Electrostatics

Introduction

Electromagnetism is a science of the combination of electrical and magnetic phenomenon. Electromagnetism can be divided into 2 parts:

(1) **Electrostatics:** It deals with the study of charges at rest.

(2) **Electrodynamics:** It deals with the study of charges in motion (discusses magnetic phenomenon)

In this chapter we will be dealing with charges at rest i.e. Electrostatics.

CHARGE: The word electricity comes from the Greek work electron, which means "amber". Amber is petrified tree resin, and the ancients knew that if you rub an amber rod with a piece of cloth, the amber attracts small pieces of leaves or dust. A piece of hard rubber, a glass rod, or a plastic ruler rubbed with a cloth will also display this "amber effect or **static electricity** as we call it today. You can readily pick up small pieces of paper with a plastic comb or ruler that you've just vigorously rubbed with even a paper towel. You have probably experienced static electricity when combing your hair or when taking a synthetic blouse or shirt from a clothes dryer. And you may have felt a shock when you touched a metal doorknob after sliding across a car seat or walking across a nylon carpet. In each case, an object becomes "charged" due to a rubbing process and is said to possess a net electric charge.

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made. Charge is the physical property of certain fundamental particles (like electron, proton) by virtue of which they interact with the other similar fundamental particles. To distinguish the nature of interaction, charges are divided into two parts (i) positive (ii) negative. Like charges repel and unlike charges attract. Sl unit of charge is coulomb and CGS unit is esu. $1C = 3 \times 10^9$esu.

Magnitude of the smallest known charge is $e = 1.6 \times 10^{-19}$C (charge of one electron or proton).

Note: True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body.

Structure of Atom

An atom consists of two parts (i) nucleus and (ii) extra nuclear part. Nucleus consists of neutrons and protons and extra nuclear part has electrons revolving around nucleus.

In a neutral atom ⇒ Number of electrons = number of protons.

Charge of electron = charge of proton = 1.602×10^{-19} coulomb.

Normally positive charges are positron, proton and positive ions. In nature practically free existing positive charges are positive ions and negative charges are electrons.

Conductors and Insulators

An object can be broadly classified in either of the following two categories:

(i) Conductors (ii) Insulators

Conductors: These are the materials that allow flow of charge through them. This category generally comprises of metals but may sometimes contain non–metals too. (Carbon in form of graphite)

Insulators: These are the materials which do not allow movement of charge through them.

Charging of Bodies

An object can be charged by addition or removal of electrons from it. In general an object can either be a conductor or insulator. Thus we are going to discuss charging of a conductor and charging of an insulator.

Charging of Conductors

Conductors can be charged by

(a) Bringing it in contact with another charged conductor (charging by conduction)

(b) Bringing a charged body (conductor or insulator) near the conductor to be charged and there by inducing charge on it. This process is called induction.

Charging of Insulators

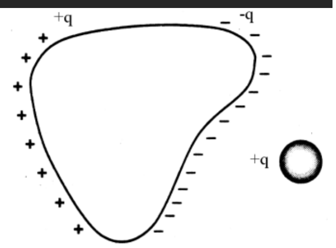
Since charge cannot flow through insulators, neither conduction nor induction can be used to charge insulators, so in order to charge an insulator friction is used. Whenever an insulator is rubbed against a body, exchange of electrons takes place between the two. This results in appearance of equal and opposite charges on the insulator and the other body. Thus the insulator is charged.

For example: rubbing of plastic with fur, silk with glass causes charging of these things.

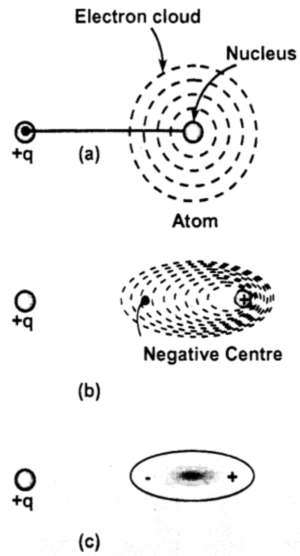
Note:

- To charge the bodies through friction one of them has to be an insulator.
- The cause of charging is the actual transfer of electrons from one material to another during rubbing. Protons are not transferred during rubbing.
- The material with lower work function loses electrons and becomes positively charged.
- As an electron has a finite mass, therefore, there always occurs some change in mass during charging.

Charged Induction (Elementary Concept)



We know every charge particle attracts another particle which is having opposite charge. This concepts leads to charge separation in extended bodies. We consider different cases of charge separation in conducting and non–conducting bodies. Consider the case of a conducting body placed near a charge as shown in figure. Due to the force exerted on free electrons of conducting body by external charge +q, these electrons start drifting toward +q. Due to this on the surface of body facing +q, negative charge –q is developed due to excess of electrons and on the opposite face of body a similar positive charge +q; is developed due to deficiency of electrons as shown in figure. This charge separation in a body is called "charge induction".

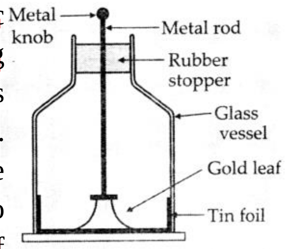


The alignment of dipoles in a non–conducting body appear due to an external charge is called induction in non–conducting bodies or polarization of non–conducting bodies. Thus in both the cases of conducting and non–conducting bodies when a positive charge is brought closer, opposite charges are developed in the bodies due to charge induction and the negative, induced charge is close to the external positive charge.

This phenomenon also explains why always a charged body attracts a neutral body. It is also obvious that in conductor's induction is more due to drift of free electrons hence attraction force on neutral conducing body will be more due to any external charge.

Note:

- **Gold–leaf electroscope:** It is a device used for detecting an electric charge and identifying its polarity. It consists of a vertical conducting rod passing through a rubber stopper fitted in the mouth of a glass vessel. Two thin gold leaves are attached to lower end of the rod. When a charged object touches the metal knob at the outer end of the rod, the charge flows down to the leaves. The leaves diverge due to repulsion of the like charges they have received. The degree of divergence of the leaves gives a measure of the amount of charge.



Various characteristics of charge are:

(i) Like charges repel while unlike charges attract.

(ii) Charge is invariant. (iii) Total charge is always conserved

(iv) Charge can be added algebraically (v) With transfer of charge mass is also transferred

(vi) Charge is quantized and minimum value of charge is 1.6×10^{-19}C.

Quantization of Charge: Charge exists in discrete packets rather than in continuous amount.

i.e. charge on any body is the integral multiple of the charge on an electron

$\Rightarrow$ $Q = \pm ne$, where n = 0, 1, 2, …………..

Note:

- The smallest amount of charge or basic quantum of charge is the charge on the electron or a proton.
  It is = 1.602192×10^{-19}C
- Recent discoveries in high energy physics have indicated that the elementary particles like protons and neutrons are themselves built out of more elementary units, called **quarks,** which have charges (2/3) e and (–1/3) e.
- Quantization is a universal law of nature. Like charge, energy and angular momentum of an electron are also quantized.

Conservation of Charge

Charge is conserved, i.e. total charge on an isolated system is constant. By isolated system, we mean here a system through the boundary of which no charge is allowed to escape or enter. This does not require that the amount of positive and negative charges separately conserved; only their algebraic sum is constant.

Note:

- Conservation of charge implies that electric charges can be created or destroyed always in the form of equal and opposite pairs but never in isolation.
- The law of conservation of charge is an exact law of nature. It is valid in all domains of nature.

Charges on a Conductor: Static charges reside on the surface of the conductor.

Subjective Assignment – I

Q.1 A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+ 19.2 \times 10^{-19}$.

(i) Find the number of electrons lost by glass rod.

(ii) Find the negative charge acquired by silk.

(iii) Is there transfer of mass from glass to silk? Given, $m_e = 9 \times 10^{-31}$ kg

Q.2 Estimate number of electrons in 100g of water. How much is total negative charge on these electrons?

Q.3 How much positive and negative charge is there in a cup of water?

Q.4 How many electrons will have a total electric charge of 1 coulomb?

Q.5 It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark, (denoted by U) of charge + (2/3) e and the 'down' quark (denoted by d) of charge (–1/3) e together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter). Suggest a possible quark composition of a proton and neutron.

Q.6 A comb drawn through person's hair on a dry day causes 10^{22} electrons to leave the person's hair and stick to the comb. Calculate the charge carried by the comb.

Q.7 If a body gives out 10^9 electrons every second, how much time is required to get total charge of 1C from it?

Answers

1. (i) 12 (ii) $– 19.2 \times 10^{-19}$ C (iii) 1.08×10^{-29} kg

2. 3.35×10^{25}, 5.35×10^6 C

3. 1.338×10^7 C

4. 6.25×10^{18}

5. p = UUd, n = Udd

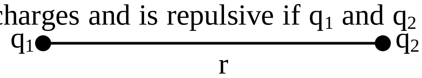
6. 1.6×10^3C

7. 198.18 year

Coulomb's Law

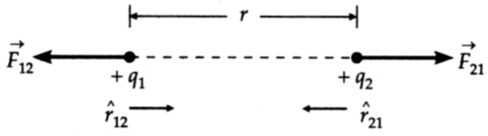
Coulomb, through his experiments found out that the two charges 'q_1' and 'q_2' kept at distance 'r' in a medium as shown in figure exert a force 'F' on each other. The value of force F is given by $F = k\dfrac{q_1 q_2}{r^2}$.

This law gives the net force experienced by q_1 and q_2 taking in account the medium surrounding them. Where F gives the magnitude of electrostatic force. q_1 and q_2 are the magnitudes of the two interacting charges. K is electrostatic constant which depends upon the medium surrounding the two charges. This force F acts along the line joining the two charges and is repulsive if q_1 and q_2 are of same sign and is attractive if they are of opposite sign.



Coulomb's Law in Vector Form

As shown in figure, consider two positive point charges q_1 and q_2 placed in vacuum at distance r from each other. They repel each other.



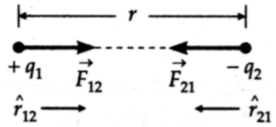
In vector form, Coulomb's law may be expressed as $\vec{F}_{21}$ = Force on charge q_2 due to q_1

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad \text{where } \hat{r}_{12} = \frac{\vec{r}_{12}}{r}, \text{ is a unit vector in the direction from } q_1 \text{ and } q_2.$$

Similarly, $\vec{F}_{12}$ = Force on charge q_1 due to q_2

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \qquad \text{where } \hat{r}_{21} = \frac{\vec{r}_{21}}{r}, \text{ is a unit vector in the direction from } q_2 \text{ to } q_1.$$

The coulombian forces between unlike charges ($q_1 q_2 < 0$) are attractive, as shown in figure.



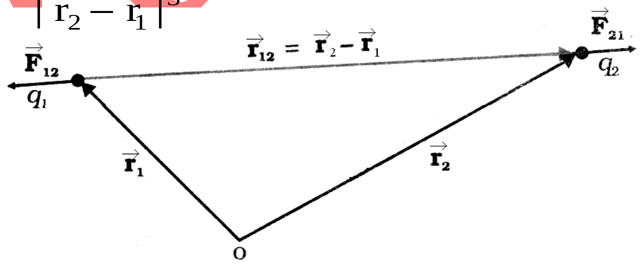
As $\hat{r}_{21} = -\hat{r}_{12}$, therefore $\vec{F}_{21} = -\vec{F}_{12}$. This means that the two charges exert equal and opposite force on each other. So *Coulombian forces obey Newton's third law of motion.*

Force acting on q_1 because of q_2 $\qquad \vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2}$, but $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

To write force as a vector this magnitude is to be multiplied by a unit vector in direction of force.

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \times \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} \qquad \text{or} \qquad \overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

Very similarly $\qquad \overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$



Force Depending on Medium

When two charges are placed in vacuum or when the same set of charges is placed in a medium, the net force experienced by the charges will be different. The effect of presence of medium is accounted in the proportionality constant K. This electrostatic constant K is defined as

$$K = \frac{1}{4\pi\varepsilon} \text{ where } \in = \in_0 \in_r$$

Here $\in$ = absolute permittivity of medium $\in_0$ = permittivity of free space, having a constant value = 8.85×10^{-12} coul^2/N – m^2, $\in_r = \in / \in_0$ = Relative permittivity of medium with respect to free space, also termed as dielectric constant.

For free space $\in_r = 1$ and $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ \frac{N-m^2}{coul^2}$

Here we can say that when two charges are placed in vacuum (or air) the force experienced by the charges can be given as

$$F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

When these charges are submerged in medium, having dielectric, constant $\in_r$, the force becomes

$$F_{med} = \frac{1}{4\pi\varepsilon_0 \in_r}\frac{q_1q_2}{r^2} \quad \text{Or} \quad F_{med} = \frac{F_{air}}{\in_r} \quad \text{as} \quad \in_r > 1 \Rightarrow F_{med} < F_{air}$$

Limitation of Coulomb's Law

Coulomb's Law have some limitations. These are

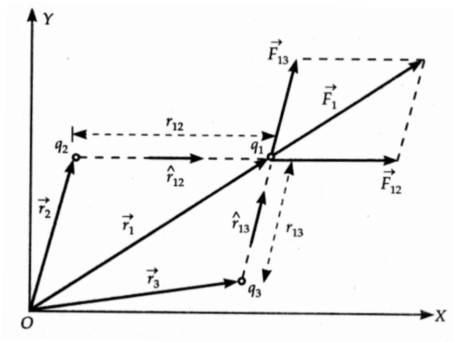
(1) It is only valid for point charges i.e. if we have two large conducting spheres having charges q_1 and q_2, then we cannot use coulomb's law to find the force between the two.

(2) Coulomb's Law is only valid for static charges.

Coulomb's Law gives the net force of interaction between the two charges. If the two charges are moving, both the charges will have an associated magnetic field. The net force will then be the vector sum of electrostatic force and the magnetic force.

Principle of Superposition

This principle tells us that if charge Q is acted upon by several charges q_1, q_2 … q_n, then the force on Q can be found out by calculating separately the forces F_1, F_2 …, F_n, exerted by q_1, q_2, … q_n respectively on Q and then adding these forces vectorially. Their resultant F is the total force on Q due to the collection of charges.



As shown in figure, consider N point charges q_1, q_2, q_3, ….., q_N placed in vacuum at points whose position vectors w.r.t. origin O are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots\ldots, \vec{r}_N$ respectively. According to the principle of superposition, the total force on charge q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots + \vec{F}_{1N}$$

where $\vec{F}_{12}, \vec{F}_{13}, \ldots, \vec{F}_{1N}$ are the forces exerted on charge q_1 by individual charges q_2, q_3, …., q_N respectively.

Subjective Assignment – II

Q.1 The electrostatic force of repulsion between two positively charged ions carrying equal charges is 3.7×10^{-9}N, when they are separated by distance of 5Å. How many electrons are missing from each ion?

Q.2 A free pith–ball A of 8g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to a second pith–ball B fixed 5 cm below the former ball so that the upper ball is stationary?

Q.3 Two similarly equally charged identical metal spheres A and B repel each other with a force of 2.0×10^{-5} N. A third identical uncharged sphere C is touched to A, then placed at the mid–point between A and B. Calculate the net electrostatic force on C.

Q.4 Two 'free' point charges +4e and +e are placed a distance 'a' apart. Where should a third point charge q be placed between them such that the entire system maybe in equilibrium? What should be the magnitude and sign of q? What type of a equilibrium will it be?

Q.5 A charge Q is to be divided on two objects. What should be the values of the charges on the two objects so that the force between the objects can be maximum?

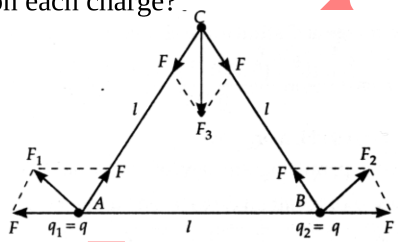
Q.6 A small brass sphere having a positive charge of 1.7×10^{-8} C is made to touch another sphere of the same radius having a negative charge of 3.0×10^{-9} C. Find the force between them when they are separated by a distance of 20 cm. What will be the force between them when they are immersed in an oil of dielectric constant 3?

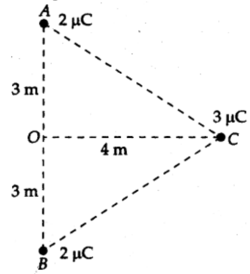
Q.7 The sum of two point charges is 7μC. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge.

Q.8 Two pith–balls each weighing 10^{-3} kg are suspended from the same point by means of silk threads 0.5m long. On charging the balls equally, they are found to repel each other to a distance of 0.2 m. Calculate the charge on each ball.

Q.9 An infinite number of charges each equal to 4 μC are placed along x–axis at x = 1m, x = 2m, x = 4m, x = 8m and so on. Find the total force on a charge of 1 C placed at the origin.

Q.10 Consider the charges q, q and – q placed at the vertices of an equilateral triangle, as shown in figure. What is the force on each charge?





Q.11 Two equal positive charges, each of 2μC interact with a third positive charge of 3μC situated as shown in figure. Find the magnitude and direction of the force experienced by the charge of 3μC.

Q.12 Four charges +q, + q, –q and – q are placed respectively at the four corners A, B, C and D of a square of side a. Calculate the force on a charge Q placed at the centre of the square.

Q.13 Five electrons have been removed from each atom to form ions. Find the electrostatic force between two such ions, when separated by a distance of 4Å in air.

Q.14 Two extremely small charged copper spheres have their centres separated by a distance of 50 cm, in vaccum.

(a) What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C?

(b) What will be the force of repulsion if

(i) the charge on each sphere is doubled and their separation is halved?

(ii) the two spheres are placed in water? (Dielectric constant of water = 80).

Answers

1.	2	2.	4.36×10^{-7} C (positive)	3.	2.0×10^{-5} N, along BC
4.	2a/3, 4e/9, unstable	5.	Q/2, Q/2		
6.	1.1×10^{-5} N; 0.367×10^{-5} N			7.	5 μC, 2μC
8.	2.357×10^{-6} C	9.	4.8×10^{4} N	10.	$F\hat{BC}$, $F\hat{AC}$, $\sqrt{3}\,F\,\hat{n}$
11.	3.456×10^{-3} N, along OC produced				

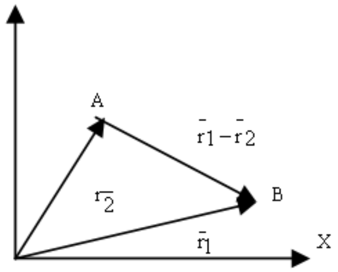
12. $\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{4\sqrt{2}\,Qq}{a^2}, \text{ parallel to AD or BC}\right)$

13. 3.6×10^{-8}N

14. (a) 15.21×10^{-3} N, (b) (i) $15.21\times 16 \times 10^{-3}$ N, (ii) 1.9×10^{-4} N

Electric Field

Electric field due to a point charge is the space surrounding it, within which electric force can be experienced by the another charge. Electric field strength or Electric intensity $(\vec{E})$ at a point is the electric force experienced by a unit test charge at that point.



Mathematically, $\vec{E} = \dfrac{\vec{F}}{q_o}$, where q_o is positive test charge.

SI unit is Newton per coulomb (NC^{-1}) or volt per metre (Vm^{-1}).

The dimensions for $\vec{E}$

$$[E] = \frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{C} = \frac{MLT^{-2}}{I.T} = [MLT^{-3}I^{-1}]$$

Physical Significance of Electric Field: The force experienced by the test charge q_0 is different at different points. So $\vec{E}$ also varies from point to point.

i.e., **Electrostatic force= Charge × Electric field.** Thus an electric field plays an intermediary role in the forces between two charges:

$$\text{Charge} \rightleftharpoons \text{Electric field} \rightleftharpoons \text{Charge.}$$

Electric field is a characteristic of the system of charges and is independent of the test charge that we place at a point to determine the field.

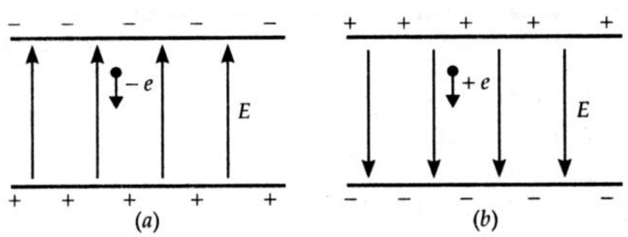
Subjective Assignment – III

Q.1 Calculate the electric field strength required to just support a water drop of mass 10^{-3} kg and having a charge 1.6×10^{-19} C.

Q.2 How many electrons should be removed from a coin of mass 1.6g, so that it may just float in an electric field of intensity 10^9 NC^{-1}, directed upward?

Q.3 A pendulum of mass 80 milligram carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 2×10^4 Vm^{-1}. Find the tension in the thread of the pendulum and the angle it makes with the vertical.

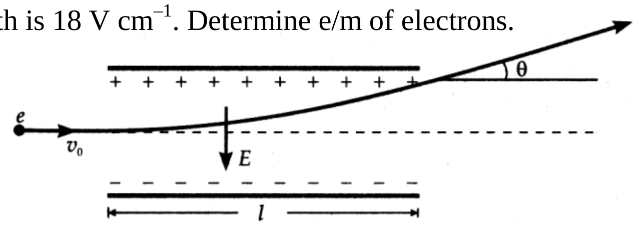
Q.4 An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 NC^{-1}. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance. Compute the time of fall in each



case. Contrast the situation (a) with that of 'free fall under gravity.

Q.5 A stream of electrons moving with a velocity of 3×10^7 ms^{-1} is deflected by 2 mm in traversing a distance of 0.1m in a uniform electric field of strength is 18 V cm^{-1}. Determine e/m of electrons.

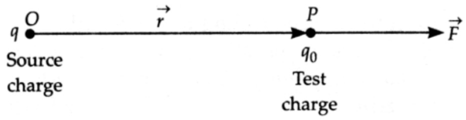
Q.6 An electric field E is set up between the two parallel plates of a capacitor, as shown in figure. An electron enters the field symmetrically between the plates with a speed v_0. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field.



Answers

1.	6.125×10^{12} NC^{-1}	2.	9.8×10^7	3.	8.81×10^{-4} N, $\theta = 27^o$
4.	2.9×10^{-9} s, 1.25×10^{-7} s	5.	2×10^{11} C kg^{-1}	6.	$\theta = \tan^{-1}\dfrac{eEl}{mv_0^2}$

Electric field due to a point charge



According to Coulomb's Law, the force on charge q_0 is $\vec{F} = \dfrac{1}{4\pi\varepsilon_0} . \dfrac{qq_0}{r^2}\hat{r}$

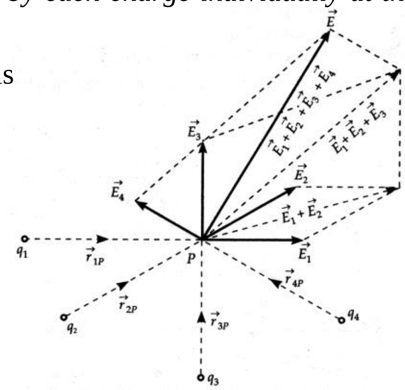
where $\hat{r}$ is a unit vector in the direction from q to q_0. Electric field at point P is $\vec{E} = \dfrac{\vec{F}}{q_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$

The magnitude of the field $\vec{E}$ is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

Electric field due to a system of point charges

According to ***principal of superposition of electric field,*** *the electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.*

Hence, the electric field at point P due to the system of N charges is



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ...+ \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + ...+ \frac{q_N}{r_{NP}^2}\hat{r}_{NP}\right]$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{qi}{q_{ip}^2}\hat{r}_{iP}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r} - \hat{r}_i|^2} . \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r} - \hat{r}_i|^3}(\vec{r} - \vec{r}_i)$$

Subjective Assignment – IV

Q.1 Assuming that the charge on an atom is distributed uniformly in a sphere of radius 10^{-10}m, what will be the electric field at the surface of the gold atom? For gold, Z = 79.

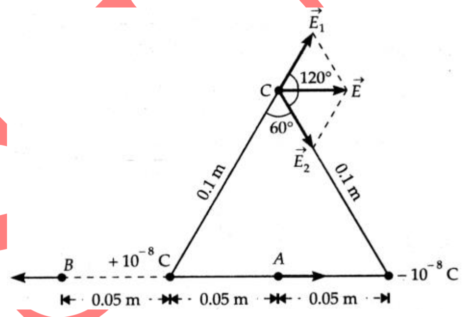
Q.2 Two point charges of 2.0×10^{-7} C and 1.0×10^{-7} C are 1.0 cm apart. What is the magnitude of the field produced by either charge at the site of the other?

Q.3 Two point charges of $+5\times10^{-19}$ C and $+20\times10^{-19}$C are separated by a distance of 2m. Find the point on the line joining them at which electric field intensity is zero.

Q.4 Two point charges of + 16 μC and – 9μC are placed 8 cm apart in air. Determine the position of the point at which the resultant field is zero.

Q.5 Two point charges q_1 = + 0.2 C and q_2 = + 0.4 C are placed 0.1 m apart. Calculate the electric field at
(a) the mid–point between the charges
(b) a point on the line joining q_1 and q_2 such that it is 0.05 m away from q_2 and 0.15 m away from q_1.

Q.6 Two point changes q_1 and q_2 of 10^{-8} C and -10^{-8} C respectively are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in figure.



Q.7 ABCD is a square of side 5 m. Charges of + 50C, – 50C and + 50C are placed at A, C and D respectively. Find the resultant electric field at B.

Q.8 Four charges +q, +q, –q, –q are placed respectively at the four corners A, B, C and D of a square of side 'a'. Calculate the electric field at the centre of the square.

Answers

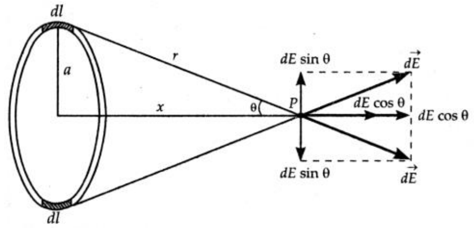
1. 1.138×10^{13} NC^{-1}
2. 1.8×10^{7} NC^{-1}, 9×10^{6} NC^{-1}
3. at 2/3 m to the right of q_1 charge
4. at a point 24 cm to the right of – 9μC
5. (a) 7.2×10^{11} NC^{-1}, acting along BO, (b) 1.52×10^{12} NC^{-1}, acting along AP
6. 7.2×10^{4} NC^{-1}, 3.2×10^{4} NC^{-1}, 9×10^{3} NC^{-1}
7. 2.7×10^{10} NC^{-1}, 25.5° with x–axis
8. $4\sqrt{2}\,k\dfrac{q}{a^2}$, 45° with AC

Electric field due to charged ring along its axis

Consider a differential element of the ring of length ds. Charge on this element is $dq = \left(\dfrac{q}{2\pi a}\right)dl$.

This element sets up a differential element field $d\bar{E}$ at point P.

The resultant field $\vec{E}$ at P is found by integrating the effects of all the elements that make up the ring. From symmetry this resultant field must lie along the right axis. Thus, only the component of $d\bar{E}$ parallel to this axis contributes to the final result.



$$\vec{E} = \int d\vec{E} \qquad \Rightarrow \qquad \vec{E} = \int d\vec{E}\cos\theta$$

$$dE = \frac{1}{4\pi \in_o}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon \in_o}\left(\frac{qds}{2\pi a}\right)\cdot\frac{1}{(a^2+x^2)}$$

$$\cos\theta = \frac{x}{(a^2+x^2)^{1/2}}$$

To find the total x–component E_x of the field at P, we integrate this expression over all segment of the ring.

$$E_x = \int d\vec{E}\cos\theta = \frac{1}{4\pi\epsilon_o}\frac{x}{2\pi a.(a^2+x^2)^{3/2}}\int dl$$

The integral is simply the circumference of the right $2\pi a$. $$E = \frac{1}{4\pi\epsilon_o}\frac{qx}{(a^2+x^2)^{3/2}}$$

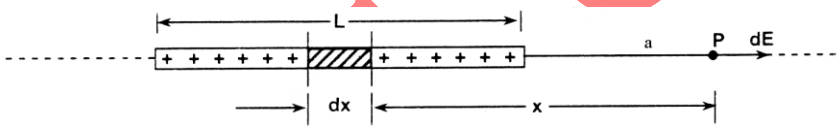
As q is positive charge, field is directed away from the centre of ring, along its axis.

Electric field intensity due to uniformly charged rod along it axis.

Consider an element, dx at a distance, x from the point, P, where we seek to find the electric field. The elemental charge, $dq = \lambda dx$

Then, $dE = k.\frac{\lambda dx}{x^2}$ or $E = k\lambda\int_a^{a+L}\frac{1}{x^2}dx = k\lambda\left[-\frac{1}{x}\right]_a^{a+L}$

$$= k\lambda\left[\frac{-1}{a+L}+\frac{1}{a}\right]$$

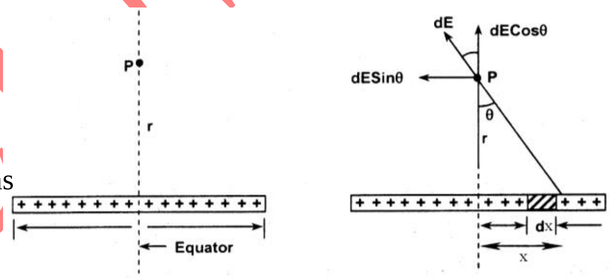


$$\vec{E} = \frac{k\lambda L}{a(a+L)}$$

Electric field intensity at a equatorial point due to a uniformly charged rod

The charge on small element is

$$dq = \frac{Q}{L}dx$$



If the strength of electric field at point P due to this charge dq is dE, then it can be given as

$$dE = \frac{Kdq}{(r^2+x^2)}$$

The component dE sin θ will get cancelled and net electric field at point P will be due to integration of dE cosθ only.

Thus net electric field strength at point P can be given as

$$E_p = \int dE\cos\theta = \int_{-L/2}^{+L/2}\frac{KQdx}{(r^2+x^2)}\times\frac{r}{\sqrt{r^2+x^2}} = \frac{KQr}{L}\int_{-L/2}^{+L/2}\frac{dx}{(r^2+x^2)^{3/2}}$$

Now substituting $x = r\tan\theta$ $dx = r\sec^2\theta\, d\theta$

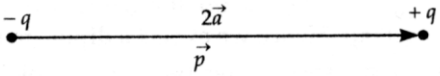
We get $$E_p = \frac{KQr}{L}\int\frac{r\sec^2\theta\, d\theta}{r^3\sec^3\theta} = \frac{KQ}{Lr}\int\cos\theta\, d\theta = \frac{KQ}{Lr}[\sin\theta]$$

We get $$E_p = \frac{KQr}{Lr}\left[\frac{x}{\sqrt{x^2+r^2}}\right]_{-L/2}^{+L/2} = \frac{KQ}{Lr}\left[\frac{L/2}{\sqrt{\frac{L^2}{4}+r^2}}+\frac{L/2}{\sqrt{\frac{L^2}{4}+r^2}}\right]$$ $$E_p = \frac{2kQ}{r\sqrt{L^2+4r^2}}$$

Electric Dipole

Electric dipole: *A pair of equal and opposite charges separated by a small distance is called an electric dipole.*

Dipole moment: It measures the strength of an electric dipole. *The dipole moment of an electric dipole is a vector whose magnitude is product of either charge times the separation between the two opposite charges and the direction is along the dipole axis from the negative to the positive charge.*



Dipole moment = Either charge × a vector drawn from negative to positive charge

or $\vec{p} = q \times 2\,\vec{a}$

The SI unit of dipole moment is *coulomb metre* (Cm).

Examples of electric dipoles. Dipoles are common in nature. In molecules like H_2O, HCl, C_2H_5OH, CH_3COOH, etc., the centre of positive charges does not fall exactly over the centre of negative charges. Such molecules are electric dipoles. They have a permanent dipole moment.

Ideal or point dipole. *Dipole of negligibly small size is called an ideal or point dipole.*

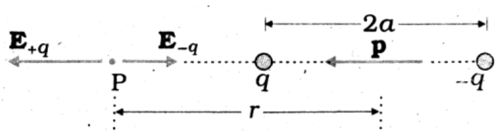
(i) Electric field due to electric dipole for points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in figure. Then

$$E_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{p}$$

where $\hat{p}$ is the unit vector along the dipole axis (from –q to q). Also

$$E_{+q} = -\frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p}$$



The total field at P is

$$E = E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p} = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

For r > > a

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} \quad (r >> a) \ \text{ but } 2aq = P$$

$$\therefore\ \vec{E}_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}\hat{P}$$

(ii) Electric field due to electric dipole for points on the equatorial plane

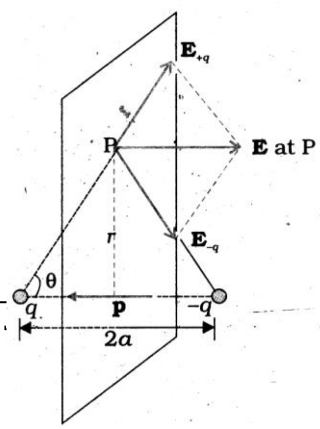
The magnitudes of the electric fields due to the two charges +q and –q are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$ and are equal.

The directions of E_{+q} and E_{-q} are as shown in figure. Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to $\hat{p}$.

We have

$$E = -(E_{+q} + E_{-q})\cos\theta$$



$$= -\frac{2qa}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}} \hat{p}$$

At large distances (r > > a), this reduces to

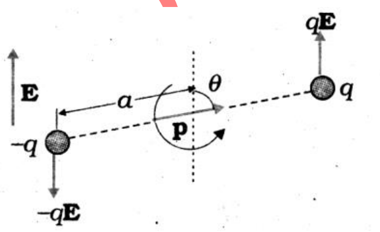
$$E = -\frac{2qa}{4\pi\varepsilon_0 r^3}\hat{p} \quad (r >> a) \qquad \text{But} \qquad p = q \times 2a$$

$$\vec{E}_{eq} = -\frac{P}{4\pi\varepsilon_0 r^3} \hat{P}(r >> a)$$

Hence $\vec{E}_{ax} = -2\vec{E}_{eq}$

Dipole in a Uniform External Field

There is a force qE on q and a force – qE on –q. The net force on the dipole is zero, since E is uniform. The forces act at different points, resulting in a torque on the dipole.



Magnitude of torque $= qE \times 2a\sin\theta = 2qaE\sin\theta$

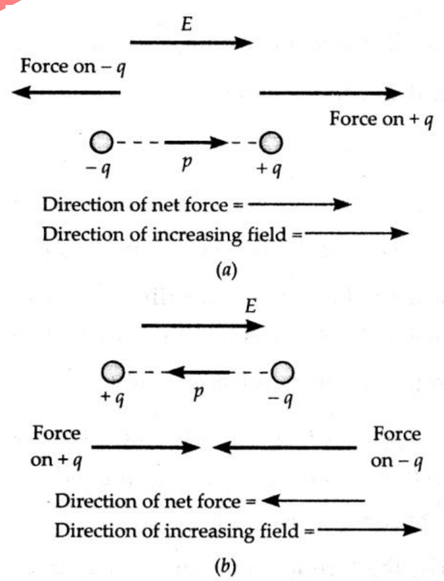
Its direction is normal to the plane of the paper, coming out of it.

But 2aq = P

$\therefore \quad \tau = PE\sin\theta \qquad$ Thus, $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

Dipole in a non–uniform electric field: In a non–uniform electric field, the +q and –q charges of a dipole experience different forces (not equal and opposite) at slightly different positions in the field and hence a net force $\vec{F}$ acts on the dipole in a non–uniform field. Also, a net torque acts on the dipole which depends on the location of the dipole in the non–uniform field. $\qquad \vec{\tau} = \vec{p} \times \vec{E}(\vec{r})$

where $\vec{r}$ is the position vector of the centre of the dipole.

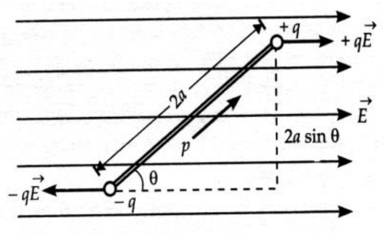


When the dipole is parallel or antiparallel to $\vec{E}$. In a non–uniform field, if $\vec{p}$ is parallel to $\vec{E}$ or antiparallel to $\vec{E}$, the net torque on the dipole is zero (because the forces on charges $\pm$ q become linear) However, there is a net force on the dipole. As shown in figure, when $\vec{p}$ is parallel to $\vec{E}$, a net force acts on the dipole in the direction of increasing $\vec{E}$. When $\vec{p}$ is antiparallel to $\vec{E}$, a net force acts in the direction of decreasing $\vec{E}$.

Potential energy of a dipole placed in a uniform electric field.

As shown in figure, consider an electric dipole placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ making an angle θ with the field. Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends. The two forces form a couple. The torque exerted by the couple will be $\qquad \tau = qE \times 2a\sin\theta = pE\sin\theta$

where q × 2a = p, is the dipole moment.



If the dipole is rotated through a small angle dθ against the torque acting on it, then the small work done is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

The total work done in rotating the dipole from its orientation making an angle θ_1, with the direction of the field to θ_2 will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE[-\cos\theta]_{\theta_1}^{\theta_2} = pE[\cos\theta_1 - \cos\theta_2]$$

This work done is stored as the potential energy U of the dipole.

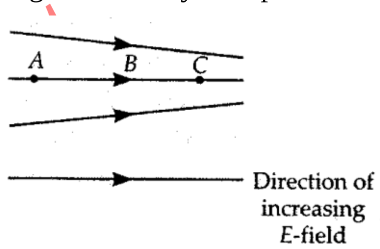
$\therefore \quad U = pE(\cos\theta_1 - \cos\theta_2)$

If initially the dipole is oriented perpendicular to the direction of the field ($\theta_1 = 90^o$) and then brought to some orientation making an angle θ with the field ($\theta_2 = \theta$), then potential energy of the dipole will be

$U = pE(\cos 90^o - \cos\theta) = pE(0 - \cos\theta)$ or $U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

Note:

- In a uniform electric field, an electric dipole experiences no net force but a non zero torque.
- As the net force on a dipole in a uniform electric field is zero, therefore, no linear acceleration is produced.
- Torque on a dipole becomes zero when it aligns itself parallel to the field.
- Torque on a dipole is maximum when it is held perpendicular to the field $\vec{E}$.
- In a non–uniform electric field, a dipole experiences a non zero force and non zero torque. In the special case when the dipole moment is parallel or antiparallel to the field, the dipole experiences a zero torque and a non zero force.
- A non–uniform or specifically an increasing E–field may be represented by field lines as shown. Clearly, $E_A < E_B < E_C$



- The direction of the electric field at an axial point of an electric dipole is same as that of its dipole moment and at an equatorial point it is opposite to that of dipole moment.
- The strength of electric field at an axial point of a short dipole is twice the strength at the same distance on the equatorial line.
- At larger distances, the dipole field ($E \propto 1/r^3$) decreases more rapidly than the electric field of a point charge ($E \propto 1/r^2$)

Subjective Assignment – V

Q.1 Two charges, one $+5\mu C$ and another $-5\mu C$ are placed 1 mm apart. Calculate the dipole moment.

Q.2 An electric dipole, when held at 30^o with respect to a uniform electric field of $10^4\ NC^{-1}$ experiences a torque of 9×10^{-26} Nm. Calculate dipole moment of the dipole.

Q.3 An electric dipole consists of two opposite charges of magnitude $1/3 \times 10^{-7}$ C, separated by 2cm. The dipole is placed in an external field of 3×10^{7} NC^{-1}. What maximum torque does the electric field exert on the dipole?

Q.4 Calculate the electric field due to an electric dipole of length 10 cm having charges of 1 μC at an equatorial point 12 cm from the centre of the dipole.

Q.5 Two point charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity at a point distant 4 cm from the midpoint on the axial line of the dipole.

Q.6 Two charges $\pm$ 10 μC are placed 5.00 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole.

Q.7 The force experienced by a unit charge when placed at a distance of 0.10m from the middle of an electric dipole on its axial line is 0.025 N and when it is placed at a distance of 0.2 m, the force is reduced to 0.002 N. Calculate the dipole length.

Q.8 An electric dipole of moment 5×10^{-8} C–m is aligned in a uniform electric field of 1.44×10^{4} N/C. Calculate potential energy of the dipole to hold the dipole at 60^{o} with the direction of electric field.

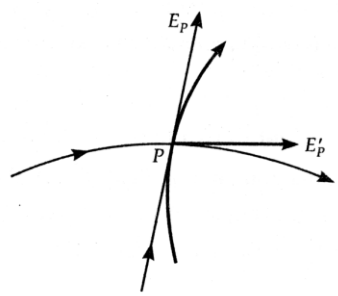
Q.9 An electric dipole consists of two opposite charges of magnitude $q = 1 \times 10^{-6}$ C separated by 2.0 cm. The dipole is placed in an external field of 1×10^{5} NC^{-1}. What maximum torque does the field exert on the dipole? How much work must an external agent do to turn the dipole from one end to another end, starting from position of alignment ($\theta = 0^{o}$)?

Q.10 Two charges of – 4 μC and + 4μC are placed at the points A (1, 0, 4) and B (2, –1, 5) located in an electric field $\vec{E} = 0.02\hat{i}$ V/cm. Calculate the torque acting on the dipole.

Answers

1.	5×10^{-9} Cm	2.	1.8×10^{-29} Cm	3.	0.02 Nm
4.	4.096×10^{5} NC^{-1}	5.	10^{8} NC^{-1}		
6.	(a) 2.66×10^{5} NC^{-1}, (b) 1.33×10^{5} NC^{-1}	7.	0.10 m	8.	3.6×10^{-4} J
9.	2×10^{-3} N–m, 4×10^{-3} J	10.	1.131×10^{-5} N–m		

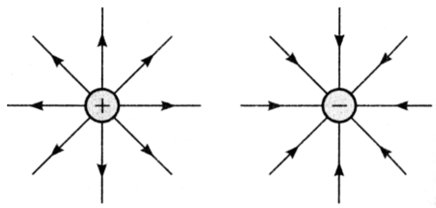
Lines of Force



It has been found quite convenient to visualize the electric field pattern in terms of lines of force. The electric field pattern vector at a point is related to imaginary lines of force in two ways. The line of force in an electric field is a curve such that the tangent at any point on it gives the direction of the resultant electric field strength at that point.

(i) The lines of force are continuous smooth curves without any breaks.

(ii) Tangent to the line of force at a point gives the direction of $\vec{E}$.



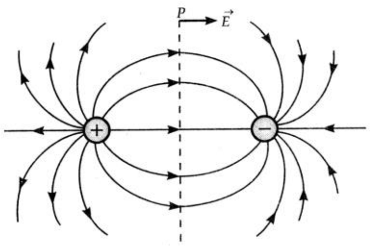
(iii) These lines of force are so drawn that their number per unit cross–sectional area in a region is proportional to intensity of electric field.

(iv) Electric line of force can never form closed loops.

(v) Lines of force are imaginary.

(vi) They emerge from a positive charge and terminate on a negative charge.

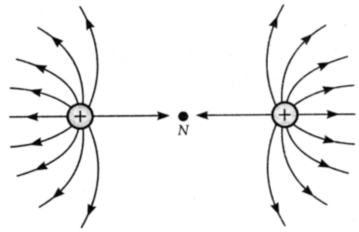
(vii) Lines of force do not intersect.



Reason: If they intersect, then there will be two tangents at the point of intersection (figure) and hence two directions of the electric field at the same point, which is not possible.

(viii) The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

(ix) The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.



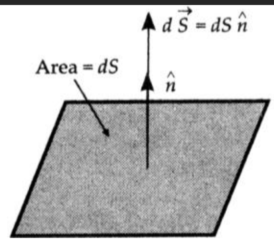
(x) The lines of force have a tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.

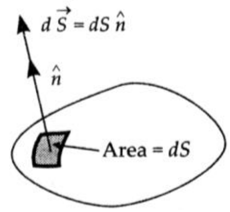
(xi) The relative closeness of the lines of force gives a measure of the strength of the electric field in any region.

(xii) The lines of force do not pass through a conductor because the electric field inside a charged conductor is zero.

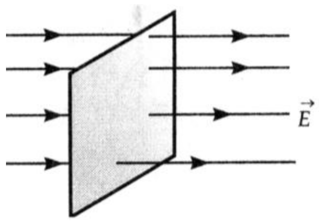
Area Vector

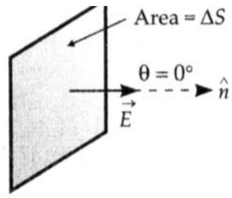
The direction of a planar area vector is specified by the normal to the plane. In figure, a planar area element dS has been represented by a normal vector $\overrightarrow{dS}$. The length of vector $\overrightarrow{dS}$ represents the magnitude dS of the area element. If $\hat{n}$ is a unit vector along the normal to the planar area, then $\overrightarrow{dS} = dS\,\hat{n}$





*The **electric flux** through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.*

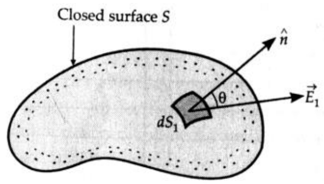




As shown in figure, if an electric field $\vec{E}$ passes normally through an area element ΔS, then the electric flux through this area is $\Delta\phi_E = E\ \Delta S$

In case the field $\vec{E}$ is non–uniform, we consider a closed surface S lying inside the field, as shown in figure. We can divide the surface S into small area elements: $\Delta\vec{S}_1, \Delta\vec{S}_2, \Delta\vec{S}_3, \ldots.., \Delta\vec{S}_N$. Let the corresponding electric fields at these elements be $\vec{E}_1, \vec{E}_2, \ldots., \vec{E}_N$

Then the electric flux through the surface S will be



$$\phi_S = \vec{E}_1 . \Delta\vec{S}_1 + \vec{E}_2 . \Delta\vec{S}_2 + \ldots.. + \vec{E}_N . \Delta\vec{S}_N$$

$$= \sum_{i=1}^{N} \vec{E}_i . \Delta\vec{S}_i$$

When the number of area elements becomes infinitely large $(N \to \infty)$ and $\Delta S \to 0$, the above sum approaches a surface integral taken over the closed surface. Thus

$$\phi_E = \lim_{\substack{N\to\infty \\ \Delta S\to 0}} \sum_{i=1}^{N} \vec{E}_i . \overrightarrow{\Delta S_i} = \oint_S \vec{E}. \overrightarrow{dS}$$

Hence electric flux may be defined as the surface integral of electric field of any closed surface.

Electric flux is a *scalar* quantity.

SI unit of electric flux = $NC^{-1} . m^2 = Nm^2 C^{-1}$ **= Vm**

Gauss's theorem

This theorem gives a relationship between the total flux passing through any *closed* surface and the net charge enclosed within the surface.

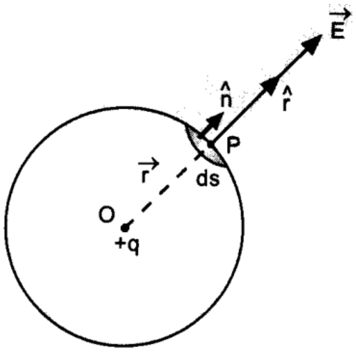
***Gauss theorem** states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.* Mathematically, it can be expressed as

$$\phi_E = \oint_S \vec{E} . \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

Gaussian surface: *Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.* It is chosen to evaluate the surface integral of the electric field produced by the charge enclosed by it, which, in turn, gives the total flux through the surface.

Proof of Gauss's Theorem: *(for spherically symmetric surfaces only)* consider a sphere of radius r, positive point charge q is situated at the centre. According to Coulomb's law, electric field intensity at any point P on the surface of the sphere is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}$$



where $\hat{r}$ is unit vector directed from O to P.

Consider a small area element ds of sphere around P. Let it be represented by vector $\overrightarrow{ds} = \hat{n}\, ds$ where $\hat{n}$ is unit vector along outdrawn normal to area element.

$$\therefore \qquad \vec{E}. \overrightarrow{ds} = \left(\frac{q}{4\pi \in_0} \frac{\hat{r}}{r^2}\right). (\hat{n}\ \overrightarrow{ds})$$

As normal to a sphere at every point is $\vec{E}$ along the radius vector at the point, therefore, $\hat{r}. \hat{n} = 1$

$$\vec{E}. \overrightarrow{ds} = \frac{q}{4\pi \in_0} \frac{ds}{r^2}$$

Integrating over the closed surface area of the sphere, we get

$$\oint_S \vec{E}. \overrightarrow{ds} = \frac{q}{4\pi \in_0 r^2} \oint_S ds = \frac{q}{4\pi \in_0 r^2} \times \text{total area of surface of sphere}$$

$$= \frac{q}{4\pi \in_0 r^2} (4\pi r^2) = \frac{q}{\in_0}$$

Hence, $\qquad \phi_E = \oint_S \vec{E}. \overrightarrow{ds} = \frac{q}{\in_0}$ $\qquad$ where proves Gauss's theorem.

Note:

1. Remember that Gauss's theorem holds good for any closed surface, regardless of its shape or size.
2. The surface that we choose for the application of Gauss's law is called the Gaussian surface.
3. Take care to see that the Gaussian surface chosen does not pass through any discrete charge. This is because electric field is not well defined at the location of the charge.
4. In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field $\vec{E}$ (whose flux is calculated) is due to all the charges, both inside and outside the closed surface. However, the term (q) represents only the total charge inside the closed surface.
5. Gauss's theorem is based on inverse square dependence of $\vec{E}$ on distance.

Coulomb's Law from Gauss's Theorem

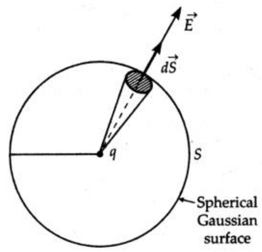
Deduction of Coulomb's Law from Gauss's Theorem

By symmetry, $\vec{E}$ has same magnitude at all points on S. Also $\vec{E}$ and $\overrightarrow{dS}$ at any point on S are directed radially outward. Hence flux through area $\overrightarrow{dS}$ is $d\phi_E = \vec{E}.\overrightarrow{dS} = EdS\cos 0^o = EdS$

Net flux through closed surface S is

$$\phi_E = \oint_S \vec{E}.\overrightarrow{dS} = \oint_S E\,dS = E\oint_S dS = E \times \text{total surface area of } S = E \times 4\pi r^2$$

Using Gauss's theorem,



$$\phi_E = \frac{q}{\varepsilon_0} \qquad \text{or} \qquad E\times 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad \text{or} \qquad E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$$

The force on the point charge q_0 if placed on surface S will be

$$E = q_0\ E = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

This proves the Coulomb's law.

Subjective Assignment – VI

Q.1 If $\vec{E} = 6\hat{i} + 3\hat{j} + 4\hat{k}$, calculate the electric flux through a surface of area 20 units in Y–Z plane.

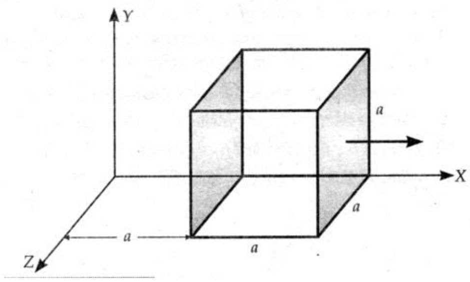
Q.2 A circular plane sheet of radius 10 cm is placed in a uniform electric field of 5×10^5 NC^{-1}, making an angle of 60^o with the field. Calculate electric flux through the sheet.

Q.3 Calculate the number of electric lines of force originating from a charge of 1 C.

Q.4 A positive charge of 17.7 μC is placed at the centre of a hollow sphere of radius 0.5m. Calculate the flux density through the surface of the sphere.

Q.5 Calculate the electric flux through each of the six faces of a closed cube of length *l*, if a charge q is placed (a) at its centre and (b) at one of its vertices.

Q.6 The electric field components in figure are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 800$ N/Cm^2. Calculate (i) the flux ϕ_E through the cube and (ii) the charge within the cube. Assume that a = 0.1 m.



Q.7 An electric field is uniform, and in the positive x direction for positive x and uniform with the same magnitude in the negative x direction for negative x. It is given that

$$\vec{E} = 200\hat{i}\ NC^{-1}\ \text{for}\ x > 0 \qquad \text{and} \qquad \vec{E} = -200\hat{i}\ NC^{-1}\ \text{for}\ x < 0$$

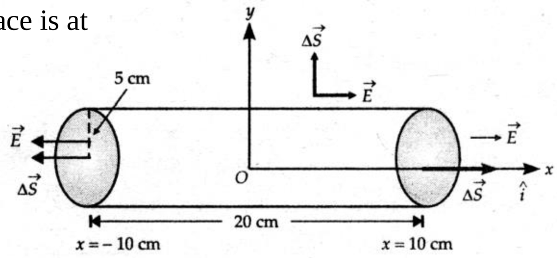
A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x–axis so that one face is at x = + 10 cm and the other is at x = – 10 cm.

(i) What is the net outward flux through each flat face?

(ii) What is the flux through the side of the cylinder?

(iii) What is the net outward flux through the cylinder?

(iv) What is the net charge inside the cylinder?

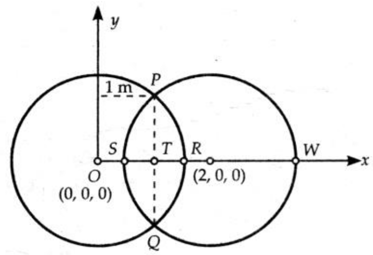


Q.8 You are given a charge + Q at the origin O. Consider a sphere S with centre (2, 0, 0) of radius $\sqrt{2}$m. Consider another sphere of radius $\sqrt{2}$m centered at the origin. Consider the spherical caps (i) PSQ (ii) PRQ (iii) PWQ, with normals outward to the respective spheres, and (iv) the flat circle PTQ with normal along the x–axis.

(a) What is sign of electric flux through each of the surface (i) – (iv)?

(b) What is the relation between the magnitudes of fluxes through surfaces (i) – (iv)?

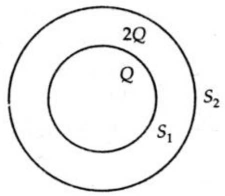
(c) Calculate the flux through the surface (ii) directly. Assume that the area of the cap (ii) is A.



Q.9 S_1 and S_2 are two hollow concentric spheres enclosing charges Q and 2Q respectively as shown in figure

(i) What is the ratio of the electric flux through S_1 and S_2?

(ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air?



Answers

1.	120 units	2.	$1.36 \times 10^4\ Nm^2\ C^{-1}$	3.	1.129×10^{11}

4. $6.4 \times 10^5\ NC^{-1}$ 5. $\frac{q}{6\varepsilon_0}, \frac{q}{24\varepsilon_0}$ 6. $1.05\ Nm^2\ C^{-1}$, 9.27×10^{-12} C

7. (i) + $1.57\ Nm^2\ C^{-1}$, (ii) 0 (iii) $3.14\ Nm^2\ C^{-1}$, (iv) 2.78×10^{-11} C

8. (a) The outward drawn normal on cap PSQ points towards left while it points towards right for caps PRQ, PWQ and circle PTQ. So the flux is negative for (i) and positive for the rest.

(b) The same electric field lines crossing (i) also cross (ii), (iii). Also, by Gauss's law, the fluxes through (iii) and (iv) add upto zero. Hence, all magnitudes of fluxes are equal.

(c) 4.5×10^9 QA $NC^{-1}m^2$ 9. (i) 1 : 3, (ii) $Q/5\varepsilon_0$

Field Due to an Infinitely Long Charged Wire

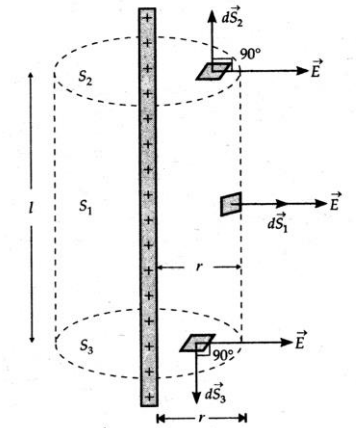
To determine the field at a distance r from the line charge, we choose a cylindrical Gaussian surface of radius r, length *l* and with its axis along the line charge. As shown in figure, it has curved surface S_1 and flat circular ends S_2 and S_3. Obviously, $\vec{dS}_1 \parallel \vec{E}$, $\vec{dS}_2 \perp \vec{E}$ and $\vec{dS}_3 \perp \vec{E}$. So only curved surface contributes towards the total flux.

$$\phi_E = \oint_S \vec{E}.\vec{dS}$$

$$= \oint_{S_1} \vec{E}.\vec{dS}_1 + \int_{S_2} \vec{E}.\vec{dS}_2 + \int_{S_3} \vec{E}.\vec{dS}_3$$

$$= \int_{S_1} EdS_1 \cos 0^o + \int_{S_2} EdS_2 \cos 90^o + \int_{S_3} EdS_3 \cos 90^o$$

$$= E\int dS_1 + 0 + 0$$



= E × area of the curved surface

or $\phi_E = E \times 2\pi r l$

Charge enclosed by the Gaussian surface, $q = \lambda l$

Using Gauss's theorem, $\phi_E = q/\varepsilon_0$, we get

or $E.2\pi rl = \frac{\lambda l}{\varepsilon_0}$ or $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ $\Rightarrow E \propto \frac{1}{r}$

Electric Field Due to a Uniformly Charged Infinite Plane Sheet

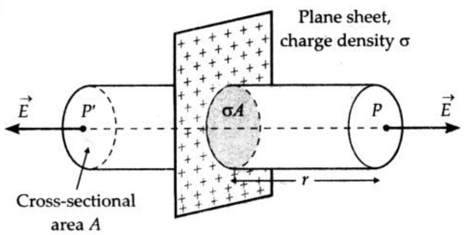
Consider a thin, infinite plane sheet of charge with uniform surface charge density σ. By symmetry, electric field E points outwards normal to the sheet. Also, it must have same magnitude and opposite direction at two point P and P'. We choose cylindrical Gaussian surface of cross–sectional area A and length 2r with its axis perpendicular to the sheet.

As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane–end faces of the cylinder is

$$\phi_E = EA + EA = 2\ EA$$

Charge enclosed by the Gaussian surface, $q = \sigma A$

According to Gauss's theorem,



$$\phi_E = \frac{q}{\varepsilon_0}$$

$\therefore \quad 2\,EA = \frac{\sigma A}{\varepsilon_0} \quad$ or $\quad E = \frac{\sigma}{2\varepsilon_0}$

<u>If the plane sheet has a finite thickness</u> then charges on both sides of the sheet are to be considered. Thus, electric field $E = \frac{\sigma}{\varepsilon_0}$.

<u>If plane sheet is conducting</u> then charges on both sides of sheet are to be considered. Thus, electric field

$$E = \frac{\sigma}{\varepsilon_0}$$

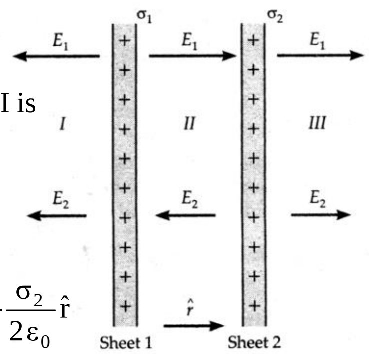
Solved Question

Q.1 *Two infinite parallel planes have uniform charge densities of σ_1 and σ_2. Determine the electric field at points (i) to the left of the sheets, (ii) between them, and (iii) to the right of the sheets.*

Electric field of two positively charged parallel plates. Figure shows two thin plane parallel sheets of charge having uniform charge densities σ_1 and σ_2 with $\sigma_1 > \sigma_2 > 0$. Suppose $\hat{r}$ is a unit vector pointing from left to right.

In the region I: Fields due to the two sheets are

$$\vec{E}_1 = -\frac{\sigma_1}{2\varepsilon_0}\hat{r}, \qquad \vec{E}_2 = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$$



From principle of superposition, total electric field at any point of region I is

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

In the region II: Fields due to the two sheets are $\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}, \qquad \vec{E}_2 = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$

$\therefore \quad$ Total field, $\vec{E}_{II} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 - \sigma_2)$

In the region III: Fields due to the two sheets are $\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}, \qquad \vec{E}_2 = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$

$\therefore \quad$ Total field, $\vec{E}_{III} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$

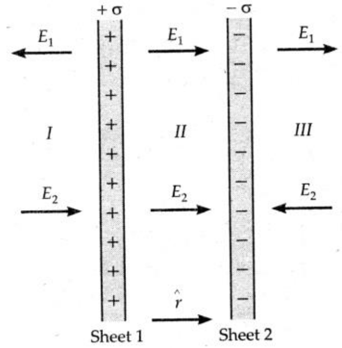
Q.2 *Two infinite parallel planes have uniform charge densities $\pm$ σ. Determine the electric field in (i) the region between the planes, and (ii) outside it.*

Electric field of two oppositely charged plane parallel plates. As shown in figure, consider tow plane parallel sheets having uniform surface charge densities of $\pm$ σ. Suppose $\hat{r}$ be a unit vector pointing from left to right.

In the region I: Fields due to the two sheets are

$$\vec{E}_1 = -\frac{\hat{r}}{2\varepsilon_0}\sigma, \quad \vec{E}_2 = \frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field, $\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{\hat{r}}{2\varepsilon_0}\sigma + \frac{\hat{r}}{2\varepsilon_0}\sigma = 0$



In the region II: Fields due to the two sheets are

$$\vec{E}_1 = \frac{\hat{r}}{2\varepsilon_0}\sigma, \quad \vec{E}_2 = \frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field,

$$\vec{E}_{II} = \frac{\hat{r}}{2\varepsilon_0}\sigma + \frac{\hat{r}}{2\varepsilon_0}\sigma = \frac{\sigma}{\varepsilon_0}\hat{r}$$

In the region III: Fields due to the two sheets are

$$E_1 = \frac{\hat{r}}{2\varepsilon_0}\sigma, \; E_2 = -\frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field $\vec{E}_{III} = 0$

Field Due to a Uniformly Charged thin Spherical Shell or Field Due to Conducting Sphere

Consider a thin spherical shell of charge of radius R with uniform surface charge density σ.

(a) *When point P lies outside the spherical shell.* the total charge q inside the Gaussian surface is the charge on the shell of radius R and area $4\pi R^2$.

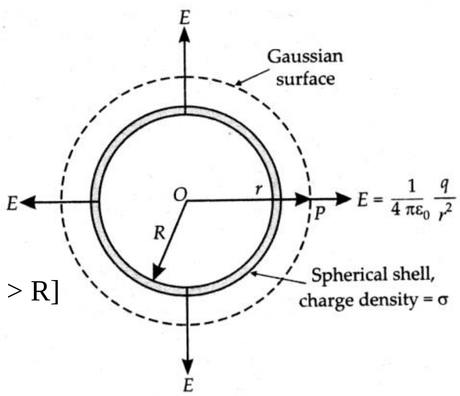
∴ $q = 4\pi R^2\sigma$

Flux through the Gaussian surface,

$$\phi_E = E \times 4\pi r^2$$

By Gauss's theorem, $\phi_E = \frac{q}{\varepsilon_0}$

∴ $E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$ or $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ [For r > R]



(b) *When point P lies on the spherical shell.* The Gaussian surface just encloses the charged spherical shell.

Applying Gauss's theorem,

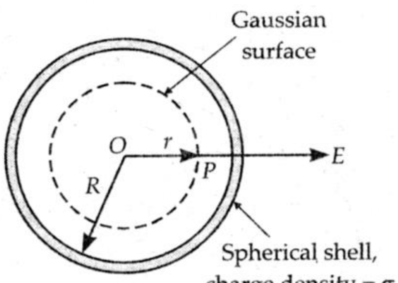
$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

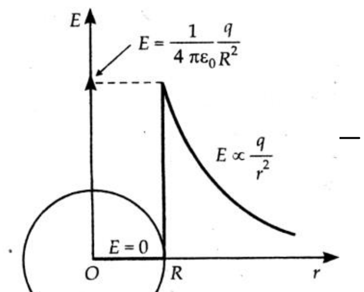
or $E = \frac{q}{4\pi\varepsilon_0 R^2}$ [Fro r = R] or $E = \frac{q}{\varepsilon_0}$ [$\because q = 4\pi R^2\sigma$]

(c) *When point P lies inside the spherical shell.* As is clear from figure, the charge enclosed by the Gaussian surface is zero, *i.e.*,

$q = 0$

Flux through the Gaussian surface,





$\phi_E = E \times 4\pi r^2$

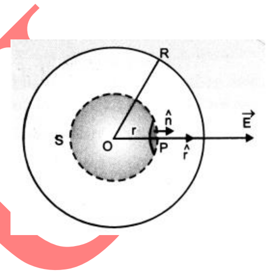
Applying Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$E \times 4\pi r^2 = 0$

or $E = 0$

Electric Field Intensity Due to a Non–conducting Charged Solid Sphere



Suppose a non conducting solid sphere of radius **R and centre O** has uniform volume density of charge ρ. We have to calculate electric field intensity $\vec{E}$ at any point P, where OP = r. With O as centre and r as radius, imagine a sphere S, which acts as a Gaussian surface. At every point of S, magnitude of $\vec{E}$ is same, directed radially outwards. It q' is the charge enclosed by the sphere S, then according to Gauss's law.

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\, ds = E\oint_S ds = \frac{q'}{\in}$$

where ∈ is electrical permittivity of the material of the insulating sphere.

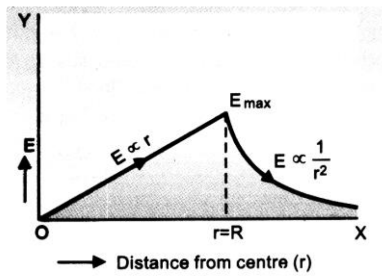
$\therefore \quad E(4\pi r^2) = \frac{q'}{\in} \quad$ or $E = \frac{q'}{4\pi \in r^2}$ ….. (i)

Now, charge inside S. i.e. q' = volume of S × volume density of charge $q' = \frac{4}{3}\pi r^3 \times \rho$

From (i), $E = \frac{4}{3}\frac{\pi r^3 \rho}{4\pi \in r^2} = \frac{r\rho}{3\in}$,

i.e., $E = \frac{r\rho}{3\in}$ …. (ii)

Clearly, $E \propto r$.



i.e. electric intensity at any point inside a non–conducting charged solid sphere varies directly as the distance of the point from the centre of the sphere.

At the **centre** of the sphere, r = 0,

$\therefore \quad E = 0$

At the surface of the sphere,

r = R

$\therefore \quad E = \frac{R\rho}{3\in} \Rightarrow$ **maximum**

We have already proved that outside the sphere, $E \propto 1/r^2$

All these results are plotted in figure, which represents the variation of electric field intensity E with distance (r) from the centre of a non–conducting uniformly charged solid sphere.

Electric Field Strength due to a long Charged Cylinder

Case – I: Conducting Cylinder

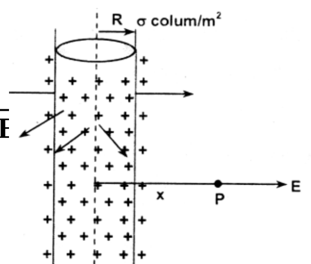
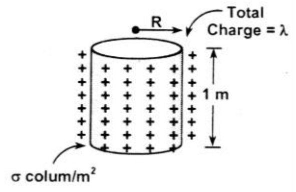


Figure shows a long conducting cylinder charged uniformly on its surface with surface with surface density σ coul/m^2. Here due to uniform charge distribution, the electric field direction can be taken radially outward as shown. Here for outer points it can be taken that electric field is originated from the central axis of the cylinder. Thus here we can use the result of a uniformly charged long wire. Here if we calculate the electric field strength at point P at a distance x from the axis of cylinder, it can be given as $E_p = \frac{2K\lambda}{x}$

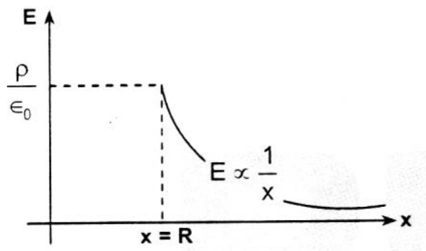


Here λ is charge per unit length on cylinder which can be calculated as

$$\lambda = \sigma \times 2\pi R \times 1$$

Thus E_p can be given as $E_p = \frac{2 \times \sigma . 2\pi R}{4\pi\epsilon_0\, x}$

$$= \frac{\sigma R}{\epsilon_0\, x}$$

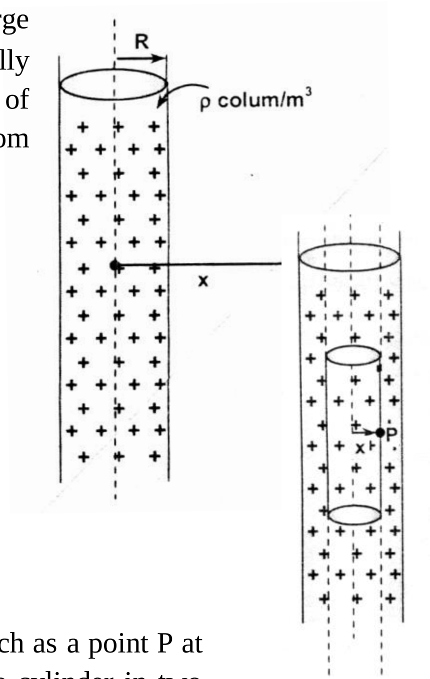


For points on surface, we have x = R thus electric field strength on the surface point can be given as $E_S = \frac{\sigma}{\epsilon_0}$

For interior points, we know inside a metal body due to charges electric field strength is always zero.

Case – II: Non–conducting Cylinder:

For a non–conducting cylinder uniformly charged with a volume charge density ρ coul/m^3 as shown in figure, electric field strength is also in radially outward direction in such a way that it is originating from the axis of cylinder. Thus electric field strength at exterior points at a distance x from the axis of cylinder can be given as



$$E_p = \frac{2K\lambda}{x}$$

Hence λ, the charge per unit on the cylinder can be given as

$$\lambda = \rho \times \pi R^2 \times 1$$

Thus we have $$E_p = \frac{2K(\rho\pi R^2)}{x} = \frac{\rho R^2}{2\epsilon_0\, x}$$

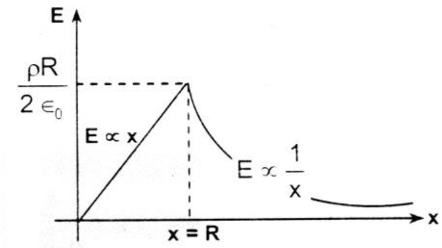
Electric field strength on the surface of this cylinder i.e. …..

$$x = R \Rightarrow \quad E_s = \frac{\rho R}{2\epsilon_0}$$

Now if we find electric field strength at interior points of the cylinder, such as a point P at a distance x from the axis of cylinder as shown in figure, we divide the cylinder in two parts, one is an inner solid cylinder of radius x, point P is on outer surface. Another is a cylindrical shell of inner radius x and outer radius R, point P is an interior point.

Now due to uniform charge distribution in a hollow cylinder no electric field exist at an interior point, the electric field at point P will only be due to the inner cylinder of radius x. For finding this we assume again the line charge and whole charge of inner cylinder is concentrated at its axis, whose charge per unit length λ can be given as $\lambda = \rho \times \pi x^2 \times 1$

Thus electric field strength at point P can be given as

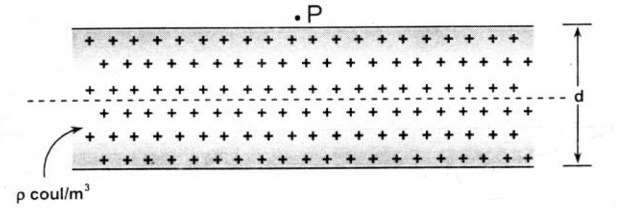


$$E = \frac{2K\lambda}{x} = \frac{1}{2\pi \in_0} \times \frac{\rho \pi x^2}{x}$$

$$E_{in} = \frac{\rho x}{2 \in_0}$$

Electric Field due to a Large Thick Charged Sheet

Figure shows a large sheet of thickness d, uniformly charged at charge density ρ coul/m^3. On both sides of sheet due to it, electric field strength is direct, away from the sheet.

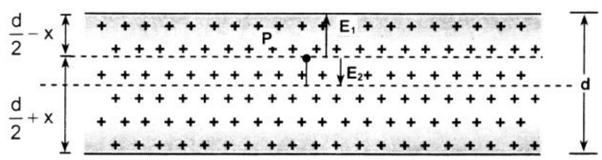


The electric field strength at a point P in front of the sheet as shown in figure can be given by $E = \frac{\sigma}{2\in_0}$

Where σ is the charge per unit surface area of the sheet which can be given as $\sigma = \rho d$

Thus $$E = \frac{\rho d}{2 \in_0} \qquad \ldots (1)$$

To find electric field strength at an interior point P of the sheet at a distance x from its central place as shown in figure we divide sheet in two sheets. One of thickness $\left(\frac{d}{2} - x\right)$ and other of thickness $\left(\frac{d}{2} + x\right)$



Due to thinner sheet electric field at point P is in downward direction, say it is E_2 which is given as

$$E_2 = \frac{\rho\left(\frac{d}{2} - x\right)}{2\in_0} \qquad \text{[using equation –(1)]}$$

Similarly due to thickness sheet electric field at P is in upward direction, say it is E_1 which is given

$$E_1 = \frac{\rho\left(\frac{d}{2} + x\right)}{2\in_0} \qquad \text{[using equation –(1)]}$$

Net electric field at point P can be given as

$$E_p = E_1 - E_2$$

$$= \frac{\rho\left(\frac{d}{2}+x\right)}{2\epsilon_0} - \frac{\rho\left(\frac{d}{2}-x\right)}{2\epsilon_0} \qquad \therefore \qquad E_p = \frac{\rho x}{\epsilon_0}$$

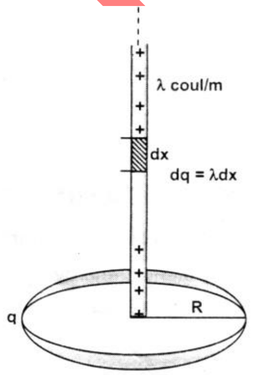
Ex.1 A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its end coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

Sol. Force dF on the element dx of wire due to ring = $dq\vec{E}$

$$dF = \frac{Kqx}{(x^2+R^2)^{3/2}}\lambda dx$$

$$F = Kq\lambda\int_0^{\infty}\frac{x\,dx}{(R^2+x^2)^{3/2}}$$

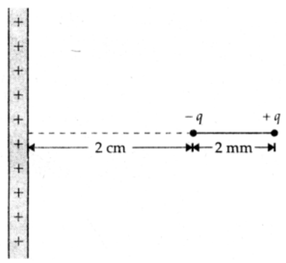
$$F = \frac{\lambda q}{4\pi\varepsilon_0 R}$$



Subjective Assignment – VII

Q.1 Two long straight parallel wires carry charges λ_1 and λ_2 per unit length. The separation between their axes is d. Find the magnitude of the force exerted on unit length of one due to the charge on the other.

Q.2 An electric dipole consists of charges $\pm$ 2 × 10^{-8} C, separated by a distance of 2 mm. It is placed near a long line charge of density 4.0 × 10^{-4} Cm^{-1}, as shown in figure, such that the negative charge is at distance of 2 cm from the line charge. Calculate the force acting on the dipole.



Q.3 A charge of 17.7 × 10^{-4}C is distributed uniformly over a large sheet of area 200 m^2. Calculate the electric field intensity at a distance of 20 cm from it in air.

Q.4 A charged particle having a charge of – 2.0 × 10^{-6} C is placed close to a non–conducting plate having a surface charge density of 4.0 × 10^{-6} Cm^{-2}. Find the force of attraction between the particle and the plate.

Q.5 A particle of mass 9×10^{-5} g is kept over a large non conducting sheet of charge density 5×10^{-5} Cm^{-2}. What charge should be given to the particle, so that if released, it does not fall?

Q.6 A large non conducting sheet of charge having surface charge density 5.0×10^{-16} Cm^{-2} lies in the X–Y plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60^o with the Z–axis.

Q.7 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly over its surface. What is the electric field (i) inside the sphere, (ii) just outside the sphere, (iii) at a point 18 cm from the centre of the sphere?

Answers

1. $f = \dfrac{\lambda_1\lambda_2}{2\pi\varepsilon_0 d}$
2. 0.7 N, acting towards the line charge
3. 10^6 NC^{-1}
4. 0.45 N
5. 3.12×10^{-13} C
6. 4.44×10^{-7} Nm^2 C^{-1}
7. (i) 0, (ii) 10^5 NC^{-1}, (iii) 4.44×10^4 NC^{-1}

Electrostatic Potential Energy

Potential energy of a system of particle is defined only in conservative fields. As electrostatic field is also conservative, we define potential energy for it. Before proceeding further, we should keep in mind the following points, which are useful in understanding potential energy in electric field.

(i) Doing work implies supply of energy

(ii) Energy can neither be transferred nor be transformed into any other form without doing work.

(iii) Kinetic energy implies utilization; of energy where as potential energy implies storage of energy.

(iv) Whenever work is done on a system of bodies, the supplied energy to the system is either used in form of KE of its particles or it will be stored in system in form of increase in the potential energy of system.

(v) When all particles of a system are separated far apart by infinite distance there will be no interaction between them. This state we take as reference of zero potential energy. Now potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles.

Electrostatic potential energy in defined in two ways.

(i) Interaction energy of charged particles of a system

(ii) Self energy of a charged object (will be discussed later)

Electrostatic Interaction Energy

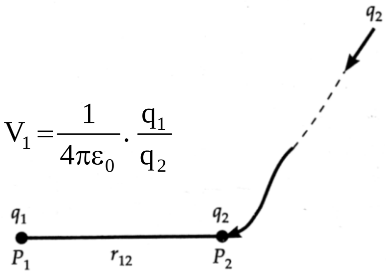
Electrostatic interaction energy of a system of charged particles is defined as the external work required assembling the particles from infinity to a given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system hence final potential energy of system will be positive. If the force between the particles is attractive, work will be done by the system and final potential energy of system will be negative.

Let us take some illustrations to understand this concept in detail.

Interaction Energy of a System of Two charged Particles

The **electric potential** energy of a system of point charges may be defined as the amount of work done in assembling the charges at their locations by bringing them in, from infinity.

Potential energy of a system of two point charges. Suppose a point charge q_1 is at rest at a point P_1 in space, as shown in figure. It takes no work to bring the first charge q_1 because there is no field yet to work against.



$\therefore \quad W_1 = 0$

Electric potential due to charge q_1 at a point P_2 at distance r_{12} from P_1 will be $V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{q_2}$

If charge q_2 is moved in from infinity to point P_2, the work required is

$$W_2 = \text{Potential} \times \text{charge} = V_1 \times q_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r_{12}}$$

As the work done is stored as the potential energy *U* of the system $(q_1 + q_2)$, so $U = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r_{12}}$

Potential energy of a system of N point charges. The expression for the potential energy of N point charges can be written as

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_i\,q_j}{r_{ij}} = \frac{1}{2}.\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i \neq j}}^{N}\frac{q_i\,q_j}{r_{ij}}$$

As double summation counts every pair twice, to avoid this the factor 1/2 has been introduced.

The amount of work done to move a point charge q from point A having electric potential V_A to point B having electric potential V_B.

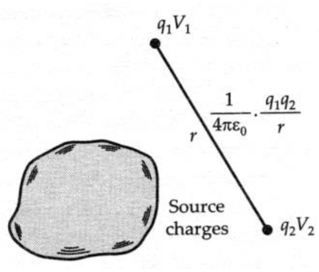
$$= \text{charge moved} \times \text{potential difference}$$
$$= q\,(V_B - V_A)$$

Potential energy of a system of two point charges in an electric field. Let V_1 and V_2 be the electric potentials of the field $\vec{E}$ at the points where q_1 and q_2 are located. Then potential energy of the two charges in the field $\vec{E}$ is

U = P.E. due to interaction between q_1 and $\vec{E}$

+ P.E. due to interaction between q_2 and $\vec{E}$

+ P.E. due to interaction between q_1 and q_2



or $\quad U = q_1V_1 + q_2V_2 + \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r}$

Electron volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt. $\quad 1\text{ eV} = 1.6 \times 10^{-19}$ J

Multiples and Submultiples of eV

1 meV (milli electron volt) = 10^{-3} eV = 1.6×10^{-22} J

1 keV (kilo electron volt) = 10^{3} eV = 1.6×10^{-16} J

1 MeV (million electron volt) = 10^6 eV = 1.6×10^{-13} J

1 GeV (giga electron volt) = 10^9 eV = 1.6×10^{-10} J

1 TeV (tera electron volt) = 10^{12} eV = 1.6×10^{-7} J

Free and bound charges

In metallic conductor, the electrons of the outer shells of the atoms are loosely bound to the nucleus. They get detached from the atoms and move almost freely inside the metal. In an external electric field, these free inside the metal. In an external electric field, these free electrons drift in the opposite direction of the electric field. The positive ions which consist of nuclei and electrons of inner shells remain held in their fixed positions. These immobile charges constitute the bound charges.

In insulators, the electrons are tightly bound to the nuclei and cannot be detached from the atoms, i.e., charges in insulators are bound charges. Due to the absence of free charges, insulators are poor conductors of electricity.

Electric Potential

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, electric potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at the point can be given as

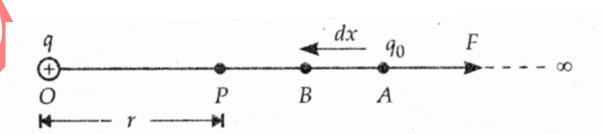
$$V = \frac{U}{q_0} \text{ joule/coul}$$

As potential energy of a charge with electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can defined electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

Electric Potential due to a Point Charge in its Surrounding

Suppose a test charge q_0 is placed at point A at distance x from O. By Coulomb's law, the electrostatic force acting on charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2}$$



The force $\vec{F}$ acts away from the charge q. the small work done in moving the test charge q_0 from A to B through small displacement $\overrightarrow{dx}$ against the electrostatic force is

$$dW = \vec{F} \cdot \overrightarrow{dx} = Fdx \cos 180^o = -Fdx$$

The total work done in moving the charge q_0 from infinity to the point P will be

$$W = \int dW = -\int_{\infty}^{r} Fdx = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2} dx$$

$$= -\frac{qq_0}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}\, dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r} \quad = -\frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r}$$

Hence work done in moving a unit test charge from infinity to the point P, or the electric potential at point P is

$$V = \frac{W}{q_0} \text{ or } V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

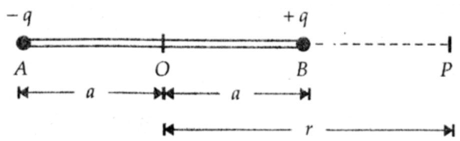
Electric Potential Due to a Dipole

Electric potential at an axial point of a dipole

As shown in figure, consider an electric dipole consisting of two point charge –q and +q and separated by distance 2a. Let P be a point on the axis of the dipole at a distance r from its centre O.

Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}.\frac{q}{BP}$$



$$= -\frac{1}{4\pi\varepsilon_0}.\frac{q}{r+a} + \frac{1}{4\pi\varepsilon_0}.\frac{q}{r-a}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(r+a)-(r-a)}{r^2-a^2}\right] = \frac{1}{4\pi\varepsilon_0}.\frac{q\times 2a}{r^2-a^2} \quad \text{or}$$

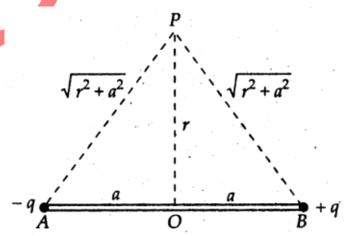
$$V = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2-a^2} \qquad [\because p = q\times 2a]$$

For a short dipole, $a^2 << r^2$, so $V = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$

Electric potential at an equatorial point of a dipole

Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}.\frac{q}{BP}$$

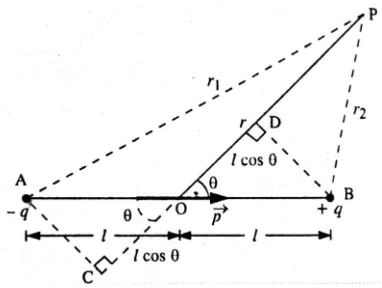


$$= -\frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{r^2+a^2}} = 0$$

Electric Potential at Any Point Due to an Electric Dipole

Consider any point P at a distance r from the centre (O) of the electric dipole AB. Let OP makes an angle θ with the dipole moment vector, $\vec{p}$ and r_1, r_2 be the distances of point P from –q charge and +q charge respectively.

Potential at P due to – q charge, $V_1 = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{r_1} = -\frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$



Potential at P due to +q charge, $V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$

∴ Potential at P due to the dipole,

$V = V_1 + V_2$

or $$V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q}{r_2} \quad \text{or} \quad V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_2}\right] \quad \dots \text{(i)}$$

Draw a perpendicular from A which meets the line OP at C when produced backward. Also draw BD ⊥ on OP.

Then $r_1 = AP \simeq CP = OP + OC = r + l\cos\theta$ ($\because$ from ΔAOC, $OC = l\cos\theta$)

and $r_2 = BP = DP = OP - OD = r - l\cos\theta$ ($\because$ from ΔBOD, $OD = l\cos\theta$)

Substituting the values of r_1 and r_2 in equation (i), we get,

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r - l\cos\theta)} - \frac{1}{(r + l\cos\theta)}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{r + l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right) = \frac{q\ 2l\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}$$

i.e., $$V = \frac{p\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}$$ … (ii) ($\because$ dipole moment, $p = q.\,2l$)

If $r >> l$, then equation (ii) becomes $V = \dfrac{p\cos\theta}{4\pi\epsilon_0 r^2}$

Since $p\cos\theta = \hat{p}.\hat{r}$, where $\hat{r}$ is unit vector directed along OP.

$$V = \frac{\hat{p}.\hat{r}}{4\pi\epsilon_0 r^2} \quad \text{for } r >> l$$

Special Cases:

1. If point P lies on the axial line of the dipole i.e. $\theta = 0^o$

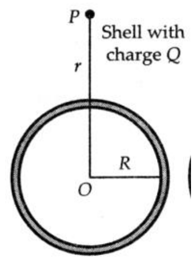
   then equation, becomes $V_{ax} = \dfrac{p}{4\pi\epsilon_0 r^2}$ or $V \propto \dfrac{1}{r^2}$ [$\because \cos 0^o = 1$]

2. If point P lies on the equatorial line of the dipole i.e., $\theta = 90^o$

   then $V_{eq} = 0$ [$\because \cos 90^o = 0$]

   Potential due to a dipole is zero at all points on the equatorial line of the dipole.

Electric potential due to uniformly charged thin spherical shell

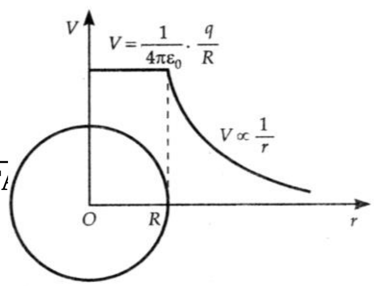


(a) *When the point P lies outside the shell.* We know that for a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Hence electric potential at an outside point is equal to that of a point charge located at the centre, which is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad [\text{For } r > R]$$

(b) *When point P lies on the surface of the shell.* Here r = R. Hence potential on the surface of the shell is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad [\text{For } r = R]$$



(c) *When point P lies inside the shell.* The electric field at any point inside the shell is zero. Hence electric potential due to a uniformly charged spherical shell is constant everywhere inside the shell and its value is equal to that on the surface. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad [\text{For } r < R]$$

Subjective Assignment – VIII

Q.1 If 100 J of work has to be done in moving an electric charge of 4C from a place, where potential is – 10V to another place, where potential is V volt, find the value of V.

Q.2 Determine the electric potential at the surface of a gold nucleus. The radius is 6.6×10^{-15} m and the atomic number Z = 79. Given charge on a proton = 1.6×10^{-19} C.

Q.3 (i) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away. (ii) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the chare is brought?

Q.4 A metal wire is bent in a circle of radius 10 cm. It is given a charge of 200 μC which spreads on it uniformly. Calculate the electric potential at its centre.

Q.5 Electric field intensity at point 'B' due to a point charge 'Q' kept at point 'A' is 24 NC^{-1} and the electric potential at point 'B' due to same charge is 12 JC^{-1}. Calculate the distance AB and also the magnitude of charge Q.

Q.6 To what potential we must charge an insulated sphere of radius 14 cm so that the surface charge density is equal to 1μ Cm^{-2}?

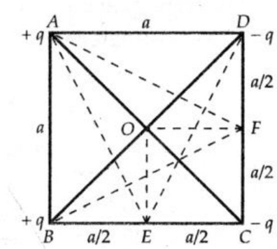
Q.7 A charge of 24 μC is given to a hollow metallic sphere of radius 0.2 m. Find the potential
(i) at the surface of the sphere, and (ii) at a distance of 0.1 cm from the centre of the sphere.

Q.8 Twenty seven drops of same size are charged at 220 V each. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.

Q.9 Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Q.10 Calculate the electric potential at the centre of a square of side $\sqrt{2}$ m, having charges 100 μC, – 50 μC, 20μC, and – 60μc at the four corners of the square.

Q.11 Four charges +q, +q, –q and –q are placed respectively at the corners A, B, C and D of a square of side 'a' arranged in the given order. Calculate the electric potential at the centre O. If E and F are the midpoints of sides BC and CD respectively, what will be the work done in carrying a charge 'e' from O to E and from O to F?



Q.12 A short electric dipole has dipole moment of 4×10^{-9} Cm. Determine the electric potential due to the dipole at a point distance 0.3m from the centre of the dipole situated (a) on the axial line (b) on equatorial line and (c) on a line making an angle of 60° with the dipole axis.

Q.13 Two point charges of +3μC and –3μC are placed 2×10^{-3} m apart from each other. Calculate (i) electric field and electric potential at a distance of 0.6 m from the dipole in broad–side–on position (ii) electric field and electric potential at the same point after rotating the dipole through 90^o.

Q.14 Two charges –q and +q are located at points A(0, 0, –a) and B(0, 0, +a) respectively. How much work is done in moving a test charge from point P(7, 0, 0) to Q (–3, 0, 0)?

Answers

1.	15 V	2.	1.7×10^7 V	3.	4×10^4 V, 8×10^{-5} J
4.	18×10^6 V	5.	0.667×10^{-9} C	6.	15840 V
7.	(i) 1.08×10^6 V, (ii) 1.08×10^6 V			8.	1980 V
9.	9 cm from 3×10^{-8}C, 45 cm	10.	9×10^4 V	11.	$\frac{qe}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}} - 1\right)$
12.	400 V, 200 V	13.	250 NC^{-1}, 150 V	14.	0

Relation between Electric Field and Potential

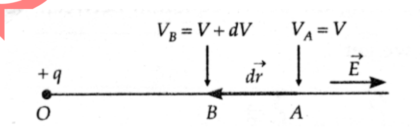
Let A and B be two adjacent points separated by distance dr. The two points are so close that electric field $\vec{E}$ between them remains almost constant.

The external force required to move test charge q_0 (without acceleration) against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0 \vec{E}$$

The work done to move the test charge from A to B is

$$W = \vec{F} . \vec{dr} = -q_0 \vec{E}.\vec{dr} = -q_0 E\, dr \cos 0^o$$



Also, the work in moving the test charge from A to B is

W = Charge × potential difference

$= q_0 (V_B - V_A) = q_0\, dV$

Equating the two works done, we get

$$-q_0 E\, dr = q_0\, dV \quad \text{or} \quad E = -\frac{dV}{dr}$$

The quantity dV/dr is the rate of charge of potential with distance and is called *potential gradient*. Thus *the electric field at any point is equal to the negative of the potential gradient at that point.*

From the above relation between electric field and potential, we can draw the following important conclusions:

(i) Electric field is in that direction in which the potential decrease is steepest.

(ii) The magnitude of electric field is equal to the change in the magnitude of potential per unit displacement (called potential gradient) normal to the equipotential surface at the given point.

Computing Electric Field from Electric Potential

As $E = -\frac{dV}{dr}$

$$E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad E_z = -\frac{dV}{dz}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} \quad \therefore \quad E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

Computing electric potential from electric field

The relation between electric field and potential is

$$\vec{E} = -\frac{dV}{\vec{dr}} \qquad \text{or} \qquad dV = -\vec{E}.\vec{dr}$$

Integrating the above equation between points $\vec{r_1}$ and $\vec{r_2}$, we get

$$\int_{V_1}^{V_2} dV = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E} \,.\, \vec{dr} \qquad \text{or} \qquad V_2 - V_1 = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E}.\,\vec{dr}$$

where V_1 and V_2 are the potentials at $\vec{r_1}$ and $\vec{r_2}$ respectively. If we take $\vec{r_1}$ at infinity, then $V_1 = 0$ and put $\vec{r_2} = \vec{r}$, we get $V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}.\,d\vec{r}$

Certain facts

- ***Electric potential*** is a property of an electric field, whether a charged particle is placed in that field or not. It is measured in joule/coulomb or volt.
- *Electric potential energy* is the energy of a charged particle in an external electric field; rather, it is the energy of the system consisting of charged object and the external electric field. It is measured in joule.

Subjective Assignment – IX

Q.1 What is potential gradient at a distance of 10^{-12} m from the centre of the platinum nucleus? What is the potential gradient at the surface of the nucleus? Atomic number of platinum is 78 and radius of platinum nucleus is 5×10^{-15} m.

Q.2 The electric potential V(x) along the X–axis varies with distance x (in metre), according to the relation $V = 4x^2$. Calculate the force experienced by a 1 μC charge placed at point x = 1m.

Q.3 A uniform electric field of 2 kN/C is in the x direction. A point charge = 3μC initially at rest at the origin is released. What is K.E. of this charge at x = 4m? Also, calculate V(4m) – V (0).

Q.4 If the potential in region of space around the point (–1m, 2m, 3m) is given by $V = (10x^2 + 5y^2 - 3z^2)$, calculate the three components of electric field at this point.

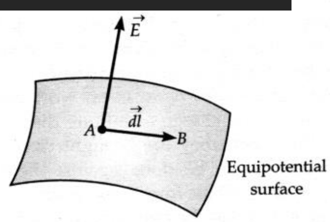
Q.5 The electric field outside a charged long straight wire is given by $E = \frac{1000}{r}$ V m^{-1}, and is directed outwards. What is the sign of the charge on the wire? If two points A and B are situated such that $r_A = 0.2$ m and $r_B = 0.4$m, find the value of $(V_B - V_A)$.

Answers

1.	1.123×10^{17} Vm^{-1}; 4.5×10^{21} Vm^{-1}	2.	-8×10^{-6} N
3.	24×10^{-3} J; -8×10^{3} V	4.	$E_x = 20$ V m^{-1}; $E_y = -20$ V m^{-1}; $E_z = 18$ V m^{-1}
5.	+ charge; – 693.1 V		

Equipotential Surface:

Any surface that has same electric potential at every point on it is called an equipotential surface. The surface may be surface of a body or a surface in space.



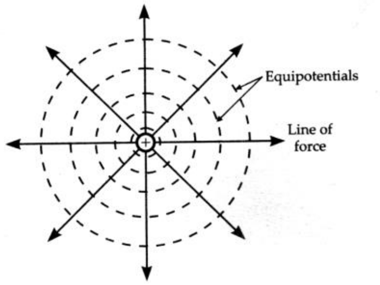
Properties of equipotential surfaces:

1.No work is done in moving a test charge over an equipotential surface.

2. Electric field is a always normal to the equipotential surface at every point.

3. Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field.

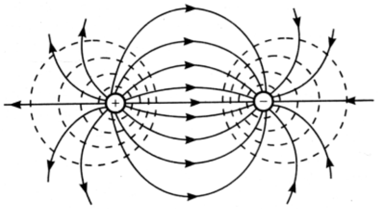
4. No two equipotential surfaces can intersect each other.

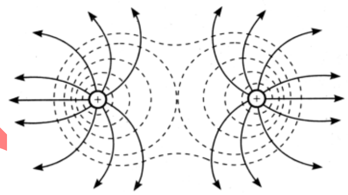


(i) Equipotential surfaces of a positive point charge. The electric potential due to a point charge q at distance r from it is given by

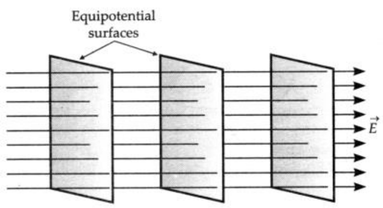
$$V=\frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

(ii) Equipotential surface of two equal and opposite point charges:





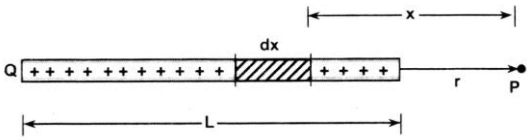
(iii) Equipotential surfaces of two equal positive charges.



(iv) Equipotential surfaces for a uniform electric field.

Electric Potential due to a Charge Rod

Figure shows a charged rod of length L, uniformly charged with a charge Q, due to this we will find electric potential at a point P at a distance r from one end of the rod shown in figure.



For this we consider an element of width dx at a distance x from the point P. Charge on this element is

$$dq = \frac{Q}{L}dx$$

The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{Kdq}{x}$$

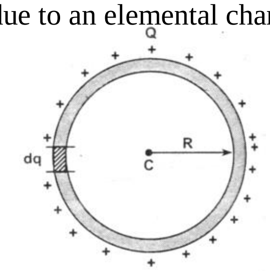
Net electric potential at point P can be given as $V=\int dV=\int_r^{r+L}\frac{KQ}{Lx}dx$

$$V=\int dV=\int_r^{r+L}\frac{KQ}{Lx}dx \qquad =\frac{KQ}{L}\ell_n\left(\frac{r+L}{\rho}\right)$$

Electric Potential due to a Charged Ring

Case – I: At its centre

To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as



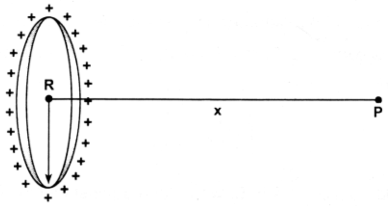
$$dV = \frac{Kdq}{R}$$

Total potential at C is $\quad V = \int dV$

$$= = \int \frac{Kdq}{R} = \frac{KQ}{R}$$

Case – II: At a point on Axis of Ring

If we wish a find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2 + R^2}$ from point P, thus the potential at P can be given as

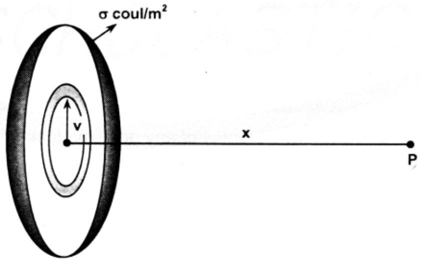


$$V_p = \frac{KQ}{\sqrt{R^2 + x^2}}$$

Electric Potential due to a Uniformly Charged Disc

Figure shows a uniformly charged disc of radius R with surface charge density ρ coul/m^2. To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is

$$dq = \sigma.\ 2\pi y\ \ dy$$



Due to this ring, the electric potential at point P can be given as

$$dV = \frac{Kdq}{\sqrt{x^2 + y^2}} = \frac{K.\sigma.2\pi y\ dy}{\sqrt{x^2 + y^2}}$$

Net electric potential at point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\epsilon_0}.\frac{y.dy}{\sqrt{x^2 + y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2 + y^2}\right]_0^R$$

Electric Potential due to non–conducting uniformly charged sphere

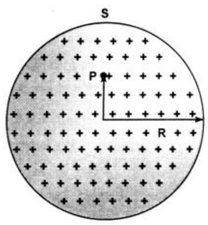
For outer and inner surface points here also we can say that potential remains same as that of a conducting sphere as

$$V_{out} = \frac{KQ}{x} \qquad \text{(for } x > R\text{)}$$

$$V_s = \frac{KQ}{R} \qquad \text{(for } x = R\text{)}$$

For an interior point unlike to a conducting sphere, potential will not remain uniform as electric field exists inside region. We know inside a uniformly charged sphere electric field is in radially outward direction thus as we move away from centre, in the direction of electric potential decreases. As shown in figure, if there is a point P at a distance x from the centre of sphere, the potential difference between point P and S can be given as

$$V_P - V_S = \int_x^R \frac{KQx}{R^3} dx$$



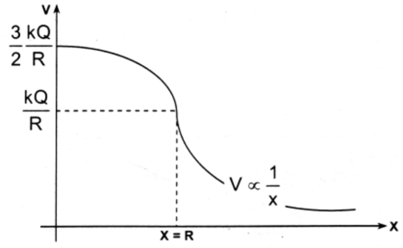
or $$V_P - \frac{KQ}{R} = \frac{KQ}{2R^3}(R^2 - x^2)$$

or $$V_P = \frac{KQ}{2R^3}(R^2 - x^2) + \frac{KQ}{R}$$

$$V_P = \frac{KQ}{2R^3}(3R^2 - x^2)$$

Here at x = 0, we have potential at centre of sphere is

$$V_C = \frac{3KQ}{2R} = \frac{3}{2}V_S$$



Subjective Assignment – X

Q.1 A particle of charge + 3 × 10^{-9} C is in a uniform field directed toward left. It is released from rest and moves a distance of 5cm after which its kinetic energy is found to be 4.5 × 10^{-5} J.
(a) What work was done by the electric force?
(b) What is the magnitude of the electric field?
(c) What is the potential of starting point w.r.t. the end point?

Q.2 Two positive point charges each of magnitude q are placed on the y axis at the point (O, a) and (O, – a). If a positively charged particle of charge q_0 and mass m is slightly displaced from origin in the direction of negative x–axis.
(i) What will be its speed at infinity?
(ii) If the particle is projected towards the left along the x–axis from a point at a large distance on the right of the origin with a velocity half that acquired in part (i), at what distance from origin will it come to rest?

Q.3 Three concentric metallic shells A, B and C of radii a, b and c (a < b < c) have surface charge densities σ, – σ and σ respectively.
(i) Find the potential of three shells A, B and C
(ii) If the shells A and C are at the same potential, obtain the relation between the radii, a, b and c.

Q.4 A circular ring of radius R with uniform positive charge density λ per unit length is located in the y – z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(R\sqrt{3}, 0, 0)$ on the positive x–axis directly towards O, with initial velocity v. Find the smallest (non–zero) value of the speed v such that the particle does not return to P.

Answers

1. (a) 4.5 × 10^{-5} J, (b) 3 × 10^5 N/C, (c) 1.5 × 10^4 volts

2. $v = 2\sqrt{\frac{Kqq_0}{ma}}$, $\left(\sqrt{15}a,\ 0\right)$

3. (i) $V_A = \frac{\sigma}{\epsilon_0}[a - b + c]$, $V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right]$, $V_C = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - \frac{b^2}{c} + c\right]$ (ii) c = (a + b)

4. $v = \sqrt{\left(\frac{\lambda q}{2\epsilon_0 m}\right)}$

Board Oriented Questions

Q.1 A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity)

Q.2 Ordinary rubber is an insulator. But special rubber tyres of aircraft are made slightly conducting. Why is this necessary?

Q.3 Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?

Q.4 A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Q.5 A boy brings the palm of his hand near the disc of a charged gold leaf electroscope. The leaves of the electroscope are observed to collapse slightly. But when the boy moves his hand away from the gold leaf electroscope, leaves resume their original position. How do you explain the behaviour of leaves?

Q.6 In defining electric field due to a point charge, the test charge has to be vanishingly small. How this condition can be justified, when we know that charge less than that on an electron or a proton is not possible?

Q.7 An electron and a proton are kept in the same electric field. Will they experience same force and have same acceleration?

Q.8 A charged particle is free to move in an electric field. Will it always move along an electric field?

Q.9 Why do charges reside on the surface of the conductor?

Q.10 Do the electric lines of force really exist? What is about the field they represent?

Q.11 An electric dipole free to move is placed in a uniform electric field. Explain alongwith diagram its motion when it is placed,

(a) parallel to the field (b) perpendicular to the field

Q.12 An electric dipole is a pair of equal and opposite charges, separated by a small fixed distance between them. The dipole is free to move. What is the action on it, when it is placed in

(i) a uniform electric field, and (ii) a non–uniform electric field?

Q.13 A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blow up; how does E vary for points (i) inside the balloon, (ii) on the surface of the balloon and

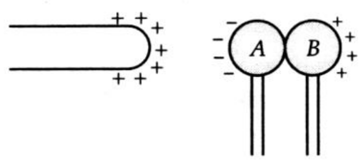
(iii) outside the balloon?

Q.14 A glass rod rubbed with silk is brought close to two uncharged spheres in contact with each other, inducing charges on them as shown in figure. Describe what happen when

(i) The spheres are slightly separated, and

(ii) The glass rod is subsequently removed, and finally

(iii) The spheres are separated far apart?



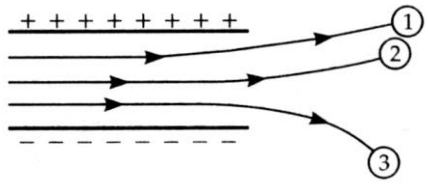
Answer the following questions:

(a) If glass rod has a charge +Q and the right sphere has charge +q, what is the charge on the left sphere? Which property of charge has been used?

(b) In the stages (ii) and (iii), roughly sketch how charges on the two spheres will be distributed?

(c) At what stage: (i), (ii) or (iii) will the force between the two spheres be largest? Smallest?

Q.15 Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



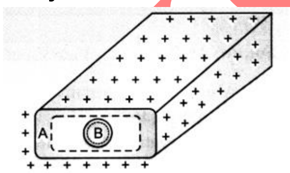
(a) Suppose that a particle is attracted towards the positive plate: what must the charge on it be?

(b) Suppose, two particles have identical curved trajectories. Which of following are necessarily true?

(i) They have same charge; (ii) They have same mass; (iii) The charges have the same sign; (iv) They have the same e/m ratio.

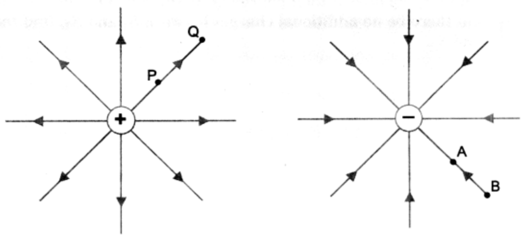
(c) You are given the initial velocity v of a beam particle and the length of the capacitor l. What other measurement would enable one to find e/m?

Q.16 What is the electric field in the cavity, if a conductor having a cavity is charged. Does the result depend on the shape and size of cavity of the conductor?



Q.17 A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire?

Q.18 Figure show the field lines of a single positive and negative charge respectively.



(a) Give the signs of potential difference ($V_P — V_Q$) and ($V_B — V_A$).

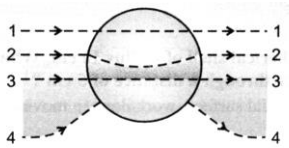
(b) Give the sign of potential energy difference of a small negative charge between the points Q and P ; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

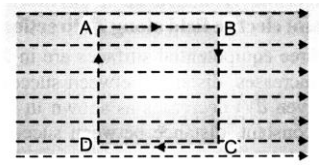
(d) Give the sign of the work done by external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Q.19 A metallic solid sphere is placed in a uniform electric field. Which path is followed by lines of force?

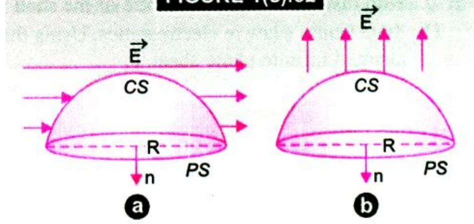


Q.20 Can we create an electric field in which all the lines of force are parallel but their density increases continuously in a direction perpendicular to the lines of force, figure?



Q.21 A hemispherical body is placed in a uniform electric field E. What is the flux linked with the curved surface, it field is (a) parallel to base, figure 1(c)62(a) and (b) perpendicular to base, figure 1(c).62(b).

FIGURE 1(c).62



Q.22 In which orientation, a dipole placed in a uniform fields is in (i) stable (ii) unstable equilibrium?

Q.23 What is nature of symmetry of field due to a point charge?

Q.24 At what points, dipole field intensity is parallel to the line joining the charges?

Q.25 What does $(q_1 + q_2) = 0$ signify?

Q.26 Define the term electric dipole moment. Is it scalar or vector?

Q.27 Derive an expression for electric field intensity at a point due to

(i) A point charge (ii) A group of charges (iii) Continuous charge distribution

Q.28 If electric field intensity is zero at a given point, will electric potential be necessarily zero at that point?

Q.29 Can two equipotential surfaces intersect each other? Give reasons.

Q.30 If a point charge be rotated in a circle of radius r around a charge q, what will be the work done?

Q.31 Draw an equipotential surface for a uniform electric field.

Q.32 Name the physical quantity which has its unit joule coulomb^{-1}. Is it a scalar or vector?

Q.33 Electric potential due to an electric dipole is cylindrically symmetric. Comment.

Q.34 Suppose that the earth has a net charge that is not zero. It is still possible to adopt the earth as standard reference point of potential and assign the potential V = 0 to it?

Q.35 Is electric flux a scalar or a vector?

Q.36 What are the units of electric flux?

Q.37 What is the shape of equipotential surface for a point charge?

Q.38 What is the electrostatic potential due to electric dipole at an equatorial point?

Q.39 What is the work done in moving a test charge q through a distance of 1 cm along the equatorial axis of an electric dipole?

Q.40 Define the term potential energy of charge q at a distance r in an external electric field.

Q.41 Write an expression for potential energy of two charges q_1 & q_2 at $\vec{r}_1$ & $\vec{r}_2$ in a uniform electric field $\vec{E}$.

Q.42 Equipotential surfaces are perpendicular to field lines. Why?

Q.43 Electric charge is distributed uniformly on the surface of a spherical rubber balloon. Show how the value of electric intensity and potential vary (i) on the surface (ii) inside and (iii) outside?

Q.44 Draw 3 equipotential surface corresponding to a field that uniformly increases in magnitude but remains constant along Z–direction. How are these surfaces different from that of a constant electric field along Z–direction.

Q.45 Define electric flux. Write its SI unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change?

Q.46 State Gauss's theorem in electrostatics. Apply this theorem to derive an expression for electric field intensity at a point near an infinitely long straight charged wire.

Q.47 A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell. Draw a graph of electric field E(r) with distance r from the centre of the shell for $0 \le r \le \infty$.

Q.48 State Gauss's law in electrostatics. Using this law, derive an expression for electric field due to a uniformly charged infinite plane sheet.

Objective Assignment – I

Q.1 An isolated conducting sphere is given positive charge. Its mass:
(a) remains unchanged (b) decreases (c) increases (d) may increase or decrease

Q.2 A charge conductor has charge on its:
(a) outer surface (b) inner surface (c) middle point (d) surrounding

Q.3 When a bird sits on a very high voltage cable:
(a) its feathers tend to spread (b) its feathers tend to compress
(c) it receives an electric shock (d) neither its feathers have any effect nor electric shock is received by it

Q.4 If a glass rod is rubbed with silk, it acquires a positive charge because:
(a) protons are added to it (b) protons are removed from it
(c) electrons are added to it (d) electrons are removed from it

Q.5 An isolated solid metallic sphere is given + Q charge. The charge will be distributed on the sphere:
(a) uniformly but only on surface (b) only on surface but not uniformly
(c) uniformly inside the volume (d) non uniformly inside the volume

Q.6 A soap bubble is given a negative charge then its radius:
(a) decreases (b) increases (c) remains unchanged
(d) nothing can be predicted as information is insufficient

Q.7 There are two charges + 1μC and + 5 μC. The ratio of the forces acting on them will be:
(a) 1 : 5 (b) 1 : 1 (c) 5 : 1 (d) 1 : 25

Q.8 A sure test of electrification is:
(a) attraction (b) repulsion (c) friction (d) induction

Q.9 Two metallic spheres carry equal charges. The distance between the spheres cannot be considered large in comparison with the diameters of the spheres. In which case, will the force of interaction between the spheres be greater?

(a) like charges (b) unlike charges (c) one is neutral and other is charged

(d) none of the above

Q.10 Mark correct option or options.

(a) Like charged bodies always repel each other (b) Like charged bodies always attract each other

(c) Like charged bodies may attract each other (d) None of these

Q.11 Two identically charged spheres when suspended by strings of equal length make an angle of 30^o with each other. When they are immersed in a liquid of density less than density of material of the spheres:

(a) the electric force between them increases (b) the electric force between them decreases

(c) the net downward force will increase (d) the net downward force will remain unchanged

Q.12 Two negative charges of unit magnitude and a positive charge q are placed along a straight line. The charge q is placed between negative charges as such the system of charges is in equilibrium. This system is in:

(a) stable equilibrium for displacement of charge q in normal direction of line joining negative charges

(b) unstable equilibrium for displacement of charge q in normal direction of line joining negative charges

(c) stable equilibrium for the displacement of charge q in the direction of line joining the negative charges

(d) neutral equilibrium for the displacement of charge q along the line joining the negative charges

Q.13 Mark correct option. Electrostatic experiment is:

(a) affected on the humid day (b) not affected on humid day

(c) independent of medium (d) none of the above

Q.14 Two identical pendulums A and B are suspended from the same point. The bobs are given positive charges, with A having more charge than B. They diverge and reach at equilibrium, with A and B making angles θ_1 and θ_2 with the vertical respectively:

(a) $\theta_1 > \theta_2$ (b) $\theta_1 < \theta_2$

(c) $\theta_1 = \theta_2$ (d) the tension in A is greater than that in B

Q.15 Two small identical balls A and B lying on a horizontal smooth plane are connected by a massless spring. Ball A is fixed but ball B is free to move. When both balls are charged identically, then:

(a) at the time of maximum separation between balls, magnitude of acceleration will be maximum

(b) at the equilibrium position of B, velocity of ball B will be maximum

(c) the ball B executes simple harmonic motion (d) all of the above

Q.16 Three charged particles are collinear and are in equilibrium. Then:

(a) all the charged particles have the same polarity (b) the equilibrium is unstable

(c) all the charged particles cannot have the same polarity (d) both (b) and (c) are correct

Q.17 How many electrons must be removed from an electrically neutral metal plate to give it a positive charge of 1×10^{-7} coulomb?

(a) 6.25×10^{11} (b) 6.45×10^{13} (c) 6.25×10^{-11} (d) 6.45×10^{-13}

Q.18 In a hydrogen atom, the distance between the electron and proton is 2.5×10^{-11}m. The electrical force of attraction between them will be

(a) 2.8×10^{-7} N (b) 6.2×10^{-7} N (c) 3.7×10^{-7} N (d) 9.1×10^{-7} N

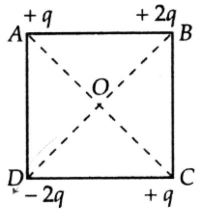
Q.19 Two charged spheres separated by a distance d exert some force F on each other. If they are immersed in a liquid of dielectric constant 2, then the force exerted by them, if all other conditions are same, is

(a) 4 F (b) 2 F (c) F (d) F/2

Q.20 Charge 4Q, q and Q are placed along X–axis at positions x = 0, x = l/2 and x = l, respectively. Find the value of q so that force on charge Q is zero.

(a) Q (b) Q/2 (c) – Q/2 (d) – Q

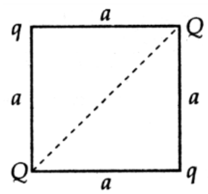
Q.21 Four charges are arranged at the corners of a square ABCD as shown in figure. The force on the charge kept at the centre O is



(a) zero (b) along the diagonal BD

(c) along the diagonal AC (d) perpendicular to side AB

Q.22 Four charges as shown in figure are placed at the corners of a square of side length a. What is the ratio of (Q/a) if net force on Q is zero?



(a) $\frac{1}{2\sqrt{2}}$ (b) $-2\sqrt{2}$

(c) 1/2 (d) $\frac{1}{\sqrt{2}}$

Q.23 The unit of permittivity of free space (ε_0) is

(a) $CN^{-1}m^{-1}$ (b) Nm^2C^{-2} (c) $C^2N^{-1}m^{-2}$ (d) $C^2N^{-2}m^{-2}$

Q.24 An electron is moving round the nucleus of a hydrogen atom in a circular orbit of radius r, the coulomb force $\vec{F}$ between the two is

(a) $-k\frac{e^3}{r^3}\hat{r}$ (b) $k\frac{e^2}{r^3}\vec{r}$ (c) $-k\frac{e^2}{r^3}\vec{r}$ (d) $k\frac{e^2}{r^3}\hat{r}$

Q.25 Point charges + 4q, – q and + 4q are kept on the X–axis at points x = 0, x = a and x = 2a respectively.

(a) Only –q is in stable equilibrium (b) None of these charges is in equilibrium

(c) All the charges are in unstable equilibrium (d) All the charges are in stable equilibrium

Q.26 A body can be negatively charged by

(a) giving excess of electrons to it (b) removing some electrons from it

(c) giving some protons to it (d) removing some neutrons from it

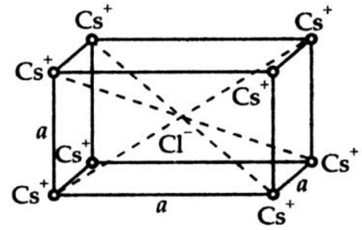
Q.27 The number of electrons for one coulomb of charge is

(a) 6.25×10^{18} (b) 6.25×10^{19} (c) 6.25×10^{21} (d) 6.25×10^{23}

Q.28 Using mass (M), length (L), time (T) and current (A) as fundamental quantities, the dimension of permittivity is

(a) $[ML^{-2}T^2A]$ (b) $[M^{-1}L^{-3}T^4A^2]$ (c) $[MLT^{-2}A]$ (d) $[ML^2T^{-1}A^2]$

Q.29 In the basic CsCl crystal structure, Cs^+ and Cl^- ions are arranged in a bcc configuration as shown in the figure. The net electrostatic force exerted by the eight Cs^+ ions on the Cl^- ion is



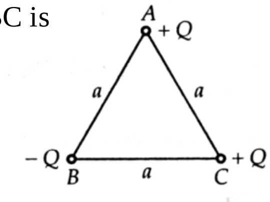
(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{4e^2}{3a^2}$

(b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{16e^2}{3a^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{32e^2}{3a^2}$

(d) zero

Q.30 Three charges are placed at the vertices of an equilateral triangle of side a as shown in the figure. The force experienced by the charge placed at the vertex A in a direction normal to BC is



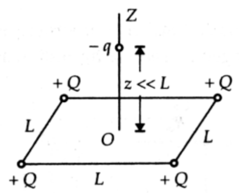
(a) $\frac{Q^2}{(4\pi\varepsilon_0 a^2)}$

(b) $-Q^2(4\pi\varepsilon_0 a^2)$

(c) zero

(d) $\frac{Q^2}{(2\pi\varepsilon_0 a^2)}$

Q.31 Four point +ve charge of same magnitude (Q) are placed at four corners of a rigid square frame as shown in figure. The plane of the frame is perpendicular to Z–axis. If a –ve point charge is placed at a distance z away from the above frame, the



(a) –ve charge oscillates along the Z–axis

(b) it moves away from the frame

(c) it moves slowly towards the frame and stays in the plane of the frame

(d) it passes through the frame only once

Q.32 Two identical conductors of copper and aluminium are placed in an identical electric field. The magnitude of induced charge in the aluminium will be

(a) zero (b) greater than in copper

(c) less than in copper (d) equal to that of copper

Q.33 A comb run through one's dry hair attracts small bits of paper. This is due to

(a) comb is a good conductor (b) paper is a good conductor

(c) the atoms in the paper get polarized by the charged comb

(d) the comb possesses magnetic properties

Q.34 Two charges of equal magnitudes and at a distance r exert a force F on each other. If the charges are halved and distance between them is doubled, then the new force acting on each charge is

(a) F/8 (b) F/4 (c) 4 F (d) F/16

Q.35 A and B are two identical spherical charged bodies which repel each other with force F, kept at a finite distance. A third uncharged sphere of the same size is brought in contact with sphere B and removed. It is then kept at mid–point of A and B. Find the magnitude of force on C.

(a) F/2 (b) F/8 (c) F (d) zero

Q.36 A charge Q is divided in two parts q and Q – q. What is value of q for maximum force between them?

(a) 3Q/4 (b) Q/3 (c) Q (d) Q/2

Q.37 Identify the wrong statement in the following, Coulomb's law correctly described the electric force that

(a) binds the electrons of an atom to its nucleus

(b) binds the protons and neutrons in the nucleus of an atom

(c) binds atoms together to form molecules (d) binds atoms and molecules to form solids

Q.38 An infinite number of charges, each of charge 1μC are placed on the x–axis with co–ordinates x = 1, 2, 4, 8, … ∞. If a charge of 1C is kept at the origin, then what is the net force acting on 1 C charge?

(a) 9000 N (b) 12000 N (c) 24000 N (d) 36000N

Q.39 Two equal charges are separated by a distance d. A third charge placed on a perpendicular bisector at x distance from centre will experience maximum coulomb force, when

(a) $x = \frac{d}{\sqrt{2}}$ (b) x = d/2 (c) $x = \frac{d}{2\sqrt{2}}$ (d) $x = \frac{d}{3}$

ANSWERS

1	b	2	a	3	a	4	d	5	**a**
6	b	7	b	8	b	9	b	10	c
11	b	12	a	13	a	14	c	15	d
16	d	17	a	18	c	19	d	20	d
21	b	22	b	23	c	24	c	25	c
26	a	27	a	28	b	29	d	30	c
31	a	32	d	33	c	34	d	35	c
36	d	37	b	38	b	39	c		

Objective Assignment – II

Q.1 Deuteron and alpha particle in air are at separation 1Å. The magnitude of electric field intensity on α–particle due to deuteron is

(a) 5.76×10^{11} N/C (b) 1.44×10^{11} N/C (c) 2.828×10^{11} N/C (d) zero

Q.2 Two charges + 5 μ C and + 10μC are placed 20 cm apart. The electric field at the midpoint between the two charges is

(a) 4.5×10^{6}N/C towards + 5μC (b) 13.5×10^{6}N/C towards + 5μC

(c) 4.5×10^{6}N/C towards + 10 μC (d) 13.5×10^{6} N/C towards + 10 μC

Q.3 Two small charged spheres A and B have charges 10 μC and 40 μC respectively, and are held at a separation of 90 cm from each other. At what distance from A, electric intensity would be zero?

(a) 22.5 cm (b) 18 cm (c) 36 cm (d) 30 cm

Q.4 A simple pendulum has a length *l* and the mass of the bob is m. The bob is given a charge of q coulomb. The pendulum is suspended between the vertical plates of a charged parallel plate capacitor. If E is the electric field strength between the plates, the time period of the pendulum is given by

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $\sqrt{\frac{l}{\sqrt{g+\frac{qE}{m}}}}$ (c) $2\pi\sqrt{\frac{l}{\sqrt{g-\frac{qE}{m}}}}$ (d) $\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

Q.5 A particle of mass m and charge q is placed at rest in a uniform electric field E and then released, the kinetic energy attained by the particle after moving a distance y, will be

(a) q^2Ey (b) qEy (c) $q\,E^2y$ (d) $q\,Ey^2$

Q.6 In a non–uniform electric field, electric dipole experiences

(a) torque only (b) torque as well as net force

(c) force only (d) none of these

Q.7 Two equal and opposite charges of 2×10^{-10} C are placed at a distance of 1 cm forming a dipole and are placed in an electric field of 2×10^5 N/C. The maximum torque on dipole is

(a) $2\sqrt{2}\times10^{-6}$ Nm (b) 8×10^{8} Nm (c) 4×10^{-9} Nm (d) 4×10^{-7} Nm

Q.8 If σ = surface charge density, ε = electric permittivity, the dimensions of σ/ε are same as

(a) electric force (b) electric field intensity

(c) pressure (d) electric charge

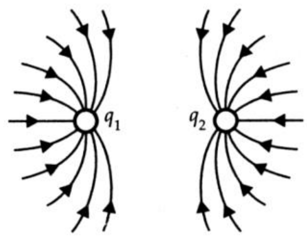
Q.9 Out of the following is not a property of field lines

(a) Field lines are continuous curves without any breaks

(b) Two field lines cannot cross each other

(c) Field lines start at positive charges and end at negative charges

(d) They form closed loops



Q.10 Figure gives electric lines of force due to two charges q_1 and q_2. What are the signs of the two charges?

(a) Both are negative (b) Both are positive

(c) q_1 is positive but q_2 is negative (d) q_1 is negative but q_2 is positive

Q.11 There is an electric field in the X–direction. If the work done in moving a charge of 0.2 C though a distance of 2m along a line making an angle of 60^{o} with X–axis is 4J, then what is the value of E?

(a) $\sqrt{3}\,NC^{-1}$ (b) $4\,NC^{-1}$ (c) $5\,NC^{-1}$ (d) $20\,NC^{-1}$

Q.12 Three point charges +q, – 2q and +q are placed at points (x = 0, y = a, z = 0), (x = 0, y = 0, z = 0) and (x = a, y = 0, z = 0) and (x = a, y = 0, z = 0) respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are

(a) $\sqrt{2}$ qa along the line joining the points (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0)

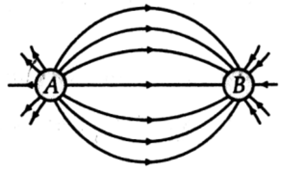
(b) qa along the line joining thepoints (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0).

(c) $\sqrt{2}$ qa along + x direction (d)) $\sqrt{2}$ qa along + y direction

Q.13 A semi–circular arc of radius a is charged uniformly and the charge per unit length is λ. The electric field at the centre is

(a) $\frac{\lambda}{4\pi\varepsilon_0 a}$ (b) $\frac{\lambda}{\pi\varepsilon_0 a^2}$ (c) $\frac{\lambda}{2\pi\varepsilon_0 a}$ (d) $\frac{\lambda}{2\pi\varepsilon_0 a^2}$

Q.14 The spatial distribution of the electric field due to two charges (A, B) is shown in figure. Which one of the following statements is correct?



(a) A is +ve and B –ve and | A | + | B |

(b) A is –ve and B +ve and | A | = | B |

(c) Both are + ve but A > B

(d) Both are – ve but A > B

Q.15 The point charges Q and – 2Q are placed some distance apart. If the electric field at the location of Q is E, then the electric field at the location of – 2Q will be

(a) – E/2 (b) – 3E/ 2 (c) – E (d) – 2 E

Answers

1.	b	2.	a	3.	d	4.	d	5.	b
6.	b	7.	d	8.	b	9.	d	10.	a
11.	d	12.	a	13.	c	14.	a	15.	A

Objective Assignment – III

Q.1 Unit of electric flux are

(a) $NC^{-1} m^2$ (b) JC^{-1} (c) V (d) Vm

Q.2 Positive electric flux from a closed surface indicates that electric lines of force are directed

(a) outwards (b) inwards (c) outwards or inwards (d) none of these

Q.3 A surface encloses an electric dipole. The net flux through the surface is

(a) zero (b) positive (c) negative (d) infinite

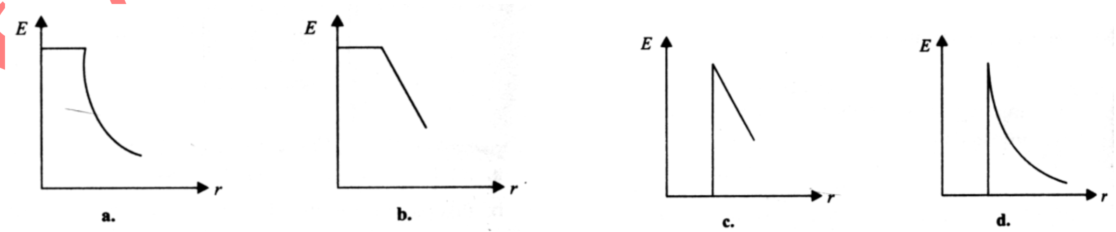
Q.4 In a region with a uniform electric field, the number of lines of force unit area is E. If a spherical metallic conductor is placed in the area, the field inside the conductor will be

(a) zero (b) E (c) more than E (d) less than E

Q.5 An insulated sphere of radius R has a uniform volume charge density ρ. The electric field at a point P inside the sphere at a distance f from the centre is

(a) $\frac{R\rho}{3\varepsilon_0}$ (b) $\frac{r\rho}{3\varepsilon_0}$ (c) zero (d) $\frac{2}{3}\left(\frac{r\rho}{\varepsilon_0}\right)$

Q.6 Which one of the following graphs shows the variation of electric field strength E with distance r from the center of a hollow conducting sphere?



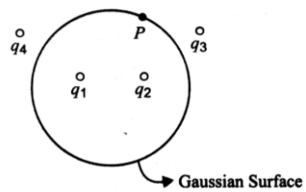
Q.7 A cylinder of length L and radius b has its axis coincident with the x–axis coincident with the x–axis. The electric field in this region is $\vec{E} = 200\hat{i}$. Find the flux through the left end of cylinder.

(a) 0 (b) 200 πb^2 (c) 100 πb^2 (d) – 200 πb^2

Q.8 Consider two infinite parallel charged metal plates with equal and opposite charge densities +σ and –σ. Determine the electric field in the region between the plates.

(a) σ/ε_0 (b) 0 (c) $\sigma/2_0$ (d) $2\sigma/\varepsilon_0$

Q.9 Consider the Gaussian surface that surrounds part of the charge distribution shown in figure. Then, the contribution to the electric field at point P arises from charges



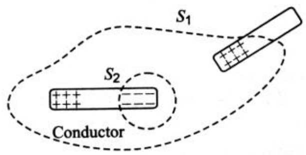
(a) q_1 and q_2 only

(b) q_3 and q_4 only

(c) q_1, q_2, q_3 and q_4

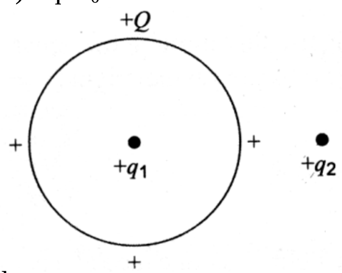
(d) none of the above

Q.10 Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as in figure. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then, flux through surface S_1 is



(a) 0 (b) q/ϵ_0 (c) $-q/\epsilon_0$ (d) $2q/\epsilon_0$

Q.11 A thin metallic spherical shell contains a charge Q on its surface. A point charge q1 is placed at the centre of the shell and another charge q_2 is placed outside the shell. All the three charges are positive. Then, the force on charge q_1 is



(a) towards right (b) towards left

(c) zero (d) none of these

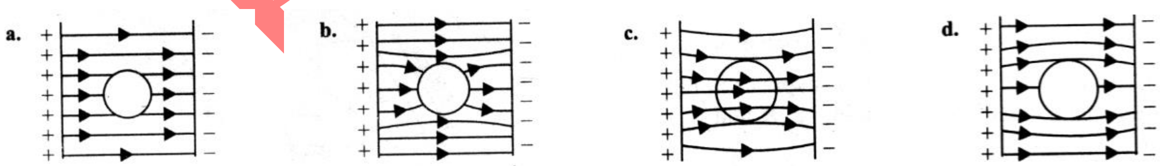
Q.12 If one penetrates a uniformly charged spherical cloud, electric field strength

(a) decreases directly as the distance from the center

(b) increases directly as the distance from the center

(c) remains constant (d) none of the above

Q.13 An uncharged metal sphere is placed between two equal and oppositely charged metal plates. The nature of lines of force will be



Q.14 A hollow metallic sphere of radius 10 cm is given a charge of 3.2×10^{-9} C. The electric intensity at a point 4 cm from the center is

(a) 9×10^{-9} NC^{-1} (b) 288 NC^{-1} (c) 2.88 NC^{-1} (d) zero

Q.15 The surface density on a copper sphere is σ. The electric field strength on the surface of the sphere is

(a) σ (b) σ/2 (c) $\sigma/ 2\varepsilon_0$ (d) σ/ε_0

Q.16 A cylinder of radius R and length l is placed in a uniform electric field E parallel to the axis of the cylinder. The total over the curved surface of the cylinder is

(a) zero (b) $\pi R^2 E$ (c) $2\pi R^2 E$ (d) $E/\pi R^2$

Q.17 A cube of side 10 cm encloses a charge of 0.1 μC at its centre. Calculate the number of lines of force through each face of the cube.

(a) 1.113×10^{11} (b) 1.13×10^{4} (c) 1.13×10^{9} (d) 1883

Q.18 Number of electric lines of force from 0.5 C of positive charge in a dielectric medium of constant 10 s

(a) 5.65×10^{9} (b) 1.13×10^{11} (c) 9×10^{9} (d) 8.85×10^{-12}

Q.19 The electric flux from a cube of edge l is ϕ. What will be its value if edge of cube is made 2l and charge enclosed in halved?

(a) 4ϕ (b) 2ϕ (c) $\phi/2$ (d) ϕ

Q.20 In a certain region of space, there exists a uniform electric field of $2 \times 10^{3}\ \hat{k}\ Vm^{-1}$. A rectangular coil of dimensions 10 cm × 20 cm is placed in x – y plane. The electric flux through the coil is

(a) zero (b) 30 Vm (c) 40 Vm (d) 50 Vm

Q.21 Which of the following may be discontinuous across a charged conducting surface?

(a) Electric potential (b) Electric intensity

(c) Both electric potential and intensity (d) None of the above

Q.22 Consider two concentric spherical surfaces, S_1 with radius a and S_2 with radius 2a, both centered on the origin. There is a charge +q at the origin, and no other charges. Compare the flux ϕ_1 through S_1 with the flux ϕ_2 through S_2.

(a) $\phi_1 = 4\phi_2$ (b) $\phi_1 = 2\phi_2$ (c) $\phi_1 = \phi_2$ (d) $\phi_1 = \phi_2/2$

Q.23 Under what conditions can the electric flux ϕ_E be found through a closed surface?

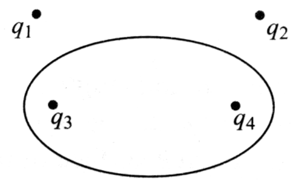
(a) If the magnitude of electric field is known everywhere on the surface.

(b) If the total charge inside the surface is specified.

(c) If the total charge outside the surface is specified.

(d) Only if the location of each point charge inside the surface is specified.

Q.24 Figure shows four charges q_1, q_2, q_3, q_4 fixed in space. Then, the total flux of electric field through a closed surface S, due to all charges q_1, q_2, q_3 and q_4, is



(a) not equal to the total flux through S due to charges q_3 and q_4

(b) equal to the total flux through S due to charges q_3 and q_4

(c) zero if $q_1 + q_2 = q_3 + q_4$

(d) twice total flux through S due to charges q_3 and q_4 if $q_1 + q_2 = q_3 + q_4$

Q.25 If the flux of the electric field through a closed surface is zero, then

(a) the electric field must be zero everywhere on the surface.

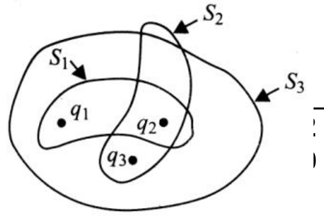
(b) the total charge inside the surface must be zero

(c) the electric field must be uniform throughout the closed surface

(d) the charge outside the surface must be zero

Q.26 Eight charges, 1 μC, –7μC, 10μC, 2μC, –5μC, –3μC and 6μC are situated at the eight corners of a cube of side 20 cm. A spherical surface of radius 80 cm encloses this cube. The center of the sphere coincides with the centre of the cube. Then, the total outgoing flux from the spherical surface (in units of Vm) is

(a) $36\pi \times 10^{3}$ (b) $684\pi \times 10^{3}$ (c) zero (d) none of these



Q.27 Three charges of $q_1 = 1 \times 10^{-6}$ C, $q_2 = 2 \times 10^{-6}$ C and $q_3 = -3 \times 10^{-6}$ have been placed as shown. Then, the net electric flux will be maximum for the surface

(a) S_1 (b) S_2

(c) S_3 (d) same for all three

Q.28 A charge q is distributed uniformly on a ring of radius 'a'. A sphere of equal radius 'a' is constructed with its center at the periphery of the ring. Calculate the flux of the electric field through the surface of the sphere.

(a) $\frac{q}{3\varepsilon_0}$ (b) $\frac{2q}{3\varepsilon_0}$ (c) $\frac{q}{4\varepsilon_0}$ (d) $\frac{3q}{4\varepsilon_0}$

Q.29 In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. The electric flux through a surface of area of 100 units in x – y plane is

(a) 800 units (b) 300 units (c) 400 units (d) 1500 units

Q.30 A spherical shell of radius R = 1.5 cm has a charge q = 20μC uniformly distributed over it. What is the force exerted by one half over the other half?

(a) zero (b) 10^{-2} N (c) 500 N (d) 2000 N

Answers

1	a	2	a	3	a	4	a	5	b
6	d	7	d	8	a	9	c	10	b
11	c	12	a	13	b	14	d	15	d
16	a	17	d	18	a	19	c	20	c
21	b	22	c	23	b	24	b	25	b
26	c	27	a	28	a	29	b	30	d

Objective Assignment – IV

Q.1 How much work is required to carry a 6 μC charge from negative to the positive terminal of a 9 battery?

(a) 54×10^{-3} J (b) 54×10^{-9} J (c) 54×10^{-6} J (d) 54×10^{-12} J

Q.2 The potential at a point, due to a positive charge of 100 μC at a distance of 9 m, is

(a) 10^4 V (b) 10^6 V (c) 10^5 V (d) 10^7 V

Q.3 The potential at the cente of the sphere, if hollow metallic sphere of radius 10 cm is charged such that potential of its surface is 70 V, is

(a) 100 V (b) 35 V (c) 70 V (d) 7 V

Q.4 A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at a distance of 2 cm from the centre of the sphere is

(a) zero (b) 10 V (c) 4 V (d) 10/3 V

Q.5 The electric potential at the surface of an atomic nucleus (Z = 50) of radius 9.0×10^{-13} cm is

(a) 9×10^5 volt (b) 8×10^6 volt (c) 80 volt (d) 9 volt

Q.6 At a point A, there is an electric field of 500 V/m and potential difference of 3000 V. The distance between the point charge and A is

(a) 6 m (b) 36 m (c) 12 m (d) 144 m

Q.7 If an electron is brought towards an another electron, then electric potential energy of the system

(a) increases (b) becomes zero (c) decreases (d) remains the same

Q.8 If we carry a charge once around an equipotential path, then work done by the charge is

(a) infinity (b) positive (c) negative (d) zero

Q.9 Two points charge of + 10μC and – 10μC are placed at points A and B. If P and Q are the two points lying on the perpendicular bisector of the line AB, then work done in taking a charge of 5μC from P to Q will be equal to

(a) distance between P and Q (b) zero

(c) distance between A and B (d) either (a) and (c)

Q.10 In a certain charge distribution, all points having zero potential can be joined by a circle S. Points inside S have positive potential and points outside S have negative potential. A positive charge, which is free to move, is placed inside S.

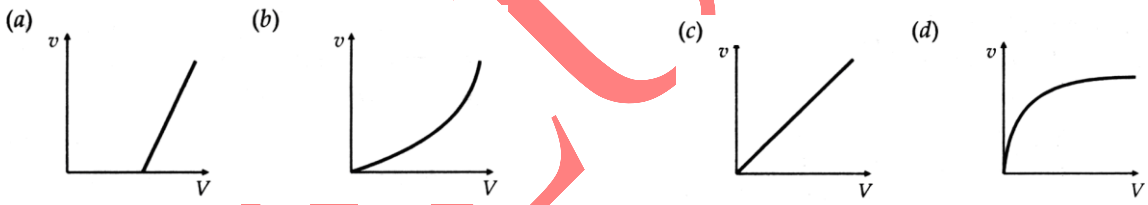
(a) It will remain in equilibrium

(b) It can move inside S, but it cannot cross S

(c) It must cross S at some time

(d) It may move, but will ultimately return to its starting point

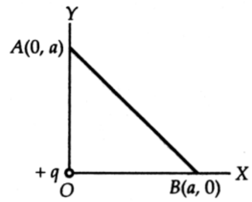
Q.11 The velocity *v* acquired by an electron starting from rest and moving through potential difference V is shown by which of the following graphs?



Q.12 A point charge +q is placed at the origin O as shown in the figure. Work done in taking another point charge –Q from the point A(0,a) to another point B(a, 0) along the straight path AB is

(a) $\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2}\right)\sqrt{2}a$ (b) $\left(-\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2}\right)\sqrt{2}a$

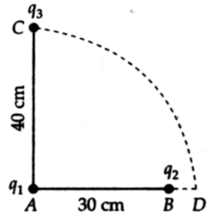
(c) $\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2}\right)\frac{a}{\sqrt{2}}$ (d) zero



Q.13 Identical charge (–q) are placed at each corner of a cube of side b. Then, the electrostatic potential energy of charge (+q) placed at the centre of the cube will be

(a) $\frac{-4\sqrt{2}q^2}{\pi\varepsilon_0}$ (b) $\frac{8\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (c) $-\frac{4q^2}{\sqrt{3}\pi\varepsilon_0 b}$ (d) $\frac{8\sqrt{2}q^2}{4\pi\varepsilon_0 b}$

Q.14 Two charges q_1 and q_2 are placed 30 cm apart as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is



(a) 8 q_1 (b) 6 q_1

(c) 8 q_2 (d) 6 q_2

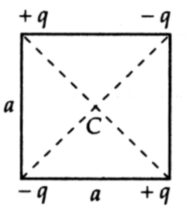
Q.15 The voltage of clouds is 4×10^6 V with respect to ground. In a lightning strike lasting 0.1s, a charge of 4 C is delivered to the ground. The power of the lightning strike is
(a) 160 MW (b) 80 MW (c) 20 MW (d) 500 MW

Q.16 It is possible to have a positively charged body at
(a) zero potential (b) negative potential (c) positive potential (d) all of these

Q.17 Electric potential of earth is taken to be zero, because earth is a good
(a) insulator (b) conductor (c) semi–conductor (d) dielectric

Q.18 Equipotential surfaces associated with an electric field, which is increasing in magnitude along the X–direction, are
(a) planes parallel to YZ–plane (b) planes parallel to XY–plane
(c) planes parallel to XZ–plane (d) coaxial cylinders of increasing radii around the X–axis

Q.19 The potential to which a conductor is raised, depends on
(a) the amount of charge (b) geometry and size of the conductor
(c) both (a) and (b) (d) only on (a)



Q.20 What is the electric potential at the centre of the square?
(a) zero (b) $\frac{kq}{a\sqrt{2}}$ (c) $\frac{kq}{a^2}$ (d) none of these

Q.21 Three charges 1 μC, 2 μC, 3 μC are kept at vertices of an equilateral triangle of side 1 m. If they are brought nearer, so that they now form an equilateral triangle of side 0.5 m, then work done is
(a) 11 J (b) 1.1 J (c) 0.1 J (d) 0. 11 J

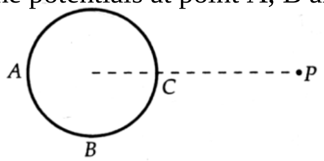
Q.22 A solid sphere and a hollow sphere of equal diameters are raised to the same potential. Then,
(a) hollow sphere has more change (b) both have equal charge
(c) only hollow sphere has charge (d) solid sphere has more charge

Q.23 What is angle between electric field and equipotential surface?
(a) 90^o always (b) 0^o always (c) 0^o to 90^o (d) 0^o to 180^o

Q.24 A dipole is placed in a uniform electric field, its potential energy will be minimum when the angle between its axis and field is
(a) zero (b) π (c) $\pi/2$ (d) 2π

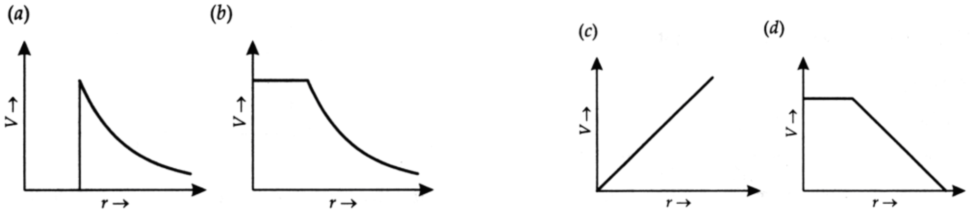
Q.25 A dipole is placed parallel to the electric field. If W is the work done in rotating the dipole by 60^o; then work done in rotating it by 180^o is
(a) 2 W (b) 3W (c) 4W (d) W/2

Q.26 A hollow conducting sphere is placed in an electric field produced by a point charge placed at P as shown in figure. Let V_A, V_B, V_C be the potentials at point A, B and C respectively. Then



(a) $V_C > V_B$ (b) $V_B > V_C$ (c) $V_A > V_B$ (d) $V_A = V_C$

Q.27 In the case of a charged metallic sphere, potential (V) changes with respect to distance (r) from centre as



Q.28 n small metal drops of same size are charged to V volt each. If they coalesce to form a single large drop, then its potential will be

(a) V/n (b) Vn (c) $Vn^{1/3}$ (d) $Vn^{2/3}$

Q.29 A uniform electric field pointing in positive X–direction exists in a region. Let A be the origin, B be the point on the X–axis at x = + 1 cm and C be the point on the Y–axis at y = + 1 cm. Then the potential at points A, B and C satisfy.

(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$

Q.30 A hollow metal sphere of radius 5 cm is charged, such that the potential on its surface is 10 V. The potential at the centre of the sphere is

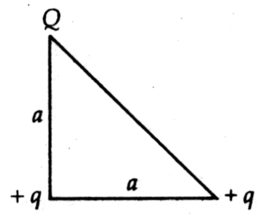
(a) 0 V (b) 10 V

(c) same as at a point 5 cm away from the surface (d) same as at a point 25 cm away from surface

Q.31 Two equal charges are fixed at x = – a and x = + a on the X–axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q when it is displaced by a small distance x along the X–axis, is approximately proportional to

(a) x (b) x^2 (c) x^3 (d) 1/x

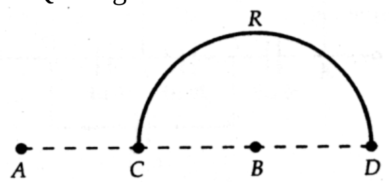
Q.32 Three charges Q, +q and +q are placed at the vertices of a right angled isosceles triangle as shown. Net electrostatic energy of configuration is zero, if Q is equal to



(a) $\frac{-q}{1+\sqrt{2}}$ (b) $\frac{-2q}{2+\sqrt{2}}$

(c) – 2q (d) + q

Q.33 Charges +q and –q are placed at points A and B respectively which are at a distance 2L apart, C is the midpoint between A and B. The work done in moving a charge +Q along the semicircle CRD is



(a) $\frac{qQ}{2\pi\varepsilon_0 L}$ (b) $\frac{qQ}{6\pi\varepsilon_0 L}$

(c) $-\frac{qQ}{6\pi\varepsilon_0 L}$ (d) $\frac{qQ}{4\pi\varepsilon_0 L}$

Answers

1	c	2	c	3	c	4	b	5	b
6	a	7	a	8	d	9	b	10	c
11	d	12	d	13	c	14	c	15	a
16	d	17	b	18	a	19	c	20	a
21	c	22	a	23	a	24	a	25	c
26	d	27	b	28	d	29	b	30	b
31	b	32	b	33	c				

Objective Assignment – V

Q.1 Eight dipoles of charges of magnitude $\underline{+}$ e are placed inside a cube. What will be the total flux coming out of the cube?

(a) 0 (b) ∞ (c) e (d) – e

Q.2 Three small spheres each having a charge +q are placed on the circumference of a circle such that they form an equilateral triangle. Pick out the correct statement.

(a) V = E = 0. (b) $V = 27 \times 10^9 \dfrac{q}{r}$ volt and E = 0

(c) $E = 27 \times 10^9\ q/r^2\ N\ C^{-1}$ and V = 0 (d) $E = 27 \times 10^9\ q/r^2\ N\ C^{-1}$ and $V = 27 \times 10^9 \dfrac{q}{r}$ volt.

Q.3 Which of the following is not the property of equipotential surfaces?

(a) They do not cross each other.

(b) They do not concentric spheres for uniforms electric field.

(c) Rate of change of potential with the distance on them is zero.

(d) They can be imaginary spheres.

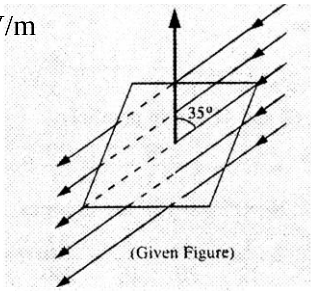
Q.4 The electric potential at a point (x, y) in the x – y plane is given by V = – k × y. the field intensity at a distance r from the origin varies as:

(a) r^2 (b) r (c) r^{-1} (d) r^{-2}

Q.5 The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9)$ volt. Value of electric field at x = 1 m is:

(a) – 20 V/m (b) 6 V/m (c) 11 V/m (d) – 23 V/m

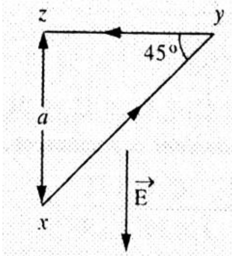
Q.6 The square surface shown in figure measures 3.2mm on each side. It is immersed in a uniform electric field with E = 1800 N/C. The field lines make an angle of 35^o with the outward pointing normal. Calculate flux through the surface.



(a) 1.5×10^{-2}N m^2/C (b) -1.5×10^{-2} N m^2/C

(c) -1.5×10^{-2}N m/c (d) $1 \times 10_{-2}$N m^2/C

Q.7 The unit positive charges moves in an electric field E along the path xyz. The voltage between points x and y is:



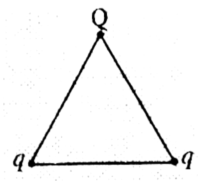
(a) zero

(b) Ea

(c) Ea/2

(d) Ea/4

Q.8 Three charge Q, +q and +q are placed at the vertices of an equilateral triangle of side l. If the net electrostatic energy of the system is zero, then Q is equal to



(a) $-q^2/2$ (b) – q

(c) + q^2 (d) zero

Q.9 Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then:

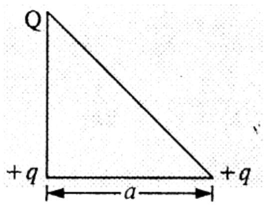
(a) negative and distributed uniformly over the surface of the sphere.

(b) negative and appears only at the point on the sphere closest to the point charge.
(c) negative and distributed non–uniformly over the entire surface of the sphere. (d) zero

Q.10 Two point charges +q and –q are held fixed at (–d, 0) and (d, 0) respectively of a (x, y) co–ordinate system, then
(a) the electric field $\vec{E}$ at all points on the X–axis has the same direction.
(b) $\vec{E}$ at all points on the y–axis is along $\hat{i}$.
(c) work has to be done in bringing a test charge from infinity to the origin.
(d) the dipole moment is 2qd directed along $\hat{i}$.

Q.11 A non conducting ring of radius 0.5m carries a total charge of 1.1×10^{-10} C distributed non uniformly on its circumference producing an electric field E everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -E.\,dl$ (l – 0 being centre of the ring) in volt is:
(a) + 2 (b) – 1 (c) – 2 (d) zero

Q.12 Three charges Q, +q and +q are placed at the vertices of a right angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero, if Q is equal to:



(a) $\frac{-q}{1+\sqrt{2}}$ (b) $\frac{-2q}{2+\sqrt{2}}$ (c) $-2q$ (d) + 2

Answers

1.	a	2.	b	3.	c	4.	b	5.	a
6.	b	7.	b	8.	a	9.	d	10.	b
11.	a	12.	b						

Higher Order Assignment

Q.1 Two equal charges are placed at a separation of 1.0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person?

Q.2 Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons in a nucleus remain at a separation of this order.

Q.3 Two charges 2.0×10^{-6} C and 1.0×10^{-6} C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges?

Q.4 Suppose the second charge in the previous problem is -1.0×10^{-6} C. Locate the position where a third charge will not experience a net force.

Q.5 Two charged particles are placed at a distance 1.0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge?

Q.6 Estimate number of electrons in 100 g of water. How much is total negative charge on these electrons?

Q.7 Suppose all the electrons of 100g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively 10.0 cm away from each other, find the force of attraction between them. Compare it with your weight.

Q.8 Consider a gold nucleus to be a sphere of radius 6.9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion?

Q.9 Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0.1 N, how many electrons were transferred from one sphere to the other during rubbing?

Q.10 NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between ions to be 2.75×10^{-8} cm, find the force of attraction between them. State the assumptions (if any) that you have made.

Q.11 Suppose an attractive nuclear force acts between two protons which may be written as $F = Ce^{-kr}/r^2$. (a) Write down the dimensional formulae and appropriate SI units of C and k. (b) Suppose that k = 1 fermi^{-1} and that the repulsive electric force between the protons is just balanced by the attraction nuclear force when the separation is 5 fermi. Find the value of C.

Q.12 Three equal charges, 2.0×10^{-6} C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.

Q.13 Four equal charges 2.0×10^{-6} C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.

Q.14 A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0.53 angstrom (1 angstrom = 10^{-10} m and is abbreviated as Å) with the proton at the centre. The hydrogen atom is said to be in the ground state in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.

Q.15 Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.

Q.16 Ten positively charged particles are kept fixed on x–axis at points x = 10 cm, 20 cm, 30 cm, ..., 100cm. The first particle has a charge 1.0×10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the acting on a 1 C charge placed at the origin.

Q.17 Two charged particles having charge 2.0×10^{-8} C each are joined by an insulating string of length 1m and the system is kept on a smooth horizontal table. Find the tension in the string.

Q.18 Two identical balls, each having a charge of 2.00×10^{-7} C and a mass of 100 g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5.0 cm apart and then released. Find (a) the electric force on one of the charged balls (b) the components of the resultant force on it along and perpendicular to the string (c) the tension in the string (d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.

Q.19 Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude 2.0×10^{-8} C.

Q.20 Two small spheres, each having a mass of 20g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at equilibrium is found to be 4 cm. Find the charge on each sphere.

Q.21 Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length *l*. Find the mass of each ball if the angle between the strings is 2θ in equilibrium.

Q.22 A particle having a charge of 2.0×10^{-4} C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100g. What charge should the bob be given so that the string becomes loose?

Q.23 Two particles A and B having charges q and 2q respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?

Q.24 Two identically charged particles are fastened to the two ends of a spring of spring constant 100 N m^{-1} and natural length 10 cm. The system rests on a smooth horizontal table. If the charge on each particle is 2.0×10^{-8} C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assumption after you solve the problem.

Q.25 A particle A having a charge of 2.0×10^{-6} C is held fixed on a horizontal table. A second charged particle of mass 80g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is $\mu = 0.2$. Find the range within which the charge of this second particle may lie.

Q.26 A particle A having a charge of 2.0×10^{-6} C and a mass of 100 g is placed at the bottom of a smooth inclined plane in inclination 30^{o}. Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

Q.27 Two particles A and B, each having a charge Q, are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of this maximum force?

Q.28 Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having mass m and charge q is kept at the middle point of the line AB. (a) If it is displaced through a distance x perpendicular to AB, what would be the electric force experienced by it. (b) Assuming $x << d$, show that this force is proportional to x. (c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement? Find the time period of the oscillations if these conditions are satisfied.

Q.29 Repeat the previous problem if the particle C is displaced through a distance x along the line AB.

Q.30 The electric force experienced by a charge of 1.0×10^{-6} C is 1.5×10^{-3} N. Find the magnitude of the electric field at the position of the charge.

Q.31 Two particles A and B having charges of $+ 2.00 \times 10^{-6}$ C and of $- 4.00 \times 10^{-6}$ C respectively are held fixed at a separation of 20.0 cm. Locate the point(s) on the line AB where (a) the electric field is zero (b) the electric potential is zero.

Q.32 A point charge produces an electric field of magnitude 5.0 N C^{-1} at a distance of 40 cm from it. What is the magnitude of the charge?

Q.33 A water particle of mass 10.0 mg and having a charge of 1.50×10^{-6} C stays suspended in a room. What is the magnitude of electric field in the room? What is its direction?

Q.34 Three identical charges, each having a value 1.0×10^{-8} C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the centre of the triangle.

Q.35 Positive charge Q is distributed uniformly over a circular ring of radius R. A particle having a mass m and a negative charge q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming x < < R, find the time period of oscillation of the particle if it is released from there.

Q.36 A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

Q.37 A 10 cm long rod carries a charge of + 50 μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

Q.38 Consider a uniformly charged ring of radius R. Find the point on the axis where the electric field is maximum.

Q.39 A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the centre? You may answer this part without making any numerical calculations.

Q.40 A circular wire–loop of radius a carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the centre due to the remaining wire.

Q.41 A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

Q.42 A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a uniform, horizontal electric field of 20 kVm^{-1}. Find the tension in the thread.

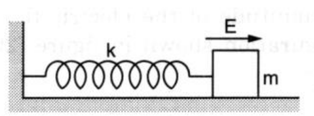
Q.43 A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest?

Q.44 A particle of mass 1 g and charge 2.5×10^{-4} C is released from rest in an electric field of 1.2×10^4 NC^{-1}. (a) Find the electric force and the force of gravity acting on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis? (b) How long will it take for the particle to travel a distance of 40 cm? (c) What will be the speed of the particle after travelling this distance? (d) How much is the work done by the electric force on the particle during this period?

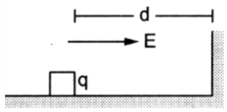
Q.45 A ball of mass 100g and having a charge of 4.9×10^{-5} C is released from rest in a region where a horizontal electric field of 2.0×10^4 NC^{-1} exists. (a) Find the resultant force acting on the ball. (b) What will be the path of the ball? (c) Where will the ball be at the end of 2s?

Q.46 The bob of a simple pendulum has a mass of 40g and a positive charge of 4.0×10^{-6} C. It makes 20 oscillations in 45s. A vertical electric field pointing upward and of magnitude 2.5×10^4 NC^{-1} is switched on. How much time will it now take to complete 20 oscillations?

Q.47 A block of mass m having a charge q is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant k as shown in figure. A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block.



Q.48 A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure. The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion?



Q.49 A uniform electric field of 10 NC^{-1} exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

Q.50 12J of work has to be done against an existing electric field to take a charge of 0.01 C from A to B. How much is the potential difference $V_B - V_A$?

Q.51 Two equal charges, 2.0×10^{-7} C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude moved to a point 20 cm from both the charges. How much work is done by the electric field during the process?

Q.52 An electric field of 20 NC^{-1} exists along the x–axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,

(a) A = (0, 0); B = (4m, 2m) (b) A = (4m, 2m); B = (6m, 5m)

(c) A = (0, 0); B = (6m, 5m)

Do you find any relation between the answers of parts (a), (b) and (c)?

Q.53 Consider the situation of the previous problem. A charge of -2.0×10^{-4} C is moved from the point A to the point B. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

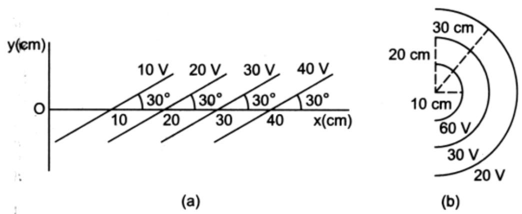
Q.54 An electric field $\vec{E} = (\vec{i}\,20 + \vec{j}\,30)\,NC^{-1}$ exists in the space. If the potential at the origin is taken to be zero, find the potential at (2m, 2m).

Q.55 An electric field $\vec{E} = \vec{i}\,Ax$ exists in the space, where A = 10 V m^{-2}. Take the potential at (10, 20m) to be zero. Find the potential at the origin.

Q.56 The electric potential existing in space is V(x, y, z) = A(xy + yz + zx). (a) Write the dimensional formula of A. (b) Find the expression for the electric field. (c) If A is 10 SI units, find the magnitude of the electric field at (1m, 1m, 1m).

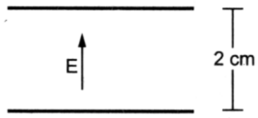
Q.57 Two charged particles, having equal charges of 2.0×10^{-5} C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

Q.58 Some equipotential surface are shown in figure. What can you say about the magnitude and the direction of the electric field?



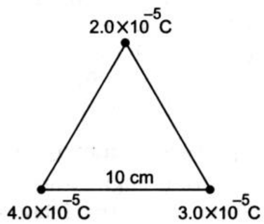
Q.59 Consider a circular ring of radius r, uniformly charged with linear charge density λ. Find the electric potential at a point on the axis at a distance x from the centre of the ring. Using this expression for the potential, find the electric field at this point.

Q.60 An electric field of magnitude 1000 NC^{-1} is produced between two parallel plates having a separation of 2.0 cm as shown in figure. (a) What is the potential difference between the plates? (b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate? (c) Suppose the electron is projected from the lower plate with the speed calculated in part. (d) The direction of projection makes an angle of 60^{o} with the field. Find the maximum height reached by the electron.



Q.61 A uniform field of 2.0 NC^{-1} exists in space in x–direction. (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z). (b) At which points, the potential is 25V? (c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at a general point? (d) What will be the potential at the origin if the potential at infinity is taken to be zero? Is it practical to choose the potential at infinity to be zero?

Q.62 How much work has to be done in assembling three charged particles at the vertices of an equilibrium triangle as shown in figure?



Q.63 The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.

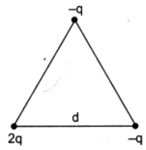
Q.64 Two identical particles, each having a charge of 2.0×10^{-4} C and mass of 10g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large?

Q.65 Two particles have equal masses of 5.0 g each and opposite charges of $+ 4.0 \times 10^{-5}$ C and $- 4.0 \times 10^{-5}$C. They are released from rest with a separation of 1.0 cm between them. Find the speeds of the particles when the separation is reduced to 50 cm.

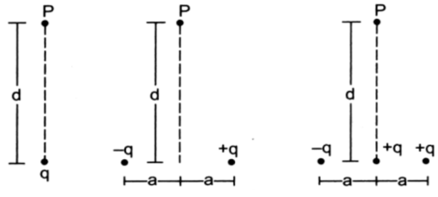
Q.66 A sample of HCl gas is placed in an electric field of 2.5×10^{4} N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Find the maximum torque that can act on a molecule.

Q.67 Two particles A and B, having opposite charges 2.0×10^{-6} C and $- 2.0 \times 10^{-6}$ C, are placed at a separation of 1.0 cm. (a) Write down the electric dipole moment of this pair. (b) Calculate the electric field at a point on the axis of the dipole 1.0 cm away from the centre. (c) Calculate the electric field at a point on the perpendicular bisector of the dipole and 1.0 m away from the centre.

Q.68 Three charges are arranged on the vertices of an equilateral triangle as shown in figure. Find the dipole moment of the combination.



Q.69 Find the magnitude of the electric field at the point P in the configuration shown in figure for d > > a. Take 2qa = p.



(a) (b) (c)

Q.70 Two particles, carrying charges – q and + q and having equal masses m each, are fixed at the ends of a light rod of length *a* to form a dipole. The rod is clamped at an end and is placed in a uniform electric field E with the axis of the dipole along the electric field. The rod is slightly titled and then released. Neglecting gravity find the time period of small oscillations.

Q.71 Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6.4 g (take the atomic weight of copper to be 64g mol^{-1}).

Answers

1. 2.3×10^{-4} C
2. 230 N
3. 5.9 cm from the larger charge in between the two charges
4. 34.1 cm from the larger charge on the line joining the charge in the side of the smaller charge
5. 2.3×10^{-24} N
6. 3.35×10^{25}, 5.35×10^{6} C
7. 2.56×10^{25} N
8. 1.2 N
9. 2×10^{11}
10. 3.05×10^{-9} N
11. (a) ML^3T^{-2}, L^{-1}, Nm^2, m^{-1}, (b) 3.4×10^{-26} Nm^2
12. 24.9 N at 30° with the extended sides from the charge under consideration
13. 27.5 N at 45° with the extended sides of the square from the charge under consideration
14. 8.2×10^{-8} N
15. 2.18×10^{6} ms^{-1}
16. 4.95×10^{5} N
17. 3.6×10^{-6} N
18. (a) 0.144 N, (b) zero, 0.095 N away from the other charge, (c) 0.986 N and (d) 0.95 ms^{-2} perpendicular to the string and going away from the other charge
19. 8.2g, 8.2×10^{-2} N
20. 4.17×10^{-8} C
21. $\dfrac{q^2 \cot\theta}{16\pi\varepsilon_0 gl^2 \sin^2\theta}$
22. 5.4×10^{-9} C
23. $-(6-4\sqrt{2})q$, between q and 2q at a distance of $(\sqrt{2}-1)d$ *from q*
24. 3.6×10^{-6} m
25. between $\pm$ 8.71×10^{-8} C
26. 27cm from the bottom
27. $d/2\sqrt{2}$, $3.08\dfrac{Qq}{4\pi\varepsilon_0 d^2}$
28. $(a)\ \dfrac{Qqx}{2\pi\varepsilon_0\left(x^2+\dfrac{d^2}{4}\right)^{3/2}}$, $(c)\left[\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}\right]^{\frac{1}{2}}$
29. time period = $\left[\dfrac{\pi^3\varepsilon_0 md^3}{2Qq}\right]^{\frac{1}{2}}$
30. 1.5×10^{3} NC^{-1}

31. (a) 48.3 cm from A along BA, (b) 20 cm from A along BA and $\frac{20}{3}$ cm from A along AB

32. 8.9×10^{-11} C

33. 65.3 NC^{-1}, upward

34. zero, 2.3×10^3 V

35. $\left[\frac{16\pi^3\varepsilon_0 mR^3}{Qq}\right]^{1/2}$

36. $\frac{Q}{2\varepsilon_0 L^2}$

37. 5.2×10^7 NC^{-1}

38. $R/\sqrt{2}$

39. zero

40. $\frac{QdL}{8\pi^2 \varepsilon_0 a^3}$

41. $\frac{q}{4\pi\varepsilon_0 d^2}$ towards the charge q

42. 8.8×10^{-4} N

43. $\frac{mu^2}{2qE}$

44. (a) 3.0 N, 9.8×10^{-3} N, (b) 1.63×10^{-2} s, (c) 49.0 ms^{-1}, (d) 1.20 J

45. (a) 1.4 N making an angle of 45^o with $\vec{g}$ and $\vec{E}$, (b) straight line along the resultant force, (c) 28m from the starting point on the line of motion

46. 52 s

47. qE/ k

48. $\sqrt{\frac{8md}{qE}}$

49. 5 V

50. 1200 volts

51. 3.6×10^{-3} J

52. (a) – 80 V, (b) – 40 V, (c) – 120 V

53. 0.016 J, 0.008 J, 0.024 J

54. – 100 V

55. 500 V

56. (a) $MT^{-3} I^{-1}$, (b) $-A\{\vec{i}\,(y+z)+\vec{j}\,(z+x)+\vec{k}\,(x+y)\}$, (c) 35 NC^{-1}

57. 36 J

58. (a) 200Vm^{-1} making an angle 120^o with x–axis (b) radially outward, decreasing with distance as $E=\frac{6Vm}{r^2}$

59. $\frac{r\lambda}{2\varepsilon_0 (r^2+x^2)^{1/2}}, \frac{r\lambda x}{2\varepsilon_0 (r^2+x^2)^{3/2}}$

60. (a) 20 V, (b) 2.65×10^6 ms^{-1}, (c) 0.50 cm

61. (a) – (2.0 V m^{-1})x, (b) points on the plane x = – 12.5 m, (c) 100 V – (2.0 V m^{-1})x, (d) infinity

62. 234 J

63. 0.1 C

64. 600 m s^{-1}

65. 54 m s^{-1} for each particle

66. 8.5×10^{-26} Nm

67. (a) 2.0×10^{-8} Cm, (b) 360 N C^{-1}, (c) 180 N C^{-1}

68. $qd\sqrt{3}$, along the bisector of the angle at 2q, away from the triangle

69. $(a)\ \frac{q}{4\pi\varepsilon_0 d^2}, (b)\ \frac{p}{4\pi\varepsilon_0 d^3}, (c)\ \frac{1}{4\pi\varepsilon_0 d^3}\sqrt{q^2d^2+p^2}$

70. $2\pi\sqrt{\frac{ma}{qE}}$

71. 6×10^{22}

Higher Order Assignment – II

Q.1 The electric field in a region is given by $\vec{E}=\frac{3}{5}E_o\vec{i}+\frac{4}{5}E_o\vec{j}$ and $E_o = 2.0\times10^3\,NC^{-1}$. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the y – z plane.

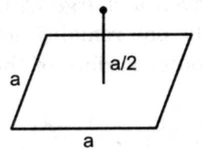
Q.2 A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

Q.3 Show that there can be no net charge in a region in which the electric field is uniform at all points.

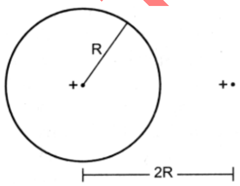
Q.4 The electric field in a region is given by $\vec{E} = \frac{E_o x}{l}\vec{i}$. Find the charge contained inside a cubical volume bounded by surfaces x = 0, x = a, y = 0, z = 0 and z = a. Take $E_o = 5 \times 10^3\ NC^{-1}$, $l = 2$ cm and a = 1 cm.

Q.5 A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

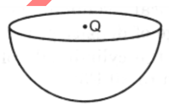
Q.6 A charge Q is placed at a distance a/2 above the centre of a horizontal, square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.



Q.7 Find the flux of the electric field through a spherical surface of radius R due to a charge of 10^{-7} C at the centre and another equal charge at a point 2R away from the centre.



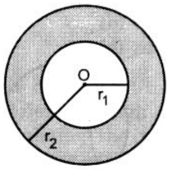
Q.8 A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure).



Q.9 A spherical volume contains a uniformly distributed charge of density $2.0 \times 10^{-4}\ C\ m^{-3}$. Find the electric field at a point inside the volume at a distance 4.0 cm from the centre.

Q.10 The radius of a gold nucleus (Z = 79) is about 7.0×10^{-15} m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface?

Q.11 A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 (figure). Find the electric field at a point P distance x away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of x for $0 < x < 2r_2$ (figure).



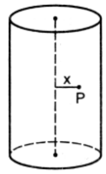
Q.12 A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a. (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

Q.13 Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius 10^{-15} m. The two 1s electrons make a spherical charge cloud at an average distance of 1.3×10^{-11} m from the nucleus, whereas the two 2s electrons make another spherical cloud at an average distance of 5.2×10^{-11} m from the nucleus. Find the electric field at (a) a point just inside the 1s cloud and (b) a point just inside the 2s cloud.

Q.14 Find the magnitude of electric field at a point 4cm away from a line charge of density 2×10^{-6} C m^{-1}.

Q.15 A long cylindrical wire carries a positive charge of linear density 2.0×10^{-8} C m^{-1}. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron.

Q.16 A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis (figure)



Q.17 A non–conducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the centre plane. Draw a qualitative graph of E against x for $0 < x < d$.

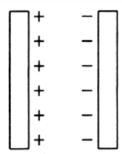
Q.18 A charged particle having a charge of -2.0×10^{-6} C is placed close to a non–conducting plate having a surface charge density 4.0×10^{-6} C m^{-2}. Find the force of attraction between the particle and the plate.

Q.19 One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged non–conducting plate and the other end is fastened to a small ball having a mass of 10g and a charge of 4.0×10^{-6} C. In equilibrium, the thread makes an angle of 60^{o} with the vertical. Find the surface charge density on the plate.

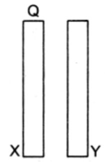
Q.20 Consider the situation of the previous problem (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

Q.21 Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces.

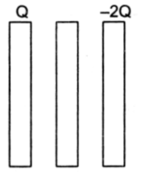
Q.22 Two large conducting plates are placed parallel to each other and they carry equal and opposite charge with surface density σ as shown in figure. Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.



Q.23 Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate X, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.



Q.24 Three identical metal plates with large surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge Q, the rightmost a charge –2Q and the middle one remains neutral. Find the charge appearing on the other surface of the rightmost plate.



Answer

1. 240 N m^2 C^{-1} 2. $Q/(2\varepsilon_0)$ 4. 2.2×10^{-12} C

5. Q/ε_0 6. $Q/(6\varepsilon_0)$ 7. 1.1×10^4 Nm^{-2} C^{-1}

8. $Q/(2\varepsilon_0)$ 9. 3.0×10^5 NC^{-1}

10. (a) 2.32×10^{21} NC^{-1}, (b) 1.16×10^{21} NC^{-1} 11. $\dfrac{Q(x^3-r_1^3)}{4\pi\varepsilon_0 x^2 (r_2^3-r_1^3)}$

12. $(a)\ -\dfrac{Q}{4\pi a^2}, \dfrac{Q}{4\pi a^2},\ (b)\ -\dfrac{Q}{4\pi a^2}, \dfrac{Q+q}{4\pi a^2},\ (c)\ \dfrac{Q}{4\pi\varepsilon_0 x^2}$ in both situations

13. (a) 3.4×10^{13} NC^{-1}, (b) 1.1×10^{12} NC^{-1} 14. 9×10^5 NC^{-1}

15. 2.88×10^{-17} J 16. $\rho x/(2\varepsilon_0)$ 17. $\rho x/\varepsilon_0$

18. 0.45 N 19. 7.5×10^{-7} C m^{-2} 20. (a) 0.20 N, (b) 0.45 s

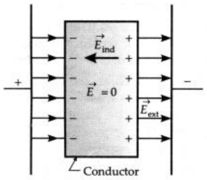
21. 0.505×10^{-12} C m^{-2} 22. (a) zero, (b) σ/ε_0, (c) zero

23. $(a)\ \dfrac{Q}{2A}$, $(b)\ \dfrac{Q}{2A\varepsilon_0}$ towards left, (c) $\dfrac{Q}{2A\varepsilon_0}$ towards right, (d) $\dfrac{Q}{2A\varepsilon_0}$ towards right 24. – Q/2

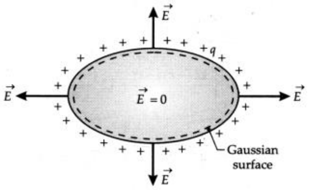
CAPACITOR

Behaviour of Conductors in Electrostatic Fields

1. Net electrostatic field is zero in the interior of a conductor.

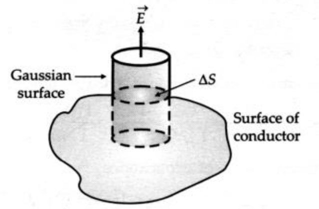


2. Just outside the surface of a charged conductor, electric field is normal to the surface.
3. The net charge in the interior of a conductor is zero and any express charge resides at its surface.

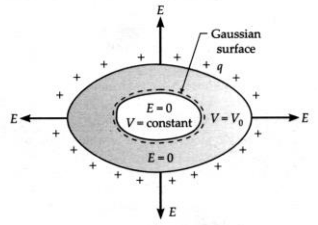


4. Potential is constant within and on the surface of a conductor.

5. Electric field at the surface of a charged conductor is proportional to the surface charge density.



6. Electric field is zero in the cavity of a hollow charged conductor.



Electrostatic Shielding

The cavity inside the conductor remains shielded from outside electric influence. This is known as electrostatic shielding. Such a field free region is called a *Faraday cage.*

The phenomenon of making a region free from any electric field is called **electrostatic shielding.** It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

Applications of electrostatic shielding

1. In a thunderstorm accompanied by lightning, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lightning.
2. Sensitive components of electronic devices are protected or shielded from external electric disturbances by placing metal shields around them.
3. In a coaxial cable, the outer conductor connected to ground provides an electrical shield to the signals carried by the central conductor.

Note:

- In the interior of a conductor, the electric, the electric field and the volume charge density both vanish. Therefore, charges in a conductor can only be at the surface.
- Electric field at the surface of a charged conductor must be normal to the surface at every point.
- For a conductor without any surface charge, electric field is zero even at the surface.
- The entire body of each conductor, including its surface, is at a constant potential.
- If we have conductors of arbitrary size, shape and charge configuration, then each conductor will have a characteristic value of constant potential which may differ from one conductor to another.
- A cavity inside a conductor is shielded from outside electrical disturbances. However, the electric shielding does not work the other way round. That is, if we place charges inside the cavity, the exterior of the conductor cannot be shielded from the electric fields of the inside charges.

Electrical capacitance of a conductor

The electrical capacitance of a conductor is the measure of its ability to hold electric charge. If we increase the charge on a conductor, its potential also increases. If a charge Q put on an insulated conductor increases its potential by V, then

$$Q \propto V \qquad \text{or} \qquad Q = CV$$

The proportionality constant C is called the capacitance of the conductor. Thus

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential}}$$

Hence the **capacitance** of a conductor may be defined as the charge required to increase the potential of the conductor by unit amount.

It depends upon the following factors:

1. Size and shape of the conductor.
2. Nature (permittivity) of the surrounding medium.
3. Presence of the other conductors in its neighborhood.

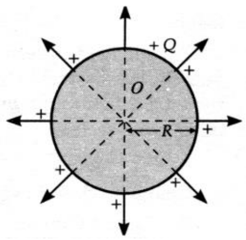
Units of Capacitance: The SI unit of capacitance is farad (F).

Dimensions of capacitance: The unit of capacitance is $F = \frac{C}{V} = \frac{C}{J/C} = \frac{C^2}{J} = \frac{(As)^2}{Nm}$

$\therefore$ Dimensions of capacitance $= \frac{A^2T^2}{MLT^{-2}.L} = [M^{-1}L^{-2}T^4A^2]$

Capacitance of an isolated spherical conductor

Consider an isolated spherical conductor of radius R. The charge Q is uniformly distributed over its entire surface. The potential at any point on the surface of the spherical conductor will be



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

$\therefore$ Capacitance of the spherical conductor situated in vacuum is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}}$$

Or $C = 4\pi\varepsilon_0 R$

Clearly, the capacitance of a spherical conductor is proportional to its radius.

Subjective Assignment – I

Q.1 Find the capacitance of a conducting sphere of radius 10 cm situated in air. How much charge is required to raise it to a potential of 1000 volt?

Q.2 If the capacitance of conductor carrying a charge of 8 C is 0.005 F, calculate its potential.

Q.3 125 drops of water each of radius 2 mm and carrying charge of 1 nC are made to form a bigger drop. Find the capacitance and potential of the bigger drop.

Q.4 N drops of mercury of equal radii and possessing equal charges combine to form a big drop. Compare charge, capacitance and potential of bigger drop with the corresponding quantities of individual drops.

Q.5 Twenty seven spherical drops of radius 3 mm and carrying 10^{-12} C of charge are combined to form a single drop. Find the capacitance and the potential of the bigger drop.

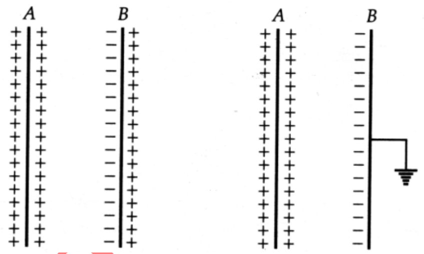
Q.6 A charged spherical conductor has a surface charge density of 0.07 C cm^{-2}. When the charge is increased by 4.4 C, the surface charge density changes by 0.084 C cm^{-2}. Find the initial charge and capacitance of the spherical conductor.

Answers

1.	11 pF, 11×10^{-8} C	2.	1600 V	3.	1.1 pF, 113.6 V
4.	N, $N^{1/3}$, $N^{2/3}$	5.	1 pF, 27 V	6.	22 C, 5.56×10^{-12} F

Principle of a capacitor

Consider a positively charged metal plate A and place an uncharged plate B close to it, as shown in figure. Due to induction, the closer face of plate B acquires negative charge and its farther face acquires a positive charge. The negative charge on plate B tends to reduce the potential on plate A, while the positive charge on plate B tends to increase the potential on A. As the negative charge of plate B is closer to plate A than its positive charge, so the net effect is that the potential of A decreases by a small amount and hence its capacitance increases by a small amount.

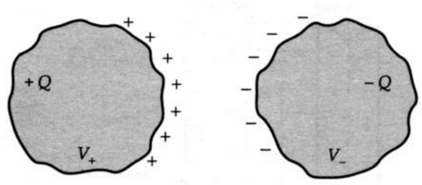


Now if the positive face of plate B is earthed, its positive charge gets neutralized due to the flow of electrons from the earth to the plate B. Hence increases its capacitance by a large amount. The capacitance of an insulated conductor is considerably increased when we place an earthed connected conductor near it. Such a system of two conductors is called a **capacitor.**

Capacitor: *A capacitor is an arrangement of two conductors separated by an insulating medium that is used to store electric charge and electric energy.*

A capacitor, in general, consists of two conductors of any size and shape carrying different potentials and charges, and placed closed together in some definite positions relative to one another.

Capacitance of a capacitor: As shown in figure usually a capacitor consists of two conductors having charges +Q and –Q. The potential difference between them is $V = V_+ - V_-$. Here Q is called the charge on the capacitor.



Note: *that the charge on capacitor does not mean the total charge given to the capacitor which is +Q – Q = 0*

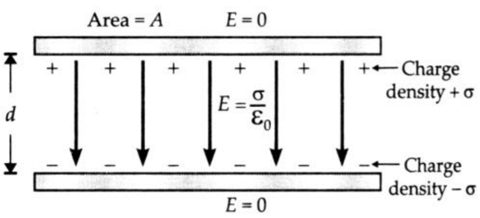
Parallel plate capacitor

It consists of two large plane parallel conducting plates, separated by a small distance.

Let A = area of each plate, d = distance between the two plates

$\pm\sigma$ = uniform surface charge densities on the two plates

$\pm Q = +\sigma A$ = total charge on each plate.



In the outer regions above the upper plate and below the lower plate, the electric fields due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In inner region between two capacitor plates, electric fields due to the two charged plates add up. The net field is

$$E=\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$$

P.D. between the plates = Electric field × distance between the plates or $V = Ed = \dfrac{\sigma d}{\varepsilon_0}$

Capacitance of the parallel plate capacitor is

$$C=\frac{Q}{V}=\frac{\sigma A}{\sigma d/\varepsilon_0} \qquad \text{or} \qquad C=\frac{\varepsilon_0 A}{d}$$

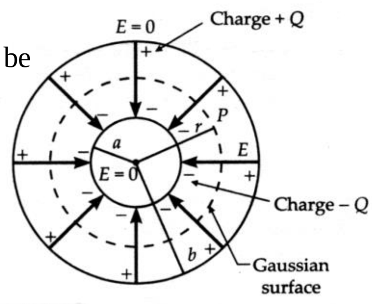
Factors on which the capacitance of a parallel plate capacitor depends

1. Area of the plates ($C \propto A$).
2. Distance between the plates ($C \propto 1/d$)
3. Permittivity of the medium between the plates ($C \propto \varepsilon$).

Spherical capacitor

A spherical capacitor consists of two concentric spherical shells of inner and outer radii a and b. The two shells carry charges –Q and +Q respectively. Since the electric field inside a hollow conductor is zero, so $\vec{E} = 0$ for r < a. Also the field is zero outside the outer shell, i.e., $\vec{E} = 0$ for r > b. A radial field $\vec{E}$ exists in the region between the two shells due to the charge on the inner shell only. To determine the electric field at any point P at distance r from the centre, consider a concentric sphere of radius r as Gaussian surface. Using Gauss's theorem,

$$\phi_E = E.\,4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad \text{or} \qquad E=\frac{Q}{4\pi\varepsilon_0 r^2}$$



The potential difference (caused by inner sphere alone) between two shells will be

$$V=-\int_a^b \vec{E}.\,d\vec{r} = \int_a^b E dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_a^b r^{-2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

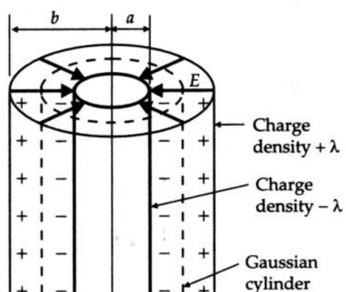
[∵ $\vec{E}$ points radially inward and $d\vec{r}$ points outward so $\vec{E}.\,d\vec{r} = Edr\,180^o = -Edr$]

$$C=\frac{Q}{V}=\frac{Q}{\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{a}-\dfrac{1}{b}\right]} \qquad \text{or} \qquad C=\frac{4\pi\varepsilon_0\, ab}{b-a}$$

Cylindrical Capacitor

A cylindrical capacitor consists of two coaxial conducting cylinders of inner and outer radii a and b. Let the two cylinders have uniform linear charge densities of $\pm\lambda$. The length L of the capacitor is so large (L >> radii a or b) that the edge effect can be neglect. The electric field in the region between the two cylinders comes only from the inner cylinder. The electric field E at any point P in between the two cylinders at a distance r from the central axis, we consider a coaxial Gaussian cylinder of radius r. Using Gauss's theorem, the flux through Gaussian surface must be

or $E.\,2\pi r\,L = \dfrac{\lambda L}{\varepsilon_0}$ $\qquad E = \dfrac{\lambda}{2\pi\varepsilon_0\, r}$



∴ Potential difference between the two cylinders is

$$V = -\int_a^b \vec{E}.\vec{dr} = \int_a^b E dr \quad [\because \vec{E} \text{ and } d\vec{r} \text{ are in opposite directions}]$$

$$= \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r} dr \quad = \frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b = \frac{\lambda}{2\pi\varepsilon_0}[\ln b - \ln a]$$

or $$V = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{a}$$

Total charge on each cylinder is $Q = L\lambda$

∴ Capacitance of cylindrical capacitor is $C = \frac{Q}{V} = \frac{L\lambda}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}}$ or $C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$

Subjective Assignment – II

Q.1 When 1.0×10^{12} electrons are transferred from one conductor to another of a capacitor, a potential difference of 10V develops between the two conductors. Calculate the capacitance of the capacitor.

Q.2 When the potential difference across a capacitor is reduced by 120 V, the charge on the capacitor charges from 360 μC to 120 μC. What is the capacitance of the capacitor?

Q.3 A parallel plate capacitor has plate area of 25.0 cm^2 and a separation of 2.0 mm between its plates. The capacitor is connected to 12 V battery. (i) Find the charge on the capacitor. (ii) If the plate separation is decreased by 1.0 mm, what extra charge is given by the battery to the positive plate?

Q.4 A sphere of radius 0.03m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 volt and outer sphere is earthed, find the capacitance and the charge on the inner sphere.

Q.5 The thickness of air layer between the two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the sphere of 1.2 m diameter. Find the radii of the surfaces.

Q.6 The negative plates of a parallel plate capacitor is given a charge of -20×10^{-8} C. Find the charges appearing on the four surfaces of the capacitor plates.

Q.7 Calculate the capacitance of a parallel plate capacitor having circular discs of radii 5.0 cm each. The separation between the discs is 1.0 mm.

Q.8 A parallel–plate capacitor has plates of area 200 cm^2 and separation between the plates 1.0 mm. (i) What potential difference will be developed if a charge of 1.0 nC is given to the capacitor? (ii) If the plate separation is now increased to 2.0 mm, what will be the new potential difference?

Q.9 A spherical capacitor has an inner sphere of radius 9 cm and an outer sphere of radius 10 cm. The outer sphere is earthed and the inner sphere is charged. What is the capacitance of the capacitor?

Q.10 A charge of $+ 2.0 \times 10^{-8}$ C is placed on the positive plate and a charge of -1.0×10^{-8} C on the negative plate of a parallel plate capacitor of capacitance 1.2×10^{-3} μF. Calculate the potential difference developed between the plates.

Answers

1. 1.6×10^{-8} F
2. 2 μF
3. (i) 1.32×10^{-10} C, (ii) 1.32×10^{-10} C
4. 8.33 pF, 1.25×10^{-8} C
5. a = 10 cm and b = 12 cm
6. (1) = -10×10^{-8}C, (2) = $+ 10 \times 10^{-8}$ C, (3) = $- 10 \times 10^{-8}$ C, (4) = $- 10 \times 10^{-8}$ C

7.	0.69×10^{-10}	8.	5.65 V, 11.3 V	9.	0.1 nF	10.	12.5 V

Combination of Capacitors in Series and in Parallel

Capacitors in series: When the negative plate of one capacitor is connected to the positive plate of the second, and the negative of the second to the positive of third and so on, the capacitors are said to be connected in series.

The potential differences across the various capacitors are

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

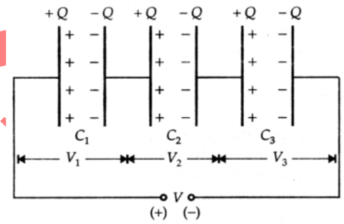
For the series circuit, the sum of these potential differences must be equal to the applied potential difference.

$$\therefore \quad V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad \text{or} \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \ldots (1)$$

Clearly, the combination can be regarded as an effective capacitor with charge Q and potential difference V. If C_s is the equivalent capacitance of the series combination, then

$$C_s = \frac{Q}{V} \quad \text{or} \quad \frac{1}{C_s} = \frac{V}{Q} \quad \ldots. (2)$$

From equations (1) and (2), we get $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$



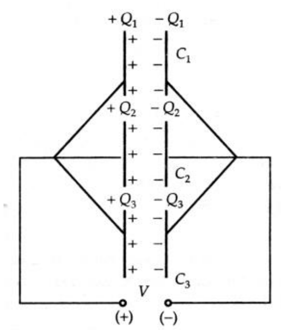
For a series combination of n capacitors, we can write

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots. + \frac{1}{C_n}$$

1. The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances.
2. The equivalent capacitance is smaller than the smallest individual capacitance.
3. The charge on each capacitors is same.
4. The potential difference across any capacitor is inversely proportional to its capacitance.

Capacitors in parallel

When the positive plates of all capacitors are connected to one common point and the negative plates to another common point, the capacitors are said to be connected in parallel.



Total charge stored in the combination is

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3) V \quad \ldots (1)$$

If C_p is the equivalent capacitance of the parallel combination, then

$$Q = C_p V \quad \ldots (2)$$

From equations (1) and (2), we get

$$C_p V = (C_1 + C_2 + C_3) V \quad \text{or} \quad C_p = C_1 + C_2 + C_3$$

For a parallel combination of n capacitors, we can write $C_p = C_1 + C_2 + \ldots. + C_n$

For parallel combination of capacitors

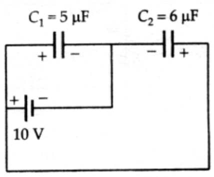
1. The equivalent capacitance is equal to sum of the individual capacitances.
2. The equivalent capacitance is larger than largest individual capacitance.
3. The potential difference across each capacitor is same.
4. The charge on each capacitor is proportional to its capacitance.

Subjective Assignment – III

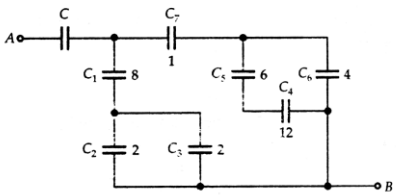
Q.1 Two capacitors of capacitance of 6 μF and 12 μF are connected in series with a battery. The voltage across the 6μF capacitance is 2V. Compute the total battery voltage.

Q.2 Two capacitors of capacitances 3 μF and 6 μF, are charged to potentials of 2 V and 5V respectively. These two charged capacitors are connected in series. Find the potential across each of the two capacitors now.

Q.3 Calculate the charge supplied by the battery in the arrangement shown in figures.



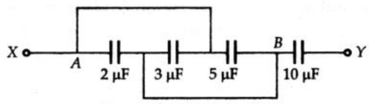
Q.4 From the network shown in figure, find the value of the capacitance C if the equivalent capacitance between points A and B is to be 1 μF. All the capacitances are in μF.



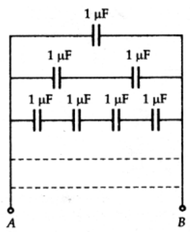
Q.5 Connect three capacitors of 3 μF, 3μF and 6μF such that their equivalent capacitance is 5μF.

Q.6 Seven capacitors, each of capacitance 2 μF are to be connected in a configuration to obtain an effective capacitance of 10/11 μF. Suggest a suitable combination to achieve the desired result.

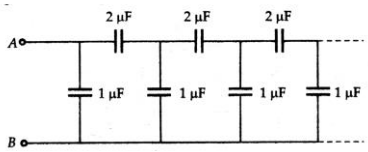
Q.7 Four capacitors are connected as shown in figure. Calculate the equivalent capacitance between the points X and Y.



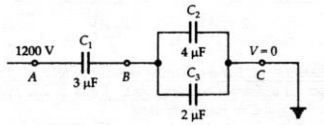
Q.8 There are infinite number of capacitors, each of capacitance 1 μF. They are connected in rows, such that the number of capacitors in the first row, second row, third row, fourth row, are respectively 1, 2, 4, 8, …. The rows of these capacitors are then connected between points A and B, as shown in figure. Determine the equivalent capacitance of the network between the points A and B.



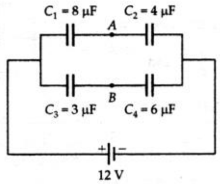
Q.9 Find the equivalent capacitor of the ladder (figure) between points A and B.



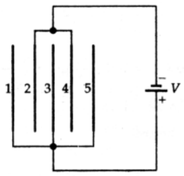
Q.10 In the circuit shown in figure, if the point C is earthed and point A is given a potential of + 1200 V, find the charge on each capacitor and the potential at the point B.



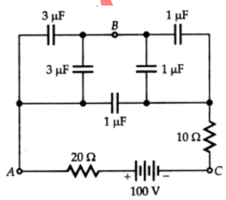
Q.11 Four capacitors C_1, C_2, C_3 and C_4 are connected to a battery of 12V, as shown in figure. Find the potential difference between the points A and B.



Q.12 Five identical capacitor plates, each of area A are arranged such that the adjacent plates are at distance d apart. The plates are connected to a source of emf V, as shown in figure. Find the charges on the various plates.



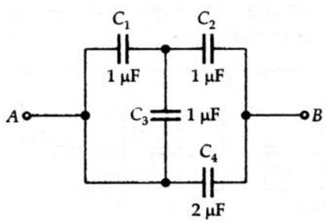
Q.13 For the network shown in figure, find the potential difference between points A and B, and that between B and C in the steady state.



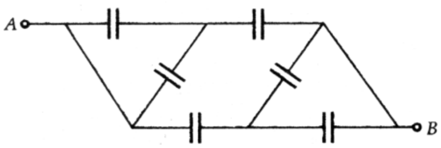
Q.14 The capacities of three capacitors are in the ratio 1 : 2 : 3. Their equivalent capacity in parallel is greater than the equivalent capacity in series by 60/11 pF. Calculate the individual capacitances.

Q.15 How would you connect 8, 12 and 24 μF capacitors to obtain (i) minimum capacitance (ii) maximum capacitance? If a potential difference of 100 volt is applied across the system, what would be the charges on the capacitors in each case?

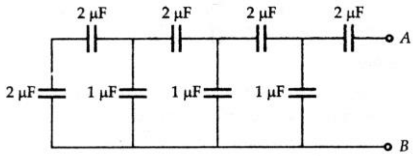
Q.16 Find the equivalent capacitance between point A and B for the network shown in figure.



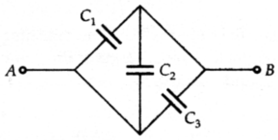
Q.17 A network of six identical capacitors, each of value C is made, as shown in figure. Find the equivalent capacitance between the points A and B.



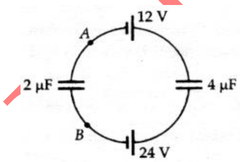
Q.18 Find the equivalent capacitance between points A and B of the network of capacitors shown in figure.



Q.19 In figure, $C_1 = 1\mu F$, $C_2 = 2\mu F$ and $C_3 = 3\ \mu F$. Find the equivalent capacitance between points A and B.



Q.20 Find the potential difference between the points A and B of the arrangement shown in figure.

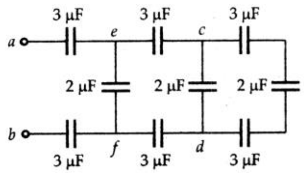


Q.21 For the network shown in figure, compute

(i) the equivalent capacitance between points a and b.

(ii) charge on each of the capacitors nearest to a and b when $V_{ab} = 900$ V

(iii) V_{cd}, when $V_{ab} = 900$ V



Answers

1. 3 V
2. 12 V, 6 V
3. 110 μC
4. 32/23 μF
5. series combination of 3 μF & 6 μF in parallel with 3 μF
6. parallel combination of 5 capacitors connected in series with 2 capacitors
7. 5 μF
8. 2 μF
9. 2 μF
10. 2.4×10^{-3} C, 1.6×10^{-3} C, 0.8×10^{-3} C, 400 V
11. 4 V
12. $\frac{\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}, \frac{\varepsilon_0 AV}{d}$
13. 25 V, 75 V
14. 1 pF, 2 pF, 3 pF
15. (i) In series, C_{min} =4μF, q =400μC, (ii) In parallel, C_{max}= 44μF, q_1 =800 μC, q_2 =1200 μC, q_3 =2400 μC
16. $\frac{8}{3}$μF
17. 4C/3
18. 1 μF
19. 6 μF
20. 8 V
21. (i) 1 μF, (ii) 900 μC, (iii) 100 v

Energy stored in a capacitor

A capacitor is a device to store energy. The process of charging up a capacitor involves the transferring of electric charges from its one plate to another. The *work done in charging the capacitor is stored as its electrical potential energy*. This energy is supplied by the battery at the expense of its stored chemical energy and can be recovered by allowing the capacitor to discharge.

Expression for the energy stored in a capacitor: consider a capacitor of capacitance C. Initially, its two plates are uncharged. Suppose the positive charge is transferred from plate 2 to plate 1 bit by bit. In this process, external work has to be done because at any stage plate 1 is at higher potential than the plate 2. Suppose at any instant the plates 1 and 2 have charges Q' and –Q' respectively, as shown in figure. Then the potential difference between the two plates will be $V' = \frac{Q'}{C}$

Suppose now a small additional charge dQ' be transferred from plate 2 to plate 1. The work done will be

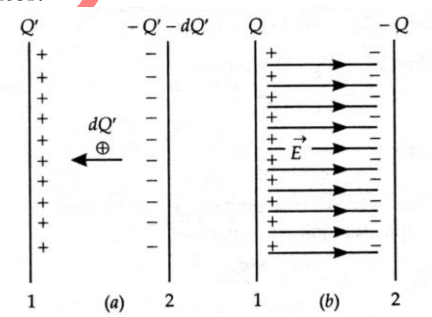
$$dW = V' .dQ' = \frac{Q'}{C} .dQ'$$

Total work done in transferring a charge Q from plate 2 to plate 1 [fig.] will be

$$W = \int dW = \int_0^Q \frac{Q'}{C}.dQ' = \left[\frac{Q'^2}{2C}\right]_0^Q = \frac{1}{2}.\frac{Q^2}{C}$$

This work done is stored as electrical potential energy U of the capacitor.

$$U = \frac{1}{2}.\frac{Q^2}{C} = \frac{1}{2}.CV^2 = \frac{1}{2}QV$$



Energy stored in a series combination of capacitors.

For a series combination, Q = constant

Total energy,

$$U = \frac{Q^2}{2}.\frac{1}{C} = \frac{Q^2}{2}.\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...\right]$$

$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + ... \quad \text{or} \quad U = U_1 + U_2 + U_3 + \ldots$$

Energy stored in a parallel combination of capacitors.

For a parallel combination, V = constant

Total energy,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}[C_1 + C_2 + C_3 + ...]V^2$$

$$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + ... \quad \text{or} \quad U = U_1 + U_2 + U_3 + ...$$

Hence total energy is additive both in series and parallel combinations of capacitors.

Energy density of an electric field

Consider a parallel plate capacitor, having plate area A and plate separation d. Capacitance of the parallel plate capacitor is given by $C = \frac{\varepsilon_0 A}{d}$

If σ is the surface charge density on the capacitor plates, then electric field between the capacitor plates will be

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad \sigma = \varepsilon_0 E$$

Charge on either plate of capacitor is $Q = \sigma A = \varepsilon_0 EA$

∴ Energy stored in the capacitor is

$$U = \frac{Q^2}{2C} = \frac{(\varepsilon_0 EA)^2}{2.\frac{\varepsilon_0 A}{d}} = \frac{1}{2}\varepsilon_0 E^2 \ Ad$$

But Ad = volume of the capacitor between its two plates. Therefore, the energy stored per unit volume of the energy density of the electric field is given by

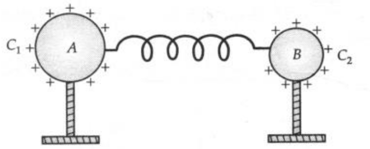
$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Note: An electric field E can be regarded as a seat of energy with energy density equal to $\frac{1}{2}\varepsilon_0 E^2$. Similarly, energy is also associated with a magnetic field.

Redistribution of Charges

Consider two insulated conductors A and B of capacitances C_1 and C_2, and carrying charges Q_1 and Q_2 respectively. Let V_1 and V_2 be their respective potentials. Then

$$Q_1 = C_1 V_1 \quad \text{and} \quad Q_2 = C_2 V_2$$



Now, if the two conductors are joined by a thin conducting wire, then the positive charge will flow from the conductor at higher potential to that at lower potential till their potentials become equal. Thus the charges are redistributed. But the total charge still remains $Q_1 + Q_2$.

$$\text{Common potential} = \frac{\text{Total charge}}{\text{Total capacitance}} \quad \text{or} \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

If after redistribution charges on A and B are Q'_1 and Q'_2 respectively, then

$$Q'_1 = C_1V \quad \text{and} \quad Q'_2 = C_2V \qquad \therefore \quad \frac{Q'_1}{Q'_2} = \frac{C_1}{C_2}$$

Thus, after redistribution, the charges on the two conductors are in the ratio of their capacitances.

Loss of energy in redistribution of charges

Let C_1 and C_2 be the capacitances and V_1 and V_2 be the potentials of the two conductors before they are connected together. Potential energy before connection is

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

After connection, let V be their common potential. Then

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Potential energy after connection is

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2 \quad = \frac{1}{2}\cdot\frac{(C_1V_1 + C_1V_2)^2}{(C_1 + C_2)}$$

Loss in energy,

$$U = U_i - U_f$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2(C_1 + C_2)}[C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2]$$

$$= \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}[V_1^2 + V_2^2 - 2V_1V_2] \qquad = \frac{1}{2}\cdot\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}$$

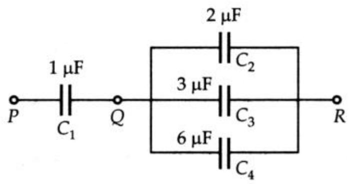
This is always positive whether $V_1 > V_2$ or $V_1 < V_2$. So when two charged conductors are connected, charges flow from higher potential side to lower potential side till the potentials of the two conductors get equalized. In doing so, there is always some loss of potential energy in the form of heat due to the flow of charges in connecting wires.

Subjective Assignment – IV

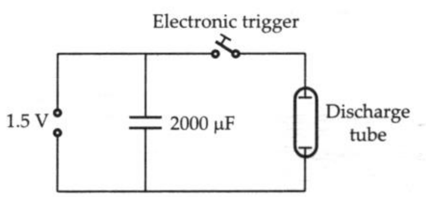
Q.1 How much work must be done to charge a 24 μF capacitor when the potential difference between the plates is 500V?

Q.2 A capacitor is charged through a potential difference of 200 V, when 0.1 C charge is stored in it. How much energy will it release, when it is discharged?

Q.3 In figure, the energy stored in C_4 is 27J. Calculate the total energy stored in the system.



Q.4 In a camera–flash circuit (figure), a 2000 μF capacitor is charged by a 1.5 V cell. When a flash is required, the energy stored in the capacitor is discharged by means of a trigger T through a discharge tube in 0.1 millisecond. Find the energy stored in the capacitor and the power of the flask.



Q.5 A 80 μF capacitor is charged by a 50 V battery. The capacitor is disconnected from the battery and then connected across another uncharged 320 μF capacitor. Calculate charge on the second capacitor.

Q.6 (i) A 900 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor?

(ii) The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system? (iii) Where has remainder of the energy gone?

Q.7 A capacitor is charged to potential V_1. The power supply is disconnected and the capacitor is connected in parallel to another uncharged capacitor.

(i) Derive the expression for the common potential of the combination of capacitors.

(ii) Show that total energy of the combination is less than the sum of the energy stored in them before they are connected.

Q.8 A capacitor of capacitance 6 μF is charged to a potential of 150 V. Its potential falls to 90 V, when another capacitor is connected to it. Find the capacitance of the second capacitor and the amount of energy lost due to the connection.

Q.9 A battery of 10 V is connected to a capacitor of capacitor 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further, compare this energy with the initial energy stored in the first capacitor.

Q.10 A capacitor charged from a 50V d.c. supply is found to have charge of 10 μC. What is the capacitance of the capacitor and how much energy is stored in it?

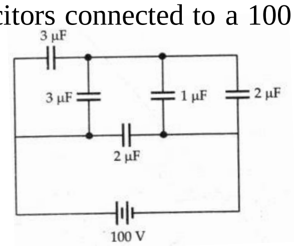
Q.11 A 4μF capacitor is connected to another 8 μF capacitor. Combination is charged at 300 V. Calculate

(i) total charge on the combination (ii) total energy stored in the combination

Q.12 Two capacitors are connected in parallel and the energy stored is 18 J, when a potential difference of 6000 V is applied across the combination. With the same capacitors connected in series, the energy stored is 4 J for the same potential difference. What are the individual capacitances?

Q.13 Two capacitors of capacitances 25 μF and 100 μF are connected in series and are charged by a battery of 120 V. The battery is then removed. The capacitors are now separated and connected in parallel. Find (i) p.d. across each capacitor (ii) energy–loss in the process.

Q.14 Figure shows a network of five capacitors connected to a 100 V supply. Calculate the total charge and energy stored in the network.



Q.15 The radii of two charged metallic spheres are 5 cm and 10 cm. Both have a charge of 75 μC. Both the spheres are connected together with a conducting wire. Calculate (i) the quantity of charge transferred through the wire and (ii) the common pot entail of the spheres after connecting them.

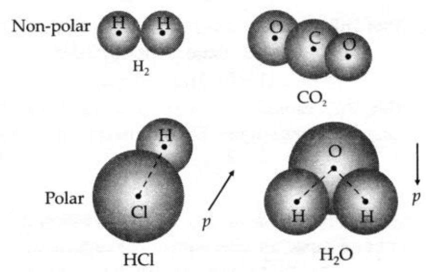
Answers

1.	3 J	2.	10 J	3.	594.0 J
4.	22.5 W, 2.25×10^{-3} J	5.	3.2×10^{-3} C		
6.	(i) 4.5×10^{-6} J, (ii) 2.25×10^{-6} J	7.	(i) $V = \dfrac{C_1 V}{C_1 + C_2}$, (ii) $\dfrac{C_1 C_2 V_1^2}{(C_1 + C_2)}$		
8.	4 μF, 0.027 J	9.	2.5 J, 1 : 2	10.	0.2 μF, 2.5×10^{-4}
11.	(i) 36×10^{-4} C, (ii) 0.54 J	12.	$\frac{2}{3}\mu F, \frac{1}{3}\mu F$	13.	38.4 V, 0.05184 J
14.	4×10^{-4} C, 0.02 J	15.	25 μC, 9×10^{6} V		

Dielectrics and Their Polarization

Dielectrics

A dielectric is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another through it. A dielectric is essentially an insulator which can be polarized through small localized displacements of its charges.

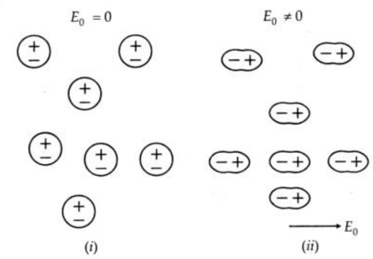


Examples. Glass, wax, water, air, wood, rubber, stone, plastic, etc.

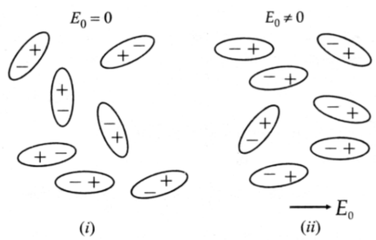
Polar and non–polar dielectrics: A dielectric may consist of either polar or non–polar molecules. A molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons) is called a polar molecule.

A molecule in which the centre of mass of positive charges coincides with the centre of mass of negative charges is called a non–polar molecule. The dielectrics made of non–polar molecules are called non–polar dielectrics.

Polarization of a non–polar dielectric in an external electric field



In the absence of any electric field, the centres of, positive and negative charges of the molecules of a non–polar dielectric coincide. The dipole moment of each molecule is zero. In the presence of an external electric field E_0, the centres of positive charges are displaced in the direction of external field while the centres of negative charges are displaced in the opposite direction. The displacement of the charges stops when the force exerted on them by the external field is balanced by the restoring force due to the internal fields in the molecules. This induces dipole moment in each molecule i.e., each non–polar molecule becomes an induced dipole. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the direction of the external field.



Polarization of a polar dielectric in an external electric field

The molecules of a polar dielectric have permanent dipole moments. In the absence of any external electric field, the dipole moments of different molecules are randomly oriented due to thermal agitation in the material. So the total dipole moment is zero. When an external field is applied, the dipole moments of different molecules tend to align with the field. As a result, there is a net dipole moment in the direction of the field. The extent of polarization depends on relative values of two opposing energies:

1. The potential energy of the dipole in the external field which tends to align the dipole with the field.

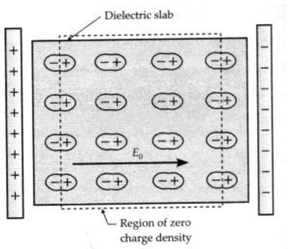
2. Thermal energy of agitation which tends to randomize the alignment of the dipole.

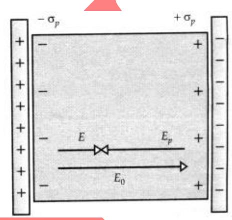
Hence both polar and non–polar dielectrics develop a net dipole moment in the presence of an external electric field. This fact is called **polarization of the dielectric.**

The polarization $\vec{P}$ is defined as the dipole moment per unit volume and its magnitude is usually referred to as the polarization density. The direction of $\vec{P}$ is same as that of the external field $\vec{E}_0$.

Reduction of electric field by the polarization of a dielectric

Consider a rectangular dielectric slab placed in a uniform electric field $\vec{E}_0$ acting parallel to two of its faces, as shown in figure. Its molecular dipoles align themselves in the direction of $\vec{E}_0$. This results in uniform polarization of the dielectric, i.e., every small volume of the slab has a dipole moment in the direction of $\vec{E}_0$.





The polarized dielectric is equivalent to two charged surfaces with induced surface charge densities $\pm\sigma_p$.

Reduced field inside a dielectric and dielectric constant

The induced surface charges set up an electric field $\overrightarrow{E_p}$ (field due to polarization) inside the dielectric in a direction opposite to that of external field $\overrightarrow{E_0}$, thus tending to reduce the original field in the dielectric. The resultant field $\vec{E}$ in the dielectric will be equal to $\overrightarrow{E_0} - \overrightarrow{E_p}$ and directed in the direction of $\overrightarrow{E_0}$.

The ratio of the original field $\overrightarrow{E_0}$ and the reduced field $\overrightarrow{E_0} - \overrightarrow{E_p}$ in the dielectric is called dielectric constant (κ) or relative permittivity (ε_r). Thus $\kappa = \frac{\overrightarrow{E_0}}{\vec{E}} = \frac{\overleftarrow{E_0}}{\overrightarrow{E_0} - \overrightarrow{E_p}}$

Polarization density

The induced dipole moment developed per unit volume of a dielectric when placed in an external electric field is called polarization density. It is denoted by P.

Hence $$P = \frac{Q_p d}{Ad} = \frac{Q_p}{A} = \sigma_p$$

Electric susceptibility

If field $\vec{E}$ is not large, then the polarization $\vec{P}$ is proportional to resultant field $\vec{E}$ existing in the dielectric, i.e.,

$$\vec{P} \propto \vec{E} \qquad \text{or} \qquad \vec{P} = \varepsilon_0 \chi \vec{E}$$

where χ (chi) is a proportionality constant called electric susceptibility. The multiplicative factor ε_0 is used to keep χ dimensionless. Clearly, $\chi = \frac{\vec{P}}{\varepsilon_0 \vec{E}}$

Thus the ratio of the polarization to ε_0 times the electric field is called the electric susceptibility of the dielectric. It describes the electrical behaviour of dielectric.

Relation between κ and χ

The net electric field in a polarized dielectric is

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

But $\vec{E}_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$

$\therefore$ $\vec{E} = \vec{E}_0 - \frac{P}{\varepsilon_0}$ or $\vec{E} = \vec{E}_0 - \frac{\varepsilon_0 \chi \vec{E}}{\varepsilon_0}$ $[\vec{P} = \varepsilon_0 \chi \vec{E}]$

Dividing both sides by $\vec{E}$, we get

$1 = \frac{\vec{E}_0}{\vec{E}} - \chi$ or $1 = \kappa - \chi$ or $\kappa = 1 + \chi$

Dielectric Strength

Dielectric strength: When a dielectric is placed in a very high electric field, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is called **dielectric breakdown.**

*The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called **dielectric strength** of the material.*

The unit of dielectric strength is same as that of electric field is Vm^{-1}.

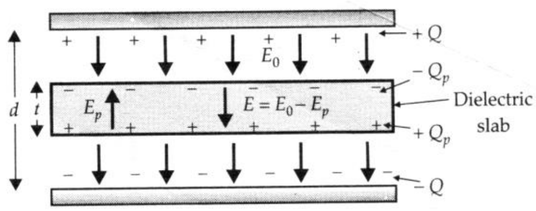
Capacitance of a Parallel Plate Capacitor with a Dielectric Slab

The capacitance of a parallel plate area A and plate separation d with vacuum between its plates is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

When a dielectric slab of thickness t < d is placed between the plates, the field E_0 polarises the dielectric. This induces charge $-Q_p$ on the upper surface and $+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0} [\sigma_p = \frac{Q}{P} = P, \text{ polarization density}]$$



The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{\kappa} \quad \left[\because \frac{E_0}{E_0 - E_p} = \kappa\right]$$

where κ is the dielectric constant of the slab. So between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance (d – t). Hence the potential difference between the capacitor plates is

$$V = E_0 (d - t) + Et = E_0 (d - t) + \frac{E_0}{\kappa} t \left[\because \frac{E_0}{E} = \kappa\right] = E_0\left(d - t + \frac{t}{\kappa}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{\kappa}\right)$$

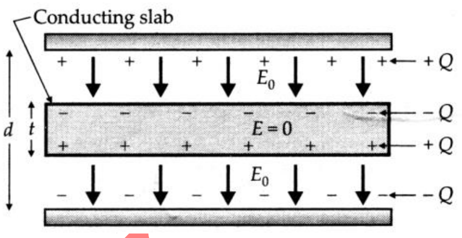
The capacitance of the capacitor on introduction of dielectric slab becomes $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}$

If the dielectric fills the entire space between the plates, then t = d, and we get $C = \frac{\varepsilon_0 A}{d} . \kappa = \kappa C_0$

Capacitance of Parallel Plate Capacitor with a Conducting Slab

Consider a parallel plate capacitor of plate area A and plate separation d. If the space between the plates is vacuum, its capacitance is given by $C_0 = \frac{\varepsilon_0 A}{d}$

When a conducting slab of thickness t < d is placed between the capacitor plates, free electrons flow inside it so as to reduce the field to zero inside the slab, as shown in figure. Charges – Q and +Q appear on the upper and lower faces of the slab. Now the electric field exists only in the vacuum regions between the plates of the capacitor on the either side of the slab, i.e., the field exists only in thickness d – t, therefore, potential difference between the plates of the capacitor is



$$V = E_0 (d - t) = \frac{Q}{\varepsilon_0 A}(d - t)$$

∴ Capacitance of the capacitor in the presence of conducting slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{(d-t)} = \frac{\varepsilon_0 A}{d}.\frac{d}{d-t} \quad \text{or} \quad C = \left(\frac{d}{d-t}\right).C_0$$

Clearly, $C > C_0$. *Thus the introduction of a conducting slab of thickness t in a parallel plate capacitor increases its capacitance by a factor of* $\frac{d}{d-t}$.

Uses of Capacitors

Capacitors are very useful circuit elements in any of the electric and electronic circuits. Some of their uses are:

1. To produce electric field of desired patterns, e.g., for Millikan's experiment.
2. In radio circuits for tuning.
3. In power supplies for smoothing the rectified current.
4. For producing rotating magnetic fields in induction motors.
5. In the tank circuit of oscillators.
6. They store not only charge, but also energy in the electric field between their plates.

Effect of Dielectric on Various Parameters

A parallel–plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates. Explain the changes, if any, that occur in the values of (i) charge on the plates, (ii) electric field between the plates, (ii) p.d. between the plates, (iv) capacitance and (v) energy stored in capacitor.

Effect of dielectric when the battery is kept disconnected from the capacitor. Let Q_0, C_0, V_0, E_0 and U_0 be the charge, capacitance, potential difference, electric field and energy stored respectively before the dielectric slab is inserted. Then

$$Q_0 = C_0 V_0, E_0 = \frac{V_0}{d}, \; U_0 = \frac{1}{2} C_0 V_0^2$$

(i) **Charge:** The charge on the capacitor plates remains Q_0 because the battery has been disconnected before the insertion of the dielectric slab.

(ii) **Electric field:** When the dielectric slab is inserted between the plates, the induced surfaced charge on the dielectric *reduces the field* to a new value given by $E=\frac{E_0}{\kappa}$

(iii) **Potential difference:** The reduction in the electric field results in the *decrease in potential difference.*

$$V=Ed=\frac{E_0 d}{\kappa}=\frac{V_0}{\kappa}$$

(iv) **Capacitance:** As a result of the decrease in potential difference, the *capacitance increases* κ times.

$$C=\frac{Q_0}{V}=\frac{Q_0}{V_0/\kappa}=\kappa\frac{Q_0}{V_0}=\kappa C_0$$

(v) **Energy stored:** The energy stored decreases by factor of κ.

$$U=\frac{1}{2}CV^2=\frac{1}{2}(\kappa C_0)\left(\frac{V_0}{\kappa}\right)^2=\frac{1}{\kappa}.\frac{1}{2}C_0V_0^2=\frac{U_0}{\kappa}$$

A parallel plate capacitor is charged by a battery. When battery remains connected, a dielectric slab is inserted between the plates. Explain what changes, if any, occur in the values of (i) p.d. between the plates, (ii) electric field between the plates, (iii) capacitance, (iv) charge on the plates and (v) energy stored in the capacitor?

Effect of dielectric when battery remains connected across the capacitor. Let Q_0, C_0, V_0, E_0 and U_0 be the charge, capacitance, potential difference, electric field and energy stored respectively, before the introduction of the dielectric slab. Then

$$Q_0=C_0V_0, E_0=\frac{V_0}{d}, U_0=\frac{1}{2}C_0V_0^2$$

(i) **Potential difference:** As the battery remains connected across the capacitor, so the *potential difference remains constant at V_0* even after the introduction of dielectric slab.

(ii) **Capacitance:** The *capacitance increases* from C_0 to C.

$$C = \kappa C_0$$

(iii) **Charge:** The charge on the capacitor plates *increases* from Q_0 to Q.

$Q = CV = \kappa C_0V_0 = \kappa Q_0$

(iv) **Electric field:** As the potential difference remains unchanged, so the electric field E_0 between the capacitor plates remains unchanged.

$$E=\frac{V}{d}=\frac{V_0}{d}=E_0$$

(v) **Energy stored:** The *energy* stored in the capacitor *increases* κ times.

$$U=\frac{1}{2}CV^2=\frac{1}{2}(\kappa C_0)V_0^2=\kappa.\frac{1}{2}C_0V_0^2=\kappa U_0$$

Subjective Assignment – V

Q.1 A parallel–plate capacitor having plate area 100 cm^2 and separation 1.0 mm holds a charge of 0.12 μC when connected to a 120 V battery. Find the dielectric constant of the materials filling the gap.

Q.2 A parallel–plate capacitor consists of 26 metal strips, each of 3 cm × 4 cm, separated by mica sheets of dielectric constant 6 and uniform thickness 0.2 mm. Find the capacitance.

Q.3 A parallel–plate capacitor of capacity 0.5 μF is to be constructed using paper sheets of thickness 0.04 mm as dielectric. Find how many circular metal foils of diameter 0.1 m will have to be used. Take the dielectric constant of paper used as 4.

Q.4 The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes 2/3 rd of its original value. What is the new distance between the plates?

Q.5 A parallel plate capacitor with plate separation 5 mm is charged by a battery. It is found that on introducing a mica sheet 2 mm thick, while keeping the battery connections intact, the capacitor draws 25% more energy from the battery than before. Find the dielectric constant of mica.

Q.6 In a parallel plate capacitor, the capacitance increases from 4 μF to 80 μF, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?

Q.7 A parallel plate capacitor with air between the plates has a capacitance of 8 μF. The separation between the plates is now reduced by half and the space between them is filed with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.

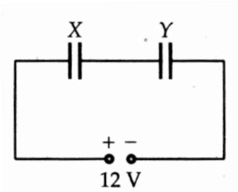
Q.8 An ebonite plate ($\kappa = 3$), 6 mm thick, is introduced between the parallel plates of a capacitor of plate area 2×10^{-2} m^2 and plate separation 0.01 m. Find the capacitance.

Q.9 Two parallel plate capacitors, X and Y, have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.

(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate potential difference between the plates of X and Y.

(iii) What is the ratio of electrostatic energy stored in X and Y?



Q.10 An electric field $E_0 = 3 \times 10^4$ Vm^{-1} is established between the plates, 0.05 m apart, of a parallel plate capacitor. After removing the charging battery, an uncharged metal plate of thickness t = 0.01 m is inserted between the capacitor plates. Find the p.d. across the capacitor (i) before, (ii) after the introduction of the plate. (iii) What would be the p.d. if a dielectric slab ($\kappa = 2$) were introduced in place of metal plate?

Q.11 The area of parallel plates of an air–filled capacitor is 0.20 m^2 and the distance between them is 0.01m. The p.d. across the plates is 3000V. When a 0.01 m thick dielectric sheet is placed between the plates, the p.d. decreases to 1000 V. Determine (i) capacitance of the capacitor before placing the sheet
(ii) charge on each plate (iii) dielectric constant of the material (iv) capacitance of the capacitor after placing the dielectric (b) permittivity of the dielectric. Given $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}.

Q.12 The capacitance of a parallel plate capacitor is 50 pF and the distance between the plates is 4 mm. It is charged to 200 V and then the charging battery is removed. Now a dielectric slab ($\kappa = 4$) of thickness
2 mm is placed. Determine (i) final charge on each plate (ii) final potential difference between the plates (iii) final energy in the capacitor and (iv) energy loss.

Q.13 A parallel plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1 mm. A dielectric of dielectric constant 5 and dielectric strength 1.9×10^7 Vm^{-1} is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

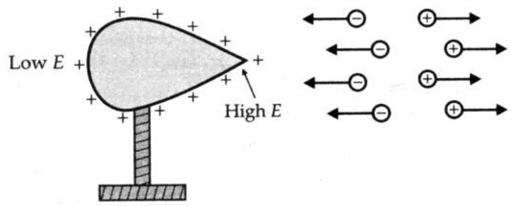
Answers

1.	11.3	2.	7.97×10^{-9} F	3.	73	4.	8 mm
5.	2	6.	20	7.	80 μF	8.	29.5 pF
9.	(i) 5 μF, 20 μF, (ii) 9.6 V, 2.4 V, (iii) 4 : 1			10.	(i) 1500 V, (ii) 1200 V, (iii) 1350 V		
11.	(i) 1.77×10^{-10} F, (ii) 5.31×10^{-7} C, (iii) 3, (iv) 5.31×10^{-10} F, (v) 2.65×10^{-11} Fm^{-1}						
12.	(i) same, (ii) 125 V, (iii) 6.25×10^{-7} J, (iv) 3.75×10^{-7} J					13.	8.4×10^{-6} C

Discharging Action of Sharp Points: Corona Discharge

When a spherical conductor of radius r carries a charge q, its surface charge density is

$$\sigma = \frac{q}{A} = \frac{q}{4\pi r^2}$$



Electric field on the surface is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi r^2}$$

The pointed end of a conductor is highly curved and its radius of curvature r very small. If the conductor is given a charge q, then the charge density σ at the pointed end will be very high. Consequently, the electric field near the pointed end will be very high which may cause the ionization or electrical breakdown of the surrounding air.

This process by which the charge at the pointed end of a conductor gets discharged is called ***corona discharge.***

Collecting Action of a Hollow Conductor

Consider a small sphere of radius r placed inside a large spherical shell of radius R. Let the spheres carry charges q and Q, respectively. Total potential on the outer sphere,

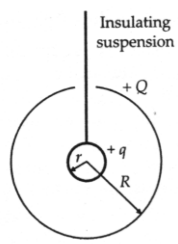
V_R = Potential due to its own charge Q + potential due to the charge q on the inner sphere $= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{R}\right]$

Potential on the inner sphere due to its own charge is $V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

As the potential at every point inside a charged sphere is the same as that on its surface, so potential on the inner sphere due to charge Q on outer sphere is

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

∴ Total potential on inner sphere $V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{Q}{R}\right]$



Hence the potential difference is $V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$

So if q is positive, the potential of the inner sphere will always be higher than that of the outer sphere. Now if the two spheres are connected by a conducting wire, the charge q will flow entirely to the outer sphere, irrespective of the charge Q already present on the outer sphere. In fact this is true for conductors of any shape.

Van de Graff Generator

The Van de Graff generator is a device, that produces intense electric field (*building up high voltage of a few million volt.*)

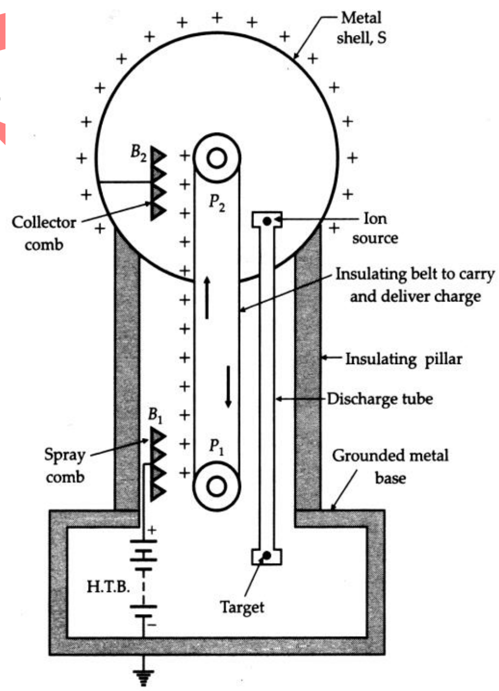
Principle: Its working is based on following two electrostatic phenomena:

(i) *Discharging action of sharp points (corona discharge) i.e., electric discharge takes place in air or gases readily at the pointed ends of conductors.*

(ii) *If a charge conductor is brought into internal contact with a hollow conductor, all of its charge transfers to the hollow conductor, howsoever high the potential of the latter may be.*

Construction:

A large spherical conducting shell (of few metres radius) is supported at a height several metres above the ground on an insulating column. A long narrow belt of insulating material, like rubber or silk, is wound around two pulleys, P_1 at ground level and P_2 at the centre of the shell.



This belt is kept continuously moving by an electric motor attached to the lower pulley P_1. Near the bottom and the top of its run, the belt passes close to two sharply pointed brass combs B_1 and B_2, pointing towards the belt. The comb B_1, called *spray comb* is given a positive potential of 10 kV with respect to the earth by means of a battery; while comb B_2, called collecting comb, is connected to the spherical shell S.

Working

Due to the high electric field at the pointed ends of comb B_1, the air of the neighbourhood gets ionized and its positive charge repelled or sprayed passes close to comb B_2, it induces a negative charge at the pointed ends of comb B_2 and a positive charge on the shell S. the positive charge spreads uniformly on the pointed ends of comb B_2 ionizes air there and repels negative charges on to belt which neutralize its positive charge. This process continues. As more and more positive charge is given to shell, its potential continues to rise. This way, a high potential of 6 to 8 million volts can be built upon sphere

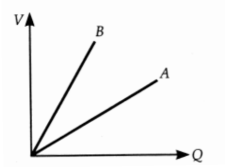
A discharge tube is placed with its upper end inside the hollow sphere and lower end earthed. The ion source is placed at the upper end of the tube. The high potential on the sphere repels the charged particles downward with large acceleration, where t hey hit the target atoms to bring about the nuclear disintegration.

Use:

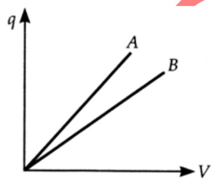
The high potential difference set up in a Van de Graaff generator is used to accelerate charged particles like protons, deutrons, α–particles, etc. to high energies of about 10 MeV, needed for experiments to probe the small scale structure of matter.

Board Oriented Questions

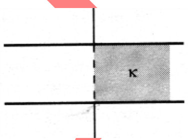
Q.1 The graph shows the variation of voltage V across the plates of two capacitors A and b versus increase of charge Q stored on them. Which of capacitors has higher capacitance? Give reason for your answer.



Q.2 The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2. The two capacitors have same plate separation but the plate area of C_2 is double than that of C_1. Which of the lines in the graph corresponding to C_1 and C_2 and why?

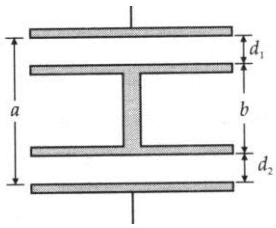


Q.3 As shown in figure, a dielectric material of dielectric constant κ is inserted in half portion between the plates of a parallel–plate capacitor. If its initial capacitance is C, what is the new capacitance?



Q.4 Find the capacitance of there parallel plates, each of area A metre^2 and separated by d_1 and d_2 metre. The in–between spaces are filled with dielectrics of relative permittivity ε_1 and ε_2. The permittivity of free space is ε_0.

Q.5 Figure shows two capacitors joined in series, the rigid central sect ion of length b being movable. Prove that the equivalent capacitance of the combination is independent on the position of the central section.



Q.6 A parallel plate capacitor, when there is vacuum between the plates, has capacitance C_0. What will be its capacitance, when

(i) distance between the plates is doubled,

(ii) a sheet of thickness t of a dielectric of relative permittivity κ is introduced between the plates assume t = d.

Q.7 Two metal plates form a parallel plate capacitor. The distance between the plates is d. A metal sheet of thickness d/2 and of the same area is introduced between the plates. What is the ratio of the capacitances in the two cases?

Q.8 A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential 'V'. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant 10. State with reason, how does (i) electric field between the plates, and (ii) energy stored in the capacitor charges.

Q.9 A parallel plate capacitor is charged to a potential difference 'V' by a d.c. source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change:

(i) electric field between the plates, (ii) capacitance and (iii) energy stored in the capacitor.

Q.10 Capacitors P, Q and R are of same capacity C. A battery can charge the capacitor P to a potential difference V. If after charging P, the battery is disconnected from it and the charged capacitor P is connected in following separate instances to Q and R:

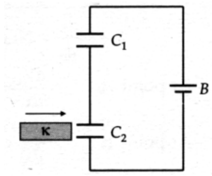
(i) to Q in parallel and (ii) to R is series, then what will be the potential differences between the plates of P in the two instances?

Q.11 The plates of a parallel plate capacitor are drawn apart keeping them connected to a battery. Next the same plates are drawn apart from the same initial condition keeping the battery disconnected. In which case is more done? Give reason.

Q.12 An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.

Q.13 A capacitor is connected across a battery. (i) Why does each plate receive a charge of exactly the same magnitude? (ii) is this true even if the plates are of different sizes?

Q.14 Two identical capacitors C_1 and C_2 are connected to a battery B, as shown in figure. A dielectric slab is slipped between the plates of C_2, the battery remaining connected. What happens to the charge, the capacitance, the potential difference and stored energy of each capacitor?



Q.15 Explain the underlying principle of working of a parallel plate capacitor. If two similar plates, each of area A having surface charge densities + σ and –σ are separated by a distance d in air, write expressions for: (i) The electric field between the two plates (ii) The potential difference between the plates
(iii) The capacitance of the capacitor so formed.

Q.16 During lighting, you are safer inside a house than under a tree. Why?

Q.17 The safest way to protect yourself from lightning is to be inside a car. Comment.

Q.18 Two spheres of radii R and 2R are charged, so that both of these have same surface charge density σ. The spheres are located away from each other and are connected by a thin conducting wire. Find the new charge density on the two spheres.

Q.19 Two conductors identical in shape and size, but one of copper and the other of aluminium (which is less conducting) are both placed in an identical electric field. In which metal, more charge will be induced.

Q.20 Sketch a graph to show how charge Q given to a capacitor of capacity C varies with the potential difference V.

Q.21 Is it possible for a metal sphere of 1 cm radius to hold a charge of 1C?

Q.22 Is there any conductor which can be given almost unlimited charge?

Q.23 Two spheres of silver of same radii, one solid and the other hollow are charged to the same potential, which one has greater charge?

Q.24 Can you place a parallel plate capacitor of one farad capacity in your house?

Q.25 If the plates of a charged capacitor be suddenly connected to each other by a wire, what will happen?

Q.26 Why does the electric conductivity of earth's atmosphere increase with altitude?

Q.27 A spherical shell of radius b with charge Q is expanded to radius a. Find the work done by the electrical forces in the process?

Q.28 When a battery is connected across a capacitor, are the charges on the plates always equal and opposite, even for plates of different sizes?

Q.29 Why is a space ship entering the ionosphere not sufficiently heated, though temperature at the top of ionosphere is nearly 700 K?

Q.30 Can we give as much charge to a capacitor as we wish?

Q.31 By what factor does the capacity of a metal sphere increase if its volume is tripled?

Q.32 A metal foil of negligible thickness is introduced between two plates of a capacitor at the centre. What will be the new capacitance of the capacitor?

Q.33 An uncharged insulated conductor A is brought near a charged insulated conductor B. What happens to charge and potential of B?

Q.34 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the plates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the new value of capacitance.

Q.35 Derive an expression for energy stored in a parallel plate capacitor of capacitance C with air as medium between the plates having charges Q and – Q. Show that this energy can be expressed in terms of electric field as $\frac{1}{2}\epsilon_0 E^2$ Ad, where Ad, where A is area of each plate and d is the separation between the plates. How will the energy stored in a fully charged capacitor change capacitor change when the separation between the plates is doubled and the dielectric medium of constant 4 is introduced between the plates?

Q.36 Explain what is meant by dielectric constant.

Q.37 Derive an expression for the capacitance of a spherical capacitor.

Q.38 Explain the underlying principle of working of a parallel plate capacitor. If two similar plates, each of area A having surface charge densities + σ and – σ are separated by a distance d in air, write expressions for

(i) the electric field at points between the two plates

(ii) the potential difference between the plates.

(iii) the capacitance of the capacitor so formed.

Q.39 Three capacitors of capacitances C_1, C_2 and C_3 are connected (i) in series, (ii) in parallel. Show that the energy stored in the series combination is the same as that in parallel combination.

Q.40 Explain what is meant by dielectric polarization. Hence establish the relation $K = 1 + \chi$

Q.41 Deduce the effect of introducing (i) a conducting slab (ii) a dielectric slab in between the plates of a parallel plate condenser on the capacitance of the condenser.

Q.42 Explain the principle on which Van–de–Graaff generator operates. Draw a labelled schematic sketch and write briefly it's working.

NCERT Questions

Q.1 Two charges 5×10^{-8}C and -3×10^{-8}C are located 16 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Q.2 A regular hexagon of side 10 cm has a charge of 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Q.3 Two charges + 2μC and –2 μC are placed at point A and B, 6 cm apart. (i) Identify an equipotential surface of the system. (ii) What is the direction of the electric field at every point on the surface.

Q.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field

(a) inside the sphere, (b) just outside the sphere, (c) at a point 18 cm from the centre of the sphere?

Q.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates be reduced by half, the space between them is filled with a substance of dielectric constant, $\kappa = 6$?

Q.6 Three capacitors each of capacitance 9 pF are connected in series. (a) What is the total capacitance of the combination? (b) What is the potential difference across each capacitor when the combination is connected to a 120 V supply?

Q.7 Three capacitors of capacitances 2 pF, 3pF and 4 pF are connected in parallel. (a) What is the total capacitance of the combination? (b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Q.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If the capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Q.9 Explain what would happen if in the capacitor given in question 8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates, (i) while the voltage supply remains connected (ii) after the supply was disconnected.

Q.10 A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in capacitor?

Q.11 A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Q.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0) via a point R (0, 6 cm, 9 cm).

Q.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Q.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential (a) at the mid–point of the line joining the two charges, and (b) at a point 10 cm from this mid–point in a plane normal t o the line and passing through the mid–point.

Q.15 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero even if the shell is not spherical, but has any irregular shape? Explain.

Q.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $(\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$

where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at the point. (The direction of $\hat{n}$ is from side 1 to side 2)

Hence show that just outside a conductor, the electric field is $\sigma\hat{n}/\varepsilon_0$

(b) Show that the tangential component of electric field is continuous from one side of a charged surface to another.

Q.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co–axial conducting cylinder. What is the field in the space between the two cylinders?

Q.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53Å.

(i) Estimate the potential energy of the system in eV, taking the zero of potential energy at infinite separation of the electron from proton.

(ii) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (i)?

(iii) What are the answers to (i) and (ii) above if the zero of potential energy is taken at 1.06 Å separation?

Q.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+ ions, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Q.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than that on its flatter portions.

Q.21 Two charges –q and +q are located at points (0, 0, –a) and (0, 0, a) respectively.

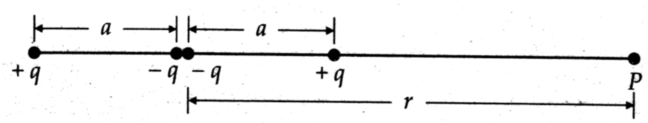
(i) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0)?

(ii) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> a$

(iii) How much work is done in moving a small test charge from the point (5, 0, 0) to (–7, 0, 0) along the x–axis?

Does the answer change if the path of the test charge between the same points is not along the x–axis?

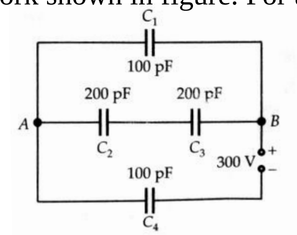
Q.22 Figure, shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r > > a. Contrast your result with that due to an electric dipole and an electric monopole (i.e. a single charge).



Q.23 An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires a minimum number of capacitors.

Q.24 What is the area of the plates of a 2F parallel plate capacitor? Given that the separation between the plates is 0.5 cm.

Q.25 Obtain the equivalent capacitance of the network shown in figure. For a 300 V supply, determine the charge and voltage across each capacitor.



Q.26 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

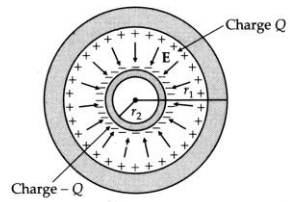
(i) How much energy is stored by the capacitor.

(ii) View this energy stored in the electrostatic field between the plates and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Q.27 A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost ion the form of heat and electromagnetic radiation?

Q.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}$qE, where q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor $\frac{1}{2}$.

Q.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (figure). Show that the capacitance of a spherical capacitor is given by



$$C = \frac{4\pi\varepsilon_0 \, r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Q.30 A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the co–centric spheres is filled with a liquid of dielectric constant 32. (a) Determine the capacitance of the capacitor. (b) What is the potential of the inner sphere? (c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Q.31 Answers carefully:

(i) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$, where r is the distance between their centres?

(ii) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss' law be still true?

(iii) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force passing through the point?

(iv) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(v) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(vi) What meaning would you give to the capacity of a single conductor?

(vii) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Q.32 A cylindrical capacitor has two co–axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Q.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. For safety, we would like the field never to exceed say 10% of dielectric strength. What minimum area of plates is required to have a capacitance of 50 pF?

Q.34 Describe schematically the equipotential surfaces corresponding to

(i) a constant electric field in the Z–direction.

(ii) a field that uniformly increases in magnitude but remains in a constant (say, Z) directions.

(iii) a single positive charge at the origin.

(iv) a uniform grid consisting of long equally spaced parallel charged wires in a plane

Q.35 In a Van de Graaff type generator, a spherical metal shell is to be a 1.5×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required?

Q.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what is charge q_2 on the shell is.

Q.37 Answer the following:

(i) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why do then we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside).

(ii) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?

(iii) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(iv) What are the forms of energy into which the electric energy of the atmosphere is dissipated during a lighting?

Answers

1. 10 cm, 40 cm
2. 2.7×10^6 V
4. (a) 0, (b) 10^5 NC^{-1}, (c) 4.44×10^4 NC^{-1}
5. 96 pF
6. (a) 3pF, (b) 40V
7. (a) 9 pF, (b) 2×10^{-10} C, 3×10^{-10} C, 4×10^{-10} C
8. 18 pF, 1.8×10^{-9} C
9. (i) 100 V, 108 pF, 1.08×10^{-8} C, (ii) 108 pF, 16.6 V, 1.08×10^{-8} C
10. 1.5×10^{-8} J
11. 6×10^{-6} J
12. 1.2 J
13. $V=\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}, E=0$
14. 2.4×10^5 V, 2×10^5 V
15. (a) $\sigma_{in} = \frac{-q}{4\pi r_1^2}, \sigma_{out}=\frac{Q+q}{4\pi r_2^2}$, (b) zero
17. $E=\frac{\lambda}{2\pi\varepsilon_0 r}$
18. (i) $\simeq -27.2$ eV, (ii) 13.6 eV, (iii) – 13.6 eV, 13.6 eV
19. – 19.2 eV
20. b/a
21. (i) $V_2=\frac{1}{4\pi\varepsilon_0}\frac{P}{(z^2-a^2)}$, v=0 (ii) $V=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$, (iii) W=0
22. $V_{quad}\,\alpha\frac{1}{r^3}, V_{dipole}\,\alpha\frac{1}{r^2}$
23. $3 \times 6 = 18$
24. 1130 km^2
25. V_1 = 100V, V_4 = 200 V, $V_2 = V_3$ = 50 v
26. (i) 2.55×10^{-6} J, (ii) 0.113 Jm^{-3}
27. 2.67×10^{-2} J
30. (a) 5.5×10^{-9} F, (b) 4.5×10^2 V, (c) 1.3×10^{-11} F
31. (i) No, (ii) No, (iii) Not necessarily, (iv) Zero, (v) No, (vi) A single conductor is a capacitor with one plate at infinity. It also possesses capacitance, (vii) Because of its bent shape and the presence of two highly polar O – H bonds, a water molecule possesses a large permanent dipole moment about 0.6×10^{-29} Cm. Hence water has a large dielectric constant.
32. 1.2×10^{-10} F, 2.9×10^4 V
33. 19 cm^2
35. 30 cm
37. (ii) Yes, (iv) electrical energy is lost as (a) light energy involved in lightning (b) heat and sound energy in the accompanying thunder.

Objective Assignment – I

Q.1 The ratio of charge to potential of a body is known as

(a) capacitance (b) inductance (c) conductance (d) resistance

Q.2 If P.D. across a capacitor is changed from 15V to 30 V, work done is W. What will be the work done when P.D. is changed from 30V to 60V?

(a) W (b) 4 W (c) 3W (d) 2 W

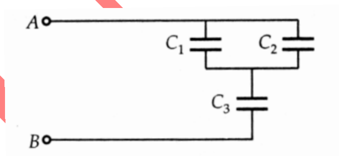
Q.3 Capacity of a parallel plate condenser is 10 μF when the distance between its plates is 8 cm. If the distance between the plates is reduced to 4 cm, its capacity will be

(a) 10 μF (b) 20 μF (c) 15 μF (d) 40 μF

Q.4 The equivalent capacity of two capacitors in series is 3μF and in parallel is 16μF. Their individual capacities are

(a) 12, 4 (b) 8, 8 (c) 10, 16 (d) 12, 2

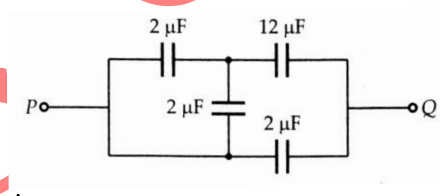
Q.5 In the given network capacitance, C_2 = 10μF, C_1 = 5μF and C_3 = 4μF. What is the resultant capacitance between A and B?



(a) 2.2μF (b) 1.2 μF

(c) 3.2 μF (d) 4.7 μF

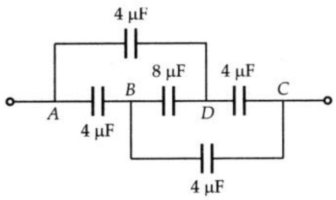
Q.6 Four capacitors are connected in a circuit as shown in figure. The effective capacitance in between P and Q will be



(a) 10 μF (b) 5 μF

(c) 2 μF (d) 7.5 μF

Q.7 Equivalent capacitance of the given combination of five capacitors is

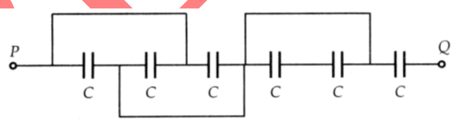


(a) 4 μF (b) 10 μF

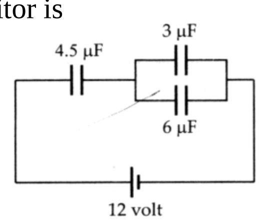
(c) 8 μF (d) 120 μF

Q.8 For circuit the equivalent capacitance between P and Q is

(a) 6 C (b) 4 C (c) 3C/4 (d) 6C/11



Q.9 In the circuit shown in the figure, the potential difference across 4.5μF capacitor is



(a) 8/3 volt

(b) 4 volt

(c) 6 volt

(d) 8 volt

Q.10 The capacity of a condenser is 4×10^{-6} farad and its potential is 100 volts. The energy released on discharging it fully will be

(a) 0.04 J (b) 0.02 J (c) 0.025 J (d) 0.05 J

Q.11 A 4μF capacitor is charged to 400 V. If its plates are joined through a resistance, then heat produced in the resistance is

(a) 0.16 J (b) 0.64 J (c) 0.32 J (d) 1.28 J

Q.12 If the potential of a capacitor having capacity 8 μF is increased from 10 V to 20V, then increase in its energy will be

(a) 4×10^{-4} J (b) 12×10^{-4} J (c) 4×10^{-6} J (d) 12×10^{-6} J

Q.13 A capacitor of 20 μF is charged up to 500 V is connected in parallel with another capacitor of 10 μF which is charged upto 200 V. The common potential is

(a) 500 V (b) 300 V (c) 400 V (d) 200 V

Q.14 Two insulated metallic spheres of 3 μF and 5 μF capacitances are charged to 300 V and 500 V respectively. The energy loss, when they are connected by a wire, is

(a) 0.012 J (b) 0.0375 J (c) 0.0218 J (d) 3.75 J

Q.15 When a dielectric material is introduced between the plates of a charged condenser, then electric field between the plates

(a) decreases (b) remains constant (c) increases (d) first (c) then (a)

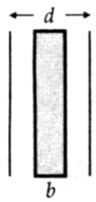
Q.16 If the distance between the plates of parallel plate capacitor is halved and the dielectric constant is doubled, then its capacity will

(a) increase by 16 times (b) increase by 4 times (c) increase by 2 times (d) remain the same

Q.17 A parallel plate condenser with oil between the plates (dielectric constant of oil k = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes

(a) $\sqrt{2}\,C$ (b) $\frac{C}{\sqrt{2}}$ (c) 2 C (d) C/2

Q.18 A copper plate of thickness b is placed inside a parallel plate capacitor of plate distance d and are A as shown in figure. The capacitance of capacitor is



(a) $\frac{A\varepsilon_0}{d}$ (b) $\frac{A\varepsilon_0}{b}$

(c) $\frac{A\varepsilon_0}{d-b}$ (d) ∞

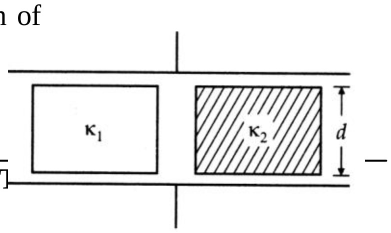
Q.19 If a dielectric plate of thickness t is placed between the plates of a parallel plate capacitor of plate distance d, the capacitance becomes half of the original value. The dielectric constant of the plate will be

(a) $\frac{2t}{2d+t}$ (b) $\frac{2t}{2d-t}$ (c) $\frac{t}{d+t}$ (d) $\frac{t}{d-t}$

Q.20 A parallel plate capacitor has capacitance C. If it is equally filled with parallel layers of materials of dielectric constants k_1 and k, its capacity becomes C_1. The ratio of C_1 to C is

(a) $k_1 + k_2$ (b) $\frac{k_1 k_2}{k_1 + k_2}$ (c) $\frac{k_1 + k_2}{k_1 k_2}$ (d) $\frac{2k_1 k_2}{k_1 + k_2}$

Q.21 Two dielectric of dielectric constants k_1 and k_2 are filled in the gap of parallel plate capacitor as shown in figure. The capacitor has plate each of area A and separation d. The capacitance of the capacitor is



(a) $\frac{\varepsilon_0 A(k_1+k_2)}{2d}$ (b) $\frac{\varepsilon_0 A}{2d}\left(\frac{k_1+k_2}{k_1 k_2}\right)$

(c) $\frac{\varepsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$ (d) $\frac{2\varepsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$

Q.22 Across each of two capacitors of capacitance 1 μF and 4 μF, a potential difference of 10 V is applied. Then positive plate of one is connected to the negative plate of the other, and negative plate of one is connected to the positive plate of the other. After contact,

(a) charge on each is zero (b) charge on each is same but non–zero

(c) charge on each is different but non–zero (d) none of these

Q.23 A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2 C is similarly charged to a potential difference 2V. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) 3/2 CV^2 (c) 25/6 CV^2 (d) 9/2 CV^2

Q.24 Top of the stratosphere has an electric field E (in units of V/m) nearly equal to

(a) 0 (b) 10 (c) 100 (d) 1000

Q.25 The surface charge density (in C/2) of the earth is about

(a) 10^{-9} (b) -10^{9} (c) 10^{9} (d) -10^{-9}

Answers

1.	a	**2.**	b	**3.**	b	**4.**	a
5.	c	**6.**	b	**7.**	a	**8.**	c
9.	d	**10.**	b	**11.**	c	**12.**	b
13.	c	**14.**	b	**15.**	a	**16.**	b
17.	d	**18.**	c	**19.**	c	**20.**	d
21.	a	**22.**	c	**23.**	b	**24.**	c
25.	d						

Objective Assignment – II

Q.1 8 drops of Hg are combined to form a bigger single drop. The capacitance of a single small drop and that of the single big drop will be in the ratio of

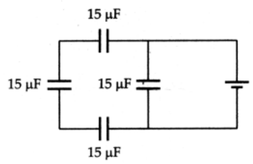
(a) 1 : 2 (b) 1 : 8 (c) 8 : 1 (d) none of these

Q.2 Three capacitors of capacitances 1 μF, 2 μF and 3 μF are connected in series and a p.d. of 11 V is applied across the combination. Then, the p.d., across the plates of 1 μF capacitor is

(a) 2 V (b) 4 V (c) 1 V (d) 6 V

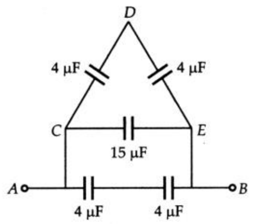
Q.3 The equivalent capacitance is

(a) 15 μF (b) 20 μF (c) 25 μF (d) 30 μF



Q.4 Equivalent capacitance between A and B is

(a) 19 µF (b) 6 µF (c) 268 µF (d) 10/ 38 µF



Q.5 A parallel plate air capacitor is charged and then isolated. When a dielectric material is inserted between the plates of the capacitor, then which of the following does not change?

(a) Electric field between the plates (b) Potential difference across the plates

(c) Charge on the plates (d) Energy stored in the capacitor

Q.6 A parallel plate air capacitor has a capacitance C. When it is half filled with a dielectric of dielectric constant 5, the percentage increase in the capacitance will be

(a) 400% (b) 66.6% (c) 33.3% (d) 200 %

Q.7 Two capacitors of capacitance C are connected in series. If one of them is filled with dielectric substance k, what is the effective capacitance?

(a) $\frac{kC}{(1+k)}$ (b) C (k + 1) (c) $\frac{2kC}{1+k}$ (d) none of these

Q.8 A 10 micro farad capacitor is charged to 500 V and then its plates are joined together through a resistance of 10 ohm. The heat produced in the resistance is

(a) 500 J (b) 250 J (c) 125 J (d) 1.25 J

Q.9 If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to

(a) CV (b) $\frac{1}{2}nCV^2$ (c) CV^2 (d) $\frac{1}{2n}CV^2$

Q.10 A capacitor is charged by connecting a battery across its plates. It stores energy u. Now the battery is disconnected and another identical capacitor is connected across it, then the energy stored by both capacitors of the system will be

(a) u (b) u/2 (c) 2 u (d) 3/2 u

Q.11 An air capacitor is charged with an amount of charge q and dipped into an oil tank. If the oil is pumped out, the electric field between the plates will

(a) increase (b) decrease (c) remain the same (d) become zero

Q.12 A parallel plate capacitor is charged. If the plates are pulled apart,

(a) the capacitance increases (b) the potential difference increases

(c) the total charge increases (d) charge and potential difference remains the same

Answer Sheet

1.	a	2.	d	3.	b	4.	a	5.	c
6.	b	7.	a	8.	d	9.	b	10.	b
11.	a	12.	b						

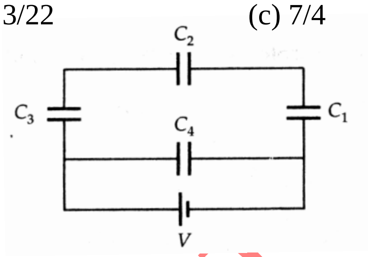
Objective Assignment – III

Q.1 Three capacitors each of capacitance 4 μF are to be connected in such a way that the effective capacitance is 6μF. This can be done by connecting

(a) all of them in series
(b) all of them in parallel
(c) two in parallel and one in series
(d) two in series and one in parallel

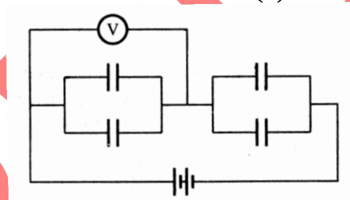
Q.2 A network of four capacitors of capacitances equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is

(a) 4/7 (b) 3/22 (c) 7/4 (d) 22/3



Q.3 Four capacitors of 25 μF each are connected as shown in figure. If the d.c. voltmeter reads 200 V, charge on each plate of the capacitor is

(a) 2×10^{-3} C (b) 5×10^{-3} C (c) 2×10^{-2} C (d) 5×10^{-2} C



Q.4 A parallel plate capacitor with oil between the plates (dielectric constant of oil, k = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes

(a) $\sqrt{2}C$ (b) 2 C (c) $\frac{C}{\sqrt{2}}$ (d) $\frac{C}{2}$

Q.5 A parallel plate air capacitor is charged to a potential difference of V volt. After disconnecting the charging battery, the distance between the plates of the capacitor is increased using an insulating handle. As a result, the potential difference between the plates

(a) decreases (b) increases (c) does not change (d) becomes zero

Q.6 A capacitor of capacitor C_1 is charged upto potential V and then connected in parallel to an uncharged capacitor of capacitance C_2. The final potential difference across each capacitor will be

(a) $\frac{C_2V}{C_1+C_2}$ (b) $\frac{C_1V}{C_1+C_2}$ (c) $\left(1+\frac{C_2}{C_1}\right)V$ (d) $\left(1-\frac{C_2}{C_1}\right)V$

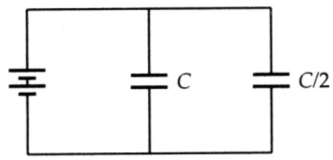
Q.7 Energy per unit volume for a capacitor having area A and separation d kept at potential difference V is given by

(a) $\frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$ (b) $\frac{1}{2\varepsilon_0}\frac{V^2}{d^2}$ (c) $\frac{1}{2}CV^2$ (d) $\frac{Q^2}{2C}$

Q.8 A capacitor is charged by connecting a battery across its plates. It stores energy u. Now the battery is disconnected and another identical capacitor is connected across it. Then the energy stored by both capacitors of the system will be

(a) u (b) u/2 (c) 3u/2 (d) u/4

Q.9 Two condensers, one of capacity C and the other of capacitor C/2 are connected to a V volt battery, as shown. The work done in charging fully both the condensers is



(a) $\frac{1}{4}CV^2$ (b) $\frac{1}{2}CV^2$

(c) $\frac{3}{4}CV^2$ (d) $2\ CV^2$

Q.10 The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give on an average
2 eV energy to an electron in the metal will be in units of V/m

(a) 5×10^{11} (b) 8×10^{-11} (c) 5×10^{7} (d) 8×10^{7}

Q.11 The electric potential at a point (x, y, z) is given by $V = -x^2y - xz^3 + 4$. The electric field $\vec{E}$ at that point is

(a) $\vec{E} = \hat{i}2xy + \hat{j}(x^2 + y^2) + \hat{k}(3xz - y^2)$ (b) $\vec{E} = \hat{i}z^3 + \hat{j}xyz + \hat{k}z^2$

(c) $\vec{E} = \hat{i}(2xy - z^3) + \hat{j}xy^2 + \hat{k}3z^2x$ (d) $\vec{E} = \hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}3xz^2$

Q.12 Three concentric spherical shells have radii a, b and c (a < b < c) and have surface charge densities $\sigma, -\sigma$ and σ respectively. If V_A, V_B and V_C denote potentials of three shells, then for c = a + b, we have

(a) $V_C = V_B \neq V_A$ (b) $V_C \neq V_B \neq V_A$ (c) $V_C = V_B = V_A$ (d) $V_C = V_A \neq V_B$

Q.13 Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be

(a) 3 C, V/3 (b) C/3, 3V (c) 3C, 3V (d) C/3, V/3

Q.14 When a capacitor is connected to a battery,

(a) a current flows in the circuit for some time, then decreases to zero.

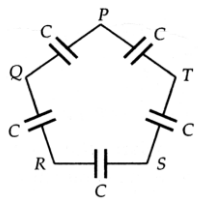
(b) no current flows in the circuit at all

(c) an alternating current flows in the circuit (d) none of the above

Q.15 What is the area of the plates of a 3F parallel plate capacitor, if the separation b/w the plates is 5mm?

(a) 1.694×10^9 m^2 (b) 4.529×10^9 m^2 (c) 9.281×10^9 m^2 (D) 12.281×10^9 m^2

Q.16 Five capacitors, each of capacitance value C are connected as shown in the figure. The ratio of capacitance between P and R; and the capacitance between P and Q is



(a) 3 : 1 (b) 5 : 2

(c) 2 : 3 (d) 1 : 1

Q.17 Given a number of capacitors labeled as 8 μF – 250 V. Find the minimum number of capacitors needed to get an arrangement equivalent to 16 μF –1, 000 V.

(a) 4 (b) 16 (c) 32 (d) 64

Q.18 A 40F capacitor in a defibrillator is charged to 3000 V. The energy stored in the capacitor is sent through the patient during a pulse of duration 2 ms. The power delivered to the patient is

(a) 45 kW (b) 90 kW (c) 180 kW (d) 360 kW

Answer Sheet

1.	d	2.	b	3.	b	4.	d	5.	b
6.	b	7.	a	8.	b	9.	c	10.	c
11.	d	12.	d	13.	b	14.	a	15.	a
16.	c	17.	c	18.	b				

Objective Assignment – IV

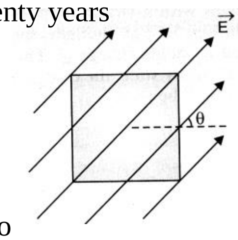
Q.1 A parallel plate capacitor has an electric field of 10^5 V/m between the plates. If the charge on the capacitor plate is 1 μC, the force on each capacitor plate is:

(a) 0.5 N (b) 0.05 N (c) 0.005 N (d) None of these

Q.2 If 10^{10} electrons are acquired by a body every second, the time required by the body to get a total charge of 1 C would be

(a) two hours (b) two days (c) two years (d) twenty years

Q.3 A square surface of side L meters in the plane of the paper is placed in a uniform electric field E (volt/m) acting along the same plane at an angle θ with the horizontal side of the square as shown in figure. The electric flux linked to the surface, in units of volt. m, is



(a) EL^2 (b) $EL^2 \cos\theta$ (c) $EL^2 \sin\theta$ (d) zero

Q.4 A parallel plate condenser with a dielectric of constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from the plates and then reinserted. The net work done by the system in the process is:

(a) zero (b) $\frac{1}{2}(K-1)CV^2$ (c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

Q.5 Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

(a) negative and distributed uniformly over the surface of the sphere

(b) negative and appears only at the point on the surface closest to the point charge

(c) negative and distributed non–uniformly over the entire surface of the sphere (d) zero

Q.6 Positive and negative point charges of equal magnitude are kept at $\left(0,0,\frac{a}{2}\right)$ and $\left(0,0,\frac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from (–a, 0, 0) to (0, a, 0) is

(a) positive (b) negative (c) zero

(d) depends on the path connecting the initial and final positions

Q.7 An electron of mass M_e initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass M_p also initially at rest take time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio $\frac{t_2}{t_1}$ is nearly equal to:

(a) 1 (b) $\sqrt{\frac{M_p}{M_e}}$ (c) $\sqrt{\frac{M_e}{M_p}}$ (d) 1836

Q.8 Three point charges + q, – 2 q and + q are placed at points (x = 0, y = a, z = 0); (x = 0, y = 0, z = 0) and (x = a, y = 0, z = 0) respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are:

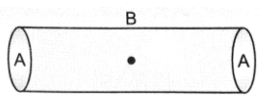
(a) $\sqrt{2}$ q a along the line joining points (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0)

(b) q a along the line joining points (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0)

(c) $\sqrt{2}q$ a along + x direction (d) $\sqrt{2}q$ a along + y direction

Q.9 A hollow cylinder has a charge q coulomb within it. If ϕ is electric flux in units of voltmeter associated with the curved surface B, the flux linked with the plane surface A in units of volt–meter will be

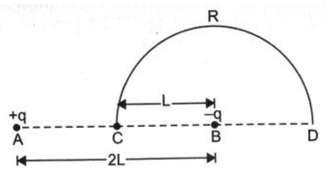
(a) $\frac{q}{2\epsilon_0}$ (b) $\frac{\phi}{3}$ (c) $\left(\frac{q}{\epsilon_0}-\phi\right)$ (d) $\frac{1}{2}\left(\frac{q}{\epsilon_0}-\phi\right)$



Q.10 Charges + q and – q are placed at points A and B respectively, which are distance 2L apart. C is the mid–point of A and B (figure). The work done in moving a charge + Q along the semicircle CRD is

(a) $\frac{qQ}{2\pi\epsilon_0 L}$ (b) $\frac{qQ}{6\pi\epsilon_0 L}$

(c) $\frac{-qQ}{6\pi\epsilon_0 L}$ (d) $\frac{qQ}{4\pi\epsilon_0 L}$

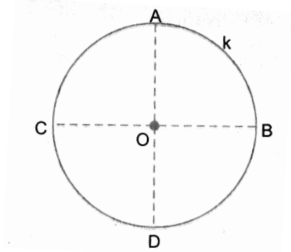


Q.11 The potential at a point distant x (measured in μm) due to some charges situated on the x–axis is given by $V(x)=\frac{20}{x^2-4}$ Volt. The electric field at x = 5 μ m is given by

(a) $\frac{5}{3}volt/\mu m$ and in + x direction (b) $\frac{10}{9}volt/\mu m$ and in neg. x direction

(c) $\frac{10}{9}volt/\mu m$ and in + x direction (d) $\frac{10}{9}volt/\mu m$ and in neg. x direction

Q.12 A thin conducting ring of radius R is given a charge + Q (figure). The electric field at the center O of the ring due to the charge on the part AKB of the ring is E. The electric field at the centre due to the charge on part ACDB of the ring is



(a) 3 E along KO (b) E along OK

(c) E along KO (d) 3 E along OK

Q.13 A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and work done by the battery will be

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) 1 (d) $\frac{1}{4}$

Q.14 An electric charge 10^{-3} μC is placed at the origin (0, 0) of XY Co–ordinate system. Two points A and B are situated at $\left(\sqrt{2}, \sqrt{2}\right)$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 4.5 V (b) 9 V (c) zero (d) 2 V

Q.15 A charge Q is placed at each of two opposite corners of a square. A charge q is placed at each of the other two corners. If the net electric force on Q is zero, then Q/q equals

(a) – 1 (b) 1 (c) $-\frac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

Q.16 The electric potential at a point in free space due to a charge Q coulomb is $Q \times 10^{11}$ V. The electric field at the point is

(a) $4\pi\epsilon_0 Q \times 10^{22}$ V/m (b) $12\pi\epsilon_0 Q \times 10^{20}$ V/m

(c) $4\pi\epsilon_0 Q \times 10^{20}$ V/m (d) $12\pi\epsilon_0 Q \times 10^{22}$ V/m

Q.17 Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be

(a) 3C, 3V (b) C/3, V/3 (c) 3C, V/3 (d) C/3, 3V

Q.18 The electric potential at a point (x, y, z) is given by $V = -x^2y - xz^3 + 4$. The electric field at the point is

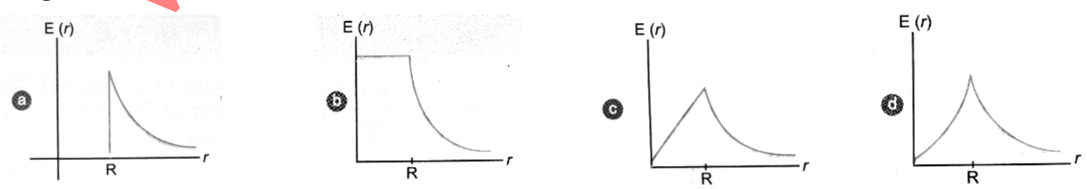
(a) $\vec{E} = \hat{i}(2xy - z^3) + \hat{j}xy^2 + \hat{k}3z^2x$ (b) $\vec{E} = \hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}3xz^2$

(c) $\vec{E} = \hat{i}2xy + \hat{j}(x^2 + y^2) + \hat{k}(3xz - y^2)$ (d) $\vec{E} = \hat{i}z^3 + \hat{j}xyz + \hat{k}z^2$

Q.19 A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness d/3, while the other has dielectric constant $K_2 = 6$ and thickness 2d/3. Capacitance of the capacitor is now

(a) 1.8 pF (b) 45 pF (c) 40.5 pF (d) 20.25 pF

Q.20 A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs (figure) most closely represents the electric field E(r) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?



Q.21 Three concentric spherical shells have radii a, b and c (a < b < c) and have surface charge density σ, – σ + σ respectively. If V_A, V_B and V_C denote the potentials of the three shells, then for c = a + b, we have

(a) $V_C = V_B = V_A$ (b) $V_C = V_A \neq V_B$ (c) $V_C = V_B \neq V_A$ (d) $V_C \neq V_B \neq V_A$

Q.22 Two positive ions, each carrying a charge q are separated by a distance d. If F is the force of repulsion between the ions, the number of electrons missing from each ion will be (e being the charge of an electron)

(a) $\frac{4\pi \epsilon_0 Fd^2}{e^2}$ (b) $\sqrt{\frac{4\pi \epsilon_0 Fd^2}{d^2}}$ (c) $\sqrt{\frac{4\pi \epsilon_0 Fd^2}{e^2}}$ (d) $\frac{4\pi \epsilon_0 Fd^2}{q^2}$

Q.23 A series combination of n_1 capacitors, each of value C is charged by a source of potential difference 4V. When another parallel combination of n_2 capacitors each of value C_2 is charged by a source of potential difference V, it has the same total energy stored in it as the first combination has. The value of C_2 is terms of C_1 is then

(a) $\frac{2C_1}{n_1 n_2}$ (b) $16\frac{n_2}{n_1}C_1$ (c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

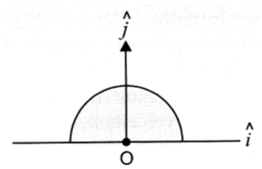
Q.24 Let there be a spherically symmetric charge distribution with charge density varying as $\rho(x)=\rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto r = R, and ρ (r) = 0 for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by

(a) $\frac{\rho_0 r}{4\epsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (b) $\frac{4\rho_0 r}{3\epsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (c) $\frac{\rho_0 r}{3\epsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (d) $\frac{4\pi\rho_0 r}{3\epsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

Q.25 A thin semi–circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is

(a) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

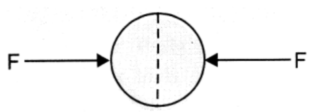


Q.26 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6g cm^{-3}, the dielectric constant of the liquid is

(a) 3 (b) 2 (c) 1 (d) 4

Q.27 A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (figure). F is proportional to

(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$ (c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$



Q.28 A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5 V m^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity 2 × 10^{-3} ms^{-1}. Given, g = 9.8 ms^{-2}, viscosity of the air = 1.8 × 10^{-5} Ns m^{-2} and the density of oil = 9000 kg m^{-3}, the magnitude of q is

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C (c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

Q.29 Let $\rho(r) = \dfrac{Qr}{\pi R^4}$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at a distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) $\dfrac{Q}{4\pi \in_0 r_1^2}$ (b) $\dfrac{Q r_1^2}{4\pi \in_0 R^4}$ (c) $\dfrac{Q r_1^2}{3\pi \in_0 R^4}$ (d) zero

Answers

1	b	2	d	3	d	4	a	5	d
6	c	7	b	8	a	9	d	10	c
11	c	12	b	13	a	14	c	15	d
16	a	17	d	18	b	19	c	20	a
21	b	22	c	23	d	24	a	25	b
26	b	27	a	28	d	29	c		

Higher Order Assignment

Q.1 When 1.0×10^{12} electrons are transferred from one conductor to another, a potential difference of 10V appears between the conductors. Calculate the capacitance of the two–conductor system.

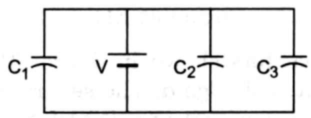
Q.2 The plates of a parallel – plate capacitor are made of circular discs of radii 5.0 cm each. If the separation between the plates is 1.0 mm, what is the capacitance?

Q.3 Suppose, one wishes to construct a 1.0 farad capacitor using circular discs. If the separation between the discs be kept at 1.0 mm, what would be the radius of the discs?

Q.4 A parallel–plate capacitor having plate area 25 cm^2 and separation 1.00 mm is connected to a battery of 6.0 V. Calculate the charge flown through the battery. How much work has been done by the battery during the process?

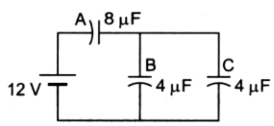
Q.5 A parallel–plate capacitor has plate area 25.0 cm^2 and a separation of 2.00 mm between the plates. The capacitor is connected to a battery of 12.0 V. (a) Find the charge on the capacitor. (b) The plate separation is decreased to 1.00 mm. Find the extra charge given by the battery to the positive plate.

Q.6 Find the charges on the three capacitors connected to a battery as shown in figure. Take $C_1 = 2.0$ μF, $C_2 = 4.0$ μF, $C_3 = 6.0$ μF and V = 12 volts.

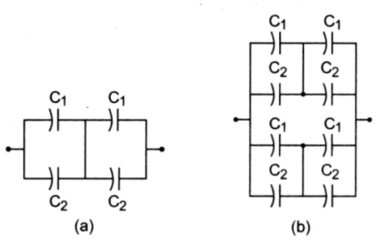


Q.7 Three capacitors having capacitances 20μF, 30μF and 40μF are connected in series with a 12V battery. Find the charge on each of the capacitors. How much work has been done by the battery in charging the capacitors?

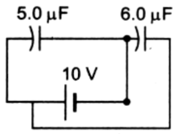
Q.8 Find the charge appearing on each of the three capacitors shown in figure.



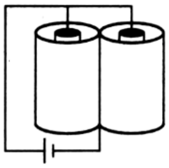
Q.9 Take $C_1 = 4.0$ μF and $C_2 = 6.0$ μF in figure. Calculate the equivalent capacitance of the combination between the points indicated.



Q.10 Find the charge supplied by the battery in the arrangement shown in figure.

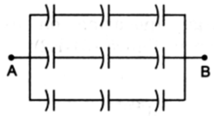


Q.11 The outer cylinders of two cylindrical capacitors of capacitance 2.2 μF each, are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected as shown in figure. Find the total charge supplied by the battery to the inner cylinders.



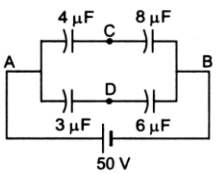
Q.12 Two conducting spheres of radii R_1 and R_2 are kept widely separated from each other. What are their individual capacitances? If the spheres are connected by a metal wire, what will be the capacitance of the combination?

Q.13 Each of the capacitors shown in figure has a capacitance of 2 μF. Find the equivalent capacitance of the assembly between the points A and B. Suppose, a battery of emf 60 volts is connected between A and B. Find the potential difference appearing on the individual capacitors.

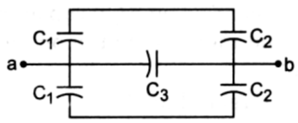


Q.14 It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they can withstand only 50 V. Design a combination which can yield the desired result.

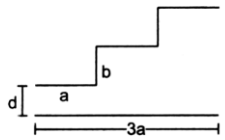
Q.15 Take the potential of the point B in figure to be zero. (a) Find the potentials at the points C and D. (b) If a capacitor is connected between C and D, what charge will appear on this capacitor?



Q.16 Find the equivalent capacitance of the system shown in figure between the points a and b.



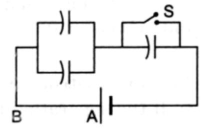
Q.17 A capacitor is made of a flat plate of area A and a second plate having a stair–like structure as shown in figure. The width of each stair is a and the height is b. Find the capacitance of the assembly.



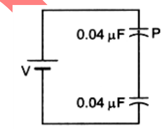
Q.18 A cylindrical capacitor is constructed using two coaxial cylinders of the same length 10 cm and of radii 1mm and 4mm. (a) Calculate the capacitance. (b) Another capacitor of the same length is constructed with cylinders of radii 4mm and 8mm. Calculate the capacitance.

Q.19 A 100 pF capacitor is charged to a potential difference of 24V. It is connected to an uncharged capacitor of capacitance 20 pF. What will be the new potential difference across the 100 pF capacitor?

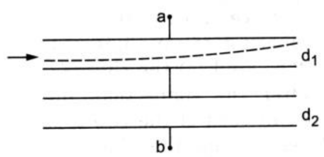
Q.20 Each capacitor shown in figure has a capacitance of 5.0 μF. The emf of the battery is 50 V. How much charge will flow through AB if the switch S is closed?



Q.21 The particle P shown in figure has a mass of 10 mg and a charge of – 0.01 μC. Each plate has a surface area 100 cm^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium?

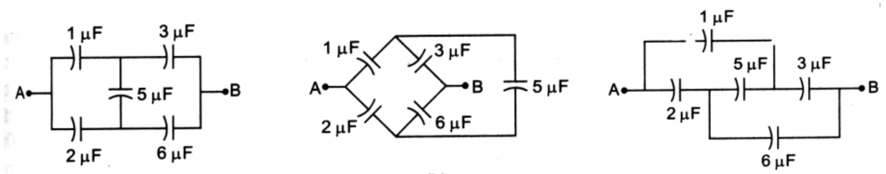


Q.22 Both the capacitors shown in figure are made of square plates of edge a. The separations between the plates of the capacitors are d_1 and d_2 as shown in the figure. A potential difference V is applied between the points a and b. An electron is projected between the plates of the upper capacitor along the central line. With what minimum speed should the electron be projected so that it does not collide with any plate? Consider only the electric forces.



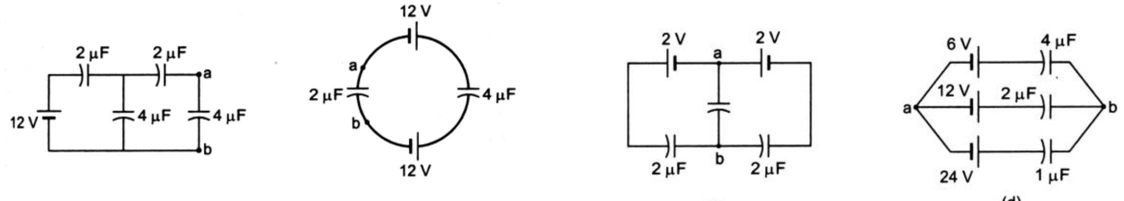
Q.23 The plates of a capacitor are 2.00 cm apart. An electron proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as the electron reaches the positive plate. At what distance from the negative plate was the pair released?

Q.24 Convince yourself that parts (a), (b) and (c) of figure are identical. Find the capacitance between the points A and B of the assembly.



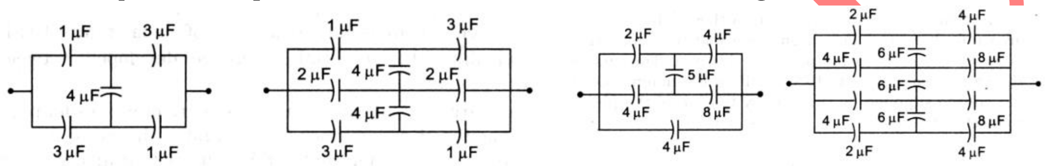
(a) (b) (c)

Q.25 Find the potential difference $V_a - V_b$ between the points a and b shown in each part of the figure.



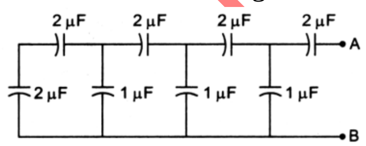
(a) (b) (c) (d)

Q.26 Find the equivalent capacitances of the combinations shown in figure between the indicated points.

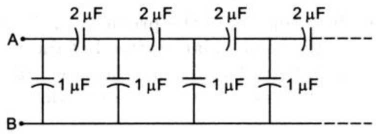


(a) (b) (c) (d)

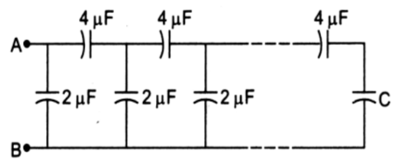
Q.27 Find the capacitance of the combination shown in figure between A and B.



Q.28 Find the equivalent capacitance of the infinite ladder shown in figure between the points A and B.



Q.29 A finite ladder is constructed by connecting several sections of 2μF, 4μF capacitor combinations as shown in figure. It is terminated by a capacitor of capacitance C. What value should be chosen for C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between?

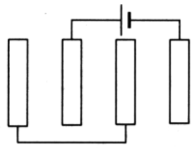


Q.30 A charge of + 2.0 × 10^{-8} C is placed on the positive plate and a charge of – 1.0 × 10^{-8} C on the negative plate of a parallel – plate capacitor of capacitance 1.2 × 10^{-3} μF. Calculate the potential difference developed between the plates.

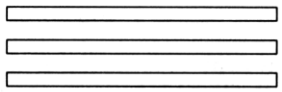
Q.31 A charge of 20 μC is placed on the positive plate of an isolated parallel–plate capacitor of capacitance 10μF. Calculate the potential difference developed between the plates.

Q.32 A charge of 1μC is given to one plate of a parallel–plate capacitor of capacitance 0.1 μF and a charge of 2μC is given to the other plate. Find the potential difference developed between the plates.

Q.33 Each of the plates shown in figure has surface area $(96/\varepsilon_0) \times 10^{-12}$ Fm on one side and the separation between the consecutive plates is 4.0 mm. The emf of the battery connected is 10 volts. Find the magnitude of the charge supplied by the battery to each of the plates connected to it.



Q.34 The capacitance between the adjacent plates shown in figure is 50 nF. A charge of 1.0 μC is placed on the middle plate. (a) What will be the charge on the outer surface of the upper plate? (b) Find the potential difference developed between the upper and the middle plates.

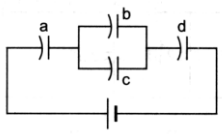


Q.35 Consider the situation of the previous problem. If 1.0 μC is placed on the upper plate instead of the middle, what will be the potential difference between (a) the upper and the middle plates and (b) the middle and the lower plates?

Q.36 Two capacitors of capacitances 20.0 pF and 50.0 pF are connected in series with a 6.00 V battery. Find (a) the potential difference across each capacitor and (b) the energy stored in each capacitor.

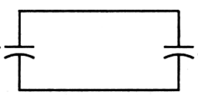
Q.37 Two capacitors of capacitances 4.0 μF and 6.0 μF are connected in series with a battery of 20 V. Find the energy supplied by the battery.

Q.38 Each capacitor in figure has a capacitance of 10 μF. The emf of the battery is 100 V. Find the energy stored in each of the four capacitors.



Q.39 A capacitor with stored energy 4.0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in the two capacitors.

Q.40 A capacitor of capacitance 2.0 μF is charged to a potential difference of 12V. It is then connected to an uncharged capacitor of capacitance 4.0 μF as shown in figure. Find (a) the charge on each of the two capacitors after the connection, (b) the electrostatic energy stored in each of the two capacitors and (c) the heat produced during the charge transfer from one capacitor to the other.



Q.41 A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centred at the origin.

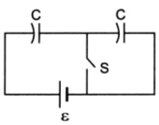
Q.42 A metal sphere of radius R is charged to a potential V. (a) Find the electrostatic energy stored in the electric field within a concentric sphere of radius 2R. (b) Show that the electrostatic field energy stored outside the sphere of radius 2R equals that stored within it.

Q.43 A large conducting plane has a surface charge density 1.0×10^{-4} C m^{-2}. Find the electrostatic energy stored in a cubical volume of edge 1.0 cm in front of the plane.

Q.44 A parallel–plate capacitor having plate area 20 cm^2 and separation between the plates 1.00 mm is connected to a battery of 12.0 V. The plates are pulled apart to increase the separation to 2.0 mm. (a) Calculate the charge flown through the circuit during the process. (b) How much energy is absorbed by the battery during the process? (c) Calculate the stored energy in the electric field before and after the process. (d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart. (e) Show and justify that no heat is produced during this transfer of charge as the separation is increased.

Q.45 A capacitor having a capacitance of 100 μF is charged to a potential difference of 24 V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12 V with the positive plate of the capacitor joined with the positive terminal of the battery. (a) Find the charges on the capacitor before and after the reconnection. (b) Find the charge flown through the 12 V battery. (c) Is work done by the battery or is it done on the battery? Find its magnitude. (d) Find the decrease in electrostatics field energy. (e) Find the heat developed during the flow of charge after reconnection.

Q.46 Consider the situation shown in figure. The switch S is open for a long time and then closed. (a) Find the charge flown through the battery when the switch S is closed. (b) Find the work done by the battery.



(c) Find the change in energy stored in the capacitors. (d) Find the heat developed in the system.

Q.47 A capacitor of capacitance 5.00 μF is charged to 24.0 V and another capacitor of capacitance 6.0 μF is charged to 12.0 V. (a) Find the energy stored in each capacitor. (b) The positive plate of the first capacitor is now connected to the negative plate of the second and vice–versa. Find the new charges on capacitors. (c) Find the loss of electrostatic energy during the process. (d) Where does this energy go?

Q.48 A 5.0 μF capacitor is charged to 12 V. The positive plate of this capacitor is now connected to the negative terminal of a 12 V battery and vice versa. Calculate the heat developed in the connecting wires.

Q.49 The two square faces of a rectangular dielectric slab (dielectric constant 4.0) of dimensions 20 cm × 20 cm × 1.0 mm are metal–coated. Find the capacitance between the coated surfaces.

Q.50 If the above capacitor is connected across a 6.0 V battery, find (a) the charge supplied by the battery, (b) the induced charge on the dielectric and (c) the net charge appearing on one of the coated surfaces.

Q.51 The separation between the plates of a parallel–plate capacitor is 0.500 cm and its plate area is 100 cm^2. A 0.400 cm thick metal plate is inserted into the gap with its faces parallel to the plates. Show that the capacitance of assembly in independent of position of the metal plate within the gap and find its value.

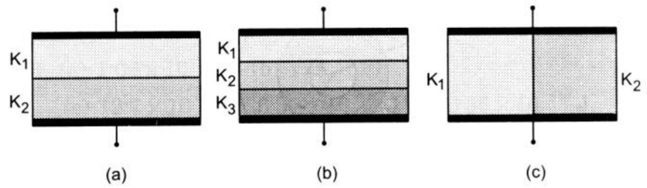
Q.52 A capacitor stores 50 μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 100 μC flows through the battery. Find the dielectric constant of the material inserted.

Q.53 A parallel–plate capacitor of capacitance 5μF is connected to a battery of emf 6 V. The separation between the plates is 2mm. (a) Find the charge on the positive plate. (b) Find the electric field between the plates. (c) A dielectric slab of thickness 1 mm and dielectric constant 5 is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted?

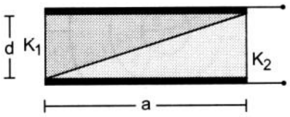
Q.54 A parallel–plate capacitor has plate area 100 cm^2 and plate separation 1.0 cm. A glass plate (dielectric constant 6.0) of thickness 6.0 mm and an ebonite plate (dielectric constant 4.0) are inserted one over the other to fill the space between the plates of the capacitor. Find the new capacitance.

Q.55 A parallel–plate capacitor has plate area 400 cm^2 and separation between the plates 1.0 mm is connected to a power supply of 100 V. A dielectric slab of thickness 0.5 mm and dielectric constant 5.0 is inserted into the gap. (a) Find the increase in electrostatic energy. (b) If the power supply is now disconnected and the dielectric slab is taken out, find the further increase in energy. (c) Why does the energy increase in inserting the slab as well as in taking it out?

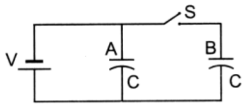
Q.56 Find the capacitances of the capacitors shown in figure. The plate area is A and the separation between the plates is d. Different dielectric slabs in a particular part of the figure are of the same thickness and the entire gap between the plates is filled with the dielectric slabs.



Q.57 A capacitor is formed by two square metal – plate of edge a, separated by a distance d. Dielectrics of dielectric constants K_1 and K_2 are filled in the gap as shown in figure. Find the capacitance.



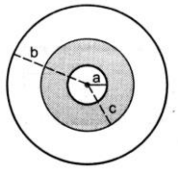
Q.58 Figure shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.



Q.59 A parallel–plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. Find the work done on the system in the process of inserting the slab.

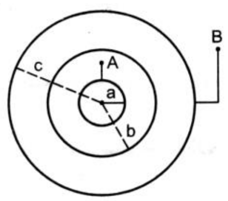
Q.60 A capacitor having a capacitance of 100 μF is charged to a potential difference of 50 V. (a) What is the magnitude of the charge on each plate? (b) The charging battery is disconnected and a dielectric of dielectric constant 2.5 is inserted. Calculate the new potential difference between the plates. (c) what charge would have produced this potential difference in absence of the dielectric slab. (b) Find the charge induced at a surface of the dielectric slab.

Q.61 A spherical capacitor is made of two conducting spherical shells of radii a and b. The space between the shells is filled with a dielectric of dielectric constant K upto a radius c as shown in figure. Calculate the capacitance.



Q.62 Consider an assembly of three conducting concentric spherical shells of radii a, b and c as shown in figure. Find the capacitance of the assembly between the points A and B.

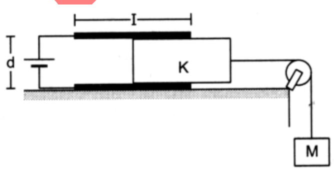
Q.63 Suppose the space between the two inner shells of the previous problem is filled with a dielectric of dielectric constant K. Find the capacitance of the system between A and B.



Q.64 An air – filled parallel – plate capacitor is to be constructed which can stored 12 μC of charge when operated at 1200 V. What can be the minimum plate area of the capacitor? The dielectric strength of air is 3×10^6 V m^{-1}.

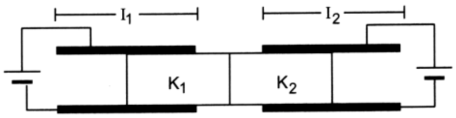
Q.65 A parallel–plate capacitor with the plate area 100 cm^2 and the separation between the plates 1.0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.

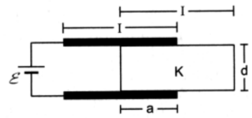
Q.66 Consider the situation shown in figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf ε. All surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium.



Q.67 Figure shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width b and lengths l_1 and l_2. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2. Neglecting any friction, find the ratio of the emf of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.

Q.68 Consider the situation shown in figure. The plates of the capacitor have plate area A and are clamped in the laboratory. The dielectric slab is released from rest with a length a inside the capacitor. Neglecting any effect of friction or gravity, show that slab will execute periodic motion and find its time period.





Answers

1. 1.6×10^{-8} F
2. 6.95×10^{-5} μF
3. 6 km
4. 1.33×10^{-10} C, 8.0×10^{-10} J
5. (a) 1.33×10^{-10} C, (b) 1.33×10^{-10} C
6. 24 μC, 48 μC, 72 μC
7. 110 μC on each, 1.33×10^{-3} J
8. 48 μC on the 8 μF capacitor and 24 μC on each of the 4 μF capacitors
9. (a) 5μF, (b) 10 μF
10. 110 μC
11. 44 μC
12. $4\pi\varepsilon_0 R_1$, $4\pi\varepsilon_0 R_2$; $4\pi\varepsilon_0 (R_1 + R_2)$
13. 2 μF, 20 V
15. (a) 50/3 μV at each point (b) zero
16. $C_3 + \dfrac{2C_1C_2}{C_1 + C_2}$
17. $\dfrac{\varepsilon_0 A(3d^2 + 6bd + 2b^2)}{3d(d+b)(d+2b)}$
18. (a) 8 pF, (b) same as in (a)
19. 20 V
20. 3.3×10^{-4} C
21. 43 mV
22. $\left(\dfrac{Vea^2}{md_1(d_1+d_2)}\right)^{1/2}$
23. 1.08×10^{-3} cm
24. 2.25 μF
25. $(a)\ \dfrac{12}{11}V\ (b) -8V\ (c)\ zero\ (d) -10.3V$
26. $(a)\ \dfrac{11}{6}\mu F, (b) \dfrac{11}{4}\mu F, (c)\ 8\mu F, (d)\ 8\mu F$
27. 1 μF
28. 2 μF
29. 4 μF
30. 12.5 V
31. 1 V
32. 5 V
33. 0.16 μC
34. (a) 0.50 μC, (b) 10 V
35. (a) 10 V, (b) 10 V
36. (a) 1.71 V, 4.29 V, (b) 184 pJ, 73.5 pJ
37. 960 μJ
38. 8 mJ in (a) and (d), 2 mJ in (b) and (c)
39. 2.0 J
40. (a) 8 μC, 16 μC, (b) 16 μJ, 32 μJ, (c) 96 μJ
41. $\dfrac{Q^2}{8\pi\varepsilon_0 R}$
42. (a) $\pi\varepsilon_0 RV^2$
43. 5.6×10^{-4} J
44. (a) 1.06×10^{-10} C, (b) 12.7×10^{-10} J, (c) 12.7×10^{-10} J, 6.35×10^{-10} J, (d) 6.35×10^{-10} J
45. (a) 2400 μC, 1200 μc, (b) 1200 μC, (c) 14.4 mJ, (d) 21.6 mJ, (e) 7.2 mJ
46. (a) Cε/2, (b) $C\varepsilon^2/2$, (c) $C\varepsilon^2/4$, (d) $C\varepsilon^2/4$
47. (a) 1.44 mJ, 0.432 mJ, (b) 21.8 μC, 26.2 μC, (c) 1.77 mJ
48. 1.44 mJ
49. 1.42 nF
50. (a) 8.5 nC (b) 6.4 nC, (c) 2.1 nC
51. 88 pF
52. 3
53. (a) 30 μC, (b) 3×10^3 Vm^{-1}, (c) 8.3 μF, (d) 20 μC
54. 44 pF
55. (a) 1.18 uJ, (b) 1.97 μJ

56. (a) $\frac{2K_1K_2\varepsilon_0 A}{d(K_1+K_2)}$, (b) $\frac{3\varepsilon_0 A\, K_1K_2K_3}{d(K_1K_2+K_2K_3+K_3K_1)}$, (c) $\frac{\varepsilon_0 A}{2d}(K_1+K_2)$ 57. $\frac{\varepsilon_0 K_1K_2 a^2 \ln\frac{K_1}{K_2}}{(K_1-K_2)d}$

58. 3 : 5   59. $\frac{\varepsilon_0 AV^2}{2d}\left(\frac{1}{K}-1\right)$

60. (a) 5 mC, (b) 20 V, (c) 2 mC, (d) 3 mC   61. $\frac{4\pi\varepsilon_0 Kabc}{Ka(b-c)+b(c-a)}$ 62. $\frac{4\pi\varepsilon_0 ac}{c-a}$

63. $\frac{4\pi\varepsilon_0 Kabc}{Ka(c-b)+c(b-a)}$ 64. 0.45 m^2 65. 2.5×10^{-7} N

66. $\frac{\varepsilon_0 b\, \mathcal{E}^2 (K-1)}{2\,dg}$ 67. $\sqrt{\frac{K_2-1}{K_1-1}}$ 68. $8\sqrt{\frac{(l-a)\,lmd}{\varepsilon_0 A\mathcal{E}^2 (K-1)}}$