NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 13
NUCLEI

13.1 (a) Two stable isotopes of lithium ${}_{3}^{6}\text{Li}$ and ${}_{3}^{7}\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512u and 7.01600u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}_{5}^{10}\text{B}$ and ${}_{5}^{11}\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811u. Find the abundances of ${}_{5}^{10}\text{B}$ and ${}_{5}^{11}\text{B}$.

Sol. (a) Atomic weight = weighted average of the isotopes

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{(7.5 + 92.5)} = \frac{45.1134 + 648.98}{100} = 6.941\text{u}$$

(b) Let relative abundance ${}_{5}^{10}\text{B}$ be x%.

$\therefore$ Relative abundance of ${}_{5}^{11}\text{B}$ = (100 – x)%

Proceeding as above, $10.811 = \dfrac{10.01294x + 11.00931 \times (100 - x)}{100}$

x = 19.9% and (100 – x) = 80.1%.

13.2 The three stable isotopes of neon: ${}_{10}^{20}\text{Ne}$, ${}_{10}^{21}\text{Ne}$ and ${}_{10}^{22}\text{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Sol. The average atomic mass of neon is

m (Ne) = [90.51 × 19.99 + 0.27 × 20.99 + 9.22 × 21.99] × 10^{-2} = 20.18 u

13.3 Obtain the binding energy (in MeV) of a nitrogen nucleus $({}_{7}^{14}\text{N})$, given m $({}_{7}^{14}\text{N})$ = 14.00307 u.

Sol. ${}_{7}^{14}\text{N}$ nucleus is made up of 7 protons and 7 neutrons.

Mass of nucleons forming nucleus = $7m_p + 7m_n$ = Mass of 7 protons + Mass of 7 neutrons

= 7 × 1.00783 + 7 × 1.00867u = 7.05431 + 7.06069 = 14.11550 u

Mass of nucleus, m_N = 14.00307 u

Mass defect = 14.11550 – 14.00307 = 0.11243 amu

Energy equivalent to mass defect = 0.11243 × 931 = 104.67 MeV

$\therefore$ Binding energy = 104.67 MeV

13.4 Obtain the binding energy of the nuclei and in units of MeV from the following data:

m $({}_{26}^{56}\text{Fe})$ = 55.934939 amu; m $({}_{83}^{209}\text{Bi})$ = 208.980388 amu.

Sol. (i) ${}_{26}^{56}\text{Fe}$ nucleus contains 26 protons and (56 – 26) = 30 neutrons

Mass of 26 protons = 26 × 1.007825 = 26.20345 amu

Mass of 30 neutrons = 30 × 1.008665 = 30.25995 amu

Total mass of 56 nucleons = 56.46340 amu

Mass of ${}_{26}^{56}\text{Fe}$ nucleus = 55.934939

$\therefore$ Mass defect, Δm = 56.46340 – 55.934939 = 0.528461amu

Total binding energy = 0.528461 × 931.5 MeV = 492.26MeV

Average binding energy per nucleon = $\dfrac{492.26}{56}$ = 8.790 MeV

(ii) ${}^{209}_{83}\text{Bi}$ nucleus contains 83 protons and (209 – 83) = 126 neutrons

Mass of 83 protons = 83 × 1.007825 = 83.649475 amu

Mass of 126 neutrons = 126 × 1.008665 = 127.091790amu

Total mass of nucleons = 210.741260 amu

Mass of nucleus = 208.980388 amu

∴ Mass defect, Δm = 210.741260 – 208.980388 = 1.760872

Total binding energy = 1.760872 × 931.5 MeV =1640.26 MeV

Average binding energy per nucleon = $\dfrac{1640.26}{209}$ = 7.848 MeV

Hence, ${}^{56}_{26}\text{Fe}$ has greater B.E. per nucleon than ${}^{209}_{83}\text{Bi}$.

13.5 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}\text{Cu}$ atoms (of mass 62.92960u).

Sol. Mass of atom = 62.92960u

Mass of 29 elecrons = 29 × 0.000548 u = 0.015892 u

Mass of nucleus = (62.92960 – 0.015892) u = 62.913708 u

Mass of 29 protons = 29 × 1.007825 u = 29.226925 u

Mass of (63 – 29) i.e., 34 neutrons = 34 × 1.008665 u = 34.29461 u

Total mass of protons and neutrons = (29.226925 + 34.29461) = 63.521535 u

Binding energy = (63.521535 – 62.913708) × 931.5 MeV = 0.607827 × 931.5 MeV

Required energy = $\dfrac{6.023\times10^{23}}{63}$ × 3 × 0.607827 × 931.5MeV = 1.6 × 10^{25} MeV = 26 × 10^{12} J

13.6 Write nuclear reaction equations for

(i) α-decay of ${}^{226}_{88}\text{Ra}$ (ii) α-decay of ${}^{242}_{94}\text{Pu}$ (iii) β^--decay of ${}^{32}_{15}\text{P}$

(iv) β^--decay of ${}^{210}_{83}\text{Bi}$ (v) β^+-decay of ${}^{11}_{6}\text{C}$ (vi) β^+-decay of ${}^{97}_{43}\text{Tc}$

(vii) Electron capture of

Sol. (i) ${}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$ (ii) ${}^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$

(iii) ${}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^{-1} + \bar{\nu}$ (iv) ${}^{210}_{83}\text{Bi} \longrightarrow {}^{210}_{84}\text{Po} + e^{-1} + \bar{\nu}$

(v) ${}^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^{+} + \nu$ (vi) ${}^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{Mo} + e^{+} + \nu$

(vii) ${}^{120}_{54}\text{Xe} + e^{-} \longrightarrow {}^{120}_{53}\text{I} + \nu$

13.7 A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to (a) 3.125%, (b) 1% of its original value?

Sol. (a) The fraction of the original sample left = $\dfrac{3.125}{100} = \dfrac{1}{32} = \left(\dfrac{1}{2}\right)^5$

Hence, there are 5 half lives of T years spent. Thus, the time taken is 5T years.

(b) The fraction of the original sample left = $\dfrac{1}{100} = \left(\dfrac{1}{2}\right)^n$

or $2^n = 100 \Rightarrow n \log 2 = \log 100$

Hence, $n = \dfrac{\log 100}{\log 2} = \dfrac{2}{0.301} = 6.64$

Hence, there are 6.64 half lives of T years spent. Thus, the time taken is 6.64 T years.

13.8 The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Sol. Here, normal activity, $R_0 = 15$ decays/min.

Present activity R = 9 decays/min., T = 5730 years, Age t=?

As activity is proportional to the number of radioactive atoms, therefore,

$$\frac{N}{N_0} = \frac{R}{R_0} = \frac{9}{15}$$

But $\frac{N}{N_0} = e^{-\lambda t} \quad \therefore\ e^{-\lambda t} = \frac{9}{15} = \frac{3}{5} \quad ; \quad e^{+\lambda t} = \frac{5}{3}$

$$\lambda t \log_e e = \log_e \frac{5}{3} = 2.3026 \log 1.6667$$

$$\lambda t = 2.3026 \times 0.2218 = 0.5109 \ ; \quad t = \frac{0.5109}{\lambda}$$

But $\lambda = \frac{0.693}{T} = \frac{0.693}{5730} Yr^{-1}$

$$t = \frac{0.5109}{0.693/5730} = \frac{0.5109 \times 5730}{0.693} = 4224.3 \text{ years}$$

13.9 Obtain the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}Co$ is 5.3 years.

Sol. Strength of radioactive source = 8.0 mCi = 8.0×10^{-3} Ci

$= 8.0 \times 10^{-3}$ C $\times 3.7 \times 10^{10}$ disintegrations s^{-1} = 29.6×10^{7} disintegrations s^{-1}

Since the strength of the source decreases with time,

$\therefore \quad \frac{dN}{dt} = -29.6 \times 10^7$. But $\frac{dN}{dt} = -\lambda N \quad \therefore -\lambda N = -29.6 \times 10^7$ or $\lambda N = 29.6 \times 10^7$

or $\quad N = \frac{29.6 \times 10^7}{\lambda} = \frac{29.6 \times 10^7 \times T}{0.693} \quad \left(\because \lambda = \frac{0.693}{T}\right)$

$$= \frac{29.6 \times 10^7 \times 5.3 \times 365 \times 24 \times 60 \times 60}{0.693} = 7.137 \times 10^{16}$$

Number of atoms in 60g of cobalt = 6.023×10^{23}

Mass of 1 atom of cobalt $= \frac{60}{6.023 \times 10^{23}} \times 7.139 \times 10^{16}\ g = 7.11\ \mu g$

13.10 The half-life of is 28 years. What is the disintegration rate of 15mg of this isotope ?

Sol. Since, $\lambda = \frac{0.693}{T} = \frac{0.693}{28 \times 365 \times 24 \times 60 \times 60} = 7.85 \times 10^{-10}\ s^{-1}$

90g of Sr contains 6.023×10^{23} atoms

15mg of Sr contains, atoms

$$N_0 = \frac{6.023\times10^{23}\times15\times10^{-3}}{90} = 1.0038 \times 10^{20} \text{ atoms}$$

Disintegration rate, $\frac{dN}{dt} = -\lambda N_0 = 7.85 \times 10^{-10} \times 1.0038 \times 10^{20} = 7.88 \times 10^{-10}$ dps or Bq

$$= \frac{7.88\times10^{10}}{3.7\times10^{10}} \text{Ci} = 2.13 \text{ Ci}$$

13.11 Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}\text{Au}$ and the silver isotope $^{107}_{47}\text{Ag}$.

Sol. As, $R \approx A^{1/3}$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = (1.84)^{1/3}$$

$$\Rightarrow \quad \log_{10}\left(\frac{R_1}{R_2}\right) = \log_{10}(1.84)^{1/3}$$

$$\Rightarrow \quad \log_{10}\left(\frac{R_1}{R_2}\right) = \frac{1}{3}\log_{10}(1.84) = \frac{1}{3}\times 0.2648 = 0.08827$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \text{antilog } (0.08827) = 1.23$$

13.12 Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) $^{226}_{88}\text{Ra}$ and (b) $^{220}_{86}\text{Rn}$. Given : m ($^{226}_{88}\text{Ra}$) = 226.02540u, m ($^{222}_{86}\text{Rn}$) = 222.01750 u,

m ($^{220}_{86}\text{Ra}$) = 220.01137u, m ($^{216}_{84}\text{Po}$) = 216.00189 u.

Sol. (a) The difference in mass between the original nucleus and the decay products
= 226.02540u – (222.01750 u + 4.00260u) = + 0.0053u
Energy equivalent = 0.0053 × 931.5 MeV = 4.93695 MeV = 4.94 MeV
The decay products would emerge with total kinetic energy 4.94 MeV. Momentum is conserved. If the parent nucleus is at rest, the daughter and the α-particle have momenta of equal magnitude p but opposite direction. Kinetic energy, $K = p^2/2m$.
Since p is the same for the two particles therefore the kinetic energy divides inversely as their mases.
The α-particle $\frac{222}{222+4}$ gets of the total i.e. $\frac{222}{226} \times 4.94$ MeV or 4.85 MeV.

(b) The difference in mass between the original nucleus and the decay products
= 220.01137u – (216.00189u + 4.00260u) = 0.00688u
Energy equivalent = 0.00688u × 931.5 MeV = 6.41 MeV

$$E_\alpha = \frac{216}{216+4} \times 6.41 \text{ MeV} = 6.29 \text{ MeV}$$

13.13 The radionuclide ^{11}C decays according to $^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^+ + \nu$; $T_{1/2}$ = 20.3 min
The maximum energy of the emitted positron is 0.960 MeV.
Given the mass values: m ($^{11}_{6}\text{C}$) = 11.011434 u and m ($^{11}_{5}\text{B}$) = 11.009305 u.
Calculate Q and compare it with the maximum energy of the positron emitted.

Sol. Mass defect = [m ($^{11}_{6}\text{C}$) – 6m_e] – [m ($^{11}_{5}\text{B}$) – 5m_e + m_e]

= m ($^{11}_{6}\text{C}$) – m ($^{11}_{5}\text{B}$) – 2m_e = 11.011434 u – 11.009305 u – 2 × 0.000548u = 0.001033 u

Q = 0.001033 × 931.5 MeV = 0.962 MeV

$Q = E_d + E_e + E_v$

The daughter nucleus is too heavy compared to e^+ and v. So, it carries negligible energy ($E_d \approx 0$). If the kinetic energy (E_v) carried by the neutrino is minimum (i.e., zero), the positron carries maximum energy, and this is practically all energy Q. Hence, maximum $E_e \approx Q$.

13.14 The nucleus ${}^{23}_{10}\text{Ne}$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

m (${}^{23}_{10}\text{Ne}$) = 22.994466 u, ${}^{23}_{11}\text{Na}$) = 22.989770 u.

Sol. The β-decay of ${}^{23}_{10}\text{Ne}$ may be represented as

$${}^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} - {}^{0}_{-1}e + \overline{\nu} + Q$$

Ignoring the rest mass of antineutrino $(\overline{\nu})$ and electron.

Mass defect, $\Delta m = m({}^{23}_{10}\text{Ne}) - m({}^{23}_{11}\text{Na})$ = 22.994466 – 22.989770 = 0.004696 amu

$\Rightarrow$ Q = 0.004696 × 931 MeV = 4.372 MeV.

As ${}^{23}_{10}\text{Ne}$ is very massive, this energy of 4.372 MeV, is shared by e– and $\overline{\nu}$ pair.

The maximum K.E. of e^- = 4.372 MeV, when energy carried by $\overline{\nu}$ is zero.

13.15 The Q value of a nuclear reaction : A + b → C + d is defined by $Q = [m_A + m_b - m_C - m_d]\,c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$ (ii) ${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$

Atomic masses are given to be

m (${}^{2}_{1}\text{H}$) = 2.014102 u ; m (${}^{3}_{1}\text{H}$) = 3.016049 u

m (${}^{12}_{6}\text{C}$) = 12.000000 u ; m (${}^{20}_{10}\text{Ne}$) = 19.992439 u

Sol. (i) ${}^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$

$Q = \Delta m \times 931.5$ MeV = [m (${}^{1}_{1}\text{H}$) + m (${}^{3}_{1}\text{H}$) – 2m (${}^{2}_{1}\text{H}$) × 931 MeV

= [1.007825 + 3.016049 – 2 × 2.014102] × 931 MeV = – 4.03 MeV

∴ The reaction is endothermic.

(ii) ${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$

$Q = \Delta m \times 931.5$ MeV = [m (${}^{12}_{6}\text{C}$) – m (${}^{20}_{10}\text{Ne}$) – m (${}^{4}_{2}\text{He}$) × 931 MeV

= [24.000000 – 19.992439 – 4.002603] × 931 MeV = + 4.61 MeV

∴ The reaction is exothermic.

13.16 Suppose, we think of fission of a ${}^{56}_{26}\text{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\text{Al}$. Is the fission energetically possible Argue by working out Q of the process.

Given m (${}^{56}_{26}\text{Fe}$) = 55.93494 u and m (${}^{28}_{13}\text{Al}$) = 27.98191 u.

Sol. Q = m (${}^{56}_{26}\text{Fe}$) – 2m (${}^{28}_{13}\text{Al}$) × 931.5 MeV = [55.93494 – 2 × 27.98191] × 931.5 MeV

Q = – 0.02886 × 931.5 MeV= –26.88 MeV, which is negative.

The fission is not possible energetically.

13.17 The fission properties of ${}^{239}_{94}\text{Pu}$ are very similar to those of ${}^{235}_{92}\text{U}$. The average energy released per fission is 180MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission?

Sol. Energy released per fission of ${}^{239}_{94}\text{Pu}$ = 180 MeV

Quantity of fissionable material = 1 kg

In 239gm, Pu, number of fissional atom or nuclei = 6.023×10^{23}

In 1g of Pu, number of fissionable atom or nuclei = $\frac{6.023 \times 10^{23}}{239}$

In 1000gm of Pu, number of fissionable atom or nuclei = $\frac{6.023 \times 10^{23}}{239} \times 1000 = 25.2 \times 10^{23}$

Energy released in fission of single Pu nucleus = 180 MeV.

Energy released in fission of 25.2×10^{23} Pu nucleus or in fission of 1 kg pure Pu

$= 180 \times 25.2 \times 10^{23} = 4536 \times 10^{23}$ MeV $= 4.5 \times 10^{26}$ MeV

13.18 A 1000 MW fission reactor consumes half of its fuel in 5.00y. How much ${}^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Sol. Power of reactor = 1000 MW = 10^3 MW = 10^9W = $10^9 Js^{-1}$.

Energy generated by reaction 5 years $= 5 \times 365 \times 24 \times 60 \times 60 \times 10^9$ J

Energy generated per fission = 200 MeV = $200 \times 1.6 \times 10^{-13}$ J

Number of fission taking place or number of U^{235} nuclei required = $\frac{5 \times 365 \times 24 \times 60 \times 60 \times 10^9}{200 \times 1.6 \times 10^{-13}}$

$= 8.2125 \times 10^{26} \times 6 = 49.275 \times 10^{26}$

Mass of 6.023×10^{23} nuclei of U = 235gm = 235 2 × 10^{-3}kg

Mass of 8.2125×10^{26} nuclei of U = $\frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 6 \times 8.2125 \times 10^{26} = 1932$ kg

$\frac{1}{2}$ of fuel = 1932 kg

Total fuel = 3864 kg

13.19 How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as : ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + n + 3.2MeV$

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Sol. When two nuclei of deuterium fuse together,

Energy released = 3.2 MeV

Number of deuterium atoms in 2 kg = $\frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$

When 6.023×10^{26} nuclei of deuterium fuse together, energy released

$= \frac{3.2}{2} \times 6.023 \times 10^{26}$ MeV $= \frac{3.2}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-13}$ J $= 1.54 \times 10^{14}$ J or Ws

Power of electric lamp = 100W

If the lamp glows for time t, then the electrical energy consumed by the lamp is 100t.

$\therefore$ 100t = 1.54×10^{14} J or t = 1.54×10^{12} s = $\frac{1.54 \times 10^{12}}{3.154 \times 10^7}$ years = 4.88×10^4 years.

13.20 Calculate the height of the potential barrier for a head on collision of two deuterons. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Sol. Suppose the two particles are fired at each other with the same kinetic energy K so that they are brought to rest by their mutual Coulomb repulsion when they are just touching each other. We can take this value of K as a representative measure of the height of Coulomb barrier.

$$2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{(2R)}$$

$$K = \frac{e^2}{16\pi\varepsilon_0 R} = \frac{(1.6 \times 10^{-19})^2}{16 \times 3.14 \times 8.85 \times 10^{12} \times 2 \times 10^{-15}} J = 2.8788 \times 10^{-14} J$$

$$= \frac{2.8788 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^3} keV = 179.9 keV$$

13.21 From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Sol. It is found that a nucleus of mass number A has a radius $R = R_0A^{1/3}$

where, $R_0 = 1.2 \times 10^{-15}$ m

This implies that the volume of the nucleus, which is proportional to R^3 is proportional to A.

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{Density of nucleus} = \frac{\text{mass of nucleus}}{\text{volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Above derived equation shows that density of nucleus is constant, independent of A, for all nuclei and density of nuclear matter is approximately 2.3×10^7 kg m^{-3} which is very large as compared to ordinary matter, say water which is 10^3 kg m^{-3}.

13.22 For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted). $e^+ + {}^A_ZX \to {}^A_{Z-1}Y + \nu$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice versa.

Sol. Consider the two competing processes:

$${}^A_ZX \to {}^A_{Z-1}Y + e^+ + \nu_e + Q_1 \qquad \text{(positron emission)}$$

$$e^- + {}^A_ZX \to {}^A_{Z-1}Y + \nu_e + Q_2 \qquad \text{(electron capture)}$$

$$Q_1 = [m_N({}^A_ZX) - m_N({}^A_{Z-1}Y) - m_e]c^2 = [m({}^A_ZX) - Zm_e - m({}^A_{Z-1}Y) + (Z-1)m_e - m_e]c^2$$

$$= [m({}^A_ZX) - m({}^A_{Z-1}Y) - 2m_e]c^2$$

$$Q_2 = [m_N({}^A_ZX) + m_e - m_N({}^A_{Z-1}Y)]c^2 = [m({}^A_ZX) - m({}^A_{Z-1}Y)]c^2$$

This means $Q_1 > 0$ implies $Q_2 > 0$ but $Q_2 > 0$ does not necessarily mean $Q_1 > 0$. Hence the result.

13.23 In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}Mg$ (23.98504u), ${}^{25}_{12}Mg$ (24.98584u) and ${}^{26}_{12}Mg$ (25.98259u). The natural abundance of ${}^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Sol.

Isotopes	Abundance Y	Atomic mass (Z)
${}_{12}Mg^{24}$	78.99	23.98504
${}_{12}Mg^{25}$	x	24.98584
${}_{12}Mg^{26}$	100 – (78.99 + x) = 21.1 – x	23.98504
	ΣY = 100	

Mean atomic mass = 24.312

$$\text{Average atomic mass} = \frac{\Sigma YZ}{\Sigma Y}$$

$\Rightarrow \quad 24.312 = \dfrac{78.99 \times 23.98504 + x \times 24.98584 + (21.01 - x)\ 25.98254}{100}$

or $\quad 2431.2 = 1894.58 + 24.98584x + 545.89 - 25.98254x$

or $\quad 2431.2 = 2440.47 - 0.99675x$

or $\quad 0.99675x = 2440.47 - 2431.2 = 9.27$

or $\quad x = \dfrac{9.27}{0.99675} = 9.30$

$\therefore \quad 21.01 - x = 21.01 - 9.30 = 11.71$

Relative abundance of ${}_{12}Mg^{25} = 9.30\%$

Relative abundance of ${}_{12}Mg^{26} = 11.71\%$

13.24 The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}Ca$ and ${}^{27}_{13}Al$ from the following data: $m({}^{40}_{20}Ca) = 39.962591$ u, $\quad m({}^{41}_{20}Ca) = 40.962278$ u

${}^{26}_{13}Al = 25.986895$ u, $\quad m({}^{27}_{13}Al) = 26.981541$ u

Sol. The equation for the neutron separation in first case can be written as, ${}^{41}_{20}Ca + Q_1 \longrightarrow {}^{40}_{20}Ca + {}^{0}_{1}n$

$\Delta m = m({}^{40}_{20}Ca) + m({}^{0}_{1}n) - m({}^{41}_{20}Ca) = 39.962591 + 1.008655 - 40.962278 = 0.008978$ u

But, $1u \equiv 931.5$ MeV

Hence, $0.008978 \equiv 0.008978 \times 931.5$ MeV $= 8.363$ MeV

${}^{27}_{13}Al + Q_2 \longrightarrow {}^{26}_{13}Al + {}^{0}_{1}n$

$\Delta m = m({}^{26}_{13}Al) + m({}^{0}_{1}n) - m({}^{27}_{13}Al)$

But, $1u \equiv 931.5$ MeV

Hence, $0.014019 \equiv 0.014019 \times 931.5$ MeV $= 13.06$ MeV

13.25 A source contains two phosphorous radio nuclides ${}^{32}_{15}P$ ($T_{1/2}$ = 14.3d) and ${}^{33}_{15}P$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}P$. How long one must wait until 90% do so?

Sol. We know that $-\dfrac{dN}{dt} \propto N$

So, clearly the initial ratio of the amounts of ${}^{33}_{15}P$ and ${}^{32}_{15}P$ is 1 : 9. We have to find the time after which the ratio is 1:9.

Initially, if the amount of ${}^{33}_{15}P$ is x, the amount of ${}^{32}_{15}P$ is 9x.

Finally, if the amount of ${}^{33}_{15}P$ is 9y, the amount of ${}^{32}_{15}P$ is y.

Using, $N = \dfrac{N_0}{2^{t/T}} \quad ; \quad 9y = \dfrac{x}{2^{t/25.3}} \quad ; \quad y = \dfrac{9x}{2^{t/14.3}}$

Dividing, $9 = \dfrac{x}{2^{t/25.3}} \times \dfrac{2^{t/14.3}}{9x}$ or $81 = 2^{\frac{t}{14.3} - \frac{t}{25.3}}$ or $81 = 2^{\frac{11t}{361.79}}$

or $\quad \log_{10} 81 = \dfrac{11t}{361.79}\log_{10} 2 = \dfrac{11 \times 0.3010t}{361.79} = 9.15 \times 10^{-3}\ t$

$9.15 \times 10^{-3}\ t = 1.91 \qquad$ or $\qquad t = \dfrac{1.91 \times 1000}{9.15} d = 208.7\ d$

13.26 Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$${}^{223}_{88}\text{Ra} \rightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

$${}^{223}_{88}\text{Ra} \rightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Sol. (i) For decay process ${}^{223}_{88}\text{Ra} \rightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C} + Q$

Mass defect, Δm = mass of Ra^{223} – (mass of Pb^{209} + mass of C^{14})

= 223.01850 – (208.98107 + 14.00324) = 0.03419 u

∴ Q = 0.03419 × 931 MeV = 31.83 MeV

(ii) For decay process ${}^{223}_{88}\text{Ra} \rightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He} + Q$

Mass defect, Δm = mass of Ra^{223} – (mass of Rn^{219} + mass of He^4)

= 223.01850 – (219.00948 + 4.00260) = 0.00642 u

∴ Q = 0.00642 × 931 MeV = 5.98 MeV

As Q-values are positive in both the cases, therefore both the decays are energetically possible.

13.27 Consider the fission of ${}^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}\text{Ce}$ and ${}^{99}_{44}\text{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are

m (${}^{238}_{92}\text{U}$) =238.05079 u, m (${}^{140}_{58}\text{Ce}$) =139.90543 u, m (${}^{99}_{44}\text{Ru}$) = 98.90594 u

Sol. Fission reaction is : ${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{44}\text{Ru} + Q$

Q-value = (mass of U^{238} + mass of ${}^{1}_{0}\text{n}$ – mass of Ce^{140} – mass of Ru^{99}) × 931.5 MeV

= (238.05079 + 1.00867 – 139.90543 – 98.90594) × 931.5 MeV = 231.1 MeV

13.28 Consider the D-T reaction (deuterium-tritium fusion)

$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + \text{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:

m (${}^{2}_{1}\text{H}$) = 2.014102 u

m (${}^{3}_{1}\text{H}$) = 3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

Sol. (a) For the process : ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + \text{n} + Q$

Q-value = [mass of ${}^{2}_{1}\text{H}$ + mass of ${}^{3}_{1}\text{H}$ – mass of ${}^{4}_{2}\text{He}$ – mass of n] × 931 MeV

= (2.014102 + 3.016049 – 4.002603 – 1.00867) × 931 MeV = 0.018878 × 931 = 17.58 MeV

(b) Repulsive potential energy of two nuclei when they almost touch each other is

$$\frac{q^2}{4\pi\varepsilon_0 (2r)} = \frac{9\times10^9\ (1.6\times10^{-19})^2}{2\times2\times10^{-15}}\ \text{Joule}\ \ 5.76 \times 10^{-14}\ \text{J}$$

Classically, K.E. at least equal to this amount is required to overcome coulomb repulsion.

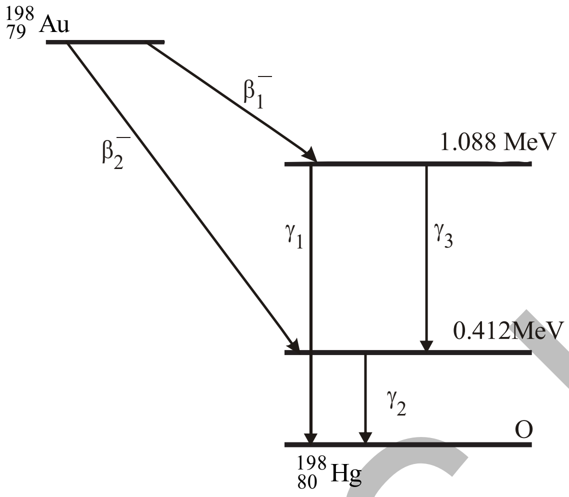
Using relation $KE = 2\times\frac{3}{2}kT$; $T = \frac{KE}{3k} = \frac{5.76\times10^{-14}}{3\times1.38\times10^{-23}} = 1.39 \times 10^9$ K

In actual practice, the temperature required for trigerring the reaction is somewhat less.

13.29 Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in figure. You are given that

m (^{198}Au) = 197.968233 u

m (^{198}Hg) = 197.966760 u



Sol. $\nu(\gamma_1) = \dfrac{(1.088-0)\times 1.6\times 10^{-19}}{6.62\times 10^{-34}}$ = 2.63 × 10^{20} s^{-1}

$\nu(\gamma_2) = \dfrac{(0.412-0)\times 1.6\times 10^{-13}}{6.62\times 10^{-34}}$ = 9.96 × 10^{20} s^{-1}

$\nu(\gamma_3) = \dfrac{(1.088-0.412)\times 1.6\times 10^{-13}}{6.62\times 10^{-34}}$ = 1.63 × 10^{20} s^{-1}

The emission of β_1^- decay may be represented as:

$$^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + E(\beta_1^-) + E(\gamma_1),$$ where, E (γ_1) = 1.088 MeV

Now, $E(\beta_1^-) = [m({}^{198}_{79}\text{Au}) - m({}^{198}_{80}\text{Hg}) - m_e]$ × 931.5 MeV – E (γ_1)

where m ($^{198}_{79}$Au) and m ($^{198}_{80}$Hg) are masses of the $^{198}_{79}$Au and $^{198}_{79}$Au nuclei.

∴ $E(\beta_1^-) = [\{M({}^{198}_{79}\text{Au}) - 79m_e\} - \{M({}^{198}_{80}\text{Hg}) - 80m_e\} - m_e]$ × 931.5 – 1.088

= $[M({}^{198}_{79}\text{Au}) - M({}^{198}_{80}\text{Hg})]$ × 931.5 – 1.088

= 197.968233 – 197.966760) × 931.5 – 1.088 = 1.372 – 1.088 = 0.284 MeV

The emission of β_2^- decay may be represented as:

$$^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + E(\beta_2^-) + E(\gamma_2)$$

As in case of β_1^- decay, it can be deducted that

∴ $E(\beta_2^-) = [M({}^{198}_{79}\text{Au}) - M({}^{198}_{80}\text{Hg})]$ × 931.5 – E (γ_2) = 1.372 – 0.412 = 0.960 MeV

13.30 Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within Sun and (b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Sol. In sun, four hydrogen nuclei fuse to form a helium nucleus with the release of 26 MeV energy.

∴ Energy released by fusion of 1 kg of hydrogen = $\dfrac{6\times 10^{23}\times 26}{4}\times 10^{3}$ MeV

E_1 = 39 × 10^{26} MeV

As energy released in fission of one atom of $_{92}U^{235}$ = 200MeV

$\therefore$ Energy released by fission of 1 kg of ${}_{92}U^{235} = \dfrac{6\times10^{23}\times1000}{235}\times 200$ MeV

$E_2 = 5.1 \times 10^{26}$ MeV

$$\frac{E_1}{E_2} = \frac{39\times10^{26}}{5.1\times10^{26}} = 7.65$$

i.e., energy released in fusion is 7.65 times the energy released in fission.

13.31 Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Sol. Target of producing electric power = 100,000 MW.

Required electric power from nuclear plants = $100000 \times \dfrac{10}{100}$ = 10,000 MW

Therefore, required electric energy from nuclear plants per year

$= (10,000 \times 10^6 \text{ W}) \times 365 \times 24 \times 60 \times 60 = 3.1536 \times 10^{17}$ J

Electrical energy recovered from the fission of one U^{235} nucleus = $200 \times \dfrac{25}{100}$ = 50 MeV

$= 50 \times 1.6 \times 10^{-13} = 8 \times 10^{-12}$ J

$\therefore$ Number of fissions of U^{235} nucleus required = $\dfrac{3.1536\times10^{7}}{8\times10^{-12}} = 3.942 \times 10^{28}$

Number of moles of U^{235} required per year = $\dfrac{3.942\times10^{28}}{6.023\times10^{23}} = 6.5449 \times 10^4$

Therefore, mass of U^{235} required per year = $6.5449 \times 10^4 \times 235$ = 1538.054g = 1.538054 kg