

ATOMS AND NUCLEI

LEARNING OBJECTIVES

1. Alpha- Particle Scattering Experiments.
2. Rutherford's Model of Atom.
3. Bohr Model, Energy Levels, Hydrogen Spectrum.
4. Composition and Size of Nucleus, Atomic Masses, Isotopes, Isobars and Isotones.
5. Radioactivity- Alpha, Beta and Gamma Particles/ Rays and their Properties, Decay Law.
6. Mass-Energy Relation, Mass Defect.
7. Binding Energy per Nucleon and Its Variation with Mass Number.
8. Nuclear Fission and Fusion.



KEY CONCEPTS

SECTION - 1 : ATOMIC STRUCTURE

9.1

DALTON'S ATOMIC THEORY

- * Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed.
- * All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties.
- * The atoms of different elements are made up of hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen).
- * The atom is stable and electrically neutral.

9.2

THOMSON'S ATOMIC MODEL

- * The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere.

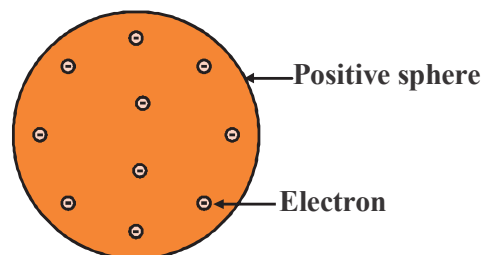


Figure : Thomson's Model of an Atom

- * Atom is a positively charged sphere of radius 10^{-10}m in which electron are embedded in between.
- * The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.
- * Shortcomings of Thomson's model
 - (i) The spectrum of atoms cannot be explained with the help of this model.
 - (ii) Scattering of α -particles cannot be explained with the help of this model.

9.3

RUTHERFORD MODEL

- * In Rutherford experiment α -particles particle are emitted by some radioactive material (polonium), kept inside a thick lead box.
- * A very fine beam of α -particles pass through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil.
- * While passing through the gold foil, α -particles are scattered through different angles.

- * The scattered alpha-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope.
- * The scattered alpha-particles on striking the screen produced brief light flashes or scintillations.
- * These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.

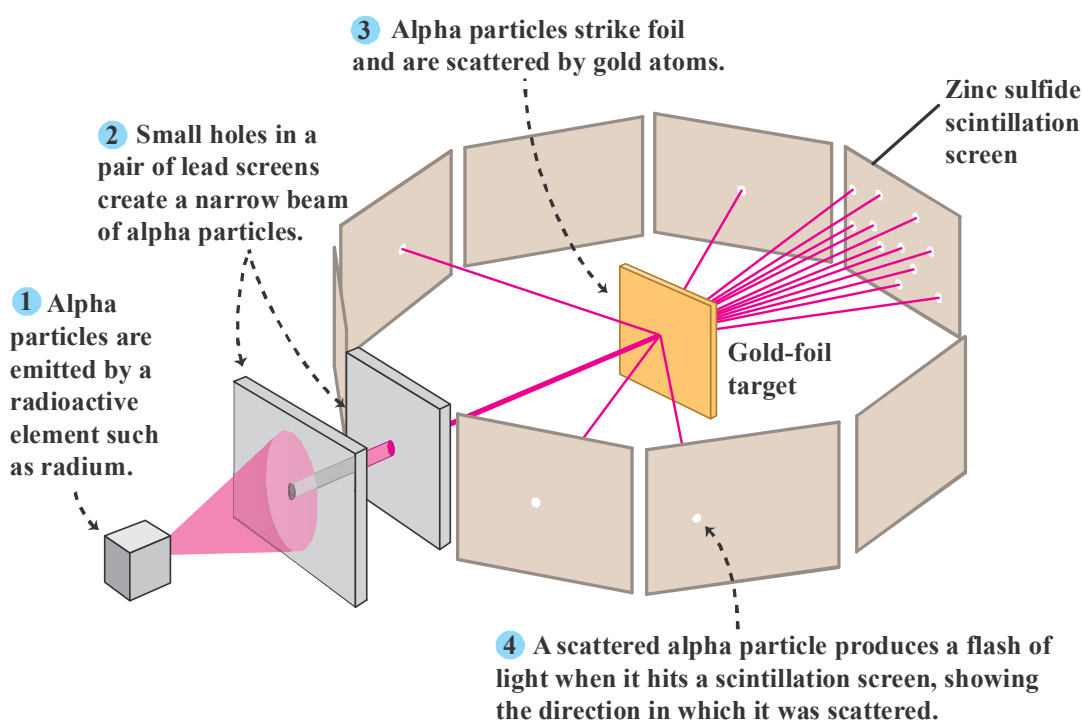


Figure : The Rutherford scattering experiments investigated what happens to alpha particles fired at a thin gold foil. The results of this experiment helped reveal the structure of atoms.

Rutherford's α -Particle Scattering

Experiment Observations

- (i) Most of the α -particles passed through the gold foil undeflected.
- (ii) A small fraction of the α -particles was deflected by small angles.
- (iii) A very few α -particles (~ 1 in 20,000) bounced back, that is, were deflected by nearly 180° .
- * The centre portion of the atom where all the mass is concentrated is called the nucleus.

- * An atom is extremely hollow with a lot of space that is empty. Almost all the atoms are due to the presence of protons & neutrons in the nucleus.
- * The electrons move extremely rapidly about the nucleus and the space they occupy as they move defines the volume of the atom.
- * The radius of the nucleus is of the order of 10^{-13} cm or 10^{-15} m.
- * According to Rutherford scattering formula, the number of α -particles scattered at angle θ by a target is : $N(\theta) \propto \text{cosec}^4(\theta/2)$

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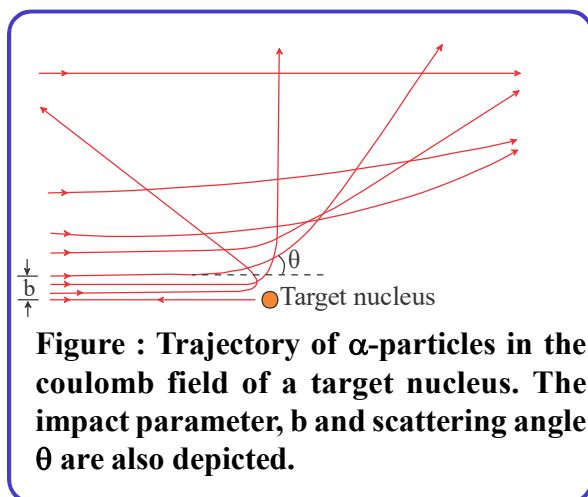
- * **The impact parameter** is the perpendicular distance of the initial velocity vector of the α -particle from the centre of the nucleus.
- (ii) If electron will continuously radiate energy then its spectra should be continuous but this is not the case. Atoms give line spectra.

$$b = \frac{2Ze^2 \cot(\theta/2)}{4\pi\epsilon_0 mv^2}$$

- * **Distance of closest approach :**
When α particle is turned the kinetic energy must be converted to electric potential energy since collision is elastic

$$\frac{1}{2}mv^2 = \frac{K(2e)(Ze)}{d}$$

$$\text{distance of closest approach } d = \frac{4K Ze^2}{mv^2}$$



A Comparison of Thomson's and Rutherford's Models of the Atom

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.

(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).

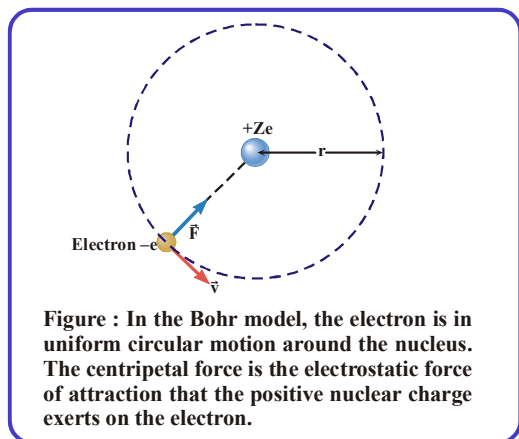
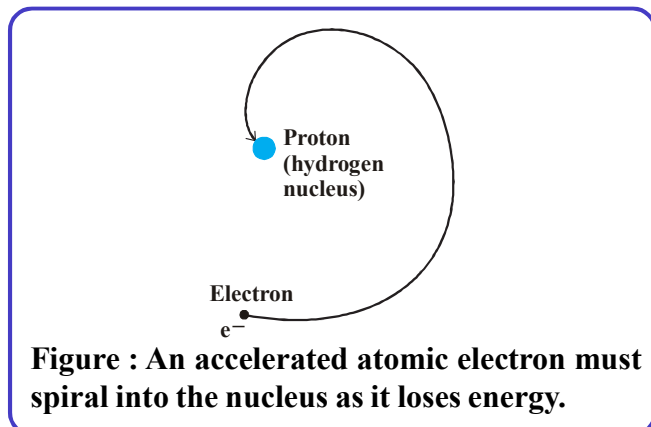
9.4 BOHR'S MODEL

- * Attractive coulomb force between electron and nucleus provide necessary centripetal force.

$$\frac{mv^2}{r} = K \frac{Ze^2}{r^2}$$

Defects in Rutherford's Model of Atom

- (i) When electron revolve continuously around the positively charged nucleus then energy would be lost and due to the attraction of nucleus electron will merge with the nucleus.



- * **Rule of Stable Orbits :** Electron orbits around nucleus in only those orbits where angular momentum is an integral multiple of $\frac{h}{2\pi}$.

$$mvr = n \frac{h}{2\pi}$$

$$\left[\because \hbar = \frac{h}{2\pi} \right]$$

- * Electromagnetic radiations are emitted if an electron jumps from stationary orbit of higher energy E_2 to another stationary orbit of lower energy E_1 .

The frequency ν of the emitted radiation is related by the equation.

$$E_2 - E_1 = h\nu$$

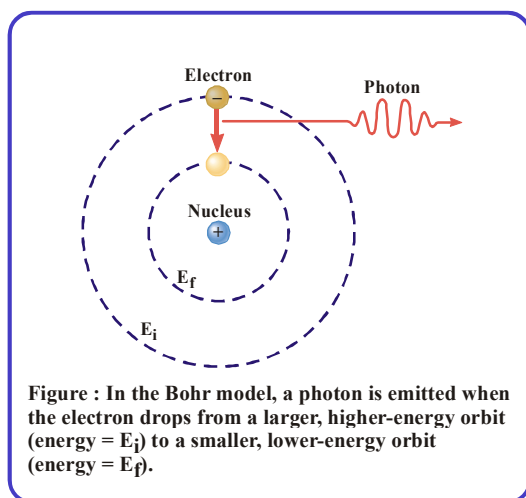


Figure : In the Bohr model, a photon is emitted when the electron drops from a larger, higher-energy orbit (energy = E_i) to a smaller, lower-energy orbit (energy = E_f).

Defects of Bohr Model

- * This model could not explain the fine structure of spectral lines, Zeeman effect and Stark effect.
- * This model is valid only for single electron systems.
- * An orbit of the electron in the Bohr model is the circular path of motion of an electron around the nucleus. But according to quantum mechanics, we cannot associate a definite path with the motion of the electrons in an atom. We can only talk about the probability of finding an electron in a certain region of space around the nucleus. This probability can be inferred from the one-electron wave function called the **orbital**. This function depends only on the coordinates of the electron.
- * This model could not explain the intensity of spectral lines.
- * It could not explain the doublets obtained in the spectra of some of the atoms.

Application of Bohr's Theory

(i) The Radius of n^{th} Orbit

$$(a) r_n = \frac{n^2 h^2}{4\pi^2 k Z e^2 m} ; (b) r_n \propto \frac{n^2}{mZ}$$

$$(c) \text{ For hydrogen, } Z = 1, r_n = 0.529 n^2 \text{ \AA}$$

$$r_1 : r_2 : r_3 = 1 : 4 : 9$$

$$r_H : r_{He^+} : r_{Li^{2+}} = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 1$$

n - order of orbit or principal quantum number

Z - Atomic number of element

m - Mass of particle like electron, Meuon, etc. rotating about nucleus.

(ii) The Velocity of Electron in n^{th} Orbit

$$(a) V_n = \frac{2\pi K Z e^2}{nh} ; (b) V_n = \frac{Z}{n}$$

$$(c) V_n = \frac{c}{137 n}$$

(d) For hydrogen,

$$v_n = \frac{2.188 \times 10^6}{n} = \frac{c}{137 n} \text{ m/s}$$

$$v_1 : v_2 : v_3 = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

(iii) Frequency (f_n) of Electron in n^{th} Orbit

$$(a) f_n = \frac{4\pi^2 K^2 Z^2 e^4 m}{n^3 h^3} ; (b) f_n \propto \frac{Z^2 m}{n^3}$$

$$(c) f_n = \frac{6.62 \times 10^{15} Z^2}{n^3} \text{ Hz}$$

(iv) The period (T_n) of an electron in n^{th} orbit

$$(a) T_n = \frac{n^3 h^3}{4\pi^2 m e^4 K^2 Z^2} ; (b) T_n \propto \frac{n^3}{Z^2 m}$$

$$(c) T_n = \frac{1.5 \times 10^{-16} n^3}{Z^2} \text{ sec}$$

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(v) Current (I_n) due to Orbital Motion

$$(a) I_n = ef_n = \frac{4\pi^2 K^2 Z^2 e^5 m}{n^3 h^3}$$

$$(b) I_n \propto \frac{Z^2 m}{n^3}; (c) I_n = \frac{1.06 Z^2}{n^3} \text{ mA}$$

(vi) Magnetic Field (B_n) at Nucleus due to Orbital Motion of Electron

$$(a) B_n = \frac{\mu_0 I_n}{2r_n} = \frac{8\pi^4 K^3 Z^3 e^7 m^2}{n^5 h^5}$$

μ_0 = Magnetic permeability in vacuum

$$(b) B_n \propto \frac{Z^3 m^2}{n^5}; (c) B_n = \frac{12.58 Z^3}{n^5} \text{ T}$$

(vii) Magnetic Moment

$$(a) M_n = I_n A_n = \pi r_n^2 I_n$$

$$(b) M_n = \frac{eh}{4\pi m} n$$

(c) If $n = 1$, then

$$M = \frac{eh}{2m} = 9.26 \times 10^{-24} \text{ A-m.}$$

It is called **Bohr Magnetron**

(viii) Potential Energy (U_n) in n^{th} Orbit

$$U_n = \frac{-KZe^2}{r_n} = \frac{-27.2}{n^2} Z^2 \text{ eV}$$

$$\text{For H-atom, } U_n = \frac{-Ke^2}{r_n}$$

(ix) Kinetic Energy (E_{kn}) in n^{th} Orbit

$$E_{kn} = \frac{KZe^2}{2r_n} = \frac{13.6 Z^2}{n^2} \text{ eV}$$

$$\text{For H-atom, } E_{kn} = \frac{Ke^2}{2r_n}$$

(x) Total Energy in n^{th} Orbit

= Kinetic energy + Potential energy

$$E_n = U_n + E_{kn}$$

$$(a) E_n = \frac{-KZe^2}{2r_n} = \frac{-2\pi^2 k^2 me^4 Z^2}{n^2 h^2}$$

$$(b) E_n = \frac{-RChZ^2}{n^2},$$

$$R = \text{Rydberg constant} = \frac{2\pi^2 K^2 me^4}{ch^3}$$
$$= 1.1 \times 10^7 \text{ m}^{-1}$$

$$Rhc = 1 \text{ Rydberg energy} = 13.6 \text{ eV}$$

$$(c) E_n = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

(d) For H atom $E_1 = -13.6 \text{ eV}$,
 $E_2 = -3.40 \text{ eV}$, $E_3 = -1.51 \text{ eV}$

$$(e) TE = -KE = \frac{PE}{2}$$

(xi) Ionization Energy of Electron E_{ion}

$$(a) E_{ion.} = E_{\infty} - E_n$$

$$(b) E_{ion.} = E_n = \frac{13.6 Z^2}{n^2} \text{ eV}$$

Ionisation energy of H-atom = 13.6 eV

Ionisation energy of He^+ = 54.4 eV

(xii) Ionization Potential of Electron V_{ion}

$$(a) V_{ion.} = \frac{E_n \text{ (in J)}}{e} = \frac{13.6 Z^2 \text{ (in V)}}{n^2}$$

$$(b) V_{ion.} \propto \frac{Z^2}{n^2}$$

(xiii) Excitation Energy of Electron E_{ext}

$$E_{ext.} = E_{high} - E_{low}$$

For hydrogen atom,

E_{ext} of 1st excited state

$$= E_2 - E_1 = (-3.4) - (-13.6) = 10.2 \text{ eV}$$

$$E_{\text{ext}} \text{ of 2nd excited state} \\ = E_3 - E_1 = (-1.51) - (-13.6) = 12.09 \text{ eV}$$

$$E_{\text{ext}} \text{ of 3rd excited state} \\ = E_4 - E_1 = (-0.85) - (-13.6) = 12.75 \text{ eV}$$

Quantum State	Energy
$n = \infty$ _____	0
$n = 5$ _____	$-13.6/5^2 = -0.54$
$n = 4$ _____	$-13.6/4^2 = -0.85$
$n = 3$ _____ 2nd Excited State	$-13.6/3^2 = -1.51$
$n = 2$ _____ 1st Excited State	$-13.6/2^2 = -3.4$
$n = 1$ _____ Ground State	-13.6 eV

Figure : Energy levels of Hydrogen atoms

(xiv) Binding Energy of Electron E_{BE}

$$E_{BE} = -E_n$$

BE of e^- of H-atom in $n = 4$ level is 0.85 eV
 BE of 1st excited state of H-atom is 3.4 eV
 BE of 1st excited state of He^+ atom is 13.6 eV.

Wavelength of Radiation

* When an electron jumps from the state n_2 to the lower state n_1 (i.e. $n_2 > n_1$). The loss in energy ($E_{n_2} - E_{n_1}$) is emitted as a photon of radiation.

* The wavelength of the radiation is given by

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad [\text{For H-like atom}]$$

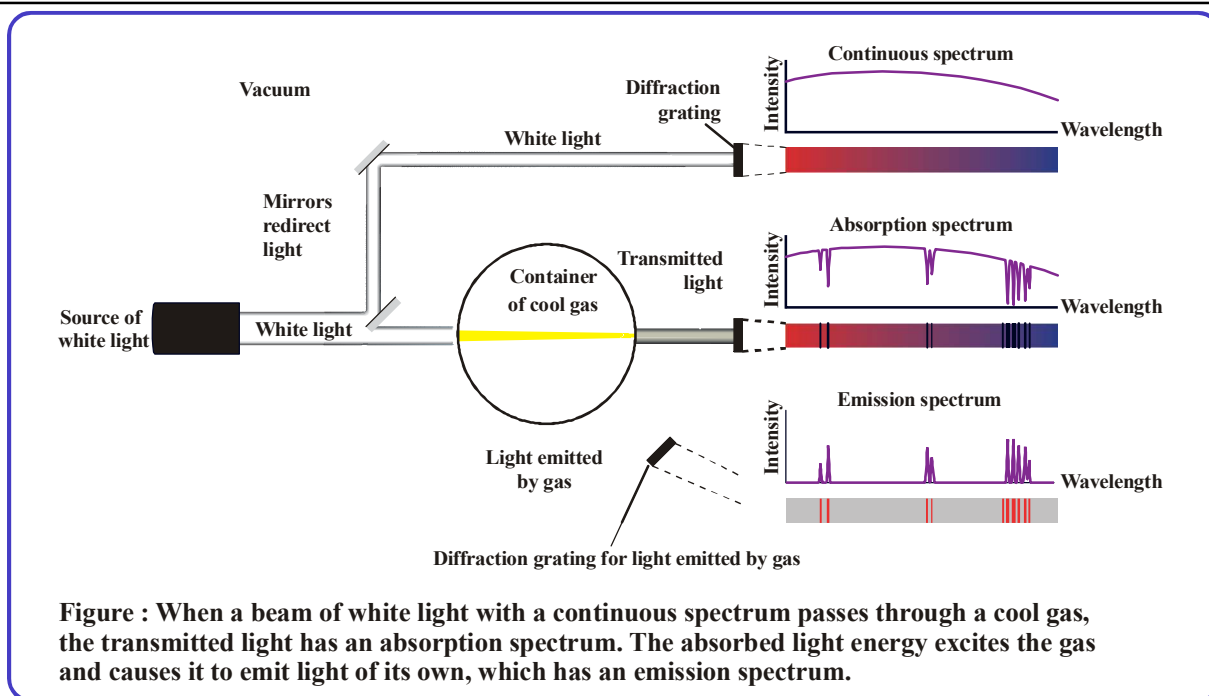
* $1/\lambda$ is called **wave number**.

$$R = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1}$$

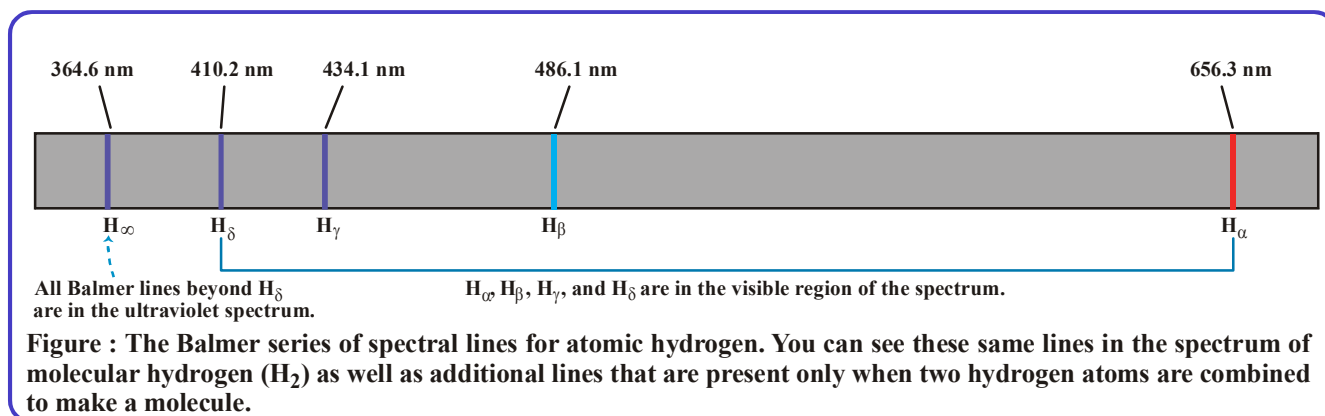
9.5

EMISSION AND ABSORPTION SPECTRA

- * When atoms or molecules absorb certain portions of radiant energy, the remaining portion of the radiation produces **absorption spectrum**.
- * When a white light is allowed to pass through a tube containing sodium vapour and then through a prism, an absorption spectrum results.
- * When a solid or liquid or gas is heated or subjected to electric field, electrons in the atoms are excited. When excited electrons return to their normal or ground state, they give off light of characteristic colour. When this light is passed through a prism, an **emission spectrum** results.
- * The emission spectrum may be continuous or discontinuous.
- * When gases or vapours are subjected to an electric discharge, light is emitted. When this emitted light is passed through a prism and then analysed with a spectroscope, a **discontinuous spectrum** consisting of series of sharp lines with dark areas in between is obtained. It is called line spectrum. In other words, a spectrum, which consists of discrete lines is called **line spectrum**.
- * The line spectra are characteristic of atoms and so line spectrum is also called **atomic spectrum**.
- * A spectrum which consists of a series of very closely spaced lines is called **band spectrum**.
- * The spectrum of sunlight has dark lines called **Fraunhofer lines**.
- * Oil flame spectrum is a continuous emission spectrum.
- * Hydrogen spectrum is a line spectrum. It is the simplest of the spectra of gases because hydrogen has only one electron.
- * The spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way. Each of these sets is called a **spectral series**.



- * In 1885, the first such series was observed by a Balmer in the visible region of the hydrogen spectrum. This series is called **Balmer series**.
- * The line with the longest wavelength, 656.3 nm in the red is called H_α ; the next line with wavelength 486.1 nm in the blue green is called H_β , the third line 434.1 nm in the violet is called H_γ ; and so on.
- * As the wavelength decreases, the lines appear closer together and are weaker in intensity.



9.6 VARIOUS SERIES OF HYDROGEN SPECTRUM

(i) **Lyman series** $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n_2^2} \right]$,

where $n_2 = 2, 3, 4, \dots$

$n_2 = 2$ for first member of Lyman series.

$n_2 = 3$ for second member of Lyman series.

.....

This series lies in ultraviolet region of the spectrum.

Series limit or Minimum wavelength

$$\lambda_{\min} = \frac{1}{R} = 912 \text{ \AA}$$

Maximum wavelength

(First member $n_1 = 1$ and $n_2 = 2$)

$$\lambda_{\max} = \frac{4}{3} R = 1216 \text{ \AA}$$

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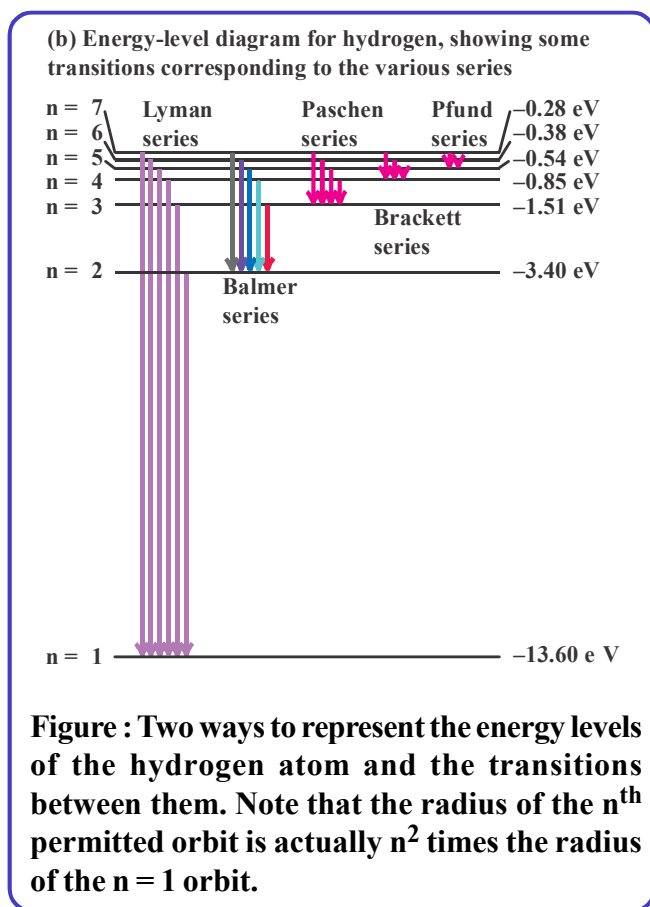
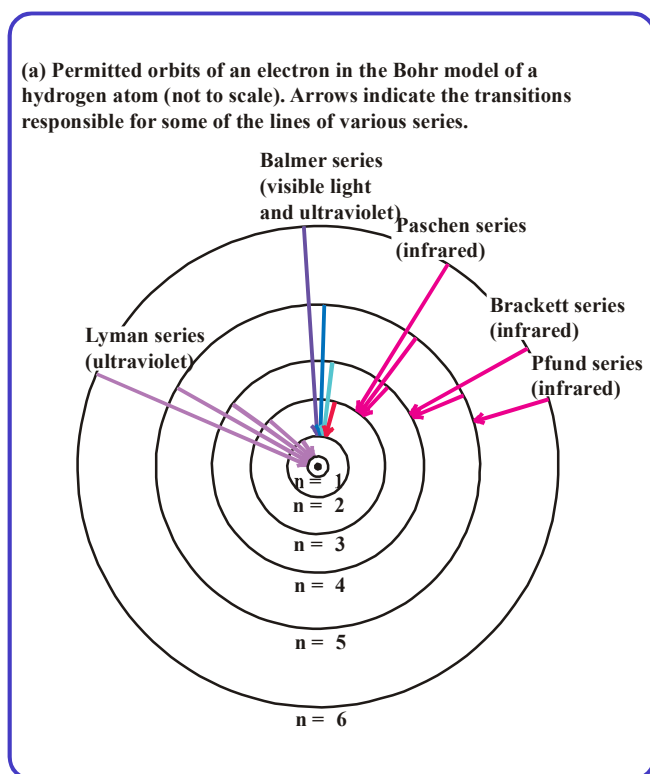


Figure : Two ways to represent the energy levels of the hydrogen atom and the transitions between them. Note that the radius of the n^{th} permitted orbit is actually n^2 times the radius of the $n = 1$ orbit.

(ii) **Balmer series** $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right]$,

where $n_2 = 3, 4, 5 \dots$

This series lies in the visible part of the spectrum.

Minimum wavelength $\lambda_{\min} = \frac{4}{R} = 3646 \text{ \AA}$

Maximum wavelength $\lambda_{\max} = \frac{36}{5R} = 6563 \text{ \AA}$

(iii) **Paschen series** $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{n_2^2} \right]$,

where, $n_2 = 4, 5, 6 \dots$

This series lies in the infra red region of the spectrum.

Minimum wavelength $\lambda_{\min} = \frac{9}{R} = 8204 \text{ \AA}$

Maximum wavelength $\lambda_{\max} = \frac{144}{7R} = 18752.4 \text{ \AA}$

(iv) **Brackett series**

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{4^2} - \frac{1}{n_2^2} \right]$$

where, $n_2 = 5, 6, 7$

(v) **Pfund series** $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{5^2} - \frac{1}{n_2^2} \right]$,

where, $n_2 = 6, 7, 8 \dots$

(vi) Number of lines in emission spectrum

$$= \frac{n(n-1)}{2}$$

9.7 HYDROGEN-LIKE ATOMS

* We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}) and so on. Such atoms are called hydrogenlike atoms.

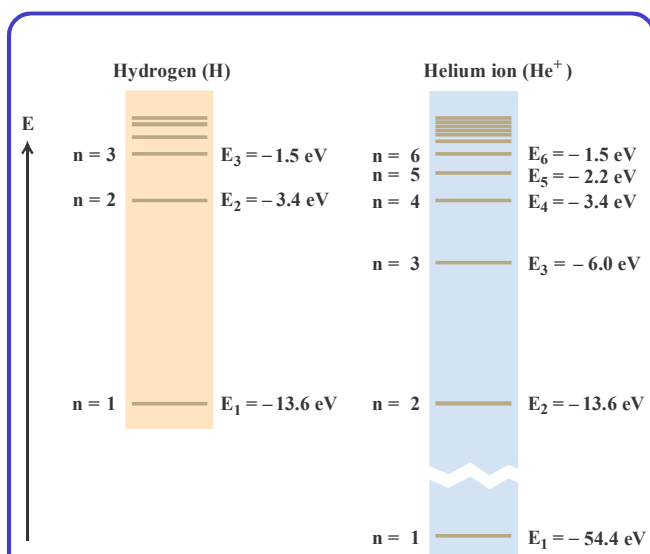


Figure : Energy levels of H and He⁺. The energy expression, is multiplied by $Z^2 = 4$ for He⁺ so the energy of an He⁺ ion with a given n is almost exactly four times that of an H atom with the same n. (There are small differences of the order of 0.05% because the reduced masses are slightly different.)

- * In such atoms, the nuclear charge is not e but Ze where Z is the atomic number, equal to the number of protons in the nucleus.
- * The effect in the previous analysis is to replace e^2 everywhere by Ze^2 .
- * In particular, the orbital radii become smaller by a factor of Z, and the energy levels are multiplied by Z^2 .

9.8

FRANCK – HERTZ EXPERIMENT

- * The existence of discrete energy levels in an atom was directly verified in 1914 by James Franck and Gustav Hertz. They studied the spectrum of mercury vapour when electrons having different kinetic energies passed through the vapour.
- * The electrons collide with the mercury atoms and can transfer energy to the mercury atoms. This can only happen when the energy of the electron is higher than the energy difference between an energy level of Hg occupied by an electron and a higher unoccupied level.
- * By direct measurement, Franck and Hertz found that the emission spectrum of mercury has a line corresponding to this wavelength.

For this experimental verification of Bohr's basic ideas of discrete energy levels in atoms and the process of photon emission, Frank and Hertz were awarded the Nobel prize in 1925.

9.9

LASER LIGHT

- * LASER stands for Light Amplification by Stimulated Emission of Radiation.
- * Light is emitted from a source in the form of packets of waves.
- * Light coming out from an ordinary source contains a mixture of many wavelengths. There is also no phase relation between the various waves. Therefore, such light, even if it is passed through an aperture, spreads very fast and the beam size increases rapidly with distance.
- * In the case of laser light, the wavelength of each packet is almost the same. Also the average length of the packet of waves is much larger. This means that there is better phase correlation over a longer duration of time. This results in reducing the divergence of a laser beam substantially.
- * If there are N atoms in a source, each emitting light with intensity I, then the total intensity produced by an ordinary source is proportional to NI, whereas in a laser source, it is proportional to N^2I . Considering that N is very large, we see that the light from a laser can be much stronger than that from an ordinary source.



- * Ground state energy of H-atom = -13.6 eV
- * Ground state energy of He⁺ atom = -54.4 eV
- * Ground state energy of Li⁺⁺ atom = -122.4 eV
- * If the frequency of emitted photon from an atom of mass M is 'f' then the recoil energy of the

$$\text{atom is } \frac{h^2 f^2}{2Mc^2}.$$

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* For faster calculation remember,

$$\lambda = \frac{hc}{eE} \approx \frac{12400}{E \text{ (eV)}} \text{ \AA}$$

Different Series of Hydrogen Spectrum

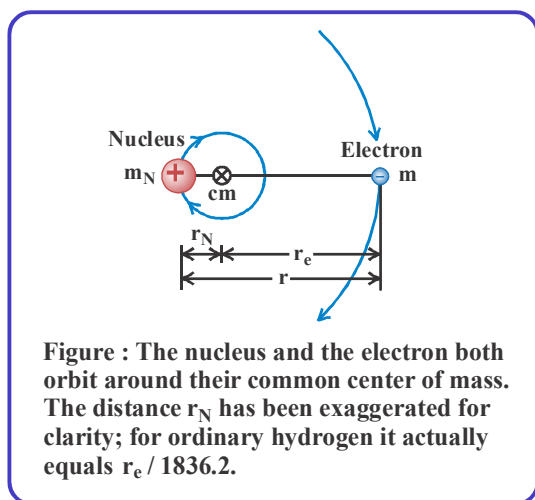
Series	n_1	n_2	Region
Lyman	1	2, 3, 4	Ultra violet
Balmer	2	3, 4, 5	Visible
Paschen	3	4, 5, 6, ..	Infra red
Brackett	4	5, 6, 7...	Infra red
Pfund	5	6, 7, 8 ..	Infra red

Effect of mass of nucleus on Bohr model :

In Bohr model it is assumed that the nucleus (a proton) remains at rest.

* However, as Fig. shows, the proton and electron both revolve in circular orbits about their common center of mass.

It turns out that we can take this motion into account very simply by using in Bohr's equations not the electron rest mass m but a quantity called the reduced mass μ of the system.



We can analyze the motion of electron with respect to nucleus by assuming nucleus to be at rest and the mass of electron replaced by its

reduced mass μ , given as
$$\mu = \frac{m_N m_e}{m_N + m_e}$$

Expression of energy of electron in n^{th} orbit of

Bohr model :
$$E'_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \left(\frac{\mu}{m} \right)$$

EXAMPLE 1

A hydrogen atom in the ground state is excited by radiations of wavelength 975 \AA . Find :

- (a) the energy state to which the atom is excited.
 (b) how many lines will be possible in emission spectrum

SOLUTION:

(a) $\lambda = 975 \text{ \AA} = 975 \times 10^{-10} \text{ m}$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n^2} \right]$$

$$\frac{1}{975 \times 10^{-10}} = 1.1 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{n^2} \right] \text{ or } n = 4$$

(b) $n = 4$

$$\text{Number of spectral lines (N)} = \frac{n(n-1)}{2}$$

$$N = \frac{4 \times (4-1)}{2} = 6$$

Possible transition $4 \rightarrow 3, 4 \rightarrow 2, 4 \rightarrow 1,$
 $3 \rightarrow 2, 3 \rightarrow 1, 2 \rightarrow 1$

EXAMPLE 2

Find the first and second excitation potentials of an atom when its ionisation potential is 122.4 V .

SOLUTION:

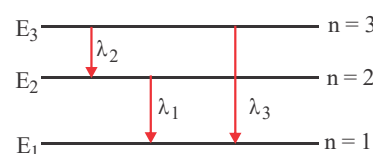
I.P. = 122.4 V

$$E_{\text{ex1}} = 122.4 - \frac{122.4}{4} = 91.8 \text{ V}$$

$$E_{\text{ex2}} = 122.4 - \frac{122.4}{9} = 108.8 \text{ V}$$

EXAMPLE 3

For the given transition of electron, obtain the relation between λ_1, λ_2 & λ_3 .



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SOLUTION:

For given condition,

$$E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$$

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_1}; \quad \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

EXAMPLE 4

Find the maximum wavelength of Brakett series of hydrogen atom.

SOLUTION:

$$n_1 = 4 \text{ and } n_2 = 5$$

$$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{4^2} - \frac{1}{5^2} \right]$$

$$\text{or } \lambda_{\max} = \frac{25 \times 16 \times 10^{10}}{9 \times 1.1 \times 10^7} = 40400 \text{ \AA}$$

EXAMPLE 5

Find the ratio of wavelength of first line of Lyman series of doubly ionised lithium atom to that of the first line of Lyman series of deuterium (${}_1\text{H}^2$).

SOLUTION:

For deuterium (${}_1\text{H}^2$)

$$\frac{1}{\lambda_D} = R \times 1^2 \times \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

For lithium (Li^{+2})

$$\frac{1}{\lambda_{\text{Li}}} = R \times 3^2 \times \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\frac{\lambda_{\text{Li}}}{\lambda_D} = \frac{1}{9} = 1:9$$

EXAMPLE 6

Find the ratio of equivalent current due to electron motion in first and second orbits of hydrogen atom.

SOLUTION:

$$I_n \propto \frac{1}{n^3} \quad \therefore \frac{I_1}{I_2} = \left[\frac{n_2}{n_1} \right]^3 = \left[\frac{2}{1} \right]^3 = 8:1$$

EXAMPLE 7

Find the ratio of the area of orbit of first excited state of electron to the area of orbit of ground level for hydrogen atom.

SOLUTION:

$$A \propto r^2 \propto n^4$$

$$\frac{A_2}{A_1} = \left[\frac{2}{1} \right]^4 = \frac{16}{1} = 16:1$$

EXAMPLE 8

If the ionisation potential in the ground state for hydrogen is 13.6 e.V., then find the excitation potential of third orbit.

SOLUTION:

$$\text{I.P.} = 13.6 \text{ eV}$$

$$E_4 - E_3 = \frac{13.6}{4^2} - \left[\frac{-13.6}{3^2} \right] = 0.66 \text{ eV}$$



Checkup 1

Q.1 Which one of the following statements is true?

(a) An atom is less easily ionized when its outermost electron is in an excited state than when it is in the ground state. (b) An atom is more easily ionized when its outermost electron is in an excited state than when it is in the ground state. (c) The energy state (excited state or ground state) of the outermost electron in an atom has nothing to do with how easily the atom can be ionized.

Q.2 An electron in the hydrogen atom is in the $n=4$ energy level. When this electron makes a transition to a lower energy level, is the wavelength of the photon emitted in (a) the Lyman series only, (b) the Balmer series only, (c) the Paschen series only, or (d) could it be in the Lyman, the Balmer, or the Paschen series?

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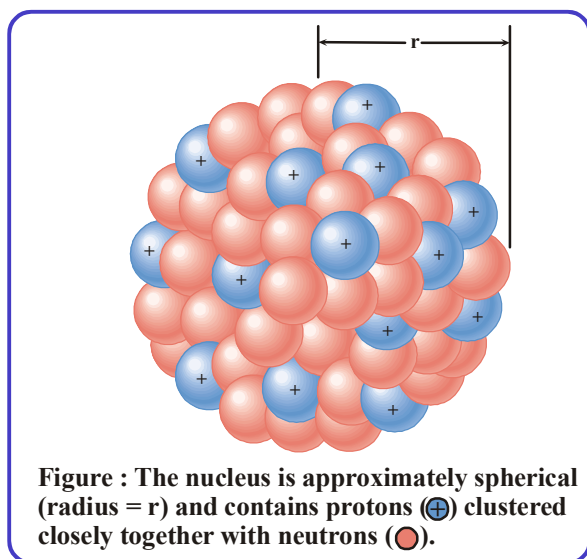
- Q.3** A tube contains atomic hydrogen, and nearly all of the electrons in the atoms are in the ground state or $n = 1$ energy level. Electromagnetic radiation with a continuous spectrum of wavelengths (including those in the Lyman, Balmer, and Paschen series) enters one end of the tube and leaves the other end. The exiting radiation is found to contain strong absorption lines. To which one or more of the series do the wavelengths of these absorption lines correspond? Assume that once an electron absorbs a photon and jumps to a higher energy level, it does not absorb yet another photon and jump to an even higher energy level.
- Q.4** When an electron jump from fourth orbit to ground state of hydrogen atom then calculate the wavelength of emitted photon.
- Q.5** Find the atomic number of atom when given that its ionisation potential is equal to 122.4 V.
- Q.6** The acceleration of an electron in first orbit of H-atom is : $\left(\bar{h} = \frac{h}{2\pi} \right)$
- (A) $\frac{m^3 r^3}{\hbar^2}$ (B) $\frac{\hbar^2}{m^2 r^3}$
(C) $\frac{\hbar^2}{mr^3}$ (D) $\frac{mr^3}{\hbar^2}$
- Q.7** The radius of first orbit of an electron in hydrogen atom is r_0 . The radius of first orbit of helium atom will be :
(A) $r_0/2$ (B) r_0
(C) $2r_0$ (D) $4r_0$

SECTION - 2 : NUCLEAR PHYSICS

9.10

COMPOSITION OF NUCLEUS

- * Rutherford proposed the existence of a nucleus in 1911 to explain the results of his α scattering experiment.
- * Nucleus is the central core of an atom in which the entire positive charge and almost the entire mass of an atom is concentrated.



- * The nucleus is made of elementary particles called neutrons and protons.
- * All nuclei except hydrogen are made up of neutrons and protons.
- * Hydrogen nucleus contains a single proton.
- * **Neutron** is a neutral particle carrying no charge
- Mass of neutron $m_n = 1.6749 \times 10^{-27}$ kg
 $= 1.008665$ amu
- They are not deflected by external electric and magnetic fields
- Neutrons have high penetrating power and low ionizing power
- Neutrons are stable inside the nucleus. Outside the nucleus they are unstable with a half life of about 13 minutes
- Neutron was discovered by James Chadwick in 1932 when he tried to explain results of collision of α particles with Beryllium.
 ${}_2\text{Be}^4 + {}_2\text{He}^4 \rightarrow {}_6\text{C}^{13} \rightarrow {}_6\text{C}^{12} + {}_0n^1 + Q$
- The spin angular momentum of a neutron is $\frac{1}{2} \left(\frac{h}{2\pi} \right)$.
- Depending on speed they are classified as fast and slow (thermal) neutrons.

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- * **Proton** is a charged particle carrying unit positive charge.
 - Mass of proton $m_p = 1.6726 \times 10^{-27} \text{ kg}$
 $= 1.007825 \text{ amu}$
 - Proton was discovered by Goldstein in 1919.
 - The number of protons present inside the nucleus of an atom is called atomic number (Z) of an element.
 - As atom is electrically neutral so number of protons inside the nucleus is equal to number of electrons in an atom.
 - According to Heisenberg a proton and neutron can be regarded as two different charge states of same particle called nucleon.
 - The total number of protons and neutrons present inside the nucleus is known as mass number (A) of an element.
Number of nucleons or Mass number (A)
 $=$ proton number (Z) + neutron number (N)
 - In lighter nuclei the number of neutrons and protons are equal while in heavier nuclei number of neutrons is greater than number of protons.
 - A nuclide is a specific nucleus of an atom characterized as ${}_Z X_N^A$ where A is mass number, Z is proton number and N is neutron number.

Types of Nuclei

Isotopes

These are nuclei of same element having same Z but different A

e.g. ${}_8\text{O}^{16}$, ${}_8\text{O}^{17}$, ${}_8\text{O}^{18}$; ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_1\text{H}^3$;
 ${}_{92}\text{U}^{234}$, ${}_{92}\text{U}^{235}$, ${}_{92}\text{U}^{238}$

All isotopes of an element have same chemical properties.

They occupy same place in periodic table. They cannot be separated by chemical analysis. They can be separated by mass spectrometers.

Isotones

These are nuclei of different elements having same N but different A.

e.g. C_7^{13} & N_7^{14} ; Be_5^9 & B_5^{10}

Isotones are different elements with different chemical properties. They occupy different positions in periodic table. They can be separated by chemical analysis and mass spectrometers.

Isobars

These are nuclei of different elements having same A but different N and Z.

e.g. ${}_6\text{C}^{14}$ and ${}_7\text{N}^{14}$; ${}_{18}\text{Ar}^{40}$ and ${}_{20}\text{Ca}^{40}$

Isobars are different elements with different chemical properties. They occupy different positions in periodic table. They can be separated by chemical analysis but cannot be separated by mass spectrometers.

9.11 NUCLEAR FORCES

The strong forces of attraction which firmly hold the nucleons in the small nucleus and account for stability of nucleus are called as nuclear forces.

Characteristics of Nuclear Force

1. **The nuclear force is a short range force.**

The force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.

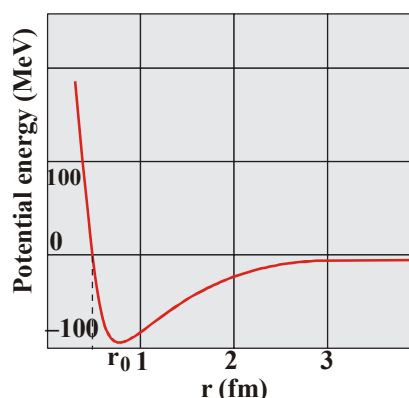


Figure : Potential energy of a pair of nucleons as a function of their separation. For a separation greater than r_0 , the force is attractive and for separations less than r_0 , the force is strongly repulsive.

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2. Nuclear forces are strongest force in nature

Nature of force	Relative strength	Interaction time
Nuclear	$1-10^{39}$	10^{-22} sec
Electromagnetic	$10^{-3}-10^{36}$	10^{-15} sec
Weak	$10^{-13}-10^{26}$	10^{-8} sec
Gravitational	$10^{-39}-1$	10^{-2} sec

3. Nuclear forces are charge independent

* Force between a pair of protons, a pair of neutrons and a pair of neutron and proton is equal. $F(n-n) = F(p-p) = F(n-p)$

* The net force between pair of neutrons and a pair of neutron and proton is equal. This is slightly greater than force between pair of protons because force between protons is reduced due to electrostatic repulsion

Net force $F(n-n) = \text{Net force } F(n-p) > \text{Net force } F(p-p)$

4. Nuclear forces are spin dependent

* Nuclear force depends on relative orientation of spins between two interacting nucleons

* The force of attraction between two nucleons with parallel spin is greater than force between nucleons with antiparallel spin.

* Deuteron is formed in a bound state only if spins of neutron and proton are parallel.

5. Nuclear forces show saturation property

* The nucleon in nucleus interacts with its nearest neighbour only.

* It remains unaffected by the presence of other surrounding nucleons.

* The nuclear force between a pair of nucleons in light and heavy nucleus is equal.

6. Nuclear forces are non-central forces

* They do not act along line joining the centre of two nucleons.

* The non-central component depends on orientation of spins relative to line joining the centre of two nucleons.

7. Nuclear forces are exchange forces

* The nuclear forces originate by exchange of mesons (π^+ , π^0 , π^-) between the nucleons.

* Mass of meson = 0.15 amu

$$= 140 \text{ MeV} = 280 \times \text{mass of electron}$$

* $p-p$ force $p + \pi^0 \longleftrightarrow p$

$n-n$ force $n + \pi^0 \longleftrightarrow n$

$n-p$ force $p + \pi^- \longleftrightarrow n$

$n + \pi^+ \longleftrightarrow p$

* The theory of exchange forces was given by Yukawa.

* The potential energy of a particle in this force field is given by Yukawa potential

$$U(r) = U_0 e^{-r/r_0} \text{ where } r_0 \text{ \& } U_0 \text{ are constants.}$$

9.12

STABILITY OF THE NUCLEUS

* The limited range of action of the strong nuclear force plays an important role in the stability of the nucleus. For a nucleus to be stable, the electrostatic repulsion between the protons must be balanced by the attraction between the nucleons due to the strong nuclear force. But one proton repels all other protons within the nucleus, since the electrostatic force has such a long range of action.

* In contrast, a proton or a neutron attracts only its nearest neighbors via the strong nuclear force.

* As the number Z of protons in the nucleus increases under these conditions, the number N of neutrons has to increase even more, if stability is to be maintained.

* Figure shows a plot of N versus Z for naturally occurring elements that have stable nuclei.

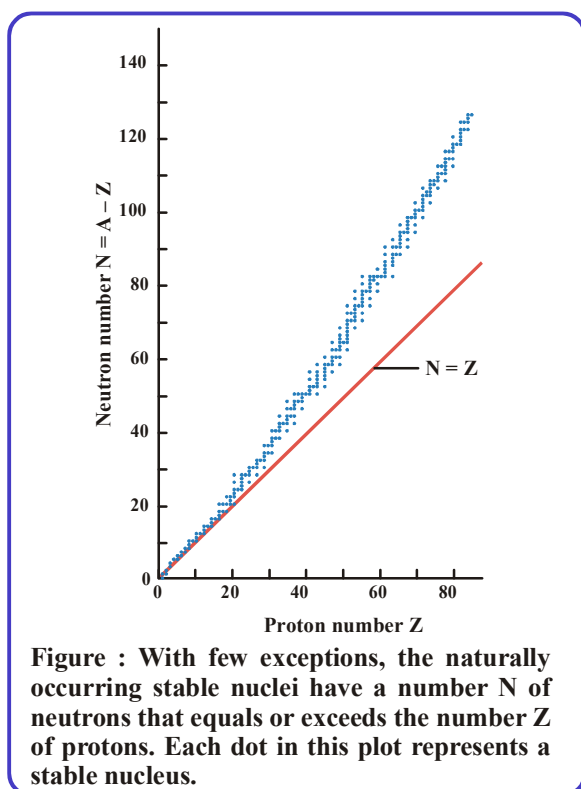
* For reference, the plot also includes the straight line that represents the condition $N = Z$.

* With few exceptions, the points representing stable nuclei fall above this reference line, reflecting the fact that the number of neutrons becomes greater than the number of protons as the atomic number Z increases.

* The stable nucleus with the largest number of protons ($Z = 83$) is that of bismuth, ${}_{83}^{209}\text{Bi}$, which contains 126 neutrons.

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- * All nuclei with more than 83 protons (e.g., uranium, $Z=92$) are unstable and spontaneously break apart or rearrange their internal structures as time passes.

9.13 SIZE OF NUCLEUS

- * Radius of nucleus is related to mass number as $R = R_0 A^{1/3}$ where R_0 is constant & $R_0 = 1.25 \times 10^{-15} \text{m}$

DENSITY OF NUCLEUS

- * Volume of nucleus $V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi R_0^3 A$,
so volume $V \propto A$
Mass of nucleus
= mass of protons + mass of neutrons
= mA where m is mass of one nucleon
Density of nucleus

$$\rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}} = \frac{mA}{\frac{4}{3} \pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

- * The nuclear density is independent of mass number A .
- * The nuclear density is nearly constant and is equal to

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.25 \times 10^{-15})^3}$$

$$= 2.04 \times 10^{17} \text{ kg / m}^3$$

Nuclear density is of order $10^{17} \text{ kg m}^{-3}$

- * The nuclear density is maximum at centre of nucleus and decreases as one moves away from the centre.
The distance from the centre of nucleus where density becomes 50% of its density at centre is called nuclear radius. The high density of nucleus indicates compactness of nucleus.

ATOMIC MASS UNIT

- * 1 atomic mass unit (amu)
= $\frac{1}{12}$ of mass of carbon (${}_{6}\text{C}^{12}$) atom
 $1 \text{ amu} = \frac{1}{12} \left(\frac{12}{6.023 \times 10^{23}} \right) = 1.66 \times 10^{-24} \text{g}$
= $1.66 \times 10^{-27} \text{ kg}$
Energy equivalent to 1 amu mass
 $E = mc^2 = 1.66 \times 10^{-27} (3 \times 10^8)^2 \text{ joule}$
= $1.49 \times 10^{-10} \text{ joule} = 931.5 \text{ MeV}$
 $1 \text{ amu} = 1.49 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$

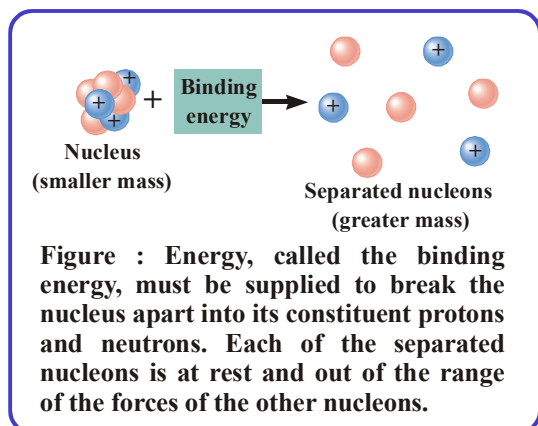
9.14 MASS DEFECT

- * The mass of the nucleus is always less than the sum of masses of nucleons composing the nucleus.
- * The difference between the rest mass of nucleus and sum of rest masses of nucleons constituting the nucleus is known as mass defect.
Mass defect

$$\Delta m = [Zm_p + (A - Z)m_n] - M({}_Z\text{X}^A)$$

9.15 BINDING ENERGY

- * The energy required to break a nucleus into its constituent nucleons and place them at infinite distance is called binding energy.



- * The energy equivalent to mass defect is called binding energy.
- * This is the energy with which the nucleons are held together.
- * The binding energies (\sim MeV) are very large as compared to molecular binding energies (\sim eV)
- * Binding energy

$$BE = (\Delta m) c^2$$

$$= c^2 [Zm_p + (A - Z)m_n - M(ZX^A)]$$
 Rest mass of protons + rest mass of neutrons
 = rest mass of nucleus + BE.

BINDING ENERGY PER NUCLEON

- * The binding energy per nucleon (E_{bn}) of a nucleus is the average energy required to extract a nucleon from the nucleus.
- * Binding energy per nucleon

$$\bar{B} = \frac{\text{Total binding energy}}{\text{Total number of nucleons}} = \frac{BE}{A} = \frac{\Delta mc^2}{A}$$

$$= \frac{c^2}{A} [Zm_p + (A - Z)m_n - M(ZX^A)]$$

- * The plot of binding energy per nucleon with mass number A is shown as :

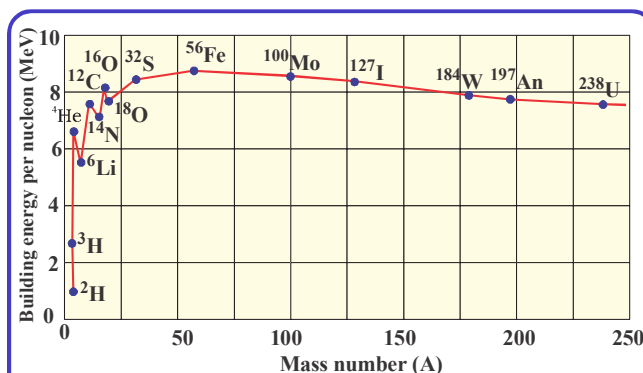
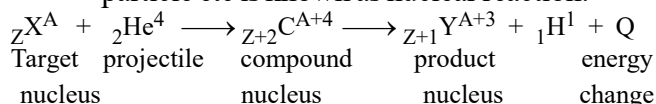


Figure : The binding energy per nucleon as a function of mass number.

- * Binding energy per nucleon gives a measure of stability of nucleus. More is binding energy per nucleon more is the stability of nucleus.
- * Binding energy per nucleon is small for lighter nuclei i.e. ${}^1_1\text{H}$, ${}^2_1\text{H}$ etc.
- * For $A < 28$ at $A = 4n$ the curve shows some peaks at ${}^4_2\text{He}$, ${}^8_4\text{Be}$, ${}^{16}_6\text{C}$, ${}^{16}_8\text{O}$, ${}^{20}_{10}\text{Ne}$, ${}^{24}_{12}\text{Mg}$.
- * This represents extra stability of these elements with respect to their neighbours.
- * The binding energy per nucleon is maximum about 8.75 MeV for Fe^{56} .
- * E_{bn} is lower for both light nuclei ($A < 30$) and heavy nuclei ($A > 170$).
- * The constancy of the binding energy in the range $30 < A < 170$ is a consequence of the fact that the nuclear force is short-ranged.
- * The binding energy per nucleon decreases for $A > 200$ They become less stable and exhibit radioactivity.
- * In fusion lighter nuclei fuse to form heavier nuclei.
- * The process is accompanied by increase in binding energy per nucleon.
- * In fission a heavy nucleus splits into two lighter nuclei. Here also increase in binding energy per nucleon takes place. The heaviest stable nuclide is ${}^{209}_{83}\text{Bi}$.

9.16 NUCLEAR REACTION

- * The transformation of one stable nucleus into another nucleus by bombardment with suitable high energy particles like proton, neutron, α particle etc is known as nuclear reaction.

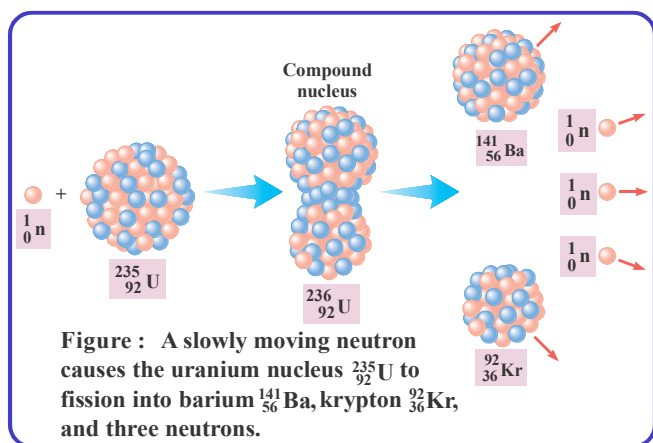


- e.g.
 ${}_{13}^{27}\text{Al}^{27} + {}_2^4\text{He}^4 \rightarrow {}_{15}^{31}\text{P}^{31} \rightarrow {}_{14}^{30}\text{Si}^{30} + {}_{+1}^0\text{e}^0 + {}_0^1\text{n}^1 + Q$
 The nuclear reactions obey following conservation laws

- conservation of linear momentum
- conservation of total energy
- conservation of charge
- conservation of number of nucleons.
- conservation of angular momentum.

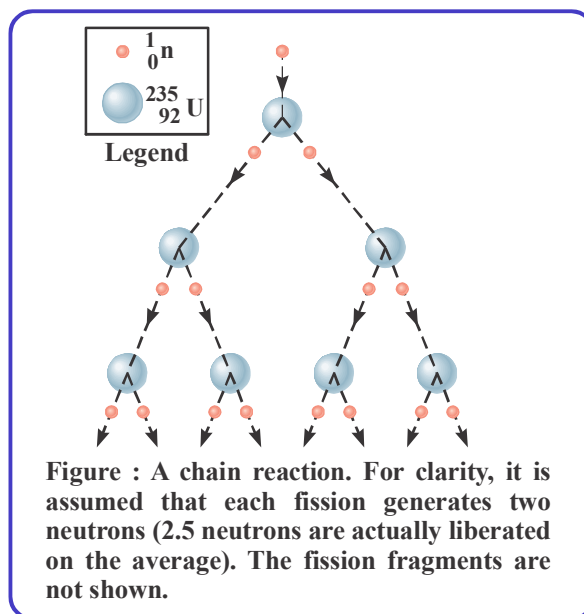
NUCLEAR FISSION

- * Nuclear fission was discovered by Otto Hahn and Strassman.



- * The process of splitting of a heavy nucleus into two nuclei of comparable size and release of large energy is called **fission**.
- * U^{235} nucleus captures a thermal neutron.
- * This forms a compound nucleus U^{236} in excited state
- * The shape of nucleus is distorted and nucleus splits into two fragments emitting several neutrons.
- * The neutrons emitted in fission are fast neutrons.

Their energy is about 2MeV. On an average 2.5 neutrons are emitted per fission.



- * The binding energy per nucleon of products is greater than the reactants.
 - * The energy released in fission of Uranium is about 200 MeV.
 - * The fission energy released per nucleon is about 0.84 MeV.
 - * The fission of U^{235} may take place by different routes but amount of energy released per fission is nearly equal.
- $${}_{92}\text{U}^{235} + {}_0^1\text{n}^1 \rightarrow {}_{92}\text{U}^{236}$$
- $$\rightarrow {}_{38}\text{Sr}^{95} + {}_{54}\text{Xe}^{139} + 2{}_0^1\text{n}^1$$
- $$\rightarrow {}_{41}\text{Nb}^{99} + {}_{51}\text{Sb}^{133} + 4{}_0^1\text{n}^1 \text{ etc.}$$
- * The fission fragments are highly radioactive. Nuclear fission can be explained on basis of **liquid drop model**. The natural Uranium has following isotopes ${}_{92}\text{U}^{234}$ (0.006%) ; ${}_{92}\text{U}^{235}$ (0.72%) ; ${}_{92}\text{U}^{238}$ (99.27%)
 - * ${}_{92}\text{U}^{238}$ is not fissionable. This can be converted to plutonium which is fissionable by neutrons.
- $${}_{92}\text{U}^{238} + {}_0^1\text{n}^1 \rightarrow {}_{93}\text{Np}^{239} + {}_{-1}^0\text{e}^0 + \bar{\nu}$$
- $${}_{93}\text{Np}^{239} \rightarrow {}_{94}\text{Pu}^{239} + {}_{-1}^0\text{e}^0 + \bar{\nu}$$
- * The Uranium in which fraction of U^{235} is increased from 0.7% to 2.3% is called enriched Uranium.

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Energy released per gm of Uranium :

$$\begin{aligned} & \text{Energy released per gm of Uranium} \\ &= \frac{\text{Avogadro number}}{\text{mass number}} \\ & \quad \times \text{energy released per fission} \end{aligned}$$

$$= \frac{6.023 \times 10^{23}}{235} \times 200 = 5.12 \times 10^{23} \text{ MeV}$$

energy released by 1 gm of U^{235}

$$= 5.12 \times 10^{23} \text{ MeV}$$

$$= 8.2 \times 10^{10} \text{ J} = 2.28 \times 10^4 \text{ kWh}$$

$$= 2 \times 10^{10} \text{ calorie}$$

This energy is equivalent to

(i) Energy obtained by burning 2560 kg of coal.

(ii) energy obtained by burning 20 tonne of explosive TNT.

* The energy is released in form of kinetic energy of fission fragments, γ -rays, heat, sound and light energy.

* The fission process can take place at normal pressure and temperature.

CHAIN REACTION

* In fission of uranium atom 2.5 neutrons are produced on an average.

* In favourable conditions these may produce fission of other uranium nuclei.

* At each stage number of neutrons available for fission gets multiplied which can cause further fission of uranium nuclei.

* The process once started continues by itself till entire uranium is consumed. This process is called nuclear chain reaction.

* The neutrons produced in each fission may be lost due to following reasons :

(a) Leakage of neutrons from the system.

(b) absorption of neutrons by U^{238} .

(c) absorption of neutrons by impurities.

Neutron Multiplication Factor

$$K = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(a) If $K = 1$ chain reaction is sustained.

* This leads to a controlled chain reaction.

* **Controlled chain reaction** : When only one of the neutron produced in each fission is able to produce fission then reaction is called controlled chain reaction.

* The energy is produced at a uniform rate.

* This forms working principle of nuclear reactor

* The size of fissionable material is called critical size and its mass as critical mass.

* The minimum mass of Uranium for which chain reaction is possible is called **critical mass**.

(b) If $K > 1$ chain reaction is accelerated because number of neutrons available for fission at each stage increases rapidly

* This leads to uncontrolled chain reaction

* Energy is produced at rapidly increasing rate.

* This is working principle of atom bomb.

* The size of material is super critical.

(c) If $K < 1$ the chain reaction stops because number of neutrons available for fission decreases at each stage.

* The energy produced decreases

* The size of material is sub critical

* The rate of decay of neutrons is proportional to surface area of Uranium block.

* The rate of production of neutrons is proportional to number of nuclei present in Uranium block or volume of the block.

NUCLEAR REACTOR

* Nuclear reactor is a device in which controlled nuclear chain reaction is initiated and maintained to produce energy.

* Reactors are used

(i) For power generation

(ii) To produce radioactive isotopes used in medicine, industry and agriculture

(iii) To produce Plutonium Pu^{239} used in making atom bomb.

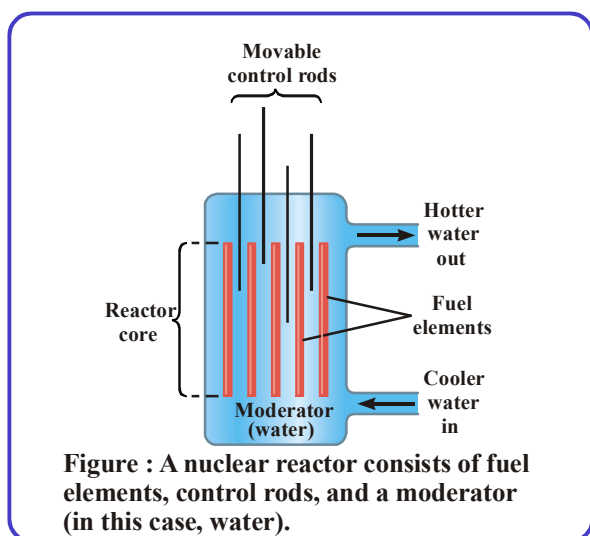


Figure : A nuclear reactor consists of fuel elements, control rods, and a moderator (in this case, water).

- * **Thermal reactor :** Reactor in which energy is produced by fission of U^{235} by slow neutron is called thermal reactor.

Main parts of Reactor :

(a) Fuel

It is fissionable material used for fission. The common fuels are U^{233} , U^{235} , Pu^{239} .

(b) Moderator

These slow down the fast neutrons to thermal neutrons. e.g. Heavy water, graphite, beryllium oxide.

(c) Control Rods

These help in controlling the rate of fission by absorbing the neutrons produced in fission. e.g. Cadmium rods.

(d) Coolant

A substance which is used to remove the heat produced and transfer it from core of nuclear reactor to the surrounding is called coolant. e.g. air, water or carbondioxide.

(e) Shield

The whole reactor is protected with concrete walls 2 to 2.5 m thick so that radiations emitted during nuclear reactions do not produce harmful effects.

Very small fraction of about 1.2% of natural Uranium fuel is used.

NUCLEAR FUSION

- * The process in which two or more lighter nuclei combine to form a heavy nucleus is known as nuclear fusion.

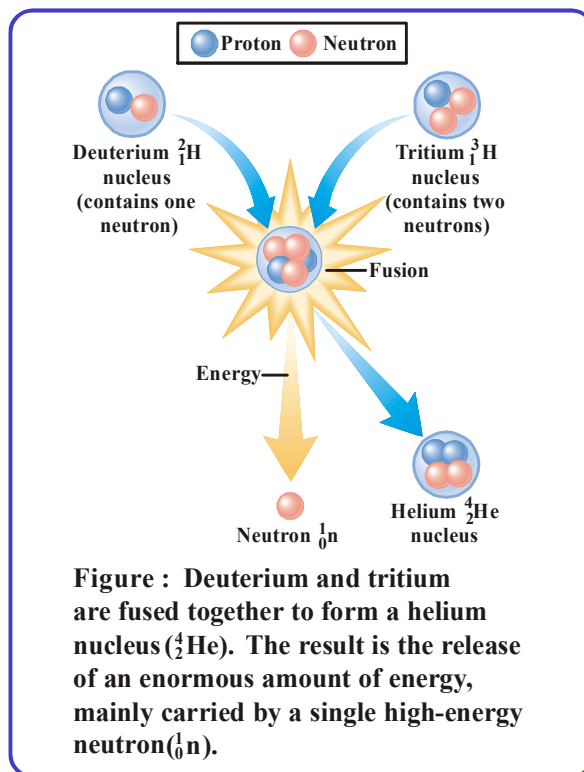
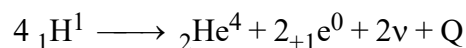


Figure : Deuterium and tritium are fused together to form a helium nucleus (${}^4_2\text{He}$). The result is the release of an enormous amount of energy, mainly carried by a single high-energy neutron (${}^1_0\text{n}$).

- * The binding energy per nucleon of product is greater than the reactants.
- * The energy released per nucleon is large ~ 6.75 MeV.

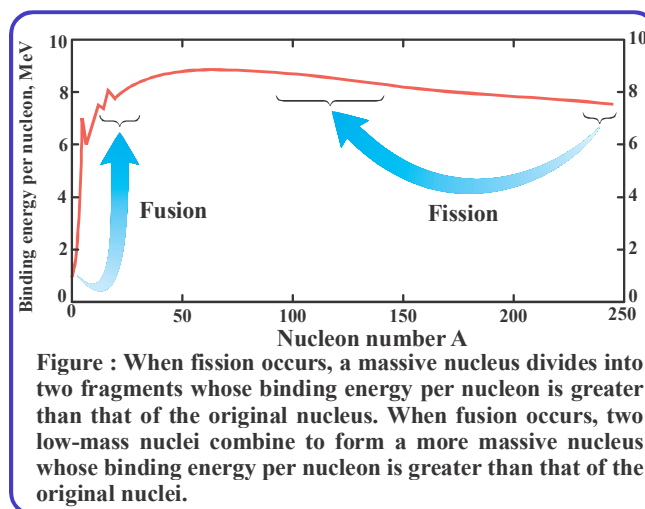


Figure : When fission occurs, a massive nucleus divides into two fragments whose binding energy per nucleon is greater than that of the original nucleus. When fusion occurs, two low-mass nuclei combine to form a more massive nucleus whose binding energy per nucleon is greater than that of the original nuclei.

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- * Fusion is possible at high pressure ($\sim 10^6$ atm.) and high temperature ($\sim 10^8$ °C). The proton-proton cycle happens at lower temperature as compared to carbon-nitrogen cycle.
- * Nuclear fusion is possible in a place which has reactants in large quantity.
- * Hydrogen bomb works on principle of nuclear fusion.
- * The explosion of a hydrogen bomb needs an explosion of atom bomb to generate required temperature. No harmful radiations are produced in fusion.



- * If binding energy per nucleon is more for a nucleus, then it is more stable.

For example, if $\left(\frac{BE_1}{A_1}\right) > \left(\frac{BE_2}{A_2}\right)$

then nucleus 1 would be more stable.

- * The coolant must be a substance that absorbs neutrons very lightly.
- * The energy of neutrons emitted in fission is of the order of 1 MeV or higher.

EXAMPLE 9

Determine the ratio of radius of nuclei ${}_{13}\text{Al}^{27}$ and ${}_{52}\text{Te}^{125}$.

SOLUTION:

$$As R \propto A^{1/3}$$

$$\frac{R_{Al}}{R_{Te}} = \left(\frac{A_{Al}}{A_{Te}}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \left(\frac{3^3}{5^3}\right)^{1/3} = \frac{3}{5}$$

EXAMPLE 10

Calculate the mass defect of a deuteron (${}_1\text{H}^2$).

Given $M({}_1\text{H}^2) = 2.014102$ amu,
 $m_n = 1.008665$ amu, $m_p = 1.007825$ amu.

SOLUTION:

Mass defect

$$\Delta m = [Zm_p + (A - Z)m_n] - M({}_1\text{H}^2)$$

$$= (1 \times 1.007825 + (2 - 1) 1.008665) - 2.014102$$

$$\Delta m = 0.002388 \text{ amu}$$

EXAMPLE 11

Calculate the binding energy per nucleon for ${}_{17}\text{Cl}^{35}$. Given $M(\text{Cl}^{35}) = 34.9800$ amu,
 $m_n = 1.008665$ amu and $m_p = 1.007825$ amu.

SOLUTION:

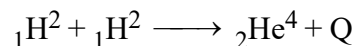
$$BE = Zm_p + (A - Z)m_n - M(\text{Cl}^{35})$$
$$= 17 \times 1.007825 + 18 \times 1.008665 - 34.9800$$
$$= 0.308995 \text{ amu}$$

$$BE = 0.308995 \times 931.5 = 287.83 \text{ MeV}$$

$$\bar{B} = \frac{BE}{A} = \frac{287.75}{35} = 8.22 \text{ MeV/nucleon}$$

EXAMPLE 12

Determine the energy released in the process



Given $M({}_1\text{H}^2) = 2.01471$ amu

$M({}_2\text{He}^4) = 4.00388$ amu

SOLUTION:

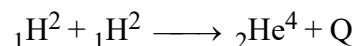
$$\text{Mass defect } \Delta m = 2 \times 2.01471 - 4.00388$$
$$= 0.02554 \text{ amu}$$

$$\text{energy liberated} = 0.02554 \times 931.5 \text{ MeV}$$
$$= 23.79 \text{ MeV}$$

EXAMPLE 13

The binding energies per nucleon for deuteron (${}_1\text{H}^2$) and helium (${}_2\text{He}^4$) are 1.1 MeV and 7 MeV respectively. Determine energy released when two deuterons fuse to form a helium nucleon.

SOLUTION:



Binding energy of deuteron = $2 \times 1.1 = 2.2$ MeV

Binding energy of helium

$$= 4 \times 7 \text{ MeV} = 28 \text{ MeV}$$

$$\text{Energy released} = (28 - 2 \times 2.2) = 23.6 \text{ MeV}$$

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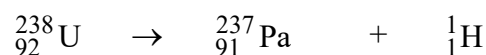


Checkpoint 2

- Q.1** Two nuclei differ in their numbers of protons and their numbers of neutrons. Which one or more of the following statements is/are true? (a) They are different isotopes of the same element. (b) They have the same electric charge. (c) They could have the same radii. (d) They have approximately the same nuclear density.
- Q.2** A material is known to be an isotope of lead, although the particular isotope is not known. From such limited information, which of the following quantities can you specify? (a) Its atomic number (b) Its neutron number (c) Its atomic mass number.
- Q.3** Two nuclei have different nucleon numbers A_1 and A_2 . Are the two nuclei necessarily isotopes of the same element?
- Q.4** Can two nuclei have the same radius, even though they contain different numbers of protons and different numbers of neutrons?
- Q.5** Rank the following nuclei in ascending order according to the binding energy per nucleon (smallest first): (a) Phosphorus ${}_{15}^{31}\text{P}$ (b) Cobalt ${}_{27}^{59}\text{Co}$ (c) Tungsten ${}_{74}^{184}\text{W}$ (d) Thorium ${}_{90}^{232}\text{Th}$.
- Q.6** The following table gives values for the mass defect Δm for four hypothetical nuclei, A, B, C, and D. Which statement is true regarding the stability of these nuclei? (a) Nucleus D is the most stable, and A is the least stable. (b) Nucleus C is stable, whereas A, B, and D are not. (c) Nucleus A is the most stable, and D is not stable. (d) Nuclei A and B are stable, but B is more stable than A.

	A	B
Mass defect	$+6.0 \times 10^{-29}$ kg	$+2.0 \times 10^{-29}$ kg
Δm	C	D
	0 kg	-6.0×10^{-29} kg

- Q.7** Uranium ${}_{92}^{238}\text{U}$ decays into thorium ${}_{90}^{234}\text{Th}$ by means of α decay. Another possibility is that the ${}_{92}^{238}\text{U}$ nucleus just emits a single proton instead of an α particle. This hypothetical decay scheme is shown below, along with the pertinent atomic masses:



Uranium Protactinium Proton
238.050 78 u 237.051 14 u 1.007 83 u

For a decay to be possible, it must bring the parent nucleus toward a more stable state by allowing the release of energy. Compare the total mass of the products of this hypothetical decay with the mass of ${}_{92}^{238}\text{U}$ and decide whether the emission of a single proton is possible for ${}_{92}^{238}\text{U}$.

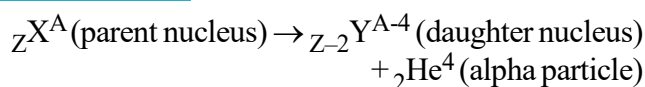
- Q.8** Which one or more of the following statements correctly describe differences between fission and fusion? (a) Fission involves the combining of low-mass nuclei to form a more massive nucleus, whereas fusion involves the splitting of a massive nucleus into less massive fragments. (b) Fission involves the splitting of a massive nucleus into less massive fragments, whereas fusion involves the combining of low-mass nuclei to form a more massive nucleus. (c) More energy per nucleon is released when a fission event occurs than when a fusion event occurs. (d) Less energy per nucleon is released when a fission event occurs than when a fusion event occurs.
- Q.9** Which one or more of the following statements concerning fission and fusion are true? (a) Both fission and fusion reactions are characterized by a mass defect. (b) Both fission and fusion reactions always obey the conservation laws of physics. (c) Both fission and fusion take advantage of the fact that the binding energy per nucleon varies with the nucleon number of the nucleus.

SECTION - 3 : RADIOACTIVITY

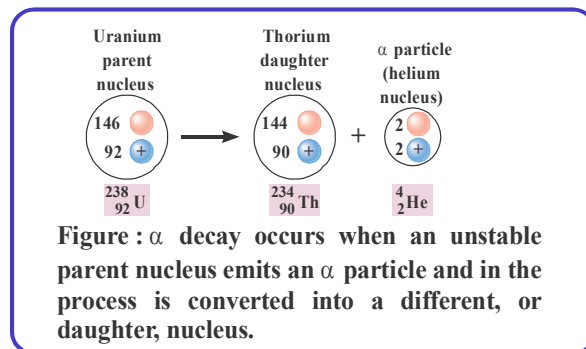
- * The phenomenon of spontaneous emission of radiations from a substance is called **radioactivity**.
- * Radioactivity was discovered by Henry Becquerel in 1896 in Uranium salts.
- * The substances like Uranium, Radium, Thorium, Polonium etc. which show radioactivity are called radioactive substances.
- * Nuclei with $Z > 83$ spontaneously disintegrate with emission of α and β particles.
- * In radioactivity emission of alpha (α), Beta (β) and gamma (γ) radiation takes place. These are called radioactive radiations or Becquerel rays.
- * The simultaneous emission of α and β particles is not possible. Only one particle is emitted at a time.
- * Radioactivity is a nuclear phenomenon. α , β and γ radiations originate from the nucleus.
- * The electronic configuration is unaffected in radioactivity.
- * The emission of radiations causes a change in structure of nucleus. This causes transformation of an atom to new lighter atom or changes a radioactive element into element of lower atomic weight.
- * All heavier radioactive elements emit radiations till they are converted to stable ${}_{82}\text{Pb}^{206}$.
- * Radioactivity is a statistical process, so it is governed by the laws of probability.
- * The disintegration of all atoms has equal probability.
- * Radioactivity is a spontaneous process which is independent of all external conditions.
- * It is not affected by temperature, pressure, electric or magnetic field.

9.17 TYPE OF RADIOACTIVE PROCESSES

Alpha Decay



- * Alpha particle consists of 2 neutrons, 2 protons & carries positive charge in magnitude 2 electrons.



- * It is doubly ionized helium nuclei. α emission takes place when size of nucleus becomes too large. The decay reduces the size of nucleus.
- * α emission is explained on basis of quantum mechanical tunnel effect.
- * The energy released in α decay

$$Q = (M_X - M_Y - M_\alpha) c^2$$

The kinetic energy of α particle

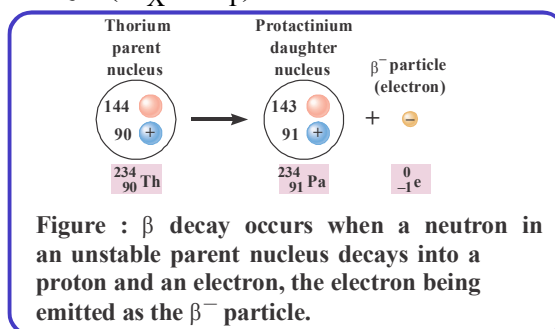
$$E_\alpha = \left(\frac{A-4}{A} \right) Q, \text{ where } A \text{ is mass number}$$

and Q is disintegration energy

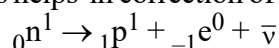
Beta Decay

(a) Electron Emission (β^-)

- * ${}_Z\text{X}^A \rightarrow {}_{Z+1}\text{Y}^A + {}_{-1}e^0 \text{ (}\beta^- \text{ particle)} + \bar{\nu}$
- e.g. ${}_6\text{C}^{14} \rightarrow {}_7\text{N}^{14} + {}_{-1}e^0 + \bar{\nu}$ (antineutrino)
- * $Q = (M_X - M_Y) c^2$



- * β^- particles are fast moving electrons carrying negative charge. β^- particles are emitted when nucleus has too many neutrons relative to number of protons i.e. N/Z ratio is larger than required.
- * The emission of electron takes place when a neutron is converted to proton inside the nucleus. This helps in correction of N/Z ratio.

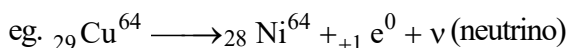
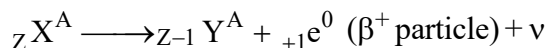


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* The interaction responsible for β decay is weak interaction.

(b) Positron Emission



* $Q = (M_X - M_Y - 2m_e) c^2$

$$M_{\text{nucl}}({}_Z X^A) = M_{\text{nucl}}({}_{Z-1} Y^A) + m_e + Q/c^2$$

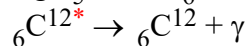
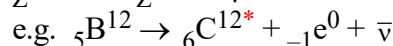
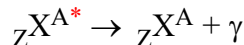
In this case, only $(Z-1)m_e$ is needed for the daughter atomic mass which gives us a remaining mass of $2m_e$.

* β^+ particles are positrons with mass equal to an electron but carry a unit positive charge.

* β^+ particles are emitted when nucleus has too many protons relative to number of neutrons i.e. N/Z ratio is smaller than required.

* The emission of positron takes place when a proton is converted to neutron inside the nucleus. This increases N/Z ratio. ${}_1 p^1 = {}_0 n^1 + {}_{+1} e^0 + \nu$

(c) Gamma Decay



* γ rays are electromagnetic radiations which are chargeless and massless.

* γ rays are emitted when nucleus has excess energy. γ rays are emitted when nucleus jumps from excited state to lower level or ground state. This reduces the energy of nucleus.

* γ rays are electromagnetic radiations of short wavelength ($\sim 10^{-12}\text{m}$) which travel with speed of light.

Comparison of Properties of α , β and γ Radiations

Property	α - rays	β - rays	γ - rays
1. Nature	These are doubly ionized helium atom ${}_2\text{He}^4$ charge $q = +2e = 3.2 \times 10^{-19}\text{C}$ mass $m = 2p + 2n = 4\text{amu}$ $= 4 \times 1.6 \times 10^{-27}\text{kg}$	These are beam of fast moving electrons (β^-) and positrons (β^+) charge $\beta^- = -e = -1.6 \times 10^{-19}\text{C}$ $\beta^+ = +e = 1.6 \times 10^{-19}\text{C}$ $m(\beta^-) = m(\beta^+) = 9.1 \times 10^{-31}\text{kg}$	These are electromagnetic radiations of high frequency and travel in form of photons. charge $q = 0$ (chargeless) rest mass = 0 effective mass = $\frac{h\nu}{c^2} = \frac{h}{\lambda c}$
2. Velocity	Speed ranges between 1.4×10^7 to $2.20 \times 10^7\text{m/s}$ $v_\alpha \sim 0.05c$	Speed ranges from 1% to 90% of velocity of light $v_\beta \sim 0.9c$	Speed equals velocity of light $v_\gamma = c$
3. Ionising power	These have maximum ionizing power (1000)	There ionizing power is less than α particles and more than γ rays (100)	There ionizing power is least (1)
4. Penetration power	The penetration power is smallest. Can only penetrate through 0.01 mm thick Al sheet (1)	Penetration power is about 100 times that of α rays, can penetrate through 1 mm thick Al sheet (100)	Penetration power is very large. Can penetrate about 30 cm thick Al sheet (10000)
5. Range	Range is very small (few cms in air)	Range is more than α rays. (few meters in air)	Range is very large (many hundreds of meter in air)
6. Nature of spectrum	Line spectrum	continuous spectrum	line spectrum
7. Interaction with matter	produces heat	produces heat	produces photoelectric effect Compton effect, pair production
8. Effect of electric and magnetic field	Suffers small deflection	suffers large deflection	pass undeflected
9. Effect of photographic plate and ZnS	Affects photographic plate and produces fluorescence	Affects photographic plate and produces fluorescence	Affects photographic plate and produces fluorescence.

Behaviour in Electric and Magnetic Field

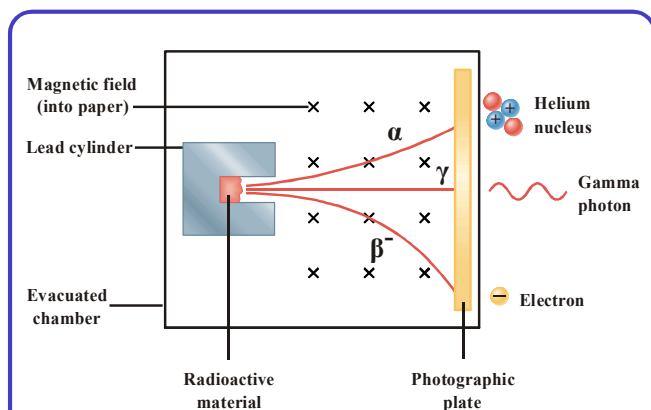


Figure: α and β rays are deflected by a magnetic field and, therefore, consist of moving charged particles. γ rays are not deflected by a magnetic field and, consequently, must be uncharged.

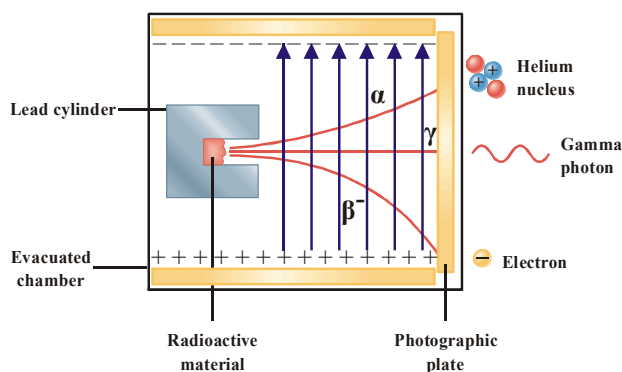


Figure: α and β rays are deflected by a electric field and, therefore, consist of moving charged particles. γ rays are not deflected by a electric field and, consequently, must be uncharged.

9.18

RADIOACTIVE DECAY LAW

* The rate of decay (number of disintegrations per second) is proportional to number of radioactive atoms (N) present at that time t .

* Rate of decay $\frac{-dN}{dt} \propto N$

$$\text{or } \frac{dN}{dt} = -\lambda N \quad \text{or } \boxed{N = N_0 e^{-\lambda t}} \quad \dots(1)$$

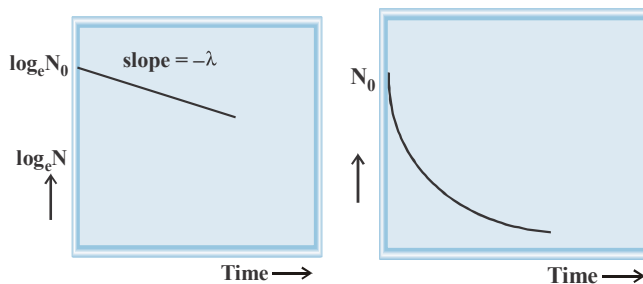
where λ is disintegration constant,
 N_0 = number of active atoms at $t = 0$.
 Equation one is the radioactive decay law.

It shows that number of active nuclei decreases exponentially with time.

* The fraction of active atoms remaining at time t

$$\text{is } \frac{N}{N_0} = e^{-\lambda t}$$

* The number of atoms that have decayed in time t is $N_0 - N = N_0 (1 - e^{-\lambda t})$



* The fraction of atoms that have decayed in time

$$t \text{ is } \frac{N_0 - N}{N_0} = 1 - e^{-\lambda t}$$

DECAY CONSTANT

* Decay constant is rate of decay of radioactive atoms per active atom.

$$\lambda = \frac{-dN/dt}{N} = \frac{\text{rate of decay}}{\text{number of active atoms}}$$

* At $t = \frac{1}{\lambda}$; $N = \frac{N_0}{e}$

The decay constant of radioactive element is equal to reciprocal of the time after which number of remaining active atoms reduce to $1/e$ times of original value.

* At $t = \frac{1}{\lambda}$, fraction of active nuclei left

$$\frac{N}{N_0} = \frac{1}{e} = 0.37 \quad \text{or } 37\%$$

Fraction of decayed nuclei

$$1 - \frac{N}{N_0} = 0.63 = 63\%$$

* $\lambda = \frac{dN/N}{dt}$

The decay constant is the probability of decay per active atom per unit time.

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- * The decay constant depends on nature of radioactive substance and is independent of temperature, pressure, force etc.
- * The decay constant for a stable substance is zero
- * Unit of decay constant is second^{-1} & dimension is T^{-1}
- * If there are more than one radioactive elements in a group then the resultant decay constant is equal to sum of individual decay constants

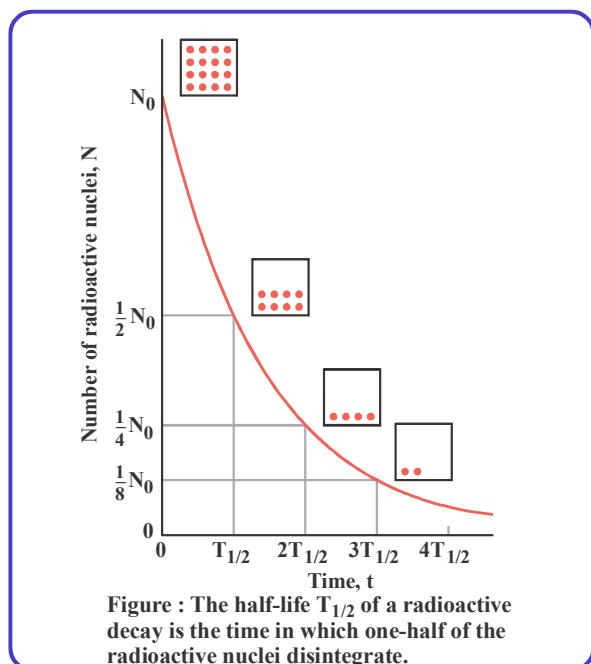
$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \dots$$

$$\text{or } \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} + \dots$$

HALF LIFE

- * The time in which number of radioactive atoms reduce to half of its initial value is known as half

$$\text{life i.e. at } t = T, N = \frac{N_0}{2}$$



From radioactive decay law

$$\frac{N_0}{2} = N_0 e^{-\lambda T} \quad \text{or} \quad T = \frac{0.693}{\lambda}$$

- * The half life depends on nature of radioactive elements.
- * The half life of an element indicates the rate of decay. When half life is large rate of decay is small.

- * After $t = nT$ number of active atoms left

$$N = \frac{N_0}{2^n} = \frac{1}{2^{t/T}} \cdot N_0$$

where $T =$ half life and $n =$ number of half lives.

- * Number of radioactive atoms decayed in n half

$$\text{lives } N_0 - \frac{N_0}{2^n} = N_0 \left(\frac{2^n - 1}{2^n} \right)$$

- * Half life for a given radioactive substance is constant. It does not change with time. It is unaffected by pressure, temperature etc.

AVERAGE OR MEAN LIFE (τ)

- * The life time of various atoms in a radioactive substance ranges from 0 to infinity. The mean life of an atom in a radioactive substance is called average life of radioactive substance.

- * Mean life,

$$\tau = \frac{\text{the sum of lives of all active atoms}}{\text{total number of active atoms}}$$

$$\text{or } \tau = \frac{\int_0^{N_0} t dN}{N_0} = \frac{N_0 \lambda \int_0^{N_0} t e^{-\lambda t} dt}{N_0} = \frac{1}{\lambda}$$

- * Mean life is equal to reciprocal of decay constant ($\tau = 1/\lambda$).

$$\text{Half life } T = \frac{0.693}{\lambda} = 0.693\tau$$

$$\text{Average life, } \tau = \frac{T}{0.693} = 1.44T$$

- * $\tau > T$ i.e. average life is greater than half life.
- * Mean life of a radioactive substance is constant. It does not change with temperature or pressure.
- * From $N = N_0 e^{-\lambda t}$

$$\text{At } t = \tau = \frac{1}{\lambda}; \quad N = \frac{N_0}{e} = 0.37N_0$$

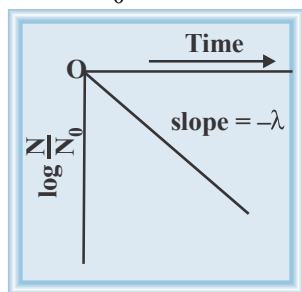
So mean life is the time in which –

- (a) Number of active atoms reduces to 37% of its initial value.
- (b) Number of decayed atoms is 63%.

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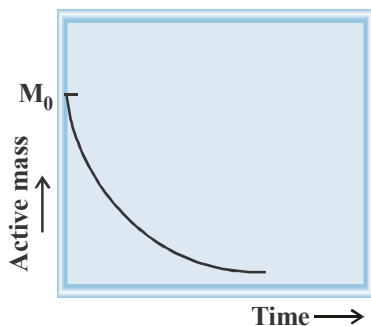
- * Mean life is equal to magnitude of reciprocal of slope of $\log \frac{N}{N_0}$ v/s t curve.



DECAY OF ACTIVE MASS

- * The mass of radioactive substance (m) \propto number of active atoms (N). So $N = N_0 e^{-\lambda t}$ becomes $M = M_0 e^{-\lambda t}$
- * Mass of radioactive substance decreases exponentially with time. The time in which mass of active substance is reduced to half is known

as half life. $\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T}$



- * Mean life is the time in which mass reduces to 37% of its initial value.
- * Number of active atoms is given mass M in

grams is $n = \frac{6.023 \times 10^{23} \times M}{A}$,

where A is mass number

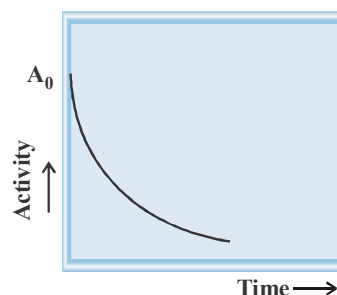
ACTIVITY

- * The number of decays per unit time or decay rate is called **activity**.

$$A = \frac{dN}{dt} = N_0 \lambda e^{-\lambda t} = A_0 e^{-\lambda t} = N \lambda$$

where $N_0 \lambda = A_0$ is initial activity.

$A = A_0 e^{-\lambda t}$ is the activity law which shows activity decreases exponentially with time.



- * Activity is proportional to number of active atoms ($A \propto N$) which depends on mass of radioactive sample.
- * The activity of one gram of radioactive substance called **specific activity**.
- * Half life is the time in which activity of radioactive substance is reduced to half.
- * Mean life is the time in which the activity reduces to 37% of the original value.
- * The variation of Activity with time is

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T}, \text{ where T is half life.}$$

Units of activity

- * **Curie** : The specific activity of 1 gm of Radium 226 is called one curie.
1 curie = 3.7×10^{10} disintegrations per second
- * **Rutherford** :
1 rutherford = 10^6 disintegrations per second
- * **Becquerel** :
1 Becquerel = 1 disintegration per second

9.19

RADIOACTIVE SERIES

- * The heavy natural nuclides can decay to stable end products by four paths. The four paths have mass numbers given as $4n, 4n + 1, 4n + 2, 4n + 3$ where n is integer.
- * Last element of series is stable and has a decay constant zero.

Series	Mass number	Starting isotope	Stable end product	
Thorium	4n	${}_{90}\text{Th}^{232}$	${}_{82}\text{Pb}^{208}$	Natural
Neptunium	4n+1	${}_{93}\text{Np}^{237}$	${}_{83}\text{Bi}^{209}$	Artificial
Uranium	4n+2	${}_{92}\text{U}^{238}$	${}_{82}\text{Pb}^{206}$	Natural
Actinium	4n+3	${}_{92}\text{U}^{235}$	${}_{82}\text{Pb}^{207}$	Natural

9.20

CARBON DATING

- * Radioactive dating is the process of determination of time interval which has passed by making use of radioactive decay of a sample containing radioactive substance.
- * It helps in calculating age of geological specimens like rocks, biological specimens like bones of animals or trunk of trees and age of earth.
- * The isotope of carbon ${}^6\text{C}^{14}$ is radioactive.
- * It is formed in atmosphere by bombardment of nitrogen atoms with cosmic rays
 ${}^7\text{N}^{14} + {}^0_1\text{n}^1 \rightarrow {}^6\text{C}^{14} + {}^1_1\text{H}^1$
- * The ${}^6\text{C}^{14}$ combines with oxygen to form carbon dioxide which is absorbed by plants so concentration of ${}^6\text{C}^{14}$ is constant with time.
- * The living plants and animals have a fixed ratio of ${}^6\text{C}^{14}$ to ordinary carbon ${}^6\text{C}^{12}$.
- * When a plant or animal dies the content of ${}^6\text{C}^{14}$ decreases while that of ${}^6\text{C}^{12}$ remains constant.
- * The ratio of two indicates the time that has passed since death of plant or animal.
- * The time interval is calculated from the laws of radioactive disintegration

$$t = \frac{1}{\lambda} \log_e \frac{N_0}{N} = \frac{2.303}{\lambda} \log_{10} \frac{N_0}{N} \quad \left(\frac{N_0}{N} = \frac{A_0}{A} \right)$$

where N_0 is number of ${}^6\text{C}^{14}$ nuclei at time of death, λ is decay constant of ${}^6\text{C}^{14}$ and N is number of ${}^6\text{C}^{14}$ nuclei currently present in sample.

9.21

USES OF RADIOACTIVE ISOTOPES

1. **In Medicine**
 - * Co^{60} for treatment of cancer
 - * Na^{24} for circulation of blood
 - * I^{131} for thyroid
 - * Sr^{90} for treatment of skin & eye
 - * Fe^{59} for location of brain tumor
 - * Radiographs of castings and teeth
2. **In Industries**
 - * For detecting leakage in water and oil pipe lines
 - * For investigation of wear & tear, study of plastics & alloys, thickness measurement.

3. **In Agriculture**
 - * C^{14} to study kinetics of plant photosynthesis
 - * P^{32} to find nature of phosphate which is best for given soil & crop
 - * Co^{60} for protecting potato crop from earth worm
 - * Sterilization of insects for pest control.

4. **In Scientific research**
 - * K^{40} to find age of meteorites
 - * S^{35} in factories

5. **Carbon dating**
 - * It is used to find age of earth and fossils
 - * The age of earth is found by Uranium disintegration and fossil age by disintegration of C^{14} .
 - * The estimated age of earth is about 5×10^9 years.
 - * The half life of C^{14} is 5700 years.

6. **As Tracers**
 - * A very small quantity of radio isotope present in any specimen is called tracer.
 - * This technique is used to study complex biochemical reactions, in detection of cracks, blockages etc, tracing sewage or silt in sea.

7. **In Geology**
 - * For dating geological specimens like ancient rocks, lunar rocks using Uranium
 - * For dating archaeological specimens, biological specimens using C^{14} .



- * Relation of half life with different intervals of time

$$a \xrightarrow{t_{1/2}} \frac{a}{2} \xrightarrow{t_{1/2}} \frac{a}{4} \xrightarrow{t_{1/2}} \frac{a}{8} \xrightarrow{t_{1/2}} \frac{a}{16}$$
- * If a radioactive nucleus decay by two processes a and b, then the effective decay constant $\lambda = \lambda_a + \lambda_b$ and the effective half life is

$$t_{1/2} = \frac{t_{1/2}(a) \cdot t_{1/2}(b)}{t_{1/2}(a) + t_{1/2}(b)}$$

*

Time	Fraction of active atoms remained (N/N ₀)	Fraction of atoms decayed $\frac{N_0 - N}{N_0}$
0	1 (100%)	0 (0%)
T	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
2T	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
3T	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)

where, T is half life

* The number of undecayed nuclei present after n

$$\text{mean life is } N = (0.37)^n N_0 = \left(\frac{1}{e}\right)^n N_0$$

EXAMPLE 14

In a old rock, ratio of nuclei of uranium and lead is 1 : 1. Half life of uranium is 4.5×10^9 years. Let initially it contains only uranium nuclei. How old is the rock ?

SOLUTION:

Let present active nuclei of uranium is N then initial active nuclei is 2N.

Present active fraction of uranium = 1/2

$$\frac{1}{2} = \frac{1}{2^{t/T_{1/2}}} \text{ or } \frac{t}{T_{1/2}} = 1$$

$$t = T_{1/2} = 4.5 \times 10^9 \text{ years}$$

EXAMPLE 15

The amount of active substance reduces to 1/64 of its initial value in 15 hours. What is the half life?

SOLUTION:

$$\text{Using } \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \text{ or } \frac{1}{64} = \left(\frac{1}{2}\right)^{15/T}$$

$$\text{or } \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{15/T} \text{ so } \frac{15}{T} = 6 \text{ or } T = 2.5 \text{ hour.}$$

EXAMPLE 16

The half lives of X and Y are 3 minutes and 27 minutes respectively. At some instant activity of both are same, then the ratio of active nuclei of X and Y at that instant is ?

SOLUTION:

$$A_1 = \lambda_1 N_1 \text{ and } A_2 = \lambda_2 N_2$$

$$A_1 = A_2 \Rightarrow \frac{0.693}{T_1} N_1 = \frac{0.693}{T_2} N_2$$

$$\Rightarrow \frac{N_1}{T_1} = \frac{N_2}{T_2} \Rightarrow \frac{N_1}{N_2} = \frac{3}{27} = \frac{1}{9}$$

$$\Rightarrow N_1 : N_2 = 1 : 9$$

EXAMPLE 17

One gram of Radium emits 3.7×10^{10} α particles per second. Calculate half life and mean life of Radium. Given Atomic mass of Radium = 226

SOLUTION:

Rate of decay of Radium = rate of emission of α particles.

$$\text{or } \frac{-dN}{dt} = \lambda N = 3.7 \times 10^{10} \text{ per second}$$

$$\text{Number of active atoms } N = \frac{6.023 \times 10^{23} \times 1}{226}$$

$$\therefore \lambda N = \frac{0.693}{T} \times \frac{6.023 \times 10^{23}}{226} = 3.7 \times 10^{10}$$

$$\text{or } T = 1583 \text{ years}$$

$$\text{Mean life } \tau = 1.44 T = 1.44 \times 1580 = 2279 \text{ years}$$

EXAMPLE 18

If a radioactive material contains 0.1 mg of Th²³⁴ how much of it will remain unchanged after 120 days. Given Half life is 24 days.

SOLUTION:

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{120/24} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

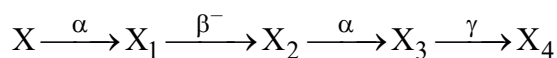
$$M = \frac{M_0}{32} = \frac{0.1 \text{ mg}}{32} = 3.125 \text{ } \mu\text{g.}$$

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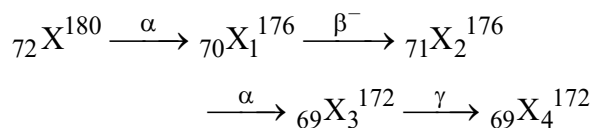
EXAMPLE 19

A radioactive nucleus decays as



If mass number and charge number of X are 180 and 72 then find these values of X_4 .

SOLUTION:



Checkup 3

- Q.1** The thallium ${}_{81}^{208}\text{Tl}$ nucleus is radioactive, with a half-life of 3.053 min. At a given instant, the activity of a certain sample of thallium is 2400 Bq. Using the concept of a half-life, and without doing any written calculations, determine whether the activity 9 minutes later is
- (a) a little less than $\frac{1}{8}$ (2400 Bq) = 300 Bq,
(b) a little more than $\frac{1}{8}$ (2400 Bq) = 300 Bq,
(c) a little less than $\frac{1}{3}$ (2400 Bq) = 800 Bq, or
(d) a little more than $\frac{1}{3}$ (2400 Bq) = 800 Bq.
- Q.2** The half-life of indium ${}_{49}^{115}\text{In}$ is 4.41×10^{14} yr. Thus, one-half of the nuclei in a sample of this isotope will decay in this time, which is very long. Is it possible for any single nucleus in the sample to decay after only one second?
- Q.3** Is it possible for two different samples of the same radioactive element to have different activities?
- Q.4** To which one or more of the following objects, each about 1000 year old, can the radiocarbon dating technique not be applied? (a) A wooden box (b) A gold statue (c) Some well-preserved animal fur.
- Q.5** Suppose there were a greater number of carbon ${}^{14}_6\text{C}$ atoms in a plant living 5000 year ago than is currently believed. When the seeds of this plant are tested using radiocarbon dating, is the age obtained too small or too large?
- Q.6** Tritium is an isotope of hydrogen and undergoes β^- decay with a half-life of 12.33 year. Like carbon ${}^{14}_6\text{C}$, tritium is produced in the atmosphere because of cosmic rays and can be used in a radioactive dating technique. Can tritium dating be used to determine a reliable date for a sample that is about 700 year old?
- Q.7** The half life of a radioactive substance is _____ than its mean life by _____.
- (A) smaller, 69.13% (B) smaller, 30.7%
(C) larger, 30.7% (D) larger, 69.3%
- Q.8** The half life of ${}^{131}\text{I}$ is 8 days. Given a sample of ${}^{131}\text{I}$ at time $t = 0$, we can assert that
- (A) no nucleus will decay before $t = 4$ days.
(B) no nucleus will decay before $t = 8$ days.
(C) all nuclei will decay before $t = 16$ days.
(D) a given nucleus may decay at any time $t = 0$.
- Q.9** A sample contains 16 gm of radioactive material, the half life of which is two days. After 32 days, the amount of sample is
- (A) less than 1 milligram (B) 1/4 gm
(C) 1/2 gm (D) 1 gm
- Q.10** The half life Po-218 is 3 minutes. What fraction of a 10 gram sample of Po-218 will remain after 15 min?
- (A) 1/32 (B) 1/64
(C) 1/25 (D) 1/15
- Q.11** There are three lumps of a radioactive substance. Their activities are in the ratio of 1 : 2 : 3. What will be the ratio of their activities at any future time?
- (A) 1 : 2 : 3 (B) 2 : 1 : 3
(C) 3 : 2 : 1 (D) 2 : 3 : 1

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IMPORTANT POINTS

Bohr Atomic Model :

- * The electron in a stable orbit does not radiate

$$\text{energy . i.e. } \frac{mv^2}{r} = \frac{kze^2}{r^2}$$

- * A stable orbit is that in which the angular momentum of the electron about nucleus is an

$$\text{integral (n) multiple of } \frac{h}{2\pi}.$$

$$\text{i.e. } mvr = n \frac{h}{2\pi}; n = 1, 2, 3, \dots (n \neq 0).$$

- * The electron can absorb or radiate energy only if the electron jumps from a lower to a higher orbit or falls from a higher to a lower orbit.

- * **For hydrogen atom (Z = 1)**

For n^{th} orbit –

- $L_n = \text{angular momentum} = n \frac{h}{2\pi}$
- $r_n = (0.529 \text{ \AA}) n^2$
- $E_n = \frac{-13.6 \text{ eV}}{n^2}$
- Binding Energy (BE) $_n = -E_n = \frac{13.6 \text{ eV}}{n^2}$.
- $E_{n_2} - E_{n_1} = \text{Energy emitted when an electron jumps from } n_2^{\text{th}} \text{ orbit to } n_1^{\text{th}} \text{ orbit } (n_2 > n_1).$

$$\Delta E = (13.6 \text{ eV}) \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\Delta E = hv$$

$v = \text{frequency of spectral line emitted.}$

$$\frac{1}{\lambda} = v = \text{wave number} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right].$$

- For hydrogen like atom/species of atomic number Z:

$$r = (0.529 \text{ \AA}) \frac{n^2}{Z}; E = (-13.6) \frac{Z^2}{n^2} \text{ eV}$$

- * Nuclides with the same atomic number Z, but different neutron number N are called isotopes. Nuclides with the same A are isobars and those with the same N are isotones.

- * Radii of nuclei : $R = R_0 A^{1/3}$, where $R_0 = \text{a constant} = 1.2 \text{ fm.}$

Nuclear density is independent of A. It is of the order of 10^{17} kg/m^3 .

- * The difference in mass of a nucleus and its constituents is called the **mass defect**,

$$\Delta M = (Z m_p + (A - Z) m_n) - M;$$

$$\Delta E_b = \Delta M c^2; 1 \text{ amu} = 931 \text{ MeV}$$

$$\text{B.E. per nucleon} = \frac{(\Delta M) c^2}{A}$$

Greater the B.E., greater is the stability of the nucleus.

- * In the mass number range $A = 30$ to 170 , the binding energy per nucleon is nearly constant, about 8 MeV/nucleon.

- * Law of radioactive decay : $N = N_0 e^{-\lambda t}$.

- * Activity = $\frac{dN}{N} = -\lambda N$ (unit is Becquerel)

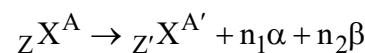
- * Half time period, $T_{1/2} = \frac{0.693}{\lambda}$

- * Mean life of an atom

$$= \frac{\Sigma \text{ life time of all atoms}}{\text{total number of atoms}}; T_{av} = \frac{1}{\lambda}$$

- * **Curie** : The unit of activity of any radioactive substance in which the number of disintegration per second is 3.7×10^{10} .

- * For calculating number of α and β particles

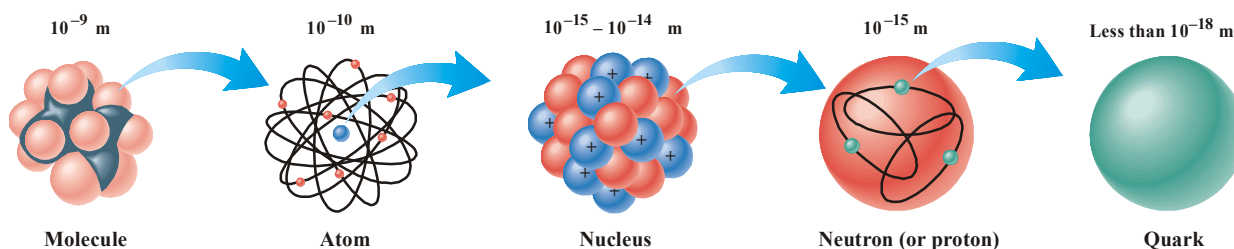


$$A = A' + 4n_1; Z = Z' + 2n_1 - n_2$$

	Electron	Proton	Neutron
Discovery	Sir. J. J. Thomson	Goldstein	Chadwick
Nature of charge	Negative	Positive	Neutral
Amount of charge	-1.6×10^{-19} Coloumb	1.6×10^{-19} Coloumb	zero
Mass	$9.10939 \times 10^{-31} \text{ kg}$	$1.67262 \times 10^{-27} \text{ kg}$	$1.67493 \times 10^{-27} \text{ kg}$
Mass (amu)	0.00054	1.00727	1.00867

* **Results of Bohr's Theory**

Physical quantity	Formula	Ratio
Radius of Bohr orbit (r_n)	$r_n = 0.53 \frac{n^2}{Z} \text{ \AA}$	$r_1 : r_2 : r_3 \dots r_n = 1 : 4 : 9 \dots n^2$
Velocity of electron in n^{th} Bohr orbit (v_n)	$v_n = \frac{2\pi KZe^2}{nh}$	$v_1 : v_2 : v_3 \dots v_n = 1 : \frac{1}{2} : \frac{1}{3} \dots \frac{1}{n}$
Angular velocity of electron (ω_n)	$\omega_n = \frac{8\pi^3 K^2 Z^2 mc^4}{n^3 h^3}$	$\omega_1 : \omega_2 : \omega_3 \dots \omega_n = 1 : \frac{1}{8} : \frac{1}{27} \dots \frac{1}{n^3}$
Time Period of electron (T_n)	$T_n = \frac{n^3 h^3}{4\pi K^2 Z^2 me^4}$	$T_1 : T_2 : T_3 \dots T_n = 1 : 8 : 27 \dots n^3$
Orbital current (I_n)	$I_n = \frac{4\pi^2 K^2 Z^2 m}{n^3 h^3}$	$I_1 : I_2 : I_3 \dots I_n = 1 : \frac{1}{8} : \frac{1}{27} \dots \frac{1}{n^3}$
Angular momentum (J_n)	$J_n = \frac{nh}{2\pi}$	$J_1 : J_2 : J_3 \dots J_n = 1 : 2 : 3 \dots n$
Centripetal acceleration (a_n)	$a_n = \frac{16\pi^4 K^3 Z^3 me^6}{n^4 h^4}$	$a_1 : a_2 : a_3 \dots a_n = 1 : \frac{1}{16} : \frac{1}{81} \dots \frac{1}{n^4}$
Kinetic energy (E_{K_n})	$E_{K_n} = \frac{RchZ^2}{n^2}$	$E_{K_1} : E_{K_2} \dots E_{K_n} = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$
Potential energy (U_n)	$U_n = \frac{-2RChZ^2}{n^2}$	$U_1 : U_2 : U_3 \dots U_n = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$
Total energy (E_n)	$E_n = \frac{-RChZ^2}{n^2}$	$E_1 : E_2 : E_3 \dots E_n = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$



* **Figure :** The current view of how matter is composed of basic units, starting with a molecule and ending with a quark. The approximate sizes of each unit are also listed.

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SOLVED EXAMPLES

EXAMPLE 1

What is the shortest wavelength present in the Paschen series of spectral lines?

SOLUTION:

The wavelength of the spectral lines forming Paschen series is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n_1^2} \right)$$

For shortest wavelength, $n_1 = \infty$

$$\therefore \frac{1}{\lambda} = \frac{R}{9} \text{ or } \lambda = \frac{9}{R}. \text{ Since, } \frac{1}{R} = 911 \text{ \AA}$$

$$\therefore \lambda = 9 \times 911 = 8199 \text{ \AA}.$$

EXAMPLE 2

A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

SOLUTION:

$$E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J} \\ = 3.68 \times 10^{-19} \text{ J}$$

$$\nu = \frac{E}{h} = \frac{3.68 \times 10^{-19}}{6.626 \times 10^{-34}} = 5.55 \times 10^{14} \text{ Hz}.$$

EXAMPLE 3

The ground state energy of hydrogen atom is -13.6 eV . What are the kinetic and potential energies of the electron in this state

SOLUTION:

$$\text{The PE of electron } E_p = - \frac{1}{4\pi \epsilon_0} \frac{e^2}{r}$$

$$\text{KE is } E_k = \frac{1}{2} \times \frac{1}{4\pi \epsilon_0} \frac{e^2}{r} = -\frac{1}{2} E_p$$

$$\text{Since } E_p + E_k = -13.6 \text{ eV}$$

$$E_p - \frac{1}{2} E_p = -13.6 \text{ eV i.e., } E_p = -27.2 \text{ eV}$$

$$\therefore E_k = -\frac{1}{2} E_p = \frac{27.2}{2} = 13.6 \text{ eV}$$

EXAMPLE 4

A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon,

SOLUTION:

We know, energy of an electron in the n th orbit

$$\text{of hydrogen atom is given by } E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\text{When } n = 1 \text{ (ground level), } E_1 = -13.6 \text{ eV}$$

$$\text{When } n = 4, E_4 = -\frac{13.6}{16} = -0.85 \text{ eV}$$

$$\therefore \text{Energy difference between } n = 1 \text{ and } n = 4 \\ = E_4 - E_1 = -0.85 + 13.6 = 12.75 \text{ eV}$$

$$\therefore \text{The hydrogen atom will go to } n = 4$$

$$\text{From } n = 1 \text{ if the photon energy} = 12.75 \text{ eV} \\ = 12.75 \times 1.6 \times 10^{-19} \text{ J} = 20.4 \times 10^{-19} \text{ J}$$

$$\text{or } h\nu = 20.4 \times 10^{-19}$$

$$\text{or } \nu = \frac{20.4 \times 10^{-19}}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}} \\ = 3.08 \times 10^{15} \text{ Hz}$$

$$\therefore \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.08 \times 10^{15}} = 9.74 \times 10^{-8} \text{ m}.$$

EXAMPLE 5

Obtain the binding energy of the nuclei ${}^{56}_{26}\text{Fe}$

$$m({}^{56}_{26}\text{Fe}) = 55.934939 \text{ amu}.$$

SOLUTION:

${}^{56}_{26}\text{Fe}$ nucleus contains 26 protons and

$$(56 - 26) = 30 \text{ neutrons}$$

$$\text{Mass of 26 protons} = 26 \times 1.007825 \\ = 26.20345 \text{ amu}$$

$$\text{Mass of 30 neutrons} = 30 \times 1.008665 \\ = 30.25995 \text{ amu}$$

$$\text{Total mass of 56 nucleons} = 56.46340 \text{ amu}$$

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Mass of ${}_{26}^{56}\text{Fe}$ nucleus = 55.934939

$$\therefore \text{Mass defect, } \Delta m = 56.46340 - 55.934939 \\ = 0.528461 \text{ amu}$$

Total binding energy
= $0.528461 \times 931.5 \text{ MeV} = 492.26 \text{ MeV}$
Average binding energy per nucleon

$$= \frac{492.26}{56} = 8.790 \text{ MeV}$$

EXAMPLE 6

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to 3.125% of its original value.

SOLUTION:

The fraction of the original sample left

$$= \frac{3.125}{100} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

Hence, there are 5 half lives of T years spent. Thus, the time taken is 5T years.

EXAMPLE 7

The electron energy in hydrogen atom is given by $E = (-2.18 \times 10^{-18})/n^2$ joules. Calculate the energy required to remove an electron completely from the $n = 2$ orbit. What is the longest wavelength (in cm) of light that can be used to cause this transition?

SOLUTION:

$$\text{We know, } E_n = \frac{-2.18 \times 10^{-18}}{n^2} \text{ J}$$

Energy required to remove an electron from $n = 2$,

$$\Delta E = E_\infty - E_2 = -2.18 \times 10^{-18} \text{ J} \left(\frac{1}{\infty^2} - \frac{1}{2^2} \right) \\ = \frac{2.18 \times 10^{-18} \text{ J}}{4} = 5.42 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{5.42 \times 10^{-19} \text{ J}}$$

$$\lambda = 3.667 \times 10^{-7} \text{ m} = 3.667 \times 10^{-5} \text{ cm.}$$

EXAMPLE 8

Obtain the amount of ${}_{27}^{60}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength.

The half-life of ${}_{27}^{60}\text{Co}$ is 5.3 years.

SOLUTION:

$$\text{Strength of radioactive source} \\ = 8.0 \text{ mCi} = 8.0 \times 10^{-3} \text{ Ci} \\ = 8.0 \times 10^{-3} \times 3.7 \times 10^{10} \text{ disintegrations s}^{-1} \\ = 29.6 \times 10^7 \text{ disintegrations s}^{-1}$$

Since the strength of the source decreases with

$$\text{time, } \frac{dN}{dt} = -29.6 \times 10^7.$$

$$\text{But } \frac{dN}{dt} = -\lambda N$$

$$\therefore -\lambda N = -29.6 \times 10^7$$

$$\text{or } \lambda N = 29.6 \times 10^7 \text{ or } N = \frac{29.6 \times 10^7}{\lambda}$$

$$\text{or } N = \frac{29.6 \times 10^7 \times T}{0.693} \left(\because \lambda = \frac{0.693}{T} \right)$$

$$= \frac{29.6 \times 10^7 \times 5.3 \times 365 \times 24 \times 60 \times 60}{0.693} \\ = 7.137 \times 10^{16}$$

Number of atoms in 60g of cobalt

$$= 6.023 \times 10^{23}$$

Mass of 1 atom of cobalt

$$= \frac{60}{6.023 \times 10^{23}} \times 7.139 \times 10^{16} \text{ g} = 7.11 \mu\text{g}$$

EXAMPLE 9

Obtain approximately the ratio of the nuclear radii of the gold isotope ${}_{79}^{197}\text{Au}$ and the silver isotope ${}_{47}^{107}\text{Ag}$.

SOLUTION:

$$\text{As, } R \approx A^{1/3}$$

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2} \right)^{1/3} = \left(\frac{197}{107} \right)^{1/3} = (1.84)^{1/3}$$

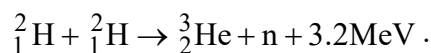
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EXAMPLE 10

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium?

Take the fusion reaction as



SOLUTION:

When two nuclei of deuterium fuse together, energy released = 3.2 MeV

Number of deuterium atoms in 2 kg

$$= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$$

When 6.023×10^{26} nuclei of deuterium fuse together.

Energy released

$$= \frac{3.2}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.2}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J}$$

$$= 1.54 \times 10^{14} \text{ J}$$

Power of electric lamp = 100W

If the lamp glows for time t , then the electrical energy consumed by the lamp is $100t$.

$$\therefore 100t = 1.54 \times 10^{14} \text{ J or } t = 1.54 \times 10^{12} \text{ s}$$

$$= \frac{1.54 \times 10^{12}}{3.154 \times 10^7} \text{ years} = 4.88 \times 10^4 \text{ years.}$$

EXAMPLE 11

A sample has 4×10^{16} radioactive nuclei of half life 10 days. The number of atoms decaying in 30 days is –

(A) 3.9×10^{16}

(B) 5×10^{15}

(C) 10^{16}

(D) 3.5×10^{16}

SOLUTION:

$$\text{(D). } N = 4 \times 10^{16} \left(\frac{1}{2}\right)^{30/10} = \frac{1}{2} \times 10^{16}$$

$$\begin{aligned} \text{Atoms decayed} &= 4 \times 10^{16} - \frac{1}{2} \times 10^{16} \\ &= 3.5 \times 10^{16} \end{aligned}$$

EXAMPLE 12

A sample of radioactive element has a mass of 10gm at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives

(A) 6.30gm

(B) 1.35gm

(C) 2.50gm.

(D) 3.70gm

SOLUTION:

(B). Using the relation for mean life.

$$\text{Given : } t = 2\tau = 2 \left(\frac{1}{\lambda}\right) \quad \left(\because \tau = \frac{1}{\lambda}\right)$$

$$M = M_0 e^{-\lambda t} = 10 e^{-\lambda \times \frac{2}{\lambda}} = 10 \left(\frac{1}{e}\right)^2 = 1.35\text{g}$$

EXAMPLE 13

If in nuclear fusion process the masses of the fusing nuclei be m_1 and m_2 and the mass of the resultant nucleus be m_3 , then –

(A) $m_3 > (m_1 + m_2)$

(B) $m_3 = m_1 + m_2$

(C) $m_3 = |m_1 - m_2|$

(D) $m_3 < (m_1 + m_2)$

SOLUTION:

(D). $m_3 < (m_1 + m_2)$ ($\because m_1 + m_2 = m_3 + E$)
as $E = [m_1 + m_2 - m_3] C^2$

EXAMPLE 14

The half life of radium is about 1600 years. Of 100g of radium existing now, 25g will remain unchanged after –

(A) 3200 years

(B) 4800 years

(C) 6400 years

(D) 2400 years

SOLUTION:

(A). 100g will become 25g in two half lives so it is 3200 years.

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EXAMPLE 15

A radio isotope X with a half life T years decays to Y which is stable. A sample of the rock from a cave was found to contain X and Y in the ratio (1 : A), then find the age of the rock.

SOLUTION:

$$X : Y = 1 : A ; X : (X + Y) = 1 : (A+1)$$

$$\text{If } (A + 1) = 2^n$$

then age of the rock will be $t = nT_{1/2} = nT$

For example, if $a = 7$, $n = 3$, $t = 3T_{1/2}$.

EXAMPLE 16

The half life of a radioactive nucleus is T days. The time interval $(t_2 - t_1)$ between the time t_2 when 2/3 of it has decayed and the time t_1 when 1/3 of it had decayed is T days.

SOLUTION:

$$N_1 = N_0 e^{-\lambda t}; N_1 = \frac{1}{3} N_0$$
$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \quad \dots\dots (1)$$

$$N_2 = \frac{2}{3} N_0 ; \frac{2}{3} N_0 = N_0 e^{-\lambda t_1} \quad \dots\dots (2)$$

From eq. (1) and (2)

$$\frac{1}{2} = e^{-\lambda (t_2 - t_1)} ; \lambda (t_2 - t_1) = \ln 2$$

$$t_2 - t_1 = \frac{\ln 2}{\lambda} = T_{1/2} = T$$

EXAMPLE 17

The mean life of a radioactive material for alpha decay and beta decay are, respectively, 1620 years and 520 years. What is the half life of the sample (in years) ?

SOLUTION:

There exists two channels for the decay. Therefore, the mean life is obtained from

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \quad \text{or} \quad t = \frac{t_1 t_2}{t_1 + t_2} = \frac{1620 \times 520}{1620 + 520}$$
$$= 394 \text{ years}$$

The half life $T = 0.693 t$

$$= 0.693 \times 394 = 273 \text{ years.}$$

EXAMPLE 18

A freshly prepared radioactive sample has activity which is 64 times the standard background activity considered as safe. If the half life of the sample is 2 hours, than after how much time the activity falls below the safety levels?

SOLUTION:

Let us assume that the safe activity is R_s .

Then $R_0 = 64 R_s$ and we require time in which R becomes R_s .

$$\text{Now use } \frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T} \quad \text{or} \quad \frac{R_s}{64R_s} = \left(\frac{1}{2}\right)^{t/2}$$

$$\text{or} \quad \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/2} \quad \text{or} \quad 6 = t/2 \quad \text{or} \quad t = 12 \text{ h}$$



QUESTION BANK

EXERCISE-1 (LEVEL-1)

SECTION - 1 (VOCABULARY BUILDER)

Choose one correct response for each question.

For Q.1-Q.5 : Match the column I with column II.

Q.1 For atomic model of hydrogen atom given by Niels Bohr, match the following proportionalities

Column I

- (a) Angular momentum
- (b) Velocity of electron
- (c) Radius of orbit
- (d) Energy of electron

Column II

- (i) $1/n$
- (ii) n^2
- (iii) $1/n^2$
- (iv) n

Codes

- (A) (a) - i, (b)-ii, (c)-iii, (d)-iv
- (B) (a) - iv, (b)-iii, (c)-ii, (d)-i
- (C) (a) - iv, (b)-i, (c)-iii, (d)-ii
- (D) (a) - iv, (b)-i, (c)-ii, (d)-iii

Q.2 For spectral series of hydrogen atom, match the following

Column I

- (a) $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$ (i) Lyman series
- (b) $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$ (ii) Paschen series
- (c) $\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right)$ (iii) Brackett series
- (d) $\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$ (iv) Pfund series

Column II

Codes

- (A) (a) - iv, (b)-iii, (c)-ii, (d)-i
- (B) (a) - iii, (b)-ii, (c)-iv, (d)-i
- (C) (a) - ii, (b)-i, (c)-iv, (d)-i
- (D) (a) - i, (b)-ii, (c)-iii, (d)-iv

Q.3 Match the Column

Column I

- (a) Fission
- (b) Fusion
- (c) β -decay
- (d) Exothermic nuclear

Column II

- (i) Matter-energy
- (ii) In atoms of high atomic number only
- (iii) In atoms of low atomic number only
- (iv) Involves weak nuclear forces

- (A) (a) - i, ii ; (b)-i, iii ; (c)-iv ; (d)-i
- (B) (a) - i, iii ; (b)-i, ii ; (c)-iv ; (d)-ii
- (C) (a) - ii ; (b)-i, iv ; (c)-i ; (d)-ii, iii
- (D) (a) - iii ; (b)-i, ii ; (c)-i, ii ; (d)-iii

Q.4 Match column I of the nuclear processes with column II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

Column I

- (a) Alpha decay
- (b) β^+ decay
- (c) Fission
- (d) Proton emission

Column II

- (i) ${}^8_{15}\text{O} \rightarrow {}^7_{15}\text{N} + \dots$
- (ii) ${}^{92}_{238}\text{U} \rightarrow {}^{90}_{234}\text{Th} + \dots$
- (iii) ${}^{83}_{185}\text{Bi} \rightarrow {}^{82}_{184}\text{Pb} + \dots$
- (iv) ${}^{94}_{239}\text{Pu} \rightarrow {}^{57}_{140}\text{La} + \dots$

Codes:

- (A) (a) - iv, (b) - ii, (c) - i, (d) - iii
- (B) (a) - i, (b) - iii, (c) - ii, (d) - iv
- (C) (a) - ii, (b) - i, (c) - iv, (d) - iii
- (D) (a) - iv, (c) - iii, (c) - ii, (d) - i

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- | Q.5 | Column I
(Scientist name) | Column II
(Discovery) | |
|------------|--------------------------------------|--|--|
| (a) | A.H. Becquerel | (i) Discovered radioactivity of uranium. | (c) R. A. Millikan (iii) Showed that electrons have a negative charge. |
| (b) | J. J. Thomson | (ii) Measured the electronic charge. | (d) E. Rutherford (iv) Established the nuclear model of the atom. |
| | | | (A) (a) - ii, (b)-iii, (c)-iv, (d)-i |
| | | | (B) (a) - i, (b)-ii, (c)-iv, (d)-iii |
| | | | (C) (a) - i, (b)-iii, (c)-ii, (d)-iv |
| | | | (D) (a) - iii, (b)-i, (c)-ii, (d)-iv |

SECTION - 2 (BASIC CONCEPTS BUILDER)

For Q.6 to Q.25 : Choose one word for the given statement from the list.

Visible, ultraviolet, Rutherford's model, Heavy water, Balmer, Lyman, No different from, 1:4:9, 1:2:3, $0.16 \mu\text{C}$ 10.2eV , antineutrino, True, False, 13.6eV , line, light, heavy, continuous, $1 : 2^{1/3}$, liquid drop model of nucleus, 100, neutrino, Thomson's model.

- Q.6** A free neutron decays into a proton, an electron and _____.
- Q.7** When the electron jumps from any of the outer orbits to the first orbit, the spectral lines emitted are in the _____ region of the spectrum and they are said to form a series called _____ series.
- Q.8** When the electron jumps from any of the outer orbits to the second orbit, we get a spectral series called the _____ series. The lines of this series in hydrogen have their wavelength in the _____ region.
- Q.9** The ratio of the radii of the first three Bohr orbit is _____.
- Q.10** The first excitation potential energy or the minimum energy required to excite the atom from ground state of hydrogen atom is _____.
- Q.11** According to Rutherford atom model, the spectral lines emitted by an atom is _____ spectrum.
- Q.12** The ratio of the angular momentum of the electron in first three Bohr orbit is _____.
- Q.13** A nucleus breaks into two parts whose velocity is in ratio 2:1. The ratio of their radius is _____.
- Q.14** The size of the atom in Thomson's model is _____ the atomic size in Rutherford's model. (much greater than/no different from/much less than.)
- Q.15** In the ground state of _____ electrons are in stable equilibrium, while in _____ electrons always experience a net force. (Thomson's model/ Rutherford's model.)
- Q.16** A classical atom based on _____ is doomed to collapse. (Thomson's model/ Rutherford's model.)
- Q.17** An atom has a nearly continuous mass distribution in a _____ but has a highly non-uniform mass distribution in _____. (Thomson's model/ Rutherford's model.)
- Q.18** The positively charged part of the atom possesses most of the mass in _____. (Rutherford's model/both the models.)
- Q.19** _____ is used as a moderator in nuclear reactors.
- Q.20** In one α and 2β emissions atomic number reduces by 2. (True/False)
- Q.21** Nuclear fission can be explained by _____.

- Q.22** A nuclear decay is expressed as ${}_6\text{C}^{11} \rightarrow {}_5\text{B}^{11} + \beta^+ + \text{X}$, X is –
- Q.23** Half-life period of a radioactive substance is 6h. After 24 hrs. activity is $0.01 \mu\text{C}$, initial activity was ____
- Q.24** Half-life of a radioactive substances is 12.5h and its mass is 256g. After ____ hr, the amount of remaining substance is 1g.
- Q.25** ____ (light/heavy) nuclei are suitable for fusion process.

SECTION - 3 (ENHANCE PROBLEM SOLVING SKILLS)

Choose one correct response for each question.

**PART
1**

RUTHERFORD'S MODEL

- Q.26** Rutherford's experiments suggested that the size of the nucleus is about
(A) 10^{-14} m to 10^{-12} m
(B) 10^{-15} m to 10^{-13} m
(C) 10^{-15} m to 10^{-14} m
(D) 10^{-15} m to 10^{-12} m
- Q.27** For scattering of α -particles Rutherford suggested that –
(A) mass of atom and its positive charge were concentrated at centre of atom.
(B) only mass of atom is concentrated at centre of atom.
(C) Only positive charge of atom is concentrated at centre of atom.
(D) Mass of atom is uniformly distributed throughout its volume.
- Q.28** An α -particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of –
(A) 10^{-12} cm (B) 10^{-10} cm
(C) 1 \AA (D) 10^{-15} cm
- Q.29** In the Geiger-Marsden scattering experiment, in case of head-on collision the impact parameter should be
(A) maximum (B) minimum
(C) infinite (D) zero
- Q.30** Rutherford's atomic model could account for –
(A) concept of stationary orbits
(B) the positively charged central core of an atom
(C) origin of spectra
(D) stability of atoms

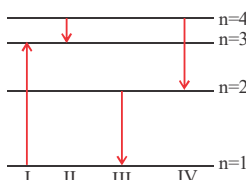
**PART
2**

BOHR'S MODEL

- Q.31** Ionisation energy of an electron in ground state of a hydrogen atom is –
(A) 13.6 eV
(B) -13.6 eV
(C) more than 13.6 eV
(D) less than 13.6 eV
- Q.32** The radius of hydrogen atom in its ground state is 5.3×10^{-11} m. After collision with an electron it is found to have a radius of 21.2×10^{-11} m. What is the principal quantum number n of the final state of the atom
(A) $n = 4$ (B) $n = 2$
(C) $n = 16$ (D) $n = 3$
- Q.33** The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be
(A) $\frac{16}{25} \lambda_0$ (B) $\frac{20}{27} \lambda_0$
(C) $\frac{27}{20} \lambda_0$ (D) $\frac{25}{16} \lambda_0$

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- Q.34** In the Bohr model of the hydrogen atom, let R , v and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which quantity is proportional to the quantum number n
- (A) R/E (B) E/v
(C) RE (D) vR
- Q.35** For the Bohr's first orbit of circumference $2\pi r$, the de-Broglie wavelength of revolving electron will be
- (A) $2\pi r$ (B) πr
(C) $1/2\pi r$ (D) $1/4\pi r$
- Q.36** The radius of first orbit of hydrogen atom is 0.53\AA . The radius of its fourth orbit will be-
- (A) 0.193\AA (B) 4.24\AA
(C) 2.12\AA (D) 8.48\AA
- Q.37** The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?
- 
- (A) III (B) IV
(C) I (D) II
- Q.38** Which of the following transitions in hydrogen atoms emit photons of highest frequency?
- (A) $n = 2$ to $n = 6$ (B) $n = 6$ to $n = 2$
(C) $n = 2$ to $n = 1$ (D) $n = 1$ to $n = 2$
- Q.39** As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion –
- (A) Kinetic energy, potential energy and total energy decrease.
(B) Kinetic energy decreases, potential energy increases but total energy remains same.
(C) Kinetic energy and total energy decrease but potential energy increases.
(D) Its kinetic energy increases but potential energy and total energy decrease.
- Q.40** In which of the following systems will the radius of first orbit ($n = 1$) be minimum?
- (A) doubly ionized lithium.
(B) singly ionized helium.
(C) deuterium atom.
(D) hydrogen atom.
- Q.41** The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because –
- (A) of the electrons not being subject to a central force.
(B) of the electrons colliding with each other.
(C) of screening effects.
(D) the force between the nucleus and an electron will no longer be given by coulomb's law.
- Q.42** The angular speed of the electron in the n^{th} orbit of Bohr's hydrogen atom is –
- (A) directly proportional to n .
(B) inversely proportional to $n^{1/2}$.
(C) inversely proportional to n^2 .
(D) inversely proportional to n^3 .
- Q.43** An electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are –
- (A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 2$
- Q.44** When electron jumps from $n = 4$ level to $n = 1$ level, the angular momentum of electron changes by
- (A) $4h/2\pi$ (B) $3h/2\pi$
(C) $2h/2\pi$ (D) $h/2\pi$

PART 3

ATOMIC SPECTRA

- Q.45** In a sample of hydrogen like atoms all of which are in ground state, a photon beam containing photons of various energies is passed. In absorption spectrum, five dark lines, are observed. The number of bright lines in the emission spectrum will be (assume that all transitions takes place).
 (A) 5 (B) 10
 (C) 15 (D) None of these
- Q.46** The ratio of the largest to shortest wavelengths in Lyman series of hydrogen spectra is
 (A) 25/9 (B) 17/6
 (C) 9/5 (D) 4/3
- Q.47** The energy of the highest energy photon of Balmer series of hydrogen spectrum is close to
 (A) 13.6 eV (B) 3.4 eV
 (C) 1.5 eV (D) 0.85 eV
- Q.48** If ν_1 is the frequency of the series limit of Lyman series, ν_2 is the frequency of the first line of Lyman series and ν_3 is the frequency of the series limit of the Balmer series, then
 (A) $\nu_1 - \nu_2 = \nu_3$ (B) $\nu_1 = \nu_2 - \nu_3$
 (C) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$ (D) $\frac{1}{\nu_1} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$
- Q.49** Spectrum of sunlight is an example for
 (A) Line absorption spectrum
 (B) Continuous emission spectrum
 (C) Continuous absorption spectrum
 (D) Band emission spectrum
- Q.50** Pick out the **INCORRECT** statement from the following:
 (A) Mercury vapour lamp produces line emission spectrum.
 (B) Oil flame produces line emission spectrum.
 (C) Band spectrum helps us to study molecular structure.
 (D) Sunlight spectrum is an example for line absorption spectrum.
- Q.51** Which of the following spectral series of hydrogen atom is lying in visible range of electromagnetic wave?
 (A) Paschen series (B) Pfund series
 (C) Lyman series (D) Balmer series

PART 4

NUCLEUS

- Q.52** Correct order is –
 (A) $F_{\text{gravitation}} > F_{\text{electrostatic}} > F_{\text{nuclear}}$
 (B) $F_{\text{nuclear}} > F_{\text{gravitation}} > F_{\text{electrostatic}}$
 (C) $F_{\text{nuclear}} > F_{\text{electrostatic}} > F_{\text{gravitation}}$
 (D) $F_{\text{gravitation}} > F_{\text{nuclear}} > F_{\text{electrostatic}}$
- Q.53** Nuclear radius of ${}^8\text{O}^{16}$ is 3×10^{-15} m. Find the density of nuclear matter.
 (A) $7.5 \times 10^{17} \text{ kg m}^{-3}$ (B) $5.7 \times 10^{17} \text{ kg m}^{-3}$
 (C) $2.3 \times 10^{17} \text{ kg m}^{-3}$ (D) $1.66 \times 10^{17} \text{ kg m}^{-3}$
- Q.54** The ratio of the radii of the nuclei ${}^{27}_{13}\text{Al}$ and ${}^{52}_{25}\text{Te}^{125}$ is approximately –
 (A) 6 : 10 (B) 13 : 52
 (C) 40 : 177 (D) 14 : 73
- Q.55** The radius of the ${}_{30}\text{Zn}^{64}$ nucleus is nearly (in fm)–
 (A) 1.2 (B) 2.4
 (C) 3.7 (D) 4.8
- Q.56** A nucleus ${}_Z\text{X}^A$ emits 9α -particles and 5β particle. The ratio of total protons and neutrons in the final nucleus is
 (A) $\frac{Z-13}{A-Z-23}$ (B) $\frac{(Z-18)}{(A-36)}$
 (C) $\frac{(Z-13)}{(A-36)}$ (D) $\frac{(Z-13)}{(A-Z-13)}$

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- Q.57** Determine the ratio of speed of electrons in hydrogen atom in its 3rd & 4th orbit
(A) 1 : 2 (B) 1 : 3
(C) 1 : 4 (D) 4 : 3
- Q.58** When ${}_3\text{Li}^7$ nuclei are bombarded by protons, and the resultant nuclei are ${}_4\text{Be}^8$, the emitted particles will be –
(A) gamma photons (B) neutrons
(C) alpha particles (D) beta particles
- Q.59** A nucleus with $Z=92$ emits the following in a sequence : $\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is –
(A) 78 (B) 82
(C) 74 (D) 76
- Q.60** O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms
(A) is not important because nuclear forces are short-ranged.
(B) is as important as electrostatic force for binding the two atoms.
(C) cancels the repulsive electrostatic force between the nuclei.
(D) is not important because oxygen nucleus have equal number of neutrons & protons.
- Q.61** Masses of nuclei of hydrogen, deuterium and tritium are in ratio –
(A) 1 : 2 : 3 (B) 1 : 1 : 1
(C) 1 : 1 : 2 (D) 1 : 2 : 4
- Q.62** The ratio of the nuclear radii of the gold isotope ${}_{79}^{197}\text{Au}$ and silver isotope ${}_{47}^{107}\text{Ag}$ is
(A) 1.23 (B) 0.216
(C) 2.13 (D) 3.46
- Q.63** Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}\text{Ne}$ nucleus and M_2 the mass of a ${}_{20}^{40}\text{Ca}$ nucleus. Then –
(A) $M_2 = M_1$ (B) $M_2 > 2M_1$
(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$
- Q.64** A force between two protons is same as the force between proton and neutron. The nature of the force is
(A) Weak nuclear force
(B) Strong nuclear force
(C) Electrical force
(D) Gravitational force
- Q.65** Binding energy per nucleon is
(A) energy required to separate proton from the nucleus.
(B) energy required to separate a neutron from the nucleus.
(C) energy required to separate nucleons of a nucleus.
(D) energy required to separate a proton or a neutron (on an average) from the nucleus.
- Q.66** If mass equivalent to one mass of proton is completely converted into energy then determine the energy produced?
(A) 931.49 MeV (B) 731.49 MeV
(C) 911.49 MeV (D) 431.49 MeV
- Q.67** If mass equivalent to one mass of electron is completely converted into energy then determine the energy liberated.
(A) 1.51 MeV (B) 0.51 MeV
(C) 3.12 MeV (D) 2.12 MeV
- Q.68** If the binding energy of deuterium is 2.23 MeV, then the mass defect will be- (in a.m.u.)
(A) 0.0024 (B) – 0.0024
(C) – 0.0012 (D) 0.0012
- Q.69** The mass defect for the nucleus of helium is 0.0303 a.m.u. What is the binding energy per nucleon for helium in MeV
(A) 28 (B) 7
(C) 4 (D) 1

PART
5

BINDING ENERGY

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- Q.70** If the binding energy per nucleon in Li^7 and He^4 nuclei are respectively 5.60 MeV and 7.06 MeV, then energy of reaction $\text{Li}^7 + \text{p} \rightarrow 2\text{He}^4$ is
(A) 19.6 MeV (B) 2.4 MeV
(C) 8.4 MeV (D) 17.3 MeV

- Q.71** The mass defect in a particular nuclear reaction is 0.3 grams. The amount of energy liberated in kilowatt hours is (Velocity of light = $3 \times 10^8 \text{m/s}$)
(A) 1.5×10^6 (B) 2.5×10^6
(C) 3×10^6 (D) 7.5×10^6

- Q.72** The mass of ${}^7_3\text{Li}$ is 0.042 amu less than the sum of masses of its constituents. The binding energy per nucleon is –
(A) 5.586 MeV (B) 10.522 MeV
(C) 2.433 MeV (D) 3.739 MeV

PART 6 NUCLEAR ENERGY

- Q.73** How much mass has to be converted into energy to produce electric power of 500 MW for one hour?
(A) $2 \times 10^{-5} \text{kg}$ (B) $1 \times 10^{-5} \text{kg}$
(C) $3 \times 10^{-5} \text{kg}$ (D) $4 \times 10^{-5} \text{kg}$

- Q.74** Commonly used moderators are
I. water. II. heavy water (D_2O)
III. graphite IV. sodium chloride (NaCl).
(A) I, II and III (B) I and II
(C) I, II and IV (D) All of these

- Q.75** Fast neutrons can easily be slowed down by
(A) the use of lead shielding.
(B) passing them through water.
(C) elastic collisions with heavy nuclei.
(D) applying a strong electric field.

- Q.76** From fission reaction of ${}^{235}_{92}\text{U}$, on an average number of neutrons (per fission) released is –
(A) 1 (B) 2
(C) 3 (D) 2.5

PART 7

RADIOACTIVITY

- Q.77** SI unit for activity is –
(A) Curie (B) Rutherford
(C) Pascal (D) Becquerel
- Q.78** The half life of a radioactive substance is 20 s, the time taken for the sample to decay by $7/8^{\text{th}}$ of its initial value is
(A) 20 s (B) 40 s
(C) 60 s (D) 80 s
- Q.79** For a radioactive sample half life $T_{1/2}$ and disintegration constant λ are related as
(A) $T_{1/2} = \ln 2 \cdot \lambda$ (B) $T_{1/2} = \frac{\ln 2}{\lambda}$
(C) $T_{1/2} \times \ln 2 = \lambda$ (D) None of these
- Q.80** If $t_{1/2}$ is the half life of a substance then $t_{3/4}$ is the time in which substance
(A) Decays $(3/4)^{\text{th}}$ (B) Remains $(3/4)^{\text{th}}$
(C) Decays $(1/2)$ (D) Remains $(1/2)$
- Q.81** The half-life period of radium is 1600 years. Its average life time will be
(A) 3200 years (B) 4800 years
(C) 2319 years (D) 4217 years
- Q.82** Three α -particles and one β -particle decaying takes place in series from an isotope ${}_{88}\text{Ra}^{236}$. Finally the isotope obtained will be
(A) ${}_{84}\text{X}^{220}$ (B) ${}_{86}\text{X}^{222}$
(C) ${}_{83}\text{X}^{224}$ (D) ${}_{83}\text{X}^{215}$
- Q.83** The counting rate observed from a radioactive source at $t=0$ second was 1600 counts per second and at $t=8$ seconds it was 100 counts per second. The counting rate observed, as counts per second at $t=6$ seconds, will be
(A) 400 (B) 300
(C) 200 (D) 150

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- Q.84** A radio isotope has a half life of 75 years. The fraction of the atoms of this material that would decay in 150 years will be
(A) 66.6% (B) 85.5%
(C) 62.5% (D) 75%
- Q.85** An atomic nucleus ${}_{90}\text{Th}^{232}$ emits several α and β radiations and finally reduces to ${}_{82}\text{Pb}^{208}$. It must have emitted
(A) 4α and 2β (B) 6α and 4β
(C) 8α and 24β (D) 4α and 16β
- Q.86** In a mean life of a radioactive sample
(A) About 1/3 of substance disintegrates
(B) About 2/3 of the substance disintegrates
(C) About 90% of the substance disintegrates
(D) Almost all the substance disintegrates
- Q.87** The radioactivity of an element becomes $1/64^{\text{th}}$ of its original value in 60 sec. Then the half life period is
(A) 5 sec (B) 10 sec
(C) 20 sec (D) 30 sec
- Q.88** A radioactive material has a half life of 10 days. What fraction of the material would remain after 30 days
(A) 0.5 (B) 0.25
(C) 0.125 (D) 0.33
- Q.89** Two radioactive nuclei A and B are taken with their disintegration constant λ_A and λ_B and initially N_A and N_B number of nuclei are taken then the time after which their undisintegrated nuclei are same is
(A) $\frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$
(B) $\frac{1}{(\lambda_A + \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$
(C) $\frac{1}{(\lambda_B - \lambda_A)} \ln \left(\frac{N_B}{N_A} \right)$
(D) $\frac{1}{(\lambda_A - \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$
- Q.90** Consider α and β particles and γ -rays each having an energy of 0.5 MeV. In the increasing order of penetrating power, the radiation are respectively
(A) α, β, γ (B) α, γ, β
(C) β, γ, α (D) γ, β, α
- Q.91** In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because
(A) they will break up.
(B) elastic collision of neutrons with heavy nuclei will not slow them down.
(C) the net weight of the reactor would be unbearably high.
(D) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.
- Q.92** When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom
(A) do not change for any type of radioactivity.
(B) change for α and β radioactivity but not for γ -radioactivity.
(C) change for α -radioactivity but not for others.
(D) change for β -radioactivity but not for others.
- Q.93** Complete the series ${}^6\text{He} \rightarrow e^+ + {}^6\text{Li}^+$
(A) neutrino (B) antineutrino
(C) proton (D) neutron

EXERCISE-2 (LEVEL-2)

Choose one correct response for each question.

- Q.1** A beam of α -particles of velocity $2.1 \times 10^7 \text{ m/s}$ is scattered by a gold ($z = 79$) foil. Find out the distance of closest approach of the α -particle to the gold nucleus. The value of charge/mass for α -particle is $4.8 \times 10^7 \text{ c/kg}$.
(A) $2.5 \times 10^{-14} \text{ m}$ (B) $1.5 \times 10^{-14} \text{ m}$
(C) $5 \times 10^{-12} \text{ m}$ (D) $3 \times 10^{-11} \text{ m}$
- Q.2** The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
(A) 1 (B) 8
(C) 4 (D) 16
- Q.3** Which sample contains greater number of nuclei: a 5.00- μCi sample of ^{240}Pu (half-life 6560y) or a 4.45- μCi sample of ^{243}Am (half-life 7370y)
(A) ^{240}Pu (B) ^{243}Am
(C) Equal in both (D) None of these
- Q.4** The energy required to knock out the electron in the third orbit of a hydrogen atom is equal to
(A) 13.6 eV (B) $+\frac{13.6}{9} \text{ eV}$
(C) $-\frac{13.6}{3} \text{ eV}$ (D) $-\frac{3}{13.6} \text{ eV}$
- Q.5** In which of the following process the number of protons in the nucleus increases –
(A) α -decay (B) β^- decay
(C) β^+ decay (D) k-capture
- Q.6** An electron jumps from the 4th orbit to the 2nd orbit of hydrogen atom. Given the Rydberg's constant $R = 10^5 \text{ cm}^{-1}$. The frequency in Hz of the emitted radiation will be
(A) $\frac{3}{16} \times 10^5$ (B) $\frac{3}{16} \times 10^{15}$
(C) $\frac{9}{16} \times 10^{15}$ (D) $\frac{3}{4} \times 10^{15}$
- Q.7** A hydrogen atom (ionisation potential 13.6 eV) makes a transition from third excited state to first excited state. The energy of the photon emitted in the process is
(A) 1.89 eV (B) 2.55 eV
(C) 12.09 eV (D) 12.75 eV
- Q.8** The wavelength of the first line of Balmer series is 6563 Å. The Rydberg constant for hydrogen is about
(A) $1.09 \times 10^7 \text{ per m}$ (B) $1.09 \times 10^8 \text{ per m}$
(C) $1.09 \times 10^9 \text{ per m}$ (D) $1.09 \times 10^5 \text{ per m}$
- Q.9** An alpha nucleus of energy $\frac{1}{2} mv^2$ bombards a heavy nuclear target of charge Ze . Then the distance of closest approach for the alpha nucleus will be proportional to –
(A) $1/v^4$ (B) $1/Ze$
(C) v^2 (D) $1/m$
- Q.10** The weight based ratio of U^{238} and Pb^{226} in a sample of rock is 4 : 3 . If the half life of U^{238} is 4.5×10^9 years, then the age of rock is –
(A) 9.0×10^9 years (B) 6.3×10^9 years
(C) 4.5×10^9 years (D) 3.78×10^9 years
- Q.11** Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be –
(A) 2 (B) 3
(C) 5 (D) 6
- Q.12** Assume that a neutron breaks into a proton and an electron. The energy released during this process is –
(Mass of neutron = $1.6749 \times 10^{-27} \text{ kg}$,
Mass of proton = $1.6725 \times 10^{-27} \text{ kg}$,
Mass of electron = $9 \times 10^{-31} \text{ kg}$)
(A) 0.73 MeV (B) 7.10 MeV
(C) 6.30 MeV (D) 5.4 MeV

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- Q.13** In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to –
(A) $1/n$ (B) $1/n^2$
(C) $1/n^{3/2}$ (D) $1/n^3$
- Q.14** A nucleus with mass number 220 initially at rest emits an α -particle. If the Q value of the reaction is 5.5 MeV, the KE of the α particle is
(A) 4.4 MeV (B) 5.4 MeV
(C) 5.6 MeV (D) 6.5 MeV
- Q.15** The wavelength of the first line of Lyman series is 1215 Å, the wavelength of first line of Balmer series will be –
(A) 4545 Å (B) 5295 Å
(C) 6561 Å (D) 6750 Å
- Q.16** Tritium is an isotope of hydrogen whose nucleus Triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + e^- + \bar{\nu}$. If one of the neutrons in Triton decays, it would transform into He^3 nucleus. This does not happen. This is because
(A) Triton energy is less than that of a He^3 nucleus.
(B) the electron created in the beta decay process cannot remain in the nucleus.
(C) both the neutrons in triton have a decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus.
(D) because free neutrons decay due to external perturbations which is absent in a triton nucleus.
- Q.17** A fraction f_1 of a radioactive sample decays in one mean life, and a fraction f_2 decays in one half-life. Then
(A) $f_1 > f_2$ (B) $f_1 < f_2$
(C) $f_1 = f_2$ (D) None of these
- Q.18** Radon has 3.8 days as its half-life. How much radon will be left out of 15 mg mass after 38 days?
(A) 1.05 mg (B) 0.015 mg
(C) 0.231 mg (D) 0.50 mg
- Q.19** For the ground state, the electron in the H-atom has an angular momentum = \hbar , according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,
(A) because Bohr model gives incorrect values of angular momentum.
(B) because only one of these would have a minimum energy.
(C) angular momentum must be in the direction of spin of electron.
(D) because electrons go around only in horizontal orbits.
- Q.20** The energy required to excite an electron in hydrogen atom to its first excited state is
(A) 8.5 eV (B) 10.2 eV
(C) 12.7 eV (D) 13.6 eV
- Q.21** A spectral line results from the transition $n = 2$ to $n = 1$ in the single electron system given below. Which one of these will produce the shortest wavelength emission ?
(A) H (B) He^+
(C) Li^{++} (D) Deuterium atom
- Q.22** The wavelength of the first line of the Lyman series of a ten times ionized Na atom ($Z = 11$) is nearest to
(A) 0.1 Å (B) 10 Å
(C) 100 Å (D) 1000 Å
- Q.23** Which of the following statement is correct in connection with hydrogen spectrum
(A) The longest wavelength in the Balmer series is longer than the longest wavelength in Lyman series.
(B) The shortest wavelength in the Balmer series is shorter than the shortest wavelength in the Lyman series.
(C) The longest wavelength in both Balmer and Lyman series are equal.
(D) The longest wavelength in Balmer series is shorter than the longest wavelength in the Lyman series.


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- Q.24** N^{th} level of Li^{2+} has the same energy as the ground state energy of the hydrogen atom. If r_N and r_1 be the radius of the N^{th} Bohr orbit of Li^{2+} and first orbit radius of H atom respectively, then the ratio (r_N/r_1) is
(A) 9 (B) 1/9
(C) 3 (D) None
- Q.25** In a hydrogen like atom, energy required to excite the electron from its first excited state to second excited state is 7.55 eV. The energy required to remove the electron from its ground state is
(A) 72.6 eV (B) 67.9 eV
(C) 58.6 eV (D) 54.4 eV
- Q.26** The ratio of the binding energies of the hydrogen atom in the first and the second excited states is
(A) 1/4 (B) 4
(C) 4/9 (D) 9/4
- Q.27** An α -particle and a free electron, both initially at rest combine to form a He^+ ion in its ground state with the emission of a single photon. the energy of the photon is :
(A) 54.4 eV (B) 27.2 eV
(C) 13.6 eV (D) 40.8 eV
- Q.28** An electron orbiting around the nucleus of an atom
(A) has a magnetic dipole moment.
(B) exerts an electric force on the nucleus equal to that on it by the nucleus.
(C) does produce a magnetic induction at the nucleus.
(D) all of these
- Q.29** A radioactive sample S_1 having the activity A_1 has twice the number of nuclei as another sample S_2 of activity A_2 . If $A_2 = 2A_1$, then the ratio of half life of S_1 to the half life of S_2 is –
(A) 4 (B) 2
(C) 0.25 (D) 0.75
- Q.30** When a neutron is disintegrated to give a β -particle–
(A) a neutrino alone is emitted
(B) a proton and neutrino are emitted
(C) a proton alone is emitted
(D) a proton and an antineutrino are emitted
- Q.31** When an electron jumps from the orbit $n = 2$ to $n = 4$, then wavelength of the radiations absorbed will be – (R is Rydberg's constant).
(A) $3R/16$ (B) $5R/16$
(C) $16/5R$ (D) $16/3R$
- Q.32** The ratio of minimum wavelength of Lyman and Balmer series will be –
(A) 10 (B) 5
(C) 0.25 (D) 1.25
- Q.33** The fraction of the initial number of radioactive nuclei which remain undecayed after half of a half-life of the radioactive sample is –
(A) $1/\sqrt{2}$ (B) 1/2
(C) $1/2\sqrt{2}$ (D) 1/4
- Q.34** 1 curie represents
(A) 1 disintegration per second
(B) 10^6 disintegrations per second
(C) 3.7×10^{10} disintegrations per second
(D) 3.7×10^7 disintegrations per second
- Q.35** The ratio of the magnetic dipole moment to the angular momentum of the electron in the 1st orbit of hydrogen atom is –
(A) e/m (B) $2m/e$
(C) m/e (D) $e/2m$
- Q.36** If n is the orbit number of the electron in a hydrogen atom, the correct statement among the following is
(A) hydrogen emits infrared rays for the electron transition from $n = \infty$ to $n = 1$.
(B) electron energy is zero for $n = 1$
(C) electron energy varies as n^2
(D) electron energy increases as n increases
- Q.37** The radius of ${}_{29}\text{Cu}^{64}$ nucleus in Fermi is (given $R_0 = 1.2 \times 10^{-15}$ m)
(A) 1.2 (B) 7.7
(C) 9.6 (D) 4.8

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- Q.38** In a radioactive decay, an element ${}_Z X^A$ emits four α -particles, three β -particles and eight gamma photons. The atomic number and mass number of the resulting final nucleus are –
(A) $Z - 5, A - 13$ (B) $Z - 5, A - 16$
(C) $Z - 8, A - 13$ (D) $Z - 11, A - 16$
- Q.39** A radioactive nucleus has specific binding energy ' E_1 '. It emits an α -particle. The resulting nucleus has specific binding energy ' E_2 '. Then –
(A) $E_2 < E_1$ (B) $E_2 > E_1$
(C) $E_2 = 0$ (D) $E_2 = E_1$
- Q.40** In hydrogen atom, electron excites from ground state to higher energy state and its orbit velocity is reduced to $1/3^{\text{rd}}$ of its initial value. The radius of the orbit in the ground state is R . The radius of the orbit in that higher energy state is –
(A) $3R$ (B) $27R$
(C) $9R$ (D) $2R$
- Q.41** Decay constants of two radio-active samples A and B are $15x$ and $3x$ respectively. They have equal number of initial nuclei. The ratio of the number of nuclei left in A and B after time $1/6x$ is –
(A) e^2 (B) e^{-1}
(C) e^{-2} (D) e
- Q.42** Mass numbers of the elements A, B, C and D are 30, 60, 90 and 120 respectively. The specific binding energy of them are 5 MeV, 8.5 MeV, 8 MeV and 7 MeV respectively. Then, in which of the following reaction/s energy is released?
(a) $D \rightarrow 2B$ (b) $C \rightarrow B + A$
(c) $B \rightarrow 2A$
(A) in (b), (c) (B) in (a), (c)
(C) in (a), (b) and (c) (D) only in (a)
- Q.43** The ionisation energy of an electron in the ground state of helium atom is 24.6 eV. The energy required to remove both the electrons is –
(A) 51.8 eV (B) 79 eV
(C) 38.2 eV (D) 49.2 eV
- Q.44** The figure shows the energy level of certain atom. When the electron deexcites from $3E$ to E , an electromagnetic wave of wavelength λ is emitted. What is the wavelength of the electromagnetic wave emitted when the electron deexcites from $5E/3$ to E ?
- 
- (A) 3λ (B) 2λ
(C) 5λ (D) $3\lambda/5$
- Q.45** Pick out the correct statements from the following:
(a) Electron emission during β -decay is always accompanied by neutrino.
(b) Nuclear force is charge independent.
(c) Fusion is the chief source of stellar energy.
(A) (a), (b) correct (B) (a), (c) are correct
(C) only (a) is correct (D) (b), (c) are correct
- Q.46** A nucleus ${}_Z X^A$ emits an α -particle with velocity v . The recoil speed of the daughter nucleus is
(A) $\frac{A-4}{4v}$ (B) $\frac{4v}{A-4}$
(C) v (D) $v/4$
- Q.47** A radioactive substance emits 100 beta particles in the first 2 seconds and 50 beta particles in the next 2 seconds. The mean life of the sample is –
(A) 4 seconds (B) 2 seconds
(C) $(2/0.693)$ seconds (D) 2×0.693 seconds
- Q.48** In the sun about 4 billion kg of matter is converted to energy each second. The power output of the sun in watt is
(A) 3.6×10^{26} (B) 0.36×10^{26}
(C) 36×10^{26} (D) 0.036×10^{26}
- Q.49** What is the energy of the electron revolving in third orbit expressed in eV?
(A) 1.51 eV (B) 3.4 eV
(C) 4.53 eV (D) 4 eV

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- Q.50** A radioactive decay can from an isotope of the original nucleus with the emission of particles –
(A) one α and one β (B) one α four β
(C) four α and one β (D) one α and two β
- Q.51** The half life of a radioactive substance is 20 minutes. The time taken between 50 % decay and 87.5% decay of the substance will be
(A) 25 minutes (B) 30 minutes
(C) 10 minutes (D) 40 minutes
- Q.52** A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio
(A) 4 : 1 (B) 8 : 1
(C) 2 : 1 (D) 6 : 1
- Q.53** If an electron in hydrogen atom jumps from an orbit of level $n = 3$ to an orbit of level $n = 2$, the emitted radiation has a frequency
(R = Rydberg constant., C = velocity of light)
(A) $8RC/9$ (B) $3RC/27$
(C) $5RC/36$ (D) $RC/25$
- Q.54** Total energy of electron in an excited state of hydrogen atom is -3.4 eV. The kinetic and potential energy of electron in this state
(A) $K = +10.2$ eV ; $U = -13.6$ eV
(B) $K = -6.8$ eV ; $U = +3.4$ eV
(C) $K = 3.4$ eV ; $U = -6.8$ eV
(D) $K = -3.4$ eV ; $U = -6.8$ eV
- Q.55** A radioactive sample of half-life 10 days contains 1000x nuclei. Number of original nuclei present after 5 days is
(A) 250 x (B) 500 x
(C) 750 x (D) 707 x
- Q.56** There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After n half lives of A, rate of disintegration of both are equal. The value of n is
(A) 4 (B) 2
(C) 1 (D) 5
- Q.57** After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000dps after another 140 days. The initial activity of the sample in dps is
(A) 6000 (B) 9000
(C) 3000 (D) 24000
- Q.58** If a star can convert all the He nuclei completely into oxygen nuclei. The energy released per oxygen nuclei is :
[Mass of the nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994amu]
(A) 7.6 MeV (B) 56.12 MeV
(C) 10.24 MeV (D) 23.4 MeV
- Q.59** The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is
(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm
- Q.60** In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is
(A) $E\left({}^{236}_{92}\text{U}\right) > E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}\text{Y}\right) + 2E(n)$
(B) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}\text{Y}\right) + 2E(n)$
(C) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Kr}\right) + 2E(n)$
(D) $E\left({}^{236}_{92}\text{U}\right) = E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Kr}\right) + 2E(n)$
- Q.61** For a radioactive sample the counting rate changes from 6520 counts/minute to 3260 counts/minute in 2 minutes. Determine the decay constant.
(A) 1.78×10^{-2} per sec (B) 0.78×10^{-3} per sec
(C) 2.78×10^{-6} per sec (D) 5.78×10^{-3} per sec
- Q.62** What is the decay constant of a radioactive substance whose half life is 5 hours
(A) 1.85×10^{-5} per sec
(B) 0.85×10^{-5} per sec
(C) 3.85×10^{-5} per sec
(D) 38.5×10^{-5} per sec

EXERCISE-3 (LEVEL-3)

Choose one correct response for each question.

Q.1 An electron revolves round a nucleus of charge Ze . In order to excite the electron from the state $n = 3$ to $n = 4$, the energy required is 66.0 eV . Z will be -

- (A) 25 (B) 10
(C) 4 (D) 5

Q.2 Determine the power output of a ${}_{92}\text{U}^{235}$ reactor if it takes 30 days to use 2kg of fuel. Energy released per fission is 200 MeV and $N = 6.023 \times 10^{26}$ per kilomole.

- (A) 63.28 MW (B) 3.28 MW
(C) 0.6 MW (D) 50.12 MW

Q.3 The nuclide ${}^{131}\text{I}$ is radioactive, with a half-life of 8.04 days. At noon on January 1, the activity of a certain sample is 60089. The activity at noon on January 24 will be

- (A) 75 Bq (B) Less than 75 Bq
(C) More than 75 Bq (D) 150 Bq

Q.4 A hydrogen atom emits a photon corresponding to an electron transition from $n = 5$ to $n = 1$. The recoil speed of hydrogen atom is almost (mass of proton $\approx 1.6 \times 10^{-27} \text{ kg}$).

- (A) 10 ms^{-1} (B) $2 \times 10^{-2} \text{ ms}^{-1}$
(C) 4 ms^{-1} (D) $8 \times 10^2 \text{ ms}^{-1}$

Q.5 An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

- (A) 79.0 (B) 51.8
(C) 49.2 (D) 38.2

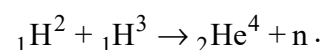
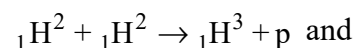
Q.6 The ionisation potential of H-atom is 13.6 V . When it is excited from ground state by monochromatic radiations of 970.6 \AA , the number of emission lines will be (according to Bohr's theory)

- (A) 10 (B) 8
(C) 6 (D) 4

Q.7 The radius of first orbit of hydrogen atom is 0.53 \AA and the electron is executing 6.54×10^{15} revolutions per second. The magnetic moment of electron will be-

- (A) $9.3 \times 10^{-24} \text{ Amp-m}^2$
(B) $6.54 \times 10^{-27} \text{ Amp-m}^2$
(C) $6.54 \times 10^{-24} \text{ Amp-m}^2$
(D) $5.3 \times 10^{-24} \text{ Amp-m}^2$

Q.8 A star initially has 10^{40} deuterons. It produces energy via the processes



If the average power radiated by the star is 10^{16} W , the deuteron supply of the star is exhausted in a time of the order of -

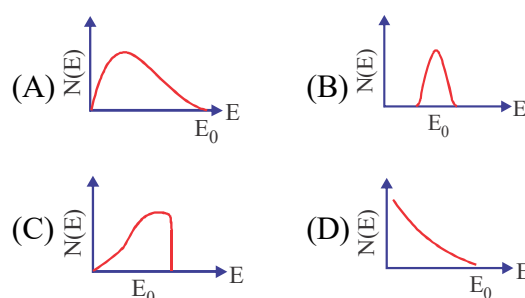
(The masses of nuclei are:

$$m({}_1\text{H}^2) = 2.014 \text{ amu}, m(\text{p}) = 1.007 \text{ amu},$$

$$m(\text{n}) = 1.0084 \text{ amu}, m({}_2\text{He}^4) = 4.001 \text{ amu})$$

- (A) 10^6 s (B) 10^8 s
(C) 10^{12} s (D) 10^{16} s

Q.9 The energy spectrum of β -particles [number $N(E)$ as a function of β -energy E] emitted from a radioactive source is -



For Q. 10– 11

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is c .

Q.10 The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2 . Then

- (A) $E_2 = 2E_1$ (B) $E_1 > E_2$
(C) $E_2 > E_1$ (D) $E_1 = 2E_2$

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- Q.11** The speed of daughter nuclei is –
- (A) $c \frac{\Delta m}{M + \Delta m}$ (B) $c \sqrt{\frac{2\Delta m}{M}}$
(C) $c \sqrt{\frac{\Delta m}{M}}$ (D) $c \sqrt{\frac{\Delta m}{M + \Delta m}}$
- Q.12** Hydrogen (${}_1\text{H}^1$), Deuterium (${}_1\text{H}^2$), singly ionised Helium (${}_2\text{He}^4$)⁺ and doubly ionised lithium (${}_3\text{Li}^6$)⁺⁺ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively then approximately which one of the following is correct?
- (A) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(B) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$
(C) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(D) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
- Q.13** The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of $3 \times 10^{-4}\text{T}$. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to –
- (A) 0.8 eV (B) 1.6 eV
(C) 1.8 eV (D) 1.1 eV
- Q.14** The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is $B = -\frac{me^4}{8n^2 \epsilon_0^2 h^2}$ ($m = \text{electron mass}$). If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be $B = -\frac{Me^4}{8n^2 \epsilon_0^2 h^2}$ ($M = \text{proton mass}$)
This last expression is not correct because
(A) n would not be integral.
(B) Bohr-quantisation applies only to electron.
(C) the frame in which the electron is at rest is not inertial.
(D) the motion of the proton would not be in circular orbits, even approximately.
- Q.15** The half life of ${}^{90}_{38}\text{Sr}$ is 28 years. The disintegration rate of 15 mg of this isotope is of the order of –
(A) 9.877×10^{11} Bq (B) 7.877×10^{10} Bq
(C) 3.877×10^7 Bq (D) 5.877×10^9 Bq
- Q.16** The difference between the longest wavelength line of the Balmer series and shortest wavelength line of the Lyman series for a hydrogenic atom (Atomic no. Z) equal to $\Delta\lambda$. The value of the Rydberg constant for the given atom is
(A) $\frac{5}{31} \frac{1}{\Delta\lambda \cdot Z^2}$ (B) $\frac{5}{36} \frac{Z^2}{\Delta\lambda}$
(C) $\frac{31}{5} \frac{1}{\Delta\lambda \cdot Z^2}$ (D) none
- Q.17** Two radioactive nuclei A and B are present in equal numbers to begin with. Three day later, number of A nuclei are 3 times number of B nuclei. Choose the correct statement.
(A) $\lambda_B - \lambda_A = \frac{\ln 3}{3 \text{ days}}$
(B) $\lambda_A - \lambda_B = \frac{\ln 3}{3 \text{ days}}$
(C) the ratio of activity rate of A and B after 3 days is less than 3:1.
(D) Both (A) and (C)
- Q.18** The activity of a fresh radioactive solution, of volume 1 litre, is 1200 Bq. A volume ΔV of the same liquid has an activity 120 Bq after three half lives. Then ΔV must be
(A) 600 c.c. (B) 800 c.c.
(C) 400 c.c. (D) 880 c.c.

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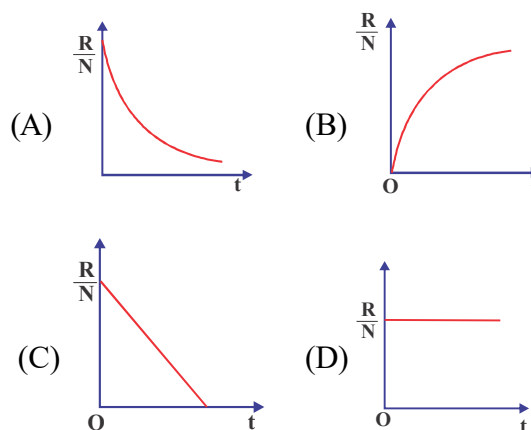
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- Q.19** Two radioactive samples X and Y having half life 3 years and 2 years respectively have been decaying for many years. Today both samples have equal number of atoms. The number of atoms in the sample X will be twice of the number of atoms in the sample Y after
 (A) 6/5 years (B) 5/6 years
 (C) 6 years (D) 2 years
- Q.24** One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the 2nd line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is
 (A) $n = 4 \rightarrow n = 2$ (B) $n = 8 \rightarrow n = 2$
 (C) $n = 8 \rightarrow n = 4$ (D) $n = 12 \rightarrow n = 6$

Paragraph for Question Nos. 20 to 21

In a mixture of H-He⁺ gas (He⁺ is singly ionized He atom), H atoms and He⁺ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He⁺ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

- Q.20** The quantum number n of the state finally populated in He⁺ ions is –
 (A) 2 (B) 3
 (C) 4 (D) 5
- Q.21** The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He⁺ ion is –
 (A) 1/4 (B) 1/2
 (C) 1 (D) 2
- Q.22** The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming bohr's model to be applicable, write variation of r_n with n , n being the principal quantum number?
 (A) $r_n \propto n$ (B) $r_n \propto 1/n$
 (C) $r_n \propto n^2$ (D) $r_n \propto 1/n^2$
- Q.23** A hydrogen atom and a Li^{2+} ion are both in the second excited state. If ℓ_{H} and ℓ_{Li} are their respective electronic angular momenta, and E_{H} and E_{Li} their respective energies, then –
 (A) $\ell_{\text{H}} > \ell_{\text{Li}}$ and $|E_{\text{H}}| > |E_{\text{Li}}|$
 (B) $\ell_{\text{H}} = \ell_{\text{Li}}$ and $|E_{\text{H}}| < |E_{\text{Li}}|$
 (C) $\ell_{\text{H}} = \ell_{\text{Li}}$ and $|E_{\text{H}}| > |E_{\text{Li}}|$
 (D) $\ell_{\text{H}} < \ell_{\text{Li}}$ and $|E_{\text{H}}| < |E_{\text{Li}}|$
- Q.25** In a hydrogen atom following the Bohr's postulates the product of linear momentum and angular momentum is proportional to $(n)^x$ where 'n' is the orbit number. Then 'x' is –
 (A) 0 (B) 2
 (C) -2 (D) 1
- Q.26** The photon radiated from hydrogen corresponding to 2nd line of Lyman series is absorbed by a hydrogen like atom X in 2nd excited state. As a result the hydrogen like atom X makes a transition to n^{th} orbit. Then –
 (A) X = He⁺, $n = 4$ (B) X = Li⁺⁺, $n = 6$
 (C) X = He⁺, $n = 6$ (D) X = Li⁺⁺, $n = 9$
- Q.27** A radioactive sample has N_0 active atoms at $t = 0$. If the rate of disintegration at any time is R and the number of atoms is N , then the ratio R/N varies with time as –



- Q.28** If radiation corresponding to first line of "Balmer series" of He⁺ ion knocked out electron from 1st excited state of H atom, the kinetic energy of ejected electron from H atom would be (eV)

Given $E_n = -\frac{Z^2}{n^2} (13.6 \text{ eV})$

- (A) 4.155 eV (B) 8.310 eV
 (C) 2.515 eV (D) 5.550 eV

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ANSWER KEY

CHECK UP 1

- (1) (b) (2) (d)
 (3) The absorption lines belong only to the Lyman series, since very few electrons are present with $n = 2$ or $n = 3$.
 (4) $\approx 122 \text{ nm}$. (5) $Z = 3$
 (6) (B) (7) (A)

CHECK UP 2

- (1) (c), (d) (2) (a)
 (3) No (4) Yes
 (5) d, c, a, b (6) (c)
 (7) It is not possible, because the total mass of the decay products is greater than the mass of the parent nucleus ${}_{92}^{238}\text{U}$, indicating that energy would not be released.
 (8) (b) and (d) (9) (a), (b) and (c)

CHECK UP 3

- (1) (b) (2) Yes (3) Yes
 (4) (b) (5) Too small
 (6) No (7) (B) (8) (D)

- (9) (A) (10) (A) (11) (A)

EXERCISE - 1

- (1) (D) (2) (D) (3) (A) (4) (C)
 (5) (C)
 (6) Antineutrino

$${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu}$$
 neutron proton electron antineutrino
 (7) Ultraviolet, Lyman (8) Balmer, visible
 (9) 1 : 4 : 9 (10) 10.2 eV
 (11) Continuous (12) 1 : 2 : 3
 (13) $1 : 2^{1/3}$
 (14) No different from
 (15) Thomson's model ; Rutherford's model.
 (16) Rutherford's model.
 (17) Thomson's model, Rutherford's model.
 (18) Both the models.
 (19) Heavy water
 (20) False
 (21) Liquid drop model of nucleus.
 (22) Neutrino (23) $0.16 \mu\text{C}$
 (24) 100 hr (25) Light

EXERCISE 1 (SECTION-3)

Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	C	A	B	B	A	B	B	D	A	D	A	C	D	A	A	D	A	B	C	D	B	A	A	B
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	C	C	A	D	A	D	A	A	A	A	A	D	B	D	A	B	A	B	D	D	A	A	A	B
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93							
A	D	D	C	B	A	C	C	C	D	B	B	B	C	C	A	B	B	A							

EXERCISE-2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	B	C	B	B	C	B	A	D	D	D	A	D	B	C	A	A	B	A	B	C	B	A	C	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	A	D	A	D	D	C	A	C	D	D	D	B	B	C	C	D	B	A	D	B	C	B	A	D
Q	51	52	53	54	55	56	57	58	59	60	61	62													
A	D	B	C	C	D	C	C	C	B	A	D	C													

EXERCISE-3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
A	B	A	C	C	A	C	A	C	A	C	B	A	D	C	B	C	D	B	C	C	A	A	B	D	A	D	D	A