NCERT SOLUTIONS
PHYSICS XII CLASS
CHAPTER - 9
RAY OPTICS AND OPTICAL INSTRUMENTS

9.1 A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Sol. Using $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign conventions, we get, $-\frac{1}{27}+\frac{1}{v}=-\frac{1}{(36/2)}$

or $\frac{1}{v}=-\frac{1}{18}+\frac{1}{27}=\frac{-3+2}{54}=-\frac{1}{54}$ or v = – 54 cm

⇒ Image is formed at the distance of 54 cm from the mirror on the same side as the object. This is the position where a screen should be placed.

Now magnification, $m=\frac{I}{O}=\frac{-v}{u}$ $\therefore \frac{I}{2.5}=\frac{-(-54)}{-27}=-2$ $\therefore$ I = – 2 × 2.5 = – 5 cm

⇒ The image is real, inverted and magnified. If the candle is brought closer to the mirror, the screen would have to be moved away from the mirror. If the distance of the candle from mirror is less than 18 cm, the image becomes virtual and is not observable on the screen.

9.2 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Sol. Using $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign conventions, we get, $-\frac{1}{12}+\frac{1}{v}=\frac{1}{15}$ ⇒ $\frac{1}{v}=\frac{1}{15}+\frac{1}{12}=\frac{4+5}{60}$

⇒ $\frac{1}{v}=\frac{9}{60}$ or $v=\frac{60}{9}$ cm or v = 6.7 cm

⇒ The image is formed at 6.7 cm behind the mirror.

As magnification $m=\frac{I}{O}=-\frac{v}{u}$, we get ,

$\frac{I}{4.5}=\frac{-\frac{60}{9}}{-12}=\frac{5}{9}$ $\therefore$ $I=\frac{5\times 4.5}{9}$ = 2.5 cm

⇒ The image is virtual, erect and diminished. As we move the needle away from the mirror, the image goes on decreasing in size and move towards the principal focus on the other side.

9.3 A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Sol. As $\frac{\text{Real depth}}{\text{Apparent depth}}={}_a\mu_w$,

we get $\frac{12.5}{9.4}={}_a\mu_w$ or ${}_a\mu_w=1.33$

If water is replaced by liquid, then ${}_a\mu_\ell=1.63$

So apparent depth = $\frac{12.5}{1.63}$ = 7.66 cm

$\therefore$ Distance through which the microscope has to be moved = 9.4 – 7.66 = 1.74 cm

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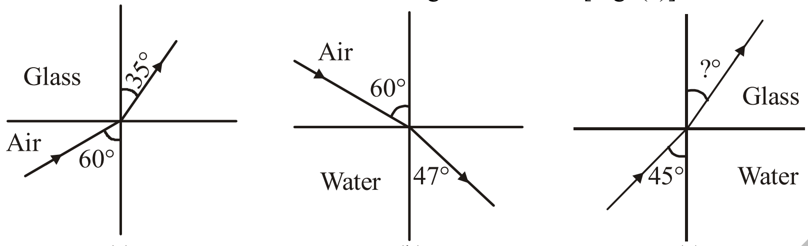
9.4 Figures (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45º with the normal to a water-glass interface [Fig. (c)].



(a) (b) (c)

Sol. From fig (a), $_a\mu_g = \dfrac{\sin 60°}{\sin 35°} \quad \left(\because \mu = \dfrac{\sin i}{\sin r}\right)$

$$= \frac{\sqrt{3}/2}{0.5736} = 1.51$$

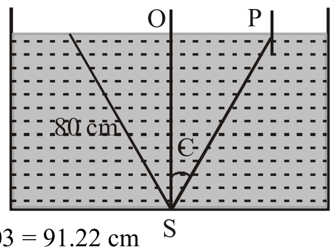
From fig. (b), $_a\mu_w = \dfrac{\sin 60°}{\sin 40°} = \dfrac{\sqrt{3}/2}{0.6561} = 1.32$

$$_w\mu_g = \frac{\sin i}{\sin r} = \frac{1.51}{1.32} = 1.14$$

$\Rightarrow \dfrac{\sin 45°}{\sin r} = 1.14 \quad$ or $\sin r = \dfrac{1}{\sqrt{2} \times 1.14} = 0.6203$

$\Rightarrow r = \sin^{-1} 0.6203 = 38°21'$

9.5 A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)



Sol. $\therefore$ Using $\mu = \dfrac{1}{\sin C}$ or $\sin C = \dfrac{1}{\mu}$

$\Rightarrow \sin C = \dfrac{1}{1.33} = 0.7518$ or $C = 48°45'$

Also, as shown in the figure if S is the source and P is the point from where total internal reflection starts to occur, then in triangle OSP,

$= \tan C$ or $OP = OS \tan C = 80 \tan 48°45' = 80 \times 1.1403 = 91.22$ cm

therefore, area of surface of water = Area of the circle of radius OP

$= \pi \times (OP)^2 = 3.14 \times (91.22)^2 = 26128\ cm^2 = 2.613\ m^2$

9.6 A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Sol. Using $_a\mu_g = \dfrac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}$

we get $_a\mu_g = \dfrac{\sin \dfrac{60° + 40°}{2}}{\sin \dfrac{60°}{2}} = \dfrac{\sin 50°}{\sin 30°} = \dfrac{0.7660}{0.5000} = 1.53$

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When the prism is placed in water, then

$$_w\mu_g = \frac{_a\mu_g}{_a\mu_w} = \frac{1.53}{1.33} = 1.15$$

$$\therefore \quad _w\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow 1.15 = \frac{\sin\frac{60^\circ+\delta_m}{2}}{\sin\frac{60^\circ}{2}}$$

or $\sin\frac{60^\circ+\delta_m}{2} = 1.15 \sin 30^\circ = 0.5750$

$\therefore \quad \frac{60^\circ+\delta_m}{2} = \sin^{-1} 0.5750 = 35^\circ 6'$

or $60^\circ + \delta_m = 70^\circ 12'$ or $\delta_m = 10^\circ 12'$

9.7 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?

Sol. Using lens maker formula and sign convention we get

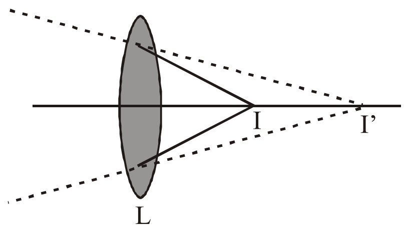
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right] = 0.55 \times \frac{2}{R} \quad \text{(By sign convention)}$$

or $R = \frac{1.10}{1} \times 20 = 22$ cm

9.8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?

Sol. (a) The formation of image with and without the placement of a convex lens shown, I' represents a virtual object with real image at I'.



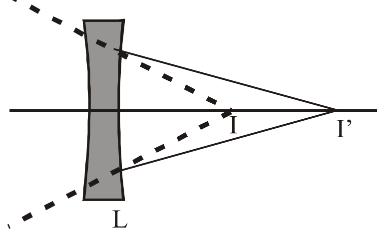
Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \quad \frac{1}{20} = -\frac{1}{12} + \frac{1}{v}$ or $\frac{1}{v} = \frac{1}{20} + \frac{1}{12}$

or $\frac{1}{v} = \frac{3+5}{60} = \frac{8}{60} = \frac{2}{15} \quad \therefore v = \frac{15}{2} = 7.5$ cm

i.e. image is formed 7.5 cm away from the lens and it is a real image.

(b) For a concave lens, the image formation is as shown.



$\therefore$ Using $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow -\frac{1}{12} + \frac{1}{v} = -\frac{1}{16}$

or $\frac{1}{v} = -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48}$ or $v = 48$ cm

i.e. the image formed is at a distance of 48 cm from the lens and it is a real image.

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9.9 An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Sol. $\therefore$ Using $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and sign convention we get $-\frac{1}{-14}+\frac{1}{v}=\frac{1}{-21}$

or $\frac{1}{v}=-\frac{1}{21}-\frac{1}{14}=\frac{-2-3}{42}=\frac{-5}{42}$ $\quad \therefore$ $v=-\frac{42}{5}=-8.4$ cm

Also, the magnification, $m=\frac{I}{O}=\frac{v}{u}\Rightarrow\frac{I}{3.0}=\frac{-8.4}{-14}=0.6\Rightarrow I=1.8$ cm.

The image is virtual, erect, diminished and is formed on the same side of the lens at a distance of 8.4cm from the lens. If the object is moved away from the lens, the image moves towards the principal focus and goes on decreasing in size.

9.10 What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Sol. Here, $f_1 = 30$ cm and $f_2 = -20$ cm

For the combination of two thin lenses, the focal length of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{30}+\frac{1}{(-20)}=\frac{2-3}{60}=-\frac{1}{60}\Rightarrow f=-60 \text{ cm}$$

Since the focal length is negative, the combination is a diverging lens.

9.11 A compound microscope consists of an objective lens of focal length 2.0cm and an eyepiece of focal length 6.25cm separated by a distance of 15cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

Sol. Here, $f_0 = 2.0$ cm, $f_e = 6.25$ cm,

Distance between object lens and eye piece = 15 cm

(a) For the formation of image at the least distance of distinct vision.

$v_e = -25$cm : $u_e = ?$

$\therefore$ From the relation, $-\frac{1}{u_e}+\frac{1}{v_e}=\frac{1}{f_e}$, we have $-\frac{1}{u_e}+\frac{1}{-25}=\frac{1}{6.25}$

or $-\frac{1}{u_e}=\frac{1}{6.25}+\frac{1}{25}=\frac{4+1}{25}\Rightarrow u_e=-5$ cm

Negative sign tells that the object for the object lens lies to the left of the lens.

$\therefore$ Distance of the image from the object lens = 15 – 5 = 10 cm

$\therefore$ $v_0 = 10$ cm

Using $-\frac{1}{u_0}+\frac{1}{v_0}=\frac{1}{f_0}$, we get $-\frac{1}{u_0}+\frac{1}{10}=\frac{1}{2}$ or $-\frac{1}{u_0}=\frac{1}{2}-\frac{1}{10}=\frac{5-1}{10}=\frac{4}{10}$

$\therefore$ $u_0=-\frac{10}{4}$ or $u_0=-2.5$ cm

Magnifying power, $M=\frac{v_0}{-u_0}\left(1+\frac{D}{f_e}\right)=\frac{10}{-(-2.5)}\left(1+\frac{25}{6.25}\right)=4\,(5)=20$

(b) When the final image is formed at infinity then the object for the eye lens must be placed at its principal focus. Therefore, $u_e=-f_e=-6.25$ cm and $v_0=L-|u_e|=15-6.25=8.75$ cm

Now using $-\frac{1}{u_0}+\frac{1}{v_0}=\frac{1}{f_0}\Rightarrow-\frac{1}{u_0}+\frac{1}{8.75}=\frac{1}{2.0}$

or $\frac{1}{u_0}=\frac{1}{8.75}-\frac{1}{2}=\frac{2-8.75}{8.75\times2}\Rightarrow u_0=-2.59$ cm $\quad\therefore\quad M=-\frac{v_0}{u_0}\frac{D}{f_e}=-\frac{8.75}{-2.59}\times\frac{25}{6.25}=13.5$

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9.12 A person with a normal near point (25cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Sol. Here $f_0 = 0.8$ cm ; $f_e = 2.5$ cm

$D = 25$ cm, $v_e = -D = -25$ cm

$u_0 = -9.0$ mm $= -0.90$ cm

Linear magnification of the eye lens is given by

$$m_e = \frac{v_e}{u_e} = 1 + \frac{D}{f_e} \Rightarrow \frac{-25}{u_e} = 1 + \frac{25}{2.5} = 11 \quad \therefore u_e = -\frac{25}{11} \text{ cm}$$

Using $-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0}$ for objective lens we get $-\frac{1}{-0.90} + \frac{1}{v_0} = \frac{1}{0.80}$

or $\frac{1}{v_0} = \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9 - 0.8}{0.72}$ $\quad \therefore v_0 = $ cm $= 7.2$ cm

$\therefore$ Separation between two lenses $= |u_e| + |v_0| = \frac{25}{11} + 7.2 = 2.27 + 7.2 = 9.47$ cm

Magnifying power, $M = \frac{v_0}{-u_0}\left(1 + \frac{D}{f_0}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right) = 88$

9.13 A small telescope has an objective lens of focal length 144cm and an eyepiece of focal length 6.0cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Sol. Here, focal length of objective lens, $f_0 = 144$ cm

Focal length of eye-piece, $f_e = 6.0$ cm

Therefore, using $m = \frac{f_0}{f_e} \Rightarrow m = \frac{144}{6.0} = 24$

Also, the distance between objective and eye piece is $= f_0 + f_e = 144 + 6 = 150$ cm

9.14 (a) A giant refracting telescope at an observatory has an objective lens of focal length 15m. If an eyepiece of focal length 1.0cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6m, and the radius of lunar orbit is 3.8×10^8m.

Sol. (a) Angular magnification $= \frac{f_0}{f_e} = \frac{1500}{1} = 1500$

(b) Let D be the diameter of moon's image

Then $\frac{D}{1500} = \frac{\text{Diameter of moon}}{\text{Radius of lunar orbit}} = \frac{3.48 \times 10^6 \times 100}{3.8 \times 10^8 \times 100}$

$\Rightarrow D = 1500 \times \frac{3.48}{380} = 13.7$ cm

9.15 Use the mirror equation to deduce that:

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Sol. (a) For a concave mirror, the mirror formula is given by

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$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$ ....(i), where both f and u are negative i.e. $f < 0$ and $u < 0$.

Then, for an object between f and 2f, we have, $2f < u < f$ or $\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$

Multiplying by –1, the inequality becomes $-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f} \Rightarrow \frac{1}{f} = -\frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$

Using equation (i), we have $\frac{1}{2f} < \frac{1}{v} < 0$

It means that if v is negative, the image formed is real and beyond 2f.

(b) For a convex mirror, using the mirror formula $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, where $f > 0$ and $u < 0$ we find that v is always positive i.e. the image lies on the right of the mirror and is thus a virtual image.

(c) The magnification of mirror, $m = \frac{f}{f-u}$

For a convex mirror f is positive whereas u is negative. Therefore $f - u$ is always greater than f i.e. $f - u > f$ $\therefore$ $m < 1$ i.e., the image is diminished.

(d) For a concave mirror, $f < 0$, $u < 0$

$\therefore$ For an object placed in between pole and principal focus, $f < u < 0 \Rightarrow \frac{1}{f} - \frac{1}{u} > 0$

But using mirror formula, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, we find, $\frac{1}{v} > 0$ i.e. v is positive and lies to the right of pole.

$\Rightarrow$ The image is virtual. Also, magnification, $m = -\frac{v}{u} = \frac{v}{|u|}$

Since $\frac{1}{v} < \frac{1}{|u|}$ i.e., $v > |u|$

$\therefore$ m is positive and greater than 1.

So, the image is enlarged.

9.16 A small pin fixed on a table top is viewed from above from a distance of 50cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

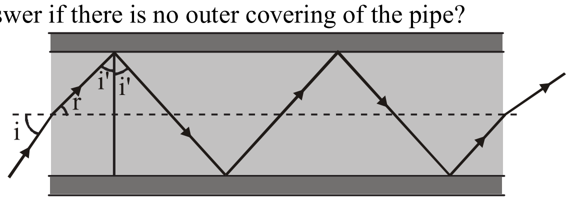
Sol. The lateral displacement $t = d\left[1-\frac{1}{\mu}\right]$

i.e. $t = 15\left(1-\frac{1}{1.5}\right) = 15 \times \frac{0.5}{1.5} = 5$ cm

For small angles of incidence, the answer does not depend upon the location of the slab.

9.17 (a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?



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Sol. (a) The maximum value of the acceptance angle $i = \theta_a$ is given by $\sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}$

where μ_1 is the refractive index of core and μ_2 that of the cladding.

Here $\mu_1 = 1.68$ and $\mu_2 = 1.44$

$\therefore \sin\theta_a = \sqrt{(1.68)^2 - (1.44)^2} = \sqrt{0.7488} = 0.8653 \Rightarrow \theta_a = 59°55'$

Therefore, the range of angle of incidence is $0 < i < 59°55'$

(b) In case there is no outer coating, then $\mu_2 = 1$ (for air)

$\therefore$ Critical angle for reflection, $i_c' = \sin^{-1}\left(\frac{1}{\mu_1}\right) = \sin^{-1}\left(\frac{1}{1.68}\right) = 36°32'$

Now for $i = 90°$, $\frac{\sin i}{\sin r} = \mu_1 \Rightarrow \frac{\sin 90°}{\sin r} = 1.68$ or $r = 36°32'$

$\therefore$ Using $r + i = 90° \Rightarrow i = 90° - r = 90° - 36°32' = 53°28'$

Clearly, $i > i_c'$ (36°32')

Therefore, all incident rays in the range 0 to 90° suffer total internal reflection.

9.18 Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

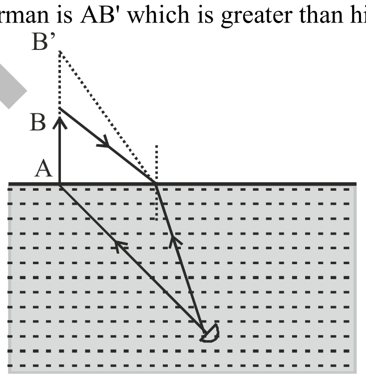
(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Sol. (a) Yes. They can produce real images if the object is a virtual object.

(b) There is no contradiction in this case. The virtual image of the object acts as an object for the convex lens of our eye and the lens of our eye make a real image of this object on the retina.

(c) Let AB the fisherman standing on the bank of the lake. The rays of light from the head of the fisherman bends towards the normal on refraction at the interface separating water and air. The refracted rays appear to come from point B' instead of point the B for the diver. Thus, for a diver the height of fisherman is AB' which is greater than his actual height AB.



(d) The apparent depth of a pond of water decreases when viewed obliquely. This is due to the refraction of light from the surface of water.

(e) The refractive index of diamond is 2.5 which gives, the critical angle at 24°. The faces of the diamond are cut in such a way that whenever light falls on any of the faces, the angle of incidence is greater than the critical angle i.e. 24°. So when light falls on the diamond, it suffers repeated internal reflections. The light which finally emerges out from few places in certain directions makes the diamond sparking.

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9.19 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Sol. Using the lens formula, $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ ......(1)

Taking the sign convention into consideration, we have

$-u + v = 3 \Rightarrow v = 3 + u$ ....(2)

Using this value of υ in eqn. (1),

$$-\frac{1}{u}+\frac{1}{3+u}=\frac{1}{f} \text{ or } \frac{-3-u+u}{3u+u^2}=\frac{1}{f} \Rightarrow f=\frac{-(3u+u^2)}{3}$$

For f to be maximum, $\frac{df}{du} = 0$

$$\therefore \quad \frac{d}{du}\left[-\left(\frac{3u+u^2}{3}\right)\right]=0 \Rightarrow \frac{-1}{3}(3+2u)=0$$

or $2u = -3 \Rightarrow u = -3/2$

$\therefore$ from eqn. (2), $v = 3 - \frac{3}{2} = \frac{3}{2}$

Using these values of u and v in eqn. (1),

$$-\frac{1}{-3/2}+\frac{1}{3/2}=\frac{1}{f} \text{ or } \frac{1}{f}=\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$$

$\therefore \quad f = \frac{3}{4} = 0.75$ m

Alt.: For a image, least distance between object and image should be four times the focal length $\Rightarrow 4f = 300$ i.e. f = 75 cm i.e. 0.75 m.

9.20 A screen is placed 90cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20cm. Determine the focal length of the lens.

Sol. The lens formula is $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ ......(1)

We know that by convention u is negative

$\therefore \quad -u + v = 90$ ......(2)

$\Rightarrow \quad v = 90 + u$

$\therefore$ from eqn. (1), $-\frac{1}{u}+\frac{1}{90+u}=\frac{1}{f} \Rightarrow \frac{-90-u+u}{90u+u^2}=\frac{1}{f}$ or $u^2 + 90u + 90f = 0$ .....(3)

Equation (3) is quadratic in u and gives two values of u say u_1 and u_2 in terms of f.

$\therefore$ sum of roots $u_1 + u_2 = -90$ .....(4)

and product of roots $u_1u_2 = 90f$

Also, $u_1 - u_2 = 20$ cm .....(5)

$\therefore$ Adding (4) and (5), we get,

$2u_1 = -90 + 20 = 70 \Rightarrow u_1 = -35$ cm

and subtracting (4) (5),

$2u_2 = -90 - 20 = -110$ or $u_2 = -55$cm

Now $u_1u_2 = +90f \Rightarrow 35 \times 55 = 90f$

or $f = \frac{55 \times 35}{90} = 21.39$ cm.

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9.21 (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident?
Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40cm. Determine the magnification produced by the two-lens system, and the size of the image.

Sol. (a) As $-\frac{1}{u_1}+\frac{1}{v_1}=\frac{1}{f_1}$ we have $-\frac{1}{-\infty}+\frac{1}{v}=\frac{1}{30}$ i.e. $v_1 = 30$ cm

This image formed by convex lens appears as an object placed at 30 – 8 cm = 22 cm from the convex lens on the right hand side.

$\therefore$ Using $-\frac{1}{u_2}+\frac{1}{v_2}=\frac{1}{f_2}$, we have, $-\frac{1}{22}+\frac{1}{v_2}=\frac{1}{-20}$

i.e. $\frac{1}{v_2}=\frac{-1}{20}+\frac{1}{22}=\frac{-11+10}{220}=\frac{-1}{220}$ $\therefore$ $v_2 = -220$ cm

$\Rightarrow$ The parallel beam of light appears to diverge from (220 – 8) = 212 cm from the convex lens from the left.

Now let the parallel beam of light fall on the concave lens after which the convex lens is placed at 8 cm. Then $\frac{-1}{u_1}+\frac{1}{v_1}=\frac{1}{f_1}$ $\Rightarrow$ $\frac{-1}{-\infty}+\frac{1}{v_1}=\frac{1}{-20}$ or $v_1 = -20$ cm

$\Rightarrow$ The image is formed – 20 cm to the left of concave length and $\therefore$ it is 20 + 8 = 28 cm to the left of convex lens.

Again, $-\frac{1}{u_2}+\frac{1}{v_2}=\frac{1}{f_2}\Rightarrow\frac{-1}{-28}+\frac{1}{v_2}=\frac{1}{30}$ i.e., $\frac{1}{v_2}=\frac{1}{30}-\frac{1}{28}=\frac{14-15}{420}=\frac{-1}{420}$

i.e., $v_2 = -420$ cm

$\Rightarrow$ The light appears to diverge from a point at 420 cm to the left of convex lens.

Clearly the answer depends upon the side of incidence so the notion of effective focal length is not useful in this case.

(b) Using $-\frac{1}{u_1}+\frac{1}{v_1}=\frac{1}{f_1}$ we have $\frac{1}{40}+\frac{1}{v_1}=\frac{1}{30}$

i.e. $\frac{1}{v_1}=\frac{1}{30}-\frac{1}{40}=\frac{4-3}{120}=\frac{1}{120}$ $\therefore$ $v_1 = 120$ cm

$\therefore$ Magnification produced by convex lens.

$$m_1=\frac{v_1}{|u_1|}=\frac{120}{40}\Rightarrow m_1=3$$

Now $u_2 = 120 - 8 = 112$ cm (i.e. object is to the right of concave lens)

$f_2 = -20$ cm

$\therefore$ $-\frac{1}{u_2}+\frac{1}{v_2}=\frac{1}{f_2}\Rightarrow-\frac{1}{112}+\frac{1}{v_2}=\frac{1}{-20}$

or $\frac{1}{v_2}=\frac{1}{20}+\frac{1}{112}=\frac{-28+5}{560}=\frac{-23}{560}$ $\therefore$ $v_2=-\frac{560}{23}$

$\therefore$ Magnification produced by concave lens.

$$m_2=\frac{|v_2|}{u_2}\Rightarrow m_2=\frac{\frac{560}{23}}{112}=\frac{5}{23}$$

$\therefore$ Net magnification $m = m_1 \times m_2 = 3 \times \frac{5}{23} = 0.65$

$\therefore$ Size of image = m × size of object = 0.65 × 1.5 = 0.98 cm

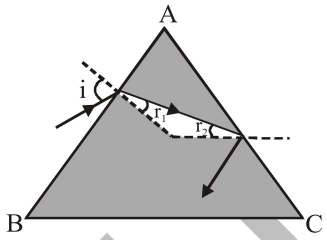
VPLUSU

9.22 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Sol. Let the ray of light be incident on the face AB at angle i so that it is totally internally reflected at face AC.

Then the critical angle is given by

$$\sin C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6562 \qquad \therefore\ C = 41°$$



$\therefore$ For total internal reflection $r_2 = C = 41°$

$\therefore$ Using $A = r_1 + r_2 \Rightarrow r_1 = A - r_2 = 60° - 41° = 19°$

$\therefore$ Using $\frac{\sin i}{\sin r_1} = \mu \Rightarrow \frac{\sin i}{\sin 19°} = 1.524$

or $\sin i = 1.52 \sin 19° = 0.4962 \qquad \therefore\ i \approx 30°$

9.23 You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion,

(b) disperse (and displace) a pencil of white light without much deviation.

Sol. (a) Angular dispersion produced by two prisms i.e. crown glass and flint glass should be zero in this case i.e. $(\mu_{blue} - \mu_{red})A + (\mu'_{blue} - \mu'_{red})A' = 0$

A' < A because $(\mu'_{blue} - \mu'_{red})$ for flint glass prism is more than $(\mu_{blue} - \mu_{red})$ for crown glass. Thus crown glass prism of larger angle is to be combined with a smaller angled flint glass prism.

(b) For no deviation, $(\mu_{yel} - 1)A + (\mu'_{yel} - 1)A' = 0$

$$\mu'_y > \mu_y$$

In the combination of prisms, flint glass prism of greater and greater angle may be tried but in any case still this angle will be smaller than the angle of the crown glass prism.

9.24 For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Sol. When the object is placed at infinity, the eye makes use of the least converging power,

Therefore, total converging power of cornea and the eye lens = 40 + 20 = 60 dioptre.

$\therefore$ Using $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$; $u = -\infty$, $f = \frac{1}{P} = \frac{1}{60}$ m

$$\therefore \quad \frac{-1}{-\infty} + \frac{1}{v} = 60 \Rightarrow v = \frac{1}{60}\text{ m} = \frac{100}{60}\text{ cm} \text{ i.e., } v = \frac{5}{3}\text{ cm}$$

To focus the object at the near point,

$u = -25$ cm ; $v = \frac{5}{3}$ cm

$$\therefore \quad -\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{-1}{-25} + \frac{1}{5/3} = \frac{1}{25} + \frac{3}{5}$$

or $\frac{1}{f} = \frac{1+15}{25} = \frac{16}{25} \qquad \therefore$ Power $P = \frac{100}{f\text{ (cm)}}$ dioptre $= \frac{100}{25} \times 16 = 64$ dioptre

$\Rightarrow$ Power of eye lens = 64 – 40 = 24 dioptre.

Therefore, the rough range of eye lens is 20 to 24 dioptre.

VPLUS U

9.25 Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Sol. Name three defects are basically different type of deflects.

(a) Hyperopia (Hypermetropia) i.e. long sightedness.
It is long sightedness (or far sightedness) of eye i.e. near objects are not clearly seen but the objects which are far away (i.e. distant objects) from the eye are seen clearly. This deflect is removed by using suitable convergent lens (Figure A)

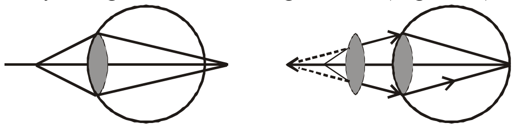


Figure (A)

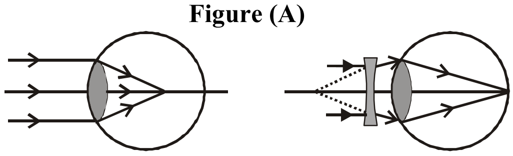


Figure (B)

(b) Myopia i.e. short sightedness. It is short sightedness (near sightedness) of eye i.e. far off objects are not clearly seen but the near objects are clearly seen. This defect can be corrected by using suitable divergent lens (Figure B).

(c) Presbyopia i.e. loss of power of accommodation. In this deflect both far off and nearer objects are not clearly seen. This deflect is generally corrected by using bi-focal lenses.

9.26 A myopic person has been using spectacles of power –1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power + 2.0dioptres. Explain what may have happened.

Sol. For –1 dioptre, the far point for eyes is 1 m i.e. 100 cm. The near point is 25 cm. The objects lying at infinity are brought at 100 cm from his eyes using the concave lens and the objects lying in between 25 cm and 100 cm are brought to focus using the ability of accommodation of the eye lens. In the old age, this ability of accommodation is reduced and the near point reaches 50 cm from his eyes.

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get, $-\frac{1}{50} + \frac{1}{25} = \frac{1}{f}$ = i.e. f = 50 cm

and $P = \frac{100}{f} = \frac{100}{50} = 2$ dioptre

The person requires glasses of + 2 dioptre.

9.27 A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Sol. This is due to the defect of lenses called astigmatism. The defect arises because of the fact that curvature of the eye-lens and the cornea is not same in different planes. This defect is removed by using cylindrical lens with vertical axis.

9.28 A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Sol. (a) To see the object at a closest distance, the image of object should be formed at the least distance of distinct vision.

Using $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ we have $-\frac{1}{u} - \frac{1}{25} = \frac{1}{5}$

VPLUSU

i.e. $-\frac{1}{u}=\frac{1}{5}+\frac{1}{25}=\frac{5+1}{25}=\frac{6}{25}$ $\quad\therefore$ u $=\frac{-25}{6}$ = – 4.2 cm

$\Rightarrow$ The object is to be placed at 4 cm from the magnifying glass.
Also, to see the object at the farthest point, its image must be formed at infinity.

$\therefore$ $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ i.e. $-\frac{1}{u}+\frac{1}{-\infty}=\frac{1}{5}$ i.e. u = – 5 cm

i.e. the object is to be placed at 5 cm from the magnifying glass.

(b) Angular magnification, m = $\frac{D}{|u|}$

$\therefore$ Maximum angular magnification = $\frac{25}{25/6}$ = 6

and minimum angular magnification = $\frac{25}{5}$ = 5.

9.29 A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.
(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?
(b) What is the angular magnification (magnifying power) of the lens?
(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Sol. (a) Using the lens formula, $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

When f = 10 cm, u = – 9 cm, we get

$\therefore$ $-\frac{1}{-9}+\frac{1}{v}=\frac{1}{10}$ i.e. $\frac{1}{\upsilon}=\frac{1}{10}-\frac{1}{9}=\frac{9-10}{90}$ i.e. v = – 90 cm

Linear magnification = $\frac{v}{u}=\frac{-90}{-9}=10$

$\therefore$ Area of each square in the image

$\therefore$ (1mm × 10) × (1mm × 10) = 100 mm^2 = 1 cm^2

(b) Angular magnification = $\frac{D}{|u|}=\frac{25}{9}\approx 2.8$

(c) Clearly magnification and power magnification are not equal to each other unless the image is located near the least distance of distinct vision i.e. v = D.

9.30 (a) At what distance should the lens be held from the figure in Q. 9.29 in order to view the squares distinctly with the maximum possible magnifying power?
(b) What is the magnification in this case?
(c) Is the magnification equal to the magnifying power in this case? Explain.

Sol. (a) The magnifying power is maximum if the image is formed at the least distance of distinct point from the eye, i.e. if v = –25 cm

Also, f = 10 cm

$\therefore$ $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $\Rightarrow$ $-\frac{1}{u}+\frac{1}{-25}=\frac{1}{10}$

i.e., $\frac{1}{u}=-\frac{1}{10}-\frac{1}{25}=\frac{-5-2}{50}=\frac{-7}{50}$

$\therefore$ u = – $\frac{50}{7}$ cm = – 7.14 cm

(b) Linear magnification = $\frac{v}{u}=\frac{-25}{-50/7}$ = 3.5

VPLUSU

(c) Angular magnification = $\frac{D}{|u|} = \frac{25}{50/7} = 3.5$

$\Rightarrow$ The linear magnification and magnifying power is equal in this case.

9.31 What should be the distance between the object in Q. 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

Sol. The new area to be observed = 6.25 mm^2.

$\therefore$ Length of each side = $\sqrt{6.25}$ = 2.5 mm.

Now linear magnification $= \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u} \Rightarrow \frac{v}{u} = \frac{2.5\text{mm}}{1\text{mm}} = 2.5 \Rightarrow v = 2.5u$

Now using $\frac{1}{f} = -\frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{10} = -\frac{1}{u} + \frac{1}{2.5u} = \frac{-2.5+1}{2.5u} = \frac{-15}{2.5u} \Rightarrow u = -6$ cm

Therefore, v = 2.5 u = – 2.5 × 6 = – 15 cm

Since | v | = 15cm < 25cm i.e. the image lies at distance less than the least distance of distinct vision from the eye, it can not be observed distinctly.

9.32 Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Sol. (a) The magnifying glass produces the virtual image of the object at the least distance of distinct vision. Since the object is placed close to the eye, the object has larger angular size if it were placed at the least distance of distinct vision.

(b) Yes. The angular magnification decreases slightly because angle subtended at eye is somewhat less than the angle subtended at the lens.

(c) The aberrations start croping up if the convex lens of smaller and smaller focal length is made.

(d) Angular magnification of eye piece is given by $\left(1+\frac{D}{f_e}\right)$ and angular magnification of objective is approximately given by v/f_0.

Clearly for better magnification focal length of eye piece f_e and focal length of objective f_0 should be short.

(e) If we position our eye close to the eyepiece, the whole light will not fall on our eye and the field of view will decrease. So we placed our eye a short distance away from the eye-piece to collect the large amount of light refracted through the eyepiece to increase the field of view.

9.33 An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?

Sol. For the image formed at the least distance of distinct vision, the magnifying power is given by

$m = m_0 m_e$

Here $m_0 = \frac{v_0}{-u_0}$ and $m_e = 1 + \frac{D}{f_e}$

Using $m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$

$\therefore$ $m = m_0 m_e$ i.e. $m_0 = \frac{m}{m_e} = \frac{30}{6} = 5.$

$m_0 = \frac{v_0}{-u_0} = 5$ i.e., $v_0 = -5u_0$

$\therefore$ Using $-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0}$, we get, $-\frac{1}{u_0} - \frac{1}{5u_0} = \frac{1}{f_0}$ or $\frac{-5-1}{5u_0} = \frac{1}{1.25}$

or $u_0 = -\frac{6}{5} \times 1.25 = -1.5$ cm

$\therefore$ $v_0 = -5u_0 = -5 \times (-1.5) = 7.5$ cm

Again using $-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$, we get $-\frac{1}{u_e} + \frac{1}{-25} = \frac{1}{5}$ (image is formed at 25 cm)

$\therefore$ $\frac{-1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{5+1}{25} = \frac{6}{25}$ or $u_e = -\frac{25}{6}$ cm $= -4.17$ cm

$\therefore$ $|u_e| = 4.17$ cm

Thus, the distance between the objective and the eyes lens = $v_0 + |u_e| = 7.5 + 4.17 = 11.67$

Also, the position of the object from the objective lens = $|u_0| = 1.5$ cm

9.34 A small telescope has an objective lens of focal length 140cm and an eyepiece of focal length 5.0cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25cm)?

Sol. (a) In normal adjustment, $|m| = \frac{f_0}{|f_e|} = \frac{140}{5} = 28,$

(b) When the image is formed at the least distance of distinct vision.

$$|m| = \frac{f_0}{|f_e|}\left(1+\frac{f_e}{D}\right) = \frac{140}{5}\left(1+\frac{5}{25}\right) = 28 \times \frac{6}{5} = 33.6$$

9.35 (a) For the telescope described in Q. 9.34 (a), what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25cm?

Sol. (a) Since the final image is formed at infinity, the distance between the object lens and the eye-piece is $f_0 + f_e = 140 + 5 = 145$ cm

(b) The angle subtended by the tower is $\alpha = \frac{\text{Height of tower}}{\text{Distnace of tower}} = \frac{100}{3000} = \frac{1}{30}$

Also, if h is the size of image formed by the object, then

$$\alpha = \frac{h}{f_0} = \frac{h}{140} \quad \therefore \frac{h}{140} = \frac{1}{30} \Rightarrow h = \frac{140}{30} = 4.7\text{cm}$$

(c) Angular magnification of eye piece is $= \left(1+\frac{D}{f_e}\right) = 1+\frac{25}{5} = 6$

$\therefore$ Height of the final image = 6h = 6 × 4.7 = 28.2 cm

VPLUSU

9.36 A Cassegrain telescope uses two mirrors as shown in Fig. Such a telescope is built with the mirrors 20mm apart. If the radius of curvature of the large mirror is 220mm and the small mirror is 140mm, where will the final image of an object at infinity be?

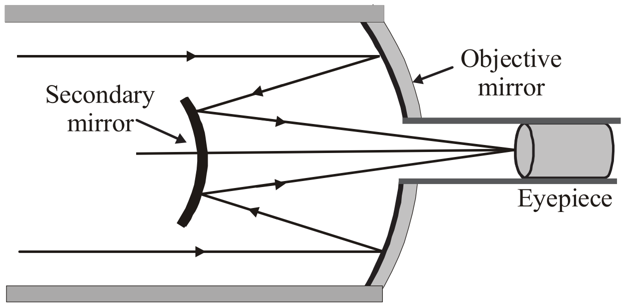


Figure : Schematic diagram of a reflecting telescope (Cassegrain).

Sol. $f_0 = \frac{220}{2} = 110$ mm

Object at infinity will form an image at 110 mm.

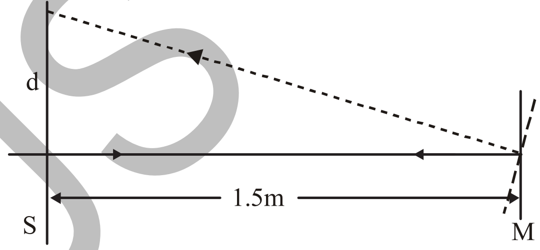
This image will act as virtual object for the smaller mirror.

Distance of virtual object for small mirror, u = 110 – 20 = 90mm, $f_s = \frac{140}{2} = 70$ mm

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get $\frac{1}{v}=\frac{1}{f_s}=\frac{1}{u}=\frac{1}{70}-\frac{1}{90}=\frac{1}{315}$ mm

Distance of final image is 315 mm from the small mirror on the right side.

9.37 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

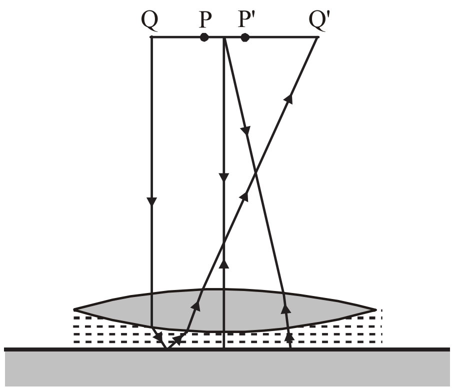


Sol. We know that if the mirror is turned by an angle θ, the reflected ray turns by an angle 2θ. The current in the coil has produced a deflection of 3.5° in the mirror, therefore, the reflected ray will be deflected by an angle 2 × 3.5° = 7°.

From the figure, we have, $\frac{d}{1.5} = \tan 7°$ $\left[\tan\theta = \frac{\text{perpendicular}}{\text{base}}\right]$

or d = 1.5 tan 7° = 1.5 × 0.123 = 0.184m = 18.4cm.

9.38 Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0cm. What is the refractive index of the liquid?



Sol. In the presence of the liquid, the distance of the needle from the lens is equal to the focal length f of the combnation of the convex lens and the plano concave lens formed by the liquid below it i.e. f = 45 cm. Also μ = 1.5

In the absence of the liquid, the distance of the needle and the lens is equal to the focal length of the convex lens only i.e. $f_1 = 30$ cm

∴ If f_2 is the focal length of plane concave lens formed by the liquid, then $\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$

or $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{45} - \frac{1}{30} = \frac{2-3}{90} = \frac{-1}{90}$ $\Rightarrow f = -90$ cm

Using the lens maker's formula, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

we get $\frac{1}{30} = (1.5 - 1)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right)$ $(\because R_1 = R \text{ and } R_2 = -R)$

$= 0.5 \times \frac{2}{R} = \frac{1}{R} \Rightarrow R = 30$ cm

For the plano convex lens formed by the liquid $f_2 = -90$ cm, $\mu = ?$ $R_2 = \infty$ and $R_1 = -30$ cm

$\therefore$ Using $\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, we get $\frac{1}{-90} = (\mu - 1)\left(-\frac{1}{30} - \frac{1}{\infty}\right)$

i.e. $\mu - 1 = \frac{30}{90} = \frac{1}{3}$ i.e. $\mu = 1 + \frac{1}{3} = \frac{4}{3} = 1.33$

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