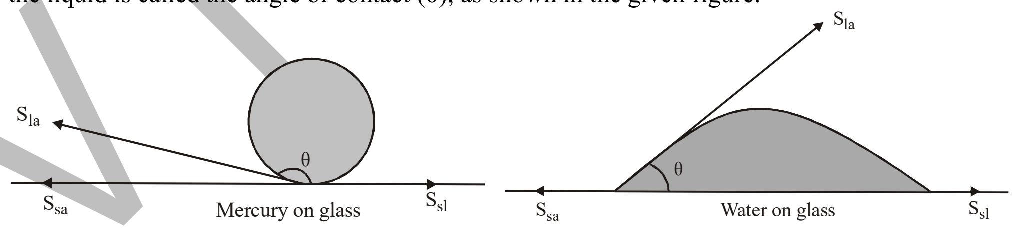
NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 10
MECHANICAL PROPERTIES OF FLUIDS

10.1 Explain why
(a) The blood pressure in humans is greater at the feet than at the brain.
(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100km
(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

Sol. (a) The pressure of a liquid is given by the relation: $P = h\rho g$
Where, P = Pressure, h = Height of the liquid column, ρ = Density of the liquid,
g = Acceleration due to the gravity
It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.
(b) Density of air is the maximum near the sea level. Density of air decreases with increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at the sea level. Atmospheric pressure is proportional to density.
Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at the sea level.
(c) When force is applied on a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

10.2 Explain why
(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)
(c) Surface tension of a liquid is independent of the area of the surface.
(d) Water with detergent dissolved in it should have small angles of contact.
(e) A drop of liquid under no external forces is always spherical in shape.

Sol. (a) The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact (θ), as shown in the given figure.



S_{la}, S_{sa}, and S_{sl} are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e., $\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}$

The angle of contact θ, is obtuse if $S_{sa} < S_{la}$ (as in the case of mercury on glass). This angle is acute if $S_{sl} < S_{la}$ (as in the case of water on glass).

(b) Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops.

On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.

(c) Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.

(d) Water with detergent dissolved in it has small angles of contact (θ). This is because for a small θ, there is a fast capillary rise of the detergent in the cloth. The capillary rise of a liquid is directly proportional to the cosine of the angle of contact (θ). If θ is small, then $\cos\theta$ will be large and the rise of the detergent water in the cloth will be fast.

(e) A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is the minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

10.3 Fill in the blanks using the word(s) from the list appended with each statement:

(a) Surface tension of liquids generally .... with temperatures (increases /decreases)

(b) Viscosity of gases .... with temperature, whereas viscosity of liquids .... with temperature (increases / decreases)

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to ...., while for fluids it is proportional to .... (shear strain / rate of shear strain)

(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows ...
(conservation of mass / Bernoulli's principle)

(e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)

Sol. (a) decreases
The surface tension of a liquid is inversely proportional to temperature.

(b) increases; decreases
Most fluids offer resistance to their motion. This is like internal mechanical friction, known as viscosity. Viscosity of gases increases with temperature, while viscosity of liquids decreases with temperature.

(c) Shear strain; Rate of shear strain
With reference to the elastic modulus of rigidity for solids, the shearing force is proportional to the shear strain. With reference to the elastic modulus of rigidity for fluids, the shearing force is proportional to the rate of shear strain.

(d) Conservation of mass/Bernoulli's principle
For a steady-flowing fluid, an increase in its flow speed at a constriction follows the conservation of mass/Bernoulli's principle.

(e) Greater
For the model of a plane in a wind tunnel, turbulence occurs at a greater speed than it does for an actual plane. This follows from Bernoulli's principle and different Reynolds' numbers are associated with the motions of the two planes.

10.4 Explain why

(a) To keep a piece of paper horizontal, you should blow over, not under, it.

(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.

(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(e) A spinning cricket ball in air does not follow a parabolic trajectory.

Sol. (a) When air is blown under a paper, the velocity of air is greater under the paper than it is above it. As per Bernoulli's principle, atmospheric pressure reduces under the paper.
This makes the paper fall. To keep a piece of paper horizontal, one should blow over it.
This increases the velocity of air above the paper. As per Bernoulli's principle, atmospheric pressure reduces above the paper and the paper remains horizontal.

(b) According to the equation of continuity:

Area × Velocity = Constant

For a smaller opening, the velocity of flow of a fluid is greater than it is when the opening is bigger. When we try to close a tap of water with our fingers, fast jets of water gush through the openings between our fingers. This is because very small openings are left for the water to flow out of the pipe. Hence, area and velocity are inversely proportional to each other.

(c) The small opening of a syringe needle controls the velocity of the blood flowing out. This is because of the equation of continuity. At the constriction point of the syringe system, the flow rate suddenly increases to a high value for a constant thumb pressure applied.

(d) When a fluid flows out from a small hole in a vessel, the vessel receives a backward thrust. A fluid flowing out from a small hole has a large velocity according to the equation of continuity:

Area × Velocity = Constant

According to the law of conservation of momentum, the vessel attains a backward velocity because there are no external forces acting on the system.

(e) A spinning cricket ball has two simultaneous motions – rotatory and linear. These two types of motion oppose the effect of each other. This decreases the velocity of air flowing below the ball. Hence, the pressure on the upper side of the ball becomes lesser than that on the lower side. An upward force acts upon the ball. Therefore, the ball takes a curved path. It does not follow a parabolic path.

10.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Sol. Mass of the girl, m = 50 kg

Diameter of the heel, d = 1 cm = 0.01 m

Radius of the heel, r = d/2 = 0.005 m

Area of the heel = $\pi r^2 = \pi (0.005)^2 = 7.85 \times 10^{-5}$ m^2

Force exerted by the heel on the floor:

$F = mg = 50 \times 9.8 = 490$ N

Pressure exerted by the heel on the floor:

$$P = \frac{\text{Force}}{\text{area}} = \frac{490}{7.85 \times 10^{-5}} = 6.24 \times 10^6 \text{ N m}^{-2}$$

Therefore, the pressure exerted by the heel on the horizontal floor is 6.24×10^6 Nm^{-2}.

10.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.

Sol. Density of mercury, $\rho_1 = 13.6 \times 10^3$ kg/m^3

Height of the mercury column, $h_1 = 0.76$ m

Density of French wine, $\rho_2 = 984$ kg/m^3

Height of the French wine column = h_2

Acceleration due to gravity, g = 9.8 m/s^2

The pressure in both the columns is equal, i.e.,

Pressure in the mercury column = Pressure in the French wine column

$$\rho_1 h_1 g = \rho_2 h_2 g \ ; \quad h_2 = \frac{\rho_1 h_2}{\rho_2} = \frac{13.6 \times 10^3 \times 0.76}{984} = 10.5\text{m}$$

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

10.7 A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

Sol. The maximum allowable stress for the structure, $P = 10^9$ Pa

Depth of the ocean, d = 3 km = 3×10^3 m, Density of water, $\rho = 10^3$ kg/m^3

Acceleration due to gravity, g = 9.8 m/s^2

The pressure exerted because of the sea water at depth, d = $\rho dg = 3 \times 10^3 \times 10^3 \times 9.8 = 2.94 \times 10^7$ Pa

The maximum allowable stress for the structure (10^9 Pa) is greater than the pressure of the sea water (2.94×10^7 Pa). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

10.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?

Sol. Here, mass of car = 3000 kg

Area of cross section of large piston = 425 cm^2 = 425×10^{-4} m^2.

$\therefore$ The maximum pressure that the smaller piston would have to bear

$$= \frac{\text{weight of car}}{\text{area of cross-section}} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5 \text{ Nm}^{-2}.$$

10.9 A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?

Sol. Here, h_w = 10.0 cm,

h_s = 12.5 cm. ρ_w = 1 gm cm^{-3}.

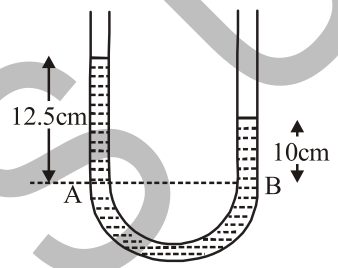
As the mercury column in the two arms is in level.

Pressure due to water = Pressure due to spirit

$$h_w \rho_w g = h_s \rho_s g$$

or $$\rho_s = \frac{h_w \rho_w}{h_s} = \frac{10.0 \times 1}{12.5} = 0.8 \text{ gm cm}^{-3}$$

$\therefore$ R.D. of spirit = 0.8



10.10 In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

Sol. When we pour 15 cm. of water and spirit in the respective arms, then

h_w = 10 + 15 = 25 cm., h_s = 12.5 + 15 = 27.5 cm.

Pressure due to water column, $P_1 = 0.25 \times 10^3 \times g$ Nm^{-2}.

Pressure due to mercury column, $P_2 = 0.275 \times 0.8 \times 10^3 \times g$ Nm^{-2}

Since $P_1 > P_2$ the mercury will rise in the spirit arm. If h is the difference in the levels of mercury in the two arms then $P_1 - P_2 = h\rho g$

$$0.25 \times 10^3 \times g - 0.275 \times 0.8 \times 10^3 \times g = h \times 13.6 \times 10^3 g$$

or $(0.25 - 0.275 \times 0.8) = h \times 13.6$

or $$h = \frac{0.25 - 0.2200}{31.6} = \frac{0.03}{13.6} \text{m} = 0.22 \text{ cm}$$

10.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

Sol. No. Bernoulli's equation cannot be used to describe the flow of water through a rapid in a river because of the turbulent flow of water. This principle can only be applied to a streamline flow.

10.12 Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

Sol. No. It does not matter if one uses gauge pressure instead of absolute pressure while applying Bernoulli's equation. The two points where Bernoulli's equation is applied should have significantly different atmospheric pressures.

10.13 Glycerine flows steadily through a horizontal tube of length 1.5m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}. What is the pressure difference between the two ends of the tube?

Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine 0.83 Ns m^{-2}.

Sol. Here $\ell = 1.5\text{m}$, r = 1.0 cm = 10^{-2} m, $\rho = 1.3 \times 10^3$ kg m^{-3}, $\eta = 0.83$ Ns m^{-2}.

mass collected per second = 4.0×10^{-3} kg s^{-1}

$\therefore$ volume of the liquid flowing per second

$$V = \frac{\text{mass}}{\text{density}} = \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$$

From Poiseuill's equation,

$$V = \frac{\pi P r^4}{8\eta\ell}; \quad P = \frac{8\eta\ell V}{\pi r^4} = \frac{8 \times 0.83 \times 1.5 \times 4 \times 10^{-3}}{3.14\,(1.0 \times 10^{-2})^4 \times 1.3 \times 10^3} = 9.8 \times 10^2 \text{ Nm}^{-2}.$$

10.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m/s and 63 m/s respectively. What is the lift on the wing if its area is 2.5m^2? Take the density of air to be 1.3 kg m^{-3}.

Sol. Let v_1 be the speed and P_1 be the pressure on the upper surface of the wing, and corresponding values on the lower surface be v_2 and P_2 respectively.

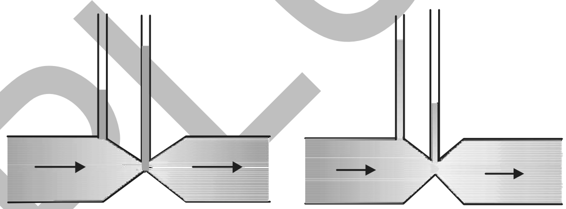
$\therefore$ $v_1 = 70$ m/s, $v_2 = 63$ m/s, A = 2.5 m^2, r = 1.3 kg m^{-3}.

According to Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2\ ; \quad P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)\ ; \quad P_2 - P_1 = \frac{1}{2} \times 3 \times (70^2 - 63^2)$$

Force (lift) on the wing $= A\,(P_2 - P_1) = 2.5 \times \frac{1}{2} \times 1.3 \times (70^2 - 63^2) = 2.5 \times \frac{1}{2} \times 1.3 \times 133 \times 7 = 1.5 \times 10^3$ N

10.15 Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?



(a) (b)

Sol. Figure (a) is correct.

According to continuity equation, av = constant as $v \propto 1/a$

Velocity is more at the narrow portion of the tube.

According to Bernouli's theorem, $P + \frac{1}{2}\rho v^2$ = Constant

Where v is less, pressure is more.

10.16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?

Sol. Area of cross-section of the spray pump, A_1 = 8 cm^2 = 8×10^{-4} m^2, Number of holes, n = 40

Diameter of each hole, d = 1 mm = 1×10^{-3} m, Radius of each hole, r = d/2 = 0.5×10^{-3} m

Area of cross-section of each hole, a = $\pi r^2 = \pi\,(0.5 \times 10^{-3})^2$ m^2

Total area of 40 holes, $A_2 = \pi \times a = 40 \times \pi\,(0.5 \times 10^{-3})^2$ m^2 = 31.41×10^{-6} m^2

Speed of flow of liquid inside the tube, V_1 = 1.5 m/min = 0.025 m/s

Speed of ejection of liquid through the holes = V_2

According to the law of continuity, we have: $A_1V_1 = A_2V_2$

$$V_2 = \frac{A_1V_2}{A_2} = \frac{8\times10^{-4}\times0.025}{31.61\times10^{-6}} = 0.633 \text{ m/s}$$

Therefore, the speed of ejection of the liquid through the holes is 0.633 m/s.

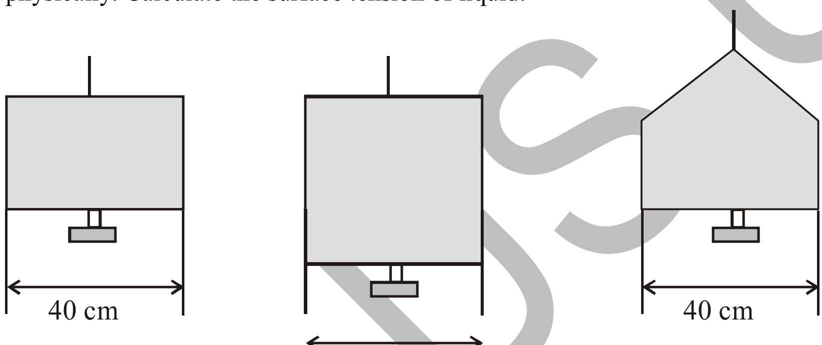
10.17 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30cm. What is the surface tension of the film?

Sol. Here, $F = 1.5 \times 10^{-2}$ N, L = 30cm = 0.3m

From formula, F = T . 2L

or $T = \frac{F}{2L} = \frac{1.5\times10^{-2}}{2\times0.3} = 2.5 \times 10^{-2}\ \text{Nm}^{-2}$

10.18 Figure (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically. Calculate the surface tension of liquid.



(a) **(b)** **(c)**

Sol. Length of the film = 40.0 cm. = 0.4 m

$\therefore$ Total weight supported = 4.5×10^{-2} N

F = S × 2L

$$\therefore\quad S = \frac{F}{2L} = \frac{4.5\times10^{-2}}{2\times0.4} = 5.625\times10^{-2}\ \text{Nm}^{-1}$$

As length of the film supporting the weight is same and temperature is also same, the weight supported by the film will also remain same i.e., 4.5×10^{-2} N.

10.19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} N m^{-1}.
The atmospheric pressure is 1.01×10^5 Pa.

Sol. Here $r = 3.00$ mm $= 3 \times 10^{-3}$ m, $S = 4.65 \times 10^{-1}$ Nm^{-1}.

Pressure inside the drop = atmospheric pressure + excess pressure =

$$= P + \frac{2S}{R} = 1.01\times10^5 + \frac{2\times4.65\times10^{-1}}{3.00\times10^{-3}}$$

$$= 1.01 \times 10^5 + 2 \times 1.55 \times 10^2 = 1.0131 \times 10^5\ \text{NM}^{-2}.$$

10.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20°C) is 2.50×10^{-2} N m^{-1}? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble?
(1 atmospheric pressure is 1.01×10^5 Pa).

Sol. Excess pressure inside the soap bubble is 20 Pa;

Pressure inside the air bubble is 1.06×10^5 Pa

Soap bubble is of radius, $r = 5.00$ mm $= 5 \times 10^{-3}$ m

Surface tension of the soap solution, $S = 2.50 \times 10^{-2}$ Nm^{-1}

Relative density of the soap solution = 1.20

$\therefore$ Density of the soap solution, $\rho = 1.2 \times 10^3$ kg/m^3

Air bubble formed at a depth, h = 40 cm = 0.4 m

Radius of the air bubble, $r = 5$ mm $= 5 \times 10^{-3}$ m

1 atmospheric pressure $= 1.01 \times 10^5$ Pa

Acceleration due to gravity, $g = 9.8$ m/s^2

Hence, the excess pressure inside the soap bubble is given by the relation:

Therefore, the excess pressure inside the soap bubble is 20 Pa. The excess pressure inside the air bubble is given by the relation: $P = \frac{4S}{r} = \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 20\text{Pa}$

Therefore, the excess pressure inside the air bubble is 10Pa.

At a depth of 0.4 m, the total pressure inside the air bubble = Atmospheric pressure + hρg + P'

$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10 = 1.057 \times 105$ Pa $= 1.06 \times 10^5$ Pa

Therefore, the pressure inside the air bubble is 1.06×10^5 Pa

10.21 A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door close.

Sol. Here $h_\omega = 4.0$ m, $\rho_\omega = 1.0 \times 10^3$kgm^{-3}, $h_a = 4.0$m, $\rho_a = 1.7 \times 10^3$ kg m^{-3}, $A = 20\text{cm}^2 = 20 \times 10^{-4}$ m^2.

Pressure due to water column $P_\omega = h_\omega \rho_\omega g$

Pressure due to acid column $P_a = h_a \rho_a g$

$\therefore \quad \Delta P = P_a - P_\omega = (h_a\rho_a - h_\omega \rho_\omega)\, g$

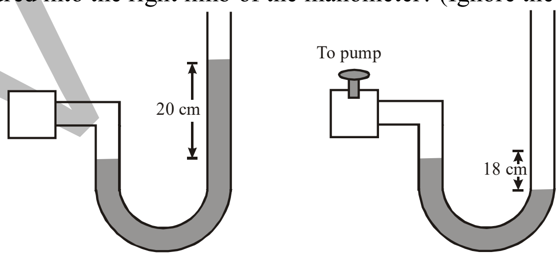
$\therefore \quad \Delta P = (4.0 \times 1.7 \times 10^3 - 4.0 \times 1 \times 10^3) \times 9.8$ Nm^{-2}.

$\therefore$ Force required to keep the door close $= A.\Delta P = 20 \times 10^{-4} \times 4.0 \times 0.7 \times 10^3 \times 9.8 = 55$ N

10.22 A manometer reads the pressure of a gas in an enclosure as shown in figure (a) When a pump removes some of the gas, the manometer reads as in figure (b). The liquid used in the manometers in mercury and the atmospheric pressure is 76 cm of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).



(a) (b)

Sol. (a) **For figure (a):** Atmospheric pressure, $P_0 = 76$ cm of Hg

Difference between the levels of mercury in the two limbs gives gauge pressure. Hence, gauge pressure is 20 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure = 76 + 20 = 96 cm of Hg

For figure (b) : Difference between the levels of mercury in the two limbs = –18 cm

Hence, gauge pressure is –18 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure = 76 cm – 18 cm = 58 cm

(b) 13.6 cm of water is poured into the right limb of figure (b).
Relative density of mercury = 13.6
Hence, a column of 13.6 cm of water is equivalent to 1cm of mercury.
Let h be the difference between the levels of mercury in the two limbs.
The pressure in the right limb is given as:
P_R = Atmospheric pressure + 1 cm of Hg = 76 + 1 = 77 cm of Hg ...... (1)
The mercury column will rise in the left limb.
Hence, pressure in the left limb, $P_L = 58 + h$ ...... (2)
Equating equations (1) and (2), we get: $77 = 58 + h \quad \therefore \quad h = 19$ cm
Hence, the difference between the levels of mercury in the two limbs will be 19 cm.

10.23 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

Sol. Two vessels having the same base area have identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has non-zero vertical components. When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel. Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

10.24 During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein?

Sol. Gauge pressure, P = 2000 Pa
Density of whole blood, $\rho = 1.06 \times 10^3$ kg m^{-3}
Acceleration due to gravity, g = 9.8 m/s^2
Height of the blood container = h
Pressure of the blood container, P = hρg

$$\therefore \quad h = \frac{P}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925\text{m}$$

The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2 m.

10.25 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

Sol. (a) Diameter of the artery, d = 2×10^{-3} m, Viscosity of blood, $\eta = 2.084 \times 10^{-3}$ Pa s
Density of blood, $\rho = 1.06 \times 10^3$ kg/m^3
Reynolds number for laminar flow, $N_R = 2000$
The largest average velocity of blood is given as:

$$V_{arg} = \frac{N_R \eta}{\rho d} = \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 2 \times 10^{-3}} = 1.966 \text{ m/s}$$

Therefore, the largest average velocity of blood is 1.966 m/s.

(b) As the fluid velocity increases, the dissipative forces become more important. This is because of the rise of turbulence. Turbulent flow causes dissipative loss in a fluid.

10.26 (a) What is the largest average velocity of blood low in an artery of radius 2×10^{-3} m if the flow must remain laminar?
(b) What is the corresponding flow rate? (Take viscosity of blood to be 2.084×10^{-3} pas)

Sol. Here, $r = 2 \times 10^{-3}$ m, $\rho = 1.06 \times 10^3$ kg m^{-3}.
(a) For laminar flow $R_e = 2000$

$R_e = \rho VD/\eta \qquad \therefore\ V = \dfrac{\eta R_e}{\rho D} = \dfrac{2000 \times 2.084 \times 10^{-3}}{(1.06 \times 10^3) \times 4 \times 10^{-3}} = 0.98\ \text{ms}^{-1}$

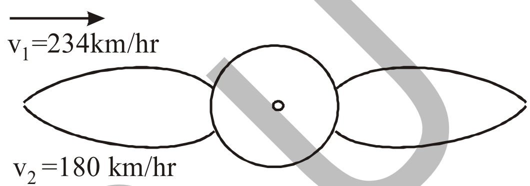
(b) Volume flowing per sec $Q = AV$

$Q = \pi r^2 v = \pi\,(2 \times 10^{-3})^2 \times 0.98 = 1.23 \times 10^{-5}\ \text{m}^3\text{s}^{-1}$.

10.27 A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg m^{-3}).

Sol. Here, $V_1 = 234 \times \dfrac{5}{18} = 65\ \text{ms}^{-1}$, $V_2 = 180 \times \dfrac{5}{18} = 50\ \text{ms}^{-1}$, $A = 25\ \text{m}^2$

Applying Bernouli's theorem above and below the wings



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$(P_2 - P_1) = \frac{1}{2}\rho\,(v_1^2 - v_2^2) = \frac{1}{2} \times (65^2 - 50^2)$$

$$= \frac{1}{2}(65 - 50)\,(65 + 50) = 862.5\ \text{Nm}^{-2}$$

$\therefore$ Upward thrust on the two wings = 862.5 × (25 × 2)

This force supports the weight of the aeroplane

i.e., $m \times 9.8 = 862.5 \times 50$ or $m = \dfrac{862.5 \times 50}{9.8} = 4400$ kg

10.28 In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0 × 10^{-5} m and density 1.2 × 10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8 × 10^{-5} Pa s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Sol. Here, $r = 2.0 \times 10^{-5}$ m, $\rho = 1.2 \times 10^3$ kg m^{-3}, $\eta = 1.8 \times 10^{-5}$ Nm^{-2}.

From formula, terminal velocity

$$V = \frac{2}{9}\frac{r^2(\rho - \sigma)\,g}{\eta} = \frac{2 \times (2 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}} = 5.8 \times 10^{-2}\ \text{m/s}$$

Now viscous force on the drop

$$F = 6\pi\eta r v = 6 \times \frac{22}{7} \times (1.8 \times 10^{-5}) \times (2 \times 10^{-5}) \times 5.8 \times 10^{-2} = 3.93 \times 10^{-19}\ \text{N}.$$

10.29 Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6 × 10^3 kg m^{-3}.

Sol. Here $\theta = 140°$, $r = 1.00$ mm $= 10^{-3}$ m, $S = 0.465$ Nm^{-1}, $\rho = 13.6 \times 10^3$ kg m^{-3}.

From formula $h = \dfrac{2S\cos\theta}{r\rho g} = h = \dfrac{2 \times 0.465 \times \cos 140°}{10^{-3} \times 13.6 \times 10^3 \times 9.8} = \dfrac{2 \times 0.465\,(-\sin 50°)}{13.6 \times 9.8}$

$\cos 140° = \cos(90 + 50)° = -\sin 50° = -\dfrac{2 \times 0.465 \times 0.7660}{13.6 \times 9.8} = -5.34 \times 10^{-3}\ \text{m} = -5.34\ \text{mm}$

– ve sign shows that mercury dips down in the tube relative to outside liquid surface.

10.30 Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3 × 10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0 × 10^3 kg m^{-3} ($g = 9.8$ m s^{-2}) .

Sol. Here, $r_1 = \frac{3.0}{2}\text{mm} = 1.5\times10^{-3}\,\text{m}$, $r_2 = \frac{6.0}{2}\text{mm} = 3 \times 10^{-3}$ m, S = 7.3 × 10^{-2} m,

$\rho = 1.0 \times 10^{-3}$ kg m^{-3}, θ = 0°

$$\therefore \quad h_1 = \frac{2S\cos\theta}{r_1\rho g} = \frac{2S}{r_1\rho g},\; h_2 = \frac{2S\cos\theta}{r_2\rho g} = \frac{2S}{r_2\rho g} \text{ as } \theta = 0°$$

$\therefore$ The difference in water level in the two limbs $= h_2 - h_1 = \frac{2S}{r_1\rho g} - \frac{2S}{r_2\rho g} = \frac{2S}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$$= \frac{2\times7.3\times10^{-2}}{10^3\times9.8}\left[\frac{1}{1.5\times10^{-3}} - \frac{1}{3\times10^{-3}}\right] = \frac{2\times7.3\times10^{-2}}{9.8}\left[\frac{3-1.5}{1.5\times3}\right]$$

$$= \frac{2\times7.3}{9.8}\times10^{-2}\times\frac{1}{3} = 4.97 \times 10^{-3} \text{ m } 5.0 \text{ mm}$$

10.31 (a) It is known that density ρ of air decreases with height y as $\rho = \rho_0 e^{-y/y_0}$ where $\rho_0 = 1.25$ kgm^{-3} is the density at sea level, and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m^3 is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?
[Take y_0 = 8000 m and ρ_{He} = 0.18 kg m^{-3}].

Sol. **(a)** $\frac{d\rho}{dy} \propto \rho \quad \therefore \quad \frac{d\rho}{dy} = -K\rho$

K is proportionality constant. Negative sign indicates that ρ decreases with increase in the value of y

$$\frac{d\rho}{\rho} = -K.dy \quad \text{or} \quad \int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = -\int_0^y K.dy$$

$$(\log\rho) - \log\rho_0 = -K\,(y)_0^y + K'$$

or $\quad \log\rho - \log\rho_0 = -Ky + K'$

when $y = 0$, $\rho = \rho_0$ at $y = 0$

$\therefore \quad \log\rho - \log\rho_0 = +K' \quad ; \quad K' = 0$

$\therefore \quad \log\rho - \log\rho_0 = -Ky$

or $\quad \log\left(\frac{\rho}{\rho_0}\right) = -Ky$ or $\frac{\rho}{\rho_0} = e^{-Ky}$ or $\rho = \rho_0 e^{-Ky}$

K is a constant suppose y_0 is a constant such that $K = 1/y_0$ then $\rho = \rho_0 e^{-y/y_0}$

(b) The balloon will rise till its density becomes equal to the density of air at that height.

$$\text{Density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{Pay load + mass o He}}{\text{volume}} = \frac{(400+1425\times0.18)}{1425}\text{kg m}^{-3}$$

Now, $\rho = \rho_0 e^{-y/y_0}$ $\qquad \frac{400+1425\times0.18}{1425} = \rho_0 e^{-y/y_0}$ ..... (i)

In part (a) $\rho_0 = 1.25$ kg m^{-3} and in part (b) y_0 = 8000 m

$$\therefore \quad \frac{400+1425\times0.18}{1425} = 1.25\,e^{-y/8000} \quad \text{or} \quad e^{-y/8000} = 0.36856$$

Taking log on both sides

$$-\frac{y}{8000} = \log_e(0.36856) = -1 \qquad \text{or} \qquad y = 8000 \text{ m} = 8 \text{ km}.$$