NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 6
WORK, ENERGY & POWER

6.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) work done by gravitational force in the above case,
(c) work done by friction on a body sliding down an inclined plane,
(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Sol. (a) Positive
In the given case, force and displacement are in the same direction. Hence, the sign of work done is positive. In this case, the work is done on the bucket.

(b) Negative
In the given case, the direction of force (vertically downward) and displacement (vertically upward) are opposite to each other. Hence, the sign of work done is negative.

(c) Negative
Since the direction of frictional force is opposite to the direction of motion, the work done by frictional force is negative in this case.

(d) Positive
Here the body is moving on a rough horizontal plane. Frictional force opposes the motion of the body. Therefore, in order to maintain a uniform velocity, a uniform force must be applied to the body. Since the applied force acts in the direction of motion of the body, the work done is positive.

(e) Negative
The resistive force of air acts in the direction opposite to the direction of motion of the pendulum. Hence, the work done is negative in this case.

6.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1. Compute the

(i) work done by the applied force in 10s
(ii) work done by friction in 10s
(iii) work done by the net force on the body in 10s
(iv) change in kinetic energy of the body in 10s.

Sol. Here, $m = 2$ kg, $u = 0$, $F = 7N$, $\mu = 0.1$, $t = 10s$

Force of kinetic friction

$$f_k = \mu mg = 0.1 \times 2 \times 9.8 = 1.96 \text{ N}$$

$$\therefore \quad f_{net} = F - f_k = 7 - 1.96 = 5.04 \text{ N}$$

$$\therefore \quad \text{Acceleration } a = \frac{f_{net}}{m} = \frac{5.04}{2} = 2.52 \text{ m/s}^2$$

Distance covered in 10s

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times 10 \times 10 = 126\text{m}$$

(i) Work done by the applied force

$$W = F \times s = 7 \times 126 = 882 \text{ J}$$

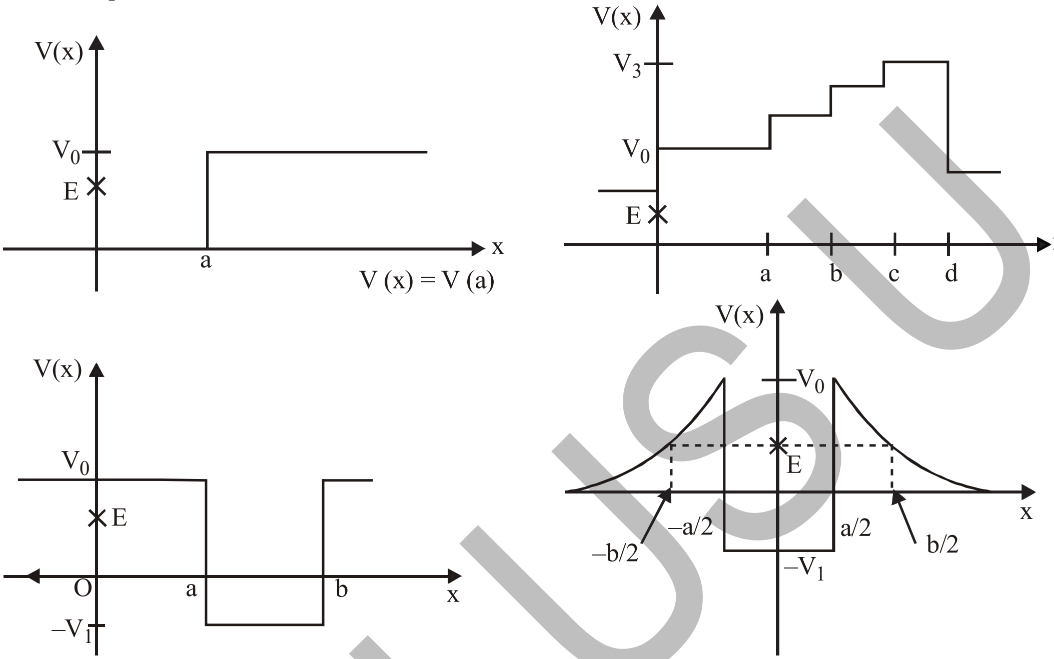
(ii) Work done by friction

$$W = f_k \times s = -1.96 \times 126 = -246.96 \text{ J}$$

(iii) Work done by net force $= F_{net} \times s = 5.04 \times 126 = 635.04$ J

(iv) Change in kinetic energy = Work done by net force = 635.04 J

6.3 Given in figure are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.



Sol. Total energy E = K.E. + P.E.

$\therefore$ KE = E – PE

The particle can exist in such a region in which its KE is positive.

(a) For x > a P.E. $(V_0) > E$

It gives K.E. negative so the particle cannot exist in the region x > a.

(b) For x < a and x > b, P.E. $(V_0) > E$

It gives K.E. negative so the particle cannot exist in the region x < a and x > b.

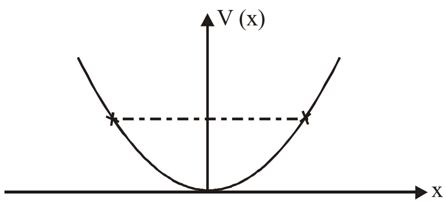
(c) In this diagram, in every region P.E. $(V_0) > E$.

$\therefore$ K.E. is negative. The particle cannot be found in any region.

(d) For $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$ P.E. (V) > E.

K.E. is negative. The particle cannot be present in these regions.

6.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\ \text{Nm}^{-1}$, the graph of V(x) versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.



Sol. Since, K.E. + P.E. = T.E.

$\therefore$ (P.E.)$_{max}$ = T.E. [when (K.E.) is min = 0] i.e., $\frac{1}{2}kx^2 \le$ T.E. or $\frac{1}{2} \times 0.5x^2 \le 4$ or $x^2 \le 4$

or $-2 \le x \le +2$ $\quad\quad$ [$\because$ $|x| \le 2$]

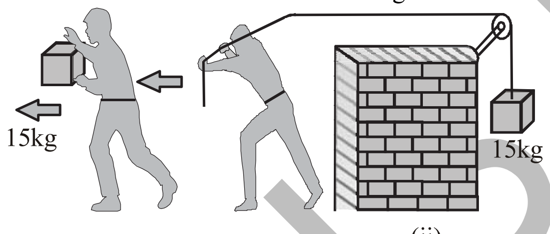
6.5 Answer the following:

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

(d) In Fig. (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig.(ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



(i) (ii)

Sol. (a) Rocket : The burning of the casing of a rocket in flight (due to friction) results in the reduction of the mass of the rocket. According to the conservation of energy:

$$\text{T.E.} = \text{P.E.} + \text{K.E.} = mgh + \frac{1}{2}mv^2$$

The reduction in the rocket's mass causes a drop in the total energy. Therefore, the heat energy required for the burning is obtained from the rocket.

(b) Gravitational force is a conservative force. Since the work done by a conservative force over a closed path is zero, the work done by the gravitational force over every complete orbit of a comet is zero.

(c) When an artificial satellite, orbiting around earth, moves closer to earth, its potential energy decreases because of the reduction in the height. Since the total energy of the system remains constant, the reduction in P.E. results in an increase in K.E. Hence, the velocity of the satellite increases. However, due to atmospheric friction, the total energy of the satellite decreases by a small amount.

(d) In the second case

Case (i) : Mass, m = 15 kg, Displacement, s = 2 m

Work done, $W = Fs \cos \theta$, Where θ = Angle between force and displacement

$= mgs \cos \theta = 15 \times 2 \times 9.8 \sin 90° = 0$

Case (ii) : Mass, m = 15 kg, Displacement, s = 2m

Here, the direction of the force applied on the rope and the direction of the displacement of the rope are same.

Therefore, the angle between them, $\theta = 0°$

Since $\cos 0° = 1$

Work done, $W = Fs \cos \theta = mgs = 15 \times 9.8 \times 2 = 294$ J

Hence, more work is done in the second case.

6.6 Underline the correct alternative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many particle system is proportional to the external force/sum of the internal forces on the system.
(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Sol. (a) Decreases (b) Kinetic energy
(c) External force (d) Total linear momentum

Explanation:

(a) A conservative force does a positive work on a body when it displaces the body in the direction of force. As a result, the body advances toward the centre of force. It decreases the separation between the two, thereby decreasing the potential energy of the body.
(b) The work done against the direction of friction reduces the velocity of a body. Hence, there is a loss of kinetic energy of the body.
(c) Internal forces, irrespective of their direction, cannot produce any change in the total momentum of a body. Hence, the total momentum of a many- particle system is proportional to the external forces acting on the system.
(d) The total linear momentum always remains conserved whether it is an elastic collision or an inelastic collision.

6.7 State if each of the following statements is true or false. Give reasons for your answer.
(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.
(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
(c) Work done in the motion of a body over a closed loop is zero for every force in nature.
(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Sol. (a) False. In an elastic collision, the total energy and momentum of both the bodies, and not of each individual body, is conserved.
(b) False. Although internal forces are balanced, they cause no work to be done on a body. It is the external forces that have the ability to do work. Hence, external forces are able to change the energy of a system.
(c) False. The work done in the motion of a body over a closed loop is zero for a conservation force only.
(d) True. In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system. This is because in such collisions, there is always a loss of energy in the form of heat, sound, etc.

6.8 Answer carefully, with reasons:
(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?
(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?
(c) What are the answers to (a) and (b) for an inelastic collision?
(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Sol. (a) No
In an elastic collision, the total initial kinetic energy of the balls will be equal to the total final kinetic energy of the balls. This kinetic energy is not conserved at the instant the two balls are in contact with each other. In fact, at the time of collision, the kinetic energy of the balls will get converted into potential energy.
(b) Yes
In an elastic collision, the total linear momentum of the system always remains conserved.
(c) No; Yes
In an inelastic collision, there is always a loss of kinetic energy, i.e., the total kinetic energy of the billiard balls before collision will always be greater than that after collision.
The total linear momentum of the system of billiards balls will remain conserved even in the case of an inelastic collision.

(d) Elastic
In the given case, the forces involved are conservation. This is because they depend on the separation between the centres of the billiard balls. Hence, the collision is elastic.

6.9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to –
(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol. **(ii).** From v = u + at = 0 + at
As power P = Fv = ma × at = ma^2t.
As m and a are constant $P \propto t$.

6.10 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to –
(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol. **(iii).** We know, P = W/t or W = P × t

Also, $W = \frac{1}{2}mv^2$

$\therefore \quad \frac{1}{2}mv^2 = P \times t \quad \text{or} \quad v^2 = \frac{2P\,.\,t}{m} \quad \text{or} \quad v = \frac{ds}{dt} = \left(\frac{2P\,.\,t}{m}\right)^{1/2}$

On integrating, $s = \frac{2}{3}\left(\frac{2P}{m}\right)^{1/2} t^{3/2}$ i.e., $s \propto t^{3/2}$

6.11 A body constrained to move along the z-axis of a coordinate system is subject to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4m along the z-axis?

Sol. Here, $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

$\vec{d} = 4\hat{k}$ m

$\therefore \quad W = \vec{F}.\vec{d} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{k}) = 12J$

6.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass = 9.11×10^{-31} kg, Proton mass = 1.67×10^{-27}kg, 1eV = 1.60×10^{-19}J).

Sol. Let E_1 and E_2 be the energy of electron and proton respectively

$\therefore \quad E_1 = \frac{1}{2}m_e v_e^2$ .... (1) ; $\quad E_2 = \frac{1}{2}m_p v_p^2$ .... (2)

$\frac{m_e v_e^2}{m_p v_p^2} = \frac{1}{10}$

$\therefore \quad \frac{v_e}{v_p} = \sqrt{\frac{1}{10} \times \frac{m_p}{m_e}} == \sqrt{\frac{1}{10} \times \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}}} = 10^2 \sqrt{\frac{1.67}{91.1}} = 13.53$

6.13 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1}?

Sol. Here, r = 2mm = 2×10^{-3}m, h = 500m
Work done by the gravitational force on the drop in each half of its journey

$= F \times s = mg \times \text{(half of the height)} = \frac{4}{3}\pi r^3 . \rho . g . \frac{h}{2}$

$= \frac{4}{3} \times \frac{22}{7}(2 \times 10^{-3})^3 \times 10^3 \times 9.8 \times 250 = 0.082 \text{ J}$

6.14 A molecule in a gas container hits a horizontal wall with speed 200 m/s and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Sol. We know momentum is conserved in all collisions.

Let us now check its K.E.

Let m in mass of the gas molecules and mass of wall in M (M >> m)

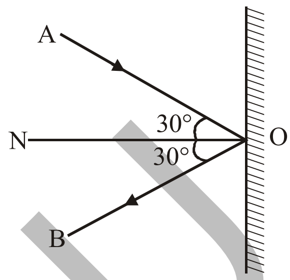
Total K.E. after collision

$E_2 = \frac{1}{m}(200)^2 + \frac{1}{2}M\,(0)^2 = 2 \times 10^4 \text{ m Joule}$

Where velocity of wall in zero.

Which is equal to K.E. of molecule before collision.

Hence, collision in elastic.



6.15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Sol. Here, $V = 30 \text{ m}^3$, $t = 15\text{m} = 15 \times 60 = 900\text{s}$

Height h = 40m, Efficiency $\eta = 30\%$, $\rho = 10^3 \text{ kg m}^{-3}$.

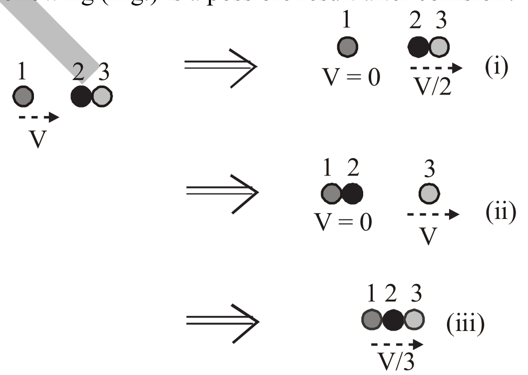
Mass of water pumped $m = V \times \rho = 30 \times 10^3$ kg.

Power consumed $= \frac{mgh}{t}$

$P_0 = \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 1.30 \times 10^4 \text{ W}$

If ρ_i is input power then $P_i = \frac{P_0}{\eta} = \frac{1.30 \times 10^4}{30} \times 100 \cong 43.33 \text{ kW}$

6.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig.) is a possible result after collision?



Sol. Let m be the mass of each ball.

Total K.E. of the system before collision $= \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv^2$

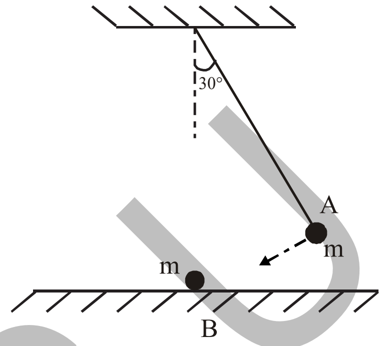
Case I: K.E. of the system after collision $= 0 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 \times 2 = \frac{mv^2}{4}$

Case II: K.E. of the system after collision = $0+\frac{1}{2}mv^2=\frac{1}{2}mv^2$

Case III: K.E. of the system after collision = $\frac{1}{2}m\left(\frac{v}{3}\right)^2\propto 3=\frac{mv^2}{6}$

In, elastic collision total energy of the system remains conserved. Here, case (II) satisfies the condition.

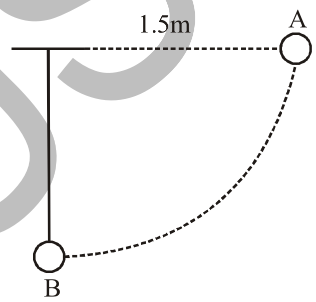
6.17 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.



Sol. Since, bob A and B are identical and collision is elastic they will interchange their velocities, i.e., bob A would come to rest and bob B would begin to move with velocity equal to the initial velocity of A.

6.18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Sol. Here h = 1.5m
Velocity of body at B = v
Energy dissipated = 5%



$\therefore$ K.E. at B = (P.E. at A) × $\frac{95}{100}$

$$\frac{1}{2}mv^2=(mgh)\frac{95}{100}$$

or $v=\sqrt{\frac{95}{100}\times 9.8\times 2\times 1.5}=5.3$ m/s

6.19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty?

Sol. The sand bag is placed on a trolley that is moving with a uniform speed of 27 km/h. The external forces acting on the system of the sandbag and the trolley is zero. When the sand starts leaking from the bag, there will be no change in the velocity of the trolley.
This is because the leaking action does not produce any external force on the system. This is in accordance with Newton's first law of motion. Hence, the speed of the trolley will remain 27 km/h.

6.20 A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m?

Sol. Here, m = 0.5 kg, $v = ax^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$.
Initial velocity at x = 0, $v_1 = 0$
Final velocity at x = 2, $v_2 = a\,(2)^{3/2} = 5 \times 2^{3/2}$
According to work energy theorem

$$W=\frac{1}{2}m\,(v_2^2-v_1^2)=\frac{1}{2}\times 0.5\,[(5\times 2^{3/2})^2-0]=50J$$

6.21 The blades of a windmill sweep out a circle of area A.

(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?
(b) What is the kinetic energy of the air ?
(c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, $v = 36$ km/h and the density of air is $1.2\ kg\ m^{-3}$. What is the electrical power produced?

Sol. (a) Volume of the wind blowing per second = Av

$\therefore$ Mass of the wind = $Av\rho$

Mass of the air passing in time t = $Av\rho t$

(b) K.E. of air = $\frac{1}{2}mv^2 = \frac{1}{2}(Av\rho t)\ v^2 = \frac{Av^3\rho t}{2}$

(c) Electrical energy produced = $\frac{25}{100} \times$ K.E. of air = $\frac{1}{4} \times \frac{Av^3\rho t}{2} = \frac{Av^3\rho t}{8}$

$\therefore$ Electric power = $\frac{W}{t} = \frac{Av^3\rho t}{8t} = \frac{Av^3\rho}{8} = \frac{1}{8} \times 30\,(10)^3 \times 1.2 = 4500$ W

6.22 A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Sol. Here, m = 10 kg, h = 0.5 m, n = 1000

(a) Work done against gravitational force

W = n (mgh) = 1000 (10 × 9.8 × 0.5) = 49,000 J

(b) Mechanical energy supplied by 1 kg of fat

$= 3.8 \times 10^7 \times \frac{20}{100} = 0.76 \times 10^7$ J/kg

$\therefore$ Fat used by the dieter = $\frac{1}{0.76 \times 10^7} \times 49,000 = 6.45 \times 10^{-3}$ kg

6.23 A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.

Sol. Let the area be Am^2.

$\therefore$ Total power = 200 A

Electrical energy produced per second = 200A × $\frac{20}{100}$ = 40A

or 40A = 8000 (8kw = 8000 watt/sec.)

$\therefore$ A = $\frac{8000}{40} = 200\ m^2$

This area is comparable to the roof of a large house of $250\ m^2$.

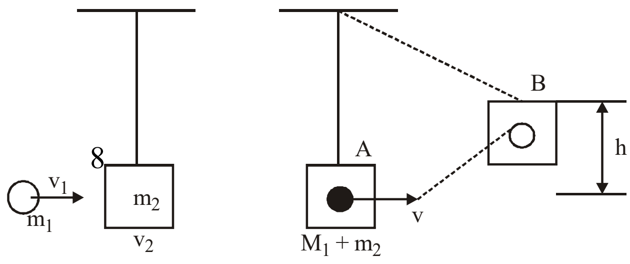
6.24 A bullet of mass 0.012 kg and horizontal speed 70 m/s strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Sol. Here mass of bullet $m_1 = 0.012$ kg

Velocity of bullet $v_1 = 70$ m/s

Mass of block $m_2 = 0.4$ kg

velocity of block $v_2 = 0$ (at rest)



Let V is the velocity of combination,
when bullet get embedded in block.
Applying law of conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\ V$$

$$\therefore \quad V = \frac{m_1v_1 + m_2v_2}{(m_1 + m_2)} = \frac{0.012 \times 70 + 0}{(0.012 + 0.4)} = 2.04 \text{ m/s}$$

Let the block rises to height h
P.E. of combination = K.E. of combination

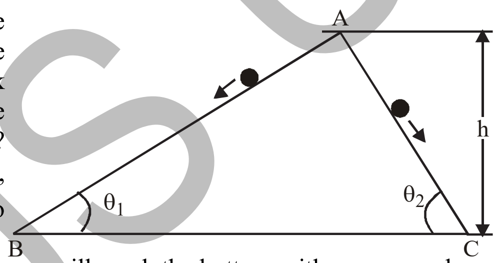
$$(m_1 + m_2)\ gh = \frac{1}{2}(m_1 + u_2)\ u^2$$

$$\text{or} \quad h = \frac{1}{2}\frac{v^2}{g} = \frac{(2.04)^2}{2 \times 9.8} \quad \text{or} \quad h = 0.212 \text{ m}$$

To calculate heat energy produced, let us calculate energy lost

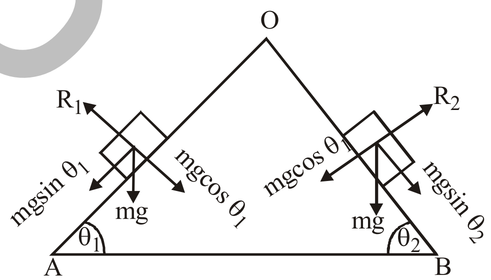
$$= \frac{1}{2}mu_1^2 - \frac{1}{2}(m_1 + m_2)\ V^2 = \frac{1}{2}(0.012)\ (70)^2 - \frac{1}{2}(0.412)\ (2.04)^2 = 28.50 \text{ J}$$

6.25 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig.). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given θ_1 = 300, θ_2 = 600, and h = 10 m, what are the speeds and times taken by the two stones?



Sol. Since the height of the two planes is same both the stones will reach the bottom with same speed.

$$v = \sqrt{2gh}$$



Here, acceleration in the first block

$$a_1 = g \sin \theta_1$$

Acceleration in the second block

$$a_2 = g \sin \theta_2$$

as $\theta_2 > \theta_1$

$\therefore$ $a_2 > a_1$

Using first equation of motion

$$v = u + at$$

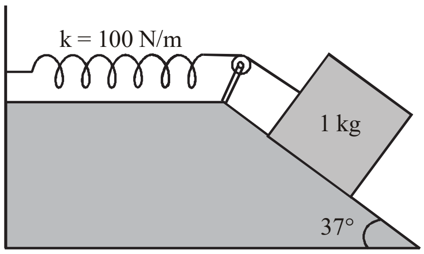
$$v = 0 + at \quad \text{or} \quad t = v/a$$

Since, $v_1 = v_2$; $t \propto 1/a$

$$\text{or} \quad \frac{t_1}{t_2} = \frac{a_2}{a_1} \Rightarrow \quad t_2 < t_1 \text{ as } a_2 > a_1$$

Second stone will take lesser time and reach the bottom first.

6.26 A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



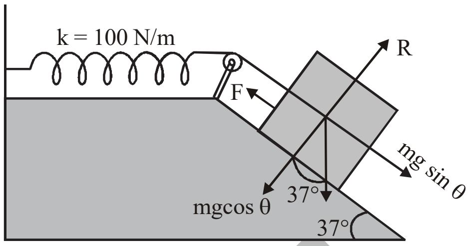
Sol. From figure, R = mg cos θ,

$$f = \mu R = \mu\ mg \cos \theta$$

Net force on the block in the downward direction

$= mg \sin \theta - f = mg \sin \theta - \mu\, mg \cos \theta = mg (\sin \theta - \mu \cos \theta)$

Distance moved = 10cm = 0.1m



In equilibrium,

Work done = P.E. of stretched spring

or $mg (\sin \theta - \mu \cos \theta)\, x = \frac{1}{2} kx^2$

or $2mg\ (\sin \theta - \mu \cos \theta) = kx$

or $2 \times 1 \times 10 (\sin 37 - \mu \cos 37) = 100 (0.1)$

or $\sin 37 - \mu \cos 37 = \frac{10}{20} = 0.5$

$0.601 - \mu \times 0.798 = 0.5$

or $\mu = \frac{0.601 - 0.5}{0.798} - \frac{0.101}{0.798} = 0.126$

6.27 A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 m s^{-1}. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

Sol. Mass of the bolt, m = 0.3 kg, Speed of the elevator = 7 m/s, Height, h = 3 m

Since the relative velocity of the bolt with respect to the lift is zero, at the time of impact, potential energy gets converted into heat energy.

Heat produced = Loss of potential energy = mgh = $0.3 \times 9.8 \times 3 = 8.82$ J

The heat produced will remain the same even if the lift is stationary. This is because of the fact that the relative velocity of the bolt with respect to the lift will remain zero.

6.28 A trolley of mass 200 kg moves with a uniform speed of 36km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Sol. Mass of the trolley, M = 200 kg

Speed of the trolley, v = 36 km/h = 10 m/s

Mass of the boy, m = 20 kg

Initial momentum of the system of the boy and the trolley

$= (M + m)v = (200 + 20) \times 10 = 2200$ kg m/s

Let v' be the final velocity of the trolley with respect to the ground.

Final velocity of the boy with respect to the ground = v'–4

Final momentum = $Mv' + m (v' - 4) = 200v' + 20v' - 80 = 220v' - 80$

As per the law of conservation of momentum:

Initial momentum = Final momentum

$2200 = 220v' - 80$

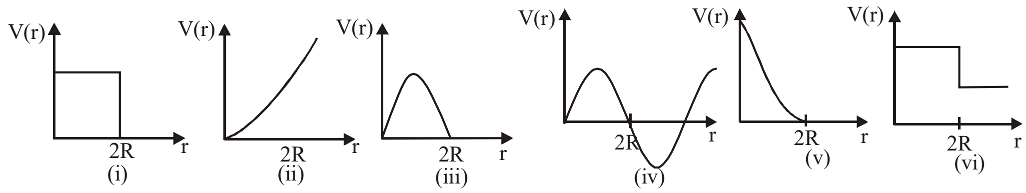
$\therefore \quad v' = \frac{2280}{220} = 10.36$ m/s

Length of the trolley, $\ell = 10$ m ; Speed of the boy, v" = 4 m/s

Time taken by the boy to run,

$\therefore$ Distance moved by the trolley = $v'' \times t = 10.36 \times 2.5 = 25.9$ m

6.29 Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billiard balls? Here r is the distance between centres of the balls.

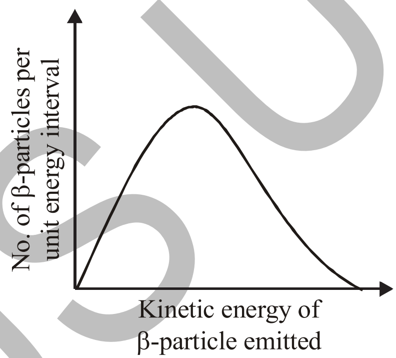


Sol. (i), (ii), (iii), (iv), and (vi)

The potential energy of a system of two masses is inversely proportional to the separation between them. In the given case, the potential energy of the system of the two balls will decrease as they come closer to each other. It will become zero (i.e., $V(r) = 0$) when the two balls touch each other, i.e., at $r = 2R$, where R is the radius of each billiard ball. The potential energy curves given in figures (i), (ii), (iii), (iv), and (vi) do not satisfy these two conditions. Hence, they do not describe the elastic collisions between them.

6.30 Consider the decay of a free neutron at rest: $n \rightarrow p + e^-$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus (Fig.).



Sol. In this decay process

$n = p + e$

The energy of electron is given by Δmc^2.

where, Δm = mass defect = constant.

Therefore, the two body decay of this type cannot explain the given continuous energy distribution is the β decay of a neutron or a nucleus.