**Subject: Physics** 

**Topic: Atoms and Nuclei** 

[SOLUTIONS]

(1) **(D).** For shortest orbit n = 1  $r_n = n^2 r_1$ 

$$\frac{(1)^2}{z} \times 0.529 \text{Å} = 18 \times 10^{-2} \text{ Å}$$

z = 0(2) (A). :: A

(A).  $\therefore A_n = \pi r_n^2$  &  $r \propto n^2$ 

$$\therefore A_n \propto n^4 \qquad \therefore \frac{A_2}{A_3} = \frac{(2)^4}{(3)^4} = \frac{16}{81}$$

(3) (D). Perimeter =  $2\pi r$ 

$$\therefore \frac{2\pi r_2}{2\pi r_3} = \frac{r_2}{r_3} = \frac{r_2}{r_3} = \frac{4}{9}$$

(4) (D).  $(r_m) = \left(\frac{m^2}{z}\right)(0.53\text{Å}) = (n \times 0.53)\text{Å}$ 

$$\therefore \frac{m^2}{z} = n \; ; \; m = 5 \text{ for } _{100} \text{Fm}^{257}$$

(the outermost shell) and z = 100

$$\therefore$$
  $n = \frac{(5)^2}{100} = \frac{1}{4}$ 

**(5) (B).** 
$$\frac{v_3}{v_4} = \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}$$

**(6) (D).**  $E_n = -\frac{Rch}{n^2}$ 

Given  $E_1 = -13.6 \text{ eV} = -\text{Rch}$ 

$$E_4 = \text{Energy of 4}^{\text{th}} \text{ state} = -\frac{\text{Rch}}{4^2} = \frac{E_1}{16},$$

$$E_4 = -\frac{13.6}{16} = -0.85 \text{ eV}$$

(7) **(B).** A = 238 - 4 = 234, Z = 92 - 2 = 90

**(8) (C).** The energy produced per second is

= 
$$1000 \times 10^3 \text{ J} = \frac{10^6}{1.6 \times 10^{-19}} \text{ eV}$$
  
=  $6.25 \times 10^{24} \text{ eV}$ 

The number of fissions should be, thus

number = 
$$\frac{6.25 \times 10^{24}}{200 \times 10^6}$$
 = 3.125 × 10<sup>16</sup>

(9) (A).  $\lambda = \lambda_1 + \lambda_2 \Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$ 

$$\therefore T = \frac{T_1 T_2}{T_1 + T_2} = \frac{810 \times 1620}{810 + 1620} = 540 \text{ years}$$

Hence 1/4<sup>th</sup> of material remain after 1080 years.

(10) (C). 
$$E = -13.6 \frac{Z^2}{n^2} = -13.6 \times \frac{1}{4} = -3.4 \text{eV}$$

Required energy = +3.4 eV

(11) **(B).** Mass of two different nuclei is different so recoiling energy is different, so energy of two photons is different.

(12) (D). First excited state of Li<sup>++</sup> means second orbit.

$$E = -\frac{13.6 \times 9}{4} = 30.6eV$$

(13) (A).  $F = \frac{k}{r}$ ;  $\frac{mv^2}{r} = \frac{k}{r} \Rightarrow v = constant$ 

$$mvr = \frac{2h}{2\pi} \implies v \propto \frac{n}{r}; \frac{n}{r} = constant; r \propto n$$

and v is constant so KE is also constant.

(14) **(D).** Energy of I - R radiation  $\leq$  energy of U - V radiation

(15) (C). 
$$\frac{3}{2}$$
 KT × 2 = 7.7 × 10<sup>-14</sup>  
 $\frac{3}{2}$  × 1.38 × 10<sup>-23</sup> × 2 × T = 7.7 × 10<sup>-14</sup>  
T  $\approx 10^9$  K

(16) (A). Nuclear density is constant hence,  $mass \propto volume \text{ or } m \propto V$ 

(17) **(B).** Given that  $K_1 + K_2 = 5.5 \text{MeV}$  ..... (1) From conservation of linear momentum,  $P_1 = P_2$  or  $\sqrt{2K_1(216\text{m})} = \sqrt{2K_2(4\text{m})}$ 

as 
$$P = \sqrt{2Km}$$
  
 $\therefore K_2 = 54 K_1$  ...... (2)  
Solving eqs. (1) and (2), we get  
 $K_2 = KE$  of  $\alpha$ -particle = 5.4 MeV

(18) (B). 
$$\Delta E = \Delta M \times C^2$$
  
 $\Delta M = (8M_P + 9M_n - M_o)$   
There are 8 protons and 9 neutrons in  $_8O^{17}$ 

(19)**(B).** γ-ray have no charge and no mass.

**(20) (A).** 
$$R = R_0 \left(\frac{1}{2}\right)^n$$
 ......(1)

Here R = activity of radioactive substance

after n half-lives = 
$$\frac{R_0}{16}$$
 (given)

Substituting in eq. (1), we get n = 4

$$\therefore$$
 t = (n)t<sub>1/2</sub> = (4) (100 \text{ } \text{s}) = 400 \text{ } \text{s}

(21) (A). 
$$n = \frac{15}{3} = 3$$
 half lives

$$N = \frac{N_0}{2^n} = \frac{N_0}{2^3} = \frac{N_0}{8}$$

(22) **(D).** 
$$T = \frac{\ell n 2}{\lambda} \Rightarrow \lambda = \frac{\ell n 2}{T}$$
;  $N = \frac{N_0}{2^n}$ 

$$1250 = \frac{5000}{2^{n}} \Rightarrow \frac{1}{2^{n}} = \frac{1}{2^{2}} \Rightarrow n = 2$$

$$\frac{t}{T} = 2 \Rightarrow \frac{5}{T} = 2 \Rightarrow T = 2.5$$
 minute

$$\lambda = \frac{\ln 2}{2.5} = 0.4 \ln 2$$

(23)**(D).** Protons are not emitted in radioactive

(24) **(B).** 
$$_{92}U^{238} \rightarrow _{2}He^{4} + _{90}Pu^{234}$$
  
Momentum remains consered

$$(4m) u = (234 m)$$
;  $v = \frac{4}{234} u$ 

(25)(C). During  $\gamma$ -decays atomic number (Z) and mass number (A) does not change. So, the correct option is (C) because in all other options either Z, A or both is/are changing.

(26) (C). 
$$E_1 = -\frac{13.6(3)^2}{(1)^2}$$
;  $E_3 = -\frac{13.6(3)^2}{(3)^2}$ 

$$\Delta E = E_3 - E_1 = 13.6 (3)^2 \left[ 1 - \frac{1}{9} \right]$$

$$=\frac{13.6\times9\times8}{9}=108.8 \text{ eV}$$

(27)(A). In inelastic collision kinetic energy is not conserved so some part of K.E. is lost.

atoms in ground state = 13.6 eV

Reduction in K.E. = K.E. before collision–K.E. after collision. Now, since initial K.E. of each of two hydrogen

Total K.E. of both Hydrogen atom before collision =  $2 \times 13.6 = 27.2 \text{ eV}$ If one H atom goes over to first excited state  $(n_1 = 2)$  and other remains in ground state  $(n_2 = 1)$  then their combined K.E. after

collision is 
$$\frac{13.6}{(2)^2} + \frac{13.6}{(1)^2} = 3.4 + 13.6 = 17eV$$

Reduction in K.E. = 27.2 - 17 = 10.2 eV

(A). Number of atoms in 1 kg of pure 230 Pu (28)

$$=\frac{6.023\times10^{23}}{239}\times1000=2.52\times10^{24}$$

As average energy released per fission is 180

: Total energy released  $= 2.52 \times 10^{24} \times 180 \text{ MeV}$  $= 4.53 \times 10^{26} \text{ MeV}$ 

(29)(A).  $\beta^-$  decay is represented as

$$_{Z}X^{A} \rightarrow _{Z+1}Y^{A} + _{-1}e^{0} + \overline{\nu} + Q_{1}$$

$$\begin{array}{ll} \therefore & Q_1 = [m_N (_Z X^A) - m_N (_{Z+1} Y^A) - m_e] c^2 \\ & = [m_N (_Z X^A) + Z m_e - m_N (_{Z+1} Y^A) \\ & - (Z+1) m_e] c^2 \\ & = [m (_Z X^A) - m (_{Z+1} Y^A)] c^2 = (M_x - M_y) c^2 \\ \beta^+ \text{ decay is represented as} \end{array}$$

$$_{Z}X^{A} \rightarrow _{Z-1}Y^{A} + _{1}e^{0} + v + Q_{2}$$

(30)(C). During  $\beta$ -decay, a neutron is transformed into a proton and an electron.

This is why atomic number (Z = number of protons) increases by one and mass number (A = number of protons + neutrons) remainsunchanged during beta decay.

(C).  $\lambda = 0.3465 \text{ day}^{-1}$ ; t = 4 days(31)

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = 2 \text{ days}$$

$$n = \frac{t}{T_{1/2}} = \frac{4}{2} = 2$$

Hence, sample left undecayed after a period

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%$$

- Sample decayed = 75%
- (32)**(D).** Here,  $\Delta m = 0.3\%$  of 1 kg

$$\frac{0.3}{100}$$
 kg = 3 × 10<sup>-3</sup> kg

$$E = (\Delta m) c^2 = 3 \times 10^{-3} \times (3 \times 10^8)^2$$
$$= 27 \times 10^{13} J$$

(33)**(B).** One mol of any isotope contains Avogardo's number of atoms and so 1 g of  $^{238}_{92}$ U contains

$$\frac{1}{238} \times 6.025 \times 10^{23} = 25.3 \times 10^{20} \text{ atoms}$$

The decay rate R is

$$R = \lambda N = \frac{0.693}{T_{1/2}} N$$

$$=\frac{0.693\times25.3\times10^{20}}{1.42\times10^{17}}\mathrm{s}^{-1}$$

$$= 1.23 \times 10^4 \text{ s}^{-1} = 1.23 \times 10^4 \text{ Bg}$$

[As 1 yr = 
$$3.16 \times 10^7$$
 s

[As 1 yr = 
$$3.16 \times 10^7$$
 s.  
 $T_{1/2} = 4.5 \times 10^9 \times 3.16 \times 10^7$   
=  $1.42 \times 10^{17}$  s]

(D). The reaction rate is controlled through control-(34)rods made out of neutron-absorbing material such as cadmium. In addition to control rods, reactors are provided with safety rods which, when required, can be inserted into the reactor and K can be reduced rapidly to less than unity.

- (35)(A). In simple Bohr's model, there is only one electron and so it cannot be applied for multielectron atoms.
- (C). Here,  $a_0 = 53$  pm, n = 1 for ground state For Li<sup>++</sup> ion, Z = 3(36)Radius of nth orbit

$$r = \frac{n^2h^2}{4\pi^2mKZe^2} = \frac{a_0n^2}{Z}$$

$$r = \frac{53 \times (1)^2}{3} = 17.66 \approx 18 \text{ pm}$$

**(37)** (A). Angular momentum is also called a moment of momentum.

For second orbit, n = 2

$$L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

- (A). A set of atoms in an excited state decays in (38)general to any of the states with lower energy.
- (39)(C). In  $\beta$ -emission, a neutron of nucleus decays into a proton, a β-particle and an anti-neutrino  $n \rightarrow p + e^- + \overline{\nu}$
- (40)(**D**). The force that determines the motion of atomic electrons is the familiar Coulomb force. For average mass nuclei the binding energy per nucleon is approximately 8MeV, which is much larger than the binding energy in atoms. Therefore, to bind a nucleus together there must be a strong attractive force of a totally different kind. It must be strong enough to overcome the repulsion between the (positively charged) protons and to bind both protons and neutrons into the tiny nuclear volume.
- (41)**(B).** Heavy stable nuclei have more neutrons than protons. This is because electrostatic force between protons are repulsive.
- (C). According to radioactive decay (42)

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$
. Here,  $t = \frac{T_{1/2}}{2}$ 

$$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$$

- (43)**(C).** Since half life of the material is 1 year, therefore, after 1 year, half the number of atoms will decay on an average. The containers will in general have different numbers of the atoms of the material but their average will be close to 5000.
- (C).  $T_X = T_{Y(avg)} = 1.44 T_Y$ i.e.  $T_X > T_Y \therefore \lambda_X < \lambda_Y$  $\therefore R = \lambda N \text{ and } N \text{ is same for both,}$ **(44)** 

  - $R_X < R_Y \text{ or } R_Y > R_X$ i.e. Y will decay faster than X.
- (45)(B). Repulsion of protons or positively charge nuclei imposes a potential barrier, which hinder fusion.