NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 14
OSCILLATIONS

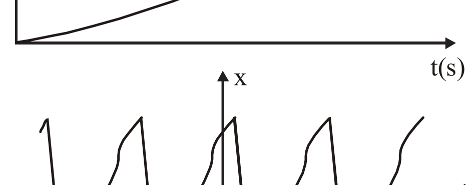
14.1 Which of the following examples represent periodic motion?
(a) A swimmer completing one (return) trip from one bank of a river to the other and back.
(b) A freely suspended bar magnet displaced from its N-S direction and released.
(c) A hydrogen molecule rotating about its center of mass.
(d) An arrow released from a bow.

Sol. **(b) and (c)**
(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.
(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.
(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.
(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

14.2 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(a) the rotation of earth about its axis.
(b) motion of an oscillating mercury column in a U-tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(d) general vibrations of a polyatomic molecule about its equilibrium position.

Sol. (a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.
(b) An oscillating mercury column in a U-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.
(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.
(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

14.3 Figure depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion) ?

(a)


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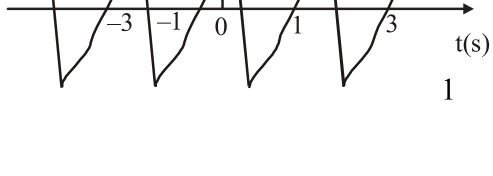
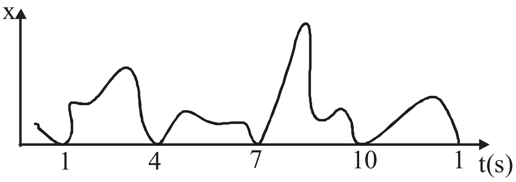
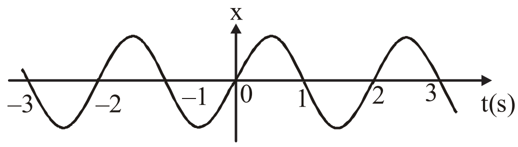


(c) 

(d) 

Sol. (a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.
(b) In this case, the motion of the particle repeats itself after 2s. Hence, it is a periodic motion, having a period of 2s.
(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.
(d) In this case, the motion of the particle repeats itself after 2s. Hence, it is a periodic motion, having a period of 2s.

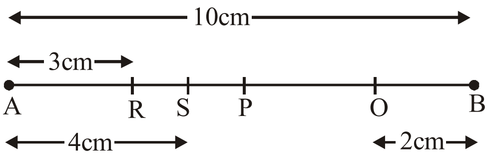
14.4 Which of the following functions of time represent
(a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion?
Give period for each case of periodic motion (ù is any positive constant):

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^3 \omega t$ (c) $3 \cos (\pi/4 - 2\omega t)$
(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$ (e) $\exp (-\omega^2 t^2)$ (f) $1 + \omega t + \omega^2 t^2$

Sol. (a) It represents S.H.M.
Time period $T = 2\pi/\omega$
(b) Periodic motion but not S.H.M.
Time period $T = 2\pi/\omega$
(c) Simple harmonic motion
Time period $T = \pi/\omega$
(d) Periodic motion but not S.H.M.
Time period $T = 2\pi/\omega$
(e) Non-periodic motion.
(f) Non-periodic motion.

14.5 A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is
(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

Sol. In figure, A and B represents the extreme positions of S.H.M. For velocity, the direction from A to B is taken positive. For acceleration and the force, the direction is taken positive along AP and negative if directed along BP.



(a) At the end A: Being at extreme position velocity is zero. Acceleration and force both are directed from A to P so they are positive.

(b) At the end B: Velocity is zero. The force and acceleration are directed along BP, i.e., along negative direction so both are negative.

(c) At the mid point of AB going towards A: The particle is moving along the direction PA, i.e., negative direction. Hence, velocity is negative while acceleration and force both are zero.

(d) At 2 cm. away from B going towards A: The particle has a tendency to move along QA which is negative direction so, velocity, force and acceleration all are negative.

(e) At 3cm away from A going towards B: The particle is at R moving towards P which is in positive direction. Therefore, velocity, acceleration and force all are +ve.

(f) At 4cm away from A going towards A: The direction of velocity is along SA which is negative direction. Therefore, velocity is negative but, acceleration and force points towards mean position so, they are positive.

14.6 Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$ (b) $a = -200x^2$ (c) $a = -10x$ (d) $a = 100x^3$

Sol. (a) $a = 0.7x$

Comparing it with $a = -\omega^2 x$

We find that motion is not SHM.

(b) $a = -200x^2$

Motion is not SHM.

(c) $a = -10x$

Comparing it with $a = -\omega^2 x$

The motion is SHM.

(d) $a = 100x^3$

Motion is not SHM.

14.7 The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \phi)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle ? The angular frequency of the particle is πs^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Sol. Initially, at $t = 0$: Displacement, $x = 1$ cm, Initial velocity, $v = \omega$ cm/sec.

Angular frequency, $\omega = \pi$ rad/s^{-1}

It is given that: $x(t) = A\cos(\omega t + \phi)$

$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$

$A\cos\phi = 1$ .... (1)

Velocity, $v = dx/dt$

$\omega = -A\omega\sin(\omega t + \phi)$

$1 = -A\sin(\omega \times 0 + \phi) = -A\sin\phi$

$A\sin\phi = -1$ .... (2)

Squaring and adding equations (1) and (2), we get

$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$

$A^2 = 2$

$\therefore \quad A = \sqrt{2}$cm.

Dividing equation (ii) by equation (i), we get $\tan\phi = -1$

$\therefore \quad \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, ...$

SHM is given as x = B sin (ωt + α)

Putting the given values in this equation, we get

$1 = B \sin [\omega \times 0 + \alpha]$

$B \sin \alpha = 1 \qquad .... (3)$

Velocity, $v = \omega B \cos (\omega t + \alpha)$

Substituting the given values, we get

$\pi = \pi B \sin \alpha$

$B \sin \alpha = 1 \qquad .... (4)$

Squaring and adding equations (iii) and (iv), we get:

$B^2 (\sin^2\phi + \cos^2\phi) = 1 + 1$

$B^2 = 2$

$\therefore \quad B = \sqrt{2}\text{cm}.$

Dividing equation (3i) by equation (4), we get:

$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1}$; $\tan\alpha = 1 = \frac{\pi}{4}$

$\therefore \quad \alpha = \frac{\pi}{4}, \frac{5\pi}{4}, ...$

14.8 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

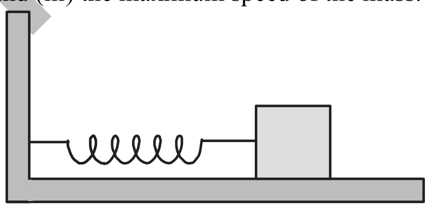
Sol. Here, increase in the length of the spring for 50 kg is 0.20m. Therefore,

$K = \frac{F}{x} = \frac{50 \times 9.8}{0.20} = 50 \times 49 \text{ Nm}^{-1}.$

Now, $T = 2\pi\sqrt{\frac{m}{K}} \qquad \therefore \; 0.6 = 2\pi\sqrt{\frac{m}{50 \times 49}}$ or $0.36 = \frac{4\pi^2 \times m}{50 \times 49}$ or $m = \frac{0.36 \times 50 \times 49}{4\pi^2}\text{kg}$

Therefore, weight of body = mg = $\frac{0.36 \times 50 \times 49 \times 9.8}{4\pi^2} \simeq 220$ N

14.9 A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.



Sol. Here, K = 1200 N/m, m = 3 kg, a = 2 × 10^{-2} m.

(i) Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{1200}{3}} = \frac{10}{\pi}\text{Hz}$

(ii) Maximum acceleration, $\alpha_0 = a\omega^2 = 2 \times 10^{-2}\left(2\pi \times \frac{10}{\pi}\right)^2 = 2 \times 10^{-2} (20)^2 = 8 \text{ ms}^{-2}$

(iii) Maximum speed, $v_0 = a\omega = a \times 2\pi\nu = 2 \times 10^{-2} \times \left(2\pi \times \frac{10}{\pi}\right) = 0.4$ m/s

14.10 In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is
(a) at the mean position,
(b) at the maximum stretched position, and
(c) at the maximum compressed position.
In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Sol. Here, a = 2.0cm, $\omega = \sqrt{\frac{R}{m}} = \sqrt{\frac{1200}{3}} = 20s^{-1}$

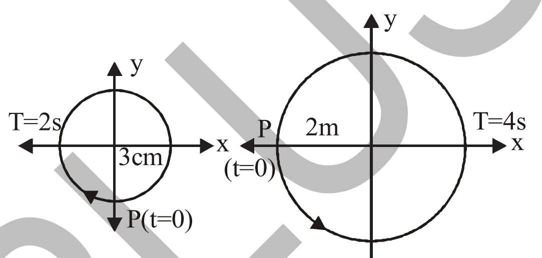
(a) As time is noted from the mean position, hence using
$x = a \sin \omega t$, we have $x = 2 \sin 20t$

(b) At maximum stretched position, the body is at the extreme right position, with an initial phase of $\pi/2$ rad. Then $x = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos \omega t = 2\cos 20t$

(c) At maximum compressed position, the body is at the extreme left position, with an initial phase of $3\pi/2$ rad. Then $x = a \sin\left(\omega t + \frac{3\pi}{2}\right) = -\,a \cos \omega t = -\,2\cos 20t$

The function neither differ in amplitude nor in frequency. They differ in initial phase.

14.11 Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



(a) (b)

Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Sol. (a) When perpendicular is dropped on the x-axis of the circle from position (t = 0), we get x = 0. Therefore initial phase is zero. With increase in time particle moves towards left say to P_1 now when we drop the perpendicular on the x-axis, we get negative value of x. The radius of the circle is 3cm, so amplitude of SHM is 3 cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi\ s^{-1}$

Equation of SHM is $y = A \sin \omega t = -3 \sin \pi t$ cm.

(b) Here A = 2m, T = 4s

or $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\ s^{-1}$

$\phi = \frac{3\pi}{2}$ radian

$\therefore \quad x = A \sin(\omega t + \phi) = 2\sin\left(\frac{\pi}{2}t + \frac{3\pi}{2}\right) = -2\cos\frac{\pi}{2}t$ m

14.12 Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t =0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2 \sin(3t + \pi/3)$ (b) $x = \cos(\pi/6 - t)$ (c) $x = 3 \sin(2\pi t + \pi/4)$ (d) $x = 2 \cos \pi t$

Sol. (a) $x = A \cos(\omega t + \phi)$ .... (1)

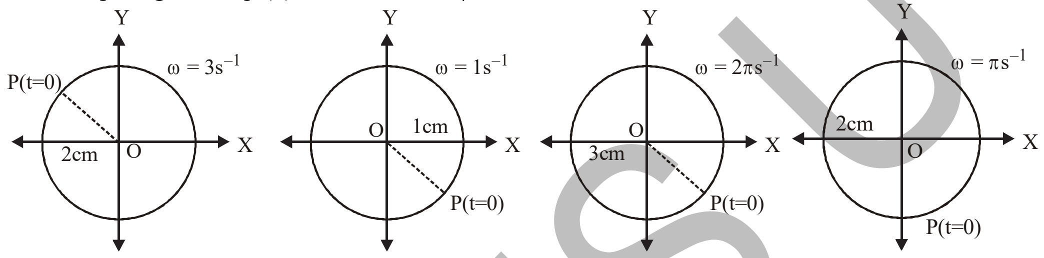
Comparing equation (1) with the given equation

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = 2\cos\left[\frac{\pi}{2} + 3t + \frac{\pi}{3}\right] = 2\cos\left[3t + \frac{5\pi}{6}\right]$$

$A = 2$, $\omega = 3$, $\phi = 5\pi/6$

(b) Here, $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$

Comparing with eq. (1), $A = 1$, $\omega = 1$, $\phi = -\pi/6$



(a) (b) (c) (d)

(c) Here, $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right) = 3\cos\left(2\pi t + \frac{3\pi}{2} + \frac{\pi}{4}\right)$

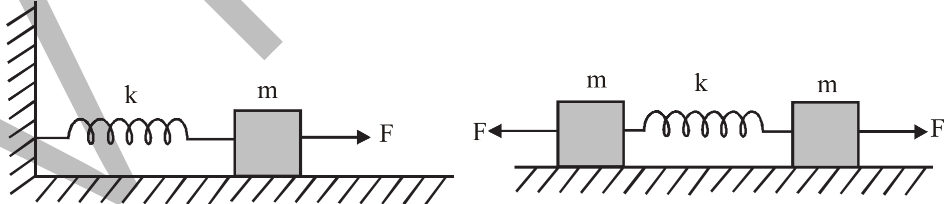
Comparing it with eq. (1)

$$A = 3,\ \omega = 2\pi,\ \phi = \frac{3\pi}{2} + \frac{\pi}{4}$$

(d) $x = 2 \cos \pi t$

Comparing with eq. (1), $A = 2$, $\omega = \pi$, $\phi = 0$

14.13 Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.



(a) (b)

(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

Sol. (a) Maximum extension = F/k, where k is spring constant.

(b) Let x_1 and x_2 be the positions of the two masses from reference point O.

Forces by the two springs on the two masses are F and –F (magnitude of each in kx)

From Newton's second law

$$m\frac{d^2x_1}{dt^2} = -kx \text{ .... (1)} \quad \text{and} \quad m\frac{d^2x_2}{dt^2} = kx \text{ .... (2)}$$

Subtracting eq. (2) from eq. (1)

$$m\left[\frac{d^2x_1}{dt^2}-\frac{d^2x_2}{dt^2}\right]=-2kx \;;\quad m\frac{d^2}{dt^2}(x_1-x_2)=-2kx \quad \text{.......... (3)}$$

The change is the length of the spring is

$$x=(x_1-x_2)-\ell \quad \text{.......... (4)}$$

Since ℓ is constant.

$$\frac{d^2x}{dt^2}=\frac{d^2}{dt^2}(x_1-x_2)$$

$\therefore$ Eq. (3) becomes $m\dfrac{d^2x}{dt^2}+2kx=0$ or $\dfrac{d^2x}{dt^2}+\dfrac{2k}{m}x=0$

Comparing with $a+\omega^2y=0$

$$\omega=\sqrt{\frac{2k}{m}} \quad \therefore \quad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{2k}} \quad \text{.......... (5)}$$

In case (a) time period is given by, $T=2\pi\sqrt{\dfrac{m}{2k}}$

14.14 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200rad/min, what is its maximum speed?

Sol. Here $a=(1/2)\text{m}$, $\omega=200$ min

$\therefore \quad v_{max}=a\omega=\dfrac{1}{2}\times 200=100$ m/min.

14.15 The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s ? (g on the surface of earth is 9.8 m s^{-2})

Sol. Here, $g_m=1.7$ m s^{-2}, $g_e=9.8$ m s^{-2}

$T_e=3.5$ sec, $T=2\pi\sqrt{\dfrac{\ell}{g}}$, i.e., $T\propto\dfrac{1}{\sqrt{g}}$

$\therefore \quad \dfrac{T_m}{T_e}=\sqrt{\dfrac{g_e}{g_m}}$; $T_m=\sqrt{\dfrac{9.8}{1.7}}\times 3.5=2.40\times 3.5=8.4\text{s}$

14.16 Answer the following questions:

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T=2\pi\sqrt{\dfrac{m}{k}}$.
A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\ell/g}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Sol. (a) In case of simple pendulum, $K\propto$ mass m
So, m cancels out.

(b) In case $\sin\theta < \theta$, if $mg\sin\theta$ is replaced by $mg\theta$, there will be an effective reduction in the value of g which gives rise increase in time period according to formula

$T = 2\pi\sqrt{\ell / g}$ (which is true when $\sin\theta \approx \theta$)

(c) The wrist watch gives correct time. The working of wrist watch depends on spring action which is independent of g.

(d) We know, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

For a freely falling body $g = 0$ $\therefore \nu = 0$

14.17 A simple pendulum of length *l* and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Sol. We know centripetal acceleration

$a_c = v^2/R$ (In horizontal direction)

Acceleration due to gravity = g (in vertical direction)

$\therefore$ Effective value of acceleration, $g' = \sqrt{g^2 + (v^2/R)^2}$

From formula, $T = 2\pi\sqrt{\frac{\ell}{g}}$; $T = 2\pi\sqrt{\frac{\ell}{(g^2 + (v^2/R^2))^{1/2}}}$

14.18 A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_ℓ. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_\ell g}}$, where ρ is the density of cork.

(Ignore damping due to viscosity of the liquid).

Sol. Base area of the cork = A, Height of the cork = h

Density of the liquid = ρ_ℓ, Density of the cork = ρ

In equilibrium: Weight of the cork = Weight of the liquid displaced by the floating cork.

Let the cork be depressed slightly by x.

As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Up-thrust = Restoring force, F = Weight of the extra water displaced F = – (Volume × Density × g)

Volume = Area × Distance through which the cork is depressed

Volume = Ax

$\Delta F = -A \times \rho_\ell g$ .... (1)

According to the force law, $F = kx \Rightarrow k = F/x$

Where, k is a constant

$k = \frac{F}{x} = -A\rho_\ell g$ .... (2)

The time period of the oscillations of the cork:

$T = 2\pi\sqrt{m/k}$ .... (3)

Where, m = Mass of the cork = Volume of the cork × Density

= Base area of the cork × Height of the cork × Density of the cork

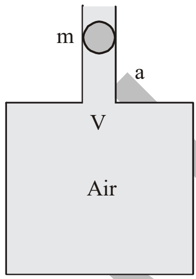
= Ahρ

Hence, the expression for the time period becomes: $T = 2\pi\sqrt{\frac{h\rho}{\rho_\ell g}}$

14.19 One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Sol. When both the ends are open to the atmosphere and the difference in levels of the liquid in the two arms is h, the net force on the liquid column is $Ah\rho g$ where A is the area of cross-section of the tube and ρ is the density of the liquid. Since restoring force is proportional to h, motion is simple harmonic.

14.20 An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.



Sol. Let ΔV is the decrease in volume when the ball is pressed down, then $\Delta V = ay$

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

Let P be the excess pressure, then Bulk modulus of elasticity

$$B = \frac{\text{Stress}}{\text{Volume Strain}} \; ; \quad B = -\frac{P}{\Delta V / V} \quad \text{or} \quad P = -B \times \frac{\Delta V}{V}$$

Negative sign shows that with increase in pressure volume decreases.

$$\text{Force } F = P \times a = -B \times \frac{\Delta V}{V} a = -B \frac{(a\,.\,y)}{V} a \; ; \quad P = -\frac{Ba^2}{V} y$$

If m is the mass of the ball and α is the acceleration then according to Newton's second law,

$$F = m\alpha = -\frac{Ba^2}{V} y \quad \text{or} \quad \alpha = -\frac{Ba^2}{mv} y$$

As Ba^2/mv is constant $\alpha \propto -y$

Thus, the motion of ball is S.H.M. time period of which is given by

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{mv}{Ba^2}}$$

14.21 You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Sol. (a) Weight of automobile = $3000 \times 10 = 3 \times 10^4$ N

$$\text{Weight supported by each spring} = \frac{3 \times 10^4}{4} = 7500\text{N}$$

Due to this force spring is compressed by 15cm.

$\therefore$ Spring constant $k = 7500/0.15 = 5 \times 104\ \text{Nm}^{-1}$.

(b) In case of damped oscillation, amplitude reduction factor is given by

$$\frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4} = e^{bt/2m} \qquad .... (1)$$

A_1, A_2, ... are amplitudes of the first, second, third .... oscillations b is damping constant, t is period of damped oscillations.

ω_0 = natural angular frequency

ω = angular frequency of damped oscillations.
k = spring constant.

then $\omega_0 = \sqrt{\frac{k}{m}}$ and $\omega_0 = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$$\therefore \qquad t = 2\pi / \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \qquad \text{........ (2)}$$

From eq. (1) and (2)

$$\frac{A_1}{A_2} = \exp\left[\frac{b}{2m} \cdot \frac{2\pi}{\sqrt{k/m - (b/2m)^2}}\right] = \exp\frac{\pi b}{m\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}} = \exp\frac{2\pi b}{\sqrt{4km - b^2}}$$

Taking log of both sides

$$\log_e 2 = \frac{2\pi b}{\sqrt{4 \times 50000 \times 750 - b^2}} \quad \text{or} \quad 0.6932 = \frac{2\pi b}{\sqrt{4 \times 50000 \times 750 - b^2}}$$

On squaring both sides and solving, b = 1343 kg s^{-1}.

14.22 Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Sol. In SHM the displacement is given by y = A sin ωt

$$\therefore \quad v = \frac{dy}{dt} = A\omega\cos\omega t \qquad \text{........ (1)}$$

$$\text{K.E.} = \frac{1}{2}Mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t \qquad \text{........ (2)}$$

Average K.E. over one complete cycle

$$\bar{K} = \frac{1}{T}\int_0^T \frac{1}{2}mv^2 dt = \frac{1}{T}\int_0^T \frac{1}{2}mA^2\omega^2\cos^2\omega t\, dt = \frac{mA^2\omega^2}{2T}\int_0^T \cos^2\omega t\, dt$$

$$= \frac{mA^2\omega^2}{2T}\int_0^T \frac{(1+\cos 2\omega t)}{2}dt = \frac{mA^2\omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{mA^2\omega^2}{4T}\left[T - 0 + \frac{1}{2\omega}\left(\sin 2\times\frac{2\pi}{T}.T - \sin\frac{2\pi}{T}.0\right)\right] = \frac{mA^2\omega^2}{4T}.T = \frac{1}{4}mA^2\omega^2$$

Now potential energy, $U = \frac{1}{2}m\omega^2 y^2$

Average P.E. over one complete cycle

$$\bar{U} = \frac{1}{T}\int_0^t \frac{1}{2}m\omega^2 y^2 dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2\sin^2\omega t\, dt$$

$$= \frac{1}{2T}.m\omega^2 A^2\int_0^T\left[\frac{1-\cos 2\omega t}{2}\right]dt = \frac{m\omega^2 A^2}{4T}[T] = \frac{m\omega^2 A^2}{4}$$

i.e., $\bar{K} = \bar{U}$ i.e., average K.E. equals the average P.E.

14.23 A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation J = $-\alpha\,\theta$, where J is the restoring couple and θ the angle of twist).

Sol. The time period of torsional oscillation of a body is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad \text{or} \qquad C = \frac{4\pi^2 I}{T^2}$$

I = Moment of inertia, C = torsional constant

Here, $T = 1.5s$, $\text{M.I.} = \frac{1}{2}MR^2 = \frac{1}{2}\times 10 \times (0.15)^2$

$$\therefore \quad C = \frac{4\,(3.14)^2 \times 5 \times (0.15)^2}{(1.5)^2} = 1.97 \text{ Nm rad}^{-1}.$$

14.24 A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5cm, (b) 3 cm, (c) 0 cm.

Sol. Here, $r = 5cm = 0.05m$, $T = 0.2s$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi \text{ rad/s}$$

When displacement is y_2, then acceleration, $a = -\omega^2 y$

Velocity, $V = \omega\sqrt{r^2 - y^2}$

Case (a), when $y = 5cm = 0.05m$

$a = (-10\pi)^2 \times 0.05 = -5\pi^2 \text{ m/s}^2$

$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0$

Case (b), when $y = 3cm = 0.03m$

$a = (-10\pi)^2 \times 0.03 = -3\pi^2 \text{ m/s}^2$

$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi \text{ m/s}$

Case (c), when $y = 0$

$a = (-10\pi)^2 \times 0 = 0$

$V = 10\pi\sqrt{(0.05)^2 - 0^2} = 10\pi \times 0.05 = 0.5\pi \text{ m/s}$

14.25 A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.

Sol. $x = A\cos(\omega t + \theta)$; Velocity, $dx/dt = -A\,\omega \sin(\omega t + \theta)$

When $t = 0$, $x = x_0$ and $dx/dt = -v_0$

$\therefore \quad x_0 = A\cos\theta$ .... (1)

$-v_0 = A\omega\sin\theta$ or $A\sin\theta = v_0/\omega$ .... (2)

On squaring and adding eq. (1) and (2), we get

$A^2(\sin^2\theta + \cos^2\theta) = (v_0^2/\omega^2) + x_0^2$

$A = [(v_0^2/\omega^2) + x_0^2]^{1/2}$