NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 1
ELECTRIC CHARGES AND FIELDS

1.1 What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30cm. apart in air?

Sol. Given, $q_1 = 2 \times 10^{-7}$ C, $q_2 = 3 \times 10^{-7}$ C, r = 30cm = 0.3m

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{9\times10^9 \times 2\times10^{-7} \times 3\times10^{-7}}{(0.3)^2} = 6 \times 10^{-3} \text{ N (Repulsive)}$$

1.2 The electrostatic force on a small sphere of charge 0.4μC due to another small sphere of charge –0.8μC in air is 0.2 N.
(a) What is the distance between the two spheres ?
(b) What is the force on the second sphere due to the first ?

Sol. (a) Given, F = 0.2 N, $q_1 = 0.4\ \mu C = 0.4 \times 10^{-6}$ C, $q_2 = 0.8\ \mu C = 0.8 \times 10^{-6}$ C

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}. \text{ Thus, } r^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{F}$$

$$r^2 = \frac{9\times10^9 \times 0.4\times10^{-6} \times 0.8\times10^{-6}}{0.2} = 36 \times 4 \times 10^{-4} = 144 \times 10^{-4}$$

$r = 12 \times 10^{-2}$ m = 12cm.

(b) $F_{12} = F_{21}$, i.e. 0.2 N and force is attractive as charges are unlike.

1.3 Check out the ratio $\frac{ke^2}{Gm_em_p}$ is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does the ratio signify?

Sol. $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} ; $m_e = 9.1 \times 10^{-31}$ kg; $m_p = 1.67 \times 10^{-27}$ kg e $= 1.1 \times 10^{-31}$ kg

and $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$

$$\text{Now, } \frac{ke^2}{Gm_em_p} = \frac{9\times10^9 \frac{Nm^2}{C^2} \times 1.6\times10^{-19} C \times 1.6\times10^{-19} C}{6.67\times10^{-11} Nm^2kg^{-2} \times 9.1\times10^{-31} kg \times 1.67\times10^{-27}} = 2.4 \times 10^{39}$$

which is dimensionless.
This is the ratio of electric force to the gravitational force (at the same distance) between an electron and a proton.
It also establishes that the electrostatic force is about 10^{39} times stronger than the gravitational force.

1.4 (a) Explain the meaning of the statement 'electric charge of a body is quantised'.
(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Sol. (a) Quantisation of electric charge : The electric charge is always an integral multiple of e which is termed as quantization of charge. i.e., $q = \pm ne$
Here +e is taken as charge on a proton while –e is taken as charge on the electron. The charge on a proton and electron are numerically equal i.e., 1.6×10^{-19} but opposite in sign. "Quantization is a property due to which charge exists in discrete packets rather than in continuous quantity."

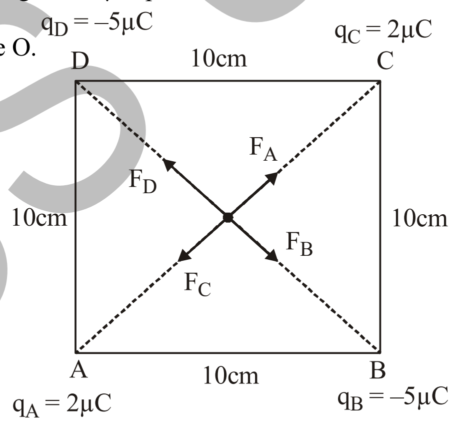
(b) Based on many practical phenomena, we may ignore quantisation of electric charge and consider the charge to be continuous. Larger scale electric charge may be considered as integral multiple of the basic unit e. The "graininess" of charge can be ignored and it can be imagined that this large scale (macroscopic level) can be charged continuously and its quantisation is insignificant and can be ignored.

1.5 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Sol. When a glass rood is rubbed with silk cloth, glass rod becomes positively charged while silk cloth becomes negatively charged. The amount of positive charge on the glass rod is found to be exactly the same as negative charge on silk cloth. Thus, the system of glass rod and silk cloth, which was neutral before rubbing, still possesses no net charge after rubbing.
Similar phenomenon is observed with other pairs of bodies. Thus, in an isolated system the total charge is neither created nor destroyed, the change is simply transferred from one body to the other. This observation is consistent with the law of conservation of charges.

1.6 Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$, and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?

Sol. Suppose a square ABCD with each side of 10cm. and centre O.
At the centre, the charge of 1μC is placed.
$q_D = -5\mu C$, $q_C = 2\mu C$, $q_A = 2\mu C$, $q_B = -5\mu C$
As $q_A = q_C$, the charge of 1μC experiences equal and opposite forces F_A and F_C due to charges q_A and q_C.
At the same time, the charge 1μC experiences equal and opposite forces.
F_B and F_D due to charges q_B and q_D. Thus the net force on charge of 1μC due to the given charges is zero.

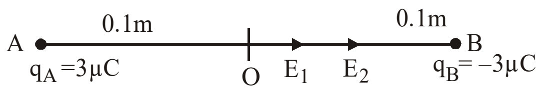


1.7 (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?
(b) Explain why two field lines never cross each other at any point?

Sol. (a) The direction of electric field at a point is displayed by the tangent at that point on a line of force. Generally the direction of electric field changes from point to point. Therefore, the lines of force are generally, curved lines. Further, they are continuous curves and cannot have sudden breaks. Even if it is so, the absence of electric field at the break points will be indicated by it.
(b) Intersection of two lines will result into two directions of the electric field at one point which is not possible.

1.8 Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.
(a) What is the electric field at the midpoint O of the line AB joining the two charges?
(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

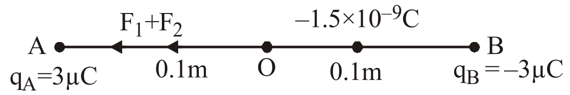
Sol. $q_A = 3\ \mu C = 3 \times 10^{-6}$ C, $q_B = -3\ \mu C = -3 \times 10^{-6}$ C and d = 20cm.
(a) Let us assume that a unit positive test charge is placed at O. q_A will repel this test charge while q_B will attract.



$\vec{E} = \vec{E}_1 + \vec{E}_2$; $E = E_1 + E_2$ (as E_1 and E_2 are in same direction)

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_A}{r^2} + \frac{1}{4\pi\varepsilon_0}.\frac{q_B}{r^2} = \frac{9\times10^9}{(0.1)^2}\,[3 \times 10^{-6} + 3 \times 10^{-6}] = 5.4 \times 10^6\ NC^{-1} \text{ along OB}$$

(b) As a negative test charge of 1.5×10^{-19}C is placed at O. q_A will attract it while q_B will repel.



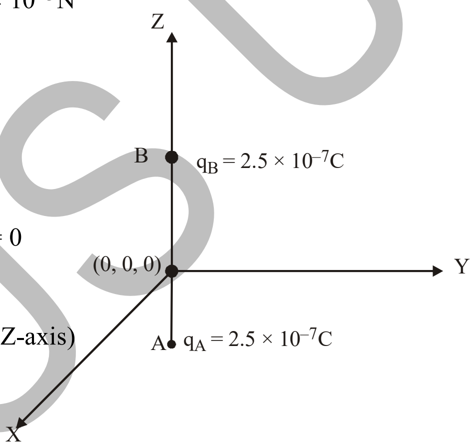
Therefore, the net force

$$F = F_1 + F_2 = \frac{9\times10^9\times3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2} + \frac{9\times10^9\times3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2}$$

$$= \frac{9\times10^9\times3\times10^{-6}\times1.5\times10^{-9}\times2}{(0.1)^2} = 8.1 \times 10^{-3}\ N$$

Alt. $F = q_0E = 1.5 \times 10^{-19} \times 5.4 \times 10^6 = 8.1 \times 10^{-3}$N

1.9 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, −15 cm) and B: (0,0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?



Sol. Total charge, $q = q_A + q_B = 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Distance between charges = 15 + 15 = 30cm = 0.3m

Electric dipole moment,

$p = q\,.\,a = 2.5 \times 10^{-7}$ (0.3m) $= 7.5 \times 10^{-5}$ Cm (along Z-axis)

1.10 An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4\ NC^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Sol. Given, $p = 4 \times 10^{-9}$ Cm, $\theta = 30°$; $E = 5 \times 10^4\ NC^{-1}$

Torque, $\tau = \vec{p} \times \vec{E} = p.E\sin\theta = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$

or $\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \frac{1}{2}$ or $\tau = 10^{-4}$ Nm

1.11 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Sol. (a) Given $q = -3 \times 10^{-7}$ C, $e = -1.6 \times 10^{-19}$ C

$\therefore$ Number of electrons transferred $n = \frac{q}{e} = \frac{-3\times10^{-7}}{-1.6\times10^{-19}} = 1.875\times10^{12}$

Electrons are transferred from wool to polythene during rubbing as polythene has negative charge.

(b) Yes, but of negligible amount because mass of an electron is very-very small.

1.12 (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Sol. (a) $q_1 = 6.5 \times 10^{-7}$ C; $q_2 = 6.5 \times 10^{-7}$ C, r = 50cm = 0.50cm.

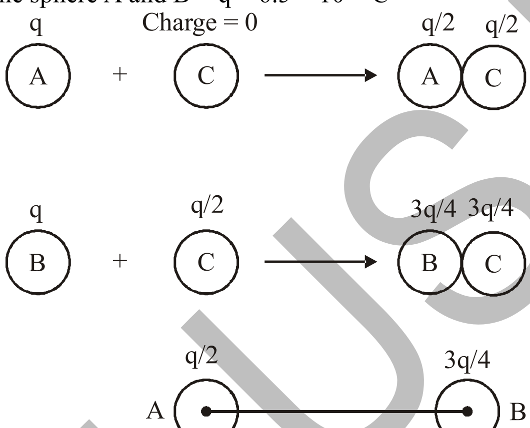
$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,\text{Nm}^2\text{C}^{-2},\ F = ?$$

Coulomb's law, $F = k\dfrac{q_1q_2}{r^2} = \dfrac{9\times10^{9}\times6.5\times10^{-7}\times6.5\times10^{-7}}{(0.50)^2} = 1.5 \times 10^{-2}$ N

(b) Now, if each sphere is charged double, and the distance between them is halved then the force of repulsion is : $F = k\dfrac{2q_1\,2q_2}{(r/2)^2}$; $F = 16k\dfrac{q_1q_2}{r^2} = 16 \times 1.5 \times 10^{-2} = 24 \times 10^{-2}$ N = 0.24 N

1.13 Suppose the spheres A and B in previous question have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Sol. Charge on each of the sphere A and B = q = 6.5×10^{-7} C



When a similar but uncharged sphere C is placed in contact with sphere A, each sphere shares a charge q/2, equally. (Charge redistribute till potential become same, same size sphere have same potential when charges are same).

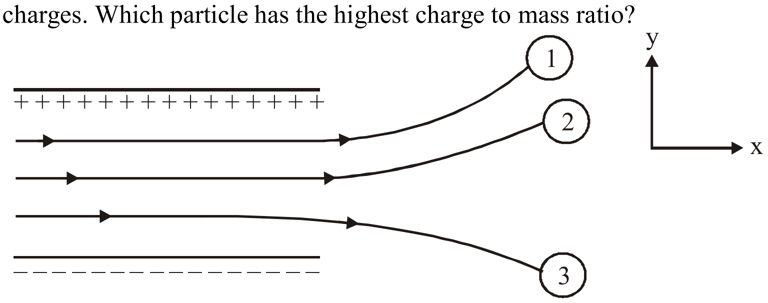
Now, if the sphere C is placed in contact with sphere B, the charge is equally redistributed, so that

Charge on sphere B or C = $\dfrac{1}{2}\left(q+\dfrac{q}{2}\right) = \dfrac{3q}{4}$

Thus, the force of repulsion between A and B is

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{3q}{4}\cdot\frac{q}{2}}{\left(\frac{r}{2}\right)^2} = \frac{3}{8}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} = \frac{3}{8}\times1.5\times10^{-2}\,\text{N} = 0.5625 \times 10^{-2}\ \text{N} = 5.7 \times 10^{-3}\ \text{N}$$

1.14 Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Sol. Particles (1) and (2) are negatively charged and particle (3) is positively charged, since the charged particles are deflected towards oppositely charged plates.
Further, as the displacement y $\propto$ (e/m) therefore, paritcle (3) having maximum value of y has the highest charge to mass ratio.

1.15 Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{i}\ NC^{-1}$. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

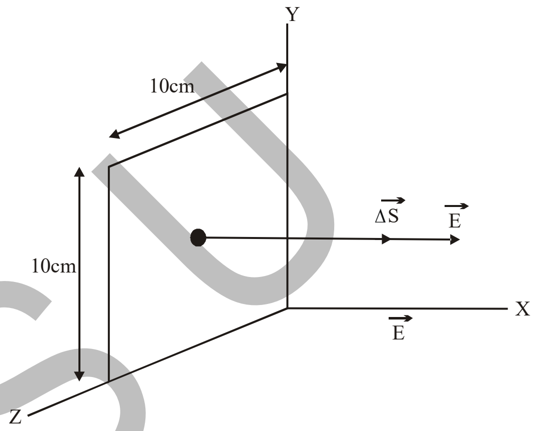
Sol. Given, $\vec{E} = 3 \times 10^3 \hat{i}\ NC^{-1}$

(a) ΔS (Area of the square = 10 × 10 = 100cm^2 = 10^{-2} m^{-2}
The area of a surface can be represented as a vector along normal to the surface.
Since normal to the square is along x-axis, we have

$$\Delta\vec{S} = 10^{-2}\hat{i}\ m^{-2}$$

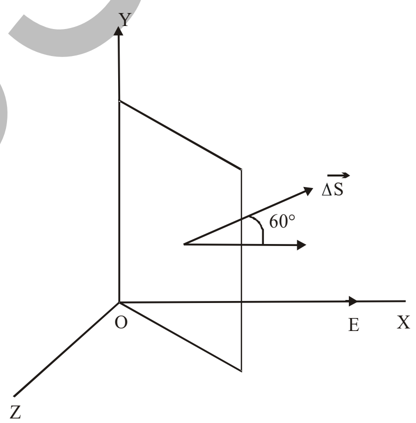
Electric flux through the square

$$\phi = \vec{E}.\Delta\vec{S} = (3 \times 10^3 \hat{i}).(10^{-2}\hat{i}) = 30\ Nm^2C^{-1}$$



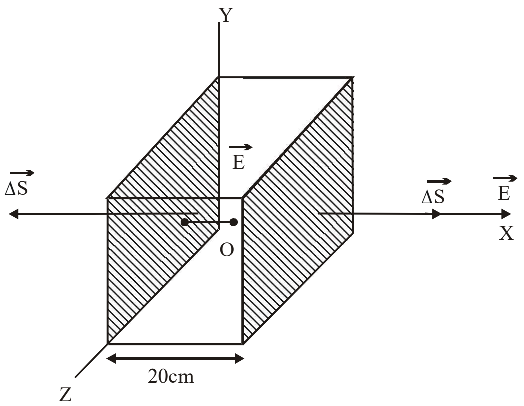
(b) Given, the angle between area vector and the electric field is 60°. Therefore,

= 3 × 10^3 × 10^{-2} × = 15 Nm^{2}C^{-1}.



1.16 What is the net flux of the uniform electric field of previous question through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Sol. Zero, $\because$ Total incoming flux = Total outgoing flux.



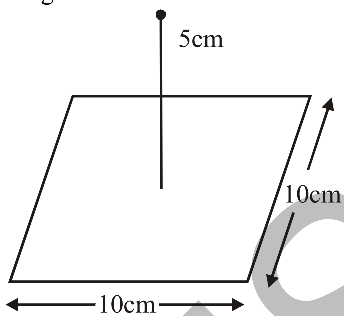
1.17 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 Nm^2/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Sol. Given, $\phi = 8.0 \times 10^3$ Nm^2/C

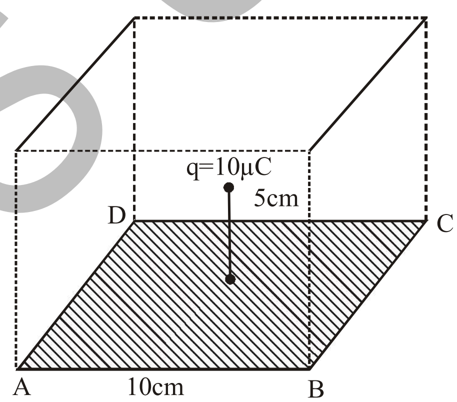
(a) As $\phi = \dfrac{q}{\varepsilon_0}$. Hence, $q = \phi \,.\, \varepsilon_0$ or $q = 8.0 \times 10^3 \text{ N} \times 8.85 \times 10^{-12} = 70.8 \times 10^{-9} = 0.07\ \mu C$

(b) No, it cannot be said so because there may be equal number of positive and negative elementary charges inside the box. It can only be said that net charge inside the box is zero.

Q.18 A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Figure. What is the magnitude of the electric flux through the square?



Sol. Let us assume that the charge $q = \pm 10\ \mu C = 10^{-5}$ C is placed at a distance of 5cm. from the square ABCD of each side 10cm.
The square ABCD can be considered as one of the six faces of a cube of each side 10cm.
Now, the total electric flux through the faces of the cube as per Gaussian theorem.
Therefore, the total electric flux through the square ABCD will be



$$\phi_E = \frac{1}{6} \times \phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{1}{6} \times \frac{10^{-5}}{8.854 \times 10^{-12}} = 1.88 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

1.19 A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Sol. Given, $q = 2.0\mu C = 2.0 \times 10^{-6}$ C
The total flux through the surface of the cube (using Gaussian theorem) is given by

$$\phi = \frac{q}{\varepsilon_0} = \frac{2.0 \times 10^{-6}}{8.854 \times 10^{-12}} = 2.26 \times 10^5 \text{ Nm}^2\text{C}^{-1}.$$

1.20 A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Sol. (a) Doubling the radius of Gaussian surface will not affect the electric flux since the charge enclosed is the same in the two cases.
Thus, the fluxs will remain be the same i.e., -1.0×10^3 Nm^2/C

(b) $\phi = \dfrac{q}{\varepsilon_0}$ $\therefore q = \phi \,.\, \varepsilon_0$ or, $q = -1.0 \times 10^3 \times 8.8 \times 10^{-12} = -8.85 \times 10^{-9}$C

1.21 A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is
1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Sol. Using, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, $q = E.\,4\pi\varepsilon_0\,.\,r^2$

or, $q = 1.5 \times 10^3 \times \left(\frac{1}{9\times10^9}\right)\times(0.2)^2$

or $q = \frac{6}{9}\times10^{-8} = \frac{60}{9}\times10^{-9} = 6.67 \times 10^{-9}$ C

Here, q is negative since electric field is directed inward.
Thus, $q = 6.67 \times 10^{-9}$ C = – 6.67 nC.

1.22 A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0\mu C/m^2$. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Sol. Given, $r = \frac{2.4}{2} = 1.2m$; $\sigma = 80 \times 10^{-6}\ C/m^2$

(a) Charge on sphere, $q = \sigma\,.\,A = \sigma\,.\,4\pi r^2$
$q = 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2 = 1.45 \times 10^{-3}$ C
(b) The total electric flux leaving the surface of the sphere

$$\phi = \frac{q}{\varepsilon_0} = \frac{1.45\times10^{-3}}{9\times10^{-12}} = 1.6 \times 10^8\ Nm^2/C$$

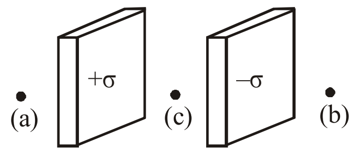
1.23 An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Sol. Given, $E = 9 \times 10^4$ N/C, $r = 2 \times 10^{-2}$ m

As, $E = \frac{\lambda}{2\pi r\varepsilon_0}$ $\therefore \lambda = E.2\pi r\varepsilon_0$ or $\lambda = \frac{9\times10^4\times2\pi\times2\times10^{-2}}{4\pi\times9\times10^9} = 10^{-7}\ Cm^{-1}$

1.24 Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\ C/m^2$. What is **E**: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

Sol. (a) To the left of the plates, electric field is zero.
(b) To the right of the plates, electric field is zero.
(c) Electric field between the plates



$E = \frac{\sigma}{\varepsilon_0} = \sigma \times 4\pi \times 9 \times 10^9$

or, $E = 17.0 \times 10^{-22} \times 4 \times 3.14 \times 9 \times 10^9$
or, $E = 1921.7 \times 10^{-13} = 1.92 \times 10^{-10}$ N/C

ADDITIONAL EXERCISES

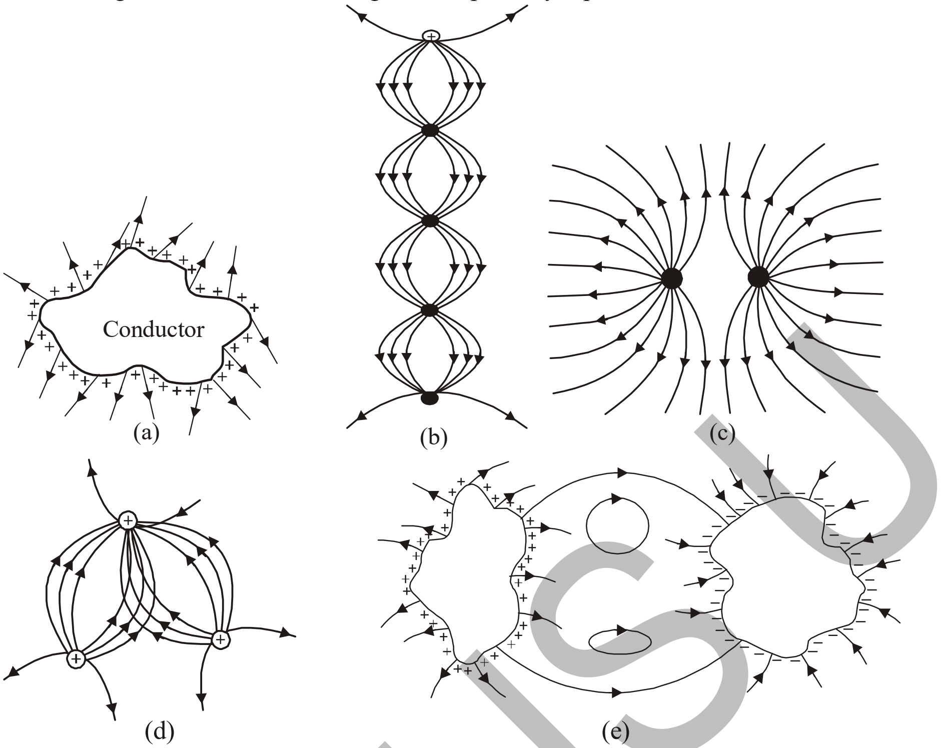
1.25 An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81\ m\ s^{-2}$; $e = 1.60 \times 10^{-19}$ C).

Sol. Given, $E = 2.25 \times 10^4\ NC^{-1}$; $n = 12$, $\rho = 1.26$ gm cm^{-3} or 1.26×10^3 kg m^{-3}
Since, the droplet is stationary weight of the droplet = force due to the electric field.

$\therefore \frac{4}{3}\pi r^3\rho g = qE = neE$ or $r^3 = \frac{3Ene}{4\pi\rho g} = \frac{3\times2.55\times10^4\times12\times1.6\times10^{-19}}{4\times3.14\times1.26\times10^3\times9.81} = 0.9 \times 10^{-18}$

$r = (0.9 \times 10^{-18})^{1/3}$; $r = 9.81 \times 10^{-7}$ m

1.26 Which among the curves shown in Fig. cannot possibly represent electrostatic field lines?



Sol. (a) Figure (a) cannot represent electrostatic field lines since electrostatic field lines start or end only at 90° to the surface of the conductor.

(b) Figure (b) cannot represent electrostatic field lines as electrostatic field lines do not start from a negative charge.

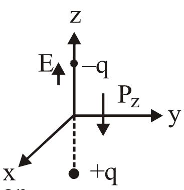
(c) Electrostatic field lines are represented by figure (c).

(d) Figure (d) cannot represent electrostatic field lines since no two such lines of force can intersect each other.

(e) As electrostatic field lines cannot form closed loop, therefore figure (e) does not represent electrostatic field lines.

1.27 In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} cm in the negative z-direction?

Sol. Force acting on a electric dipole in the positive z-direction which is placed in a non-uniform electric field. $F = P_x \dfrac{\partial E}{\partial x} + P_y \dfrac{\partial E}{\partial x} + P_z \dfrac{\partial E}{\partial z}$



As, the electric field changes uniformly in the positive z-direction,

Thus, $\dfrac{\partial E}{\partial z} = +10^5\,NC^{-1}m^{-1}$

As, the system has the total dipole moment equal to 10^{-7} Cm in the negative z-direction,

Thus, $P_x = 0, P_y = 0, P_z = -10^{-7}$ cm

$\therefore$ $F = 0 + 0 - 10^{-7} \times 10^5 = -10^{-2}$ N

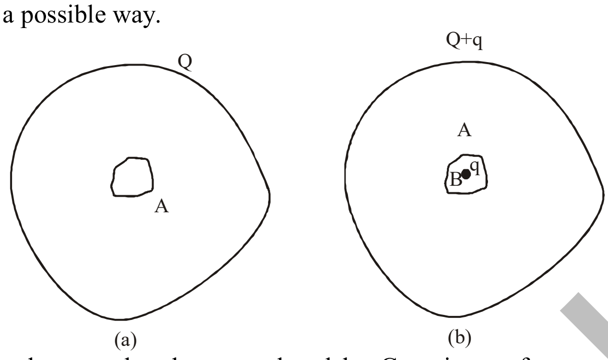
It is indicated by the negative sign that the force 10^{-2} N acts in the negative z-direction. In an electric field $\vec{E}$, the torque on dipole moment P is given by $\vec{\tau} = \vec{P} \times \vec{E}$; $|\vec{\tau}| = PE\sin\theta$

As are acting in opposite direction, $\theta = 180°$. So, $|\vec{\tau}| = PE \sin 180° = 0$

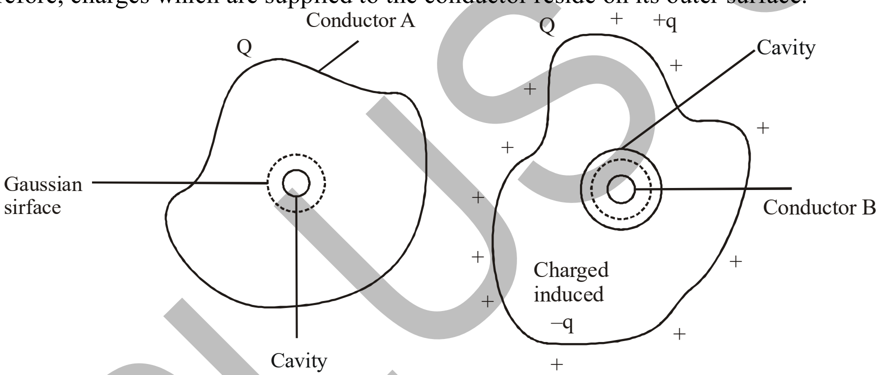
1.28 (a) A conductor A with a cavity as shown in figure (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor.

(b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Fig. (b)].

(c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.



Sol. (a) Gaussian theorem the charge enclosed by Gaussian surface must be zero as electric field vanishes everywhere inside a conductor. Thus, electric field vanishes inside the cavity. Therefore, charges which are supplied to the conductor reside on its outer surface.



(b) Let us take a Gaussian surface inside the conductor which is quite close to the cavity.

According to the Gaussian theorem, $\oint \phi_E = \int E.ds = \dfrac{\text{total charge}}{\varepsilon_0}$

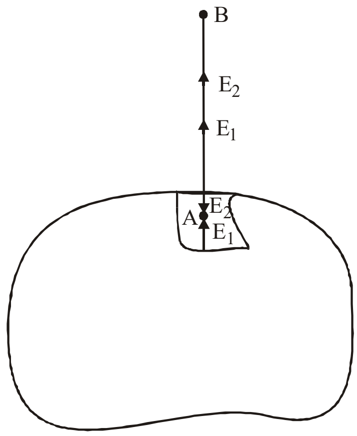
(as the electric field inside the conductor is zero)

The total charge enclosed by the Gaussian surface must be zero. This requires a charge of -q units to be induced on the inner surface of the hollow conductor A. But an equal and opposite charge +q units must appear on the outer surface of conductor A, so that the total charge on the outer surface of A is Q + q.

(c) Use a metallic surface to enclose the sensitive instrument fully. Because of the electrostatic shielding, the electric field inside the metal surface vanishes.

1.29 A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\dfrac{\sigma}{2\varepsilon_0}\right)\hat{n}$, where is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Sol. Let us take a charged conductor with the hole filled up, as shown by shaded portion in the figure.



We find with the application of Gaussian theorem that field inside is zero and just outside is $\frac{\sigma}{\varepsilon_0}\hat{n}$.

This field can be viewed as the superposition of the field E_2 due to the filled up hole plus the field E_1 due to the rest of the charged conductor.

The two fields (E_1 and E_2) must be equal and opposite as the field vanishes inside the conductor.

Thus, $E_1 - E_2 = 0$

Now, the field outside the conductor is given by

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \quad \therefore \quad 2E_1 = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad E_1 = \frac{\sigma}{2\varepsilon_0}$$

Therefore, field in the hole (due to the rest of the conductor) is given as: $E_1 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

(unit vector in the outward normal direction)

1.30 Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Use Coulomb's law directly and evaluate the necessary integral.]

Sol. Uniform line of charge

Charge per unit length = λ

(1) Symmetry considered: The E-field from +z and – z directions cancel along z-direction,

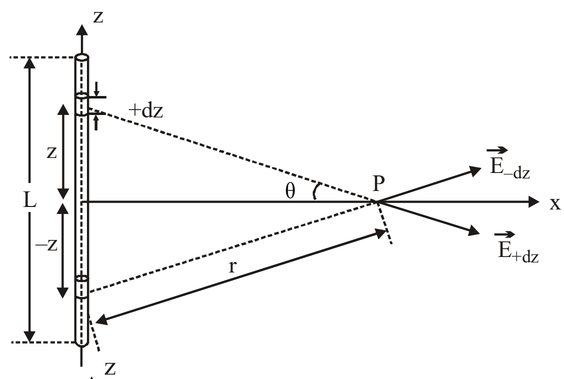
$\therefore$ Only horizontal E-field components need to be considered.

(2) For each element of length dz, charge

$$dq = \lambda\, dz$$

$\therefore$ Horizontal E-field at point P due to element dz

$$|d\vec{E}|\cos\theta = \underbrace{\frac{1}{4\pi\varepsilon_0}.\frac{\lambda dz}{r^2}}_{dE_{dz}}\cos\theta$$



$\therefore$ E-field due to entire line charge at point P

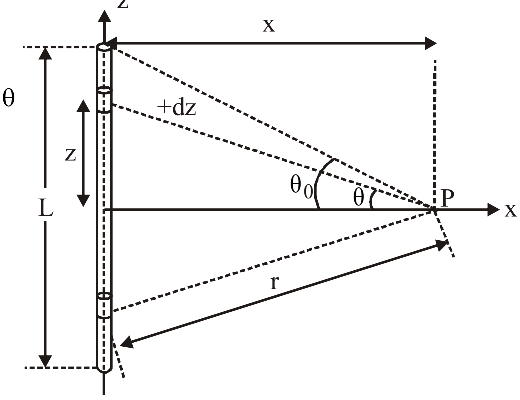
$$E = \int_{-L/2}^{L/2} \frac{1}{4\pi\varepsilon_0}.\frac{\lambda dz}{r^2}\cos\theta = 2\int_0^{L/2} \frac{\lambda}{4\pi\varepsilon_0}.\frac{dz}{r^2}\cos\theta$$

To calculate this integral:

(i) First, notice that x is fixed, but z; r; θ all varies.

(ii) Change of variable (from z to θ)

(1) $z = x\tan\theta \quad \therefore \quad dz = x\sec^2\theta\, d\theta$



$x = r\cos\theta \quad \therefore\ r^2 = x^2\sec^2\theta$

$z = 0, \qquad \theta = 0°$

(2) When $z = L/2$, $\theta = \theta_0$ where $\tan\theta_0 = \frac{L/2}{x}$

$$E = 2.\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\theta_0}\frac{x\sec^2\theta d\theta}{x^2\sec^2\theta}.\cos\theta = 2.\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\theta_0}\frac{1}{x}.\cos\theta\, d\theta = 2.\frac{\lambda}{4\pi\varepsilon_0}\frac{1}{x}(\sin\theta)\Big|_0^{\theta_0}$$

$$= 2.\frac{\lambda}{4\pi\varepsilon_0}\frac{1}{x}\sin\theta_0 = 2.\frac{\lambda}{4\pi\varepsilon_0}\frac{1}{x}\frac{L/2}{\sqrt{x^2+\left(\frac{L}{2}\right)^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{x\sqrt{x^2+\left(\frac{L}{2}\right)^2}} \text{ along x-direction}$$

Limiting case:

$L >> x :\ ;\ E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{x.\frac{L}{2}}\ ;\ E_x = \frac{\lambda}{2\pi\varepsilon_0 x}$

1.31 It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3)e, and the 'down' quark (denoted by d) of charge (–1/3)e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Sol. Charge on 'up' quark, (u) = $+\frac{2}{3}e$; Charge on 'down' quark, (d) = $-\frac{1}{3}e$

Charge on proton = e ; Charge on a neutron = 0

Let a proton contains x 'up' quarks and (3 – x) 'down' quarks. Then total charge on a point is

$$+\frac{2}{3}ex - \frac{1}{3}e\,(3-x) = e \quad\text{or}\quad +\frac{2}{3}x - 1 + \frac{x}{3} = 1 \quad\text{or,}\ x = 2$$

and $3 - x = 3 - 2 = 1$

i.e., protons contain 2 'up' quarks and 1 'down' quark. It quark composition should be 'uud'.

Let a neutron contains y 'up' quarks and (3 – y) 'down' quarks.

Then total charge on a neutron is $+\frac{2}{3}ey - \frac{1}{3}e\,(3-y) = 0$ or $+\frac{2}{3}y - 1 + \frac{y}{3} = 0$ or $y = 1$

and $3 - y = 3 - 1 = 2$

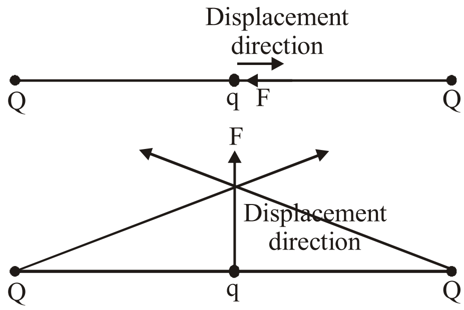
i.e., neutrons contain 1 'up' quark and 2 'down' quarks. Its quark composition should be 'uud'.

1.32 (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where E = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Sol. (a) It can be proved by contradiction. Assume that the test charge placed at null point be in stable equilibrium. The test charge displaced slightly in any direction will experience a restoring force towards the null-point as the stable equilibrium requires restoring force in all directions. That is, all field lines near the null point should be directed inwards towards the null point. This indicates that there is a net inward flux of electric field through a closed surface around the null point. But, according to Gauss law, the flux of electric field through a surface enclosing no charge must be zero. This contradicts our assumption. Therefore, the test charge placed at null point must be necessarily in unstable equilibrium.

(b) On the mid-point of the line joining the two charges, the null point lies. The test charge will experience a restoring force if it is displaced slightly on either side of the null point along this line. While the net force takes it away from the null point if it is displaced normal to this line. That is no restoring force acts in the normal direction. But restoring force in all directions is demanded by stable equilibrium, therefore, test charge placed at null point will not be in stable equilibrium.

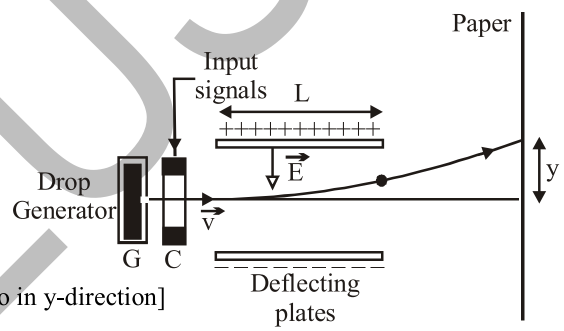


1.33 A particle of mass m and charge – q enters the region between the two charged plates initially moving along x-axis with speed v_x. The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2mv_x^2)$.

Sol. The particle is moving along x-axis in a uniformly charged electric field between two oppositely charged metallic plates of length L. The motion of an electron in an electric field is analogous to the motion of a projectile in the gravitational field. The only difference is that here the constant electric field is upward and is limited to the region between the plates.

Since x-component of the electric force is zero therefore, acceleration along x-axis is zero. So, the velocity v_x along x-axis is constant. If x is the horizontal distance covered in time t, then

$$x = v_x t \text{ or } t = \frac{x}{v_x}$$



Force acting along y-axis, $F_y = qE$

Acceleration along y-axis, $a_y = \frac{qE}{m}$

where m is the mass of particle (electron)

If y is the vertical distance covered by the particle in time t, then

$$y = \frac{1}{2} a_y t^2 \quad [\because \text{Initial velocity is zero in y-direction}]$$

or $$y = \frac{1}{2}\frac{qE}{m}\left(\frac{x}{v_x}\right)^2 \qquad \therefore y = \frac{qE}{2mv_x^2} x^2$$

So, within the electric field, the particle follows a parabolic path.

Let y_1 be the vertical deflection suffered by the particle inside the electric field.

When x = L, then $y = y_1$.

$$\therefore \; y_1 = \frac{qEL^2}{2mv_x^2}$$

1.34 Suppose that the particle in previous question is an electron projected with velocity $v_x = 2.0 \times 10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate? (| e | = 1.6×10^{-19} C, $m_e = 9.1 \times 10^{-31}$ kg.)

Sol. Using $y = \frac{qEL^2}{2mv_x^2}$, we get $\frac{0.5 \times 10^{-2}}{2} = \frac{(1.6 \times 10^{-19})(9.1 \times 10^2)(L^2)}{2 \times 9.1 \times 10^{-31} \times 2 \times 10^6 \times 2 \times 10^6}$

i.e., $L^2 = \frac{2}{1.6} \times 10^{-4}$ i.e., $L = 1.12 \times 10^{-2} = 1.12$ cm.