NCERT SOLUTIONS
PHYSICS XI CLASS
CHAPTER - 9
MECHANICAL PROPERTIES OF SOLIDS

9.1 A steel wire of length 4.7 m and cross section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross section 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?

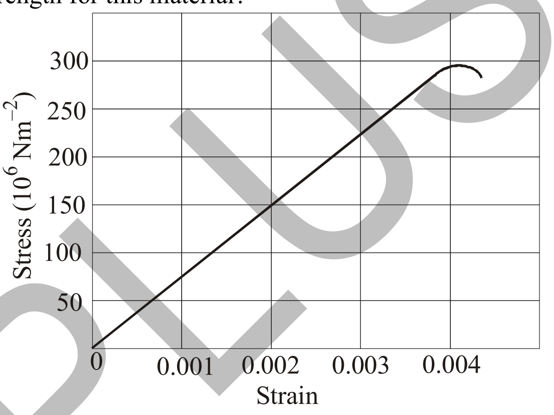
Sol. Here $L_s = 4.7$ m, $L_c = 3.5$ m, $a_s = 3.0 \times 10^{-5}$ m^2, $a_c = 4.0 \times 10^{-5}$ m^2

$$\Delta L_s = \Delta L_c\ ;\ F_s = F_c$$

$$Y_s = \frac{F_s L_s}{a_s \Delta L_s} \text{ and } Y_c = \frac{F_c L_c}{a_c \Delta L_c}$$

$$\frac{Y_s}{Y_c} = \frac{L_s}{a_s} \cdot \frac{a_c}{L_c} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4 \times 10^{-5}}{3.5} = 1.79$$

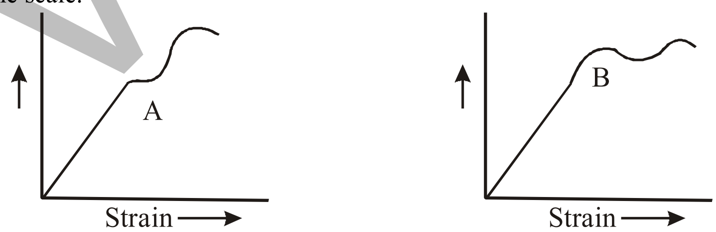
9.2 Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



Sol. (a) Young's modulus $Y = \frac{\text{Stess}}{\text{Strain}} = \frac{150 \times 10^6}{0.002} = 7.5 \times 10^{10}$ Nm^{-2}

(b) Yield strength = Maximum stress = 300×10^6 Nm^{-2}

9.3 The stress versus strain graph for two materials A and B are shown in figure. The graphs are on the same scale.



(a) Which material has greater Young's modulus?
(b) Which material is more ductile?
(c) Which is more brittle?
(d) Which of the two is stronger material?

Sol. (a) For a given strain, stress is more for material A.

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

Hence, Young's modulus is greater for A than that of B.

(b) For material A, there is a larger stress strain variation beyond elastic point, therefore A is more ductile, than B.

(c) For material B, there is a small stress-strain variation beyond elastic point, therefore, B is more brittle than A.

(d) The strength of a material, is the measure of the amount of stress required to cause fracture, therefore A is stronger than B.

9.4 Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

Sol. (a) False. For a given stress, the strain in rubber is more than it is in steel.

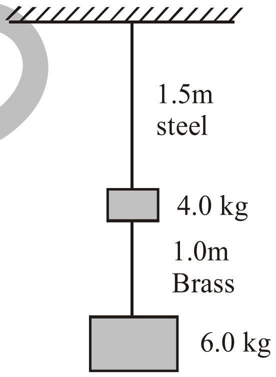
Young's modulus, Y = Stress/strain

For a constant stress: $Y \propto 1/\text{strain}$

Hence, Young's modulus for rubber is less than it is for steel.

(b) True. Shear modulus is the ratio of the applied stress to the change in the shape of a body. The stretching of a coil changes its shape. Hence, shear modulus of elasticity is involved in this process.

9.5 Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.



Sol. Here, for brass wire $F_b = 6 \times 9.8$ N, $L_b = 1.0$ m

$$r_b = \frac{0.25}{2} \times 10^{-2} \text{m}, \; Y_b = 0.91 \times 10^{11} \text{Nm}^{-2}$$

$$\therefore \quad \Delta L_b = \frac{F_b . L_b}{Y_b . \pi r_b^2} = \frac{6 \times 9.8 \times 1.0}{0.91 \times 10^{11} \times 3.14 \times (0.125 \times 10^{-2})^2}$$

$$= 1.3 \times 10^{-4} \text{ m}$$

For steel wire, $F_s = (6 + 4.0) \times 9.8 = 98$ N, $L_s = 1.5$

$$r_s = \frac{0.25}{2} \times 10^{-2} \text{m}, \; Y_s = 2 \times 10^{11} \text{Nm}^{-2}$$

$$\Delta L_b = \frac{F_s . L_s}{Y_s . \pi r_s^2} = \frac{98 \times 1.5}{2 \times 10^{11} \times 3.14 \times (0.125 \times 10^{-2})^2} = 1.5 \times 10^{-4} \text{ m}$$

9.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Sol. Here, $a = (0.1)^2 = 0.01 \text{ m}^2$

$L = 10$ cm $= 0.10$ m, $m = 100$ kg

$\therefore \quad F = 100 \times 10 = 10^3$ N

$\eta = 2.5 \times 10^9 \text{ Nm}^{-2}$, $\Delta x = ?$

From formula, $\Delta x = \dfrac{F.L}{a.\eta}$ or $\Delta x = \dfrac{10^3 \times 0.1}{0.01 \times 25 \times 10^9} = 4 \times 10^{-7}$ m

9.7 Four identical hollow cylindrical columns of steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 and 60 cm. respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Nm^{-2}.

Sol. Here, M = 50000 kg

Area of cross sectional of 4 cylindrical columns

$a = 4\pi\,(0.6^2 - 0.3^2)$; $Y = 2.0 \times 10^{11}$ Nm^{-2}.

$$\therefore \quad Y = \frac{MgL}{a\Delta L}$$

$$\therefore \quad \frac{\Delta L}{L} = \frac{Mg}{aY} = \frac{50000 \times 9.8}{4 \times 3.14 \times (0.6^2 - 0.3^2) \times 2 \times 10^{11}} = 7.2 \times 10^{-7}$$

9.8 A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

Sol. Length of the piece of copper, $\ell = 19.1$ mm $= 19.1 \times 10^{-3}$ m

Breadth of the piece of copper, b = 15.2 mm = 15.2×10^{-3} m

Area of the copper piece:

$A = \ell \times b = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4}\ m^2$

Tension force applied on the piece of copper, F = 44500 N

Modulus of elasticity of copper, $\eta = 42 \times 10^9\ N/m^2$.

Modulus of elasticity, $\eta = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\text{Strain}}$

$$\therefore \text{Stress} = \frac{F}{A\eta} = \frac{44500}{2.9 \times 10^{-4} \times 42 \times 10^9} = 3.65 \times 10^{-3}.$$

9.9 A steel cable with a radius of 1.5cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?

Sol. Radius of the steel cable, r = 1.5 cm = 0.015 m

Maximum allowable stress = 10^8 N m^{-2}

Maximum stress = $\frac{\text{Maximum force}}{\text{Area of cross-section}}$

$\therefore$ Maximum force = Maximum stress × Area of cross-section = $10^8 \times N\,(0.015)^2$

$= 7.065 \times 10^4$ N

Hence, the cable can support the maximum load of 7.065×10^4 N.

9.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Sol. The tension force acting on each wire is the same. Thus, the extension in each case is the same. Since the wires are of the same length, the strain will also be the same.

The relation for Young's modulus is given as:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\text{Strain}} = \frac{4F/\pi d^2}{\text{Strain}}$$

Where, F = Tension force, A = Area of cross-section, d = Diameter of the wire

It can be inferred from equation (i) that $Y \propto 1/d^2$.

Young's modulus for iron, $Y_1 = 190 \times 10^9$ Pa

Diameter of the iron wire = d_1

Young's modulus for copper, $Y_2 = 110 \times 10^9$ Pa

Diameter of the copper wire = d_2

Therefore, the ratio of their diameters is given as: $\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.31 : 1$

9.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Sol. Mass, m = 14.5 kg ; Length of the steel wire, ℓ = 1.0 m

Angular velocity, ω = 2 rev/s

Cross-sectional area of the wire, a = 0.065 cm^2

Let $\delta\ell$ be the elongation of the wire when the mass is at the lowest point of its path.

When the mass is placed at the position of the vertical circle, the total force on the mass is:

$$F = mg + m\ell\omega^2 = 14.5 \times 9.8 + 14.5 \times 1 \times (2)^2 = 200.1 \text{ N}$$

$$\text{Young modulus} = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell} \quad \therefore \quad \Delta\ell = \frac{F\ell}{AY}$$

Young's modulus for steel = 2×10^{11} Pa

$$\therefore \Delta\ell = \frac{200.1 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} = 1539.23 \times 10^{-7} = 1.539 \times 10^{-4} \text{ m}$$

Hence, the elongation of the wire is 1.539×10^{-4} m.

9.12 Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, Pressure increase = 100.0atm. Final volume = 100.5 litre.

Sol. Here $\Delta P = 100.0 \text{ atm} = 100.0 \times 1.013 \times 10^5 \text{ Nm}^{-2}$.

$\Delta V = 100.5 - 100.0 = 0.5$ litre

V = 100.0 litre

$$\therefore \quad B = \frac{V\Delta P}{\Delta V} = \frac{100.0 \times 1.013 \times 10^7}{0.5} = 2.03 \times 10^9 \text{ Nm}^{-2}.$$

9.13 What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{ kg m}^{-3}$?

Sol. Let the given depth be h.

Pressure at the given depth, $p = 80.0 \text{ atm} = 80 \times 1.01 \times 10^5$ Pa

Density of water at the surface, $\rho_1 = 1.03 \times 10^3 \text{ kg m}^{-3}$

Let ρ_2 be the density of water at the depth h.

Let V_1 be the volume of water of mass m at the surface.

Let V_2 be the volume of water of mass m at the depth h.

Let ΔV be the change in volume.

$$\Delta V = V_1 - V_2 = m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$$\therefore \quad \text{Volumetric strain} = \frac{\Delta V}{V_1} = m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \times \frac{\rho_1}{m}$$

$$\therefore \quad \frac{\Delta V}{V_1} = 1 - \frac{\rho_1}{\rho_2} \quad \text{......... (1)}$$

$$\text{Bulk modulus,} \quad B = \frac{pV_1}{\Delta V}; \frac{\Delta V}{V_1} = \frac{p}{B}$$

Compressibility of water = $1/B = 45.8 \times 10^{-11} \text{ Pa}^{-1}$

$$\therefore \frac{\Delta V}{V_1} = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.71 \times 10^{-3} \quad \text{.......... (2)}$$

For eq. (1) and (2), we get, $1 - \frac{\rho_1}{\rho_2} = 3.71 \times 10^{-3}$

$$\rho_2 = \frac{1.03 \times 10^3}{1 - (3.71 \times 10^{-3})} = 1.034 \times 10^3 \text{ kg m}^{-3}.$$

Therefore, the density of water at the given depth (h) is 1.034×10^3 kg m^{-3}.

9.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10atm.

Sol. Hydraulic pressure exerted on the glass slab, p = 10 atm = $10 \times 1.013 \times 10^5$ Pa

Bulk modulus of glass, B = 37×10^9 Nm^{-2}

Bulk modulus, $B = \frac{p}{\Delta V / V}$, where ΔV/V = Fractional change in volume

$$\therefore \quad \frac{\Delta V}{V} = \frac{p}{B} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = 2.73 \times 10^{-5}$$

Hence, the fractional change in the volume of the glass slab is 2.73×10^{-5}.

9.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Sol. Length of an edge of the solid copper cube, ℓ = 10 cm = 0.1 m

Hydraulic pressure, p = 7.0×10^6 Pa

Bulk modulus of copper, B = 140×10^9 Pa

Bulk modulus, $B = \frac{p}{\Delta V / V}$, where ΔV/V = Volumetric strain

ΔV = Change in volume, V = Original volume.

$$\Delta V = \frac{pV}{B}$$

Original volume of the cube, V = ℓ^3

$$\therefore \quad \Delta V = \frac{p\ell^3}{B} = \frac{7 \times 10^6 \times (0.1)^3}{140 \times 10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^{-3}$$

Therefore, the volume contraction of the solid copper cube is 5×10^{-2} cm^{-3}.

9.16 How much should the pressure on a litre of water be changed to compress it by 0.10%?

Sol. Volume of water, V = 1 L

It is given that water is to be compressed by 0.10%.

$$\therefore \quad \text{Fractional change} = \frac{\Delta V}{V} = \frac{0.1}{100 \times 1} = 10^{-3}$$

Bulk modulus, $B = \frac{\rho}{\Delta V / V}$,

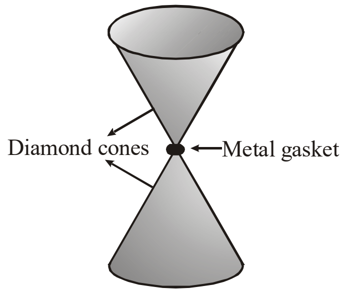
$$\rho = B \times \frac{\Delta V}{V}$$

Bulk modulus of water = B = 2.2×10^9 Nm^{-2}.

$\rho = 2.2 \times 10^9 \times 10^{-3} = 2.2 \times 10^6$ Nm^{-2}.

Therefore, the pressure on water should be 2.2×10^6 Nm^{-2}.

9.17 Anvils made of single crystals of diamond, with the shape as shown in figure, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000N. What is the pressure at the tip of the anvil?

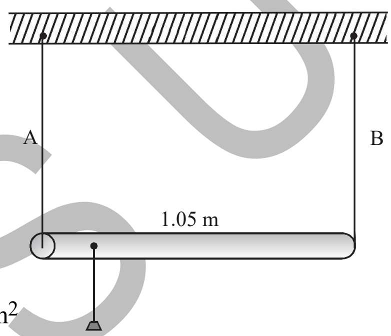


Sol. Here F = 50,000 N, d = 0.5 mm = 5×10^{-4} m

$$\therefore \quad A = \frac{\pi d^2}{4} = \frac{3.14\,(5 \times 10^{-4})}{4} m^2$$

$$\therefore \quad P = \frac{F}{A} = \frac{50000 \times 4}{3.14 \times (5 \times 10^{-4})^2} = 2.5 \times 10^{11}\,\text{Pa}$$

9.18 A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig.. The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2, respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.



Sol. (a) 0.7 m from the steel-wire end

(b) 0.432 m from the steel-wire end

Cross-sectional area of wire A, a_1 = 1.0 mm^2 = 1.0×10^{-6} m^2

Cross-sectional area of wire B, a_2 = 2.0 mm^2 = 2.0×10^{-6} m2

Young's modulus for steel, $Y_1 = 2 \times 10^{11}$ Nm^{-2}

Young's modulus for aluminium, $Y_2 = 7.0 \times 10^{10}$ Nm^{-2}

(a) Let a small mass m be suspended to the rod at a distance y from the end where wire A is attached.

Stress in the wire = $\frac{\text{Force}}{\text{Area}} = \frac{F}{a}$

If the two wires have equal stresses, then:

Where, F_1 = Force exerted on the steel wire

F_2 = Force exerted on the aluminum wire

The situation is shown in the following figure.

Taking torque about the point of suspension, we have:

$$F_1 y = F_2\,(1.05 - y)$$

$$\frac{F_1}{F_2} = \frac{(1.05 - y)}{y} \qquad \text{.......... (2)}$$

Using eq. (1) and (2), we can write

$$\frac{(1.05 - y)}{y} = \frac{1}{2}\ ; \quad 2\,(1.05 - y) = y$$

$$2.1 - 2y = y\ ; \ 3y = 2.1 \ \therefore \ y = 0.7\text{m}$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of 0.7m from the end where wire A is attached.

(b) Young modulus = $\frac{\text{Stress}}{\text{Strain}}$

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/a}{Y}$$

If the strain in the two wires is equal, then

$$\frac{F_1/a_1}{Y_1} = \frac{F_2/a_2}{Y_2}$$

$$\frac{F_1}{F_2} = \frac{a_1 Y_1}{a_2 Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7} \quad \text{....... (3)}$$

Taking torque about the point where mass m, is suspended at a distance y_1 from the side where wire A attached, we get:

$$F_1 y_1 = F_2 (1.05 - y_1)$$

$$\frac{F_1}{F_2} = \frac{(1.05 - y_1)}{y_1} \quad \text{....... (4)}$$

Using eq. (3) and (4), we get

$$\frac{(1.05 - y_1)}{y_1} = \frac{10}{7}$$

$$7 (1.05 - y_1) = 10 y_1$$

$$17 y_1 = 7.35 \quad \therefore \quad y_1 = 0.432 \text{ m}$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of 0.432m from the end where wire A is attached.

9.19 A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

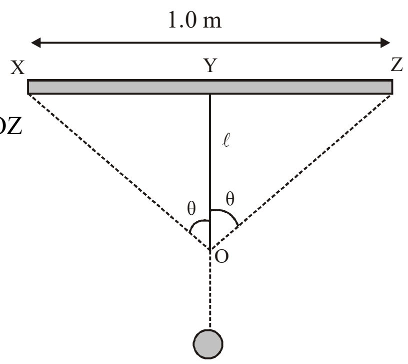
Sol. Length of the steel wire = 1.0 m

Area of cross-section, $A = 0.50 \times 10^{-2}$ cm^2 $= 0.50 \times 10^{-6}$ m^2

A mass 100g is suspended from its midpoint.

m = 100 g = 0.1 kg

Hence, the wire dips, as shown in the given figure.



Original length = XZ, Depression = ℓ

The length after mass m, is attached to the wire = XO + OZ

Increase in the length of the wire:

$$\Delta\ell = (XO + OZ) - XZ$$

Where, $XO = OZ = [(0.5)^2 + \ell^2]^{1/2}$

$$\therefore \quad \Delta\ell = 2\,[(0.5)^2 + \ell^2]^{1/2} - 1.0$$

$$= 2 \times 0.5 \left[1 + \left(\frac{\ell}{0.5}\right)^2\right]^{1/2} - 1.0$$

Expanding and neglecting higher terms, we get

$$\Delta\ell = \frac{\ell^2}{0.5} \ ; \qquad \text{Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

Let T be the tension in the wire $\therefore$ $mg = 2T\cos\theta$

Using the figure, it can be written as:

$$\cos\theta = \frac{\ell}{((0.5)^2 + \ell^2)^{1/2}} = \frac{\ell}{(0.5)\left(1 + \left(\frac{\ell}{0.5}\right)^2\right)^{1/2}}$$

Expanding the expression and eliminating the higher terms:

$$\cos\theta = \frac{\ell}{(0.5)\left(1+\frac{\ell^2}{2\,(0.5)^2}\right)}$$

$$\left(1+\frac{\ell^2}{0.5}\right) \simeq 1 \text{ for small } \ell \qquad \therefore \quad \cos\theta = \frac{\ell}{0.5}$$

$$\therefore \quad T = \frac{mg}{2\left(\frac{\ell}{0.5}\right)} = \frac{mg \times 0.5}{2\ell} = \frac{mg}{4\ell}$$

$$\text{Young modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{mg \times 0.5}{4\ell \times A \times \ell^2} \quad ; \quad \ell = \sqrt[3]{\frac{mg \times 0.5}{4YA}}$$

Young's modulus of steel, $Y = 2 \times 10^{11}$ Pa

$$\therefore \quad \ell = \sqrt{\frac{0.1 \times 9.8 \times 0.5}{4 \times 2 \times 10^{11} \times 0.50 \times 10^{-6}}} = 0.0106 \text{ m}$$

Hence, the depression at the midpoint is 0.0106 m.

9.20 Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.

Sol. Diameter of the metal strip, $d = 6.0 \text{ mm} = 6.0 \times 10^{-3}$ m

Maximum shearing stress = 6.9×10^7 Pa

Maximum force = Maximum stress × Area = $6.9 \times 10^7 \times \pi \times (r)^2$

$= 6.9 \times 10^7 \times \pi \times (3 \times 10^{-3})^2 = 1949.94$ N

Each rivet carries one quarter of the load.

$\therefore$ Maximum tension on each rivet = 4 × 1949.94 = 7799.76 N

9.21 The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

Sol. Water pressure at the bottom, $p = 1.1 \times 10^8$ Pa

Initial volume of the steel ball, $V = 0.32 \text{ m}^3$

Bulk modulus of steel, $B = 1.6 \times 10^{11} \text{ Nm}^{-2}$

The ball falls at the bottom of the Pacific Ocean, which is 11 km beneath the surface.

Let the change in the volume of the ball on reaching the bottom of the trench be 4V.

$$\text{Bulk modulus, } B = \frac{p}{\Delta V / V}$$

$$\Delta V = \frac{B}{pV} = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}} = 2.2 \times 10^{-4} \text{ m}^3$$

Therefore, the change in volume of the ball on reaching the bottom of the trench is $2.2 \times 10^{-4} \text{ m}^3$.