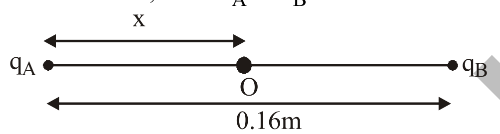
NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 2
ELECTROSTATIC POTENTIAL AND CAPACITANCE

2.1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Sol. **Case I:** Let the potential be zero at O, then $V_A + V_B = 0$

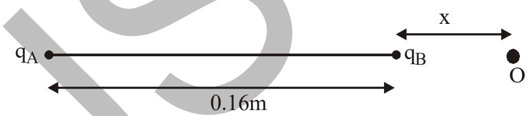


where V_A is electric potential due to charge q_A and V_B is the electric potential due to charge q_B.

i.e. $9 \times 10^9 \dfrac{q_A}{x} + 9 \times 10^9 \dfrac{q_B}{r - x} = 0$ i.e., $9 \times 10^9 \left[\dfrac{5 \times 10^{-8}}{x} + \dfrac{(-3 \times 10^{-8})}{(0.16 - x)}\right] = 0 \Rightarrow \dfrac{5}{x} = \dfrac{3}{0.16 - x}$

$\Rightarrow$ 5 (0.16 – x) = 3x $\Rightarrow$ x = 0.1 = 10cm.

Case II : $9 \times 10^9 \dfrac{q_B}{x} + 9 \times 10^9 \dfrac{q_A}{0.16 + x} = 0$



$\Rightarrow \dfrac{3}{x} = \dfrac{5}{0.16 + x} \Rightarrow$ 3 (0.16 + x) = 5x

or 2x = 0.8 i.e., x = 0.4m = 0.4 × 100 = 40cm.

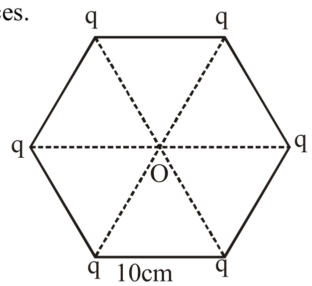
i.e., electric potential is zero at a distance of 40cm. from the charge q_B.

2.2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Sol. Total potential at O is given by,

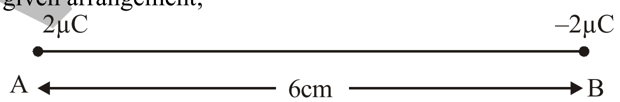
$$V = 6 \times \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)$$

$$= 6 \times (9 \times 10^9) \times \frac{5 \times 10^{-6}}{0.1} = 2.7 \times 10^6 \text{ V}$$



2.3 Two charges 2 μC and –2 μC are placed at points A and B, 6 cm apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?

Sol. (a) For given arrangement,



An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.

(b) The direction of the electric field at every point on this surface is normal to the plane in the direction of AB. (From A to B)

2.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field
(a) inside the sphere
(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere?

Sol. (a) Inside a conductor, the electric field is zero because the charge resides on the surface of a conductor.

(b) Electric field just outside the sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-7}}{(12 \times 10^{-2})^2} = 10^5\ NC^{-1}$$

(c) Electric field at a distant point is given by

$$E' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = (9 \times 10^9) \times \frac{1.6 \times 10^{-7}}{(18 \times 10^{-2})^2} = 4.44 \times 10^4\ NC^{-1}.$$

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Sol. Using $C = \frac{\varepsilon_0 A}{d}$; $C' = \frac{\varepsilon_0 \varepsilon_r A}{d/2} = 2\varepsilon_r\left(\frac{\varepsilon_0 A}{d}\right)$; $C' = 12\,(8 \times 10^{-12}) = 96 \times 10^{-12}\ F = 96\ pF$

2.6 Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Sol. (a) Here, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = 3 \times \frac{1}{C}$

i.e., $\frac{1}{C_{eq}} = 3 \times \frac{1}{9 \times 10^{-12}} = \frac{1}{3 \times 10^{-12}}$ or $C_{eq} = 3 \times 10^{-12}\ F = 3pF$

(b) Since three equal capacitors are across 120V,

So, p.d. across each capacitor $= \frac{120}{3} = 40V$

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Sol. (a) Total capacitance = $C_1 + C_2 + C_3 = 2 + 3 + 4 = 9$ pF

(b) Using q = CV

$\therefore$ $q_1 = C_1V = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10}$ C,

$q_2 = C_2V = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10}$ C

$q_3 = C_3V = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10}$ C

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Sol. Using, $C = \frac{\varepsilon_0 A}{d} = \frac{(8.854 \times 10^{-12})\,(6 \times 10^{-3})}{3 \times 10^{-3}} = 17.708 \times 10^{-12}\ C = 18pF$

Using, C = q/V, we get q = CV i.e., q = 18 × 100 = 1.8 × 10^{-9}C

2.9 Explain what would happen if in the capacitor given in previous question, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) while the voltage supply remained connected. (b) after the supply was disconnected

Sol. (a) (i) $C' = \varepsilon_r C = 6 \times 18 = 108$ pF (ii) $q = C'V = 108 \times 100 = 1.08 \times 10^{-8}$ C

(b) q remains = 1.8×10^{-9} C, Capacitance = 108 pF

$$\therefore \quad V = \frac{q}{C} = \frac{1.8 \times 10^{-9}}{108 \times 10^{-12}} = 16.6V$$

2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Sol. $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times 50 \times 50 = 1.5 \times 10^{-8}$ J.

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Sol. $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 600 \times 10^{-12} \times 200 \times 200 = 12 \times 10^{-6}$ J.

When connected to another capacitor of same capacity the voltage will be equally shared i.e. halved, but the capacitance will be doubled, so the energy is halved.

$\therefore$ Energy lost = 6×10^{-6} J.

ADDITIONAL EXERCISES

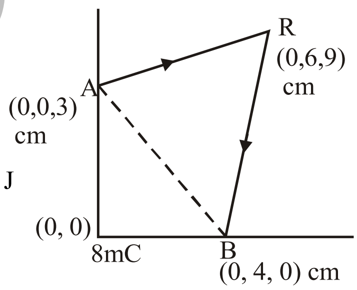
2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point A (0, 0, 3 cm) to a point B (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Sol. As electrostatic field is conservative in nature work done is independent on path followed.

Work done = $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_B} - \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_A}$

i.e., Work done = $\frac{1}{4\pi\varepsilon_0} qq_0 \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

i.e., Work done = $(9 \times 10^9)(8 \times 10^{-3}) \times (-2 \times 10^{-9}) \left(\frac{1}{0.04} - \frac{1}{0.03}\right) = 1.2$ J



2.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol. (i) Distance of the centre of the cube from vertex is half of the diagonal of the cube

i.e., $r = \frac{1}{2}\sqrt{b^2 + b^2 + b^2} = \frac{\sqrt{3}}{2}b$

Using $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ for 8 charges at 8 vertices,

$$V' = 8V = 8 \times \frac{q}{4\pi\varepsilon_0 r} \; ; \quad \frac{8q}{4\pi\varepsilon_0 \frac{\sqrt{3}}{2}} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

(ii) From symmetry, it is clear that electric field at centre of the cube is zero.

2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

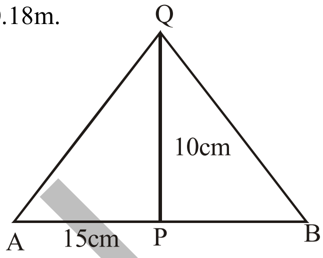
Sol. (a) (i) Potential, $V = V_A + V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{r} + \frac{q_B}{r}\right) = 9 \times 10^9 \left(\frac{1.5 \times 10^{-6}}{0.15} + \frac{2.5 \times 10^{-6}}{0.15}\right)$

$= 2.4 \times 10^5$ V

(ii) Electric field, $E = E_B - E_A = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_B}{r^2} - \dfrac{q_A}{r^2}\right) = 9\times10^9\left(\dfrac{2.5\times10^{-6}}{0.15^2} - \dfrac{1.5\times10^{-6}}{0.15^2}\right)$

$= 4 \times 10^5\ NC^{-1}$ towards 1.5 μC charge.

(b) (i) In this case distance AQ becomes $\sqrt{15^2+10^2} = \sqrt{325}$cm i.e., 0.18m.



Now potential, $V = V_A + V_B = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_A}{0.18} + \dfrac{q_B}{0.18}\right)$

$= \dfrac{9\times10^9}{0.18}[(1.5\times10^{-6}) + (2.5\times10^{-6})] = 2 \times 10^5$ V

(ii) Electric field, $E = \sqrt{E_A^2 + E_B^2 + 2E_AE_B\cos\theta}$

Here, $\theta = 2\alpha$. where $\tan\alpha = \dfrac{15}{10}$ or $\alpha = 56.3°$ and $\theta = 56.3 \times 2 = 112.6°$

$E_A = 9\times10^9 \times \dfrac{1.5\times10^{-6}}{0.182} = 4.167 \times 10^5\ NC^{-1}$

$E_B = 9\times10^9 \times \dfrac{2.5\times10^{-6}}{0.182} = 6.944 \times 10^5\ NC^{-1}$

∴ $E = 6.58 \times 10^5\ NC^{-1}$

For direction, $\tan\beta = \dfrac{E_A \sin\theta}{E_B + E_A\cos\theta}$. We get $\tan\beta = 0.7143$ or $\beta = 35.7°$

Resultant electric field is inclined at an angle $(90 - \alpha) + \beta$
i.e., $(90 - 56.3) + 35.7 = 69.4°$ with AB.

2.15 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Sol. (a) Charge placed at the centre of a shell is +q. Hence, a charge of magnitude –q will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is –q. Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \qquad \text{.......... (1)}$$

A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is Q+q. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \qquad \text{.......... (2)}$$

(b) Yes. The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $(\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$, where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\frac{\sigma\,\hat{n}}{\varepsilon_0}$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

Sol. (a) Electric field on one side of a charged body is $\vec{E}_1$ and electric field on the other side of the same body is $\vec{E}_2$. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n} \qquad \text{.......... (1)}$$

Where, $\hat{n}$ = Unit vector normal to the surface at a point,

σ = Surface charge density at that point

Electric field due to the other surface of the charged body,

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n} \qquad \text{.......... (2)}$$

Electric field at any point due to the two surfaces,

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$(\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{\varepsilon_0} \qquad \text{.......... (3)}$$

Since inside a closed conductor, $\vec{E}_1 = 0$

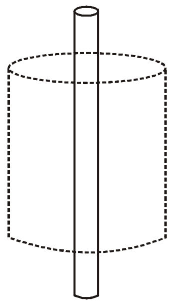
$\therefore$ $\vec{E} = \vec{E}_2 = -\frac{\sigma}{2\varepsilon_0}\hat{n}$. Therefore, the electric field just outside the conductor is $\frac{\sigma}{\varepsilon_0}\hat{n}$

(b) When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Sol. Consider a cylinder having charge density λ.

Electric flux is given by ϕ = E × area of curved surface of cylinder of radius r and length ℓ



i.e., $\phi = E \times 2\pi r\ell$

According to Gauss's law, $\phi = \frac{q}{\varepsilon_0}$

$\therefore$ $E.2\pi r\ell = \frac{q}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0}$ i.e., $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Sol. (a) $$\text{P.E., } U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9\times(-1.6\times10^{-19})\,(1.6\times10^{-19})}{0.53\times10^{-10}} = -43.47\times10^{-19}\text{ J}$$

$$= -\frac{43.47\times10^{-19}}{1.6\times10^{-19}}\text{eV} = -27.17\text{eV}$$

Taking zero at infinity, P.E. = –27.17 – 0 = –27.17 eV

(b) K.E. of electron is half of P.E.

$$\therefore\ \text{K.E.} = \frac{27.12}{2} = 13.585 \text{ (K.E. is always positive)}$$

Total energy of electron = –27.17 + 13.585 = – 13.585 eV

Work required to free the electron = 0 – (–13.585) = 13.585 eV

(c) P.E. at 1.06×10^{-10} m separation,

$$U' = \frac{9\times10^9\times(-1.6\times10^{-19})\,(1.6\times10^{-19})}{1.06\times10^{-10}} = -21.74\times10^{-19}\text{ J}$$

$$= -\frac{21.74\times10^{-19}}{1.06\times10^{-10}} = -13.585\text{ eV}$$

Taking –13.585 eV as zero of PE., then P.E. of the system = –27.17 – (–13.585) = –13.585eV

2.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

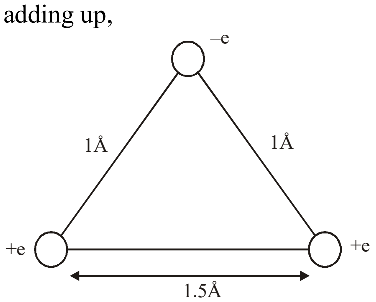
Sol. Using, $\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ for each pair of charges and then adding up,

we get the total P.E.

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{e\times e}{1.5\times10^{-10}} - \frac{e\times e}{10^{-10}} - \frac{e\times e}{10^{-10}}\right]$$

$$= \frac{(9\times10^9)\times(1.6\times10^{-19})^2}{10^{-10}}\left[\frac{1}{1.5} - 1 - 1\right]$$

$$= -19.2\text{ eV}$$



Zero of PE. is taken as infinity.

2.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Sol. Let a be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere.

Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere.

Since the two spheres are connected with a wire, their potential (V) will become equal.

Let E_A be the electric field of sphere A and E_B be the electric field of sphere B.

Therefore, their ratio, $\dfrac{E_A}{E_B} = \dfrac{Q_A}{4\pi\varepsilon_0 \times a^2} \times \dfrac{b^2 \times 4\pi\varepsilon_0}{Q_B}$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \qquad \text{....... (1)}$$

However, $\dfrac{Q_A}{Q_B} = \dfrac{C_A V}{C_B V}$ and $\dfrac{C_A}{C_B} = \dfrac{a}{b}$ $\quad \therefore \dfrac{Q_A}{Q_B} = \dfrac{a}{b}$ .... (2)

Putting the value of eq. (2) in eq. (1), we obtain $\dfrac{E_A}{E_B} = \dfrac{ab^2}{ba^2} = \dfrac{b}{a}$.

Therefore, the ratio of electric fields at the surface is b/a.

Alternatively: Charge transfer till potential become same.

$V = E_1 a = E_2 b$ (Using $V = Ed$)

$$\frac{E_1}{E_2} = \frac{b}{a}$$

2.21 Two charges –q and +q are located at points (0, 0, –a) and (0, 0, a), respectively.

(a) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0)?

(b) Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.

(c) How much work is done in moving a small test charge from the point (5,0,0) to (–7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Sol. (a) Charge –q is located at (0, 0, –a) and charge + q is located at (0, 0, a). Hence, they form a dipole. Point (0, 0, z) is on the axis of this dipole and point (x, y, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (x, y, 0) is zero.

Electrostatic potential at point (0, 0, z) is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z-a}\right) + \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{z+a}\right) = \frac{2qa}{4\pi\varepsilon_0\,(z^2-a^2)} = \frac{p}{4\pi\varepsilon_0\,(z^2-a^2)}$$

where, ε_0 = Permittivity of free space, p = Dipole moment of the system of two charges = 2qa

(b) Distance r is much greater than half of the distance between the two charges.

Hence, the potential (V) at a distance r is inversely proportional to square of the distance

$V \propto 1/r^2$

(c) **Zero.** A test charge is moved from point (5, 0, 0) to point (–7, 0, 0) along the x-axis.

Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$$V_1 = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(5-0)^2+a^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0\sqrt{25^2+a^2}} + \frac{q}{4\pi\varepsilon_0\sqrt{25^2+a^2}} = 0$$

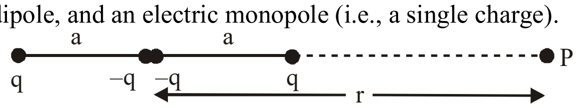
Electrostatic potential, V_2, at point (–7, 0, 0) is given by,

$$V_2 = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}} + \frac{1}{\sqrt{(-7)^2+(a)^2}} = \frac{-q}{4\pi\varepsilon_0\sqrt{49+a^2}} + \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{49+a^2}} = 0$$

Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (–7, 0, 0) along the x-axis.

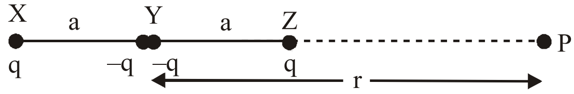
The answer does not change because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

2.22 Figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Sol. Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.

A point is located at P, which is r distance away from point Y.



The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.

Charge +q placed at point X ; Charge –2q placed at point Y ; Charge +q placed at point Z

XY = YZ = a ; YP = r ; PX = r + a ; PZ = r – a

Electrostatic potential caused by the system of three charges at point P is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a^2}{r(r^2 - a^2)}\right] = \frac{2qa^2}{4\pi\varepsilon_0 r^3\left(1 - \frac{a^2}{r^2}\right)}$$

Since, $\frac{r}{a} >> 1 \quad \therefore \frac{a}{r} << 1$; $\frac{a^2}{r^2}$ is taken as negligible. $\therefore \quad V = \frac{2qa^2}{4\pi\varepsilon_0 r^3}$

It can be inferred that potential, $V \propto 1/r^3$

However, it is known that for a dipole, $V \propto 1/r^2$ And, for a monopole, $V \propto 1/r$

2.23 An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Sol. Let N capacitors be used in m rows when each row has n capacitor i.e. N = mn

In series, $400 \times n = 10^3$

i.e., $n = \frac{10^3}{400} = 2.5$ i.e., 3.

In parallel, $\frac{1}{C} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$ i.e., $C = \frac{1}{3}\mu F$

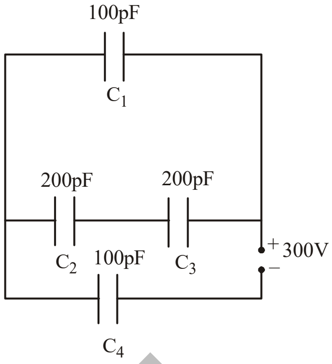
Total capacitance of m rows, $C_{eq} = mC$ i.e., $m = \frac{C_{eq}}{C} = \frac{2}{1/3} = 6$

Total capacitors = m × n = 6 × 3 = 18.

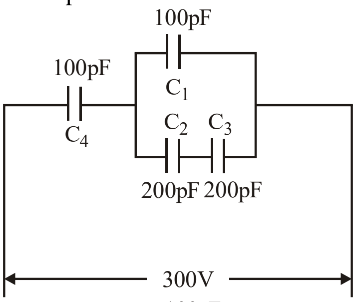
2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Sol. Using, $C = \frac{\varepsilon_0 A}{d}$, we get $A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}} = 1.13 \times 10^9\ m^2$

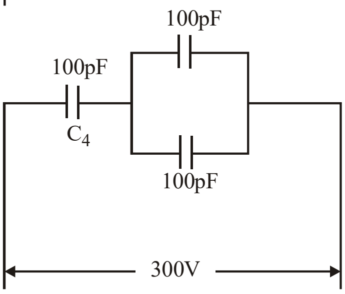
2.25 Obtain the equivalent capacitance of the network in Fig. For a 300 V supply, determine the charge and voltage across each capacitor.



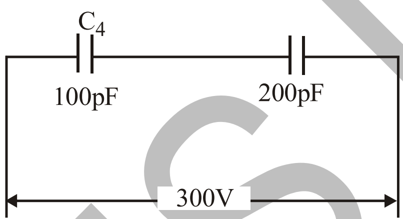
Sol. The equivalent circuit is as shown :



$\Rightarrow$



$\Rightarrow$



Potential difference across C_4 is in the ratio 2 : 1 i.e., 200 V

$\therefore$ Charge on $C_4 = C_4V_4 = 100 \times 200 \times 10^{-12} = 2 \times 10^{-8}$ C

Potential difference across C_1 = 100V

Charge on $C_1 = C_1 \times V = 100 \times 100 \times 10^{-12} = 1 \times 10^{-8}$ C

Potential difference across C_2 and C_3 is 50V each.

$\therefore$ Charge on C_2 or $C_3 = C_2V_2 = 200 \times 50 \times 10^{-12} = 10^{-8}$ C

2.26 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume U. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Sol. Using, $C = \dfrac{\varepsilon_0 A}{d}$ We get $C = \dfrac{8.854 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} = 3.187 \times 10^{-11}$F

Work done, $W = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 3.187 \times 10^{-11} \times 400^2 = 2.55 \times 10^{-6}$ J

Energy per unit volume, $U = 0.113$ J m^{-3}

Energy per unit volume, $U = \dfrac{1}{2}\dfrac{CV^2}{Ad}$ But, $E = \dfrac{V}{d}$ i.e., $V = Ed$

$\therefore$ Energy per unit volume, $U = \dfrac{1}{2}\dfrac{CE^2d^2}{Ad} = \dfrac{1}{2}\dfrac{\varepsilon_0 A}{d}\dfrac{E^2d^2}{Ad}$

Relation between U and E is, $U = \dfrac{1}{2}\varepsilon_0 E^2$

2.27 A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Sol. Energy stored, $U = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 200^2 = 8 \times 10^{-2}$ J

On connection to an uncharged capacitor,

Total charge, $q = C_1V_1 + C_2V_2 = 4 \times 10^{-6} \times 200 + 0 = 8 \times 10^{-4}$ C

Total capacitance, $C = C_1 + C_2 = 4 + 2 = 6$ μF

Common potential, $V = \frac{q}{c} = \frac{8 \times 10^{-4}}{6 \times 10^{-6}} = \frac{400}{3}$ V

Energy stored, $U_2 = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times \left(\frac{400}{3}\right)^2 = 5.33 \times 10^{-2}$ J

Energy lost $= U_1 - U_2 = 8 \times 10^{-2} - 5.33 \times 10^{-2} = 2.67 \times 10^{-2}$ J

2.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

Sol. Let the distance between the plates be increased by Δx.

∴ Work done by external agency = F Δx

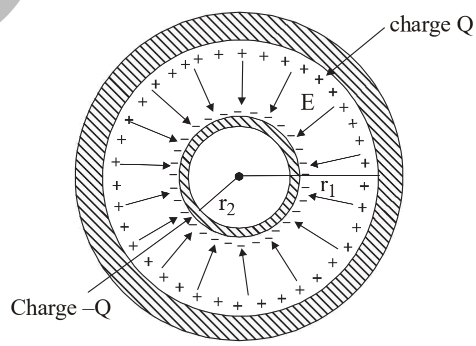
If U is energy density, the increase in P.E. is given by U (A Δx)

$F\,\Delta x = UA\,\Delta x\ ;\ F = UA$

$U = \frac{1}{2}\varepsilon_0 E^2$ i.e., $F = \frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}\varepsilon_0 AEE = \frac{1}{2}QE$ $\quad \left(Q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE\right)$

Outside the conductor, the field is E but inside it is zero. Thus, average value i.e. E/2 decides the force.

2.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig.). Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$, where r_1 and r_2 are the radii of outer and inner spheres, respectively.



Sol. Radius of the outer shell = r_1

Radius of the inner shell = r_2

The inner surface of the outer shell has charge +Q.

The outer surface of the inner shell has induced charge –Q.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

where, ε_0 = Permittivity of free space

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{Q\,(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}$$

Capacitance of the given system is given by,

$$C = \frac{\text{Charge (Q)}}{\text{Potential difference (V)}} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

2.30 A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Sol. (a) Using $C = 4\pi\varepsilon_0 k \dfrac{ab}{b-a}$,

we get $C = \dfrac{1}{9\times10^9}\dfrac{32\times12\times10^{-2}\times13\times10^{-2}}{(13\times10^{-2})-(12\times10^{-2})} = 5.547 \times 10^{-9}$ F

(b) $V = \dfrac{q}{C} = \dfrac{2.5\times10^{-6}}{5.547\times10^{-9}} = 450.7\text{V}$

(c) $C' = 4\pi\varepsilon_0 r = \dfrac{1}{9\times10^9}\times12\times10^{-12} = 1.33 \times 10^{-11}$ F

$\dfrac{C}{C'} = \dfrac{5.547\times10^{-9}}{1.333\times10^{-11}} = 416$

Clearly C' is small because there is no nearby earthed conducting plate.

2.31 Answer carefully:

(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres?

(b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(f) What meaning would you give to the capacitance of a single conductor?

(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Sol. (a) The force between two conducting spheres is not exactly given by the expression, $Q_1Q_2/4\pi\varepsilon_0 r^2$, because there is a non-uniform charge distribution on the spheres.

(b) Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of $1/r^2$, on r.

(c) Yes, If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration and not of velocity.

(d) Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero.

(e) No, Electric field is discontinuous across the surface of a charged conductor. However, electric potential is continuous.

(f) The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

(g) Water has an unsymmetrical space as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica. Mica does not have polar molecules.

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5μ. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Sol. Length of a co-axial cylinder, $\ell = 15$ cm = 0.15 m

Radius of outer cylinder, $r_1 = 1.5$ cm = 0.015 m

Radius of inner cylinder, $r_2 = 1.4$ cm = 0.014 m

Charge on the inner cylinder, $q = 3.5\ \mu C = 3.5 \times 10^{-6}$ C

Capacitance of a co-axial cylinder will radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi\varepsilon_0 \ell}{\log_e \frac{r_1}{r_2}}, \quad \varepsilon_0 = \text{where Permittivity of free space} = 8.85 \times 10^{-12}\ N^{-1}M^{-2}C^{-2}.$$

$$\therefore \quad C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.303 \log_{10}\left(\frac{0.15}{0.14}\right)} = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.303 \times 0.0299} = 1.2 \times 10^{-10}\,F$$

Potential difference of the inner cylinder is given by, $V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.89 \times 10^4$ V

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about $10^7\ Vm^{-1}$. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Sol. Potential rating of a parallel plate capacitor, V = 1 kV = 1000 V

Dielectric constant of a material, $\varepsilon_r = 3$

Dielectric strength = 10^7 V/m

For safety, the field intensity never exceeds 10% of the dielectric strength.

Hence, electric field intensity, E = 10% of $10^7 = 10^6$ V/m

Capacitance of the parallel plate capacitor, C = 50 pF = 50×10^{-12} F

Distance between the plates is given by, $d = \frac{V}{E} = \frac{1000}{10^6} = 10^{-3}\,m$

Capacitance is given by the relation, $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

Where, A = Area of each plate, ε_0 = Permittivity of free space = $8.85 \times 10^{-12}\ N^{-1}M^{-2}C^{-2}$.

$$A = \frac{Cd}{\varepsilon_0 \varepsilon_r} = \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19\text{cm}^2.$$ Hence, the area of each plate is about 19 cm^2.

2.34 Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the z-direction,

(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(c) a single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Sol. (a) Equidistant planes parallel to the x-y plane are the equipotential surfaces.

(b) Planes parallel to the x-y plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.

(c) Concentric spheres centered at the origin are equipotential surfaces.

(d) A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

2.35 In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Sol. Potential difference, $V = 15 \times 10^6$ V

Dielectric strength of the surrounding gas = 5×10^7 V/m

Electric field intensity, E = Dielectric strength = 5×10^7 V/m

Minimum radius of the spherical shell required for the purpose is given by,

$$r = \frac{V}{E} = \frac{15 \times 10^6}{5 \times 10^7} = 0.3\text{m} = 30\text{cm}. \qquad \left(V = \frac{kq}{r}, \quad E = \frac{kq}{r^2}\right)$$

Hence, the minimum radius of the spherical shell required is 30 cm.

2.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Sol. According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V_1 between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.

2.37 Answer the following:

(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m^2. Will he get an electric shock if he touches the metal sheet next morning?

(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

Sol.

(a) We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential.

(b) Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.

(c) The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.

(d) During lightning and thunderstorm, light energy, heat energy, and sound energy are dissipated in the atmosphere.