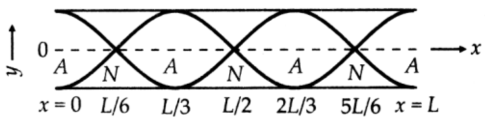
(*b*)

Frequency, $\nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}} = 2\nu$

This frequency is called *first overtone or second harmonic.*

(iii) **Third mode of vibration:** Here the antinodes at the open ends are separated by three nodes and two antinodes.

$$L = 6.\frac{\lambda_3}{4} \qquad \text{or} \qquad \lambda_3 = \frac{2L}{3}$$



(*c*)

$\therefore$ Frequency, $\nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$

This frequency is called *second overtone or third harmonic.*

Hence various frequencies of an open organ pipe are in the ratio 1 : 2 : 3 : 4 : .... These are called harmonics.

V PLUS U

Analytical Treatment of Stationary Waves in a Closed Organic Pipe

Consider a cylindrical pipe of length L lying along the x–axis, with its closed end at x = 0 and open end at x = L. The sound wave sent along the pipe may be represented as

$$y_1 = A \sin(\omega t + kx)$$

The wave reflected from the closed end may be represented as

$$y_2 = -A \sin(\omega t - kx)$$

The negative sign before A is due to reversal of phase at the closed end. By the principle of superposition, the resultant stationary wave is given by

$$y = y_1 + y_2 = A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$

$$= 2A \cos \omega t \sin kx = (2A \sin kx) \cos \omega t.$$

Clearly, t = 0 at x = 0 i.e., a node is formed at the closed. The resultant displacement at x = L will be maximum (+ve or –ve) because the open end is a free or loose boundary. This condition is satisfied if $\sin kL = \pm 1$

or $\quad kL = (2n-1)\dfrac{\pi}{2}$ $\quad$ where n = 1, 2, 3, ... $\quad$ or

$$\frac{2\pi}{\lambda}L = (2n-1)\frac{\pi}{2} \quad \text{or} \quad \lambda_n = \frac{4L}{2n-1}$$

The corresponding frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{(2n-1)v}{4L} = \frac{(2n-1)}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

For n = 1,

$$\nu_1 = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \ \text{(say)}$$

This is the smallest frequency of the stationary waves produced in the closed pipe. It is called *fundamental frequency or first harmonic.*

For n = 2, $\quad \nu_2 = \dfrac{3v}{4L} = 3\nu$ $\quad$ [First overtone or third harmonic]

For n = 3, $\quad \nu_3 = \dfrac{5v}{4L} = 5\nu$ $\quad$ [Second overtone or fifth harmonic] and so on.

Normal modes of a closed organ pipe

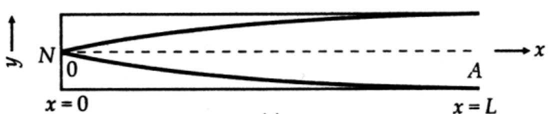
In a closed organ pipe, one end of the pipe is open and the other end is closed. As the wave is reflected from the closed end, the direction of motion of the particles changes. The displacement is zero at the closed end i.e., a node is formed at the closed end. The displacement of the particles is maximum at the open end, so an antinode is formed at the open end. The displacement of the particles is maximum at the open end, so an antinode is formed at the open end. The different modes of vibration of closed pipe are shown in figure.

(i) ***First mode of vibration***

In this simplest mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then

$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency,



(a)

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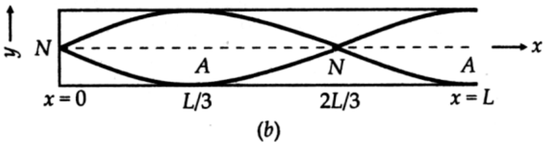
$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

This frequency is called *first harmonic or fundamental frequency.*

(ii) ***Second mode of vibration***

In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end.

$\therefore \quad L = \frac{3\lambda_2}{4} \quad$ or $\quad \lambda_2 = \frac{4L}{3}$



(b)

Frequency,

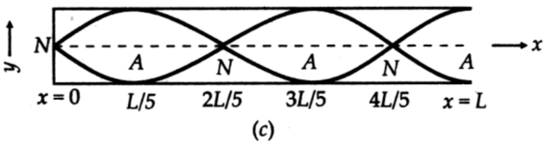
$$\nu_2 = \frac{v}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called *first overtone or third harmonic.*

(iii) ***Third mode of vibration***

In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end.

$\therefore \quad L = \frac{5\lambda_3}{4} \quad$ or $\quad \lambda_3 = \frac{4L}{5}$



(c)

Frequency,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

Hence different frequencies produced in a closed organ pipe are in the ratio 1 : 3 : 5 : 7 : ...... i.e., only odd harmonics are present in a closed organ pipe.

Assignment Subject – VIII

Q.1 What should be minimum length of an open organ pipe for producing a note of 110 Hz? The speed of sound is 330 ms^{-1}.

Q.2 The length of an organ pipe open at both ends is 0.5 m. Calculate the fundamental frequency of the pipe, if the velocity of sound in air be 350 ms^{-1}. If one end of the pipe is closed, then what will be the fundamental frequency?

Q.3 A pipe 30.0 cm long is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of sound in air as 330 ms^{-1}.

Q.4 Find the ratio of the length of a closed pipe to that of an open pipe in order that the second overtone of the former is in unison with the fourth overtone of the latter.

Q.5 An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of the open pipe?

Q.6 The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound in air = 330 ms^{-1}.

Q.7 A well with vertical sides and water at the bottom resonates at 7 Hz and at no other lower frequency. The air in the well has density 1.10 kg m^{-3} and bulk modulus of 1.33×10^5 Nm^{-2}. How deep is the well?

Q.8 A well with vertical sides and water at the bottom resonates at 7 Hz and at no other lower frequency. The air in the well has density 1.10 kg m^{-3} and bulk modulus of 1.33×10^5 Nm^{-2}. How deep is the well?

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Q.9 A resonance air column resonates with a tuning fork of 512 Hz at length 17.4 cm. Neglecting the end correction, deduce the speed of sound in air.

Q.10 A resonance tube is resonated with tuning fork of frequency 512 Hz. Two successive lengths of the resonated air–column are 16.0 cm and 51.0 cm. The experiment is performed at the room temperature of 40^oC. Calculate the speed of sound at 0^oC and the end correction.

Q.11 A brass rod 1 metre long is firmly clamped in the middle and one end is stroked by a resined cloth. What is pitch of the note you will hear? Young's modulus for brass=10^{12} dyn cm^{-2} and density=9g cm^{-3}

Q.12 An open organ pipe produces a note of frequency 512 Hz at 15^oC, calculate the length of the pipe. Velocity of sound at 0^oC is 335 ms^{-1}.

Q.13 Find the frequencies of the fundamental note and the first overtone in an open air column and a closed air column of length 34 cm. The velocity of sound at room temperature is 340 ms^{-1}.

Q.14 Prove that a pipe of length 2L open at both ends has same fundamental frequency as another pipe of length L closed at the other end. Also, state whether the total sound will be identical for two pipes.

Q.15 The fundamental frequency of a closed organ pipe is equal to the first overtone of an open organ pipe. If the length of the open pipe is 60 cm, what is the length of the closed pipe?

Q.16 The fundamental tone produced by an organ pipe has a frequency of 110 Hz. Some other frequencies in the notes produced by this pipe are 220, 440, 550, 660 Hz. Is this pipe open at both ends or open at one end and closed at the other? Calculate the effective length of the pipe. Speed of sound = 330 ms^{-1}.

Q.17 The fundamental frequency of an open organ pipe is 300 Hz. The frequency of the first overtone of another closed organ pipe is the same as the frequency of the first overtone of open pipe. What are the lengths of the pipes? The speed of sound = 330 ms^{-1}.

Q.18 Find the ratio of length of a closed organ pipe to that of open pipe in order that the second overtone of the former is in unison with fourth overtone of the latter.

Q.19 A tuning fork of frequency 341 Hz is vibrated just over a tube of length 1 m. Water is being poured gradually in the tube. What height of water column will be required for resonance? The speed of sound in air is 341 ms^{-1}.

Q.20 A resonance air column shows resonance with a tuning fork of frequency 256 Hz at column lengths 33.4 cm and 101.8 cm. Find (i) end–correction and (ii) the speed of sound in air.

Answers

1.	1.5 m	2.	350 Hz, 175 Hz	3.	II harmonic, no
4.	1 : 2	5.	200 Hz	6.	99 cm
7.	75 cm, 99 cm	8.	12.41 m	9.	356.35 ms^{-1}
10.	334 ms^{-1}, 1.5 cm	11.	1666.67 Hz	12.	0.336 m
13.	(i) 500 Hz, 1000 Hz, (ii) 250 Hz, 750 Hz	14.	No	15.	15 cm
16.	Pipe is open at both ends, 1.5 m	17.	55.0 cm, 41.25 cm	18.	3 : 2
19.	25 cm or 75 cm	20.	(i) 0.8 cm, (ii) 350.2 ms^{-1}		

Beats

When two sound waves of slightly different frequencies travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls (technically known as waxing and waning of sound) alternately with time. *These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequencies are called beats.* One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.

Beat frequency = Difference in frequencies of the two superposing waves $\nu_{beat} = \nu_1 = \nu_2$

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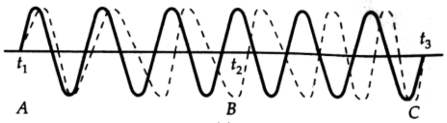
Essential condition for the formation of beats

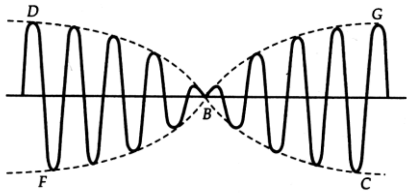
For beats to be audible, the difference in the frequency of the two sound waves should not exceed 10.

Formation of beats by graphical method

The full line curve is the displacement time curve of a wave of frequency v_1 and the dashed curve is for a wave of frequency v_2. Hence v_1 is slightly greater than v_2, so the first wave is slightly smaller than second.



(a)



(b)

At time t_1, the two waves meet in the same phase at a given point. They reinforce to produce maximum sound intensity. With the passage of time, the phase difference between the two waves increases and so the two curves gradually get more and more out of step.

At time t_2, the two waves are in exactly opposite phases. This happens when one wave gains half a vibration over the other. Now they produce minimum sound intensity. Now the phase difference goes on decreasing with time. At time t_3 one wave gains one full vibration on the other and the two waves are again in same phase and produce maximum intensity, and so on. The resultant wave, obtained by the algebraic sum of the displacements of the two waves, is shown by full line curve in figure. The dashed envelopes above and below it show how the amplitude of the resultant wave varies with time. The time interval from t_1 to t_3 is *one beat period*, because during this interval only one beat is formed. Moreover, during this interval, the first wave completes $(v + 1)$ oscillations while the second wave completes v oscillations. Thus beat frequency is equal to the difference in frequencies of the two superposing waves.

Analytical treatment of beats

Consider two harmonic waves of frequencies v_1 and v_2 (v_1 being slightly greater than v_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented as

$$y_1 = A \sin \omega_1 t = A \sin 2\pi v_1 t$$

$$y_2 = A \sin \omega_2 t = A \sin 2\pi v_2 t$$

By the principle of superposition, the resultant displacement at the given point will be

$$y = y_1 + y_2 = A \sin 2\pi v_1 t + A \sin 2\pi v_2 t$$

$$= 2A \cos 2\pi\left(\frac{v_1 - v_2}{2}\right) t.\ \sin 2\pi\left(\frac{v_1 - v_2}{2}\right) t$$

If we write $v_{mod} = \dfrac{v_1 - v_2}{2}$ and $v_{av} = \dfrac{v_1 + v_2}{2}$

$\therefore$ $y = 2A \cos 2\pi v_{mod}\, t \sin 2\pi v_{av}\, t$ or $y = R \sin (2\pi v_{av} t)$

where $R = 2A \cos (2\pi v_{mod} t)$ is the amplitude of the resultant wave. As v_1 is slightly greater than v_2, so $\underline{v}_{mod} << v_{av}$ i.e., R varies very slowly with time. Hence the above equation represents a wave of periodic rapid oscillation of average frequency v_{av} 'modulated' by a slowly varying oscillation of frequency v_{mod}.

The amplitude R of the resultant wave will be maximum, when $\cos 2\pi v_{mod} t = \pm 1$

or $2\pi v_{mod} t = n\pi$ or $\pi(v_1 - v_2)t = n\pi$

or $t = \dfrac{n}{v_1 - v_2} = 0, \dfrac{1}{v_1 - v_2}, \dfrac{2}{(v_1 - v_2)}, ...$

$\therefore$ The time interval between successive maxima $= \dfrac{1}{v_1 - v_2}$

Similarly, the amplitude R will be minimum, when $\cos 2\pi \nu_{mod} t = 0$

or $2\pi\nu_{mod} t = (2n + 1)\pi/2$ or $\pi(\nu_1 - \nu_2)t = (2n + 1)\pi/2$

or $t = \frac{(2n+1)}{2(\nu_1 - \nu_2)} = \frac{1}{\nu_1 - \nu_2}, \frac{3}{2(\nu_1 - \nu_2)}, \frac{5}{2(\nu_1 - \nu_2)}, ...$

∴ The time interval between successive minima $= \frac{1}{\nu_1 - \nu_2}$

Clearly, both maxima and minima of intensity occur alternately. Technically, one maximum of intensity followed by a minimum is called *beat.* Hence time interval between two successive beats or the *beat period* is

$$t_{beat} = \frac{1}{\nu_1 - \nu_2}$$

The *number of beats produced per second is called* ***beat frequency.***

$$\nu_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \nu_{beat} = \nu_1 - \nu_2$$

∴ Beat frequency = Difference between the frequencies of two superposing waves.

Practical Applications of Beats

(i) Determination of unknown frequency

Suppose ν_1 is the known frequency of tuning fork A and ν_2 is the unknown frequency of turning fork B. When the two tuning forks are sounded together, suppose they produce b beats per second. Then

$$\nu_2 = \nu_1 + b \quad \text{or} \quad \nu_1 - b$$

The exact frequency may be determined by any of the following two methods:

(a) Loading Method: Attach a little wax to the prong of the tuning fork B. Again, find the number of beats produced per second. If the frequency of B is greater than that of A i.e., (ν_1 + b), then the attaching of a little wax lowers its frequency and reduces the difference in frequencies of A and B. This would decrease the beat frequency.

Hence, if the beat frequency decrease on loading the prong of the tuning fork of the unknown frequency, then the unknown frequency is greater than the known frequency. That is,

$$\nu_2 = \nu_1 + b$$

On the other hand, if the frequency of B is less than that of A i.e., (ν_1 – b), then the attaching of a little wax further lowers its frequency and increases the difference in frequencies of A and B. This would increase the beat frequency.

Hence, *if the beat frequency increase on loading the prong of the tuning fork of the unknown frequency, then the unknown frequency is less than the known frequency*. That is, $\nu_2 = \nu_1 - b$

(b) Filing method: If a prong of the tuning fork B is filed, its frequency increases. Again, note the number of beats produced per second. If on filing the prong of B, the beat frequency decreases, then $\nu_2 = \nu_1 - b$

If on filing the prong of B, the beat frequency increases, then $\nu_2 = \nu_1 + b$

(ii) For tuning musical instruments: Musicians use the beat phenomenon in tuning their musical instruments. If an instrument is sounded against a standard frequency and tuned until the beats disappear, then the instrument is in tune with the standard frequency.

(iii) For producing colourful effect in music: Sometimes, a rapid succession of beats is knowingly introduced in music. This produces an effect similar to that of human voice and is appreciated by the audience.

(iv) **For detection of marsh gas in mines:** Here we use two small organ pipes tuned to the same frequency. One pipe contains pure dry air and the other ordinary mine air.

If any marsh gas or methane appears in the mine, the density of the mine air in the second pipe decreases which slightly changes its frequency of vibration. When sounded with the first organ pipe, it gives rise to beats. The miners are thus warned well in advanced of the explosive marsh gas.

Subjective Assignment – IX

Q.1 The points of the prongs of a tuning fork B originally in unison with a tuning fork A of frequency 384 are filed and the fork produces 3 beats per second, when sounded together with A. What is the pitch of B after filling?

Q.2 A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. What is the frequency of the unknown fork?

Q.3 A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.

Q.4 A fork of unknown frequency when sounded with one of frequency 288 Hz gives 4 beats per second and when loaded with a piece of wax again gives 4 beats per second. How do you account for this and what was the unknown frequency?

Q.5 Two tuning forks A and B produce 4 beats/ second. On loading B with wax, 6 beats/second were heard. If the quantity of wax is reduced, the number of beats per second again becomes 4. Find the frequency of B if the frequency of A is 256 Hz.

Q.6 A tuning fork produces 4 beats/s when sounded with a tuning fork of frequency 512 Hz. The same tuning fork when sounded with another tuning fork of frequency 514 Hz produces 6 beats/s. Find the frequency of the tuning fork.

Q.7 A tuning fork of known frequency of 256 Hz makes 5 beats s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. What was the frequency of the piano string before increasing the tension?

Q.8 When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

Q.9 A set of 24 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats per second with the preceding one and the last sounds the octave of the first, find the frequencies of the first and the last forks.

Q.10 In an experiment, it was found that a tuning fork and a sonometer wire gave 5 beats per second both when the length of the wire was 1m and 1.05 m. Calculate the frequency of the fork.

Q.11 A tuning fork of frequency 200 Hz is in unison with a sonometer wire. How many beats per second will be heard if the tension of the wire were increased by 2%?

Q.12 The two parts of sonometer wire divided by a movable knife differ by 2 mm and produce one beat per second when sounded together. Find their frequencies if the whole length of the wire is one metre.

Q.13 Two similar sonometer wires of the same material produce 2 beats per second. The length of one is 50 cm and that of the other is 50.1 cm. Calculate the frequencies of the two wires.

Q.14 A tuning fork of unknown frequency vibrates in unison with a wire of certain length stretched under a tension of 5 kg f. It produces 6 beats per second with the same wire, when tension is changed to 4.5 kg f. Find the frequency of tuning fork.

Q.15 Two air columns (of resonance tubes) 100 cm and 101 cm long give 17 beats in 20 seconds, when each is sounding its fundamental mode. Calculate the velocity of sound.

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Q.16 Two tuning forks A and B give 5 beats per second. A resounds with a closed column of air 15 cm long and B with an open column 30.5 cm long. Calculate their frequencies. Neglect end correction.

Q.17 At 16^oC, two open end organ pipes, when sounded together produce 34 beats in 2 seconds, How many beats per second will be produced, if temperature rises to 51^oC? Neglect increase in length of the pipes.

Q.18 A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15^oC. When the temperature falls to 10^oC, the two produce 3 beats per second. Find the frequency of the fork.

Q.19 A set of 25 tuning forks is arranged in order of decreasing frequency. Each fork gives 3 beats with succeeding one. The first fork is octave of the last. Calculate the frequency of the first and 16^{th} fork.

Q.20 A tuning fork when vibrating along with a sonometer produces 6 beats per second when the length of the wire is either 20 cm or 21 cm. Find the frequency of the tuning fork.

Q.21 A and B are two wires whose fundamental frequencies are 256 and 382 Hz respectively. How many beats in two seconds will be heard by the third harmonic of A and second harmonic of B?

Answers

1.	387 Hz	2.	292 cps	3.	306 Hz	4.	292 Hz
5.	260 Hz	6.	508 Hz	7.	251 Hz	8.	196 Hz
9.	92 Hz, 184 Hz	10.	205 Hz	11.	2 Hz		
12.	249.5 Hz, 250.5 Hz	13.	1002 Hz, 1000 Hz	14.	116.72 Hz		
15.	343.4 ms^{-1}	16	305 Hz, 300 Hz	17.	18 Hz	18.	110 Hz
19.	144 Hz, 99 Hz	20.	246 Hz	21.	8		

Doppler Effect

Whenever there is a relative motion between the source of sound, the observer and the medium; the frequency of sound as received by the observer is different from the frequency of sound emitted by the source. *The apparent change in the frequency of sound when the source, the observer and the medium are in relative motion is called Doppler effect.*

Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves such as microwaves, radiowaves and visible light. However, Doppler effect is noticeable only when the relative velocity between the source and the observer is an appreciable fraction of the wave velocity.

Apparent frequency when both the source and observer are in motion

As shown in figure, consider the case when both the source and the observer are moving towards each other with speeds v_s and v_0 respectively. If ν is the frequency of the source, it sends out compression pulses through the medium at regular intervals of $T = 1/\nu$.

At time t = 0, the observer is at O_1 and the source at S_1 and the distance between them is L when the source emits the first compression pulse. Since the observer is also moving towards the source, so the speed of the wave relative to the observer is $(v + v_0)$. Therefore, the observer will receive the first compression pulse at time,

$$t_1 = \frac{L}{v+v_0}$$

At time t = T, both the source and observer have moved towards each other covering distances $S_1 S_2 = v_s T$ and $O_1 O_2 = v_0 T$ respectively. The new distance between the source and the observer is $S_2 O_2 = L - (v_s + v_0) T$

The second compression pulse will reach the observer at time,

$$t_2 = T + \frac{L-(v_s+v_0)T}{v+v_0}$$

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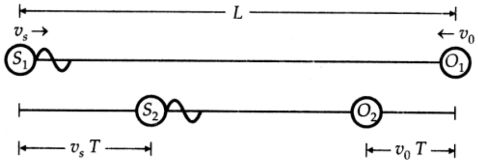
The time interval between two successive compression pulses or the period of the wave as recorded by the observer is $T' = t_2 - t_1$



$$= T + \frac{L-(v_s+v_0)T}{v+v_0} - \frac{L}{v+v_0}$$

$$= \left(1 - \frac{v_s+v_0}{v+v_0}\right)T = \left(\frac{v-v_s}{v+v_0}\right)T$$

Hence the apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v+v_0}{v-v_s} \cdot \frac{1}{T} \qquad \text{or} \qquad \nu' = \frac{v+v_0}{v-v_s}\nu \qquad \text{... (1)}$$

When the source moves towards the observer and the observer moves away from the source

In this case, the apparent frequency can be obtained by replacing v_0 by $-v_0$ in equation (1). Thus

$$\nu' = \frac{v-v_0}{v-v_s}\nu \qquad \text{.... (2)}$$

If the medium moves with a velocity v_m in the direction of propagation of sound, then

$$\nu' = \frac{v+v_m-v_0}{v+v_m-v_s}\nu$$

If the medium moves with a velocity v_m in the opposite direction of sound, then

$$\nu' = \frac{v-v_m-v_0}{v-v_m-v_s}\nu$$

Doppler effect in sound is asymmetric

Suppose a source of sound moves towards a stationary observer with the speed v'. Then the observed frequency will be $\nu' = \frac{v}{v-v'}\nu$

Now if the observer moves towards the stationary source with the same speed v', then the observer frequency will be $\nu' = \frac{v+v'}{v}\nu$

Clearly, $v' \neq v$. Thus the observed frequency in not same when the observer is stationary and the source moves towards it or when the source is stationery and the observer moves towards it with the same speed. For this reaction, *the Doppler effect in sound is said to asymmetric. However, the Doppler effect in light is symmetric.*

NOTE

- The observer frequency depends on the actual velocities of the source and the observer and not on their relative velocities.
- The motion of the source brings about a change in the wavelength of the sound waves and hence there is a change in the observed frequency.
- The motion of the observer merely changes the rate at which the sound waves are received him. The observer intercepts more waves (when he approaches) or fewer waves (when he recedes) each second. The wavelength of the sound waves remains unaffected.
- The apparent frequency is larger than the actual frequency, if the separation between the source and the observer is decreasing and is smaller if the separation is increasing.
- **No Doppler effect is observed** i.e., there is no shift in frequency in the following situations: (i) When both the source and the observer move in the same direction with the same speed. (ii) When

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either the source or the observer is at the centre of a circle and the other is moving along it with an uniform speed. (iii) when both the source and the observer are at rest and the wind alone is blowing.

- The Doppler formula for apparent frequency is applicable when $v_0 < v$ and $v_s < v$. It does not hold when the speed of the source or the observer becomes equal to greater than the speed of the wave.
- When the observer is at rest and the source moves with a supersonic speed (a speed greater than the peed of sound), the resultant wave motion is a conical wave called a **shock wave.** It is due to the shock wave that we hear a sudden and violet sound, called **sonic boom,** when a supersonic jet plane passes by.
- The ratio of the speed of the source to the speed of sound (v/v_0) is called **Mach number.** Shock waves are produced by objects moving with Mach number greater than one.

Subjective Assignment – X

Q.1 A source and an observer are approaching one another with the relative velocity 40 ms^{-1}. If the true source frequency is 1200 Hz, deduce the observed frequency under the following conditions:
(i) All velocity is in the source alone (ii) All velocity is in the observer alone
(iii) The source moves in air at 100 ms^{-1} towards the observer, but the observer also moves with the velocity v_0 in the same direction.

Q.2 A railway engine end a car are moving on parallel tracks in opposite directions with sped of 144 kmh^{-1} and 72 kmh^{-1}, respectively. The engine is continuously sounding a whistle of frequency 500 Hz. The velocity of sound is 340 ms^{-1}. Calculate the frequency of sound heard in the car when
(i) the car and the engine are approaching each other, (ii) the two are moving away from each other

Q.3 The sirens of two fire engines have a frequency of 600 Hz each. A man hears the sirens from the two engines, one approaching him with a speed of 36 km h^{-1} and the other going away from him at a speed of 54 km h^{-1}. What is the difference in frequency of two sirens heard by the man? Take the speed of sound to be 340 ms^{-1}.

Q.4 An observer moves towards a stationary source of sound with a velocity one–fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

Q.5 An observer standing on a railway crossing receives frequencies of 2.2 kHz and 1.8 kHz when the train approaches and recedes from observer. Find velocity of the train. The speed of sound in air is 300 ms^{-1}.

Q.6 On a quiet day, two persons A and B, each sounding a note of frequency 580 Hz, are standing a few metres apart. Calculate the number of beats heard by each in one second when A moves towards B with a velocity of 4 ms^{-1}. (Speed of sound in air = 330 ms^{-1}).

Q.7 A train stands at a platform blowing a whistle of frequency 400 Hz in still air.
(i) What is the frequency of the whistle heard by a man running (a) towards the engine at 10 ms^{-1}, (b) away from the engine at 10 ms^{-1}?
(ii) What is speed of sound in each case?
(iii) What is the wavelength of sound received by the running man in each case?
Take speed of sound in still air = 340 ms^{-1}

Q.8 Consider a source moving towards an observer at the speed $v_s = 0.95$ v. Deduce the observed frequency if the original frequency is 500 Hz. Think what would happen if $v_s > v$. (Jet planes moving faster than sound are so common). Here v is the velocity of sound.

Q.9 A machine gun is mounted on an armoured car. The gun can point (a) in the direction of motion or against the direction of motion of the car. The muzzle speed of the bullet equals the speed of sound in still air i.e., 340 ms^{-1}. If the car moves with a speed of 20 ms^{-1}, find out the sign and magnitude of the time difference between the bullet's arrival (t_b) and the arrival of the sound of firing (t_s) at a

target
500 m away from the car at the instant of firing in each case.

Q.10 An observer is moving towards a wall at 2 ms^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the wall. Calculate the beat frequency between these two sounds, if the true frequency of the source is 680 Hz. Velocity of sound = 340 ms^{-1}.

Q.11 A rocket is moving at a speed of 200 ms^{-1} towards a stationary target. While moving, it emits a sound wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (a) the frequency of the sound wave as detected by a detector attached to the target and (b) the frequency of the echo as detected by a detector attached to the rocket.

Q.12 A siren is fitted on a car going towards a vertical wall at a speed of 36 kmh^{-1}. A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after refection from the wall. Calculate the apparent frequency of the wave

(a) coming directly from the siren to the person, and

(b) coming after reflection. Take the speed of sound to be 340 ms^{-1}. Actual frequency of siren = 500 Hz

Q.13 If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 330 ms^{-1}.

Q.14 A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15 rad s^{-1}. What is the lowest and highest frequency heard by a listener a long distance away at rest w.r.t. centre of the circle? Can the apparent frequency be ever equal to the actual frequency? Take v = 330 ms^{-1}.

Q.15 A train approaches a stationary observer, the velocity of train being 1/20 of the velocity of sound. A sharp blast is blown with the whistle of the engine at equal intervals of a second. Find the interval between the successive blasts as heard by the observer.

Q.16 A car passing a check post gives sound of frequency 1000 cps. If the velocity of the car is 72 kmh^{-1} and of sound is 350 ms^{-1}, find the change in apparent frequency as it crosses the post.

Answer

1. (i) 1360 Hz, (ii) 1341 Hz, (iii) 1400 Hz
2. (i) 600 Hz, (ii) 421 Hz
3. 618.2 Hz, 576.6 Hz, 41.6 Hz
4. 20%
5. 30 ms^{-1}
6. 7, 7
7. (i) (a) 411.8 Hz, (b) 388.2 Hz, (ii) (a) 350 ms^{-1}, (b) 330 ms^{-1}, (iii) 0.85 m
8. 10,000 Hz
9. (a) – 0.0817 s, (b) 0.0919 s
10. 8 Hz
11. (a) 2538.5 Hz (b) 4077 Hz
12. (a) 485.7 Hz, (b) 515.2 Hz
13. 33 ms^{-1}
14. 495 Hz, 594 Hz
15. 19/20 s
16. 114.66 Hz

Characteristics of Musical Sounds

Music: The sound which has a pleasing sensation to the ears is called music. It is produced by regular and periodic vibrations, without any sudden change in loudness. Musical sound can be represented by a periodic wave function and can be splited into various harmonics.

Example: The sound produced by plucking the string of a sitar, by bowing the string of a violin, sound from a tabla etc.

Noise: The sound which has non–pleasing or jarring effect on the ears is called noise. It is produced at irregular intervals and there is sudden change in loudness. The components of wave function have no definite regularities.

Example: The sound produced by an explosion, sound from a market, etc.

Characteristics of Musical Sounds

The *three* characteristics of musical sounds are (i) Loudness or intensity (ii) Pitch (iii) Quality or timbre.

(i) **Loudness:** *Loudness is the amount of energy crossing unit area around a point in one second.* Loudness depends on:

(a) Intensity which depends on amplitude and is proportional to the square of the amplitude.
(b) The surface area of the sound body.
(c) Density of the medium
(d) The presence of other resonant objects around the sounding body.
(e) The distance of the source from the listener.
(f) The motion of air

(ii) **Pitch:** *Pitch is a sensation which helps a listener to distinguish between a high and a grave note. Pitch depends on frequency.*

The voice of a child or a lady is shriller than that of a man i.e., the pitch of a lady's sound is higher than that of a man. The mosquito's sound is of high pitch and hence high frequency. The pitch of a sound heard can change due to Doppler effect.

(iii) **Quality or timbre:** *Quality of sounds enables us to distinguish between two sounds of same pitch and loudness. It is due to the quality of sound that one can recognize one's friend without seeing him.* Quality of sound depends on the number of overtones present in it. It is due to different overtones present in musical instruments t hat we are able to recognize them by their sounds.

Threshold of hearing or Threshold Intensity

The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing. For a sound of frequency 10 kHz, the threshold of hearing is 10^{-12} Wm^{-2}.

Relation between loudness and intensity

According to Weber–Fechner law, the loudness of a sound of intensity I is given by

$$L = \text{Log}_{10}\frac{I}{I_0},$$ where I_0 is the threshold of hearing.

Unit of sound level is decibel (dB)

Musical Scale

A series of notes whose fundamental frequencies have definite ratios and which produce a pleasing effect on the ear when sounded in succession constitute a musical scale. The simplest musical scale called the diatonic scale has eight notes comprising an octave. The frequency ratio of the eighth and the first note is 2 : 1. Conventionally, the fundamental frequency of the first note is taken to be 256 Hz and that of the last 512 Hz. The frequencies of the intermediate notes with their Indian names are given below:

Symbol	Indian Name	Frequency in the base 256 Hz
C	Sa	256
D	Re	288
E	Ga	320
F	Ma	341.3
G	Pa	384
A	Dha	426.7
B	Ni	480
C_1	Sa	512

Reverberation

The persistence of audible sound after the source has ceased to emit sound is called reverberation. When sound waves suffer multiple reflections from the walls, ceiling and other materials present in the hall. The intensity of the sound heard is the combined effect of direct waves and reflected waves. Due to this, the

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sound persists for some time even after the source has stopped producing sound. This is the cause of reverberation.

Reverberation time: *It is defined as the time which sound takes to fall in intensity to one millionth (10^{-6}) part of its original intensity after it was stopped.* Optimum time of reverberation depends upon the size of the hall, absorption material and the nature of programme to be heard. Reverberation time should neither be too high nor too low for quality of sound. Sabine formula for reverberation time of a hall is

$$T = \frac{0.16\ V}{\Sigma\ as}$$

Here V = volume of the hall in cubic metre.

$\sum as = a_1 s_1 + a_2 s_2 + a_3 s_3 + ...$ = Total absorption of the hall

$s_1, s_2, s_3, ...$ = Areas of the various surfaces in m^2

$a_1, a_2, a_3, ...$ = Absorption coefficients of the various surfaces

Usually a reverberation period varying between 0.5 to 1.5 seconds is quite good enough in hearing depending upon the nature of auditorium and programme.

Conceptual Problems

Q.1 Given below are some examples of wave motion. State in each case, if the wave motion is transverse, longitudinal or a combination of both:

(i) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.

(ii) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(iii) Waves produced by a motor boat sailing in water.

(iv) Light waves travelling from the sun to the earth.

(v) Ultrasonic waves in air produced by a vibrating quartz crystal.

Q.2 Why is the sound produced in air not heard by a person deep inside the water?

Q.3 In summer, the sound of a siren is heard louder in night than in the day to a person on the earth. Why?

Q.4 If two waves of same frequency but of different amplitudes travelling in opposite directions through a medium superpose upon each other, will they form stationary wave? Is energy transferred? Are there any nodes?

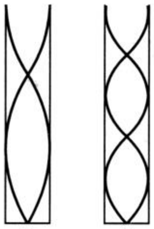
Q.5 What are overtones and harmonics? The presence of which makes a sound musical? OR Distinguish between harmonics and overtones.

Q.6 All harmonics are overtones but all overtones are not harmonics. How?

Q.7 The fundamental frequency of a source of sound is 200 Hz and the source produces all the harmonics. State, with reasons, with which of the following frequencies this source will resonate: 150, 200, 300 and 600 Hz?

Q.8 An organ pipe is in resonance with a tuning fork. What change will have to be done in the length L to maintain the resonance, if (i) the temperature increases, (ii) hydrogen is filled in place of air and (iii) pressure becomes higher?

Q.9 Figure shows two vibrating modes of an air column. Find the ratio of frequencies of the two modes.



Q.10 Two progressive sound waves each of frequency 170 Hz and travelling in opposite directions in air superpose to produce stationary waves. The speed of sound in air is 340 ms^{-1}. What is the separation between (i) two successive nodes, (ii) two successive antinodes and (iii) a node and its nearest antinode?

Q.11 A sonometer wire resonates with a tuning fork. If the length of the wire between the bridge is made twice even then it can resonate with the same tuning fork. How?

Q.12 Why does a tuning fork have two prongs? Would the tuning fork be of any use, if one of the prongs is cut off?

Q.13 Why is a tunning fork used as a standard oscillator? On what factors does pitch of a tuning fork depend?

Q.14 A sitar wire and a tabla, when sounded together, produce 5 beats per second. What can be concluded from this? If the tabla membrane is tightened, will the beat rate increase or decrease?

Q.15 Doppler effect is asymmetric in sound whereas in case of light it is symmetric. Explain.

Q.16 Distinguish between transverse and longitudinal waves.

Q.17 What is red shift? What does it indicate?

Q.18 An incident wave is represented by y = 20 sin (2x – 4t). Write the expression for reflected wave:
(i) from a rigid boundary (ii) from an open boundary

Q.19 State the principle of superposition of waves. Distinguish between conditions for the production of stationary waves and beats.

Q.20 Differentiate between Stationary waves and Progressive waves.

Answer

1. (i) transverse, (ii) longitudinal, (iii) combination of both, (iv) transverse, (v) longitudinal

9. 3 : 5 10. (i) 1.0 m, (ii) 1. 0 m, (iii) 0.5 m 13. $v \propto \frac{d}{\ell^2}\sqrt{\frac{Y}{\rho}}$

18. (i) y = – 20 sin (2x + 4t), (ii) y = 2.0 sin (2x + 4t)

NCERT Exercise

Q.1 A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If a transverse jerk is struck at one end of the string, how long does disturbance take to reach the other end?

Q.2 A stone dropped from the top of a tower 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top? Speed of sound in air = 340 ms^{-1}; g = 9.8 ms^{-2}.

Q.3 A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals speed of sound in dry air at 20°C which is 343 ms^{-1}?

Q.4 Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air
(a) is independent of pressure, (b) increases with temperature, (c) increases with humidity

Q.5 You have learnt that a travelling wave in one dimension is represented by a function y = f (x, t) where x and t must appear in the combination x– vt or x + vt, i.e., y = F (x $\pm$ v t). It the converse true? Examine if the following functions for y can possibly represent a travelling wave:
(i) $(x - vt)^2$ (ii) log [(x + v t)/x_0] (iii) ex p [– (x + v t)/x_0] (iv) 1/ (x + v t)

Q.6 A bat emits ultrasonic wound of frequency 100 kHz in air. If this sound meets a water surface, what is the wavelength of (i) the reflected sound, (ii) the transmitted sound? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.

Q.7 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz.

Q.8 A transverse harmonic wave on a string is described by y (x, t) = 3.0 sin (36 t + 0.018x + π/4), where x, y are in cm and t in s. The positive direction of x is from left to right.

(i) Is this a travelling or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(ii) What are its amplitude and frequency?

(iii) What is the initial phase at the origin?

(iv) What is the least distance between two successive crests in the wave?

Q.9 For the wave described in above question displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Q.10 For a travelling harmonic wave y = 2.0 cos (10 t – 0.0080x + 0.35) where x and y are in cm and t in s. What is the phase difference between oscillatory motion at two points separated by a distance, (i) 4m, (ii) 0.5 m, (iii) λ/2, (iv) 3λ/4?

Q.11 The transverse displacement of a string (clamped at its two ends) is given by

$$y\,(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos (120\,\pi t)$$

where x, y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following:

(a) Does the function represent a travelling or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite direction. What are the wavelength, frequency and speed of propagation of each wave?

(c) Determine the tension in the string.

Q.12 The transverse displacement of a string (clamped at its two ends) is given by

$$y(x,t)=0.06\sin\frac{2\pi}{3}x\cos 120\pi t,$$

where x, y are in m and t is in s.

(i) Do all the points on the string oscillate with the same

(a) frequency, (b) phase, (c) amplitude?

(ii) What is the amplitude of a point 0.375 m away from one end?

Q.13 Given below are some functions of x and t to represent the displacement (transverse of longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all:

(a) y = 2 cos (23x) sin (10t) (b) $y=2\sqrt{x-vt}$

(c) y = 3 sin (5x – 0.5t) + 4 cos (5x – 0.5t) (d) y = cos x sin t + cos 2x sin 2t

Q.14 A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-1}. What is

(i) the speed of a transverse wave on the string, and

(ii) the tension in the string?

Q.15 A metre–long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz), when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.

Q.16 A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

Q.17 A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open?

(Speed of sound = 340 ms^{-1}).

Q.18 Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

Q.19 Explain why (or how):

(a) in a sound wave, a displacement node is a pressure antinode and vice versa,

(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any 'eyes".

(c) a violin note and sitar note may have same frequency, yet we can distinguish between two notes

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

Q.20 A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air.

(i) what is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 ms^{-1}, (b) recedes from the platform with a speed of 10 ms^{-1}?

(ii) What is the speed of sound in each case?

Q.21 A train standing in a station yard blows a whistle of frequency 400 Hz in still air. (a) A wind starts blowing in the direction from the yard to the station with a speed of 10 ms^{-1}. What are the frequency, wavelength and speed of the sound for an observer standing on the station platform? (b) Is the situation exactly equivalent to the case, when the air is still and the observer runs towards the yard at a speed of 10 ms^{-1}? Take speed of sound in still air = 340 ms^{-1}.

Q.22 A travelling harmonic wave on a string is described by $y = 7.5 \sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$

(i) What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation?

(ii) Locate the points of the string, which have the same transverse displacement and velocity as the x = 1 cm point at t = 2s, 5s, 11s.

Q.23 A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite: (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?

Q.24 One end a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y–direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

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Q.25 A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An energy submarine moves towards the SONAR with a speed of 360 kmh^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.

Q.26 Earthquakes generate sound wave inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, how far away does the earthquake occur?

Q.27 A bat is flitting about in a cave, navigating via ultrasonic bleeps. Assume that the second emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Answers

1. 0.5 s 2. 8.7 s 3. 2.06×10^4 N
4. only (iii) represent, travelling wave
6. (i) 3.4×10^{-3} m, (ii) 1.49×10^{-2} m 7. 4.047×10^{-4} m
8. (i) 20 ms^{-1}, (ii) 5.73 s^{-1}, (iii) $\pi/4$ rad, (iv) 3.49 m 9. initial phase
10. (i) 3.2 rad, (ii) 0.40 rad, (iii) π rad, (iv) $3\pi/2$ rad
11. (a) stationery waves (b) 180 ms^{-1}, 60 Hz, 3m (c) 648 N 12. 0.042 m
14. (i) 78.75 ms^{-1}, (ii) 248 N 15. 346.8 ms^{-1} 16. 5.06 km s^{-1}
17. No 18. 318 Hz 20. (i) 412.12 Hz, 388.57 Hz (ii) 340 ms^{-1}
21. (a) 350 ms^{-1},0.875m, frequency remain unchanged (b) 411.8 Hz, 350 ms^{-1}, wave length remain unchanged
22. (i) 1.67 cm, 87.76 cm s^{-1}, 2400 cm s^{-1}, (ii) 1256.64 cm
23. (i) No, (b) no it is the frequency of pulse repetition 24. $y = 0.05 \sin (16.1 \times 10^2 t - 4.84 x)$
25. 45.93 kHz 26. 1920 km 27. 42.47 kHz

Objective Assignment – I [IIT Entrance Exam]

Multiple Choice Questions with One Correct Answers

Q.1 A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer, who is standing in air, is

(a) 200 Hz (b) 3000 Hz (c) 120 Hz (d) 600 Hz

Q.2 The ratio of the speed of sound in nitrogen gas to that in the helium gas at 300 K is

(a) $\sqrt{2/7}$ (b) $\sqrt{1/7}$ (c) $\sqrt{3}/5$ (d) $\sqrt{6}/5$

Q.3 Two monatonic ideal gases 1 and 2 of molecular masses M_1 and M_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas
2 is given by

(a) $\sqrt{\frac{M_1}{M_2}}$ (b) $\sqrt{\frac{M_2}{M_1}}$ (c) $\frac{M_1}{M_2}$ (d) $\frac{M_2}{M_1}$

Q.4 The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of sound will be

(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

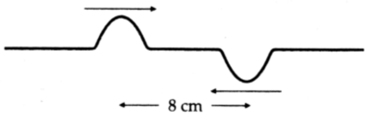
Q.5 A travelling wave in a stretched string is described by the equation y = Asin (kx – ωt). The maximum particle velocity is

(a) Aω (b) ω/ k (c) dω/ dk (d) x / t

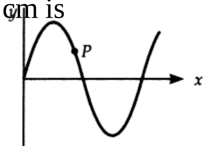
Q.6 Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be

(a) zero (b) purely kinetic

(c) purely potential (d) partly kinetic and partly potential.



Q.7 A transverse sinusoidal wave moves along a string in the positive x–direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap–shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$ (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$ (c) $\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$ (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$

Q.8 A wave represented by the equation y = a cos(kx – ωt)

is superposed with another wave to form a stationary wave such that point x = 0 is a node. The equation for the other wave is

(a) a cos (kx – ωt) (b) – a cos (kx – ωt) (c) – a cos (kx + ωt) (d) – a sin (kx – ωt)

Q.9 Two vibrating strings of the same material but lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1/ f_2 is given by

(a) 2 (b) 4 (c) 8 (d) 1

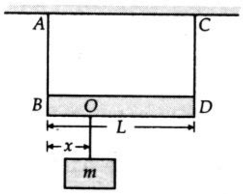
Q.10 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same turning fork forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) (1/25) kg

Q.11 An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ (c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

Q.12 A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to x. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. X is



(a) L/5 (b) 4L/5 (c) 3L/ 5 (d) L/ 4

Q.13 In the experiment to determine the speed of sound using a resonance column,

(a) prongs of the tuning fork are kept in a vertical plane

(b) prongs of the tuning fork are kept in a horizontal plane

(c) in one of two resonances observed, the length of the resonating air column is close to the wavelength of sound in air

(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air.

Q.14 An open pipe is in resonance in 2^{nd} harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased of f_2 such that resonance again occurs in nth harmonic. Choose the correct option

(a) $n=3, f_2=\frac{3}{4}f_1$ (b) $n=3, f_2=\frac{5}{4}f_1$ (c) $n=5, f_2=\frac{3}{4}f_1$ (d) $n=5, f_2=\frac{5}{4}f_1$

Q.15 In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is

(a) 51.2 cm/s (b) 102.4 cm/s (c) 204.8 cm/s (d) 153.6 cm/s

Q.16 An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

(a) 200 Hz (b) 300 Hz (c) 240 Hz (d) 480 Hz

Q.17 In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

(a) 0.012 m (b) 0.025 m (c) 0.05 m (d) 0.024 m

Q.18 A pipe of length l_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal.

(a) $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$ (c) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$

Q.19 The ends of a stretched wire of length L, are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A \sin (\pi x/ L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin (2\pi x/ L) \sin 2\omega t$ and energy is E_2. Then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$ (c) $E_2 = 4 E_1$ (d) $E_2 = 16 E_1$

Q.20 Two plane harmonic sound waves are expressed by the equations:

$$y_1 (x, t) = A \cos (0.5 \pi x - 100 \pi t) \quad \text{and} \quad y_2 (x, t) = A \cos (0.46\pi x - 92 \pi t)$$

All parameters are in mks system. How many times does an observer hear maximum intensity in one second?

(a) 4 (b) 6 (c) 8 (d) 10

Q.21 A vibrating string of length *l* under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to 340 m/s, the frequency n of the tuning fork in Hz is

(a) 344 (b) 336 (c) 117.3 (d) 109.3

Q.22 A whistle giving out 450 Hz approaches a stationary observer with speed 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)

(a) 409 (b) 429 (c) 517 (d) 500

Q.23 A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is

(a) 18/19 (b) 1/2 (c) 2 (d) 19/18

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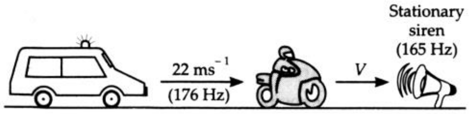
Q.24 A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is

(a) 242/252 (b) 2 (c) 5/6 (d) 11/6

Q.25 A police car moving at 22 ms^{-1}, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motor–cycle, if it is given that he does not observe any beats.



(a) 33 ms^{-1} (b) 22 ms^{-1} (c) 11 ms^{-1} (d) zero

Multiple Choice Questions with One or More than One Correct Answer

Q.26 A wave equation which gives the displacement along the y–direction is given by $y = 10^{-4}$ sin (60t + 2x), where x and y are in metre and t is time in second. This represents a wave

(a) travelling with a velocity of 30 m/s in the negative x–direction

(b) of wavelength π m

(c) of frequency (30/π) hertz

(d) of amplitude 10^{-4} m travelling along the negative x–direction.

Q.27 A transverse wave is described by the equation

$$y = y_0 \sin 2\pi \left(ft - \frac{x}{\lambda}\right).$$

The maximum particles velocity is equal to four times the wave velocity is

(a) $\lambda = \pi\frac{y_0}{4}$ (b) $\lambda = \pi\frac{y_0}{2}$ (c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

Q.28 The displacement of particles in a string stretched in the x–direction is represented by y. Among the following expressions for y, those describing wave motion are

(a) cos kx sin ωt (b) $k^2x^2 - \omega^2t^2$ (c) $\cos^2 (kx + \omega t)$ (d) $\cos (k^2x^2 - \omega^2t^2)$

Q.29 A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\lambda t + \frac{\pi}{3}\right)$$

where x is in metre and t is in second. The expression represents

(a) a wave travelling in the position x–direction with a velocity 1.5 m/s

(b) a wave travelling in the negative x–direction with a velocity 1.5 m/s

(c) a wave travelling in the negative x–direction having a wavelength 0.2 m

(d) a wave travelling in the positive x–direction having a wavelength 0.2 m.

Q.30 In a wave motion y = asin (kx – ωt), y can represents

(a) electric field (b) magnetic field (c) displacement (d) pressure

Q.31 y(x, t) = $0.8/[4x + 5t)^2 + 5]$ represents a moving pulse, where x and y are in metre and t in second. Then

(a) pulse is moving in + x direction

(b) in 2s it will travel a distance of 2.5 m

(c) its maximum displacement is 0.16 m

(d) it is a symmetric pulse

Q.32 A transverse simusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. If a = 10^{-3} m and v = 10 ms^{-1}, then λ and f are given by

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(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m (c) $f = 10^3/ 2\pi$ Hz (d) $f = 10^4$ Hz

Q.33 As a wave propagates,

(a) the wave intensity remains constant for a plane wave

(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(c) the wave intensity decreases as inverse square of the distance from source for a spherical wave

(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all time

Q.34 Standing waves can be produced

(a) on a string clamped at both the ends (b) on a string clamped at one end free at the other

(c) when incident wave gets reflected from a wall

(d) when two identical waves with a phase difference of π are moving in the same direction.

Q.35 An air column in a pipe, which is closed at one end, will be in resonance with a vibrating turning fork of frequency 264 Hz if the length of the column in cm is

(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

Q.36 A tube closed at one end and containing air produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)

(a) 1024 (b) 512 (c) 256 (d) 128

Q.37 A student performed the experiment to measure the speed of sound in air using resonance air–column method. Two resonances in the air–column were obtained by lowering the water level. The resonance with the shorter air–column is the first resonance and that with the longer air–column is the second resonance. Then

(a) the intensity of the sound heard at the first resonance was more than that at the second resonance

(b) the prongs of the tuning fork, were kept in a horizontal plane above the resonance tube

(c) the amplitude of the vibration of the end of the prongs is typically round 1 cm

(d) the length of air–column at the first resonance was somewhat shorter than $1/4^{th}$ of the wavelength of the sound in air.

Q.38 An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

(a) 8/3 (b) 3/8 (c) 1/6 (d) 1/3

Q.39 Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency

(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 hz

Q.40 A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is

(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0.40 s

Q.41 Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tensions in the strings, then it could be said that while making the above changes in tension.

(a) T_2 was decreased (b) T_2 was increased (c) T_1 was decreased (d) T_1 was increased

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Q.42 The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000\,t)$$

This expression may be considered to be a result of the superposition of

(a) two (b) three (c) four (d) five

Q.43 A wave disturbance in a medium is described by $y(x, t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metre and t is in second.

(a) A node occurs at x = 0.15 m (b) An antinode occurs at x = 0.3 m

(c) The speed of wave is 5 ms^{-1} (d) The wavelength is 0.2 m

Q.44 A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c.

(a) the number of waves striking the surface per second if $f\frac{(c+v)}{c}$

(b) the wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$

(c) The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$

(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$.

Q.45 The (x, y) co–ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression for u is (are) (a = positive constant)

(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$ (b) $a\sin(\pi x/L)\sin(\pi y/L)$

(c) $a\sin(\pi x/L)\sin(2\pi y/L)$ (d) $a\cos(2\pi x/L)\sin(\pi y/L)$

Answers

1	d	2	c	3	b	4	c	5	a
6	b	7	a	8	c	9	d	10	a
11	a	12	a	13	a	14	d	15	c
16	a	17	b	18	b	19	c	20	a
21	a	22	d	23	d	24	b	25	b
26	a, b, c, d	27	b	28	a	29	b, c		
30	a, b, c, d	31	b, c, d	32	a, c	33	a, c, d	34	a, b, c
35	a, c	36	a	37	a, d	38	d	39	a, b, d
40	b	41	b, c	42	b	43	a, b, c, d	44	a, b, c
45	b, c								

Objective Assignment – II [AIEEE Exercise]

Q.1 The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$

The amplitude of the particle is

(a) – 4 (b) 4 (c) $4\sqrt{2}$ (d) 8

Q.2 The displacement y of a particle in a medium can be expressed as $y = 10^{-6} \sin (100t + 20x + \pi/4)$

where t is in second and x in metre. The speed of the wave is

(a) 2, 000 ms^{-1} (b) 5 ms^{-1} (c) 20 ms^{-1} (d) 5π ms^{-1}

Q.3 The displacement y of a wave travelling in the X–direction is given by $y = 10^{-4} \sin (600t - 2x + \pi/3)$

where x is expressed in metres and t is seconds. The speed of the wave motion (in ms^{-1}) is

(a) 300 (b) 600 (c) 1200 (d) 200

Q.4 A wave $y = a \sin (\omega t - kx)$, on a string meets with another wave producing a node at x = 0. Then, the equation of the unknown wave is

(a) $y = a \sin (\omega t + kx)$ (b) $y = -a \sin (\omega t + kx)$

(c) $y = a \sin (\omega t - kx)$ (d) $y = -a \sin (\omega t - kx)$

Q.5 A wave travelling along the x–axis is described by the equation $y(x, t) = 0.005 \cos (\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\pi, \beta = \pi$ (c) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

Q.6 The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)

(a) 460 ms^{-1} (b) 500 ms^{-1} (c) 650 ms^{-1} (d) 1420 ms^{-1}

Q.7 Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is

(a) 20 (b) 80 (c) 40 (d) 120

Q.8 A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then the lowest resonant frequency for this string is

(a) 1,050 Hz (b) 10.5 Hz (c) 105 Hz (d) 1.05 Hz

Q.9 A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance, when carrying an alternating current of frequency *v*. The frequency *v* of the alternating source is

(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz

Q.10 Tube A has both ends open, while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

Q.11 When temperature increases, the frequency of a tuning fork

(a) increases (b) decreases (c) increases or decreases depending on the material

(d) remains the same

Q.12 A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. The frequency of the unknown fork is

(a) 286 cps (b) 292 cps (c) 294 cps (d) 288 cps

Q.13 When two tuning forks (fork 1 and fork 2) sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork?

(a) 200 Hz (b) 202 Hz (c) 196 Hz (d) 204 Hz

Q.14 A tuning fork of known frequency of 256 Hz makes 5 beats s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(a) (256 + 2) Hz (b) (256 – 2) Hz (c) (256 – 5) Hz (d) (256 + 5) Hz

Q.15 An observer moves towards a stationary source of sound with a velocity one–fourth of the velocity of sound. What is the percentage increase in the apparent frequency?

(a) zero (b) 0.5 % (c) 5 % (d) 20%

Q.16 A whistle producing sound waves of frequencies 9,500 Hz and above is approaching a stationary person with speed u ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of u upto which he can hear the whistle is

(a) $15\sqrt{2}\,ms^{-1}$ (b) $15/\sqrt{2}\,ms^{-1}$ (c) 15 ms^{-1} (d) 30 ms^{-1}

Q.17 A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}).

(a) 49 m (b) 98 m (c) 147 m (d) 196 m

Q.18 A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(a) 100 (b) 1000 (c) 10000 (d) 10

Answers

1	c	2	b	3	a	4	b	5	b
6	d	7	b	8	c	9	a	10	c
11	b	12	b	13	c	14	c	15	d
16	c	17	b	18	a				

Objective Assignment – III [AIIMS Entrance Exam]

Q.1 The waves, in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion, is known as

(a) transverse waves (b) longitudinal waves

(c) propagated waves (d) none of these

Q.2 The waves produced by a motor boat sailing in water are

(a) transverse (b) longitudinal

(c) longitudinal and transverse (d) stationary

Q.3 For a wave propagating in a medium, identify the property that is independent of the others.

(a) velocity (b) wavelengths

(c) frequency (d) all these depend on each other

Q.4 A boat at anchor is rocked by waves, whose crests are 100 m apart and velocity is 25 ms^{-1}. The boat bounces up once in every

(a) 2,500 s (b) 75 s (c) 4 s (d) 0.25 s

Q.5 Newton's formula for the velocity of sound in gases is

(a) $v=\sqrt{P/\rho}$ (b) $v=\sqrt{\rho/P}$ (c) $v=\sqrt{\rho/2P}$ (d) $v=\sqrt{2P/\rho}$

Q.6 The velocities of sound at the same temperature in two monoatomic gases of densities of ρ_1 and ρ_2 are v_1 and v_2 respectively. If $\rho_1/\rho_2 = 4$, then the value of v_1/v_2 is
(a) 1/4 (b) 1/2 (c) 2 (d) 4

Q.7 An earthquake generates both transverse (S) and longitudinal (P) sound waves in the earth. The speed about of S waves is about 4.5 km s^{-1} and that of P waves is about 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4.0 min before the first S wave. The epicenter of the earthquake is located at a distance of about
(a) 25 km (b) 250 km (c) 2,500 km (d) 5,000 km

Q.8 If equation of a sound wave is $y = 0.0015 \sin(62.8x + 314t)$ then its wavelength will be
(a) 0.1 unit (b) 0.2 unit (c) 0.3 unit (d) 2 unit

Q.9 A transverse wave passes through a string with the equation $y = 10 \sin \pi (0.02x - 2t)$ where x is in metres and t in seconds. The maximum velocity of the particles in wave motion is
(a) 63 ms^{-1} (b) 78 ms^{-1} (c) 100 ms^{-1} (d) 121 ms^{-1}

Q.10 The plane wave is described by the equation $y = 3 \cos(x/4 - 10t - \pi/2)$, where x and y are in meters and t in seconds. The maximum velocity of the particles of the medium due to this is
(a) $30\,ms^{-1}$ (b) $\frac{3\pi}{2}ms^{-1}$ (c) $\frac{3}{4}ms^{-1}$ (d) 40 ms^{-1}

Q.11 The equation of wave is given by $y = 10 \sin(2\pi t/30 + \alpha)$
If the displacement is 5 cm at t = 0, then the total phase at t = 7.5 s will be
(a) $\pi/3$ rad (b) $\pi/2$ rad (c) $2\pi/5$ rad (d) $2\pi/3$ rad

Q.12 If two sound waves having a phase difference of 60^o, then they will have a path difference of
(a) $\lambda/6$ (b) $\lambda/3$ (c) λ (d) 3λ

Q.13 Energy is not carried by which of the following wave?
(a) stationary (b) progressive (c) transverse (d) electromagnetic

Q.14 Standing waves are produced in 10 m long stretched string. If the string vibrates in 5 segments and wave velocity is 20 ms^{-1}, its frequency is
(a) 2 Hz (b) 4 Hz (c) 5 Hz (d) 10 Hz

Q.15 If vibrations of a string are to be increased by a factor 2, tension in the string must be made
(a) half (b) twice (c) four times (d) eight times

Q.16 The tension in piano wire is 10 N. What should be the tension in the wire to produce a note of double the frequency?
(a) 5 N (b) 20 N (c) 40 N (d) 80 N

Q.17 A string in a musical instrument is 50 cm long and its fundamental frequency is 800 Hz. If a frequency of 1,000 Hz is to be produced, then required length of string is
(a) 62.5 (b) 50 cm (c) 40 cm (d) 37.5 cm

Q.18 The frequency of a tuning fork is 256. It will not resonate with a fork of frequency
(a) 256 (b) 512 (c) 738 (d) 768

Q.19 An organ pipe closed at one end has fundamental frequency of 1,500 Hz. The maximum number of overtones generated by this pipe, which a normal person can hear is
(a) 12 (b) 9 (c) 6 (d) 4

Q.20 A tube closed at one end containing air produces fundamental note of frequency 512 Hz. If the tube is open at both the ends, the fundamental frequency will be
(a) 256 Hz (b) 768 Hz (c) 1,024 Hz (d) 1,280 Hz

Q.21 A closed organ pipe and an open pipe of the same length produce four beats per second, when sounded together. If the length of the closed pipe is increased, then the number of beats will
(a) increase (b) decrease (c) remain the same (d) first (b) then (c)

V PLUS U

Q.22 A resonance air column of length 20 cm resonates with a tuning fork of frequency 450 Hz. Ignoring end correction, the velocity of sound in air is
(a) 720 ms^{-1} (b) 820 ms^{-1} (c) 920 ms^{-1} (d) 360 ms^{-1}

Q.23 A sings with a frequency v and B sings with a frequency $1/8^{th}$ that of A. If the energy remains the same and the amplitude of A is a, then the amplitude of B is
(a) a (b) 2 a (c) 8 a (d) 16 a

Q.24 If fundamental frequency is 50 and next successive frequencies are 150 and 250, then it is
(a) a pipe closed at both ends (b) a pipe closed at one end
(c) an open pipe (d) a stretched string

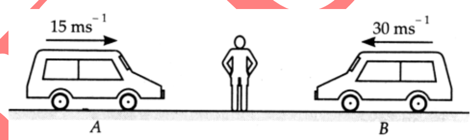
Q.25 A stone thrown into still water, creates a circular wave pattern moving radially outwards. If r is the distance measured from the centre of the pattern, the amplitude of the wave varies as
(a) $r^{-1/2}$ (b) r^{-1} (c) $r^{-3/2}$ (d) r^{-2}

Q.26 A siren emitting sound of frequency 800 Hz is going away from a static listener with a speed of 30 ms^{-1}. Frequency of the sound to be heard by the listener is (Take velocity of sound as 330 ms^{-1})
(a) 733.3 Hz (b) 644.8 Hz (c) 481.2 Hz (d) 286.5 Hz

Q.27 An observer standing by the side of a road hears the siren of an ambulance, which is moving away from him. If the actual frequency of the siren is 2,000 Hz, then the frequency heard by the observer will be
(a) 1,990 Hz (b) 2,000 Hz (c) 2,100 Hz (d) 4, 000 Hz

Q.28 A source of frequency 240 Hz is moving towards an observer with a velocity of 20 ms^{-1}. The observer is now moving towards the source with a velocity of 20 ms^{-1}. Apparent frequency heard by observer, if velocity of sound is 340 ms^{-1}, is
(a) 240 Hz (b) 270 Hz (c) 330 Hz (d) 360 Hz

Q.29 Two cars approach stationary observer from opposite sides as shown in figure. The observer hears no beats. If the frequency of the horn of car B is 504 Hz, the frequency of the horn of the car A will be



(a) 529.2 Hz (b) 440.5 Hz (c) 295.2 Hz (d) none of these

Assertion & Reason

Directions: *In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as*

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false.
(d) If both assertion and reason are false.

Q.30 **Assertion:** Sound wave cannot propagate through vacuum but light can.
Reason: Sound wave can not be polarized but light can.

Q.31 **Assertion:** Sound wave cannot propagate fastest in solids.
Reason: Sound wave can propagate slightly in vacuum.

Q.32 **Assertion:** Speed of wave $= \dfrac{wavelength}{time\ period}$
Reason: Wavelength is the distance between two nearest particles in phase.

Q.33 **Assertion:** Ocean waves hitting a beach are always found to be nearly normal to the shore.
Reason: Ocean waves hitting a beach are assumed to be plane waves.

Q.34 **Assertion:** When a beetle moves along the sand within a few tens of centrimetres of a sand scorpion, the scorpion immediately turns towards the beetle and dashes towards it.

Reason: When a beetle disturbs the sand, it sends pulses along the sand's surface. One set of pulses is longitudinal, while the other set is transverse.

Q.35 **Assertion:** A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.

Reason: The same tuning fork will not be in resonance with an open pipe of the same length due to end correction.

Q.36 **Assertion:** When two vibrating tuning forks having frequencies 256 Hz and 512 Hz are held near each other, beats cannot be heared.

Reason: The principle of superposition is valid if the frequencies of the oscillations are nearly equal.

Answers

1	a	2	c	3	c	4	c	5	a
6	b	7	c	8	a	9	a	10	a
11	d	12	a	13	a	14	c	15	c
16	c	17	c	18	c	19	c	20	c
21	a	22	d	23	c	24	b	25	a
26	a	27	a	28	b	29	a	30	b
31	c	32	b	33	a	34	a	35	c
36	c								

CBSE PMT Prelims Exam

Q.1 With the propagation of a longitudinal wave through a material medium, the quantities transmitted in the propagation direction are

(a) energy, momentum and mass (b) energy
(c) energy and mass (d) energy and linear momentum

Q.2 Which one of the following statements is true?

(a) both light and sound waves can travel in vacuum
(b) both light and sound waves in air are transverse
(c) the sound waves in air are longitudinal while the light waves are transverse
(d) both light and sound waves in air are longitudinal

Q.3 The velocity of sound in any gas depends upon

(a) wavelength of sound only (b) density and elasticity of gas
(c) intensity of sound waves only (d) amplitude and frequency of sound

Q.4 A hospital uses an ultrasonic scanner to locate tumours in a tissue. The operating frequency of the scanner is 4.2 MHz. The speed of sound in a tissue is 1.7 km/s. The wavelength of sound in the tissue is close to

(a) 4×10^{-3} m (b) 8×10^{-3} m (c) 4×10^{-4} m (d) 8×10^{-4} m

Q.5 A 5.5 metre length of string has a mass of 0.035 kg. If the tension in the string in 77 N, the speed of a wave on the string is

(a) 110 ms^{-1} (b) 165 ms^{-1} (c) 77 ms^{-1} (d) 102 ms^{-1}

Q.6 The temperature at which the speed of sound becomes double as was at 27^oC is

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(a) 273°C (b) 0°C (c) 927°C (d) 1027°C

Q.7 Velocity of sound waves in air is 330 m/s. For a particular sound wave in air, a path difference of 40 cm is equivalent to phase difference of 1.6 π. The frequency of this wave is
(a) 165 Hz (b) 150 Hz (c) 660 Hz (d) 330 Hz

Q.8 Two sound waves having a phase difference of 60° have path difference of
(a) λ/6 (b) λ/3 (c) 2λ (d) λ/2

Q.9 In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacement is 0.170 sec. The frequency of wave is
(a) 0.73 Hz (b) 0.36 Hz (c) 1.47 Hz (d) 2.94 Hz

Q.10 Which of the following represents a wave?
(a) y = A sin (ωt – kx) (b) y = A cos (at – bx + c)
(c) y = A sin kx (d) y = A sin ωt

Q.11 A wave travelling in positive X–direction with a = 0.2 m, velocity = 360 m/s and λ = 60 m, then correct expression for the wave is
(a) $y=0.2\sin\left[2\pi\left(6t+\frac{x}{60}\right)\right]$ (b) $y=0.2\sin\left[\pi\left(6t+\frac{x}{60}\right)\right]$
(c) $y=0.2\sin\left[2\pi\left(6t-\frac{x}{60}\right)\right]$ (d) $y=0.2\sin\left[\pi\left(6t-\frac{x}{60}\right)\right]$

Q.12 The frequency of sinusoidal wave y = 0.40 cos [200t + 0.80x] would be
(a) 1000 π Hz (b) 2000 Hz (c) 20 Hz (d) $\frac{1000}{\pi}\,Hz$

Q.13 The equation of a sound wave is y = 0.0015 sin (62.4x + 316 t)
The wavelength of this wave is
(a) 0.3 unit (b) 0.2 unit (c) 0.1 unit (d) cannot be calculated

Q.14 A transverse wave propagating along x–axis is represented by y(x, t) = 8.0 sin (0.5πx – 4πt – π/4), where x is in metres and t is in seconds. The speed of the wave is
(a) 8 m/s (b) 4π m/s (c) 0.5π m/s (d) π/4 m/s

Q.15 The equation of a wave is represented by $y = 10^{-4}\sin\left(100t-\frac{x}{10}\right)$ m, then velocity of wave will be
(a) 100 m/s (b) 4 m/s (c) 1000 m/s (d) 10 m/s

Q.16 A wave in a string has an amplitude of 2 cm. The wave travels in the +ve direction of x–axis with a speed of 128 m/s and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is
(a) y = 0.02 m sin (15.7x – 2010 t) (b) y = 0.02m sin (15.7x + 2010t)
(c) y = 0.02 m sin (7.85x – 1005 t) (d) y = 0.02m sin (7.85x + 1005 t)

Q.17 Equation of progressive wave is given by $y=4\sin\left[\pi\left(\frac{t}{5}-\frac{x}{9}\right)+\frac{\pi}{6}\right]$ where y, x are in cm and t is in second. Then which of the following is correct?
(a) v = 5 cm/s (b) λ = 18 cm (c) a = 0.04 cm (d) f = 50 Hz

Q.18 A transverse wave is represented by the equation $y = y_0 \sin\frac{2\pi}{\lambda}(vt-x)$. For what value of λ, is the maximum particle velocity equal to two times the wave velocity?
(a) $\lambda=\frac{\pi y_0}{2}$ (b) $\lambda=\frac{\pi y_0}{3}$ (c) $\lambda=2\pi y_0$ (d) $\lambda = \pi y_0$

Q.19 The phase difference between two waves, represented by

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$y_1 = 10^{-6} \sin [100t + (x/50) + 0.5]$ m
$y_2 = 10^{-6} \cos [100t + (x/50)]$m
where x is expressed in metres and t is expressed in seconds, is approximately
(a) 1.07 radians (b) 2.07 radians (c) 0.5 radian (d) 1.5 radians

Q.20 Two waves have equations,
$x_1 = a \sin (\omega t - kx + \phi_1)$
$x_2 = a \sin (\omega t - kx + \phi_2)$
If in the resultant wave the frequency and amplitude remain equal to those of superimposing waves, the phase difference between them is
(a) $\pi/6$ (b) $2\pi/3$ (c) $\pi/4$ (d) $\pi/3$

Q.21 The equations of two waves acting in perpendicular direction are given as
$x = a \cos (\omega t + \delta)$ and $y = a \cos (\omega t + \alpha)$
where $\delta = \alpha + \frac{\pi}{2}$, the resultant wave represents
(a) parabola (b) a circle (c) an ellipse (d) a straight line

Q.22 A stationary wave is represented by
$y = A \sin (100t) \cos (0.01x)$,
where y and A are in millimeters, t is in seconds and x is in metres. The velocity of the wave is
(a) 10^4 m/s (b) not derivable (c) 1 m/s (d) 10^2 m/s

Q.23 A standing wave having 3 nodes and 2 antinodes is formed between two atoms having a distance 1.21Å between them. The wavelength of the standing wave is
(a) 6.50Å (b) 2.42Å (c) 1.21Å (d) 3.63Å

Q.24 Standing waves are produced in 10 m long stretched string. If the string vibrates in 5 segments and wave velocity is 20 m/s, the frequency is
(a) 5 Hz (b) 10 Hz (c) 2 Hz (d) 4 Hz

Q.25 A stretched string resonates with tuning fork of frequency 512 Hz when length of the string is 0.5 m. The length of string required to vibrate resonantly with a tuning fork of frequency 256 Hz would be
(a) 0.25 m (b) 0.5 m (c) 1 m (d) 2 m

Q.26 If the tension and diameter of a sonometer wire of fundamental frequency n are doubled and density is halved, then its fundamental frequency will become
(a) $\frac{n}{4}$ (b) $\sqrt{2}\,n$ (c) n (d) $\frac{n}{\sqrt{2}}$

Q.27 A wave of frequency 100 Hz travels along a string towards its fixed end. When this wave travels back, after reflection, a node is formed at a distance of 10 cm from the fixed end. The speed of the wave (incident and reflected) is
(a) 20 m/s (b) 40 m/s (c) 5 m/s (d) 10 m/s

Q.28 The length of a sonometer wire AB is 110 cm. Where should the two bridges be placed from A to divide the wire in 3 segments whose fundamental frequencies are in the ratio of 1 : 2 : 3?
(a) 60 cm and 90 cm (b) 30 cm and 60 cm (c) 30 cm and 90 cm (d) 40 cm and 80 cm

Q.29 A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has
(a) three nodes and three antinodes (b) three nodes and four antinodes
(c) four nodes and three antinodes (d) four nodes and four antinodes

Q.30 A cylindrical tube, open at both ends has fundamental frequency f in air. The tube is dipped vertically in water, so that half of it is in water. The fundamental frequency of air column is now

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(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

Q.31 A string is cut into three parts, having fundamental frequencies n_1, n_2, n_3 respectively. Then original fundamental frequency n is related by the expression as

(a) $\frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}$ (b) $n = n_1 \times n_2 \times n_3$ (c) $n=n_1+n_2+n_3$ (d) $n=\frac{n_1+n_2+n_3}{3}$

Q.32 For production of beats the two sources must have

(a) different frequencies and same amplitude (b) different frequencies

(c) different frequencies, same amplitude and same phase

(d) different frequencies and same phase

Q.33 A source of frequency *v* gives 5 beats/second when sounded with a source of frequency 200 Hz. The second harmonic of frequency 2*v* of source gives 10 beats/second when sounded with a source of frequency 420 Hz. The value of *v* is

(a) 205 Hz (b) 195 Hz (c) 200 Hz (d) 210 Hz

Q.34 A source of sound gives 5 beats per second, when sounded with another source of frequency 100 second^{-1}. The second harmonic of the source, together with a source of frequency 205 sec^{-1} gives 5 beats per second. What is the frequency of the source?

(a) 105 second^{-1} (b) 205 second^{-1} (c) 95 second^{-1} (d) 100 second^{-1}

Q.35 Two waves of lengths 50 cm and 51 cm produced 12 beats per sec. The velocity of sound is

(a) 340 m/s (b) 331 m/s (c) 306 m/s (d) 360 m/s

Q.36 Two sound waves with wavelengths 5.0 m and 5.5 m respectively, each propagates in a gas with velocity 330 m/s. We expect the following number of beats per second

(a) 6 (b) 2 (c) 0 (d) 1

Q.37 The wave described by $y = 0.25 \sin (10\pi x - 2\pi t)$ where x and y are in metres and t in seconds, is a wave travelling along the

(a) – ve x–direction with frequency 1 Hz

(b) – ve x–direction with frequency π Hz and wavelength λ – 0.2 m

(c) + ve x–direction with frequency 1 Hz and wavelength λ = 0.2 m

(d) – ve x–direction with amplitude 0.25 m and wavelength λ = 0.2 m

Q.38 Each of the two strings of length 51.6 cm and 49.1 km are tensioned separately by 20 N force. Mass per unit length of both the strings is same and equal to 1 g/m. When both the strings vibrate simultaneously the number of beats is

(a) 3 (b) 6 (c) 7 (d) 8

Q.39 The driver of a car travelling with speed 30 m/s towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by driver is

(a) 500 Hz (b) 550 Hz (c) 555.5 Hz (d) 720 Hz

Q.40 Two vibrating tuning forks produce waves given by $y_1 = 4 \sin 500 \pi t$ and $y_2 = 2 \sin 506\pi t$. Number of beats produced per minute is

(a) 360 (b) 180 (c) 60 (d)3

Q.41 Two trains move towards each other with the same speed. The speed of sound is 340 m/s. If the height of the tone of the whistle of one of them heard ion the other changes to 9/8 times, then the speed of each train should be

(a) 20 m/s (b) 2 m/s (c) 200 m/s (d) 2000 ms/

Q.42 A car is moving towards a high cliff. The driver sounds a horn of frequency f. The reflected sound heard by the driver has frequency 2f. If v is the velocity of sound, then the velocity of the car, in the same velocity units, will be

(a) $v/\sqrt{2}$ (b) v/3 (c) v/4 (d) v/2

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Q.43 An observer moves towards a stationary source of sound with a speed 1/5th of the speed of sound. The wavelength and frequency of the source emitted are λ and f respectively. The apparent frequency and wavelength recorded by the observer are respectively

(a) 1.2 f, 1.2 λ (b) 1.2 f, λ (c) f, 1.2. λ (d) 0.8 f, 0.8 λ

Q.44 A vehicle, with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency n + n_1. Then (if the sound velocity in air is 300 m/s)

(a) $n_1 = 0.1$ n (b) $n_1 = 0$ (c) $n_1 = 10n$ (d) $n_1 = -0.1$ n

Q.45 A whistle revolves in a circle with angular speed $\omega = 20$ rad/sec using a string of length 50 cm. If the frequency of sound from the whistle is 385 Hz, then what is the minimum frequency heard by an observer which is far away from the centre (velocity of sound = 340 m/s)?

(a) 385 Hz (b) 374 Hz (c) 394 Hz (d) 333 Hz

Q.46 Two sound sources each emitting waves of wavelength λ are fixed a given distance apart and an observer moves from one source to another with velocity u. Then number of beats heard by him

(a) $\frac{2u}{\lambda}$ (b) $\frac{u}{\lambda}$ (c) $\sqrt{u\lambda}$ (d) $\frac{u}{2\lambda}$

Q.47 A star, which is emitting radiation at a wavelength of 5000Å is approaching the earth with a velocity of 1.5×10^6 m/s. The change in wavelength of the radiation as received on the earth is

(a) 25 Å (b) 100 Å (c) zero (d) 2.5 Å

Q.48 If the amplitude of sound is doubled and the frequency reduced to one forth, the intensity of sound at the same point will be

(a) increasing by a factor of 2 (b) decreasing by a factor of 2
(c) decreasing by a factor of 4 (d) unchanged

Q.49 A point source emits sound equally in all directions in a non–absorbing medium. Two points P and Q are at distances of 2 m and 3m respectively from the source. The ratio of the intensities of the waves at P and Q is

(a) 3 : 2 (b) 2 : 3 (c) 9 : 4 (d) 4 : 9

Q.50 The time of reverberation of a room A is one second. What will be the time (in seconds) of reverberation of a room, having all the dimensions double of those of room A?

(a) 1 (b) 2 (c) 4 (d) 1/2

Q.51 Wave has simple harmonic motion whose period is 4 seconds while another wave which also possesses simple harmonic motion has its period 3 sec. If both are combined, then the resultant wave will have the period equal to

(a) 4 sec (b) 5 sec (c) 12 sec (d) 3 sec

Answers

1	b	2	c	3	b	4	c	5	a
6	c	7	c	8	a	9	c	10	
	ab								
11	c	12	d	13	c	14	a	15	c
16	c	17	b	18	d	19	a	20	b
21	b	22	a	23	c	24	a	25	c
26	c	27	a	28	a	29	d	30	a
31	a	32	b	33	a	34	a	35	c
36	a	37	c	38	c	39	d	40	b
41	a	42	b	43	b	44	b	45	b
46	a	47	a	48	c	49	c	50	b

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51 c

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