NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 15
WAVES

15.1 A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Sol. Here, M = 2.50 kg, T = 200 N, ℓ = 20.0 m

$$\therefore \quad m = \frac{M}{\ell} = \frac{2.50}{20.0}\frac{kg}{m}$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200 \times 20}{2.5}} = 40 \text{ m/s}$$

Therefore, time taken by jerk to reach the other end of string.

$$t = \frac{L}{v} = \frac{20.0}{40} = 0.5 \text{ sec}$$

15.2 A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1}? (g = 9.8 m s^{-2})

Sol. Let t_1 and t_2 be the time taken by stone to reach the pond and time take by sound to reach the top of tower.

For t_1: $S = ut + \frac{1}{2}gt^2$

$$300 = 0 + \frac{1}{2} \times 9.8\, t_1^2 \quad \text{or} \quad t_1^2 = \frac{600 \times 2}{9.8}$$

or $$t_1 = \sqrt{\frac{600 \times 2}{9.8}} = 7.82 \text{ s}$$

For t_2: Time taken by sound $t_2 = \frac{S}{V} = \frac{300}{340} = 0.88$ s

$\therefore$ Total time taken = 7.82 + 0.88 = 8.7s

15.3 A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 m s^{-1}.

Sol. Here, ℓ = 12.0m, v = 343 m s^{-1}, M = 2.10 kg

$$m = \frac{M}{\ell} = \frac{2.10}{12}\frac{kg}{m}$$

From formula, $v = \sqrt{\frac{T}{m}}$

$$T = mv^2 = \frac{M}{\ell}.v^2 = \frac{210}{12} \times (343)^2 = 2.1 \times 10^4 \text{ N}$$

15.4 Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(a) is independent of pressure,

(b) increases with temperature,
(c) increases with humidity.

Sol. (a) Take the relation: $v = \sqrt{\frac{\gamma P}{\rho}}$ .... (1)

Where, Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$

M = Molecular weight of the gas, V = Volume of the gas

Hence, eq. (1) reduces to $v = \sqrt{\frac{\gamma PV}{M}}$ .... (2)

Now from the ideal gas equation for n = 1:

$PV = RT$

For constant T, PV = Constant

Since both M and γ are constants, v = Constant

Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation: $v = \sqrt{\frac{\gamma P}{\rho}}$

For one mole of an ideal gas, the gas equation can be written as: PV = RT

$P = \frac{RT}{V}$ … (3)

Substituting equation (3) in equation (1), we get:

$v = \sqrt{\frac{\gamma RT}{V\rho}} = \sqrt{\frac{\gamma RT}{M}}$ … (4)

Where, Mass, $M = \rho V$ is a constant, γ and R are also constants.

We conclude from equation (4) that $v \propto \sqrt{T}$.

Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.

(c) Let v_m and v_d be the speeds of sound in moist air and dry air respectively.

Let ρ_m and ρ_d be the densities of moist air and dry air respectively.

Take the relation: $v = \sqrt{\frac{\gamma P}{\rho}}$

Hence, the speed of sound in moist air is: $v_m = \sqrt{\frac{\gamma P}{\rho_m}}$ .... (1)

And the speed of sound in dry air is: $v_d = \sqrt{\frac{\gamma P}{\rho_d}}$ .... (2)

On dividing equations (i) and (ii), we get:

$\frac{v_m}{v_d} = \sqrt{\frac{\gamma P}{\rho_s} \times \frac{\rho_d}{\gamma P}} = \sqrt{\frac{\rho_d}{\rho_s}}$

However, the presence of water vapour reduces the density of air, i.e., $\rho_d < \rho_m$

$\therefore \quad v_m > v_d$

Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.

15.5 You have learnt that a travelling wave in one dimension is represented by a function y = f (x, t) where x and t must appear in the combination x – v t or x + v t, i.e. y = f (x ± v t). Is the converse true? Examine if the following functions for y can possibly represent a travelling wave:
(a) $(x - vt)^2$ (b) $\log [(x + vt)/x_0]$ (c) $1/(x + vt)$

Sol. The converse is not true. A obvious requirement for an acceptable function for a travelling wave is that it should be finite everywhere and at all times. Only function (c) satisfies this condition, the remaining functions cannot possibly represent a travelling wave.

15.6 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340m s^{-1} and in water 1486 m s^{-1}.

Sol. Here, $\nu = 1000$ KHz $= 10^6$ Hz, $v_w = 1476$ m/s, $v_a = 340$ m/s

From formula, $v = \nu\lambda$

$$\lambda_a = \frac{v_a}{\nu} = \frac{340}{10^6} = 3.4 \times 10^{-4}\,\text{m}$$

$$\lambda_\omega = \frac{v_\omega}{\nu} = \frac{1486}{10^6} \approx 1.49 \times 10^{-3}\,\text{m}$$

15.7 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Sol. Here, $v = 1.7 \times 10^3$ ms^{-1}, $\nu = 4.2 \times 10^6$ Hz

From formula = $v = \nu\,\lambda$

$$\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.0 \times 10^{-4}\ \text{m}$$

15.8 A transverse harmonic wave on a string is described by
$y (x, t) = 3.0 \sin (36t + 0.018x + \pi/4)$, where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave?
If it is travelling, what are the speed and direction of its propagation?
(b) What are its amplitude and frequency?
(c) What is the initial phase at the origin?
(d) What is the least distance between two successive crests in the wave?

Sol. (a) Travelling wave, $y(x,t) = 3.0 \sin 2\pi\left(\frac{t}{2\pi/36} + \frac{x}{2\pi/0.018} + \frac{\pi}{4 \times 2\pi}\right)$ .... (1)

Comparing it with standard equation $y = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda} + \phi\right)$ .... (2)

Direction of propagation : +ve direction of x-axis.

Comparing eq. (1) and (2), $T = \frac{2\pi}{36}$ sec ; $\lambda = \frac{2\pi}{0.018}$ cm

$\therefore\ v = \nu\lambda = \frac{\lambda}{T} = \frac{2\pi}{0.018} \times \frac{36}{2\pi} = 2000$ cm s^{-1} = 20m/s

(b) Amplitude a = 3.0 cm

Frequency, $\nu = \frac{36}{2\pi} = \frac{18}{\pi} = 5.7$ Hz

(c) Initial phase $\phi = \frac{\pi}{4}$

(d) $\lambda = \frac{2\pi}{0.018} = 348.8\text{cm} \simeq 3.5\text{m}$

15.9 For the wave described in Exercise 15.8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Sol. All the waves have different phases.

The given transverse harmonic wave is: $y\,(x, t) = 3.0 \sin\left(36t + 0.018\pi + \frac{\pi}{4}\right)$ .... (1)

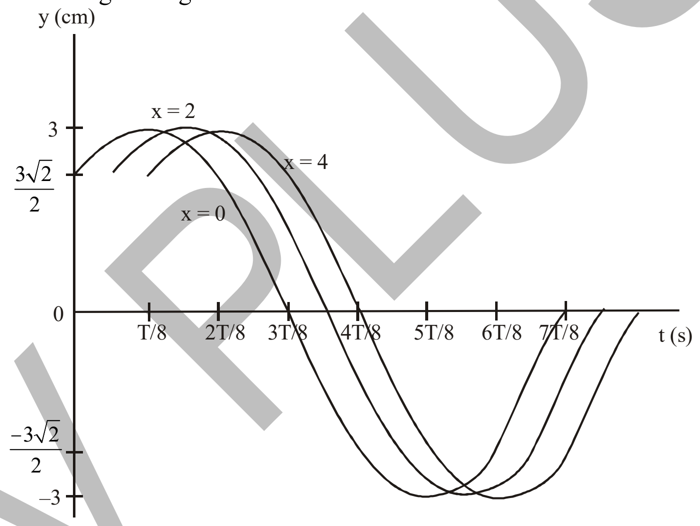
For x = 0, the equation reduces to: $y\,(0, t) = 3.0 \sin\left(36t + \frac{\pi}{4}\right)$

Also, $\omega = \frac{2\pi}{T} = 36$ rad/s $\quad \therefore \quad T = \frac{\pi}{8}$ s

Now, plotting y as t graphs using the different values of t, as listed in the given table.

t (s)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$
y (cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0
t (s)	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
y (cm)	$\frac{-3\sqrt{2}}{2}$	−3	$\frac{-3\sqrt{2}}{2}$	0

For x = 0, x = 2, and x = 4, the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in x. The y-t plots of the three waves are shown in the given figure.



15.10 For the travelling harmonic wave: y (x, t) = 2.0 cos 2π (10t – 0.0080 x + 0.35)
where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of (a) 4 m, (b) 0.5 m, (c) λ/2, (d) 3λ/4

Sol. Standard equation of plane progressive wave is $y = a\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right]$

(a) Phase difference = $\frac{2\pi}{\lambda} \times \Delta x = 6.4\pi$ rad (b) Phase difference = 0.8π rad

(c) Phase difference = $\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad (d) Phase difference = $\frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad

15.11 The transverse displacement of a string (clamped at its both ends) is given by

$y\,(x, t) = 0.060 \sin\left(\frac{2\pi}{3}x\right)\cos\,(120\pi t)$, where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

Answer the following:

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

Sol. $$y = 0.060 \sin\frac{2\pi x}{3}\cos 120\pi t$$

Comparing it with standard equation of stationary wave

$$y = 2a \sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T}$$

(a) Stationary wave

(b) $\frac{2\pi x}{\lambda} = \frac{2\pi x}{3}$ i.e., $\lambda = 3$m

$$120\pi t = \frac{2\pi t}{T}$$

or $T = \frac{1}{60}$ sec or $\nu = 60$ Hz

Now, $v = \nu\,\lambda = 60 \times 3 = 180\ ms^{-1}$

(c) Now, $v = \sqrt{\frac{T}{m}}$ or $T = mv^2 = \left(\frac{3.0 \times 10^{-2}}{1.5}\right)(180)^2 = 648$ N

15.12 (i) For the wave on a string $y\,(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos\,(120\pi t)$, where x and y are in m & t in s.

The length of the string is 1.5m and its mass is 3.0×10^{-2} kg. , do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers.

(ii) What is the amplitude of a point 0.375 m away from one end?

Sol. (i) All the point on the string have the same frequency and phase but not the same amplitude.

(ii) The amplitude at distance x is $2A \sin\frac{2\pi x}{\lambda}$

Here, 2A = 0.06 m, λ = 3m, x = 0.375 m

$\therefore$ Required amplitude = 0.06 sin $\frac{2\pi}{3}$ (0.375) = 0.042 m.

15.13 Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all (a) y = 2 cos (3x) sin (10t) (b) $y = 2\sqrt{x - vt}$

(c) y = 3 sin (5x – 0.5t) + 4 cos (5x – 0.5t) (d) y = cos x sin t + cos 2x sin 2t

Sol. (a) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation.

(b) The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave.

(c) The given equation represents a travelling wave as the harmonic terms kx and ωt are in the combination of kx – ωt.

(d) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves.

15.14 A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

Sol. $\nu = 45$ Hz, $M = 3.5 \times 10^{-2}$ kg, $m = 4.0 \times 10^{-2}$ kg/m

We know, $m = \frac{M}{\ell}$ $\quad \therefore \ \ell = \frac{M}{m} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = \frac{7}{8}$m

(a) $\therefore \ v = \nu\,\lambda = \nu \times 2\ell = (45)\left(2 \times \frac{7}{8}\right) = 78.75$ m/s

(b) Now, $v = \sqrt{\frac{T}{m}}$ $\quad \therefore \ T = m \,.\, v^2 = 4.0 \times 10^{-2}\,(78.75)^2 = 248$ N

15.15 A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5cm or 79.3cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

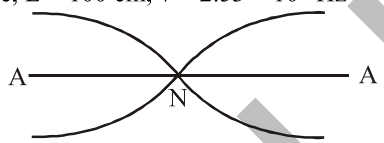
Sol. The tube will act as closed organ pipe.

$\therefore \quad \lambda = 4 \times 0.255 = 1.02$m

The speed of sound is given by the relation: $\quad v = \nu\,\lambda = 340 \times 1.02 = 346.8$ m/s

15.16 A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?

Sol. Here, L = 100 cm, $\nu = 2.53 \times 10^3$ Hz



From Figure, $L = \lambda/2$ or $\lambda = 2L = 2 \times 100 = 200$ cm.

$\therefore \ v = \nu\,\lambda = 2.53 \times 10^3 \times 200 \times 10^{-2} = 5.06$ km s^{-1}.

15.17 A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

Sol. $\quad \nu_n = (2n-1)\frac{v}{4\ell}$

$n \approx 1$

Hence, the first mode of vibration frequency is resonantly excited by the given source.

In a pipe open at both ends, the nth mode of vibration frequency is given by the relation:

$n = \frac{2\ell\nu_n}{v} \simeq 0.5$

Since the number of the mode of vibration (n) has to be an integer, the given source does not produce a resonant vibration in an open pipe.

15.18 Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

Sol. Frequency of string A, $f_A = 324$ Hz

Frequency of string B = f_B

Beat's frequency, n = 6 Hz

Beat's frequency is given as : $n = |\, f_A \pm f_B \,|$

$6 = 324 \pm f_B$

$f_B = 330$ Hz or 318 Hz

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$\nu \propto \sqrt{T}$

Hence, the beat frequency cannot be 330 Hz.

$\therefore \quad f_B = 318$ Hz

15.19 Explain why (or how):

(a) in a sound wave, a displacement node is a pressure antinode and vice versa,

(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",

(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

Sol. (a) A node is a point where the amplitude of vibration is the minimum and pressure is the maximum. On the other hand, an antinode is a point where the amplitude of vibration is the maximum and pressure is the minimum.

Therefore, a displacement node is nothing but a pressure antinode, and vice versa.

(b) Bats emit very high-frequency ultrasonic sound waves. These waves get reflected back toward them by obstacles. A bat receives a reflected wave (frequency) and estimates the distance, direction, nature, and size of an obstacle with the help of its brain senses.

(c) The overtones produced by a sitar and a violin, and the strengths of these overtones, are different. Hence, one can distinguish between the notes produced by a sitar and a violin even if they have the same frequency of vibration.

(d) Solids have shear modulus. They can sustain shearing stress. Since fluids do not have any definite shape, they yield to shearing stress. The propagation of a transverse wave is such that it produces shearing stress in a medium. The propagation of such a wave is possible only in solids, and not in gases.

Both solids and fluids have their respective bulk moduli. They can sustain compressive stress. Hence, longitudinal waves can propagate through solids and fluids.

(e) A pulse is actually is a combination of waves having different wavelengths. These waves travel in a dispersive medium with different velocities, depending on the nature of the medium. This results in the distortion of the shape of a wave pulse.

15.20 A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 m s^{-1}, (b) recedes from the platform with a speed of 10 m s^{-1}? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as 340 m s^{-1}.

Sol. (i) (a) Here air is still and listener is standing on the platform so $v_m = 0$, $v_0 = 0$

From formula, $\nu' = \left(\dfrac{v + v_m - v_0}{v + v_m - v_s}\right)\nu$; $\nu' = \left(\dfrac{v}{v - v_s}\right)\nu$

$\nu = 400$ Hz, $v_s = 10 \text{ ms}^{-1}$, $v = 340 \text{ ms}^{-1}$.

$\therefore \quad \nu' = \left(\dfrac{340}{340 - 10}\right) 400 = 412.12 \text{ Hz}$

(b) When the train recedes from the platform the frequency of the whistle

$$\nu' = \left(\frac{v}{v + v_s}\right)\nu = \left(\frac{340}{340 + 10}\right) 400 = 388.57 \text{ Hz}$$

(ii) The speed of sound in each case will be same.

15.21 A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with at a speed of 10 ms^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 m s^{-1}? The speed of sound in still air can be taken as 340 m s^{-1}.

Sol. $\nu = 400$ Hz, $v_m = 10$ m/s and $v = 340$ m/s

Since the wind is blowing in the direction of sound, therefore the effective speed of sound will be

$= v + v_m = 340 + 10 = 350$ m/s

As the source and observer both are at rest, therefore, frequency remains unchanged.

$$\therefore \quad \nu = 400 \text{ Hz and } \lambda = \frac{v + v_m}{\nu} = \frac{350}{340} = 0.875\text{m}$$

Now, when air is not blowing, $v_m = 0$

$\therefore \quad v_0 = 10$ m/s and $v_s = 0$

$\because \quad$ The source is at rest, wavelength does not change to $\lambda' = \lambda = 0.875$ m

and speed of sound $= v + v_m = 340 + 0 = 340$ m/s

The situation in the two cases given in the question is different.

15.22 A travelling harmonic wave on a string is described by

$y(x, t) = 7.5 \sin(0.0050x + 12t + \pi/4)$

(a) What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.

Sol. $$y = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

(a) At x = 1, t = 1

$$y = 7.5\sin\left(0.005 + 12 + \frac{\pi}{4}\right) = 7.5 \sin(0.005 + 12 + 0.785)$$

$= 7.5 \sin(12.56 + 0.23) = 7.5 \sin 0.23 \approx 1.7$ cm

Now, $v = dy/dt = 7.5 \times 12 \cos(0.0050x + 12t + \pi/4)$

For t = 1, x = 1

$v = 90 \cos(0.0050 + 12 + \pi/4) = 90 \cos 0.23 = 87.75$ cm s^{-1}

To calculate the velocity of propagation of the wave compare it with equation.

$$y = A\sin\frac{2\pi}{\lambda}(x - vt + \phi)$$

$$\frac{2\pi}{\lambda} = 0.005 \quad \text{or} \quad \lambda = \frac{2\pi}{0.005} = 12.60\text{m}$$

$$\text{and} \quad \frac{2\pi}{\lambda}.V = -12 \quad \text{or} \quad V = \frac{-12}{(2\pi/\lambda)} = \frac{-12}{0.005} = -24 \text{ m/s}$$

(b) Now, all the points which an at a distance of $\pm \lambda$, $\pm 2\lambda$, $\pm 3\lambda$ etc. from x = 1 will have same displacement and velocity where λ is the wavelength of wave (12.60m)

15.23 A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20s), is the frequency of the note produced by the whistle equal to
1/20 or 0.05 Hz ?

Sol. A pulse has a definite speed of propagation but it does not have a definite wavelength and frequency. The frequency of the note produced by the whistle is not 0.05 Hz. The frequency of the note will depend upon the length of the whistle and various other factors.

15.24 One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

Sol. Here, T = 90 × 9.8 N = 882 N

$m = 8.0 \times 10^{-3}$ kg m^{-1} , $\nu = 256$Hz, $a = 5.0 \times 10^{-2}$ m

Velocity of transverse wave

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{882}{8 \times 10^{-3}}} = 3.32 \times 10^2 \text{ m/s}$$

$$\omega = 2\pi\nu = 2 \times 3.14 \times 256 = 1.6 \times 10^2 \text{ rad s}^{-1}.$$

Now, $v = \frac{\omega}{k}$ or $k = \frac{\omega}{v} = \frac{1.6 \times 10^2}{3.32 \times 10^2} = 4.84$ rad/m

$\therefore$ Equation of wave, $y = a \sin(\omega t - kx)$

$y = 0.05 \sin(16.1 \times 10^2 t - 4.8x)$

15.25 A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 m s^{-1}.

Sol. Here source is the SONAR.

Frequency of it $\nu = 40$ KHz $= 40 \times 10^3$ Hz, $v_s = 0$

Energy submarine is moving toward SONAR

$\therefore$ $V_0 = -360$ km/h $= -360 \times (5/18) = -100$ m/s

$$\nu = \left(\frac{v + v_m - v_0}{v + m_m - v_s}\right) \nu$$

$$\nu' = \frac{1450 + 100}{1450} \times 40 \times 10^3 = 42.8 \times 10^3 \text{ Hz} \approx 42.8 \text{ kHz}$$

For reflected sound submarine becomes source and SONAR is listener (observer)

$$\nu'' = \frac{1450}{1450 - 100} \times 42.8 = 45.9 \text{ kHz}$$

15.26 Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Sol. Let L be the distance and t_1 be the time taken by the transverse wave to each seismograph recorder

$$t_1 = \frac{L}{v_1} = \frac{L}{4 \text{ kms}^{-1}}$$

Let t_2 be the time taken by the longitudinal wave

$$\therefore \quad t_2 = \frac{L}{v_2} = \frac{L}{8 \text{ kms}^{-1}}$$

Given t = difference between the two recordings

$$60 \times 4 = L\left(\frac{1}{4} - \frac{1}{8}\right)$$

or $L = 4 \times 60 \times 8 = 1920$ km.

15.27 A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Sol. Here, $\nu = 40$ KHz, $v_{air} = 340$ m/s, $v_{bat} = 0.03\ v_{air}$

When the bat moves towards stationary wall, apparent frequency at the observer

$$\nu' = \frac{\nu \cdot v_{air}}{v_{air} - v_{bat}} = \frac{40\ \text{KHz} \times 340}{340 - 0.03 \times 340} = 41.23\ \text{KHz}$$

This frequency acts as source and the moving bat acts as observer

$$\nu'' = \nu'\left[\frac{v_{air} + v_{bat}}{v_{air}}\right] = 41.23\left[\frac{340 + 0.03 \times 340}{340}\right] = 42.47\ \text{kHz}$$