NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 8
GRAVITATION

8.1 Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull.
However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

Sol. (a) No (b) Yes

(a) Gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.

(b) If the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity(g).

(c) Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

8.2 Choose the correct alternative:

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-G\,Mm\,(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Sol. (a) Decreases (b) Decreases
(c) Mass of the body (d) More

(a) Acceleration due to gravity at depth h is given by the relation: $g_h = \left(1 - \frac{2h}{R_e}\right) g$

Where, R_e = Radius of the Earth
g = Acceleration due to gravity on the surface of the Earth
It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

(b) Acceleration due to gravity at depth d is given by the relation: $g_d = \left(1 - \frac{d}{R_e}\right) g$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

(c) Acceleration due to gravity of body of mass m is given by the relation: $g = GM/R^2$
Where, G = Universal gravitational constant, M = Mass of the Earth, R = Radius of the Earth
Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

(d) Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by: $V(r_1) = -\frac{GmM}{r_1}$; $V(r_2) = -\frac{GmM}{r_2}$

$\therefore$ Difference in potential energy, $V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

Hence, this formula is more accurate than the formula $mg(r_2 - r_1)$.

8.3 Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

Sol. Here, $T_e = 1$ year, $r_e = 1$ AU

$$T_p = \frac{T_e}{2} \text{ year}, \ r_p = ?$$

From Kepler's third law

$$\frac{T_e^2}{T_p^2} = \frac{R_e^3}{R_p^3} \ ; \ R_p = R_e\left(\frac{T_p}{T_e}\right)^{2/3} = 1 \times \left[\frac{T_e/2}{T_e}\right]^{2/3} = 1 \times \left[\frac{1}{2}\right]^{2/3} = 0.63 \text{ AU}$$

8.4 One of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Sol. For a satellite of Jupiter $T^2 \times R^3$

$$T_1 = 1.769 \text{ days} = 1.769 \times 24 \times 60 \times 60\text{s} \ \text{ or } \ T^2 = \frac{4\pi^2}{GM}R^3$$

Radius of the Orbit $r_i = 4.22 \times 10^8$ m

Mass of Jupiter $M_1 = \frac{4\pi^2 r_1^3}{GT_1^2}$ or $M = \frac{4\pi^2 R^3}{GT^2}$

or $M_1 = \frac{4(3.14)^2 \times (4.22 \times 10^8)^3}{(6.67 \times 10^{-11}) \times (1.769 \times 24 \times 60 \times 60)^2}$

Orbital period of earth around the Sun, $T = 1$ year $= 365.25 \times 24 \times 60 \times 60$

Orbital radius $r = 1$ AU $= 1.496 \times 10^{11}$ m

$$\therefore \ \text{Mass of Sun } M = \frac{4\pi^2 r^3}{GT^2} = \frac{4(3.14)^2 \times (1.496 \times 10^{11})^3}{(6.67 \times 10^{-11}) \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$\therefore \ \frac{M}{M_1} = \frac{1}{1046} \cong \frac{1}{1000} \ \text{ or } M_1 = 1000\text{ M}$$

8.5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5ly.

Here $r = 50{,}000$ ly $= 50{,}000 \times 9.46 \times 10^{15}$ m $= 4.73 \times 10^{20}$ m

$M = 2.5 \times 10^{11}$ solar mass $= 2.5 \times 10^{11} \times 2 \times 10^{30} = 5.0 \times 10^{41}$ kg

From formula, $M = \frac{4\pi^2 r^3}{GT^2}$ or $T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2} = \left[\frac{4 \times (3.14)^2 \times (4.73 \times 10^{20})^3}{6.67 \times 10^{-11} \times 5 \times 10^{41}}\right]^{1/2} = 1.12 \times 10^{16}$ s

8.6 Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Sol. (a) Kinetic energy (b) Less

(a) Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative. Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

(b) An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

8.7 Does the escape speed of a body from the earth depend on
(a) the mass of the body,
(b) the location from where it is projected,
(c) the direction of projection,
(d) the height of the location from where the body is launched?

Sol. Escape velocity is given by $v_e = \sqrt{\frac{2Gm}{R}}$

(a) Escape velocity is independent of the mass m of the body, projected.
(b) Gravitational potential V = Gm/R depends slightly on the latitude of the point, so escape velocity also depends on the latitude of the location of point of projection.
(c) Escape velocity does not depend on the direction of projection.
(d) The escape velocity depends on height of location as escape velocity is also given by $v_e = \sqrt{2gR}$ and g decreases with height.

8.8 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

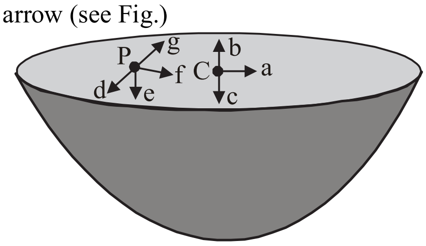
Sol. (a) As a consequence of Kepler's second law, of planetary motion, a planet moves fasten where it is close to the sun and moves slower when away from the sun. Thus, linear speed of the planet is not constant.
(b) The angular momentum remains constant as the planet moves under the effect of a pure radial force.
(c) Kinetic energy of the planet is given by $\frac{1}{2}mv^2$ where v is linear speed, which is not constant so K.E. of the planet keeps on changing.
(d) The distance of the planet keeps on changing from the sun, its P.E. also keeps on changing.
(e) The total energy of the planet remains constant.

8.9 Which of the following symptoms is likely to affect an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem. In the following two exercises, choose the correct answer from among the given ones

Sol. (a) Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.
(b) A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.
(c) Headaches are caused because of mental strain. It can affect the working of an astronaut in space.

(d) Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

8.10 The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig.)



(i) a, (ii) b, (iii) c, (iv) 0.

Sol. The gravitational potential at all points in side a spherical shell is constant. As gravitational intensity is equal to negative of the gravitational potential gradient, it will be zero, at all points inside a hollow spherical shell. It means the gravitational force acting on a particle at any point inside a spherical shell will be symmetrical placed.

Thus, if we remove the upper hemispherical shell the net gravitational force acting on the particle at the centre Q pr at any other point P will be acting downward which will be direction of gravitational intensity too, because gravitational intensity is defined as the gravitational force per unit mass. Hence at Q the direction of gravitational intensity will be along C, options (iii) is correct.

8.11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow

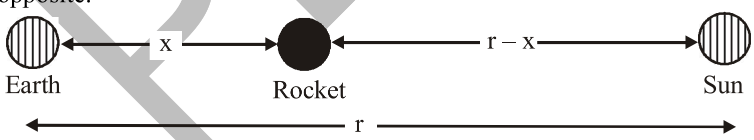
(i) d, (ii) e, (iii) f, (iv) g.

Sol. As per above explanation, the gravitational intensity at P will be along e i.e., option (ii) is correct.

8.12 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero ? Mass of the sun = 2×10^{30} kg, Mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Sol. Here, $M_s = 2\times 10^{30}$ kg, $M_e = 6\times 10^{24}$ kg, $r = 1.5 \times 10^{11}$ m

Let x be the distance from earth, where gravitational force on rocket due to Sun and earth becomes equal and opposite.



$$\therefore \quad \frac{GM_s m}{(r-x)^2} = \frac{GM_e m}{x^2} \quad \text{or} \quad \frac{r-x}{x} = \sqrt{\frac{M_s}{M_e}} = \sqrt{\frac{2\times10^{30}}{6\times10^{24}}} \quad \text{or} \quad \frac{r}{x} - 1 = \frac{10^3}{\sqrt{3}}$$

$$\text{or} \quad \frac{r}{x} = 1 + \frac{10^3}{\sqrt{3}} = \frac{\sqrt{3}+10^3}{\sqrt{3}} \quad \text{or} \quad x = \frac{1.5\times10^{11}\times\sqrt{3}}{\sqrt{3}+10^3} = 2.59 \times 10^8 \text{ m}$$

8.13 How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is

1.5×10^8 km.

Sol. Here T = 365 days = 365 × 24 × 60 × 60s

The gravitational force provides the necessary centripetal force

$$\frac{GM_s m_e}{R^2} = m_e R\omega^2 \quad \text{or} \quad M_s = \frac{R^3\omega^2}{G}$$

or $\quad M_s = \dfrac{(1.5\times10^{11})^3}{6.67\times10^{11}}\left(\dfrac{2\pi}{365\times24\times60\times60}\right)^2 = 2\times10^{30}$ kg

8.14 A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun?

Sol. Here, $T_s = 29.5\ T_e$

$R_e = 1.5\times10^8$ km, $R_s = ?$

From relation, $\dfrac{T^2}{R_e^3} = \dfrac{T_s^2}{R_s^3}$

$$R_s = R_e\left(\frac{T_s}{T_e}\right)^{2/3} = 1.5\times10^8\left(\frac{295T_e}{T_e}\right)^{2/3} = 1.43\times10^9 \text{ km.}$$

8.15 A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?

Sol. At a height acceleration due to gravity is given by :

$$g' = \frac{g}{(1+h/R)^2} \qquad \text{Here } h = R/2$$

$$g' = \frac{g}{\left(1+\frac{R/2}{R}\right)^2} \quad \text{or} \quad g' = g\times\frac{4}{9} \quad \text{or} \quad mg' = mg\times\frac{4}{9} = 63\times\frac{4}{9} = 28\text{N}$$

8.16 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Sol. Here, mg = 250 N, d = R/2

$$g' = g\left(1-\frac{d}{R}\right) \quad \text{or} \quad mg' = mg\left(1-\frac{d}{R}\right) = 250\left(1-\frac{R/2}{R}\right) = 250\times\frac{1}{2} = 125 \text{ N}$$

8.17 A rocket is fired vertically with a speed of 5 km/s from the earth's surface. How far from the earth does the rocket go before returning to the earth?

Mass of the earth = 6.0×10^{24} kg; Mean radius of the earth = 6.4×10^6 m;

G = 6.67×10^{-11} N m^2 kg^{-2}.

Sol. Let the rocket be fired with velocity v from the surface of earth and it reaches a height h from the surface of earth.

Total energy of rocket at the surface of earth = K.E. + P.E. = $\dfrac{1}{2}mv^2 + \left(\dfrac{-GMm}{R}\right)$

At highest point, the K.E. of body becomes zero, and potential energy is $-\left(\dfrac{GMm}{R+h}\right)$

Applying law of conservation of energy

Total energy of the surface of earth = total energy at highest point

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0 + \left(-\frac{GMm}{R+h}\right)$$

or $\quad \dfrac{v^2}{2} = \dfrac{GM}{R} - \dfrac{GM}{R+h} = \dfrac{gR^2}{R} - \dfrac{gR^2}{R+h} = gR\left(1-\dfrac{R}{R+h}\right) = gR\left(\dfrac{h}{R+h}\right)$

or $\quad v^2(R+h) = 2gRh$

or $\quad h = \dfrac{Rv^2}{2gR - v^2} = \dfrac{(6.4 \times 10^6)\,(5 \times 10^3)^2}{2\,(9.8 \times 6.4 \times 10^6) - (5 \times 10^3)^2}$ = 1.6 × 10^6 m

R + h = 8 × 10^6 m

8.18 The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Sol. Let the velocity of body far away from the earth is V', then

Total energy on the surface of earth = Total energy far away from the earth.

$$\frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv'^2 + 0$$

But v = $3v_e$.

$$\therefore \quad \frac{1}{2}m\,(3v_e)^2 - \frac{GMm}{R} = \frac{1}{2}mv'^2$$

or $\quad (3v_e)^2 - \dfrac{2GMm}{R} = v'^2$

$(3v_e)^2 - v_e^2 = v'^2$ or $v'^2 = 8v_e^2$

or $\quad v'^2 = 2\sqrt{2}\ v_e = 2\sqrt{2} \times 11.2 = 31.7\ \text{km/s}$

8.19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence?

Mass of the satellite = 200 kg; Mass of the earth = 6.0 × 10^{24} kg;

Mean radius of the earth = 6.4 × 10^6 m; G = 6.67 × 10^{-11} N m^2 kg^{-2}.

Sol. Here, h = 400 km = 4 × 10^5 m

m = 200 kg, M = 6.0 × 10^{24} kg, R_e = 6.4 × 10^6 m

Total energy of orbiting satellite at a height h

$$= -GM\,\frac{m}{(R+h)} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + \frac{1}{2}m\frac{GM}{(R+h)} = \frac{-GMm}{2\,(R+h)}$$

Energy expended to rocket the satellite out of the earths gravitational field

$$= \frac{GMm}{2\,(R+h)} = \frac{(6.67 \times 10^{-11})\,(6 \times 10^{24})\,(200)}{2\,(6.4 \times 10^6 + 4 \times 10^5)} \approx 5.9 \times 10^9\ \text{J}$$

8.20 Two stars each of one solar mass (= 2 × 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Sol. Here M = 2 × 10^{30} kg,

r = initial distance between two stars = 10^9 km = 10^{12} m.

$\therefore$ Initial potential energy of the system = $-\dfrac{GMm}{r}$

K.E. of the system = $\left(\dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2\right) = mv^2$

v is the velocity of each star just before collision. Also the distance between their centres r' = 2R

$\therefore$ Final potential energy of two stars = $-\dfrac{GMm}{2R}$

Initial total energy = Final total energy

$$-\frac{GMm}{r} = mv^2 - \frac{GMm}{2R} \quad \text{or} \quad mv^2 = \frac{GMm}{2R} - \frac{GMm}{r} \quad \text{or} \quad v^2 = \frac{GM}{2R} - \frac{GM}{r}$$

$$= 6.67 \times 10^{-11} \times 2 \times 10^{30} \left(\frac{1}{2\times10^7} - \frac{1}{10^{12}}\right) = 6.67 \times 2 \times 10^{19} \left(\frac{10^5 - 2}{2\times10^{12}}\right) = 6.67 \times 10^{12}$$

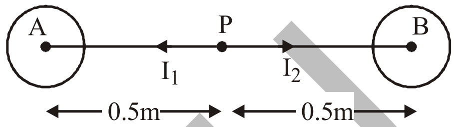
or $v = \sqrt{6.67\times10^{12}} = 2.58 \times 10^6$ m/s

8.21 Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Sol. Let A and B are two spheres and P be the mid point of AB.

Gravitational field at P due to mass at A

$$I_1 = \frac{GM}{r^2} = \frac{G\times100}{(0.5)^2}$$



Gravitational field at P due to mass at B

$$I_2 = \frac{G\times100}{(0.5)^2} \text{ along PB}$$

As I_1 and I_2 are equal in magnitude but point along opposite direction, net gravitational field intensity at P is zero.

Gravitational potential is a scalar quantity so net potential at P is

$$V = V_A + V_B = -\frac{GM}{r} - \frac{GM}{r} = -\frac{2GM}{r} = \frac{-2\times(6.67\times10^{-11})\times100}{0.5} = -2.67 \times 10^{-8} \text{ J kg}^{-1}.$$

8.22 A geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero).

Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

Sol. Here, $M = 6\times 10^{24}$ kg, R = 6400 km, h = 36,000 km.

$\therefore$ r = R + h = 36000 + 6400 = 42,400 km = 4.24×10^7 m

From formula, $V = -\frac{GM}{r} = -\frac{6.67\times10^{-11}\times6\times10^{24}}{4.24\times10^7} = -9.43 \times 10^6 \text{ J kg}^{-1}$

8.23 A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun=2×10^{30} kg).

Sol. For the object to remain struck to the surface of star due to gravity, the acceleration due to gravity should be greater than the centrifugal acceleration due to its rotation.

Here, $M = 2.5 \times 2 \times 10^{30}$ kg, R = (12,000) m, ν = 1.2 rps

Acceleration due to gravity, $g = \frac{GM}{R^2} = -\frac{6.67\times10^{-11}\times2.5\times2\times10^{30}}{(12000)^2} = 2.3 \times 10^{12} \text{ ms}^{-2}$

Centrifugal force = $R\omega^2 = R(2\pi\nu)^2 = (12,000)(2\pi \times 1.2)^2 = 1.1 \times 10^6 \text{ ms}^{-2}$.

Since $g > R\omega^2$ the body will remain stuck with the surface of star.

8.24 A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^8 km; G = 6.67×10^{-11} N m^2 kg^{-2}.

Sol. Energy of the spaceship in the orbit E = K.E. + P.E.

$$= \frac{1}{2}mv^2 - \frac{GMm}{R+h} = \frac{1}{2}m\left(\sqrt{\frac{GM}{R+h}}\right)^2 - \frac{GMm}{R+h} = -\frac{GMm}{2\,(R+h)}$$

But m = 1000 kg, M = 2 × 10^{30} kg; R = 2.28 × 10^8 km = 2.28 × 10^{11} m; h = 3395 × 10^3 m

$$\therefore \quad E = -\frac{6.67\times10^{-11}\times2\times10^{30}\times1000}{2\,(2.28\times10^{11}+3.395\times10^6)} = -2.9\times10^{11}\text{ J}$$

∴ Energy needed to rocket the spaceship out of solar system ≈ 3 × 10^{11} J

8.25 A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4 × 10^{23} kg; Radius of mars = 3395 km; G = 6.67× 10^{-11} N m^2 kg^{-2}.

Sol. Total energy of the rocket = K.E. + P.E. = $\frac{1}{2}mv^2 - \frac{GMm}{R}$

Remaining energy of rocket = $\left[\frac{80}{100}\times\frac{1}{2}mv^2 - \frac{GMm}{R}\right]$

Let the rocket rise to height h, where it will have only P.E.

Therefore, $\left[\frac{80}{100}\times\frac{1}{2}mv^2 - \frac{GMm}{R}\right] = -\frac{GMm}{R+h}$

Simplifying, $h = \frac{R}{\frac{GM}{0.4v^2R}-1} = 495$ km