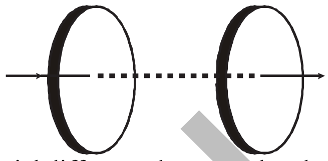
NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 8
ELECTROMAGNETIC WAVE

8.1 Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 mm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.



(a) Calculate the capacitance and the rate of charge of potential difference between the plates.
(b) Obtain the displacement current across the plates.
(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Sol. (a) Using $C = \frac{\varepsilon_0 A}{d}$

we get $C = \frac{8.854\times10^{-12}\times\pi r^2}{d} = \frac{8.854\times10^{-12}\times3.14\times(12\times10^{-2})^2}{5\times10^{-3}} = 80.1\ \text{pF}$

Now using, $V = \frac{Q}{C}$ we get $\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt}$ or $\frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{80.1\times10^{-12}} = 1.875\times10^9\ \text{Vs}^{-1}$.

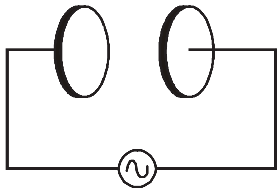
(b) Displacement current, $I_d = \varepsilon_0\frac{d\phi_E}{dt} = \varepsilon_0\frac{d\,(EA)}{dt} = \varepsilon_0 A\frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$

$= (8\times10^{-12})\,(1.875\times10^{10}) = 0.15\ \text{A}$ (Across capacitor $\phi_E = EA$, ignoring end correction)

Alt. : Displacement current = Conduction current

(c) Yes. Kirchhoff's first rule is valid at each plate of the capacitor provided that we take the sum of conduction and displacement currents.

8.2 A parallel plate capacitor (Fig.) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.



(a) What is the rms value of the conduction current?
(b) Is the conduction current equal to the displacement current?
(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

Sol. Here, $V_{rms} = 230\text{V}$, $C = 100\ \text{pF} = 100\times10^{-12} = 10^{-10}\ \text{F}$, $\omega = 300\ \text{rad s}^{-1}$

(a) Using, $I_{rms} = \frac{V_{rms}}{X_C}$, we get $I_{rms} = \frac{V_{rms}}{1/C\omega} = V_{rms}\times C\omega = 230\times10^{-10}\times300 = 69\times10^{-7} = 6.9\ \mu\text{A}$

(b) Yes, because $I_d = \varepsilon_0\frac{d\phi_E}{dt} = \varepsilon_0\frac{d\,(EA)}{dt}$; $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$; $I_d = \varepsilon_0 A\frac{d}{dt}\left(\frac{Q}{\varepsilon_0 A}\right) = \frac{\varepsilon_0 A}{\varepsilon_0 A}\frac{dQ}{dt} = I_c$

(c) Consider the figure given :

Using Ampere's circuital law, we get $\oint \vec{B}.d\vec{\ell} = \varepsilon_0\mu_0\oint\frac{\partial\vec{E}}{\partial t}.d\vec{S}$

or $B\times2\pi r = \mu_0\varepsilon_0\frac{dE}{dt}\pi r^2 \quad \therefore\ B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}$ .... (1)

But $E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} \quad \therefore\ \frac{dE}{dt} = \frac{dq/dt}{A\varepsilon_0} = \frac{I}{\pi R^2\varepsilon_0}$, where I is the rms value of current.

V PLUS U

$$\therefore \quad B = \frac{\mu_0 \varepsilon_0 r}{2} \times \frac{I}{\pi R^2 \varepsilon_0} = \frac{\mu_0 r I}{2\pi R^2}$$

$$\Rightarrow \text{ Amplitude, } B_0 = \frac{\mu_0 r}{2\pi R^2} I_0$$

$$\text{i.e., } B_0 = \frac{\mu_0 r \sqrt{2} I}{2\pi R^2} \quad (\because I_0 = \sqrt{2} I)$$

$$= \frac{(4\pi \times 10^{-7})(3 \times 10^{-2})\, 1.414\, (6.9 \times 10^{-6})}{2\pi \times (6 \times 10^{-2})^2} = 1.626 \times 10^{-11} \text{ T}$$

Alternatively: The formula $B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_d$ goes through even if i_d (and therefore B) oscillates in time. The formula shows they oscillate in phase.

Since $i_d = i_0$, we have, $B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_0$, where B_0 and i_0 are the amplitudes of the oscillating magnetic field and current, respectively. $i_0 = \sqrt{2} I_{rms} = 9.76$ μA.

For r = 3 cm, R = 6 cm, $B_0 = 1.63 \times 10^{-11}$ T.

8.3 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800Å and radio-waves of wavelength 500m?

Sol. The speed of light (3×10^8 m/s) in a vacuum is the same for all wavelengths. It is independent of the wavelength in the vacuum.

8.4 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Sol. The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (H) are in the x-y plane. They are mutually perpendicular.

Frequency of the wave, $\nu = 30 \text{ MHz} = 30 \times 10^6 \text{ s}^{-1}$

Speed of light in a vacuum, $c = 3 \times 10^8$ m/s

Wavelength of a wave is given as:

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{30 \times 10^6} = 10\text{m}$$

8.5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Sol. Using $\nu = \frac{c}{\lambda}$ we get $\lambda = \frac{c}{v}$

$$\text{i.e., } \lambda = \frac{3 \times 10^8}{7.5 \times 10^6} = 40\text{m} \; ; \; \lambda' = \frac{3 \times 10^8}{12 \times 10^6} = 25\text{m}$$

$\Rightarrow$ The corresponding wavelength band is 40m to 25m.

8.6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Sol. The frequency of an electromagnetic wave produced by the oscillator is the same as that of a charged particle oscillating about its mean position i.e., 10^9 Hz.

8.7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

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Sol. Using $\frac{E_0}{B_0} = c$, we get $E_0 = B_0 c$ i.e., $E = (510 \times 10^{-9})(3 \times 10^{8}) = 153\ NC^{-1}$.

8.8 Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine, B_0, ω, λ and k. (b) Find expressions for E and B.

Sol. (a) (i) Using , $\frac{E_0}{B_0} = c$ we get $B_0 = \frac{E}{c} = \frac{120}{3 \times 10^{8}} = 40 \times 10^{-8}\,T$ i.e. $B_0 = 400$ nT

(ii) $\omega = 2\pi\nu = 2 \times \pi \times 50 \times 10^{6} = 3.14 \times 10^{8}$ rad s^{-1}

(iii) $c = \lambda\nu$ i.e., $\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{50 \times 10^{6}} = 6m$

(iv) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{6} = \frac{2 \times 3.14}{6} = 1.05$ rad m^{-1}

(b) $\vec{E} = E_0 \sin(kx - \omega t)\,\hat{j} = 120 \sin(1.05x - 3.14 \times 10^{8} \times t)$, with usual units and

$\vec{B} = B_0 \sin(kx - \omega t)\,\hat{k} = 400 \times 10^{-9} \sin(1.05x - 3.14 \times 10^{8} \times t)$ s, with usual units.

8.9 Use the formula E = hν (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Sol. Using the relation for photon energy (in eV),

$$E = \frac{h\nu}{e}, \text{ we get } E = \frac{hc}{\lambda e} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{\lambda \times 1.6 \times 10^{-19}} = \frac{1.24 \times 10^{-6}}{\lambda\ (\text{in m})} eV$$

(a) For γ-rays ($\lambda < 10^{-3}$nm) let λ be 10^{-12} m then $E = \frac{1.24 \times 10^{-6}}{10^{-12}} = 1.24 \times 10^{6}\,eV = 1.24$ MeV

(b) For X-rays (1nm to 10^{-3}nm) let λ be 1mm then $E = \frac{1.24 \times 10^{-6}}{10^{-9}} = 1.24 \times 10^{3}\,eV = 1240$ eV

(c) For visible light (700nm to 400nm) let λ be 700nm, then $E = \frac{1.24 \times 10^{-6}}{700 \times 10^{-9}} = 1.77eV$

(d) For microwaves (0.1m to 1mm) let λ be 10cm, then $E = \frac{1.24 \times 10^{-6}}{10^{-1}} = 1.24 \times 10^{-5}\,eV$

(e) For radiowaves (> 0.1m) let λ be 1 km = 1000m, then $E = \frac{1.24 \times 10^{-6}}{1000} = 1.24 \times 10^{-9}\,eV$

Conclusions: The above result indicates that the different wavelengths in the electromagnetic spectrum can be obtained by multiplying roughly the powers of ten.
The visible wavelengths are spaced by a few eV.
The nuclear energy levels (from γ rays) are spaced about 1 MeV

8.10 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude $48Vm^{-1}$.

(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^{8}\ m\ s^{-1}$.]

Sol. (a) Using, $c = \lambda\nu$, we get $\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{2 \times 10^{10}} = 1.5 \times 10^{-2}\,m$

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(b) Using $c = \frac{E_0}{B_0}$, we get $B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8} = 1.6 \times 10^{-7}$ T

(c) Average energy density of the electric field, $U_E = \frac{1}{2}\varepsilon_0 E^2$

and average density of the magnetic field, $U_B = \frac{B^2}{2\mu_0}$

Also, $c = \frac{E}{B}$ and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ i.e. $\frac{E}{B} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Then, $\frac{U_E}{U_B} = \frac{\varepsilon_0 E^2}{\frac{B^2}{\mu_0}} = \mu_0\varepsilon_0 \frac{E^2}{B^2} = \mu_0\varepsilon_0 \frac{1}{\mu_0\varepsilon_0} = 1$

ADDITIONAL EXERCISES

8.11 Suppose that the electric field part of an electromagnetic wave in vacuum is

$\vec{E} = \{(3.1 \text{ N/C}) \cos [(1.8 \text{ rad/m}) y + (5.4 \times 10^6 \text{ rad/s}) t]\}$.

(a) What is the direction of propagation?
(b) What is the wavelength λ?
(c) What is the frequency ν?
(d) What is the amplitude of the magnetic field part of the wave?
(e) Write an expression for the magnetic field part of the wave.

Sol. Here $\vec{E} = [3.1 \cos (1.8\, y + 5.4 \times 10^6 t)]\ \hat{i}$.

Comparing it with standard equation, $\vec{E} = [E_0 \cos (ky + \omega t)]\ \hat{i}$ we get the following answers

(a) Wave is propagating along $-\hat{j}$ direction i.e. negative y direction because coefficient of y is positive.

(b) Using, $k = \frac{2\pi}{\lambda}$ we get $\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.8} = 3.5\text{m}$

(c) Frequency, $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.5} = 86$ MHz

(d) $c = \frac{E_0}{B_0}$ or $B_0 = \frac{E_0}{c} = \frac{3.1}{3 \times 10^8} = 10\text{nT}$

(e) Using, $\vec{B} = -B_0 \cos (ky + \omega t)\ \hat{k}$ = 10nT (cos 1.8y rad m^{-1} + 5.4 × 10^6 rad s^{-1})

[E is along $\hat{i}$, c is along $-\hat{j}$, c is in direction of $\vec{E} \times \vec{B}$
$-\hat{j} = \hat{i} \times \hat{k}$ $\therefore$ $\vec{B}$ is along $\hat{k}$]

8.12 About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) at a distance of 1m from the bulb?
(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

Sol. (a) Visible power = 5W

$\therefore$ Average intensity of radiation at 1m = $\frac{\text{Power}}{4\pi r^2} = \frac{5}{4 \times \pi \times 1}$ = 0.4 W m^{-2}

(b) Average intensity of radiation at 10m = $\frac{\text{Power}}{4\pi r'^2} = \frac{5}{4 \times \pi \times 10^2}$ = 0.004 W m^{-2}

8.13 Use the formula $\lambda_m T = 0.29$cmK to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

Sol. Using $\lambda_m T = 0.29$ cm K, we get $T = \frac{0.29}{\lambda_m} K$

(a) For $\lambda_m = 10^{-10}$ cm, we get $T = \frac{0.29}{10^{-10}} = 2.9 \times 10^9$ K

(b) For $\lambda_m = 1\text{nm} = 10^{-7}$ cm, we get $T = \frac{0.29}{\lambda_m} = \frac{0.29}{10^{-7}} = 2.9 \times 10^6$ K

(c) For $\lambda_m = 5 \times 10^{-7}$ cm, we get $T = \frac{0.29}{5 \times 10^{-7}} = 6000$ K

(d) For $\lambda_m = 1\mu\text{m} = 10^{-4}$ cm, we get $T = \frac{0.29}{10^{-4}} = 2.9 \times 10^3$ K = 2900 K

(e) For $\lambda_m = 1\text{m} = 100$ cm we get, $T = \frac{0.29}{100} = 2.9 \times 10^{-3}$ K

(f) For $\lambda_m = 1\text{km} = 10^5$ cm we get, $T = \frac{0.29}{10^5} = 2.9 \times 10^{-6}$ K

The above results tell us the range of temperature required to obtain the different parts of the spectrum. For example, to obtain a wavelength of 1 μm, a temperature of 2900 K is required.
To obtain visible radiation, say $\lambda = 5 \times 10^{-7}$ m, the source should have a temperature of about 6000K.
Note: a lower temperature will also produce this wavelength but not the maximum intensity.

8.14 Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.
(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).
(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].
(d) 5890 Å - 5896 Å [double lines of sodium]
(e) 14.4keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mossbauer spectroscopy)].

Sol. (a) Given wavelength is of the order of 10^{-2}m i.e. short radio wave.
(b) Frequency is of the order of 10^9 Hz i.e. short radio wave.
(c) $\lambda_m T = 0.29$ or $\lambda_m = \frac{0.29}{T} = \frac{0.29}{7} = 0.09\text{cm} = 0.0009\text{m}$.
Wavelength is of the order of 10^{-4}m i.e., microwave.
(d) Given wavelengths are of the order of 10^{-7}m i.e., visible radiations (Yellow light)
(e) Using, $E = \frac{hc}{\lambda e}$ we get $\lambda = \frac{hc}{\text{Energy} \times e} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{(1.44 \times 10^3)(1.6 \times 10^{-19})}$
i.e., $\lambda = 0.86 \times 10^{-10} = 8.6 \times 10^{-11}$ m.
The wavelength of the order of 10^{-10}m corresponding to X-rays or soft γ-rays.

8.15 Answer the following questions:
(a) Long distance radio broadcasts use short-wave bands. Why?
(b) It is necessary to use satellites for long distance TV transmission. Why?
(c) Optical and radio telescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?
(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?

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(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?

(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

Sol. (a) Long distance radio broadcasts use shortwave bands because only these bands can be refracted by the ionosphere.

(b) It is necessary to use satellites for long distance TV transmissions because television signals are of high frequencies and high energies. Thus, these signals are not reflected by the ionosphere. Hence, satellites are helpful in reflecting TV signals. Also, they help in long distance TV transmissions.

(c) With reference to X-ray astronomy, X-rays are absorbed by the atmosphere. However, visible and radio waves can penetrate it. Hence, optical and radio telescopes are built on the ground, while X-ray astronomy is possible only with the help of satellites orbiting the Earth.

(d) The small ozone layer on the top of the atmosphere is crucial for human survival because it absorbs harmful ultraviolet radiations present in sunlight and prevents it from reaching the Earth's surface.

(e) The temperature of the earth would be lower because the Greenhouse effect of the atmosphere would be absent.

(f) The clouds produced by global nuclear war would perhaps cover substantial parts of the sky preventing solar light from reaching many parts of the globe. This would cause a 'winter'.

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