NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 10
WAVE OPTICS

10.1 Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

Sol. Given, $\lambda = 589\text{nm}$, $c = 3 \times 10^8$ m/s, $\mu = 1.33$

(a) For reflected light

Wavelength $\lambda = 589\text{nm} = 589 \times 10^{-9}$ m

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{589\times10^{-9}} = 5.09 \times 10^{14} \text{ Hz}$$

Speed $v = c = 3 \times 10^8$ m/s

(b) For refracted light, $\lambda' = \frac{\lambda}{\mu} = \frac{589\times10^{-9}}{1.33} = 4.42 \times 10^{-7}$ m

As frequency remains unaffected on entering another medium, therefore,

$\nu' = \nu = 5.09 \times 10^{14}$ Hz

Speed, $v' = \frac{c}{\mu} = \frac{3\times10^8}{1.33} = 2.55 \times 10^8$ m/s

10.2 What is the shape of the wavefront in each of the following cases:

(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Sol. (a) Spherical

(b) Plane

(c) Plane (Because a small area on the surface of a large sphere is nearly planar)

10.3 (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 m s^{-1})

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Sol. (a) Refractive index, $\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$

$\therefore$ Speed of light in glass $= \frac{\text{speed of light in vacuum}}{\mu_g} = \frac{3.0\times10^6}{1.5} = 2.0 \times 10^6$ m/s

(b) The speed of light in glass is not independent of the colour of light.

The refractive index of a violet component of white light is greater than the refractive index of a red component. Hence, the speed of violet light is less than the speed of red light in glass. Hence, violet light travels slower than red light in a glass prism.

10.4 In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

Sol. Given, $D = 1.4$m, $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m

Fringe width, $\omega = \dfrac{1.2}{4} = 0.3 \times 10^{-2}$ m, Using formula, Fringe width $\omega = \dfrac{D\lambda}{d}$

$$\therefore \quad \lambda = \frac{\omega d}{D} = \frac{0.3 \times 10^{-2} \times 0.28 \times 10^{-3}}{1.4} = 0.06 \times 10^{-5} = 600 \times 10^{-9} = 600 \text{ nm.}$$

10.5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Sol. Phase difference corresponding to λ is 2π and phase difference corresponding to $\lambda/3$ is $2\pi/3$.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Let that $I_1 = I_2 = I_0$

In the first case, $K = I_0 + I_0 + 2I_0 \cos 2\pi = 4I_0$

In the second case, $K' = I_0 + I_0 + 2I_0 \cos(2\pi/3) = I_0 + I_0 - 2I_0 (1/2) = I_0$

Now, $\dfrac{K'}{K} = \dfrac{I_0}{4I_0} = \dfrac{1}{4}$ or $K' = \dfrac{K}{4}$

10.6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Sol. Here, $\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9}$ m

$\lambda_2 = 520 \text{ nm} = 520 \times 10^{-9}$ m

Suppose, d = distance between two slits

(a) For third bright fringe, n = 3

$$x = n\lambda_1 \frac{D}{d} = 3 \times 650 \times 10^{-7} \times \frac{120}{0.2} = 0.117 \text{ cm} = 1.17 \text{ mm}$$

(b) Let n fringes of wavelength 650nm coincide with (n + 1) fringes of wavelength 520nm.

$$x = n\lambda_1 \frac{D}{d} = \frac{(n+1)\,D}{d} \times \lambda_2 \quad \text{or} \quad x = n \times 650 = (n+1) \times 520$$

$$\text{or} \quad \frac{n+1}{n} = \frac{650}{520} = \frac{5}{4} \quad \text{or} \quad 1 + \frac{1}{n} = \frac{5}{4} \Rightarrow \frac{1}{n} = \frac{5}{4} - 1 = \frac{1}{4} \quad \text{or} \quad n = 4$$

Hence, $x = n\lambda_1 \dfrac{D}{d} = 4 \times 650 = 10^{-7} \times \dfrac{120}{0.2} = 1.56$mm

10.7 In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

Sol. Angular fringe separation, $\theta = \dfrac{\lambda}{d}$ or $d = \dfrac{\lambda}{\theta}$

In water, $d = \dfrac{\lambda'}{\theta'}$ $\quad \therefore \dfrac{\lambda}{\theta} = \dfrac{\lambda'}{\theta'}$ or $\dfrac{\theta'}{\theta} = \dfrac{\lambda'}{\lambda} = \dfrac{1}{\mu} = \dfrac{3}{4}$ or $\theta' = \dfrac{3}{4}\theta = \dfrac{3}{4} \times 0.2° = 0.15°$

10.8 What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

Sol. Given, $\mu = 1.5$. Using formula, $\tan i_p = \mu$ or $i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5)$

Thus, $i_p = 56.3°$

10.9 Light of wavelength 5000Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Sol. The wavelength and frequency of the reflected light are the same as that of the incident light.

$\therefore$ Wavelength of reflected light = 5000 Å

Frequency of reflected light = $c/\lambda = \frac{3 \times 10^8}{5000 \times 10^{-10}}$ Hz = 6×10^{14} Hz

According to law of reflection, i = r

The reflected ray is normal to the incident ray.

i + r = 90°

i + i = 90°

2i = 90°

or i = 45°

10.10 Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400nm.

Sol. Here, a = 4mm = 4×10^{-3} m

λ = 400nm = 400×10^{-9} m = 4×10^{-7} m

Ray optics is good approximation upto distances equal to Fresnel's distance (Z_F).

$$Z_F = \frac{a^2}{\lambda} = \frac{(4 \times 10^{-3})^2}{4 \times 10^{-7}} = 40\text{m}$$

10.11 The 6563 Å H_α line emitted by hydrogen in a star is found to be red shifted by 15Å. Estimate the speed with which the star is receding from the Earth.

Sol. λ = 6563 Å = 6563×10^{-10} m

$\lambda' - \lambda$ = 15Å = 15×10^{-10} m

$$\lambda' - \lambda = \frac{V_s \lambda}{c}$$

$$V_s = \frac{c}{\lambda}(\lambda' - \lambda) = \frac{3.0 \times 10^8}{6563 \times 10^{-10}} \times 15 \times 10^{-10} = 6.8566 \times 10^5 \text{ ms}^{-1}$$

or $V_s = 6.86 \times 10^5$ ms^{-1}

10.12 Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

Sol. No; Wave theory

Newton's corpuscular theory of light states that when light corpuscles strike the interface of two media from a rarer (air) to a denser (water) medium, the particles experience forces of attraction normal to the surface. Hence, the normal component of velocity increases while the component along the surface remains unchanged.

Hence, we can write the expression: c sin i = v sin r … (i)

Where, i = Angle of incidence, r = Angle of reflection, c = Velocity of light in air,

v = Velocity of light in water

We have the relation for relative refractive index of water with respect to air as:

$$\mu = \frac{v}{c}$$

Hence, equation (i) reduces to $\frac{v}{c} = \frac{\sin i}{\sin r} = \mu$ .... (ii)

But, $\mu > 1$

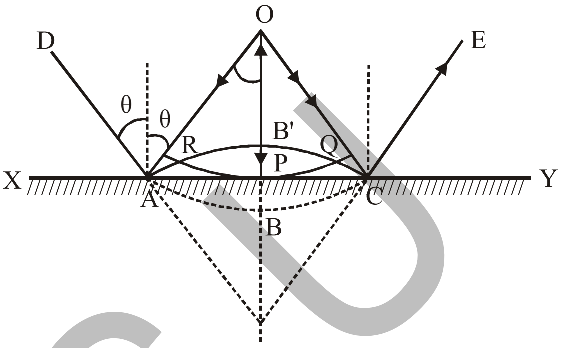
Hence, it can be inferred from equation (ii) that v > c.

This is not possible since this prediction is opposite to the experimental results of c > v.

The wave picture of light is consistent with the experimental results.

10.13 You have learnt in the text how Huygens' Principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

Sol. Let O be a point object in front of plane mirror XY at a normal distance OP from it. Spherical wavefront starts from it. A part RPQ of the wavefront touches the plane mirror at P. Whereas disturbance from R and Q continues moving forward along normals (rays) OR and OQ, that from P reflects back. When disturbances from R and Q reach the mirror at A and C respectively, that from P reaches B'. This gives rise to reflected spherical wavefront AB'C. ABC is the virtual position (position in basence of mirror) of the wavefront. The reflected wavefront AB'C, appears to start from I.



I becomes virtual image for O as real point object. Draw AN normal to XY, hence parallel to PQ. Now, OA is incident ray (being normal to incident wavefront ABC) and AD is reflected ray (being normal to reflected wavefront AB'C).

Hence, $\angle$ OAN = $\angle$ DAN = θ [i = r]
But $\angle$ OAN = alternate $\angle$ AOP
and $\angle$ DAN = corresponding $\angle$ AIP
$\angle$ AOP = $\angle$ AIP

Now, in Δ AIP and Δ AOP
$\angle$ AIP = $\angle$ AOP (each θ)
$\angle$ API = $\angle$ APO (each 90°)

AP is common to both, Δ_s become congruent. Hence, PI = PQ

i.e., normal distance of image from the mirror = normal distance of object from the mirror.
Thus, virtual image is formed as much behind the mirror as the object in front of it.

10.14 Let us list some of the factors, which could possibly influence the speed of wave propagation:
(i) nature of the source. (ii) direction of propagation.
(iii) motion of the source and/or observer. (iv) wavelength. (v) intensity of the wave.
On which of these factors, if any, does (a) the speed of light in vacuum, (b) the speed of light in a medium (say, glass or water), depend?

Sol. (a) The speed of light in a vacuum i.e., 3×10^8 m/s (approximately) is a universal constant. It is not affected by the motion of the source, the observer, or both. Hence, the given factor does not affect the speed of light in a vacuum.
(b) Out of the listed factors, the speed of light in a medium depends on the wavelength of light in that medium.

10.15 For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

Sol. Sound waves can propagate only through a medium. The two given situations are not scientifically identical because the motion of an observer relative to a medium is different in the two situations. Hence, the Doppler formulas for the two situations cannot be the same.

In case of light waves, sound can travel in a vacuum. In a vacuum, the above two cases are identical because the speed of light is independent of the motion of the observer and the motion of the source. When light travels in a medium, the above two cases are not identical because the speed of light depends on the wavelength of the medium.

10.16 In double-slit experiment using light of wavelength 600nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Sol. Angular fringe width, $\beta_\theta = \frac{\lambda}{d} \Rightarrow d = \frac{\lambda}{\beta_\theta}$

Now, $\lambda = 600 \times 10^{-9}$ m,

$$\beta_\theta = 0.1° = \frac{0.1 \times \pi}{180} \text{ radian} = \frac{0.1 \times 3.14}{180} \text{ radian}$$

$$\therefore \quad d = \frac{600 \times 10^{-9} \times 180}{0.1 \times 3.14} \text{m} = 3.44 \times 10^{-4} \text{ m.}$$

10.17 Answer the following questions:

(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

(d) Two students are separated by a 7m partition wall in a room 10m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.

(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

Sol. (a) In a single slit diffraction experiment, if the width of the slit is made double the original width, then the size of the central diffraction band reduces to half and the intensity of the central diffraction band increases up to four times.

(b) The interference pattern in a double-slit experiment is modulated by diffraction from each slit. The pattern is the result of the interference of the diffracted wave from each slit.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. This is because light waves are diffracted from the edge of the circular obstacle, which interferes constructively at the centre of the shadow. This constructive interference produces a bright spot.

(d) Bending of waves by obstacles by a large angle is possible when the size of the obstacle is comparable to the wavelength of the waves.
On the one hand, the wavelength of the light waves is too small in comparison to the size of the obstacle. Thus, the diffraction angle will be very small. Hence, the students are unable to see each other. On the other hand, the size of the wall is comparable to the wavelength of the sound waves. Thus, the bending of the waves takes place at a large angle. Hence, the students are able to hear each other.

(e) The justification is that in ordinary optical instruments, the size of the aperture involved is much larger than the wavelength of the light used.

10.18 Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

Sol. Size of aperture, a = 50m

Distance of aperture from tower, $Z_F = \frac{40}{2} = 20 \text{ km} = 20 \times 10^3$ m

Fresnel distance, $Z_F = \frac{a^2}{\lambda} \Rightarrow \lambda = \frac{a^2}{Z_F} = \frac{(50)^2}{20 \times 10^3}$ or $\lambda = 125 \times 10^{-3}$ m = 12.5cm.

10.19 A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

Sol. Given, D = 1m, n = 1

$x = 2.5\text{mm} = 2.5 \times 10^{-3}$ m

$\lambda = 500\text{nm} = 500 \times 10^{-9} \text{ m} = 5 \times 10^{-7}$ m

Using formula, $x = n\frac{\lambda D}{d} \Rightarrow d = \frac{n\lambda D}{x}$ or $d = \frac{1 \times 5 \times 10^{-7} \times 1}{2.5 \times 10^{-3}} = 2 \times 10^{-4}$ m = 0.2 mm

10.20 Answer the following questions:

(a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.

(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Sol. (a) Weak radar signals sent by a low flying aircraft can interfere with the TV signals received by the antenna. As a result, the TV signals may get distorted. Hence, when a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen.

(b) The principle of linear superposition of wave displacement is essential to our understanding of intensity distributions and interference patterns. This is because superposition follows from the linear character of a differential equation that governs wave motion. If y_1 and y_2 are the solutions of the second order wave equation, then any linear combination of y_1 and y_2 will also be the solution of the wave equation.

10.21 In deriving the single slit diffraction pattern, it was stated that the 1intensity is zero at angles of nλ/a. Justify this by suitably dividing the slit to bring out the cancellation.

Sol. Let the slit width 'a' be dividing into n equal parts of width 'a' so that, a' = a/n or a = na'

Then angle, $\theta = \frac{n\lambda}{a} = \frac{n\lambda}{na'}$ or $\theta = \frac{\lambda}{a'}$

At this angle, each slit part will make first diffraction minimum. Hence, resultant intensity to all slits will be zero in that direction.