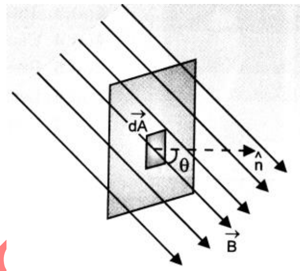
# Electromagnetic Induction




## Introduction

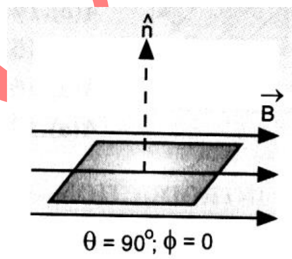
This phenomenon of generating current/ e.m.f. by changing magnetic fields is called Electromagnetic Induction (EMI). The e.m.f. so developed is called induced e.m.f. If the conductor is in the form of a closed circuit, a current flows in the circuit. This is called induced current. This phenomenon of EMI is the basis of working of power generators, dynamos, transformers etc.

## Magnetic Flux

The magnetic flux $\phi$ through any surface held in a magnetic field $\vec{B}$ is measured by the total number of magnetic lines of force crossing the surface.

For a uniform magnetic field $\vec{B}$ crossing the plane of area $\vec{A}$ at an angle $\theta$ with the normal to the plane, magnetic flux $\phi$ is given by

$$\phi = \vec{B}.\vec{A} = BA\cos\theta \qquad \text{... (1)}$$




where $\theta$ is smaller angle between $\vec{B}$ and $\vec{A}$, i.e., $\theta$ is the angle, which normal to the surface area makes with $\vec{B}$. This is shown in figure. Here $\hat{n}$ represents unit vector along the outdrawn normal to the area element.

Equation (1) can be rewritten to include magnetic flux over curved surfaces in non–uniform magnetic fields, as

$$\phi = \vec{B}_1.d\vec{A}_1 + \vec{B}_2.d\vec{A}_2 + .... = \sum_{all} \vec{B}_i.d\vec{A}_i = \int \vec{B}.d\vec{A} \qquad \text{... (2)}$$

where all stands for summation over *all* the element $dA_i$ comprising the surface and $B_i$ is magnetic field at the area element $dA_i$.

The integration is over the entire surface area.

When the magnetic field is touching the surface tangentially,

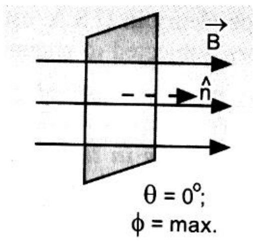



Figure $\qquad \theta = 90^o$

$\therefore \qquad \phi = BA\cos 90^o = 0$

When the magnetic field is normal to the surface, figure $\theta = 0^o$

$\therefore \qquad \phi = BA\cos 0^o = BA = \text{max. value}$

This means $B = \phi/A$, i.e., magnetic field strength B is magnetic flux per unit area and is called **magnetic flux density or magnetic induction.**

If the coil has N turns, total amount of magnetic flux linked with the coil is

$$\phi = N(\vec{B}.\vec{A}) = NBA\cos\theta$$

### Units of Magnetic Flux

The SI unit of magnetic flux is weber (Wb). One weber is the amount of magnetic flux over an area of 1 metre$^2$ held normal to a uniform magnetic field of one tesla. Thus

$$1 \text{ weber} = 1 \text{ tesla} \times 1 \text{ m}^2$$

The c.g.s. unit of $\phi$ is Maxwell (Mx), where

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

It should be clearly understood that ***magnetic flux is a scalar quantity*** (represented by the dot product of two vectors $\vec{B}$ and $\vec{A}$).

The **dimensional formula** of $\phi$ can be deduced from

$$\phi = BA\cos\theta$$

from $F = Bqv$, $B = \dfrac{F}{qv}$ $\quad\therefore\quad$ $\phi = \dfrac{F}{qv}A\cos\theta = \dfrac{(MLT^{-2})(L^2)}{(AT)(LT^{-1})} = [ML^2T^{-2}A^{-1}]$

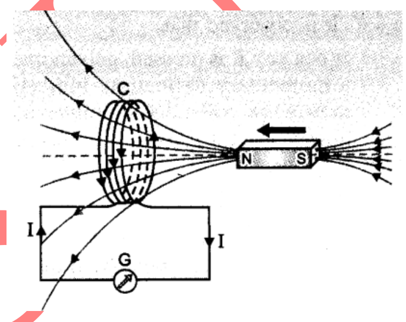
In this dimensional formula, A stands for ampere, SI unit of current. As $[ML^2T^{-1}]$ corresponds to energy, ***SI unit of magnetic flux will be***

$$\text{weber} = \frac{\text{joule}}{\text{ampere}} = \frac{\text{joule}}{\text{coulomb/sec}} = \frac{\text{joule}-\text{sec}}{\text{coulomb}} = \text{volt}-\text{sec}.$$

***NOTE:*** When a body is present in a uniform or non–uniform magnetic field, outward flux is taken to be positive, while inward flux is taken as negative.

## The Experiments of Faraday

### Experiment 1: Current induced by a magnet




Figure, shows a coil or loop C of a few turns of conducting material insulated from one another. It is connected to a sensitive galvanometer G. Faraday and Henry observed that

(i) When north pole of a bar magnet is pushed towards the coil, the galvanometer shows a sudden deflection, indicating that current is induced in the coil.

(ii) The galvanometer deflection is temporary. It lasts as long as the bar magnet is in motion.

(iii) When the magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, indicating reversal in the direction of induced current.

(iv) When south pole of bar magnet is moved towards or away from the coil, the galvanometer deflections are opposite to those observed with the north pole for similar movements.

(v) The galvanometer deflection (and hence induced current) is found to be **larger** when magnet is pushed towards or pulled away from the coil **faster.**

(vi) When the bar magnet is held stationary and coil C is moved towards or away from the magnet, the same effects are observed. It shows that *relative motion between the coil and the magnet is responsible for induction of electric current in the coil.*

### Experiment 2: Current induced by current

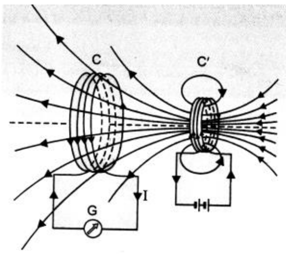



Figure shows a coil or loop C of a few turns of conducting material insulated from one another connected to a sensitive galvanometer G. C' is another such coil connected to a battery. A steady current through coil C' produces a uniform magnetic field along its axis.

(i) When coil C' is moved towards the coil C, the galvanometer shows a sudden deflection. This indicates that electric current is induced in coil C.

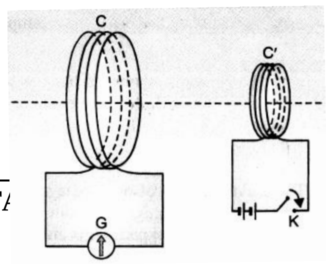
(ii) When coil C' is moved away from coil C, the galvanometer shows a deflection in the opposite direction. This indicates that direction of current induced in coil C is reversed.

(iii) The deflection is temporary. It lasts so long as there is relative motion between the two coils.

(iv) The galvanometer deflection (and hence induced current) is found to be larger when coils are moved faster towards/away from each other.

We observe that coil C' carrying current in Experiment 2 behaves as a bar magnet as in Exp. 1

### Experiment 3: Current induced by changing current



In Experiment 1 relative motion between a coil and a bar magnet produced induced current. In Experiment 2, relative motion between one coil and another coil carrying current produced induced current. Through the present experiment. Faraday showed that relative motion is not an absolute requirement. Current can be induced even without relative motion. Figure shows two coil C and C' held stationary.

Coil C is connected to a sensitive galvanometer G and coil C' is connected to a battery through a tapping key K. It is observed that

(i) When key K is pressed, galvanometer in coil C shows a momentary deflection, indicating that current is induced in coil C. The pointer in the galvanometer, however, returns to zero immediately.

(ii) When the key K is kept pressed continuously, there is no deflection in the galvanometer.

(iii) When the key K is released, the galvanometer shows again a momentary deflection, but in the opposite direction. The pointer in the galvanometer returns to zero almost instantly.

(iv) The galvanometer deflection increases dramatically, when an iron rod is inserted into the coils along their axis, and the key K is pressed/ released.

## Faraday's Laws of Electromagnetic Induction

**First Law**

*Whenever the amount of magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit. The induced e.m.f. lasts so long as the change in magnetic flux continues.*

**Second Law**

*The magnitude of e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.*

**Explanation**

***First Law:*** In Faraday's experiment, when magnet is moved towards the coil, number of magnetic lines of force linked with the coil increases, i.e., magnetic flux increases. When the magnet is moved away, the magnetic flux linked with the coil decreases. In both the cases, galvanometer shows deflection indicating that e.m.f. is induced in the coil.

When there is *no relative motion* between the magnet and the coil, magnetic flux linked with the coil remains constant. That is why galvanometer shows no deflection. Thus, induced e.mf. is produced when magnetic flux changes and induced e.m.f. continues so long as the change in magnetic flux continues. This is first law. The same results follow from Faraday's second experiment.

**Second Law**

In Faraday's experiment, when magnetic is moved faster, the magnetic flux linked with the coil changes at a faster rate. Therefore, galvanometer deflection is more. However, when the magnet is moved slowly, rate of change of magnetic flux is smaller. Therefore, galvanometer deflection is smaller, i.e., induced e.m.f. is smaller. Hence magnitude of e.m.f. induced varies directly as the rate of change of magnetic flux linked with the coil. This is second law.

If $\phi_1$ is amount of magnetic flux linked with a coil at any time and $\phi_2$ is the magnetic flux linked with the coil after t sec., then

Rate of change of magnetic flux = $\dfrac{\phi_2-\phi_1}{t}$

According to Faraday's second law, induced e.m.f.

$e \propto \dfrac{(\phi_2-\phi_1)}{t}$ or $e=\dfrac{k(\phi_2-\phi_1)}{t}$ where k is a constant of proportionality.

As k = 1 (in all systems of units) $\therefore$ $e=\dfrac{\phi_2-\phi_1}{t}$

if d $\phi$ is small change in magnetic flux in a small time dt, then $e = \frac{-d\phi}{dt}$

*Negative sign is taken because induced e.m.f. always opposes any change in magnetic flux associated with the circuit.* This follows from Lenz's law discussed. In case of a closely wound coil of N turns, change in magnetic flux associated with each turn is the same. Therefore, total induced e.m.f. is given by $e = -N\frac{d\phi}{dt}$

By increasing number of turns N in the coil, we can increase the induced e.m.f.

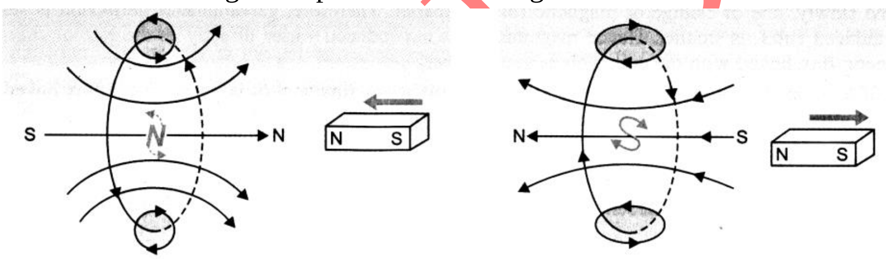
## Lenz's Law

This law gives us the direction of current induced in a circuit.

*According to **Lenz's law**, the polarity of the induced e.m.f. is such that it opposes the change in magnetic flux responsible for its production.*

For example, in figure, when north pole of a bar magnet is being pushed towards the coil, the amount of magnetic flux linked with the coil increases. Current is induced in the coil in such a direction that it *opposes the increase in flux.* This is possible only when current induced in the coil is in *anticlockwise direction* with respect to an observer on the side of the bar magnet. The magnetic moment $\vec{M}$ associated with this induced current has north polarity towards the north pole of the approaching bar magnet, as shown in figure.

Similarly, when north pole of the bar magnet is moved away from the coil, figure, the magnetic flux linked with the coil decreases. To counter this decrease in magnetic flux, current induced in the coil is in clockwise direction so that its south pole faces the receding north pole of the bar magnet.




This would result in an attractive force, which opposes the motion of the magnet and the corresponding decrease in magnetic flux.

## Experimental Verification of Lenz's Law

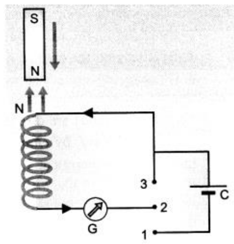



Figure shows the experimental set up for verifying Lenz's law. A coil of a few turns is connected to a cell C and a sensitive galvanometer G through a two way key 1, 2, 3.

Put the plug of key between 1 and 2. Cell sends current through the coil. At the upper face of the coil, the current is *anticlockwise,* which would produce *north pole* on this face. Suppose the galvanometer deflection is to the *right.* Obviously, if galvanometer deflection were to the left, current would be clockwise at the upper face, which would behave as south pole.

Remove the plug of key from 1 and 2. Insert the plug of key between 2 and 3. Now, move N–pole of a bar magnet towards the coil. The galvanometer shows a sudden deflection to the right indicating that current induced in the coil is anticlockwise and upper end of the coil behaves as north. It opposes the inwards motion of N–pole of the bar magnet, which is the cause of induced current.

Similarly, when N–pole of the bar magnet is moved away from the coil, the galvanometer shows a sudden deflection to the left, indicating that current induced in the coil is clockwise and upper end of the coil behaves as south. It opposes the outward motion of N–pole of the bar magnet, i.e., cause of induced e.m.f. is opposed. Exactly similar results follow when S–pole of magnet is moved instead of N–pole.

Hence *induced current always opposes the change in magnetic flux which produces it.* This verifies Lenz's law.

## Lenz's Law and Energy Conservation

In the experimental verification of Lenz's law, when N–pole of magnet is moved towards the coil, the upper face of the coil acquires north polarity. Therefore, work has to be done **against** the force or repulsion, in bringing the magnet closer to the coil. Similarly, when N pole of magnet is moved away, south polarity develops on the upper face of the coil. Therefore, work has to be done against the force of attraction, in taking the magnet away from the coil.

*It is this mechanical work done in moving the magnet w.r.t. the coil has changes into electrical energy producing induced current.* Thus, energy is being transformed only.

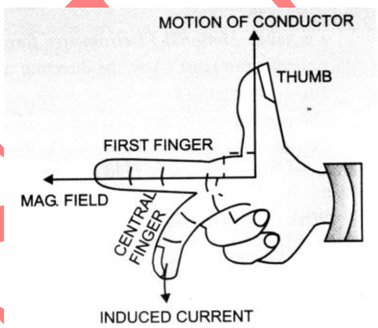
*When we do not move the magnet, work done is zero. Therefore, induced current is also not produced.*

Hence Lenz's law obeys the principle of energy conservation. Conversely, Lenz's law can be treated as a consequence of the principal of energy conservation.

## Fleming's Right Hand Rule

Fleming's right hand rule also gives us the direction of induced e.m.f./ current, in a conductor moving in a magnetic field. According to this rule,

*If we stretch the first finger, central finger and thumb of our right hand in mutually perpendicular directions such that first finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current (figure)*




The direction of induced current given by **Lenz's Law and fleming's right hand rule** will obviously be the same.

## Subjective Assignment – I

Q.1 A rectangular loop of area 20 cm × 30 cm is placed in a magnetic field of 0.3 T with its plane (i) normal to the field (ii) inclined 30° to the field and (iii) parallel to the field. Find the flux linked with the coil in each case.

Q.2 A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 s. The magnetic flux between pole pieces is known to be $8 \times 10^{-4}$ Wb. Estimate the emf induced in the wire.

Q.3 The magnetic flux through a coil perpendicular to the plane is varying according to the relation:

$$\phi = (5t^3 + 4t^2 + 2t - 5) \text{ Wb}$$

Calculate the induced current through the coil at t = 2s, if the resistance of the coil is 5 Ω.

Q.4 A square loop of side 10 cm and resistance of 0.70Ω is placed vertically in the east–west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north–east direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time–interval.

Q.5 A 10Ω resistance coil has 1000 turns and at a time $5.5 \times 10^{-4}$ Wb flux passes through it. If the flux falls to $0.5 \times 10^{-4}$ Wb in 0.1 second, find the emf generated in volts and the charge flown through the coil in coulomb.

Q.6 A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V when the magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil?

Q.7 A wire 88 cm long bent into a circular loop is placed perpendicular to the magnetic field of flux density 2.5 $Wbm^{-2}$. Within 0.5 s, the loop is changed into 22 cm square and flux density is increased to 3.0 $Wbm^{-2}$. Calculate the value of the emf induced.

Q.8 A coil of mean area 500 $cm^2$ and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through $180^o$ in 1/10 second. Calculate the average induced emf.

Q.9 A circular coil of radius 10 cm, 500 turns and resistance $2\Omega$ is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through $180^o$ in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is $3.0 \times 10^{-5}$ T.

Q.10 A coil of cross–sectional area A lies in a uniform magnetic field B with its plane perpendicular to the field. In this position the normal to the coil makes an angle of $0^o$ with the field. The coil rotates at a uniform rate to complete one rotation in time T. Find the average induced emf in the coil during the interval when the coil rotates:

(i) from $0^o$ to $90^o$ position (ii) from $90^o$ to $180^o$ position

(iii) from $180^o$ to $270^o$ and (iv) from $270^o$ to $360^o$

Q.11 A conducting circular loop is placed in a uniform transverse magnetic field of 0.02 T. Somehow, the radius of the loop begins to decrease at a constant rate of 1.0 mm/s. Find the emf induced in the loop at the instant when the radius is 2 cm.

Q.12 Find the magnetic flux linked with a rectangular coil of size 6 cm × 8 cm placed at right angle to a magnetic field of 0.5 $Wbm^{-2}$.

Q.13 A square coil of 600 turns, each side 20 cm, is placed with its plane inclined at $30^o$ to a uniform magnetic field of $4.5 \times 10^{-4}$ $Wbm^{-2}$. Find the flux through the coil.

Q.14 The magnetic flux threading a coil changes from $12 \times 10^{-3}$ Wb to $6 \times 10^{-3}$ Wb in 0.01 s. Calculate the induced emf.

Q.15 A magnetic field of flux density 1.0 $Wbm^{-2}$ acts normal to a 80 turn coil of 0.01 $m^2$. Find the emf induced in it, if this coil is removed from the field in 0.1 s.

Q.16 A 70 turn coil with average diameter of 0.02 m is placed perpendicular to magnetic field of 9000 T. If the magnetic field is changed to 6000 T in 3s, what is the magnitude of the induced emf?

Q.17 A magnetic field of flux density 10 T acts normal to a 50 turn coil of 100 $cm^2$ area. Find the emf induced in it if the coil is removed from the field in 1/20 s.

Q.18 A coil has 1000 turns and 500 $cm^2$ as its area. it is placed at right angles to a magnetic field of $2 \times 10^{-5}$ Wb $m^{-2}$. It is rotated through $180^o$ in 0.2 s. Calculate the average emf induced in it.

Q.19 A wire 40 cm long bent into a rectangular loop 15 cm × 5 cm is placed perpendicular to the magnetic field whose flux density is 0.8 $Wbm^{-2}$. Within 1.0 second, the loop is changed into a 10 cm square and flux density increase to 1.4 $Wbm^{-2}$. Calculate the value of induced emf.

Q.20 An air–cored solenoid of length 50 cm and area of cross–section 28 $cm^2$ has 200 turns and carries a current of 5.0 A. On switching off, the current decreases to zero within a time interval of 1.0 ms. Find the average emf induced across the ends of the open switch in the circuit.

Q.21 A closed coil consists of 500 turns on a rectangular frame of area 4.0 $cm^2$ and has a resistance of $50\Omega$. It is kept with its plane perpendicular to a uniform magnetic field of 0.2 Wb $m^{-2}$. Calculate the amount of charge flowing through the coil when it is turned over (rotated through $180^o$). Will this answer depend on the speed with which the coil is rotated.

Q.22 The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to relation $\phi = (5t^2 + 10t + 5)$ milliweber. Calculate the emf induced in the loop at t = 5s.

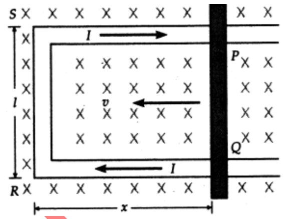
## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | (i) $1.8 \times 10^{-2}$ Wb, (ii) $0.9 \times 10^{-2}$ Wb, (iii) zero | | | 2. | $1.6 \times 10^{-3}$ V |
| 3. | 15.6 A | 4. | 1 mV, 1.4 mA | 5. | 5 V, 0.05 C |
| 6. | 28 | 7. | 0.0176 V | 8. | 0.04 V |
| 9. | $3.8 \times 10^{-3}$ V, $1.9 \times 10^{-3}$ A | 10. | (i) $\frac{4BA}{T}$, (ii) $\frac{4BA}{T}$, (iii) $-\frac{4BA}{T}$, (iv) $-\frac{4BA}{T}$ | | |

| | | | | | |
|---|---|---|---|---|---|
| 11. | 2.5 μV | 12. | $2.4 \times 10^{-3}$ Wb | 13. | $5.4 \times 10^{-3}$ wb |
| 14. | 0.6 V | 15. | 8 V | 16. | 2.2 V |
| 17. | 100 V | 18. | 10 mV | 19. | – 0.008 V |
| 20. | 1.4 V | 21. | $1.6 \times 10^{-3}$ C, No | 22. | 0.06 V |

## Motional Electromotive Force

*The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf.* Figure shows a rectangular conducting loop PQRS in the plane of paper. The conductor PQ is free to move without any loss of energy due to friction etc. The crosses represent a uniform magnetic field B which is perpendicular to the plane of the paper and directed inwards.




Let the conductor PQ be moved towards the left with a constant velocity v. The area enclosed by the loop PQRS decreases. Therefore, amount of magnetic flux linked with the loop decreases. An e.m.f. is induced in the loop as detected by deflection in the galvanometer G connected in the loop. At any time,

If the length RQ = x and PQ = RS = ℓ, then magnetic flux linked with the loop PQRS

$\phi = B\,\ell x$

As x is changing with time, the amount of magnetic flux linked with the loop changes. Therefore, an e.m.f. is induced in the loop, given by

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(B\ell x) \quad ; \quad e = B\ell\left(\frac{-dx}{dt}\right) = B\ell v$$

where $\frac{-dx}{dt} = v$ is the velocity of the conductor PQ towards the left.

**e** is called ***Motional Electromotive force.***

According to Lenz's law, the direction of induced current is given by *If we stretch the first finger, central finger and thumb of our right hand in mutually perpendicular directions such that first finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current.*

**Alternative Method**

The expression for motional e.m.f. can also be obtained using Lorentz force equation.

Consider any arbitrary charge +q in the conductor PQ. As the conductor moves, charge q also moves with speed v in the magnetic field B. Lorentz force on this charge = q v B. According to Fleming's left hand rule, the direction of this force is towards Q.

Work done in moving the charge from P to Q is $W = q\,v\,B \times \ell$

As e.m.f. is work done per unit charge. Therefore, $e = \frac{W}{q} = \frac{qvB \times \ell}{q}$

$e = B\,\ell\,v$

If R is resistance of the loop at a given instant, the induced current I at that instant would be given by

$$I = \frac{e}{R} = \frac{B\ell v}{R}$$

The direction of induced current is given by *Fleming's right hand rule,* or *Lenz's law.*

***NOTE:***

The magnetic flux linked with a loop does not change with time when

(i) magnet and loop are moving with the same velocity in the same direction,

(ii) magnet is rotated around its axis without changing its distance from the loop,

(iii) loop is moved in a uniform magnetic field and the whole of the loop remains in the field.

***Obviously, no e.m.f. will be induced in the loop under these conditions.***

## Energy Consideration in Motional E.M.F.

When a conductor of length $\ell$ is moved with a velocity v in a perpendicular magnetic field B, the motional e.m.f. produced in the conductor is $e = B\ell v$

Let r be the resistance of movable arm PQ of the rectangular conductor. If we assume that the resistance of remaining arms QR, RS and SP is negligible compared to r, then overall resistance of the rectangular loop would remain r even when PQ is moved.

$\therefore$ Current induced in the loop $I = \frac{e}{r} = \frac{B\ell v}{r}$

The magnitude of force on the conductor PQ moving in the magnetic field is

$$F = BI\ell = B\left(\frac{B\ell v}{r}\right)l = \frac{B^2\ell^2 v}{r}$$

The direction of this force is opposite to the velocity of the conductor.

Power required to push the conductor

$$P = F \times v = \frac{B^2\ell^2 v}{r} \times v = \frac{B^2\ell^2 v^2}{r}$$

which is the same as the power required to push the conductor.

Hence, mechanical energy required to move the conductor PQ is converted into electrical energy first, (i.e., the induced e.m.f.) and then to thermal energy.

$\therefore$ Heat energy produced/sec $= \frac{B^2\ell^2 v^2}{r}$

***NOTE:***

We can calculate the amount of charge induced from the change in magnetic flux.

As $e = \frac{d\phi}{dt}$ and also, $e = Ir = \left(\frac{dQ}{dt}\right)r$

where r is resistance of the circuit,

$\therefore$ $\left(\frac{dQ}{dt}\right)r = \frac{d\phi}{dt}$ or $dQ = \frac{d\phi}{r}$

i.e., amount of charge induced is equal to change in magnetic flux divided by the resistance of the circuit.

Clearly, the induced charge depends on the net change in the magnetic flux and not on the time interval $\Delta t$ of the flux change. Thus the induced charge does not depend on the rate of change of magnetic flux.

## Subjective Assignment – II

Q.1 An aircraft with a wing span of 40m flies with a speed of 1080 $kmh^{-1}$ in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is $1.75 \times 10^{-5}$ T. Find the emf that develops between the tips of the wings.

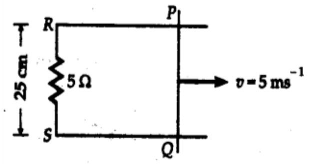
Q.2 A jet plane is travelling west at 450 $ms^{-1}$. If the horizontal component of earth's magnetic field at that place is $4 \times 10^{-4}$ tesla and the angle of dip is $30^o$, find the emf induced between the ends of wings having a span of 30 m.

Q.3 A railway track running north–south has two parallel rails 1.0 m apart. Calculate the value of induced emf between the rails, when a train passes at a speed of 90 $kmh^{-1}$. The horizontal component of earth's magnetic field at that place is $0.3 \times 10^{-4}$ $Wbm^{-2}$ and angle of dip is $60^o$.

Q.4 A conductor of length 1.0 m falls freely under gravity from a height of 10 m so that it cuts the lines of force of the horizontal component of earth's magnetic field of $3 \times 10^{-5}$ $Wbm^{-2}$. Find the emf induced in the conductor.

Q.5 Twelve wires of equal lengths (each 10 cm) are connected in the form of a skeleton–cube (i) If the cube is moving with a velocity of 5 $ms^{-1}$ in the direction of a magnetic field of 0.05 $Wbm^{-2}$, find the emf induced in each arm of the cube. (ii) If the cube moves perpendicular to the field, what will be the induced emf in each arm?

Q.6 Figure, shows a conducting rod PQ in contact with metal rails RP and SQ which are 25 cm apart in a uniform magnetic field of flux density 0.4 T acting perpendicular to the plane of the paper. Ends R and S are conducted through a 5Ω resistance. What is the emf when the rod moves to the right with a velocity of 5 $ms^{-1}$? What is the magnitude and direction of the current through 5Ω resistance? If the rod moves to the left with the same speed, what will be the new current and its direction?




Q.7 A metallic rod of length L is rotated at an angular speed ω normal to a uniform magnetic field B. Derive expressions for the (i) emf induced in the rod (ii) current induced and (iii) heat dissipation, if the resistance of the rod is R.

Q.8 A metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to its plane passing through its centre in a magnetic field B acting perpendicular to the plane of the disc. Calculate the induced emf between the rim and the axis of the disc.

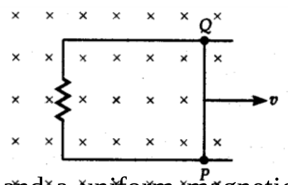
Q.9 A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field $B_H$ at a place. If $B_H$ = 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel?

Q.10 When a wheel with metal spokes 1.2 m long rotates in a magnetic field of flux density $5 \times 10^{-5}$ T normal to the plane of the wheel, an emf of $10^{-2}$ V is induced between the rim and the axle of the wheel. Find the rate of revolution of the wheel.

Q.11 A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1m, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field of 1 T and parallel to the axis is present everywhere. What is the emf between the centre and the metallic ring?

Q.12 A circular copper disc 10 cm in radius rotates at 20π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc. (i) Calculate the potential difference developed between the axis of the disc and the rim. (ii) What is the induced current, if the resistance of the disc is 2 ohm?

Q.13 A 0.5 m long metal rod PQ completes the circuit as shown in figure. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 $ms^{-1}$.




Q.14 A straight conductor 1 metre long moves at right angles to both, its length and a uniform magnetic field. If the speed of the conductor is 2.0 $ms^{-1}$ and the strength of the magnetic field is $10^4$ gauss, find the value of induced emf in volt.

Q.15 If a 10 m long metallic bar moves in a direction at right angle to a magnetic field with a speed of 5.0 $ms^{-1}$, 25 V emf is induced in it. Find the value of the magnetic field intensity.

Q.16 A 0.4 m long straight conductor is moved in a magnetic field of induction 0.9 $Wbm^{-2}$ with velocity of 7 $ms^{-1}$. Calculate the maximum emf induced in the conductor.

Q.17 A horizontal telephone wire 1 km long is lying east–west in earth's magnetic field. It falls freely to the ground from a height of 10 m. Calculate the emf induced in the wire on striking the ground. Given $B_H$ = 0.32 G.

Q.18 A horizontal wire 24 cm long falls in the field of flux density 0.8 T. Calculate the emf induced in it at the end of 3s, after it was dropped from rest. Suppose the wire moves perpendicular to its length as well as to magnetic field. Take g = 9.8 $ms^{-2}$.

Q.19 The two rails of a railway track insulated from each other and the ground are connected to a millivolmeter. What is the reading of the voltmeter when a train travels at a speed of 180 $kmh^{-1}$ along the track, given that the vertical component of the earth's magnetic field is $0.2 \times 10^{-4}$ Wb $m^{-2}$ and the rails are separated by 1m?

Q.20 A metre gauge train is running due north with a constant speed of 90 km $h^{-1}$ on a horizontal track. If the vertical component of earth's magnetic field is $3 \times 10^{-5}$ Wb $m^{-2}$, calculate the emf induced across the axle of the train of length 1.25 m.

Q.21 A jet plane is moving at a speed of 1000 km $h^{-1}$. What is the potential difference across the ends of its wings 20 m long. Given total intensity of earth's magnetic field is $3.5 \times 10^{-4}$ tesla and angle of dip at the place is $30^o$.

Q.22 A straight rod 2 m long is placed in an aeroplane in the east–west direction. The aeroplane lifts itself in the upward direction at a speed of 36 km $h^{-1}$. Find the potential difference between the two ends of rod if the vertical component of earth's magnetic field is $\frac{1}{4\sqrt{3}}$ gauss and angle of dip = $30^o$.

Q.23 When a wheel with metal spokes 1.0 m long is rotated in a magnetic field of flux density $2 \times 10^{-4}$ T is normal to the plane of wheel, an emf of $\pi \times 10^{-2}$ V is induced between the rim and the axle. Find the rate of rotation of the wheel.

Q.24 A fan blade of length 2a rotates with frequency f cycles per second perpendicular to a magnetic filed B. Find the p.d. between the center and the end of the blade.

Q.25 In a ceiling fan, each blade rotates in a circle of radius 0.5 m. If the fan makes 20 revolutions per second and if the vertical component of earth's field is $8 \times 10^{-5}$ Wb $m^{-2}$, calculate the p.d. developed between the ends of each blade.

Q.26 A metal disc of radius 200 cm is rotated at a constant angular speed of 60 rad $s^{-1}$ in a plane at right angles to an external field of magnetic induction 0.05 $Wbm^{-2}$. Find the emf induced between the centre and a point on the rim.

Q.27 A copper disc of radius 10 cm placed with its plane normal to a uniform magnetic field completes 1200 rotations per minute. If induced emf between the intensity of the magnetic field. Take $\pi$ = 3.142

Q.28 A gramophone disc of brass of diameter 30 cm rotates horizontally at the rate of 100/3 rpm. If the vertical component of earth's magnetic field be 0.01 Wb $m^{-2}$, then calculate the emf induced between the centre and the rim of the disc.

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | 0.21 V | 2. | 3.12 V | 3. | $1.3 \times 10^{-3}$ V |
| 4. | $4.2 \times 10^{-4}$ V | 5. | (i) 0 V, (ii) $2.5 \times 10^{-2}$ V | 6. | 0.1 A |
| 7. | $\frac{1}{4}\frac{B^2L^2\omega}{R}$ | 8. | $\frac{1}{2}BR^2\omega$ | 9. | $6.28 \times 10^{-5}$ V |
| 10. | 44.2 rps | 11. | 157 V | 12. | (i) 0.0628 V, (ii) 0.0314 A |
| 13. | 0.00375 N | 14. | 2 V | 15. | 0.5 T |
| 16. | 2.52 V | 17. | 0.448 V | 18. | 5.6 V |
| 19. | 1 mV | 20. | $9.375 \times 10^{-4}$ V | 21. | 0.97 volt |
| 22. | $5 \times 10^{-4}$ V | 23. | 50 rps | 24. | $-\pi Ba^2 f$ |
| 25. | 0.001 T | 26. | 6 V | 27. | $10^{-2}$ T |
| 28. | $3.9 \times 10^{-4}$ V | | | | |

## Various Methods of producing Induced E.M.F.

As $\phi = BA\cos\theta$, the magnetic flux $\phi$ can be changed by changing B, A or $\theta$. Hence there are *three methods* of producing induced e.m.f.

1. By changing the magnitude of magnetic field B,

2. By changing the area A, i.e., by shrinking or stretching or changing the shape of the coil.
3. By changing angle θ between the direction of B and normal to the surface area A i.e. changing the relative orientation of the surface area and the magnetic field.

***(a) Induced e.m.f. by changing the magnetic field***

In faraday's experiment 1, figure, motion of the magnet towards the coil increases the magnetic field B at any point on the wire loop and *vice–versa*. In either case, galvanometer shows deflection. Thus, e.m.f. is induced by changing B.

In Faraday's experiment 3, figure, changing current in coil C' (on pressing or releasing key K), changes the magnetic field at any point on coil C. This results in the production of inducted e.m.f. in coil C.

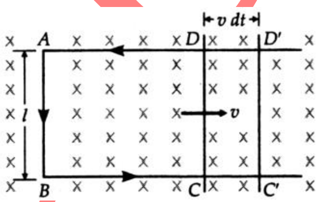
***(b) Induced e.m.f. by changing area A***

Consider a conductor CD of length ℓ moving with a velocity v towards right on U–shaped conducting rails situated in a magnetic field B, as shown in fig. Field is uniform and points into the plane of paper. As the conductor slides, the area of the circuit changes from ABCD to ABC' D' in time dt.

The increase in flux,

$d\phi = B \times \text{change in area}$

$= B \times \text{area } CDD'C' = B.\ \ell.\ vdt$




This sets up induced emf in the loop of magnitude,

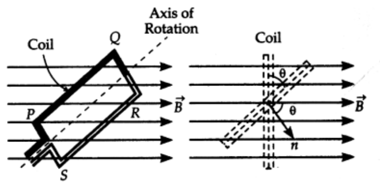
$$|e| = \frac{d\phi}{dt} = Blv$$

According to Fleming's right rule, the induced current flows in the anticlockwise direction.

***(c) Induced emf by changing relative orientation of the coil and the magnetic field: Theory of AC generator***

Consider a coil PQRS free to rotate in a uniform magnetic field $\vec{B}$. The axis of rotation of the coil is perpendicular to the field $\vec{B}$. The flux the coil, when its normal makes an angle θ with field, is given by

$$\phi = BA\cos\theta$$



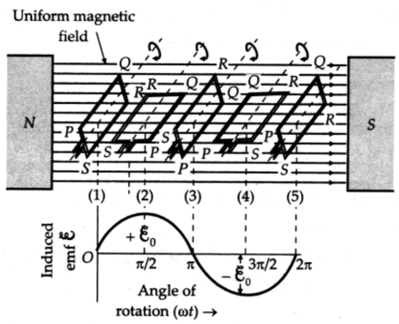

where A is the face area of the coil.

If the coil rotates with an angular velocity ω and turns through an angle θ in time t, then

$\theta = \omega t \qquad \therefore \qquad \phi = BA\cos\omega t$

As the coil rotates, the magnetic flux linked with it changes. An induced emf set up in the coil which is given by

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t) = BA\omega\sin\omega t$$




If the coil has N turns, then the total induced emf will be

$e = NBA\,\omega\sin\omega t$

Thus the induced emf varies sinusoidally with time t. The value of induced emf is maximum when $\sin\omega t = 1$ or $\omega t = 90^o$, i.e., when the plane of the coil is *parallel* to the field $\vec{B}$. Denoting this maximum value by $e_0$, we have

$e_0 = NBA\,\omega$

$\therefore \qquad e = e_0 \sin\omega t = e_0 \sin 2\pi ft$

where f is the frequency of rotation of the coil.

Figure shows how the induced emf e between the two terminals of the coil varies with time. We consider the following specific cases:

1. When $\omega t = 0^o$, the plane of the coil is perpendicular to $\vec{B}$. $\sin \omega t = \sin 0^o = 0$ so that $e = 0$

2. When $\omega t = \pi/2$, the plane of the coil is parallel to field $\vec{B}$, $\sin \omega t = \sin \frac{\pi}{2} = 1$, so that $e = e_0$

3. When $\omega t = \pi$, the plane of the coil is again perpendicular to B, $\sin \omega t = \sin \pi = 0$, so that $e = 0$

4. When $\omega t = \frac{3\pi}{2}$ the plane of the coil is again parallel to $\vec{B}$, $\sin \omega t = \sin \frac{3\pi}{2} = -1$ so that $e = -e_0$

5. When $\omega t = 2\pi$, the plane of the coil again becomes perpendicular to $\vec{B}$ after completing one rotation,
   $\sin \omega t = \sin 2\pi = 0$ so that $e = 0$

As the coil continues to rotate in the same sense, the same cycle of changes repeats again and again. As shown in figure, the graph between emf e and time t is a sine curve. Such an emf is called *sinusoidal or alternating emf*. Both the magnitude and direction of the emf change regularly with time.

The fact that an induced emf is set up in a coil when rotated in a magnetic field forms the basic principle of a dynamo or a generator.

**NOTE**

- The magnetic flux linked with a surface is maximum when it is held perpendicular to the direction of the magnetic field and the flux linked is zero when the surface is held parallel to the direction of the magnetic field.
- Induced emf is set up whenever the magnetic flux linked with a circuit changes even if the circuit is open. However, the induced current flows only when the circuit is closed.
- No emf is induced when a coil and a magnet move with the same velocity in the same direction.
- No emf is induced when a magnet is rotated about its own axis. However, emf is induced when a magnet is rotated about an axis perpendicular to its length.
- No emf is induced when a closed loop moves totally inside a uniform magnetic field.
- Just as a changing magnetic field produces an electric field, a changing electric field also sets up a magnetic field.
- The electric fields created by stationary charges have vanishing and path independent loop integrals. Such fields are called *conservative fields*.

$$\oint \vec{E}.\, d\vec{\ell} = 0$$

- The electric fields created by time–varying magnetic fields have non–vanishing loop integrals and are called *non–conservative fields*. Their loop integrals are path dependent.

$$\oint \vec{E}.\, d\vec{\ell} = \frac{d\phi}{dt}$$

- Electric potential is meaningful only for electric fields produced by stationary charges. It has no meaning for electric fields set up by magnetic induction.
- The heart beating induces a.c. in the surrounding tissues. The detection and study of these currents is called *elecrocardiography* which provides valuable information regarding the pathology of the heart.
- **Migration of birds:** Every winter birds from Siberia fly unerringly to water spots in the Indian subcontinent. It is believed that migratory birds make use of earth's magnetic field to determine their direction. As birds contain no ferromagnetic material, so electromagnetic induction appears to be the only mechanism to determine direction. However, very small emfs induced across the bodies of these birds create a doubt about the validity of this hypothesis. So the migration pattern of birds is still a mystery.

Q.1 A circular coil of area 300 $cm^2$ and 25 turns rotates about its vertical diameter with an angular speed of 40 $s^{-1}$ in a uniform horizontal magnetic field of magnetic 0.05 T. Obtain the maximum voltage induced in the coil.

Q.2 A rectangular coil of length 1 m and width 0.5 m, and 10 turns is rotated at 50 revolutions per second. The magnetic field within which the coil is rotated is B = 0.5 T. Calculate the peak value of the voltage generated across the ends of the coil.

Q.3 A flat coil of 500 turns, each of area $5 \times 10^{-3}$ $m^2$, rotates in a uniform magnetic field of 0.14 T at an angular speed of 150 rad $s^{-1}$. The coil has a resistance of 5Ω. The induced emf is applied to an external resistance of 10Ω. Calculate the peak current through the resistance.

Q.4 A rectangular coil of wire has dimensions 0.2 m × 0.1m. The coil has 2000 turns. The coil rotates in a magnetic field about an axis parallel to its length and perpendicular to the magnetic field of 0.02Wb $m^{-2}$. The speed of rotation of the coil is 4200 r.p.m. Calculate (i) the maximum value of the induced emf in the coil (ii) the instantaneous value of induced emf when the plane of the coil has rotated through an angle of $30^o$ from the initial position.

Q.5 A rectangular coil of 200 turns of wire, 15 cm × 40 cm makes 50 revolutions/ second about an axis perpendicular to the magnetic field of 0.08 weber/$m^2$. What is the instantaneous value of induced emf when the plane of the coil makes an angle with magnetic lines of (i) $0^o$, (ii) $60^o$ and (iii) $90^o$?

Q.6 A closely wound rectangular coil of 200 turns and size 0.3 m × 0.1 m is rotating in a magnetic field of induction 0.005 Wb $m^{-2}$ with a frequency of revolution 1800 rpm about an axis normal to the field. Calculate the maximum value of induced emf.

Q.7 A rectangular coil of dimensions 0.1m × 0.5 m consisting of 2000 turns rotates about an axis parallel to its longer side, making 2100 revolutions per minute in a field of 0.1 T. What is the maximum emf induced in the coil? Also find the instantaneous emf, when the coil is $60^o$ to field.

Q.8 The armature coil of a generator has 20 turns and its area is 0.127 $m^2$. How fast should it be rotated in a magnetic field of 0.2 $Wbm^{-2}$, so that the peak value of induced emf is 160 V?

Q.9 A 50 turns coil of area 500 $cm^2$ is rotating at a rate of 50 rounds per second perpendicular to a magnetic field of 0.5 $Wbm^{-2}$. Calculate the maximum value of induced emf.

Q.10 Calculate the maximum emf induced in a coil of 100 turns and 0.01 $m^2$ area rotating at the rate of 50 rps about an axis perpendicular to a uniform magnetic field of 0.05 T. If the resistance of the coil is 30Ω, what is the maximum power generated by it?

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | 1.5 V | 2. | 785 V | 3. | 3.5 A |
| 4. | 3520 V, 1760 V | 5. | (i) 301.6 V, (ii) 150.8 V, (iii) 0 | | |
| 6. | 5.65 V | 7. | 2200 V, 1100 V | 8. | 50 rps |
| 9. | 392.5 V | 10. | 15.7 V, 8.23 W | | |

## Eddy Currents

Eddy Currents are the currents induced in the bulk pieces of conductors when the amount of magnetic flux linked with the conductor changes.

*However, their flow patterns resemble swirling eddies in water. That is why they are called eddy currents. These were discovered by Foucault in the year 1985 and hence they are also called Foucault currents.*

*For example*, when we move a metal plate out of a magnetic field, the relative motion of the field and the conductor again induces a current in the conductor. The conduction electrons making up the induced current whirl about within the plate as if they were caught in an eddy (or whirlpool) of water. This is called the eddy current.

The magnitude of eddy current is $i = \frac{\text{induced e.m.f.}}{\text{resistance}} = \frac{e}{R}$

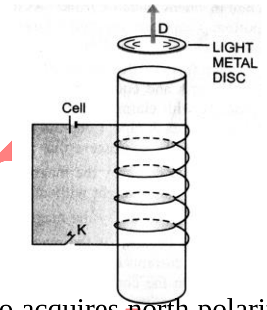
But $\quad e = -\dfrac{d\phi}{dt} \quad \therefore \quad i = -\dfrac{d\phi/dt}{R}$

The direction of eddy currents is given by Lenz's law, or Fleming's right hand rule.

***NOTE:***

Eddy currents are basically the currents induced in the body of a conductor due to change in magnetic flux linked with the conductor.

**Experimental Demonstration**




Hold a light metallic disc D at top of the cross–section of an electromagnet connected to a cell and a tap key K. When key K is pressed, the disc is thrown up into the air. This is due to eddy currents developed in the disc.

As current through the solenoid increases from zero to maximum, the magnetic flux along the axis of the solenoid increases. Therefore, magnetic flux linked with the disc increases. Induced currents or eddy currents develop in the disc and magnetise it.

If upper end of solenoid initially acquires north polarity, the lower face of disc D also acquires north polarity in accordance with Lenz's law. The force of repulsion between the two throws the disc up in the air. Later, the disc falls down due to gravity. Things are repeated when key K is released.

## Applications of Eddy Currents

Eddy currents are useful in many ways though they have some disadvantages too.

Some of the important applications of eddy currents are:

**(a)** **Electro–magnetic damping:** When a steady current is passed through the coil of a galvanometer, it is deflected. Normally, the coil oscillates about its equilibrium position for some time before coming to rest. We cannot read the galvanometer deflection until the coil comes to rest.

To avoid the delay in reading galvanometer deflection due to these oscillations, the coil is wound over a non magnetic *metallic frame*. As the coil is deflected, eddy currents set up in the metallic frame oppose its motion. Therefore, the coil attains its equilibrium position almost instantly. Thus, the motion of coil is damped. This is called electromagnetic damping.

For further illustration of electromagnetic damping, we take two hollow thin cylindrical pipes of equal length and equal internal diameter, one made of aluminium and other of PVC. Fix the two pipes vertically with clamps on retort stands. Take a small cylindrical magnet whose diameter is less than the diameter of each pipe. Drop the magnet through each pipe such that it does not touch the sides of the pipe during the fall. We observe that:

(i) Time taken by the magnet of fall through PVC pipe is equal to time taken by the magnet to fall through the same height without the pipe.

(ii) Time taken by the magnet to fall through aluminium pipe is comparatively longer.

This is because as the magnet falls through the aluminium pipe, eddy currents are generated in the pipe. These currents oppose the change in magnetic flux (i.e., motion of the magnet). This is electromagnetic damping. On the contrary, material of PVC pipe is insulating. Therefore, no eddy currents are generated and there is no opposition to the motion of the falling magnet through the PVC pipe.

**(b)** **Induction Furnace:** It is used to produce high temperatures which are utilized in preparing alloys by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the constituent metals. The large eddy currents generated in the metals produce high temperatures sufficient to melt the metals.

**(c)** **Magnetic Brakes:** In some electrically powered trains, strong electromagnets are situated in the train, just above the rails. When electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are so mechanical linkages, therefore, the braking action is smooth.

**(d)** **Electric Power Meters:** In the power meter of your house, you must have observed a rotating shiny disc. This is a metal disc which rotates due to eddy currents developed in the disc, by magnetic fields produced by alternating current.

**(e)** **Induction Motor :** An induction motor or a.c. motor is yet another important application of eddy currents. A rotating magnetic field produces strong eddy currents in a rotor, which starts rotating in the direction of the rotating magnetic field.

**(f)** In **speedometers** of automobiles and **energy metres.**

(g) Eddy currents are also used in **dia–thermy i.e., in deep heat treatment of the human body.**

Some of the **undesirable effects** of eddy current are:

(i) They oppose the relative motion.

(ii) They involve loss of energy in the form of heat.

(iii) The excessive heating may break the insulation in the appliances and reduce their life.

**To minimize the eddy currents,** the metal core to be used in an appliance like dynamo, transformer, choke coil, motor etc. is taken in the form of *thin sheets*. Each sheet is electrically insulated from the other by insulating varnish like lacquer. Such a core is called ***a laminated core.*** The planes of these sheets are arranged parallel to the magnetic field so that they cut across the eddy current paths.

Large resistance between the thin sheets confines the eddy currents to the individual sheets. Hence the eddy currents are reduced to a large extent.

As dissipation of electric energy into heat varies directly as the square of the strength of electric current, therefore, heat loss is greatly reduced.

**NOTE:**

- ♦ Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes.
- ♦ Eddy currents tend to follow the path of least resistance inside a conductor. So they form irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.
- ♦ Eddy currents have both undesirable effects and practically useful applications.
- ♦ Eddy currents can be induced in biological tissues. For example, the cavity of the eye is filled with a conducting fluid. A large transient magnetic field of 1 T alternating at a frequency of 60 Hz then induces such a large current in the retina that it produces a *sensation of intense brightness.*

## Induced Electric Field

The electric field produced by stationary charges is called electrostatic field, and for such a field $\oint \vec{E}.\vec{dl}=0$ i.e. electrostatic field is conservative.

In case of electromagnetic induction, line integral of induced field $\vec{E}$ round a closed path is not zero i.e. induced electric field is *non–conservative*. In such a field, work done in moving a charge round a closed path is *not zero.*

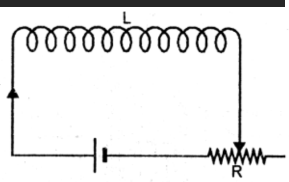
Just as a changing magnetic field produces an electric field, similarly, a changing electric field also produces a magnetic field.

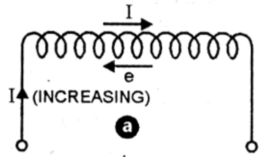
## Self Induction

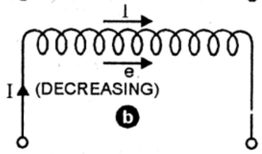
*Self Induction is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself.*

For the reason, self induction is also called the inertia of electricity.

In figure, if current in a coil L is changed by varying the contact position on a variable resistor, a self induced e.m.f. appears in the coil–while the current is changing.



When current (I) is increasing, the self induced e.m.f. (e) appears across the coil in a direction such that *it opposes the increase*. Therefore, it would be in a direction opposite to I, figure.

When current (I) is decreasing, the self induced e.m.f. (e) appears across the coil in a direction, such that it *opposes the decrease*. Therefore, it would be in the direction of (I), figure.

***Coefficient of Self Induction of Self Inductance***

Suppose I = strength of current flowing through a coil at any time.

$\phi$ = amount of magnetic flux linked with all the turns of the coil at that time.

It is found that $\phi \propto I$ or $\phi = L I$

where L is a constant of proportionality and is called *coefficient of self induction or self inductance of* the coil. The *value of* L *depends on number of turns, area of cross–section and nature of material of the core on which the coil is wound.*

If I = 1, $\phi = L \times 1$ or $L = \phi$. Therefore,

***Coefficient of self induction*** *of a coil is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil.*

Now, the e.m.f. induced in the coil is given by

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) \quad \text{or} \quad e = -L \times \frac{dI}{dt}$$

If $\frac{dI}{dt} = 1$, then $e = -L \times 1$ or $L = -e$

**Coefficient of self induction** *of a coil is equal to the e.m.f. induced in the coil when rate of change of current through the coil is unity. Inductance is a scalar quantity.*

*The SI unit of L is henry.*

*As,* $L = \frac{-e}{dI/dt}$ $\quad \therefore \quad$ $1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/sec}} = \frac{1 \text{ volt} - \text{sec}}{\text{ampere}}$

*Self inductance of a coil is said be* ***one henry (H)*** *when a current change at the rate of 1 ampere/sec through the coil induces an e.m.f. of 1 volt in the coil.*

Thus, 1 henry = 1 volt–sec/ampere = 1 weber/ampere

Smaller units of L are 1 millihenry (1 mH) = $10^{-3}$ henry,

1 microhenry (1 $\mu$H) = $10^{-6}$ henry

***Dimensions of self inductance L***

As $L = \frac{e\,dt}{dI}$ and $e = \frac{\text{work}(W)}{\text{charge}(q)}$ $\quad \therefore \quad L = \frac{W}{q}\frac{dt}{dI}$

$$L = \frac{[M^1 L^2 T^{-2}][T]}{[AT][A]} = [M^1 L^2 T^{-2} A^{-2}]$$

## Self Inductance of a Long Solenoid

A long solenoid is that whose length is very large as compared to its radius of cross–section. The magnetic field B at any point inside such a solenoid is practically constant and is given by $B = \frac{\mu_0 N I}{\ell}$

where $\mu_0$ is absolute magnetic permeability of free space/air, which forms the core of the solenoid, $\ell$ is length of the solenoid, N is total number of turns in the solenoid.

$\therefore$ Magnetic flux through each turn of the solenoid coil = B × area of each turn = $\left(\mu_0 \frac{N}{\ell} I\right) A$

where A is area of each turn of the solenoid.

Total magnetic flux linked with the solenoid = flux through each turn × total number of turns

i.e., $$\phi = \mu_0 \frac{N}{\ell} I\, A \times N$$

If L is coefficient of self inductance of the solenoid, then $\phi = LI$

$$\therefore \quad LI = \mu_0 \frac{N}{\ell} I\, A \times N \quad \text{or} \quad L = \frac{\mu_0 N^2 A}{\ell}$$

If core is of any other magnetic material, $\mu_0$ is replaced by $\mu$, where $\mu = \mu_0\, \mu_r$

$$\therefore \quad L = \frac{\mu_0\, \mu_r\, N^2 A}{\ell}$$

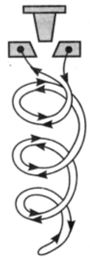
**Factors on which self–inductance depends:**

1. *Number of turns:* Larger the number of turns in the solenoid, larger its self–inductance. $L \propto N^2$
2. *Area of cross–section:* Larger the area of cross section of the solenoid, larger its self–inductance.
   $L \propto A$
3. *Permeability of the core material:* The self inductance of a solenoid increases $\mu_r$ times if it is wound over an iron core of relative permeability $\mu_r$.

***Phenomena associated with self–induction***

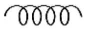
***1.*** ***Sparking:*** The break of a circuit is very sudden. When the circuit is switched off, a large self induced emf is set up in circuit in same direction as the original emf. This causes a big spark across the switch.

***2.*** ***Non–inductive winding:*** In resistance boxes and post office boxes, different resistance coils have to be used. Here the wire is first doubled over itself and then wound in the form of a coil over a bobbin. Due to this, the currents in the two halves of the wire flow in opposite directions as shown in figure. The inductive effects of the two halves of the wire, being in opposite directions, cancel each other. The net self–inductance of the coil is minimum. Such a winding of coils is called *non–inductive winding*. The resistance coils having no self–inductance are called *non–inductive resistance.*

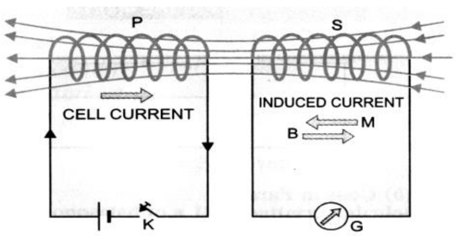


***3.*** ***Electromagnetic damping:***

**NOTE**

- Inductance is a measure of the ratio of induced flux $\phi$ to the current I. It is a scalar quantity having the dimensions of magnetic flux divided by current. Its dimensions in terms of the fundamental quantities are $[ML^2T^{-2}A^{-2}]$. Its SI unit is $WbA^{-1}$ or $VsA^{-1}$ which is called *henry* (H).
- Inductance plays the role of electrical inertia. The analogue of self–inductance in mechanics is mass.
- A solenoid made from a thick wire has a negligible resistance but a sufficiently large self–inductance. Such an element is called ***ideal inductor,*** denoted by 
- A wire itself cannot act as an inductor because the magnetic flux linked with the wire of negligible cross–sectional area is zero. Only a wire bent into the form of a coil can act as an inductor. Moreover, the self–induced emf appears only during the time the current through it is changing.
- The inductance of a coil depends on its geometry and the intrinsic properties of the material that fills up the space inside it. In this sense, it bears similarity to capacitance and resistance. The capacitance of a parallel plate capacitor depends on the plate area and plate separation (geometry) and the dielectric constant $\kappa$ of the interposing medium (intrinsic material property). Similarly, the resistance of a conductor depends on its length and cross–sectional area (geometry) and resistivity (intrinsic material property).
- The capacitance, resistance, inductance and diode constitute the ***four passive elements*** of an electrical circuit. In fact, these are the ***four alphabets*** of electrical/ electronic engineering.

## Mutual Induction




***Mutual induction** is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.*

Let us take two coils P and S held closely. P is connected to a cell through a key K. S is connected to a sensitive galvanometer G (figure). On pressing or releasing K, galvanometer shows a sudden temporary deflection. This is due to mutual induction.

*On pressing K,* current in P increases from zero to maximum value. It takes some time. During this time (or make M), current in P is increasing. Therefore, magnetic flux linked with P is increasing. As S is close by, magnetic flux associated with S also increases. An e.m.f. is induced in S.

*On releasing K,* current in P decreases from maximum to zero value. It takes some time. During this time (or break B), current in P is decreasing. Therefore, magnetic flux linked with P is decreasing. As S is held close by ; magnetic flux associated with S also decreases. An e.m.f. is induced in S. According to Lenz's law, induced current in S. during break flows in the direction of the cell current in P so as to *oppose the decrease in current* in P i.e. it prolongs the decay of current.

***Coefficient of Mutual Induction or Mutual Inductance***

Suppose, $I$ = strength of current in one coil

$\phi$ = total amount of magnetic flux linked with all the turns of the neighbouring coil.

It is found that $\phi \propto I$ or $\phi = MI$

where M is a constant of proportionality and is called *coefficient of mutual induction or mutual inductance* of the two coils.

If I = 1, $\phi = M \times I$ or $M = \phi$. Thus,

***Coefficient of mutual induction or mutual inductance** of two coils is numerically equal to the amount of magnetic flux linked with one coil when unit current flows through the neighbouring coil.*

The e.m.f. induced in the neighbouring coil is given by $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(MI)$

$$e = -M\frac{dI}{dt}$$

***Coefficient of mutual induction or mutual inductance** of two coils is equal to the e.m.f. induced in one coil when rate of change of current through the other coil is unity.*

The SI unit of M is henry.

*Coefficient of mutual induction or mutual inductance of two coils is said to be one henry, when a current change at the rate of the rate of one ampere/sec in one coil induces an e.m.f. of one volt in the other coil.*

As stated in case of self inductance, 1 henry = 1 weber/ampere = 1 volt/ ampere

*The dimensions of mutual inductance are the same as the dimensions of self–inductance.*

***The coefficient of mutual induction or mutual inductance of two coils depends on:***

(i) geometry of two coils, i.e., size of coils, their shape, number of turns, nature of material on which two coils are wound, (ii) distance between two coils,

(iii) relative placement of two coils (i.e. orientation of the two coils)

## Mutual Inductance of Two Long Coaxial Solenoids

Figure shows two long co–axial solenoids, each of length ($l$). Let $n_1$ be the number of turns per unit length of inner solenoid $S_1$ or radius $r_1$, $n_2$ is number of turns per unit length of outer solenoid $S_2$ or radius $r_2$.

Imagine a time varying current $I_2$ through $S_2$, which sets up a time varying magnetic flux $\phi_1$ through $S_1$.

$\therefore \quad \phi_1 = M_{12}\,(I_2)$

where $M_{12}$ is coefficient of mutual inductance of solenoid $S_1$ w.r.t. solenoid $S_2$. The magnetic field due to current $I_2$ in $S_2$ is

$$B_2 = \mu_0 n_2 I_2$$

$\therefore$ The magnetic flux through $S_1$ is

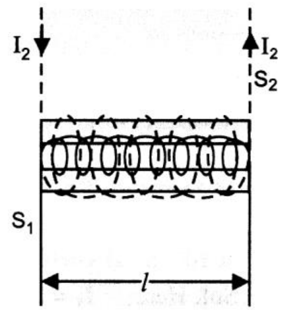
$$\phi_1 = B_2 A_1 N_1$$

where $N_1 = n_1 \ell$ = total no. of turns of $S_1$

$$\phi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 \ell)$$

$$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 \ell I_2$$

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 \ell$$




If a material of relative magnetic permeability $\mu_r$ fills the space inside $S_2$, we get

$$M_{12} = \mu r\, \mu_0 n_1 n_2 \pi r_1^2 \ell$$

Note that we have neglected the **edge effects;** and considered the magnetic field to be uniform throughout the length and width of the solenoid $S_2$. This approximation is valid for $\ell >> r_2$.

Let us consider the reverse case.

A time varying current $I_1$ passed through solenoid $S_1$ results in the development of magnetic flux $\phi_2$ through solenoid $S_2$, where

$$\phi_2 = M_{21} I_1$$

where $M_{21}$ is coefficient of mutual inductance of solenoid $S_2$ w.r.t. solenoid $S_1$.

The magnetic flux due to $S_1$ is confined solely inside $S_1$, as the solenoids are assumed to be very long. There is no magnetic field outside $S_1$ due to current in $S_1$. Therefore, magnetic flux linked with $S_2$ is

$$\phi_2 = B_1 A_1 N_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 \ell)$$

$$\phi_2 = \mu_0 n_1 n_2 \pi r_1^2 \ell I_1$$

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 \ell$$

Hence $$M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 \ell$$

Thus coefficient of mutual inductance of two coils depends on their geometry, their separation and relative orientation.

$$\therefore \quad M = \mu_0 \left(\frac{N_1}{\ell}\right)\left(\frac{N_2}{\ell}\right)\pi r_1^2 \times \ell \qquad M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

**NOTE:**

Remember that in this equation.

$N_1$ = total number of turns in solenoid $S_1$ : $N_2$ = total number of turns in solenoid $S_2$

l = length of longer solenoid (when two solenoids happen to be of different lengths)

$A = \pi r_1^2$ = area of cross section of inner solenoid.

## Factors on which mutual inductance depends

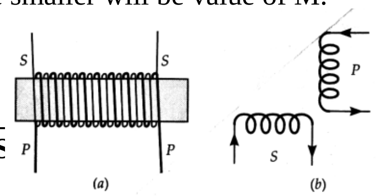
**1.** ***Number of turns:*** Larger the number of turns in the two solenoids, larger will be their mutual inductance.

$$M \propto N_1 N_2$$

**2.** ***Common cross–sectional area:*** Larger the common cross–sectional area of two solenoids, larger will be their mutual inductance.

**3.** ***Relative separation:*** Larger the distance between two solenoids, smaller will be the magnetic flux linked with the secondary coil due to current in the primary coil. Hence smaller will be value of M.

**4.** ***Relative orientation of the two coils:*** M is maximum when the entire flux of the primary is linked with the secondary, i.e., when the primary



coil completely envelopes the secondary coil. M is minimum when the two coils are perpendicular to each other, as shown in figure.

**5.** ***Permeability of the core material:*** If the two coils are wound over an iron core of relative permeability $\mu_r$, their mutual inductance increases $\mu_r$, times.

**Coefficient of coupling**

*The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together.* If $L_1$ and $L_2$ are the self–inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K=\frac{M}{\sqrt{L_1 L_2}}$$ The value of K lies between 0 and 1.

When the coupling is perfect i.e., the entire flux of primary is linked with the secondary, M is maximum and K = 1. When there is no coupling, M = 0 and K = 0.

Thus K is maximum for the coupling shown in figure(a) and minimum for the coupling shown in figure(b) above.

**NOTE:**

- ♦ When two coils are inductively coupled, in addition to the emf produced due to mutual induction, induced emf is set up in the two coils due to self–induction also.
- ♦ The mutual inductance of two coils is a property of their combination. The value of M remains unchanged irrespective of the fact that current is passed through one coil or the other.
- ♦ While calculating the mutual inductance of two long c o–axial solenoids, the cross–sectional area of the inner solenoid is to be considered.
- ♦ While calculating the mutual inductance of two co–axial solenoids of different lengths, the length of the larger solenoid is to be considered.

## Grouping of Inductances

***Inductances in series***

(i) Let the series connection be such that the current flows in the same sense in the two coils as shown in figure.

Let $L_{eq}$ be the equivalent inductance of the two self–inductances $L_1$ and $L_2$ connected in series. For the series combination, the emfs induced in the two coils get added up. Thus

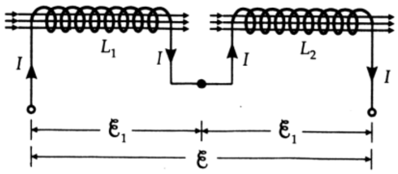
$$e_{eq} = e_1 + e_2$$

If the rate of change of current in the series circuit is $\frac{dI}{dt}$, then

$$e_1=-L_1\frac{dI}{dt}-M\frac{dI}{dt},$$

$$e_2=-L_2\frac{dI}{dt}-M\frac{dI}{dt}$$

and $$e_{eq}=-L_{eq}\frac{dI}{dt}$$




The negative sign throughout indicates that both self and mutual induced emfs are opposing the applied emf. Using the above equations, we have

$$e_{eq} = e_1 + e_2$$

or $$-L_{eq}\frac{dI}{dt}=-(L_1+M+L_2+M)\frac{dI}{dt}$$

or $$L_{eq} = L_1 + L_2 + 2M$$

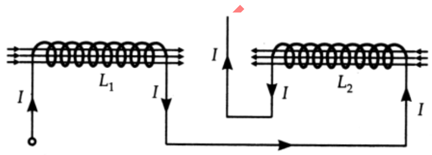
(ii) Let the series combination be such that the current flows in opposite senses in the two coils, as shown in figure.

The emf induced in the two coils will be

$$e_1 = -L_1\frac{dI}{dt} + M\frac{dI}{dt}, \quad e_2 = -L_2\frac{dI}{dt} + M\frac{dI}{dt}$$

Here the mutual emfs act in the direction of applied emf and hence positive. For this series combination also, the emfs induced in the two coils get added up.

Hence $$e_{eq} = e_1 + e_2 = -[L_1 - M + L_2 - M]\frac{dI}{dt}$$




But $$e_{eq} = -L_{eq}\frac{dI}{dt}$$

$\therefore$ $$-L_{eq}\frac{dI}{dt} = -[L_1 + L_2 - 2M]\frac{dI}{dt}$$

or $L_{eq} = L_1 + L_2 - 2M$

**Inductances in Parallel**

For the parallel combination, the total current I divides up through the two coils as

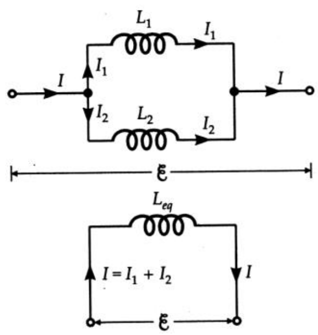
$$I = I_1 + I_2$$

$\therefore$ $$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

For parallel combination, induced emf across the combination is equal to the induced emf across each inductance. Thus

$$e = -L_1\frac{dI_1}{dt} \quad \text{or} \quad \frac{e}{L_1} = -\frac{dI_1}{dt}$$

$$e = -L_2\frac{dI_2}{dt} \quad \text{or} \quad \frac{e}{L_2} = -\frac{dI_2}{dt}$$




This is because the mutual inductance M is negligible. If $L_{eq}$ is the equivalent inductance of the parallel combination, then

$$e = -L_{eq}\cdot\frac{dI}{dt} = -L_{eq}\left[\frac{dI_1}{dt} + \frac{dI_2}{dt}\right]$$

$$= L_{eq}\left[-\frac{dI_1}{dt} - \frac{dI_2}{dt}\right]$$

or $$\frac{e}{L_{eq}} = \left[\frac{e}{L_1} + \frac{e}{L_2}\right] \quad \text{or} \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

If there is any mutual inductance M between the coils, then $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm M}$$

**Subjective Assignment – IV**

Q.1 What is the self–inductance of a coil, in which magnetic flux of 40 milliweber is produced when 2A current flows through it?

Q.2 A 200 turn coil of self–inductance 20 mH carries a current of 4 mA. Find the magnetic flux linked with each turn of the coil.

Q.3 If a rate of change of current of 4 $As^{-1}$ induces an emf of 10 mV in a solenoid, what is the self–inductance of the solenoid?

Q.4 A 12 V battery connected to a 6Ω, 10 H coil through a switch drives a constant current through the circuit. The switch is suddenly opened. If it takes 1ms to open the switch, find the average emf induced across the coil.

Q.5 An inductor of 5 H inductance carries a steady current of 2A. How can a 50 V self–induced emf be made to appear in the inductor?

Q.6 What is the self–inductance of an air core solenoid 50 cm long and 2 cm radius of it has 500 turns?

Q.7 An air–cored solenoid with length 30 cm, area of cross–section 25 $cm^2$ and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of $10^{-3}$ s. How much is the average back emf induced across the ends of the open switch in the circuit?

Q.8 A large circular coil, of radius R, and a small circular coil, of radius r, are put in vicinity of each other. If the coefficient of mutual induction, for this pair, equals 1 mH, what would be the flux linked with the larger coil when a current of 0.5 A flows through the smaller coil?

Q.9 What is the mutual inductance of a pair of coils if a current change of six ampere in one coil causes the flux in the second coil of 2000 turns to change by $12 \times 10^{-4}$ Wb per turn?

Q.10 An emf of 0.5 V is developed in the secondary coil, when current in primary coil changes from 5.0 A to 2.0 A in 300 millisecond. Calculate the mutual inductance of the two coils.

Q.11 If the current in the primary circuit of a pair of coils changes from 5 A top 1 A in 0.02 s, calculate (i) induced emf in the secondary coil if the mutual inductance between the two coils is 0.5 H and (ii) the change of flux per turns in the secondary, if it has 200 turns.

Q.12 Over a solenoid of 50 cm length and 2 cm radius and having 500 turns, is wound another wire of 50 turns near the center. Calculate the (i) mutual inductance of the two coils (ii) induced emf in the second coil when the current in the primary changes from 0 to 5 A in 0.02 s.

Q.13 A solenoid coil has 50 turns per centimeter along its length & a cross–sectional area of $4 \times 10^{-4}$ $m^2$. 200 turns of another wire are wound round the first solenoid coaxially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils.

Q.14 A solenoid of length 50 cm with 20 turns per cm and area of cross–section 40 $cm^2$ completely surrounds another co–axial solenoid of the same length, area of cross–section 25 $cm^2$ with 25 turns per cm. Calculate the mutual–inductance of the system.

## Answers

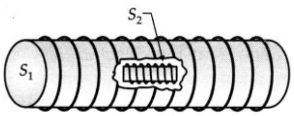
| | | | | | |
|---|---|---|---|---|---|
| 1. | $2 \times 10^{-2}$ Wb | 2. | $4 \times 10^{-7}$ Wb | 3. | 5 mH |
| 4. | 20,000V | 5. | by reducing the current to zero in 0.2s | | |
| 6. | $7.89 \times 10^{-4}$ H | 7. | 6.542 V | 8. | $5 \times 10^{-4}$ Wb |
| 9. | 0.4 H | 10. | 0.05 H | 11. | (i) 100 V (ii) 0.01 Wb |
| 12. | (i) 78.96 μH, (ii) – 19.74 mV | 13. | $5.027 \times 10^{-4}$ H | 14. | 7.85 mH |

## Subjective Assignment – V

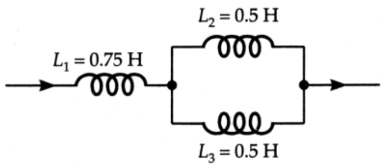
Q.1 (a) A toroidal solenoid with an air–core has an average radius of 15 cm, area of cross–section 12$cm^2$ and 1200 turns. Obtain the self–inductance of the toroid. Ignore field variation across the cross–section of the toroid.

(b) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced emf in the second coil.

Q.2 Figure shows a short solenoid of length 4 cm, radius 2.0 cm and number of turns 100 lying inside on the axis of a long solenoid, 80 cm length and number of turns 1500. What is the flux through the long solenoid if a current of 3.0 A flows through the short solenoid? Also obtain the mutual inductance of the two solenoids.




Q.3 Three inductances are connected as shown in figure. Find the equivalent inductance.




Q.4 Magnetic flux of 5 microweber is linked with a coil, when a current of 1 mA flows through it. What is the self–inductance of the coil?

Q.5 Calculate induced emf in a coil of 10 H inductance in which current changes from 8A to 3A in 0.2 s.

Q.6 A magnetic flux of $8 \times 10^{-4}$ Wb is linked with each turns of a 200–turn coil when there is an electric current of 4 A in it. Calculate the self–inductance of the coil.

Q.7 The self inductance of an inductor coil having 100 turns is 20 mH. Calculate the magnetic flux through the cross–section of the coil corresponding to a current of 4 mA. Also, find the total flux.

Q.8 A coil of inductance 0.5 H is connected to a 18 V battery. Calculate the rate of growth of current.

Q.9 An average emf of 25 V is induced in an inductor when the current in it is changed from 2.5 A in one direction to the same value in the opposite direction in 0.1 s. Find the self–inductance of the inductor.

Q.10 A coil has a self–inductance of 10 mH. What is the maximum magnitude of the induced emf in the inductor, when a current I = 0.1 sin 200 t ampere is sent through it.

Q.11 What is the self–inductance of a solenoid of length 40 cm, area of cross–section 20 $cm^2$ and total number of turns 800?

Q.12 The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 $As^{-1}$. Find the emf induced in it.

Q.13 Calculate the mutual inductance between two coils when a current of 2 A changes to 6 A in 2 s and induces an emf of 20 mV in the secondary coil.

Q.14 The mutual inductance between two coils is 2.5 H. If the current in one coil is changed at the rate 2.0 $As^{-1}$, what will be the emf induced in the other coil?

Q.15 In a carspark coil, an emf of 40,000 V is induced in the secondary when the primary current changes from 4A to zero in 10 μs. Find the mutual inductance between the primary and secondary windings of this spark coil.

Q.16 If the current in the primary circuit of a pair of coils changes from 10A to 0 in 0.1 s, calculate
(i) the induced emf in the secondary if the mutual inductance between the two coils is 2H, and
(ii) the change of flux per turn in the secondary if it has 500 turns.

Q.17 A conducting wire of 100 turns is wound over 1 cm near the centre of a solenoid of 100 cm length and 2 cm radius having 1000 turns. Calculate the mutual inductance of the two coils.

Q.18 A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross–section is $1.2 \times 10^{-3}$ $m^2$. Around its central section a coil of 300 turns is closely wound. If an initial current of 2 A is reversed in 0.25 s, find the emf induced in the coil.

Q.19 Calculate the mutual inductance between two coils if a current 10 A in the primary coil changes the flux by 500 Wb per turn in the secondary coil of 200 turns. Also determine the induced emf across the ends of the secondary coil in 0.5 s.

## Answers

| | | | |
|---|---|---|---|
| 1. | (a) 2.3 mH, (b) 0.023 V | 2. | $2.96 \times 10^{-4}$ H, $8.9 \times 10^{-4}$ Wb |
| 3. | 1 H | 4. | 5 mH |
| 5. | 250 V | 6. | $4 \times 10^{-2}$ H |
| 7. | $8 \times 10^{-7}$ Wb, $8 \times 10^{-5}$ Wb | 8. | 36 $As^{-1}$ |
| 9. | 0.5 H | 10. | 0.2 V |
| 11. | 4.02 mH | 12. | $6 \times 10^{-4}$ V |
| 13. | 10 mH | 14. | 5.0 V |

15. 0.1 H  16. (i) 200 V, – 0.04 Wb
17. $1.58 \times 10^{-4}$ H  18. 48 mV  19. $10^4$ H, $2 \times 10^5$ V

## Growth and Decay of Current in an Inductor

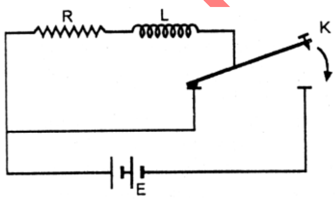
**(a) Growth of Current**

Consider an ohmic resistance R and a coil of inductance connected to a battery E through a more key K as shown in figure. On pressing K, the battery is connected. Current grows in the R – L circuit. Due to self induction, an induced e.m.f. is set up across L. By Lenz's law, this induced e.m.f. opposes the growth of current.

If I is strength of current at any instant t and dI/dt is rate of growth of current at this instant, then

Potential difference across R = IR

Potential difference across L = $L\dfrac{dI}{dt}$




As E is the e.m.f. of the battery,

$\therefore \quad E = IR + L\dfrac{dI}{dt}$ ... (1)

When current reaches its maximum value

i.e., $I = I_0;\ \dfrac{dI}{dt} = 0$

$\therefore \quad E = I_0 R$, using this value in (1), we get

$$I_0R = IR + L\frac{dI}{dt}$$

$$R(I_0 - I) = L\frac{dI}{dt} \quad \text{or} \quad \int_0^I \frac{dI}{I_0 - I} = \int_0^t \frac{R}{L}dt$$

$$-\left[\log_e(I_0 - I)\right]_0^I = \frac{R}{L}t$$

$$-\log_e(I_0 - I) + \log_e I_0 = -\frac{R}{L}t \quad \text{or} \quad \log_e(I_0 - I) - \log_e I_0 = \frac{R}{L}t$$

or $\log_e \dfrac{I_0 - I}{I_0} = e^{-\frac{R}{L}t}$ or $1 - \dfrac{I}{I_0} = e^{-\frac{R}{L}t}$

or $\dfrac{I}{I_0} = 1 - e^{-\frac{R}{L}t}$ or $I = I_0(1 - e^{-\frac{R}{L}t})$

$$I = I_0(1 - e^{-\frac{t}{\tau}}) \quad \text{....(2)} \quad \text{where} \quad \tau = L/R$$

This is **Helmholtz equation** governing growth of current in LR circuit. It shows that growth of current in an inductor is exponential.

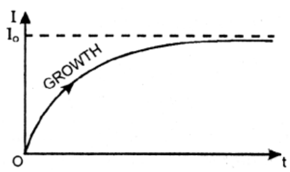
**Time Constant:**

The quantity $\tau = L/R$ is called time constant or inductive time constant of LR circuit. This is because dimensions of $\tau = L/R$ are those of time and for a given LR circuit, its value is constant.

If $\tau = L/R$, then from (2),

$$I = I_0(1 - e^{-1}) = I_0\left(1 - \frac{1}{e}\right) = I_0\left(1 - \frac{1}{2.718}\right)$$

$$I = I_0(1 - 0.368) = 0.632\, I_0 = 63.2\%\ I_0$$



*We may define time constant of LR circuit as the time in which current in the circuit grows to 63.2% of the maximum value of current.*

Again, from equation (2), we find that for

$I = I_0 \,.\, e^{-t/\tau} = 0 \qquad \text{or} \qquad t = \infty$

i.e. current in LR circuit would attain maximum value only after infinite time. However, practically, current reaches its maximum value after a time which is roughly five times the time constant.

The growth of current with time in LR circuit is shown in figure.

**(b) Decay of Current**

In figure, when morse key K is released, the battery is cut off. The current in the LR circuit decays. If I is the current at any time t during break, then as battery is disconnected, putting E = 0 in equation (1), we get

$$IR + L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = -IR \qquad \text{or} \qquad \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating both sides, we get

$$\int_{I_0}^{I}\frac{dI}{I} = -\int_0^t \frac{R}{L}dt \qquad \text{or} \qquad [\log_e I]_{I_0}^{I} = -\frac{R}{L}t$$

or $$\log_e I - \log_e I_0 = -\frac{R}{L}t \qquad \text{or} \qquad \log_e \frac{I}{I_0} = -\frac{R}{L}t$$

or $$\frac{I}{I_0} = e^{-\frac{R}{L}t} \qquad \text{or} \qquad I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau} \qquad ....(3)$$

where $\tau = L / R$

This is the **Helmholtz equation** for decay of current in LR circuit.

**Time constant:** The quantity $\tau = L/R$ is the time constant of LR circuit, as said already.

If $t = L/R$, then from (3),

$$I = I_0 e^{-1} = \frac{I_0}{e} = \frac{I_0}{2.718} = 0.368\ I_0 = 36.8\%\ I_0$$

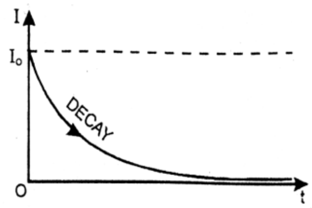
Hence we may define *time constant of LR circuit as the time in which current decays to 36.8% of the maximum value.*

Again, from (3), for I = 0

$e^{-t/\tau} = 0 \qquad \text{or} \qquad t = \infty$

i.e. current reduces to zero only after infinite time.

**Physical Significance of Time Constant**




If $\tau = L/R$ is small, then from (2), I attains its final value $I_0$ more rapidly; and from (3), current decays to zero also more rapidly. If L/R is large, both the growth and decay of current in LR circuit are slow. This time constant of LR circuit determines the rate at which current grows or decay in the LR circuit. Smaller the values of time constant faster are the growth as well as decay of current in the circuit. The reverse is also true.

**Subjective Assignment – VI**

Q.1 What resistance must be connected in series with an inductor of 5 millihenry so that the circuit has a time constant of $2 \times 10^{-3}$ sec?

Q.2 When a current of 10 ampere is flowing through a resistance of 20 ohm and inductance of 10 henry, the battery is switched off. Find (i) current after 0.4 sec. (ii) the time the current takes to fall to 60% of its initial value.

Q.3 A circuit containing a 30 mH inductor in series with a 60 ohm resistance is connected to a d.c. supply. Determine the time constant of the circuit.

Q.4 A coil of resistance 20 ohm and inductance 0.5 H is connected to direct current supply of 200 V. Calculate the rate of increase of current at

(i) the instant of closing the switch, (ii) at t = L/R seconds after the switch is closed

Also, calculate steady value of current in the circuit.

Q.5 A potential difference of 1 volt is applied to a coil of resistance 1 Ω and inductance 1 H. What is the current after 1 second? How long does it take the current to reach half its final value?

Q.6 A solenoid having a resistance of 10Ω and inductance 10 H is connected to a battery of 20 V and negligible internal resistance. After how long will the current in it rise to 1 ampere?

Q.7 An inductor (I = 20 mH), a resistor (R = 100Ω) and a battery (E = 10 V) are connected in series. Find (a) the time constant, (b) the maximum current and (c) the time elapsed before the current reaches 99% of the maximum value.

Q.8 A coil of inductance 8.4 mH and resistance 6 ohm is connected to a 12 V battery. Find the time in which current in the coil will grow to 1 A.

## Answers

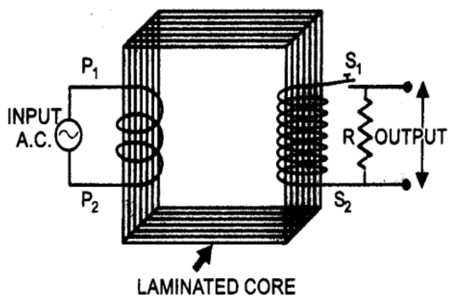
| | | | |
|---|---|---|---|
| 1. | 2.5 Ω | 2. | (i) 4.49 A, (ii) 0.255 s |
| 3. | $5 \times 10^{-4}$ s | 4. | 400 A $s^{-1}$; 147.17 A $s^{-1}$; 10 A |
| 5. | 0.632A; 0.693 s | 6. | 0.693 s |
| 7. | $2 \times 10^{-4}$ s; $10^{-1}$ A; $0.92 \times 10^{-3}$ s | 8. | $9.7 \times 10^{-4}$ s |

## Transformer

A transformer is an electrical device which is used for changing the a.c. voltages. A transformer which increases the a.c. voltages is called a step up transformer. A transformer which decreases the a.c. voltages is called a step down transformer.

**Principle :** A transformer is based on the principle of mutual induction, i.e., whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

**Construction :** A transformer consists of a rectangular soft iron core made of laminated sheets, well insulated from one another, fig. Two coils $P_1$ and $P_2$ (the primary coil) and $S_1$ and $S_2$ (the secondary coil) are wound on the same core, but are well insulated from each other. Note that both the coils are also insulated from the core. The source of alternating e.m.f. (to be transformed) is connected to the primary coil $P_1$ $P_2$ and a load resistance R is connected to secondary coil $S_1$ $S_2$ through an open switch S. Thus, there can be no current through secondary coil so long as the switch is open.




**Theory and working :** Let the alternating e.m.f. supplied by the a.c. source connected to primary be

$$E_p = E_0 \sin \omega t$$

As we have assumed the primary to be a pure inductance with zero resistance, the sinusoidal primary current $I_p$ lags the primary voltage $E_p$ by 90°. The primary's power factor, $\cos\phi = \cos 90° = 0$. Therefore, no power is dissipated in primary. The alternating primary current induces an alternating magnetic flux $\phi_B$ in the iron core. Because the core extends through the secondary winding, the induced flux also extends through the turns of secondary. According to Faraday's law of electromagnetic induction, the induced e.m.f. per turn ($E_{trun}$) is same for both, the primary and secondary. Also, the voltage $E_p$ across the primary is equal t the e.m.f. induced in the primary, and the voltage $E_s$ across the secondary is equal to the e.m.f. induced in the secondary. Thus,

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_p}{n_p} = \frac{E_s}{n_s}$$

Here, $n_p$ ; $n_s$ represent total number of turns in primary and secondary coils respectively.

$$\therefore \qquad \boxed{E_s = E_p \frac{n_s}{n_p}} \qquad \text{... (1)}$$

If $n_s > n_p$ ; $E_s > E_p$, the transformer is a step up transformer. Similarly, when $n_s < n_p$ ; $E_s < E_p$. The device is called a step down transformer. $\frac{n_s}{n_p} = K$ represents transformation ratio.

Note that this relation (1) is based on three assumptions.

**(i)** The primary resistance and current are small.

**(ii)** There is no leakage of magnetic flux. The same magnetic flux links both, the primary and secondary coils.

**(iii)** The secondary current is small.

Now, the rate at which the generator/source transfers energy to the primary = $I_pE_p$. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is $I_s E_s$. As we assume that no energy is lost along the way, conservation of energy requires that

$$\boxed{I_p E_p = I_s E_s} \quad \therefore \; I_s = I_p \frac{E_p}{E_s} \quad \text{from (1), } \frac{E_p}{E_s} = \frac{n_p}{n_s}$$

$$\therefore \qquad \boxed{I_s = I_p \cdot \frac{n_p}{n_s} = \frac{I_p}{K}} \qquad \text{....(2)}$$

For a step up transformer, $E_s > E_p$ ; $K > 1$ so, $I_s < I_p$

i.e., secondary current is weaker when secondary voltage is higher, i.e., whatever we gain in voltage, we lose in current in the same ratio.

The reverse is true for a step down transformer.

From equation (2), $\qquad I_p = I_s\left(\frac{n_s}{n_p}\right) = \frac{E_s}{R}\left(\frac{n_s}{n_p}\right)$

Using equation (2), we get $\qquad I_p = \frac{1}{R} \cdot E_p \left(\frac{n_s}{n_p}\right)\left(\frac{n_s}{n_p}\right)$

$$I_p = \frac{1}{R}\left(\frac{n_s}{n_p}\right)^2 E_p \qquad \text{....(3)}$$

This equation has the form $I_p = \frac{E_p}{R_{eq}}$, where the equivalent resistance $R_{eq}$ is $R_{eq} = \left(\frac{n_p}{n_s}\right)^2 R \qquad$ ....(4)

Thus $R_{eq}$ is the value of load resistance as seen by the source/generator, i.e., the source/generator produces current $I_p$ and voltage $E_p$ as if it were connected to a resistance $R_{eq}$.

**Efficiency** of a transformer is defined as the ratio of output power to the input power.

i.e., $$\boxed{\eta = \frac{\text{output power}}{\text{Input power}} = \frac{E_s\, I_s}{E_p\, I_p}}$$

**Note**

A transformer is essentially an a.c. device. It cannot work on d.c. A transformer changes a.c. voltages/currents. It does not affect the frequency of a.c.

**Energy losses in a transformer :** Following are the major sources of energy loss in a transformer.

**1.** Copper loss is the energy loss in the form of heat in the copper coils of a transformer. This is due to Joule heating of conducting wires. These are mainmised using thick wires.

**2.** Iron loss is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimized by taking laminated cores.

**3.** Leakage of magnetic flux occurs inspite of best insulations. Therefore, rate of change of magnetic flux linked with each turn of $S_1S_2$ is less than the rate of change of magnetic flux linked with each turn of $P_1$ $P_2$. It can be reduced by winding the primary and secondary coils one over the other.

**4.** Hysteresis loss. This is the loss of energy due to repeated magnetization and demagnetization of the iron core when a.c. is fed to it. The loss is kept to a minimum by using a magnetic material which has a low hysteresis loss.

**5.** Magnetostriction, i.e., humming noise of a transformer. Therefore, output power in the best transformer may be roughly 90% of the input power.

**Uses of transformer :** A transformer is used in almost all a.c. operations e.g.

**(i)** In voltage regulators for T.V., refrigerator, computer, air conditioner etc.

**(ii)** In the induction furnaces.

**(iii)** A step down transformer is used for welding purposes.

(iv) In the transmission of a.c. over long distances. The loss of power in the transmission lines of I2R, where I is strength of current and R is the resistance of the wires. To reduce the power loss, a.c. is transmitted over long distances at extremely high voltages. This reduces I in the same ratio. Therefore, $I^2$ R becomes negligibly low. As I has been reduced sufficiently, $I^2R$ remains negligible even when R is not very small. This means we can use even thin line wires of large resistance R instead of thick ones. This saves us a lot of material (copper). Therefore, cost of transmission is reduced considerably.

## Subjective Assignment – VII

1 The primary coil of an ideal step–up–transformer has 100 turns and the transformation ratio is also 100. The input voltage and the power are 220 V and 1100 W respectively. Calculate

(i) number of turns in the secondary (ii) the current in the primary

(iii) voltage across the secondary (iv) the current in the secondary

(v) power in the secondary

2 How much current is drawn by the primary of a transformer which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω?

3 A transformer has 500 turns in the primary and 1000 turns in its secondary winding. The primary voltage is 200 V and the load in the secondary is 100 Ω. Calculate the current in the primary, assuming it to be an ideal transformer.

4 In an ideal transformer, number of turns in the primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V, calculate (i) output voltage and (ii) current in primary.

5 The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24 V, 24 W, calculate the current in the primary coil of the circuit.

6 A transformer of 100% efficiency has 200 turns in the primary and 40, 000 turns in the secondary. It is connected to a 220 V a.c. mains and the secondary feeds to a 100 kΩ resistance. Calculate the output potential difference per turn and the power delivered to the load.

7 A step down transformer is used to reduce the main supply of 220 V to 11 V. If the primary draws a current of 5A and the secondary 90 A, what is the efficiency of the transformer?

8 Calculate the current drawn by the primary of a transformer, which steps down 200 V to 20V to operate a device of resistance 20 Ω. Assume the efficiency of the transformer to be 80%.

9 A 10 kW transformer has 20 turns in the primary and 100 turns in the secondary circuit. An a.c. voltage $e_1$ = 600 sin 314 t is applied to the primary. Find (i) the maximum value of flux and (ii) the maximum value of the secondary voltage.

10 (i) The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 kW, calculate the primary voltage. (ii) If the resistance of the primary is 0.2Ω and that of the secondary is 2.0Ω and the efficiency of the transformer is 90%, calculate the heat losses in the primary and the secondary coils.

11 A transformer has 300 primary turns and 2400 secondary turns. If the primary supply voltage is 230 V, what is the secondary voltage?

12 A transformer has 200 primary turns and 150 secondary turns. If the operating voltage for the load connected to the secondary is measured to be 300 V, what is the voltage supplied to the primary?

13 The ratio of the number of turns in the primary and the secondary coils of a step up transformer is 1 : 200. It is connected to a..c. mains of 200 V. Calculate the voltage developed in the secondary. Determine value of maximum current in secondary, when a current of 2.0 A flows through the primary.

14 A transformer of 100% efficiency has 500 turns in the primary and 10, 000 turns in the secondary coil. If the primary is connected to 220 V supply, what is the voltage across the secondary coil?

15 When a voltage of 120 V is impressed across the primary of a transformer, the current in the primary is 1.85 A. Find the voltage across the secondary, when it delivers 150 mA. The transformer has an efficiency of 95%.

16 The primary of a transformer has 200 turns and the secondary has 1000 turns. If the power output from the secondary at 1000 V is 9 kW, calculate (i) the primary voltage and (ii) the heat loss in the primary coil if the resistance of primary is 0.2 Ω and the efficiency of the transformer is 90%.

17 A town situated 20 km away from a power plant generating power at 440 V, requires 6000 kW of electric power at 200 V. The resistance of two wire line carrying power is 0.4 Ω per km. The town gets power from the line through a 3000 – 220 V step down transformer at a substation in town.

(i) Find the line power losses in the form of heat

(ii) How much power must the plant supply, assuming there is negligible power loss due to leakage?

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | (i) 10, 000, (ii) 5A, (iii) 22,000 V, (iv) 0.05 A, (v) 1100 W | | | 2. | 0.01 |
| 3. | 8 A | 4. | (i) 1000 V, (ii) 50 A | 5. | 0.1 A |
| 6. | 1.1 V, 19.36 kW | 7. | 90% | 8. | 0.125 A |
| 9. | (i) 0.0955 Wb, (ii) 3000 V | | | 10. | (i) 220 V, (ii) 746.6 W, 242 W |
| 11. | 1.84 kV | 12. | 400 V | 13. | 40, 000 V, 0.01 A |
| 14. | 4400 V | 15. | 1406 V | 16. | 200 V, 500 W |
| 17. | (i) 640 kW, 1240 kW | | | | |

## Conceptual Problems

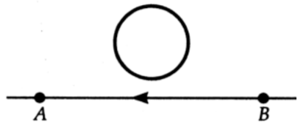
Q. 1 A train is moving with uniform speed from north to south. (i) Will any induced emf appear across the ends of its axle? (ii) Will the answer be affected if the train moves from east to west?

Q.2 A wire kept along north–south direction is allowed to fall freely. Will an emf be induced in wire?

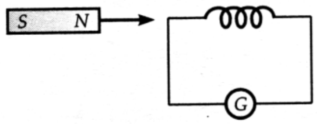
Q.3 A cylindrical bar magnet is kept along the axis of a circular coil. Will there be a current induced in the coil if the magnet is rotated about its axis? Give reason.

Q.4 A vertical metallic pole falls down through the plane of the magnetic meridian. Will any emf be produced between its ends? Give reason for your answer.

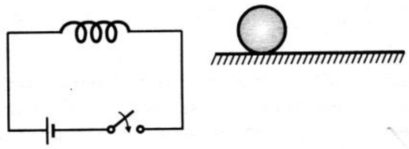
Q.5 The electric current flowing in a wire in the direction B to A is decreasing. What is the direction of induced current in the metallic loop kept above the wire as shown in figure.



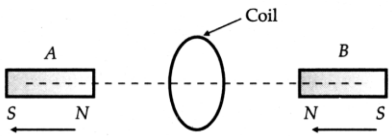

Q.6 Will there be any current induced in the coil shown in figure, if a bar magnet is swiftly moved towards or away from the coil. If yes, what will be the direction of current?



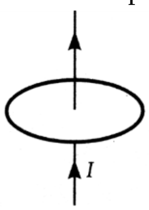

Q.7 Figure shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. What happens to the ring, as the switch is closed?



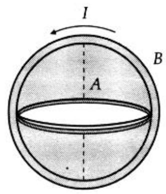
Q.8 In the given figure, A and B are identical magnets. Magnet A is moved away from the coil with a given speed. Magnet B is moved towards the coil with the same speed. What is the induced emf in the coil?




Q.9 The current I in a wire passing normally through the centre of a conducting loop is increasing at a constant rate. Will any current be induced in the loop?




Q.10 The planes of two circular conductors are perpendicular to each other, as shown in figure. If the current in conductor B is changed, will any current be induced in conductor A?

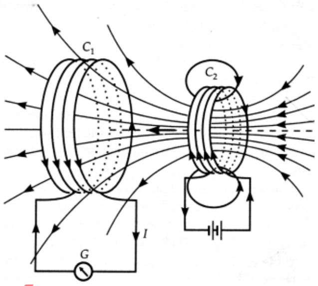



Q.11 Why do birds fly off a high tension wire when the current is switched on?

Q.12 An artificial satellite with a metal surface, is orbiting the earth around the equator. Will the earth's magnetism induce a current in it?

Q.13 A piece of metal and a piece of non–metallic stone are dropped from the same height near the surface of the earth. Which will reach the ground earlier?

Q.14 Two similar circular co–axial loops carry equal currents in the same direction. If the loops be brought nearer, what will happen to the currents in them?



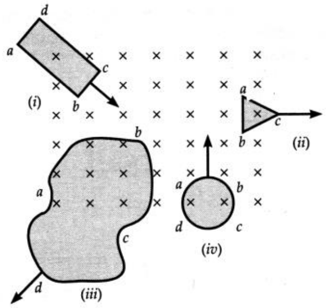

Q.15 Figure shows two coils $C_1$ and $C_2$ placed facing each other. The steady current in coil $C_2$ produces a steady magnetic field. Whenever the coil $C_2$ is moved towards $C_1$ or away from it, a current is induced in $C_1$ as indicted by the deflection in the galvanometer.

Answer the following questions:

(a) What would you do to obtain a large deflection of the galvanometer?

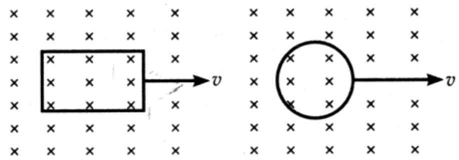
(b) How would you demonstrate the presence of an induced current in the absence of a galvanometer?

Q.16 Figure below shows planar loops of different shapes moving out of or into a region of a magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law.



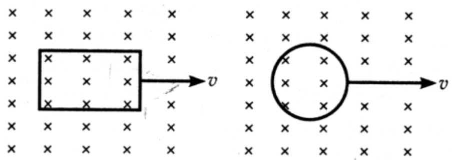

Q.17 Answer the following questions:

(a) A closed loop is held stationary in the magnetic field between the north and south poles of the permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnets?

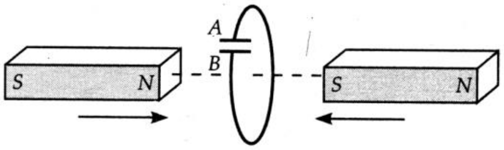



(b) A closed conducting loop moves normal to the electric field between the plates of a large capacitor. Is a current induced in the loop when it is (i) wholly inside the capacitor (ii) partially outside the plates of capacitor? The electric field is normal to the plane of the loop.

(c) A rectangular loop and a circular loop are moving out of a uniform magnetic field region (figure) to a field free region with a constant velocity. In which loop do you expect the induced emf to be constant during the passage out of the field region? The field is normal to the loops.

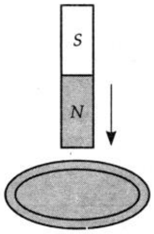



(d) Predict the polarity of the capacitor in the situation described by figure.

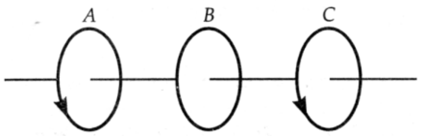



Q.18 (a) A bar magnet falls from height 'h' through a metal ring (figure). Will its acceleration be equal to 'g'? Give reason for your answer.

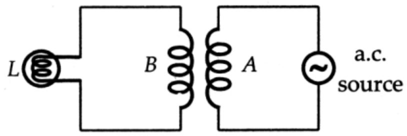
(b) What happens if the ring, in the above case is cut so as not to form a complete loop?




Q.19 Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry currents, as shown in figure. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any current induced in B? If yes, mark its direction.



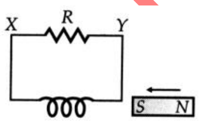

Q.20 In figure, a coil 'B' is connected to low voltage bulb L and placed parallel to another coil 'A' as shown. Explain the following observations:



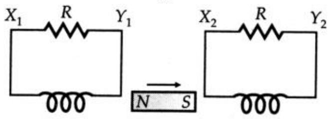

(i) Bulb lights, and

(ii) Bulb gets dimmer if the coil 'B' is moved upwards

Q.21 Predict the direction of induced current in resistance R in figures. Give reason for your answer.




(a)



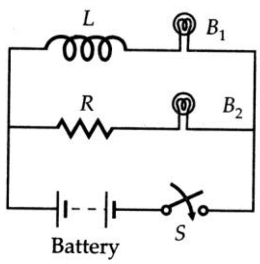

(b)

Q.22 Figure shows an inductor L and a resistor R connected in parallel to a battery through a switch. The resistance of R is the same as that of the coil that makes L. Two identical bulbs are put in each arm of the circuit.

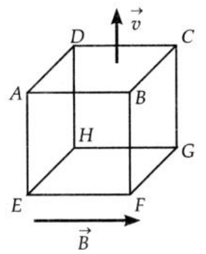
(i) Which of the bulbs lights up bright when S is closed?

(ii) Will the two bulbs be equally bright after some time?

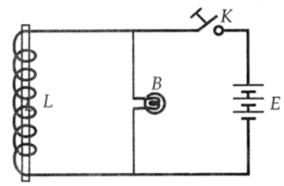
Give reason for your answer.



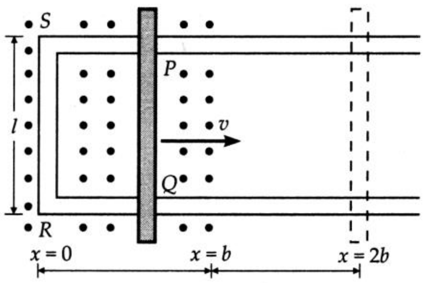

Q.23 Twelve wires of equal length are connected to form a skeleton cube which moves with a velocity 'v' perpendicular to the magnetic field $\vec{B}$. What will be the induced emf in each arm of the cube?




Q.24 How does the self inductance of an air core coil change, when (i) the number of turns in the coil is decreased, (ii) an iron rod is introduced in the coil? A copper coil L wound on a soft iron core and a lamp B are connected to a battery E through a tapping key K. When the key is closed, the lamp glows dimly. But when the key is suddenly opened, the lamp flashes for an instant to much greater brightness. Explain.

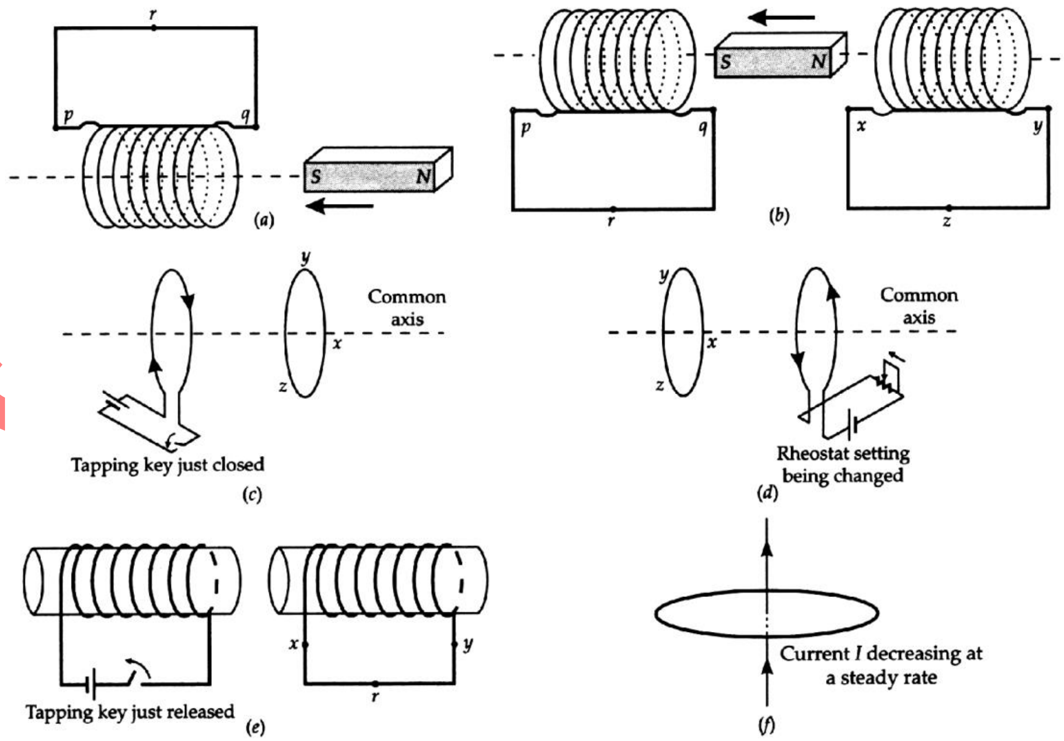



Q.25 Refer to figure. The arm PQ of the rectangular conductor is moved from x = 0 to the right side. The uniform magnetic field is perpendicular to the plane and extends from x = 0 to x = b and is zero for x > b. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from x = 0 to x = 2b and is then moved back to x = 0 with constant speed v. Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as joule heat. Sketch the variation of these quantities with time.




## NCERT Exercise

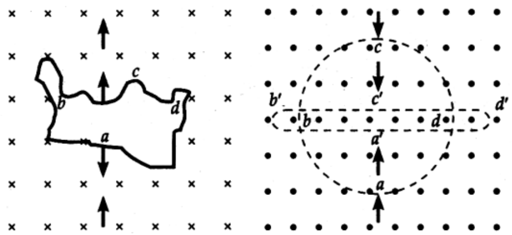
Q.1 Predict the direction of induced current in the situations described by the flowing figures a to f



Q.2 Use Lenz's law to determine the direction of induced current in the situations described by figure.

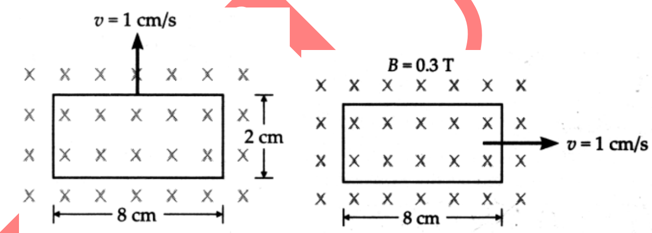
(a) A wire of irregular shape turning into a circular shape.

(b) A circular loop being deformed into a narrow straight wire.




Q.3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm$^2$ placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1s, what is the induced voltage in the loop while the current is changing?

Q.4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cms$^{-1}$ in a direction normal to the (i) longer side (ii) shorter side of the loop?




For how long does the induced voltage last in each case?

Q.5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s$^{-1}$ about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Q.6 A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 rad s$^{-1}$ in a uniform horizontal magnetic field of magnitude 3.0 × 10$^{-2}$ T. Obtain the maximum and the average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Q.7 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s$^{-1}$ at right angles to horizontal component of the earth's magnetic field, 0.30 × 10$^{-4}$ Wb m$^{-2}$.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Q.8 Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self–inductance of the circuit.

Q.9 A jet plane is travelling west at the speed of 1800 km h$^{-1}$. What is the voltage difference developed between the ends of the wing 25 m long, if the earth's magnetic field at the location has a magnitude of 5.0 × 10$^{-4}$ T and the dip angle is 30$^o$?

Q.10 Suppose the loop in question 4 is stationary but current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 Ts$^{-1}$. If the cut is joined and the loop has a resistance of 16Ω, how much power is dissipated by the loop as heat? What is the source of this power?

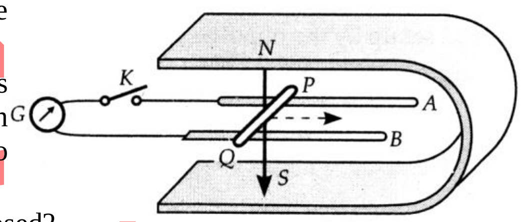
Q.11 A square loop of side 12 cm with its sides parallel to x–and y–axes moves with a velocity of 8 cms$^{-1}$ in the positive x–direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform is space nor constant in time. It has a gradient of 10$^{-3}$ T cm$^{-1}$

along the negative x–direction, and it is decreasing in time at the rate of $10^{-3}Ts^{-1}$. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 mΩ.

Q.12 It is desired to measure the magnitude of field between the poles of a powerful loudspeaker magnet. A small flat search coil of area 2.0 $cm^2$ with 25 closely wound turns is positioned normal to the field direction and then quickly snatched out of the field region. (Equivalently, one can give it a quick $90^o$ turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to the coil) is 7.5 mC. The resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of the magnet.

Q.13 Figure shows a metal rod PQ resting on the rails AB and positioned between the poles of a permanent magnet. The rails, the rod and the magnetic field are in three mutually perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Go through the following questions and answer them.

(a) Suppose K is open and the rod moves with a speed of 12 $cms^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf.




(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

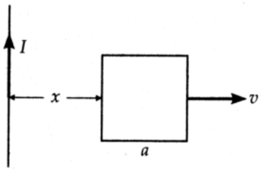
(e) How much power is required (by an external agent) to keep the rod moving at the same speed (= 12 $cms^{-1}$) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of the power?

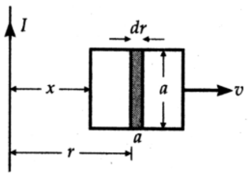
(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Q.14 An air–cored solenoid with length 30 cm, area of cross–section 25 $cm^2$ and number of turns 500 carries a current of 2.5 A. The current is suddenly switched off in a brief time of $10^{-3}$s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Q.15 (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in figure.




(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s. Calculate the induced emf in the loop at the instant when x = 0.2m. Take a = 0.1 m and assume that the loop has a large resistance.



Q.16 A line charge $\lambda$ per unit length is lodged uniformly onto the wheel of mass M and radius R. The wheel has light non–conducting spokes and is free to rotate without friction about its axis (figure). A uniform magnetic field extends over a circular region within the rim. It is given by

$$\vec{B} = -B_0\hat{k} \quad [r \le a,\ a < R]$$
$$= 0 \quad \text{(otherwise)}$$

What is the angular velocity of the wheel after the field is suddenly switched off?

## Answers

1. (a) qrpq, (b) prq in one coil and along yzx in other coil, (c) yzx, (d) zyx, (e) xry, (f) no current
2. (a) adcb, (b) a' d' c' b'
3. $7.5 \times 10^{-6}$ V
4. (i) $2.4 \times 10^{-4}$ V, 2 sec, (ii) $0.6 \times 10^{-4}$ V, 8 s
5. 100 V
6. $e_{max} = 0.603$ V, $e_{av} = 0$, P = 0.018 W; external agent
7. (a) $1.5 \times 10^{-3}$ V, (b) west to east, (c) western end
8. 4 H
9. 3.1 V
10. $6.4 \times 10^{-10}$ W
11. $2.9 \times 10^{-2}$ A
12. 0.75 Wb $m^{-2}$
13. (a) P = + ve and Q = – ve, (b) (i) excess charge build (ii) it is maintained by the flow of current, (c) rod is in equilibrium, (d) $75 \times 10^{-3}$ N, (e) $9 \times 10^{-3}$ w; zero power, (f) $9 \times 10^{-3}$ w; external agent, (g) zero
14. 6.54 V
15. (a) $\dfrac{\mu_0 Ia}{2\pi}\ln\left(1+\dfrac{a}{x}\right)$, (b) $1.7 \times 10^{-5}$ V
16. $\vec{\omega} = -\dfrac{B\pi a^2\lambda}{mR}\hat{k}$

## CBSE PMT Prelims Exam

Q.1 A metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is

(a) equal to g (b) less than g (c) more than g (d) either (a) or (c)

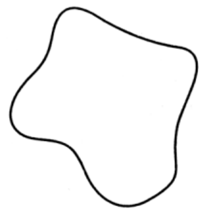
Q.2 A magnetic field of $2 \times 10^{-2}$ T acts at right angles to a coil of area 100 $cm^2$ with 50 turns. The average emf induced in the coil is 0.1 V, when it is removed from the field in the time t. The value of t is

(a) 0.1 s (b) 0.01 s (c) 1 s (d) 10 s

Q.3 A straight line conductor of length 0.4 m is moved with a speed of 7 $ms^{-1}$ perpendicular to magnetic field of intensity 0.9 $Wbm^{-2}$. The induced emf across the conductor is

(a) 1.26 V (b) 2.52 V (c) 5.24 V (d) 25.2 V

Q.4 As a result of change in the magnetic flux linked with the closed loop shown in the figure, an emf of V volt is induced in the loop. The work done (in joule) in taking a charge Q coulomb once along the loop is



(a) Q V (b) 2 QV (c) QV/2 (d) zero

Q.5 If N is the number of turns in a coil, the value of self–inductance varies as

(a) $N^0$ (b) N (c) $N^2$ (d) $N^{-2}$

Q.6 The current in a self inductance L = 40 mH is to be increased uniformly from 1 A to 11 A in 4 millisecond. The emf induced in the inductor during the process is

(a) 100 V (b) 0.4 V (c) 40 V (d) 440 V

Q.7 In an inductor of self inductance L = 2mH, current changes with time according to relation $I = t^2e^{-t}$. At what time, emf is zero?

(a) 4 s (b) 3 s (c) 2 s (d) 1 s

Q.8 Two coils have a mutual inductance 0.005 H. The current change in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0$ = 10A and $\omega = 100\pi$ rad $s^{-1}$. The maximum value of emf in the second coil is

(a) $2\pi$ (b) $5\pi$ (c) $6\pi$ (d) $12\pi$

Q.9 Two coils of self inductance 2 mH and 8mH are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is

(a) 16 mH (b) 10 mH (c) 6 mH (d) 4 mH

Q.10 In a region of uniform magnetic induction B = $10^{-2}$ T, a circular coil of radius 30 cm and resistance $\pi^2$ ohm is rotated about an axis, which is perpendicular to the direction of B and which forms a diameter of coil. If the coil rotates at 200 r.p.m., amplitude of alternating current induced in the coil is

(a) $4\pi^2$ mA (b) 30 mA (c) 6 mA (d) 200 mA

Q.11 A rectangular, a square, a circular and an elliptical loop, all in the (x–y) plane, are moving out of a uniform magnetic field with a constant velocity, $\vec{V} = v\hat{i}$. The magnetic field is directed along the negative z–axis. The induced emf, during the passage of these loops, out of the field region, will not remain constant for:

(a) the circular and the elliptical loops (b) only the elliptical loop
(c) any of the four loops (d) the rectangular, circular and elliptical loops

Q.12 A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s. The induced emf in the loop when the radius is 2 cm is

(a) $4.8\pi$ μV (b) 0.8 π μV (c) 1.6 π μV (d) $3.2\pi$ μV

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | b | 2. | a | 3. | b | 4. | a | 5. | c |
| 6. | a | 7. | c | 8. | b | 9. | d | 10. | c |
| 11. | a | 12. | d | | | | | | |

## Delhi PMT and VMMC Entrance Exam

Q.1 S.I. unit of magnetic flux is

(a) tesla (b) oersted (c) weber (d) gauss

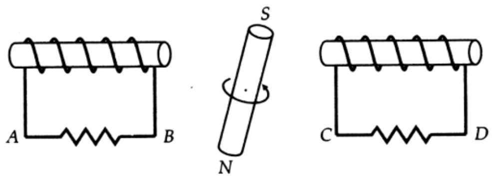
Q.2 A moving conductor coil produces an induced emf. This is in accordance with

(a) Lenz's law (b) Coulomb's law (c) Faraday's law (d) Ampere's law

Q.3 The dimensional formula for emf e in MKS system will be

(a) $[ML^2T^{-2}Q^{-1}]$ (b) $[ML^2T^{-1}]$ (c) $[ML^{-2}Q^{-1}]$ ( d) $[MLT^{-2}Q^{-2}]$

Q.4 The magnet in figure rotates as shown on a pivot through its centre. At the instant shown, what are the directions of the induced currents?




(a) A to B and C to D (b) B to A and C to D
(c) A to B and D to C (d) B to A and D to C

Q.5 Suppose the number of turns in a coil be tripled, the value of magnetic flux linked with it

(a) remains unchanged (b) becomes 1/3
(c) is tripled (d) none of these

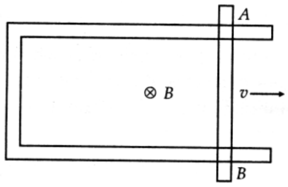
Q.6 A magnetic field $2 \times 10^{-2}$ T acts at right angles to a coil of area 100 $cm^2$ with 50 turns. The average emf induced in the coil is 0.1 V, when it is removed from the field in time t. The value of t is

(a) 0.01 sec (b) 0.5 sec (c) 0.1 sec (d) 1 sec

Q.7 If a 2m wire is moving with a velocity of 1 m/s perpendicular to a magnetic field of 0.5 $Wb/m^2$, then emf induced in it, will be

(a) 0.2 V (b) 1 V (c) 0.5 V (d) 2 V

Q.8 A 50 cm long bar AB is moved with a speed of 4 $ms^{-1}$ in a magnetic field B = 0.01 T as shown in figure. The emf generated is

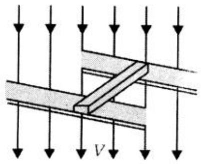



(a) 0.01 V (b) 0.02 V (c) 0.03 V (d) 0.04 V

Q.9 An aeroplane having a wing span of 35m flies due north with the speed of 90 m/s, given B = $4 \times 10^{-5}$ T. The potential difference between the tips of the wings will be

(a) 0.126 V (b) 1.26 V (c) 12.6 V (d) 0.013 V

Q.10 If the vertical component of earth's magnetic field be $6 \times 10^{-5}$ Wb/$m^2$, then what will be the induced potential difference produced between the rails of a meter–gauge when a train is running on them with a speed of 36 km/hr?




(a) $2 \times 10^4$ V (b) $6 \times 10^{-4}$ V (c) $3 \times 10^{-4}$ V (d) $9 \times 10^{-4}$ V

Q.11 In a coil of self inductance 5 henry, the rate of change of current is 2 amp per second. The emf induced in the coil is

(a) – 5 V (b) 5 V (c) – 10 V (d) 10 V

Q.12 If a current of 10 flows in one second through a coil and the induced emf is 10 V, then the self inductance of the coil will be

(a) 1 H (b) 2 H (c) 4 H ( d) 2/5 H

Q.13 The average emf induced in which a current changes from 0 to 2 A in 0.05 sec in 8V. The self–inductance of the coil is

(a) 0.1 H (b) 0.4 H (c) 0.2 H (d) 0.8 H

Q.14 What is the unit of self–inductance of a coil

(a) volt $s^{-1}A^{-1}$ (b) volt $s^{-1}$ A (c) volt $^{-1}sA^{-1}$ (d) volt s $A^{-1}$

Q.15 If number of turns per unit length of a coil of a solenoid is doubled, its self–inductance will

(a) remain constant (b) be doubled (c) be halved (d) be four times

Q.16 What is the coefficient of mutual inductance, when the magnetic flux changes by $2 \times 10^{-2}$ Wb and change in current is 0.01 A?

(a) 2 H (b) 4 H (c) 3 H (d) 8 H

Q.17 Two inductors each of inductance 'L' are joined in parallel. What is their equivalent inductance?

(a) 2 L (b) L/2 (c) L (d) zero

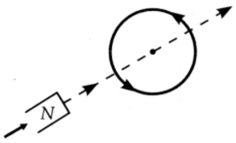
Q.18 If an inductor having inductance L is joined to another identical inductor with its one joined, the resultant inductance would become

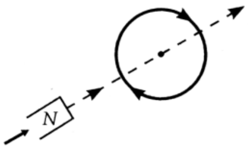
(a) 2 L (b) L/2 (c) zero (d) L/4

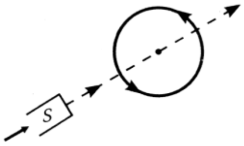
Q.19 Two inductors of inductance L, each are connected in series with opposite mutual inductance. What is the resultant inductance?

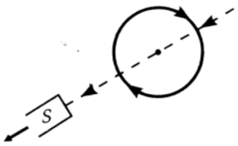
(a) zero (b) L (c) 2 L (d) 3 L

Q.20 Which of the following figurers correctly depicts the Lenz's law? The arrows show the movement of the labeled pole of a bar magnet into a closed circular loop and the arrows on the circle show the direction of the induced current.

(a) 


(b) 


(c) 


(d) 


## Answers

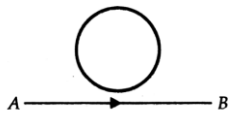
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | c | 3. | a | 4. | a | 5. | c |
| 6. | c | 7. | b | 8. | b | 9. | a | 10. | b |
| 11. | c | 12. | a | 13. | c | 14. | d | 15. | d |
| 16. | a | 17. | b | 18. | a | 19. | a | 20. | a |

## AIIMS Entrance Exam

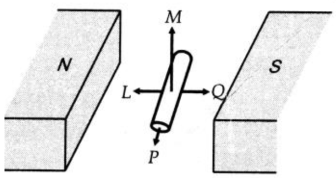
Q.1 The magnetic flux linked with a coil (in Wb) is given by the equation: $\phi = 5t^2 + 3t + 16$
The induced emf in the coil in the fourth second will be
(a) 10 V (b) 108 V (c) 145 V (d) 210 V

Q.2 In Lenz's law, there is conservation of
(a) charge (b) momentum (c) energy (d) current

Q.3 The current flows from A to B as shown in the figure. The direction of the induced current in the loop is
(a) clockwise (b) anticlockwise
(c) straight line (d) none of these

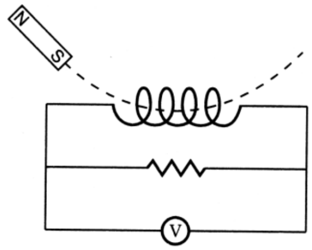



Q.4 A potential difference will be induced between the ends of the conductor shown in the figure, when the conductor moves along

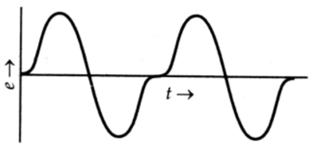


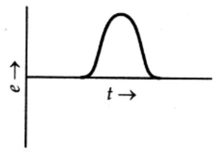

(a) P (b) Q (c) L (d) M

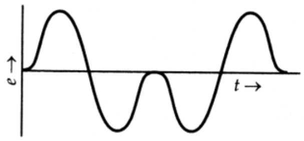
Q.5 A magnet is made to oscillate with a particular frequency, passing through a coil as shown in the figure:

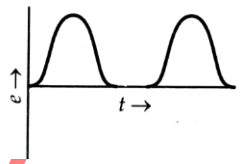



The time variation of the magnitude of emf generated across the coil during one cycle is

(a) 


(b) 


(c) 


(d) 


Q.6 In a coil of self–induction 5 H, the rate of change of current is 2 A $s^{-1}$. Then, emf induced in the coil is

(a) 10 V (b) –10 V (c) 5 V (d) – 5 V

## ***Assertion and Reasons***

***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R) Mark the correct choice as:

(a) if both assertion and reason are true and reason is the correct explanation of the assertion.

(b) if both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) if assertion is true, but reason is false

(d) both assertion and reason are false statements

Q.7 **Assertion:** Faraday's laws are consequence of the conservation of energy.

**Reason:** In a purely resistive a.c. circuit, the current lags behind the emf in phase

Q.8 **Assertion:** The presence of large magnetic flux through a coil maintains a current in the coil, if the circuit is continuous.

**Reason:** Only a change in magnetic flux will maintain an induced current in the coil.

Q.9 **Assertion:** An emf is induced in a closed loop, where magnetic flux is varied. The induced $\vec{E}$ is not a conservative field.

**Reason:** The line integral $\vec{E}.\vec{dl}$ around the closed loop in non–zero

Q.10 **Assertion:** If current is flowing through a machine of iron, eddy currents are produced.

**Reason:** Change in magnetic flux through an area causes eddy current.

Q.11 **Assertion:** No power loss is associated with a pure capacitor in an a.c. circuit.

**Reason:** No current is flowing in this circuit.

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | a | 2. | c | 3. | a | 4. | d | 5. | a |
| 6. | b | 7. | c | 8. | d | 9. | a | 10. | a |
| 11. | c | | | | | | | | |

## AIEEE

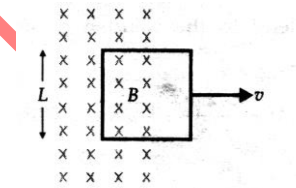
Q.1 A coil having n turns and resistance RΩ is connected with a galvanometer of resistance 4 RΩ. This combination is moved in time t seconds from a magnetic flux $\phi_1$ weber to $\phi_2$ weber. The induced current in the circuit is

(a) $\frac{\phi_2-\phi_1}{5R\,nt}$ (b) $-\frac{n(\phi_2-\phi_1)}{5Rt}$ (c) $-\frac{(\phi_2-\phi_1)}{Rnt}$ (d) $-\frac{n(\phi_2-\phi_1)}{Rt}$

Q.2 In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is

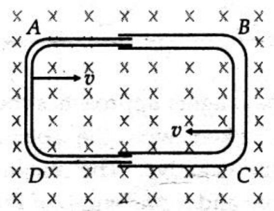
(a) $\frac{B\pi r^2\,\omega}{2R}$ (b) $\frac{(B\pi r^2\,\omega)^2}{8R}$ (c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

Q.3 A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is




(a) zero (b) R v B (c) v BL/ R (d) v B L

Q.4 One conducting U tube can slide inside another as shown in figure. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be




(a) Blv (b) – Blv
(c) zero (d) 2 Blv

Q.5 A metal conductor of length 1m rotates vertically about one of its ends at angular velocity 5 rad $s^{-1}$. If the horizontal component of the earth's magnetic field is $0.2 \times 10^{-4}$ T, then emf developed between the two ends of the conductor is

(a) 5 μV (b) 50 μV (c) 5 mV (d) 50 mV

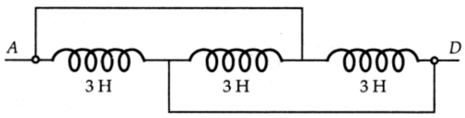
Q.6 When the current changes from +2 A to – 2 A in 0.05 s, an emf of 8 V induced in the coil. The coefficient of self–induction of the coil is

(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H

Q.7 Two coils are placed closed to each other. The mutual inductance of the pair of coils depends upon

(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the material of the wires of the coils (d) the currents in the two coils

Q.8 The inductance between A and D is



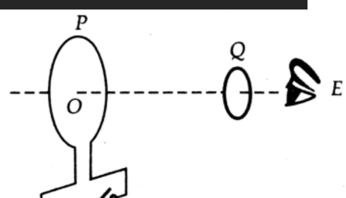

(a) 3.66 H (b) 9 H (c) 0.66 H (d) 1 H

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | b | 2. | b | 3. | d | 4. | d | 5. | b |
| 6. | d | 7. | b | 8. | d | | | | |







Q.1 As shown in figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current $I_P$ flows in P (as seen by E) and an induced current $I_{Q1}$ flows in Q. The switch remains closed for a long time. When S is opened, a current $I_{Q2}$ flows in Q. Then directions of $I_{Q1}$ and $I_{Q2}$ (as seen by E) are

(a) both clockwise
(b) both anticlockwise
(c) respectively clockwise and anticlockwise
(d) respectively anticlockwise and clockwise

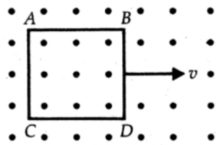
Q.2 Two identical circular loops of metal wire are lying on a table without touching each other. Loop – A carries a current, which increases with time. In response, loop – B.

(a) remains stationary
(b) is attracted by the loop A
(c) is repelled by the loop A
(d) rotates about its CM, with CM fixed

Q.3 A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following:

(a) the entire rod is at the same electric potential
(b) there is an electric field in the rod
(c) the electric potential is highest at the centre of the rod and decreases towards its ends
(d) the electric potential is lowest at the centre of the rod and increases towards its ends

Q.4 A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Electric field is induced.




(a) in AD, but not in BC
(b) in BC, but not in AD
(c) neither in AD nor in BC
(d) in both AD and BC

Q.5 The two rails of a railway track insulated from each other and the ground are connected to millivoltmeter. What is the reading of the millivoltmeter, when a train passes at a speed of 180 km $h^{-1}$ along the track, given that the vertical component of earth's magnetic field is $0.2 \times 10^{-4}$ Wb $m^{-2}$ and rails are separated by 1 m?

(a) $10^{-2}$ V
(b) 0.10 mV
(c) 1 V
(d) 1 mV

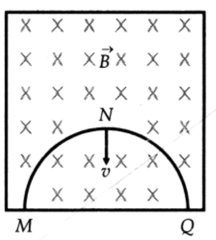
Q.6 An infinitely long cylinder is kept parallel to a uniform magnetic field B directed along positive Z–axis. The direction of induced current as seen from the Z–axis will be

(a) clockwise of the +ve Z–axis
(b) anticlockwise of the +ve Z–axis
(c) zero
(d) along the magnetic field

Q.7 A thin circular ring of area A is held perpendicular to a uniform field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing though the galvanometer is

(a) $\frac{BR}{A}$
(b) $\frac{AB}{R}$
(c) ABR
(d) $\frac{B^2 A}{R^2}$

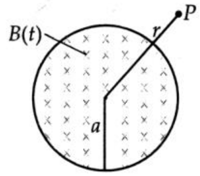
Q.8 A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ as shown in the figure. At the position MNQ the speed of the ring is v. Then, the potential difference developed across the ring is




(a) zero
(b) $\frac{1}{2}$Bv $R^2$ and M is at higher potential

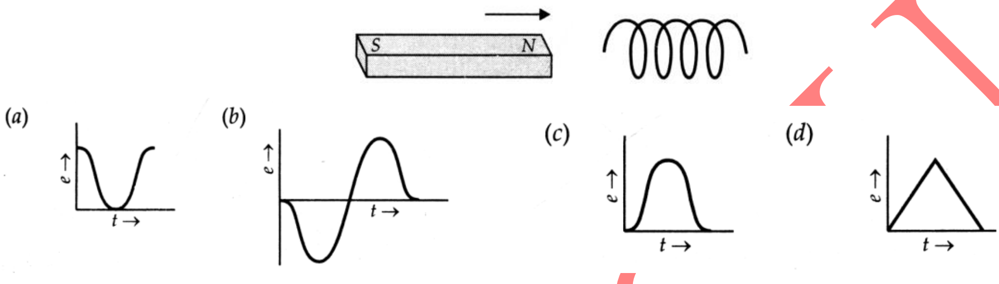
(c) $\pi$ R Bv and Q is at higher potential (d) 2 R Bv and Q is at higher potential

Q.9 A uniform but time–varying magnetic field B(t) exists within a circular region of radius a and is directed into the plane of the paper as shown in the figure. The magnitude of the induced electric field at the point P at a distance r from the centre of the circular region.




(a) is zero (b) decreases as 1/r (c) increases as r (d) decreases as $1/r^2$

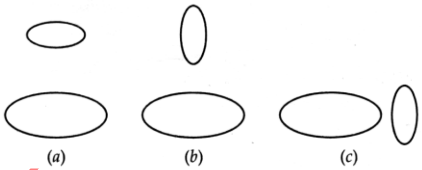
Q.10 The variation of induced emf (e) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as




Q.11 A short–circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved (b) the same (c) doubled (d) quadrupled

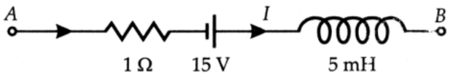
Q.12 Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be




(a) maximum in situation (a) (b) maximum in situation (b)

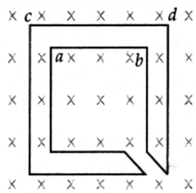
(c) maximum in situation (c) (d) same in all situations

Q.13 The network shown in figure is part of a complete circuit. If at a certain instant the current (I) is 5A, and is decreasing at a rate of $10^3$ A/s, then $V_B - V_A$ is




(a) 5 V (b) 10 V (c) 15 V (d) 20 V

Q.14 The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. $I_1$ and $I_2$ are the currents in the segments ab and cd. Then

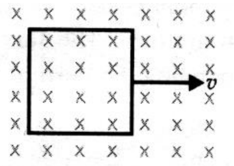



(a) $I_1 > I_2$ (b) $I_1 < I_2$

(c) $I_1$ is in direction ba and $I_2$ is in direction cd (d) $I_1$ is in direction ab and $I_2$ is in direction dc

***Multiple Choice Questions with One or More than One Correct Answers***

Q.15 A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. The current induced in the loop is




(a) $\frac{BLv}{R}$ clockwise (b) $\frac{BLv}{R}$ anticlockwise (c) $\frac{2BLv}{R}$ anticlockwise (d) zero

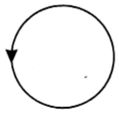
Q.16 The SI unit of inductance, the henry, can be written as

(a) weber/ampere (b) volt–second/ampere (c) joule/(ampere)$^2$ (d) ohm–second

Q.17 A small square loop of wire of side l is placed inside a large square loop of wire of side L(L > > 1). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

(a) $\frac{l}{L}$ (b) $\frac{l^2}{L}$ (c) $\frac{L}{l}$ (d) $\frac{L^2}{l}$

Q.18 A field line is shown in the figure. This field cannot represent



(a) magnetostatic field (b) electrostatic field

(c) induced electric field (d) gravitational field

Q.19 Two metallic rings A and B, identical in shape and size but having different resistivities $\rho_A$ and $\rho_B$ are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights $h_A$ and $h_B$, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses $m_A$ and $m_B$ is (are)

(a) $\rho_A > \rho_B$ and $m_A = m_B$ (b) $\rho_A < \rho_B$ and $m_A = m_B$

(c) $\rho_A > \rho_B$ and $m_A > m_B$ (d) $\rho_A < \rho_B$ and $m_A < m_B$

## Match the following

Q.20 Column I gives certain situations in which a straight metallic wire of resistance R is used and column II gives some resulting effects. Match the statements in column I with the statements in column II.

| S. No. | Column I | Sr. No. | Column II |
|---|---|---|---|
| (a) | A charged capacitor is connected to the ends of the wire | (p) | A constant current flows through the wire |
| (b) | The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion. | (q) | Thermal energy is generated in the wire |
| (c) | The wire is placed in a constant electric field that has a direction along the length of the wire | (r) | A constant potential difference develops between the ends of the wire |
| (d) | A battery of constant emf is connected to | (s) | Charges of constant magnitude appear at |

| | | | |
|---|---|---|---|
| | the ends of the wire | | the ends of the wire |

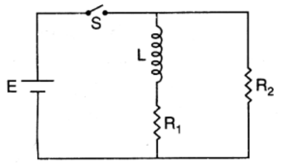
## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | d | 2 | c | 3 | b | 4 | d | 5 | b |
| 6 | c | 7 | b | 8 | d | 9 | b | 10 | b |
| 11 | b | 12 | a | 13 | c | 14 | d | 15 | d |
| 16 | abcd | 17 | b | 18 | bd | 19 | bd | | |

20. a – q; b – r, s; c – r, s; d – p, q, r

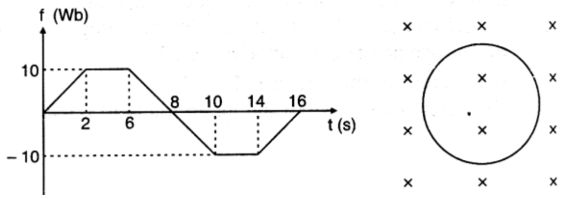
## Match the Column

Q.1 In the circuit shown in figure E = 18 V, L = 2H, $R_1$ = 3Ω, $R_2$ = 6Ω. Switch S is closed at t = 0. Match the following.



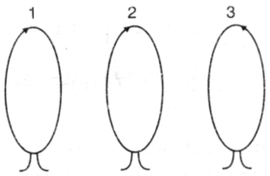

| | **Table – I** | | **Table – II** |
|---|---|---|---|
| (A) | Current through $R_1$ at t = 0 | (P) | 6 A |
| (B) | Current through $R_1$ at t = ∞ | (Q) | 3 A |
| (C) | Current through $R_2$ at t = 0 | (R) | Zero |
| (D) | Current through $R_2$ at t = ∞ | (S) | Infinite |

Q.2 Magnetic flux in a circular coil of resistance 10Ω changes with time as shown in figure. ⊗ direction indicates a direction perpendicular to paper inwards. Match the following table



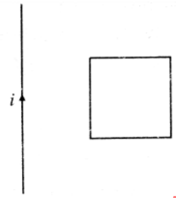

| | **Table – I** | | **Table – II** |
|---|---|---|---|
| (A) | At 1 s induced current is | (P) | clockwise |
| (B) | At 5 s induced current is | (Q) | anticlockwise |
| (C) | At 9 s induced current is | (R) | zero |
| (D) | At 15 s induced current is | (S) | 2 A |
| | | (T) | None |

Q.3 Three coils are placed infront of each other as shown. Currents in 1 and 2 are in same direction, while that in 3 is in opposite direction. Match the following table




| | Table – I | | Table – II |
|---|---|---|---|
| (A) | When current in 1 is increased | (P) | current in 1 will increase |
| (B) | When current in 2 is increased | (Q) | current in 2 will increase |
| (C) | When current in 3 is increased | (R) | current in 3 will increase |
| | | (S) | none |

Q.4 A square loop is placed near a long straight current carrying wire as shown. Match following table




| | Table – I | | Table – II |
|---|---|---|---|
| (A) | If current is increased | (P) | Induced current in loop is clockwise |
| (B) | If current is decreased | (Q) | Induced current in loop is anticlockwise |
| (C) | If loop is moved away from the wire | (R) | Wire will attract the loop |
| (D) | If loop is moved towards the wire | (S) | Wire will repel the loop |

## Assertion and Reason

Q.5 **Assertion:** Lenz's law violates the principle of conservation of energy.

**Reason:** Induced emf always opposes the change in the magnetic flux responsible for its production.

(a) A (b) B (c) C (d) D (e) E

Q.6 **Assertion:** The induced emf and current will be same in two identical loops of copper and aluminium, when rotated with same speed in the same magnetic field.

**Reason:** Induced emf is proportional to rate of change of magnetic field while induced current depends on resistance of wire.

(a) A (b) B (c) C (d) D (e) E

Q.7 **Assertion:** A bulb connected in series with a solenoid in connected to AC source. If a soft iron core is induced in the solenoid, the bulb will glow brighter.

**Reason:** On introducing soft iron cure in the solenoid, the inductance increases.

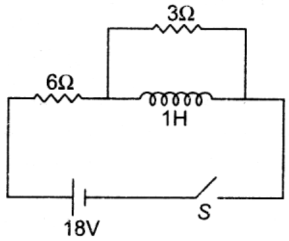
(a) A (b) B (c) C (d) D (e) E

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1 | (A) R, (B) P, (C) Q, (D) Q | 2 | (A) Q, (B) R, (C) P, (D) Q | | |
| 3 | (A) R, (B) R, (C) P, Q | 4 | (A) Q, S (B) P, R (C) P, R (D) Q, S | | |
| 5 | d | 6 | d | 7 | d |

## Comprehensions Type Questions

### Passage – 1




In an L – R circuit current growth takes place according to the law:

$$i = i_0 (1 - e^{-t/\tau})$$

Here $i_0$ is the steady state current (at $t \to \infty$) given by V/R. V is the applied voltage and R the resistance of circuit. $\tau_L$ = L/R is called time constant. Potential difference across inductor is given by $V_L = L\frac{di}{dt}$, where L is the inductance.

At time t = 0, an inductor offers infinite resistance (in dc) and at t = ∞ if offers zero resistance. Time constant of a circuit can be obtained by $\tau_L = \frac{L}{R_{net}}$, *where* $R_{net}$ is the net resistance across inductor after short circuiting the battery.

In the circuit shown in figure switch S is closed at time t = 0.

Q.1 Current i from the battery at time t is given by

(a) $3(1 - e^{-2t})$ (b) $3 + e^{-2t}$ (c) $3(1 - e^{-t/9})$ (d) $3 - e^{-2t}$

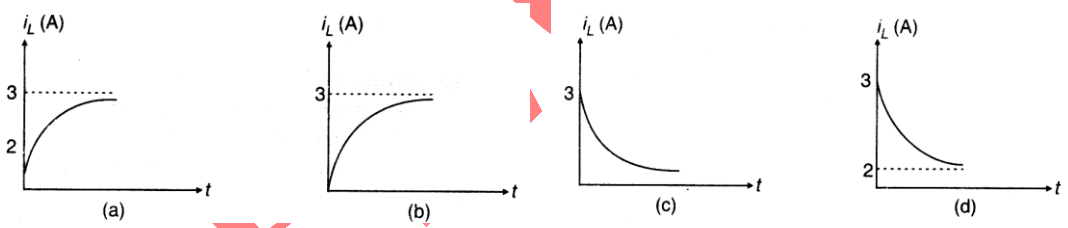
Q.2 Potential difference across 3Ω resistance at time t is given by

(a) $9e^{-2t}$ (b) $6e^{-2t}$ (c) $3e^{-2t}$ (d) $18(1 - e^{-t/9})$

Q.3 At what time current through 3Ω resistance and 1H inductor are equal?

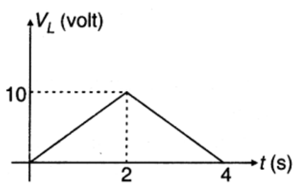
(a) $ln\sqrt{\frac{5}{3}}$ (b) $ln\left(\frac{8}{3}\right)$ (c) $ln\left(\frac{5}{3}\right)$ (d) $ln\sqrt{\frac{8}{3}}$

Q.4 Taking left to right current through the inductor as positive current, current through inductor varies with time t as




### Passage – 2

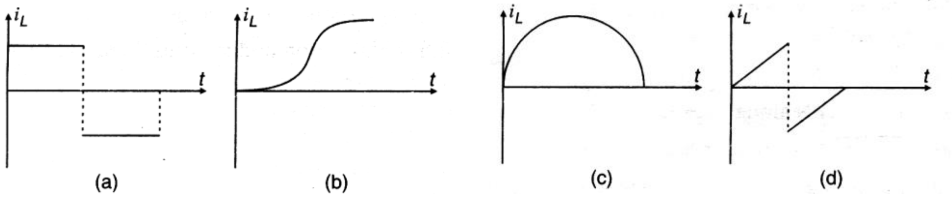
The potential different across a 2H inductor as a function of time is shown in fig. At time t=0, current is zero




Q.5 Current at t = 2 s is

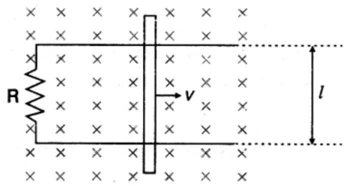
(a) 1 A (b) 3 A (c) 4 A (d) 5 A

Q.6 current versus time graph across the inductor will be

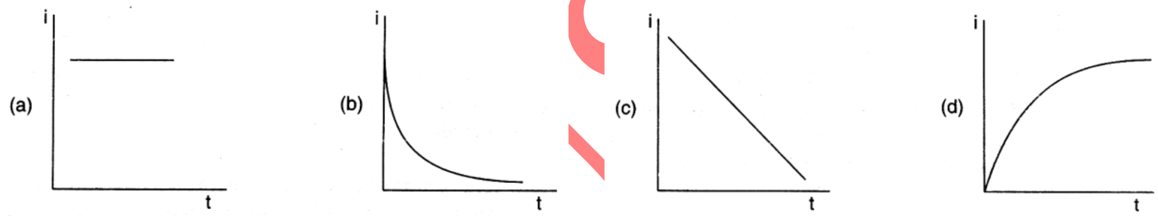



## Passage – 3

A conducting bar is slid at a constant velocity v along two conducting rods. The rods are separated by a distance l and connected across a resistor R. The entire apparatus is placed in an external magnetic field B directed into the page.




Q.7 Which of the following represents the current i generated by the apparatus?

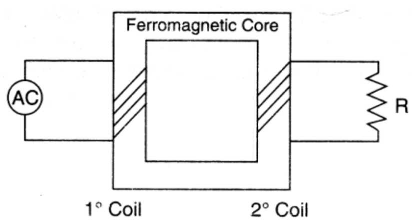



Q.8 An increase in which of the following would NOT increase the current generated by the apparatus?
(a) v (b) l (c) R (d) B

Q.9 How will the current in the apparatus shown above the generated?
(a) Sinusoidally (b) Clockwise (c) Counterclockwise
(d) There is not enough information to determine the direction of the current

## Passage – 4

- A physics lab is designed to study the transfer of electrical energy from one circuit to another by means of a magnetic field using simple transformers. Each transformer has two coils of wire electrically insulated from each other but wound around a common core of ferromagnetic material. The two wires are close together but do not touch each other.




- The primary (1°) coil is connected to a source of alternating (AC) current. The secondary (2°) coil is connected to a resistor such as a light bulb. The AC source produces an oscillating voltage and current in primary coil that produces an oscillating voltage and current in the primary coil that produces an oscillating magnetic field in the core material. This in turn induces an oscillating voltage and AC current in the secondary coil.

- Students collected the following data comparing the number of turns per coil (N), the voltage (V) and the current (I) in the coils of three transformers.

| | **Primary Coil** | | | **Secondary Coil** | | |
|---|---|---|---|---|---|---|
| | $N_1^o$ | $V_1^o$ | $I_1^o$ | $N_2^o$ | $V_2^o$ | $I_2^o$ |
| Transformer 1 | 100 | 10 V | 10 A | 200 | 20 V | 5 A |
| Transformer 2 | 100 | 10 V | 10 A | 50 | 5 V | 20 A |
| Transformer 3 | 200 | 10 V | 10 A | 100 | 5 V | 20 A |

Q.10 The primary coil of a transformer has 100 turns and is connected to a 120 V AC source. How many turns are in the secondary coil if there's a 2400 V across it?
(a) 5 (b) 50 (c) 200 (d) 2000

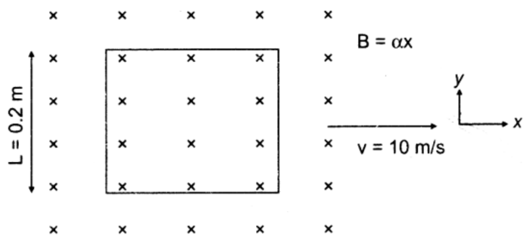
Q.11 A transformer with 40 turns in its primary coil is connected to a 120 V AC source. If 20 W of power is supplied to the primary coil, how much power is developed in the secondary coil?
(a) 10 W (b) 20 W (c) 80 W (d) 160 W

Q.12 Which of the following is a correct expression for R, the resistance of the load connected to the secondary coil?
(a) $\left(\frac{V_1^o}{I_1^o}\right)\left(\frac{N_2^o}{N_1^o}\right)$ (b) $\left(\frac{V_1^o}{I_1^o}\right)\left(\frac{N_2^o}{N_1^o}\right)^2$ (c) $\left(\frac{V_1^o}{I_1^o}\right)\left(\frac{N_1^o}{N_2^o}\right)$ (d) $\left(\frac{V_1^o}{I_1^o}\right)\left(\frac{N_1^o}{N_2^o}\right)^2$

Q.13 A 12 V battery is used to supply 2.0 mA of current to the 300 turns in the primary coil of a given transformer. What is the current in the secondary coil if $N_2$ = 150 turns?
(a) zero (b) 1.0 mA (c) 2.0 mA (d) 4.0 mA

**Passage – 5**

- According to Faraday's laws of EMI, an induced emf appears in a moving conductor due to change in flux associated with it. An arrangement with a square loop of mass m = 0.1 kg, length L = 0.2m and resistance R = 80mΩ lies with its length along the x–axis as shown in the figure. It moves with speed v = 10 $ms^{-1}$ in the positive x–direction in a magnetic field which is into the page and has a magnitude that varies with x according to B = αx, where α = 0.2 T $m^{-1}$.



Q.14 If loop start moving with the same velocity in x–direction then
(a) induced emf in the loop will remain the same in magnitude but direction will be opposite
(b) induced emf in the loop will remain the same both in magnitude and direction
(c) induced emf in the loop will change in magnitude but direction will be the same
(d) induced emf in the loop will change both in magnitude and direction

Q.15 Find the direction and magnitude of induced current
(a) 1 A clockwise (b) 1 A anticlockwise
(c) 2 A clockwise (d) 2 A anticlockwise

Q.16 Find the net magnetic force on the loop
(a) $8 \times 10^{-3}$ N along + x axis (b) $8 \times 10^{-3}$ N along – x axis
(c) $16 \times 10^{-3}$ N along + x axis (d) $16 \times 10^{-3}$ N along – x axis

Q.17 Suppose an external force is applied to keep the loop moving with constant velocity. Find the rate at which external force does work on the loop.
(a) $40 \times 10^{-3}$ W (b) $80 \times 10^{-3}$ W (c) $160 \times 10^{-3}$ W (d) $360 \times 10^{-3}$ W

Q.18 If loop start moving with the same velocity in + y direction then:
(a) induced emf in the loop will be clockwise
(b) induced emf in the loop will be anticlockwise
(c) there will be no induced emf in the loop
(d) data given in the questions are insufficient

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | d | 2 | b | 3 | a | 4 | b | 5 | d |
| 6 | b | 7 | a | 8 | c | 9 | c | 10 | d |
| 11 | b | 12 | b | 13 | d | 14 | b | 15 | b |
| 16 | b | 17 | b | 18 | c | | | | |

# ALTERNATING CURRENTS

## The alternating current

Most of the electric power generated and used in the world is in the form of a.c., i.e., alternating current. This is because

**(i)** Alternating voltages can be easily and efficiently converted from one value to the other by means of transformers.

**(ii)** The alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

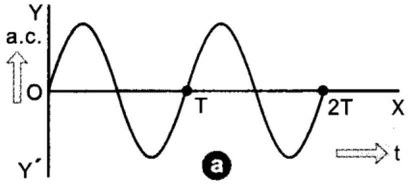
The magnitude of alternating current changes continuously with time and its direction is reversed periodically. It is represented by

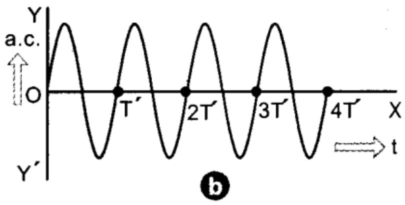
$$\boxed{I = I_0 \sin \omega t} \quad \text{or} \quad \boxed{I = I_0 \cos \omega\, t}$$

here, I is instantaneous value of current, i.e., magnitude of current at any instant of time *t* to $I_0$ is the peak value or maximum value of a.c. It is also called amplitude of a.c., ω is called angular frequency of a.c.

Also, $$\boxed{\omega = \frac{2\pi}{T} = 2\pi \nu}$$

where T is the time period or period of a.c. It is equal to the time taken by the a.c. to go through one complete cycle of variation. Again, ν is the frequency of a.c. It is equal to the number of complete cycles of variation gone through by the a.c. in one second.




Alternating e.m.f. which may be represented by

$$\boxed{E = E_0 \sin \omega t} \quad \text{or} \quad \boxed{E = E_0 \cos \omega t}$$

**Amplitude**

*The maximum value attained by an alternating current in either direction is called its* ***amplitude*** or ***peak value*** and is denoted by $I_0$.

**Time Period**

*The time taken by an alternating current to complete one cycle of its variations is called* its time period and is denoted by T. This time is equal to the time taken by the coil to complete one rotation in the magnetic field. As angular velocity of the coil is $\omega$ and its angular displacement in one complete cycle is $2\pi$, so

$$\text{Time period} = \frac{\textit{Angular displacement in a complete cycle}}{\textit{Angular velocity}} \qquad \text{or} \qquad T = \frac{2\pi}{\omega}$$

**Frequency**

*The number of cycles completed per second by an alternating current is called its frequency and is denoted by* v. The frequency of an alternating current is same as the frequency of rotation of the coil in the magnetic field. Thus

$$v = \frac{1}{T} = \frac{\omega}{2\pi}$$

The alternating current supplied to our houses has a frequency of 50 cps or 50 Hz.

## Mean value or average value of Alternating current

The mean or average value of a.c. over any half cycle is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle (i.e., T/2) as in sent by the a.c . through the same circuit, in the same time.

To calculate the mean or average value, let an alternating current be represented by

$$I = I_0 \sin \omega t \qquad \text{....(1)}$$

If the strength of current is assumed to remain constant for a small time, dt, then small amount of charge sent in a small time dt is

$$dq = I\,dt$$

Let q be the total charge sent by a.c. in the first half cycle (i.e., $0 \rightarrow T/2$)

$$\therefore \qquad q = \int_0^{\frac{T}{2}} I\,dt$$

Using (1), we get $q = \int_0^{\frac{T}{2}} I_0 \sin \omega t \,.\, dt = I_0 \left[ -\frac{\cos \omega t}{\omega} \right]_0^{\frac{T}{2}} = -\frac{I_0}{\omega} \left[ \cos \omega \frac{T}{2} - \cos 0^o \right]$

$$= -\frac{I_0}{\omega} \left[ \cos \pi - \cos 0^0 \right] \qquad [\because \omega T = 2\pi]$$

$$q = -\frac{I_0}{\omega}[-1-1] = \frac{2I_0}{\omega} \qquad \text{....(2)}$$

If $I_m$ represents the mean or average value of a.c. over the 1st half cycle, then

$$q = I_m \times \frac{T}{2} \qquad \text{....(3)}$$

From (2) and (3), we get $I_m \times \frac{T}{2} = 2\frac{I_0}{\omega} = \frac{2I_0 \,.\, T}{2\pi}$ or $I_m = \frac{2}{\pi} I_0 = 0.637\, I_0$

Hence, mean or average value of a.c. over positive half cycle is 0.637 times the peak value of a.c., i.e., 63.7% of the peak value.

Similarly, the mean or average value of a.c. over the negative half cycle is obtained by integrating within the limit T/2 to T. It comes out to be – 0.637 $I_0$. Hence the mean or average value of a.c. over one complete cycle is 0.637 $I_0$ – 0.637 $I_0$ = zero. This can also be derived directly by integrating within the limits 0 to T.

## Mean value or average value of alternating E.M.F.

The mean or average value of alternating e.m.f. over a half cycle is that value of constant e.m.f. which would send the same amount of charge through a circuit in the time of half cycle (T/2), as is sent by alternating e.m.f. through the same circuit in the same time.

Let as alternating e.m.f. be represented by $E = E_0 \sin\omega t$.

If I is the value of current at any instant t, then $I = \frac{E}{R} = \frac{E_0}{R}\sin\omega t$, where R is resistance of the circuit.

If this current remains constant for a small time dt, then small amount of charge sent by alternating e.m.f. in the small time dt is

$$dq = I\,dt = \frac{E_0}{R}\sin\omega t\,dt$$

$\therefore$ Total charge sent by alternating e.m.f. in the first half cycle (0 $\rightarrow$ T/2) would be

$$q = \int_0^{\frac{T}{2}} \frac{E_0}{R}\sin\omega t\,dt = \frac{E_0}{R}\left[-\frac{\cos\omega t}{\omega}\right]_0^{\frac{T}{2}} = \frac{-E_0}{\omega R}\left[\cos\omega\frac{T}{2} - \cos 0^0\right]$$

$$= -\frac{E_0}{\omega R}\left[\cos\pi - \cos 0^0\right] \qquad [\because \omega T = 2\pi]$$

$$q = -\frac{E_0}{\omega r}(-1-1) = \frac{2E_0}{\omega R} \qquad \text{....(1)}$$

If $E_m$ is mean or average value of alternating e.m.f. over the first half cycle, then

$$q = \frac{E_m}{R}\times\frac{T}{2} \qquad \text{....(2)}$$

From (1) and (2), $\frac{E_m}{R}\frac{T}{2} = \frac{2E_0}{\omega R} = \frac{2E_0}{\left(\frac{2\pi}{T}\right)R} = \frac{2E_0 T}{2\pi R}$ or $E_m = \frac{2}{\pi}E_0 = 0.637\,E_0 = 63.7\%\,E_0$

Hence, mean or average value of alternating e.m.f. over positive half cycle is 0.637 times the peak value of alternating e.m.f. Similarly, over the negative half cycle (T/2 $\rightarrow$ T)

$$\boxed{E_m = \frac{-2E_0}{\pi} = -0.637\,E_0}$$

Hence average value of alternating e.m.f. over the full cycle = zero. This can also be derived directly by integrating within the limits 0 to T.

**Note :** That ordinary d.c. ammeter and d.c. voltmeter cannot measure alternating currents / voltages. They record zero reading, when used in a.c. circuits, because average value of alternating current/voltage over a full cycle is zero.

## Root mean square value of alternating current

The root mean square (r.m.s) value of a.c. is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c. when passed through the same resistance for the same time.

The r.m.s. value is also called effective value of a.c. or virtual value of a.c. It is represented by $I_{rms}$ or $I_{eff}$ or $I_v$.

To calculate it, suppose , an alternating current is represented by

$$I = I_0 \sin\omega t$$

Let this current flow though a resistance R. In a small time dt, the amount of heat produced in resistance R is

$$dH = I^2 R\,dt$$

In one complete cycle (time $0 \to T$), the total amount of heat produced in the resistance R would be

$$H = \int_0^T I^2\,R\,dt$$

$\therefore$ $$H = \int_0^T (I_0^2 \sin^2 \omega t) R\,dt = I_0^2\,R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\,.dt\right] = \frac{I_0^2\,R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 R}{2}\left[(T-0) - \left(\frac{\sin 2\omega T}{2\omega} - \sin 0^0\right)\right] = \frac{I_0^2\,R}{2}\left[T - \frac{\sin 2\times 2\pi}{2\omega}\right] \quad [\because \omega T = 2\pi]$$

$\therefore$ $$H = \frac{I_0^2 RT}{2} \qquad [\because \sin 4\pi = 0] \qquad ....(1)$$

If r.m.s. value or virtual value of a.c. is represented by $I_v$, then the amount of heat produced in the same resistance R in the same time T would be

$$H = I_v^2\,RT \qquad ....(2)$$

From (1) and (2), we get $I_v^2\,RT = \dfrac{I_0^2\,RT}{2}$

$$\boxed{I_v = \frac{I_0}{\sqrt{2}} = 0.707\,I_0}$$

Hence the r.m.s. value or effective value or virtual value of a.c. is 0.707 times the peak value of a.c. i.e., 70.7% of the peak value of a.c.

## *Root mean square value of alternating E.M.F.*

The root mean square (r.m.s.) value of alternating e.m.f. is defined as that value of steady voltage, which would generate the same amount of heat in a given resistance in a given time as is done by the alternating e.m.f. , when applied to the same resistance for the same time. The r.m.s. value is also called effective value or virtual value of alternating e.m.f. If is represented by $E_{rms}$ or $E_{eff}$ or $E_v$.

To calculate it, suppose the alternating e.m.f. is represented by $E = E_0 \sin\omega t$. Small amount of heat produced when this alternating e.m.f. is applied to a resistance R for a small time dt is

$$dH = \frac{E^2}{R}\,dt = \frac{(E_0 \sin\omega t)^2}{R}\,dt$$

$$dH = \frac{E_0^2}{R}\sin^2 \omega t\,dt$$

In one complete cycle ($0 \to T$), total amount of heat produced in resistance R is

$$H = \int_0^T \frac{E_0^2}{R}\sin^2 \omega t\,dt = \frac{E_0^2}{R}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt = \frac{E_0^2}{2R}\int_0^T 1\,dt - \frac{E_0^2}{2R}\int_0^T \cos 2\omega t\,dt$$

$$H = \frac{E_0^2}{2R}\left[T\right]_0^T - \text{Zero} \qquad \left[\because \int_0^T \cos 2\omega t\,dt = 0\right]$$

$$H = \frac{E_0^2}{2R}(T-0) = \frac{E_0^2 T}{2R} \qquad \text{....(1)}$$

If $E_v$ is the rms value of alternating e.m.f., then amount of heat produced in the same resistance R in the same time T is

$$H = \frac{E_v^2}{R}T \qquad \text{....(2)}$$

From (1) and (2), we get $\frac{E_v^2}{R}T = \frac{E_0^2}{2R}$

$$E_v^2 = \frac{E_0^2}{2} \qquad \text{or} \qquad E_v = \frac{E_0}{\sqrt{2}}$$

i.e., $$E_v = \frac{E_0}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{E_0 \times 1.414}{2} = 0.707\,E_0$$

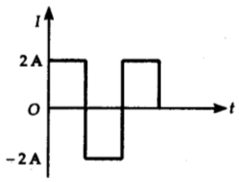
Hence, r.m.s. value of alternating e.mf. is 0.707 times the peak value of alternating e.m.f.

**Note :** 220 V a.c. is more dangerous than 220 V d.c., because 220 V ac. has peak value $E_0 = \pm\sqrt{2}$ $E_v = \pm 1.414 \times 220V = \pm 311V$ , but 220 V d.c. has value fixed at 220 V only.

Alternating current/voltage are measured by a.c. ammeter/voltmeter respectively. These instruments are called hot wire instruments and they measure only virtual values of alternating currents/voltages. The values of alternating voltages/currents quoted anywhere are virtual values only. For example, 220 V a.c. means $E_v = 220$ volt. An a.c. of 1 A means $I_v$ = 1A.

## Subjective Assignment – I

1. The electric mains in a house are marked 220V, 50Hz. Write down the equation for instantaneous.
2. An electric bulb operates 12V d.c. If this bulb is connected to an a.c. source and gives normal brightness, what would be the peak value of the source?
3. The peak value of an alternating voltage applied to a 50Ω resistance is 10V. Find the rms current. If the voltage frequency is 100Hz, write the equation for the instantaneous current.



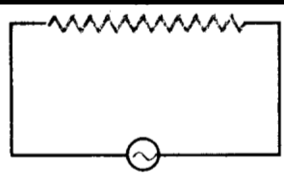

4. Calculate the rms value of the alternating current shown in figure.
5. The electric current in a circuit is given by $i = i_0(t/\tau)$ for some time. Calculate the rms current for the period t = 0 to t = τ.
6. If effective value of current in 50 Hz a.c. circuit is 5.0 A, what is (i) the peak value of current (ii) the mean value of current over half a cycle and (iii) the value of current 1/300s after it was zero?
7. The instantaneous value of an alternating voltage in volts is given by the expression $\xi_t = 140\sin 300t$, where t is in second. What is (i) peak value of the voltage, (ii) its rms value and (iii) frequency of the supply?

8. A resistance of 40Ω is connected to a.c. source of 220V, 50Hz. Find (i) the rms current (ii) the maximum instantaneous current in the resistor and (iii) the time taken by the current to change from its maximum value to the rms value.
9. The instantaneous emf of an a.c. source is given by $\xi = 300 \sin 314t$. What is the rms value of the emf?
10. The emf of an a.c. source is given by the expression $\xi = 300 \sin 314t$. Write the value of peak voltage and frequency of the source.
11. The instantaneous current from an a.c. source is $I = 5\sin 314t$. What is the rms value of current?
12. An alternating current in amperes is given by $I = 50\sin(400\pi t + \phi)$. Find the frequency and the rms value of the current.
13. An alternating emf of peak value 350 V is applied across an a.c. ammeter of resistance 100Ω. What is the reading of the ammeter?
14. The effective value of current in a 50 cycle a.c. circuit is 5A. What is the value of current 1/300 second after it was zero?
15. The peak value of an alternating current of frequency 50Hz is 14.14A. Find its rms value. How much time will the current take in reaching from 0 to maximum value?
16. A 100Ω iron is connected to a 200V, 50 cycles wall plug. What is (i) peak potential difference (ii) average potential difference and (iii) rms current?
17. The equation of a.c. in a circuit is $I = 50\sin 100\pi t$. Find (i) frequency of a.c., (ii) mean value of a.c. over positive half cycle, (iii) rms value of current and (iv) the value of current 1/300 second after it was zero.

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | $311\sin 314t$ volt | 2. | 17 V | 3. | 141.4 mA ; $200\sin 200\pi t$ mA |
| 4. | 2 A | 5. | $\frac{i_0}{\sqrt{3}}$ | 6. | (i) 7.07 A (ii) 4.5 A (iii) 6.12 A |
| 7. | (i) 140 V (ii) 100 V (iii) 50 Hz | | | 8. | (i) 5.5 A (ii) 7.8 A (iii) 2.5 ms |
| 9. | 212 V | 10. | 300 V, 50 Hz | 11. | 3.54 A |
| 12. | 200 Hz, $25\sqrt{2}$ A | 13. | 2.47 A | 14. | 6.123 A |
| 15. | 10 A, 5 ms | 16. | (i) 311 V (iii) 198 V (iiii) 2.2 A | | |
| 17. | 50Hz, 31.8 A, 35.35 A, 43.3A | | | | |

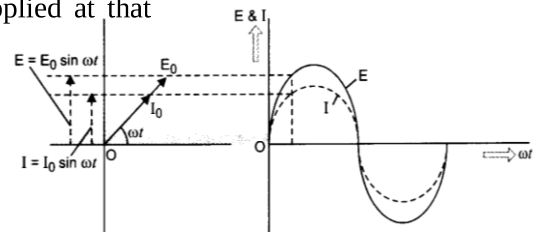
## *A.C. circuit containing resistance only*



Let a source of alternating e.m.f. be connected to a pure resistance R. Suppose the alternating e.m.f. supplied is represented by

$$E = E_0 \sin \omega t \qquad ....(1)$$

Let I be the current in the circuit at any instant *t*. The potential difference developed across R with be IR. This most be equal to e.m.f. applied at that instant, i.e.,




$$IR = E = E_0 \sin\omega t$$

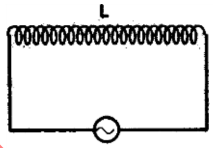
or $$I = \frac{E_0}{R}\sin\omega t = I_0 \sin\omega t \qquad ....(2)$$

where $I_0 = E_0 / R$ maximum value of current.

From equation (1) and (2), we note that both E and I are functions of sin ωt. Hence the emf E and current I are in *same phase* in a purely resistive circuit. This means that both e and I attain their zero, minimum and maximum values at the same respective times. This phase relationship is shown graphically.

The phasor diagram for a resistive a.c. circuit. Both the phasors $\vec{E}$ and $\vec{I}$ are in the same direction, making same angle ωt with x–axis. The phase angle between them is zero.

## A.C. Circuit containing inductance only




Alternating e.m.f. be connected to a circuit containing a pure inductance L. Suppose the alternating e.m.f. supplied is represented by

$$E = E_0 \sin\omega t \qquad ....(1)$$

If dI/dt is the rate of change of current through L at any instant, then induced e.m.f. in the inductor the same instant is = $-L\, dI/dt$. The negative sign indicates that induced e.m.f. opposes the change of current.

Usually, inductors have some resistance in their windings. But we shall assume that resistance of this inductor is negligible. The circuit is therefore, a purely inductive circuit. To maintain the flow of current in the is circuit., applied voltage must be equal and opposite to the induced voltage.

i.e., $$E = -\left(-L\frac{dI}{dt}\right) = E_0 \sin\omega t \quad \text{or} \quad L\frac{dI}{dt} = E_0 \sin\omega t$$

or $$dI = \frac{E_0}{L}\sin\omega t\; dt$$

Integrating both sides, we get , $I = \frac{E_0}{L}\int \sin\omega t\, dt$

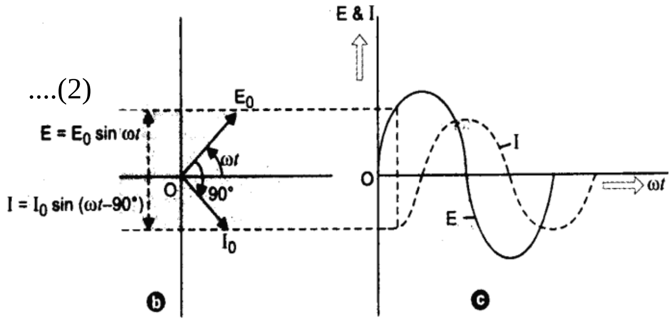
$$I = \frac{E_0}{L}\left(\frac{-\cos\omega t}{\omega}\right) \quad \text{or} \quad I = \frac{-E_0}{\omega L}\cos\omega t$$

$$I = \frac{-E_0}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \quad \text{or} \quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2)$$

The current will be maximum , i.e., $I = I_0$, when $\sin(\omega t - \pi/2) = \text{maximum} = 1$

$\therefore$ $$I_0 = \frac{E_0}{\omega L}\times 1$$

Hence $$I = I_0 \sin(\omega t - \pi/2) \qquad ....(2)$$




this is the form of alternating current developed.

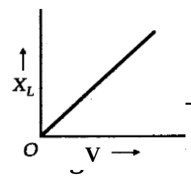
comparing (1) with (2), we find that in an a.c. circuit containing L only, alternating current I lags behind the alternating voltage E by a phase angle of 90°, i.e., by one fourth of a period. Conversely, volt age across L leads the current by a phase angle of 90°.

By the Ohm's law equation, viz, current = Voltage/Resistance, we find that (ωL) represents the effective resistance offered by inductance L. This is called inductive reactance and is denoted by $X_L$.

Thus $X_L = \omega L = 2\pi v L$ where v is the frequency of a.c. supply.

The inductive reactance limits the current in a purely inductive circuit in the same way as resistance limits the current in a purely resistive circuit. Clearly, the inductive reactance ($X_L$) is directly proportional to the inductance (L) and also to the frequency (v) of the alternating current.

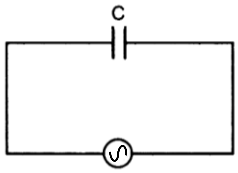
In dc circuits, v = 0 $\quad\therefore\; X_L = 0$



i.e., a pure inductance offers zero resistance to dc. It means a pure inductor cannot reduce dc. The units of inductive reactance is ohm.

Hence $X_L$ is measured in ohm, just as resistance is measured in ohm. The dimensions of inductive reactance are the same as those of resistance.

## A.C. Circuit Containing Capacitance Only




Let a source of alternating e.m.f. be connected to a capacitor only of capacitance C. Suppose the alternating e.m.f. supplied is

$$E = E_0 \sin \omega t \qquad .....(1)$$

The current flowing in the circuit transfers charge to the plates of the capacitor. This produces a potential difference between the plates. The capacitor is alternately charged and discharged as the current reverses each half cycle. At any instant t, suppose q is the charge on the capacitor. Therefore, potential difference across the plates of capacitor V = q/C.

At every instant, the potential difference V must be equal to the e.m.f. applied i.e.

$$V = \frac{q}{C} = E = E_0 \sin \omega t \qquad \text{or} \qquad q = CE_0 \sin \omega t$$

If I is instantaneous value of current in the circuit at instant t, then $I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t)$

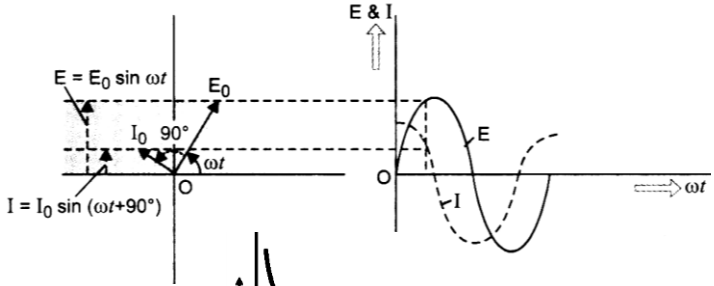
$$I = CE_0 (\cos \omega t)\omega \qquad \text{or} \qquad I = \frac{E_0}{\frac{1}{\omega C}} \cos \omega t$$

$$I = \frac{E_0}{1/\omega C} \sin (\omega t + \pi/2) \qquad \text{or} \qquad I_0 = \frac{E_0}{1/\omega C}$$

$\therefore$ $$I = I_0 \sin (\omega t + \pi/2) \qquad ....(2)$$

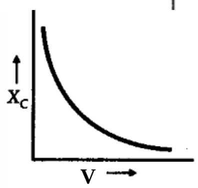
This is the form of alternating current developed.

Comparing (1) with (2), we find that in an a.c. circuit containing C only, alternating current I leads the alternating e.m.f. by a phase angle of 90°.




By Ohm's law equation, viz current = voltage/resistance, we find that (1/ω C) represents effective resistance offered by the capacitor. This is called *capacitative reactance* and is denoted by $X_C$.

Thus $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$$




where v is frequency of a.c. supply. The capacitative reactance limits the amplitude of current in a purely capacitative circuit in the same way as the resistance limits the current in a purely resistive circuit. Clearly, capacitative reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the condenser.

In d.c. circuit, v = 0, $\therefore X_C = \infty$. Hence, a condenser will block d.c.

The units of $X_C$ is ohm. Hence $X_C$ is measured in ohms, just like resistance R. The dimensions of capacitative reactance are same as that of resistance.

It should be clearly understood that the physical processes involved in capacitative and resistive circuits are quite different. In a resistive circuit, the resistance is due to the obstruction to the passage of electrons through the resistor. But in a capacitative circuit, resistance to the flow of current is offered by the charged capacitor.

1. A 100 Hz a.c. is flowing in a 14 mH coil. Find its reactance.
2. A pure inductor of 25.0 mH is connected to a source of 220V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50Hz.
3. Find the maximum value of current when an inductance of one henry is connected to an a.c. source of 200 volts, 50 Hz.
4. A coil has an inductance of 1H. (i) At what frequency will it have a reactance of 3142Ω? (ii) What should be the capacity of a capacitor when has the same reactance at that frequency?
5. An a.c. circuit consists of only an inductor of inductance 2H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference ($V_{eff}$) across the inductor.
6. Alternating emf, $\xi = 220 \sin 100\pi t$ is applied to a circuit containing an inductance of $1/\pi$ H. Write an equation for instantaneous current through the circuit. What will be the reading of an a.c. ammeter if connected in the circuit?
7. An inductor of inductance 200mH is connected to an a.c. source of peak emf 210V and frequency 50 Hz. Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value?
8. A 1.50μF capacitor is connected to a 220V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?
9. A capacitor of 1μF is connected to an a.c. source of emf $\xi = 250 \sin 100\pi t$. Write an equation for instantaneous current through the circuit and give reading of a.c. ammeter connected in the circuit.
10. What is the inductive reactance of a coil if current through it is 800 mA and the voltage across it is 40V?
11. Find the value of current through an inductance of 2.0H and negligible resistance, when connected to an a.c. source of 150V and 50 Hz.
12. An inductance of neglibible resistance, whose reactance is 22Ω at 200Hz is connected to a 220V, 50Hz power line. What is the value of inductance and reactance?
13. A coil of self–inductance has inductive reactance of 88Ω. Calculate the self–inductance of the coil if the frequency is 50Hz.
14. Find the maximum current through an inductance of 2H connected to an a.c. source of 150V, 50Hz.
15. Calculate the frequency at which the inductive reactance of 0.7 H inductor is 220Ω.
16. What is the capacitive reactance of a 5μF capacitor when it is a part of a circuit whose frequency is (i) 50Hz (ii) $10^6$ Hz?
17. A capacitor has a capacitance of $1/\pi$ μF. Find its reactance for a frequency of (i) 50 Hz and (ii) $10^6$ Hz.
18. A 1.5 μF capacitor has a capacitive reactance of 12Ω. What is the frequency of the source? If the frequency of the source is doubled, what will be the capacitive reactance?
19. A capacitor of capacitance 10μF is connected to an oscillator giving an output voltage, $\xi = 10 \sin \omega t$ volt. If $\omega = 10\ \text{rad}\,s^{-1}$, find the peak current in the circuit.
20. A capacitor has a reactance of 100Ω at 50 Hz. What will be its reactance at 125 Hz?

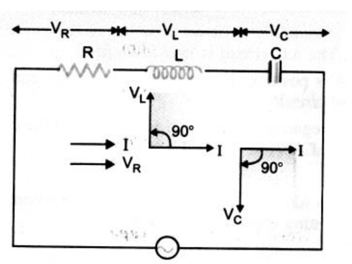
**Answers**

| | | | | | |
|---|---|---|---|---|---|
| 1. | 8.8 Ω | 2. | 7.85 Ω , 28.03 A | 3. | 0.9 A |
| 4. | (i) 500 Hz (ii) 0.11 μF | 5. | 133.2 V | 6. | 1.556 A |
| 7. | 3.3 A | 8. | 212 Ω, 1.04 A; 1.47 A | 9. | 0.06 A |
| 10. | 50 Ω | 11. | 0.239 A | 12. | 0.0175H, 5.5 Ω |
| 13. | 0.28 H | 14. | 0.337 A | 15. | 50 Hz |
| 16. | 636.6Ω, $3.18 \times 10^{-2}$Ω | 17. | 10Ω, 0.5 Ω | 18. | 8846 Hz, 6Ω |

19. 1.0 mA 20. 40 Ω

## A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (LCR Circuit)

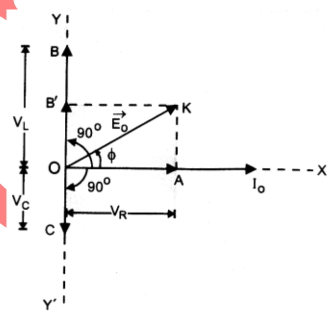
**(a) Phasor Treatment**




Let a pure resistance R, a pure inductance L and an ideal capacitor of capacitance C be connected in series to a source of alternating e.m.f. As R, L, C are in series, therefore, current at any instant through the three elements has the *same amplitude and phase.* Let it be represented by

$I = I_0 \sin \omega t$

However, voltage across each element bears a different phase relationship with the current. Now,




(i) The maximum voltage across R is $\vec{V}_R = \vec{I}_0 R$, current phasor $\vec{I}_0$ is represented along OX.

As $\vec{V}_R$ is in phase with current, it is represented by the vector $\overrightarrow{OA}$, along OX.

(ii) The maximum voltage across L is $\vec{V}_L = \vec{I}_0 X_L$

As voltage across the inductor leads the current by 90°, it is represented by $\overrightarrow{OB}$ along OY, 90° ahead of $\vec{I}_0$.

(iii) The maximum voltage across C is $\vec{V}_C = \vec{I}_0 X_C$

As voltage across the capacitor lags behind the alternating current by 90°, it is represented by $\overrightarrow{OC}$ rotated clockwise through 90° from the direction of $\vec{I}_0$. $\overrightarrow{OC}$ is along OY'.

As the voltages across L and C have a phase difference of 180°, the net reactive voltage is $(\vec{V}_L - \vec{V}_C)$, assuming that $\vec{V}_L > \vec{V}_C$.

In figure, it is represented by $\overrightarrow{OB'}$. The resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ is the diagonal $\overrightarrow{OK}$ of the rectangle OAKB'. Hence the vector sum of $\vec{V}_R, \vec{V}_L$ and $\vec{V}_C$ is phasor $\vec{E}_0$ represented by $\overrightarrow{OK}$, making an angle $\phi$ with current phasor $\vec{I}_0$.

As $$OK = \sqrt{OA^2 + OB'^2}$$

∴ $$E_0 = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

The total effective resistance of LCR circuit is called impedance of the circuit. It is represented by Z, where

$$Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

From figure, it is clear that in an a.c circuit containing R, L, C, the voltage leads the current by a phase angle $\phi$, where

$$\tan \phi = \frac{AK}{OA} = \frac{OB'}{OA} = \frac{V_L - V_C}{V_R} = \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

or $$\tan \phi = \frac{X_L - X_C}{R}$$

$\therefore$ The alternating e.m.f. in the LCR circuit would be represented by

$$E = E_0 \sin(\omega t + \phi)$$

Three cases arise:

(i) When $X_L = X_C$ then $\tan\phi = 0$ $\therefore$ $\phi = 0^o$

Hence voltage and current are in the same phase. The a.c. circuit is non–inductive.

(ii) When $X_L > X_C$ than $\tan\phi$ is positive. Therefore, $\phi$ is positive. Hence voltage *leads* the current by a phase angel $\phi$. The a.c. circuit is *inductance dominated circuit.*

(iii) When $X_L < X_C$ than $\tan\phi$ is negative. Therefore $\phi$ is negative. Hence voltage lags behind the current by a phase angle $\phi$. The a.c. circuit is *capacitance dominated circuit.*

## (b) Analytical Treatment of LCR Series Circuit

Let a pure resistance R, a pure inductance L and an ideal condenser of capacity C be connected in series to a source of alternating e.m.f. Suppose the alternating e.m.f. supplied is

$$E = E_0 \sin\omega t \qquad \text{....(1)}$$

At any instant of time t, suppose q = charge on capacitor , I = current in the circuit, $\frac{dI}{dt}$ = rate of change of current in the circuit

$\therefore$ Potential differences across the condenser = q/C, potential difference across inductor = $L\frac{dI}{dt}$

potential difference across resistance = RI

$\therefore$ The voltage equation of the circuit is $L\frac{dI}{dt} + RI + \frac{q}{C} = E = E_0 \sin\omega t$

As $I = \frac{dq}{dt}$, therefore, $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

$\therefore$ The voltage equation becomes $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0 \sin\omega t$ ....(2)

This is like the equation of a forced, damped oscillator. Let the solution of equation (2) be

$$q = q_0 \sin(\omega t + \theta)$$

$\therefore$ $\frac{dq}{dt} = q_0\,\omega\cos(\omega t + \theta)$ $\qquad$ $\frac{d^2q}{dt^2} = -q_0\,\omega^2 \sin(\omega t + \theta)$

Substituting these values in equation (2), we get

$$L[-q_0\,\omega^2 \sin(\omega t + \theta)] + R\,q\omega\cos(\omega t + \theta) + \frac{q_0}{C}\sin(\omega t + \theta)\,] = E_0 \sin\omega t$$

$$q_0\,\omega\,[R\cos(\omega t + \theta) - \omega L\sin(\omega t + \theta) + \frac{1}{\omega C}\sin(\omega t + \theta) = E_0\sin\omega t$$

As $\omega L = X_L$ and $\frac{1}{\omega C} = X_C$, therefore,

$$q_0\,\omega\,[R\cos(\omega t + \theta) + (X_C - X_L)\sin(\omega t + \theta)] = E_0\sin\omega t$$

Multiplying and dividing above equation by *Z*, we get

$$q_0\,\omega Z\left[\frac{R}{Z}\cos(\omega t + \theta) + \frac{X_C - X_L}{Z}\sin(\omega t + \theta)\right] = E_0\sin\omega t \qquad \text{....(3)}$$

Let $\frac{R}{Z} = \cos\phi$ and $\frac{X_C - X_L}{Z} = \sin\phi$ ....(4)

so that $\tan\phi = \dfrac{X_C - X_L}{R}$ ....(5)

Substituting equation (4) in equation (3), we get

$$q_0\,\omega\,Z\,[\cos(\omega t+\theta)\cos\phi + \sin(\omega t+\theta)\sin\phi] = E_0 \sin\omega t$$

or $$q_0\,\omega\,Z\cos(\omega t+\theta-\phi) = E_0\sin\omega t = E_0\cos(\omega t - \pi/2)$$

Comparing the two sides of this equation, we find that

$$E_0 = q_0\,\omega Z = I_0 Z, \quad \text{where } I_0 = q_0\,\omega$$

and $$\omega t + \theta - \phi = \omega t - \pi/2 \quad \therefore \quad \theta - \phi = -\frac{\pi}{2} \text{ or } \theta = -\frac{\pi}{2}+\phi$$

$\therefore$ Current in the circuit is $I = \dfrac{dq}{dt} = \dfrac{d}{dt}[q_0\sin(\omega t+\theta)] = q_0\omega\cos(\omega t+\theta)$

$$I = I_0\cos(\omega t+\theta))$$

$\therefore$ $$I = I_0\cos(\omega t+\phi-\pi/2)$$

$$I = I_0\sin(\omega t+\phi)$$ ....(6)

From (5), we get $\phi = \tan^{-1}\dfrac{(X_C - X_L)}{R}$

As $\cos^2\phi + \sin^2\phi = 1$

$\therefore$ $\left(\dfrac{R}{Z}\right)^2 + \left(\dfrac{X_C - X_L}{Z}\right)^2 = 1$ [Using equation (4)]

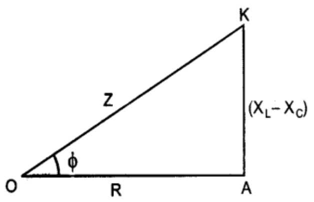
or $R^2 + (X_C - X_L)^2 = Z^2$

or $Z = \sqrt{R^2 + (X_C - X_L)^2}$ ....(7)

The equations (5), (6) and (7) for current, phase angle and impedance agree perfectly with the corresponding equations obtained by the technique of phasors.

## ***Impendence Triangle***

Resistance (R) stands for ohmic resistance. It arises on account of material of the conductor ($R = \rho\, l/a$). The reactance (X) is the resistance that arises on account of opposing e.m.f. induced due to change in the strength of current. The inductive reactance, $X_L = \omega L$ is the resistance offered by the inductor coil. The capacitate reactance $X_C = \dfrac{1}{\omega C}$ is the resistance offered by the condenser. As voltage across the inductor leads the current by a phase angle of $90^o$ and voltage across the capacitor lags behind the current by a phase angle of $90^o$, therefore $X_L$ and $X_C$ have opposite signs. Total reactance is taken to be $\pm(X_L - X_C)$.




The total effective resistance offered by the LCR circuit is called Impedance. It is represented by Z.

From the three phasors, $\vec{V}_R = \vec{I}_0\ R$ ; $\vec{V}_L = \vec{I}_0\ X_L$ and $\vec{V}_C = \vec{I}_0\ X_C$, we obtain, what is known as Impedance triangle, as shown in figure. The base OA of this triangle represents ohmic resistance R, the perpendicular AK represents reactance $(X_L - X_C)$ and diagonal OK represents the impendance (Z) of the a.c. circuit. Therefore impendance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $\angle AOK = \phi$ is the phase angle by which voltage leads the current in the circuit, where $\tan\phi = \dfrac{X_L - X_C}{R}$.

The reciprocal of reactance is called susceptance of the a.c. circuit and reciprocal of impendance is called admittance of a.c. circuit. Both, susceptance and admittance are measured in mho, i.e., $\text{ohm}^{-1}$ or Siemen.

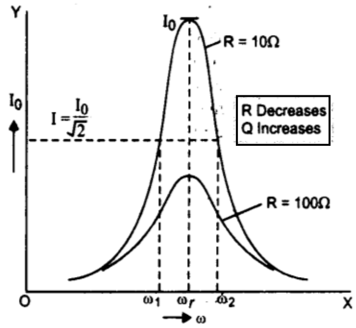
## Electric Resonance

The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the natural frequency of oscillation of the system. If such a system is driven by an energy source, whose frequency is equal to the natural frequency of the system, the amplitude of oscillation becomes large and resonance is said to occur.

**(a) Series Resonance Circuit :** A circuit in which inductance L, capacitance C and resistance R are connected in series, and the circuit admits maximum current corresponding to a given frequency of a.c., is called series resonance circuit.

The impedance (Z) of an LCR circuit is given by $z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At very low frequencies, inductive reactance $X_L = \omega L$ in negligible, but capacitiative reactance $\left(X_C = \frac{1}{\omega C}\right)$ is very high. As frequency of alternating e.m.f. applied to the circuit is increased, $X_L$ goes on increasing and $X_C$ goes on decreasing. For a particular value of $\omega\ (= \omega_0$, say).




$$X_L = X_C$$

i.e.,
$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi v_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad v_0 = \frac{1}{2\pi\sqrt{LC}}$$

At this particular frequency $V_0$, as $X_L = X_C$, therefore, $Z = \sqrt{R^2 + 0} = R = \min$ i.e., impedance of LCR circuit is minimum and hence the current $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ becomes maximum.

This frequency is called series resonance frequency. $\frac{1}{2\pi\sqrt{LC}}$ is the natural frequency of oscillation of an LC circuit containing no resistance. The variation of circuit peak current with the changing frequency of applied voltage is shown in fig. It is clear that for frequencies greater than or less than $\omega_0$ the values of peak current are less than the maximum value ($I_0$). Further, $\omega = \omega_0$, value of peak current is maximum ($I_0$). The maximum value of peak current is inversely proportional to R. For lower R values, $I_0$ is large and vice–versa.

As series resonance circuit admits maximum peak current through it. At resonance, it is called acceptor circuit. The particle application of series resonance circuit is in radio and T.V. receiver sets. The antenna of a radio/T.V. intercepts signals from many broadcasting stations. To receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the desired station. Therefore, resonance occurs. The amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

**Note :** The phenomenon of Resonance is exhibited by a circuit when both *L* and *C* are present in the circuit. Only then, the voltages across *L* and *C* may cancel each other, being opposite in phase. The total source voltage appears across R giving maximum current $I_0 = E_0 / R$. Obviously, we cannot obtain resonance in RL or RC circuit.

## Q Factor of resonance circuit or sharpness of resonance

The characteristic of a series resonant circuit is determined by the Q factor or Quality factor of the circuit. It defines sharpness of tuning at resonance.

The Q factor of series resonant circuit is defined as the ratio of the voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across R.

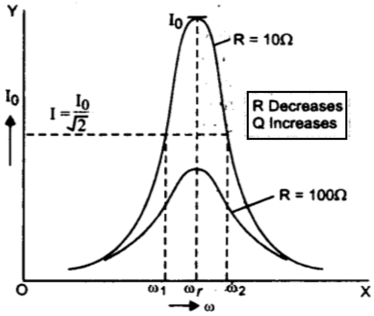
i.e.,
$$Q = \frac{\text{voltage across L or C}}{\text{applied voltage} (= \text{voltage across R})}$$

$$Q = \frac{(\omega_0 L)I}{RI} = \frac{\omega_0 L}{R} \quad \text{or} \quad Q = \frac{\left(\frac{1}{\omega_0 c}\right)I}{RI} = \frac{I}{RC\,\omega_0}$$

Using $\omega_0 = \frac{1}{\sqrt{LC}}$, we get

$$Q = \frac{L}{R}\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{or} \quad Q = \frac{1\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$




It can be shown that Q is just a number having no dimensions. Thus Q factor may also be taken as voltage multiplication factor of the circuit.

As R is increased, Q factor of the circuit decreases. This is clear from as shown in fig. The electronic circuits with high Q values would respond to a very narrow range of frequencies and vice–versa. The values of Q usually vary from 10 to 100. However, the electronic circuits dealing with very high frequencies may have Q = 200. It should be clearly understood that higher the value of Q, the narrower and sharper is the resonance.

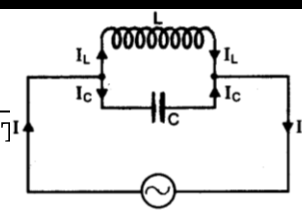
**Another Definition of Q Factor**

Suppose we choose a value of ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value. From the above figure , we find that there are two such values of ω, symmetrical about $\omega_0$ say $\omega_1$ and $\omega_2$, one smaller and the other greater than $\omega_0$. We may write $\omega_1 = (\omega_0 - \Delta\omega)$ and $\omega_2 = (\omega_0 + \Delta\omega)$. The difference $(\omega_2 - \omega_1) = 2\Delta\omega$ is often called the band width of the circuit. The quantity $\left(\frac{\omega_0}{2\Delta\omega}\right)$ is regarded as a measure of sharpness of resonance, i.e., Q factor of resonance circuit is the ratio of resonance angular frequency to band width of the circuit (which is difference in angular frequencies at which power is half the maximum power or current is $I_0/\sqrt{2}$).

Clearly, smaller the band width, sharper or narrower is the resonance. If the resonance is less sharp, not only is the maximum current less, the circuit is close to resonance for larger range (2Δω) of frequencies. Therefore, tuning of the circuit will not be good. Thus, less sharp is resonance, lesser is the selectivity of the circuit and vice–versa.

## Parallel Resonance Circuit

A parallel resonance circuit consists of a coil of inductance L and a condenser of capacity C, joined in parallel to a source of alternating e.m.f. as shown in fig.



Let the alternating emf. supplied by the source

$$E = E_0 \sin \omega t$$

As current through L lags behind the applied alternating e.m.f. by a phase angle π/2, therefore, we may write the value of current through *L* at a particular instant of time *t* as

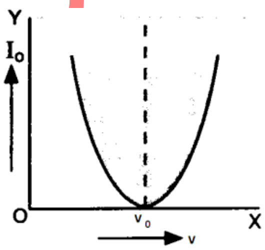
$$I_L = \frac{E_0}{X_L} \sin(\omega t - \pi/2)$$

Again, as current through C leads the applied alternating e.m.f. by a phase angle π/2, therefore, we may write the value of current through C at same instant t as

$$I = \frac{E_0}{X_C} \sin(\omega t + \pi/2)$$

The total current I in the circuit at this instant is, therefore, $I = I_L + I_C$

$$I = \frac{E_0}{X_L} \sin(\omega t - \pi/2) + \frac{E_0}{X_C} \sin(\omega t + \pi/2)$$




$$I = \frac{E_0}{X_L}(-\cos\omega t) + \frac{E_0}{X_C}\cos\omega t = E_0 \cos\omega t\left[-\frac{1}{X_L} + \frac{1}{X_C}\right]$$

$$I = E_0 \cos\omega t\left[-\frac{1}{\omega L} + \omega C\right]$$

We find that I = 0, when $\omega C - \frac{1}{\omega L} = 0$, i.e., $\omega C = \frac{1}{\omega L}$ or $\omega^2 = \frac{1}{LC}$

or $\omega = \frac{1}{\sqrt{LC}}$ or $2\pi v = \frac{1}{\sqrt{LC}}$ or $v = \frac{1}{2\pi\sqrt{LC}}$

i.e., frequency of applied alternating e.m.f. becomes equal to natural frequency of oscillation of the circuit. Hence resonance occurs. The circuit is called parallel resonance circuit and this frequency v is called parallel resonance frequency. Therefore, at parallel resonance frequency, I = 0, i.e., the resonance circuit does not allow any current to flow through it, as shown in fig. The impedance of parallel resonance circuit at this frequency must obviously be maximum.

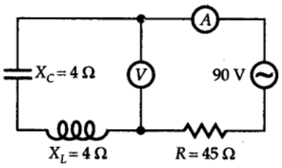
The parallel resonance circuits are used in the transmitting circuits. They reject the currents corresponding to parallel resonance frequencies and allow other frequencies to pass through. Such circuits are, therefore, called filter circuits or rejector circuits or even anti resonance circuits.

## Subjective Assignment – III

1 Determine the impedance of a series LCR–circuit if the reactance of C and L are 250 Ω and 220 Ω respectively and R is 40Ω.

2 A resistor of 50 ohm, an inductor of (20/π) H and a capacitor of (5/ π) μF are connected in series to a voltage source 230 V, 50 Hz. Find the impedance of the circuit.

3 What will be the readings in the voltmeter and ammeter of the circuit shown in figure.




4 A 0.3 H inductor, 60 μF capacitor and a 50Ω resistor are connected in series with a 120 V, 60 Hz supply. Calculate (i) impedance of the circuit, (ii) current flowing in the circuit

5 A resistor of 12 ohm, a capacitor of reactance 14 ohm and a pure inductor of inductance 0.1 henry are joined in series and placed across a 200 volt, 50 Hz a.c. supply. Calculate

(i) the current in the circuit and (ii) the phase angle between the current and the voltage.

6 A 100 mH inductor, a 20 μF capacitor and a 10 ohm resistor are connected in series to a 100 V, 50 Hz a.c. source. Calculate:

(i) Impedance of the circuit at resonance (ii) Current at resonance (iii) Resonant frequency

7 A series LCR circuit consists of a resistance of 10Ω, a capacitor of reactance 60Ω and an inductor coil. The circuit is found to resonate when put across 300 V, 100 Hz supply. Calculate:

(i) the inductance of the coil (ii) current in the circuit at resonance

8 A resistance of 2 ohms, a coil of inductance 0.01 H are connected in series with a capacitor, and put across a 200 volt, 50 Hz supply. Calculate:

(i) the capacitance of the capacitor so that the circuit resonates.

(ii) the current and voltage across the capacitor at resonance.

9 An inductor coil joined to a 6V battery draws a steady current of 12 A. This coil is connected in series to a capacitor and a.c. source of alternating emf 6 V. If the current in the circuit is in phase with the emf, find the rms current.

10 A radio wave of wavelength 300 m can be transmitted by a transmission centre. A condenser of capacity 2.4 μF is available. Calculate the inductance of the required coil for resonance.

11 A 25.0 mF capacitor, a 0.10 henry inductor and a 25.0 ohm resistor are connected in series with an A.C. source whose emf is given by $\xi = 310 \sin 314\, t$ (volt).

(i) What is the frequency of the emf? (ii) What is the reactance of the circuit?

(iii) What is the impedance of the circuit? (iv) What is the current of the circuit?

(v) What is the phase angle of the current by which it leads or lags the applied emf?

(vi) What is the expression for the instantaneous value of current in the circuit?

(vii) What are the effective voltages across the capacitor, the inductor and the resistor?

(viii) Construct a vector diagram for these voltages.

(ix) What value of inductance will make the impedance of circuit minimum?

12 A 2 μF capacitor, 100Ω resistor and 8 H inductor are connected in series with an a.c. source. What should be the frequency of the a.c. source, for which the current drawn in the circuit is maximum? If the peak value of emf of the source is 200 V, find for maximum current:

(i) the inductive and capacitive reactive reactance of the circuit,

(ii) total impedance of the circuit,

(iii) peak value of current in the circuit,

(iv) the phase difference between voltages across inductor and resistor, and

(v) the phase difference between voltages across inductor and capacitor.

13 In a series LCR–circuit, the resonant frequency is 800 Hz. The half power points are obtained at frequencies 745 and 855 Hz. Calculate the Q–factor of the circuit. Also calculate the bandwidth.

14 A resistor of 50Ω, an inductor of 20/π H and a capacitor of 5/π μF are connected in series to a voltage supply of 230 V – 50 Hz. Find the impedance of the circuit.

15 A 40 Ω resistor, 3 mH inductor and 2 μF capacitor are connected in series in a 110 V, 5000 Hz a.c. source. Calculate the value of the current in the circuit.

16 An inductor 200 mH, a capacitor C and a resistor 10 ohm are connected in series with a 100 V, 50 $s^{-1}$ a.c. source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor.

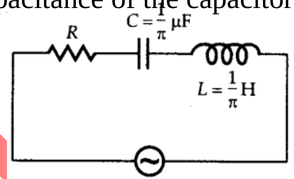
17 A 50 μF capacitor, 0.05 H inductor and a 48Ω resistor are connected in series with an a.c. source of emf, $\xi = 310 \sin 314\, t$. Calculate the reactance of the circuit and tell its nature. What is the phase angle between the current and the applied emf?

18 An LCR–series circuit with L = 100 mH, C = 100 μF, R = 120 Ω is connected an a.c. source of emf $\xi = 30 \sin 100\, t$ volt. Find the impedance, peak current and resonant frequency of the circuit.

19 A 12Ω resistance and an inductance of 0.05/ πH with negligible resistance are connected in series. Across the ends of this is connected a 130 V alternating voltage of frequency 50 Hz. Calculate the alternating current in the circuit and the potential difference across the resistance and across the inductance.

20 A capacitor, resistor of 5Ω and an inductor of 50 mH are in series with an a.c. source marked 100 V, 50 Hz. It is found that voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the circuit.

22 In the a.c. circuit shown in figure, the main supply has constant voltage but variable frequency. For what frequency will the voltage across the resistance R be maximum?




23 An a.c. source of frequency 50 hertz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit, so that the bulb glows with maximum brightness.

24 A 200 km long telegraph wire has a capacity of 0.014 μF per km. If it carries an alternating current of 50 kHz, what should be the value of an inductance required to be connected in series so that impedance is minimum?

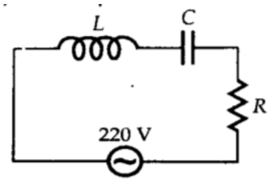
25 Figure shows a series LCR circuit connected to a variable frequency 200 V source:

L = 4.0 H, C = 100 μF and R = 40Ω

(i) Calculate the resonant frequency of the circuit

(ii) Obtain the impedance of the circuit and the amplitude of the current at resonating frequency

(iii) Determine r.m.s. potential drop across L




26 A series circuit with L = 0.12 H, C = 0.48 mF and R = 25Ω is connected to a 220 V variable frequency power supply. At what frequency is the circuit current maximum?

27 Find the capacitance reactance of 10 μF capacitor at 1000 cycles $s^{-1}$. Calculate the inductance required to produce series resonance with the capacitor at this frequency.

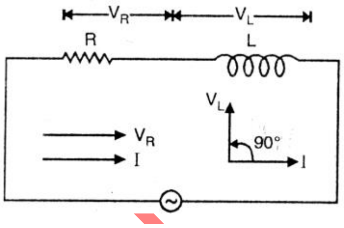
28 Compute the resonant frequency and the Q–factor of a series LCR–circuit having L = 4.0 H, C = 36 μF and R = 10/3Ω. How can the sharpness of the resonance of the circuit be improved by a factor of 2 by reducing its full width at half maximum?

## Answers

1. 50Ω 2. 50Ω 3. 2A, 0
4. (i) 85Ω, (ii) 1.41 A 5. (i) 10 A, (ii) 53.1°
6. (i) 10 Ω, (ii) 10 A, (iii) 112.6 Hz 7. (i) 0.095 H, (ii) 30 A
8. (i) $11 \times 10^{-4}$ F, (ii) 100 A, 303.03 V 9. 12 A
10. $1.055 \times 10^{8}$ H
11. (i) 50 Hz, (ii) 95.9Ω, (iii) 99. 1Ω, (iv) 2.22 A, (v) 1.31 rad, (vi) 3.13 sin (314t + 1.31), (vii) 282.6 V, 69.7 V, 55.5 V, (ix) 0.405 H
12 39.8 Hz, (i) 2000 Ω, (ii) 100 Ω, (iii) 2A, (iv) 90°, (v) 180° 13. (i) 7.27, (ii) 110 Hz
14. 50 Ω 15. 1.25 A 16. 50.6 μF
17. 48Ω (capacitive), 45° (current leads voltage) 18. 150Ω, 0.2 A, 50 Hz

| | | | | | |
|---|---|---|---|---|---|
| 19. | 10 A, 120 V, 50 V | 21. | $2.02 \times 10^{-4}$ F, 5Ω | 22. | 500 Hz |
| 23. | $2.02 \times 10^{-4}$ F | 24. | 0.357 mH | | |
| 25. | (i) 50 rad $s^{-1}$, (ii) 40Ω, 7.78 A, (iii) 1100 V | | | 26. | 21 Hz |
| 27. | 15.92Ω, $2.534 \times 10^{-3}$ H | 28. | 250/3 rad $s^{-1}$, to double Q, R should be halved | | |

## A.C. circuit containing resistance and inductance




Alternating e.m.f. be connected to an ohmic resistance R and a coil of inductance L, in series as shown in fig. Proceeding as in A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (LCR Circuit) in fig. and ignoring $\vec{V}_c$ as there is no capacitor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_L^2} \quad \text{(since by putting } X_c = 0)$$

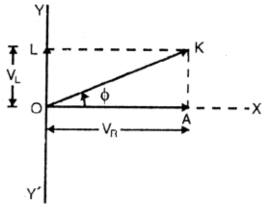


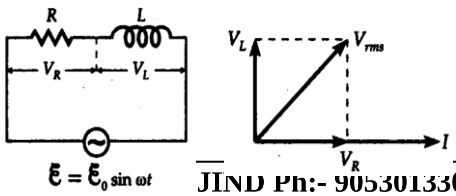

Fig. represents phasor diagram of RL circuit. We find that in RL circuit, voltage leads the current by a phase angle ϕ, where

$$\tan\phi = \frac{AK}{OA} = \frac{OL}{OA} = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R}$$

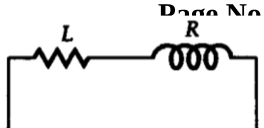
$$\tan\phi = \frac{X_L}{R} \quad \text{(since by putting } X_c = 0)$$

## Subjective Assignment– IV

1. When an inductor L and a resistor R in series are connected across a 12V, 50Hz, supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by π/3 radian. Calculate the value of R.
2. A bulb of resistance 10Ω, connected to an inductor of inductance L, is in series with an a.c. source marked 100V, 50Hz. If the phase angle between the voltage and current is π/4 radian, calculate the value of L.
3. A coil of resistance 300Ω and inductance 1.0 H is connected across an alternating voltage of frequency 300/2π Hz. Calculate the phase difference between the voltage and current in the circuit.
4. A coil when connected across a 10 V d.c. supply draws a current of 2A. When it is connected across a 10V – 50Hz a.c. supply, the same coil draws a current of 1A. Explain why it draws lesser current in the second case. Hence determine the self inductance of the coil.
5. An 80V, 800 W heater be operated on a 100V, 50Hz supply. Calculate inductance of the choke required.
6. A student connects a long air core coil of manganin wire to a 100 V d.c. source and record a current of 1.5 A. When the same coil is connected across 100V, 50Hz a.c. source current reduces to 1.0 A.
   (i) Give reason for this observation, (ii) Calculate the value of the reactance of the coil.
7. When 200 volts d.c. are applied across a coil, a current of 2 ampere flows through it. When 200 volts a.c. of 50 cps are applied to the same coil, only 1.0 ampere flows. Calculate the resistance, impedance and inductance of the coil.
8. A 60 V – 10W electric lamp is to be run on 100V – 60Hz mains. (i) Calculate the inductance of the choke required. (ii) If a resistor is to be used in place of choke coil to achieve the same result, calculate its value.
9. A 12 Ω resistance and an inductance of 0.05/π H are connected in series. Across the ends of the circuit is connected a 130V a.c. supply of 50 Hz. Calculate (i) the current in the circuit and (ii) phase difference between the current and voltage.
10. The a.c. circuit shown in fig. has a choke L and a resistance R. The potential difference across the resistance R is $V_R$ = 160 V and that across the choke is $V_L$ = 120V. Find the virtual value of the applied voltage. If the virtual current in

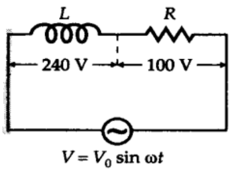




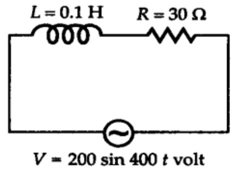



the circuit be 1.0 A, then calculate the total impedance of circuit. If the direct current be passed in the circuit, then what will be the potential difference in the circuit?

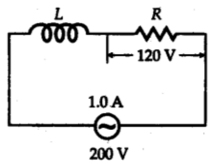
11. In the circuit shown in fig. the potential difference across the indicator L and resistor R are 120V and 90V respectively and the rms value of current is 3A. Calculate (i) the impedance of the circuit and (ii) the phase angle between the voltage and current.
12. Calculate the impendance of a coil of resistance 3Ω and reactance 4Ω.
13. An inductance coil has a resistance of 100 ohm. When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by $45^o$. Calculate the self–inductance of the coil.
14. An a.c. source of 100V r.m.s., 50 Hz is connected across a 20Ω resistor and 2mH inductor in series. Calculate (i) impedance of the circuit and (ii) r.m.s. current in the circuit.
15. A 100V, 50 Hz a.c. source is connected to a series combination of an inductance of 100 mH and a resistance of 20Ω. Calculate the magnitude and phase of the current.
16. A current of 11A flows through a coil, when connected to a 110V d.c. source. When 110V a.c. of 50 Hz is applied to the coil, only 0.5 A current flows. Calculate the (i) resistance (ii) impedance and (iii) inductance of the coil.
17. An arc takes a current of 10A at 80V. Find the inductance, which should be put in series to work the arc from 220V, 50Hz supply.
18. A current of 2.0 A is flowing in the LR-circuit shown in fig. Find the voltage across the a.c. source and the impedance of the circuit.




19. Calculate the impedance and the rms current in the a.c. circuit shown in fig.



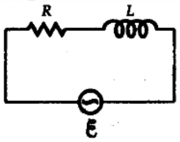

20. The virtual current in the a.c. circuit shown in fig. is 1.0 A. Find (i) virtual voltage across the coil L, (ii) impedance of the circuit and (iii) reactance of the coil.




21. A circuit containing a resistance of 50Ω and an inductance of (1/π) H in series is connected to a 200V a.c. line of frequency 60 Hz. Find the reactance, the impedance, the current in the circuit and the phase difference between the alternating voltage and current.
22. An a.c. circuit consists of a 220V, 50Hz supply connected across a 100Ω resistor. What inductance should be connected in the circuit in series with resistance so that the current is reduced to half?
23. A long solenoid connected to a 12V d.c. source passes a steady current of 2A. When the solenoid is connected to an a.c. source of 12V at 50Hz, the current flowing is 1A. Calculate the inductance of the solenoid.
24. A choke coil and a resistance are connected in series in an a.c. circuit. a potential difference of 130V is applied to the circuit. If the potential difference across the resistance is 50V, what should be the potential difference across the choke coil?
25. An emf, ξ = 200 sin 377t volt is applied across an inductance L having a resistance of 1.0Ω. The maximum current is found to be 10A. Find the value of L.

26. An electric circuit containing an inductance L and resistance R in series has an impedance of 50Ω at 100Hz and an impedance of 100Ω at 500Hz. Find the values of L and R.

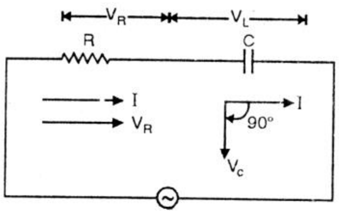
27. In the RL–circuit shown in fig. resistance R = 30Ω, reactance $X_L$ = 40Ω and peak emf = 220V. Calculate the (i) impedance Z, (ii) phase difference between the emf and current and (iii) the peak current $I_0$ in the circuit.




## Answers

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 12 Ω | 2. | 0.0318 H | 3. | $45^o$ | 4. | 0.0288 H |
| 5. | 0.019 H | 6. | 74.53 Ω | 7. | (i) 100 Ω , (ii) 200 Ω (iii) 0.55 H | | |
| 8. | 1.273 H (ii) 240 Ω | 9. | 10 A, $22.6^o$ | 10. | 200V, 200 Ω, 160 V | | |
| 11. | 50 Ω, $53.1^o$ | 12. | 5Ω | 13. | 0.016 H | | |
| 14. | 20Ω, 5A | 15. | 2.68 A, current lags behind voltage by $57.5^o$ | | | | |
| 16. | (i) 10Ω (ii) 220 Ω (iii) 0.7 H | | | 17. | 0.065 H | | |
| 18. | 260V, 130Ω | 19. | 50Ω, $2\sqrt{2}$ A | 20. | (i) 160 V (ii) 200 Ω (iii) 160 Ω | | |
| 21. | 120 Ω, 130Ω, 1.538 A, $\tan^{-1} 24$ | | | 22. | 0.55 H | | |
| 23. | 33 mH | 24. | 120 V | 25. | 53 mH | | |
| 26. | 28.13 mH, 46.77 Ω | 27. | (i) 50Ω, (ii) $53.1^o$ , (iii) 4.4 A | | | | |

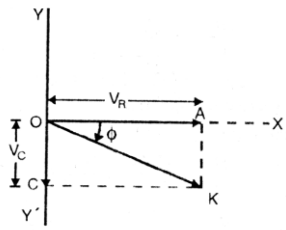
## A.C. circuit containing resistance and capacitance




Let a source of alternating e.m.f. be connected to an ohmic resistance R and a condenser of capacity C, in series as shown in fig. Proceeding as in a.c. circuit containing resistance, inductance and capacitance in series (LCR circuit) in fig. and ignoring $\vec{V}_L$ as there is no inductor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_C^2} \quad \text{(since by putting } X_L = 0)$$

Fig. represents phasor diagram of RC circuit. We find that in RC circuit, voltage lags behind the current by a phase angle φ, where




$$\tan\phi = \frac{AK}{OA} = \frac{OC}{OA} = \frac{V_C}{V_R} = \frac{I_0 X_C}{I_0 R}$$

$$\tan\phi = \frac{X_C}{R} \quad \text{(since by putting } X_L = 0)$$

**Note :** In all a.c. circuits, the relation that holds in $\frac{E_v}{I_v} = Z$, where

$Z = R$ , in case of a.c. circuit containing R only

$Z = X_L$ in case of a.c. circuit containing L only.

$Z = X_C$ in case of a.c. circuit containing *C* only

$Z = \sqrt{R^2 + X_L^2}$ , in case of a.c. circuit containing R & L.

$Z = \sqrt{R^2 + X_C^2}$ , in case of a.c. circuit containing R and C.

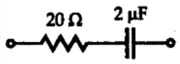
$Z = X_L - X_C$ , in case of a.c. circuit containing L and C.

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ , in case of a.c. circuit containing R, L and C.

The phase relation between alternating voltage and current in any a.c. circuit is given by $\tan\phi = \frac{X_L - X_C}{R}$

## Subjective Assignment – V

1. What is the value of current in the a.c. circuit containing R = 10Ω, C = 50μF in series across 200V, 50Hz a.c. source?
2. When an alternating voltage of 220 V is applied across a device X, a current of 0.5A flows through the circuit and is in phase with the applied voltage. When same voltage is applied across another device Y, the same current flows through the circuit but it leads the applied voltage by π/2 radian. (i) Name the devices X and Y. (ii) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y.
3. A series circuit contains a resistor of 20Ω, a capacitor and an ammeter of negligible resistance. It is connected to a source of 220V – 50Hz. If the reading of the ammeter is 2.5A, calculate the reactance of the capacitor.
4. An alternating current of 1.5mA rms and angular frequency ω = 100 rad $s^{-1}$ flows through a 10kΩ resistor and 0.50 μF capacitor in series. Calculate the value of rms voltage across the capacitor and the impendence of the circuit.
5. A 20V – 5W lamp is to run on 200V – 50Hz a.c. mains. Find the capacitance of a capacitor required to run the lamp.
6. A resistor of 200Ω and a capacitor of 15.0 μF are connected in series to a 220V, 50Hz as source. (a) Calculate the current in the circuit : (b) Calculate the voltage (rms) across the resistor and the capacitor, Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.
7. In a series R – C circuit, R = 30Ω, C = 0.25 μF, V = 100 V and ω = 10,000 rad $s^{-1}$. Find the current in the circuit and calculate the voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.
8. An a.c. circuit consists of a series combination of circuit elements 'X' and 'Y'. The current is ahead of the voltage in phase by π/4. if element 'X' is a pure resistor of 100Ω, (i) name the circuit element 'Y' and (ii) calculate the rms value of current, if rms value of voltage is 141V.
9. A circuit containing a 20Ω resistor and 0.1μF capacitor in series is connected to 230V a.c. supply of angular frequency 100 rad $s^{-1}$. What is the impedance of the circuit?
10. A circuit consists of a resistance of 10Ω and a capacitance 0.1μF. If an alternating emf of 100V, 50Hz is applied, find the current in the circuit.
11. A 20W, 50V, filament is connected in series to an a.c. mains of 250V, 50Hz. Calculate the value of the capacitor required to run the lamp.
12. Find the impedance of the circuit shown in fig. for (i) direct current and (ii) alternating current of frequency 10/π kHz.




13. A 1 μF capacitor is connected to a 220 V –50 Hz a.c. source. Find the virtual value of current through circuit. What is the peak value of voltage across the capacitor?
14. A capacitor in series with a resistance of 30 Ω is connected to a.c. mains. The reactance of the capacitor is 40 Ω. Calculate the phase difference between the current and the supply voltage.
15. A circuit has a resistance of 10Ω and its impedance is $100\sqrt{2}\,\Omega$. Find the reactance of the circuit.

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | 3.10 A | 2. | (i) x is resistor and y is capacitor, (ii) 0.35 A | | |
| 3. | 85.7 Ω | 4. | $2.23 \times 10^4\ \Omega$ ; 30V | 5. | 4.0 μF |
| 6. | (a) 0.755 A (b) 151 V , 160.3 V | | | 7. | 0.25 A , 7.5 V , 100 V |
| 8. | (i) capacitor, (ii) 1 A | 9. | $10^5\ \Omega$ | 10. | 3.14 mA |
| 11. | 5.2 μF | 12. | (i) Infinite (ii) 32 Ω | 13. | 0.07 A, 311 V |
| 14. | $\tan^{-1}\frac{4}{3}$ | 15. | 100 Ω | | |

## Energy stored in an inductor

When a.c. is applied to an inductor of inductance L, the current in it grows from zero to maximum steady value $I_0$. If I is the current at any instant t, then the magnitude of induced e.m.f. developed in the inductor at that instant is

$$E = L\frac{dI}{dt} \qquad ....(1)$$

The self induced e.m.f. is also called the back e.m.f., as it opposes any change in the current in the circuit. Physically, the self inductance plays the role of inertia. It is the electromagnetic analogue of mass in mechanics. Therefore, work needs to be done against the back e.m.f. E in establishing the current. This work done is stored, in the inductor as magnetic potential energy.

For the current I at an instant t, the rate of doing work is $\frac{dW}{dt} = EI$.

Using (1), $\frac{dW}{dt} = EI = L\frac{dI}{dt} \times I$ or $dW = LI\,dI$

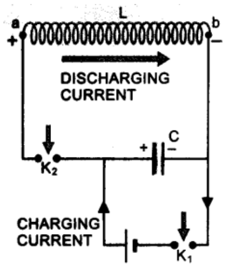
Total amount of work done in establishing the current I is $W = \int dW = \int_0^I LI\,dI = \frac{1}{2}LI^2$.

Thus energy required to build up current in an inductor = energy stored in inductor.

$$U_B = W = \frac{1}{2}LI^2$$

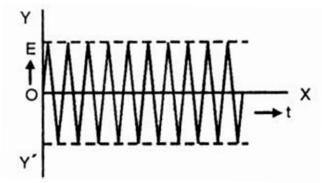
**Note :**When we compare this expression with mechanical kinetic energy of a particle of mass *m*, i.e., $KE = \frac{1}{2}mv^2$, we find that L corresponds to m. As mass m is a measure of inertia of linear motion, self inductance L is a measure of electrical inertia, which opposes both, the growth and decay of current in the circuit.

## LC Oscillations




A capacitor of capacitance C is connected to an inductor of inductance L through a key $K_2$. A cell is connected to C through key $K_1$.

When plug of $K_1$ is put in, the cell charges the capacitor to a potential V = q/C, where q is the charge on capacitor plates at any instant and V is voltage across the plates at the same instant. Some energy from the cell is stored in the dielectric medium between the plates of capacitor in the form of electrostatic energy $\left[U_E = \frac{q^2}{2C}\right]$.

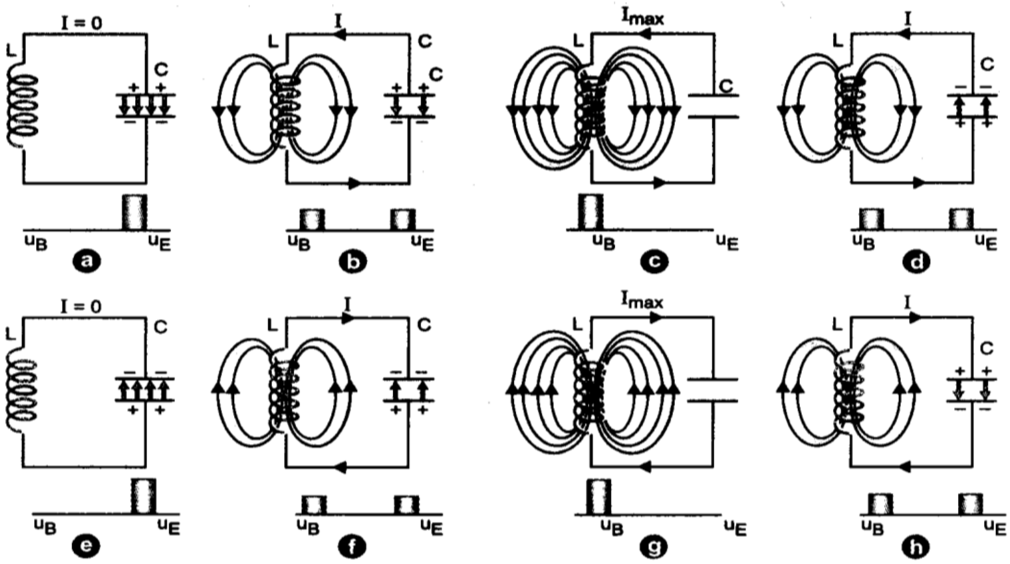



On removing plug of $K_1$ and putting in plug of $K_2$, the charged capacitor is connected to L and starts discharging through L. An induced e.m.f. develops in the circuit which opposes the growth of current in L and hence delays it. When capacitor is completely discharged, the energy stored in the capacitor appears in the form of magnetic field energy around L $\left[U_B = \frac{1}{2}LI^2\right]$.

As soon as discharge of the capacitor is complete, current stops and magnetic flux linked with L starts collapsing. Therefore an induced e.m.f. developes which starts recharging the condenser in the opposite direction. The recharging is also opposed and hence delayed. When condenser is recharged completely, the magnetic field energy around L reappears in the form of electric field energy between the plates.

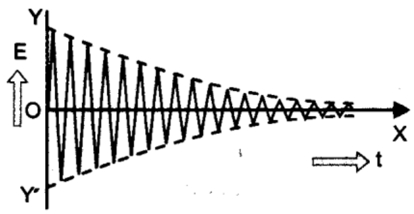
The entire process is repeated. Thus, energy taken once from the cell and given to capacitor keeps on oscillating between C and L. If the circuit has no resistance, no loss of energy would occur. The oscillation produced will be of constant amplitude, as shown in the fig. These are called undamped oscillation.

Fig. shows eight stages in a single cycle of oscillation of a resistance less LC circuit. The bar graphs in each figure represent the stored electric energy $\left(U_E = \frac{q_0^2}{2C}\right)$ and stored magnetic energy $\left(U_B = \frac{1}{2}LI_0^2\right)$. The magnetic field lines of inductor and electric field lines of capacitor are also shown .




The above fig. (a) shows capacitor with maximum charge and no current in the circuit. The above fig. (b) shows discharging of capacitor with increasing current. The above fig. (c) shows capacitor fully discharged and maximum current. The above fig.(d) shows charging of capacitor in opposite direction and current decreasing. The above fig. (e) shows capacitor recharged fully with polarity opposite to that of (a) and no current. The above fig. (f) shows discharging of capacitor, current increasing in opposite direction. The above fig. (g) shows fully discharged capacitor and maximum current in opposite direction. The above fig. (h) show charging of capacitor with current decreasing as shown. This cycle is being repeated.

In actual practice, however, there do occur some losses of energy. Therefore, amplitude of oscillations goes on decreasing. These are called damped oscillations as shown in fig. To obtain undamped oscillations, the energy loss is duly compensated in proper phase.




The frequency of electrical oscillations. It can be obtained mathematically as follows :

In **(LC Oscillation)** shown in fig. (a), the moment the circuit is completed (by putting in plug of $K_2$), charge on condenser starts decreasing, giving rise to current in the circuit. As dI/dt is positive, the induced e.m.f. in L will be such that

$$\therefore \quad \frac{q}{C} - L\frac{dI}{dt} = 0$$

As q decrease; I increases, therefore, $\quad I = -\frac{dq}{dt}$

Therefore, $\quad \frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \quad \text{or} \quad \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

This equation has the form $\frac{d^2x}{dt^2} + \omega^2 x = 0$ for a simple harmonic oscillator.

The charge, therefore, oscillates with a natural frequency,

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi v \qquad \text{so} \qquad \boxed{v = \frac{1}{2\pi\sqrt{LC}}}$$

The variation of charge with time is represented as $q = q_0 \cos \omega t$.

The current, $I = \frac{-dq}{dt} = q_0 \omega \sin \omega t = I_0 \sin \omega t$

where $I_0 = \omega q_0 =$ maximum value of current.

## Analogy between mechanical and electrical quantities

The LC oscillations can be compared to mechanical oscillations of a block of mass *m* attached to a spring of force constant *k*, whose equation is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

In case of mechanical system, $F = ma = m\frac{d^2x}{dt^2}$

For an electrical system, $E = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}$

Comparison shows that q corresponds to *x* and L corresponds to *m*.

In case of LC circuit, $\omega = \frac{1}{\sqrt{LC}}$

We find that $\frac{1}{C}$ is analogous to $k(= F/x)$. Whereas *k* tells us the (external) force required to produce a unit displacement; $\frac{1}{C} = \frac{V}{q}$ tells us the potential difference required to store a unit charge.

| S.No. | Mechanical system | S.No. | Electrical system |
|---|---|---|---|
| 1. | Mass *m* | 1. | Inductance L |
| 2. | Force constant *k* | 2. | Reciprocal capacitance 1/C |
| 3. | Displacement *x* | 3. | Charge *q* |
| 4. | Velocity, $v = \frac{dx}{dt}$ | 4. | Current, $I = \frac{dq}{dt}$ |
| 5. | Mechanical energy, $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ | 5. | Electromagnetic energy, $U = \frac{q^2}{2C} + \frac{1}{2}LI^2$ |

## Subjective Assignment – VI

1 Calculate the wavelength of radio waves radiated out by a circuit consisting of 0.02 μF capacitor and 8 μF inductor in series.

2 An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC–circuit is set oscillating at its natural frequency. Let q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that maximum value of charge q is 200 μC.

(a) When q = 100 μC, what is the value of $\frac{dI}{dt}$? (b) When q = 200 μC, what is the value of I?

(c) Find the maximum value of I.

(d) When I is equal to one–half its maximum value, what is the value of q?

3 A coil of inductance 150 mH is connected in series with a variable capacitor of capacitance 20 pF to 500 pF. Calculate the frequency range over which the circuit can be tuned.

4 A 10 μF capacitor is charged to a potential of 25 V. The battery is then disconnected and pure 100 mH coil is connected across the capacitor so that LC–oscillations are set up. Calculate the maximum current in the coil.

5 A 1.5 mH inductor in an LC–circuit stores a maximum energy of 30 μJ. What is the maximum current in the circuit?

6 In an oscillatory circuit, the self–inductance of the coil used is 10 mH. If the oscillatory frequency of the circuit is 1.0 MHz, find the capacitance of the capacitor connected in the circuit.

7 A wave of wavelength 300 m can be radiated through a transmitter. A capacitor of capacitance 2.4 μF is available. What is the inductance of the coil required for the oscillatory circuit?

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | $7.54 \times 10^2$ m | 2. | (a) $10^4$ As$^{-1}$, (b) 0, (c) 2.0 A , (d) 173.2 μC | | |
| 3. | $1.84 \times 10^4$ Hz to $9.2 \times 10^4$ Hz | 4. | 0.25 A | 5. | 0.2 A |
| 6. | 2.53 pF | 7. | $1.056 \times 10^{-8}$ H | | |

## Average power associated with resistance or non inductive circuit

In the a.c. circuit, the value of voltage and current change every instant. Therefore power in an a.c. circuit at any instant is the product of instantaneous voltage (E) and instaneous current (I). In a pure resistance, the alternating current developed is in phase with the alternating voltage applied, i.e., when $E = E_0 \sin\omega t$, then $I = I_0 \sin\omega t$

Intantaneous power = $E\,I = (E_0 \sin\omega t)\, I_0 \sin\omega t) = E_0\, I_0 \sin^2 \omega t$

If the instantaneous power remains constant for a small time *dt*, then small amount of work done in maintain the current for a small time dt is $dW = E_0\, I_0\, \sin^2 \omega t\, dt$

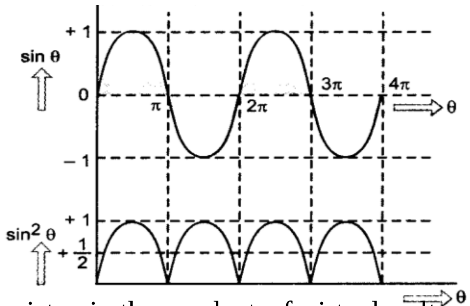
Total work done or energy spent in maintaining current over one full cycle.

$$W = \int_0^T E_0\, I_0 \sin^2 \omega t\, dt$$

$$= E_0 I_0 \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{E_0\, I_0}{2} \int_0^T dt - \frac{E_0\, I_0}{2} \int_0^T \cos 2\omega t\, dt$$

$$W = \frac{E_0\, I_0}{2}\, T - 0 \quad \left(\text{since } \int_0^T \cos 2\omega t\, dt = 0\right)$$




∴ Average power supplied to R over a complete cycle.

$$P = \frac{W}{T} = \frac{E_0\, I_0}{2} \cdot \frac{T}{T} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}}$$

$$P = E_v\, .\, I_v$$

Hence average power over a complete cycle of a.c. through the resistor is the product of virtual voltage and virtual current.

**Note :** A plot of sinθ versus θ shows that average value of sinθ over one cycle is zero. A plot of $\sin^2\theta$ various θ shows that average value of $\sin^2\theta$ over one cycle is 1/2.

Therefore , $P_{avg} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R \qquad P_{avg} = I_v^2 R = (I_v R)\, I_v = E_v\, I_v$

## Average power associated with an inductor

When a source of ac is connected to an inductor L, current in it grows from zero to maximum steady value $I_0$. An induced e.m.f. develops in the inductor, which opposes the growth of current. Work done by the external source in building up current from zero to $I_0$ is $\frac{1}{2}LI_0^2$. After the current in the inductor reaches its maximum value $I_0$, it falls from $I_0$ to zero. The energy $\frac{1}{2}LI_0^2$ supplied by the source during build up of current is returned back to the source during the fall of current. If the alternating voltage is $E = E_0 \sin\omega t$, then through L , as current lags behind E by a phase angle of $\frac{\pi}{2}$, therefore

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) = -I_0 \cos\omega t$$

As work done in small time dt would be, dw = EI dt

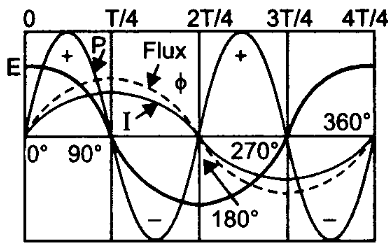
Therefore, $$W = -\int_0^T E_0 \sin\omega t .\, I_0 \cos\omega t \, dt$$

$$W = -E_0 I_0 \int_0^T \sin\omega t \cos\omega t \, dt$$

$$= \frac{-E_0 I_0}{2} \int_0^T 2\sin\omega t \cos\omega t \, dt$$

$$= \frac{-E_0 I_0}{2} \int_0^T \sin 2\omega t \, dt = \frac{-E_0 I_0}{2}\left[\left(-\frac{\cos 2\omega t}{2\omega}\right)\right]_0^T$$

$$W = \frac{E_0 I_0}{4\omega}[\cos 2\omega T - \cos 0] = 0$$




a

Therefore, average power over a complete cycle of a.c. through an ideal inductor is zero. Infact, whatever energy is needed in building up current in L is returned back during the decay of current. The magnetization and demagnetization of an inductor can be understood as follows : Fig. (a) represents one complete cycle of alternating voltage E, alternating current I and the consequent magnetic flux $\phi$ along with power P. Note that current lags behind the voltage by $90^o$. Fig. (b) shows current I entering the coil at A are increasing from zero to maximum from t = 0 to t = T/4. The magnetic fulux lines are set up as shown and core gets magnetized. As voltage and current both are positive, their product P is positive. Therefore, energy is absorbed from the source. From $t = \frac{T}{4}$ to $t = \frac{2T}{4}$, current to the coil is still positive, but is decreasing. The core gets demagnetized, fig. (c), and the net




magnetic flux becomes zero at the end of half cycle. As $\frac{dI}{dt}$ is negative, voltage E is negative.

The product of voltage and current is negative and energy is being returned to the source. From $t = \frac{2T}{4}$ to $t = \frac{3T}{4}$, current I becomes negative, i.e., it enters the coil at B and leaves at A, as shown in figure (d). As the direction of current is reversed, polarity of magnet is reversed.

Since current and voltage, both are negative, their product P is positive. The energy is absorbed from the source. From $t = \frac{3T}{4}$ to $t = T$, I decreases and reaches its zero value at $t = T$. The core is demagnetized and magnetic flux becomes zero, fig. (e). Voltage is positive and current is negative. Their product *P* is negative. Energy absorbed during $\frac{2T}{4}$ to $\frac{3T}{4}$ is returned to the source.

## Average power associated with a capacitor

When a source of a.c. is connected to a capacitor of capacitance *C*, the charge on it grows from zero to maximum steady value $Q_0$. A definite amount of work is done in charging the capacitor which is stored in the capacitor in the form of electrostatic energy. The energy stored in capacitor is $U = \frac{1}{2}CV_0^2$, where $V_0$ is maximum potential difference across the plates of the capacitor. If alternating voltage applied is

$$E = E_0 \sin \omega t$$

then through C, as current leads the e.m.f. by a phase angle of $\pi/2$.

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{or} \qquad I = I_0 \cos \omega t$$

Work done over a complete cycle is

$$W = \int EI\,dt = \int_0^T (E_0 \sin \omega t)(I_0 \cos \omega t)\,dt = \frac{E_0 I_0}{2}\int_0^T 2\sin\omega t \cos\omega t\,dt = \frac{E_0 I_0}{2}\int_0^T \sin 2\omega t\,dt$$

$$w = \frac{E_0 I_0}{2}\left[-\frac{\cos 2\omega t}{2\omega}\right]_0^T = \text{Zero}$$

$\therefore$ Average power supplied to an ideal capacitor by the source over a complete cycle of a.c. is also zero. Infact, whatever energy is needed in building up the voltage across the capacitor is returned back to the source during discharging of the capacitor.

**Note:** Average power/cycle associated each with an inductor and a capacitor is zero. But average power per cycle associated with a resistor is not zero. It is equal to $E_v \times I_v$.

## Average power in LCR circuit

Let the alternating e.m.f. applied to an LCR circuit be

$$E = E_0 \sin \omega t$$

If alternating current developed lags behind the emf applied to an LCR circuit be

$$I = I_0 \sin(\omega t - \phi)$$

Power at instant t, $\frac{dW}{dt} = EI$

$$\frac{dW}{dt} = E_0 \sin\omega t \times I_0 \sin(\omega t - \phi) = E_0 I_0 \sin\omega t(\sin\omega t \cos\phi - \cos\omega t \sin\phi)$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - E_0 I_0 \sin\omega t \cos\omega t \sin\phi$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \quad \text{(since sin2ωt = 2sin ωt cos ωt)}$$

If this instantaneous power is assumed to remain constant for a small time dt, then small amount of work done in this time is

$$dW = \left( E_0 I_0 \sin^2 \omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \right) dt$$

Total work done over a complete cycle is

$$W = \int_0^T E_0 I_0 \sin^2 \omega t \cos\phi \, dt - \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \, dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2 \omega t \, dt - \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t \, dt$$

As $\displaystyle\int_0^T \sin^2 \omega t \, dt = \frac{T}{2}$ and $\displaystyle\int_0^T \sin 2\omega t \, dt = 0$

$\therefore$ $\displaystyle W = E_0 I_0 \cos\phi \times \frac{T}{2}$

$\therefore$ Average power in the LCR circuit over a complete cycle.

$$P = \frac{W}{T} = \frac{E_0 I_0 \cos\phi}{T} \frac{T}{2} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$P = E_v I_v \cos\phi \qquad \text{... (1)}$$

Hence average power over a complete cycle in an inductive circuit is the product of virtual e.m.f. virtual current and cosine of the phase angle between the voltage and current.

**Note :** The relation (1) is applicable to all a.c. circuits. cos$\phi$ and Z will have appropriate values for different circuits.

For example : (i) In RL circuit, $Z = \sqrt{R^2 + X_L^2}$ and $\cos\phi = \dfrac{R}{Z}$

(ii) In RC circuit, $Z = \sqrt{R^2 + X_C^2}$ and $\cos\phi = \dfrac{R}{Z}$

(iii) In LC circuit, $Z = X_L - X_C$ and $\phi = 90°$

(iv) In LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\cos\phi = \dfrac{R}{Z}$

In all a.c. circuits, $I_v = \dfrac{E_v}{Z}$

## Power Factor of an a.c. circuit

That average power/cycle in an LCR circuit is

$$P = E_v I_v \cos\phi$$

Here, P is called true power, $(E_v I_v)$ is called apparent power or virtual power and cos $\phi$ is called power factor of the circuit.

Thus, $$\text{Power factor} = \frac{\text{True power (P)}}{\text{apparent power } (E_v \; I_v)} = \cos\phi$$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \text{[From impedance triangle]}$$

$$\therefore \qquad \text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$$

In a resonating circuit, $X_L = X_C$

$$\therefore \qquad \text{Power factor} = \cos\phi = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1,\ \phi = 0^\circ$$

This is the maximum value of power factor. In a pure inductor or an ideal capacitor, $\phi = 90^\circ$.

$\therefore$ Power factor $= \cos\phi = \cos 90^\circ = 0$.

Average power consumed in a pure inductor or ideal capacitor, $P = E_v I_v \cos 90^\circ = \text{Zero}$. Therefore, current through pure L or pure C, which consumes no power for its maintenance in the circuit is called Idle current or Wattless current.

In actual practice, we do not have ideal inductor or ideal capacitor. Therefore, there does occur some dissipation of energy. However, inductance and capacitance continue to be most suitable for controlling current in a.c. circuits with minimum loss of power.

**Note:** (1) At resonance, $X_L = X_C$ and $\phi = 0^\circ$.

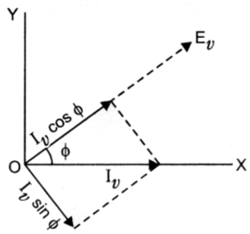
$$\therefore \qquad \cos\phi = \cos 0^\circ = 1$$

therefore, maximum power is dissipated in a circuit at resonance.

(2) That whatever the circuit, power dissipation is always through resistance R.

## Wattless current or Idle current

The current which consumes no power for its maintenance in the circuit is called wattles current or idle current. Average power over a complete cycle in an LCR circuit $P = E_v I_v \cos\phi$. In a particular circuit, suppose $E_v$ leads $I_v$ by phase angle $\phi$, as shown in fig.




$I_v$ can be resolved into two rectangular components :

$$I_v \cos\phi \text{ along } E_v \text{ and } I_v \sin\phi \perp E_v$$

Thus $I_v$ is the vector sum of two perpendicular components

$$I_v \cos\phi \text{ and } I_v \sin\phi.$$

As phase angle between $E_v$ and $I_v \cos\phi$ is zero.

$\therefore$ Average power consumed per cycle in the circuit due to component

$I_v \cos\phi$ is

$$P_{av} = E_v (I_v \cos\phi) \cos 0^\circ = E_v I_v \cos\phi$$

Again, as phase angle between $E_v$ and $I_v \sin\phi$ is $\frac{\pi}{2}$, therefore, average power consumed per cycle in the circuit due to component $I_v \sin\phi$ is

$$P_{av} = E_v (I_v \sin\phi) \cos\frac{\pi}{2} = 0$$

Thus, the average power consumed/cycle in the a.c. circuit is wholly due to component $I_v \cos\phi$ of the virtual current. The component $I_v \sin\phi$ makes no contribution to the consumption of power in the a.c. circuit. That is why this component $I_v \sin\phi$ is called the idle component or wattles component of a.c.

**Note:** At the generation stage, we try to have cosϕ = maximum, i.e., minimum of wattles current. Contrary to this, at the consumption stage, we try to have as much wattles current as possible, i.e., cos ϕ = minimum.

## Choke coil

A choke coil is an electrical appliance used for controlling current in an a.c. circuit. If we use a resistance R for the same purpose, a lot of energy would be wasted in the form of heat etc.

The choke coil consists of a number of turns of thick copper wire wound closely over a laminated soft iron core. The inductive reactance offered by the coil, $X_L = \omega L$ is large. Therefore, magnitude of a.c. is reduced, since $I_v = \dfrac{E_v}{X_L}$

In an ideal chock coil, Ohmic resistance = 0. Therefore, there is no energy dissipation. Through an ideal inductor, current lags behind the applied voltage by a phase angle of $90^o$.

∴ Average power consumed by the choke coil over a complete cycle is

$$P = E_v I_v \cos\phi = E_v I_v \cos 90° = 0$$

However, in actual practice, a choke coil of inductance L is having a small resistance r. it may be treated as a series combination of L and r. Therefore, average power consumed over a complete cycle in the practical choke coil is

$$P = E_v I_v \cos\phi$$

$$P = E_v I_v \frac{r}{\sqrt{r^2 + \omega^2 L^2}}$$

This comes out to be much smaller than the power loss ($I^2$ R) if a resistance R is used for reducing a.c. For reducing low frequency alternating currents, choke coils with laminated soft iron core are used.

This is because $I_v = \dfrac{E_v}{X_L} = \dfrac{E_v}{\omega L} = \dfrac{E_v}{2\pi \nu L}$


a.f. choke

When *v* is low, L is made high using soft iron core. These are called the a.f. chokes represented in fig. .

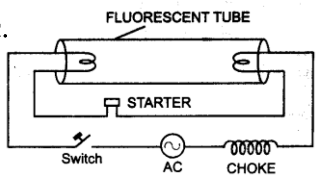
For reducing high frequency alternating currents, air cored chokes are used.

This is because $I_v = \dfrac{E_v}{X_L} = \dfrac{E_v}{\omega L} = \dfrac{E_v}{2\pi \nu L}$


r.f. choke

When v is high, L need not be made high i.e., air cored chokes are used. These are called the r.f. chokes represented in fig.

We have seen the use of choke coil in series with a fluorescent tube working on a.c. mains.



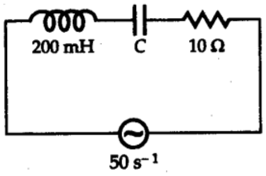

**Note :** that a choke coil can be replaced by a condenser for reducing a.c. equally effectively. This is because average power dissipated/cycle in case of a condenser is also zero.

## Subjective Assignment – VII

1 A light bulb is rated at 100 W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb.

2 A capacitor and a resistor are connected in series with an a.c. source. If the potential differences across C, R and 120 V, 90 V respectively and if the r.m.s. current of the circuit is 3 A, calculate the (i) impedance, (ii) power factor of the circuit.

3 In the following circuit, calculate, (i) the capacitance 'C' of the capacitor, if the power factor of the circuit is unity, and (ii) also calculate the Q–factor of the circuit.




4 An alternating voltage $\xi$ = 200 sin 300 t is applied across a series combination of R = 10 Ω and an inductor of 800 mH. Calculate:

(i) impedance of the circuit (ii) peak value of current in the circuit

(iii) power factor of the circuit

5 A 200 V variable frequency a.c. source is connected to a series combination of L = 5 H, C = 80 μF and R = 40 Ω. Calculate (i) angular frequency of the source to get maximum current in the circuit, (ii) the current amplitude at resonance and (iii) the power dissipated in the circuit.

6 A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which R = 3Ω, L = 25.48 mH, and C = 796 μF. Find (a) the impedance of the circuit, (b) the phase difference between the voltage across the source and the currents, (c) the power dissipated in the circuit, and (d) the power factor.

7 Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs? (b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

8 A virtual current of 4 A flows in a coil when it is connected in a circuit having alternating current of frequency 50 Hz. Power consumed in the coil is 240 W. Calculate the inductance of the coil if the virtual potential difference across it is 100 V.

9 A circuit draws a power of 550 W from a source of 220 V, 50 Hz. The power factor of the circuit is 0.8. The current in the circuit lags behind the voltage. Show that a capacitor of about $\frac{1}{42\pi}\times10^{-2}$ F will have to be connected to bring its power factor to unity.

10 An emf $\xi$ = 100sin314t is applied across a pure capacitor of 637 μF. find (i) the instantaneous current I (ii) instantaneous power P (iii) the frequency of power and (iv) the maximum energy stored in the capacitor.

11 A series LCR circuit is made by taking R = 100Ω, L = $\frac{2}{5}$H, C=$\frac{100}{\pi}$μF. This series combination is connected across an a.c. source of 220 V, 50 Hz. Calculate (i) the impedance of the circuit and (ii) the peak value of current flowing in the circuit. Calculate the power factor of this circuit and compare this value with the one at its resonant frequency.

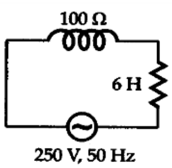
12 The current in a coil of self–inductance 2.0 H is increasing according to I = 2 sin $t^2$ ampere. Find the amount of energy spent during the period when the current changes from zero to 2 A.

13 A 100 μF capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5 A current flows through the inductance. Calculate the value of the inductance.

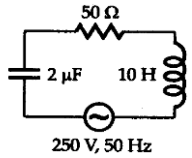
14 A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. Find the wattles component of the current in the circuit, when an alternating emf of 220 V at a frequency of 50 Hz is supplied to it.

15 An ammeter shows that an alternator is delivering 20 A. The voltmeter reads 220 V, while a wattmeter shows that 4 kW of power is being delivered. Find the power factor.

16 A 100Ω electric iron is connected to 200 V, 50 Hz a.c. source. Calculate average power delivered to iron, peak power and energy spent in one minute.

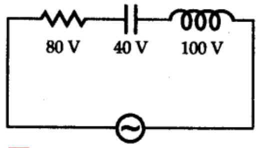


17 An a.c. circuit has a resistance and an inductance connected in series as shown in figure. Calculate the current and the power factor in the circuit.




18 Calculate the current and power factor of the circuit shown in figure.

19 In the circuit shown in figure, the potential differences across resistance, capacitance and inductance are given. Find the emf of the source of alternating current and power factor of the circuit.




20 A group of electric bulbs has a power rating of 300 W. An a.c. voltage, V = 141.4 sin (314t + π/3) is applied t o the group. Calculate the effective current.

21 An alternating voltage and the corresponding current in a circuit are given by

$$\xi = 110 \sin(\omega t + \pi/6) \text{ and } I = 5 \sin(\omega t - \pi/6)$$

respectively. Find the impedance and the average power dissipation in it.

22 An inductor 200 mH, capacitor 500 μF, resistor 10Ω are connected in series with a 100 V, variable frequency a.c. source. Calculate the

(i) frequency at which the power factor of the circuit is unity,

(ii) current amplitude at this frequency, (iii) Q–factor

23 An inductor of unknown value, a capacitor of 100 μF and a resistor of 10Ω are connected in series to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calculate the inductance of the inductor and the current amplitude.

24 A resistor of 12Ω, a capacitor of reactance 14Ω and an inductor of reactance 30Ω are joined in series and placed across a 230 V, 50 Hz supply. Calculate (i) the current in the circuit, (ii) the phase angle between the current and the voltage and (iii) the power factor.

25 A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. If the circuit has a resistance of 15Ω, obtain the average power transferred to each element of the circuit and the total power absorbed.

26 A series LCR–circuit with L = 0.24 H, C = 240 nF, R = 46Ω is connected to a 230 V variable frequency supply.

(i) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

(ii) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(iii) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(iv) What is the Q–factor of the given circuit?

## Answers

| | | | |
|---|---|---|---|
| 1. | (a) 484 Ω, (b) 311 V, (c) 0.45 A | 2. | (i) 50 Ω, (ii) 0.6 |
| 3. | (i) 50 μF, (ii) 6.32 | 4. | (i) 240.2Ω, (ii) 0.832A, (iii) 0.041 |
| 5. | (i) 50 rad $s^{-1}$, (ii) 7.07 A, (iii) 1000 W | 6. | (a) 5Ω, (b) 53.1°, (c) 4800 W, (d) 0.6 |
| 7. | (a) 35.4 Hz, (b) 66.7 A, 13.35 kW | 8. | $\frac{1}{5\pi}$ H |

10. (i) 30 cos 314 t ampere, (ii) 1000 sin 628 t watt, (iii) 100 Hz, (iv) 3.185 J

11. (i) 141.4Ω, (ii) 2.2 A, $\frac{1}{\sqrt{2}}$
12. 4 J
13. 0.01 H
14. 0.5 A
15. 0.91
16. 400 W, 800 W, 24, 000 J
17. 0.1325 A, 0.053
18. 0.16 A, 0.032
19. 100 V, 0.8
20. 3 A
21. 22Ω, 137.5 W
22. $\frac{50}{\pi}$ Hz, 14.14A, 2
23. 101.3 mH, 28.28 A
24. (i) 11.5 A, (ii) 53.1°, (iii) 0.6
25. $P_{av}^{R} = 790.6\,W, P_{av}^{L} = 0,\ P_{av}^{C} = 0,\ P_{av}^{total} = 790.6\ W$
26. (i) 663 Hz, 7.07 A, (ii) 663 Hz, 1150 W, (iii) 648 Hz and 678 Hz, 5.0 A, (iv) 21.7

## Real Resistors, inductors and capacitors

The resistors, inductors and capacitors used actually are not the pure or ideal ones. For example, a metal wire used as a resistor has some little inductance as a magnetic field is developed around the wire carrying current. Two wires in a circuit carrying current may also possess some little non–zero capacitance.

Similarly, an inductor which is a wire wound in the form of a coil does possess some Ohmic resistance. Also, there must be some capacitance between every two turns of the inductor coil.

Again, a capacitor has a very high resistance of the dielectric between the plates. Wire leads of the capacitor also possess some little inductance. Thus in actual practice, we cannot have pure or ideal resistors, inductors or capacitors.

## Advantages and drawbacks of a.c. over d.c.

**(a) Advantages of a.c. over d.c. are :**

**(i)** Alternating current can be transmitted over long distances using step up transformers. The loss of energy is negligible. Direct current cannot be transmitted as such.

**(ii)** The a.c. voltages can be easily varied using transformers.

**(iii)** The a.c. can be easily converted into d.c.

**(iv)** The magnitude of a.c. can be reduced using a choke coil, without involving loss of energy.

**(v)** The a.c. is easier and cheaper to generate than d.c. The a.c. generator are usually more robust and their efficiency is high.

**(b) Drawbacks of a.c. are :**

**(i)** It is more dangerous to work with a.c. at high voltages. The moment the insulation is faulty, one gets a severe shock.

**(ii)** The shock of a.c. is attractive, whereas that of d.c. is repulsive.

**(iii)** There are certain phenomena like electroplating , electrorefining, electrotyping etc. where a.c. cannot be used. In such cases, d.c . is needed.

**(iv)** The a.c. is transmitted more from the surface of conductor than from inside. therefore, several fine insulated wires (and not a single thick wire) are required for the transmission of a.c.

**Note :** The a.c. can be converted into d.c. with the help of a rectifier, while d.c. can be converted into a.c. with the help of an inverter. The a.c. can be stepped up or down with the help of a transformer, while d.c. cannot be.

The a.c. cannot produce chemical effects of current e.g. electrolysis : electroplating etc. This is due to large inertia of heavy ion which cannot follow the frequency of a.c.

## A.C. Generator or A.C. dynamo

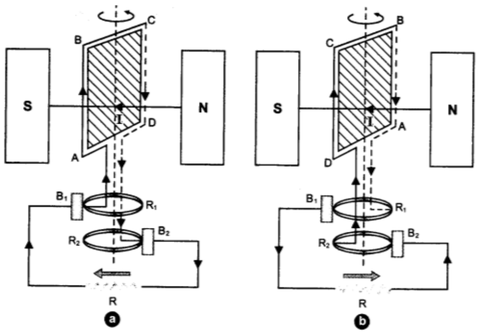
An a.c . generator/dynamo is a machine which produces alternating current energy from mechanical energy.

**Principle :** An a.c. generator/dynamo is based on the phenomenon of electromagnetic induction, i.e., whenever amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the coil. It lasts so long as the change in magnetic flux through the coil continues. The direction of current induced in given by Fleming's right hand rule.

**Construction :** The essential parts of an a.c. dynamo are shown in fig.

1. **Armature :** ABCD is a rectangular armature coil. It consists of a large number of turns of insulated copper wire wound over a laminated soft iron core, I. The coil can be rotated about the central axis.
2. **Field Magnets :** N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines. The magnetic field is of the order of 1 to 2 tesla.
3. **Slip rings :** $R_1$ and $R_2$ are two hollow metallic rings, to which two ends of armature coil are connected. These rings rotate with the rotation of the coil.
4. **Brushes :** $B_1$ and $B_2$ are two flexible metal plates or carbon rods. They are fixed and are kept in contact with $R_1$ and $R_2$ respectively. The purpose of brushes is to pass on current from the armature coil to the external load resistance R.

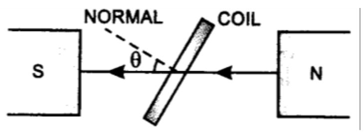
**Theory and Working**




As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes. An e.m.f. is induced in the coil. To start with, suppose the plane of the coil is perpendicular to the plane of the paper in which magnetic field is applied, with AB at front and CD at the back, fig (a). The amount of magnetic flux linked with the coil in this position is maximum. As the coil is rotated anticlockwise (or clockwise), *AB* moves inwards and CD moves outwards.

The amount of magnetic flux linked with the coil changes. According to Fleming's right hand rule, current induced in AB is from A to B and in CD, it is from C to D. In the external circuit, current flows from $B_2$ to $B_1$ fig. (a).

After half the rotation of the coil, AB is at the back and CD is at the front, fig. (b). Therefore, on rotating further, AB moves outwards and CD moves inwards. The current induced in AB is from B to A and in CD, it is from D to C. Through external circuit, current flows from $B_1$ to $B_2$; (b). This is repeated. Induced current in the external circuit changes direction after every half rotation of the coil. Hence the current induced is alternating in nature.




To calculate the magnitude of e.m.f. induced, suppose

N = number of turns in the coil,

A = area enclosed by each turn of the coil, $\vec{B}$ = strength of magnetic field

θ = angle which normal to the coil makes with $\vec{B}$ at any instant *t*.

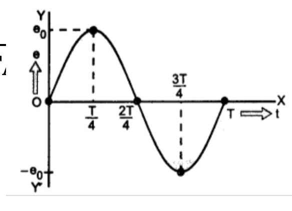
∴ Magnetic flux linked with the coil in this position

$$\phi = N(\vec{B}\cdot\vec{A}) = NBA\cos\theta = NBA\cos\omega t$$ where ω is angular velocity of the coil.

As the coil is rotated, θ changes; therefore, magnetic flux ϕ linked with the coil changes and hence an e.m.f. is induced in the coil.

At the instant *t*, if *e* is the e.m.f. induced in the coil, then

$$e = \frac{-d\phi}{dt} = -\frac{d}{dt}(NAB\cos\omega t) = -NAB\frac{d}{dt}(\cos\omega t) = -NAB(-\sin\omega t)\omega$$



$$e = NAB\,\omega \sin \omega t \qquad ....(1)$$

The induced e.m.f. will be maximum, when $\sin \omega t = \text{maximum} = 1$

$\therefore \qquad e_{max} = e_0 = NAB\omega \times 1$

Put in (2), $\qquad e = e_0 \sin \omega t \qquad ....(2)$

The variation of induced e.m.f. with time (i.e., with position of the coil) is shown in figure. The current supplied by the a.c. generator is also sinusoidal. It is given by

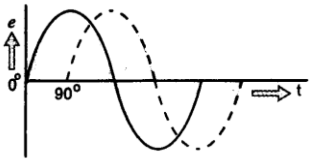
$$i = \frac{e}{R} = \frac{e_0}{R}\sin \omega t = i_0 \sin \omega t$$

where $i_0 = \frac{e_0}{R}$ = maximum value of current.

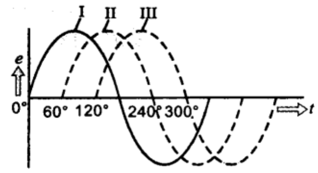
**Note :** Suppose to start with the plane of the coil is not perpendicular to the magnetic field. Therefore, at $t = 0,\ \theta \neq 0$. Let $\theta = \delta$, the phase angle. This is the angle which normal to the coil makes with the direction of $\vec{B}$. The equation (2) of e.m.f. induced in that case can be rewritten as $e = e_0 \sin(\omega t + \delta)$.

## Multi phase A.C. generator

**(a) Two phase a.c. generator :** In this generator, there are two armature coils held at 90° to each other. Each coil has its own pair of slip rings and brushes. When this pair of coils is rotated in the magnetic field, e.m.f. is induced in each coil. When e.m.f. induced in one coil is maximum, it is minimum in the other coil and vice–versa. Thus, the e.m.f. induced in the two coils differ in phase by 90°. This is called two phase a.c. as shown in figure.




**(b) Three phase a.c. generator :** In this generator, there are three armature coils equally inclined to one another at 60°. Each coil has its own pair of slip rings and brushes. When this arrangement of coils is rotated in a magnetic field, e.m.f. is induced in each coil. Thus, we obtain three alternating e.m.fs differencing in phase from one another by 60°. This is called three phase a.c. as shown in figure.




**(c)** In general, when there are a number of separate coils, each having its own pair of slip rings and brushes, the generator is called polyphase generator. The current produced is called polyphase alternating current. On end of each coil is brought to a common point through shaft of the generator. The line wire from this point is called neutral line. Separate slip rings are provided for other ends of different coils. The line wires from these rings (through their respective brushes) are called phase lines.

**Note : 1.** In most generators, the coils are held stationary and the electromagnets are rotated. In India, the frequency of rotation is 50 Hz and in countries like *USA*, the frequency of rotation is 60 Hz.

**2.** The amplitude or peak value of alt. e.m.f. depends on **(i)** number of turns in the coil **(ii)** area of the coil **(iii)** speed of rotation of the coil and **(iv)** strength of magnetic field. The frequency of a.c. generated depends only on speed off rotation of the coil.

**3. From factor** of an a.c. generator is defined as the ratio of virtual value and average value of a.c. produced by it, i.e., form factor = $\frac{I_v}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.1$

**4.** In India, we use three phase a.c. , the advantages of which are

**(i)** Output of three phase system is constant.

**(ii)** Output of three phase system is greater than the output of one and two phase systems.

**(iii)** This system is more economical.

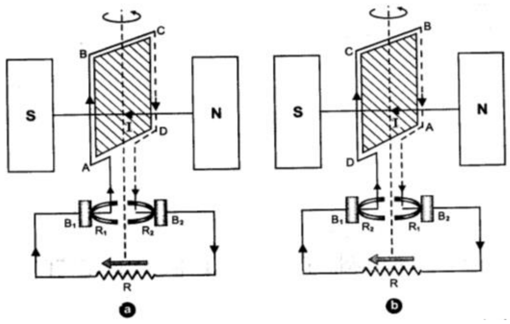
**5.** *Modern day generators produce power as high as 500 MW i.e., we can light up 5 million bulbs of 100 watt each simultaneously.*

## D.C. Generator or D.C Dynamo

It is a device which is used for producing direct current energy from mechanical energy.

**The principle** of d.c. generator is the same as that of a.c. generator. The essential parts of d.c. generator are also the same as those of a.c. generator except the slip ring arrangement. This is replaced by split rings or commutator arrangement. $R_1$ and $R_2$ are the two halves of the same metallic ring. The end of the armature coil are connected to these half rings which rotate along with the armature coil, figure (a).

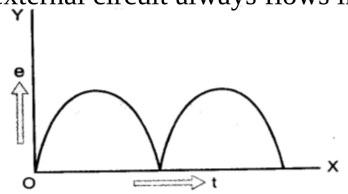
**Working :** The working of a d.c. generator is similar to that of a.c. generator and will be clear from the following two figure (b). We assume that to start with, plane of the coil is perpendicular to the plane of the paper in which magnetic field is applied.




Let us suppose that the armature coil ABCD is rotated clockwise or anticlockwise so that the arm AB moves inwards and CD moves outwards. Figure (a). Then applying Fleming's right hand rule, we see that the current flows in the armature as shown in fig. (a).

After the armature coil has rotated through $180^o$, it occupies the position shown in figure (b). On rotating the coil further in the same direction, CD is moving inwards and AB is moving outwards. Then again applying Fleming's right hand rule, we find that current flows in the armature as shown in figure (b).

Thus the direction of the induced e.m.f. and the induced current does not change in the external circuit during one complete rotation of the armature coil, i.e., the induced current in the external circuit always flows in the same direction.



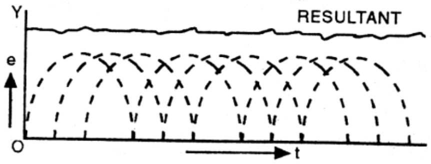

As in the case of a.c. generator, we can show that the magnitude of e.m.f. induced in d.c. generator is also.

$$\boxed{e = e_0 \sin \omega t}$$

The variation of *e* with time in d.c. generator is shown in fig.

The direction of '*e*' is not reversed in the second half cycle as explained in the working. It happens because after half the rotation of the coil, $R_1$ goes in contact with $B_2$ and $R_2$ goes in contact with $B_1$.

It we ignore the resistive losses, and consider only inductive effect, then




When we use a number of such coils equally inclined to one another, with the commutator ring divided into as many segments as the total number of ends of the coils, then each coil works independently sending its own current into the outer circuit. The resultant current/e.m.f. so obtained is shown in figure. The magnitude of resultant e.m.f/ current is almost constant and direction, of course is the same.

**Note :** The only essential difference between d.c. generator and a.c. generator is that slip ring arrangement of a.c. generator is replaced by split ring arrangement or commutator arrangement in d.c. generator.

## D.C. Motor

A d.c. motor converts direct current energy from a battery into mechanical energy of rotation.

**Principle :** It is based on the fact that when a coil carrying current is held in a magnetic field, it experiences a torque, which rotate the coil.

**Construction :** It consists of the following five parts as shown in figure.

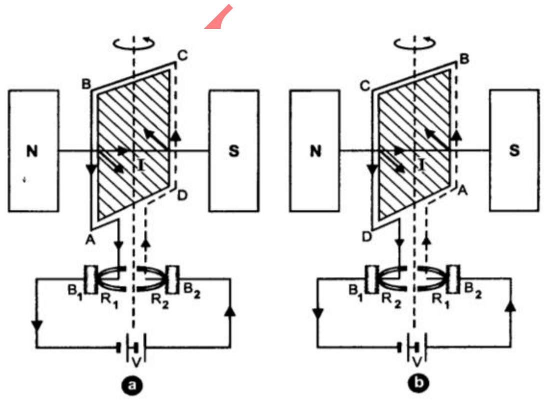
1. **Armature :** The armature coil ABCD consists of a large number of turns of insulated copper wire wound over a soft iron core.
2. **Field Magnet :** The magnetic field is supplied by a permanent magnet NS.
3. **Split rings or commutator :** These are two halves of the same ring. The ends of the armature coil are connected to these halves which also rotate with the armature.

4. **Brushes :** These are two flexible metal plates or carbon rods $B_1$ and $B_2$, which are so fixed that they constantly touch the revolving rings.
5. **Battery :** The battery consists of a few cells of voltage *V* connected across the brushes. The brushes convey the current to the rings, from where it is carried to the armature.

**Working**

The battery send current through the armature coil in the direction shown in figure. Applying Fleming's left hand rule, CD experiences a force directed inwards and perpendicular to the plane of the coil. Similarly, AB experiences a force directed outwards and perpendicular to the plane of the coil.

These two forces being equal, unlike and parallel form a couple. The couple rotates the armature coil in the anticlockwise direction. After the coil has rotated through $180^o$, $R_1$ is in contact with $B_2$ and $R_2$ is in contact with $B_1$. Therefore, the direction of current in AB and CD is reversed, fig. (b) . Now CD experiences an outward force and AB experiences an inward force. The armature coil thus continues rotating in the same i.e., anticlockwise direction.




**Back E.M.F. :** As the armature rotates in the magnetic field, the amount of magnetic flux linked with the coil changes. Therefore, an e.m.f. is induced in the coil. The direction of the induced e.m.f. is such that it opposes the battery current in the circuit. This e.m.f. is called the back e.m.f. and its magnitude goes on increasing with the speed of the armature.

Let V = e.m.f. applied across $B_1$ and $B_2$, R = resistance of the armature coil,

I = current flowing through the armature coil, at any instant *t*, E = back e.m.f. at that instant

As V and E are acting in the opposite directions, $\therefore$ effective e.m.f. across $B_1$ and $B_2$ = V– E

According to Ohm's law, $$I = \frac{V-E}{R} \qquad ....(1)$$

or $$V - IR = E \qquad ....(2)$$

**Efficiency of d.c. motor :** It is defined as the ratio of output mechanical power to the input electrical power i.e.,

$$\eta = \frac{\text{output mechanical power}}{\text{Input electric power}} \qquad ....(3)$$

Since the current I is being supplied to the armature coil by the external source of e.m.f. V, therefore, Input electric power = VI.

According to Joule's law of heating,

Power lost in the form of heat in the coil = $I^2 R$

If we assume that there is no other loss of power, then

Power converted into external work i.e., Output mechanical power = $VI - I^2R = (V - IR)I = EI$ [From (2)]

From (3), $$\eta = \frac{EI}{VI} = \frac{E}{V} \frac{\text{back e.m.f.}}{\text{applied e.m.f.}} \qquad ....(4)$$

**Maximum efficiency :** When $E = V$, from (4), $\eta = 1$ or 100%. Therefore, for $\eta$ to be maximum, i.e., 100%, back e.m.f. E should be equal to the applied e.m.f. V. But in that case, the current I flowing through the armature coil becomes zero as is clear from (1). Obviously, the motor in this case will just cease to work. This is an anomaly. Practically, the efficiency of the d.c. motor will be maximum, when output mechanical power is maximum.

i.e., EI = maximum

Using (1), $$E\,.\,\frac{(V-E)}{R} = \text{maximum}$$

Now, $\frac{E(V-E)}{R}$ will be maximum (i.e., constant), when its differential coefficient is zero.

i.e., $$\frac{d}{dE}\left[\frac{E(V-E)}{R}\right] = 0 \quad \text{or} \quad \frac{d}{dE}\left[\frac{1}{R}(VE-E^2)\right] = 0 \quad \text{or} \quad \frac{1}{R}(V-2E) = 0 \quad [\because R \neq 0]$$

$$\text{or} \quad V - 2E = 0 \quad \text{or} \quad E = \frac{V}{2}$$

From (4), $\eta = \frac{E}{V}$, When $E = \frac{V}{2}$, $\eta = \frac{V/2}{V} = \frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$

Hence a d.c. motor delivering maximum output has an efficiency of only 50%.

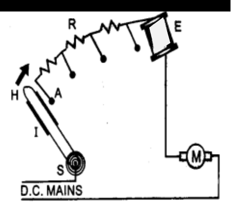
Further, when E= V/2, then from (1), Current I in the coil may become too large as R is low. Hence in practice, we do not try to get maximum output mechanical power.

**Uses : (i)** The d.c. motors are used in d.c. fans (exhaust, ceiling or table) for cooling and ventilation.

**(ii)** They are used for pumping water.

**(iii)** Big d.c. motors are used for running tram–cars and even trains.

## Motor starter




A starter is a device which is used for starting a d.c. motor safely. It function is to introduce a suitable resistance in the circuit at the time of starting of the motor. This resistance decreases gradually and reduces to zero when the motor runs at full speed.

Infact, resistance of armature of d.c. motor is kept low (to reduce to copper losses) and when armature is stationary, there is no back e.m.f.

Therefore, when full operating voltage is applied, the current through armature coil may become so large (I = V/R) that the motor may burn. A starter is needed to avoid this. The essential parts of a starter are shown in figure. To start the motor, the handle H is brought in contact with the point A, so that it entire resistance R is in series with the motor M. Therefore, the initial current becomes small. The electromagnet E gets magnetized due to the passage of current.

It attracts the iron piece I attached to the handle H. As the handle moves towards E, the series resistance of the starter decreases gradually. The current through the motor increases accordingly. When H just touches E, resistance R of the starter is out of circuit. The current becomes maximum and the armature rotates at full speed.

When the main supply is switched off, electromagnet loses its magnetism and it can no longer hold H. By the action of the spring S, the handle comes to the off position.

## Subjective Assignment – VIII

1 Kamla peddles a stationary bicycle the pedals of which are attached to a 100 turn coil of area 0.10 $m^2$. The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?

2 An a.c. generator consists of a coil of 50 turns and area 2.5 $m^2$ rotating at an angular speed of 60 rad $s^{-1}$ in a uniform magnetic field B = 0.30 T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500 Ω.

(a) What is the maximum current drawn from the generator?

(b) What is the flux through the coil when the current is zero? What is the flux when the current is maximum?

(c) Would the generator work if the coil were stationary and instead the pole pieces rotated together with the same speed as above?

3 An a.c. generator consists of a coil of 100 turns and cross–sectional area of 3 $m^2$, rotating at a constant angular speed of 60 radians/sec in a uniform magnetic field of 0.04 T. The resistance of the coil is 500 ohm. Calculate (i) maximum current drawn from the generator and (ii) maximum power dissipation in the coil.

4 A generator develops of emf of 120 V and has a terminal potential difference of 115 V, when the armature current is 25 A. What is the resistance of the armature?

5 An armature coil consists of 20 turns of wire, each of area A = 0.09 $m^2$ and total resistance 15.0 Ω. It rotates in a magnetic field of 0.5 T at a constant frequency of 150/π Hz. Calculate the value of (i) maximum (ii) average induced emf produced in the coil.

6 An a.c. generator consists of a coil of 50 turns and area 2.5 $m^2$ rotating at an angular speed of 60 rad $s^{-1}$ in a uniform magnetic field of 0.30 T. The resistance of the circuit including that of the coil is 500 Ω. (i) Find the peak value of current drawn from the generator. (ii) What is the flux through the coil when the current is zero?

7 An a.c. generator consists of a coil of 2000 turns each of area 80 $cm^2$ and rotating at an angular speed of 200 rpm in a uniform magnetic field of $4.8 \times 10^{-2}$T. Calculate the peak and r.m.s. values of emf induced in the coil.

## Answers

| | | | |
|---|---|---|---|
| 1. | 0.314 V | 2. | (a) 4.5 A, (b) 37.5 Wb, 0 (c) yes |
| 3. | (i) 1.44 A, (ii) 518.4 W | 4. | 0.2 Ω |
| 5. | (i) 270 V, (ii) 0 | 6. | 45 , 37.5 WB |
| 7. | 16.085 V, 11.375 V | | |

## Short Answer Conceptual Problems :

1. Answer the following questions :

(a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

(c) A lamp is connected in series with a capacitor. Predict your observations for d.c. and a.c. connections. What happens in each case if the capacitance of the capacitor is reduced?

2. Distinguish between resistance, reactance and impedance of an a.c. circuit.

3. Compare the important feature of resistance, reactance and impedance for an a.c. circuit.

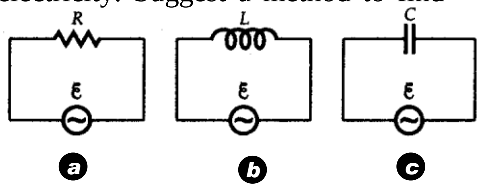
4. An ordinary moving coil ammeter used for d.c. cannot be used to measure an alternating current even if its frequency is low. Explain, why.

5. A capacitor blocks d.c. and allows a.c. to flow through it. Explain.

6. (i) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied a.c. source. (ii) Can the voltage drop across the inductor or the capacitor in a series LCR circuit be greater than the applied voltage of the a.c. source? Justify your answer.

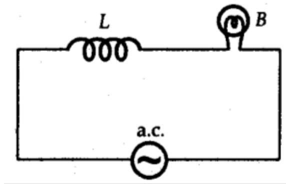
7. You are given an air coil, a bulb, an iron rod and a source of electricity. Suggest a method to find whether the given source is d.c. or a.c. Explain your answer.

8. Fig. (a), (b) and (c) show three a.c. circuits in which equal currents are flowing. If the frequency of emf be increased, how will the current be affected in these circuits? Give reason for your answer.



9. When a capacitor is connected in series with a series LR–circuit, the alternating current flowing in the circuit increases. Explain why?

10. An inductor 'L' of reactance $X_L$, is connected in series with a bulb 'B′ to an a.c. source as shown in figure. Briefly Explain how does the brightness of the bulb change, when (i) number of turns of the inductor is reduced and (ii) a capacitor of reactance $X_C = X_L$ is included in series in the same circuit.




11. At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

12. Does the current in an a.c. circuit lag, lead or remain in phase with the voltage of frequency applied to the circuit, when (i) $f = f_r$ (ii) $f < f_r$ and (iii) $f > f_r$, where $f_r$ the resonant frequency?

13. When a.c. circuit with L, C and R in series is brought into resonance, the current has large value. Why? If the capacitance C is increased, will current increase or decrease? Explain with suitable relation.

14. On the basis of power dissipation in a.c. circuit, distinguish between resistance, reactance and impedance.

15. (a) Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length *l* of the solenoid.

    (b) How does this magnetic energy compare with the electrostatic energy stored in a capacitor?

16. Show diagrammatically two different arrangements used for winding the primary and secondary coils in a transformer. Assuming the transformer to be an ideal one, write expressions for the ratio of its.

    (i) output voltage to input voltage

    (ii) output current to input current in terms of the number of turns in the primary and secondary coils. Mention two reasons for energy losses in an actual transformer.

17. A radio frequency choke is air–cored whereas an audio frequency choke is ironcored. Give reason for this difference.

18. Give two disadvantages of transmitting a.c. over long distances at low voltage and high current.

19. 11Kilowatts of power can be transmitted in two ways:

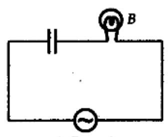
    (i) 220 volts at 50 amperes and (ii) 22,000 volts at 0.5 ampere

    Which is economical? Give reasons for your choice.

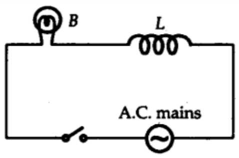
20. A simple a.c. generator having a constant magnetic field is connected to a resistive load. Explain with reasons what will be the effects of doubling its speed of rotation on the following:

    (a) the frequency of rotation, (b) the generated emf, and (c) the mechanical power required to rotate the generator?

21. A bulb B and a capacitor C are connected in series to the a.c. mains as shown in figure. The bulb glows with some brightness. How will the glow of the bulb change when a dielectric slab is introduced between the plates of the capacitor? Give reasons in support of your answer.

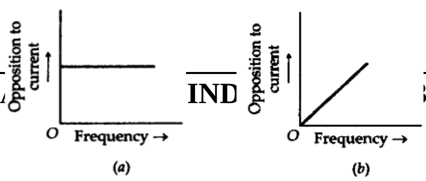



22. A light bulb and an open coil inductor are connected to an a.c. source through a key as shown in fig. The switch is closed and after some time, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increase (b) decreases (c) is unchanged, as the iron rod is inserted. Give your answer with reasons. What will be your answer if ac source is replaced by d.c.?



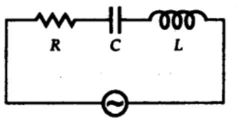

23. Prove that high frequency a.c. can pass through a pure capacitor easily but not through a pure inductor?

24. The graphs in figures (a) and (b) represent the variation of opposition offered by the circuit element to the flow of



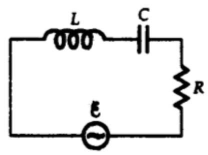
alternating current, with the frequency of the applied emf. Identify the circuit element corresponding to each graph.

25. In the circuit shown in figure, R represents an electric bulb. If the frequency of the supply is doubled, how should the values of C and L be changed so that the glow in the bulb remains unchanged?




## NCERT Exercises

1. A 100Ω resistor is connected to a 220V, 50Hz, ac supply
   (a) What is the rms value of current in the circuit.
   (b) What is the net power consumed over a full cycle?
2. (a) The peak value of an a.c. supply is 300V. What is the rms voltage?
   (b) The rms value of current in an a.c. circuit is 10A. What is the peak current?
3. A 44mH tor is connected to 220V, 50Hz a.c. supply. Determine the rms value of current in the circuit.
4. A 60 μF capacitor is connected to a 110V, 50 Hz a.c. supply. Determine rms value of current in circuit.
5. In above Q. 3 & 4 , what is net power absorbed by each circuit over a complete cycle? Explain your answer.
6. Obtain the resonant frequency $\omega_r$ of a series LCR–circuit with L = 2.0 H, C = 32 μF and R = 10Ω. What is the Q–value of the circuit?
7. A charged 30μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?
8. Suppose the initial charge on the capacitor in Q.7 is 6mC. What is the total energy stored in the circuit initially? What is the total energy at later time?
9. A series LCR–circuit with R = 20Ω, L = 15H and C = 35μF is connected to a variable frequency 200V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?
10. A radio can tune over the frequency range of portion of MW (medium wave) broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200μH, what must be the range of its variable condenser?
11. Fig. shows a series LCR–circuit connected to a variable frequency 230 V source.
    L = 5.0 H, C = 80μF, R = 40Ω.




    (a) Determine the source frequency which drives the circuit in resonance.
    (b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
    (c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC–combination is zero at the resonating frequency.
12. An LC–circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 100mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t$ = 0.
    (a) What is the total energy stored initially. Is it conserved during the LC–oscillations?
    (b) What is the natural frequency of the circuit?
    (c) At what times is the energy stored?
        (i) Completely electrical (i.e., stored in the capacitor)?
        (ii) Completely magnetic (i.e., stored in the inductor)?
    (d) At what times is the total energy shared equally between the inductor and the capacitor?
    (e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

13. A coil of inductance 0.50 H and resistance 100Ω is connected to a 240V, 50Hz a.c. supply.
    (a) What is the maximum current in the coil?
    (b) What is the time lag between the voltage maximum and the current maximum?
14. Obtain the answer to (a) and (b) in Q. 13, if the circuit is connected to a high frequency supply (240 V, 20kHz). Hence explain statement that at very high frequency, inductor in circuit amounts to open circuit. How does an inductor behave in a d.c. circuit after the steady state?
15. A 100μF capacitor in series with a 40Ω resistance is connected to a 110 V, 60Hz supply.
    (a) What is the maximum current in the circuit?
    (b) What is the time lag between current maximum and voltage maximum?
16. Obtain the answer to (a) and (b) in Q.15, if the circuit is connected to a 110V, 12kHz supply. Hence explain the statement that a capacitor is a conductor at very high frequencies. Compare this behavior with that of a capacitor in a d.c. circuit after the steady state.
17. Keeping the source frequency equal to the resonating frequency of the series LCR–circuit, if the three elements L, C and R are arranged in parallel, show that the total current in the parallel LCR–circuit is a minimum at this frequency. Obtain the current rms value in each branch of the circuit for L = 50H, C = 80μF and R = 40Ω and for the a.c. source of emf 230V for this frequency.
18. A circuit containing a 80mH inductor and a 60μf capacitor in series is connected to a 230 V, 50Hz supply. The resistance of the circuit is negligible.
    (a) Obtain the current amplitude and rms values
    (b) Obtain the rms values of potential drops across each element
    (c) What is the average power transferred to the inductor?
    (d) What is the average power transferred to the capacitor?
    (e) What is the total average power absorbed by the circuit?
19. Suppose the circuit in Q.18 has a resistance of 15Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.
20. A series LCR-circuit with L = 0.12 H, C = 480nF, R = 23Ω is connected to a 230V variable frequency supply.
    (a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.
    (b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.
    (c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?
    (d) What is the Q-factor of the given circuit?
21. Obtain the resonant frequency and Q–factor of a series LCR–circuit with L=3.0 H, C = 27μF and R = 7.4Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half-maximum' by a factor of 2. Suggest a suitable way.
22. (a) (i) In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit ?
    (ii) Is the same true for rms voltage ?
    (b) A capacitor is used in the primary circuit of an induction coil.
    (c) An applied voltage signal consists of a superposition of a d.c. voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the d.c. signal will appear across C and the a.c. signal across L.
    (d) A choke coil in series with a lamp is connected to a d.c. line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an a.c. line.

(e) Why is a choke coil needed in the use of fluorescent tubes with a.c. mains ? Why can we not use an ordinary resistor instead of the choke coil ?

23. A power transmission line feeds input power at 2300V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V?

24. At a hydroelectric power plant, the water pressure head is at a height of 300m and the water flow available is 100 $m^3s^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant. ($g = 9.8\ ms^{-2}$).

25. A small town with a demand of 800 kW of electric power at 220V is situated 15 km away from an electric plant generating power at 440V. The resistance of the two–wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000–200V step down transformer at a substation in the town.

(a) Estimate the line power loss in the form of heat

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage.

(c) Characterise the step-up transformer at the plant.

26. Do the same Q. as above with the replacement of the earlier transformer by a 40,000 – 220V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred.

## Answers

| | | | | | |
|---|---|---|---|---|---|
| 1. | (a) 2.20 A , (b) 484 W | 2. | (a) 212.1 V , (b) 14.14 A | 3. | 15.9 A |
| 4. | 2.49 A | 5. | zero in each case | 6. | 125 $s^{-1}$, 25 |
| 7. | $1.1 \times 10^3\ s^{-1}$ | 8. | 0.6 J, same at later times | 9. | 2000 W |

10. 88 pF to 198 pF

11. (a) 50 rad $s^{-1}$ (b) 40 Ω, 8.1 A (c) $V_{Lrms} = 1437.5$ V, $V_{Crms} = 14237.5$ V, $V_{Rrms} = 230$ V

12. (a) 1 J . Yes sum of the energies stored in $L$ and $C$ is conserved if $R = 0$.

(b) $\omega = 10^3$ rad $s^{-1}$, $v = 159$ Hz (c) (i) $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ....$ (ii) $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, .....$

(d) $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}, ....$ (e) 1 J

13. (a) 1.82 A (b) 3.2 ms 14. (a) $1.1 \times 10^{-2}$ A

(b) $\tan\phi = 100\pi$, $\phi$ is closed to $\pi/2$. $I_0$ is much smaller than the low frequency case (Q.13) showing thereby that at high frequencies. $L$ nearly amounts to an open circuit. In a dc circuit (after steady state) $\omega = 0$, so here $L$ acts like a pure conductor.

15. (a) 3.24 A (b) 1.55 ms

16. (a) 3.89 A (b) $\phi \approx 0.2$ and is nearly zero at high frequency. Thus, at high frequency, $C$ acts like a conductor. For a dc circuit, after steady state, $\omega = 0$ and $C$ amounts to an open circuit.

17. $I_{Rrms} = 5.75\text{A}$, $I_{Lrms} = 0.92\text{A}$, $I_{Crms} = 0.92\text{A}$, $I_{rms} = 5.75\text{A}$

18. (a) $I_0 = 11.6$ A, $I_{rms} = 8.24$ A (b) $V_{Lrms} = 207$V, $V_{Crms} = 437$ V (c) 0 (d) 0 (e) 0

19. Average power to R = 791W , Average power to L = Average power to C= 0 ,
Total power absorbed = 791 W

20. (a) $\omega_0 = 4167$ rad $s^{-1}$ , $v_0 = 663$Hz, $I_0 = 14.1$A (b) 2300 V (c) 648Hz , 678Hz, 10A (d) 21.7

21. 111 rad $s^{-1}$, 45, 3.7 Ω 23. 400 turns 24. 176 MW

25. (a) 600 kW (b) 1400 kW (c) 440 – 7000 V

26. (a) 6 kW (b) 806 kW (c) 440V – 40, 300V

## IIT Entrance Exam.

**Multiple Choice Questions with One Correct Answer**

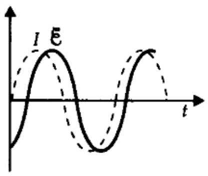
1 A coil of inductance 8.4 mH and resistance 6 $\Omega$ is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time.

(a) 500 s (b) 25 s (c) 35 ms (d) 1 ms

2 A coil of wire having inductance and resistance has a conducting ring placed coaxially with in it. The coil is connected to a battery at time t = 0, so that a time–dependent current $I_1(t)$ starts flowing through the coil. If $I_2$ (t) is the current induced in the ring, and B(t) is the magnetic field at the axis of the coil due to $I_1$ (t), then as a function of time (t > 0), product $I_2$ (t) B (t).

(a) increases with time (b) decreases with time(c) does not vary with time(d) passes through a maximum

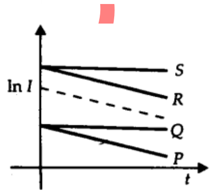
3 When an a.c. source of emf $\xi = \xi_0 \sin 100t$ is connected across a circuit, the phase difference between the emf $\xi$ and the current I in the circuit is observed to be $\pi/4$, as shown in the figure. If the circuit consists possibly only of RC or RL or LC in series, find the relationship between the two elements




(a) R = 1 k$\Omega$, C = 10 $\mu$F (b) R = 1 k$\Omega$, C = 1 $\mu$F

(c) R = 1 k$\Omega$, L = 10 H (d) R =1 k$\Omega$, L =1 H

4 A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln I with respect to time. If the resistance x is changed to 2x, the new graph will be




(a) P (b) Q (c) R (d) S

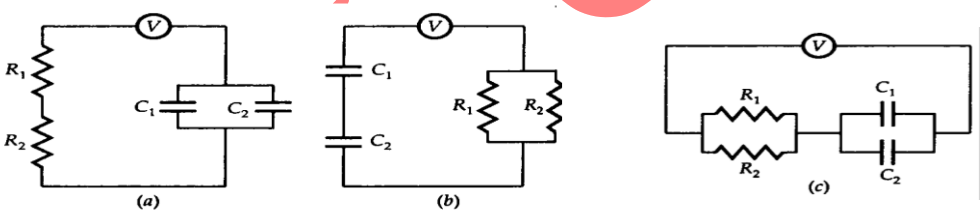
5 A 4$\mu$F capacitor and a resistance of 2.5 M$\Omega$ are in series with 12V battery. Find the time after which potential difference across the capacitor is 3 times the potential difference across the resistor.

[Given ln (2) = 0.693]

(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s

6 Find the time constant (in $\mu$s) for the given R – C circuits in the given order respectively.

$R_1 = 1\Omega$, $R_2 = 2\Omega$; $C_1 = 4\mu F$, $C_2 = 2\mu F$




(a) 18, 4, 8/9 (b) 18, 8/9, 4 (c) 4, 18, 8/9 (d) 4, 8/9, 18

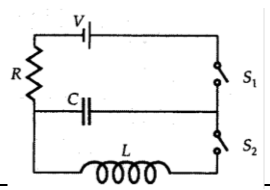
**Multiple Choice Questions with One or More Than One Correct Answer**

7 Two different coils have self–inductances $L_1$ = 8 mH and $L_2$ = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are $i_1$, $V_1$ and $W_1$ respectively. Corresponding values for the second coil at the same instant are $i_2$, $V_2$ and $W_2$ respectively. Then

(a) $\frac{i_1}{i_2}=\frac{1}{4}$ (b) $\frac{i_1}{i_2}=4$ (c) $\frac{W_2}{W_1}=\frac{1}{4}$ (d) $\frac{V_1}{V_2}=4$

**Comprehension based questions :**

**In this passage for Q. 8 to Q. 10 :** In the given circuit the capacitor (C) maybe charged through resistance R by battery V by closing switch $S_1$. Also when $S_1$ is opened and $S_2$ is closed the capacitor is connected in series with inductor (L).




8 At start, the capacitor was uncharged. When switch $S_1$ is closed and $S_2$ is kept open, time constant of this circuit $\tau$. Which of following is correct?

(a) after time interval $\tau$, charge on the capacitor is CV/2
(b) after time interval $2\tau$, charge on the capacitor of CV $(1-e^{-2})$
(c) work done by voltage source will be half of the heat dissipated when capacitor is fully charged.
(d) after time interval $2\tau$, charge on the capacitor is CV $(-e^{-1})$

9 When the capacitor gets charged completely, $S_1$ is opened and $S_2$ is closed. Then,
(a) at t = 0, energy stored in the circuit is purely in the form of magnetic energy.
(b) at any time t > 0, current in the circuit is in the same direction
(c) at t > 0, there is no exchange of energy between the inductor and capacitor.
(d) at any time t > 0, instantaneous current in the circuit may be $i\sqrt{\frac{C}{L}}$

10 Given that the total charge stored in the LC circuit is $Q_0$, for $t \geq 0$, the charge on the capacitor is
(a) $Q=Q_0\cos\left(\frac{\pi}{2}+\frac{t}{\sqrt{LC}}\right)$ (b) $Q=Q_0\cos\left(\frac{\pi}{2}-\frac{t}{\sqrt{LC}}\right)$
(c) $Q=-LC\frac{d^2Q}{dt^2}$ (d) $Q=-\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

## AIEEE

1 Alternating current cannot be measured by d.c. ammeter, because
(a) a.c. cannot pass through a.c. ammeter (b) a.c. changes direction
(c) average value of current of complete cycle is zero (d) a.c. ammeter will get damaged

2 The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?
(a) C alone (b) L alone (c) L, C (d) R, L

3 In an LCR–series a.c. circuit, the voltage across each of the components. L, C and R is 50 V. The voltage across the LC–combination will be
(a) 50 V (b) $50\sqrt{2}$V (c) 100 V (d) zero

4 In an LCR–circuit, capacitance is changed from C to 2C. For the resonant frequency to remain unchanged, the inductance should be changed from L to
(a) 4 L (b) 2 L (c) L/2 (d) L /4

5 The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of
(a) 4 μF (b) 8 μF (c) 1 μF (d) 2 μF

6 A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be
(a) 0.8 (b) 0.4 (c) 1.25 (d) 0.125

7 The power factor of an a.c. circuit having resistance R and inductance L (connected in series) and an angular velocity ω is
(a) R/ ωL (b) $R/(R^2+\omega^2L^2)^{1/2}$ (c) ωL/R (d) $R/(R^2-\omega^2L^2)^{1/2}$

8 In an oscillating LC–circuit, the maximum charge on the capacitor is Q. The charge on the capacitor, when the energy is stored equally between the electric and magnetic field is
(a) Q/2 (b) $Q/\sqrt{3}$ (c) $Q/\sqrt{2}$ (d) Q

9 In an a.c. circuit the voltage applied is $\xi = \xi_0 \sin \omega t$. The resulting current in the circuit is
$I = I_0 \sin(\omega t - \pi/2)$. The power consumption in the circuit is given by
(a) $P=\sqrt{2}\xi_0 I_0$ (b) $P=\frac{\xi_0 I_0}{\sqrt{2}}$ (c) P = 0 (d) $P=\frac{\xi_0 I_0}{2}$

10 An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5 V. 2 seconds after the connection is made, the current flowing in amperes in the circuit is

(a) $(1 - e^{-1})$ (b) $(1 - e)$ (c) $e$ (d) $e^{-1}$

11 In a transformer, number of turns in the primary is 140 and that in the secondary is 280. If current in primary is 4A, then that in the secondary is

(a) 4 A (b) 2 A (c) 6 A (d) 10 A

12 The core of any transformer is laminated, so as to

(a) reduce the energy loss due to eddy currents (b) make it light weight

(c) make it robust and strong (d) increase the secondary voltage

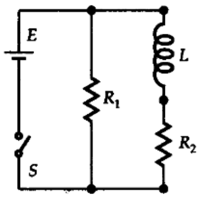
13 In an a.c. generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is

(a) N A B R (b) NA Bω (c) N A B Rω (d) N A B

14 In a series LCR – circuit, the voltage across R is 100 V and R = 1 kΩ, C = 2μF. The resonant frequency ω is 200 rad $s^{-1}$. At resonance, the voltage across L is

(a) 40 V (b) 250 V (c) $4 \times 10^{-3}$ V (d) $2.5 \times 10^{-2}$ V

15 An inductor of inductance L = 400 mH and resistors of resistances $R_1$ = 2Ω and $R_2$ = 2Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is




(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V (c) $6(1 - e^{-t/0.2})$ V (d) $12\ e^{-5t}$ V

## AIIMS Entrance Exam.

1 In a circuit, the current lags behind the voltage by a phase difference of π/2. The circuit contains which of the following?

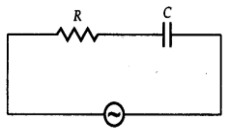
(a) only R (b) only L (c) only C (d) R and C

2 In an a.c. circuit containing only capacitor, the current

(a) leads voltage by 180° (b) remains in phase with voltage

(c) leads voltage by 90° (d) lags voltage by 90°

3 A 50 Hz a.c. source of 20 V is connected across R and C as shown in figure. The voltage across R is 12 V. The voltage across C is




(a) 8 V (b) 16 V (c) 10 V

(d) not possible to determine, unless values of R and C are given

4 In an ideal parallel LC–circuit, the capacitor is charged by connecting it to a d.c. source, which is then disconnected. The current in the circuit

(a) becomes zero instantaneously (b) grows monotonically

(c) decays monotonically (d) oscillates instantaneously

5 The power factor varies between

(a) 2 and 2.5 (b) 3.5 to 5 (c) 0 to 1 (d) 1 to 2

6 The coil of a choke in a circuit

(a) increases the current (b) decreases the current

(c) does not change the current (d) has high resistance to d.c. circuit

7 A choke coil has

(a) low inductance and high resistance (b) high inductance and low resistance

(c) low inductance and low resistance (d) high inductance and high resistance

8 A transformer works on the principal of

(a) converter (b) inverter (c) mutual induction (d) self induction

9 Quantity that remains unchanged in a transformer is

(a) voltage (b) current (c) frequency (d) none of these

10 Turn ratio in a set–up transformer is 1 : 2. If a Leclanche cell of 1.5 V is connected across the input, what is the voltage across the output?

(a) 1.5 V (b) 0 V (c) 3 V (d) 0.75 V

11 The primary winding of a transformer has 50 turns and its secondary has 500 turns. If primary is connected to a.c. supply of 20V – 50 Hz, then secondary will have an

(a) 200 V – 50 Hz (b) 200 V – 500 Hz (c) 2V – 5Hz (d) 2V – 50 Hz

12 The best material for the core of a transformer is

(a) stainless steel (b) mild steel (c) hard steel (d) soft iron

13 The working of a dynamo is based on the principle of

(a) heating effect of current (b) magnetic effect of current

(c) chemical effect of current (d) electromagnetic induction

14 If the speed of rotation of a dynamo is doubled, then the induced e.m.f. will

(a) become half (b) become double (c) become four times (d) remain unchanged

15 In an a.c. circuit, the potential differences across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference of the source is

(a) 20.0 V (b) 25.6 V (c) 31.9 V (d) 53.5 V

16 What are the dimensions of impedance?

(a) $ML^2T^{-3}I^{-2}$ (b) $M^{-1}L^{-2}T^3I^2$ (c) $ML^3T^{-3}I^{-2}$ (d) $M^{-1}L^{-3}T^3I^2$

## Reasoning types questions :

This question contains **A**(assertion) and **R** (reason).

(a) A and R are true and R is a correct explanation for A.

(b) A and R are true and R is not a correct explanation for A.

(c) A is true, R is false (d) A is false, R is true (e) A and R both false

17 **A :** The quantity L/R possesses the dimension of time.

**R :** In order to reduce the rate of increase of current through a solenoid, we should increase the time constant.

18 **A :** At resonance, the inductive reactance is equal and opposite to the capacitive reactance.

**R :** In series LCR–circuit, the inductive reactance is equal and opposite to the capacitive reactance.

19 **A :** In series LCR–circuit, the resonance occurs at one frequency only.

**R :** At resonance, the inductive reactance is equal and opposite to the capacitive reactance

20 **A :** The possibility of an electric bulb fusing is higher at the time of switching ON and OFF.

**R :** Inductive effects produce a surge at the time of switch–OFF and switch–ON.

21 **A :** We use a thick wire in secondary of a step–down transformer to reduce the production of heat.

**R :** When the plane of the armature is parallel to the lines of force of magnetic field, the magnitude of induced e.m.f. is maximum.

## CBSE PMT Prelims Exam

1. The reactance of a capacitance C is X. If both the frequency and capacitance be doubled, then new reactance will be

(a) X (b) 2X (c) 4X (d) X/4

2. What is the value of inductance L for which the current is maximum in a series LCR –circuit with C = 10μF and ω = 1000 $s^{-1}$?

(a) 100 mH (b) 1 mH (c) 10 mH (d) cannot be calculated unless R is known

3. In a circuit, L, C and R are connected in series with an alternating voltage source of frequency *f*. The current leads the voltage by 45°. The value of C is

(a) $\frac{1}{\pi f(2\pi f L - R)}$ (b) $\frac{1}{2\pi f(2\pi f L - R)}$ (c) $\frac{1}{\pi f(2\pi f L + R)}$ (d) $\frac{1}{2\pi f(2\pi f L + R)}$

4. A current $I = I_0 \sin(\omega t + \pi/2)$ flow in a circuit across which an alternating potential $\xi = \xi_0 \sin\omega t$ is applied. The power consumed in the circuit is

(a) $\xi_0 I_0/2$ (b) $\xi_0 I_0$ (c) $\xi$ (d) zero

5. In an a.c. circuit, the current flowing is $I = 5\sin(100t - \pi/2)$ (in A) and the potential difference is V = 200 sin 100t (in V). The power consumption is equal to

(a) 1000 W (b) 40 W (c) 20 W (d) 0 W

6. In an a.c. circuit, with phase voltage V and current I, the power dissipated is

(a) $\frac{1}{2}VI$ (b) $\frac{1}{\sqrt{2}}VI$ (c) VI (d) depends on the phase angle between V and I

7. For a series, LCR-circuit, the power loss at resonance is

(a) $\frac{V^2}{\omega L - 1/\omega C}$ (b) $\frac{V^2}{\omega C}$ (c) $I^3\omega C$ (d) $I^2R$

8. An inductor L having resistance R and a capacitor of capacitance C are connected to an alternating source of emf. The quality factor of the circuit is

(a) $R/\omega_0 LC$ (b) $(\omega_0 L/CR)^{\frac{1}{2}}$ (c) $(1/LCR)^{\frac{1}{2}}$ (d) $\omega_0 L/R$

9. An inductor may store energy in

(a) its electric field (b) its coil

(c) its magnetic field (d) both in electric and magnetic fields

10. A 100 mH coil carries a current of 1A. Energy stored in the form of magnetic field is

(a) 0.5 J (b) 1 J (c) 0.05 J (d) 0.1 J

11. The core of a transformer is laminated, because

(a) rusting of the core may be prevented

(b) energy losses due to eddy current may be minimised

(c) ratio of voltage in primary and secondary may be increased

(d) the weight of the transformer may be reduced

12. The primary winding of a transformer has 500 turns, whereas its secondary has 5,000 turns. The primary is connected to an a.c. supply 20V – 50 Hz. The secondary will have an output of

(a) 200V – 50 Hz (b) 200 V – 500 Hz (c) 2V – 50 Hz (d) 2V – 5Hz

13. A step up transformer operates on a 230 V line and supplies a current of 2A. The ratio of primary and secondary windings is 1: 25. The primary current is

(a) 12.5 A (b) 50 A (c) 8.8 A (d) 25 A

14. The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux $\phi$ linked with the primary coil is given by $\phi = \phi_0 + 4t$, where $\phi$ is in weber, t is time in second and $\phi_0$ is a constant, the output voltage across the secondary coil is

(a) 90 V (b) 120 V (c) 220 V (d) 30 V

15. A transformer is used to light a 100 W and 110V lamp from a 220 V mains. If the main current is 0.5A, the efficiency of the transformer is approximately

(a) 30% (b) 50% (c) 90 % (d) 10%

16. Power dissipated in an LCR series circuit connected to an a.c. source of emf $\xi$ is

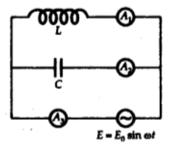
(a) $\dfrac{\xi^2\sqrt{R^2-\left(L\omega-\dfrac{1}{C\omega}\right)^2}}{R}$ (b) $\dfrac{\xi^2\left[\sqrt{R^2+\left(L\omega-\dfrac{1}{C\omega}\right)^2}\right]}{R}$

(c) $\dfrac{\xi^2 R}{\sqrt{R^2-\left(L\omega-\dfrac{1}{C\omega}\right)^2}}$ (d) $\dfrac{\xi^2 R}{\left[R^2+\left(L\omega-\dfrac{1}{C\omega}\right)^2\right]}$

17. An a.c. source is rated at 220V, 50 Hz. The time taken for voltage to change from its peak value to zero is

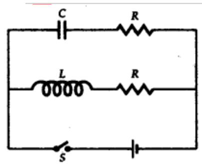
(a) 50 sec (b) 0.02 sec (c) 5 sec (d) $5\times10^{-3}$ sec.

18. An inductor L and a capacitor C are connected in the circuit as shown in the figure. The frequency of the power supply is equal to the resonant frequency of the circuit. Which ammeter will read zero ampere?




(a) $A_1$ (b) $A_2$ (c) $A_3$ (d) None of these

19. In the circuit shown in the figure, the switch S is closed at time t = 0.




$\left(\text{Given, } R=\sqrt{\dfrac{L}{C}}\right)$

The current through the capacitor and inductor will be equal at time t equals

(a) RC (b) RC ln 2 (c) $\dfrac{1}{RC\ln 2}$ (d) LR

## *Answers*

**IIT Entrance Exam.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | D | 3. | A | 4. | B | 5. | A |
| 6. | B | 7. | A, C, D | 8. | B | 9. | D | 10. | C |

**AIEEE**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | C | 2. | C, D | 3. | D | 4. | C | 5. | C |
| 6. | A | 7. | B | 8. | C | 9. | C | 10. | B |
| 11. | B | 12. | A | 13. | C | 14. | B | 15. | D |

**AIIMS Entrance Exam.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | B | 2. | C | 3. | B | 4. | D | 5. | C |
| 6. | B | 7. | B | 8. | C | 9. | C | 10. | C |
| 11. | A | 12. | D | 13. | D | 14. | B | 15. | B |
| 16. | A | 17. | C | 18. | C | 19. | A | 20. | A |
| 21. | B | | | | | | | | |

**BSE PMT Prelims Exam**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | A | 3. | D | 4. | D | 5. | D |
| 6. | D | 7. | D | 8. | D | 9. | C | 10. | C |
| 11. | B | 12. | A | 13. | B | 14. | B | 15. | C |
| 16. | D | 17. | D | 18. | C | 19. | B | | |