Subject: Physics

Topic: Electrostatics

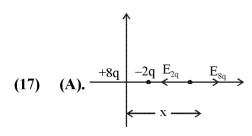
[SOLUTIONS]

- (1) (D). Charges are add algebraically (-2) + (6) = -2 + 6 = 4Mass of electron = 9.1×10^{-31} kg
 Charge on electron = -1.6×10^{-19} C
 Mass is always positive, charge can be positive or negative
 - Charge on proton = $+1.6 \times 10^{-19}$ C Mass of proton = 1.67×10^{-27} kg.
- (2) (D). 1 coulomb of charge is made of n = q/e. $= \frac{1C}{1.6 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ electrons}$
 - .. Time required = $\frac{6.25 \times 10^{18}}{10^9}$ s = 6.25×10^9 s = 200 years (: 1 year = 3.17×10^7 s)
- (3) **(B).** $250 \text{ ml} \Rightarrow 250 \text{g}$ water 18 g water has 6×10^{23} water molecules each molecule has 10 protons, 10 electrons
 - $\Rightarrow 18g \text{ water has } 6 \times 10^{23} \times 10$ $= 6 \times 10^{24} \text{ protons}$
 - $\Rightarrow 250 \text{ water has } \frac{6 \times 10^{24}}{18} \times 250 \text{ protons}$ $= \frac{1}{3} \times 250 \times 10^{24} \times 1.6 \times 10^{-19}$ $= 1.34 \times 10^{7} \text{C positive charge}$ As water is neutral, it has 1.34×10^{7} negative charge also.
- charge also. (4) (B). $U = -pE \cos \theta$ θ is max. is $\cos \theta$ is min. in case 3 & 4 U is +ve so angle is obtuse $\tau = pE \sin \theta$ max. is angle is closet to 90° $-pE \cos \theta = -V_0$ $\cos \theta_1 = \frac{V_0}{PE}$, $\cos \theta_2 = \frac{7V_0}{PE}$, $\cos \theta_3 = -\frac{3V_0}{pE}$, $\cos \theta_4 = \frac{4V_0}{pE}$
- (5) (B). The second charge is kept just outside surface.∴ Flux will not change but another charge influences the shape.

- (6) (D). $Q_{ene} = 100 \times \sigma$ $E = \frac{\sigma}{2 \in_0} \Rightarrow \sigma = 200 \times 2 \in_0 = 4 \times 10^4 \in_0 = 35.4 \times 10^{-8} \text{C}$
- (7) **(B).** Because current flows from higher potential to lower potential.
- (8) (D). May be at positive, zero or negative potential, it is according to the way one defines the zero potential.
- (9) (C). $a = \frac{qE}{m} \Rightarrow \frac{a_e}{a_p} = \frac{m_p}{m_e}$
- (10) (A). $V = E \times r \implies r = \frac{V}{E} = \frac{3000}{500} = 6 \text{ m}$
- (11) **(D).** $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$
 - $\Rightarrow 20 = \frac{10 \times 50 + C_2 \times 0}{10 + C_2}$
 - \Rightarrow 200+20C₂ = 500 \Rightarrow C₂ = 15 μ F
- (12) (D). Point charge produces non-uniform electric field.
- (13) (C). In the given condition angle between \vec{p} and \vec{E} is zero. Hence potential energy $U = -pE\cos 0 = -pE = min.$ Also in uniform electric field $F_{net} = 0$.
- **(14) (B).** $E \propto \frac{1}{r^3}$
- (15) (C). After the connection of wire $V_1 = V_2$

$$\therefore \quad \frac{Q_1}{25} = \frac{Q_2}{20} \Rightarrow \frac{Q_1}{Q_2} = \frac{25}{20} \Rightarrow Q_1 > Q_2$$

(16) (C). When the dipole is rotated through at an angle of 90° about it's perpendicular axis then given point comes out to be on equator. So field will become E / 2 at the given point.



For net electric field to be zero $E_{2q} = E_{8q}$

$$\Rightarrow \frac{K(2q)}{(x-1)^2} = \frac{K(8q)}{x^2} \Rightarrow x = 2L$$

- (18) (A). Non uniform field so torque as well as translational force.
- (19) (D). $\frac{\text{Energy stored}}{\text{Work by battery}} = \frac{\frac{1}{2}\text{CV}^2}{\text{CV}^2} = \frac{1}{2}$
- (20) **(B).** Let q_1 and q_2 be the charges and C_1 and C_2 be the capacitance of two spheres.

The charge flows from the sphere at higher potential to the other at lower potential, till their potentials becomes equal.

After sharing, the charges on two spheres

would be
$$\frac{q_1}{q_2} = \frac{C_1 V}{C_2 V} \qquad \dots (1)$$

Also,
$$\frac{C_1}{C_2} = \frac{a}{b}$$
 (2)

From eq. (1),
$$\frac{q_1}{q_2} = \frac{a}{b}$$

Ratio of surface charge on the two spheres

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1}{4\pi a^2} \cdot \frac{4\pi b^2}{q_2} = \frac{q_1}{q_2} \cdot \frac{b^2}{a^2} = \frac{b}{a}$$

[Using eq. (2)]

(29)

... The ratio of electric fields at the surfaces of two spheres

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$$

- **(21) (D).** Oxygen and hydrogen are the examples of non-polar molecules.
- (22) (A). Polar molecules have permanent electric dipole moment.

- (C). The extent of polarisation depends on the relative strength of two mutually opposite factors: the dipole potential energy in the external field tending to align the dipoles with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for nonpolar molecules, but generally the alignment effect is more important for polar molecules.
- (24) (A). The material suitable for using as a dielectric must have high dielectric strength X and large dielectric constant K.
- (25) (C). $3.9 \times 10^{-19} = n_1 e$, $6.5 \times 10^{-19} = n_2 e$, $9.1 \times 10^{-19} = n_3 e$ n_1 , n_2 and n_3 are integers for $e = 1.3 \times 10^{-19}$
- (26) **(D).** Correct statement is

 The magnetic lines of force of magnetic field produced by current carrying wire from closed loops.
- (27) **(B).** Opposite charge of rod is induced on water and it gets attracted.

(28) (B).
$$U_i = \frac{2KQq}{a}$$
;

$$U_{f} = \frac{KQq}{a+x} + \frac{KQq}{a-x} = \frac{2KQqa}{(a^{2}-x^{2})}$$

$$q$$

$$(-a,0)$$

$$Q$$

$$(a,0)$$

$$\Delta U = U_f - U_i = 2KQq \left[\frac{a}{(a^2 - x^2)} - \frac{1}{a} \right]$$
$$= 2KQqa \left[\frac{x^2}{a(a^2 - x^2)} \right]$$

$$= \frac{2KQqax^2}{a^3} \text{ (since } a^2 - x^2 \approx a^2\text{)}$$

(A). The potential V is a scalar function, whereas the field \vec{E} is a vector function. The three components of \vec{E} are given as

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(6x^2) = -12x$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(6x^2) = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(6x^2) = 0$$

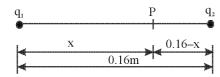
$$\vec{E} = -12x\hat{i} + 0 + 0 = -12x\hat{i}$$
at the given point, $x = 1$

$$\vec{E} = -12\hat{i}$$

(30)(D). Let the potential be zero at a point P whose distance is x metre from the charge

$$q_1 = 5 \times 10^{-8} C.$$

The distance of P from $q_2 = -3 \times 10^{-8} C$ is (0.16 - x) metre.



Potential at P due to

$$q_1 = \frac{1}{4\pi \in \Omega} \frac{q_1}{x} \,,$$

Potential at P due to

$$q_2 = -\frac{1}{4\pi \in_0} \frac{q_2}{0.16 - x}$$

But total potential at P should be zero.

$$\therefore \frac{1}{4\pi \in_0} \frac{q_1}{x} - \frac{1}{4\pi \in_0} \frac{q_2}{0.16 - x} = 0$$

or
$$\frac{0.16 - x}{x} = \frac{q_2}{q_1} = \frac{3 \times 10^{-8}}{5 \times 10^{-8}}$$

or
$$8x = 0.8$$
 or $x = 0.1$ m

(31) (C).
$$\frac{1}{4\pi\epsilon_0} \left[\frac{-qQ}{r} + \frac{(-q)(-q)}{2r} + \frac{Q(-q)}{r} \right] = 0$$

or
$$-Q + \frac{q}{2} - Q = 0$$
 or $2Q = \frac{q}{2}$ or $\frac{q}{Q} = \frac{4}{1}$

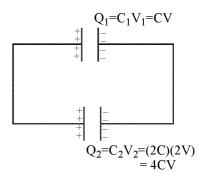
(32)(C). As we know that electrostatic field is a conservative field therefore work done does not depends upon path.

$$\Rightarrow$$
 $W_A = W_B = W_C$

(C). At any point over the spherical Gaussian sur-(33)face, net electric field is is the vector sum of electric fields due to $+q_1$, $-q_1$ and q_2 .

(B). The diagramatic representation of given problem is shown in figure.

(34)



The net charge shared between the two capacitors is

$$Q' = Q_2 - Q_1 = 4CV - CV = 3CV$$

The two capacitors will have the same potential, say V'. The net capacitance of the parallel combination of the two capacitors will be $C' = C_1 + C_2 = C + 2C = 3C$

The potential of the capacitors will be

$$V' = \frac{Q'}{C'} = \frac{2CV}{3C} = V$$

The electrostatic energy of the capacitors will

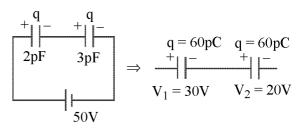
$$E' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

(35)**(D).** When S_3 is closed, due to attraction with opposite charge, no flow of charge takes places through S₃. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30V$ and $V_2 = 20V$.

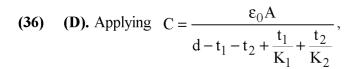
> **Alternate:** Charges on the capacitors are $q_1 = (30)(2) = 60 \text{ pC}$ and $q_2 = (20)(3)$ = 60 pC

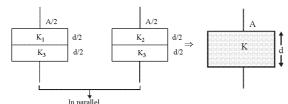
or
$$q_1 = q_2 = q$$
(say)

The situation is similar as the two capacitors in series are first charged with a battery of emf 50V and then disconnected.



When S_3 is closed, $V_1 = 30V$ and $V_2 = 20V$.





We have
$$\frac{\varepsilon_0 (A/2)}{d-d/2-d/2+\frac{d/2}{K_1}+\frac{d/2}{K_2}}$$

$$+\frac{\epsilon_0(A/2)}{d-d/2-d/2+\frac{d/2}{K_2}+\frac{d/2}{K_1}} = \frac{K\epsilon_0 A}{d}$$

Solving this equation, we get

$$\mathbf{K} = \frac{\mathbf{K}_1 \mathbf{K}_3}{\mathbf{K}_1 + \mathbf{K}_3} + \frac{\mathbf{K}_2 \mathbf{K}_3}{\mathbf{K}_2 + \mathbf{K}_3}$$

- (37) (A). Due to attraction with positive charge, the negative charge on capacitor A will not flow through the switch S.
- (38) (C). $\Delta U = \text{decrease in potential energy}$ = $U_f - U_i$ = $\frac{1}{2}C(V_1^2 + V_2^2) - \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2$ = $\frac{1}{4}C(V_1 - V_2)^2$
- (39) **(B).** Redistribution of charge will take place due to mutual attraction and hence effective distance will be less than d.

(40) (B).
$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$
; $E \propto \frac{1}{r^3} \Rightarrow F \propto \frac{1}{r^3}$

Hence, the force will become F/8.

(41) (A). Let r be the radius of each small drop and q coulomb be the charge on each.

Then potential at the surface of each drop is

Volume of big drop = $n \times volume$ of small drop

$$\frac{4}{3}\pi R^3 = n.\frac{4}{3}\pi r^3 \implies R = n^{1/3} r \dots (2)$$

From eq. (1), $q = V \times 4\pi\epsilon_0 r$

Potential of big drop V' = $\frac{1}{4\pi\epsilon_0} \frac{nq}{R}$

$$\Rightarrow V' = \frac{n4\pi\epsilon_0 \times V \times r}{4\pi\epsilon_0 \times n^{1/3}r} = n^{2/3}V$$

(42) (D). For series combination

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_s = \frac{2}{3} \mu F$$

When connected in series the maximum charge that can flow through the combination equals the lower value of charge accommodated by the first capacitor i.e. $6000 \, \mu C$

:. $Q_1 = 6000 \,\mu\text{C}$ and $8000 \,\mu\text{C}$

$$V_s = \frac{Q_1}{C_s} = \frac{6000\mu C}{(2/3)\mu C} \implies V_s = 9kV$$

(43) **(B).**
$$dV = -\vec{E}d\vec{r} = -(-2x^3\hat{i}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

= $2x^3 dx$

$$\Rightarrow \int_{0}^{v} dV = \int_{1}^{2} (2x^{3}) \times 10^{3} dx \Rightarrow V = -7.5 \times 10^{3} V$$

(44) (A). Total flux out of all six faces as per Gauss's

theorem should be
$$\frac{Q \times 10^{-6}}{\epsilon_0}$$

Therefore, flux coming out of each face

$$= \frac{Q}{6 \epsilon_0} \times 10^{-6} C$$

- (45) **(D).** Length of body diagonal = $\sqrt{3}\ell$
 - : Distance of centre of cube from each corner

$$r = \frac{\sqrt{3}}{2}\ell$$

P.E. at centre = $8 \times$ potential energy due to A

$$= 8 \times \frac{Kq \times (-q)}{r}$$

$$=8\times\frac{1}{4\pi\epsilon_0\sqrt{3}\ell}\times2\times q\times(-q)=\frac{-4q^2}{\sqrt{3}\pi\epsilon_0\ell}$$