## NCERT SOLUTIONS

## PHYSICS XII CLASS

## CHAPTER - 7

## ALTERNATING CURRENT

7.1 A $100 \Omega$ resistor is connected to a $220 \mathrm{~V}, 50 \mathrm{~Hz}$ ac supply.
(a) What is the rms value of current in the circuit?
(b) What is the net power consumed over a full cycle?

Sol. (a) $\mathrm{I}_{\mathrm{rms}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\mathrm{R}}=\frac{220}{100}=2.20 \mathrm{~A}$
(b) Net power $=\mathrm{V}_{\text {rms }} \times \mathrm{I}_{\text {rms }}=220 \times 2.20=484 \mathrm{~W}$
7.2 (a) The peak voltage of an ac supply is 300 V . What is the rms voltage?
(b) The rms value of current in an ac circuit is 10 A . What is the peak current?

Sol. (a) $\mathrm{V}_{\mathrm{rms}}=\frac{\mathrm{V}_{\max }}{\sqrt{2}}=\frac{300}{\sqrt{2}}=212.1 \mathrm{~V}$
(b) $I_{\text {rms }}=\frac{I_{\max }}{\sqrt{2}}$ or $I_{\max }=I_{r m s} \sqrt{2}$ i.e., $I_{\max }=10 \times \sqrt{2}=14.1 \mathrm{~A}$
7.3 A 44 mH inductor is connected to $220 \mathrm{~V}, 50 \mathrm{~Hz}$ ac supply. Determine the rms value of the current in the circuit.
Sol. Here, $X_{L}=2 \pi \nu \mathrm{~L}=2 \pi \times 50 \times 44 \times 10^{-3}$
$\therefore \mathrm{I}_{\mathrm{rms}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\mathrm{X}_{\mathrm{L}}}=\frac{220}{2 \pi \times 50 \times 44 \times 10^{-3}}=15.91 \mathrm{~A}$
7.4 A $60 \mu \mathrm{~F}$ capacitor is connected to a $110 \mathrm{~V}, 60 \mathrm{~Hz}$ ac supply. Determine the rms value of the current in the circuit.
Sol. Here, $X_{C}=\frac{1}{2 \pi \nu C}=\frac{1}{2 \pi \times 60 \times 60 \times 10^{-6}}$
$\therefore \quad \mathrm{I}_{\mathrm{rms}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\mathrm{X}_{\mathrm{C}}}=\frac{110}{1 /\left(2 \pi \times 60 \times 60 \times 10^{-6}\right)}=110\left(2 \pi \times 60 \times 60 \times 10^{-6}\right)=2.49 \mathrm{~A}$
7.5 In Q 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.
Sol. The net power absorbed is given by the relation,
$\mathrm{P}=\mathrm{VI} \cos \phi, \quad$ Where, $\phi=$ Phase difference between V and I.
For a pure inductive circuit, the phase difference between alternating voltage and current is $90^{\circ}$ i.e., $\phi=90^{\circ}$. Hence, $\mathrm{P}=0$ i.e., the net power is zero.
For a pure capacitive circuit, the phase difference between alternating voltage and current is $90^{\circ}$ i.e., $\phi=90^{\circ}$. Hence, $\mathrm{P}=0$ i.e., the net power is zero.
7.6 Obtain the resonant frequency $\omega_{\mathrm{r}}$ of a series LCR circuit with $\mathrm{L}=2.0 \mathrm{H}, \mathrm{C}=32 \mu \mathrm{~F}$ and $\mathrm{R}=10 \Omega$. What is the Q -value of this circuit?
Sol. Using, $\omega_{\mathrm{r}}=\frac{1}{\sqrt{\mathrm{LC}}}$,
we get $\quad \omega_{\mathrm{r}}=\frac{1}{\sqrt{2 \times 32 \times 10^{-6}}}=\frac{1}{8 \times 10^{-3}}=125 \mathrm{~s}^{-1}$.

Then, $\mathrm{Q}=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{\omega_{\mathrm{r}} \mathrm{L}}{\mathrm{R}}=\frac{125 \times 2}{10}=25$
7.7 A charged $30 \mu \mathrm{~F}$ capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?
Sol. Using, $\omega_{\mathrm{r}}=\frac{1}{\sqrt{\mathrm{LC}}}$ we get $\omega_{\mathrm{r}}=\frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}=1.1 \times 10^{3} \mathrm{~s}^{-1}$
7.8 Suppose the initial charge on the capacitor in previous question is 6 mC . What is the total energy stored in the circuit initially? What is the total energy at later time?
Sol. Using, $\mathrm{E}=\frac{1}{2} \frac{\mathrm{Q}^{2}}{\mathrm{C}}$, we get $\mathrm{E}=\frac{1}{2} \times \frac{6 \times 10^{-3} \times 6 \times 10^{-3}}{30 \times 10^{-6}}=0.6 \mathrm{~J}$
Same at later time.
7.9 A series LCR circuit with $\mathrm{R}=20 \Omega, \mathrm{~L}=1.5 \mathrm{H}$ and $\mathrm{C}=35 \mu \mathrm{~F}$ is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?
Sol. At natural frequency, $\mathrm{X}_{\mathrm{L}}=\mathrm{X}_{\mathrm{C}} \quad \therefore \mathrm{Z}=\mathrm{R}$
Using, $\mathrm{P}=\frac{\mathrm{V}^{2}}{\mathrm{R}} \quad$ we get, $\mathrm{P}=\frac{200 \times 200}{20}=2000 \mathrm{~W}$
7.10 A radio can tune over the frequency range of a portion of MW broadcast band: ( 800 kHz to 1200 kHz ). If its LC circuit has an effective inductance of $200 \mu \mathrm{H}$, what must be the range of its variable capacitor?
Sol. Using, $v=\frac{1}{2 \pi \sqrt{\mathrm{LC}}}$, we get $\mathrm{C}=\frac{1}{4 \pi^{2} v^{2} \mathrm{~L}}$ i.e., $\mathrm{C}=\frac{1}{4 \pi^{2}\left(200 \times 10^{-6}\right)\left(800 \times 10^{3}\right)^{2}}=197.8 \mathrm{pF}$
Similarly for 1200 KHz , we get $\mathrm{C}=\frac{1}{4 \pi^{2}\left(200 \times 10^{-6}\right)\left(1200 \times 10^{3}\right)^{2}}=87.9 \mathrm{pF}$
Thus the range of variable capacitor must be 88 pF to 198 pF .
7.11 Figure shows a series LCR circuit connected to a variable frequency 230 V source. $\mathrm{L}=5.0 \mathrm{H}, \mathrm{C}=80 \mu \mathrm{~F}, \mathrm{R}=40 \Omega$
(a) Determine the source frequency which drives the circuit in resonance.
(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
(c) Determine the rms potential drops across the three elements of the circuit.


Show that the potential drop across the LC combination is zero at the resonating frequency.
Sol. (a) $\omega=\frac{1}{\sqrt{\mathrm{LC}}}=\frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}=50 \mathrm{rad} \mathrm{s}^{-1}$
(b) At resonance, $\mathrm{Z}=\mathrm{R}=40 \Omega$
$\therefore \quad \mathrm{I}_{\text {max }}=\frac{\mathrm{V}_{\text {max }}}{\mathrm{R}}=\frac{\sqrt{2} \times 230}{40}=8.1 \mathrm{~A}$
(c) Across inductance $\mathrm{V}_{\mathrm{L}}=\mathrm{I} \mathrm{X}_{\mathrm{L}}=\mathrm{I} \omega \mathrm{L}=8.1 \times 50 \times 5=1437.5 \mathrm{~V}$

Across capacitance $\quad \mathrm{V}_{\mathrm{C}}=\mathrm{I} \mathrm{X}_{\mathrm{C}}=\frac{1}{\omega \mathrm{C}}=\frac{8.1}{50 \times 80 \times 10^{-6}}=1437.5 \mathrm{~V}$
Potential drop across LC combination $=\mathrm{I} \mathrm{X}_{\mathrm{L}}-\mathrm{I} \mathrm{X}_{\mathrm{C}}=1437.5-1437.5=0$

## ADDITIONAL EXERCISES

7.12 An LC circuit contains a 20 mH inductor and a $50 \mu \mathrm{~F}$ capacitor with an initial charge of 10 mC . The resistance of the circuit is negligible. Let the instant the circuit is closed be $t=0$.
(a) What is the total energy stored initially? Is it conserved during LC oscillations?
(b) What is the natural frequency of the circuit?
(c) At what time is the energy stored
(i) completely electrical (i.e., stored in the capacitor)?
(ii) completely magnetic (i.e., stored in the inductor)?
(d) At what times is the total energy shared equally between the inductor and the capacitor?
(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Sol. (a) Total initial energy $=\frac{\mathrm{Q}^{2}}{2 \mathrm{C}}=\frac{10^{-4}}{2 \times 50 \times 10^{-6}}=1 \mathrm{~J}$
Yes, if $\mathrm{R}=0$ then sum of energies for L and C is conserved.
(b) Using $\omega=\frac{1}{\sqrt{\mathrm{LC}}}$, we get $\omega=\frac{1}{\sqrt{\left(20 \times 10^{-3}\right)\left(50 \times 10^{-6}\right)}}=10^{3} \mathrm{rad} \mathrm{s}^{-1}$
and $\omega=2 \pi \nu$ or $\quad v=\frac{\omega}{2 \pi}=\frac{10^{3}}{2 \pi}=159 \mathrm{~Hz}$
(c) Using $q=q_{0} \cos \omega t$ and $U=\frac{1}{2} \frac{q^{2}}{C}$, we get
(i) Stored electrical energy at $\mathrm{t}=0, \frac{\mathrm{~T}}{2}, \mathrm{~T}, \frac{3 \mathrm{~T}}{2}, \ldots$
(ii) Electric energy as zero and purely magnetic energy at $t=\frac{T}{4}, \frac{3 \mathrm{~T}}{4}, \frac{5 \mathrm{~T}}{4} \ldots \ldots$

Here, $T=\frac{1}{v}=\frac{1}{159}=6.3 \mathrm{~ms}$
(d) The energy is shared equally between inductor and capacitor at $\frac{\mathrm{T}}{8}, \frac{3 \mathrm{~T}}{4}, \frac{5 \mathrm{~T}}{4} \ldots \ldots$. because

Energy, $E=\left(\frac{q^{2}}{2 C}\right)=\frac{1}{2} \frac{\left(q_{0} \cos \omega t\right)^{2}}{C}$
If $\omega \mathrm{t}=45^{\circ}$, then $\cos \omega \mathrm{t}=\frac{1}{\sqrt{2}}$, then $\mathrm{E}=\frac{1}{2} \frac{\mathrm{q}_{0}^{2}}{2 \mathrm{C}}=\frac{1}{2}$ of the total energy.
(e) Total initial energy of 1 J will be lost as heat due to Joule's heating effect in the resistor.
7.13 A coil of inductance 0.50 H and resistance $100 \Omega$ is connected to a $240 \mathrm{~V}, 50 \mathrm{~Hz}$ ac supply.
(a) What is the maximum current in the coil?
(b) What is the time lag between the voltage maximum and the current maximum?

Sol. (a) Current, $I_{\max }=\frac{\mathrm{V}_{\max }}{\sqrt{\mathrm{R}^{2}+\omega^{2} \mathrm{~L}^{2}}}=\frac{240 \times \sqrt{2}}{\sqrt{100^{2}+0.5^{2} \times 4 \pi^{2} \times 2500}} \quad(\because \omega=2 \pi v)$

$$
=1.82 \mathrm{~A}
$$

(b) $\quad \tan \phi=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{2 \pi v \mathrm{~L}}{\mathrm{R}}=\frac{2 \pi \times 50 \times 0.5}{100}=1.571 \Rightarrow \phi=\tan ^{-1}(1.571)=57.5^{\circ}$

Time lag $=\frac{\mathrm{T} \times 57.5}{360}=\frac{2 \pi}{\omega} \times \frac{57.5}{360}=\frac{57.5}{180 \times 2 \pi \times 50}=3.2 \mathrm{~ms}$
7.14 Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply $(240 \mathrm{~V}, 10 \mathrm{kHz})$. Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?
Sol. For the given high frequency, $\omega=2 \pi \nu=2 \pi \times 10^{4} \mathrm{rad} \mathrm{s}^{-1}$
Now, $\mathrm{I}_{0}=\frac{\mathrm{E}_{0}}{\sqrt{\mathrm{R}^{2}+\omega^{2} \mathrm{~L}^{2}}}=\frac{240 \sqrt{2}}{\sqrt{100^{2}+4 \pi^{2} \times 10^{8} \times 0.5^{2}}}=1.1 \times 10^{-2} \mathrm{~A}$
$\tan \phi=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{2 \pi v \mathrm{~L}}{\mathrm{R}}=\frac{2 \pi \times 10^{-4} \times 0.5^{2}}{100}=100 \pi$
i.e., $\phi$ is close to $\pi / 2$, then time lag $=\frac{2 \pi}{2 \pi \nu} \times \frac{\pi}{2} \times \frac{90}{360}=0.25 \times 10^{-4} \mathrm{~s}$
$\mathrm{I}_{0}$ in this case is too small, so it can be concluded that at high frequencies an inductance behaves as an open circuit. In a steady d.c. circuit $v=0$, so inductance acts as a simple conductor.
7.15 A $100 \mu \mathrm{~F}$ capacitor in series with a $40 \Omega$ resistance is connected to a $110 \mathrm{~V}, 60 \mathrm{~Hz}$ supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

Sol.

(b) $\tan \phi=-\frac{\mathrm{X}_{\mathrm{C}}}{\mathrm{R}}=-\frac{1}{\omega \mathrm{CR}}=-\frac{1}{2 \pi \times 60 \times 10^{-4} \times 40}=-0.6631$
$\therefore|\phi|=33.5^{\circ}$
Time lag $=\frac{\phi}{\omega}=\frac{33.5 \pi}{180 \times 2 \pi 60}=1.55 \mathrm{~ms}$
7.16 Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a $110 \mathrm{~V}, 12 \mathrm{kHz}$ supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.
Sol. (a) $\mathrm{I}_{\max }=\frac{\mathrm{E}_{\max }}{\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{C}}^{2}}}=\frac{110 \sqrt{2}}{\sqrt{1600+\frac{1}{4 \pi^{2} \times 144 \times 10^{6} \times 10^{-8}}}}=3.88 \mathrm{~A} \quad\left(\because \mathrm{X}_{\mathrm{C}}=\frac{1}{2 \pi \nu \mathrm{C}}\right)$
(b) $\tan \phi=-\frac{1}{\omega \mathrm{CR}}=-\frac{1}{2 \pi \times 12 \times 10^{3} \times 10^{-4} \times 40}=-\frac{1}{96 \pi}$
$\phi$ is nearly zero at high frequency. In part (a) C term is negligible at high frequency so it acts like a resistor. For a steady d.c. we have $v=0$ and $X_{C}=\infty$ so it acts like an open circuit for steady d.c.
7.17 Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.
Sol. An inductor (L), a capacitor (C), and a resistor (R) is connected in parallel with each other in a circuit where, $\mathrm{L}=5.0 \mathrm{H}, \mathrm{C}=80 \mu \mathrm{~F}=80 \times 10^{-6} \mathrm{~F}, \mathrm{R}=40 \Omega$
Potential of the voltage source, $\mathrm{V}=230 \mathrm{~V}$
Impedance $(Z)$ of the given parallel LCR circuit is given as: $\frac{1}{Z}=\sqrt{\frac{1}{R^{2}}+\left(\frac{1}{\omega L}-\omega C\right)^{2}}$
Where, $\omega=$ Angular frequency

At resonance $\frac{1}{\omega \mathrm{~L}}-\omega \mathrm{C}=0$
$\therefore \omega=\frac{1}{\sqrt{\mathrm{LC}}}=\frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}=50 \mathrm{rad} / \mathrm{s}$
Hence, the magnitude of Z is the maximum at $50 \mathrm{rad} / \mathrm{s}$. As a result, the total current is minimum.
rms current flowing through inductor $L$ is given as:

$$
\mathrm{I}_{\mathrm{L}}=\frac{\mathrm{V}}{\omega \mathrm{~L}}=\frac{230}{50 \times 5}=0.92 \mathrm{~A}
$$

rms current flowing through capacitor C is given as:

$$
\mathrm{I}_{\mathrm{C}}=\frac{\mathrm{V}}{1 / \omega \mathrm{C}}=\omega \mathrm{CV}=50 \times 80 \times 10^{-6} \times 230=0.92 \mathrm{~A}
$$

rms current flowing through resistor R is given as:

$$
\mathrm{I}_{\mathrm{R}}=\frac{\mathrm{V}}{\mathrm{R}}=\frac{230}{40}=5.75 \mathrm{~A}
$$

Total current $=5.75 \mathrm{~A}\left(\because \mathrm{I}_{\mathrm{L}}\right.$ and $\mathrm{I}_{\mathrm{C}}$ are $180^{\circ}$ out of phase $)$
7.18 A circuit containing a 80 mH inductor and a $60 \mu \mathrm{~F}$ capacitor in series is connected to a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. The resistance of the circuit is negligible.
(a) Obtain the current amplitude and rms values.
(b) Obtain the rms values of potential drops across each element.
(c) What is the average power transferred to the inductor?
(d) What is the average power transferred to the capacitor?
(e) What is the total average power absorbed by the circuit?
['Average' implies 'averaged over one cycle'.]
Sol. (a) $I_{\max }=\frac{V_{\max }}{\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}}$
Here $\mathrm{R}=0, \omega=2 \pi \times 50=100 \pi \mathrm{rad} \mathrm{s}^{-1}, \mathrm{~V}_{0}=230 \times \sqrt{2} \mathrm{~V}, \mathrm{~L}=80 \times 10^{-3}, \mathrm{C}=60 \times 10^{-6} \mathrm{~F}$
$\therefore \quad \mathrm{X}_{\mathrm{L}}=\omega \mathrm{L}=100 \pi \times 80 \times 10^{-3}=25.14 \Omega$

$$
\mathrm{X}_{\mathrm{C}}=\frac{1}{\omega \mathrm{C}}=\frac{1}{100 \pi \times 60 \times 10^{-6}}=53.03 \Omega
$$

Using these values $\mathrm{I}_{\text {max }}=11.6 \mathrm{~A}$
$\mathrm{I}_{\mathrm{rms}}=\frac{\mathrm{I}_{\max }}{\sqrt{2}}=\frac{11.6}{\sqrt{2}}=8.24 \mathrm{~A}$
(b) $\mathrm{V}_{\mathrm{L}}=\mathrm{I}_{\mathrm{rms}} \times \omega \mathrm{L}=8.24 \times 100 \pi \times 80 \times 10^{-3}=270 \mathrm{~V}$

$$
\mathrm{V}_{\mathrm{C}}=\mathrm{I}_{\mathrm{rms}} \times \frac{1}{\omega \mathrm{C}}=8.24 \times \frac{1}{100 \pi \times 60 \times 10^{-6}}=437 \mathrm{~V}
$$

(c) Here $\phi=\pi / 2$, because current lags voltage by $90^{\circ}$ in the case of an inductor $\therefore \quad \mathrm{P}_{\mathrm{L}}=\mathrm{VI} \cos (\pi / 2)=0$
(d) Here, $\phi=\pi / 2$, because current lags voltage by $90^{\circ}$ in the case of a capacitor.

$$
\therefore \quad \mathrm{P}_{\mathrm{C}}=\mathrm{VI} \cos (\pi / 2)=0
$$

(e) Total average power absorbed is zero.
7.19 Suppose the circuit in previous question has a resistance of $15 \Omega$. Obtain the average power transferred to each element of the circuit, and the total power absorbed.
Sol. Using, $\mathrm{I}_{\mathrm{rms}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\sqrt{\mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}\right)^{2}}}=\frac{230}{\sqrt{(15)^{2}+(53.03-25.14)^{2}}}=7.26 \mathrm{~A}$
Average power to inductance as well as to capacitor is zero.

Average power to resistance $=\mathrm{I}^{2}{ }_{\text {rms }} \mathrm{R}=(7.26)^{2} \times 15=791 \mathrm{~A}$
Thus, total power consumed $=791 \mathrm{~W}$
7.20 A series LCR circuit with $L=0.12 \mathrm{H}, \mathrm{C}=480 \mathrm{nF}, \mathrm{R}=23 \Omega$ is connected to a 230 V variable frequency supply.
(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.
(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.
(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?
(d) What is the Q-factor of the given circuit?
(a) $\omega_{\text {resonance }}=\frac{1}{\sqrt{\mathrm{LC}}}=\frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}=4167 \mathrm{rad} \mathrm{s}^{-1}$
$v_{\text {resonance }}=\frac{\omega_{\text {resonance }}}{2 \pi}=663 \mathrm{~Hz}$
$\mathrm{I}_{\text {max }}=\frac{\mathrm{V}_{\text {max }}}{\mathrm{R}}=\frac{230 \sqrt{2}}{23}=14.14 \mathrm{~A}$
(b) $\quad \mathrm{P}_{\mathrm{av}}=\frac{1}{2} \mathrm{I}_{\max }^{2} \mathrm{R}=\frac{1}{2}(14.1)^{2} \times 23=2300 \mathrm{~W}$
(c) $\Delta \omega=\frac{\mathrm{R}}{2 \mathrm{~L}}=\frac{23}{0.24}=958 \mathrm{rad} \mathrm{s}^{-1}$
$\Delta \omega=2 \pi \Delta v$ i.e., $\Delta v=\frac{\Delta \omega}{2 \pi}=\frac{95.8}{2 \pi}=15.2 \mathrm{~Hz}$
Thus, power absorbed is half at $v=663 \pm 15$ i.e., 648 Hz and 678 Hz .
Current amplitude $=\frac{\mathrm{I}_{0}}{\sqrt{2}}=\frac{14.1}{\sqrt{2}}=10 \mathrm{~A}$
(d) $\mathrm{Q}=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{\omega \mathrm{L}}{\mathrm{R}}=\frac{4167 \times 0.12}{23}=21.7$
7.21 Obtain the resonant frequency and Q-factor of a series LCR circuit with $\mathrm{L}=3.0 \mathrm{H}, \mathrm{C}=27 \mu \mathrm{~F}$, and $\mathrm{R}=7.4 \Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2 . Suggest a suitable way.
Sol. Inductance, $\mathrm{L}=3.0 \mathrm{H}$, Capacitance, $\mathrm{C}=27 \mu \mathrm{~F}=27 \times 10^{-6} \mathrm{~F}$, Resistance, $\mathrm{R}=7.4 \Omega$
At resonance, angular frequency of the source for the given LCR series circuit is given as:
$\omega_{\mathrm{r}}=\frac{1}{\sqrt{\mathrm{LC}}}=\frac{1}{\sqrt{3 \times 27 \times 10^{-7}}}=\frac{10^{3}}{9}=111.11 \mathrm{rad} / \mathrm{s}$
Q-factor of the series: $\mathrm{Q}=\frac{\omega_{\mathrm{r}} \mathrm{L}}{\mathrm{R}}=\frac{111.11 \times 3}{7.4}=45.0446$
To improve the sharpness of the resonance by reducing its 'full width at half maximum' by a factor of 2 without changing $\omega$, we need to reduce $R$ to half i.e., $\frac{R}{2}=\frac{7.4}{2}=3.7 \Omega$
7.22 Answer the following questions:
(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?
(b) A capacitor is used in the primary circuit of an induction coil.
(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L .
(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.
(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?
Sol. (a) Yes; the statement is not true for rms voltage It is true that in any ac circuit, the applied voltage is equal to the average sum of the instantaneous voltages across the series elements of the circuit. However, this is not true for rms voltage because voltages across different elements may not be in phase.
(b) High induced voltage is used to charge the capacitor. A capacitor is used in the primary circuit of an induction coil. This is because when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.
(c) The dc signal will appear across capacitor C because for dc signals, the impedance of an inductor (L) is negligible while the impedance of a capacitor (C) is very high (almost infinite). Hence, a dc signal appears across C. For an ac signal of high frequency, the impedance of L is high and that of C is very low. Hence, an ac signal of high frequency appears across L .
(d) For a steady state dc, L has no effect, even if it is increased by an iron core. For ac, the lamp will shine dimly because of additional impedance of the choke. It will dim further when the iron core is inserted which increases the choke's impedance.
(e) A choke coil is needed in the use of fluorescent tubes with ac mains because it reduces the voltage across the tube without wasting much power. An ordinary resistor cannot be used instead of a choke coil for this purpose because it wastes power in the form of heat.
7.23 A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V ?
Sol. Using, $\frac{\mathrm{V}_{1}}{\mathrm{~V}_{2}}=\frac{\mathrm{N}_{1}}{\mathrm{~N}_{2}}$, we get $\mathrm{N}_{2}=\frac{\mathrm{N}_{1}}{\mathrm{~V}_{1}} \times \mathrm{V}_{2}=\frac{4000 \times 230}{2300}=400$ turns
7.24 At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100 \mathrm{~m}^{3} \mathrm{~s}^{-1}$. If the turbine generator efficiency is $60 \%$, estimate the electric power available from the plant $\left(\mathrm{g}=9.8 \mathrm{~ms}^{-2}\right)$.
Sol. Hydroelectric power $=$ Column pressure $\times$ Volume of water flowing per sec. across a cross-section

$$
=\mathrm{h} \rho \mathrm{~g}(\mathrm{Av})=300 \times 9.8 \times 10^{3}(100)
$$

Electric power $=0.6 \times 300 \times 9.8 \times 10^{3} \times(100)=176 \mathrm{MW}$
7.25 A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V . The resistance of the two wire line carrying power is $0.5 \Omega$ per km . The town gets power from the line through a $4000-220 \mathrm{~V}$ step-down transformer at a substation in the town.
(a) Estimate the line power loss in the form of heat.
(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?
(c) Characterise the step up transformer at the plant.

Sol. Total resistance of line $=0.5 \times 30=15$

$$
I_{\mathrm{rms}} \text { on line }=\frac{\text { Power in watt }}{\text { Voltage }}=\frac{800 \times 1000}{4000}=200 \mathrm{~A}
$$

(a) Line power loss $=\mathrm{I}_{\text {rms }}^{2} \mathrm{R}=(200)^{2} \times 15=600 \mathrm{~kW}$
(b) Power supplied $=800+600=1400 \mathrm{~kW}$
(c) Voltage dropped $=\mathrm{I}_{\text {rms }} \mathrm{R}=200 \times 15=3000 \mathrm{~V}$
(d) The plant transformer should supply $(4000+3000)=7000 \mathrm{~V}$.

So, the step up transformer should be of $440 \mathrm{~V} / 7000 \mathrm{~V}$.
7.26 Do the same exercise as above with the replacement of the earlier transformer by a $40,000-220 \mathrm{~V}$ step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?
Sol. Distance between the town and power generating station, $\mathrm{d}=15 \mathrm{~km}$
Resistance of the two wire lines carrying power $=0.5 \Omega / \mathrm{km}$
Total resistance of the wire lines, $\mathrm{R}=(15+15) 0.5=15 \Omega$

$$
\mathrm{P}=\mathrm{V}_{1} \mathrm{I}
$$

$R m s$ current in the wire line is given as: $I=P / V_{1}=\frac{800 \times 10^{3}}{40000}=20 \mathrm{~A}$
(a) Line power loss $=\mathrm{I}_{2} \mathrm{R}=(20)^{2} \times 15=6 \mathrm{~kW}$
(b) Assuming that the power loss is negligible due to the leakage of current. Hence, power supplied by the plant $=800 \mathrm{~kW}+6 \mathrm{~kW}=806 \mathrm{~kW}$
(c) Voltage drop in the power line $=I R=20 \times 15=300 \mathrm{~V}$

Hence, voltage that is transmitted by the power plant $=300+40000=40300 \mathrm{~V}$
The power is being generated in the plant at 440 V .
Hence, the rating of the step-up transformer needed at the plant is $440 \mathrm{~V}-40300 \mathrm{~V}$.
In the previous exercise, the power loss due to the same reason is $\frac{6}{806} \times 100=0.744 \%$
Since the power loss is less for a high voltage transmission, high voltage transmissions are preferred for this purpose.

