NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 12
ATOMS

12.1 Choose the correct alternative from the clues given at the end of the each statement:
(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)
(b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/ Rutherford's model.)
(c) A classical atom based on .... is doomed to collapse. (Thomson's model/ Rutherford's model.)
(d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in .... (Thomson's model/ Rutherford's model.)
(e) The positively charged part of the atom possesses most of the mass in ....
(Rutherford's model/both the models.)

Sol. (a) No different from
(b) Thomson's model; Rutherford's model.
(c) Rutherford's model.
(d) Thomson's model, Rutherford's model
(e) Both the models.

12.2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.)
What results do you expect?

Sol. Alpha-particle is much heavier than the hydrogen nucleus. Moreover, the Coulomb's force of repulsion between α-particle and hydrogen nucleus is much smaller than the Coulomb's force of repulsion between α- particle and gold nucleus. Therefore, the scattering of α-particles by hydrogen nuclei will be quite different than the scattering of α-particle by gold nuclei. In this case, the impact parameter and the distance of closest approach will be quite small. The α-particles will be deflected slightly from their original paths. Size of hydrogen atom will not be assessed by this method.

12.3 What is the shortest wavelength present in the Paschen series of spectral lines?

Sol. The wavelength of the spectral lines forming Paschen series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_1^2}\right)$$

For shortest wavelength, $n_i = \infty$

$$\therefore \ \frac{1}{\lambda} = \frac{R}{9} \text{ or } \lambda = \frac{9}{R}$$

Since, $\frac{1}{R} = 911$ Å $\qquad \therefore \ \lambda = 9 \times 911 = 8199$ Å.

12.4 A difference of 2.3eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

Sol. $E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J} = 3.68 \times 10^{-19} \text{ J}$

Now $E = h\nu$ or $\nu = \frac{E}{h} = \frac{3.68 \times 10^{-19}}{6.626 \times 10^{-34}} = 5.55 \times 10^{14}$ Hz.

12.5 The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Sol. The potential energy of electron is given by $E_p = -\frac{1}{4\pi \in_0}\frac{e^2}{r}$

and its kinetic energy is $E_k = \frac{1}{2}\times\frac{1}{4\pi \in_0}\frac{e^2}{r} = -\frac{1}{2}E_p$

Since $E_P + E_k = -13.6$ eV

$\therefore \quad E_p - \frac{1}{2}E_p = -13.6$ eV or $\frac{1}{2}E_p = -13.6$ eV i.e., $E_p = -27.2$ eV

$\therefore \quad E_k = -\frac{1}{2}E_p = \frac{27.2}{2} = 13.6$ eV

12.6 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon

Sol. We know, energy of an electron in the nth orbit of hydrogen atom is given by $E_n = -\frac{13.6}{n^2}$ eV

When n = 1 (ground level), $E_1 = -13.6$ eV

When n = 4, $E_4 = -\frac{13.6}{16} = -0.85$ eV

$\therefore$ Energy difference between n = 1 and n = 4

$E_4 - E_1 = -0.85 + 13.6 = 12.75$ eV

$\therefore$ The hydrogen atom will go to n = 4 from n = 1 if the photon energy = 12.75 eV

$= 12.75 \times 1.6 \times 10^{-19}$ J $= 20.4 \times 10^{-19}$ J or $h\nu = 20.4 \times 10^{-19}$

or $\quad \nu = \frac{20.4\times10^{-19}}{h} = \frac{20.4\times10^{-19}}{6.63\times10^{-34}} = 3.08 \times 10^{15}$ Hz

$\therefore \quad \lambda = \frac{c}{\nu} = \frac{3\times10^8}{3.08\times10^{15}} = 9.74 \times 10^{-8}$ m.

12.7 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels.

(b) Calculate the orbital period in each of these levels.

Sol. (a) Speed of an electron in n^{th} orbit of hydrogen atom is given by

$$v_n = \frac{1}{137}\left(\frac{c}{n}\right)$$

When n = 1, $v_1 = \frac{1}{137} \times \frac{3\times10^8}{1} = 2.18 \times 10^6\ ms^{-1}$

When n = 2, $v_2 = \frac{1}{137} \times \frac{3\times10^8}{2} = 1.09 \times 10^6\ ms^{-1}$

When n = 3, $v_3 = \frac{1}{137} \times \frac{3\times10^8}{3} = 0.73 \times 10^6\ ms^{-1}$

(b) Orbital period of electron in the n^{th} orbit of hydrogen atom

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi a_0 n^2}{\frac{1}{137}\times\frac{c}{n}} = \frac{137\times 2\pi a_0 n^3}{c} \quad \text{(where } a_0 = 0.53 \times 10^{-10} \text{ m)}$$

When n = 1, $T_1 = \dfrac{137\times 2\times 3.14\times 0.53\times 10^{-10}}{3\times 10^8} = 1.52 \times 10^{-16}$ s

When n = 2, $T_2 = (2)^3 \times T_1 = 8 \times 1.52 \times 10^{-16} = 1.22 \times 10^{-15}$ s

When n = 3, $T_3 = (3)^3 \times T_1 = 27 \times 1.52 \times 10^{-16} = 4.1 \times 10^{-15}$ s.

12.8 The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n=3 orbits?

Sol. Here $r_n = Kn^2$, where $K = \dfrac{4\pi \epsilon_0 h^2}{4\pi^2 me^2}$

For n = 1, $r_1 = K(1)^2$ or $K = r_1 = 5.3 \times 10^{-11}$ m

for n = 2, $r_2 = K(2)^2 = 4K = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}$ m

Also, for n = 3, $r_3 = K(3)^2 = 9K = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}$ m

12.9 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Sol. $E = \dfrac{hc}{\lambda}$ i.e., $\lambda = \dfrac{hc}{E} = \dfrac{6.625\times 10^{-34}\times 3\times 10^8}{12.5\times 1.6\times 10^{-19}} = 0.994 \times 10^{-7} = 99$ nm.

Lyman 103 nm & 122 nm
Balmer series 656nm

12.10 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Sol. According to Bohr's postulate of quantization of angular momentum

$mvr = n\dfrac{h}{2\pi}$ $\quad\therefore n = \dfrac{2\pi mvr}{h} = \dfrac{2\times 3.14\times 6\times 10^{24}\times 3\times 10^{14}\times 1.5\times 10^{11}}{6.63\times 10^{-34}} = 2.6 \times 10^{74}$.

12.11 (a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Sol. (a) It is about the same because we are considering average deflection angle.

(b) It is much less than the predicted value because the massive core is absent in Thomoson's model.

(c) It suggests that the scattering of α-particles is primarily because of the single collision.

(d) In Thomson's model of atom. Multiple collisions are required to be considered in this model because positive charge is spread throughout in this model.

12.12 The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Sol. If electron and proton were bound by gravitational attraction, then $\frac{m_e v^2}{r} = \frac{Gm_e m_p}{r^2}$

$\therefore \quad m_e v^2 r = Gm_e m_p \qquad .....(1)$

Also, according to Bohr's condition, $m_e vr = \frac{nh}{2\pi}$

Squaring both sides, $m_e^2 v^2 r^2 = \frac{n^2h^2}{4\pi^2} \qquad ......(2)$

Dividing eqn. (2) by (1),

$$m_e r = \frac{n^2h^2}{4\pi^2 Gm_e m_p} \quad \text{or} \quad r = \frac{n^2h^2}{4\pi^2 Gm_e^2 m_p}$$

For 1st orbit, n = 1, Also, $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$,
$m_e = 9.1 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg

$$\therefore \quad r = \frac{(1)\times(6.626\times10^{-34})^2}{4\times9.87\times6.67\times10^{-11} x}$$

$(9.1 \times 10^{-31})^2 \times 1.67 \times 10^{-27} = 1.21 \times 10^{29}$ m.

It is astonishing this value of r is much greater than the size of universe.

12.13 Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n – 1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Sol. The energy of an electron in the n^{th} orbit of hydrogen atom is given by

$$E_n = \frac{-2\pi^2 me^4}{(4\pi \epsilon_0)^2 n^2 h^2} \qquad ....(1)$$

Also, for the pth orbit,

$$E_p = \frac{-2\pi^2 me^4}{(4\pi \epsilon_0)^2 p^2 h^2} \qquad ......(2)$$

The frequency of radiation emitted for the transition of electron from n^{th} to p^{th} orbit is

$$\nu = \frac{E_n - E_p}{h} = \frac{2\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3}\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \qquad \text{(Using eqns (1) \& (2))}$$

For n = n and p = n – 1, we have

$$\nu = \frac{E_n - E_{n-1}}{h} = \frac{2\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{2\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3}\left[\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right] = \frac{2\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3}\frac{(2n-1)}{n^2(n-1)^2}$$

For large value of n, 2n – 1 2n and n – 1 ≈ n.

$$\therefore \quad \nu = \frac{2\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3}.\frac{2n}{n^4} = \frac{4\pi^2 me^4}{(4\pi \epsilon_0)^2 h^3 n^3} \qquad ......(3)$$

Now the classical frequency of revolution of electron is given by

$$\nu_c = \frac{v}{2\pi r} \qquad ......(4)$$

But according to Bohr's angular momentum condition,

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr} \qquad ......(5)$$

Using eqn. (5) in eqn. (4),

$$\nu_c = \frac{nh}{4\pi^2 mr^2} \qquad ....(6)$$

But $r = \dfrac{(4\pi \in_0)\, n^2h^2}{4\pi^2 me^2}$

$$\therefore \text{ from eqn. (6), } \nu_c = \frac{4\pi^2 me^4}{(4\pi \in_0)^2 h^3 n^3} \qquad ......(7)$$

$\therefore$ for a large n, $\nu = \nu_c$ (Using (3) and (7))

i.e. frequencies are equal.

This is called the Bohr's correspondence principle.

12.14 Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10}m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Sol. (a) Here, the dimensional formula of e is A^1T^1.

The dimensional formula of m_e is M^1.

Dimensional formula of c is L^1T^{-1} and that of $\dfrac{1}{4\pi \in_0}$ is $M^1L^3T^{-4}A^{-2}$,

Therefore, the quantity $\dfrac{e^2}{4\pi \in_0 m_e c^2}$ has the dimensions of length.

Value of this quantity is $\dfrac{(1.6\times10^{-19})^2}{\left(\dfrac{1}{9\times10^9}\right)\times(9.11\times10^{-31})\times(3\times10^8)^2} = 2.81\times 10^{-15}$ m

It is very small as compared to the size of the atom.

(b) Now h has the dimensional formula ML^2T^{-1}.

Therefore, the quantity $\dfrac{4\pi \in_0 h^2}{m_e^2}$ $\left(\text{where } \hbar = \dfrac{h}{2\pi}\right)$ has the dimensions of length. It value is

$$\frac{1}{9\times10^9}\times\left(\frac{6.626\times10^{-34}}{2\times3.14}\right)^2\times\frac{1}{9.11\times10^{-31}\times(1.6\times10^{-19})^2} = 5.3\times 10^{-11} \text{ m} = 0.53 \text{ Å}$$

This is nearly equal to the size of the atom.

12.15 The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Sol. The total energy of the electron in an orbit of radius r_n is given by

$$E_n = -\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} \qquad \text{(For hydrogen atom)}$$

For first excited state, n = 2

$$\therefore \quad E_2 = -\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_2} = -3.4 \text{ eV} \quad \text{(Given)}$$

$$\therefore \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_2} = 3.4 \times 2\text{eV} = 6.8 \text{ eV} \qquad .....(1)$$

(a) The kinetic energy of the electron in the excited state is

$$\text{K.E.} = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_2} = \frac{1}{2} \times 6.8 \text{ eV} = 3.4 \text{ eV}$$

(b) The potential energy of the electron in this excited state is

$$\text{P.E.} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -6.8 \text{ eV.}$$

(c) Kinetic energy does not depend upon the choice of zero of potential energy. Therefore, its value remains unchanged. However, the potential energy gets changed with the change in the zero level of potential energy.

12.16 If Bohr's quantisation postulate (angular momentum = nh/2π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Sol. Applying the Bohr's quantization postulate, $mvr = \frac{nh}{2\pi}$

For the motion of a planet (say earth). we may consider the data : $m = 6 \times 10^{24}$ kg, $v = 30000 \text{ ms}^{-1}$ $r = 1.49 \times 10^{11}$ m and $h = 6.626 \times 10^{-34}$ Js.

$$\therefore \; n = \frac{6\times10^{24} \times 30000 \times 1.49\times10^{11} \times 2 \times 3.14}{6.626\times10^{-34}} = 2.49 \times 10^{74}, \text{ i.e. n is very large.}$$

Since n is very large, the difference between the two successive energy or angular momentum levels is very small and the levels may be considered continuous.

12.17 Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about 207m_e orbits around a proton].

Sol. Here mass of the particle revolving around the proton is

$m = 207\, m_e = 207 \times 9.1 \times 10^{-31}$ kg

$$\therefore \quad r_0 = \frac{(4\pi\epsilon_0)\,h^2}{4\pi^2 me^2} = \frac{1}{(9\times10^9)} \times \frac{(6.626\times10^{-34})^2}{4\times9.87\times207\times9.1\times10^{-31}\times(1.6\times10^{-19})^2} = 2.56 \times 10^{-13} \text{ m}$$

Also, energy of this level

$$E_0 = \frac{1}{2}\times\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_0} = -\frac{1}{2}\times\frac{9\times10^9\times(1.6\times10^{-19})^2}{2.56\times10^{-13}}$$

$$= -4.496 \times 10^{-16}\ \text{J} = -\ \frac{4.496\times10^{-16}}{1.6\times10^{-19}}\ \text{eV} = -2.81 \times 10^{3}\ \text{eV} = -2.81\ \text{KeV}.$$