NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 4
MOTION IN A PLANE

4.1 State, for each of the following physical quantities, if it is a scalar or a vector:
volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Sol. Scalar: Volume, mass, speed, density, number of moles, angular frequency.
Vector: Acceleration, velocity, displacement, angular velocity.
A scalar quantity is specified by its magnitude only. It does not require any direction to specify them. Volume, mass, speed, density, number of moles, and angular frequency are some of the scalar physical quantities.
A vector quantity is specified by its magnitude as well as the direction associated with it. Acceleration, velocity, displacement, and angular velocity belong to this category.

4.2 Pick out the two scalar quantities in the following list:
force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Sol. Work and current are scalar quantities.
Work done is given by the dot product of force and displacement. Since the dot product of two quantities is always a scalar, work is a scalar physical quantity.
Current is described only by its magnitude. Its direction is not taken into account. Hence, it is a scalar quantity.

4.3 Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Sol. Impulse is given by the product of force and time. Since force is a vector quantity, its product with time (a scalar quantity) gives a vector quantity.

4.4 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: (a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions , (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Sol.
(a) No, because only two or more scalars of same dimensions can be added.
(b) No, because a vector cannot be added to a scalar.
(c) Yes, when velocity is multiplied by mass m, we get momentum ().
(d) Yes, when volume is multiplied by density, we get mass.
(e) No, because two vectors of same dimension can be added.
(f) No, because component of a vector will be scalar and addition of a vector to a scalar is not permissible.

4.5 Read each statement below carefully and state with reasons, if it is true or false:
(a) The magnitude of a vector is always a scalar,
(b) each component of a vector is always a scalar,
(c) the total path length is always equal to the magnitude of the displacement vector of a particle.
(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,
(e) Three vectors not lying in a plane can never add up to give a null vector.

Sol.
(a) True – The magnitude of a vector is a number. Hence, it is a scalar.

(b) False – Each component of a vector is also a vector.

(c) False – Total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

(d) True – It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

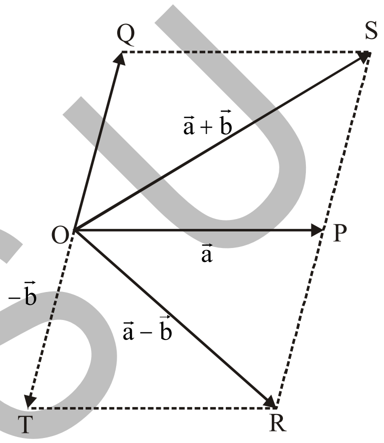
(e) True – Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

4.6 Establish the following vector inequalities geometrically or otherwise:

(a) $|a + b| \le |a| + |b|$ (b) $|a + b| \ge ||a| - |b||$

(c) $|a - b| \le |a| + |b|$ (d) $|a - b| \ge ||a| - |b||$

Sol. Consider two $\vec{a}$ and $\vec{b}$, represented by the two sides of a parallelogram, $\vec{a}+\vec{b}$ is the result which is given by diagonal,



$OP = |\vec{a}| = a$, $OQ = |\vec{b}| = b$, $OS = |\vec{a}+\vec{b}|$

(a) The length of one side of a triangle is always less than the sum of the rest two sides.
Therefore, OS < OP + PS
or OS < OP + OQ
or $|\vec{a}+\vec{b}| < |\vec{a}| + |\vec{b}|$
In case $\vec{a}$ and $\vec{b}$, are acting along a straight line then
$$|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$$
$$|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$$

(b) In Δ OPS, OS + PS > OP
or OS > | OP – PS | > | OP – OQ |
or $|\vec{a}+\vec{b}| > ||\vec{a}| - |\vec{b}||$
In case $\vec{a}$ and $\vec{b}$, are acting along a straight line in opposite direction then
$$|\vec{a}+\vec{b}| = ||\vec{a}| - |\vec{b}||$$
$\therefore$ $|\vec{a}+\vec{b}| \ge ||\vec{a}| - |\vec{b}||$

(c) In figure $\vec{a}-\vec{b} = \overrightarrow{OR}$
$$\overrightarrow{OR} < \overrightarrow{OP} + \overrightarrow{PR}$$
or $|\vec{a}-\vec{b}| < |\vec{a}| + |\vec{b}| < |\vec{a}| + |\vec{b}|$
In case two vectors are acting along a straight line
$$|\vec{a}-\vec{b}| = |\vec{a}| + |\vec{b}|$$
$$|\vec{a}-\vec{b}| \le |\vec{a}| + |\vec{b}|$$

(d) Again in Δ OPR,
OR + PR > OP
OR > | OP – PR | > | OP – OT |
$$|\vec{a}-\vec{b}| > ||\vec{a}| - |\vec{b}||$$
In case two vectors are acting along same straight line then
$$|\vec{a}-\vec{b}| = ||\vec{a}| - |\vec{b}||$$
$\therefore$ $|\vec{a}-\vec{b}| \ge ||\vec{a}| - |\vec{b}||$

4.7 Given $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$, which of the following statements are correct:

(a) $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ must each be a null vector,

(b) The magnitude of $(\vec{a}+\vec{c})$ equals the magnitude of $(\vec{b}+\vec{d})$,

(c) The magnitude of a can never be greater than the sum of the magnitudes of $\vec{b}, \vec{c}$ and $\vec{d}$

(d) $(\vec{b}+\vec{c})$ must lie in the plane of if are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear?

Sol. (a) False

(b) True $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$(\vec{a}+\vec{c})=-(\vec{b}+\vec{d})$

$|\vec{a}+\vec{c}|=|\vec{b}+\vec{d}|$

(c) True $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$\vec{a}=-(\vec{b}+\vec{c}+\vec{d})$

or $|\vec{b}+\vec{c}+\vec{d}|\le|\vec{b}|+|\vec{c}|+|\vec{d}|$

Hence, $|\vec{a}|\le|\vec{b}|+|\vec{c}|+|\vec{d}|$

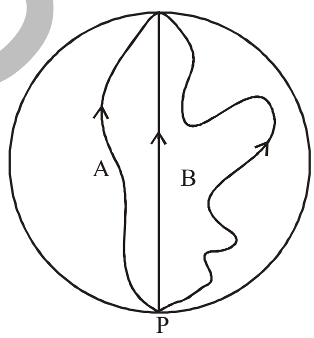
(d) True $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$(\vec{b}+\vec{c})=-(\vec{a}+\vec{d})$

$|\vec{b}+\vec{c}|=|\vec{a}+\vec{d}|$

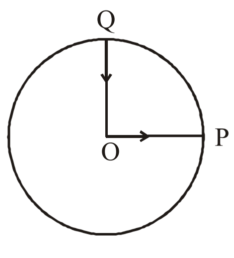
Thus, $|\vec{b}+\vec{c}|$ must be in line with $|\vec{a}+\vec{d}|$ in opposite direction. Hence line of action of $\vec{a}$ as well as $\vec{d}$ must lie in the plane containing resultant of $\vec{b}$ and $\vec{c}$.

4.8 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?



Sol. The magnitude of the displacement vector for all of them is same, which is 400m, for girl B, the magnitude of the displacement vector equals to actual length of the path.

4.9 A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



Sol. (a) Net displacement = zero

(b) Average velocity = $\dfrac{\text{Net displacement}}{\text{time taken}} = \dfrac{\text{zero}}{10/60} = 0$ km/hr

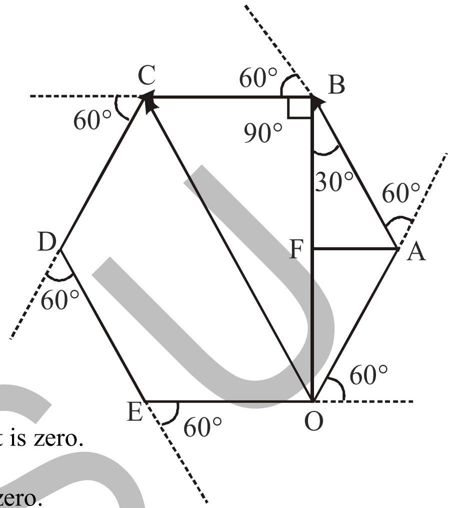
(c) Total path covered = $1 + 1 + \dfrac{\pi R}{2} = 1 + 1 + 1.57 = 3.57$km

$\therefore\ v_{ax} = \dfrac{3.57}{10/60} = 3.57\times 6 = 21.43$ km/hr.

4.10 On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Sol. (i) The path followed by the motorist will be a closed hexagonal path.

Suppose the motorist starts his journey from the point O. He takes the third turn at the point C.



Displacement = $\overrightarrow{OC}$

Clearly, $OC = \sqrt{OB^2 + BC^2} = \sqrt{(OF + FB)^2 + BC^2}$

$= \sqrt{(500\cos 30° + 500\cos 30°)^2 + 500^2}$

$= \sqrt{\left(2 \times 500 \times \frac{\sqrt{3}}{2}\right)^2 + 500^2}$

$= 500\sqrt{4}\text{m} = 1000\text{m} = 1\text{ km}$

Total path length = 500m + 500m + 500m
= 1500m = 1.5 km.

$$\frac{\text{Magntidue of displacement}}{\text{Total path length}} = \frac{1\text{ km}}{1.5\text{ km}} = \frac{2}{3} = 0.67$$

(ii) The motorist will take the sixth turn at O. Displacement is zero.
Path length is 3000m, i.e., 3 km.
Ratio of magnitude of displacement and path length is zero.

(iii) The motorist will take the 8th turn at B.
Magnitude of displacement = 2 × 500 cos 30° = $500\sqrt{3}\text{m} = \frac{\sqrt{3}}{2}\text{km}$
Path length = 8 × 500m = 4 km.

Ratio of magnitude of displacement and path length is $\frac{\sqrt{3}/2}{4}$ i.e., $\frac{\sqrt{3}}{8}$, i.e., 0.22.

4.11 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

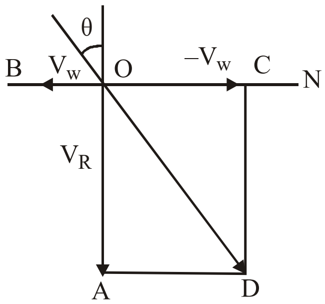
Sol. The average speed of the taxi = $\frac{\text{Actual distance travelled}}{\text{time taken}} = \frac{23\text{ km}}{28\text{ min}} = \frac{23\text{ km}}{28/60\text{h}} = 49.3\text{ km/h}$

Average velocity = $\frac{\text{displacement}}{\text{time}} = \frac{10}{28/60\text{h}} = 21.4\text{ km/h}$

The average speed is not equal to the magnitude of average velocity. The two are equal only when body moves in a straight line.

4.12 Rain is falling vertically with a speed of 30 m/s. A woman rides a bicycle with a speed of 10 m/s in the north to south direction. What is the direction in which she should hold her umbrella ?

Sol. Relative velocity of rain w.r.t. the woman, velocity of rain-velocity of the woman. In figure OA represents velocity of rain ($\overrightarrow{OA} = 30\text{m/s}$) and OB represents velocity of woman from north to south ($\overrightarrow{OB} = 10\text{m/s}$).



Relative velocity of rain w.r.t. woman = $V_R - V_w = V_R + (-V_w)$

Relative velocity of rain w.r.t. woman is represented by diagonal OD and it makes angle θ with vertical.

$$\tan\theta = \frac{AD}{OA} = \frac{10}{30} = \frac{1}{3} \text{ or } \theta = \tan^{-1}\left(\frac{1}{3}\right) = 18°26'$$

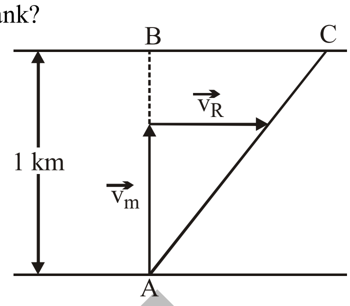
4.13 A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Sol. Time taken by the man to cross the river (AB = 1 km.)

$$t = \frac{AB}{v_m} = \frac{1}{4} = \frac{1}{4} h = 15 \text{ min}.$$

In 1/4 h the distance, through which the man goes down the river is

$BC = v_R \times t = 3$ km/h × = = 750m.



4.14 In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Sol. Relative velocity of wind w.r.t. boat (V_{wb}) = Velocity of wind (V_w) – Velocity of boat (V_b).

In figure OA represents velocity of wind V_w (m N-E direction) OB represents velocity of boat (V_b) OC represents ($-V_b$).

To find relative velocity of wind w.r.t. boat complete the parallelogram ODAC.

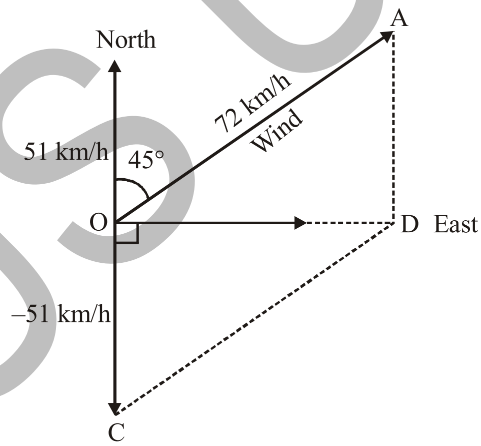
Diagonal OD represents V_{wb}. Let ∠ COD = β

$$\tan\beta = \frac{OA \sin 135°}{OC + OA \cos 135°} = \frac{72 (\sin 45°)}{51 + 72 (-\cos 45°)}$$

$$= \frac{72 (0.707)}{51 + 72 (-0.707)} = \infty$$

or β = 90° i.e., OD is along east.

Hence, flag will flutter almost due east.



4.15 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the hall ?

Sol. Maximum height h_{max} = 25 m, Horizontal range, R = ?, Velocity of projection, v = 40 m/s

We know that $h_{max} = \frac{v^2 \sin^2\theta}{2g}$

or $\sin^2\theta = \frac{25 \times 2 \times 9.8}{40 \times 40} = 0.30625$ or $\sin\theta = 0.5534$ or $\theta = \sin^{-1}(0.5534) = 33.6°$

Again, $R = \frac{v^2 \sin 2\theta}{g} = \frac{40 \times 40 \sin 67.2°}{9.8}$ or $R = \frac{1600}{9.8} \times 0.9219\text{m} = 150.5\text{m}$

4.16 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Sol. Maximum horizontal range = 100m

$\therefore \frac{v^2}{g} = 100$ .... (1)

We know that, $v(t)^2 - v(0)^2 = 2a [x(t) - x(0)]$

Now, $v(t) = 0, v(0) = v, x(t) - x(0) = h$ (say)

$\therefore 0^2 - v^2 = 2(-g)h$ or $h = \frac{1}{2} \times \frac{v^2}{g}$ or $h = \frac{1}{2} \times 100\text{m} = 50\text{m}$ [From eq. (1)]

4.17 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Sol. Here, r = 80cm = 0.8m., ν = 14/25 Hz

$$\omega = 2\pi\nu = 2\times\frac{22}{7}\times\frac{14}{25}=\frac{88}{25}\text{rad s}^{-1}$$

The centripetal acceleration, $a = r\omega^2 = \left(\frac{88}{25}\right)^2 \times 0.80 = 9.90 \text{ ms}^{-2}$

Direction of acceleration is towards the centre of circular path.

4.18 An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Sol. Here, r = 1 km = 1000 m, $v = 900 \text{ km/h} = 900 \times \frac{5}{18} = 250 \text{ m/s}$

$\therefore$ Centripetal acceleration, $a = \frac{v^2}{r} = \frac{250\times 250}{1000} = 62.5 \text{ m/s}$

Now, $\frac{\text{Centripetal acceleration}}{\text{Acceleratic due to gravity}} = \frac{62.5}{9.8} = 6.38$

4.19 Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Sol. (a) False, It is true only for uniform circular motion.

(b) True

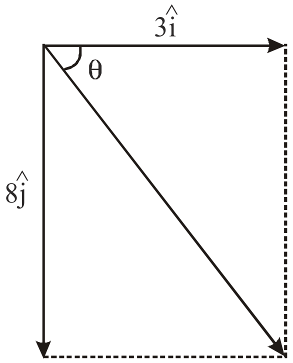
(c) True, In uniform circular motion the magnitude of the acceleration remains constant but direction changes continuously.

(d) True

4.20 The position of a particle is given by: $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}$ m, where t is in seconds and the coefficients have the proper units for to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?



Sol. (a) $\vec{v} = \frac{d}{dt}(\vec{r}) = 3\hat{i} - 4t\hat{j}$

$\vec{a} = \frac{d}{dt}(\vec{v}) = -4\hat{j}$

(b) $(\vec{v})_{t=2s} = 3\hat{i} - 8\hat{j}$

Magnitude of velocity

$= \sqrt{9+64} = \sqrt{73} \text{ m/s} = 8.544 \text{ m/s}$

$\tan\theta = \frac{8}{3} = 2.6667$ or $\theta = \tan^{-1}(2.6667) = 69.444°$

4.21 A particle starts from the origin at t = 0s with a velocity of $10.0\,\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})\ \text{ms}^{-2}$.

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

Sol. $\vec{u} = 10.0\hat{j},\ \vec{a} = 8.0\hat{i} + 2.0\hat{j},\ \vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\vec{r} = 10.0t\ \hat{j} + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})\ t^2$$

(a) x co-ordinate = $4.0\,t^2 = 16$ or $t = 2s$

y co-ordinate = $10.0 \times 2 + 1.0 \times 2 \times 2 = 24m$

(b) $\vec{v} = \frac{d}{dt}(\vec{r}) = 8t\hat{i} + (10.0 + 2t)\hat{j}$

At t = 2s, $\vec{v} = 16\hat{i} + 14\hat{j}$

$$v = \sqrt{16^2 + 14^2} = \sqrt{256 + 196} = \sqrt{452} = 21.26\ \text{ms}^{-1}$$

4.22 $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors, $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? [You may use graphical method]

Sol. (a) Magnitude of $(\hat{i} + \hat{j}) = |\hat{i} + \hat{j}| = \sqrt{2}$

Let $\hat{i} + \hat{j}$ makes an angle θ with the direction $\hat{i}$ then

$$\cos\theta = \frac{|(\hat{i} + \hat{j})|.\hat{j}}{|\hat{i} + \hat{j}||\hat{i}|} = \frac{1}{(\sqrt{2})(1)} = \frac{1}{\sqrt{2}} \text{ or } \theta = 45°$$

Magnitude of $\hat{i} - \hat{j} = \sqrt{2}$

If θ in the angle $(\hat{i} - \hat{j})$ which makes with the direction of $\hat{i}$ then

$$\cos\theta = \frac{(\hat{i} - \hat{j}).\hat{i}}{|\hat{i} - \hat{j}||\hat{i}|} = \frac{1}{\sqrt{2}} = \cos 45° \text{ or } \theta = 45°$$

Here angle θ = 45° is below x-axis i.e., θ = – 45°

(b) $\vec{A} = 2\hat{i} + 3\hat{j}$

To find to component $\vec{A}$ of along $(\hat{i} + \hat{j})$ we first find the unit vector along $(\hat{i} + \hat{j})$ say it is $\hat{a}$

Magnitude of the component of $\vec{A}$ along $(\hat{i} + \hat{j})$ = $\vec{A}.\vec{a} = (2\hat{i} + 3\hat{j}).\frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

Similarly component of $\vec{A}$ along $(\hat{i} - \hat{j})$ = $(2\hat{i} + 3\hat{j}).\frac{(\hat{i} - \hat{j})}{\sqrt{2}} = \left(\frac{2 - 3}{\sqrt{2}}\right) = \frac{-1}{\sqrt{2}}$

4.23 For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = \frac{1}{2}[\vec{v}(t_1) + \vec{v}(t_2)]$

(b) $v_{average} = \frac{[\vec{r}(t_2) - r(t_1)]}{t_2 - t_1}$

(c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$

(d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)\,t + \frac{1}{2}\vec{a}t^2$

(e) $a_{average} = \frac{[\vec{v}(t_2) - \vec{v}(t_1)]}{t_2 - t_1}$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Sol. (a) It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

(b) The arbitrary motion of the particle can be represented by this equation.

(c) The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

(d) The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.

(e) The arbitrary motion of the particle can be represented by this equation.

4.24 Read each statement below carefully and state, with reasons and examples, if it is true or false :

A scalar quantity is one that

(a) is conserved in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes.

Sol. (a) False – Energy is not conserved during inelastic collision.

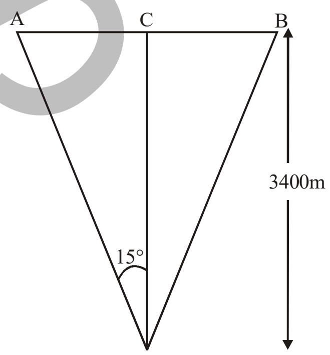
(b) False – Temperature can take negative value $\theta = -30°C$

(c) False – Speed has dimensions $[LT^{-1}]$

(d) False – Gravitational potential vary from point to point in space.

(e) True – Mass is independent of co-ordinate axes.

4.25 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0s apart is 30°, what is the speed of the aircraft?



Sol. In figure, O is observation point.

A and B are initial and final position C is the centre point.

$\therefore$ AC = OC tan θ = 3400 tan 15° = 910.86 m

$\therefore$ AB = 2 × 910.86 m

$$\text{Now speed} = \frac{\text{distance}}{\text{time}} = \frac{2 \times 910.86}{10} = 182.17 \text{ m/s}$$

4.26 A vector has magnitude and direction. Does it have a location in space ? Can it vary with time? Will two equal vectors at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Sol. (a) A vector in general has no definite location in space because a vector remains unaffected if it is displaced anywhere in space provided its magnitude and direction do not change. However, a position vector has a definite location in space.

(b) A vector can vary with time e.g., the velocity vector of an accelerated body varies with time.

(c) Two equal vectors at different locations in space do not necessarily have identical physical effects. e.g., two equal forces acting at two different points on a body which can cause the relation of a body about an axis will not produce equal turning effect.

4.27 A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?

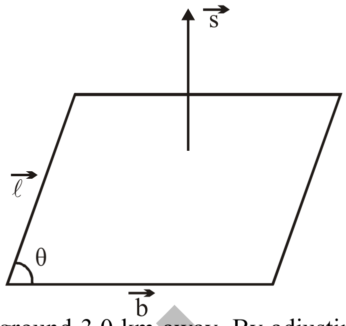
Sol. No, there are certain physical quantities which have both magnitude and direction, yet they are not vectors, and do not follow the law of vector addition. For example, rotation of a body through a finite angle about an axis, moment of inertia etc.

4.28 Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

Sol. (a) We can associate a vector with the length of a wire bent into a loop.

(b) We can associate a vector with a plane area called area vector.
The direction of area vector is always perpendicular to the plane

(c) The volume of a sphere, cannot be associated with a vector however a vector can be associated with the area of sphere.



4.29 A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away ? Assume the muzzle speed to the fixed, and neglect air resistance.

Sol. Angle of elevation, $\theta = 30°$,
Horizontal range, R = 3 km = 3000 m

We know that, $R = \dfrac{v^2 \sin 2\theta}{g}$ or $v^2 = \dfrac{Rg}{\sin 2\theta} = \dfrac{3000g}{\sin 60°} = \dfrac{6000g}{\sqrt{3}}$

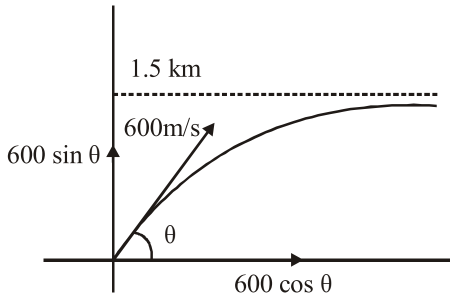
Let θ ' be the new angle of projection so that the range is 5km.

Now, $5000 = \dfrac{v^2 \sin 2\theta'}{g}$ or $5000 = \dfrac{6000g}{\sqrt{3}} \times \dfrac{\sin 2\theta'}{g}$ or $\dfrac{5\sqrt{3}}{6} = \sin 2\theta'$

Clearly, $\sin 2\theta' > 1$, This is not possible.
Thus, we can conclude that it is impossible to have a range of 5 km. merely by adjusting angle of projection and keeping the muzzle speed fixed.

4.30 A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed $600 ms^{-1}$ to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take $g = 10\ ms^{-2}$).

Sol. Here, Speed of plane 720 km/h = $720 \times \dfrac{5}{18} = 200$ m/s



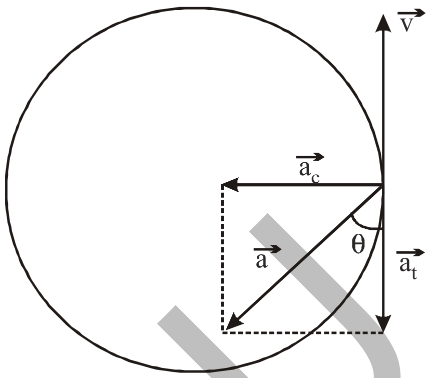
Let the gun be fired at an angle θ with horizontal, and t be the time in which the shell will hit the plane.
∴ Horizontal displacement of plane
= Horizontal displacement of shell
$200 \times 2t = 600 \cos\theta \times t$
or $\cos\theta = 1/3$ or $\theta = 70°30'$
i.e., the shell should be fired at an angle of
$(90° - 70°30' = 19°30')$ with the vertical.
Also maximum height of flight of the shell is

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g} = \frac{(600)^2(1-1/9)}{2\times 10} \simeq 16000\text{m} \simeq 16\text{ km}.$$

The pilot should fly the plane at a minimum altitude of 16km, to avoid being hit by the shell.

4.31 A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Sol. Speed, v = 27 km/h = $27 \times \frac{5}{18} ms^{-1} = 7.5 ms^{-1}$



Centripetal acceleration,

$a_c = v^2/r$ or $a_c = \frac{(7.5)^2}{80} ms^{-2} = 0.7 ms^{-2}$

P is the point at which cyclist applies brakes. At this point, tangential acceleration a_t, being negative, will act opposite to $\vec{v}$.

Total acceleration, $a = \sqrt{a_c^2 + a_t^2}$

$$a = \sqrt{(0.7)^2 + (0.5)^2} ms^{-2} = 0.86 ms^{-2}$$

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4 \; ; \; \theta = 54°28'$$