NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 11
DUAL NATURE OF MATTER AND RADIATION

11.1 Find the :
(a) maximum frequency, and (b) minimum wavelength of X-rays produced by 30 kV electrons.

Sol. (a) Using $\nu_{max} = \frac{eV}{h}$, where h = 6.63 × 10^{-34} Js and e = 1.6 × 10^{-19} C

$$\therefore \quad \nu_{max} = \frac{1.6 \times 10^{-19} \times 30000}{6.63 \times 10^{-34}} = 7.24 \times 10^{18}\ \text{Hz}$$

(b) $$\lambda_{min} = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34})\,(3 \times 10^{8})}{(1.6 \times 10^{-19})\,(3 \times 10^{4})} = 0.041 \times 10^{-19}\ \text{m} = 0.041\ \text{nm}.$$

11.2 The work function of caesium metal is 2.14 eV. When light of frequency 6 × 10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the
(a) maximum kinetic energy of the emitted electrons,
(b) Stopping potential, and
(c) maximum speed of the emitted photoelectrons?

Sol. Here W = 2.14 eV = 2.14 × 1.6 × 10^{-19} J = 3.424 × 10^{-19} J, $\nu = 6 \times 10^{14}$ Hz
Also, h = 6.626 × 10^{-34} Js.
(a) Maximum kinetic energy of photoelectrons is

$$\text{K.E.} = h\nu - W = 6.626 \times 10^{-34} \times 6 \times 10^{14} - 3.424 \times 10^{-19}$$

$$= (3.976 - 3.424) \times 10^{-19} = 0.55 \times 10^{-19}\ \text{J} = \frac{0.55 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} = 0.34\ \text{eV}$$

(b) Stopping potential is given by, $$V_0 = \frac{\text{Kinetic energy}}{e} = \frac{0.55 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{V} = 0.34\ \text{V}$$

(c) $$\frac{1}{2} m v_{max}^2 = 0.55 \times 10^{-19}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{0.55 \times 10^{-19} \times 2}{m}} = \sqrt{\frac{0.55 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}} = 3.488 \times 10^{5}\ \text{ms}^{-1} = 349\ \text{kms}^{-1}.$$

11.3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Sol. Here, V_0 = 1.5 V
Maximum kinetic energy, $(\text{K.E.})_{max} = eV_0 = 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19}$ J

11.4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42mW.
(a) Find the energy and momentum of each photon in the light beam,
(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and
(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Sol. Given, λ = 632.8nm = 632.8 × 10^{-9} m
Power, P = 9.42mW = 9.42 × 10^{-3} W

(a) $E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-14} \times 3 \times 10^8}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19}$ J

Also $p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.05 \times 10^{-27}$ kg ms^{-1}

(b) Number of photons emitted per second,

$$n = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}} = 3 \times 10^{16}$$

(c) Using mv = p

$$v = \frac{p}{m} = \frac{1.05 \times 10^{-27}}{1.67 \times 10^{-27}} = 0.63 \text{ ms}^{-1}$$

11.5 The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Sol. Given $\lambda = 550$ nm $= 550 \times 10^{-9}$ m

Energy flux $\phi = 1.388 \times 10^3$ Wm^{-2}.

Also, $h = 6.626 \times 10^{-34}$ Js and $c = 3 \times 10^8$ m s^{-1}

$\therefore$ Energy of each photon, $E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}} = 3.61 \times 10^{-19}$ J.

$\therefore$ no. of photons per square metre of earth $= \frac{\phi}{E} = \frac{1.388 \times 10^3}{3.61 \times 10^{-19}} = 3.85 \times 10^{21}$

11.6 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} Vs. Calculate the value of Planck's constant.

Sol. The slope of the graph in this case is $4.12 \times 10^{-15} = \frac{h}{e}$

$\therefore$ $h = 4.12 \times 10^{-15} \times e = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19} = 6.592 \times 10^{-34}$ Js

11.7 A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Sol. Power of sodium lamp, P = 100 W, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m.

(a) Energy of photon, $E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}} = 3.38 \times 10^{-19}$ J

(b) Rate at which the photons are delivered $= \frac{P}{E} = \frac{100}{3.38 \times 10^{-19}} = 2.96 \times 10^{20}$ photons/s.

11.8 The threshold frequency for a certain metal is 3.3×10^{14}Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff voltage for the photoelectric emission.

Sol. Given $\nu_0 = 3.3 \times 10^{14}$ Hz, $\nu = 8.2 \times 10^{14}$ Hz

Using, $eV_0 = h\nu - h\nu_0$

or $V_0 = \frac{h}{e}(\nu - \nu_0) = \frac{6.626 \times 10^{-34}}{1.6 \times 10^{-19}} \times (8.2 \times 10^{14} - 3.3 \times 10^{14}) = 2.03$Volt.

11.9 The work function for a certain metal is 4.2eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Sol. Here, $W = 4.2\text{eV} = 4.2 \times 1.6 \times 10^{-19}\ \text{J} = 6.72 \times 10^{-19}\ \text{J}$

$\therefore$ Using $h\nu_0 = W \Rightarrow \nu_0 = W/h$

$$\text{or} \quad \nu_0 = \frac{6.72\times10^{-19}}{6.626\times10^{-34}} = 1.01 \times 10^{15}\ \text{Hz}$$

$$\therefore\ \lambda_0 = \frac{c}{\nu_0} = \frac{3\times10^{8}}{1.01\times10^{15}} = 2.97 \times 10^{-7}\ \text{m} = 2.97 \times 10^{-7} \times 10^{9}\ \text{nm} = 297\ \text{nm}$$

Since $\lambda_0 < \lambda$ (330 nm) or $\nu_0 > \nu$, the photoelectric emission for this radiation can not take place.

11.10 Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Sol. Using the relation, $\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$, we get $h\nu_0 = h\nu - \frac{1}{2}mv_{max}^2$

$$\therefore\ \nu_0 = \nu - \frac{1}{2}\frac{m}{h}v_{max}^2 = 7.21 \times 10^{14} - \frac{1}{2}\times\frac{9.1\times10^{-31}\times(6\times10^{5})^2}{6.626\times10^{-34}}$$

$$= 7.21 \times 10^{14} - 2.47 \times 10^{14} = 4.74 \times 10^{14}\ \text{Hz}$$

11.11 Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Sol. $\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{488\times10^{-9}} = 6.15 \times 10^{14}\ \text{Hz}$

$\therefore$ Using $eV_0 = h\nu - W$

we get $W = h\nu - eV_0 = 6.626 \times 10^{-34} \times 6.15 \times 10^{14} - 1.6 \times 10^{-19} \times 0.38$

$$= 3.467 \times 10^{-19}\text{J} = \frac{3.467\times10^{-19}}{1.6\times10^{-19}}\ \text{eV} = 2.167\ \text{eV}$$

11.12 Calculate the : (a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Sol. Given V = 56 V.

(a) Momentum of the electron $P = \sqrt{2m_eV} = \sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times56} = \sqrt{1.631\times10^{-47}}$

$= 4.04 \times 10^{-24}\ \text{kg ms}^{-1}$.

(b) Wavelength $\lambda = \frac{12.27}{\sqrt{V}}\text{Å} = \frac{12.27}{\sqrt{56}}\text{Å} = 1.64\text{Å}$

11.13 What is the (a) momentum, (b) speed, and (c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Sol. (a) Momentum $p = \sqrt{2mE} = \sqrt{2\times9.1\times10^{-31}\times1.92\times10^{-17}} = \sqrt{3.49\times10^{-47}} = 5.91 \times 10^{-24}$ kg m/s

(b) Speed $v = \frac{p}{m} = \frac{5.91\times10^{-24}}{9.1\times10^{-31}} = 6.5 \times 10^{6}\ \text{ms}^{-1}$

(c) De Broglie wavelength $\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{5.91\times10^{-24}} = 1.13 \times 10^{-10}\ \text{m} = 1.131\ \text{Å}$

11.14 The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which (a) an electron, and (b) a neutron, would have the same de Broglie wavelength.

Sol. (a) For an electron, $\lambda = \dfrac{h}{\sqrt{2m_e E}}$

$$\therefore \quad E = \frac{h^2}{2m_e\lambda^2} = \frac{(6.626\times10^{-34})^2}{2\times9.1\times10^{-31}\times(589\times10^{-9})^2} = 7.03 \times 10^{-25}\ \text{J}$$

(b) For a neutron, $E = \dfrac{h^2}{2m_n\lambda^2} = \dfrac{(6.626\times10^{-34})^2}{2\times1.67\times10^{-27}\times(589\times10^{-9})^2} = 3.79 \times 10^{-28}$ J.

11.15 What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s,

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Sol. (a) Here, m = 0.040 kg and v = 1.0 km/s = 1000 m/s

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{0.040\times1000} = 1.66 \times 10^{-35}\ \text{m}$$

(b) Here, m = 0.060 kg and v = 1.0 m/s

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{0.060\times1.0} = 1.1 \times 10^{-32}\ \text{m.}$$

(c) Here, m = 1.0×10^{-9} kg and v = 2.2 m/s

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1.0\times10^{-19}\times2.2} = 3.01 \times 10^{-25}\ \text{m.}$$

11.16 An electron and a photon each have a wavelength of 1.00nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Sol. (a) Momentum of $P = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}}{1\times10^{-9}} = 6.626 \times 10^{-25}$ kg ms^{-1}

It is the same for both electron and photon.

(b) Energy of photon $E = \dfrac{hc}{\lambda} = \dfrac{6.626\times10^{-34}\times3\times10^{8}}{1\times10^{-9}} = 1.99 \times 10^{-16}$ J

$$= \frac{1.99\times10^{-16}}{1.6\times10^{-16}}\ \text{keV} = 1.24\ \text{keV.}$$

(c) KE of electron $= \dfrac{p^2}{2m} = \dfrac{(6.626\times10^{-25})^2}{2\times9.1\times10^{-31}} = 2.412 \times 10^{-19}\ \text{J} = \dfrac{2.412\times10^{-19}}{1.6\times10^{-19}}\ \text{eV} = 1.51$ eV.

11.17 (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10}m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

Sol. (a) Here $\lambda = 1.40 \times 10^{-10}$ m

Also, $h = 6.63 \times 10^{-34}$ Js and $m = 1.67 \times 10^{-27}$ kg

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{2m\lambda^2} = \frac{(6.63\times10^{-34})^2}{2\times1.67\times10^{-27}\times(1.4\times10^{-10})^2} = 6.7 \times 10^{-21}\ \text{J}$$

$$= \frac{6.7\times10^{-27}}{1.6\times10^{-19}}\ \text{eV} = 4.19 \times 10^{-2}\ \text{eV}.$$

(b) Since $E = \frac{3}{2}kT$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}$$

Here $k = 1.38 \times 10^{-23}\ JK^{-1}$ and $T = 300$ K

$$\therefore \lambda = \frac{(6.63\times10^{-34})}{\sqrt{3\times1.67\times10^{-27}\times1.38\times10^{-23}\times300}} = 1.456 \times 10^{-10}\ \text{m} = 1.456\ \text{Å}$$

11.18 Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Sol. The de-Broglie wavelength of photon is $\lambda_B = h/p$ ....(1)

But for a photon, $p = \frac{h\nu}{c}$

$\therefore$ From eqn. (1), $\lambda_s = \frac{h}{(h\nu/c)} = \frac{c}{\nu} = \lambda$

11.19 What is the de Broglie wavelength of a nitrogen molecule in air at 300K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Sol. For nitrogen,

$\therefore$ RMS velocity, $v = \sqrt{\frac{3kT}{m}}$

$\therefore$ de Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mkT}} = \frac{6.63\times10^{-34}}{\sqrt{3\times28.0152\times1.66\times10^{-27}\times1.38\times10^{-23}\times300}}$

or $\lambda = 0.275 \times 10^{-10}$ m. ($\therefore$ m = 2 × 14.0076 = 28.0152)

11.20 (a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The specific charge of the electron, i.e., its e/m is given to be $1.76 \times 10^{11}\ C\ kg^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong ? In what way is the formula to be modified?

Sol. (a) Here, V = 500 V; $e/m = 1.76 \times 10^{11}\ C\ kg^{-1}$

Using $v = \sqrt{\frac{2eV}{m}}$ $\left[\frac{1}{2}mv^2 = eV\right]$ or $v = \sqrt{2\times1.76\times10^{11}\times500} = 1.33 \times 10^7\ ms^{-1}$

(b) Using the same formula for $V = 10\ MV = 10 \times 10^6$ V

$v = \sqrt{2\times1.76\times10^{11}\times10\times10^6} = \sqrt{3.52\times10^{18}} = 1.88 \times 10^9$ m/s.

Since v is comparable to the velocity of light, the kinetic energy $= \frac{1}{2}mv^2$ is not exact relation.

The relativistic expression for kinetic energy (i.e. K.E. = $c^2 (m - m_0)$ must be used, so that

$$c^2 (m - m_0) = eV, \text{ where } m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

11.21 (a) A monoenergetic electron beam with electron speed of 5.20×10^6 ms^{-1} is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76×10^{11}C kg^{-1}.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

Sol. (a) $v = 5.20 \times 10^6$ ms^{-1} ; $B = 1.30 \times 10^{-4}$ T and $\frac{e}{m} = 1.76 \times 10^{11}$ C kg^{-1}

For tracing out a circle, $qvB = \frac{mv^2}{r}$ or $r = \frac{mv}{qB} = \frac{mv}{eB}$ $(\because q = e)$

$$= \frac{5.20\times 10^6}{1.76\times 10^{11} \times 1.30\times 10^{-4}} = 0.227 \text{ m}$$

(b) The formula employed in part (a) is not valid because with the increase in velocity, mass varies and in the above formula we have taken m as constant.

Instead, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is to be considered.

$$\therefore \quad r = \frac{m_0 v}{eB\sqrt{1-\frac{v^2}{c^2}}}$$

11.22 An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (-10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.

Sol. Here $r = 12.0 \text{ cm} = \frac{12}{100} \text{ m} = 0.12$ m.

Now $\frac{1}{2}mv^2 = eV \Rightarrow mv^2 = 2eV$ .....(1)

Also $\frac{mv^2}{r} = evB$.

$\therefore \frac{2eV}{r} = evB \Rightarrow v = \frac{2V}{rB}$ $\qquad \therefore v = \frac{2\times 100}{0.12\times 2.83\times 10^{-4}}$

$\therefore$ From eqn. (1), $\frac{e}{m} = \frac{v^2}{2V} = \frac{(5.89\times 10^6)^2}{2\times 100} = 1.73 \times 10^{11}$ C kg^{-1}.

11.23 (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Sol. (a) $\lambda_{min} = 0.45$ Å $= 0.45 \times 10^{-10}$ m

$\therefore \quad E_{max} = \dfrac{hc}{\lambda} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{0.45 \times 10^{-10}} = 4.42 \times 10^{-15}$ J

$= \dfrac{4.42 \times 10^{-15}}{1.6 \times 10^{-16}}$ KeV = 27.61 keV

(b) Since E = eV

$\therefore \quad V = \dfrac{E}{e} = \dfrac{4.42 \times 10^{-15}}{1.6 \times 10^{-19}}$ V = 27.61×10^{3} V = 27.61 kV

$\therefore$ The order of accelerating voltage is ≈ 30 kV.

11.24 In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^{9}eV)

Sol. Energy carried by the pair of γ-rays = 10.2 BeV

$\therefore$ Energy of each γ-ray is $E = \dfrac{10.2}{2}$ = 5.1 BeV = 5.1×10^{9} eV

$= 5.1 \times 10^{9} \times 1.6 \times 10^{-19}$ J = 8.16×10^{-10} J

$\therefore$ Using $E = \dfrac{hc}{\lambda} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{8.16 \times 10^{-10}} = 2.44 \times 10^{-16}$ m

11.25 Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radiowaves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($–10^{-10}$ W m^{-2}). Take the area of the pupil to be about 0.4 cm^{2}, and the average frequency of white light to be about 6×10^{14} Hz.

Sol. (a) Number of photons emitted per second.

$$n = \frac{\text{Power of transmitter}}{\text{Energy of each photon}}$$

Now, Power of transmitter,

P = 10 kW = 10^{4} W

λ = 500 m

$\therefore \quad E = \dfrac{hc}{\lambda} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{500} = 3.98 \times 10^{-28}$ J

$\therefore \quad n = \dfrac{P}{E} = \dfrac{10^{4}}{3.98 \times 10^{-28}} = 2.52 \times 10^{31}$

This number is a very-very large quantity.

(b) Minimum intensity, 1 = 10^{-10} Wm^{-2}

Area of pupil, A = 0.4 cm^{2}

Average frequency, ν = 6×10^{14} Hz.

$\therefore$ Energy of each photon, $E = h\nu = 6.626 \times 10^{-34} \times 6 \times 10^{14} = 3.98 \times 10^{-19}$ J.

$\therefore$ Number of photons entering into pupil of the eye per second

$= \dfrac{IA}{E} = \dfrac{10^{-10} \times 0.4 \times 10^{-4}}{3.98 \times 10^{-19}} = 1.01 \times 10^{4}$

This is quite a small number, but still large enough to be counted.

Comparison of cases (a) and (b) tells us that our eye can not count the number of photons individually.

11.26 Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is .1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity (-10^5 W m^{-2}) red light of wavelength 6328 Å produced by a He-Ne laser?

Sol. Here, $\lambda_u = 2271$ Å $= 2271 \times 10^{-10}$ m

Energy of ultraviolet light photon, $E_u = \dfrac{hc}{\lambda}$

$$\therefore \quad E_u = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10}} = 8.75 \times 10^{-19} \text{ J}$$

$\therefore$ Using the relation, $eV_0 = h\nu - W = E_u - W = 8.75 \times 10^{-19} - 1.6 \times 10^{-19} \times 1.3 = 6.67 \times 10^{-19}$ J

$$\therefore \quad \text{Work function} = 6.67 \times 10^{-19} \text{ J} = \frac{6.67 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.17\text{eV}$$

Also, for red light, $\lambda_r = 6328$ Å $= 6328 \times 10^{-10}$ m

$$\therefore \nu_r = \frac{c}{\lambda_r} = \frac{3 \times 10^8}{6328 \times 10^{-10}} = 4.74 \times 10^{14} \text{ Hz}$$

$$\therefore E_r = h\lambda_r = 6.626 \times 10^{-34} \times 4.74 \times 10^{14} = 3.14 \times 10^{-19} \text{ J} = \frac{3.14 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.96 \text{ eV}.$$

Since the energy of red photon is less than the work function for the metal, photo cell does not respond to red light.

11.27 Monochromatic radiation of wavelength 640.2 nm (1nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Sol. $\lambda = 640.2$ nm $= 640.2 \times 10^{-9}$ m

Using $eV_0 = h\nu - W = \dfrac{hc}{\lambda} - W$

$$\Rightarrow W = \frac{hc}{\lambda} - eV_0 = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{640.2 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.54$$

$$= 3.10 \times 10^{-19} - 8.64 \times 10^{-20} = [3.10 \times 0.864] \times 10^{-19} = 2.24 \times 10^{-19} \text{ J}$$

$$\therefore \quad W = \frac{2.24 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.40 \text{ eV}$$

Also, for $\lambda = 427.2$ nm $= 427.2 \times 10^{-9}$ m

using $W = \dfrac{hc}{\lambda} - eV_0$ or $eV_0 = \dfrac{hc}{\lambda} - W$

$$\text{or} \quad V_0 = \frac{hc}{e\lambda} - \frac{W}{e} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 427.2 \times 10^{-9}} - \frac{2.24 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.91 - 1.40 = 1.51 \text{ V}$$

11.28 A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used: $\lambda_1 = 3650$ Å, $\lambda_2 = 4047$ Å, $\lambda_3 = 4358$ Å, $\lambda_4 = 5461$ Å, $\lambda_5 = 6907$ Å.

The stopping voltages, respectively, were measured to be:

$V_{01} = 1.28$ V, $V_{02} = 0.95$ V, $V_{03} = 0.74$ V, $V_{04} = 0.16$ V, $V_{05} = 0$ V

Determine the value of Planck's constant h, the threshold frequency and work function for the material.

Sol. (a) Form the Einstein photoelectric equation, $eV_0 = h\nu - W$

i.e. $\frac{hc}{\lambda} - eV_0 = W$

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3650\times10^{-10}} = 8.22 \times 10^{14}\ \text{Hz}$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3\times10^8}{4047\times10^{-10}} = 7.41 \times 10^{14}\ \text{Hz}$$

$$\nu_3 = \frac{c}{\lambda_3} = \frac{3\times10^8}{4358\times10^{-10}} = 6.88 \times 10^{14}\ \text{Hz}$$

$$\nu_4 = \frac{c}{\lambda_4} = \frac{3\times10^8}{5461\times10^{-10}} = 5.49 \times 10^{14}\ \text{Hz}$$

$$\nu_5 = \frac{c}{\lambda_5} = \frac{3\times10^8}{6907\times10^{-10}} = 4.34 \times 10^{14}\ \text{Hz}$$

A graph can be plotted between V_0 and ν which comes out to be a straight line

Then, $\frac{h}{e} = \frac{\Delta V}{\Delta \nu} = \frac{1.28 - 0.16}{(8.22 - 5.50)\,10^{14}}$ i.e. $h = \frac{(1.6\times10^{-19})\,(1.12)}{2.72\times10^{14}} = 6.57 \times 10^{-34}$

(b) Now using, $\frac{hc}{\lambda_2} - eV_{02} = W$, we get $W = \frac{hc}{\lambda_2} - eV_{02}$

or $W = \frac{6.6\times10^{-34}\times3\times10^8}{4047\times10^{-10}} - 1.6 \times 10^{-19} \times 0.95 = 4.89 \times10^{-19} - 1.52 \times 10^{-19}$

$= 3.37 \times 10^{-19}\ \text{J} = \frac{3.37\times10^{-19}}{1.6\times10^{-19}}\ \text{eV} = 2.11\ \text{eV}$

Also, $\nu_0 = \frac{W}{h} = \frac{3.37\times10^{-19}}{6.6\times10^{-34}} = 5.11 \times 10^{14}\ \text{Hz}$

11.29 The work function for the following metals is given:
Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV.
Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Sol. Here, $\lambda = 3300$ Å $= 3300 \times 10^{-10}$ m
Distance, r = 1m and r' = 50 cm = 0.5 m

Using the relation, $E = h\nu = \frac{hc}{\lambda}$, we get

$E = \frac{6.626\times10^{-34}\times3\times10^8}{3300\times10^{-10}} = 6.02 \times 10^{-19}\ \text{J} = \frac{6.02\times10^{-19}}{1.6\times10^{-19}}\ \text{eV} = 3.76\ \text{eV}.$

Since the energy E of the incident photon of light is greater than the work functions of all the metals given in the question, photoelectric emission will occur in all the metals.
The distance of the source does, not increase or decrease the energy of the photon of light incident, therefore, the energy of electrons ejected will not change but the intensity of ejected electrons will increase $(11/r^2)$ and become four times.

11.30 Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2eV. What is the implication of your answer?

Sol. $A = 2cm^2 = 2 \times 10^{-4}\ m^2$

$W = 2eV = 2 \times 1.6 \times 10^{-19}\ J = 3.2 \times 10^{-19} J$

Taking the approximate radius of an atom as 10^{-10} m.

The effective area of sodium atom is $r^2 = 10^{-20}$ m.

$\therefore$ If there is one free electron per atom, then number of electron is five layers

$$= \frac{5 \times \text{Area of each layer}}{\text{Atomic area of sodium}} = \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Now Incident power, $P = 1A = 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9}$ W

$\therefore$ For absorption of the incident power equally by all electrons, energy absorbed per electron per second is $E = \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26}$ W

$\therefore$ Time required for photo electric emission to take place $= \frac{W}{E} = \frac{3.2 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^{7}$ s.

The answer obtained implies that the time of emission of electron is very large and is not in agreement with the observed time of emission, which is approximately 10^{-9} s. Thus wave -picture of radiation is not applicable for photo-electric emission.

11.31 Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1Å, which is of the order of inter-atomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31}$ kg).

Sol. Here $\lambda = 1$ Å $= 10^{-10}$ m

$\therefore$ For X ray, $E_x = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10}} = 1.99 \times 10^{-15}\ J = \frac{1.99 \times 10^{-15}}{1.6 \times 10^{-19}} = 1.24 \times 10^{4}$ eV

For the electron, $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m_eE_e}}$

Squaring and solving,

$$E_e = \frac{h^2}{2m_e\lambda^2} = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times (10^{-10})^2} = 2.41 \times 10^{-17}\ J = \frac{2.41 \times 10^{-17}}{1.6 \times 10^{-19}}\ eV = 151\ eV$$

Clearly, the energy of photon is much greater than the energy of electron.

11.32 (a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Q.11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable ? Explain. ($m_n = 1.675 \times 10^{-27}$ kg)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27ºC). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Sol. (a) $E = 150\ eV = 150 \times 1.6 \times 10^{-19}\ J = 2.4 \times 10^{-17}$ J

$\therefore$ Using $\lambda = \frac{h}{\sqrt{2m_nE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 2.4 \times 10^{-17}}} = 2.34 \times 10^{-12}$

$= 2.34 \times 10^{-12} \times 10^{10}$ Å $= 0.0234$ Å

This wavelength is about hundred times smaller than the interatomic separation of crystals. Thus, neutrons are not suitable for diffraction experiment in case of crystals.

(b) Here T = 27°C = 300 K

$$\therefore \quad \lambda = \frac{h}{\sqrt{3m_n KT}} = \frac{6.626\times10^{-34}}{\sqrt{3\times1.675\times10^{-27}\times1.38\times10^{-23}\times300}} = 1.45 \times 10^{-10}\text{ m} = 1.45\text{ Å}$$

This wavelength is comparable to the interatomic spacing of crystals. Therefore, thermal electrons are able to interact with the crystal. Since $\lambda \propto \frac{1}{\sqrt{T}}$.

Increasing the temperature decreases their de Broglie wavelength and they become unsuitable for crystal diffraction. Thus, the fast beam of neutrons needs to be thermalised with the environment for neutron diffraction experiment.

11.33 An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Sol. V = 50 kV = 50×10^3 V

$$\therefore \text{ Wavelength of electron, } \lambda_e = \frac{h}{\sqrt{2meV}} = \frac{6.626\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times50\times10^3}}$$

$= 5.49 \times 10^{-12}$ m $= 5.49 \times 10^{-12} \times 10^{10}$ Å $= 0.0549$ Å

Also, for yellow light, $\lambda_y = 5900$ Å $= 5900 \times 10^{-10}$ m

Now Resolving power $\propto \frac{1}{\lambda}$

$$\therefore \quad \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda_e} = \frac{5900\times10^{-10}}{0.0549\times10^{-10}} = 1.07 \times 10^5$$

11.34 The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Sol. Rest mass energy, $m_0c^2 = 0.511$ MeV $= 0.511 \times 1.6 \times 10^{-13}$ J $= 8.18 \times 10^{-14}$ J

$\therefore$ Using the relation, $\lambda = \frac{h}{p}$,

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{10^{-15}} = 6.626 \times 10^{-19}\text{ kg ms}^{-1}$$

$\therefore$ Total energy of the particle

$$= \sqrt{p^2c^2 + (m_0c^2)^2} = \sqrt{(6.626\times10^{-19})^2\times(3\times10^8)^2 + (8.18\times10^{-14})^2} = \sqrt{3.95\times10^{-20}}$$

$$= 1.99 \times 10^{-10}\text{ J} = \frac{1.99\times10^{-10}}{1.6\times10^{-19}}\text{ eV} = 1.24 \times 10^9\text{ eV} = 1.24\text{ BeV}$$

Thus, the energy of the proton ejected out of the linear accelerator is of the order of BeV.

11.35 Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27ºC) and 1atm pressure; and compare it with the mean separation between two atoms under these conditions.

Sol. Here T = 27ºC = 27 + 273 = 300 K

P = 1 atm = 1.01×10^5 Nm^{-2}

Also, mass of helium atom,

$$m = \frac{4}{\text{Avogadro's number}} = \frac{4}{6.02 \times 10^{23}} \text{ g} = 6.64 \times 10^{-24} \text{ g} = 6.64 \times 10^{-27} \text{ kg}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 6.64 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} = 7.3 \times 10^{-11} \text{ m}$$

$$= 7.3 \times 10^{-11} \times 10^{10} \text{ A} = 0.73 \text{ Å}$$

Also, the mean separation,

$$r_0 = \left[\frac{V}{N}\right]^{\frac{1}{3}} = \left[\frac{kT}{P}\right]^{\frac{1}{3}} = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5}\right]^{\frac{1}{3}} = [4.1 \times 10^{-26}]^{1/3} = 3.45 \times 10^{-9} \text{ m}$$

$$= 3.45 \times 10^{-9} \times 10^{10} \text{ Å} = 34.5 \text{ Å}$$

Clearly $r_0 >> \lambda$.

11.36 Compute the typical de Broglie wavelength of an electron in a metal at 27ºC and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

Sol. Here, T = 27ºC = 27 + 273 = 300 K

$$\text{Now } \lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} = 6.23 \times 10^{-9} \text{ m.}$$

$$\therefore \quad \frac{\lambda}{r_0} = \frac{6.23 \times 10^{-9}}{2 \times 10^{-10}} = 31.2$$

Clearly $\lambda >> r_0$.

11.37 Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)e ; (–1/3)e]. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations: $E = h\nu$, $p = h/\lambda$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

Sol. (a) In case of the Millikan oil drop experiment, the charge on the electron is measured. The electron revolve outside the nucleus and each has a charge e.

Thus, we do not observe the fractional charges $\left(\text{i.e.} \quad +\frac{2}{3}e \text{ or } -\frac{1}{3}e\right)$

(b) We know that the energy acquired by the electron when accelerated through V volt is

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2e}{m}V}$$

Similarly, the force on the electron in an electric field is

$$F_e = ma = eE \Rightarrow a = \frac{e}{m} E$$

and force on electron in magnetic field

$$F_m = evB = \frac{mv^2}{r} \text{ or } v = \frac{e}{m} rB$$

Clearly, the dynamics of electrons is determined by $\frac{e}{m}$ in all the above relations instead of e alone.

(c) At ordinary pressure, molecules of gas keep on colliding with each other and the ions formed do not have a chance to reach the respective electrodes to constitute a current because of their recombination. At low pressure. however, ions do not collide frequently and are able to reach the respective electrodes to constitute a current.

(d) Work function in fact is the energy required to knock out the electron from highest filled level of conduction band of an emitter. In the conduction band, there are different energy levels which collectively from a continuous band of levels. Therefore, different amounts of energy are required to bring the electrons out of the different levels. Electrons once omitted have different kinetic energies according to the energy supplied to the emitter.

(e) Since frequency for a given matter wave remains constant for different layers of the matter but wavelength λ changes so λ is more significant than v. Similarly energy $E = h\nu = \frac{1}{2} m (\lambda\nu)^2$ is also constant for a given matter wave so phase $\lambda\nu$ is also not physically significant.