XII 7

OPTICS

LEARNING OBJECTIVES

1. Reflection of Light, Spherical Mirrors, Mirror Formula. Refraction of Light, Total Internal Reflection and Its Applications Optical Fibres, Refraction at Spherical Surfaces, Lenses, Thin Lens Formula, Lens-Maker's Formula. Magnification, Power of A Lens, Combination of Thin Lenses in Contact, Combination Of a Lens and a Mirror. Refraction and Dispersion of Light Through a Prism.
2. Scattering of Light- Blue Colour of The Sky And Reddish Appearance of The Sun at Sunrise and Sunset.
3. Optical Instruments: Human Eye, Image Formation and Accommodation, Correction of Eye Defects (Myopia and Hypermetropia) Using Lenses.
4. Microscopes and Astronomical Telescopes (Reflecting and Refracting) and Their Magnifying Powers.
5. Wave Optics: Wavefront and Huygens' Principle, Reflection and Refraction of Plane Wave at a Plane Surface Using Wavefronts.
6. Proof of Laws of Reflection and Refraction Using Huygens' Principle.
7. Interference, Young's Double Hole Experiment And Expression for Fringe Width, Coherent Sources and Sustained Interference of Light.
8. Diffraction Due to a Single Slit, Width of Central Maximum.
9. Resolving Power of Microscopes and Astronomical Telescopes. Polarisation, Plane Polarized Light; Brewster's Law, Uses of Plane Polarized Light and Polaroids.

KEY CONCEPTS

PART - A : RAY OPTICS AND OPTICAL INSTRUMENTS

7.1 INTRODUCTION

* The study of optics allow us to predict what light will do in the real world.
* **Physical optics** deals primarily with the nature and properties of light itself. It is an approximation valid when energy changes are negligible. Light is treated as a wave. Allows an understanding of the "microscopic" effects such as diffraction, interference and polarization.
* **Geometrical optics -** a further approximation adequate when objects are large compared to the wavelength of light. Light is treated as rays traveling in straight lines. This is sufficiently accurate for most calculations involving eyes and lenses.
* **Quantum optics -** The best, but most complex, description we have, needed when light interacts with matter and there are significant energy changes e.g. for an understanding of the photoelectric effect, spectra, Lasers, Electro-optic devices.

NOTE

* Wave optics is used to verify wave nature of light.
* Quantum optics is used to verify particle nature of light while Geometrical optics does not ensure any particular nature of light as phenomenon in geometric optics can be shown by both particles as well as waves.

* Geometrical Optics is an idealised optics that essentially ignores the wave nature of light. It is optics for $\lambda \to 0$ in comparison to the objects encountered. It neglects interference, diffraction effects & polarization and uses rays to trace the path of light through reflecting and refracting bodies.

Optics as a Branch of Physics

- OPTICS
  - GEOMETRICAL OPTICS
    - Concern with image formation
    - Treat light as
    - Ray
      - Reflection
        - On plane mirror
        - On Curved mirror
      - Refraction
        - On plane Surface
        - Total Internal Reflection
        - Prism Theory
        - Refraction on curved surface
        - Lens theory
        - Dispersion
        - Optical Instruments
  - PHYSICAL OPTICS
    - Wave Optics
      - Treat light as
      - Wave
        - Interference
          - 1. Due to division of wave front (like Young Double Slit exp., Fresnel Biprism Exp., Lloyd's Mirror
          - 2. Due to division of amplitude (like thin films, Newton rings, Michelson's Interferrometer)
        - Diffraction
          - Fresnel
          - Fraunhoffer
    - Quantum Optics
      - Treat light as
      - Photon
        - To study Photoelectric effect, Raman effect, LASER

7.2 REFLECTION FROM PLANE SURFACES

* When a light ray strikes the surface separating two media, a part of it gets reflected, i. e., returns back in the initial medium, it is known as reflection.

Ray

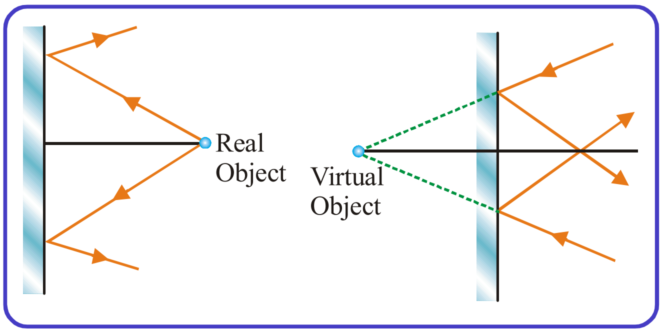
* A ray of light is the straight line path of transfer of light energy.
* It is represented by a straight line an arrow - head indicating the direction of propagation.

Mirror

* It is a highly polished smooth surface from which most of the incident light gets reflected.

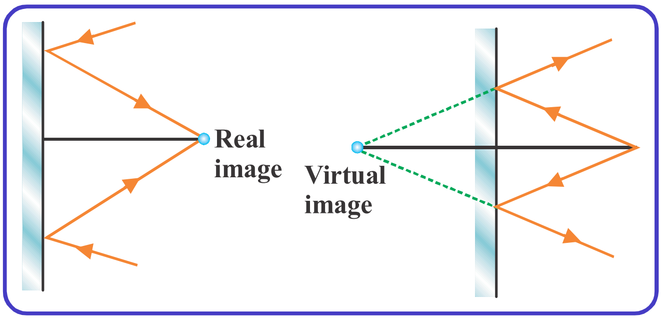
Object

* Object is defined on the basis of incident ray.
* Minimum two rays are required to show the position of object.
* Point from which incident ray actually diverge is called real object. Point at which incident rays appear to converge is called virtual object.



Image

* Point at which reflected or refracted rays actually converge is called real image. Point from which reflected or refracted rays appear to diverge is called virtual image.



* Minimum two reflected or refracted rays are required to determine the image position.

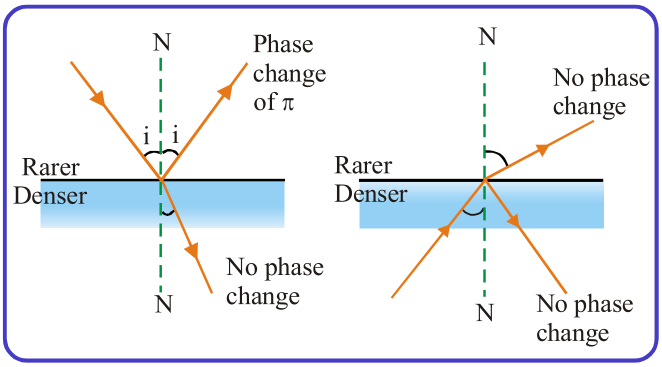
LAWS OF REFLECTION

There are three laws of reflection :

(i) The angle of incidence is equal to the angle of reflection. $\boxed{\angle i = \angle r}$

(ii) The incident ray, the normal and the reflected ray lie in the same plane.

(iii) There is a phase change of π radians when light wave is reflected by permeable denser medium surface but no phase change occurs if it is reflected by rarer medium surface.



REFLECTION BY PLANE MIRROR

* Images with mirrors are formed when many non-parallel rays from a given point on a source are reflected from the mirror surface, converge, and form a corresponding image point. When this happens, point by point for an extended object, an image of the object, point by point, is formed.

* Image formation in a plane mirror is illustrated in several sketches shown in Figure.

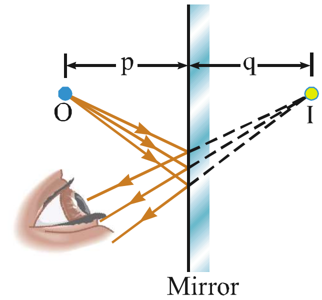


Figure : An image formed by reflection from a flat mirror. The image point I is located behind the mirror a perpendicular distance q from the mirror (the image distance). The image distance has the same magnitude as the object distance p.

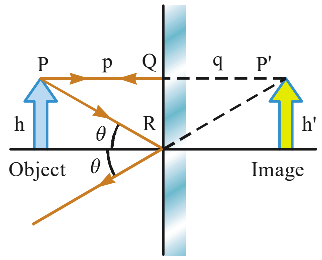


Figure : A geometric construction that is used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and P'QR are congruent, $|p| = |q|$ and $h = h'$.

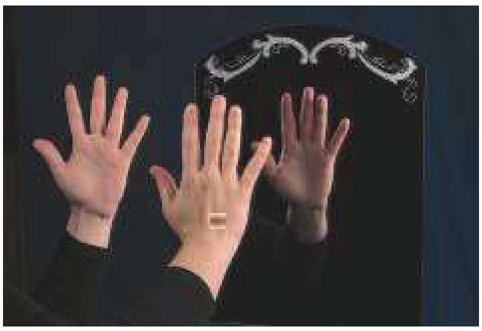


Figure : The image in the mirror of a person's right hand is reversed front to back. This makes the right hand appear to be a left hand. Notice that the thumb is on the left side of both real hands and on the left side of the image. That the thumb is not on the right side of the image indicates that there is no left-to-right reversal.

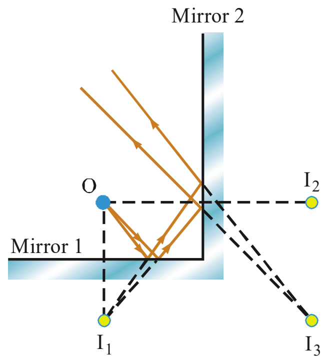
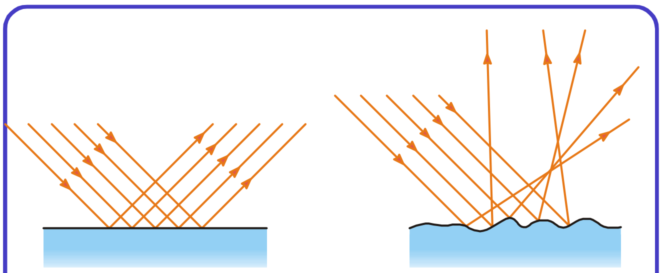


Figure : When an object is placed in front of two mutually perpendicular mirrors as shown, three images are formed.

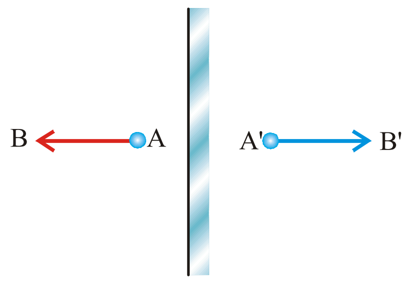
* When reflection takes place by a smooth surface, it is called regular reflection but when reflection takes place by a rough surface, it is called diffused reflection.



(a) Specular reflection (b) Diffuse reflection

Characteristics of Reflection by Plane Mirror

1. Perpendicular distance of object from mirror = Perpendicular distance of image from mirror.



2. The image is laterally inverted (better word perversion)

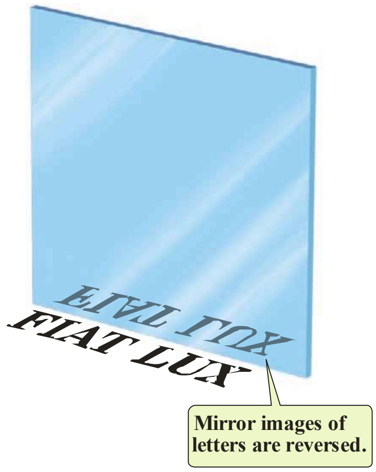
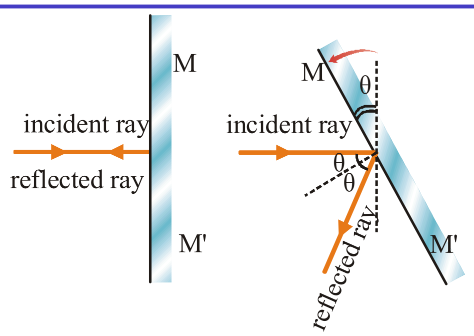


Figure : Some letters and their images in a mirror.

3. The line joining the object point with its image is normal to the reflecting surface.
4. The size of the image is the same as that of the object.
5. For a real object the image is virtual and for a virtual object the image is real.
6. If keeping the incident ray fixed, the mirror is rotated by an angle θ, about an axis in the plane of mirror, the reflected ray is rotated through an angle 2θ. This is illustrated in figure.



* When two plane mirror are held at angle θ with their reflecting surfaces facing each other and an object is placed between them, images are formed by successive reflections.

First of all we will calculate, $n = \dfrac{360}{\theta}$ then –

(i) If n is fraction, number of images will be whole part of this number. Ex. If n = 7.2, number of images = 7

(ii) If n is even whole number then number of images $= n - 1$

Ex. If $n = 6$ (for $\theta = 60°$) number of images $= 5$

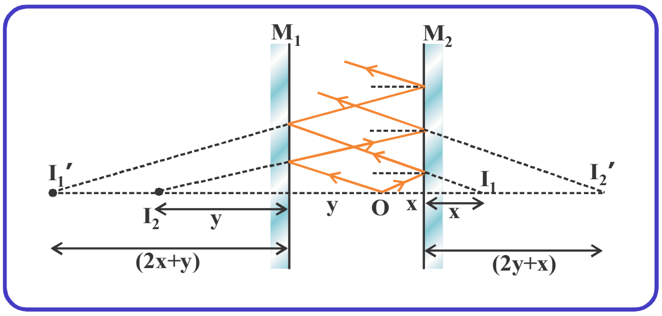
(iii) If n is odd whole number then

Number of images

$= n - 1$ (for symmetric object),

$= n$ (for asymmetric object).

* If an object is placed between two parallel mirrors ($\theta = 0°$), the number of images formed will be $(360/0) = \infty$ but of decreasing intensity in accordance with $I \propto (1/r^2)$.



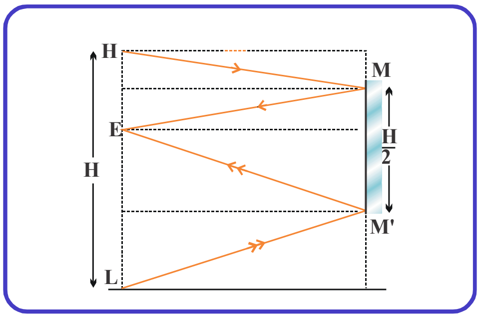
* The number of images formed by two mutually perpendicular mirrors ($\theta = 90°$) will be 3. All these three images will lie on a circle with centre at C – the point of intersection of mirrors M_1 and M_2 and whose radius is equal to the distance between C and object O.

* Two mirrors inclined to each other at different angles may provide same number of images, e.g., for any value of θ between 90° and 120° the number of maximum images formed (n) is 3. This in turn implies that if θ is given, n is unique but if n is given, θ is not unique.

* The number of images seen may be different from the number of images formed and depends on the position of observer relative to object and mirrors – e.g., if $\theta = 120°$ maximum number of images formed will be 3 (object not on bisector) but no. of images seen can only be 1, 2 or 3 depending on the position of observer.

* Though every part of a mirror forms a complete image of an object, we usually see only that part of it from which light after reflection from the mirror reaches our eye. This is why:

(a) To see his full image in a plane mirror a person requires a mirror of at least half of his height.

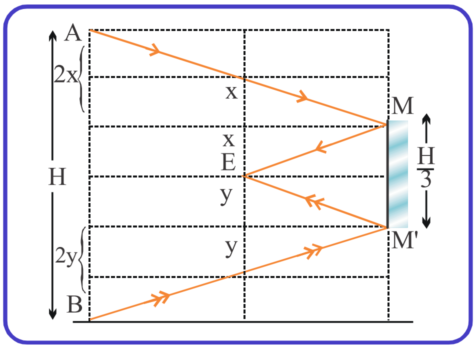


(b) To see a complete wall behind himself a person requires a mirror at least (1/3) height of wall and he must be in the middle of wall and mirror.

$H = (2x) + (x + y) + (2y)$

$H = 3\,(x + y) = 3\,MM'$

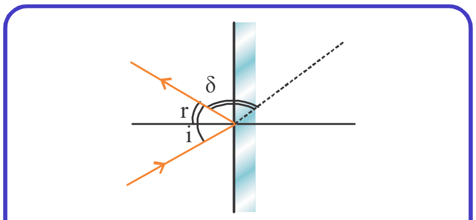
$MM' = H/3$



* Deviation δ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i,

$\delta = 180 - (\angle i + \angle r) = (180 - 2i)$

[as $\angle i = \angle r$]

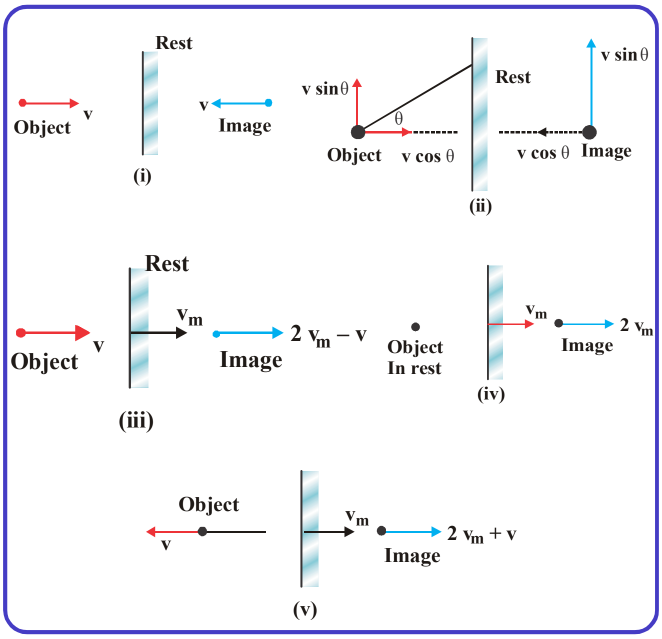


Plane Mirror

* If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at same speed v, i.e., the speed of image relative to object will be $v - (-v) = 2v$. Similarly if the mirror is moved towards (or away from) the object with a speed v the image will move towards (or away from) the object with a speed 2v.

$\vec{V}_{OM} = -\vec{V}_{IM}$, where $\vec{V}_{OM}$ = velocity of object relative to mirror and $\vec{V}_{IM}$ = velocity of image relative to mirror.

Observes the following figures :



Golden Tips

* On reflection the velocity, wavelength and frequency of light does not change. But the amplitude or intensity of the reflected ray is less than that of the incident ray.

* Time of image in plane mirror

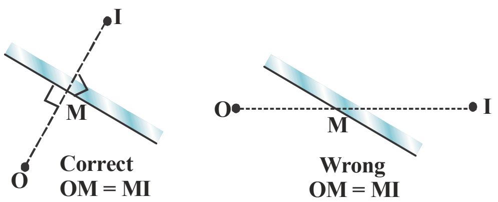
(a) Real time = X^H, Image time = $12^H - X^H$

(b) Real time = X^HY^M,
Image time = $11^H60^M - X^HY^M$

(c) Real time = $X^HY^MZ^S$,
Image time = $11^H59^M60^S - X^HY^MZ^S$

(d) If $X^HY^MZ^S > 11^H59^M60^S$ image time $= 23^H59^M60^S - X^HY^MZ^S$

* To locate of image of an object for an incline plane mirror, see the perpendicular distance of the object from the mirror.



* If the angle between the two mirrors is θ, the deviation produced by successive reflections is $\delta = \delta_1 + \delta_2 = 2\pi - 2\theta$.

* In figure, we see that

$\alpha + 2\gamma + 2\,(90° - \phi) = 180°$

$\alpha = 2\,(\phi - \gamma)$

The change in direction of the light ray is angle β, which is $180° - \alpha$

$\beta = 180° - \alpha = 180° - 2\,(\phi - \gamma)$

$= 180° - 2\,[\phi - (90° + \phi - \theta)]$

$= 360° - 2\theta$

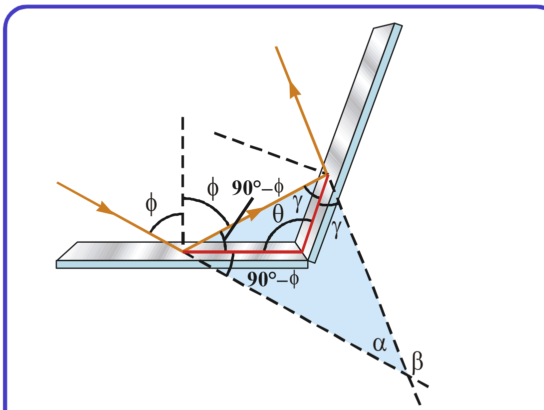


Figure : The geometry for an arbitrary mirror angle.

* If width of face is D and distance between eyes is d then the minimum length of mirror required to see complete face $= \dfrac{D-d}{2}$

* A virtual image can be photographed but it cannot be obtained on a screen.

* If mirror moves a distance x towards or away from the object, the image will move a distance 2x towards or away from the object/mirror. Similar is the case with velocity also.

* If the mirror and object are moved by a distance x, each, in opposite directions, the image will be displaced by a distance 3x in the direction of the displacement of the mirror. Same is the case with the velocity (v).

* If a plane mirror is rotated in its own plane, the incident ray and the reflected ray remain at the same positions.

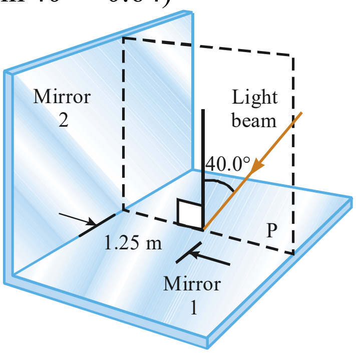
* If a luminous object is placed in front of a thick glass mirror, several images are formed due to multiple reflections.

The second image formed by the first reflection by the polished surface is much brighter than the others. The intensity of the other images rapidly fade away.

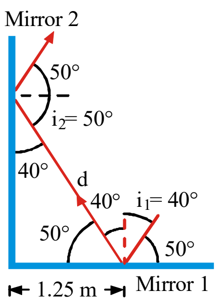
* Focal length of a plane mirror is infinity and hence its power is zero.
* The rays of light converge to real image or virtual object but diverge from virtual image or real object.
* One can see the image of an object in a plane mirror only if the line joining the image and the eye of the observer cuts the reflecting surface.

EXAMPLE 1

The two mirrors illustrated in Figure meet at a right angle. The beam of light in the vertical plane P strikes mirror 1 as shown. (a) Determine the distance the reflected light beam travels before striking mirror 2. (b) In what direction does the light beam travel after being reflected from mirror 2? (sin 40° = 0.64)



SOLUTION:

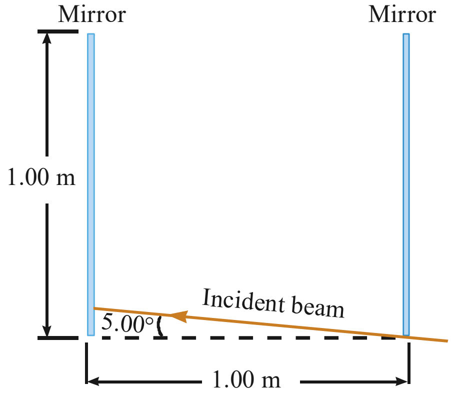


(a) From geometry, 1.25 m = d sin 40.0°
so d = 1.94 m .

(b) 50.0° above the horizontal
or parallel to the incident ray.

EXAMPLE 2

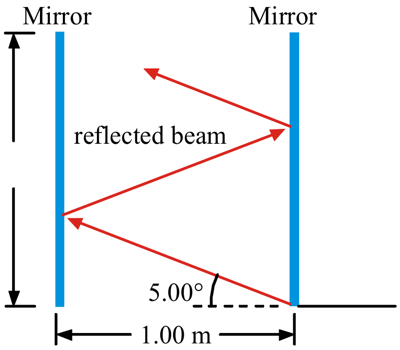
How many times will the incident beam shown in figure be reflected by each of the parallel mirrors? (tan 5° = 0.0875)



SOLUTION:

The incident light reaches the left-hand mirror at distance (1.00 m) tan 5.00° = 0.0875 m above its bottom edge. The reflected light first reaches the right-hand mirror at height

2 (0.0875 m) = 0.175 m.



It bounces between the mirrors with this distance between points of contact with either.

Since, $\dfrac{1.00\text{m}}{0.175\text{m}} = 5.72$

The light reflects five times from the right-hand mirror and six times from the left.

EXAMPLE 3

A ray of light is travelling at an angle of 20° above the horizontal. At what angle with the horizontal must a plane mirror be placed in its path so that it becomes vertically upward after reflection.

SOLUTION:

Let us first place the mirror horizontally. The reflected ray now goes at an angle of 20° above the horizontal. To make the reflected ray vertical, it has to be rotated anticlockwise by 70°. Hence the mirror must be rotated by 70°/2 = 35°.

$\Rightarrow$ the angle of the mirror with the horizontal = 35°.

Checkup 1

Q.1 Why do some emergency vehicles have the symbol ƎƆИA⅃UᙠMA written on the front?

Q.2 Images formed by an object placed between two plane mirrors whose reflecting surface make an angle of 90º with one another lie on a-

(A) Straight line (B) Zig-zag curve
(C) Circle (D) Ellipse

Q.3 Which of the following letters do not suffer lateral inversion –

(A) HGA (B) HOX
(C) VET (D) YUL

Q.4 An object is placed between two plane mirrors inclined at 30° to each other. How many images will be formed?

Q.5 Find velocity of image when object and mirror both are moving towards each other with velocity 2m/s and 3 m/s.

7.3 REFLECTION AT CURVED SURFACES

* A **curved mirror** is a smooth reflecting part (in any shape) of a symmetrical curved surface such as paraboloidal, ellipsoidal, cylindrical or spherical.

* **Concave Mirror :** If the reflection takes place from the inner surface of a spherical mirror, then the mirror is called concave mirror.

* **Convex Mirror :** If the outer surface of the spherical mirror acts as a reflector then the mirror is called convex mirror.

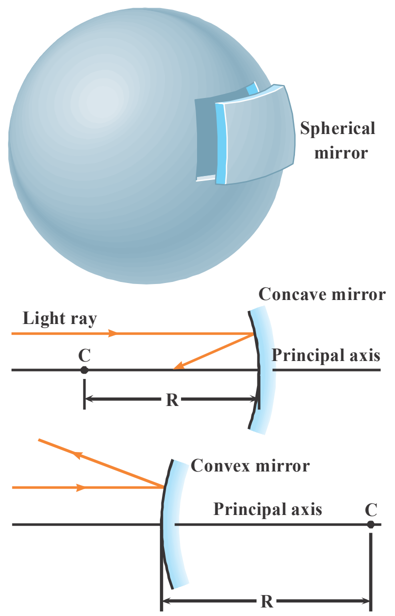


Figure : A spherical mirror has the shape of a segment of a spherical surface. The center of curvature is point C and the radius is R. For a concave mirror, the reflecting surface is the inner one; for a convex mirror it is the outer one.

Terms Related to Spherical Mirror

* **Centre of Curvature (C) :** It is the centre of sphere of which the mirror is a part.

* **Radius of Curvature (R) :** It is the radius of the sphere of which the mirror is a part.

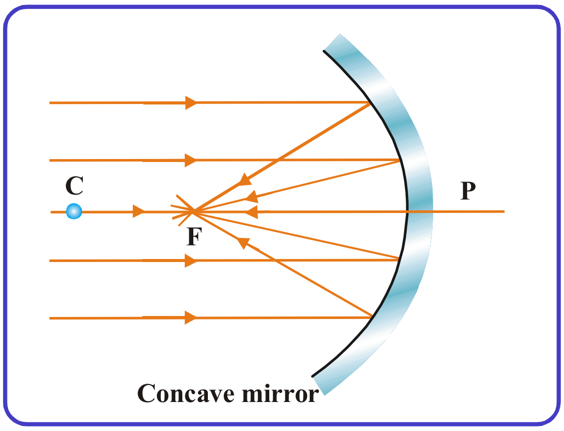
* **Pole (P) :** It is the geometrical centre of the spherical reflecting surface.

* **Principal Axis :** It is the straight line joining the curvature to the pole.

* **Focus (F) :** When a narrow beam of rays of light, parallel to the principal axis and close to it (know as paraxial rays), is incident on the surface of a mirror, the reflected beam is found to converge (concave mirror) or appear to diverge (convex mirror) from a point principal axis. This point is called **focus.**

* **Focal Length (F) :** It is the distance the pole and the principal focus. For spherical mirrors, f = R/2

Reflection through Concave Mirror



F → Principal focus
P → Pole of mirror
C → Centre of curvature.
PC = Radius of curvature.
PF = Focal length.

When a narrow beam of light travelling parallel to the principal axis is incident on the reflecting surface of the concave mirror, the beam after reflection converge at a point on the principal axis.

Rules for Ray Diagrams

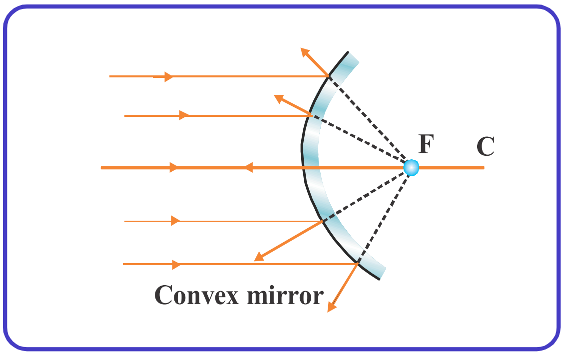
* When a ray falls in the direction of centre of curvature of mirror then it reflects back along the same path.

* A ray, parallel to the principal axis will after reflection, pass through the focus.

* A ray, passing through the focus is reflected parallel to the principal axis.

Reflection through Convex Mirror

* When a narrow beam of light travelling parallel to the principal axis is incident on the reflecting surface of the mirror, the beam after reflection appear to diverge from a point on the principal axis.

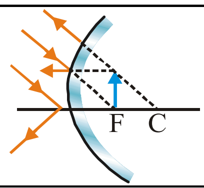
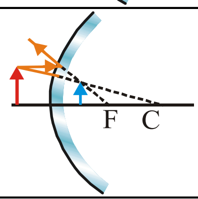


* A convex mirror forms only virtual images for all positions of the real object.

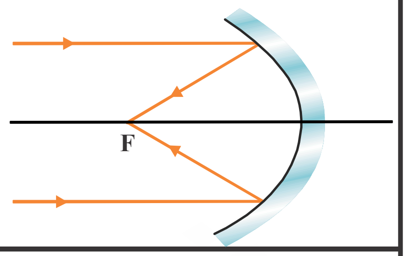
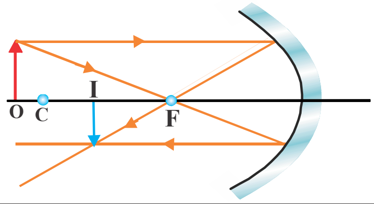
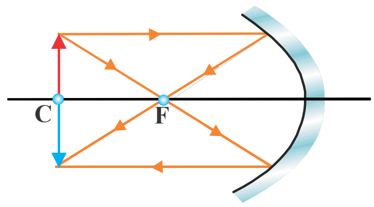
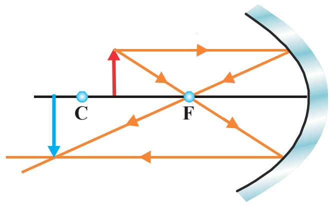
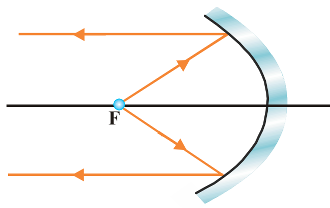
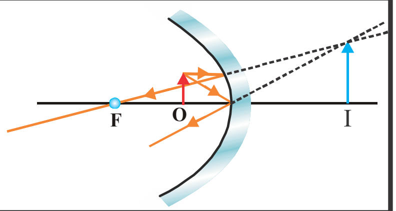
* The image is always virtual, erect, smaller than the object and is located between the pole & the focus.

* The image becomes smaller and moves closer to the focus as the object is moved away from the mirror.

* **Convex mirror**

Position of object	Figure	Position of image	Nature of image
1. At infinity		Appears at the principal focus.	Virtual, erect and extremely diminished.
2. Between infinity and the pole.		Appears between the principal focus and the pole.	Virtual, erect and diminished.

* **Image formed by the concave mirror :**

Position of object	Position of image	Nature	Figure
At infinity	At the focus	Real, inverted & diminished	
Between infinity & Centre of Curvature	Between focus & centre of curvature	Real inverted small in size	
At centre of curvature	At centre of curvature	Real, inverted and of the same size	
Between Focus & centre of curvature	Beyond centre of curvature	Real, inverted and enlarged	
At Focus	At infinity	Real, inverted and very large	
Between Focus & Pole	Behind the mirror	Erect virtual & enlarged	

Mirror Formulae

1. If an object is placed at a distance u from the pole of a mirror and its images is formed at a distance v (from the pole) then

$$\boxed{\frac{1}{v}+\frac{1}{u}=\frac{1}{f}}$$

f = x-coordinate of focus, v = x-coordinate of image, u = x-coordinate of object.
In this formula to calculate unknown, known quantities are substituted with proper sign.

Sign Convention :

* All distances are measured from the pole.
* The distance measured along the direction of propagation of light are taken as positive and the direction opposite to the propagation of light is taken as negative.

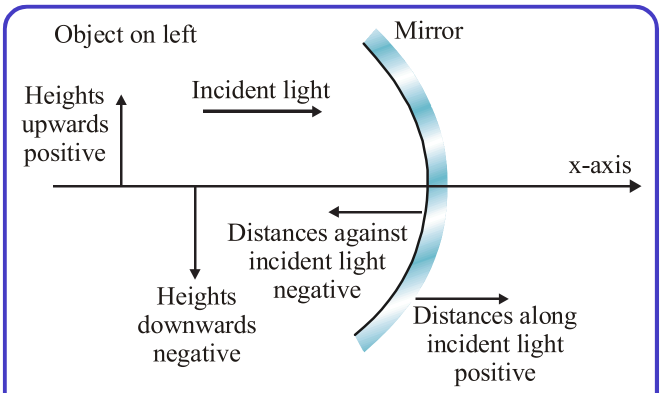


Figure : The Cartesian Sign Convention.

* The distance (heights) measured above the principal axis i.e. along positive Y axis, are taken as positive while distances below the principal axis i.e. along negative Y axis are taken as negative.

2. If a thin object linear size O situated vertically on the axis of a mirror at a distance u from the pole and its image of size I is formed at a distance v (from the pole), magnification (transverse) is defined as

$$\boxed{m = \left[\frac{I}{O}\right] = -\left[\frac{v}{u}\right]}$$

- (+ve Erect image)
- (–ve inverted image)
- (|m| >1 large image)
- (|m| < 1 Small image)

Here-ve magnification implies that image is inverted with respect to object while +ve magnification means that image is erect with respect to object

3. Other formulae of magnification

$$m = \frac{f}{f-u}, \quad m = \frac{f-v}{f}$$

4. The power of a mirror is defined as

$$\boxed{P = -\frac{1}{f(\text{in m})}} = -\frac{100}{f(\text{in cm})}$$

The unit of power is diopter.

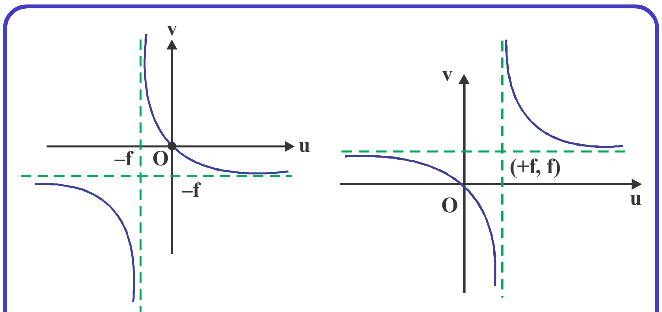
5. The focal-length of a spherical mirror of radius R is given by $\boxed{f = \frac{R}{2}}$

In sign convention, f (or R) is negative for concave or converging mirror and positive for convex or diverging mirror.

6. Newton's formula $\boxed{f^2 = x_1 x_2}$

f = focal length of mirror, x_1 = Position of object with respect to focus, x_2 = Position of image with respect to focus.

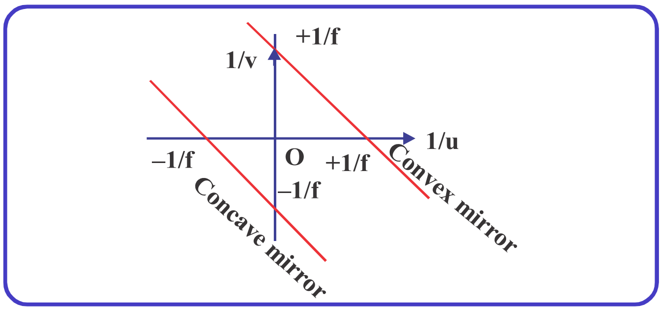
* The v versus u graph for a spherical mirror is a rectangular hyperbola.



For a concave mirror

For a convex mirror

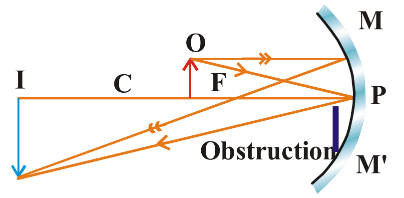
* $\frac{1}{v}$ versus $\frac{1}{u}$ graph for spherical mirror is a straight line.



Golden Tips

* **Concave paraboloid mirror :** Search lights, Motor head lights, Shaving mirror, by dentists, in solar cookers, in reflecting type telescope, etc.
* **Convex mirror** are used as rear view mirror because their field of view is large, Reflectors for street light, etc.

* As every part of a mirror forms complete image, if a part of mirror (say half) is obstructed (say converted with black paper) full image will be formed but intensity will be reduced.



* If an object moved at constant speed towards a concave mirror from infinity to focus, the image will move slower in the beginning and faster in the last, away from the mirror. This is because in the time object moves from ∞ to C the image will move from F to C and when object moves from C to F the image will move from C to ∞. At C the speed of object and image will be equal.

$$v_i = -\left(\frac{v^2}{u^2}\right) v_o \text{ [Where } v_i \text{ velocity of image]}$$

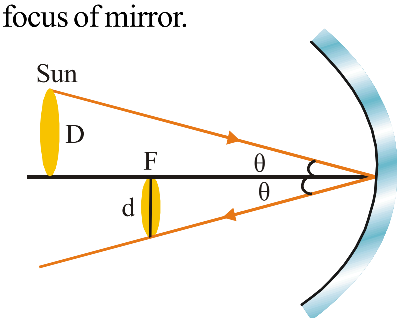
$$v_i = -\left(\frac{v}{u}\right)^2 v_0 \text{ [} v_o \text{ velocity of object]}$$

* **Identification of mirror on the basis of images :**

If the image of a real object placed near the mirror is

(i) Virtual, erect and of same size, the mirror is plane.
(ii) Virtual, erect and magnified, the mirror is concave.
(iii) Virtual, erect and diminished, the mirror is convex.

* Image of star; moon or distant object is formed at focus of mirror.



If y = the distance of sun or moon from earth.
D = diameter of moon or sun's disc.
f = focal length of the mirror
d = diameter of the image
θ = the angle subtended by sun or moon's disc

Then $\tan\theta = \theta = \frac{D}{y} = \frac{d}{f}$, Here, θ is in radian.

* The image formed by a concave mirror may be real or virtual depending on whether $u \geq f$ or $u < f$, for real object only.

* In case of a convex mirror, f is positive and u is also positive for a virtual object, hence v will be negative i.e., the image is real if $u < f$ and v will be positive, i.e., the image is virtual if $u \geq f$. Thus we conclude that a convex mirror forms always virtual image of a real object but it may form real or virtual image of a virtual object depending on whether $u < f$ or $u \geq f$.

* For no position of a real object, its image is formed between the pole and the focus of a concave mirror.

* In a concave mirror, the minimum distance between the object and its real image is zero, when $u = v = 2f = R$.

* For all positions of a real object infront of a convex mirror, the positions of images are between the pole and the focus. As the object moves from infinity to the pole, its image moves from F (focus) to P (pole). Hence $v_i << v_0$.

* Converging reflected rays form a real image and diverging reflected rays form a virtual image.

* The focal length and the radius are infinite for a plane mirror.

* Generally, a convergent mirror means a concave mirror and a divergent mirror means a convex mirror.

* A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is $\left(\frac{f}{u-f}\right)^2 b$.

➢ $$\text{Length} = m^2 b = \left(\frac{v}{u}\right)^2 b = \left(\frac{f}{u-f}\right)^2 b$$

$$\left(\because \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \therefore \frac{v}{u} = \frac{f}{u-f}\right)$$

EXAMPLE 4

A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

SOLUTION:

Using $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign conventions,

we get, $-\frac{1}{27}+\frac{1}{v}=-\frac{1}{(36/2)}$

or $\frac{1}{v}=-\frac{1}{18}+\frac{1}{27}=\frac{-3+2}{54}=-\frac{1}{54}$

or $v=-54$ cm

Image is formed at the distance of 54 cm from the mirror on the same side as the object. This is the position where a screen should be placed.

Now magnification, $m=\frac{I}{O}=\frac{-v}{u}$

$\therefore \quad \frac{I}{2.5}=\frac{-(-54)}{-27}=-2$

$\therefore \quad I=-2\times 2.5=-5$ cm

The image is real, inverted and magnified. If the candle is brought closer to the mirror, the screen would have to be moved away from the mirror. If the distance of the candle from mirror is less than 18 cm, the image becomes virtual and is not observable on the screen.

EXAMPLE 5

A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

SOLUTION:

Using $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign conventions,

$-\frac{1}{12}+\frac{1}{v}=\frac{1}{15}\Rightarrow\frac{1}{v}=\frac{1}{15}+\frac{1}{12}=\frac{4+5}{60}$

$\Rightarrow \frac{1}{v}=\frac{9}{60}$ or $v=\frac{60}{9}$ cm or $v=6.7$ cm

$\Rightarrow$ The image is formed at 6.7 cm behind the mirror.

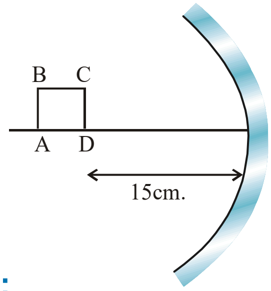
As magnification $m=\frac{I}{O}=-\frac{v}{u}$, we get ,

$\frac{I}{4.5}=\frac{-\frac{60}{9}}{-12}=\frac{5}{9} \quad \therefore \; I=\frac{5\times 4.5}{9}=2.5$ cm

$\Rightarrow$ The image is virtual, erect and diminished. As we move the needle away from the mirror, the image goes on decreasing in size and move towards the principal focus on the other side.

EXAMPLE 6

A square ABCD of side 1mm is kept at distance 15 cm. infront of the concave mirror as shown in the figure. The focal length of the mirror is 10cm. Find the length of the perimeter of its image.



SOLUTION:

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\;;\;\frac{1}{v}+\frac{1}{-15}=\frac{1}{-10}$

$v=-30,\; m=-\frac{v}{u}=-2$

$\therefore \quad A'B'=C'D'=2\times 1=2\text{mm}$

$\frac{B'C'}{BC}=\frac{A'D'}{AD}=\frac{v^2}{u^2}=4$

$\Rightarrow B'C'=A'D'=4$ mm

$\therefore \quad$ Length $=2+2+4+4=12$ mm

Checkup 2

Q.1 Consider a concave spherical mirror with a real object. Is the image always inverted? Is the image always real? Give conditions for your answers.

Q.2 Use the mirror equation to deduce that:

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Q.3 You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

Q.4 A concave mirror cannot form :
(A) virtual image of virtual object
(B) virtual image of a real object
(C) real image of a real object
(D) real image of a virtual object

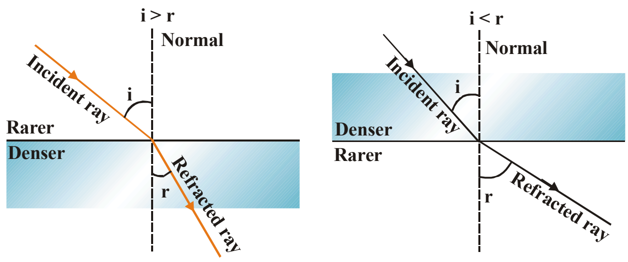
Q.5 A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at
(A) infinity
(B) pole
(C) focus
(D) 15 cm behind the mirror

Q.6 The distance of an object from a spherical mirror is equal to the focal length of the mirror. Then the image:
(A) must be at infinity (B) may be at infinity
(C) may be at the focus (D) none

7.4 REFRACTION AT PLANE SURFACES

* The bending of the light ray from its path in passing from one medium to the other medium is called **refraction of light.**

* If the refracted ray bends towards the normal relative to the incident ray, then the second medium is said to be denser than the first medium. But if the refracted ray bends away from the normal, then the second medium is said to be rarer than the first medium.
i = angle of incidence in the medium I.
r = angle of refraction in the medium II.



You must have seen bending of pencil in water beaker due to refraction.

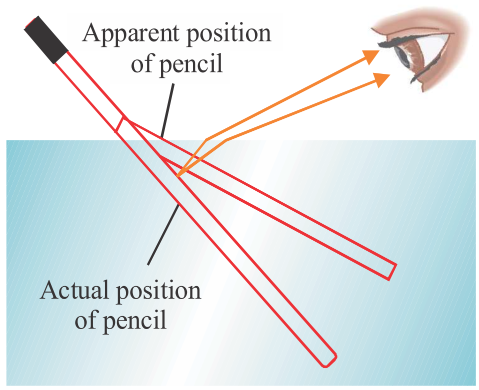


Figure : A pencil in a glass of water looks bent due to the light refraction.

REFRACTIVE INDEX

* **Absolute refractive index of medium (n or μ) :**

$$\frac{\text{Velocity of light in vaccum}}{\text{Velocity of light in medium}} = \frac{c}{v} = n \text{ or } \mu$$

For vacuum $\mu = 1$
air $\mu = 1.003$
diamond $\mu = 2.46$

* **Refractive index of second medium w.r.t. first medium**

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

$$= \frac{\text{Velocity of light in first medium}}{\text{Velocity of light in second medium}}$$

* Refractive index is the relative property of two medium. If the first medium carrying the incident ray is a vacuum, then the ratio $\frac{\sin i}{\sin r}$ is called the 'absolute refractive index of the second medium'. The relative refractive index of any two medium is equal to the ratio of their absolute refractive indices. Therefore, if the absolute refractive indices of medium 1 & 2 be n_1 & n_2 respectively, then the refractive index of medium 2 with respect to medium 1 is-

$${}_1n_2 = n_{12} = \frac{n_2}{n_1} = \frac{\sin i}{\sin r} \;;\; n_1 \sin i = n_2 \sin r$$

* $${}_1n_2 = \frac{1}{{}_2n_1}$$

* For three medium 1, 2, 3 due to successive refraction.

$${}_1n_2 \times {}_2n_3 \times {}_3n_1 = 1 \;;\; \frac{n_2}{n_1} \times \frac{n_3}{n_2} \times \frac{n_1}{n_3} = 1$$

LAW OF REFRACTION

Snell's Law

For any two medium and for light of a given wave length, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

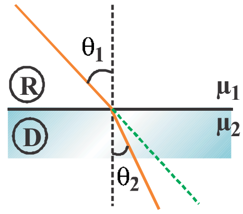
$\frac{\sin i}{\sin r} = \text{constant}$, where i = incidence angle,
r = Refraction angle.

* The incident ray, the refracted ray and the normal at the incident point all lie in the same plane.
* Frequency (and hence colour) and phase do not change (while wavelength and velocity changes)

$$\boxed{\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2} = {}_1\mu_2}$$

* Snell's law is in accordance with **Fermat's principle** according to which the ray follows the path for which time is least (precisely optimum) as compared to nearby paths.
* When light passes from rarer to denser medium it bends toward the normal.

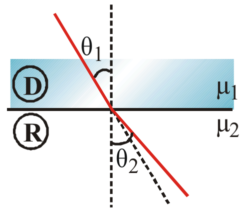
Using Snell's Law $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1}$$

Thus If $\mu_2 > \mu_1$ then $\theta_2 < \theta_1$

* When light passes from denser to rarer medium it bends away from the normal.



From Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1}$

Thus If $\mu_2 < \mu_1$ then $\theta_2 > \theta_1$

* When light propagates through a series of layers of different medium, then according to Snell's law $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \ldots\ldots\ldots = \text{constant}$
* **Conditions of no refraction**

(i) If light is incident normally on a boundary i.e., $\angle i = 0°$
Then from Snell's law
$\mu_1 \sin 0 = \mu_2 \sin r \Rightarrow \sin r = 0$
i.e. $\angle r = 0$ i.e., light passes undeviated from the boundary. (so boundary will be invisible)

(ii) If the refractive indices of two media are equal i.e., if, $\mu_1 = \mu_2 = \mu$
Then from Snell's law
$\mu_1 \sin i = \mu \sin r \Rightarrow \angle i = \angle r$ i.e., ray passes undeviated from the boundary with $\angle i = \angle r \neq 0$ and boundary will not be visible.
This is also why a transparent solid is invisible in a liquid if $\mu_s = \mu_L$

Apparent Depth and Normal Shift

* The fish seems to be nearer to the surface than it is (if the angler wants to touch the fish, she must immerse her hand down deeper than where the fish seems to be). This smaller apparent depth occurs for any flat interface:

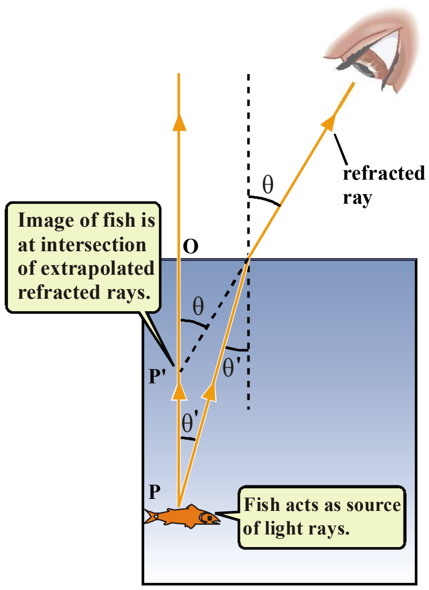
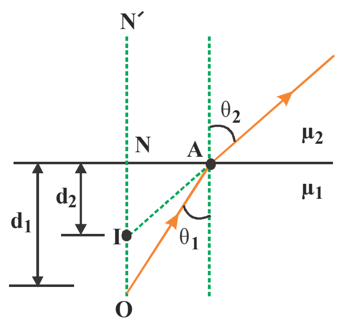


Figure : A small, shiny fish as a source of light. Note that the direction of propagation of the light is opposite to that. This does not affect the validity. The extrapolated ray (dashed) appears to come from the point P'. The angles shown are exaggerated for clarity; the location of the image of the fish is independent of angle only for small angles; that is, only when viewed from almost directly above.

* We can easily determine the relationship between the object distance and image distance. Consider the object in fig.



From triangles ONA and INA we have

$$\frac{NA}{IN} = \tan\theta_2 , \quad \frac{NA}{ON} = \tan\theta_1$$

Dividing the equations we have

$$\frac{ON}{IN} = \frac{\tan\theta_2}{\tan\theta_1}$$

ON =actual depth, IN = apparent depth

* **Paraxial Approximation :**

Assuming that the rays are all contained in a cone with a very small half angle, the value of θ is very small (Normal view), we can say that $\sin\theta = \tan\theta$,

Therefore, $\frac{ON}{IN} = \frac{\sin\theta_2}{\sin\theta_1}$

From Snell rule,

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \quad \text{or} \quad \frac{\sin\theta_2}{\sin\theta_1} = \frac{\mu_1}{\mu_2}$$

Substituting we get,

$$\frac{ON}{IN} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \frac{\mu_1}{ON} = \frac{\mu_2}{IN}$$

$$\boxed{\text{Apparent depth} = \frac{\mu_2}{\mu_1}\text{actual depth}}$$

This gives us the position of the image.

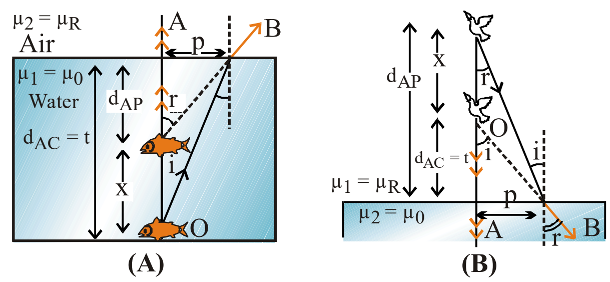
There are two possibilities :

(1) If $\mu_1 > \mu_2$ i.e., object in a denser medium is seen from a rarer medium

$$\frac{\mu_1}{\mu_2} = \frac{\mu_D}{\mu_R} = \mu(>1) \text{ So, } d_{Ap} = (d_{Ac} / \mu) ,$$

i.e., $d_{Ap} < d_{Ac}$

i.e., the image of object will appear at a lesser distance. This is why an under water object (like stone or fish) appears to be at lesser depth than in reality. The distance between object and its image, called normal shift and with $d_{Ac} = t$, will be $\boxed{x = d_{Ac} - d_{Ap} = t - (t / \mu) = t\,[1 - (1 / \mu)]}$



(2) If $\mu_1 < \mu_2$ i.e., object in a rarer medium is seen from a rarer medium

$$\frac{\mu_1}{\mu_2} = \frac{\mu_D}{\mu_R} = \mu\ (<1).$$

So, $d_{Ap} = \mu d_{Ac}$, i.e., $d_{Ap} > d_{Ac}$

i.e., the image of the object will appear to be at a greater distance. This is why a high flying object (like bird or aeroplane) appears to be higher than in reality. Normal shift in this situation will be

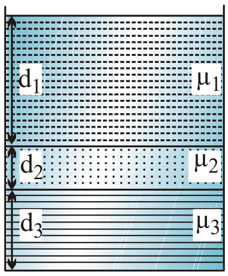
$\boxed{x = d_{Ac} - d_{Ap} = [(\mu - 1)]\, t}$ with $d_{Ac} = t$

Note : If there are number of liquids of different depths, one over the other

$$d_{Ac} = d_1 + d_2 + d_3 ......$$

and $$d_{Ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + ...$$

$$\mu = \frac{d_{Ac}}{d_{Ap}} = \frac{d_1 + d_2 + d_3 ....}{\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + ...} = \frac{\Sigma d_i}{\Sigma\,(d_i / \mu_i)}$$



In case of two liquids if $d_1 = d_2$,

$$\mu = \left[\frac{2\mu_1\mu_2}{\mu_1 + \mu_2}\right] = \text{Harmonic mean}$$

Principle of Reversibility of Light

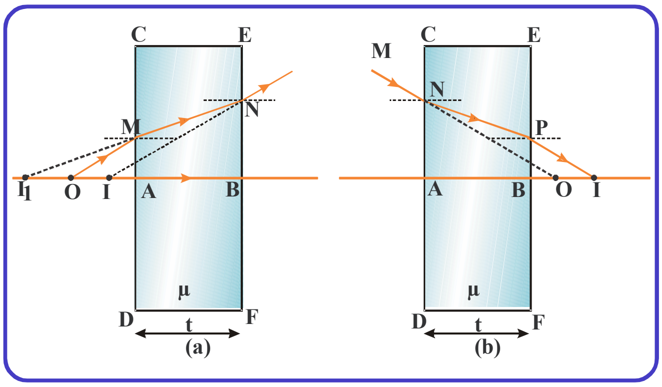
* If the path of a ray of light is reversed after suffering a number of reflections and refractions, it retraces its path. Therefore, image and object positions can be interchanged. The points corresponding to an object and its image are called "conjugate points".

Refraction through Slab

(i) Normal Shift

Two cases are possible –

Refer figure (a) : An object is placed at O. Plane surface CD forms its image (virtual) at I_1. This image acts as object for EF which finally forms the image (virtual) at I.



Distance OI is called the normal shift and its value

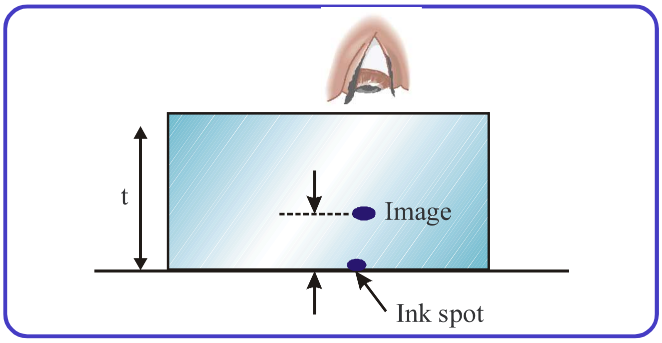
is, $\boxed{OI = \left(1 - \frac{1}{\mu}\right) t}$

Refer figure (b) : The ray of light which would had met line AB at O will now meet this line at I after two times refraction from the slab.

Here $$OI = \left(1 - \frac{1}{\mu}\right) t$$

If there is an ink spot at the bottom of a glass slab, it appears to be raised by a distance $x = t - a$

$$x = t\left(1 - \frac{1}{\mu}\right)$$



(ii) Lateral Shift

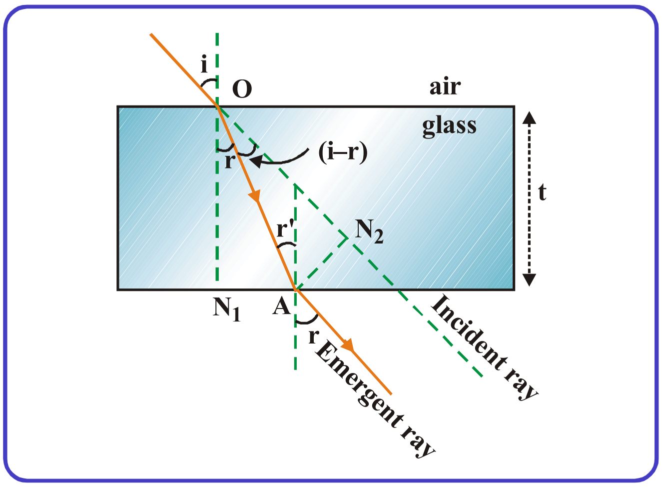
The perpendicular distance between the incident ray and the emergent ray (i.e. AN_2 in the figure) is called the "lateral shift".

$${}_a\mu_g = \frac{\sin i}{\sin r} \qquad ... (i)$$

Using principle of reversibility,

$$_a\mu_g = \frac{\sin i'}{\sin r'} = \frac{\sin i'}{\sin r} \quad [\because r' = r] \quad \text{... (ii)}$$

From equations (i) and (ii) $i = i'$



$\therefore$ Emergent ray is parallel to the incident ray

In $\Delta\, ON_1A$, $\cos r = \frac{ON_1}{OA} = \frac{t}{OA}$

or $\quad OA = \frac{t}{\cos r}$ ... (iii)

In right angle $\Delta\, ON_2A$, $\sin N_2OA = \frac{AN_2}{OA}$

or $\quad \sin(i - r) = \frac{AN_2}{(t/\cos r)}$

$\Rightarrow \quad AN_2 = \frac{t \cdot \sin(i - r)}{\cos r}$ ... (iv)

$$\boxed{\text{Lateral shift } = \frac{t \cdot \sin(i - r)}{\cos r}}$$

Special cases

(i) If $i = 0$, $r = 0$ then shift $= 0$
i.e. the ray passes through the slab without deviation and shifting.

(ii) If i is very small, r will also be small and hence

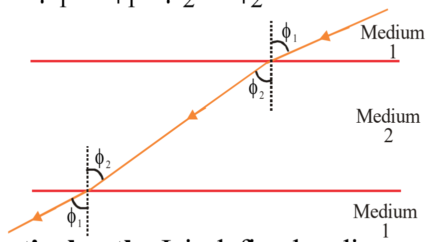
$$_a\mu_g = \mu = \frac{i}{r} \text{ or } r = \frac{i}{\mu}$$

$$\text{Lateral shift} = \frac{t \sin(i - r)}{\cos r} = t\,(i - r)$$

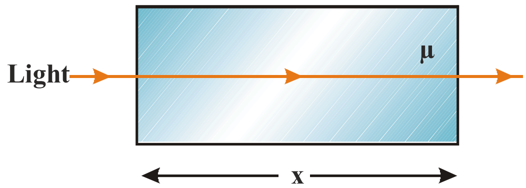
$$= t\left(i - \frac{i}{\mu}\right) = t\left(1 - \frac{1}{\mu}\right) i$$

Golden Tips

* For two medium, μ_1 & μ_2 are refractive indices with respect to vacuum, the incident and emergent rays are parallel then
$\mu_1 \sin\phi_1 = \mu_2 \sin\phi_2$.



* **Optical path :** It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium.



Time taken by light ray to pass through the medium

$= \frac{\mu x}{c}$, where x = geometrical path and

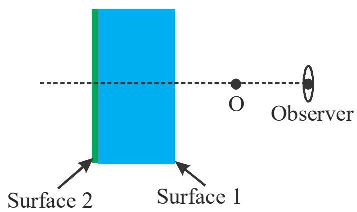
μx = optical path.

* It is refraction due to which stars appear twinkling, rivers appear shallow, Sun appears oval in the morning and evening, and pencil appears broken in water.

* To solve multiple refraction/reflection problems you can use method of interfaces or method of elements to understand both method study the following examples:

➢ The width of the silvered slab is 6 cm, refractive index of the slab is 1.5 and the object is placed 28cm in front of the slab. Locate final image.

Method I : Method of Interfaces

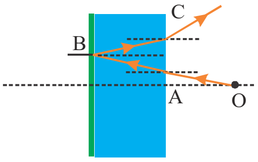


A ray of light from the object O undergoes, refraction, reflection and then refraction.

Refraction at surface 1 :

Here $\mu_1 = 1,\ \mu_2 = 1.5,\ d_1 = 28$ cm., $d_2 = ?$

Since $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$



Therefore, $d_2 = 42$ cm. from the first interface. The first image I_1 is formed 42 cm. in front of the slab.

Reflection at surface 2 :

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is

$= 42 + 6 = 48$ cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front. Therefore the second image I_2 is formed 48 cm. behind the mirror.

Second refraction at surface 1 :

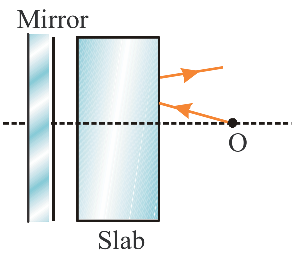
Here, $\mu_1 = 1.5,\ \mu_2 = 1,\ d_1 = 48 + 6 = 54$ cm.,

$d_2 = ?$ Since, $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$,

We get, the final image is at a distance $d_2 = 36$cm. behind the first interface.

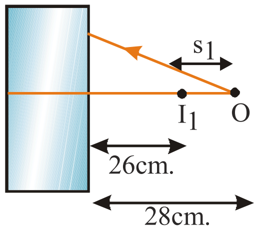
Conclusion : Final image is formed 36 cm. behind surface 1 or 30cm. behind the surface 2.

Method 2 : Method of elements : A ray of light from the object first encounters a glass slab, then a mirror and finally a glass slab again.



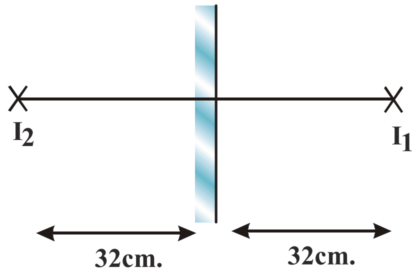
Glass slab : A slab simply shifts the object along the axis by a distance

$$s_1 = t\left(1 - \frac{1}{\mu}\right) = 2 \text{ cm.}$$



Direction of shift is towards left. Therefore the object appears to be at I_1 which is $28 - 2 = 26$ cm. from the slab.

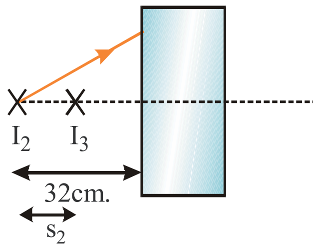
Mirror : The object for the mirror is the image I_1 formed after shift due to the slab. Therefore object distance from the mirror is $26 + 6 = 32$cm.



The image will now be formed 32 cm. behind the mirror.

Slab : The ray now travels through the slab again but this time from right to left.

Therefore it is shifted again by a distance of 2cm, but towards the right. Thus final position of the image is $32 - 2 = 30$ cm. behind the mirror.

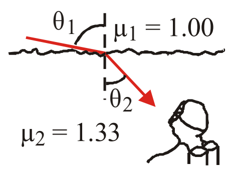


Conclusion : Final image is formed 30 cm. behind the mirror.

EXAMPLE 7

An underwater scuba diver sees the Sun at an apparent angle of 45.0° above the horizon. What is the actual elevation angle of the Sun above the horizon?

SOLUTION:



$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$
$\sin\theta_1 = 1.333 \sin 45°$
$\sin\theta_1 = (1.33)(0.707) = 0.943$
$\theta_1 = 70.5° \to 19.5°$ above the horizon

EXAMPLE 8

A rectangular tank of depth 8 meter is full of water ($\mu = 4/3$), find the apparent depth.

SOLUTION:

$$\mu = \frac{h}{h'} \Rightarrow h' = \frac{8}{4/3} = 6\text{m}$$

EXAMPLE 9

When a glass slab is placed on a dot on a paper. It appears displaced by 4 cm, viewed normally. What is the thickness of slab if the refractive index 1.5.

SOLUTION:

$$\text{Displacement} = t\left(1-\frac{1}{\mu}\right). \text{ So } 4 = t\left(1-\frac{1}{\mu}\right)$$

$$t = \frac{\mu \times 4}{\mu - 1} = \frac{1.5 \times 4}{1.5 - 1} = 12 \text{ cm}$$

EXAMPLE 10

A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

SOLUTION:

As $\dfrac{\text{Real depth}}{\text{Apparent depth}} = {}_a\mu_w$, we get

$$\frac{12.5}{9.4} = {}_a\mu_w \text{ or } {}_a\mu_w = 1.33$$

If water is replaced by liquid, then ${}_a\mu_\ell = 1.63$

$$\text{So apparent depth} = \frac{12.5}{1.63} = 7.66 \text{ cm}$$

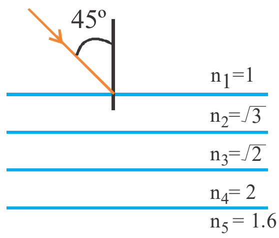
Distance through which the microscope has to be moved $= 9.4 - 7.66 = 1.74$ cm

Checkup 3

Q.1 Under certain circumstances, sound can be heard over extremely great distances. This frequently happens over a body of water, where the air near the water surface is cooler than the air higher up. Explain how the refraction of sound waves in such a situation could increase the distance over which the sound can be heard.

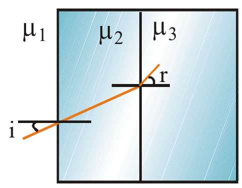
Q.2 If a solid cylinder of glass or clear plastic is placed above the words LEAD OXIDE and viewed from above as shown in Figure, the LEAD appears inverted but the OXIDE does not. Explain.

Q.3 In the figure shown the angle made by the light ray with the normal in the medium of refractive indeed $\sqrt{2}$ is:



(A) 30° (B) 60°C
(C) 90° (D) None of these

Q.4 In the figure shown $\dfrac{\sin i}{\sin r}$ is equal to :



(A) $\dfrac{\mu_2^2}{\mu_3\mu_1}$ (B) $\dfrac{\mu_3}{\mu_1}$

(C) $\dfrac{\mu_3\mu_1}{\mu_2^2}$ (D) None

Q.5 The wavelength of light in air is 6000 Å. Then the wavelength of the same light in glass [$\mu = 1.5$] relative to air will be-

(A) 4000 Å (B) 6000 Å
(C) 6001.5 Å (D) 5998.5 Å

Q.6 Time taken to cross a 4 mm window glass of refractive index 1.5 will be-

(A) 2×10^{-8} sec (B) 2×10^{8} sec
(C) 2×10^{-11} sec (D) 2×10^{11} sec

Q.7 The length of the optical path of two media in contact of length d_1 and d_2 of refractive indices μ_1 and μ_2 respectively, is –

(A) $\mu_1 d_1 + \mu_2 d_2$ (Bb) $\mu_1 d_2 + \mu_2 d_1$

(C) $\dfrac{d_1 d_2}{\mu_1 \mu_2}$ (D) $\dfrac{d_1 + d_2}{\mu_1 \mu_2}$

Q.8 When light travels from air to water and from water to glass, again from glass to CO_2 gas and finally through the air. The relation between their refractive indices will be given by –

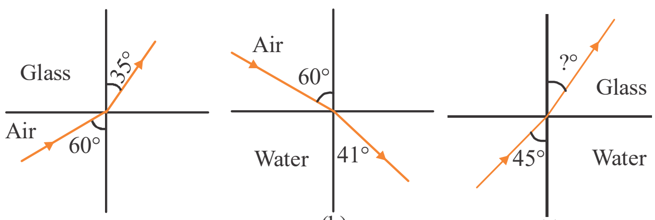
(A) ${}_a n_w \times {}_w n_{gl} \times {}_{gl} n_{gas} \times {}_{gas} n_a = 1$

(B) ${}_a n_w \times {}_w n_{gl} \times {}_{gas} n_{gl} \times {}_{gl} n_a = 1$

(C) ${}_a n_w \times {}_w n_{gl} \times {}_{gl} n_{gas} = 1$

(D) there is no such relation

Q.9 Figures (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)].



(a) (b) (c)

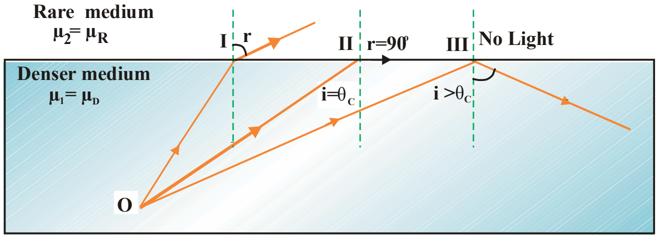
($\sin 35° = 0.5736$, $\sin 41° = 0.6561$)

7.5 TOTAL INTERNAL REFLECTION

* In case of refraction of light, from snell's law we have $\mu_1 \sin i = \mu_2 \sin r$ ....(1)
* If light is passing from denser to rarer medium through a plane boundary then $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$ so with $\mu = (\mu_D / \mu_R)$

$$\sin i = \frac{\mu_R}{\mu_D} \sin r \text{ i.e. } \sin i = \frac{\sin r}{\mu} \quad \text{....... (2)}$$

i.e. $\sin i \propto \sin r$ with $(\angle i) < (\angle r)$ (as $\mu > 1$)



* As angle of incident i increase angle of refraction r will also increase and for certain value of i (< 90°) r will become 90°.
* The value of angle of incidence for which r = 90° is called critical angle and is denoted by θ_c. From eq. (2), $\sin \theta_C = \dfrac{\sin 90}{\mu}$

$$\text{i.e., } \boxed{\sin \theta_C = \frac{1}{\mu}} \quad \text{.......... (3)}$$

And hence eqn. (2) in terms of critical angle can be written as $\sin i = \sin r \times \sin \theta_c$ i.e. ,

$$\sin r = \frac{\sin i}{\sin \theta_C} \quad \text{........... (4)}$$

So if $i > \theta_c$ $\sin r > 1$.

This means that r is imaginary (as the value of sin of any angle can never be greater than unit) physically this situation implies that refracted ray does not exist.

* The total light incident of the boundary will be reflected back in to the same medium from the boundary. This phenomena is called **total internal reflection.**

NOTE

* For total internal reflection to take place light must pass from denser to rarer medium.

* Total internal reflection will take place only if angle of incidence is greater than critical angle.

* In case of total internal reflection as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case reflection from mirror or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted. This is why image formed by TIR are much bright than formed by mirror or lenses.

* For a given pair of medium critical angle depends on wavelength of light used and critical angle is maximum for red and minimum for violet rays.

$$\left[\text{as} \;\; \mu \propto \frac{1}{v} \propto \frac{1}{\lambda}\right]$$

* For a given light is depends on nature of pair of medium lesser the μ greater will the critical angle and vice–versa. [$\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$]

* **Glass air :**

$$\text{As } \mu_g = \frac{3}{2} \text{ and } \mu_A = 1 \text{ i.e., } \mu = \frac{\mu_G}{\mu_A} = \frac{3}{2}$$

$$\text{So } (\theta_C)_{GA} = \sin^{-1}\left[\frac{2}{3}\right] = 42^o$$

* **Water air :**

$$\text{As } \mu_W = \frac{4}{3} \text{ and } \mu_A = 1 \;\; \text{i.e., } \mu = \frac{\mu_W}{\mu_A} = \frac{4}{3}$$

$$\text{So } (\theta_C)_{WA} = \sin^{-1}\left[\frac{3}{4}\right] = 49^o$$

* **Glass Water :**

$$\text{As } \mu_G = \frac{3}{2} \text{ and } \mu_W = \frac{4}{3} \text{ i.e., } \mu = \frac{\mu_G}{\mu_W} = \frac{9}{8}$$

$$\text{So, } (\theta_C)_{GW} = \sin^{-1}\left(\frac{8}{9}\right) \approx 63^o$$

Application of TIR

1. Sparking of Diamond

(μ = 2.46, critical angle wr.t. air = 24°) :

A diamond sparkles because, when it is held a certain way, the intensity of the light coming from it is greatly enhanced. A ray of light striking a bottom facet of the diamond at an angle of incidence exceed the critical angle, are totally reflected back into the diamond, eventually exiting the top surface to give the diamond its sparkle.

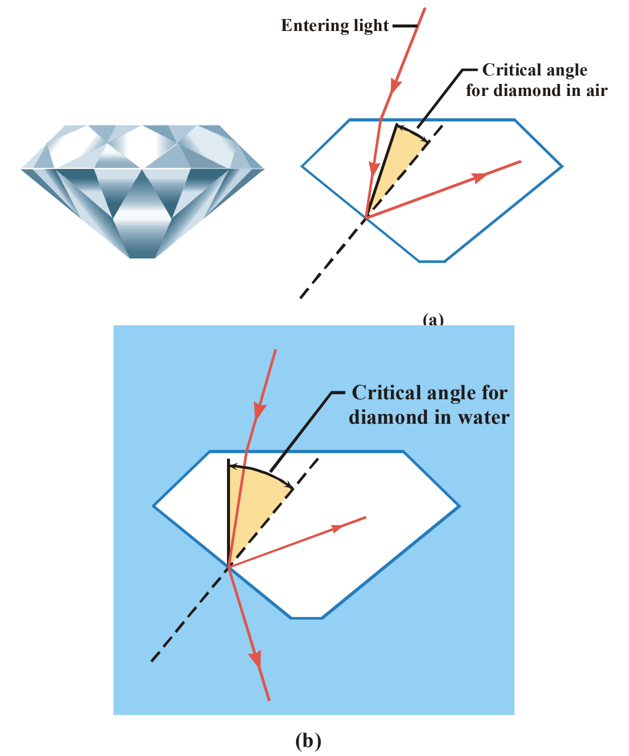


Figure : (a) Near the bottom of the diamond, light is totally internally reflected, because the incident angle exceeds the critical angle for diamond and air. (b) When the diamond is in water, the same light is partially reflected and partially refracted, since the incident angle is less than the critical angle for diamond and water.

2. Optical Fibre

Take a thin solid wire (called strand) made up of glass or quartz, etc. Coat it from outside with some material whose μ is less than that of glass or quartz, etc. Now, because it is thin, hence whichever ray of light enters it from one of its end, will strike the inside surface at some angle which will definitely be greater than the critical angle. Hence, it will suffer total internal reflection again and again until it comes out from the other end. This is the principle on which optical fibres work. Hence, in effect, light can travel axially within an optical fibre, although the axis of fibre may be flexible having bent or wavy profile.

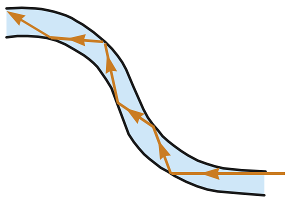


Figure : Light travels in a curved transparent rod by multiple internal reflections.

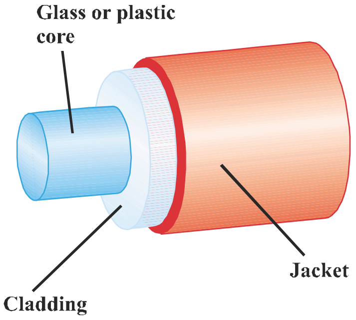


Figure : The construction of an optical fiber. Light travels in the core, which is surrounded by a cladding and a protective jacket.

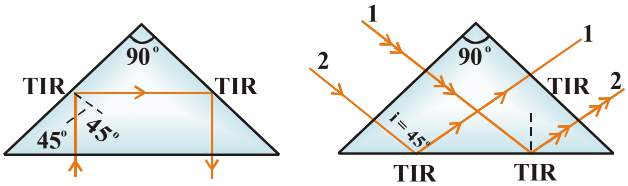
Use of Optical fibres :

(a) For medical examination inside the stomach, intensities etc., called endoscopy.

(b) The recent trend is to send even electrical signals through optical fibres, by converting them into electromagnetic radiations/light waves.

3. Optical Instruments

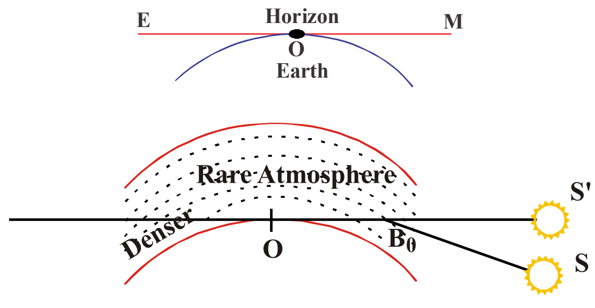
Binoculars, periscopes, and telescopes, use glass prisms and total internal reflection to turn a beam of light through 90° or 180°.



Bending of rays by 180 **Bending of Image**

4. Duration of Sun's Visibility

In absence of atmosphere the sun will be visible for its position from M to E as shown in fig. However, in presence of atmosphere (in which μ decreases with height) due to the phenomenon of TIR, the sun will become visible even when it is below the horizon (when $i > \theta_c$) and will remain visible for some time even when it goes below the horizon as shown in fig. This results in increase in duration for which the sun is visible. It is estimated that due to this effect the period of visibility of the sun increases by 2 minutes in the morning and 2 minutes in the evening.



5. Mirrage

Mirrage is an optical illusion of water observed in deserts (or in a region of high temperature) when the inverted image of an object such as a tree is observed along with the object itself on a hot day.

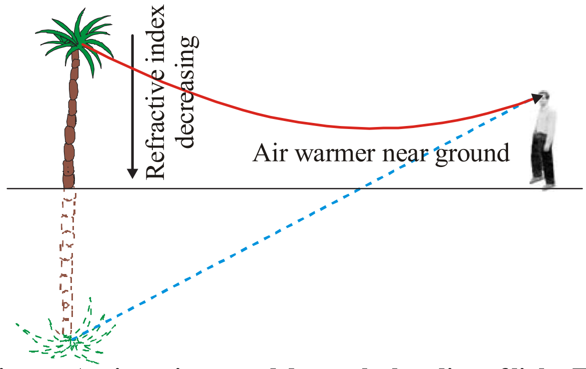


Figure : A mirage is created due to the bending of light. The index of refraction of the hot air near the ground is lower than the colder air on the top.

If atmospheric conditions are reversed i.e., the lower strata of air are cooler than the upper strata, then another sort of image called **'looming'** occurs.

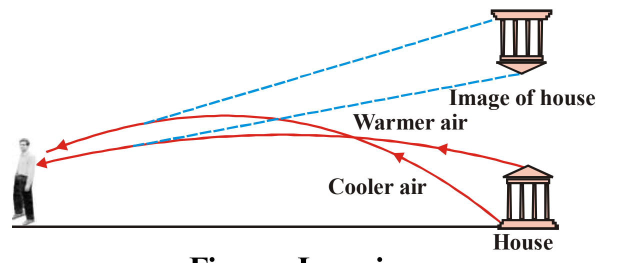


Figure : Looming

This generally takes place over a snow field or a body of cold water. The rays of light from a distant object are deviated downwards. We may see an image of a ship above the ship itself. It is also possible that the curvature of the light rays may bring into view objects normally below the horizon.

NOTE

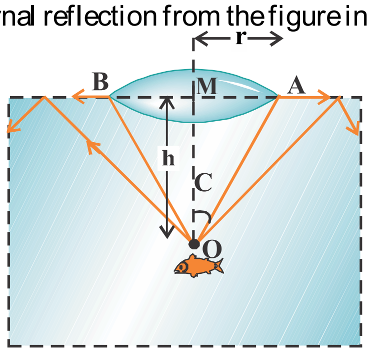
* Problem based on TIR are very simple, to solve them remember condition for TIR + apply Snell rule + Basic Mathematics and common sense.

EXAMPLE 11

A fish inside water cannot see the entire surface of the pond. Find the radius of the circular patch observed by fish.

SOLUTION:

It sees only a circular patch of light because only those light rays which are incident within a cone of semi vertex angle C (critical angle) are refracted out of the water surface all other rays suffer total internal reflection from the figure in ΔAMO



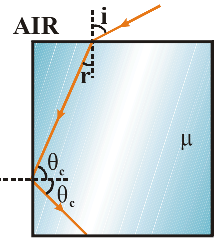
$$\sin C = \frac{r}{\sqrt{r^2+h^2}} \qquad \text{But } \sin C = \frac{1}{\mu}$$

where r is the radius of circular patch of light and h is depth of the fish.

$$\frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \qquad \therefore\ r = \frac{h}{\sqrt{\mu^2-1}}$$

EXAMPLE 12

In the figure shown, for an angle of incidence i at the top surface, find the minimum refractive index needed for total internal reflection at the vertical face.



SOLUTION:

Applying Snell's law at the top surface

$\mu \sin r = \sin i$ ...(i)

For total internal reflection the vertical face

$\mu \sin \theta_c = 1$

Using geometry, $\theta_c = 90° - r$

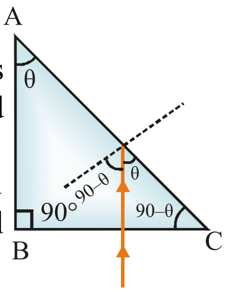
$\therefore$ $\mu \sin(90 - r) = 1$ or $\mu \cos r = 1$ ...(ii)

On squaring and adding equation (i) and (ii), we get $\mu^2 \sin^2 r + \mu^2 \cos^2 r = 1 + \sin^2 i$

or $\mu = \sqrt{1+\sin^2 i}$

EXAMPLE 13

In figure a ray of light is incident on a right-angled glass prism (inside water) at right-angle to face BC. Find the condition for total internal reflection at face AC.



SOLUTION:

For total internal reflection at face AC

$90 - \theta > \theta_c$

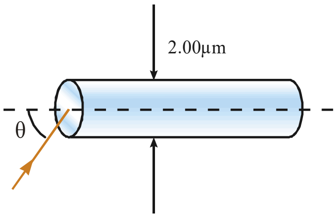
θ_c = critical angle (depend on refractive index of glass and surrounding medium)

If take $\mu_G = 1.5$ & surrounding water ($\mu_w = 4/3$)

$$\sin\theta_c = \frac{\mu_w}{\mu_G} = \frac{8}{9}\ ; \quad 90° - \sin^{-1}\left(\frac{8}{9}\right) > \theta$$

EXAMPLE 14

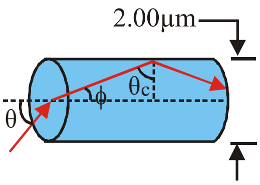
Determine the maximum angle θ for which the light rays incident on the end of the pipe in Figure are subject to total internal reflection along the walls of the pipe. Assume that the pipe has an index of refraction of 1.36 and the outside medium is air.



SOLUTION:

$$\sin\theta_c = \frac{\mu_{air}}{\mu_{pipe}} = \frac{1.00}{1.36} = 0.735 \ ; \ \theta_c = 47.3°$$

Geometry shows that the angle of refraction at the end is $\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

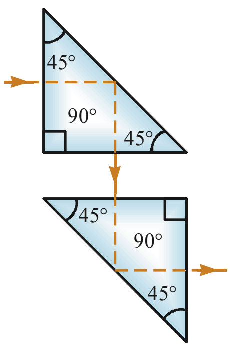


Then, Snell's law at the end,
$1.00 \sin\theta = 1.36 \sin 42.7°$ gives $\theta = 67.2°$.
The 2 μm diameter is unnecessary information.

Checkup 4

Q.1 Explain why a diamond sparkles more than a glass crystal of the same shape and size.

Q.2 Total internal reflection is applied in the periscope of a submarine to let the user "see around corners." In this device, two prisms are arranged as shown in figure, so that an incident beam of light follows the path shown. Parallel tilted silvered mirrors could be used, but glass prisms with no silvered surfaces give higher light throughput. Propose a reason for the higher efficiency.



Q.3 The refractive index of water is 4/3 and that of glass is 5/3. What will be the critical angle for the ray of light entering water from the glass –
(A) $\sin^{-1}(4/5)$ (B) $\sin^{-1}(5/4)$
(C) $\sin^{-1}(1/2)$ (D) $\sin^{-1}(2/1)$

Q.4 Relation between critical angles of water and glass is –
(A) $C_w > C_g$ (B) $C_w < C_g$
(C) $C_w = C_g$ (D) $C_w = C_g = 0$

Q.5 Critical angle for light going from medium (i) to (ii) is θ. The speed of light in medium (i) is v then speed in medium (ii) is –
(A) $v(1 - \cos\theta)$ (B) $v/\sin\theta$
(C) $v/\cos\theta$ (D) $v(1 - \sin\theta)$

Q.6 If light travels a distance x in t_1 sec. in air and 10x distance in t_2 sec. in a medium, the critical angle of the medium will be –
(A) $\tan^{-1}(t_1/t_2)$ (B) $\sin^{-1}(t_1/t_2)$
(C) $\sin^{-1}(10t_1/t_2)$ (D) $\tan^{-1}(10t_1/t_2)$

7.6 REFRACTION AT CURVED SURFACES

Refraction at a Single Spherical Surface

* Let the object is placed in a medium of the refractive index μ_1.
* The spherical curved surface (convex or concave) separate it from another medium of refractive index μ_2.
* If v, u and R are respectively, the object distance, the image distance and the radius of curvature of the refracting surface, then the formula connecting u, v and R is

$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

* This formula is applicable for convex as well as concave spherical curved surfaces.
* If we compare it with mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we find that $v \to (v/\mu_2)$ and $u \to (-u/\mu_1)$

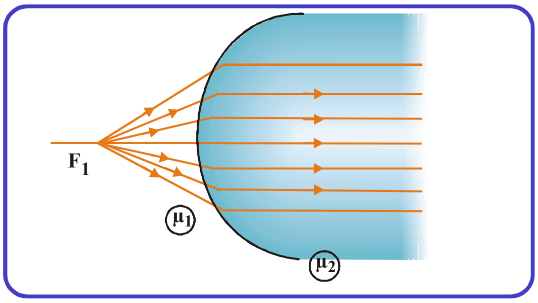
Transverse (vertical) magnification in this case

$$\boxed{m = \frac{I}{O} = -\frac{(v/\mu_2)}{(-u/\mu_1)} = \frac{\mu_1}{\mu_2}.\frac{v}{u}}$$

1. Focal length of a single spherical surface

A single spherical surface as two principal focus points –

(i) First focus: The first principal focus is the point on the axis where when an object is placed, the image is formed at infinity.

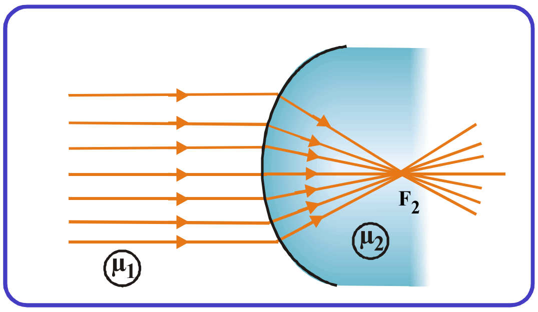


That is when, $u = f_1, v = \infty$, then from

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \left(\frac{\mu_2 - \mu_1}{R}\right)$$

$$-\frac{\mu}{(f_1)} = \frac{\mu_2 - \mu_1}{R} \quad \text{or} \quad f_1 = \frac{-\mu_1 R}{(\mu_2 - \mu_1)}$$

(ii) Second focus: Similarly, the second principal focus is the point where parallel rays focus.



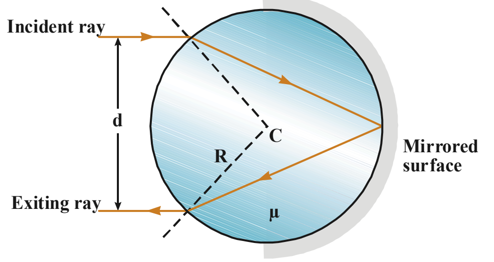
That is, $u_1 = -\infty, v_1 = f_2$, then

$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R}; \quad f_2 = \frac{\mu_2 R}{(\mu_2 - \mu_1)}$$

(iii) Ratio of Focal length : $\dfrac{f_1}{f_2} = -\dfrac{\mu_1}{\mu_2}$

EXAMPLE 15

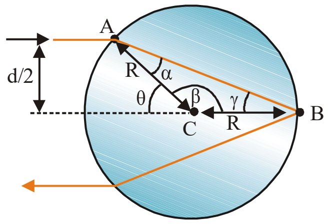
A transparent cylinder of radius R = 2.00 m has a mirrored surface on its right half, as shown in Figure. A light ray traveling in air is incident on the left side of the cylinder. The incident light ray and exiting light ray are parallel and d = 2.00 m. Determine the index of refraction of the material.



SOLUTION:

The angle of incidence at point A is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{m}}{2.00\text{m}}\right) = 30.0°$$



If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline CB of the cylinder. In the isosceles triangle ABC, $\gamma = \alpha$ and $\beta = 180° - \theta$.

Therefore, $\alpha + \beta + \gamma = 180°$ becomes

$2\alpha + 180° - \theta = 180°$ or $\alpha = \theta/2 = 15.0°$

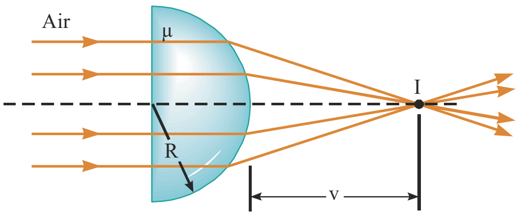
Then, applying Snell's law at point A,

$$\mu \sin\alpha = 1.00 \sin\theta$$

or $$\mu = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = 1.93$$

EXAMPLE 16

A parallel beam of light enters a glass hemisphere perpendicular to the flat face, as shown in Fig. The magnitude of the radius is 6.00 cm, and the index of refraction is 1.560. Determine the point at which the beam is focused. (Assume paraxial rays.)



SOLUTION:

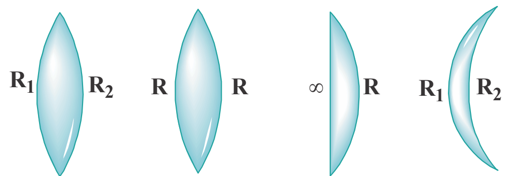
A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which R = –6.00 cm. The incident rays are parallel, so u = ∞.

Then, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

becomes $0 + \frac{1}{v} = \frac{1.00 - 1.56}{-6.00\text{ cm}}$ & v = 10.7 cm.

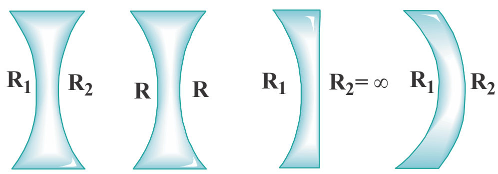
7.7 LENS THEORY

* A lens is a piece of transparent material with two refracting surface such that least one is curved and refractive index of material is different from that of the surroundings.
* A thin spherical lens with refractive index greater than that of surrounding behaves a convergent or convex lens i.e. converges parallel rays, its central (i.e. paraxial) portion is thicker than marginal one.



Bi-　Equi.-　Plano-　Concavo-
Convex lens

If $\mu_L > \mu_M$ Converging lens
If $\mu_L < \mu_M$ Diverging lens



Bi-　Equi.-　Plano-　Convexo-
Concave lens

If $\mu_L > \mu_M$ Diverging lens
If $\mu_L < \mu_M$ Converging lens

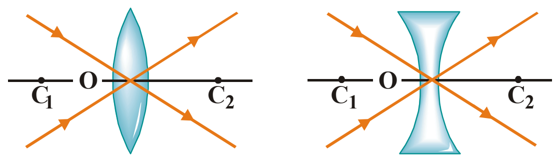
Terms Related to Thin Spherical Lens

1. Optical Centre

O is a point for given lens through which any ray passes undeviated.

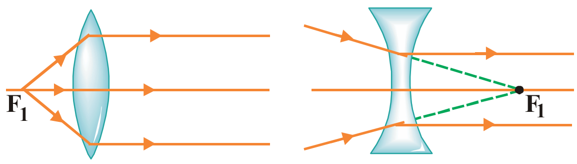
2. Principal-Axis

C_1 C_2 is a line passing through optical and perpendicular to the lens. The centre of curvature of curved surface always lie on the principal axis (as in a sphere is always perpendicular to surface)

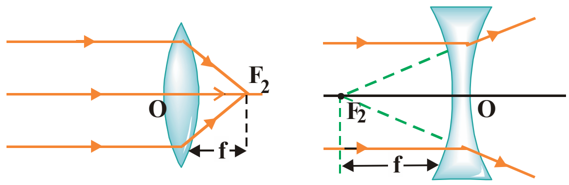


3. Principal-Focus

A lens has two surface and hence two focal points first focal point is an object position on the principal axis for which image is at infinite while



Second focal point is an image point on the principal axis for which object is at infinity.



4. Focal - Length f

It is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appear to converge.

5. Aperture

In reference to lens, aperture means effective diameter of its light transmitting area. Brightness (intensity of image formed by a lens) depends on the light passing through the lens and hence depends on the square of aperture i.e.

$$I \propto (\text{Aperture})^2$$

Thin Lens Formula

If an object is placed at a distance u from the optical centre of a lens and its images is formed at a distance v (from the optical centre) and focal length of this length is f then $\boxed{\frac{1}{v}-\frac{1}{u}=\frac{1}{f}}$

Lens Maker Formula

* **Focal length of lens**

$$\boxed{\frac{1}{f}=({}_m\mu_\ell-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}$$

where ${}_m\mu_\ell$ refractive index of lens with respect to medium.
R_1 = radius of curvature of first surface of lens,
R_2= radius of curvature of second surface of lens.

* For equibiconvex lens, $R_1 = +R$, $R_2 = -R$

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R}-\left(\frac{-1}{R}\right)\right]=\frac{2(\mu-1)}{R}$$

* For equibiconcave (or simple concave) lens, $R_1 = -R$ and $R_2 = +R$,

$$\frac{1}{f}=(\mu-1)\left(-\frac{1}{R}-\frac{1}{R}\right)=\frac{-2(\mu-1)}{R}$$

* For planoconvex lens, $R_1 = R$, $R_2 = \infty$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{\mu-1}{R}$$

Power of a Lens

* Power of lens is defined as inverse of focal length

$$\boxed{\frac{1}{f(\text{in m})}=\frac{100}{f(\text{in cm})}}$$

The unit of power is diopter.

Magnification

* If a thin object linear size O situated vertically on the axis of a lens at a distance u from the optical centre and its image of size I is formed at a distance v (from the optical centre). Magnification (transverse) is defined as

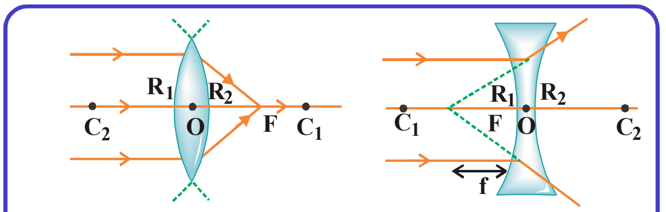
$$m=\left[\frac{I}{O}\right]=\left[\frac{v}{u}\right]$$

* Other formulae of magnification

$$\boxed{m=\frac{f}{f+u},\ m=\frac{f-v}{f}}$$

Sign - Convention

* Transverse distance measured from optical centre and are taken to be positive while those below it negative.
* Longitudinal distances are measured from optical centre and are taken to be positive if in the direction of light propagation and negative if opposite to it e.g., according to our sign convention.



* To calculate an unknown quantity the known quantities are substituted with proper sign in a given formula.

Rules for Image Formation

1. A ray passing through optical centre proceeds undeviated through the lens. (by definition of optical centre).

2. A ray passing through first focus or directed towards it, after refraction from the lens becomes parallel to the principal axis. (by definition of F_1)

3. A ray passing parallel to the principal axis after refraction through the lens passes or appear to pass through F_2 (by definition of F_2).

4. Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect is the image of the object. If they actually intersect each other the image is real and if they appear to intersect the image is said to be virtual.

SCO -16 District Shopping Centre, Jind, Haryana.
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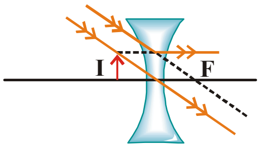
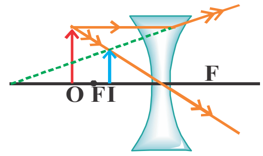
5. Image formation by a lens

(a) For Divergent or Concave Lens

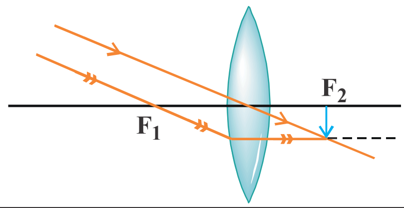
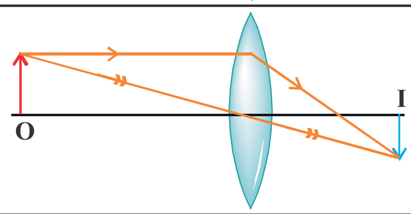
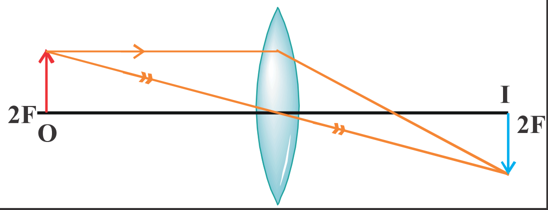
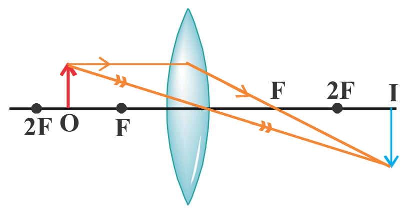
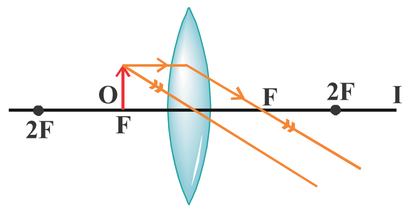
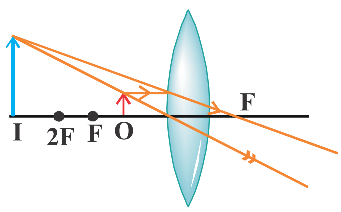


If object is at infinity, image will be formed at focus

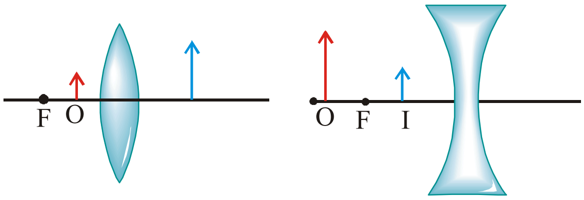


In object in front of lens, virtual, erect and diminished image is formed on same side as that of the object.

(b) For Convergent or Convex Lens

Position of object	Details of image	Figure
At infinity	Real, inverted diminished (m<<–1) At F	
Between ∞ and 2F	Real, inverted diminished (m<–1) Between F and 2F	
At 2F	Real, inverted diminished m = – 1 at 2F	
Between 2F and F	Real, inverted enlarged (m>–1) Between 2F and ∞	
At F	Real, inverted enlarged (m>>–1) At infinity	
Between focus and pole	Virtual, erect enlarged (m > +1) Between ∞ and object on same side	

* For real extended objects if the image formed by a single lens is erect (i.e., m is positive) it is always virtual. In this situation if the image is enlarged the lens is converging (i.e. convex) with object between focus and optical centre and if diminished the lens is diverging (i.e. concave) with image between focus and optical centre.



* A very small part of a lens forms complete image, if a portion (say lower half) is obstructed (say covered with black paper) Full image will be formed but brightness i.e., intensity will be reduced (to half). Also if a lens is painted with black strips and a donkey is seen through it, the donkey will not appear zebra but will remain donkey with reduced intensity.

* If L is the distance between a real image by a lens, then as

$$L = (|u| + |v|) = \left((\sqrt{u} - \sqrt{v})^2 + 2\sqrt{uv}\right)$$

So L will be minimum when

$$\left(\sqrt{u} - \sqrt{v}\right)^2 = \min = 0 \text{ i.e., } u = v$$

On substituting $u = -u$ and $v = +u$ in lens formula,

we get $\frac{1}{u} - \frac{1}{-u} = \frac{1}{f}$ i.e., $u = 2f$

So that $(L)_{min} = 2f + 2f = 4f$ [as for L_{min} $u = v$]

i.e., the minimum distance between a real object and its real image formed by a single lens is 4f.

* If an object is moved at constant towards a convex lens from infinity to focus, the image will move slower in the beginning and faster later - on, away from the lens. This is because in the time object moves from infinity to 2F, the image will move from F to 2F and when the object moves from 2F to F, The image will move from 2F to infinity. At 2F the speed of object and image will be equal.

$V_i = V_0 \left[\frac{f}{u+f}\right]^2$; where V_0 is the speed of object (u and f are to be substituted with proper sign)

* In case of **sun - goggles**, the radii of curvature of two surface are equal with centre on same side i.e., $R_1 = R_2 = R$. So

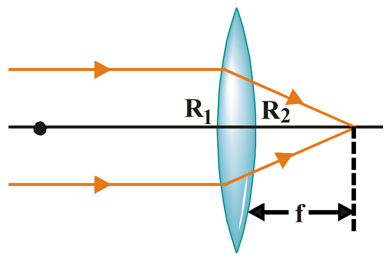
$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{+R} - \frac{1}{+R}\right] = 0$$

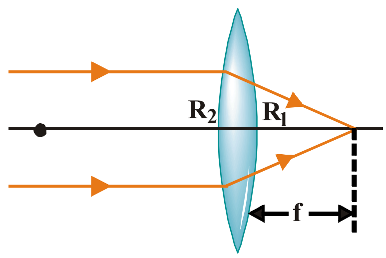
i.e., $f = \infty$ and $P = (1/f) = 0$

This is why sun-goggles have no power or infinite focal length. Same is true for a transparent sheet with the difference that here $R_1 = R_2 = \infty$

* If the two radii of curvatures of a thin lens are not equal, the focal length remains unchanged whether the light is incident on first face or the other. This is because if we substitute R_1 and R_2 with proper sign in lens - makers formula, we

always have $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$



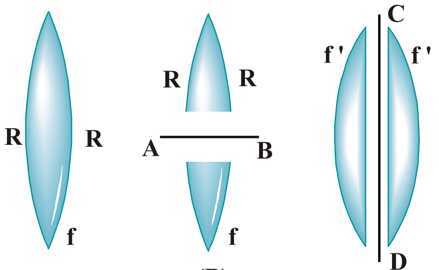


* If an equiconcave lens of focal length f is cut into equal parts by a horizontal plane AB then as none of μ, R_1 and R_2 will change the focal length of each part will be equal to that of initial lens i.e.

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

If $R_1 = R_2 = R \Rightarrow \frac{1}{f} = \frac{2(\mu - 1)}{R}$

However in this situation as light transmitting area of each part becomes half of initial so intensity will reduce to half and aperture to $\frac{1}{\sqrt{2}}$ time of its initial value (as $I \propto (\text{Aperture})^2$)



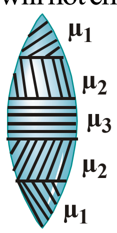
(A) (B) (C)

However if the same lens in cut into equal parts by a vertical plane CD the focal length of each part will become

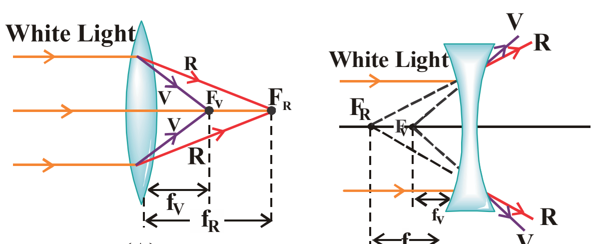
$$\frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{\mu - 1}{R} = \frac{1}{2f} \Rightarrow f' = 2f$$

i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens of remains equal to initial intensity & aperture will not change.

* If a lens is made of number of layers of different refractive indices as shown in fig., for a given wavelength of light it will have as many focal lengths and will form as many image as there are μ's as $\frac{1}{f} \propto (\mu - 1)$



* As focal length of a lens depends on μ i.e. $(1/f) \propto (\mu - 1)$ the focal length of given lens is different for different wavelengths ($\mu = A + \frac{B}{\lambda^2}$) and is maximum for red and minimum for violet whatever be the nature of lens.



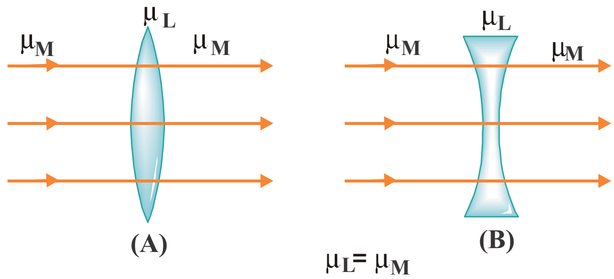
(A) (B)

* If a lens is shifted from one medium to the other depending on the refractive index of the lens and medium following three situation are possible.

(a) $\mu_m < \mu_L$ but μ_m increase :

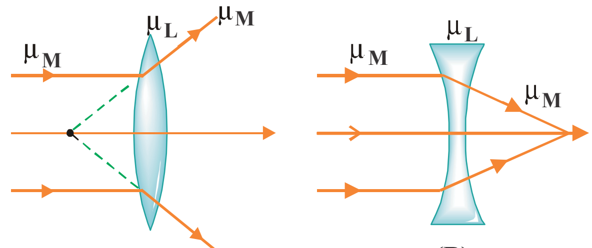
In this situation $\mu = (\mu_L / \mu_m)$ will remain greater than unity but will decrease and as $(1/f) \propto (\mu - 1)$, $(1/f)$ will decrease i.e. f will increase (without change in nature of lens).

(b) $\mu_m = \mu_L$: In this situation $\mu = (\mu_L / \mu_M) = 1$, so that $(1/f) \propto (\mu - 1) = 0$ i.e. $f = \infty$ i.e., lens will neither converge nor diverge but will behave as a plane glass plate.



$\mu_L = \mu_M$

(c) $\mu_M > \mu_L$: In this situation $\mu = (\mu_L / \mu_M) < 1$ So in lens-maker's formula sign of f and hence nature of lens will change i.e. a convergent lens will behave as divergent and vice-versa.



$\mu_M > \mu_L$

* If a lens of glass ($\mu = 3/2$) is shifted form air ($\mu = 1$) to water ($\mu = 4/3$) then as.

$$\frac{1}{f_a} = \left[\frac{3/2}{1} - 1\right] K \quad \text{and} \quad \frac{1}{f_w} = \left[\frac{(3/2)}{(4/3)} - 1\right] K$$

$$\text{With } K = \left[\frac{1}{R_1} - \frac{1}{R_2}\right] ; \quad \frac{F_w}{f_A} = \left[\frac{8}{K}\right] \times \left[\frac{K}{2}\right]$$

i.e., $F_w = 4f_A$

i.e. focal length of a lens in water becomes four times of its value in air and so power one fourth [as $P = (1/f)$].

EXAMPLE 17

An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

SOLUTION:

Using $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign convention

we get $-\frac{1}{-14}+\frac{1}{v}=\frac{1}{-21}$

or $\frac{1}{v}=-\frac{1}{21}-\frac{1}{14}=\frac{-2-3}{42}=\frac{-5}{42}$

$\therefore \quad v=-\frac{42}{5}=-8.4$ cm

Also, the magnification, $m=\frac{I}{O}=\frac{v}{u}$

$\Rightarrow \frac{I}{3.0}=\frac{-8.4}{-14}=0.6 \Rightarrow I=1.8$ cm.

The image is virtual, erect, diminished and is formed on the same side of the lens at a distance of 8.4 cm from the lens. If the object is moved away from the lens, the image moves towards the principal focus and goes on decreasing in size.

EXAMPLE 18

The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

SOLUTION:

For a image, least distance between object and image should be four times the focal length

$\Rightarrow 4f=300$ i.e. $f=75$ cm i.e. 0.75 m.

EXAMPLE 19

A screen is placed 90cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20cm. Determine the focal length of the lens.

SOLUTION:

The lens formula is $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ ......(1)

We know that by convention u is negative

$\therefore \quad -u+v=90$ ......(2)

$\Rightarrow \quad v=90+u$

$\therefore$ from eqn. (1), $-\frac{1}{u}+\frac{1}{90+u}=\frac{1}{f}$

$\Rightarrow \quad \frac{-90-u+u}{90u+u^2}=\frac{1}{f}$

or $\quad u^2+90u+90f=0$ .....(3)

Equation (3) is quadratic in u and gives two values of u say u_1 and u_2 in terms of f.

$\therefore$ sum of roots $u_1+u_2=-90$ .....(4)

and product of roots $u_1u_2=90f$

Also, $u_1-u_2=20$ cm .....(5)

$\therefore$ Adding (4) and (5), we get,

$2u_1=-90+20=-70 \Rightarrow u_1=-35$ cm

and subtracting (5) from (4),

$2u_2=-90-20=-110$ or $u_2=-55$cm

Now $u_1u_2=+90f \Rightarrow 35\times 55=90f$

or $\quad f=\frac{55\times 35}{90}=21.39$ cm

Newton Formula

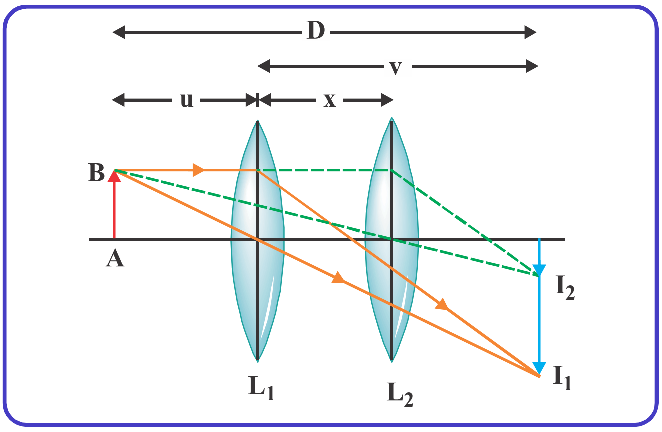
In case of a thin lens of focal length f if an object is placed at a distance x_1 from first focus and its image is formed at a distance x_2 from the second focus, then $\boxed{x_1x_2=f^2}$

Displacement Method

(To determine focal length of convex lens)

A thin converging lens of focal length f is placed between an object and a screen fixed at a distance D apart. If D > 4f, there are two positions of the lens at which a sharp image of the object is formed on the screen.

If the distance between two positions of the lens is x then, $\boxed{f=\frac{D^2-x^2}{4D}}$ .... (1)



If the magnification for two positions of the lens are m_1 and m_2 then

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \left[\frac{D+x}{D-x}\right]$$

$$\text{and} \quad m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \left[\frac{D-x}{D+x}\right] \quad \text{........ (2)}$$

(a) $m_1 \times m_2 = (I_1 I_2 / O^2) = 1$

i.e., $\boxed{O = \sqrt{I_1 I_2}}$ ...... (3)

..... (4)

(b) $m_1 - m_2 = \left[\frac{D+x}{D-x}\right] - \left[\frac{D-x}{D+x}\right] = \frac{4Dx}{D^2 - x^2}$

which in the light of eq. (1) yields

$m_1 - m_2 = \frac{x}{f}$ i.e., $\boxed{f = \frac{x}{m_1 - m_2}}$

Combination of Lenses

* When several lenses or mirrors are used coaxially, the image formation is considered one after another in steps.

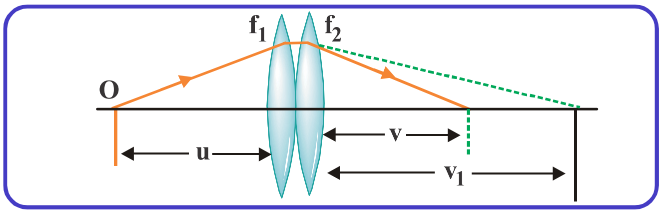
* The image formed by the lens facing the object serves as object for next lens or mirror the image formed by the second lens (or mirror) acts as object for the third and so on.

* The total magnification is such situations will be given by $m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times$...............

i.e. $m = m_1 \times m_2 \times$ ....

In case of two thin lens in contact if the first lens of focal length f_1 forms the image I_1 (of an object) at a distance v_1 from it.

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \text{..... (1)}$$



now the image I_1 will act as object for second lens and if the second lens forms image. I at a distance v from it

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \text{..... (2)}$$

So adding Eqn. (1) and (2) we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{or} \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

with $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

i.e. the combination behave as a single lens of equivalent focal length f given by

$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}$ or $P = P_1 + P_2$ ..... (3)

If the two thin lens are separated by a distance d apart F is given by $\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$,

so $P = P_1 + P_2 - P_1 P_2 d$.

Golden Tips

* If two thin lens of equal focal length but of opposite nature (i.e. one convergent and other divergent) are put in contact, the resultant focal length of the combination be

$\frac{1}{F} = \frac{1}{+f} + \frac{1}{-f} = 0$ i.e. $F = \infty$ and $P = 0$

i.e. the system will behave as a plane glass plate.

* If two thin lens of same nature are put in contact then as

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2};\ \frac{1}{F}>\frac{1}{f_1}\quad \text{and}\quad \frac{1}{F}>\frac{1}{f_2}$$

i.e. $F<f_1$ and $F<f_2$ i.e. the resultant focal length will be lesser than smallest individual.

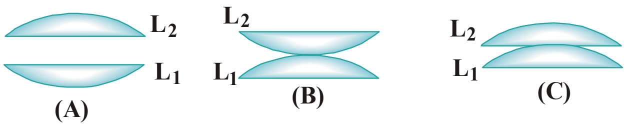
* If two thin lenses of opposite nature with different focal lengths are put in contact the resultant focal length will be of same nature as that of the lens of shorter focal length but its magnitude will be more than that of shorter focal length.

* If a lens of focal length f is divided into two equal parts as in figure (A), each part has a focal length

f′ then as $\frac{1}{f}=\frac{1}{f'}+\frac{1}{f'}$; $f'=2f$

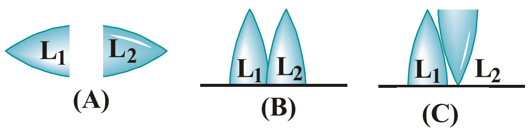
i.e. each part have focal length 2f now if these parts are put in contact as in (B) or (C) the resultant focal length of the combination will be

$$\frac{1}{F}=\frac{1}{2f}+\frac{1}{2f}\qquad \text{i.e. } F=f\,(=\text{initial value})$$



* If a lens of focal length f is cut in two equal part as shown in fig. (A) each will have focal length f. Now of these parts are put in contact as shown in fig. (B) the resultant focal length will be

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}\qquad \text{i.e. } F=(f/2)$$

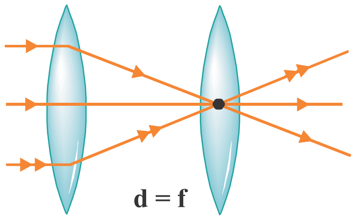


However if the two parts are put in contact as shown in fig. (C) first will behave as convergent lens of focal length f while the other divergent of same focal length (being thinner near the axis) so in this situation.

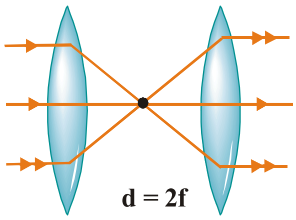
$$\frac{1}{F}=\frac{1}{+f}+\frac{1}{-f}\quad \text{i.e.}\quad F=\infty\ \text{ or }\ P=0$$

* Consider a coaxial system of two thin convex lenses of focal length f each separated by a distance d.

(a) If $d=f$ the system will behave a convex lens of focal length f.

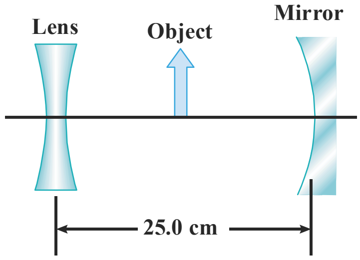


(b) If $d=2f$ the system will behave a plane glass plate of infinite focal length.



EXAMPLE 20

The object in Figure is midway between the lens and the mirror. The mirror's radius of curvature is 20.0 cm, and the lens has a focal length of –16.7 cm. Considering only the light that leaves the object and travels first toward the mirror, locate the final image formed by this system. Is this image real or virtual? Is it upright or inverted? What is the overall magnification?



SOLUTION:

$$\frac{1}{v_1}=\frac{1}{f_1}-\frac{1}{u_1}=\frac{1}{10.0\text{ cm}}-\frac{1}{12.5\text{ cm}}$$

So, $v_1=50.0$ cm (to left of mirror).

This serves as an object for the lens (a virtual object), so

$$\frac{1}{v_2}=\frac{1}{f_2}-\frac{1}{u_2}=\frac{1}{(-16.7\text{ cm})}-\frac{1}{(-25.0\text{ cm})}$$

and $v_2=-50.3$ cm,

meaning 50.3 cm to the right of the lens. Thus, the final image is located 25.3 cm to right of mirror.

$$M_1 = -\frac{v_1}{u_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

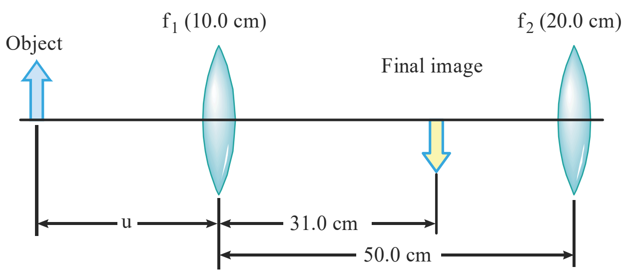
$$M_2 = -\frac{v_2}{u_2} = \frac{(-50.3\text{ cm})}{(-25.0\text{ cm})} = -2.01$$

$M = M_1 M_2 = 8.05$

Thus, the final image is virtual, upright, 8.05 times the size of object, and 25.3 cm to right of the mirror.

EXAMPLE 21

Two converging lenses having focal lengths of 10.0 cm and 20.0 cm are located 50.0 cm apart, as shown in Figure. The final image is to be located between the lenses at the position indicated. (a) How far to the left of the first lens should the object be? (b) What is the overall magnification? (c) Is the final image upright or inverted?



SOLUTION:

(a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $v_2 = -19.0$ cm). The required object distance for this lens is then

$$u_2 = \frac{v_2 f_2}{v_2 - f_2} = \frac{(-19.0\text{cm})(20.0\text{ cm})}{-19.0\text{cm} - 20.0\text{cm}} = \frac{380\text{cm}}{39.0}$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$v_1 = 50.0\text{ cm} - u_2 = 50.0\text{ cm} - \frac{380\text{cm}}{39.0} = \frac{1570\text{m}}{39.0}$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{u_1} = \frac{1}{f_1} - \frac{1}{v_1} = \frac{1}{10.0\text{cm}} - \frac{39.0}{1570\text{cm}}$$

$$\frac{157 - 39.0}{1570\text{ cm}} = \frac{118}{1570\text{ cm}}$$

or $u_1 = \dfrac{1570\text{ cm}}{118} = 13.3\text{ cm}$

(b) $$M = M_1 M_2 = \left(-\frac{v_1}{u_1}\right)\left(-\frac{v_2}{u_2}\right)$$

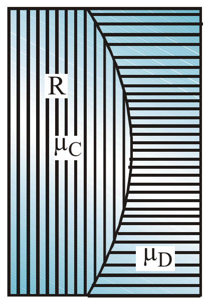
$$= \left[\left(\frac{1570\text{ cm}}{39.0}\right)\left(\frac{118}{1570\text{ cm}}\right)\right] \times \left[\frac{(-19.0\text{ cm})(39.0)}{380\text{ cm}}\right]$$

$= -5.90$

(c) Since $M < 0$, the final image is inverted.

EXAMPLE 22

A plane glass plate is constructed by combining a plano-convex lens and a plano-concave lens of different materials as shown in figure. Find the effective focal length.



SOLUTION:

As μ_C and μ_D are refractive indices of convergent and divergent lens respectively and R the radius of curvature of common interface, then by lens maker's formula,

$$\frac{1}{f_C} = (\mu_C - 1)\left[\frac{1}{\infty} - \frac{1}{-R}\right] = \frac{(\mu_C - 1)}{R} \quad \text{... (1)}$$

$$\frac{1}{f_D} = (\mu_D - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right] = \frac{-(\mu_D - 1)}{R} \quad \text{... (2)}$$

Now as the lens are in contact,

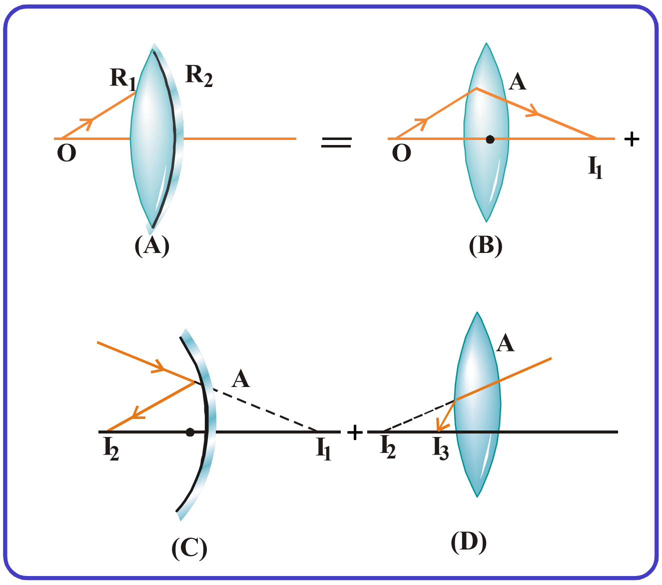
$$\frac{1}{F} = \frac{1}{f_C} + \frac{1}{f_D} = \frac{(\mu_C - \mu_D)}{R}; \quad F = \frac{R}{(\mu_C - \mu_D)}$$

As $\mu_C \neq \mu_D$, the system will act as a lens.

The system will behave as convergent lens if $\mu_C > \mu_D$ (as its focal length will be positive) and as divergent lens if $\mu_C < \mu_D$ (as F will be negative).

Lens with one Silvered Surface

* If the back surface of a lens is silvered and an object is placed in front of it then :
* First, light will pass through the lens and it will form the image I_1.
* The image I_1 will act as an object for silvered surface which acts as curved mirror and forms an image I_2 of object I_1.
* The light after reflection from silvered surface will again pass through the lens and lens will form final image I_3 of object I_2. This all is shown in Fig.



In such situation power of the silvered lens will be $P = P_L + P_M + P_L$ with $P_L = \dfrac{1}{f_L}$

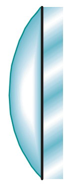
where $\dfrac{1}{f_L} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

and $P_M = -\dfrac{1}{f_M}$ where $f_M = \dfrac{R_2}{2}$

So the system will behave as a curved mirror of focal length f given by $f = -1/P$.

* When a convex lens is silvered, it behaves like a concave mirror and when a concave lens is silvered, it behaves like a convex mirror.

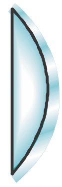
Case I : When plane face of planoconvex lens is silvered



$$f_{eff} = \frac{R}{2(\mu - 1)}$$

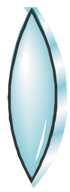
where, R = Radius of curvature
μ = Refractive index of material

Case II : When curved face of plano convex lens is silvered.



$$f_{eff} = \frac{R}{2\mu}$$

Case III : When one face of an equiconvex lens is silvered.



$$f_{eff} = \frac{R}{2(2\mu - 1)}$$

Defects of Images (Lens Aberrations)

* Our analysis of mirrors and lenses assumes that rays make small angles with the principal axis and that the lenses are thin. In this simple model, all rays leaving a point source focus at a single point, producing a sharp image. Clearly, this is not always true.
* When the approximations used in this analysis do not hold, imperfect images are formed.
* A precise analysis of image formation requires tracing each ray, using Snell's law at each refracting surface and the law of reflection at each reflecting surface. This procedure shows that the rays from a point object do not focus at a single point, with the result that the image is blurred. The departures of actual images from the ideal predicted by our simplified model are called **aberrations.**
* **Spherical Aberration**
Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens are different from the focal points of rays of the same wavelength passing near the axis. Figure illustrates spherical aberration for parallel rays passing through a converging lens.

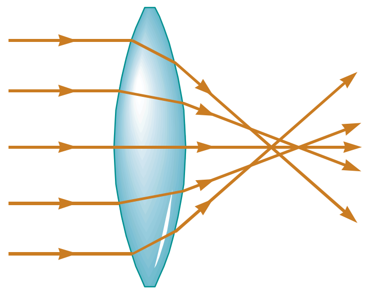


Figure : Spherical aberration caused by a converging lens.

As spherical aberration arises due to spherical nature of lens (or mirror) it can never be eliminated but can be minimise (like friction) by the following methods :

(1) Using stops : By using stops either paraxial or marginal rays are cut off, bringing the rest practically to one focus. This in turn implies that spherical aberration of a lens depends on its aperture and reducing the aperture, Spherical Aberration (SA) is reduced to a greater extent as in a microscope. However, in this method, intensity is affected adversely.

(2) Using lens of large focal length : It has been found that spherical aberration varies inversely as the cube of the focal length [i.e., $SA \propto (1/f^3)$], so if f is large, spherical aberration will reduce.

(3) Using plano-convex lens : It has been found that in case of plano-convex lens spherical aberration is minimised if its curved surface faces the incident or emergent light whichever is more parallel. This is why in telescope the curved surface of plano-convex lens faces the object while in microscope curved surface is towards the image.

(4) Using two thin lenses separated by a distance : It has been shown that in case of two thin lenses separated by a distance d, spherical aberration is minimum if: $d = f_2 - f_1$

* **Chromatic aberration**

It arises because the index of refraction of the material from which the lens is made varies with wavelength. Figure shows sunlight incident on a converging lens, in which the light spreads into its colour spectrum because of dispersion. Violet is refracted more than red, so the violet ray crosses the principal axis closer to the lens than does the red ray. Thus, the focal length of the lens is shorter for violet than for red, with intermediate values of the focal length corresponding to the colours in between. As a result of chromatic aberration, an undesirable colour fringe surrounds the image.

The difference between f_v and f_R is a measure of longitudinal chromatic aberration, i.e.

$$L.C.A. = f_R - f_V = -df \text{ with } df = f_V - f_R \quad .... (1)$$

However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...... (2)$$

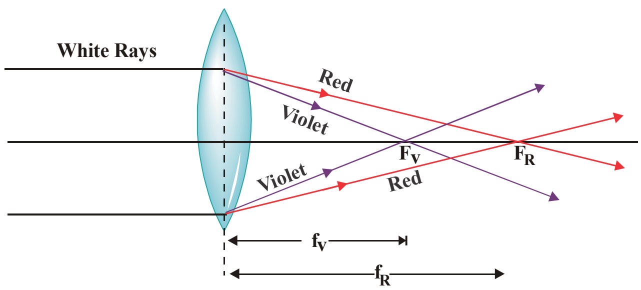
i.e., $$-\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...... (3)$$

So dividing eq. (3) by (2)

$$-\frac{df}{f} = \frac{d\mu}{(\mu-1)} = \omega \quad \left[\text{as } \omega = \frac{d\mu}{(\mu-1)}\right] \quad .... (4)$$

And hence, from eq. (1) and (4)

$$\boxed{L.C.A. = -df = \omega f} \quad ..... (5)$$



Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{i.e.,} \quad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$ i.e., $\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$

which in the light of eq. (5) reduced to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \quad \text{i.e.,} \quad \boxed{\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0} \quad .... (6)$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies his condition achromatic lens. From this condition i.e., from eq. (6) it is clear that in case of achromatic doublet :

(1) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$

i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(2) As ω_1 and ω_2 are positive quantities, for eq. (6) to hold f_1 and f_2 must be of opposite nature, i.e., if one of the lenses is convex the other must be concave.

(3) If the achromatic combination is convergent,

$$f_C < f_D \text{ and as } -\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}, \quad \omega_C < \omega_D$$

i.e., a convergent achromatic doublet convex lens has lesser focal length and dispersive power than divergent one.

EXAMPLE 23

Dispersive powers of materials used in lenses of an achromatic doublet are in the ratio 5 : 3. If the focal length of concave lens is 15 cm, then find the focal length of the other lens.

SOLUTION:

$$\frac{f}{f'} = -\frac{\omega}{\omega'} \text{ but } \frac{\omega}{\omega'} = \frac{5}{3} \text{ and } f = -15 \text{ cm}$$

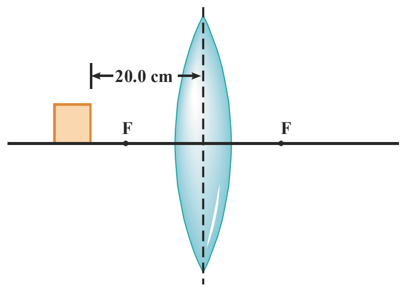
$$f' = -f\left(\frac{\omega'}{\omega}\right) = +15 \times \frac{3}{5} = 9 \text{ cm}$$

Checkup 5

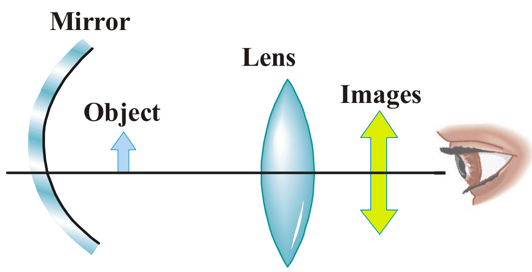
Q.1 A zip-lock plastic sandwich bag filled with water can act as a crude converging lens in air. If the bag is filled with air and placed under water, is the effective lens converging or diverging?

Q.2 A lens forms an image of an object on a screen. What happens to the image if you cover the top half of the lens with paper?

Q.3 Figure shows a thin glass ($\mu = 1.50$) converging lens for which the radii of curvature are $R_1 = 15.0$ cm and $R_2 = -12.0$ cm. To the left of the lens is a cube having a face area of 100 cm^2. The base of the cube is on the axis of the lens, and the right face is 20.0 cm to the left of the lens. (a) Determine the focal length of the lens. (b) Draw the image of the square face formed by the lens. What type of geometric figure is this? (c) Determine the area of the image.



Q.4 An observer to the right of the mirror–lens combination shown in Figure sees two real images that are the same size and in the same location. One image is upright and the other is inverted. Both images are 1.50 times larger than the object. The lens has a focal length of 10.0cm. The lens and mirror are separated by 40.0 cm. Determine the focal length of the mirror. Do not assume that the figure is drawn to scale.



Q.5 For image magnification one needs at least –
(A) two convex lens
(B) one concave and one convex lens
(C) one concave lens
(D) one convex lens

Q.6 The radii of curvatures of both the surfaces of a lens are equal (R) and it is made of a material of refractive index 1.5. Its focal length will be –
(A) ± R (B) ± 2R
(C) ± R/2 (D) zero

Q.7 What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

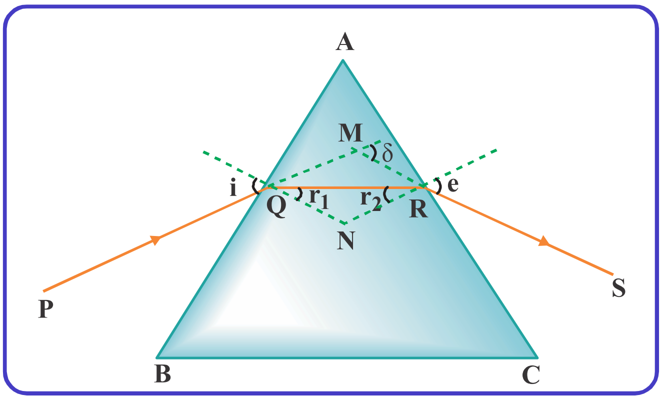
Q.8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?

Q.9 Explain why a mirror cannot give rise to chromatic aberration.

7.8 REFRACTION BY A PRISM

1. Prism

A homogeneous, transparent medium enclosed by two non-parallel refracting surfaces is called a prism. The angle between two refracting surfaces is called angle of prism.



PQ incident ray, QR refracted ray, RS emergent ray, δ angle of deviation and e angle of emergence.

2. Angle of Deviation

Angle between the incident rays & emergent ray is called angle of deviation.

In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles. Therefore, the sum of the other angles of the quadrilateral is 180º.

$$\angle A + \angle QNR = 180^\circ$$

From the triangle QNR,

$$r_1 + r_2 + \angle QNR = 180^\circ$$

Comparing these two equations, we get

$$\boxed{r_1 + r_2 = A} \quad \text{....... (1)}$$

The total deviation δ is the sum of deviations at the two faces,

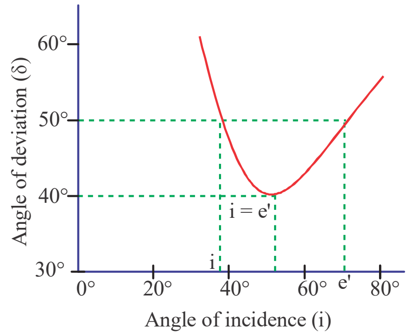
$$\delta = (i - r_1) + (e - r_2)$$

that is, $\boxed{\delta = i + e - A}$ .... (2)

3. Minimum Deviation

For a given prism, the angle of deviation depends upon the angle of incidence of the light-ray falling on the prism. It is seen from the curve that as the angle of incidence i increases, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then again increases. Thus, for one and only one, particular angle of incidence the prism produce minimum deviation.

At the minimum deviation δ_m, the refracted ray inside the prism becomes parallel to its base.



We have, $d = \delta_m$, $i = e$ which implies $r_1 = r_2$.
Eq. (1) gives $2r = A$ or $r = A/2$

Eq. (2) gives, $\delta_m = 2i - A$, or $i = (A + \delta_m)/2$

Refractive index of the prism

$$\boxed{n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$

where A is prism angle, δ_m is minimum deviation.

NOTE

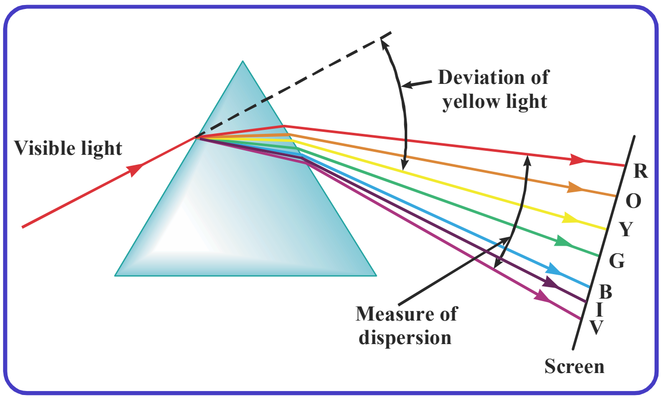
* When prism is thin, then value of A will be small ($\le 10°$) $\delta_m = (\mu - 1)A$
* Condition for maximum deviation i_1 or $i_2 = 90°$.

4. Angular Dispersion for Prism

White light splits into its constituent colours, on passing through prism. This is known as dispersion. The angle between the emergent rays of any two colours is called angular dispersion between those colours.

If deviation angle for violet & red are δ_v & δ_R respectively, then angular dispersion.

$$\theta = \delta_v - \delta_R$$



$$\delta_R = (\mu_R - 1)A, \quad \delta_v = (\mu_v - 1)A$$

$$\theta = (\mu_v - 1)A - (\mu_R - 1)A = (\mu_v - \mu_R)A.$$

5. Dispersive Power of Prism

$$\boxed{\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{(\mu_V - \mu_R)A}{(\mu_Y - 1)A}}$$

Where δ_Y is deviation angle for yellow colour.

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$$

For yellow colour refractive index is taken as mean value of violet and red colour.

$$\mu_Y = \frac{\mu_V + \mu_R}{2} = \mu$$

Dispersive power of flint glass is greater than that of crown glass.

6. Combination of Prisms

As the dispersive powers of the different materials are different, two or more prisms of different materials can be combined such that the rays of composite light on passing through the combination may suffer either dispersion without deviation or deviation without dispersion.

(i) Achromatic combination (deviation without dispersion):

Condition for achromatic combination :

$$\theta_1 + \theta_2 = 0$$

$$(\mu_v - \mu_r)A = -(\mu'_v - \mu'_r)A'$$

$$\boxed{A' = -\frac{\mu_v - \mu_r}{\mu'_v - \mu'_r}A}$$

or $\omega\delta + \omega'\delta' = 0$, where ω, ω' are dispersive powers for the two prisms and δ, δ' are the mean deviation.

Net mean deviation

$$= \left[\frac{\mu_v + \mu_R}{2} - 1\right]A + \left[\frac{\mu'_v + \mu'_R}{2} - 1\right]A'$$

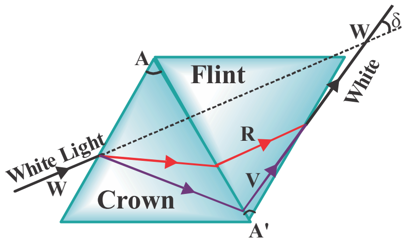
$$= (\mu - 1)A - (\mu' - 1)\frac{\mu_v - \mu_r}{\mu'_v - \mu'_r}A$$

$$= (\mu - 1)A\left[1 - \frac{\omega}{\omega'}\right] = \delta\left(1 - \frac{\omega}{\omega'}\right)$$

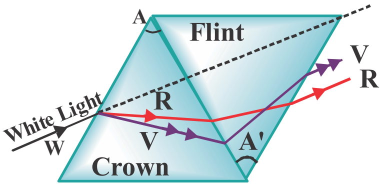
$$\boxed{\text{Net mean deviation} = \delta\left(1 - \frac{\omega}{\omega'}\right)}$$

where, μ is mean refractive index (yellow colour) for first prism and μ' is mean refractive index for second prism.

δ mean deviation of first prism, ω and ω' dispersive power of first and second prism.



(ii) Dispersion without deviation (Direct vision combination) - This combination is used for dispersion without deviation.



Condition $\delta = 0$ i.e.,

$$\left[\frac{\mu_V + \mu_R}{2} - 1\right]A + \left[\frac{\mu'_V + \mu'_R}{2} - 1\right]A' = 0$$

$$\boxed{A' = -\frac{\mu - 1}{\mu' - 1}A}$$

Sign shows that A and A' are reversed.

Net angle of dispersion

$$\theta = (\mu_v - \mu_r)A + (\mu'_v - \mu'_r)\,A'$$

$$\Rightarrow \quad \theta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)\frac{\mu - 1}{\mu' - 1}A$$

$$\Rightarrow \quad \theta = (\mu - 1)\,A\left[\frac{\mu_v - \mu_r}{\mu - 1} - \frac{\mu'_v - \mu'_r}{\mu' - 1}\right]$$

$$\Rightarrow \quad \boxed{\theta = (\mu - 1)\,A\,(\omega - \omega')}$$

Since, $\omega' > \omega$ angular dispersion is negative, hence the pattern of colours in the spectrum is reversed.

Problems solving tips :

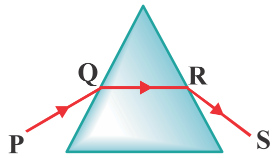
Steps 1 : Draw a ray diagram starting from the object passing through all refracting surfaces. Label all the angles.

Steps 2 : Apply the relevant formula at each of the refracting surfaces. (Snell rules)

Steps 3 : Develop any additional equations based on geometry.

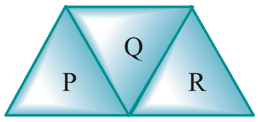
Steps 4 : Solve the simultaneous equations systematically.

* A ray of light is incident on an equilateral glass prism placed on a horizontal table.



For minimum deviation QR is horizontal.

* A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown in the figure.



The ray will suffer same deviation as before.

* A ray of light will not emerge out of a prism (whatever be the angle of incidence) if $A > 2\theta_C$ i.e., if $\mu > \text{cosec}\,(A/2)$

* In the situation of minimum deviation

(1) Angle of incidence is equal to angle of emergence and is given by
$i = \sin^{-1}[\mu \sin(A/2)]$

(2) Angle of refraction inside the prism is equal to half the angle of prism i.e., $r = (A/2)$.

* For a given material of prism, light and angle of incidence, as angle of prism increases, angle of deviation also increases, i.e. $\delta \propto A$.

* Greater the μ of the prism, greater will be the deviation as $\delta \propto (\mu - 1)$.
e.g., μ of the flint glass is more than that of crown glass, so $\delta_F > \delta_C$, for same i, A and λ.

* With increase in wavelength deviation decreases, i.e., deviation for red is least while maximum for violet.

* In the spectrum of white light produced by a prism spread of violet is most while least for red.

EXAMPLE 24

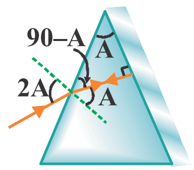
Angle of a prism is A and its one surface is silvered. Light ray falling at an angle of incidence 2A on first surface return back through the same path after suffering reflection at second silvered surface. Find the refractive index of material.

SOLUTION:

Given $i = 2A$; $r = A$

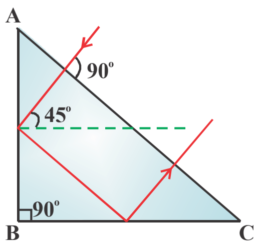
$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A} = \frac{2\sin A \ \cos A}{\sin A}$$

$= 2 \cos A$



EXAMPLE 25

A ray falls on a prism ABC (AB = BC) and travels as shown in adjoining figure. Find the minimum refraction index of the prism material.



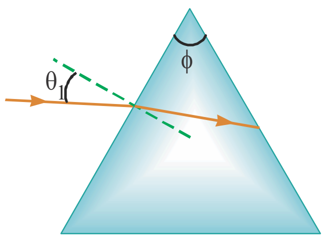
SOLUTION:

Angle of incidence $C = 45°$

$$\therefore \quad \mu = \frac{1}{\sin C} = \frac{1}{\sin 45°} = \sqrt{2}$$

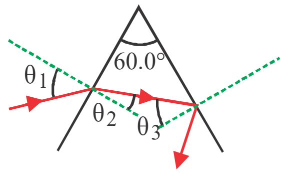
EXAMPLE 26

A triangular glass prism with apex angle ϕ has an index of refraction μ (Fig.). What is the smallest angle of incidence & θ_1 for which a light ray can emerge from the other side?



SOLUTION:

At the first refraction, $1.00 \sin \theta_1 = \mu \sin \theta_2$.



The critical angle at the second surface is given by $\mu \sin \theta_3 = 1.00$:

or $\quad \theta_3 = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 41.8°$

But $\theta_2 = 60.0° - \theta_3$

Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$)

It is necessary that $\theta_2 > 18.2°$.

Since $\sin \theta_1 = n \sin \theta_2$, this becomes

$\sin \theta_1 > 1.50 \sin 18.2° = 0.468$

or $\quad \theta_1 > 27.9°$

Checkup 6

Q.1 When two colours of light (X and Y) are sent through a glass prism, X is bent more than Y. Which colour travels more slowly in the prism?

Q.2 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Q.3 You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion,

(b) disperse (and displace) a pencil of white light without much deviation.

Q.4 Dispersion is the term used to describe –

(A) the propagation of light in straight lines.

(B) The splitting of a beam of light into component colours.

(C) The bending of a beam of light when it strikes a mirror.

(D) The change that takes place in white light after passage through red glass.

Q.5 In a glass prism –
(A) Blue light is dispersed more than red light
(B) Red light is dispersed more than blue light
(C) Both red light and blue light are equally dispersed
(D) None of these

Q.6 Deviation δ produced by a prism of refractive index μ and small angle A is given by –
(A) $\delta = (\mu - 1) A$ (B) $\delta = (\mu + 1) A$
(C) $\delta = (A - 1) \mu$ (D) $\delta = (A + 1) \mu$

Q.7 If for a given prism the angle of incidence is changed from 0° to 90°, the angle of deviation –
(A) Increases
(B) Decreases
(C) First decreases and then increases
(D) First increases and then decreases

Q.8 The refracting angle of a prism is A and the refractive index of the prism is cot A/2. The angle of minimum deviation is –
(A) 180° – 3A (B) 180° + 2A
(C) 90° – A (D) 180° – 2A

7.9 SCATTERING OF LIGHT

* As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles.
* Light of shorter wavelengths is scattered much more than light of longer wavelengths.
* The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as **Rayleigh scattering**. Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly.
* $\boxed{\text{Intensity of scattered light} \propto \dfrac{1}{\lambda^4}}$
* In fact, violet gets scattered even more than blue, having a shorter wavelength. But since our eyes are more sensitive to blue than violet, we see the sky blue.

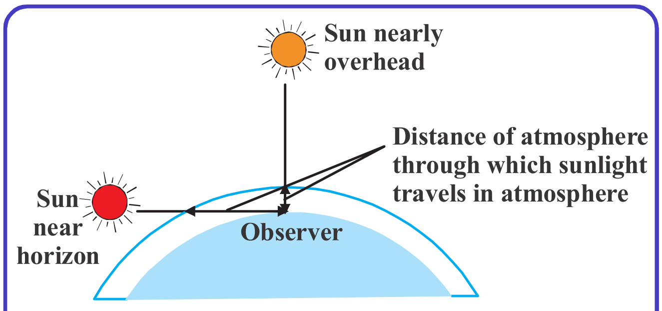


Figure : Sunlight travels through a longer distance in the atmosphere at sunset and sunrise.

* At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere. Therefore, the blue and its neighbouring colours are scattered in the long path. The light reaching the observer is predominantly red due to its least scattering owing to the larger wavelength. Hence, the sky appears red at the sunrise and at the sunset.

RAINBOW

* Rainbow is sunlight spread out into its spectrum of colours and diverted to the eye of the observer by water droplets.
* The "bow" part of the word describes the fact that the rainbow is a group of nearly circular arcs of colour all having a common center.
* Rainbows are generated through refraction and reflection of light in small rain drops.
* The sun is always behind you when you face a rainbow, and that the center of the circular arc of the rainbow is in the direction opposite to that of the sun.
* The rain, of course, is in the direction of the rainbow i.e. rain drops must be ahead of you and the angle between your line-of-sight and the sunlight will be 40°-42°.
* When a sunbeam is being refracted twice and reflected once by the droplet, a primary rainbow will form (Its inner and outer edges subtend angles of 41° and 43° with the axis of the rainbow respectively).

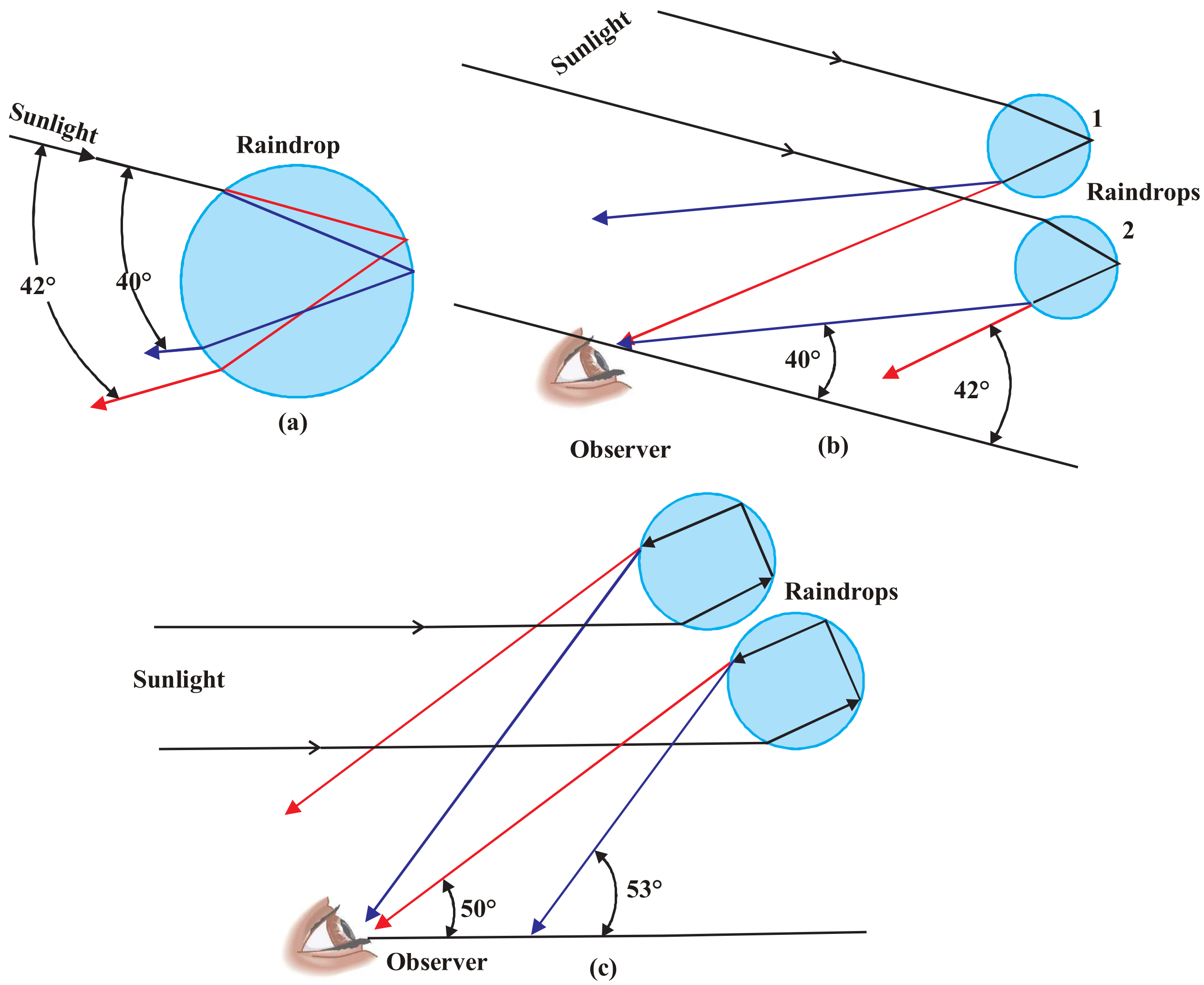


Figure : Rainbow: (a) The sun rays incident on a water drop get refracted twice and reflected internally by a drop; (b) Enlarge view of internal reflection and refraction of a ray of light inside a drop form primary rainbow; and (c) secondary rainbow is formed by rays undergoing internal reflection twice inside the drop.

* In primary rainbow the outer side is of red colour and inner side is of violet colour.
* If the beam is being refracted twice and reflected twice, a **secondary** rainbow will form (Its inner and outer edges subtend angles of 51° and 54° with the axis of the rainbow respectively).
* In secondary rainbow the outer side is of violet colour and inner side is of red colour. As the secondary rainbow is formed by one more reflection than the primary rainbow, it is much fainter and rare to see. On the other hand, since the paths of sunbeams in a primary rainbow and a secondary rainbow are different, the colours of the secondary rainbow are arranged in just the reverse order of the primary one.

7.10 OPTICAL INSTRUMENTS

THE EYE

* **Power of Accommodation :** The ability of the lens to change its shape to focus near and distant objects is called **accommodation.**
* The minimum distance, at which objects can be seen most distinctly without strain, is called the **least distance of distinct vision**.
  It is also called the **near point (N.P.)** of the eye. For a young adult with normal vision, the near point is about 25 cm.

* The **farthest point** upto which the eye can see objects clearly is called the far point (F.P.) of the eye. It is infinity for a normal eye. Thus a normal eye can see objects clearly that are between 25 cm and infinity.
* The limit of resolution of eye is one minute, i.e. two objects will not be visible distinctly to the eye if the angle subtended by them at the eye is lesser than one minute.
* The persistence of vision is (1/10) sec, i.e., if time interval between two consecutive light pulses is lesser than 0.1 sec, eye cannot distinguish them separately. This fact is taken into account in motion pictures.
* The sensitivity of eye is maximum in yellow-green light.
* **Nearsightedness:** If the eyeball is too long or the lens too spherical, the image of distant objects is brought to a focus in front of the retina and is out of focus again before the light strikes the retina. Nearby objects can be seen more easily. Eyeglasses with concave lenses correct this problem by diverging the light rays before they enter the eye. Nearsightedness is called **myopia**. Myopia most commonly develops in childhood (between 8 and 14).

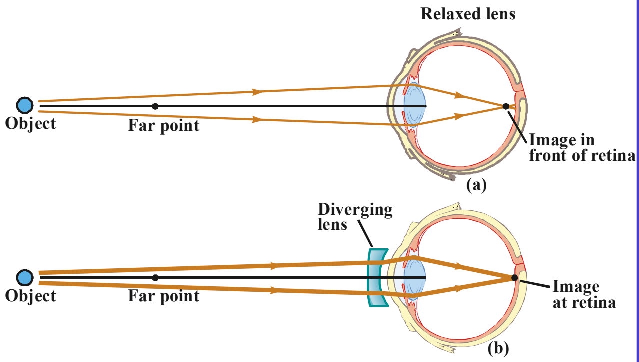


Figure : (a) When a nearsighted eye looks at an object that lies beyond the eye's far point, the image is formed in front of the retina, resulting in blurred vision. (b) Nearsightedness is corrected with a diverging lens.

If deflected far point is at a distance d from eye then focal length of use lens

$f = -d = -(\text{deflected far point})$

A person can see upto distance $\rightarrow$ x, wants to see distance $\rightarrow y(y > x)$ has to use lens of focal length: $f = \dfrac{xy}{x-y}$ or power of the lens $P = \dfrac{x-y}{xy}$.

* **Farsightedness :** If the eyeball is too short or the lens too flat or inflexible, the light rays entering the eye-particularly those from nearby objects - will not be brought to a focus by the time they strike the retina. Eyeglasses with convex lenses can correct the problem.

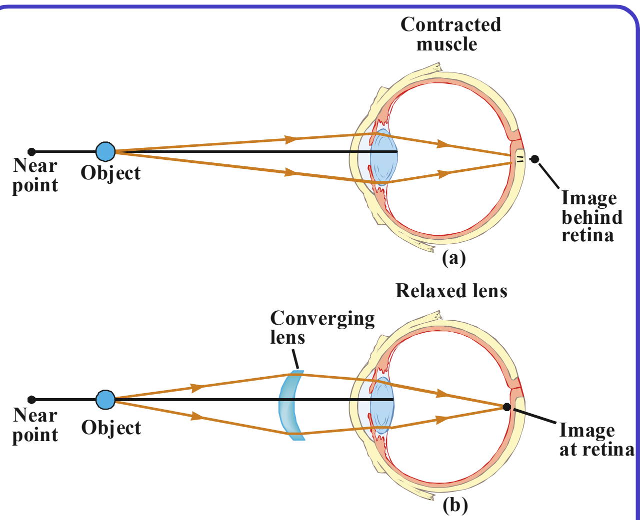


Figure : (a) When a farsighted eye looks at an object located between the near point and the eye, the image point is behind the retina, resulting in blurred vision. The eye muscle contracts to try to bring the object into focus. (b) Farsightedness is corrected with a converging lens

Farsightedness is called hypermetropia or **hyperopia**.

If a person cannot see before distance d but wants to see the object placed at distance D from eye has to use lens of focal length:

$$f = \frac{dD}{d-D} \text{ and power of the lens } P = \frac{d-D}{dD}.$$

* **Astigmatism:** Such a person cannot see all directions equally well. This is corrected by using cylindrical lenses.
* **Presbyopia :** In this defect both far off and nearer objects are not clearly seen. This is corrected by using bi-focal lens.

EXAMPLE 27

A myopic person has been using spectacles of power -1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power $+2.0$ dioptres. Explain what may have happened.

SOLUTION:

For -1 dioptre, the far point for eyes is 1 m i.e. 100 cm. The near point is 25 cm. The objects lying at infinity are brought at 100 cm from his eyes using the concave lens and the objects lying in between 25 cm and 100 cm are brought to focus using the ability of accommodation of the eye lens. In the old age, this ability of accommodation is reduced and the near point reaches 50 cm from his eyes.

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get, $-\frac{1}{50}+\frac{1}{25}=\frac{1}{f}$

i.e. $f=50$ cm

and $P=\frac{100}{f}=\frac{100}{50}=2$ dioptre

The person requires glasses of $+2$ dioptre.

EXAMPLE 28

A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

SOLUTION:

(a) To see the object at a closest distance, the image of object should be formed at the least distance of distinct vision.

Using $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ we have $-\frac{1}{u}-\frac{1}{25}=\frac{1}{5}$

i.e. $-\frac{1}{u}=\frac{1}{5}+\frac{1}{25}=\frac{5+1}{25}=\frac{6}{25}$

$\therefore\ u=\frac{-25}{6}=-4.2$ cm

The object is to be placed at $4\frac{1}{6}$ cm from the magnifying glass.

Also, to see the object at the farthest point, its image must be formed at infinity.

$\therefore\quad -\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ i.e. $-\frac{1}{u}+\frac{1}{-\infty}=\frac{1}{5}$

i.e. $u=-5$ cm i.e. the object is to be placed at 5 cm from the magnifying glass.

(b) Angular magnification, $m=\frac{D}{|u|}$

$\therefore$ Maximum angular magnification $=\frac{25}{25/6}=6$

and minimum angular magnification $=\frac{25}{5}=5.$

Camera

(a) **Pinhole Camera :** It is bases on rectilinear propagation of light and forms the so called image on the screen which is real and inverted. If an object of size O is situated at a distance u from the pinhole and its image of size I is formed at a distance v from the pin hole -

$$\theta=\frac{O}{u}=\frac{I}{v}\ \text{ i.e., }\ \frac{I}{O}=\frac{v}{u}$$

(b) **Lens-Camera :** In it a converging lens whose aperture and distance from the film can be adjusted, is used. Usually object is real and between ∞ and 2F; so the image is real, inverted diminished and between F and 2F.

Here lens formula

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ with $m=\frac{I}{O}=\frac{v}{u}$ is applicable.

In photographing an object, the image is first focused on the film by adjusting the distance between lens and film (called focusing). After focusing, aperture is set to a specific value (for desired effect) and then film is exposed to light for a given time through a shutter.

For proper exposure of a particular film, a definite amount of light energy must be incident on the film. So if I is the intensity of light, S is the light transmitting area of lens and t is the exposure time, then for proper exposure,

$$I \times S \times t = \text{constant}$$

Light transmitting area of a lens is proportional to the square of its aperture D; so above expression reduces to

$$\boxed{I \times D^2 \times t = \text{Constant}}$$

NOTE

* If aperture is kept fixed, for proper exposure,

$$I \times t = \text{constant} \quad \text{i.e.,} \quad I_1t_1 = I_2t_2$$

and if the source of light is a point

$$\frac{L_1}{r_1^2} \times t_1 = \frac{L_2}{r_2^2} \times t_2 \quad \left[\text{as } I = \frac{L}{r^2}\right]$$

* If intensity is kept fixed, for proper exposure,

$$D^2 \times t = \text{constant}$$

i.e., $\text{Time of exposure} \propto \dfrac{1}{(\text{Aperture})^2}$

* The ratio of focal length to the aperture of lens is called f-number of the camera, i.e.,

$$\text{f-number} = \frac{\text{Focal length}}{\text{Aperture}}$$

If focal length = constant

$$\text{Aperture} \propto \frac{1}{\text{f-number}}$$

$$\text{Time of exposure} \propto (\text{f-number})^2$$

EXAMPLE 29

Photograph of the ground are taken from an air-craft, flying at an altitude of 2000 m, by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any one time.

Sol. As here u = – 2000m, f = 0.50m, so from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; $\frac{1}{v} - \frac{1}{(-2000)} = \frac{1}{0.5}$

$$\frac{1}{v} = \frac{1}{0.5} - \frac{1}{2000} \cong \frac{1}{0.5} \quad \left[\text{as } \frac{1}{0.5} >> \frac{1}{2000}\right]$$

v = 0.5m = 50 cm = f

Now as in case of a lens

$$m = \frac{v}{u} = \frac{0.5}{-2000} = -\frac{1}{4} \times 10^{-3}$$

So $I_1 = (ma)(mb) = m^2A \quad [\because A = ab]$

$$A = \frac{I_1}{m^2} = \frac{18\text{cm} \times 18\text{cm}}{[(1/4) \times 10^{-3}]^2} = (720\text{m} \times 720\text{ m})$$

EXAMPLE 30

The proper exposure time for a photographic print is 20 s at a distance of 0.6 m from a 40 candle power lamp. How long will you expose the same print at a distance of 1.2 m from a 20 candle power lamp ?

SOLUTION:

In case of camera, for proper exposure

$$I_1D_1^2t_1 = I_2D_2^2t_2$$

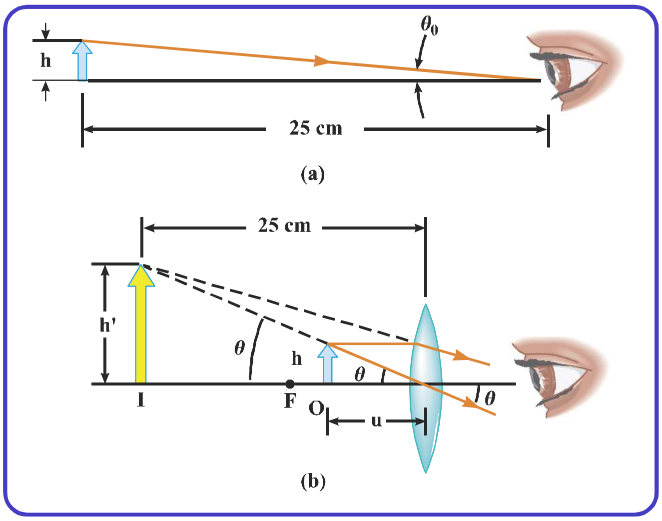
As here D is constant and $I = (L/r^2)$

$$\frac{L_1}{r_1^2} \times t_1 = \frac{L_2}{r_2^2} \times t_2 . \text{ So } \frac{40}{(0.6)^2} \times 20 = \frac{20}{(1.2)^2} t$$

i.e., t = 160 sec

Simple Microscope

* It is an optical instrument used to increase the visual angle of near objects which are too small to be seen by naked eye.
* It is also known as magnifying glass or simply magnifier and consists of a convergent lens with object between its focus and optical centre and eye close to it.
* The image formed by it is erect, virtual, enlarged and on same side of lens between object and infinity.



(a) (b)

* The magnifying power (MP) or angular magnification of a simple microscope (or an optical instrument) is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when eye is unaided (i.e., when the object is at least distance of distinct vision)

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max. visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

* If an object of size h is placed at a distance u (<D) from the lens and its image size h′ is formed at a distance v (≥ D) from the eye

$$\theta = \frac{h'}{v} = \frac{h}{u} \qquad \text{with } \theta_0 = \frac{h}{D}$$

So $$\boxed{MP = \frac{\theta}{\theta_0} = \frac{h}{u} \times \frac{D}{h} = \frac{D}{u}} \quad ......(1)$$

Now there are two possibilities

(a) If their image is at infinity [Far point]

In this situation from lens formula -

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{we have } \frac{1}{\infty} - \frac{1}{-u} = \frac{1}{f}$$

i.e., u = f. So, $$\boxed{MP = \frac{D}{u} = \frac{D}{f}} \quad ....(2)$$

As here u is maximum [as object is to be with in focus], MP is minimum and as in this situation parallel beam of light enters the eye, eye is least strained and is said to be normal, relaxed or unstrained.

(b) If the image is at D [Near point]

In this situation as v = D, from lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{we have } \frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

i.e., $$\frac{D}{u} = 1 + \frac{D}{f}$$

So $$\boxed{MP = \frac{D}{u} = \left[1 + \frac{D}{f}\right]} \quad ..... (3)$$

As the minimum value of v for clear vision is D, in this situation u is minimum and hence this is the maximum possible MP of a simple microscope and as in this situation final image is closest to eye, eye is under maximum strain.

Golden Tips

* The magnifying power (MP) have no unit. It is different from power of a lens which is expressed in diopter (D) and is equal to the reciprocal of focal length in meter.

* With increase in wavelength of light used, focal length of magnifier will increase and hence its MP will decrease.

EXAMPLE 31

A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

SOLUTION:

(a) Using the lens formula, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

When f = 10 cm, u = – 9 cm, we get

$$\therefore -\frac{1}{-9} + \frac{1}{v} = \frac{1}{10} \quad \text{i.e. } \frac{1}{v} = \frac{1}{10} - \frac{1}{9} = \frac{9-10}{90}$$

i.e. v = – 90 cm

$$\text{Linear magnification} = \frac{v}{u} = \frac{-90}{-9} = 10$$

$\therefore$ Area of each square in the image

$\therefore$ $(1\text{mm} \times 10) \times (1\text{mm} \times 10)$

$$= 100\ \text{mm}^2 = 1\ \text{cm}^2$$

(b) $\text{Angular magnification} = \frac{D}{|u|} = \frac{25}{9} \approx 2.8$

(c) Clearly magnification and power magnification are not equal to each other unless the image is located near the least distance of distinct vision i.e. $v = D$.

Compound Microscope

Construction :

* It consists of two convergent lenses of short focal lengths and apertures arranged co-axially.
* Lens (of focal length f_o) facing the object is called objective or field lens while the lens (of focal length f_e) facing the eye is called eye-piece or ocular.
* The objective has a smaller aperture and smaller focal length than eye-piece the separation between objective and eye-piece can be varied.

Image Formation :

* The object is placed between F and 2F of objective so the image formed by objective (called intermediate image) is inverted, real enlarged and at a distance greater than f_0 on the other side of the lens.
* Image formed by objective lens acts as object for eye-piece and is within its focus. So eye-piece forms final image I which is erect, virtual & enlarged with respect to intermediate image.
* The final image with respect to object is Inverted, virtual, enlarged & at a distance D to ∞ from eye.

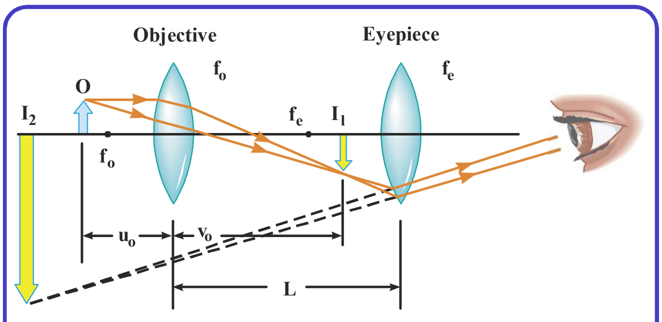


Figure : Diagram of a compound microscope, which consists of an objective lens and an eyepiece lens.

* **Magnifying power (MP) -**

Magnifying Power of an optical instrument is defined as –

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max. Visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

* If the size of object is h and least distance of distinct vision is D.

$$\theta_0 = \frac{h}{D} \quad ; \theta = \frac{h'}{u_e}$$

$$\boxed{MP = \frac{\theta}{\theta_0} = \left[\frac{h'}{u_e}\right] \times \left[\frac{D}{h}\right] = \left[\frac{h'}{h}\right]\left[\frac{D}{u_e}\right]}$$

* But for objective $m = \frac{I}{O} = \frac{v}{u}$

i.e., $\frac{h'}{h} = -\frac{v}{u}$ [as u is – ve]

So $MP = -\frac{v}{u}\left[\frac{D}{u_e}\right]$ ......(1)

with length of tube $L = v + u_e$

* Now there are two possibilities –

(a) If the final image is at infinity (far point) :

This situation is called normal adjustment as in this situation eye is least strained or relaxed. In this situation as for eye - piece $v = \infty$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e., } u_e = f_e = \text{maximum}$$

Substitution this value of u_e in Eqn. (1), we have

$$\boxed{MP = -\frac{v}{u}\left[\frac{D}{f_e}\right]} \quad \text{with } L = v + f_e \quad .....(2)$$

(b) If the final image is at D (near point) :

In this situation as for eye - piece $v = D$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e., } \frac{1}{u_e} = \frac{1}{D}\left[1 + \frac{D}{f_e}\right]$$

Substituting this value of u_e in Eqn. (1), we have

$$\boxed{MP = -\frac{v}{u}\left[1 + \frac{D}{f_e}\right]} \quad \text{with} \quad L = v + \frac{f_e D}{f_e + D}$$

....(3)

In this situation as u_e is minimum, MP is maximum and eye is most strained.

* In microscope focal length of objective lens f_o is small and object is placed very closed to objective lens so–

$$u \approx f_o \;;\; L = v + u_e \qquad [u_e << v]$$

$$L \approx v \;;\quad \boxed{|MP| = \frac{LD}{f_o f_e}}$$

Golden Tips

* For large M, f_0 and f_e should be small.
* On increasing the length L of the tube, M will increase.
* Object should be placed just away from the principal focus of the objective.
* f_0 is much smaller so that the object is very near to the objective.
* The eyepiece is also called ocular.
* Magnifying power of microscope is negative so it produces final image always inverted.
* $$m_e = \frac{v_e}{u_e} = \frac{D}{u_e} = \left[1 + \frac{D}{f_e}\right]$$

$$m_o = \frac{v_o}{u_o} = \frac{v}{u} \Rightarrow MP = m_o \times m_e$$

* $$(MP)_{min} = -\frac{v}{u}\frac{D}{f_e}; \; (MP)_{max} = -\frac{v}{u}\left[1 + \frac{D}{f_e}\right]$$
* $$|MP| = \frac{LD}{f_0 f_e}$$
* MP does not change appreciably if objective lens and eye-piece are interchanged $[MP \sim (LD/f_0 f_e)]$.
* **Resolving power:** With respect to microscope, the minimum distance between two lines at which they are just distinct is called limit of resolution and the reciprocal of limit of resolution is called resolving power. $RP = \frac{1}{RL} \propto \frac{1}{\lambda}$
* In electron microscope, $\lambda = \sqrt{(150/V)}$ Å, where V = Potential difference.

EXAMPLE 32

An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?

SOLUTION:

For the image formed at the least distance of distinct vision, the magnifying power is given by

$$m = m_0 m_e$$

Here $m_0 = \frac{v_0}{-u_0}$ and $m_e = 1 + \frac{D}{f_e}$

$$\text{Using } m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

$$\therefore \quad m = m_0 m_e \text{ i.e. } m_0 = \frac{m}{m_e} = \frac{30}{6} = 5.$$

$$m_0 = \frac{v_0}{-u_0} = 5 \text{ i.e., } v_0 = -5u_0$$

$$\therefore \quad \text{Using} -\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0}, \text{ we get,}$$

$$-\frac{1}{u_0} - \frac{1}{5u_0} = \frac{1}{f_0} \text{ or } \frac{-5-1}{5u_0} = \frac{1}{1.25}$$

$$\text{or} \quad u_0 = -\frac{6}{5} \times 1.25 = -1.5 \text{ cm}$$

$$\therefore \quad v_0 = -5u_0 = -5 \times (-1.5) = 7.5 \text{ cm}$$

Again using $-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$, we get

$$-\frac{1}{u_e} + \frac{1}{-25} = \frac{1}{5} \; (\because \text{ image is formed at 25 cm})$$

$$\therefore \quad \frac{-1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{5+1}{25} = \frac{6}{25}$$

$$\text{or} \quad u_e = -\frac{25}{6} \text{ cm} = -4.17 \text{ cm}$$

$$\therefore \quad |u_e| = 4.17 \text{ cm}$$

Thus, the distance between the objective and the eyes lens $= v_0 + |u_e| = 7.5 + 4.17 = 11.67$

Also, the position of the object from the objective lens $= |u_0| = 1.5$ cm

Telescope

* It is an optical instrument used to increase the visual angle of distant large objects.
* Object is between ∞ and 2F of objective and hence image formed by objective is real, inverted, and diminished and is between F and 2F on the other side of it. This image (called intermediate image) acts as object for eye- piece.

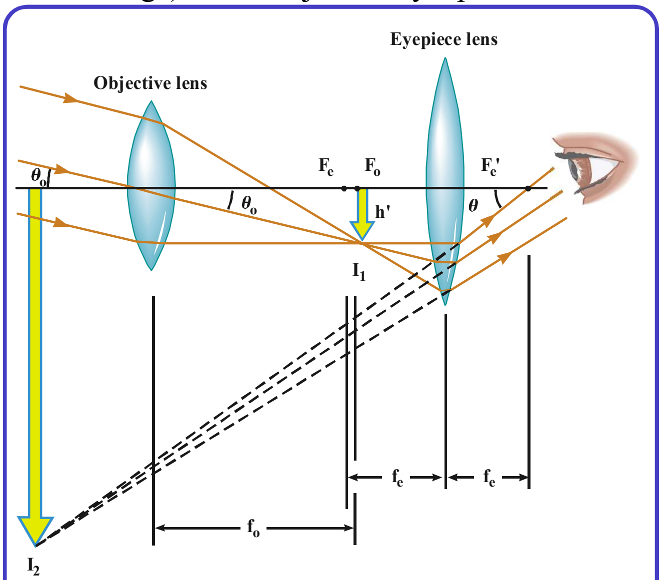


Figure : Lens arrangement in a refracting telescope, with the object at infinity.

* **Magnifying Power (MP)**

Magnifying Power of a telescope is defined as

$$MP = \frac{\text{Visual angle with instrument}}{\text{Visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

But from fig. $\theta_0 = (y/f_0)$ and $\theta = (y/-u_e)$

So $\boxed{MP = \frac{\theta}{\theta_0} = -\left[\frac{f_0}{u_e}\right]}$ with length of tube

$L = (f_0 + u_e)$ ....(1)

Now there are two possibilities –

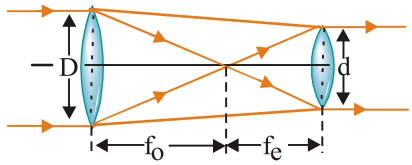
1. **If the final image is at infinity (far point)**

$$\frac{1}{-\infty} - \frac{1}{u_e} = \frac{1}{f_e} \qquad \text{i.e. } u_e = f_e$$

So substituting this value of ue in Eqn. (1)

$\boxed{MP = -(f_0/f_e)}$ and $L = (f_0 + f_e)$ ....(2)

In this case object and final image are at infinity so both total light entering and leaves the telescope are parallel to its axis as shown in fig.



2. **If the final image is at D(Near point)**

In this situation as for eye - piece v = D

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e., } \frac{1}{-u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

So substituting this value of ue in Eqn. (1),

$$\boxed{MP = \frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]}$$

with $L = f_0 + \frac{f_e D}{f_e + D}$ ....(3)

In this situation u_e is minimum so for a given telescope MP is maximum while length of tube is minimum

Note : $\frac{f_0}{f_e} = \frac{\text{Aperture of object}}{\text{Aperture of eye piece}}$

i.e., $MP = \frac{f_0}{f_e} = \frac{D}{d}$

* Resolving power of a telescope depends on the aperture of objective and wavelength of light.

$$RP \propto \frac{\text{Aperture of objective}}{\text{Wavelength}}$$

Reflecting telescopes :

* Modern telescopes use a concave mirror rather than a lens for the objective.
* Telescopes with mirror objectives are called reflecting telescopes.
* They have several advantages. First, there is no chromatic aberration in a mirror. Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed.
* Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim.

* One obvious problem with a reflecting telescope is that the objective mirror focuses light inside the telescope tube.

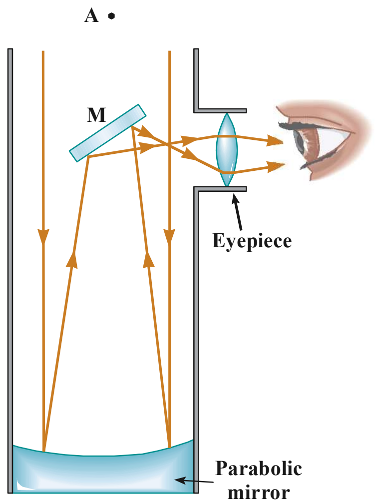


Figure : A Newtonian-focus reflecting telescope.

Golden Tips

* Magnifying power of telescope is negative so it produces always inverted image.

* $(MP)_{min} = -\left[\frac{f_0}{f_e}\right]$; $(MP)_{max} = -\frac{f_0}{f_e}\left[1+\frac{f_e}{D}\right]$

* MP becomes $(1/m^2)$ times of its initial value if objective and eye-piece are interchanged as $MP \sim [f_o / f_e]$
* MP is increased by increasing the focal length of objective lens and by decreasing the focal length of eye piece lens.
* In reflecting type telescope for high magnifying power, radius of curvature of the objective should be large.
* In reflecting type telescope final image is free from spherical and chromatic aberrations.

EXAMPLE 33

A small telescope has an objective lens of focal length 140cm and an eyepiece of focal length 5.0cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25cm)?

SOLUTION: (a) In normal adjustment,

$$|m| = \frac{f_0}{|f_e|} = \frac{140}{5} = 28,$$

(b) When the image is formed at LDDV

$$|m| = \frac{f_0}{|f_e|}\left(1+\frac{f_e}{D}\right) = \frac{140}{5}\left(1+\frac{5}{25}\right)$$

$$= 28 \times \frac{6}{5} = 33.6$$

Checkup 7

Q.1 Which glasses in Figure correct nearsightedness and which correct farsightedness?

Q.2 An optician while testing the eyes finds the vision of a patient to be 6/12. What it means.

Q.3 A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Q.4 In a compound microscope, the object is 1 cm. from the objective lens. The lenses are 30 cm. apart and the intermediate image is 5 cm. from the eye-piece. What magnification is produced?

Q.5(a) A small telescope has an objective lens of focal length 140cm and an eyepiece of focal length 5.0cm. The telescope is in normal adjustment. What is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25cm?

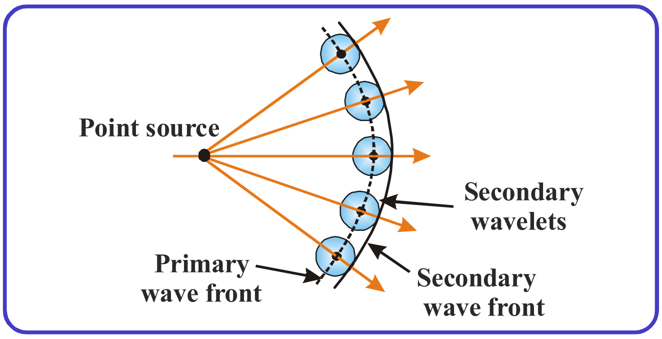
PART - B : WAVE OPTICS

7.11 WAVE FRONT

* A wavefront is defined as the locus of all adjacent points vibrating in the same phase of a physical quantity associated with the wave.
* The ray of light is considered in the direction of outward normal to the wave front.
* **Spherical wavefront :**
  (a) Light source is point source.
  (b) Effective distance should be finite.
  (c) Intensity $\propto 1/r^2$
  Cylindrical wavefront :
  (a) Light source is line source.
  (b) Effective distance should be finite.
  (c) Intensity $\propto 1/r$
  Plane wavefront :
  (a) Light source is at large distant.
  (b) Effective distance should be at infinite.
  (c) Intensity does not depends on distance.

7.12 HUYGEN'S WAVE THEORY

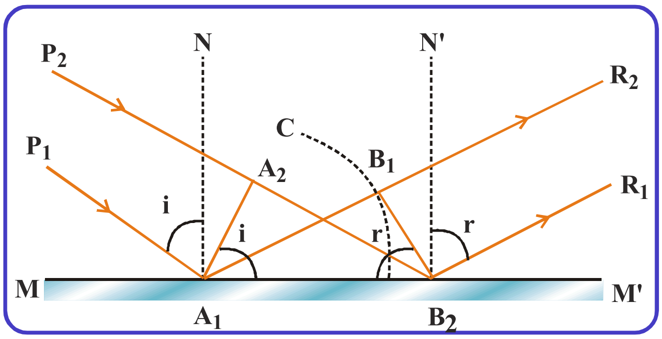
* Huygen proposed the wave theory of light. Light travels in form of wavefront in medium.
* Each point on the wavefront acts as a centre of new disturbance and emits its own set of spherical waves called secondary wavelets.



* The envelope or the locus of these wavelets in the forward direction gives the position of new wavefront at any subsequent time.
* Direction of wave propagation and wave front are mutually perpendicular to each other.
* At large distances from the source, all wavefronts appear to be plane.
* The theory explained successfully the reflection, refraction, interference, diffraction.
* In this theory, like sound, light is assumed to be a mechanical wave and in order to explain propagation of light in free space medium 'ether' is assumed. Owing to tremendous speed of light (3×10^8 m/s) ether was supposed to have very high elasticity and extremely low density. However, this theory could not explain polarisation and suffered a great set-back when the famous 'Michelson-Morley experiment' disproved the existence of ether.
* The theory predicted the presence of back wave, which proved to be failure.

Proof of Law of Reflection

Huygen's principle of wavefront construction leads to this law. In fig. P_1A_1 and P_2A_2 are the incident rays, normal to the incident wavefront A_1A_2. The angle of incidence i and the angle of reflection r are shown in figure.



From geometry, Angle between two lines
= Angle between their perpendicular lines
It can be shown that

$\angle B_2A_1A_2 = i$ and $\angle A_1B_2B_1 = r$

Also $A_2B_2 = v\Delta t$, $A_1B_1 = v\Delta t$

From the triangles $\Delta A_1A_2B_2$ and $\Delta B_2B_1A_1$,

$$\sin i = \frac{A_2B_2}{A_1B_2} = \frac{v\Delta t}{A_1B_2} = \sin r$$

Thus, $\sin i = \sin r$ or $\boxed{i = r}$

An incident ray, making an angle i with the normal, is reflected such that the reflected ray makes an angle r, which is equal to the angle i.

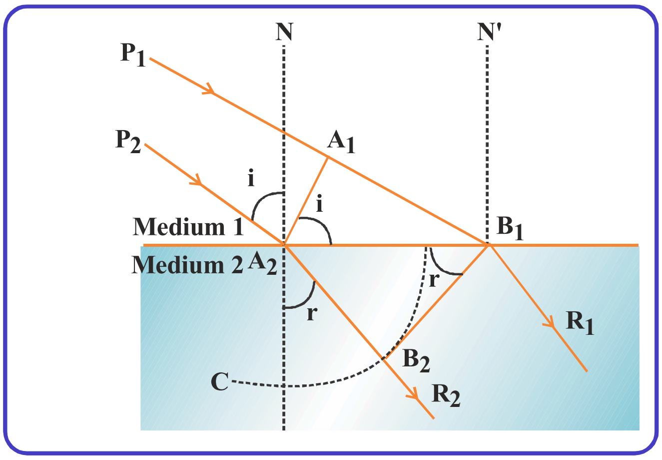
Proof of Law of Refraction

In fig., a plane wavefront A_1A_2 (of monochromatic light) impinging on the boundary of two different medium. The velocity of the wave in the medium 1 is v_1 and in the medium 2 is v_2. The relation between the angles i and r is obtained by geometry.

From fig., we can write

$$\sin i = \frac{A_1B_1}{A_2B_1}, \quad \sin r = \frac{A_2B_2}{A_2B_1}$$

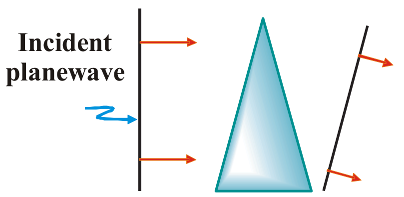
$$\text{Thus, } \frac{\sin i}{\sin r} = \frac{A_1B_1}{A_2B_2} = \frac{v_1\Delta t}{v_2\Delta t}$$



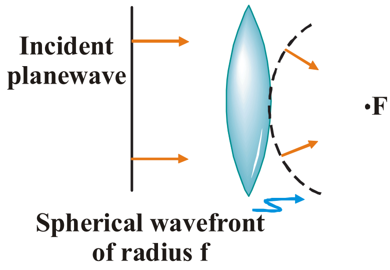
$$\text{or } \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant} \quad \text{or} \quad \boxed{\frac{\sin i}{\sin r} = \mu_{21}}$$

where μ_{21} is called the refractive index of medium 2 with respect to medium 1.

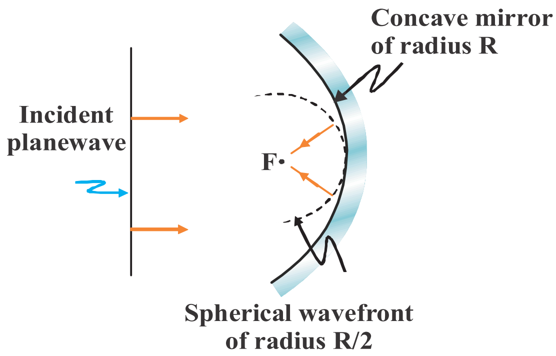
* Refraction of a plane wave by a thin prism,



* Refraction of a plane wave by a convex lens.



* Reflection of a plane wave by a concave mirror.



7.13 DOPPLER EFFECT

* It is the shift in the frequency of light when there is a relative motion between the source and the observer.

* It is given by $\boxed{\frac{\Delta f}{f} = \frac{v_r}{c} \text{ for } \frac{v}{c} << 1}$

where v_r is the radial component of the relative velocity v.

* The effect can be used to measure the speed of an approaching or receding object.

* Doppler effect occurs for sound also it is a characteristics property of waves.

* Doppler's effect in light is symmetrical but it is unsymmetrical in case of sound.

* **Red shift :** When the source moves away from the observer, v is taken positive. So from

$$\frac{\Delta f}{f} = \frac{-v}{c}, \Delta f < 0 \text{ or } \frac{\Delta\lambda}{\lambda} = \frac{v}{c}, \Delta\lambda > 0$$

i.e., the observed frequency is less than the frequency of the source.

* **Blue shift :** An increase in the frequency (or decrease in the wavelength) caused by the motion of the source towards the observer is called blue shift.

$$\frac{\Delta f}{f}=\frac{v}{c},\ \Delta f>0 \quad \text{or} \quad \frac{\Delta\lambda}{\lambda}=\frac{-v}{c},\ \Delta\lambda<0\,.$$

7.14 INTERFERENCE

* When two or more light waves exist simultaneously at the same place, the phenomenon of interference occurs.
* Among the effects that can arise because of wave interference are the multicolours often seen in a thin film, such as those displayed in this fantastic soap bubble.

Basic Terms

1. **Phase :** The argument of sine or cosine in the expression for displacement of a wave is defined as the phase.
   For displacement $y = a \sin(\omega t - kx)$, term $(\omega t - kx)$ phase or instantaneous phase.
2. **Phase difference (ϕ) :** The difference between the phases of two waves at a point is called phase difference i.e.
   if $y_1 = a_1 \sin(\omega t - kx)$ and $y_2 = a_2 \sin(\omega t - kx + \phi)$
   so phase difference $= \phi$
3. **Path difference (Δx) :** The difference in path length's of two waves meeting at a point is called path difference between the waves at that point.
   Also $\boxed{\Delta x = \frac{\lambda}{2\pi}\times\phi}$
4. **Time difference (T.D.) :** Time difference between the waves meeting at a point is

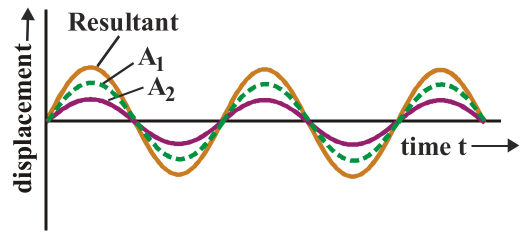
$$\text{T.D.} = \frac{T}{2\pi}\times\phi$$

$$\frac{\text{phase difference}}{\phi}=\frac{\text{path difference}}{\Delta x}=\frac{\text{Time difference}}{T}$$

5. **Coherence :** The phase relationship between two light waves can very from time to time and from point to point in space. The property of definite phase relationship is called coherence.

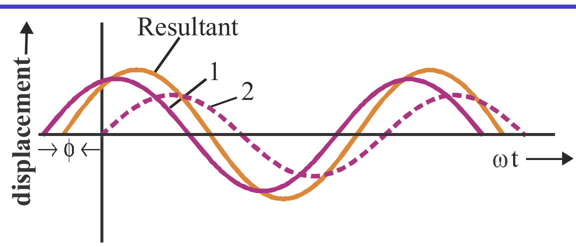
Super Position of Waves

* When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves i.e. $\vec{y} = \vec{y}_1 + \vec{y}_2$



Resultant Amplitude and Intensity

* Let us consider two waves that have the same frequency but have a certain fix (constant) phase difference between them. Their super position shown.



Let the two waves are $y_1 = a_1 \sin(\omega t - kx)$

and $y_2 = a_2 \sin(\omega t - kx + \phi)$,

where a_1, a_2 = Individual amplitudes,

ϕ = Phase difference between the waves at an instant when they are meeting a point.

1. Resultant Amplitude

The resultant wave can be written as

$y = A \sin(\omega t - kx + \theta)$

Resultant amplitude

$$\boxed{A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta}}$$

$$\theta = \tan^{-1}\left(\frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}\right)$$

2. Resultant Intensity

Intensity $\propto$ (Amplitude)^2

$\Rightarrow I_1 = ka_1^2, \Rightarrow I_2 = ka_2^2$ and

$I = kA^2$ (k is a proportionally constant).

Resultant intensity $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$

For two identical source $I_1 = I_2 = I_0$

$$\Rightarrow I_0 + I_0 + 2\sqrt{I_1 I_2}\cos\phi = 4I_0\cos^2(\phi/2)$$

$$\left[1 + \cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\boxed{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (a_1 + a_2)^2}$$

$$\boxed{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (a_1 - a_2)^2}$$

Conditions for Interference Pattern

* In order to form an interference pattern, the incident light must satisfy two conditions:

(i) The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation. For example, if two waves are completely out of phase with $\phi = \pi$, this phase difference must not change with time.

(ii) The light must be monochromatic. This means that the light consists of just one wavelength $\lambda = 2\pi/k$

Light emitted from an incandescent light bulb is incoherent because the light consists of waves of different wavelengths and they do not maintain a constant phase relationship. Thus, no interference pattern is observed.

Methods of Obtaining Coherent Sources

Two coherent sources are produced from a single source of light by two methods (i) By division of wavefront and (ii) By division of amplitude.

(i) Division of Wavefront

The wave front emitted by a narrow source is divided in two parts by reflection, refraction or diffraction. The coherent sources so obtained are imaginary. Example : Fresnel's biprism, Llyod's mirror, Young's double slit etc.

(ii) Division of Amplitude

In this arrangement light wave is partly reflected (50%) and partly transmitted (50%) to produced two light rays. The amplitude of wave emitted by an extended source of light is divided in two parts by partial reflection are partial refraction. The coherent sources obtained are real and are obtained in Newton's rings, Michelson's interferometer, colours in thin films.

Constructive and Destructive Interference

* When two waves of same frequency and constant phase difference are superposed at a certain point then the resultant intensity lie between I_{max} and I_{min}.
* When the resultant intensity is greater than $I_1 + I_2$ than the interference is said to be constructive and when the resultant intensity is less than $(I_1 + I_2)$, then the interference is said to be destructive.
* The conditions for maximum interference effects are as follows :

(i) Constructive Interference

$\phi = 0, 2\pi, 4\pi, ... = 2n\pi$

Path difference $= \frac{\phi}{2\pi}\lambda = 0, \lambda, 2\lambda, ... = n\lambda$,

Amplitude $= a_{max} = a_1 + a_2 = 2a_0$,

Intensity $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 4I_0$

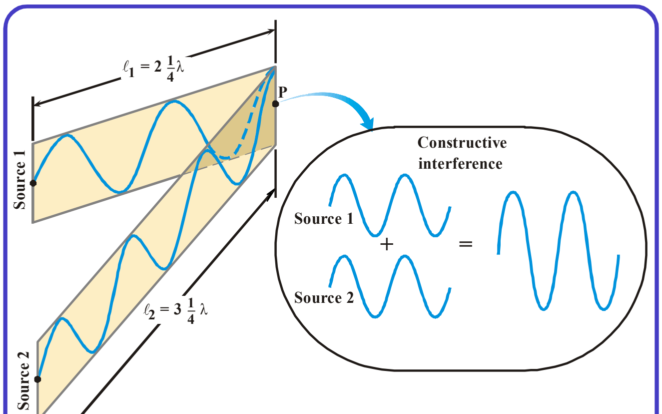


Figure : The waves emitted by source 1 and source 2 start out in phase and arrive at point P in phase, leading to constructive interference at that point.

(ii) Destructive Interference

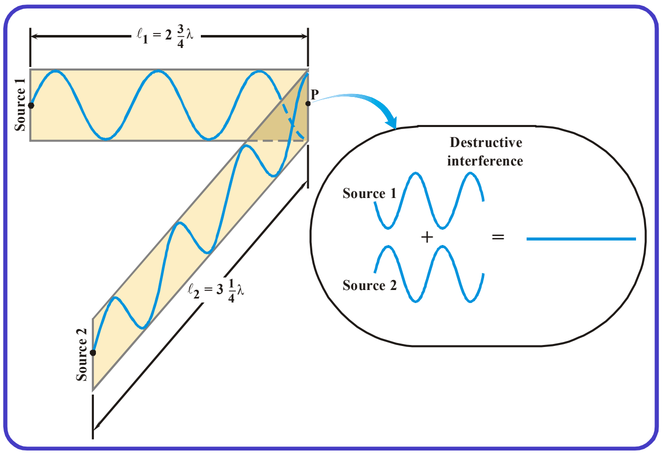
$$\phi = \pi, 3\pi, 5\pi, ... = (2n-1)\,\pi$$

Path difference

$$= \frac{\phi}{2\pi}\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ... = (2n-1)\frac{\lambda}{2},$$

Amplitude $a_{min} = a_1 - a_2 = 0 \quad [\text{if } a_1 = a_2]$

Intensity $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0 \quad [\text{If } \; I_1 = I_2]$



The intensity distribution for the interference of two waves of same frequency are shown in fig. (when amplitudes are equal) and in figure (when amplitudes are not equal)

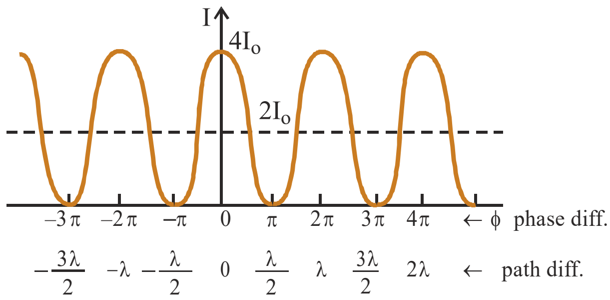


Fig. : Intensity of resultant wave (interference pattern/ when the waves have equal intensities

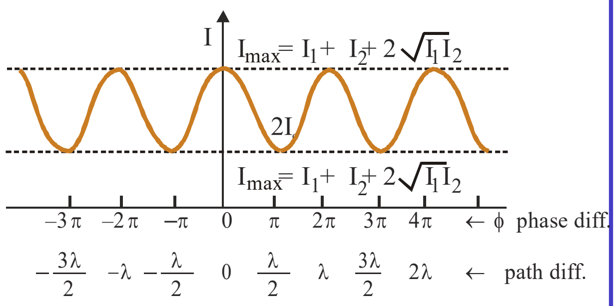


Fig. : Intensity of resultant wave (interference pattern) when the intensity of the two waves are not equal.

Note that average intensity is

$$I = \frac{I_{max} + I_{min}}{2} = I_1 + I_2$$

* Energy is conserved in interference. This indicated that energy is redistributed from destructive interference region to the constructive interference region .

* **Condition for observing sustained interference with good contrast :**

1. The initial phase difference between the interfering waves must remain constant otherwise the interference will not be sustained.
2. The frequencies and wavelength of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not be sustained.
3. The light must be monochromatic. This eliminates overlapping of patterns as each wavelength corresponds to one interference pattern.

4. The amplitudes of the interfering waves must be equal. This improves contrast with $I_{max} = 4I_0$ and $I_{min} = 0$.

7.15 YOUNG'S DOUBLE SLIT EXPERIMENT

* Figure shows one arrangement of Young's experiment, in which light of a single wavelength (monochromatic light) passes through a single narrow slit and falls on two closely spaced, narrow slits S_1 and S_2.

* These two slits act as coherent sources of light waves that interfere constructively and destructively at different points on the screen to produce a pattern of alternate bright and dark fringes.

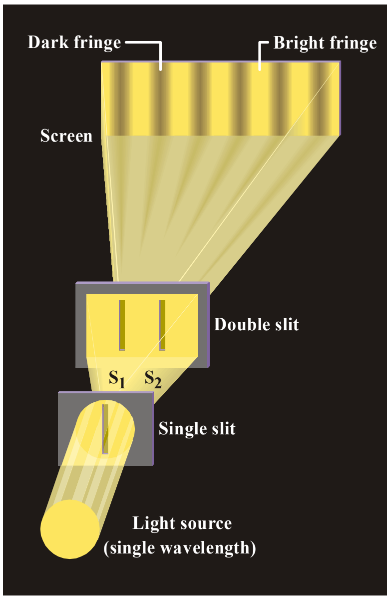


Figure : In Young's double-slit experiment, two slits S_1 and S_2 act as coherent sources of light. Light waves from these slits interfere constructively and destructively on the screen to produce, respectively, the bright and dark fringes. The slit widths and the distance between the slits have been exaggerated for clarity.

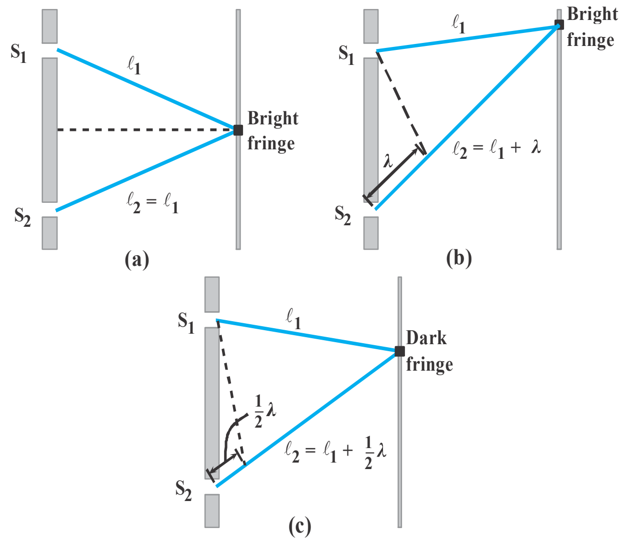


Figure : The waves that originate from slits S_1 and S_2 interfere constructively (parts a and b) or destructively (part c) on the screen, depending on the difference in distances between the slits and the screen. Note that the slit widths and the distance between the slits have been exaggerated for clarity.

Mathematical Relations

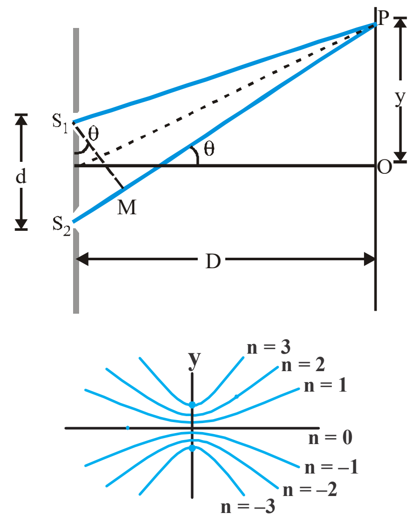


Figure : Locus of points for which $S_1P - S_2P$ is equal to zero, $\pm\lambda, \pm 2\lambda, \pm 3\lambda$.

Path difference $= S_1P - S_2P \approx S_2M$
$= \Delta x = d \sin\theta$

i.e., $\sin\theta = (\Delta x / d)$. However, for small θ,

$$\sin\theta = \tan\theta = \theta = (y / D)$$

$$\text{So, } \frac{y}{D} = \frac{\Delta x}{d} \quad \text{i.e.,} \quad y = \frac{D}{d}(\Delta x) \quad (D >> d)$$

This eq. gives the position of a point on the screen in terms of path difference Δx.
So if the point P is nth bright fringe, $\Delta x = n\lambda$ and

$$\boxed{(y_n)_{Bright} = \frac{D}{d}(n\lambda)} \quad \text{with } n = 0, 1, 2, \text{etc.}$$

Similarly, if the point P is nth minima,

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

$$\boxed{(y_n)_{Dark} = \frac{D}{d}(2n-1)\frac{\lambda}{2}}$$

with $n = 1, 2, \ldots\ldots$ etc.

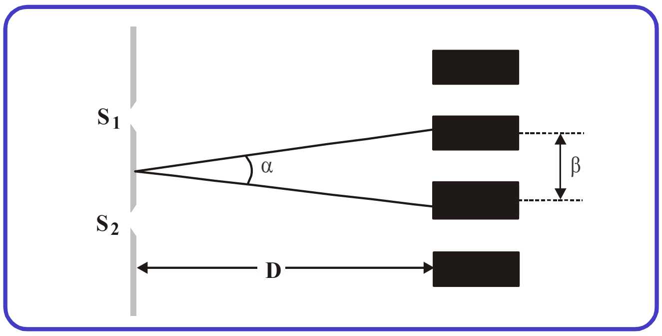
Fringe-width

Fringe-width β is defined as the distance between two consecutive maxima (or minima) on the screen, i.e.,

$$\beta = \Delta y \text{ for } \Delta x = \lambda \quad \text{So,} \quad \boxed{\beta = \frac{D}{d}(\lambda)}$$

* **Angular width of the fringe**
From fig., we notice that the angular width α, is related to the linear fringe width β by $\alpha D = \beta$

$$\text{or} \quad \alpha = \frac{\beta}{D} \quad \text{or} \quad \alpha = \frac{\lambda}{d}$$



Characteristics related to YDSE

1. Fringe width is directly proportional to the wavelength of light used, i.e., $\beta \propto \lambda$. So, fringes with red light are wider than those for blue light.
2. Fringe width is inversely proportional to the separation between the slits, i.e., $\beta \propto (1/d)$. Thus, with increase in separation between the sources, fringe width decreases.
3. With increase in distance between screen and plane of slits, fringe width β increases linearly with D. However, with increase in D, intensity of light sources and hence of interfering waves is adversely affected.
4. **Location of fringe :**
Position of n^{th} bright fringe from central maxima

$$x_n = \frac{n\lambda D}{d} = n\beta, \quad n = 0, 1, 2\ldots\ldots$$

Position of n^{th} dark fringe from central maxima

$$x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}, \quad n = 1, 2, 3\ldots\ldots$$

5. **Separation (Δx) between fringes :**
(i) Between n^{th} bright and m^{th} bright fringes
$(n > m) \quad \Delta x = (n-m)\beta$
(ii) Between n^{th} bright and m^{th} dark fringes
(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$
(b) If $n < m$ then

$$\Delta x = \left(m - n - \frac{1}{2}\right)\beta$$

6. If the interference experiment is performed in a medium of refractive index μ (say water) instead of air, the wavelength of light will change from λ to (λ/μ) and so $\beta' = \frac{D}{d}\left[\frac{\lambda}{\mu}\right] = \frac{\beta}{\mu}$
i.e., fringe width reduces the becomes $(1/\mu)$ times of its value in air.
7. If the interference experiment is repeated with bichromatic light, the fringes of two wavelengths will be coincident for the first time when :

$$y = n(\beta)_{Longer} = (n+1)(\beta)_{Shorter}$$

$$\text{i.e.,} \quad n\lambda_L = (n+1)\lambda_S$$

8. If white light is used in place of monochromatic light in Young's double slit experiment, then the central fringe is white and some coloured fringes around the central white fringe are formed. Because, $\lambda_{red} > \lambda_{violet}$ we expect that the fringe width for red should be greater than that for violet or for blue.

$$\beta_{red} > \beta_{violet}, \quad \beta_{red} > \beta_{blue}$$

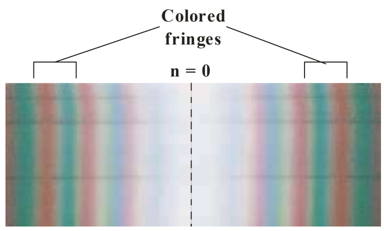


Figure : This photograph shows the results observed on the screen in one version of Young's experiment in which white light (a mixture of all colours) is used.

At the central position O, the $S_2P = S_1P$ and all colours reach point O in the same phase. Their superposition, therefore gives white fringe at the centre. Since $\beta_{red} > \beta_{violet}$, etc., the bright fringe of violet (or blue) colour forms first and that of the red forms later. Thus colours separate and few coloured fringes are obtained.

It may be noted that, the inner edge of the dark fringe is red, while the outer edge is violet (or blue). Similarly, the inner edge of the bright fringe is violet (or blue) and the outer edge is red.

9. If a filter allowing only λ_{red} is placed in front of slit S_1 and a filter allowing only λ_{blue} is placed in front of slit S_2, then there does not occur any interference. (For observing interference stationary in time, the two waves must have the same wavelength).

10. **Fringe Visibility (V) :** With the help of visibility, knowledge about coherence, fringe contrast an interference pattern is obtained.

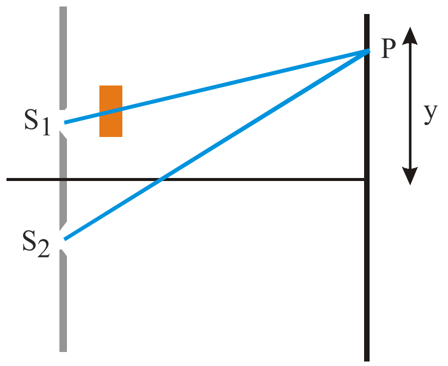
$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 2\frac{\sqrt{I_1 I_2}}{(I_1 + I_2)}$$

If $I_{min} = 0$, $V = 1$ (maximum)

i.e., fringe visibility will be best.

11. **Shifting of fringes :** Suppose a glass slab of thickness t and refractive index μ is inserted onto the path of the ray eminating from source S_1 then the whole fringe pattern shifts towards by a distance $\frac{(\mu-1)\,tD}{d}$.

This can be shown as under,



Geometric path difference between S_2P and S_1P is, $\Delta x_1 = S_2P - S_1P = \frac{yd}{D}$

Path difference produced by the glass slab,

$\Delta x_2 = (\mu - 1)\,t$

Note : Due to the glass slab path of ray 1 gets increased by Δx_2. Therefore, net path difference between the two rays is,

$$\Delta x = \Delta x_1 - \Delta x_2 = \frac{yd}{D} - (\mu-1)\,t$$

For n^{th} maxima on upper side,

$$\frac{yd}{D} - (\mu-1)\,t = n\lambda$$

$$\therefore \quad y = \frac{n\lambda D}{d} + \frac{(\mu-1)\,tD}{d}$$

Earlier it was $\frac{n\lambda D}{d}$ $\therefore$ $\boxed{\text{Shift} = \frac{(\mu-1)\,tD}{d}}$

Following three points are important with regard to Equation–

(a) Shift is independent of n, (the order of the fringe), i.e., shift of zero order maximum = shift of 7^{th} order maximum shift of 5^{th} order maximum = shift of 9^{th} order maximum and so on.

(b) Shift is independent of λ, i.e., if white light is used then, shift of red colour fringe = shift of violet colour fringe.

(c) Number of fringes shifted

$$= \frac{\text{shift}}{\text{fringe width}} = \frac{(\mu-1)tD/d}{\lambda D/d} = \frac{(\mu-1)\,t}{\lambda}$$

These numbers are inversely proportional to λ. This is because shift is same for all colours but fringe width of the colour having smaller value of λ is small, so more number of fringes will shift of this colour.

12. When waves from two coherent sources S_1 are S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2 .

7.16 INTERFERENCE IN THIN FILMS

You have probably notices the coloured bands in a soap bubble or in the film on the surface of oily water. These bands are due to the interference of light reflected from the top and bottom surfaces of the film.
The different colours arise because of variations in the thickness of the film, causing interference for different wavelengths at different points.

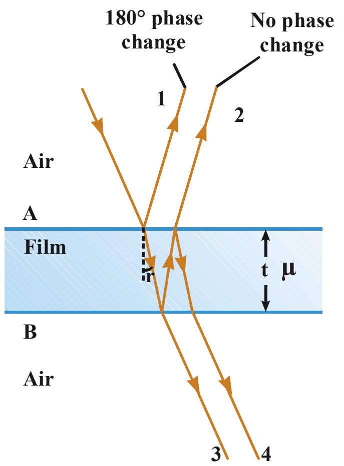


Figure : Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film. Rays 3 and 4 lead to interference effects for light transmitted through the film.

For light of wavelength λ, for reflected light, the constructive interference takes place when

$$2\mu t \cos r = (2n-1)\,\lambda/2$$

and destructive interference takes place when

$$2\mu t \cos r = n\lambda$$

For interference due to transmitted light the conditions of constructive and destructive interference are reversed, that is

$$2\mu t \cos r = n\lambda \qquad \text{(constructive)}$$

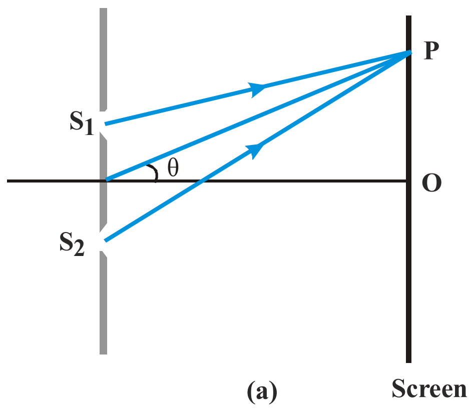
$$2\mu t \cos r = (2n-1)\,\lambda/2 \qquad \text{(destructive)}$$

NOTE

* If thickness of the film tends to zero, then due to π phase difference between the ray reflected from the upper surface and the ray reflected from the lower surface, the film appears black (dark) in reflected light. In transmitted light no such effect will occur and a thin film appears bright.

Golden Tips

* If the entire arrangement of Young's double slit experiment is immersed in water then fringe width decreases.
* If the slits are vertical [as in figure (a)] path difference is, $\Delta x = d\sin\theta$

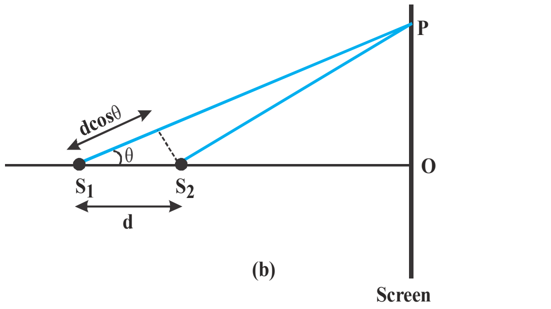


(a)

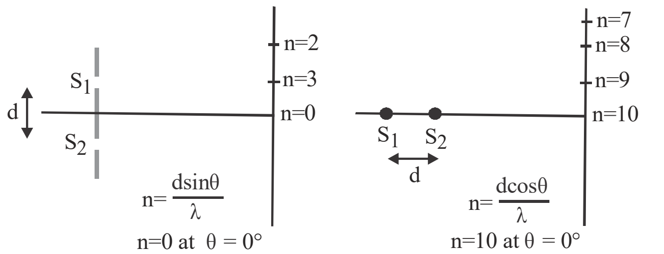
This path difference increases as θ increases. Hence, order of fringes

$\left(d\sin\theta = n\lambda \text{ or } n = \dfrac{d\sin\theta}{\lambda}\right)$ increases as we move away from point O on the screen.

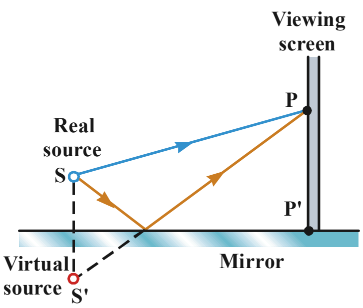
Opposite is the case when the slits are horizontal [as in figure (b)].
Here path difference is $\Delta x = d\cos\theta$



This path difference as θ increases. Hence, order of image $\left(n=\frac{d\cos\theta}{\lambda}\right)$ decreases as we move away from point O. See the figure.

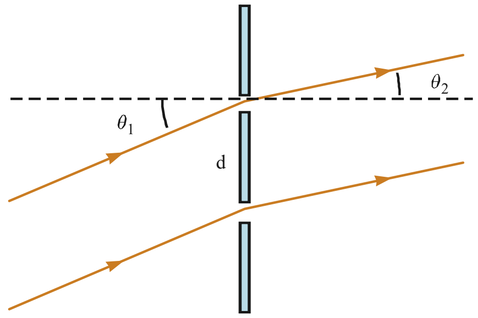


* **Lloyd's mirror :** An interference pattern is produced at point P on the screen as a result of the combination of the direct ray and the reflected ray . The reflected ray undergoes a phase change of 180°.



EXAMPLE 34

Coherent light rays of wavelength λ strike a pair of slits separated by distance d at an angle θ_1 as shown in Figure. Assume an interference maximum is formed at an angle θ_2 a great distance from the slits. Show that $d(\sin\theta_2 - \sin\theta_1) = m\lambda$, where m is an integer.



SOLUTION:

The path difference between rays 1 and 2 is:

$$\delta = d\sin\theta_1 - d\sin\theta_2.$$

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$$d(\sin\theta_1 - \sin\theta_2) = m\lambda$$

EXAMPLE 35

Maximum intensity in YDSE is I_0. Find the intensity at a point on the screen where (a) the phase difference between the two interfering beams is $\pi/3$. (b) the path difference between them is $\lambda/4$.

SOLUTION:

(a) From eq, $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

Here I_{max} is I_0 (i.e., intensity due to independent source is $I_0/4$). Therefore, at

$$\phi = \frac{\pi}{3} \text{ or } \frac{\phi}{2} = \frac{\pi}{6}, \quad I = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_0$$

(b) Phase difference corresponding to the given path difference $\Delta x = \lambda/4$ is,

$$\phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2} \quad \left(\phi = \frac{2\pi}{\lambda}\Delta x\right)$$

or $\frac{\phi}{2} = \frac{\pi}{4}$

$$\therefore \; I = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

EXAMPLE 36

White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance D >> b from the slit. At a point on the screen directly in front of one of the slits find the missing wavelength.

SOLUTION:

According to theory of interference position y of a point on the screen is given by $y = \frac{D}{d}(\Delta x)$

and as for missing wavelengths intensity will be min. (= 0),

$$\Delta x = (2n-1)\frac{\lambda}{2} \quad \text{So,} \quad y = D\frac{(2n-1)\lambda}{2d}$$

However, here d = b and y = (b/2). So,

$$\lambda = \frac{b^2}{(2n-1)D} \quad \text{with } n = 1, 2, 3, \ldots\ldots\ldots$$

i.e. wavelengths (b^2/D), $(b^2/3D)$, $(b^2/5D)$, etc., will be absent (or missing) at point P.

EXAMPLE 37

A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

SOLUTION:

Here, $\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9}$ m

$\lambda_2 = 520 \text{ nm} = 520 \times 10^{-9}$ m

Suppose, d = distance between two slits

(a) For third bright fringe, n = 3

$$x = n\lambda_1\frac{D}{d} = 3 \times 650 \times 10^{-7} \times \frac{120}{0.2}$$

$$= 0.117 \text{ cm} = 1.17 \text{ mm}$$

(b) Let n fringes of wavelength 650nm coincide with (n + 1) fringes of wavelength 520nm.

$$x = n\lambda_1\frac{D}{d} = \frac{(n+1)D}{d} \times \lambda_2$$

or $\quad x = n \times 650 = (n+1) \times 520$

or $\quad \frac{n+1}{n} = \frac{650}{520} = \frac{5}{4}$

or $\quad 1 + \frac{1}{n} = \frac{5}{4} \Rightarrow \frac{1}{n} = \frac{5}{4} - 1 = \frac{1}{4} \quad \text{or} \quad n = 4$

$$x = n\lambda_1\frac{D}{d} = 4 \times 650 = 10^{-7} \times \frac{120}{0.2} = 1.56 \text{ mm}$$

EXAMPLE 38

In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1m away. The wavelength of light used is 600nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

SOLUTION:

Angular fringe separation,

$$\theta = \frac{\lambda}{d} \quad \text{or} \quad d = \frac{\lambda}{\theta}$$

In water, $d = \frac{\lambda'}{\theta'} \quad \therefore \quad \frac{\lambda}{\theta} = \frac{\lambda'}{\theta'}$

or $\quad \frac{\theta'}{\theta} = \frac{\lambda'}{\lambda} = \frac{1}{\mu} = \frac{3}{4}$

or $\quad \theta' = \frac{3}{4}\theta = \frac{3}{4} \times 0.2° = 0.15°$

EXAMPLE 39

A thin film of oil (μ = 1.25) is located on a smooth wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no blue light at 512nm. How thick is the oil film?

SOLUTION:

Since $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require

$$2t = \frac{n\lambda_{cons}}{\mu}$$

and for destructive interference,

$$2t = \frac{[n + (1/2)]\,\lambda_{des}}{\mu}$$

$$\frac{\lambda_{cons}}{\lambda_{dest}} = 1 + \frac{1}{2n} = \frac{640 \text{ nm}}{512 \text{ nm}} = 1.25 \text{ \& } n = 2.$$

$$\text{Therefore, } t = \frac{2\,(640 \text{ nm})}{2\,(1.25)} = 512 \text{ nm}$$

Checkup 8

Q.1 Explain why two flashlights held close together do not produce an interference pattern on a distant screen.

Q.2 If Young's double-slit experiment were performed under water, how would the observed interference pattern be affected?

Q.3 In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?

Q.4 A simple way to observe an interference pattern is to look at a distant light source through a stretched handkerchief or an opened umbrella. Explain how this works.

Q.5 A certain oil film on water appears brightest at the outer regions, where it is thinnest. From this information, what can you say about the index of refraction of oil relative to that of water?

Q.6 As a soap bubble evaporates, it appears black just before it breaks. Explain this phenomenon in terms of the phase changes that occur on reflection from the two surfaces of the soap film.

Q.7 If we are to observe interference in a thin film, why must the film not be very thick (with thickness only on the order of a few wavelengths)?

Q.8 A lens with outer radius of curvature R and index of refraction n rests on a flat glass plate. The combination is illuminated with white light from above and observed from above. Is there a dark spot or a light spot at the center of the lens? What does it mean if the observed rings are noncircular?

Q.9 Why is it so much easier to perform interference experiments with a laser than with an ordinary light source?

Q.10 In double-slit experiment using light of wavelength 600nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Q.11 If the distance between the first maxima and fifth minima of a double slit pattern is 7mm and the slits are separated by 0.15 mm with the screen 50 cm. from the slits, then find the wavelength of the light used.

Q.12 A possible means for making an airplane invisible to radar is to coat the plane with an antireflective polymer. If radar waves have a wavelength of 3.00 cm and the index of refraction of the polymer is $\mu = 1.50$, how thick would you make the coating?

7.17 DIFFRACTION

* It is the spreading of waves round the corners of an obstacle, of the order of wave length.
* The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.

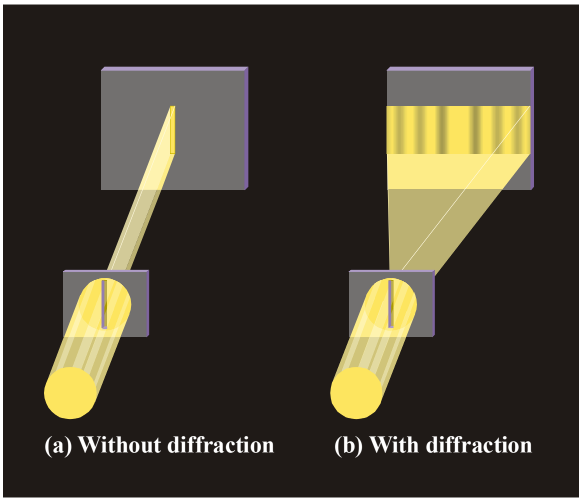


Figure : (a) If light were to pass through a very narrow slit without being diffracted, only the region on the screen directly opposite the slit would be illuminated. (b) Diffraction causes the light to bend around the edges of the slit into regions it would not otherwise reach, forming a pattern of alternating bright and dark fringes on the screen. The slit width has been exaggerated for clarity.

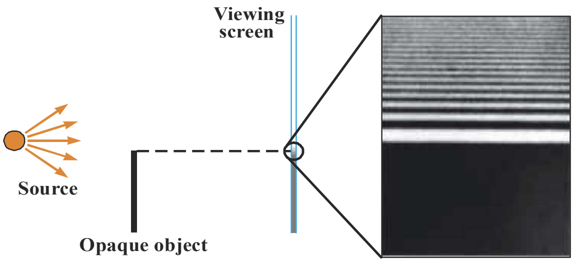


Figure : Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.

* **Necessary Conditions of Diffraction of Waves :** The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} \approx 1$$

Note : Greater the wave length of wave higher will be its degree of diffraction. This is the reason that diffraction of sound & radio waves is easily observed but for diffraction of light, additional arrangement have to be arrange.

$$\lambda_{sound} > \lambda_{light}$$

Wave length of sound is nearly equal to size of obstacle. If size of obstacle is a & wavelength of light is λ then,

S.No.	a v/s λ	Diffraction
(1)	$a << \lambda$	Not possible
(2)	$a >> \lambda$	Not possible
(3)	$a \simeq \lambda$	Possible

Interpretation of Diffraction

* As a result of diffraction, maxima & minima of light intensities are found which has unequal intensities.
* Diffraction is the result of superposing of waves from infinite number of coherent sources on the same wavefront after the wavefront has been distorted by the obstacle.

Example of Diffraction :

* When an intense source of light is views with the partially opened eye, colours are observed in the light.
* Sound produced in one room can be heard in the nearby room.
* Appearance of a shining circle around the section of sun just before sun rise.
* Coloured spectrum is observed if a light source at far distance is seen through a thin cloth.

Diffraction of Light & Sound

* Sound travels in form of waves, that's why it is also diffracted.
* Generally diffraction of sound waves is easily observed rather than light because wavelength of sound waves is the order of obstacle, but wavelength of light is very small in comparison to obstacle.
  (a) Ordinary audible sound has wavelength of the order of 1m & size of ordinary obstacle has same order that's why diffraction is easily observed.
  (b) Ordinary light has wavelength of 10^{-7} m & ordinary obstacle has greater size in comparison to its wavelength that's why diffraction pattern is not observed.
* Generally diffraction of ultrasonic waves are not observed because its wavelength has order of 1cm.

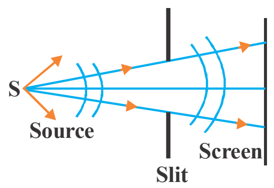
Rectilinear Motion of Light

* Rectilinear motion of light can be explained by diffraction of light.
* If size of obstacle is the order of wavelength of light, then diffraction of light takes place & its rectilinear motion of light is not possible.
* If size of obstacle is much greater than wave length of light, then rectilinear motion of light is observed.

Two Type of Diffraction

Fresnel Diffraction

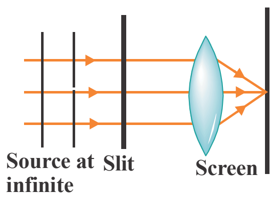
If the source or the screen or both are at finite distance from the diffracting element, it is called Fresenel diffraction.



Examples : Diffraction of a straight edge, small opaque disc and narrow wire.

Fraunhoffer Diffraction

In this type, the source and the screen are effectively at infinite distance from the diffracting element.



Examples : Diffraction of single/double slit/ diffraction grating.

FRAUNHOFFER DIFFRACTION (SINGLE SLIT)

* Let a source of monochromatic light be incident on a slit of finite width a, as shown in Figure.

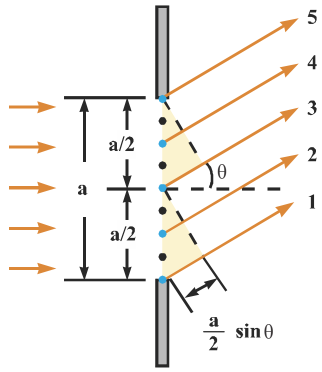


Figure : Paths of light rays that encounter a narrow slit of width a and diffract toward a screen in the direction described by angle θ. Each portion of the slit acts as a point source of light waves. The path difference between rays 1 and 3, rays 2 and 4, or rays 3 and 5 is (a/2) sin θ. (Drawing not to scale.)

* In diffraction of Fraunhofer type, all rays passing through the slit are approximately parallel.
* In addition, each portion of the slit will act as a source of light waves according to Huygens's principle.
* For simplicity we divide the slit into two halves. At the first minimum, each ray from the upper half will be exactly 180° out of phase with a corresponding ray form the lower half.
* For example, suppose there are 100 point sources, with the first 50 in the lower half, and 51 to 100 in the upper half. Source 1 and source 51 are separated by a distance a/2 and are out of phase with a path difference $\delta = \lambda/2$.
* Similar observation applies to source 2 and source 52, as well as any pair that are a distance a/2 apart. Thus, the condition for the first minimum is $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ or $\boxed{\sin\theta = \frac{\lambda}{a}}$
* Applying the same reasoning to the wavefronts from four equally spaced points a distance a/4 apart, the path difference would be $\delta = \frac{a\sin\theta}{4}$, and the condition for destructive interference is $\sin\theta = \frac{2\lambda}{a}$

* The argument can be generalized to show that destructive interference will occur when

$\boxed{a \sin\theta = n\lambda}$,

$n = \pm 1, \pm 2, \pm 3, ...$(destructive interference)

* For n^{th} secondary maximum,

$a \sin\theta_n = (2n+1)\lambda/2$

or $\boxed{\sin\theta_n \approx \theta_n = (2n+1)\dfrac{\lambda}{2a}}$

[Condition for n^{th} secondary maximum]

* The diffraction pattern due to a single slit consists of a central maximum flanked by alternate minima and secondary maxima.

* If the centre of the nth minimum is at a distance y_n from the centre of the central maximum, then

$$\sin\theta_n = \tan\theta_n = \frac{y_n}{D} = \frac{\pm n\lambda}{a} \quad \text{[For small } \theta_n)$$

or $\boxed{y_n = \dfrac{\pm n\lambda D}{a}}$ [For n^{th} minimum]

Similarly, $\boxed{y_n = \dfrac{\pm(2n+1)\lambda D}{a}}$

[For n^{th} maximum]

* Figure illustrates the intensity distribution for a single-slit diffraction.
Note that θ= 0 is a maximum.

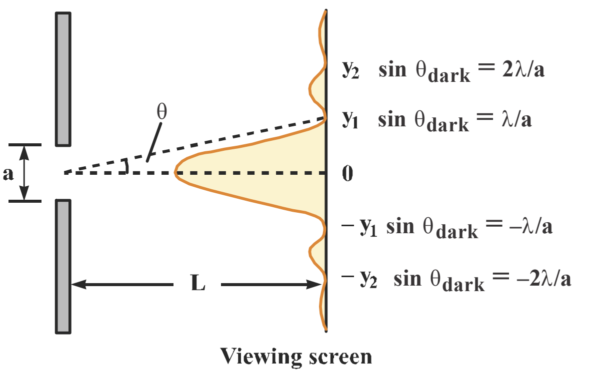


Figure : Intensity distribution for a Fraunhofer diffraction pattern from a single slit of width a. The positions of two minima on each side of the central maximum are labeled. (Drawing not to scale.)

Intensity Distribution

In case of diffraction at single slit theory shows that intensity at a point on the screen is given by:

$$I_{(\theta)} = I_m\left[\frac{\sin\alpha}{\alpha}\right]^2$$

with $\alpha = \dfrac{\phi}{2} = \dfrac{\pi}{\lambda}(a\sin\theta)$ ...... (1)

nth secondary maximum is approximately half-way between n^{th} and $(n+1)^{th}$ minima, for it

$$a\sin\theta = \frac{n\lambda + (n+1)\lambda}{2} = \left(n+\frac{1}{2}\right)\lambda$$

$$\alpha = \left(n+\frac{1}{2}\right)\pi$$

i.e, α = 1.5π, 2.5π, 3.5π, ...

$$I = I_m\left[\frac{\sin\left(n+\frac{1}{2}\right)\pi}{\left(n+\frac{1}{2}\right)\pi}\right]^2 = \frac{I_m}{\left[\left(n+\frac{1}{2}\right)\pi\right]^2}$$

$$I_1 : I_2 : I_3 :: \frac{1}{(1.5\pi)^2} : \frac{1}{(2.5\pi)^2} : \frac{1}{(3.5\pi)^2}$$

From above it is evident that the intensity in secondary maxima decreases with increase in order and in first secondary maxima is $\dfrac{1}{(1.5\pi)^2}$, i.e., 4.5% of central maximum.

Angular Width

As central maximum extends between first minima on either side, for small θ, the angular width of central maximum will be:

$\theta_0 = 2\theta_1 = (2\lambda/a)$

* The angular width of subsidiary maximum $\theta_s = (\theta_n - \theta_{n-1}) = (\lambda/a)$ is half that of central maximum $[\theta_0 = 2(\lambda/a)]$.

* The width of the central maximum

$\boxed{W = \dfrac{2\lambda D}{a}}$

Golden Tips

* With increase in distance between slit and screen angular width of maxima (either central or subsidiary [$\propto (\lambda/a)$]) does not change while linear width ($y = \theta D$) increases.
* With decrease in width of slit a, width of diffraction maxima [$\propto(\lambda/a)$] increases and subsidiary maxima shifts away from the central maxima broading the pattern.
* With decrease in wavelength λ, i.e., if red light is replaced by blue light, width of diffraction maxima ($\propto \lambda$) decreases and subsidiary minima shifts towards central maxima [without change in its position ($\theta = 0°$)], i.e., diffraction bands become narrow and crowded.
* If the experiment is performed in water instead of air, due to change in wavelength from λ to (λ/μ) he width of diffraction maxima will decrease and will become ($1/\mu$) times of its value in air and hence diffraction bands become narrow and crowded.
* The intensity at the first maximum is only $\frac{1}{22}$ of the central maximum.
* The diffraction pattern formed by an opaque disk consists of a small bright spot in the center of the dark shadow, circular bright fringes within the shadow, and concentric bright and dark fringes surrounding the shadow.

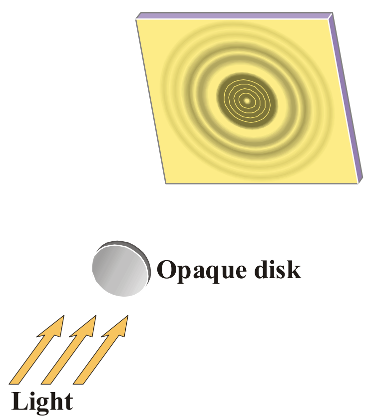


Figure : The diffraction pattern formed by an opaque disk consists of a small bright spot in the center of the dark shadow, circular bright fringes within the shadow, and concentric bright and dark fringes surrounding the shadow.

EXAMPLE 40

A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

SOLUTION:

Given, $D = 1 m, n = 1$

$x = 2.5mm = 2.5 \times 10^{-3}$ m

$\lambda = 500nm = 500 \times 10^{-9}$ m $= 5 \times 10^{-7}$ m

Using formula, $x = n\dfrac{\lambda D}{d} \Rightarrow d = \dfrac{n\lambda D}{x}$

or $d = \dfrac{1 \times 5 \times 10^{-7} \times 1}{2.5 \times 10^{-3}} = 2 \times 10^{-4}$ m $= 0.2$ mm

EXAMPLE 41

A beam of green light is diffracted by a slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the laser light.

SOLUTION:

The positions of the first-order minima are

$\dfrac{y}{L} \approx \sin\theta = \pm\dfrac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\dfrac{\lambda}{a}\right)L$ and the wavelength is $\lambda = \left(\dfrac{\Delta y}{2}\right)\left(\dfrac{a}{L}\right)$

$$= \left(\frac{4.10 \times 10^{-3} m}{2}\right)\left(\frac{0.550 \times 10^{-3} m}{2.06m}\right) = 547 \text{ nm}$$

RAYLEIGH'S CRITERION

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as Rayleigh's criterion.

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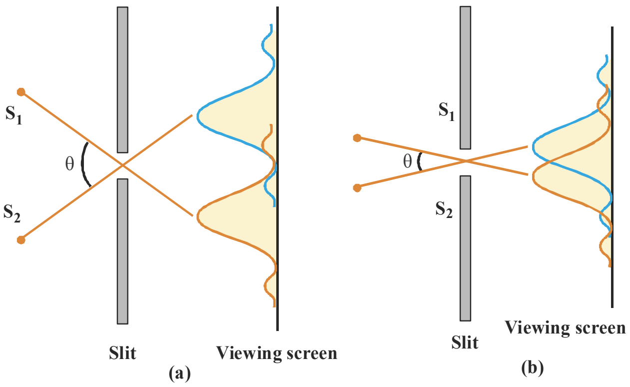


Figure : Two point sources far from a narrow slit each produce a diffraction pattern. (a) The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable. (b) The angle subtended by the sources is so small that their diffraction patterns overlap, and the images are not well resolved. (Note that the angles are greatly exaggerated. The drawing is not to scale.)

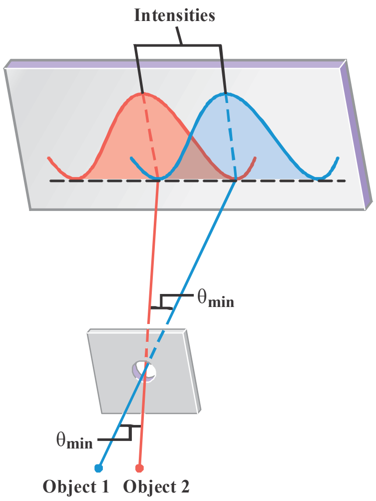


Figure : According to the Rayleigh criterion, two point objects are just resolved when the first dark fringe (zero intensity) of one image falls on the central bright fringe (maximum intensity) of the other image.

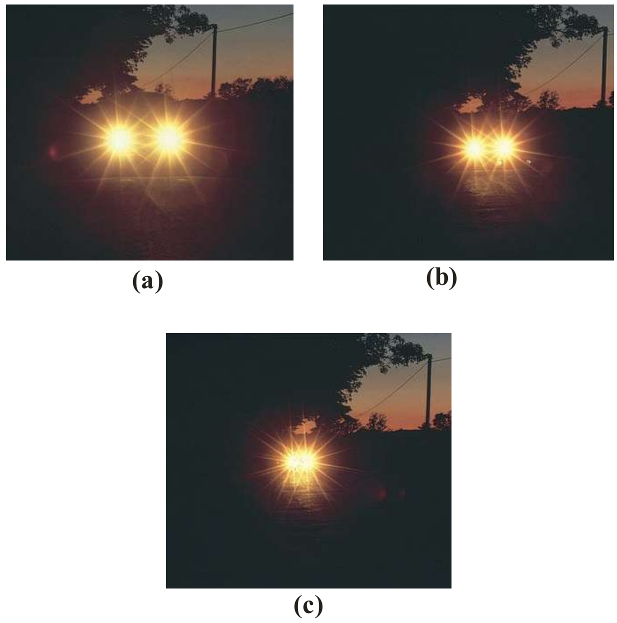


Figure : These automobile headlights were photographed at various distances from the camera, closest in part (a) and farthest in part (c). In part (c), the headlights are so far away that they are barely distinguishable.

EXAMPLE 42

Assuming that the headlights of a car are point sources, estimate the maximum distance from an observer to the car at which the headlights are distinguishable from each other.

SOLUTION:

We apply the equation $\theta_m = \dfrac{1.22\,\lambda}{D}$ for the resolution of a circular aperture, the pupil of your eye. Suppose your dark-adapted eye has pupil diameter D = 5 mm. An average wavelength for visible light is $\lambda = 550$ nm. Suppose the headlights are 2 m apart and the car is a distance L away.

Then $\theta_m = \dfrac{2\text{m}}{L} = 1.22 \times 1.1 \times 10^{-4}$ m, so

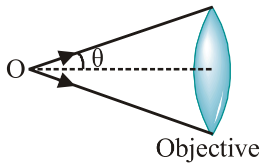
L ~ 10 km. The actual distance is less than this because the variable temperature air between you and the car makes the light refract unpredictably. The headlights twinkle like stars.

RESOLVING POWER OF OPTICAL INSTRUMENTS

* A large number of images are formed as a consequence of light diffraction from a source.
* If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved.
* Resolving power of an optical instrument is its ability to distinguish two neighbouring points.

Microscope

* The resolving power of microscope is its ability to form separate images of two point objects lying close together.
* It is determined by the least distance between the two points which can be distinguished.



* This distance is given by $d = \dfrac{1.22\,\lambda}{2\mu\sin\theta}$, where

λ is the wavelength of light used to illuminate the object and μ is the refractive index of the medium between the object and the objective.
The angle θ is the half angle of the cone of light from the point object.

* Resolving power $= \dfrac{1}{d} = \dfrac{2\mu\sin\theta}{1.22\,\lambda}$

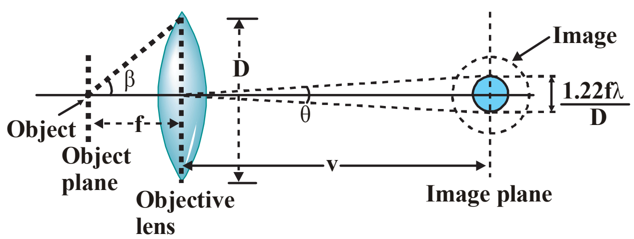
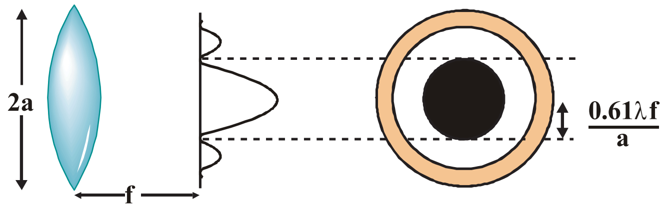


Figure : Real image formed by the objective lens of the microscope.

Telescope

* The resolving power of telescope is the reciprocal of the smallest angular separation between two distant objects whose images are separated in the telescope.
* This is given by $\theta = \dfrac{1.22\lambda}{a}$, where θ is the angle subtended by the point object at the objective, λ is the wavelength of light used and a is the diameter of the telescope objective.
* A telescope with a larger aperture objective gives a high resolving power.
* A parallel beam of light is incident on a convex lens. Because of diffraction effects, the beam gets focused to a spot of radius $\approx 0.61\ \lambda f/a$.



Diffraction Grating

Resolving power $= \lambda/d\lambda = N \times n$
(N is total number of lines & n is the order of spectrum)

Eye

The limit of resolution of human eye is 1' of arc (One minute of arc)

* A good telescope should have high magnifying power, high resolving power and large light gathering power.

Resolving power $= \dfrac{d}{1.22\lambda}$,

Light gathering power $\propto \dfrac{\pi d^2}{4}$

Brightness $\propto d^2$, where d = diameter of the aperture of the objective and λ = wavelength of the light used.
* Resolving power of electron microscope $\propto \sqrt{\text{Potential difference}}$

The Validity of Ray Optics

* **Fresnel distance (z_F) :** It is defined as the distance of the screen from the slit at which the spreading of light due to diffraction at single slit becomes equal to the width of the slit.

$$D = z_F = \frac{d^2}{\lambda}$$

* For distances much smaller than z_F, the spreading due to diffraction is smaller compared to the size of the beam.
* It becomes comparable when the distance is approximately z_F.
* For distances much greater than z_F, the spreading due to diffraction dominates over that due to ray optics (i.e., the size a of the aperture).
* Ray optics is valid in the limit of wavelength tending to zero.

Checkup 9

Q.1 Holding your hand at arm's length, you can readily block sunlight from reaching your eyes. Why can you not block sound from reaching your ears this way?

Q.2 Describe the change in width of the central maximum of the single-slit diffraction pattern as the width of the slit is made narrower.

Q.3 Answer the following questions:

(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

(d) Two students are separated by a 7m partition wall in a room 10m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.

Q.4 Light of wavelength 587.5 nm illuminates a single slit 0.750 mm in width. (a) At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850mm from the center of the principal maximum? (b) What is the width of the central maximum?

7.18 POLARISATION

* Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e., whether the light waves are longitudinal or transverse.
* The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

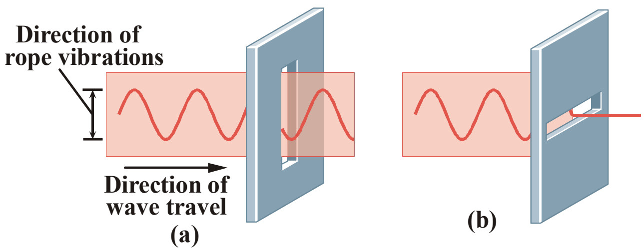


Figure : A transverse wave is linearly polarized when its vibrations always occur along one direction. (a) A linearly polarized wave on a rope can pass through a slit that is parallel to the direction of the rope vibrations, but (b) cannot pass through a slit that is perpendicular to the vibrations.

Unpolarised Light

* An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source.
* Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of $\vec{E}$ are equally probable.
* The resultant electromagnetic wave is a super position of waves produced by the individual atomic sources and it is called unpolarised light.
* In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

Polarised Light

* The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light.

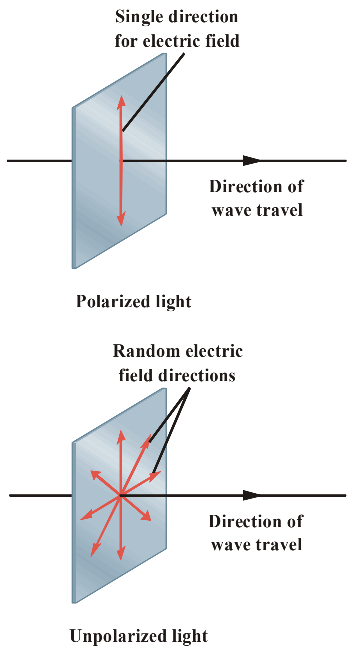


Figure : In polarized light, the electric field of the electromagnetic wave fluctuates along a single direction. Unpolarized light consists of short bursts of electromagnetic waves emitted by many different atoms. The electric field directions of these bursts are perpendicular to the direction of wave travel but are distributed randomly about it.

* In polarised light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions).
* After polarisation the vibrations become asymmetrical about the direction of propagation of light.

Polariser

Tourmaline Crystal

* When light is passed through a tourmaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism

* A nicol prism is an optical device which can be used for the production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid

* A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

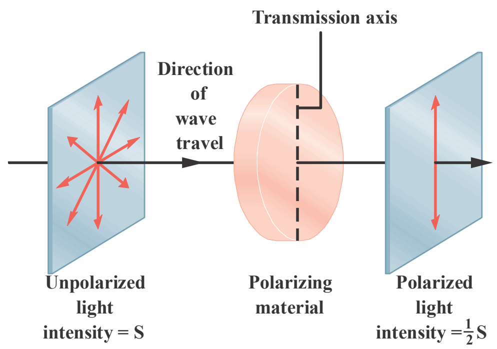
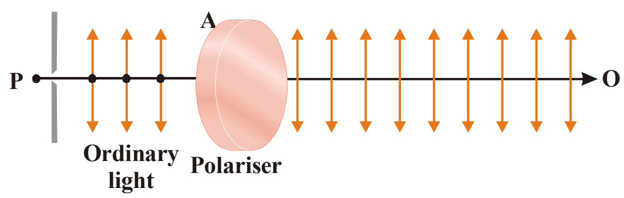


Figure : With the aid of a piece of polarizing material, polarized light may be produced from unpolarized light. The transmission axis of the material is the direction of polarization of the light that passes through the material.

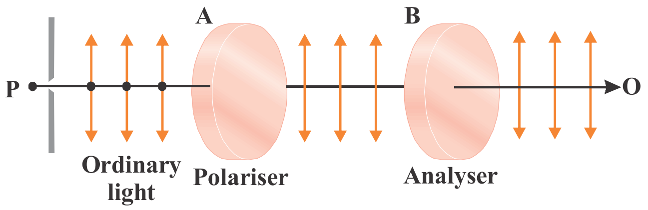
Experimental Demonstration of Polarisation of Light

* Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis).
* First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured.

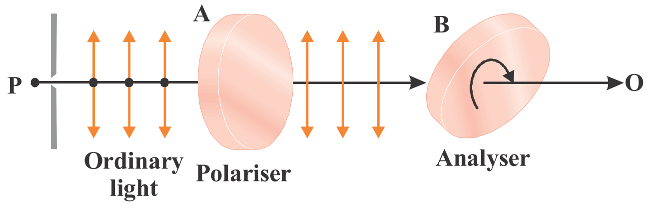


* Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light.
* Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A.

* The beam of light passes through both the crystals and outcoming light appears coloured.



* Now, rotate the crystal B about the axis PO.
* It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.



* If the crystal B is further rotated light reappears and intensity becomes maximum again when their axes are parallel. This occurs after a further rotation of B through 90°.
* This experiment confirms that the light waves are transverse in nature.
* The vibrations in light waves are perpendicular to the direction of propagation of the wave.
* First crystal A polarises the light so it is called polariser.
* Second crystal B, analyses the light whether it is polarised or not, so it is called analyser.

Methods of obtaining plane polarised Light

Polarisation by Reflection

* The simplest method to produce plane polarised light is by reflection.
* This method was discovered by Malus in 1808.
* When a beam of ordinary light is reflected from a surface, the reflected light is partially polarised.
* The degree of polarisation of the polarised light in the reflected beam is greatest when it is incident at an angle called polarising angle or Brewster's angle.

Polarising Angle

* Polarising angle is that angle of incidence at which the reflected light is completely plane polarisation.

Brewster's Law

* When unpolarised light strikes at polarising angle θ_p on an interface separating a rare medium from a denser medium of refractive index μ, such that $\mu = \tan\theta_p$ then the reflected light (light in rare medium) is completely polarised.
* Also reflected and refracted rays are normal to each other.
* The law state that the tangent of the polarising angle of incidence of a transparent medium is equal to its refractive index $\mu = \tan\theta_p$

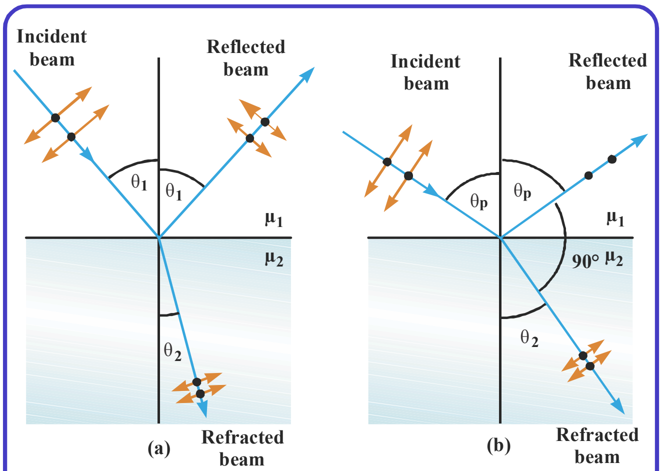


Figure : (a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p, which satisfies the equation $n = \tan\theta_p$. At this incident angle, the reflected and refracted rays are perpendicular to each other.

* In case of polarisation by reflection :

(i) For $i = \theta_p$ refracted light is partially polarised.

(ii) For $i = \theta_p$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_p$ or $i > \theta_p$ both reflected and refracted light become partially polarised.

* According to snell's law,

$$\mu = \frac{\sin\theta_p}{\sin\theta_r} \qquad \text{........ (i)}$$

But according to Brewster's law

$$\mu = \tan\theta_p = \frac{\sin\theta_p}{\cos\theta_p} \quad \text{........ (ii)}$$

From equation (i) and (ii)

$$\frac{\sin\theta_p}{\sin\theta_r} = \frac{\sin\theta_p}{\cos\theta_p} \Rightarrow \sin\theta_r = \cos\theta_p$$

$\because \quad \sin\theta_r = \sin(90° - \theta_r)$

$\Rightarrow \quad \theta_r = 90° - \theta_p \quad \text{or} \quad \theta_p + \theta_r = 90°$

Thus reflected and refracted rays are mutually perpendicular.

By Refraction

* In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle 57°.
* According Brewster law, the reflected light will be plane polarised with vibrations perpendicular to the plane of incidence and the transmitted light will be partially polarised.
* Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.
* When light travels through an amorphous material, such as glass, it travels with a speed that is the same in all directions. That is, glass has a single index of refraction. In certain crystalline materials, however, such as calcite and quartz, the speed of light is not the same in all directions. Such materials are characterized by two indices of refraction. Hence, they are often referred to as **double refracting** or birefringent materials.

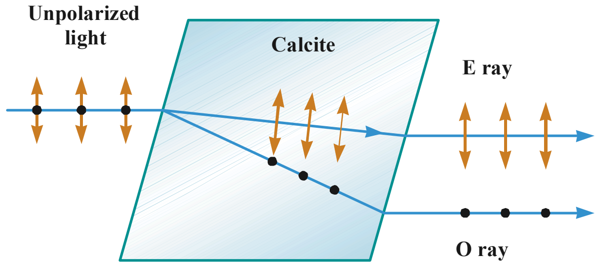


Figure : Unpolarized light incident on a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray. These two rays are polarized in mutually perpendicular directions. (Drawing not to scale.)

By Scattering

* When light is incident on small particles of dust, air molecule etc. (having smaller size as compared to the wavelength of light), it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering.
* Light scattered in a direction at right angles to the incident light is always plane-polarised.

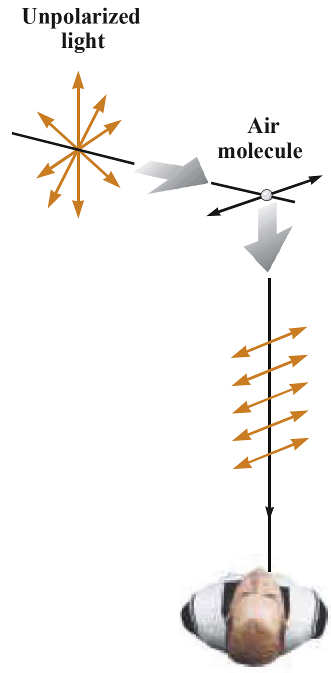


Figure : The scattering of unpolarized sunlight by air molecules. The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.

LAW OF MALUS

* When a completely plane polarised light beam is incident analyser, then intensity of emergent light varies as the square of cosine of the angle between the planes of transmission of the analyser and the polarizer.

$$I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$$

(i) If $\theta = 0°$ then $I = I_0$ maximum value (Parallel arrangement)

(ii) If $\theta = 90°$ then $I = 0$ minimum value (Crossed arrangement)

* If plane polarised light of intensity $I_0(=KA^2)$ is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be A cos θ while perpendicular to it will be A sin θ.

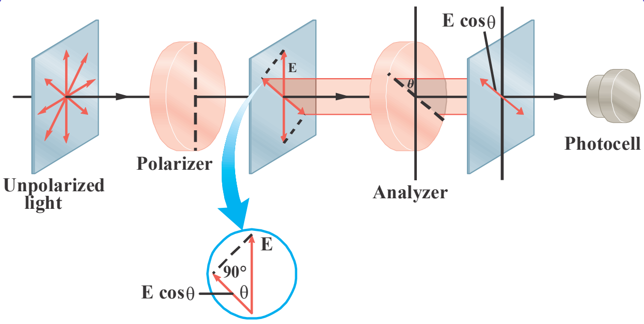


Figure : Two sheets of polarizing material, called the polarizer and the analyzer, may be used to adjust the polarization direction and intensity of the light reaching the photocell. This can be done by changing the angle between the transmission axes of the polarizer and analyzer.

* Polaroid will pass only those vibrations which are parallel to transmission axis i.e. A cos θ

  $\because \quad I_0 \propto A^2$

  So the intensity of emergent light

  $I = K(A\cos\theta)^2 = KA^2\cos^2\theta$
* If an unpolarised light is converted into plane polarised light its intensity becomes half.
* If light of intensity I_1, emerging from one polaroid called polariser is incident on a second polaroid (called analyser) the intensity of light emerging from the second polaroid is $I_2 = I_1\cos^2\theta$

  θ = angle between the transmission axis of the two polaroids.

Applications and Uses of Polariastion

* By determining the polarising angle and using Brewster's Law $\mu = \tan\theta_p$ refractive index of dark transparent substance can be determined.
* In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display (L.C.D.)
* In CD player polarised laser beam acts as needle for producing sound from compact disc.
* It has also been used in recording and reproducing three dimensional pictures.
* Polarised light is used in optical stress analysis known as photoelasticity.
* Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of optical activity.
* When Polaroid sunglasses are uncrossed, the transmitted light is dimmed due to the extra thickness of tinted plastic. However, when they are crossed, the intensity of the transmitted light is reduced to zero because of the effects of polarization.

Figure : When Polaroid sunglasses are uncrossed (left photograph), the transmitted light is dimmed due to the extra thickness of tinted plastic. However, when they are crossed (right photograph), the intensity of the transmitted light is reduced to zero because of the effects of polarization.

EXAMPLE 43

Two polaroids are crossed to each other. When one of them is rotated through 60°, then find the percentage of the incident unpolarised light that will be transmitted by the polaroids.

SOLUTION:

Initially the polaroids are crossed to each other, that is the angle between their polarising directions is 90°. When one is rotated through 60°, then the angle between their polarising directions will become 30°. Let the intensity of the incident unpolarised light $= I_0$

Then the intensity of light emerging from the first polaroid is $I_1 = \frac{1}{2}I_0$

This light is plane polarised and passes through the second polaroid. The intensity of light emerging from the second polaroid is

$I_2 = I_1\cos^2\theta$

θ = the angle between the polarising directions of the two polaroids.

$$I_1 = \frac{1}{2} I_0 \text{ and } \theta = 30°$$

$$I_2 = I_1 \cos^2 30° = \frac{1}{2} I_0 \cos^2 30° \Rightarrow \frac{I_2}{I_0} = \frac{3}{8}$$

$$\text{Transmission } \% = \frac{I_2}{I_0} \times 100 = \frac{3}{8} \times 100 = 37.5\%$$

EXAMPLE 44

If light beam is incident at polarising angle (56.3°) on air-glass interface, then find the angle of refraction in glass.

SOLUTION:

$\because \quad i_p + r_p = 90°$

$\therefore \quad r = 90° - i_p = 90° - 56.3° = 33.7°$

EXAMPLE 45

A polariser and an analyser are inclined to each other at 60°. The intensity of the polarized light emerging from the analyser is I. Find the intensity of the unpolarised light incident on the polarizer.

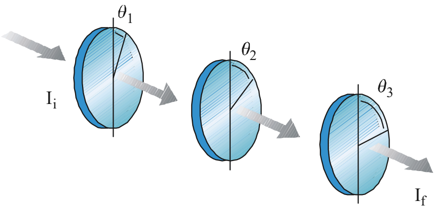
SOLUTION:

Let I_0 be the intensity of the unpolarized light incident on the polarizer. Then the intensity of the polarized light transmitted through it is $I_0/2$. Using Malus law, the intensity of the light transmitted through the analyser is

$$\left(\frac{I_0}{2}\right) \cos^2 60° = \frac{I_0}{8} \; ; \quad \frac{I_0}{8} = I \quad \therefore \; I_0 = 8I$$

EXAMPLE 46

Three polarizing disks whose planes are parallel are centered on a common axis. The direction of the transmission axis in each case is shown in Figure relative to the common vertical direction. A plane-polarized beam of light with $\mathbf{E_0}$ parallel to the vertical reference direction is incident from the left on the first disk with intensity $I_i = 10.0$ units (arbitrary). Calculate the transmitted intensity I_f when (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$; (b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, and $\theta_3 = 60.0°$.



SOLUTION:

(a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$

$I_f = I_i \cos^2(\theta_1 - 0°) \cos^2(\theta_2 - \theta_1) \cos^2(\theta_3 - \theta_2)$

$I_f = (10.0 \text{ units}) \cos^2(20.0°) \cos^2(20.0°) \cos^2(20.0°)$

$= 6.89$ units

(b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, and $\theta_3 = 60.0°$.

$I_f = (10.0 \text{ units}) \cos^2(0°) \cos^2(30.0°) \cos^2(30.0°)$

$= 5.63$ units

Checkup 10

Q.1 Is light from the sky polarized? Why is it that clouds seen through Polaroid glasses stand out in bold contrast to the sky?

Q.2 Two polaroids are oriented with their planes perpendicular to incident light and transmission axis making an angle 45° with each other. Find the fraction of incident light which is transmitted.

Q.3 For a given medium, the polarising angle is 60°. What will be the critical angle for this medium?

IMPORTANT POINTS

Reflection of Light

* Law of reflection : $\angle i = \angle r$
* Mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$ $(\because f = \frac{R}{2})$
* Power of a mirror $P = -\frac{1}{f}$

  (P is +ve for concave and –ve for convex mirror)
* Lateral or transverse or linear magnification (m_t)

$$\frac{\text{Height of the image (I)}}{\text{Height of the object (O)}} = \frac{-v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

* Longitudinal magnification

$$m_L = \frac{\text{length of image}}{\text{length of object}} = \frac{-dv}{du} = \left(\frac{v}{u}\right)^2$$

(for small objects)

* Area magnification or Superficial magnification

$$\frac{\text{Area of image}}{\text{Area of object}} = \frac{v^2}{u^2} = m^2$$

* Newton's formula $x_1 x_2 = f^2$.

Refraction of Light (at Plane Surface)

* Snell's law $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

* $\mu = \dfrac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$

* ${}_1\mu_2 = \dfrac{1}{{}_2\mu_1}$

* ${}_1\mu_2 \times {}_2\mu_3 = {}_1\mu_3$

* Cauchy's formula $\mu = A + \dfrac{B}{\lambda^2}$

* Lateral shift $= \dfrac{t\sin(i-r)}{\cos r}$

* Normal shift $= t\left(1 - \dfrac{1}{\mu}\right)$

* $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

* Apparent depth of multiple slabs $= \dfrac{t_1}{\mu_1} + \dfrac{t_2}{\mu_2} + \ldots$

* Critical angle $C = \sin^{-1}(1/\mu)$

Refraction at Curved Surface

* $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

(Object is kept in medium of μ_1)

* Thin lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

* Power $P = \dfrac{1}{f}$ (+ve for convex lens and –ve for concave lens)

* Lens Maker's formula

$$P = \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

* Thin lens lateral or transverse magnification

$$m = \frac{v}{u} = \frac{f}{u+f} = \frac{f-v}{f}$$

* Longitudinal magnification of thin lens $= (v/u)^2$

* Newton's formula $x_1 x_2 = f^2$

* Two thin lenses in contact $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

or $P = P_1 + P_2$

* Two thin lenses separated by a distance d

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

* Displacement method,

$$f = \frac{D^2 - x^2}{4D}\ ;\ O = \sqrt{I_1 I_2}$$

$$f = \frac{x}{m_1 - m_2}\quad \left(m_1 = \frac{D+x}{D-x},\ m_2 = \frac{D-x}{D+x}\right)$$

* Achromatic combination of thin lenses in contact

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

Refraction in a Prism

(a) For triangular prism :

$$r_1 + r_2 = A\ ;\ (i+e) = A + \delta$$

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2}$$

For minimum deviation $\mu = \dfrac{\sin \dfrac{A + \delta m}{2}}{\sin \dfrac{A}{2}}$

$$\left(i = e = \frac{A + \delta m}{2};\ r_1 = r_2 = r = \frac{A}{2}\right)$$

(b) For thin prism

$$\delta = (\mu - 1)A\ ;\ \theta = (\delta_V - \delta_R) = (\mu_V - \mu_R)A$$

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)}\quad \left(\mu = \frac{\mu_V + \mu_R}{2} \approx \mu_y\right)$$

* For dispersion without deviation

$(\mu - 1) A + (\mu' - 1) A' = 0$

* For deviation without dispersion

$(\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A' = 0$

Optical Instruments

(a) The Human Eye

* A myopic person can see clearly the objects lying near it only. To correct a myopic eye, the person has to use spectacles with a concave lens of focal length f = –x, where x is the distance of far point of myopic eye.

$$\frac{1}{-\text{F.P.}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

* A hypermetropic person can see clearly only the far-off objects. To correct a hypermetropic eye, the person has to use spectacle with a convex lens. $\frac{1}{-\text{N.P.}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$

(b) Camera :

Energy = Intensity × aperture area × exposure time = constant

$$\text{f-number} = \frac{\text{focal length}}{\text{diameter of aperture}}$$

(c) Simple microscope :

$m = \frac{D}{f}$ (for normal adjustment)

$m = 1 + \frac{D}{f}$ (if final image is at D)

(d) Compound microscope

$M = m_0 \times m_E$

$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_E}\right)$ (when final image is at D)

$M = \frac{v_0}{u_0} \cdot \frac{D}{f_E} \approx \frac{LD}{f_0 \cdot f_E}$ (for normal adjustment)

(e) Telescope

* **Astronomical Telescope**

- For normal adjustment

$$M = -\frac{f_o}{f_E} \text{ and } L = f_o + f_E$$

- When final image is at D

$$M = -\frac{f_o}{f_E}\left(1 + \frac{f_E}{D}\right)$$

Galilean Telescope

$$M = \frac{f_o}{f_E} \text{ and } L = f_o - |f_E|$$

Doppler Shift

$$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} = \frac{v_s}{C}$$

* Interference can take place between transverse as well as longitudinal waves.

Young's Double Slit Experiment

Position of n^{th} bright fringe $x_n = \frac{n\lambda D}{d}$ $(n = 0, 1, 2, 3, ...)$

Position of n^{th} dark fringe $x_n = \frac{(2n-1)\lambda D}{2d}$ $(n = 1, 2, 3, ...)$

Fringe width (linear) $\beta = \lambda D/d$

Fringe width (angular) $\alpha = \beta/D = \lambda/d$

Width of the central bright $w = \lambda D/d$

Inside a medium of refractive index μ the wavelength $\lambda' = \lambda/\mu$

Fringe shifts by $\frac{(\mu-1)\,tD}{d}$ on introduction of thin sheet in front of one of the slits.

Resultant wave amplitude

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Resultant wave intensity

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi \;;\; I = I_0\cos^2(\phi/2)$$

For constructive interference :

Phase difference $\phi = 2n\pi \equiv 0, 2\pi, 4\pi, 6\pi ...$

or, path difference $\Delta = n\lambda \equiv 0, \lambda, 2\lambda, 3\lambda$

$A_{max} = A_1 + A_2$

$I_{max} \propto (A_1 + A_2)^2$

$I_{max} = 4I_0$ if $I_1 = I_2 = I_0$

For destructive interference:

Phase difference $\phi = (2n-1)\pi \equiv \pi, 3\pi, 5\pi ...$

or, path difference

$$\Delta = (2n-1)\frac{\lambda}{2} \equiv \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}....$$

$A_{min} = A_1 - A_2$; $I_{min} \propto (A_1 - A_2)^2$
$I_{min} = 0$ if $I_1 = I_2 = I_0$

Relation between slit width, intensity and amplitude

$W_1 \propto A_1^2$; $W_2 \propto A_2^2$
$I_1 \propto A_1^2$; $I_2 \propto A_2^2$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

* If the YDSE apparatus is submerged in some liquid the fringe width gets reduced ($\beta' = \beta/\mu$) but if Fresnel's Biprism apparatus is submerged in a liquid (except mercury) then the fringe width increases. In water it gets increased almost three times.
* The fringe width for blue colour light will be smaller compared to that due to red colour.
* If the source behind the slits is shifted downwards, the fringe pattern shifts upwards and if source is shifted upwards then fringe pattern shifts downwards.
* In **thin films** for reflected light
  Condition for dark fringe (or a wavelength to be absent) $2\mu t \cos r = n\lambda$
  Condition for bright fringe is

  $$2\mu t \cos r = \left(n - \frac{1}{2}\right)\lambda$$

  Take r = 0, if film is viewed perpendicularly. Here μ is the refractive index of the film and t is its thickness.
* **Visibility of the interference fringes**
  Two slits separated by d and illuminated coherently by a source of angular size $\Delta\theta$, give fringes only if $\Delta\theta \le \frac{\lambda}{d}$. Further, the n^{th} fringe will be visible only if the range of wavelength $\Delta\lambda$ satisfies the relation $\Delta\lambda < \frac{\lambda}{n}$.

Diffraction

* Position of n^{th} dark fringe $x_n = \frac{n\lambda f}{a}$
  (n = 1, 2, 3 ...)
  f = focal length of lens: a = aperture
* Position of n^{th} bright $x_n = \frac{(2n+1)\lambda f}{2a}$
  (n = 1, 2 ...)
* Fringe width (linear) $\beta = \frac{\lambda f}{a}$
* Fringe width (angular) $= \frac{\beta}{f} = \frac{\lambda}{a}$
* Linear width of the central bright $w = \frac{2\lambda f}{a}$
* Angular width of the central bright $= \frac{w}{f} = \frac{2\lambda}{a}$
* Intensity of fringe at phase angle α is given by

  $$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad (I_0 = \text{intensity of central bright})$$

* Intensity of all the diffraction fringes is not the same. Central fringe is brightest (say of intensity I_0). First bright fringe has intensity only 4.5% of I_0 and second bright has intensity only 1.6% of I_0.

Polarization

* Law of Malus $I = I_0 \cos^2\theta$
* Brewster's law $\mu = \tan i_p$ (i_p = polarizing angle)
* Relation between i_p and critical angle C is

  $$\tan i_p = \frac{1}{\sin C}$$

* When unpolarized beam of light of intensity , is incident on two polarizers in contact, the angle between the axes of two polarizers being θ, then the intensity of the light finally emerging from the combination is $\frac{I}{2}\cos^2\theta$ (and not $I\cos^2\theta$). Because when unpolarized light passes the polarizer, its intensity becomes I/2 and after analyser, following the law of Malus it become $\frac{I}{2}\cos^2\theta$.
* Most interference and diffraction effects exist even for longitudinal waves like sound in air. But polarisation phenomena are special to transverse waves like light waves.

SOLVED EXAMPLES

EXAMPLE 1

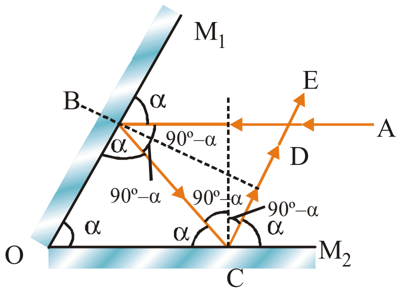
Two plane mirrors are placed at an angle α so that a ray parallel to one mirror gets reflected parallel to the second mirror after two consecutive reflections. Find the value of α

SOLUTION:

As shown in figure, ray AB goes to mirror M_1, gets reflected and travels along BC and then gets reflected by M_2 and goes in CD direction.

If the angle between M_1 and M_2 be α, then $\angle OBC$ and $\angle OCB$ are equal to α

$\therefore \quad 3\alpha = 180^\circ \; \therefore \; \alpha = 60^\circ$



EXAMPLE 2

The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

SOLUTION:

Here image can be real or virtual.

If the image is real

$f = -30,\; u = ?,\; m = -3$

$$m = \frac{f}{f-u} \quad \Rightarrow -3 = \frac{-30}{-30-u} \quad ; \; u = -40 \text{ cm.}$$

If the image is virtual

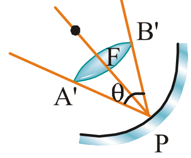
$$m = \frac{f}{f-u} \quad \Rightarrow 3 = \frac{-30}{-30-u} \Rightarrow u = -20 \text{ cm.}$$

EXAMPLE 3

The sun (diameter D) subtends an angle θ radian at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror.

SOLUTION:

Since the sun is very distant, u is very large and so 1/u is practically zero



$$\frac{1}{u} \approx 0 \;;\; \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{f} \quad ; \; v = -f$$

The image of sun will be formed at the focus and will be real, inverted and diminished

A'B' = height of image

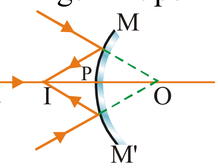
$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{A'B'}{FP} \Rightarrow \theta = \frac{d}{f} \Rightarrow d = f\theta$$

EXAMPLE 4

A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of image if the point O is -

(i) 10 cm behind the mirror

(ii) 30 cm behind the mirror



SOLUTION:

Here object is virtual

(i) $u = +10$ cm, $f = +20$ cm

$$v = \frac{uf}{u-f} = \frac{10 \times 20}{10-20} = -20 \text{ cm.}$$

$$\text{Magnification } m = -\left(\frac{v}{u}\right) = -\left(\frac{-20}{10}\right) = +2$$

(ii) $u = +30$ cm, $f = 20$cm

$$v = \frac{uf}{u-f} = \frac{30 \times 20}{30-20} = +60 \text{ cm}$$

$$m = -\left(\frac{60}{30}\right) = -2$$

EXAMPLE 5

Light waves of 5895 Å wavelength travels from vacuum to a medium of refractive index of 1.5. Velocity of light & wavelength in medium will be

(A) 2×10^8 m/sec, 3330Å

(B) 2×10^8 m/sec, 3930 Å

(C) 2×10^8 m/sec, 3390 Å

(D) None

SOLUTION:

(B). If velocity of light in vacuum is c then velocity of light in medium is

$$v = \frac{c}{\mu} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/sec}$$

Wavelength of light in medium is

$$\lambda_w = \frac{\lambda}{\mu} = \frac{5895}{1.5} = 3930 \text{ Å}.$$

EXAMPLE 6

A small pin fixed on a table top is viewed from above from a distance of 50cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

SOLUTION:

The lateral displacement $t = d\left[1-\frac{1}{\mu}\right]$

i.e. $t = 15\left(1-\frac{1}{1.5}\right) = 15\times\frac{0.5}{1.5} = 5$ cm

For small angles of incidence, the answer does not depend upon the location of the slab.

EXAMPLE 7

One light wave is incident upon a plate of refracting index μ. Incident angle i, for which refractive & reflective waves are mutually perpendicular will be

(A) $i = 45^o$ (B) $i = \sin^{-1}(\mu)$

(C) $i = \text{cosec}^{-1}(\mu)$ (D) $i = \tan^{-1}(\mu)$

SOLUTION:

(D). $\frac{\sin i}{\sin r} = \mu$

Angle between refractive & reflective waves $= 180^o - (i+r) = 90^o$

$\Rightarrow \quad i + r = 90^o;\ r = 90^o - i$

$\therefore \quad \mu = \frac{\sin i}{\sin(90-i)} = \frac{\sin i}{\cos i} = \tan i$

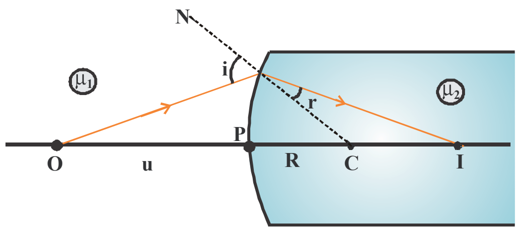
$\Rightarrow \quad i = \tan^{-1}(\mu)$

EXAMPLE 8

A small point object is placed at O, at a distance of 0.60 metre in air from a convex spherical surface of refractive index 1.5. If the radius of the curvature is 25 cm, then what is the position of the image on the principal axis.

SOLUTION:

According to sign convention, it is given that



$u = -0.6\text{m},\ R = 0.25\text{m}$

$\mu_1 = 1(\text{air}),\ \mu_2 = 1.5$

Therefore, using

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2-\mu_1}{R};\ \frac{1.5}{v} = \frac{1}{(-0.6)} + \frac{1.5-1}{0.25}$$

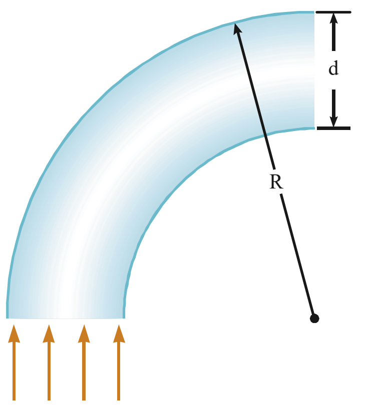
$$= -\frac{1}{0.6} + \frac{0.5}{0.25} = -\frac{5}{3} + 2 = \frac{1}{3}$$

$\Rightarrow \quad v = 4.5$ m

The image is formed on the other side of the object (i.e. inside the refracting surface).

EXAMPLE 9

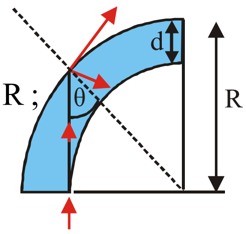
An optical fiber has index of refraction μ and diameter d . It is surrounded by air. Light is sent into the fiber along its axis, as shown in Figure. (a) Find the smallest outside radius R permitted for a bend in the fiber if no light is to escape.



(b) What If? Does the result for part (a) predict reasonable behavior as d approaches zero? As μ increases? As μ approaches 1? (c) Evaluate R assuming the fiber diameter is 100 μm and its index of refraction is 1.40.

SOLUTION:

(a) A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta = \frac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then



$\frac{n(R-d)}{R} > 1$; $nR - nd > R$;

$nR - R > nd$; $R > \frac{nd}{n-1}$

(b) As $d \to 0$, $R_{min} \to 0$. This is reasonable.

As n increases, R_{min} decreases. This is reasonable.

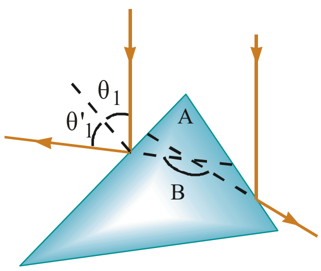
As n decreases toward 1, R_{min} increases. This is reasonable.

(c) $R_{min} = \frac{1.40\,(100\times10^{-6}\,\text{m})}{0.40}$

$= 350 \times 10^{-6}$ m

EXAMPLE 10

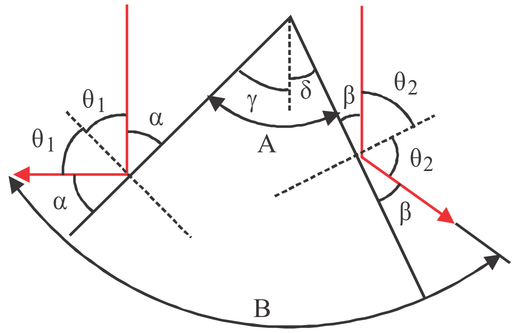
One technique for measuring the angle of a prism is shown in Figure. A parallel beam of light is directed on the angle so that parts of the beam reflect from opposite sides. Show that the angular separation of the two reflected beams is given by B = 2A.



SOLUTION:

Call θ_1 the angle of incidence and of reflection on the left face and θ_2 those angles on the right face. Let α represent the complement of θ_1 and β be the complement of θ_2.

Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles.

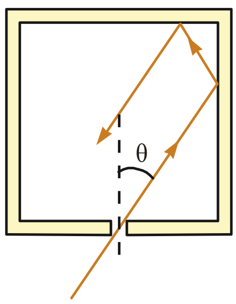


We have $A = \gamma + \delta = \alpha + \beta$

and $B = \alpha + A + \beta = \alpha + \beta + A = 2A$.

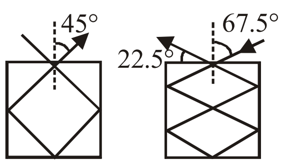
EXAMPLE 11

Figure shows a top view of a square enclosure. The inner surfaces are plane mirrors. A ray of light enters a small hole in the center of one mirror. (a) At what angle θ must the ray enter in order to exit through the hole after being reflected once by each of the other three mirrors? (b) What If? Are there other values of θ for which the ray can exit after multiple reflections? If so, make a sketch of one of the ray's paths.



SOLUTION:

(a) 45.0° as shown in the first figure to the right.



(b) Yes. If grazing angle is halved, the number of reflections from the side faces is doubled.

EXAMPLE 12

An object of length 1 cm is placed at a distance of 15 cm from a concave mirror of focal length 10 cm. The nature and size of the image are –
(A) real, inverted,1.0cm (B) real, inverted, 2.0cm
(C) virtual,erect, 0.5cm (D) virtual,erect,1.0cm

SOLUTION:

(B). Given u = – 15 cm, f = – 10 cm, O = 1 cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};\ \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-15}$$

$\therefore$ v = – 30 cm

$$\frac{I}{O}=-\frac{v}{u}=-\frac{-30}{-15}=-2$$

I = – 2 × 1 = – 2 cm
Image is inverted and on the same side (real) of size 2 cm.

EXAMPLE 13

A biconvex lens whose both the surfaces have same radii of curvature has a power of 5D. The refractive index of material of lens is 1.5. The radius of curvature of each surface is –
(A) 20 cm (B) 15 cm
(C) 10 cm (D) 5 cm

SOLUTION:

(A). $P=\frac{1}{f}$, $\therefore\ f=\frac{1}{P}=\frac{1}{5}\ m=20$ cm

For an equiconvex lens, $\frac{1}{f}=\frac{2(\mu-1)}{R}$

$\therefore$ R = 2(μ – 1) f = 2 × 0.5 × 20 = 20 cm

EXAMPLE 14

A convex lens of focal length 10.0 cm is placed in contact with a convex lens of 15.0 cm focal length. What is the focal length of the combination.

SOLUTION:

For combination of lenses

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}+\frac{1}{15}=\frac{25}{150}=\frac{1}{6}$$

Therefore, f = 6 cm

EXAMPLE 15

Focal lengths of two lens are f and f ' and dispersive powers are ω_0 and $2\omega_0$. To form achromatic combination from these–
(A) f ' = 2f (B) f ' = – 2f
(C) f ' = f/2 (D) f ' = – f/2

SOLUTION:

(B). For achromatic combination

$$\frac{\omega}{\omega'}=-\frac{f}{f'}\quad \text{but}\quad \frac{\omega}{\omega'}=\frac{\omega_0}{2\omega_0}=\frac{1}{2};\ -\frac{f}{f'}=\frac{1}{2}$$

or f ' = – 2f

EXAMPLE 16

Prism angle of a prism is 10°. Their refractive index for red & violet colour is 1.51 & 1.52 respectively, then find the dispersive power.

SOLUTION:

Dispersive power of prism, $\omega=\left(\frac{\mu_v-\mu_r}{\mu_y-1}\right)$

but $\mu_y=\frac{\mu_v+\mu_r}{2}=\frac{1.52+1.51}{2}=1.515$

Therefore $\omega=\frac{1.52-1.51}{1.515-1}=\frac{0.01}{0.515}=0.019.$

EXAMPLE 17

Prism angle & refractive index for a prism are 60° & 1.414. Find the angle of minimum deviation.

SOLUTION:

$$\mu=\frac{\sin(A+\delta_m)/2}{\sin 30^0}$$

$$\Rightarrow\ 1.414=\frac{\sin(60+\delta_m)/2}{\sin 30^0}$$

$$\Rightarrow\ \sin\left(\frac{60^\circ+\delta_m}{2}\right)=0.707=\sin 45^o$$

$$\Rightarrow\ \frac{60+\delta_m}{2}=45^o\Rightarrow\delta_m=30^o$$

EXAMPLE 18

The refracting angle of the prism is 60°. What is the angle of incidence for minimum deviation ? The refractive index of material of prism is $\sqrt{2}$.

SOLUTION:

For minimum deviation $r = A/2 = 60/2 = 30°$

From snell's law $\dfrac{\sin i}{\sin r} = \mu$

$$\sqrt{2} = \frac{\sin i}{\sin 30°} \quad \therefore \sin i = \frac{1}{2} \times \sqrt{2} = \frac{1}{\sqrt{2}}$$

$$= \sin 45° \text{ or } i = 45°$$

EXAMPLE 19

A farsighted person cannot focus clearly an objects that are less than 145cm. from his eyes. To correct this problem, the person wear eyeglasses that are located 2.0 cm. in front of his eyes. Determine the focal length that will permit this person to read a newspaper at a distance of 32.0 cm. from his eyes.

SOLUTION:

The near point is 145 cm. and eyeglasses are 2.0 cm. in front of the eyes.

Therefore, $v = -143$ cm.

The object is placed 32.0cm. from the eyes so $u = +30.0$ cm.

The focal length is obtained from equation

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{(30.0\text{cm})} + \frac{1}{-143\text{cm}}$$

$$= 0.026 \text{ cm}^{-1}.$$

Hence, $f = 38$ cm.

EXAMPLE 20

Photograph of the ground are taken from an aircraft, flying at an altitude of 2000 m, by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any one time.

SOLUTION:

As here $u = -2000$m, $f = 0.50$m, so from lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$; $\dfrac{1}{v} - \dfrac{1}{(-2000)} = \dfrac{1}{0.5}$

$$\frac{1}{v} = \frac{1}{0.5} - \frac{1}{2000} \cong \frac{1}{0.5} \left[\text{as } \frac{1}{0.5} >> \frac{1}{2000}\right]$$

$v = 0.5\text{m} = 50 \text{ cm} = f$

Now as in case of a lens

$$m = \frac{v}{u} = \frac{0.5}{-2000} = -\frac{1}{4} \times 10^{-3}$$

So $I_1 = (ma)(mb) = m^2 A \qquad [\because A = ab]$

$$\text{i.e., } A = \frac{I_1}{m^2} = \frac{18\text{cm} \times 18\text{cm}}{\left[(1/4) \times 10^{-3}\right]^2}$$

$$= (720 \text{ m} \times 720 \text{ m})$$

EXAMPLE 21

A man with normal near point (25 cm) reads a book with small print using a magnifying thin convex lens of focal length 5 cm.

(a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass ?

(b) What is the maximum and minimum MP possible using the above simple microscope?

SOLUTION:

(a) As for normal eye far and near point are ∞ and 25 cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25$ cm.

However, for a lens as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{i.e.,} \quad u = \frac{f}{(f/v) - 1}$$

So u will be minimum when

$v = \min = -25$ cm

$$\text{i.e., } (u)_{min} = \frac{5}{-(5/25) - 1} = -\frac{25}{6}$$

$$= -4.17 \text{ cm}$$

u will be maximum when $v = \max = \infty$

So the closest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) As in case of simple magnifier MP = (D/u). So MP will be minimum when
u = max = 5 cm

i.e., $(MP)_{min} = \frac{-25}{-5} = 5 \quad \left[= \frac{D}{f}\right]$

And MP will be maximum when
u = min = (25/6) cm

i.e., $(MP)_{max} = \frac{-25}{-(25/6)} = 6 \quad \left[= 1 + \frac{D}{f}\right]$

EXAMPLE 22

The length of a microscope is 14 cm and for relaxed eye the magnifying power is 25. The focal length of the eyepiece is 5cm. Calculate the distance of the object and the focal length of the objective.

SOLUTION:

Length of microscope is :
$L = v_0 + f_e, \; 14 = v_0 + 5, \; v_0 = 9$ cm

$$M = -\frac{v_0}{u_0} \times \frac{D}{f_e}$$

or $25 = -\frac{9}{u_0} \times \frac{25}{5}$ or $u_0 = -\frac{9}{5} = -1.8$ cm

Now, $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$ but u_0 is negative,

therefore $\frac{1}{f_0} = \frac{1}{9} + \frac{5}{9} = \frac{6}{9}$ or $f_0 = 1.5$ cm

EXAMPLE 23

A telescope consisting of an objective of focal length 60 cm and a single-lens eyepiece of focal length 5 cm is focussed at a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then the angular width of the image will be –

(A) 10° (B) 24°
(C) 50° (D) 1/6°

SOLUTION:

(B). $m = \frac{f_0}{f_e} = \frac{\beta}{\alpha}; \; \beta = \alpha \frac{f_0}{f_e} = 2 \times \frac{60}{5} = 24°$

EXAMPLE 24

The focal length of the objective and the eyepiece of an astronomical telescope are 16 m and 2cm respectively. A planet is viewed with its help then

(A) The angular magnification of the planet is – 800
(B) The image of the planet is inverted
(C) The distance between the objective and the eye piece is 16.02
(D) All of the above

SOLUTION:

(D). Distance between objective and eyepiece
$= f_0 + f_e = 16.02$ m.

Angular magnification

$$M = -\frac{f_0}{f_e} \text{ or } M = \frac{-16}{0.02} = -800$$

The image formed by a telescope is always inverted.

EXAMPLE 25

The two coherent sources of intensity that ratio 2 : 8 produce an interference pattern. The values of maximum and minimum intensities will be respectively.

(A) I_1 and $9\,I_1$ (B) $9I_1$ and I_1
(C) $2I_1$ and $8I_1$ (D) $8I_1$ and $2I_1$

Where I_1 is the intensity of first source

SOLUTION:

(B). $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ ....(1)

According to question

$\frac{I_1}{I_2} = \frac{2}{8} = \frac{1}{4} \quad \therefore I_2 = 4I_1$ ....(2)

From eqs. (1) and (2)

$I_{max} = I_1 + 4I_1 + 2\sqrt{4I_1^2} = 5I_1 + 4I_1$

$I_{max} = 9I_1$ ....(3)

$$I_{min} = I_1 + I_2 - 2\sqrt{4I_1 I_2} \quad ....(4)$$

From eqs. (2) and (4),

$$I_{min} = I_1 + 4I_1 - 2\sqrt{4I_1 I_2}$$

$$I_{min} = I_1$$

EXAMPLE 26

In Young's double slit experiment the two slits are illuminated by light of wavelength 5890 Å and the distance between the fringes obtained on the screen is 0.2°. If the whole apparatus is immersed in water then the angular fringe width will be, it the refractive index of water is 4/3.

(A) 0.30° (B) 0.15°
(C) 15° (D) 30°

SOLUTION:

(B). $\omega_a = \lambda/d$

$$\therefore \quad \omega_a \propto \lambda \Rightarrow \frac{(\omega_0)_{water}}{\omega_a} = \frac{\lambda_{water}}{\lambda}$$

$$\Rightarrow \frac{(\omega_0)_{water}}{\omega_a} = \frac{\lambda}{\mu_{water}\lambda} \Rightarrow (\omega_0)_{water} = 0.15°$$

EXAMPLE 27

The intensities of two sources are I and 9I respectively. If the phase difference between the waves emitted by them is π then the resultant intensity at the point of observation will be -

(A) 3I (B) 4 I
(C) 10 I (D) 82 I

SOLUTION:

(B). $I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_1 = I,\ I_2 = 9\,I,\ \phi = \pi$

$$I' = I + 9I + 2\sqrt{9I^2}\cos\pi$$

$$= 10\,I - 6\,I = 4\,I$$

EXAMPLE 28

When wave of wavelength 0.2 cm is made incident normally on a slit of width 0.004m, then the semi-angular width of central maximum of diffraction pattern will be-

(A) 60° (B) 30°
(C) 90° (D) 0°

SOLUTION:

(B). $\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$ .... (1)

According to question, $\lambda = 2 \times 10^{-3}$ m

$a = 4 \times 10^{-3}$ m .... (2)

From equation (1) and (2)

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = 30°$$

EXAMPLE 29

A polariser and an analyser are oriented so that maximum light is transmitted, what will be the intensity of outcoming light when analyser is rotated through 60°.

SOLUTION:

According to Malus Law

$$I = I_0\cos^2\theta = I_0\cos^2 60° = I_0\left[\frac{1}{2}\right]^2 = \frac{I_0}{4}$$

EXAMPLE 30

A parallel beam of monochromatic light is incident on a narrow rectangular slit of width 1mm. When the diffraction pattern is seen on a screen placed at a distance of 2m. the width of principal maxima is found to be 2.5 mm. The wave length of light is-

(A) 6250 Å (B) 6200 Å
(C) 5890 Å (D) 6000 Å

SOLUTION:

(A). Here the width of principal maxima is 2.5 mm, therefore its half width is

$$\frac{\beta}{2} = \frac{2.5}{2} = 1.25 \times 10^{-3}\text{m}$$

Diffraction angle,

$$\theta = \frac{\beta/2}{D} = \frac{1.25 \times 10^{-3}}{2}$$

$$a\,\theta = \lambda \quad ; \quad \theta = \frac{\lambda}{a} = \frac{1.25 \times 10^{-3}}{2}$$

$$\lambda = \frac{1.25 \times 10^{-3}}{2} \times a = \frac{1.25 \times 10^{-3} \times 10^{-3}}{2}$$

$$\lambda = 6.25 \times 10^{-7}\text{ m} = 6250\text{ Å}$$

EXAMPLE 31

Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?

SOLUTION:

Using lens maker formula and sign convention

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

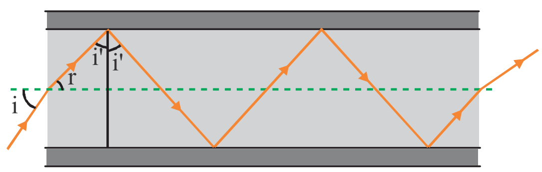
$$\therefore \quad \frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right]$$

$$= 0.55 \times \frac{2}{R} \quad \text{(By sign convention)}$$

$$\text{or} \quad R = \frac{1.10}{1} \times 20 = 22 \text{ cm}$$

EXAMPLE 32

Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.



SOLUTION:

The maximum value of the acceptance angle

$i = \theta_a$ is given by $\sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}$

where μ_1 is the refractive index of core and μ_2 that of the cladding.

Here $\mu_1 = 1.68$ and $\mu_2 = 1.44$

$$\therefore \sin\theta_a = \sqrt{(1.68)^2 - (1.44)^2} = \sqrt{0.7488}$$

$$= 0.8653$$

$$\Rightarrow \quad \theta_a = 59°55'$$

Therefore, the range of angle of incidence is

$$0 < i < 59°55'$$

EXAMPLE 33

In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

SOLUTION:

Given, $D = 1.4$m, $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m

Fringe width, $\omega = \frac{1.2}{4} = 0.3 \times 10^{-2}$ m

Using formula, Fringe width $\omega = \frac{D\lambda}{d}$

$$\therefore \quad \lambda = \frac{\omega d}{D} = \frac{0.3 \times 10^{-2} \times 0.28 \times 10^{-3}}{1.4}$$

$$= 0.06 \times 10^{-5} = 600 \times 10^{-9} = 600 \text{ nm.}$$

EXAMPLE 34

In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

SOLUTION:

Phase difference corresponding to λ is 2π and phase difference corresponding to $\lambda/3$ is $2\pi/3$.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Let that $\quad I_1 = I_2 = I_0$

In the first case,

$$K = I_0 + I_0 + 2I_0 \cos 2\pi = 4I_0$$

In the second case,

$$K' = I_0 + I_0 + 2I_0 \cos(2\pi/3)$$

$$= I_0 + I_0 - 2I_0(1/2) = I_0$$

$$\text{Now,} \quad \frac{K'}{K} = \frac{I_0}{4I_0} = \frac{1}{4} \quad \text{or} \quad K' = \frac{K}{4}$$

EXAMPLE 35

Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400nm.

SOLUTION:

Here, $a = 4mm = 4 \times 10^{-3}$ m

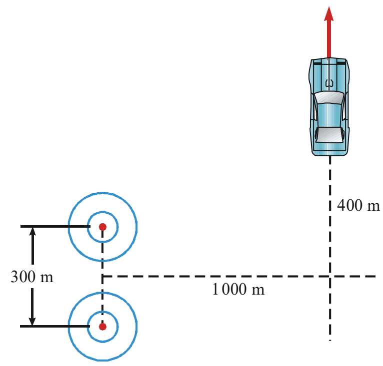
$\lambda = 400nm = 400 \times 10^{-9}$ m $= 4 \times 10^{-7}$ m

Ray optics is good approximation upto distances equal to Fresnel's distance (Z_F).

$$Z_F = \frac{a^2}{\lambda} = \frac{(4\times10^{-3})^2}{4\times10^{-7}} = 40m$$

EXAMPLE 36

Two radio antennas separated by 300 m as shown in figure simultaneously broadcast identical signals at the same wavelength. A radio in a car traveling due north receives the signals. (a) If the car is at the position of the second maximum, what is the wavelength of the signals? (b) How much farther must the car travel to encounter the next minimum in reception? (Note: Do not use the small-angle approximation in this problem.)



SOLUTION:

Note, with the conditions given, the small angle approximation does not work well. That is, sinθ, tanθ, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

(a) At the m = 2 maximum,

$$\tan\theta = \frac{400m}{1000m} = 0.400$$

$\theta = 21.8°$

So, $$\lambda = \frac{d\sin\theta}{m} = \frac{(300m)\sin 21.8°}{2}$$

$= 55.7$ m

(b) The next minimum encountered is the m = 2 minimum; and at that point,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

which becomes $d\sin\theta = \frac{5}{2}\lambda$

or $$\sin\theta = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{55.7m}{300m}\right) = 0.464$$

and $\theta = 27.7°$

so $y = (1000\text{ m})\tan 27.7° = 524$ m.

Therefore, the car must travel an additional 124 m .

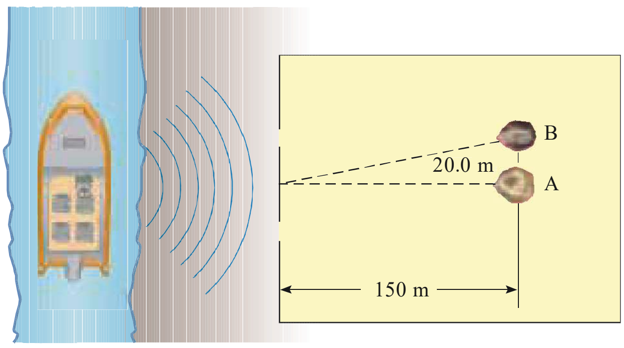
If we considered Fresnel interference, we would more precisely find

(a) $$\lambda = \frac{1}{2}\left(\sqrt{550^2 + 1000^2} - \sqrt{250^2 + 1000^2}\right)$$

$= 55.2$ m and (b) 123 m.

EXAMPLE 37

A riverside warehouse has two open doors as shown in figure. Its walls are lined with sound-absorbing material. A boat on the river sounds its horn. To person A the sound is loud and clear. To person B the sound is barely audible. The principal wavelength of the sound waves is 3.00m. Assuming person B is at the position of the first minimum, determine the distance between the doors, center to center.



SOLUTION:

Location of A = central maximum,
Location of B = first minimum.
So, $\Delta y = [y_{min} - y_{max}]$

$$= \frac{\lambda L}{d}\left(0 + \frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0 \text{ m}$$

$$\text{Thus, } d = \frac{\lambda L}{2\,(20.0 \text{ m})} = \frac{(3.00 \text{ m})\,(150\text{m})}{40.0\text{m}}$$

$$= 11.3 \text{ m}$$

EXAMPLE 38

A soap bubble ($\mu = 1.33$) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?

SOLUTION:

Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference,

we require, $\frac{\lambda_n}{2} + 2t = \lambda_n$

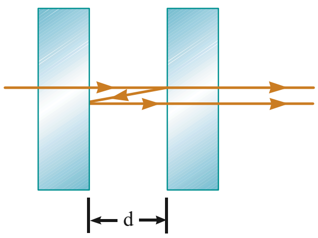
where $\lambda_n = \frac{\lambda}{\mu}$ is the wavelength in the material.

Then $2t = \frac{\lambda_n}{2} = \frac{\lambda}{2\mu}$

$\lambda = 4\mu t = 4\,(1.33)\,(115 \text{ m}) = 612 \text{ nm}$

EXAMPLE 39

A beam of 580-nm light passes through two closely spaced glass plates, as shown in Figure . For what minimum nonzero value of the plate separation d is the transmitted light bright?



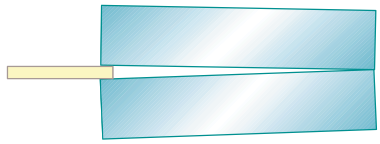
SOLUTION:

If the path length difference $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{min} = \frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = 290 \text{ nm}.$$

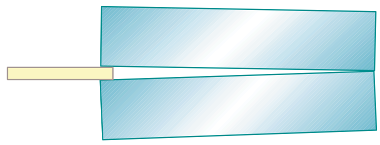
EXAMPLE 40

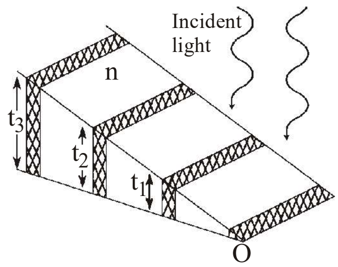
An air wedge is formed between two glass plates separated at one edge by a very fine wire, as shown in figure. When the wedge is illuminated from above by 600-nm light and viewed from above, 30 dark fringes are observed. Calculate the radius of the wire.



SOLUTION:

For destructive interference in the air, $2t = m\lambda$.





For 30 dark fringes, including the one where the plates meet,

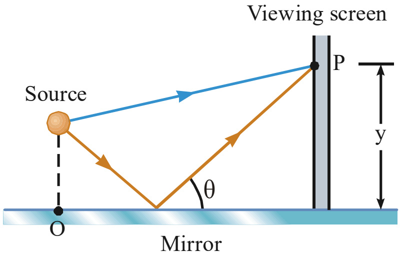
$$t = \frac{29\,(600\text{nm})}{2} = 8.70 \times 10^{-6} \text{ m}$$

Therefore, the radius of the wire is

$$r = \frac{t}{2} = \frac{8.70 \ \mu\text{m}}{2} = 4.35 \ \mu\text{m}$$

EXAMPLE 41

Interference effects are produced at point P on a screen as a result of direct rays from a 500-nm source and reflected rays from the mirror, as shown in Figure. Assume the source is 100 m to the left of the screen and 1.00 cm above the mirror. Find the distance y to the first dark band above the mirror.



SOLUTION:

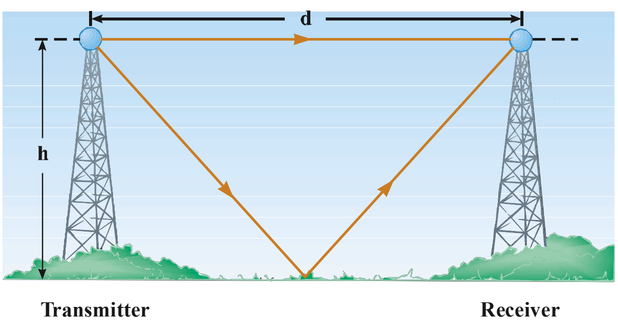
For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\pi/2$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00cm below the mirror plane.

$$y_{dark} = \frac{m\lambda L}{d} = \frac{1\,(5.00 \times 10^{-7}\,m)\,(100m)}{(2.00 \times 10^{-2}\,m)}$$

$$= 2.50 \text{ mm}$$

EXAMPLE 42

Figure shows a radio-wave transmitter and a receiver separated by a distance d and both a distance h above the ground. The receiver can receive signals both directly from the transmitter and indirectly from signals that reflect from the ground.

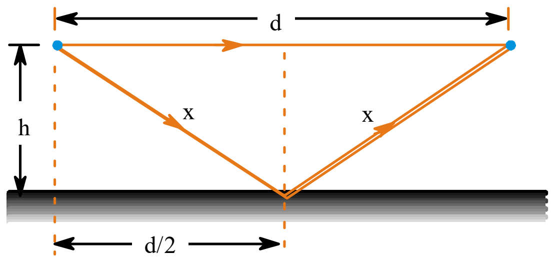


Assume that the ground is level between the transmitter and receiver and that a 180° phase shift occurs upon reflection. Determine the longest wavelengths that interfere
(a) constructively and (b) destructively.

SOLUTION:

From the sketch, observe that

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2}$$



Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \frac{\lambda}{2}$

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where $m = 0, 1, 2, 3, \ldots.$

$$2x - d = \left(m + \frac{1}{2}\right)\lambda \quad \text{or} \quad \lambda = \frac{4x - 2d}{2m + 1}$$

For the longest wavelength, $m = 0$, giving

$$\lambda = 4x - 2d = 2\sqrt{4h^2 + d^2} - 2d$$

(b) For destructive interference,

$$\delta = \left(m - \frac{1}{2}\right)\lambda, \quad \text{where } m = 0, 1, 2, 3, \ldots.$$

Thus, $2x - d = m\lambda$ or $\lambda = \frac{2x - d}{m}$

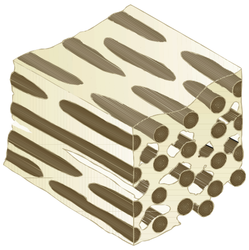
For the longest wavelength, $m = 1$ giving

$$\lambda = 2x - d = \sqrt{4h^2 + d^2} - d$$

EXAMPLE 43

Iridescent peacock feathers are shown in Figure a. The surface of one microscopic barbule is composed of transparent keratin that supports rods of dark brown melanin in a regular lattice, represented in Figure b. (Your fingernails are made of keratin, and melanin is the dark pigment giving colour to human skin.) In a portion of the feather that can appear turquoise, assume that the melanin rods are uniformly separated by 0.25μm, with air between them. (a) Explain how this structure can appear blue-green when it contains no blue or green pigment. (b) Explain how it can also appear violet if light falls on it in a different direction. (c) Explain how it can present different colours to your two eyes at the same time—a characteristic of iridescence. (d) A compact disc can appear to be any colour of the rainbow. Explain why this portion of the feather cannot appear yellow or red. (e) What could be different about the array of melanin rods in a portion of the feather that does appear to be red?


(a)


(b)

SOLUTION:

(a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\mu m$ apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d \sin\theta = m\lambda$.

The longest wavelength reflected strongly corresponds to $m = 1$:

$2\,(0.25 \times 10^{-6}\ m) \sin 90° = 1\lambda$

$\lambda = 500$ nm. This is the blue-green colour.

(b) For light incident at grazing angle 60°,

$2d \sin\theta = m\lambda$ gives

$1\lambda = 2\,(0.25 \times 10^{-6}\ m) \sin 60° = 433$ nm.

This is violet.

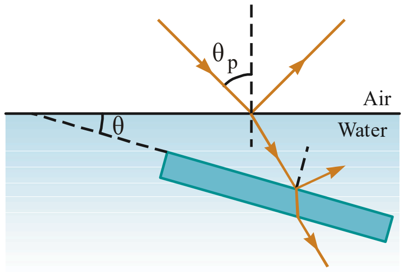
(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colours reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say 0.32 μm) they could reflect red with constructive interference.

EXAMPLE 44

Light strikes a water surface at the polarizing angle. The part of the beam refracted into the water strikes a submerged glass slab (index of refraction, 1.50), as shown in Figure. The light reflected from the upper surface of the slab is completely polarized. Find the angle between the water surface and the glass slab.



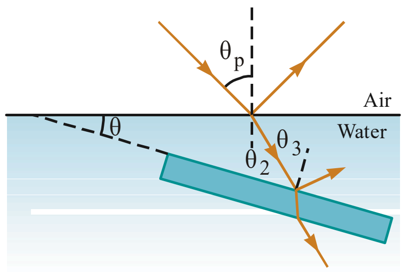
SOLUTION:

For the air-to-water interface,

$$\tan\theta_p = \frac{\mu_{water}}{\mu_{air}} = \frac{1.33}{1.00}\ ;\ \theta_p = 53.1°$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$



For the water-to-glass interface,

$$\tan\theta_p = \tan\theta_3 = \frac{\mu_{glass}}{\mu_{water}} = \frac{1.50}{1.33}\ ;\ \ \theta_3 = 48.4°$$

The angle between surfaces is $\theta = \theta_3 - \theta_2 = 11.5°$

QUESTION BANK

EXERCISE-1 (LEVEL-1)

SECTION - 1 (VOCABULARY BUILDER)

Choose one correct response for each question.
For Q.1-Q.5 : Match the column I with column II.

Q.1 In column I, some optical instruments are mentioned, while in column II, the description about nature of image they can form for real objects are given.

Column I	Column II
(a) Concave mirror	(i) Real, erect
(b) Convex mirror	(ii) Virtual, magnified
(c) Diverging lens	(iii) Real, diminished
(d) Converging lens	(iv) Virtual, diminished

Codes
(A) (a) - ii, iii ; (b)-iv ; (c)-iv ; (d)-ii, iii
(B) (a) - i, iii ; (b)-ii ; (c)-iv ; (d)-ii
(C) (a) - ii, iv ; (b)-i ; (c)-iv ; (d)-ii, iii
(D) (a) - i ; (b)-iv ; (c)-ii ; (d)-iii

Q.2 Match the following for light ray.

Column I	Column II
(a) In reflection from denser medium	(i) Amplitude may change
(b) In refraction into denser medium	(ii) Phase changes
(c) In reflection from rarer medium	(iii) No phase change
(d) In refraction into rarer medium	(iv) Velocity decreases

Codes
(A) (a) - ii, iii ; (b)-ii, iv ; (c)-iv ; (d)-i, iii
(B) (a) - i, iii ; (b)-ii ; (c)-i, iv ; (d)-ii, iii
(C) (a) - i, ii ; (b)-i, iii, iv ; (c)-i, iii ; (d)-i, iii
(D) (a) - i, ii ; (b)-ii, iv ; (c)-iii ; (d)-ii

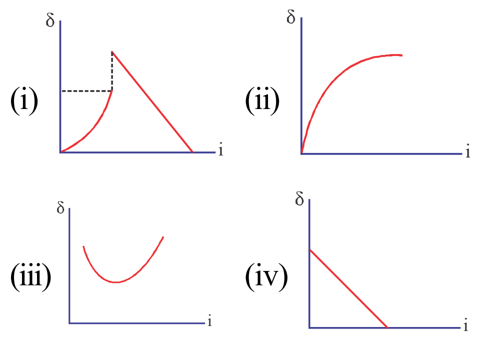
Q.3

Column I	Column II
(a) Object is between optic centre & 1st principle focus in a diverging lens.	(i) Image is inverted
(b) Object is between optic centre and 1st principle focus of a converging lens	(ii) Image is erect
(c) Object is between optic centre and 2nd principle focus of a diverging lens.	(iii) Image is of greater size than the object
	(iv) Image is of smaller size than the object
	(v) Image is real

Codes
(A) (a) - iii, v ; (b)-i, iii ; (c)-ii, iv
(B) (a) - i, iii, v ; (b)-i, iii ; (c)-iv
(C) (a) - ii, iii, iv ; (b)-ii, v ; (c)-ii, iv
(D) (a) - ii, iii, v ; (b)-ii, iii ; (c)-ii, iv

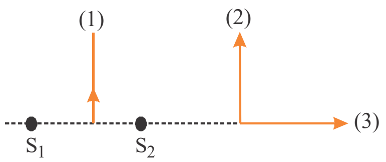
Q.4 **Column I**
(a) A ray is falling on a plane smooth mirror
(b) A ray is going from a rarer to denser medium
(c) A ray is going from a denser to rarer medium
(d) A ray is falling on a prism

Column II
(Angle of deviation V/s Angle of incidence)



Codes
(A) (a) - iv, (b)-ii, (c)-i, (d)-iii
(B) (a) - i, (b)-ii, (c)-iii, (d)-iv
(C) (a) - ii, (b)-i, (c)-iii, (d)-iv
(D) (a) - i, (b)-iii, (c)-ii, (d)-iv

Q.5 Figure shows two sources S_1 and S_2 that emit radio waves of wavelength λ in all direction. The sources are exactly in phase and are separated by a distance of 1.5λ. Line 1 is perpendicular bisector, of the line joining S_1 and S_2.



Column I	Column II
(a) Along line-1	(i) Minima all along path
(b) Along line-2	(ii) Maxima all along path
(c) Along line-3	(iii) Alternate maxima and minima with minima at point closest to S_2.
	(iv) Alternate maxima and minima with maxima at point closest to S_2.

Codes
(A) (a) - iv, (b)-ii, (c)-i
(B) (a) - ii, (b)-iii, (c)-i
(C) (a) - ii, (b)-i, (c)-iii
(D) (a) - i, (b)-iii, (c)-ii

SECTION - 2 (BASIC CONCEPTS BUILDER)

For Q.6 to Q.24 : Choose one word for the given statement from the list.

Infinite, True, False, Increases, 1.5, greater, lesser, twice, virtual, diminished, power, critical angle, red, violet, 256, small, large.

Q.6 The number of images of an object held between two parallel plane mirrors is_____.

Q.7 Light passes through a closed tube which contains a gas. If the gas inside the tube is gradually pumped out, the speed of light inside the tube_____.

Q.8 If the velocity of light in a medium is (2/3) times of the velocity of light in vacuum, then the refractive index of that medium is ___.

Q.9 A converging lens is used to form an image on a screen. When the lower half of the lens is covered by an opaque screen then, half of the image will disappear.(True/False)

Q.10 The velocity of light in rarer medium is _____. than that in a denser medium.

Q.11 For the same incident ray, when the mirror is rotated through an angle, the reflected ray is rotated through _____the angle.

Q.12 In a convex mirror irrespective of the position of the object, the image formed is always _____, erect but _____ in size.

Q.13 The angle of incidence in the denser medium at which the refracted ray just grazes the surface of separation is called the ___of the denser medium.

Q.14 The _____ of a lens is defined as the reciprocal of its focal length.

Q.15 The power of a combination of lenses in contact is the algebraic sum of the powers of individual lenses. **[True / False]**

Q.16 The refractive index is more for _____(violet/ red) rays of light than the corresponding values for_____(violet/red) rays of light.

Q.17 In primary rainbow the outer side is of _____ colour and inner side is of _____ colour.

Q.18 In secondary rainbow the outer side is of _____ colour and inner side is of _____ colour.

Q.19 If the wavelength of the light is reduced to one fourth, then the amount of scattering is increased by _____ times.

Q.20 A diffraction pattern is obtained using a beam of red light. If the red light is replaced by blue light diffraction pattern becomes broader and farther apart. **[True / False]**

Q.21 A convex lens may form real and virtual images. **[True / False]**

Q.22 A compound microscope will have large magnifying power, if both the object lens and the eye lens are of _____ focal length.

Q.23 In YDSE if white light is used, an interference pattern will consist of dark central fringe. **[True / False]**

Q.24 The resolving power of a microscope increases with decrease in wavelength. **[True / False]**

SECTION - 3 (ENHANCE PROBLEM SOLVING SKILLS)

Choose one correct response for each question.

PART 1 REFLECTION BY PLANE MIRROR

Q.25 A ray is reflected in turn by two plane mirrors mutually at right angles to each other. The angle between the incident and the reflected rays is –

(A) 90° (B) 60°
(C) 180° (D) None

Q.26 Two plane mirrors are at right angles to each other. A man stands between them and combs his hair with his right hand. In how many of the images will he be seen using his right hand

(A) None (B) 1
(C) 2 (D) 3

Q.27 A man runs towards a mirror at a speed 15 m/s The speed of the image relative to the man is

(A) 15 m/s (B) 30 m/s
(C) 35 m/s (D) 20 m/s

Q.28 Focal length of a plane mirror is

(A) Zero (B) Infinite
(C) Very less (D) Indefinite

Q.29 Two plane mirrors are inclined at 60° to each other. The no. of images formed by them will be

(A) 5 (B) 6
(C) 8 (D) None

Q.30 To get three images of a single object, one should have two plane mirrors at an angle of –

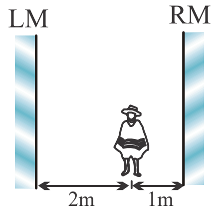
(A) 90° (B) 72°
(C) 30° (D) 60°

Q.31 A clock hung on a wall has marks instead of numbers on its dial. On the opposite wall there is a mirror, and the image of the clock in the mirror if read, indicates the time as 8:20. What is the time in the clock.

(A) 3:40 (B) 4:40
(C) 5:20 (D) 4:20

Q.32 Two mirrors, labeled LM for left mirror and RM for right mirror in the adjacent figure, are parallel to each other and 3.0 m apart. A person standing 1.0 m from the right mirror (RM) looks into this mirror and sees a series of images. How far from the person is the second closest image seen in the right mirror (RM)?

(A) 10.0 m
(B) 4.0 m
(C) 6.0 m
(D) 8.0 m



Q.33 A plane mirror approaches a stationary person with some acceleration 'a'. The acceleration of his image, as seen by the person, will be

(A) a (B) 2a
(C) a/2 (D) none

PART 2 REFLECTION BY SPHERICAL MIRROR

Q.34 The focal length of a concave mirror is 50cm. Where an object be placed so that its image is two times and inverted
(A) 75 cm (B) 72 cm
(C) 63 cm (D) 50 cm

Q.35 The minimum distance between the object and its real image for concave mirror is
(A) f (B) 2f
(C) 4f (D) Zero

Q.36 An object 2.5 cm high is placed at a distance of 10 cm from a concave mirror of radius of curvature 30 cm The size of the image is
(A) 9.2 cm (B) 10.5 cm
(C) 5.6 cm (D) 7.5 cm

Q.37 Image formed by a concave mirror of focal length 6 cm, is 3 times of the object, then the distance of object from mirror is –
(A) – 4 cm (B) 9 cm
(C) 6 cm (D) 12 cm

Q.38 A concave mirror of focal length f (in air) is immersed in water ($\mu = 4/3$). The focal length of the mirror in water will be
(A) f (B) (4/3) f
(C) (3/4) f (D) (7/3) f

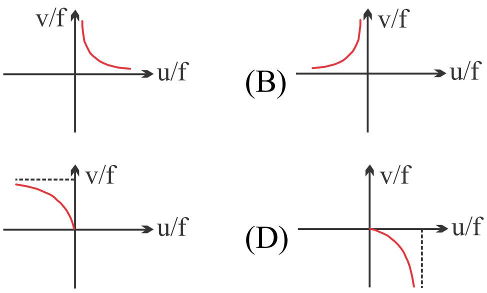
Q.39 Radius of curvature of concave mirror is 40cm & the size of image is twice as that of object, then the object distance is
(A) 60cm (B) 20cm
(C) 40cm (D) 30cm

Q.40 A concave mirror gives an image three times as large as the object placed at a distance of 20cm from it. For the image to be real, the focal length should be
(A) 10cm (B) 15cm
(C) 20cm (D) 30cm

Q.41 The distance of an object from a spherical mirror is equal to the focal length of the mirror. Then the image:
(A) must be at infinity (B) may be at infinity
(C) may be at the focus (D) none

Q.42 Find the incorrect statement/s for a concave mirror producing a virtual image of the object.
(A) The linear magnification is always greater than one, except at the pole.
(B) The linear magnification is always less than one.
(C) The magnification tends to one as the object move nearer to the pole of the mirror.
(D) The distance of the object from the pole of the mirror is less than the focal length of mirror.

Q.43 A virtual erect image in a concave mirror is represented, in the given figure, by



(A) (B)
(C) (D)

PART 3 REFRACTION AT PLANE SURFACE

Q.44 Velocity of light in a medium is 1.5×10^8 m/s. Its refractive index will be –
(A) 8 (B) 6
(C) 4 (D) 2

Q.45 Light travels through a glass plate of thickness t and having refractive index n. If c is the velocity of light in vacuum, the time taken by the light to travel this thickness of glass is
(A) t/nc (B) tnc
(C) nt/c (D) tc/n

Q.46 When a light wave goes from air into water, the quantity that remains unchanged is its
(A) Speed (B) Amplitude
(C) Frequency (D) Wavelength

Q.47 The refractive indices of glass and water w.r.t. air are 3/2 and 4/3 respectively. The refractive index of glass w.r.t. water will be
(A) 8/9 (B) 9/8
(C) 7/6 (D) None of these

Q.48 For a colour of light the wavelength for air is 6000 Å and in water the wavelength is 4500 Å. Then the speed of light in water will be
(A) 5.0×10^{14} m/s (B) 2.25×10^{8} m/s
(C) 4.0×10^{8} m/s (D) Zero

Q.49 The speed of light in air is 3×10^{8} m/s. What will be its speed in diamond whose refractive index is 2.4
(A) 3×10^{8} m/s (B) 332 m/s
(C) 1.25×10^{8} m/s (D) 7.2×10^{8} m/s

Q.50 The refractive index of a piece of transparent quartz is the greatest for
(A) Red light (B) Violet light
(C) Green light (D) Yellow light

Q.51 The wavelength of sodium light in air is 5890 Å. The velocity of light in air is 3×10^{8} m/s. The wavelength of light in a glass of refractive index 1.6 would be close to
(A) 5890 Å (B) 3681 Å
(C) 9424 Å (D) 15078 Å

Q.52 Speed of light is maximum in
(A) Water (B) Air
(C) Glass (D) Diamond

Q.53 Which one of the following statements is correct
(A) In vacuum, the speed of light depends upon frequency.
(B) In vacuum, the speed of light does not depend upon frequency but depends on wavelength.
(C) In vacuum, the speed of light is independent of frequency and wavelength.
(D) In vacuum, the speed of light depends upon wavelength.

Q.54 The distance travelled by light in glass (refractive index =1.5) in a nanosecond will be
(A) 45 cm (B) 40 cm
(C) 30 cm (D) 20 cm

Q.55 The apparent depth of a swimming pool is 1.2m. What is its real depth ? (Take $\mu_g = 4/3$)
(A) 1.6m (B) 2.5m
(C) 3.2m (D) 1m

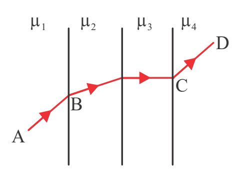
Q.56 A ray of light passes from vacuum into a medium of refractive index n. If the angle of incidence is twice the angle of refraction , then the angle of incidence is :
(A) $\cos^{-1}(n/2)$ (B) $\sin^{-1}(n/2)$
(C) $2\cos^{-1}(n/2)$ (D) $2\sin^{-1}(n/2)$

Q.57 A parallel beam of light, travelling in air, is incident at an angle of 60° on a plane boundary of refractive index $\sqrt{3}$. The angle between incident and refracted wavefronts, is
(A) 0° (B) 30°
(C) 60° (D) 150°

Q.58 A glass slab of thickness 3 cm and refractive index 3/2 is placed on ink mark on a piece of paper. For a person looking at the mark at a distance 5.0 cm above it, the distance of the mark will appear to be –
(A) 3.0 cm (B) 4.0 cm
(C) 4.5 cm (D) 5.0 cm

Q.59 A fish at a depth of 12 cm in water is viewed by an observer on the bank of a lake. To what height the image of the fish is raised.
(A) 9 cm (B) 12 cm
(C) 3.8 cm (D) 3 cm

Q.60 A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure.



The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have :
(A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$
(C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$

PART 4 TOTAL INTERNAL REFLECTION

Q.61 Critical angle of light passing from glass to air is minimum for–
(A) Red (B) Green
(C) Yellow (D) Violet

Q.62 The wavelength of light in two liquids 'x' and 'y' is 3500 Å and 7000 Å, then the critical angle of x relative to y will be
(A) 60° (B) 45°
(C) 30° (D) 15°

Q.63 For total internal reflection to take place, the angle of incidence i and the refractive index μ of the medium must satisfy the inequality
(A) $\frac{1}{\sin i} < \mu$ (B) $\frac{1}{\sin i} > \mu$
(C) $\sin i < \mu$ (D) $\sin i > \mu$

Q.64 Total internal reflection of light is possible when light enters from
(A) Air to glass (B) Vacuum to air
(C) Air to water (D) Water to air

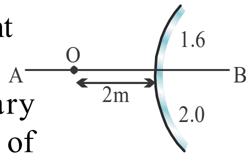
Q.65 A cut diamond sparkles because of its
(A) Hardness
(B) High refractive index
(C) Emission of light by the diamond
(D) Absorption of light by the diamond

Q.66 If the critical angle for total internal reflection from a medium to vacuum is 30°, the velocity of light in the medium is
(A) 3×10^8 m/s (B) 1.5×10^8 m/s
(C) 6×10^8 m/s (D) $\sqrt{3} \times 10^8$ m/s

Q.67 Which of the following is used in optical fibres –
(A) Total internal reflection (B) Scattering
(C) Diffraction (D) Refraction

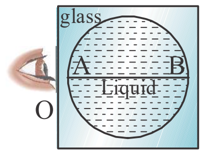
PART 5 REFRACTION AT SPHERICAL SURFACES

Q.68 In the figure shown a point object O is placed in air. A spherical boundary separates various media of radius of curvature 1.0 m. AB is principal axis. The refractive index above AB is 1.6 and below AB is 2.0. The separation between the images formed due to refraction at spherical surface is:



(A) 12 m (B) 20 m
(C) 14 m (D) 10 m

Q.69 The observer 'O' sees the distance AB as infinitely large. If refractive index of liquid is μ_1 and that of glass is μ_2, then μ_1/μ_2 is:



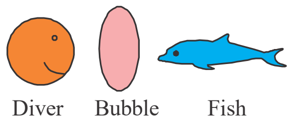
(A) 2 (B) 1/2
(C) 4 (D) None of these

Q.70 A solid transparent sphere (μ = 1.5) has a small dot at its center. When observed from outside, the apparent position of the dot will be
(A) closer to the eye than its actual position.
(B) same as its actual position.
(C) farther away from the eye than its actual position.
(D) at infinity.

PART 6 LENS

Q.71 Which of the following is not the case with the image formed by concave lens ?
(A) It may be erect or inverted.
(B) It may be magnified or diminished.
(C) It may be real or virtual.
(D) Real image may be between the pole and focus or beyond focus.

Q.72 A fish sees the smiling face of a scuba diver through a bubble of air between them, as shown. Compared to the face of the diver, the image seen by the fish will be –

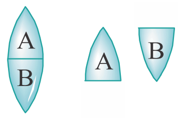


(A) smaller and erect
(B) smaller and inverted
(C) larger and erect
(D) Can be either of above depending on the distance of the diver.

Q.73 A bi-convex lens is made from glass of refractive index 1.5 and radius of curvature of both surfaces of the lens is 20cm. The incident ray parallel to principal axis will be focussed at a distance Lcm from lens on principal axis where :
(A) L = 10 (B) L = 20
(C) L = 40 (D) L = 20/3

Q.74 A convex lens of power 4D is kept in contact with a concave lens of power 3D, the effective power of combination will be :
(A) 7D (B) 4D/3
(C) 1D (D) 3D/4

Q.75 The power of a plano-convex lens is P. If this lens is cut longitudinally along its principal axis into two equal parts and then they are joined as given in the figure. The power of combination will be :



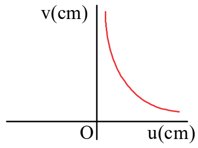
(A) P (B) 2P
(C) P/2 (D) zero

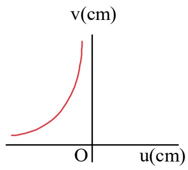
Q.76 If the focal length of a magnifier is 5 cm calculate the power of the lens.
(A) 20D (B) 10D
(C) 5D (D) 15D

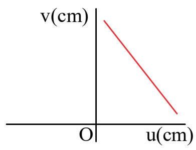
Q.77 A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object –
(A) 20 cm (B) 30 cm
(C) 60 cm (D) 80 cm

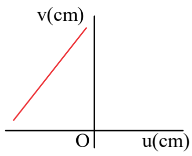
Q.78 Two lenses of power -15D and +5D are in contact with each other. The focal length of the combination is
(A) – 20 cm (B) –10 cm
(C) +20cm (D) +10 cm

Q.79 A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like

(A)


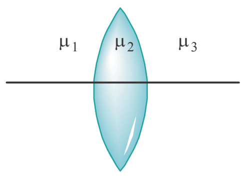
(B)


(C)


(D)


Q.80 A concave lens with unequal radii of curvature made of glass ($\mu_g = 1.5$) has a focal length of 40 cm. In air if it is immersed in a liquid of refractive index $\mu_l = 2$, then
(A) it behaves like convex lens of 80 cm focal length.
(B) it behave like a convex lens of 20 cm focal length.
(C) its focal length becomes 60 cm.
(D) nothing can be said.

Q.81 The diagram shows an equiconvex lens. What should be the condition on the refractive indices so that the lens become diverging?
(A) $2\mu_2 > \mu_1 - \mu_3$
(B) $2\mu_2 < \mu_1 + \mu_3$
(C) $2\mu_2 > 2\mu_1 - \mu_3$
(D) $2\mu_2 > \mu_1 + \mu_3$



Q.82 In the case of a converging lens, a real object is at a finite distance L from the lens. It is moving with speed 5m/s. The image is formed at one of the focus of the lens. What is the speed of the image

(A) 5 m/s (B) infinite
(C) 10 m/s (D) 20 m/s

Q.83 A converging lens is used to produce an image on a screen of an object. What change is needed for the real image to be formed nearer to the lens?

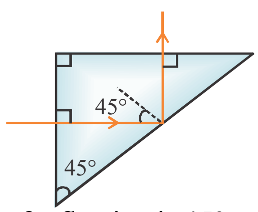
(A) increase the focal length of the lens (lens and position of object is fixed).
(B) insert a diverging lens between the lens and the screen (converging lens and position of object is fixed).
(C) increase the distance of the object from the lens.
(D) move the object closer to the lens.

PART 7 REFRACTION IN A PRISM

Q.84 A thin prism of angle $A = 6°$ produces a deviation $\delta = 3°$. Find the refractive index of the material of prism.

(A) 1.5 (B) 1.0
(C) 2.5 (D) 0.5

Q.85 A light ray is incident perpendicularly to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n –

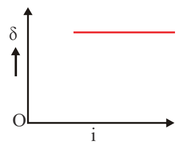


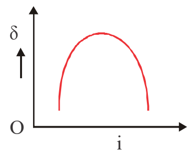
(A) $n < 1/\sqrt{2}$ (B) $n > \sqrt{2}$
(C) $n > 1/\sqrt{2}$ (D) $n < \sqrt{2}$

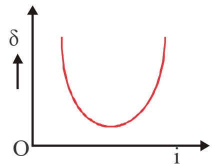
Q.86 The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then –

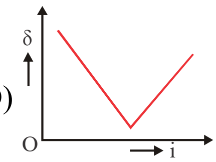
(A) D_1 can be less than or greater than D_2 depending upon the angle of prism
(B) $D_1 > D_2$
(C) $D_1 < D_2$
(D) $D_1 = D_2$

Q.87 The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by –

(A)


(B)


(C)


(D)


Q.88 Dispersion occurs when

(A) some material bend light more that other material.
(B) a material changes some frequencies more than other.
(C) light has different speeds in different materials.
(D) a material slows down some wavelengths more than others.

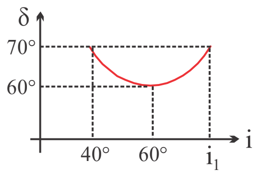
Q.89 White light is dispersed by the prism, and falls on a screen to form a visible spectrum. Which of the following is/are true?

(A) The frequency changes for each colour, but the speed stays the same.
(B) Red wavelengths are deviated through larger angles than green wavelengths.
(C) Violet light propagates at a higher speed than green light while in the prism.
(D) The speed of all colours is reduced in the prism, with maximum reduction for violet light.

Q.90 For a prism kept in air it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ' and angle of emergence 'e' become equal. Then the refractive index of the prism is

(A) 1.73 (B) 1.15
(C) 1.5 (D) 1.33

Q.91 The curve of angle of incidence versus angle of deviation shown has been plotted for prism. The value of refractive index of the prism used is



(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) $\sqrt{3}/\sqrt{2}$ (D) $2/\sqrt{3}$

Q.92 In the given curve of **above question**. Find the value of angle i_1 in degrees is
(A) 40° (B) 60°
(C) 70° (D) 90°

PART 8 SCATTERING OF LIGHT

Q.93 When light rays undergoes two internal reflection inside a raindrop, which of the rainbow is formed?
(A) Primary rainbow (B) Secondary rainbow
(C) Both (A) & (B) (D) Can't say

Q.94 At sunset or sunrise, the Sun's rays have to pass through a larger distance as
(A) shorter wavelengths are removed by scattering.
(B) longer wavelengths are removed by scattering.
(C) less frequency of scattering wavelength.
(D) Both (A) and (B).

Q.95 A passenger in an aeroplane shall
(A) never see a rainbow.
(B) may see a primary and a secondary rainbow as concentric circles.
(C) may see a primary and a secondary rainbow as concentric arcs.
(D) shall never see a secondary rainbow.

Q.96 The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as –
(A) rayleigh scattering (B) maxwell scattering
(C) oserted scattering (D) reynold scattering

Q.97 Which of the following statement is correct?
(A) At sunset or sunrise, the sun's rays have to pass through a small distance in the atmosphere.
(B) Rayleigh scattering which is proportional to $(1/\lambda)^2$.
(C) At sunset or sunrise the sun's rays have to pass through a larger distance in the atmosphere.
(D) Most of the blue and other shorter wavelengths are not removed by scattering.

Q.98 Red colour is used for danger signals because
(A) it causes fear.
(B) it undergoes least scattering.
(C) it undergoes maximum scattering.
(D) None of the above

PART 9 HUMAN EYE

Q.99 A person cannot see distinctly at the distance less than one metre. Calculate the power of the lens that he should use to read a book at a distance of 25 cm
(A) + 3.0 D (B) + 0.125 D
(C) – 3.0 D (D) + 4.0 D

Q.100 A man can see upto 100 cm of the distant object. The power of the lens required to see far objects will be
(A) + 0.5 D (B) + 1.0 D
(C) + 2.0 D (D) – 1.0 D

Q.101 For the myopic eye, the defect is cured by
(A) Convex lens (B) Concave lens
(C) Cylindrical lens (D) Toric lens

Q.102 A person can not see the objects beyond 50cm. The power of a lens to correct this vision will be
(A) + 2 D (B) – 2 D
(C) + 5 D (D) 0.5 D

PART 10 MICROSCOPE

Q.103 The focal length of the objective lens of a compound microscope is –
(A) Equal to the focal length of its eye piece
(B) Less than the focal length of eye piece
(C) Greater than the focal length of eye piece
(D) Any of the above three

Q.104 Least distance of distinct vision is 25 cm. Magnifying power of simple microscope of focal length 5 cm is
(A) 1 / 5 (B) 5
(C) 1 / 6 (D) 6

Q.105 The image formed by an objective of a compound microscope is –
(A) Real and diminished
(B) Real and enlarged
(C) Virtual and enlarged
(D) Virtual and diminished

Q.106 The magnify power of the objective of a compound microscope is 7 if the magnifying power of the microscope is 35, then the magnifying power is eyepiece will be
(A) 245 (B) 5
(C) 28 (D) 42

Q.107 In a compound microscope, the focal lengths of two lenses are 1.5 cm and 6.25 cm. If an object is placed at 2 cm from objective and the final image is formed at 25cm from eye lens, the distance between the two lenses is
(A) 6.00 cm (B) 7.75 cm
(C) 9.25 cm (D) 11.0 cm

PART 11 TELESCOPE

Q.108 An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye-piece is 36 cm and the final image is formed at infinity. Determine the focal length of objective and eye-piece.
(A) 30cm, 6cm, (B) 15cm, 12cm
(C) 25cm, 12cm (D) 8cm, 12cm

Q.109 An astronomical telescope has a large aperture to –
(A) Reduce spherical aberration
(B) Have high resolution
(C) Increase span of observation
(D) Have low dispersion

Q.110 The focal length of achromatic combination of a telescope is 90cm. The dispersive powers of lenses are 0.024 and 0.036 respectively. Their focal lengths will be -
(A) 30 cm and 60 cm (B) 45 cm and 90 cm
(C) 15 cm and 45 cm (D) 30 cm and – 45 cm

Q.111 A reflecting telescope has a large mirror for its objective with radius of curvature equal to 80 cm. The magnifying power of this telescope if eye piece used has a focal length of 1.6 cm is
(A) 100 (B) 50
(C) 25 (D) 5

Q.112 A small telescope has an objective lens of focal length 144 cm and an eye piece of focal length 6.0cm. What is the separation between the objective and the eye piece?
(A) 0.75 m (B) 1.38 m
(C) 1.0 m (D) 1.5 m

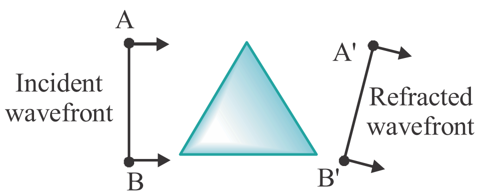
PART 12 HUYGEN'S PRINCIPLE

Q.113 The idea of secondary wavelets for the propagation of a wave was first given by
(A) Newton (B) Huygen
(C) Maxwell (D) Fresnel

Q.114 By a monochromatic wave, we mean
(A) A single ray
(B) A single ray of a single colour
(C) Wave having a single wavelength
(D) Many rays of a single colour

Q.115 A shortcoming of Huygen model could not
(A) explain the absence of the backwave.
(B) determine the shape of the wavefront for a plane wave.
(C) explain the point source emitting waves uniformly in all directions.
(D) All of the above

Q.116 Figure shows behaviour of a wavefront when it passes through a prism.



Which of the following statement{s) is/are correct?
I. Lower portion of wavefront (B') is delayed resulting in a tilt.
II. Time taken by light to reach A' from A is equal to the time taken to reach B' from B.
III. Speed of wavefront is same everywhere.
IV. A particle on wavefront A' B' is in phase with a particle on wavefront AB.
(A) I and II (B) II and III
(C) III and IV (D) I and III

Q.117 Ray diverging from a point source form a wavefront that is
(A) cylindrical (B) spherical
(C) plane (D) cubical

PART 13 INTERFERENCE OF LIGHT

Q.118 Two identical light sources S_1 and S_2 emit light of same wavelength λ. These light rays will exhibit interference if
(A) Their phase differences remain constant
(B) Their phases are distributed randomly
(C) Their light intensities remain constant
(D) Their light intensities change randomly

Q.119 The ratio of intensities of two waves are given by 4 : 1. The ratio of the amplitudes of the two waves is
(A) 2 : 1 (B) 1 : 2
(C) 4 : 1 (D) 1 : 4

Q.120 As a result of interference of two coherent sources of light, energy is
(A) Increased
(B) Redistributed and the distribution does not vary with time
(C) Decreased
(D) Redistributed and the distribution changes with time

Q.121 What causes changes in the colours of the soap or oil films for the given beam of light
(A) Angle of incidence (B) Angle of reflection
(C) Thickness of film (D) None of these

Q.122 The intensity ratio of two waves is 9 : 1. These waves produce the event of interference. The ratio of maximum to minimum intensity will be
(A) 1 : 9 (B) 9 : 1
(C) 1 : 4 (D) 4 : 1

Q.123 If the ratio of intensities of two waves is 1 : 25, then the ratio of their amplitudes will be
(A) 1 : 25 (B) 5 : 1
(C) 26 : 24 (D) 1 : 5

Q.124 If the amplitude ratio of two sources producing interference is 3 : 5, the ratio of intensities at maxima and minima is
(A) 25 : 16 (B) 5 : 3
(C) 16 : 1 (D) 25 : 9

Q.125 Two light sources are said to be coherent if they are obtained from –
(A) Two independent point sources emitting light of the same wavelength.
(B) A single point source.
(C) A wide source.
(D) Two ordinary bulbs emitting light of different wavelengths.

Q.126 Two waves of intensity I undergo Interference. The maximum intensity obtained is
(A) I / 2 (B) I
(C) 2I (D) 4I

Q.127 Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

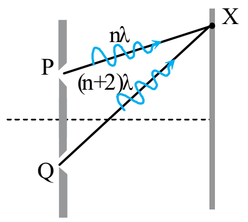
(A) 2I (B) 4I
(C) 5I (D) 7I

PART 14 YOUNG'S DOUBLE-SLIT EXPERIMENT

Q.128 Two coherent light sources S_1 and S_2 (λ= 6000 Å) are 1mm apart from each other. The screen is placed at a distance of 25 cm from the sources. The width of the fringes on the screen should be

(A) 0.015 cm (B) 0.025 cm
(C) 0.010 cm (D) 0.030 cm

Q.129 The figure shows a double slit experiment P and Q are the slits. The path lengths PX and QX are $n\lambda$ and $(n+2)\lambda$ respectively, where n is a whole number and λ is the wavelength. Taking the central fringe as zero, what is formed at X



(A) First bright (B) First dark
(C) Second bright (D) Second dark

Q.130 In the Young's double slit experiment, the spacing between two slits is 0.1 mm. If the screen is kept at a distance of 1.0m from the slits and the wavelength of light is 5000 Å, then the fringe width is

(A) 1.0 cm (B) 1.5 cm
(C) 0.5 cm (D) 2.0 cm

Q.131 In Young's double slit experiment, if L is the distance between the slits and the screen upon which interference pattern is observed, x is the average distance between the adjacent fringes and d being the slit separation. The wavelength of light is given by

(A) xd/L (B) xL/d
(C) Ld/x (D) 1/Ldx

Q.132 If yellow light in the Young's double slit experiment is replaced by red light, the fringe width will

(A) Decrease
(B) Remain unaffected
(C) Increase
(D) First increase and then decrease

Q.133 In Young's experiment, the ratio of maximum to minimum intensities of the fringe system is 4 : 1. Ratio of amplitudes of the coherent sources –

(A) 4 : 1 (B) 3 : 1
(C) 2 : 1 (D) 1 : 1

Q.134 In a Young double slit experiment, two films of thickness t_1 and t_2 having refractive indices μ_1 and μ_2 are placed in front of slits A and B respectively. If $\mu_1 t_1 = \mu_2 t_2$ the central max. will

(A) not shift
(B) shift towards A if $t_1 < t_2$
(C) shift towards B if $t_1 < t_2$
(D) shift towards A if $t_1 > t_2$

Q.135 In young's double slit experiment, the distance between two slits is made three times then the fringe width will become –

(A) 9 times (B) 1/9 times
(C) 3 times (D) 1/3 times

Q.136 A double slit experiment is performed with light of wavelength 500 nm. A thin film of thickness 2μm and refractive index 1.5 is introduced in the path of the upper beam. The location of the central maximum will –

(A) Remain unshifted
(B) Shift downward by nearly two fringes
(C) Shift upward by nearly two fringes
(D) Shift downward by 10 fringes

Q.137 Waves from two slits are in phase at the slits and travel to a distant screen to produce the second minimum of the interference pattern. The difference in the distance traveled by the waves

(A) half a wavelength
(B) a wavelength
(C) three halves of a wavelength
(D) two wavelengths

PART 15 DIFFRACTION

Q.138 The penetration of light into the region of geometrical shadow is called
(A) Polarisation (B) Interference
(C) Diffraction (D) Refraction

Q.139 A slit of size 0.15 cm is placed at 2.1 m from a screen. On illuminating it by a light of wavelength 5×10^{-5}cm, the width of central maxima will be
(A) 70 mm (B) 0.14 mm
(C) 1.4 mm (D) 0.14 cm

Q.140 Red light is generally used to observe diffraction pattern from single slit. If blue light is used instead of red light, then diffraction pattern
(A) Will be more clear (B) Will contract
(C) Will expanded (D) Will not be visualized

Q.141 A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
(A) 1.2 mm (B) 1.2 cm
(C) 2.4 cm (D) 2.4 mm

Q.142 In order to see diffraction the thickness of the film is
(A) 100 Å (B) 6,000 Å
(C) 1 mm (D) 1 cm

Q.143 Fraunhoffer diffraction pattern is observed at a distance of 2m on screen, when a plane-wavefront of 6000Å is incident perpendicularly on 0.2 mm wide slit. Width of central maxima is:
(A) 10 mm (B) 6mm
(C) 12 mm (D) None

Q.144 The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000Å. The width of the slit is-
(A) 5×10^{-5} cm (B) 1.0×10^{-4} cm
(C) 2.5×10^{-5} cm (D) 1.25×10^{-5} cm

PART 16 RESOLVING POWER

Q.145 The diameter of objective lens of a telescope is 6cm and wavelength of light used is 540nm. The resolving power of telescope is –
(A) 9.1×10^4 rad^{-1} (B) 10^5 rad^{-1}
(C) 3×10^4 rad^{-1} (D) None of the above

Q.146 For better resolution, a telescope must have a
(A) large diameter objective.
(B) small diameter objective.
(C) may be large.
(D) neither large nor small.

Q.147 The resolving power of a microscope is basically determined by the –
(A) speed of the light used.
(B) wavelength of the light used.
(C) both (A) and (B).
(D) neither (A) nor (B).

PART 17 POLARISATION

Q.148 Two Nicols are oriented with their principal planes making an angle of 60°. The percentage of incident unpolarized light which passes through the system is
(A) 50% (B) 100%
(C) 12.5% (D) 37.5%

Q.149 When light of a certain wavelength is incident on a plane surface of a material at a glancing angle 30º, the reflected light is found to be completely plane polarised. Determine refractive index of given material –
(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) $1/\sqrt{2}$ (D) 2

Q.150 The critical angle of a certain medium is $\sin^{-1}(3/5)$. The polarizing angle of the medium is
(A) $\sin^{-1}(4/5)$ (B) $\tan^{-1}(5/3)$
(C) $\tan^{-1}(3/4)$ (D) $\tan^{-1}(4/3)$

EXERCISE-2 (LEVEL-2)

Choose one correct response for each question.

Q.1 An object of length 6 cm is placed on the principle axis of a concave mirror of focal length f at a distance of 4f. The length of the image will be
(A) 2 cm (B) 12 cm
(C) 4 cm (D) 1.2 cm

Q.2 A ray of light travelling inside a rectangular glass block of refractive index $\sqrt{2}$ is incident on the glass–air surface at an angle of incidence of 45°. The refractive index of air is 1. Under these conditions the ray –
(A) Will emerge into the air without any deviation.
(B) Will be reflected back into the glass.
(C) Will be absorbed.
(D) Will emerge into the air with an angle of refraction equal to 90°.

Q.3 What is the time taken by light to cross a glass of thickness 4 mm and $\mu = 3$
(A) 4×10^{-11} sec (B) 2×10^{-11} sec
(C) 16×10^{-11} sec (D) 8×10^{-10} sec

Q.4 An under water swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer the bird appears to be at a distance from the surface of water equal to (Refractive Index of water is 4/3)
(A) 24 m (B) 12 m
(C) 18 m (D) 9 m

Q.5 In a compound microscope, the intermediate image is :
(A) virtual, erect and magnified
(B) real, erect and magnified
(C) real, inverted and magnified
(D) virtual, erect and reduced

Q.6 A plane sound wave travels from air to water. The angle of incidence is α_1 and the angle of refraction is α_2. Assuming Snell's law to be valid
(A) $\alpha_2 < \alpha_1$ (B) $\alpha_2 > \alpha_1$
(C) $\alpha_2 = \alpha_1$ (D) $\alpha_2 = 90°$

Q.7 Light passes from air into flint glass with index of refraction μ. What angle of incidence must the light have so that the component of its velocity perpendicular to the interface remains same in both medium?
(A) $\tan^{-1}(1/\mu)$ (B) $\sin^{-1}(1/\mu)$
(C) $\cos^{-1}(1/\mu)$ (D) $\tan^{-1}\mu$

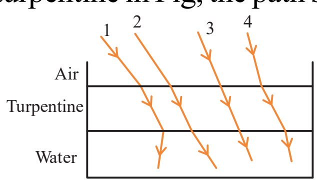
Q.8 If an observer is walking away from the plane mirror with 6m/sec. Then the velocity of the image with respect to observer will be
(A) 6m/sec (B) – 6m/sec
(C) 12 m/sec (D) 3m/sec

Q.9 An object of size 7.5cm is placed in front of a convex mirror of radius of curvature 25cm at a distance of 40cm. Size of the image should be
(A) 2.3 cm (B) 1.78 cm
(C) 1 cm (D) 0.8 cm

Q.10 The image formed by a convex mirror of focal length 30cm is a quarter of the size of the object. The distance of the object from the mirror is
(A) 30cm (B) 90cm
(C) 120cm (D) 60cm

Q.11 A concave mirror is used to focus the image of a flower on a nearby well 120cm from the flower. If a lateral magnification of 16 is desired, the distance of the flower from the mirror should be
(A) 8cm (B) 12cm
(C) 80cm (D) 120cm

Q.12 The optical density of turpentine is higher than that of water while its mass density is lower. Fig shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in Fig, the path shown is correct?



(A) 1 (B) 2
(C) 3 (D) 4

SCO -16 District Shopping Centre, Jind, Haryana.
Contact no. : 01681-245530, 9053013302.
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Q.13 A light wave has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} meters in a medium. The refractive index of the medium is
(A) 1.5 (B) 1.33
(C) 1.0 (D) 0.66

Q.14 A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :
(A) convergent lens of focal length 3.5 R.
(B) convergent lens of focal length 3 R.
(C) divergent lens of focal length 3.5 R.
(D) divergent lens of focal length 3 R.

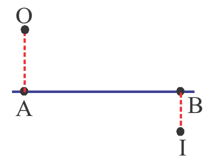
Q.15 The ratio of thickness of plates of two transparent mediums A and B is 6 : 4. If light takes equal time in passing through them, then refractive index of B with respect to A will be
(A) 1.4 (B) 1.5
(C) 1.75 (D) 1.33

Q.16 Refractive index of air is 1.0003. The correct thickness of air column which will have one more wavelength of yellow light (6000 Å) then in the same thickness in vacuum is
(A) 2 mm (B) 2 cm
(C) 2 m (D) 2 km

Q.17 A diver at a depth of 12m in water ($\mu = 4/3$) sees the sky in a cone of semi-vertical angle
(A) $\sin^{-1}$ (4/3) (B) $\tan^{-1}$ (4/3)
(C) $\sin^{-1}$ (3/4) (D) 90°

Q.18 A person wears glasses of power – 2.5 D. The defect of the eye and the far point of the person without the glasses are respectively
(A) Farsightedness, 40 cm
(B) Nearsightedness, 40 cm
(C) Astigmatism, 40 cm
(D) Nearsightedness, 250 cm

Q.19 A luminous point object is placed at O, whose image is formed at I as shown in figure. Line AB is the optical axis.



Which of the following statement is incorrect –
(A) If a lens is used to obtain the image, then it must be a converging lens and its optical centre will be the intersection point of line AB and OI.
(B) If a lens is used to obtain the image, then it must be a diverging lens and its optical centre will be the intersection point of line AB and OI.
(C) If a mirror is used to obtain the image, then the mirror must be concave and object and image subtend equal angles at the pole of the mirror.
(D) I is real image

Q.20 A student can distinctly see the object upto a distance 15cm. He wants to see the black board at a distance of 3 m. Focal length and power of lens used respectively will be
(A) – 4.8cm, –3.3D (B) – 5.8cm, –4.3D
(C) – 7.5cm, –6.3D (D) – 15.8cm, –6.3D

Q.21 If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is
(A) 150 (B) 200
(C) 250 (D) 400

Q.22 A telescope of diameter 2m uses light of wavelength 5000Å for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is
(A) 4×10^{-4} rad (B) 0.25×10^{-6} rad
(C) 0.31×10^{-6} rad (D) 5.0×10^{-3} rad

Q.23 A man can see the object between 15 cm and 30 cm. He uses the lens to see the far objects. Then due to the lens used, the near point will be
(A) (10/3) cm (B) 30 cm
(C) 15 cm (D) (100/3) cm

Q.24 An eye specialist prescribes spectacles having a combination of convex lens of focal length 40cm in contact with a concave lens of focal length 25 cm. Power of this lens combination in diopters is

(A) + 1.5 (B) – 1.5
(C) + 6.67 (D) – 6.67

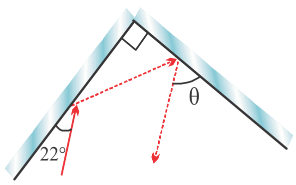
Q.25 The maximum magnification that can be obtained with a convex lens of focal length 2.5 cm is (the least distance of distinct vision is 25 cm)

(A) 10 (B) 0.1
(C) 62.5 (D) 11

Q.26 A simple magnifying lens is used in such a way that an image is formed at 25 cm away from the eye. In order to have 10 times magnification, the focal length of the lens should be –

(A) 5 cm (B) 2 cm
(C) 25 mm (D) 0.1 mm

Q.27 A ray of light is reflected by two mirrors placed normal to each other. The incident ray makes an angle of 22° with one of the mirrors. At what angle θ does the ray emerge?



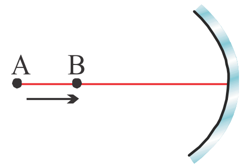
(A) 22° (B) 68°
(C) 44° (D) None

Q.28 An object of height h = 5cm is located at a distance a = 12 cm from a concave mirror with focal length 10 cm. Find the height of the image.

(A) 10 cm (B) 15 cm
(C) 20 cm (D) 25 cm

Q.29 A linear object AB is placed along the axis of a concave mirror. The object is moving towards the mirror with speed U. The speed of the image of the point A is 4U and the speed of the image of B is also 4U. If the center of the line AB is at a distance L from the mirror then find out the length of the object.

(A) 3L/2
(B) 5L/3
(C) L
(D) None



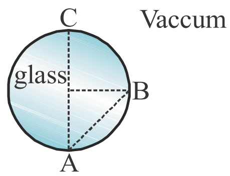
Q.30 A concave mirror of radius of curvature 40 cm forms an image of an object placed on the principal axis at a distance 45 cm in front of it. Now if the system is completely immersed in water (μ = 1.33) then

(A) the image will shift towards the mirror.
(B) the magnification will reduce.
(C) the image will shift away from the mirror and magnification will increase.
(D) the position of the image and magnification will not change.

Q.31 A pin 10 mm tall is used as an object in front of a mirror and the image formed is erect and 2mm tall.

(A) Moving the pin closer to the mirror will make the image larger.
(B) The image must be real.
(C) The mirror must be converging.
(D) Both (B) and (C) are correct.

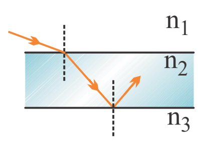
Q.32 It is found that electromagnetic signals sent inside glass sphere from A towards B reach diametrically opposite point C. The speed of electromagnetic signals in glass cannot be:



(A) 1.0×10^8 m/s (B) 2.1×10^8 m/s
(C) 2×10^7 m/s (D) 4×10^7 m/s

Q.33 Light traveling through three transparent substances follows the path shown in figure. Arrange the indices of refraction in order from smallest to largest. Note that total internal reflection does occur on the bottom surface of medium 2.

(A) $n_1 < n_2 < n_3$
(B) $n_2 < n_1 < n_3$
(C) $n_1 < n_3 < n_2$
(D) $n_3 < n_1 < n_2$



Q.34 A ray of light travelling in a medium of refractive index μ is incident at an angle θ on a composite transparent plate consisting of 50 plates of refractive indices 1.01μ, 1.02μ, 1.03μ.....1.50μ. They ray emerges from the composite plate into a medium of refractive index 1.6μ at angle x. Then

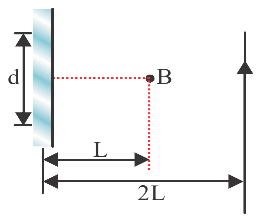
(A) $\sin x = \left(\frac{1.01}{1.5}\right)^{50} \sin\theta$

(B) $\sin x = \frac{5}{8}\sin\theta$

(C) $\sin x = \frac{8}{5}\sin\theta$

(D) $\sin x = \left(\frac{1.5}{1.01}\right)^{50} \sin\theta$

Q.35 A point source of light B, placed at a distance L in front of the centre of a plane mirror of width d, hangs vertically on a wall.

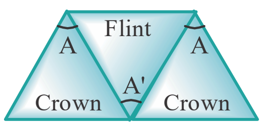


A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. The greatest distance over which he can see the image of the light source in the mirror is

(A) d/2 (B) d

(C) 2d (D) 3d

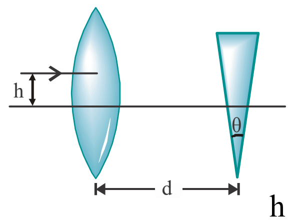
Q.36 Three thin prisms are combined as shown in figure. The refractive indices of the crown glass for red & violet rays are μ_r and μ_v respectively & those for the flint glass are μ'_r and μ'_v respectively. The ratio (A'/A) for which there is no net angular dispersion



(A) $\frac{\mu_v - \mu_r}{2(\mu'_v - \mu'_r)}$ (B) $\frac{2(\mu_v - \mu_r)}{\mu'_v - \mu'_r}$

(C) $\frac{\mu_v - \mu_r}{\mu'_v - \mu'_r}$ (D) None of these

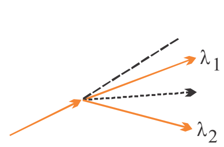
Q.37 A ray of light parallel to the axis of a converging lens (having focal length f) strikes it at a small distance 'h' from its optical centre. A thin prism having angle θ and refractive index μ is placed normal to the axis of lens at a distance 'd' from it. What should be the value of μ so that the ray emerges parallel to the lens axis.

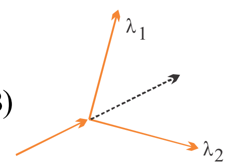


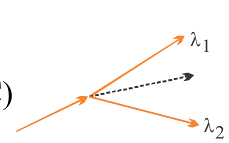
(A) $\frac{h}{f\theta}$ (B) $\frac{h}{f\theta}+1$

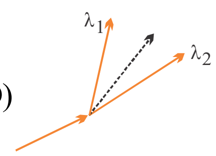
(C) $\frac{h}{(d+f)\theta}$ (D) $\frac{h}{(d+f)\theta}+1$

Q.38 Consider the four different cases of dispersion of light ray which has all the wave lengths from λ_1 to λ_2 ($\lambda_1 > \lambda_2$). The dotted represents the light ray of wave length λ_{avg}. Which ray diagram is showing maximum dispersive power?

(A)


(B)


(C)


(D)


Q.39 In Young's double slit experiment, the distance between the two slits is 0.1 mm and the wavelength of light used is 4×10^{-7}m. If the width of the fringe on the screen is 4 mm, the distance between screen and slit is

(A) 0.1 mm (B) 1 cm

(C) 0.1 cm (D) 1 m

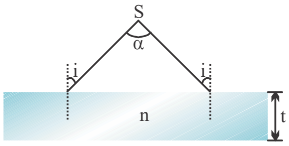
Q.40 In Young's double slit experiment, the distance between sources is 1 mm and distance between the screen and source is 1 m. If the fringe width on the screen is 0.06 cm, then $\lambda =$
(A) 6000 Å (B) 4000 Å
(C) 1200 Å (D) 2400 Å

Q.41 Two slits are separated by a distance of 0.5 mm and illuminated with light of $\lambda = 6000$Å. If the screen is placed 2.5m from the slits. The distance of the third bright image from the centre will be
(A) 1.5 mm (B) 3 mm
(C) 6 mm (D) 9 mm

Q.42 The equation of two light waves are $y_1 = 6\cos\omega t$, $y_2 = 8\cos(\omega t + \phi)$. The ratio of maximum to minimum intensities produced by the superposition of these waves will be
(A) 49 : 1 (B) 1 : 49
(C) 1 : 7 (D) 7 : 1

Q.43 In a Young's experiment , two coherent sources are placed 0.90 mm apart and the fringes are observed one metre away. f it produces the second dark fringe at a distance of 1mm from the central fringe, the wavelength of monochromatic light used would be.
(A) 60×10^{-4} cm (B) 10×10^{-4} cm
(C) 10×10^{-5} cm (D) 6×10^{-5} cm

Q.44 A diverging beam of light from a point source S having divergence angle α falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and its refractive index is μ, then the divergence angle of the emergent beam is :



(A) zero (B) α
(C) $\sin^{-1}(1/\mu)$ (D) $2\sin^{-1}(1/\mu)$

Q.45 In a certain double slit experimental arrangement interference fringes of width 1.0 mm each are observed when light of wavelength 5000Å is used. Keeping the set up unaltered, if the source is replaced by another source of wavelength 6000 Å, the fringe width will be
(A) 0.5 mm (B) 1.0 mm
(C) 1.2 mm (D) 1.5 mm

Q.46 Two parallel slits 0.6 mm apart are illuminated by light source of wavelength 6000Å. The distance between two consecutive dark fringes on a screen 1 m away from the slits is
(A) 1 mm (B) 0.01 mm
(C) 0.1 m (D) 10 m

Q.47 Young's double slit experiment is performed with light of wavelength 550 nm. The separation between the slits is 1.10 mm and screen is placed at distance of 1 m. What is the distance between the consecutive bright or dark fringes
(A) 1.5 mm (B) 1.0 mm
(C) 0.5 mm (D) None of these

Q.48 In a Young's double slit experiment, the slit separation is 1mm and the screen is 1 m from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd minima from the central maxima is –
(A) 0.50 mm (B) 1.25 mm
(C) 1.50 mm (D) 1.75 mm

Q.49 The light of wavelength 6328 Å is incident on a slit of width 0.2 mm perpendicularly, the angular width of central maxima will be
(A) 0.36° (B) 0.18°
(C) 0.72° (D) 0.09°

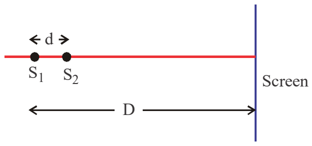
Q.50 When wave of wavelength 0.2 cm is made incident normally on a slit of width 0.004m, then the semi-angular width of central maximum of diffraction pattern will be-
(A) 60° (B) 30°
(C) 90° (D) 0°

Q.51 A screen is placed 2m away from the single narrow slit. Calculate the slit width if the first minimum lies 5mm on either side of the central maximum. Incident plane waves have a wavelength of 5000Å.
(A) 2×10^{-4} m (B) 2×10^{-3} cm
(C) 2×10^{-4} m (D) None

Q.52 In young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by :
(A) 12 (B) 18
(C) 24 (D) 30

Q.53 A beam of light of wavelength 600nm from a distant source falls on a single slit 1.00mm wide and the resulting diffraction pattern is observed on a screen 2m away. The width of the central max. is –
(A) 1.2 cm. (B) 1.2 mm.
(C) 2.4 cm. (D) 2.4 mm.

Q.54 Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be –



(A) points (B) straight lines
(C) semi-circles (D) concentric circles

Q.55 In a Young's double slit interference pattern, the intensity of the central fringe at P = I_o when one slit width is reduced to one fourth the intensity at P will be
(A) $I_o/2$ (B) $I_o/4$
(C) (9/16) I_o (D) (3/4) I_o

Q.56 Interference fringes are obtained in Young's double-slit experiment on a screen. Which of the following statements will be incorrect about the effect of introducing a thin transparent plate in the path of one of the two interfering beams.
(A) The separation between fringes remain unaffected.
(B) The entire fringe system shifts towards the side on which plate is placed
(C) The conditions for maxima and minima are reversed i.e., maxima for odd multiple of $\lambda/2$ and minima for even multiple of $\lambda/2$.
(D) Shape of the fringe also remains unaffected.

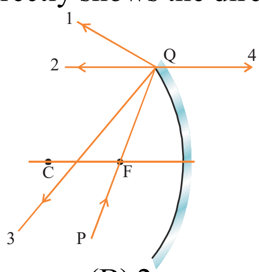
Q.57 In a Young's double–slit experiment, the central bright fringe can be identified
(A) as it has greater intensity than the other bright fringes.
(B) as it is wider than the other bright fringes.
(C) as it is narrower than the other bright fringes.
(D) by using white light instead of monochromatic light.

Q.58 A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is
(A) 7.5° (B) 5°
(C) 15° (D) 2.5°

Q.59 Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall
(A) be a fine sharp slit white in colour at the center.
(B) a bright slit white at the center diffusing to zero intensities at the edges.
(C) a bright slit white at the center diffusing to regions of different colours.
(D) only be a diffused slit white in colour.

Q.60 Which one of the following phenomena is not explained by Huygens construction of wavefront?
(A) Refraction (B) Reflection
(C) Diffraction (D) Origin of spectra

Q.61 The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (Fig). Which of the four rays correctly shows the direction of reflected ray?



(A) 1 (B) 2
(C) 3 (D) 4

SCO -16 District Shopping Centre, Jind, Haryana.
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EXERCISE-3 (LEVEL-3)

Choose one correct response for each question.

Q.1 Light of wavelength $\lambda = 5000$ Å falls normally on a narrow slit. A screen placed at a distance of 1m from the slit and perpendicular to the direction of light. The first minima of the diffraction pattern is situated at 5 mm from the centre of central maximum. The width of the slit is
(A) 0.1 mm (B) 1.0 mm
(C) 0.5 mm (D) 0.2 mm

Q.2 The height of the real image formed by a concave mirror is four times larger than the object height when the object is 30cm infront of the mirror. The radius of curvature of the mirror is –
(A) 80 cm. (B) 60cm.
(C) 48cm. (D) 20cm.

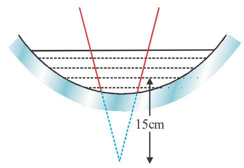
Q.3 A parallel beam of light falls normally on the first face of a prism of small angle A. At the second face it is partly transmitted and partly reflected. Then the reflected beam strike the first face again and emerges from first surface in a direction making an angle 6°30' with the normal at the first surface. The refracted beam is found to have undergone a deviation of 1°15' from original direction. Calculate the angle of prism in degree.
(A) 2 (B) 4
(C) 5 (D) 9

Q.4 Two plane mirrors are placed with reflecting surfaces parallel and facing to each other. A point object is placed between them at a distance 5cm. from first mirror and 3cm. from second mirror. The distance (in cm) between the 3rd image behind first mirror and the 3rd image behind the second mirror is X then find the value of X/12.
(A) 2 (B) 4
(C) 5 (D) 9

Q.5 The central fringe of the interference pattern produced by light of wavelength 6000Å is found to shift to the position of 4^{th} bright fringe after a glass plate of refractive index 1.5 is introduced in front of one slit in Young's experiment. The thickness of the glass plate will be –
(A) 4.8 μm (B) 8.23 μm
(C) 14.98 μm (D) 3.78 μm

Q.6 One face of a rectangular glass plate 6 cm thick is silvered. An object held 8 cm in front of the first face forms an image 12 cm behind the silvered face, then the refractive index of the glass is 6/X. Find the value of X.
(A) 2 (B) 4
(C) 5 (D) 9

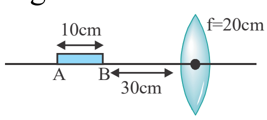
Q.7 A concave mirror of radius of curvature 10cm. is filled with water upto a very small thickness ($\mu_{water} = 4/3$). Paraxial converging rays are incident on the system, whose intersection is 15cm. behind the mirror. At how much distance from the mirror, final image will be formed. Write the answer in cm.



(A) 2 (B) 4
(C) 5 (D) 3

Q.8 A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = 2OQ. The distance PO is equal to –
(A) 5R (B) 8R
(C) 3R (D) 4R

Q.9 AB is linear object placed along optical axis as shown in figure. Tick the correct statement(s) –



(A) The length of image is smaller than the length of object.
(B) The length of image is larger than the length of object.
(C) The length of image is equal to the length of object.
(D) If middle portion of the lens is painted then the length of image is smaller than length of object.

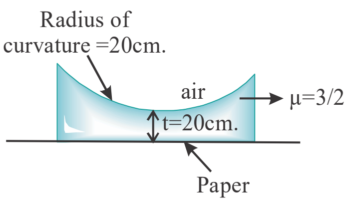
Q.10 Refractive index of a prism is $\sqrt{7/3}$ and the angle of prism is 60°. The minimum angle of incidence of a ray that will be transmitted through the prism is –

(A) 30° (B) 45°
(C) 15° (D) 50°

Q.11 A glass plate 4 mm thick is viewed from the above through a microscope. The microscope must be lowered 2.58 mm as the operator shifts from viewing the top surface to viewing the bottom surface through the glass. What is the index of refraction of the glass ?

(A) 1.61 (B) 1.55
(C) 3.24 (D) 1.21

Q.12 A planoconcave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be –



(A) 10 cm. (B) 15 cm.
(C) 50 cm. (D) None of these

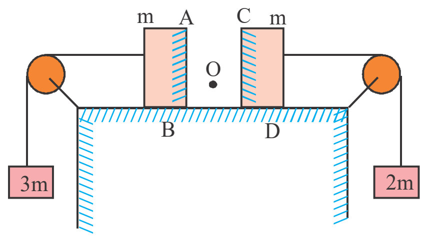
Q.13 A glass prism of refractive index 1.5 and angle of prism 6° is put in contact with another prism of refractive index 1.6 when a ray of light is made incident on this combination normally then it emerges out undeviated. The angle of second prism will be –

(A) 6° (B) 5°
(C) 4° (D) 3°

Q.14 Calculate the dispersive power for crown glass from the given data $\mu_v = 1.5230, \mu_r = 1.5145$

(A) 0.0163 (B) 0.0183
(C) 0.0142 (D) 0.0112

Q.15 Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB and CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is 17g/A then find the value of A –



(A) 2 (B) 4
(C) 5 (D) 6

Q.16 In Young's double slit experiment $\lambda = 500$nm, d = 1mm, D = 1m. Minimum distance from the central maximum for which intensity is half of the maximum intensity is –

(A) 2.5×10^{-4} m (B) 1.25×10^{-4} m
(C) 0.625×10^{-4} m (D) 0.3125×10^{-4} m

For Q.17–Q.19

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

Q.17 As the beam enters the medium, it will –

(A) diverge
(B) converge
(C) diverge near the axis and converge near the periphery
(D) travel as a cylindrical beam

Q.18 The initial shape of the wave front of the beam is

(A) convex
(B) concave
(C) convex near the axis and concave near the periphery
(D) planar

Q.19 The speed of light in the medium is –

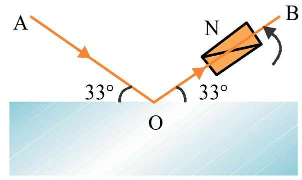
(A) minimum on the axis of the beam
(B) the same everywhere in the beam
(C) directly proportional to the intensity I
(D) maximum on the axis of the beam

Q.20 A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is -
(A) hyperbola (B) circle
(C) straight line (D) parabola

Q.21 A mixture of light, consisting of wavelength 590nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is –
(A) 393.4 nm (B) 885.0 nm
(C) 442.5 nm (D) 776.8 nm

Q.22 A parallel beam of monochromatic light of wavelength 5000Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to –
(A) 0^o (B) 15^o
(C) 30^o (D) 60^o

Q.23 A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in figure. The reflected ray OB is passed through a Nicol prism on viewing through a Nicol prism, we find on rotating the prism that



(A) The intensity is reduced down to zero and remains zero.
(B) The intensity reduces down some what and rises again.
(C) There is no change in intensity
(D) The intensity gradually reduces to zero and then again increases.

Q.24 In a Young's double slit experiment, the slits are 1mm apart are illuminated with a mixture of two wavelengths $\lambda = 750$mm and $\lambda' = 900$mm and distance between slit and screen is 2m. At what minimum distance (in mm) from the common central bright fringe on a screen the bright fringe from one interference pattern coincides with a bright fringe from the other ?
(A) 9 (B) 4
(C) 6 (D) 8

ANSWER KEY

CHECK UP 1

(1) Because when you look at the ƎƆИA⅃UᙠMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you.

(2) **(C)** **(3)** **(B)** **(4)** 11 **(5)** – 8 m/s

CHECK UP 2

(1) With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.

(2) (a) If v is negative, the image formed is real and beyond 2f.

(b) v is always positive i.e. the image lies on the right of the mirror and is thus a virtual image.

(c) f – u is always greater than f
i.e. f – u > f $\therefore$ m < 1 i.e., the image is diminished.

(d) m is positive and greater than 1.
So, the image is enlarged.

(3) Yes. They can produce real images if the object is a virtual object.

(4) **(A)** **(5)** **(D)** **(6)** **(B)**

CHECK UP 3

(1) Sound travels faster in the warmer air, and thus the sound traveling through the warm air aloft will refract much like the light refracting through the nonuniform sugar-water solution. Sound that would normally travel up over the tree-tops can be refracted back towards the ground.

(2) Both words are inverted. However OXIDE has up-down symmetry whereas LEAD does not.

(3) **(A)** **(4)** **(B)** **(5)** **(A)**
(6) **(C)** **(7)** **(A)** **(8)** **(A)**
(9) $r = \sin^{-1} 0.6203 = 38°21'$

CHECK UP 4

(1) Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

(2) Highly silvered mirrors reflect about 98% of the incident light. With a 2-mirror periscope, that results in approximately a 4% decrease in intensity of light as the light passes through the periscope. This may not seem like much, but in low-light conditions, that lost light may mean the difference between being able to distinguish an enemy armada or an iceberg from the sky beyond. Using prisms results in total internal reflection, meaning that 100% of the incident light is reflected through the periscope. That is the "total" in total internal reflection.

(3) **(A)** **(4)** **(A)** **(5)** **(B)** **(6)** **(C)**

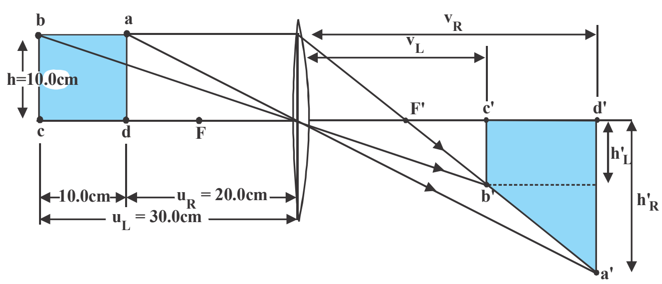
CHECK UP 5

(1) In this case, the index of refraction of the lens material is less than that of the surrounding medium. Under these conditions, a biconvex lens will be diverging.

(2) The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.

(3) (a) f = 13.3 cm.

(b) The square is imaged as a trapezoid.



(c) Area = | 224 cm^2

(4) f = 11.7 cm.

(5) **(D)** **(6) (A)**

(7) f = – 60 cm

Since the focal length is negative, the combination is a diverging lens.

(8) Iimage is formed 7.5 cm away from the lens and it is a real image.

(b) The image formed is at a distance of 48 cm from the lens and it is a real image.

(9) Chromatic aberration arises because a material medium's refractive index can be frequency dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

CHECK UP 6

(1) The light with the greater change in speed will have the larger deviation. If the glass has a higher index than the surrounding medium, X travels slower in the glass.

(2) $i \approx 30°$

(3) (a) Angular dispersion produced by two prisms i.e. crown glass and flint glass should be zero in this case $(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A' = 0$

$A' < A$ because $(\mu'_v - \mu'_r)$ for flint glass prism is more than $(\mu_v - \mu_r)$ for crown glass. Thus crown glass prism of larger angle is to be combined with a smaller angled flint glass prism.

(b) For no deviation,

$(\mu_{yel} - 1)A + (\mu'_{yel} - 1)A' = 0$; $\mu'_y > \mu_y$

In the combination of prisms, flint glass prism of greater and greater angle may be tried but in any case still this angle will be smaller than the angle of the crown glass prism.

(4) **(B)** **(5) (A)** **(6) (A)**

(7) **(C)** **(8) (D)**

CHECK UP 7

(1) The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.

(2) The person can read the letters of 6 m which the normal eye can read from 12 m.

(3) This is due to the defect of lenses called astigmatism. The defect arises because of the fact that curvature of the eye-lens and the cornea is not same in different planes. This defect is removed by using cylindrical lens with vertical axis.

(4) –125

(5) (a) $f_0 + f_e = 140 + 5 = 145$ cm

(b) 4.7 cm

(c) Height of the final image = 28.2 cm

CHECK UP 8

(1) The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no coherence between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

(2) Underwater, the wavelength of the light would decrease, $\lambda_{water} = \dfrac{\lambda_{air}}{\mu_{water}}$. Since the positions of light and dark bands are proportional to λ, the underwater fringe separations will decrease.

(3) Every colour produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With several colours, the patterns are superimposed and it can be difficult to pick out a single maximum. Using monochromatic light can eliminate this problem.

(4) The threads that are woven together to make the cloth have small meshes between them. These bits of space act as pinholes through which the light diffracts. Since the cloth is a grid of such pinholes, an interference pattern is formed, as when you look through a diffraction grating.

(5) If the oil film is brightest where it is thinnest, then $\mu_{air} < \mu_{oil} < \mu_{water}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo phase reversal. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero.

(6) As water evaporates from the 'soap' bubble, the thickness of the bubble wall approaches zero. Since light reflecting from the front of the water surface is phase-shifted 180° and light reflecting from the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus the soap film appears black.

(7) If the film is more than a few wavelengths thick, the interference fringes are so close together that you cannot resolve them.

(8) If R is large, light reflecting from the lower surface of the lens can interfere with light reflecting from the upper surface of the flat. The latter undergoes phase reversal on reflection while the former does not. Where there is negligible distance between the surfaces, at the center of the pattern you will see a dark spot because of the destructive interference associated with the 180° phase shift. Coloured rings surround the dark spot. If the lens is a perfect sphere the rings are perfect circles. Distorted rings reveal bumps or hollows on the fine scale of the wavelength of visible light.

(9) To do Young's double-slit interference experiment with light from an ordinary source, you must first pass the light through a prism or diffraction grating to disperse different colours into different directions.

With a single narrow slit you select a single colour and make that light diffract to cover both of the slits for the interference experiment. Thus you may have trouble lining things up and you will generally have low light power reaching the screen. The laser light is already monochromatic and coherent across the width of the beam.

(10) $d = 3.44 \times 10^{-4}$ m.

(11) $\beta = \dfrac{7\text{mm}}{3.5} = 2\text{mm} \Rightarrow \lambda = \dfrac{\beta D}{d} = 600\text{nm}$

(12) Treating the anti-reflectance coating like a camera-lens coating,

$$2t = \left(n + \frac{1}{2}\right)\frac{\lambda}{\mu}$$

Let $m = 0$: $t = \dfrac{\lambda}{4\mu} = \dfrac{3.00\text{ cm}}{4\,(1.50)} = 0.500$ cm.

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection.

CHECK UP 9

(1) The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

(2) The intensity of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \lambda / a$ for destructive interference on each side of the central maximum, θ increases as a decreases.

(3) (a) In a single slit diffraction experiment, if the width of the slit is made double the original width, then the size of the central diffraction band reduces to half and the intensity of the central diffraction band increases up to four times.

(b) The interference pattern in a double-slit experiment is modulated by diffraction from each slit. The pattern is the result of the interference of the diffracted wave from each slit.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. This is because light waves are diffracted from the edge of the circular obstacle, which interferes constructively at the centre of the shadow. This constructive interference produces a bright spot.

(d) Bending of waves by obstacles by a large angle is possible when the size of the obstacle is comparable to the wavelength of the waves.

On the one hand, the wavelength of the light waves is too small in comparison to the size of the obstacle. Thus, the diffraction angle will be very small. Hence, the students are unable to see each other. On the other hand, the size of the wall is comparable to the wavelength of the sound waves. Thus, the bending of the waves takes place at a large angle. Hence, the students are able to hear each other.

(4) (a) L = 1.09 m
(b) w = 1.70 mm

CHECK UP 10

(1) Light from the sky is partially polarized. Light from the blue sky that is polarized at 90° to the polarization axis of the glasses will be blocked, making the sky look darker as compared to the clouds.

(2) The fraction transmitted is 25%.

(3) $i_c = 35°16'$

EXERCISE - 1

(1) **(A)** **(2) (C)** **(3) (D)**
(4) **(A)** **(5) (B)**
(6) Infinite **(7)** Increases **(8)** 1.5
(9) **False.** Complete image will be formed
(10) Greater **(11)** Twice
(12) Virtual, Diminished **(13)** Critical angle
(14) Power **(15)** True **(16)** Violet, Red
(17) Red , Violet **(18)** Violet, Red
(19) 256 **(20)** False
(21) True **(22)** Small
(23) False **(24)** True

EXERCISE 1 (SECTION–3)

Q	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
A	C	B	B	B	A	A	A	C	B	A	D	D	A	A	D	B	B	B	D	D	C	C	B	B	C
Q	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
A	B	B	B	C	D	A	C	B	B	D	D	D	C	A	D	B	B	A	A	A	B	A	A	B	C
Q	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
A	D	A	A	B	B	A	B	D	C	A	B	C	C	D	D	A	A	D	B	A	B	A	C	B	A
Q	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124
A	D	B	B	B	D	C	B	D	A	B	D	C	D	B	C	A	A	B	A	A	B	C	D	D	C
Q	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149
A	B	D	B	A	C	C	A	C	B	B	D	C	C	C	C	B	D	B	C	B	A	A	B	C	A
Q	150																								
A	B																								

EXERCISE–2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	D	A	A	C	B	D	C	B	B	A	B	A	A	B	A	C	B	B	D	B	C	B	B	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	B	D	C	D	A	B	D	B	D	B	B	B	D	A	D	A	D	B	C	A	C	B	A	B
Q	51	52	53	54	55	56	57	58	59	60	61														
A	A	B	D	D	C	C	D	A	A	D	B														

EXERCISE–3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A	A	C	A	B	A	C	D	B	B	A	B	A	B	A	D	B	B	D	A	A	C	C	D	A