NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 6
ELECTROMAGNETIC INDUCTION

6.1 Predict the direction of induced current in the situations described by the following Figs. (a) to (f).

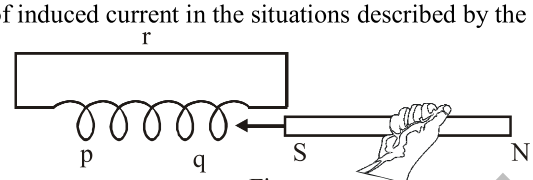


Fig. a

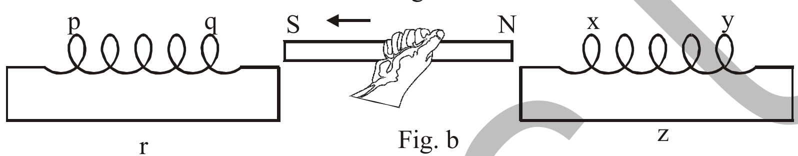
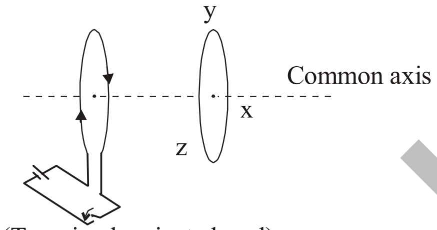


Fig. b



(Trapping key just closed)

figure (c)

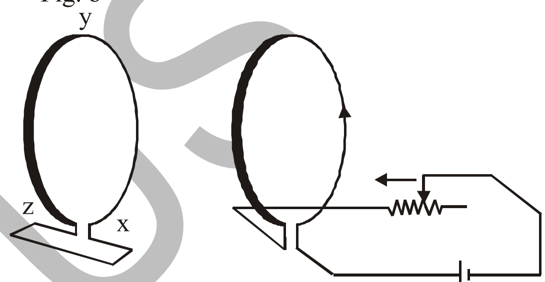


Fig. d

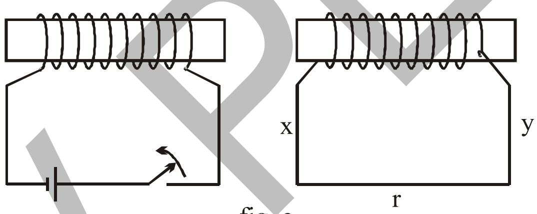


fig. e

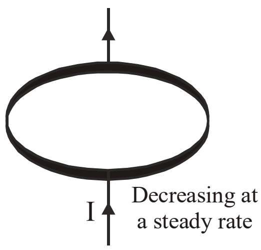


Fig.f

Sol. The direction of the induced current in a closed loop is given by Lenz's law. Direction of induced current is such that it tries to oppose cause of its own production.

(a) When South Pole approaches coil behave as South Pole (view from right towards left). The direction of the induced current is along qrpq.

(b) For coil pq: When south pole approaches coil behave as south pole (view from right towards left). The direction of the induced current is along prq.
For coil xy: When north pole moves away coil behave as south pole (view from left towards right). The direction of the induced current is along yzx.

(c) As current increase induce current tries to oppose it by producing magnetic field in opposite direction. The direction of the induced current is along yzx.

(d) As resistance decreases current increases hence induce current is such that it produces magnetic field in opposite direction. The direction of the induced current is along zyx.

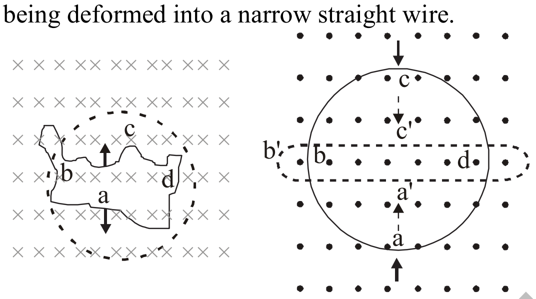
(e) As current decreases induce current tries to oppose it by producing magnetic field in same direction. The direction of the induced current is along xry.

(f) No current is induced since the field lines are lying in the plane of the closed loop.

6.2 Use Lenz's law to determine the direction of induced current in the situations described by Figure:

(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.



Sol. (a) Along adcd (flux through the surface increases during shape change, so induced current produces opposing flux).

(b) Along a'd'c'b' (flux decreases during the process)

6.3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1s, what is the induced emf in the loop while the current is changing?

Sol. Using, $e = \frac{dB}{dt}$ we get $e = NA\frac{dB}{dt}$ $(\phi = BA)$

But $B = \mu_0 \frac{N'}{\ell} I$

$\therefore \quad dB = \mu_0 \frac{N'}{\ell}(I_2 - I_1) = (4\pi \times 10^{-7})\,(1500)\,(4-2)$ ($\because$ in 1m length the no. of turns are 15 × 100)

Then, $e = \frac{1 \times 2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times 2}{0.1}$ ($\because$ loop has only one turn)

$= 7.51 \times 10^{-6}$ V

6.4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Sol. Using $e = B\ell v$ when

(i) Velocity of loop is in the direction normal to the longer side, we get

$e = (0.3 \times 10^{-2})\,(8 \times 10^{-2}) = 0.24$ mV

Using, $v = \frac{\ell}{t}$, we get $t = \frac{\ell}{v} = \frac{2}{1} = 2s$

(ii) Velocity of loop is in the direction normal to the shorter side.

Here, $e = (0.3 \times 10^{-2})\,(2 \times 10^{-2}) = 0.06$ mV

Time, $t = \frac{\ell}{v} = \frac{8}{1} = 8s$

6.5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Sol. Length of the rod, $\ell = 1$ m

Angular frequency, $\omega = 400$ rad/s

Magnetic field strength, B = 0.5 T

One end of the rod has zero linear velocity, while the other end has a linear velocity of $\ell\omega$.

Average linear velocity of the rod, $v = \frac{\ell\omega + 0}{2} = \frac{\ell\omega}{2}$

Emf developed between the centre and the ring,

$$e = B\ell v = B\ell\left(\frac{\ell\omega}{2}\right) = \frac{B\ell^2\omega}{2} = \frac{0.5\times(1)^2\times 400}{2} = 100\text{V}$$

Hence, the emf developed between the centre and the ring is 100 V.

Alternative: $|e| = \frac{d\phi}{dt} = \frac{d\,(BA)}{dt} = \frac{d}{dt}\left(\frac{B\pi\ell^2\theta}{2\pi}\right) = \frac{B\ell^2}{2}\frac{d\theta}{dt} = \frac{B\ell^2\omega}{2}$

6.6 A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Sol. (i) Flux through each turn of the loop = $\pi r^2 B\cos\omega t$

$$e = -\frac{d\phi}{dt} = -N\omega\pi r^2 B\sin\omega t$$

$$e_{max} = N\omega\pi r^2 B$$

$$|e_{max}| = 20\times 50\times\pi\times 64\times 10^{-4}\times 3\times 10^{-2} = 0.603\ \text{V}$$

(ii) e_{av} for full cycle = zero

(iii) $I_{max} = \frac{e_{max}}{2R} = \frac{0.693}{10} = 0.0603\text{A}$

(iv) Power loss = $\frac{1}{2}e_{max}I_{max} = \frac{1}{2}\times 0.603\times 0.0603 = \frac{0.036}{2} = 0.018$ W

(v) Source of power loss is the external rotor which provides the necessary torque.

6.7 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?
(b) What is the direction of the emf ?
(c) Which end of the wire is at the higher electrical potential?

Sol. (a) Using $\varepsilon = B\,v\,\ell$, we get

$$\varepsilon = (0.30 \times 164)\,(5)\,(10) = 1.5 \times 10^{-3}\ \text{V}$$

(b) Using Fleming's Right hand rule, the direction of induced emf is west to East.
(c) Since the rod will act as a source, the Eastern end will be at higher electrical potential.

6.8 Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

Sol. Using, $|\varepsilon| = L\frac{di}{dt}$, we get $200 = L\left(\frac{5-0}{0.1}\right)$

i.e., $L = \frac{200\times 0.1}{5} = 4\text{H}$

6.9 A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Sol. Using, $d\phi = MdI$, we get

$$d\phi = 1.5 \times (20 - 0) = 30\ \text{Wb}$$

6.10 A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°.

Sol. Using, $e = B_V \ell v$, we get $e = (B \sin 30°)\, \ell v$

$$e = 500 \sin 30° \,(5 \times 10^{-4})\, 25 = \frac{500}{2} \times 5 \times 10^{-4} \times 25 = 3.1\text{V}$$

ADDITIONAL EXERCISES

6.11 Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?

Sol. Induced emf, $e = \frac{d\phi}{dt} = \frac{dB}{dt}A = (0.02)\,(8 \times 2 \times 10^{-4}) = 3.2 \times 10^{-5}$ V

Induced current, $I = \frac{e}{R} = \frac{3.2}{1.6} \times 10^{-5} = 2 \times 10^{-5}$ A

Power loss = $e \times I = 3.2 \times 10^{-5}\text{ V} \times 2 \times 10^{-5}\text{ A} = 6.4 \times 10^{-10}$ W

Source of this power is the external agency which brings change in magnetic field.

6.12 A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} Tcm^{-1} along the negative x-direction (that is it increases by 10^{-3} T cm^{-1} as one moves in the negative x-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 mΩ.

Sol. Rate of change of flux due to time variation in B = $144 \times 10^{-4} \times 10^{-3} = 1.44 \times 10^{-5}$ Wb s^{-1}

Rate of change of flux due to non-uniform B = $(1.44 \times 10^{-4})\,(10^{-3})\,8 = 11.52 \times 10^{-5}$ Wb s^{-1}

The two effects add up since both causes a decrease in flux along the positive z-direction.

Total rate of change of flux = $(1.44 + 11.52)\,10^{-5} = 12.96 \times 10^{-5}$ Wb s^{-1}

i.e. $e = 12.96 \times 10^{-5}$ V and $I = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-6}} = 2.88 \times 10^{-2}$ A

Direction of induced current in such that it increase the magnetic flux linking with the loop in positive direction.

6.13 It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

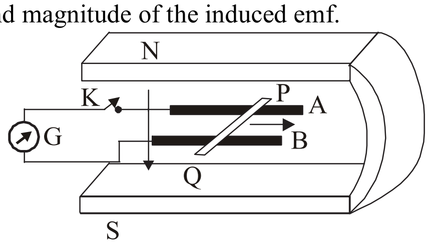
Sol. Using, $e = -N\frac{d\phi}{dt} = -N\frac{dB}{dt}A$

But $e = iR \;\therefore\; IR = -N\frac{dB}{dt}A$ i.e., $R\frac{dq}{dt} = -\frac{NA\,dB}{dt}$ or $Rq = NAB$

or $B = \frac{Rq}{NA}$ or $\phi = BA = \frac{Rq}{N} = \frac{0.5 \times 7.5 \times 10^{-3}}{25} = 0.75$ T

6.14 Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 cm s^{-1}) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Sol. Length of the rod, ℓ = 15 cm = 0.15 m

Magnetic field strength, B = 0.50 T

Resistance of the closed loop, R = 9 mΩ = 9×10^{-3} Ω

(a) Speed of the rod, v = 12 cm/s = 0.12 m/s

Induced emf is given as:

$$e = Bv\ell = 0.5 \times 0.12 \times 0.15 = 9 \times 10^{-3}\ V = 9\ mV$$

The polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

(b) Yes; when key K is closed, excess charge is maintained by the continuous flow of current.

When key K is open, there is excess charge built up at both ends of the rods.

When key K is closed, excess charge is maintained by the continuous flow of current.

(c) Magnetic force gets cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rod.

(d) Retarding force exerted on the rod, F = IBℓ,

where, I = Current flowing through the rod

$$I = \frac{e}{R} = \frac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1A$$

$\therefore$ F = $1 \times 0.5 \times 0.15 = 75 \times 10^{-3}$ N

(e) Speed of the rod, v = 12 cm/s = 0.12 m/s

Power is given as: P = Fv = $75 \times 10^{-3} \times 9 \times 10^{-3}$ W = 9mW

When key K is open, no power is expended.

(f) Power dissipated as heat = $I^2 R = (1)^2 \times 9 \times 10^{-3}$ = 9 mW

The source of this power is an external agent.

(g) Zero

In this case, no emf is induced in the coil because the motion of the rod does not cut across the field lines.

6.15 An air-cored solenoid with length 30 cm, area of cross-section $25 cm^2$ and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3}s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Sol. Here, $B = \mu_0 \frac{N}{\ell} I = (4\pi \times 10^{-7}) \frac{500}{(30 \times 10^{-2})} \times 2.5 = 52.36 \times 10^{-4}$ T

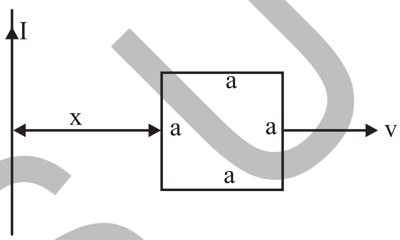
Initial flux linkage $\phi = BAN = 52.36 \times 10^{-4} \times 25 \times 10^{-4} \times 500 = 65.45 \times 10^{-4}$ Wb

Final flux linkage, $\phi_f = 0$

$$d\phi = -65.45 \times 10^{-4} \text{ Wb}$$

Then, $e = -\frac{d\phi}{dt} = \frac{65.45 \times 10^{-4}}{10^{-3}} = 6.54 \text{ V} \approx 6.5\text{V}$

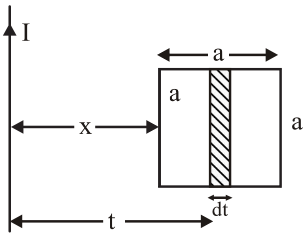
6.16 (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig.



(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s. Calculate the induced emf in the loop at the instant when x = 0.2 m. Take a = 0.1 m and assume that the loop has a large resistance.

Sol. (a) Consider a small portion of the coil of thickness dt at a distance t from the current carrying wire. Then the magnetic field strength at this portion is

$$B = \frac{\mu_0 I}{2\pi t}$$



Area of the strip, dA = a . dt

$\therefore$ Magnetic flux linked with the strip,

$$d\phi = BdA = \frac{\mu_0 Ia}{2\pi t} dt$$

$\therefore$ Total magnetic flux linked with the coil (within limits from t = x to t = a + x) is given by

$$\phi = \int_x^{a+x} \frac{\mu_0 Ia}{2\pi t} dt = \frac{\mu_0 Ia}{2\pi} \int_x^{a+x} \frac{1}{t} dt = \frac{\mu_0 Ia}{2\pi} [\log_e t]_x^{a+x}$$

$\Rightarrow$ $$\phi = \frac{\mu_0 Ia}{2\pi} [\log_e (a + x) - \ln x] = \frac{\mu_0 Ia}{2\pi} \left[\log_e \frac{a + x}{x} \right]$$

or $$\phi = \frac{\mu_0 Ia}{2\pi} \log_e \left(1 + \frac{a}{x} \right) \quad \text{.......... (1)}$$

Comparing eqn. (1) with standard equation $\phi = MI$,

we get $M = \frac{\mu_0 Ia}{2\pi} \log_e \left(1 + \frac{a}{x} \right)$

(b) The value of induced emf is given by

$$\varepsilon = \frac{\mu_0}{2\pi x} \frac{Ia^2 v}{(a + x)} = \frac{2 \times 10^{-7} \times 50 \times (0.1)^2 \times 10}{0.2\,(0.1 + 0.2)}$$

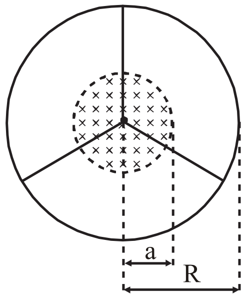
$$= 1.67 \times 10^{-5} \text{ V} \approx 1.7 \times 10^{-5} \text{ V}$$

6.17 A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig.). A uniform magnetic field extends over a circular region within the rim. It is given by,

$\vec{B} = -B_0\hat{k}$ [$r \le a$; $a < R$]

$= 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?



Sol. Change in magnetic field is given by

$$\frac{d\vec{B}}{dt} = \frac{B_0}{t}\hat{k}$$

The work done in moving the charge once around the loop due to electric field E produced by the magnetic field is given by $W = F \times \ell = qE\,(2\pi r)$

$$\therefore \quad \varepsilon = \frac{W}{q} = E.2\pi R \qquad .... (1)$$

But, from Faraday's law of electromagnetic induction,

$$\varepsilon = -\pi a^2 \frac{dB}{dt} \qquad .... (2)$$

Equating (1) and (2), $E.\pi R = -\pi a^2 \dfrac{dB}{dt}$

Multiplying both sides by q dt and rearranging, we get

$$qEdt = \frac{-\pi a^2 q}{2\pi R} dB \qquad .... (3)$$

But $qE\,dt = F\,dt$ = Impulse = Change in momentum of the wheel = $(Mv - 0) = Mv = M\omega R$

$\therefore$ From eq. (3), $M\omega R = -\pi a^2 \left(\dfrac{q}{2\pi R}\right) dB$

But $\dfrac{q}{2\pi R} = -\lambda$ and $dB = B_0$ (in magnitude)

$\therefore$ $m\omega R = -\pi a^2 \lambda B_0$ or $\omega = \dfrac{-\pi a^2 \lambda B_0}{MR}$

In vector form, $\vec{\omega} = \dfrac{-\pi a^2 \lambda B_0}{MR}\hat{k} = \dfrac{B\pi a^2 \lambda}{MR}\hat{k}$