

NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 2
UNITS AND MEASUREMENTS

2.1 Fill in the blanks –

- (a) The volume of a cube of side 1 cm is equal tom³.
 (b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to (mm)².
 (c) A vehicle moving with a speed of 18 km h⁻¹ covers....m in 1s.
 (d) The relative density of lead is 11.3. Its density isg cm⁻³ orkg m⁻³.

Sol. (a) Length of side, L = 1cm = 10⁻² m

$$\therefore \text{Volume of cube} = (10^{-2}\text{m})^3 = 10^{-6} \text{ m}^3$$

(b) Here r = 2.0cm = 20mm, h = 10.0cm = 100mm

$$\text{Surface area of solid cylinder} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 20 \times 100 = 1.26 \times 10^4 \text{ mm}^2.$$

(c) Here, speed V = 18 km hr⁻¹ = 18 × $\frac{5}{18}$ = 5 ms⁻¹

$$\therefore \text{Distance travelled in 1 second} = 5\text{m}.$$

(d) Here, R.D. = 11.3

$$\therefore \text{Density} = 11.3 \text{ g cm}^{-3} = 11.3 \times 10^3 \text{ kg m}^{-3}.$$

2.2 Fill in the blanks by suitable conversion of units

- (a) 1 kg m² s⁻² =g cm² s⁻²
 (b) 1 m = ly
 (c) 3.0 m s⁻² = km h⁻²
 (d) G = 6.67 × 10⁻¹¹ N m² (kg)⁻² = (cm)³ s⁻² g⁻¹.

Sol. (a) 10⁷; (b) 10⁻¹⁶; (c) 3.9 × 10⁴; (d) 6.67 × 10⁻⁸.

2.3 A calorie is a unit of heat or energy and it equals about 4.2J where 1J = 1 kg m² s⁻². Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γs. Show that a calorie has a magnitude 4.2 α⁻¹ β⁻² γ² in terms of the new units.

Sol. 1 cal = 4.2 kg m² s⁻².

SI	New system
n ₁ = 4.2	n ₂ = ?
M ₁ = 1 kg	M ₂ = α kg
L ₁ = 1m	L ₂ = β metre
T ₁ = 1s	T ₂ = γ second

Dimensional formula of energy is [ML²T⁻²]

Comparing with [M^aL^bT^c], we find that a = 1, b = 2, c = - 2

$$\text{Now, } n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c = 4.2 \left[\frac{1 \text{ kg}}{\alpha \text{ kg}} \right]^1 \left[\frac{1\text{m}}{\beta\text{m}} \right]^2 \left[\frac{1\text{s}}{\gamma\text{s}} \right]^{-2} = 4.2 \alpha^{-1} \beta^{-2} \gamma^2$$

2.4 Explain this statement clearly:

- (i) “To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison”.

(ii) In view of this, reframe the following statements wherever necessary :

- (a) atoms are very small objects
- (b) a jet plane moves with great speed
- (c) the mass of Jupiter is very large
- (d) the air inside this room contains a large number of molecules
- (e) a proton is much more massive than an electron
- (f) the speed of sound is much smaller than the speed of light.

Sol. (i) Statement is true. A dimensionless quantity is large or small only when it is compared to some standard.

For example, R.D. of mercury is 13.6 and R.D. of ice is 0.9. The R.D. of water is 1.0 which is more than R.D. of ice but less than R.D. of mercury.

- (ii) (a) Size of an atom is smaller than the thickness of hair.
(b) The jet plane moves faster than a superfast train.
(c) The mass of Jupiter is very large compared to the mass of the earth.
(d) The number of air molecules in a room is much larger than the number of air molecule in one mole of air.
(e) and (f) are already correct.

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Sol. Distance between Sun and Earth = Speed of light in vacuum \times time taken by light to travel from Sun to Earth = $3 \times 10^8 \text{ ms}^{-1} \times 8 \text{ min } 20\text{s} = 3 \times 10^8 \text{ ms}^{-1} \times 500\text{s} = 500 \times 3 \times 10^8 \text{ m}$

In the new system, the speed of light in vacuum is unity.

So, the new unit of length is $3 \times 10^8 \text{ m}$

\therefore distance between Sun and earth = 500 new units.

2.6 Which of the following is the most precise device for measuring length ? (i) A Vernier callipers with 20 divisions on the sliding scale, coinciding with 19 main scale divisions (ii) a screw gauge of pitch 1 mm and 100 divisions on the circular scale (iii) an optical instrument that can measure length to within a wave length of light.

Sol. (i) L.C. of Vernier callipers = 1 SD – 1 VD
 $= 1 \text{ SD} - \frac{19}{20} \text{ SD} = \frac{1}{20} \text{ SD}$
 $= \frac{1}{20} \text{ mm} = 0.005 \text{ cm}.$

(ii) L.C. of Screw gauge = $\frac{\text{pitch}}{\text{no. of divisions on C.S.}} = \frac{1}{100} \text{ mm} = 0.001 \text{ cm}$

(c) Wavelength of light = $10^{-5} \text{ cm} = 0.00001 \text{ cm}.$

The least count is least for optical instrument so it is most precise device for measuring length.

2.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?

Sol. Thickness of hair = $\frac{\text{observed width}}{\text{magnification}} = \frac{3.5}{100} = 0.035 \text{ mm}$

- 2.8** (a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?
(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

- Sol.** (a) It is done by winding a known number of turns over a pencil, turns touching each other closely. Then, the length occupied by each single turn will be equal to the diameter of the thread.
 (b) It is not possible to increase the accuracy of a screw gauge by increasing the number of divisions of the circular scale. Increasing the number divisions of the circular scale will increase its accuracy to a certain extent only.
 (c) A set of 100 measurements is more reliable than a set of 5 measurements because random errors involved in the former are very less as compared to the latter.

2.9 The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2 . What is the linear magnification of the projector-screen arrangement.

Sol. Area of the house on the slide = 1.75 cm^2

Area of the image of the house formed on the screen = $1.55 \text{ m}^2 = 1.55 \times 10^4 \text{ cm}^2$

$$\text{Areal magnification, } m_a = \frac{\text{Area of image}}{\text{Area of object}} = \frac{1.55}{1.75} \times 10^4$$

$$\text{Linear magnifications, } m_l = \sqrt{m_a} = \sqrt{\frac{1.55}{1.75} \times 10^4} = 94.11$$

2.10 State the number of significant figures in the following:

- (a) 0.007 m^2 (b) $2.64 \times 10^{24} \text{ kg}$ (c) 0.2370 g cm^{-3}
 (d) 6.320 J (e) 6.032 N m^{-2} (f) 0.0006032 m^2

Sol. (a) Answer: 1

The given quantity is 0.007 m^2 .

If the number is less than one, then all zeros on the right of the decimal point (but left to the first non-zero) are insignificant. This means that here, two zeros after the decimal are not significant. Hence, only 7 is a significant figure in this quantity.

(b) Answer: 3

The given quantity is $2.64 \times 10^{24} \text{ kg}$.

Here, the power of 10 is irrelevant for the determination of significant figures. Hence, all digits i.e., 2, 6 and 4 are significant figures.

(c) Answer: 4

The given quantity is 0.2370 g cm^{-3} .

For a number with decimals, the trailing zeroes are significant. Hence, besides digits 2, 3 and 7, 0 that appears after the decimal point is also a significant figure.

(d) Answer: 4

The given quantity is 6.320 J .

For a number with decimals, the trailing zeroes are significant. Hence, all four digits appearing in the given quantity are significant figures.

(e) Answer: 4

The given quantity is 6.032 Nm^{-2} .

All zeroes between two non-zero digits are always significant.

(f) Answer: 4

The given quantity is 0.0006032 m^2 .

If the number is less than one, then the zeroes on the right of the decimal point (but left to the first non-zero) are insignificant. Hence, all three zeroes appearing before 6 are not significant figures. All zeros between two non-zero digits are always significant. Hence, the remaining four digits are significant figures.

2.11 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Sol. Area of the sheet = $2 (\ell \times b + b \times t + t \times \ell)$

$$= 2 [4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234] \text{ m}^2$$

$$= 2 [4.255 + 0.0202 + 0.0851] \text{ m}^2$$

$$= 2 \times 4.3603\text{m}^2 = 8.7206\text{m}^2 = 8.72\text{m}^2$$

$$\text{Volume} = \ell bt = 4.234 \times 1.005 \times 0.0201\text{m}^3 = 0.0855\text{m}^3$$

- 2.12** The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

Sol. Mass of grocer's box = 2.300 kg

Mass of gold piece I = 20.15g = 0.02015 kg

Mass of gold piece II = 20.17 g = 0.02017 kg

(a) Total mass of the box = 2.3 + 0.02015 + 0.02017 = 2.34032 kg

In addition, the final result should retain as many decimal places as there are in the number with the least decimal places. Hence, the total mass of the box is 2.3kg.

(b) Difference in masses = 20.17 – 20.15 = 0.02 g

In subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

- 2.13** A physical quantity P is related to four observables a, b, c and d as follows : $P = a^3b^2 / (\sqrt{cd})$.

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result ?

Sol. $P = a^3b^2 / (\sqrt{cd})$; $\frac{\Delta P}{P} = 3 \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + \frac{\Delta d}{d}$

But $\frac{\Delta a}{a} = \frac{1}{100}$, $\frac{\Delta b}{b} = \frac{3}{100}$, $\frac{\Delta c}{c} = \frac{4}{100}$, $\frac{\Delta d}{d} = \frac{2}{100}$

$\therefore \frac{\Delta P}{P} = 3 \times \frac{1}{100} + 2 \times \frac{3}{100} + \frac{1}{2} \times \frac{4}{100} + \frac{2}{100}$

% error in P = 3% + 6% + 2% + 2% = 13%

3.763 should be rounded off to 3.8.

- 2.14** A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:

(a) $y = a \sin 2\pi t/T$ (b) $y = a \sin vt$ (c) $y = (a/T) \sin (t/a)$ (d) $y = \frac{a}{\sqrt{2}} \left[\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T} \right]$

(Here, a = max. displacement of the particle, v = speed of the particle. T = time-period of motion).

Rule out the wrong formulas on dimensional grounds.

Sol. The argument of a trigonometrical function is dimensionless.

(a) Here, argument of sine is $\frac{2\pi t}{T}$.

Dimensions of $\frac{2\pi t}{T} = \frac{[T]}{[T]} = [M^0L^0T^0] = \text{dimensionless}$.

(b) Here argument of sin is vt.

Dimension of vt = $[LT^{-1}][T] = [L] = \text{not dimensionless}$.

(c) Here argument of sine = t/a

Dimensions of $\frac{t}{a} = \frac{[T]}{[L]} = [TL^{-1}] = \text{not dimensionless}$.

(d) As above in case (a) argument of sine and cosine both are dimensionless.

\therefore Formula (b) and (c) are wrong.

- 2.15** A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity

due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c . He writes : $m = \frac{m_0}{(1-v^2)^{1/2}}$. Guess where to put the missing c .

Sol. From the given equation, $\frac{m_0}{m} = \sqrt{1-v^2}$

Since left hand side is dimensionless therefore right hand side should be also dimensionless.

So, $\sqrt{1-v^2}$ should be $\sqrt{1-\frac{v^2}{c^2}}$. The correct formula is $m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

2.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: $1 \text{ Å} = 10^{-10} \text{ m}$. The size of a hydrogen atom is about 0.5 Å . What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Sol. Volume of one hydrogen atom = $\frac{4}{3} \pi r^3 = \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3 \text{ m}^3 = 5.23 \times 10^{-31} \text{ m}^3$

According to Avogadro's hypothesis, one mole of hydrogen contains 6.023×10^{23} atoms.

\therefore Atomic volume of 1 mole of hydrogen atoms = $6.023 \times 10^{23} \times 5.23 \times 10^{-31} \text{ m}^3 = 3.15 \times 10^{-7} \text{ m}^3$

2.17 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?

Sol. Atomic volume = $\frac{4}{3} \pi r^3 \times N = \frac{4}{3} \times \frac{22}{7} (0.5 \times 10^{-10})^3 \times 6.023 \times 10^{23} = 3.154 \times 10^{-7} \text{ m}^3$

Molar volume = 22.4 litre = $22.4 \times 10^3 \text{ m}^3$

$\therefore \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{3.154 \times 10^{-7}} = 7.1 \times 10^4$.

2.18 Explain this common observation clearly : If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

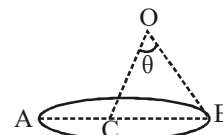
Sol. When a train moves rapidly, the line of sight changes its direction rapidly. On the other hand, in the case of far-off objects, the line of sight changes its direction extremely slowly.

2.19 The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11} \text{ m}$. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of $1''$ (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of $1''$ (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

Sol. $AB = 3 \times 10^{11} \text{ m}$

$CB = \frac{AB}{2} = 1.5 \times 10^{11} \text{ m}$

Parallax angle $\theta = 1'' = \frac{1}{60'} = \left(\frac{1}{60} \times \frac{1}{60}\right)^\circ = \left(\frac{1}{3600}\right)^\circ = \frac{\pi}{180} \times \left(\frac{1}{3600}\right) \text{ rad}$



Now angle = $\frac{\text{Arc}}{\text{radius}}$; Radius $OC = \frac{\text{Arc}}{\text{angle}} = \frac{BC}{\theta \text{ (in radian)}}$

$= \frac{1.5 \times 10^{11} \times 180 \times 3600}{\pi} = 3.1 \times 10^{16} \text{ m}$

2.20 The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun ?

Sol. Distance = 4.29 light years = $(4.29 \times 365.25 \times 86400) \times 3 \times 10^8 \text{m}$

$$= \frac{4.29 \times 365.25 \times 864 \times 3 \times 10^{10}}{3.08 \times 10^{16}} \text{par sec} \quad [∵ 1 \text{ par sec} = 3.08 \times 10^{16} \text{m}]$$

$$= 1318656.9 \times 10^{-6} \text{parsec} = 1.319 \text{parsec.}$$

Required parallax = 2θ , where θ is the annual parallax.

Since parsec is the distance corresponding to an annual parallax of one second of arc therefore θ is 1.319 second of arc.

\therefore Required parallax = 2×1.319 second of arc = 2.638 second of arc.

2.21 Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Sol. It is indeed very true that precise measurements of physical quantities are essential for the development of science. For example, ultra-shot laser pulses (time interval = 10^{-15}s) are used to measure time intervals in several physical and chemical processes.

X-ray spectroscopy is used to determine the inter-atomic separation or inter-planer spacing.

The development of mass spectrometer makes it possible to measure the mass of atoms precisely.

2.22 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

- the total mass of rain-bearing clouds over India during the Monsoon
- the mass of an elephant
- the wind speed during a storm
- the number of strands of hair on your head
- the number of air molecules in your classroom.

Sol. (a) Meteorologist has recorded about 100cm of rainfall.

i.e., $h = 100 \text{ cm} = 1 \text{ m}$

Area of India $A = 3.3$ million square km. = $3.3 \times 10^6 (10^3)^2 = 3.3 \times 10^{12} \text{m}^2$

\therefore Volume of rainfall = $A \cdot h = 3.3 \times 10^{12} \times 1 = 3.3 \times 10^{12} \text{m}^3$

Mass of rain water = Volume \times density = $3.3 \times 10^{12} \times 10^3 = 3.3 \times 10^{15} \text{kg}$

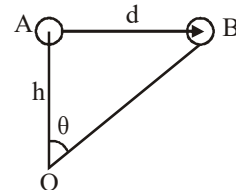
i.e., the total mass of rain bearing clouds over India during the monsoon is $3.3 \times 10^{15} \text{kg}$.

(b) Move the elephant into a boat whose base area is known, and measure the depth of boat with elephant in water. Knowing the depth of boat one can calculate volume of water displaced, by boat and elephant. From this volume subtract the volume of water displaced by empty boat. It gives the volume of water displaced by elephant. It gives the volume of water displaced by the elephant, which on being multiplied by density of water gives mass of the elephant.

(c) The wind speed can be estimated by floating a gas filled balloon in air at a known height h . $OA = h$.

As the wind blows to the right, the balloon drifts to the right to position B in one sec. So that $AB = d$ and $\angle AOB = \theta$ from figure, $\theta = d/h$ or $d = \theta \cdot h$

This is the distance travelled by the balloon in one second, i.e., speed of wind.



(d) Assuming the area of a human head to be A and radius of human hair to be r . Then area of cross section of hair = πr^2 . If distribution of hair over the head is uniform then number of

$$\text{strands of hair} = \frac{\text{Total area}}{\text{Area of C.S. each hair}} = \frac{A}{\pi r^2}$$

Consider average human head to be a circle of radius 8cm. and thickness of hair is 5×10^{-5} m.

$$\text{The no. of strands of hair} = \frac{\pi (0.08)^2}{\pi (5 \times 10^{-5})^2} = 2.5 \times 10^8$$

(e) We know 1 mole of air at NTP occupies 22.4 lt. ($22.4 \times 10^{-3} \text{ m}^3$) and occupies 6.023×10^{23} molecules.

$$\text{If volume of room is } V \text{ then no. of molecules in the room} = \left(\frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} \times V \right).$$

2.23 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data : Mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

Sol. Here, mass of Sun $M = 2.0 \times 10^{30}$ kg.

Radius of Sun $R = 7.0 \times 10^8$ m

$$\therefore \text{Density} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2.0 \times 10^{30} \times 3 \times 7}{4 \times 22 (7.0 \times 10^8)^3} \approx 1.4 \times 10^3 \text{ kg m}^{-3} \text{ s.}$$

The high density of Sun is due to inward gravitational attraction an outer layers due to the inner layers.

2.24 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72'' of arc. Calculate the diameter of Jupiter.

Sol. Angular diameter, $\theta = \frac{\text{diameter } d \text{ of planet}}{\text{distance } r \text{ of planet from the earth}}$

$$35.72 \text{ second of arc} = \frac{d}{824.7 \times 10^6 \text{ km}}$$

$$\text{or } 35.72 \times 4.85 \times 10^{-6} \text{ radian} = \frac{d}{824.7 \times 10^6 \text{ km}}$$

$$\text{or } d = 35.72 \times 4.85 \times 10^{-6} \times 824.7 \times 10^6 \text{ km} = 1.429 \times 10^5 \text{ km}$$

2.25 A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : $\tan \theta = v$ and checks that the relation has a correct limit: as $v \rightarrow 0$, $\theta \rightarrow 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Sol. No, $\tan \theta = \frac{v}{v'}$, where v' is the speed of the rainfall. $\tan \theta$ must be dimensionless.

2.26 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1s?

Sol. 100 years = 3.155×10^9 s

$$\text{Error in 1 second} = \frac{0.02}{3.155 \times 10^9} \text{ s} = 6.34 \times 10^{-12} \text{ s}$$

2.27 Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m⁻³. Are the two densities of the same order of magnitude? If so, why?

Sol. Volume of sodium atoms in 1 mole = $\left(\frac{4}{3}\pi r^3 N\right) = \frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3 (6.023 \times 10^{23})$
 $= 4.93 \times 10^{-6} \text{ m}^3.$

Average mass density of a sodium atom = $\frac{\text{mass}}{\text{volume}} = \frac{23 \times 10^{-3}}{4.93 \times 10^{-6}} = 4.67 \times 10^3 \text{ kg m}^3$

Density of solid in crystalline phase = 970 kg m³ = 0.97 × 10³ kg m⁻³.

2.28 The unit of length convenient on the nuclear scale is a Fermi: 1 f = 10⁻¹⁵ m. Nuclear sizes obey roughly the following empirical relation: $r = r_0 A^{1/3}$. Where r is the radius of the nucleus, A its mass number, and r₀ is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom.

Sol. Let A is mass number

∴ Mass of the nucleus = A × m_p

Now, density = $\frac{\text{mass}}{\text{volume}} = \frac{A \times m_p}{\frac{4}{3}\pi r^3}$

But $r = r_0 A^{1/3}$. ∴ $\rho = \frac{3A \times m_p}{4\pi r_0^3 A} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3} \approx 2.3 \times 10^{17} \text{ kg m}^{-3}.$

2.29 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances.

The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Sol. Here, t = 2.56s

Velocity of laser light in vacuum = 3 × 10⁸ m/s.

The radius of lunar orbit is the distance of moon from the earth. Let it is d.

∴ Laser beam travels a distance 2d.

(d in the forward journey and d in return journey).

∴ 2d = v × t

or $d = \frac{vt}{2} = \frac{3 \times 10^8 \times 2.56}{2} = 3.84 \times 10^8 \text{ m}$

2.30 A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s⁻¹).

Sol. If x be the distance of the enemy submarine, then the total distance to be covered by sound waves in water is 2x = velocity of sound waves × time or 2x = 1450 ms⁻¹ × 77s

or $x = \frac{1450 \times 77}{2} \text{ m} = 55825 \text{ m} = 55.825 \text{ km}$

2.31 The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us ?

Sol. Here Time taken $t = 3.0$ billion years

$$t = 3 \times 10^9 \text{ years} = 3 \times 10^9 \times 365.25 \times 24 \times 60 \times 60 \text{ sec}$$

$$\therefore \text{Distance of the quasar} = v \times t$$

$$= 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 24 \times 60 \times 60 = 2.84 \times 10^{25} \text{ m} = 2.84 \times 10^{22} \text{ km}$$

2.32 It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. Determine the approximate diameter of the moon.

Sol. Distance of moon from earth, $ME = 3.84 \times 10^8 \text{ m}$

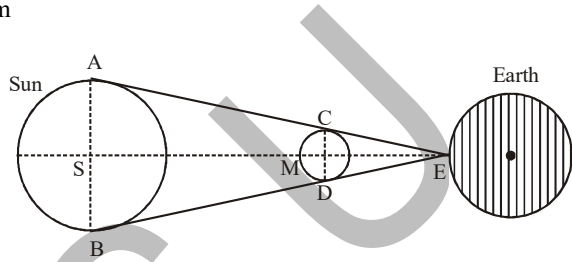
Distance of earth from Sun, $SE = 1.39 \times 10^{11} \text{ m}$

Distance of Sun $AB = 1.39 \times 10^9 \text{ m}$

$$\text{From figure, } CD = AE \times \frac{AE}{AB} = \frac{CE}{CD}$$

$$= 3.84 \times 10^8 \times \frac{1.39 \times 10^9}{1.496 \times 10^{11}}$$

$$= 3.5679 \times 10^6 \text{ m} \approx 3568 \text{ km}$$



2.33 A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c , e , mass of electron, mass of proton) and the gravitational constant G , he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~ 15 billion years). From the table of fundamental constants, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

Sol. A quantity which has the dimensions of time. It is

$$t = \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \times \frac{1}{m_p m_e^2 C^3 G}, \text{ where } e = 1.6 \times 10^{-19} \text{ C}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ NM}^2\text{C}^2$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}, m_e = 9.1 \times 10^{-31} \text{ kg}, C = 3 \times 10^8 \text{ ms}^{-1}, G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$\therefore t = (1.6 \times 10^{-19} \text{ C})^4 \times (9.1 \times 10^9)^2 \frac{1}{(1.67 \times 10^{-27}) (9 \times 10^{-31})^2 (3 \times 10^8)^3 \times 6.67 \times 10^{-11}}$$

$$= 2.18 \times 10^{16} \text{ s}$$