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ELECTROSTATICS

INTRODUCTION

* Electricity is all about charge. (Charge is the fundamental quantity of electricity.)
* The classical study of electricity is generally divided into three general areas.
  - **Electrostatics (static electricity) :** the study of the forces acting between fixed arrangements of charge in space.
  - **Electric current:** the study of the forces of energy associated with the flow of charge through circuits.
  - **Electromagnetism:** the study of the forces acting between charges in motion.
* Electric charge (often just called charge) comes in two and only two types : positive (+) and negative (–).
* The term neutral does not refer to a third type of charge, but to the presence in a region of positive and negative charges in equal amount.
* The choice of assignment of positive to one type of charge and negative to the other was completely arbitrary.
* Electrostatics deals with the study of forces, fields and potentials arising from static charges.

CONCEPT OF CHARGE

* In nature, atoms are normally found with equal numbers of protons and electrons, i.e. atom is electrically neutral.
* The charge on an electron or a proton is the smallest amount of free charge that has been discovered.
* Charges of larger magnitude are built up on an object by adding or removing electrons.
* If in a body there is excess of electrons over its neutral configuration, conventionally the body is said to be **negatively charged** and if there is deficiency of electron it is said to be **positively charged.**
* – ve charged body $\rightarrow$ Body has gained electrons
* + ve charged body $\rightarrow$ Body has lost some electrons
* + ve & – ve charge named by Benjamin Franklin.

Basic properties of electric charge :

(1) Charge is scalar and can be of two types (i.e. + ve or – ve). It adds algebraically.

(2) Charge is conserved. During any process (chemical, nuclear, decay etc.) the net electric charge of an isolated system remains constant.

* **Radioactive decay** : ${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$
  This shows that the amount of charge (92e) present before disintegration is the same as that present after the disintegration (90e + 2e = 92e).
* **Annihilation process :** An electron and a positron combine to form a photon. Before annihilation, the total charge on electron and positron is zero because the electron has a negative charge while the positron has exactly the same amount of positive charge. After annihilation they produce two gamma ray photons (energy) and still the total charge remains zero.
* **Pair production :** When higher energy gamma ray photon (charge zero) collides with the electric field inside the atom, it is converted into an electron (charge –e) and a positron (charge +e). Thus the net charge before and after the event is zero.

(3) Charge is Quantized (exists as discrete "Packets") : Robert Millikan discovered that electric charge always occurs as some integral multiple of fundamental unit of charge (e).

$q = Ne$ [N is some integer]

Charge on a body can never be $\left(\frac{1}{3}\right)e$, $\left(\frac{2}{3}\right)e$ etc. as it is due to transfer of electron.

Quark particles have charges equal to $\pm e/3$ or $\pm 2e/3$ but they are not stable in free state so we take electron only for quantization purpose.

(4) Through large number of experiments it is also well established that similar charges repel each other while dissimilar attract. (Applicable for point charges).

Here it is worth noting that true test of electrification is repulsion and not attraction as attraction may also take place between a charged and uncharged body.

(5) Charge is always associated with mass i.e. charge can not exist without mass though mass can exist without charge.

(6) Charge is transferable. Process of charge transfer is called conduction.

(7) Charge is invariant i.e. it is independent on frame of reference.

(8) Charge at rest produces $\rightarrow$ Electric effect

Charge in unaccelerated motion produces
$\rightarrow$ Electric and magnetic fields.

Accelerate charge particle$\rightarrow$ Electric & magnetic effect + radiate energy (According to electromagnetic theory)

(9) Charge resides on the outer surface of a conductor.

(10) How to express charge :

* The SI unit for measuring the magnitude of an electric charge is the coulomb (C).
* Current $\rightarrow$ drift of charge per unit time
  $I = q/t \Rightarrow q = It$
  1 coulomb $\rightarrow$ 1 ampere $\times$ 1 sec
  If a charge of 1 coulomb drift per second through cross-section of conductor, current flowing is called 1 ampere.
* Charge on electron = – 1.6×10^{-19} C,

* Charge on proton = + 1.6 × 10^{-19} C
* The coulomb is related to CGS units of charge through the basic relation. 1 Coulomb = 3 × 10^9 esu of charge

$$\text{(static coulomb or frankline)} = \frac{1}{10} \text{ emu of charge}$$

* Practical units of charge :
  Amp × hr (= 3600 coulomb), Faraday (= 96500 coulomb)
* Charge on 6.25 × 10^{18} electrons = – 1 C

Methods of Charging :

(A) Triboelectricity:

The rubbing process serves only to separate electrons already present in the materials. No electrons or protons are created or destroyed.

Materials acquiring charges on rubbing together

+ ve charged	– ve charged of
(Vitreous)	(Resinous)
Glass rod	Silk Cloth
Fur or wool	Ebonite or Rubber or Amber
Wool	Plastic
Dry hair	Plastic comb

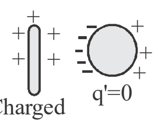
Interesting Experiment : Take any two materials from the following list and then rubbed with each other. We can always find that the former one is positively charged and the later one is negatively charged.

Fur → glass → paper → metal → silk → plastic → amber → rubber → sulfur

When a charged body is close enough to a neutral body, they attract each other. One of the applications of this effect is to use tiny paint droplets to paint the automobiles uniformly.

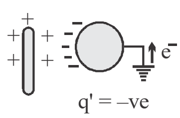
(B) Conduction : Transfer by contact with an already charged object.

(C) Induction: When a charged particle is taken near a neutral metallic object then the electrons move to one side and there is excess of electrons on that side making it negatively charged and deficiency on the other side making that side positively charged. Hence charges appear on two sides of the body (although total charge of the body is still zero). This phenomenon is called induction and the charge produced by it is called induced charge.

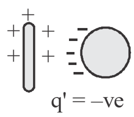


Charged body

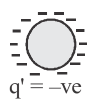
Step 1 : Charged body is brought near uncharged body



Step 2 : Uncharged body is connected to the earth



Step 3 : Uncharged body is disconnected from the earth



Step 4 : Charging body is removed

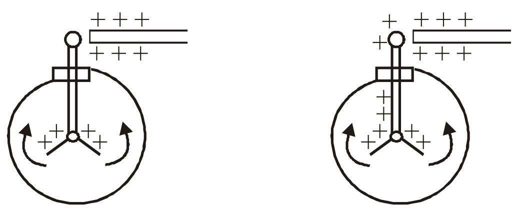
* Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by q' = – q (1 – 1/k), where 'q' is inducing charge and 'k' is the dielectric constant of the material of the uncharged body. For metals k = ∞ ⇒ q' = – q.
* A body can be charged by means of –
  (a) friction,
  (b) conduction,
  (c) induction,
  (d) thermoionic ionisation,
  (e) photoelectric effect and
  (f) field emission.

Detecting charge :

* Charge can be detected and measured with the help of gold-leaf electroscope, voltameter, ballistic galvanometer.
* Gold leaf electroscope consist of two gold leaves attached to a conducting post that has a conducting disc ball on top. The leaves are otherwise insulated from the container. Gold leaf electroscope can be used in 2 ways.
  Uncharged electroscope when uncharged, the leaves hang together vertically.

(a) If a charged body is brought near to it, charge on the ball of electroscope will be opposite to that of body & on leaves similar to that of body and leaves will diverge.

(b) If a charged body is touched : Ball & leaves both acquire similar charge and leaves will diverge.

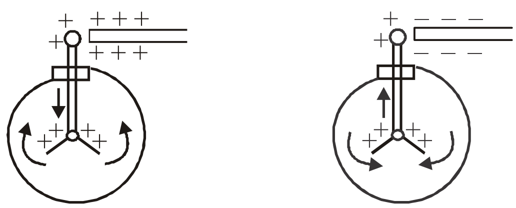


(a) **(b)**

From above method you will not be able to tell nature of charge (it may be +ve or – ve) in both case leaves will diverge.

Charged electroscope :

If a charged body is brought near a charged electroscope, the leaves will further diverge if the charge on the body is similar to that on the electroscope and will usually converge if opposite. Thus we will be able to determine nature of charge on a body.



Example 1 :

A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+ 19.2 \times 10^{-19}$ C.

(i) Find the number of electrons lost by glass rod.

(ii) Find the negative charge acquired by silk.

(iii) Is there transfer of mass from glass to silk ?

Sol. (i) Number of electrons lost by glass rod is

$$n = \frac{q}{e} = \frac{19.2 \times 19^{-19}}{1.6 \times 10^{-19}} = 12$$

(ii) Charge on silk $= - 19.2 \times 10^{-19}$ C

(iii) Since an electron has a finite mass ($m_e = 9 \times 10^{-31}$ kg), there will be transfer of mass from glass rod to silk cloth.

Mass transferred $= 12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg.

The mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

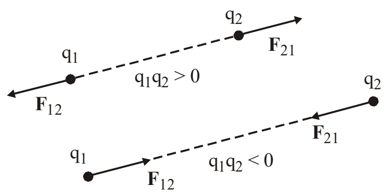
COULOMB'S LAW

* Force between two point charges (interaction force) is directly proportional to the product of magnitude of charges (q_1 and q_2) and is inversely proportional to the square of the distance between them i.e., $(1/r^2)$.

* This force is conservative in nature.

This is also called inverse square law.

* The direction of force is always along the line joining the point charges.



* If two point charges q_1 and q_2 at rest are separated by a distance r in vacuum, the magnitude of force between them is given by $F \propto \frac{q_1 q_2}{r^2}$; $\boxed{F = k\frac{q_1 q_2}{r^2}}$,where k is a constant.

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

ε_0 = permittivity of free space
$= 8.85 \times 10^{-12}$ C^2/N-m^2

* Dimension of $\varepsilon_0 = M^{-1}L^{-3}T^4A^2$

* If the point charges are kept in some other medium (say kerosene) then Coulomb's law gives $F = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$

where ε_r is known as relative permitivity of the medium and is dimensionless. (Note $\varepsilon_r > 1$).

ε_r is also denoted by K and known as **dielectric constant** of the medium. Also note that the permitivity of the medium will be $\varepsilon_0\, \varepsilon_r$.

So, ε_r or K indicates that the force between two point charges in a medium is decreased K times compared to the forces between them in vacuum. So,

$$\boxed{\varepsilon_r \text{ or } K = \frac{\text{Force in vacuum}}{\text{Force in the medium}}}$$

(between two point charges separated by a particular distance).

* For vaccum/air, $K \approx 1$

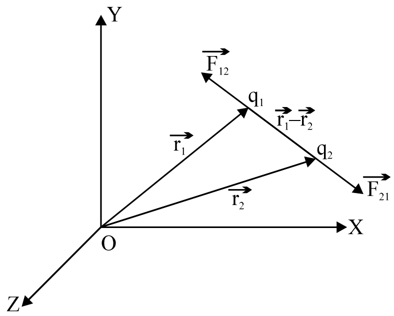
* For water, K = 80

* For Mica, K = 7 to 10

* For metal, $K = \infty$

Coulomb's Law in Vector Form

* Suppose the position vectors of two charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, then, electric force on charge q_1 due to charge q_2 is, $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$



Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Here q_1 and q_2 are to be substituted with sign. Position vector of charges q_1 and q_2 are $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively. Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

* If F_1 and F_2 are the forces acting on a charge Q due to two other charges, then the resultant force on charge Q is :

$$F^2 = F_1{}^2 + F_2{}^2 + 2F_1F_2 \cos\theta$$

where θ is the angle between $\vec{F}_1$ and $\vec{F}_2$.

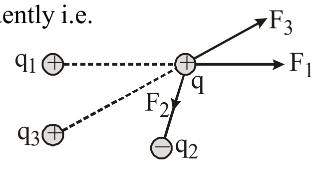
If $\vec{F}_1$ and $\vec{F}_2$ are mutually perpendicular then

$$F^2 = F_1{}^2 + F_2{}^2 \text{ or } F = \sqrt{F_1{}^2 + F_2{}^2}$$

* **Superposition Theorem :** The interaction between any two charges is independent of the presence of all other charges. Electrical force is a vector quantity therefore, the net force on any one charge is the vector sum of the all the forces exerted on it due to each of the other charges interacting with it independently i.e.

Net force on charge q,

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$



Example 2 :

The force between two point charges placed in vacuum is 18N. If a glass plate of dielectric constant 6 is now introduced between them, the electric force will be.

(A) 3 N (B) 18 N

(C) 108 N (D) Zero

Sol. **(A).** $F = \dfrac{1}{4\pi \in_0 \in_r}\left(\dfrac{q_1 q_2}{r^2}\right) = \dfrac{F_{vac}}{\in_r} = \dfrac{18}{6} = 3\text{ N}$

Example 3 :

Four charges each of $2\mu C$ are placed at x = 0, 2, 4, 8 cm on x-axis. The force exerted on the charge placed at x = 2cm will be –

(A) Zero (B) 10 N (C) 5 N (D) 10^{-3} N

Sol. **(B).** The resultant force acting on the charge at x = 2 cm due to charges at x = 0 and x = 4 cm will be zero. Hence the only force will be due to charge at x = 8 cm.

$$F = \frac{kq_1q_2}{r^2} = \frac{9\times10^9 \times 2\times 10^{-6}\times 2\times10^{-6}}{6\times10^{-2}\times 6\times10^{-2}} = 10\text{N}$$

Example 4 :

What is the smallest electric force between two charges placed at a distance of 1.0 m.

Sol. $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}$ .... (1)

For F_e to be minimum $q_1\,q_2$ should be minimum.

We know that, $(q_1)_{min} = (q_2)_{min} = e = 1.6\times10^{-19}$ C

Substituting in Eq. (1), we have

$$(F_e)_{min} = \frac{(9.0\times10^9)\,(1.6\times10^{-19})\,(1.6\times10^{-19})}{(1.0)^2}$$

$$= 2.304\times10^{-28}\text{ N.}$$

Example 5 :

Two point charges A and B have charges respectively ½C and 2C with their position vectors respectively as $(\hat{i}+\hat{j}+\hat{k})$ and $(-\hat{i}-\hat{j}+3\hat{k})$. Find the force on charge at A due to B.

Sol. $q_A = \dfrac{1}{2}C$; $\vec{r}_A = \hat{i}+\hat{j}+\hat{k}$; $q_B = 2C$; $\vec{r}_B = -\hat{i}-\hat{j}+3\hat{k}$

$$\vec{F}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q_Aq_B}{|\vec{r}_{AB}|^2}\hat{r}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q_Aq_B}{|\vec{r}_{AB}|^3}(\vec{r}_A-\vec{r}_B)$$

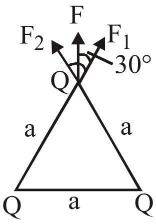
$$= (9\times10^9)\times\frac{\frac{1}{2}\times1}{|2\hat{i}+2\hat{j}-2\hat{k}|^3}(2\hat{i}+2\hat{j}-2\hat{k})$$

$$= \frac{9\times10^9\times(\hat{i}+\hat{j}-\hat{k})}{24\sqrt{3}}\text{N}$$

Example 6 :

If three equal charges (Q each) are placed on the vertices of an equilateral triangle of side a. Find the resultant force on any one charge due to the other two.

Sol. The charges are shown in figure.



The resultant force

$$F = \sqrt{F_1^2+F_2^2+2F_1F_2\cos60^\circ}$$

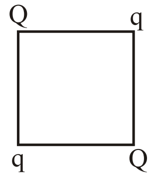
with $F_1 = F_2 = kQ^2/a^2$

$$F = \frac{\sqrt{3}kQ^2}{a^2}$$

From symmetry the direction is shown along y-axis.

Example 7 :

Four charges Q, q, Q and q are kept at the four corners of a square as shown. Find the relation between q and Q such that net force on a charge q is zero.

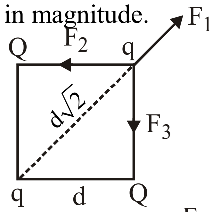


Sol. Both the q will have same sign either positive or negative. Similarly both the Q will have same sign. Let us make the force on upper right corner q equal to zero.

Lower q will apply a repelling force F_1 on upper q (because both the charges have same sign).

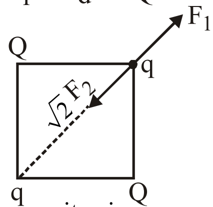
To balance this force both 'Q' must apply attractive forces $\vec{F}_2$ and $\vec{F}_3$ of equal magnitude (So, Q and q will have opposite signs). Now the resultant of $\vec{F}_2$ and $\vec{F}_3$ will be $F_2\sqrt{2}$ (Pythagoras theorem) and it will be exactly opposite to F_1 and same in magnitude.

From Coulomb's Law



$$F_1 = \frac{kq^2}{(d\sqrt{2})^2},\ F_2 = \frac{kQq}{d^2}$$

and $F_1 = \sqrt{2}\,F_2$



$$\frac{q^2}{(d\sqrt{2})^2} = \frac{\sqrt{2}Qq}{d^2}$$

$$Q = \frac{q}{2\sqrt{2}}$$

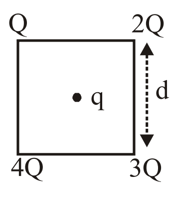
But Q and q must have opposite sign so,

$$q = -2\sqrt{2}\,Q$$

Example 8 :

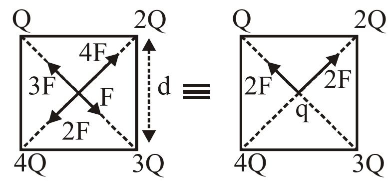
The charges on the four corners of the square are Q,

2Q, 3Q and 4Q respectively as shown in the figure, then find the force on the charge q kept at the centre of a square.



Sol. If Q applies a force F, then 2Q will apply 2F on q. Because as per Coulomb's law the force is proportional to charge, if rest of the factors are same.

Similarly 3Q will apply 3F and 4Q will apply 4F on q. The four forces along with its direction are shown below.



We can see, 3F and F are exactly opposite to each other so its net effect will be 2F towards Q and 4F and 2F are exactly opposite to each other so its effect will be 2F towards 2Q as shown in the figure. So resultant force will be (Pythagoras) of 2F and 2F equal to $2\sqrt{2}\text{ F}$.

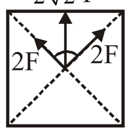
F is basically force between Q & q so that is

$$F = \frac{Qq}{4\pi\varepsilon_0(d/\sqrt{2})^2}$$

Note that the distance between Q and q is $(d/\sqrt{2})$ as the side of the square is d. So, final answer is

$$2\sqrt{2}F = \frac{2\sqrt{2}\ Qq}{4\pi\varepsilon_0(d/\sqrt{2})^2} = \frac{4\sqrt{2}\ Qq}{4\pi\varepsilon_0 d^2}$$

2√2 F

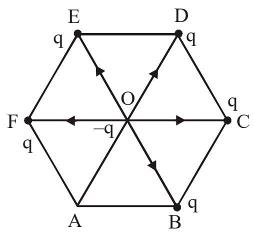


Direction is upwards.

Example 9 :

Five point charges, each of value +q are placed on five vertices of a regular hexagon of side L m. Find the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon.

Sol. If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges –q at O will be zero (as the forces due to individual charges will balance each other),



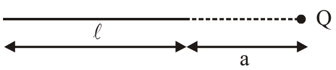
i.e., $\vec{F}_R = 0$

Now if $\vec{f}$ is the force due to sixth charge $\vec{F}$ due to remaining five charges, $\vec{F} + \vec{f} = 0$ i.e. $\vec{F} = -\vec{f}$

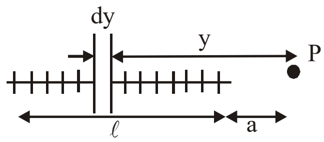
or $$F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{L}\right]^2$$

Example 10 :

A thin straight rod of length ℓ carrying a uniformly distributed charge q is located in vacuum. Find the magnitude of the electric force on a point charge Q kept as shown in the figure.



Sol. As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part, called element of length dy can be considered as point charge.



Charge on element, $dq = \lambda\ dy = \frac{q}{\ell}\ dy$

Electric force on Q due to element $= \frac{KdqQ}{y^2} = \frac{KQqdy}{y^2\ell}$

All forces are along the same direction,

$\therefore$ $F = \Sigma\ dF$. This sum can be calculated using integration,

therefore, $$F = \int_{y=a}^{a+\ell}\frac{KQqdy}{y^2\ell} = \frac{KqQ}{\ell}\left[-\frac{1}{y}\right]_a^{a+\ell}$$

$$= \frac{KQ.q}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right] = \frac{KQq}{a\ (a+\ell)}$$

Note:

(i) The total charge of the rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.

(ii) If a >> ℓ then, $F = \frac{KQq}{a^2}$

i.e. Behaviour of the rod is just like a point charge.

ELECTROSTATIC EQUILIBRIUM

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

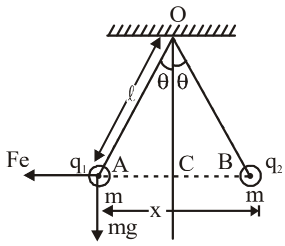
(i) **Stable Equilibrium :** A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

(ii) **Unstable Equilibrium :** If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

(iii) **Neutral Equilibrium :** If charge is displaced by a small distance and it is still in equilibrium codition then it is called neutral equilibrium.

Example 11 :

When two charged pith balls having charges q_1 and q_2 are suspended from same point with the help of silk threads then considering the equilibrium of any one ball (as shown in fig.) Find distance between the balls.



Sol. Moment of F_e about O = Moment of mg about O.

$F_e \times OC = mg \times AC$

or $\frac{F_e}{mg} = \tan\theta$ or $\frac{q_1 q_2}{4\pi \epsilon_0 x^2 . mg} = \tan\theta$

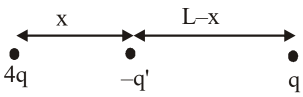
If $x << \ell$ and the charges on the pith balls are equal then it

can be easily proved that $x = \left(\frac{q_1 \ell}{2\pi \epsilon_0 \, mg}\right)^{1/3}$

Example 12 :

Two point charge +4q and +q are placed at a distance L apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and nature of third charge. Discuss also, whether the equilibrium of the system is stable, unstable or neutral.

Sol. Third charge should be placed between 4q and q so that force on third charge to be zero (let at distance x from 4q). Third charge should be –ve (let –q') for the equilibrium of other charges.



For equilibrium of third charge

$$\frac{K\,(4q)\,(q')}{x^2} = \frac{K\,(q)\,(q')}{(L - x^2)} \Rightarrow x = \frac{2L}{3}$$

For equilibrium 4q

$$\frac{K\,(4q)\,(q')}{(2L/3)^2} = \frac{K\,(4q)\,(q)}{L^2} \Rightarrow q' = \frac{4q}{9}$$

If q' is displaced along x-axis towards q then it will move in the same direction, hence, unstable equilibrium.

Example 13 :

Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density $\sigma = 0.8$ gm/cc, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is ρ=1.6 gm/cc.)

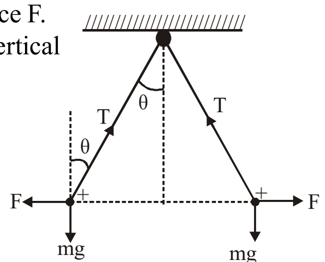
Sol. Initially as the forces acting on each ball are tension T, Weight mg and electric force F.

For its equilibrium along vertical

$T\cos\theta = m\,g$ .... (1)

and along horizontal

$T\sin\theta = F$ .... (2)



Dividing Eqn. (2) by (1),

we have $\tan\theta = \frac{F}{mg}$ .... (3)

When the balls are suspended in a liquid of density σ and dielectric constant K, the electric force will become (1/K) times, i.e., F' = (F/K) while weight

$mg' = mg - F_B = mg - V\sigma g$

[as $F_B = V\sigma g$, where σ is density of liquid]

i.e., $mg' = mg\left[1 - \frac{\sigma}{\rho}\right]$ $\left[\text{as } V = \frac{m}{\rho}\right]$

So, for equilibrium of ball,

$$\tan\theta' = \frac{F'}{mg'} = \frac{f}{Kmg\,[1 - (\sigma/\rho)]} \quad .... (4)$$

According to given information $\theta' = \theta$, so from eq. (4) and (3), we have,

$$K = \frac{\rho}{(\rho - \sigma)} = \frac{1.6}{1.6 - 0.8} = 2$$

TRY IT YOURSELF - 1

Q.1 If you rub an inflated balloon against your hair, the two materials attract each other, as shown in Figure. Is the amount of charge present in the system of the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

Q.2 Three objects are brought close to each other, two at a time. When objects A and B are brought together, they repel. When objects B and C are brought together, they also repel. Which of the following are true? (a) Objects A and C possess charges of the same sign. (b) Objects A and C possess charges of opposite sign. (c) All three of the objects possess charges of the same sign. (d) One of the objects is neutral. (e) We would need to perform additional experiments to determine the signs of the charges.

Q.3 A point charge +Q is placed at the centroid of an equilateral triangle. When a second charge +Q is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 4N. What is the magnitude of the net force on the central charge when a third charge +Q is placed at another vertex of the triangle?

(A) zero (B) 4 N

(C) $4\sqrt{2}$N (D) 8 N

Q.4 Two electrons are a certain distance apart from one another. What is the order of magnitude of the ratio of the electric force between them to the gravitational force between them?

(A) $10^8 : 1$ (B) $10^{28} : 1$

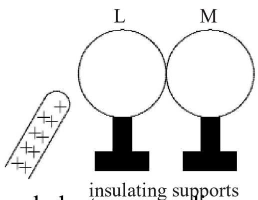
(C) $10^{31} : 1$ (D) $10^{42} : 1$

Q.5 Millikan's oil drop experiment attempts to measure the charge on a single electron, e, by measuring the charge of tiny oil drops suspended in an electrostatic field. It is assumed that the charge on the oil drop is due to just a small number of excess electrons. The charges 3.90×10^{-19} C, 6.50×10^{-19} C and 9.10×10^{-19} C are measured on three drops of oil. The charge of an electron is deduced to be,
(A) 1.3×10^{-19} C (B) 1.6×10^{-19} C
(C) 2.6×10^{-19} C (D) 3.9×10^{-19} C

Q.6 Two identical point charges are held on a smooth horizontal floor at a distance d apart by a non-conducting string with tension T. If a third identical point charge is fixed vertically above at a distance of d from both the point charges then what will be the new tension in the string.
(A) T (B) 2T (C) 3T/2 (D) none

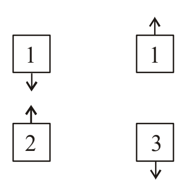
Q.7 Electric charges A and B are attracted to each other. Electric charges B and C are also attracted to each other. If A and C are held close together they will
(A) attract. (B) repel (C) not affect each other.
(D) More information is needed to answer.

Q.8 Two uncharged metal spheres, L and M, are in contact. A positively charged rod is brought close to L, but not touching it, as shown. The two spheres are slightly separated and the rod is then withdrawn. As a result:
(A) both spheres are neutral
(B) both spheres are positive
(C) both spheres are negative
(D) L is negative and M is +ve

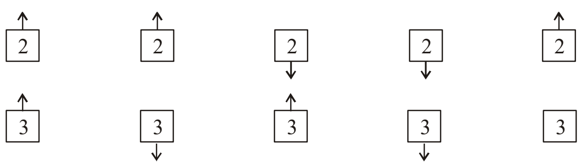


Q.9 The leaves of a positively charged electroscope diverge more when an object is brought near the knob of the electroscope. The object must be:
(A) a conductor (B) an insulator
(C) positively charged (D) negatively charged

Q.10 The diagram shows two pairs of heavily charged plastic cubes. Cubes 1 and 2 attract each other and cubes 1 and 3 repel each other.



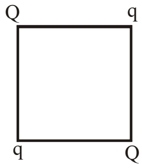
Which of the following illustrates the forces of cube 2 on cube 3 and cube 3 on cube 2?



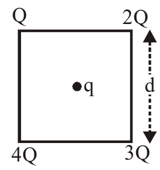
Q.11 Three objects are brought close to each other, two at a time. When objects A and B are brought together, they attract. When objects B and C are brought together, they repel. From this, we conclude that (a) objects A and C possess charges of the same sign. (b) objects A and C possess charges of opposite sign. (c) all three of the objects possess charges of the same sign. (d) one of the objects is neutral. (e) we need to perform additional experiments to determine information about the charges on the objects.

Q.12 Two fixed charges +4q and +1q are at a distance 3 m apart. At what point between the charges, a third charge +q' must be placed to keep it in equilibrium?

Q.13 Four charges Q, q, Q and q are kept at the four corners of a square as shown. What is the relation between Q and q so that the net force on a charge q is zero?



Q.14 Find the force on the charge q kept at the centre of a square of side d. The charges on the four comers of the square are Q, 2Q, 3Q and 4Q respectively as shown in the figure.



ANSWERS

(1) **(b).** **(2)** **(a, c, e).** **(3)** (B)
(4) (D) **(5)** (A) **(6)** (C)
(7) (B) **(8)** (D) **(9)** (C)
(10) (C) **(11)** **(e).**
(12) q will be placed at a distance 2 m from Q_1 and at 1 m from Q_2.
(13) $q = -2\sqrt{2}\ Q$
(14) $\dfrac{4\sqrt{2}Qq}{4\pi\varepsilon_0 d^2}$

ELECTRIC FIELD

* The physical field where a charged particle, irrespective of the fact whether it is in motion or at rest, experiences force is called an electric field.
* The concept of electric field was given by Michael Faraday. Characteristics of electric field :
(1) Electric field intensity (shortly we will call electric field).
(2) Electric potential.
(3) Electric lines of forces.

Electric field intensity $\vec{E}$:

Definition of the $\vec{E}$-field :

* One obvious disadvantage of concentrating on force is that its magnitude at every point in space depends not only on the primary charge distribution, but also on the size of the test charge q_0.
* What we really want is a map showing the field of a primary body independent of the detector, a map that could be used to compute the force at every point in space when any size charge is placed there.
* Regardless of its source, we define the electric field ($\vec{E}$) at a point in space to be the electric force experienced by a positive test-charge at that point divided by that charge

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{........ (1)}$$

Electric field has the Sl units of newtons per coulomb (N/C).

* The presence of the charge q_0 will generally change the original distribution of the other charges, particularly if the charges are on conductors. However, we may choose q_0 to be small enough so that its effect on the original charge distribution is negligible.

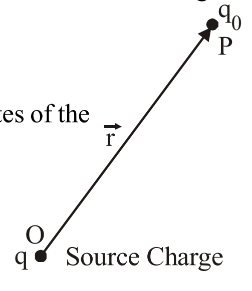
$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{E}}{q_0}$$

* Conversely, known $\vec{E}$ at any location in space (whatever the source) we can calculate the force $\vec{F}$ that would arise on any point charge q placed at that location; accordingly $\vec{F} = q\vec{E}$ .... (2)
* Notice that $\vec{F}$ and $\vec{E}$ point in the same direction when q is positive.
* A charge will never exert force on itself.

Electric field due to a point charge

* The electric field produced by a point charge q can be obtained in general terms from Coulomb's law.
* First, note that the magnitude of the force exerted by the charge q on a test charge q_0 is $F = kqq_0/r^2$. Then, divide this value by q_0 to obtained the magnitude of the field.
* Since q_0 is eliminated algebraically from the result, the electric field does not depend on the test charge:

Point charge q : $\boxed{E = \frac{kq}{r^2}}$



* If (x, y, z) are the co-ordinates of the observation point P, then

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Also, $r = (x^2 + y^2 + z^2)^{1/2}$
and $r^3 = (x^2 + y^2 + z^2)^{3/2}$

Now, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2+y^2+z^2)^{3/2}} \times (x\hat{i} + y\hat{j} + z\hat{k})$

* The three rectangular components of $\vec{E}(\vec{r})$ are as :

$$E_x(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2+y^2+z^2)^{3/2}} x,$$

$$E_y(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2+y^2+z^2)^{3/2}} y$$

and $E_z(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2+y^2+z^2)^{3/2}} z$

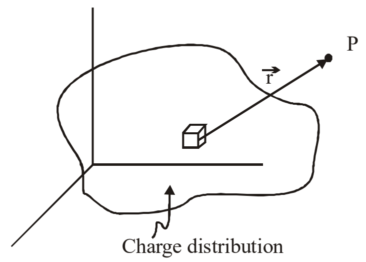
* **Electric field due to Discrete distribution of charge :**
Point charges placed at different position, use vector approach (Better term : super position rule)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ... = \sum_{i=1}^{n} \vec{E}_i \quad \text{with} \quad \vec{E}_i = \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^3} \vec{r}_i$$

* **Electric field due to continuous distribution of charge**
E-field at point P due to dq:

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} \cdot \hat{r}$$

$\therefore$ E-field due to charge distribution



$$\vec{E} = \int_{\text{Volume}} d\vec{E} = \int_{\text{Volume}} \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} \cdot \hat{r}$$

(1) In many cases, we can take advantage of the symmetry of the system to simplify the integral.
(2) To write down the small charge element dq:
1-D, $dq = \lambda\, ds$;
λ = linear charge density,
ds = small length element
2-D, $dq = \sigma\, dA$;
σ = surface charge density,
dA = small area element
3-D, $dq = \rho\, dV$;
ρ = volume charge density,
dV = small volume element

Let us consider some cases :

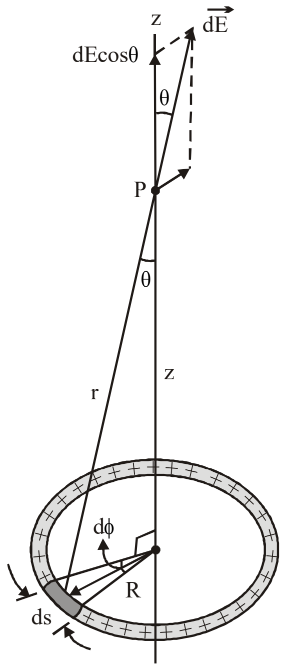
Case 1 : Charged ring

E-field at a height z above a ring of charge of radius R

NOTE : We are deriving the expression only to understand mathematical approach. For other cases you can remember the expression. We can derive some cases with the help of Gauss's law.

(1) Symmetry considered: For every charge element dq considered, there exists dq' due to which the horizontal $\vec{E}$ field components cancel.

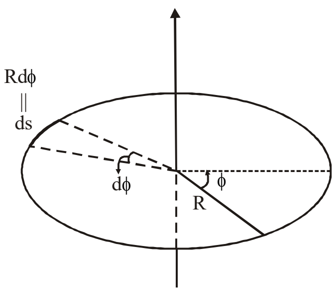
$\Rightarrow$ Overall E-field lies along z-direction.



(2) For each element of length ds, charge

$$dq = \underset{\substack{\uparrow \\ \text{Linear} \\ \text{charge density}}}{\lambda} \cdot \underset{\substack{\uparrow \\ \text{Circular} \\ \text{length element}}}{ds}$$

$dq = \lambda.R\, d\phi$, where ϕ is the angle measured on the ring plane.



$\therefore$ Net E-field along z-axis due to dq :

$$dE = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r^2}.\cos\theta$$

Total E-field $= \int dE$

$$= \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0}.\frac{\lambda R d\phi}{r^2}.\cos\theta \left(\cos\theta = \frac{z}{r}\right)$$

$360° = 2\pi$ radian, $180° = \pi$ radian

Note : Here in this, θ, R and r are fixed as ϕ varies but we want to convert, r, θ to R, z.

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda R z}{r^3}\int_0^{2\pi} d\phi$$

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda(2\pi R)z}{(z^2+R^2)^{3/2}} \quad \text{along z-axis}$$

But : $\lambda\,(2\pi R)$ = total charge on the ring.

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}.\frac{Qz}{(z^2+R^2)^{3/2}}}$$

NOTE

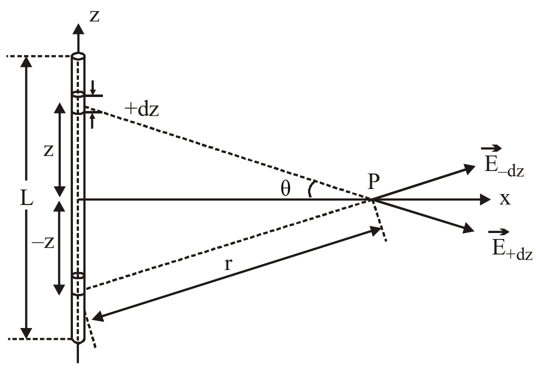
* At the centre of the ring z = 0, so E = 0
* For z >> R the ring behaves as a point charge. In this case, $E \propto \frac{1}{z^2}$
* For z << R the value of E is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{R^3} \text{ i.e. } \quad E \propto z$$

* E will be maximum at $z = \pm\frac{R}{\sqrt{2}}$. An opposite charge kept far off from the centre on the z-axis (ring axis) will execute oscillatory and periodic motion but if it is kept very close to the centre then it will execute SHM.

Case 2 : Uniform line of charge

Charge per unit length = λ



(1) Symmetry considered: The E-field from +z and – z directions cancel along z-direction,

$\therefore$ Only horizontal E-field components need to be considered.

(2) For each element of length dz, charge $dq = \lambda\, dz$

$\therefore$ Horizontal E-field at point P due to element dz

$$|d\vec{E}|\cos\theta = \underbrace{\frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dz}{r^2}}_{dE_{dz}}\cos\theta$$

After integration

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{x\sqrt{x^2+\left(\frac{L}{2}\right)^2}} \text{ along x-direction}$$

Important limiting cases :

1. $x >> L$: $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{x^2}}$

But λL = Total charge on rod

$\therefore$ System behave like a point charge

2. $L >> x$: $E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{x.\frac{L}{2}}$

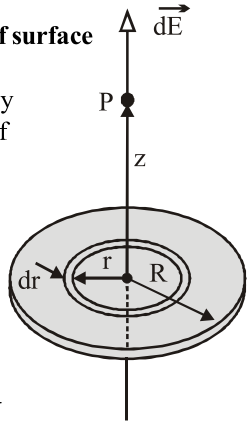
$$\boxed{E_x = \frac{\lambda}{2\pi\varepsilon_0 x}}$$

Case 3 : E-field from a disk of surface charge density σ.

We find the E-field of a disk by integrating concentric rings of charges.

Total charge of ring

$$dq = \sigma.\underbrace{(2\pi r\, dr)}_{\text{Area of the ring}}$$



E-field from ring :

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq\, z}{(z^2+r^2)^{3/2}}$$

After integration, $E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$

Very important limiting case

If R >> z, that is if we have an infinite sheet of charge with charge density σ :

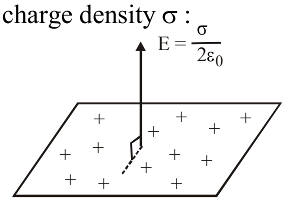


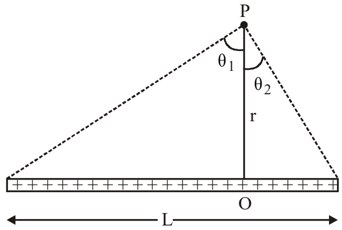
Figure : E-field due to an infinite sheet of charge, charge density = σ.

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right] \simeq \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{R}\right]; \quad \boxed{E \simeq \frac{\sigma}{2\varepsilon_0}}$$

E-field is normal to the charged surface.

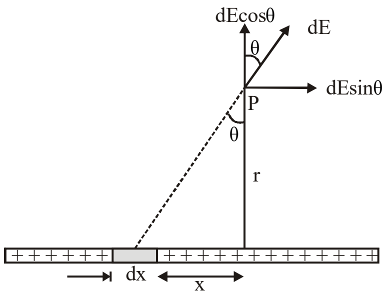
Case 4 : Electric field strength of a general point due to a uniformly charged rod :

Consider P as any general point in the surrounding of rod, to find electric field strength at P, consider an small element on rod of length dx at a distance x from point O.



dE the electric field at P due to the small element.

$$dE = \frac{kdq}{(x^2+r^2)}. \text{ Here, } dq = \frac{Q}{L}dx$$



After integration,

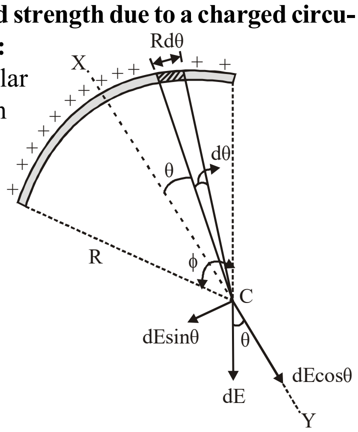
In x-direction, $E_x = \frac{kQ}{Lr}(\cos\theta_2 - \cos\theta_1)$

In y-direction, $E_y = \frac{kQ}{Lr}(\sin\theta_1 - \sin\theta_2)$

Case 5 : Electric field strength due to a charged circular arc at its centre :

Figure shows a circular arc of radius R which subtend an angle ϕ at its centre.

To find electric field strength at C, consider a small segment on arc of angular width $d\theta$ at an angle θ from the angle bisector XY as shown.



The length of elemental segment is R dθ, the charge on this element dq is $dq = \frac{Q}{\phi}.d\theta$.

Due to this dq, electric field at centre of arc C is given as

$dE = \frac{kdq}{R^2}$. After integration, $\boxed{E_C = \frac{2kQ\sin\left(\frac{\phi}{2}\right)}{\phi R^2}}$

* For semi-circular ring $\phi = \pi$.

* $E = \dfrac{2kQ}{\pi R^2}$

Case 6 : Spherical distribution of charge

(a) Conducting sphere (Hollow, solid)

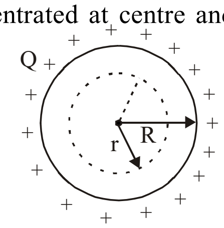
(b) Non-conducting sphere (Hollow, solid)

Case (a) Charge on surface.

Case (b) Volume distribution of charge.

(a) Hollow/solid conductor or hollow non-coductor :

Imagine a sphere passing through desired point (point where E is to be calculated), calculate charge inside it and assume it to be concentrated at centre and use point charge formula.



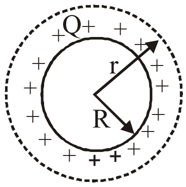
Inside sphere r < R

E = 0 (No charge inside imagined sphere)

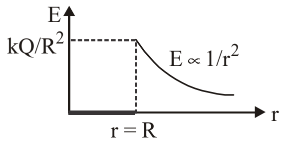
Outside r > R

$E = \dfrac{kQ}{r^2}$

Surface r = R , $E = \dfrac{kQ}{R^2}$



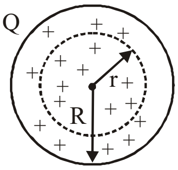
Graphically



Inside r < R : Uniform volume distribution :

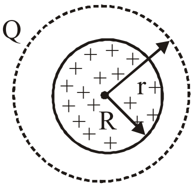
Charge inside volume

$\dfrac{4}{3}\pi r^3 \rightarrow \dfrac{Q}{\frac{4}{3}\pi R^3}\dfrac{4}{3}\pi r^3 = Q'$



or $Q' = \dfrac{Qr^3}{R^3}$; $E = \dfrac{kQ'}{r^2} = \dfrac{kQ}{R^3}r$

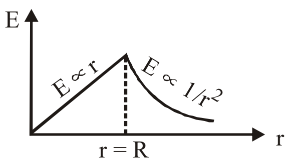
$\vec{E} = \dfrac{\rho}{3\varepsilon_0}\hat{r}$



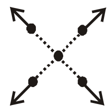
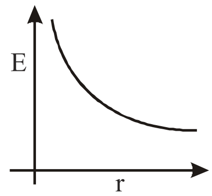
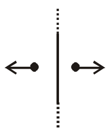
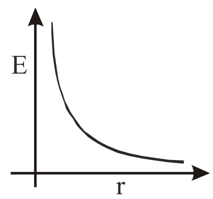
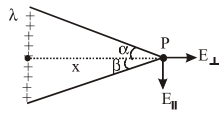
ρ = Volume charge density

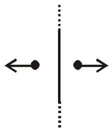
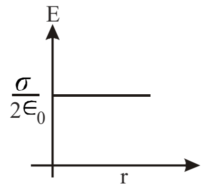
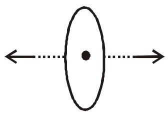
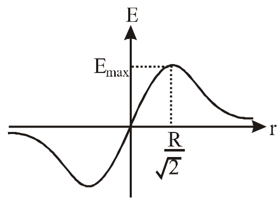
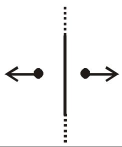
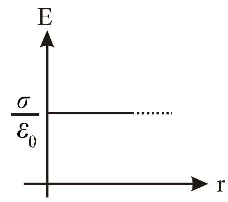
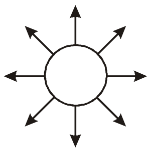
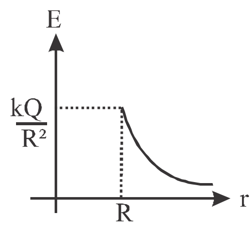
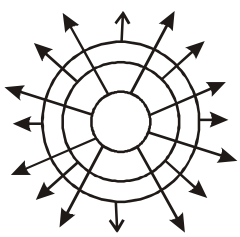
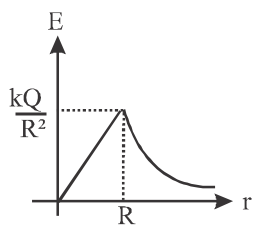
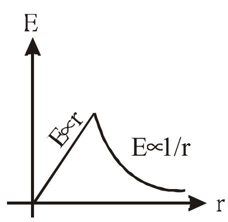
Outside, $E = \dfrac{kQ}{r^2}$; r > R ; Surface $E = \dfrac{kQ}{R^2}$,

Graphically



Electric field intensities due to various charge distributions are given in table.

Name/Type	Formula	Particular	Graph
Point charge 	$\dfrac{kq}{\lvert\vec{r}\rvert^2}.\hat{r} = \dfrac{kq}{r^3}\vec{r}$	q is source charge. $\vec{r}$ is vector drawn from source charge to the test point. Electric field is nonuniform, radially outwards due to + charges & inwards due to – charges.	
Infinitely long line charge 	$\dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \dfrac{2k\lambda\hat{r}}{r}$	λ is a linear charge density (assumed uniform) r is perpendicular distance of point from line charge. $\hat{r}$ is radial unit vector drawn from the charge to test point.	
Semi-infinite	$\dfrac{\sqrt{2}k\lambda}{r}$, $E_x = \dfrac{k\lambda}{r}, E_y = \dfrac{k\lambda}{r}$	At a point above the end of wire at an angle 45°	
Finite line of charge 	$E_x = \dfrac{k\lambda}{r}[\sin\beta + \sin\alpha]$ $E_y = \dfrac{k\lambda}{r}[\cos\alpha - \cos\beta]$ If $\alpha = \beta$ $E_\parallel = 0, E_\perp = \dfrac{\lambda}{2\pi\varepsilon_0 r}$	Where λ is the linear charge density	

Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	σ is surface charge density (assumed uniform) $\hat{n}$ is unit normal vector. Electric field intensity is independent of distance.	
Uniformly charged ring 	$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$ $E_{centre} = 0$	Q is total charge of the ring. x = distance of point on the axis from centre of the ring. Electric field is always along the axis. Maximum at $x = R/\sqrt{2}$	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	σ is the surface charge. $\hat{n}$ is unit normal vector perpendicular to the surface. Electric field intensity is independent of distance.	
Uniformly charged hollow conducting/ nonconducting/ solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r < R$ $\vec{E} = 0$	R is radius of the sphere. $\vec{r}$ is a vector drawn from centre of sphere to the point. Sphere acts like a point charge, placed at centre for points outside the sphere. $\vec{E}$ is always along radial direction. Q is total charge ($= \sigma 4\pi R^2$) (σ = surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{kQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$	$\vec{r}$ is a vector drawn from centre of sphere to the point. Sphere acts like a point charge, placed at centre for points outside the sphere. $\vec{E}$ is always along radial direction. Q is total charge $(\rho.\frac{4}{3}4\pi R^3)$ (ρ = volume charge density) Inside the sphere $E \propto r$ Outside the sphere $E \propto 1/r^2$	
Uniformly charged cylinder with a charge density ρ (R = radius of cylinder) Uniformly charged cylindrical shell with surface charge density σ is	for $r < R$, $E_{in} = \frac{\rho r}{2\varepsilon_0}$ for $r > R$, $E = \frac{\rho R^2}{2\varepsilon_0 r}$ for $r < R$, $E_{in} = 0$, for $r > R$, $E = \frac{\rho r}{\varepsilon_0 r}$		

Corona Discharge :

Dielectric strength of medium mean minimum field required for ionisation of a medium. If value of E increases above dielectric strength of medium, medium gets ionised and charge leak out into the medium from body generally it happen at the corner where E is high. This leakage process is called corona discharge.

For air dielectric strength = 3×10^6 v/m

The electric field near a high-voltage power line can be large enough to strip the electrons from air molecules, thus ionizing them and making the air a conductor. The glow resulting from the recombination of free electrons with the ions is an example of corona discharge. Break-down in air is witnessed during atmospheric lighting.

Example 14 :

If the nucleus of a hydrogen atom is considered to be a sphere of radius 10^{-15} m, then the electric field on its surface will be

(A) 14.4 V/m (B) 14.4×10^{11} V/m

(C) 14.4×10^{15} V/m (D) 14.4×10^{20} V/m

Sol. (D). $E = \dfrac{kQ}{r^2} = \dfrac{9 \times 10^9 \times 1.6 \times 10^{-19}}{(10^{-15})^2} = 14.4 \times 10^{20}$ V/m

Example 15 :

Total charge on a sphere of radius 10 cm is 1μC. The maximum electric field due to the sphere will be.

(A) 9×10^5 N/C (B) 9×10^{-5} N/C

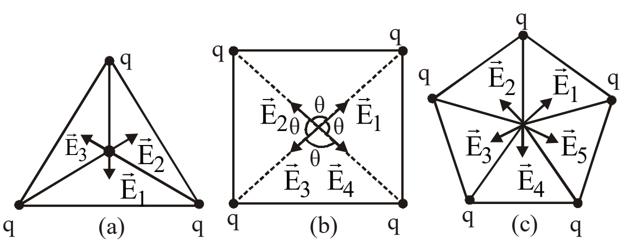
(C) 9×10^3 N/C (D) 9×10^{-3} N/C

Sol. (A). The electric field due to a charged sphere is maximum at its surface. Thus

$$E_{max} = \frac{kq}{R^2} = \frac{9 \times 10^9 \times 10^{-6}}{100 \times 10^{-4}} = 9 \times 10^5 \text{ N/C}$$

Example 16 :

If a charge q is placed at each vertex of a regular polygon, then prove that net electric field at its centre is zero.



Sol. The distance of the centre of a regular polygon from each vertex will be same. Therefore,

$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$ in (a).

$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4|$ in (b)

$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4| = |\vec{E}_5|$ in (c)

The angle between any two consecutive field vectors for each of a polygon is also same.

Hence, $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n = \vec{0}$

where n = 3, 4, 5, 6........

Note: $\vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n = 0$

$\Rightarrow \quad \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_{n-1} = -\vec{E}_n$

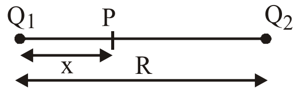
Hence, if a charge q is placed at each vertex except one vertex of regular polygon, then the net electric field at its centre, distant r from each vertex is $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r^2}$, directed towards or away from the empty vertex depending on whether q is positive or negative.

Example 17 :

Two positive charges Q_1 and Q_2 are placed on a line as shown in figure. Determine the position of point O, where the net electric field is zero.

Sol. Let position of P is at a distance x from Q_1. Then the fields at P due to Q_1 and Q_2 are in opposite directions. They will add up to give zero, only if their (electric field's) magnitude are equal. That is

$$\frac{kQ_1}{x^2} = \frac{kQ_2}{(R-x)^2}$$



$$\left(\frac{R-x}{x}\right) = \sqrt{\frac{Q_2}{Q_1}} \quad \text{or} \quad x = \frac{R}{1+\sqrt{Q_2/Q_1}}$$

The distance of point P from charge Q is

$$d = R - x = \frac{R}{1+\sqrt{Q_1/Q_2}}$$

If two negative charges are placed on a line (instead of positive charges), then the position of point P where the net electric field is zero, is again

$$x = R/\left\{1+\sqrt{Q_2/Q_1}\right\}, \; d = R/\left\{1+\sqrt{Q_1/Q_2}\right\}.$$

Example 18 :

Find out electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point $B(\sqrt{2}\text{m}, 0, 1\text{m})$.

Sol. $E = \dfrac{KQ}{|\vec{r}|^3}\vec{r} = \dfrac{KQ}{|\vec{r}|^2}\hat{r} \Rightarrow \vec{r} =$ P.V. of A – P.V. of B

(P.V. = position vector)

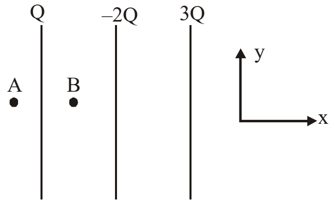
$= (-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$; $|\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$

$$E = \frac{9 \times 10^9 \times (-20 \times 10^{-6})}{8}(-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$$

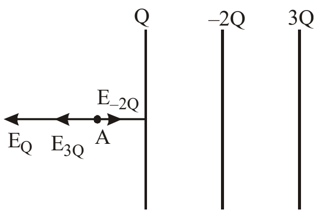
$$= -22.5 \times 10^3 \; (-\sqrt{2}\hat{i} + \hat{j} + \hat{k}) \text{ N/C}$$

Example 19 :

Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at points A and B.



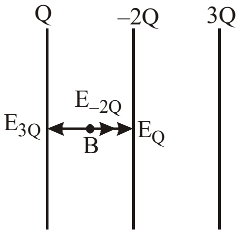
Sol. For point A :



$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q}$$

$$= -\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}j + \frac{2Q}{2A\varepsilon_0}i = -\frac{Q}{A\varepsilon_0}i$$

For point B :



$$\vec{E}_{net} = \vec{E}_{3Q} + \vec{E}_{-2Q} + \vec{E}_Q$$

$$= -\frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} + \frac{Q}{2A\varepsilon_0}i = 0$$

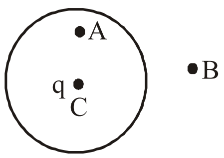
Example 20 :

Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following:

(i) Force on charge q

(ii) Electric field intensity at A.

(iii) Electric field intensity at B.



Sol. (i) Electric field at the centre of the uniformly charged hollow sphere = 0. So force on charge q = 0

(ii) Electric field at A,

$$\vec{E}_A = \vec{E}_{sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2}\ ; \ \ r = CA$$

E due to sphere = 0 ,
because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point B = $\vec{E}_{sphere} + \vec{E}_q$

$$= \frac{Kq}{r^2}\hat{r} + \frac{Kq}{r^2}\hat{r} = \frac{K\,(Q+q)}{r^2}.\hat{r}\ ;\ r = CB$$

Note: Here, we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Example 21 :

Calculate the electric field strength required to just support a water drop of mass 10^{-7} kg & having charge 1.6×10^{-19} C.

Sol. Here, $m = 10^{-7}$ kg, $q = 1.6 \times 10^{-19}$ C

Step 1 : Let E be the electric field strength required to support the water drop.

Force acting on the water drop due to electric field E is

$F = qE = 1.6 \times 10^{-19}$ E

Weight of drop acting downward,

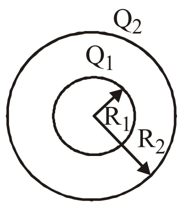
$W = mg = 10^{-7} \times 9.8$ newton.

Step 2 : Drop will be supported if F and W are equal and opposite. i.e., $1.6 \times 10^{-19}\,E = 9.8 \times 10^{-7}$

or $E = \dfrac{9.8\times10^{-7}}{1.6\times10^{-19}} = 6.125 \times 10^{12}\ N\,C^{-1}$.

Example 22 :

Two concentric uniformly charged spherical shells of radius R_1 & R_2 ($R_2 > R_1$) have total charges Q_1 & Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$

Sol. (i) For $r < R_1$,

therefore, point lies inside both the spheres

$E_{net} = E_{inner} + E_{outer} = 0 + 0$

(ii) For $R_1 \le r < R_2$

Point lies outside inner sphere but inside outer sphere:

$$\therefore\ \ E_{net} = E_{inner} + E_{outer} = \frac{KQ_1}{r^2}\hat{r} + 0 = \frac{KQ_1}{r^2}\hat{r}$$

(iii) For $r \ge R_2$,

Point lies outside inner as well as outer sphere.

Therefore, $E_{net} = E_{inner} + E_{outer}$

$$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K\,(Q_1+Q_2)}{r^2}\hat{r}$$

ELECTROSTATICS + MECHANICS CONCEPTS

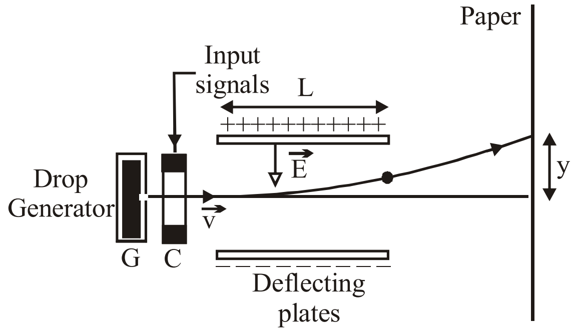
1. Motion of charged particle in uniform electric field

When we place a charge q in an E-field $\vec{E}$, the force experienced by the charge is

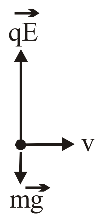
$$\vec{F} = q\vec{E} = m\vec{a}$$

Applications: Ink-jet printer, TV cathode ray tube.

Ink particle has mass m, charge q (q < 0 here). Assume that mass of inkdrop is small, what's the deflection y of the charge?



First, the charge carried by the inkdrop is negative, i.e. q<0. Note: $q\vec{E}$ points in opposite direction of $\vec{E}$.



Horizontal motion: Net force = 0

$\therefore$ L = vt .... (1)

Vertical motion: $|q\vec{E}| >> |m\vec{g}|$, q is negative,

$\therefore$ Net force = – qE = ma (Newton's 2nd Law)

$\therefore \quad a = -\dfrac{qE}{m}$ .... (2)

Vertical distance travelled: $y = \dfrac{1}{2}at^2$

2. Oscillation of charged particle in presence of electric force: In case of any oscillation first find restoring force if $F \propto -x$ $(F = -kx)$ then given oscillation called SHM and time period of oscillation equals $T = 2\pi\sqrt{\dfrac{m}{k}}$

If F is constant use kinematic equation to find time period. In all other cases use calculus approach.

Example 23 :

An electron moving in a horizontal direction with a speed of 5.0×10^7 m/sec enters a region where there is a uniform electrical field of 2000 V/m directed upwards in the plane of its motion. Find the electron's co-ordinates referred to the point of entry and the direction of its motion 4×10^{-8} second later.

Sol. $y = \dfrac{1}{2}\left(\dfrac{qE}{m}\right)t^2$, $e = q = 1.6 \times 10^{-19}$

$\therefore \quad y = \dfrac{1}{2}\left(\dfrac{2000 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right)(4 \times 10^{-8})^2 = 0.28\text{m}$

Along X-axis $x = v \times t = 5 \times 10^7 \times 4 \times 10^{-8} = 2.0\text{m}$

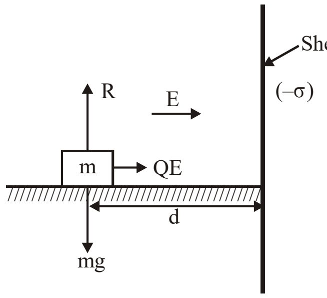
$\tan\theta = \dfrac{y}{x} = \dfrac{0.28}{2.0} = 0.14$

$\theta = 8°$ w.r.t. horizontal.

Example 24 :

A block having mass m and charge Q is resting on a frictionless plane at a distance d from fixed large non-conducting infinite sheet of uniform charge density $-\sigma$ as shown in Figure. Assuming that collision of the block with the sheet is perfectly elastic, find the time period of oscillatory motion of the block. Is it SHM?

Sol. The situation is shown in Figure.



Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to the sheet is uniform $a = \dfrac{F}{m} = \dfrac{QE}{m}$, where $E = \sigma/2\varepsilon_0$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$$d = \frac{1}{2}at^2 \text{ i.e., } t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2md}{QE}} = \sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance d in same time t. After stopping, it will again be accelerated towards the wall and so the block will execute oscillatory motion with 'span' d and time period.

$$T = 2t = 2\sqrt{\frac{2md}{QE}} = 2\sqrt{\frac{4md\varepsilon_0}{Q\sigma}}$$

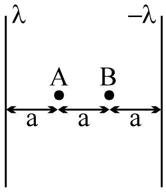
However, as the restoring force F = QE is constant and not proportional to displacement x, the motion is not simple harmonic.

TRY IT YOURSELF - 2

Q.1 A test charge of +3μC is at a point P where an external electric field is directed to the right and has a magnitude of 4×10^6 N/C. If the test charge is replaced with another test charge of –3μC, the external electric field at P (a) is unaffected (b) reverses direction (c) changes in a way that cannot be determined.

Q 2 Given that an electric field of 10 N/C has a northward direction and another electric field of 10 N/C has an eastward direction, what are the magnitude and direction of the superposition of these two electric fields?

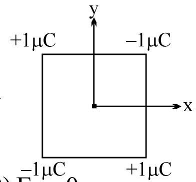
Q.3 Between two infinitely long wires having linear charge densities λ and $-\lambda$ there are two points A and B as shown in the figure. The amount of work done by the electric field in moving a point charge q_0 from A to B is equal to



(A) $\frac{\lambda q_0}{2\pi\varepsilon_0}\ln 2$ (B) $-\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$

(C) $\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$ (D) $\frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$

Q 4 Point charges are located on the corner of a square as shown. Find the components of electric field at any point on the z-axis which is axis of symmetry of the square



(A) $E_z = 0$ (B) $E_x = 0$

(C) $E_y = 0$ (D) none of these

Q.5 Four equal positive charges are located at the corners of a square.What is the magnitude of the electric field at the center of the square?

Q.6 A proton sits at coordinates (x,y) = (0, 0) and an electron at (h,d), where d>>h. At time t=0, a uniform electric field E of unknown magnitude but pointing in the positive y direction is turned on. Assuming that d is large enough that the proton-electron interaction is negligible, the y coordinates of the two particles will be equal (at equal time)

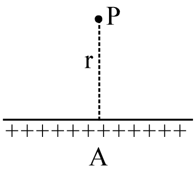
(A) at about y = d/2000

(B) at an undetermined value since E is unknown

(C) at about y = d/43

(D) nowhere: they move in opposite directions

Q.7 Electric field, due to an infinite line of charge, as shown in figure at a point P at a distance r from the line is E. If one half of the line of charge is removed from either side of point A, then–



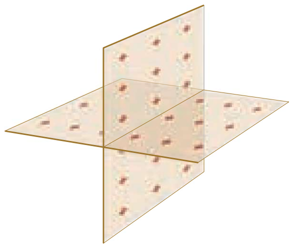
(A) electric field at P will have magnitude E/2

(B) electric field at P in x direction will be E/2

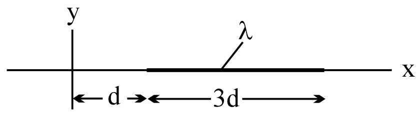
(C) electric field at P in y-direction will be E/2

(D) none of these

Q.8 Two infinitely large flat sheets with equal uniform positive charge distributions intersect at right angles (see Fig.).What is the direction of the electric field in each quadrant?



Q.9 A continuous line of charge of length 3d lies along the x-axis, extending from x = d to x = + 4d. the line carries a uniform linear charge density λ.



In terms of d, λ and any necessary physical constants, find the magnitude of the electric field at the origin.

(A) $\lambda / 5\pi\varepsilon_0 d$ (B) $\lambda / 4\pi\varepsilon_0 d$

(C) $3\lambda / 16\pi\varepsilon_0 d$ (D) $3\lambda / 8\pi\varepsilon_0 d$

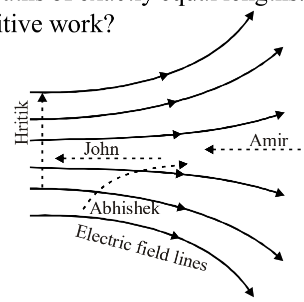
Q.10 The electric field at a distance of 1 m from a uniformly charged infinite sheet has the value E = E_0. What is the value of E at a distance of 2m? At 4 m?

Q.11 The maximum electric field at a point on the axis of a uniformly charged ring is E_0. At how many points on the axis will the magnitude of electric field be $E_0/2$

(A) 1 (B) 2

(C) 3 (D) 4

Q.12 Abhishek, Hritik, John, and Amir are assigned the tasks of moving equal positive charges slowly through an electric field, along assigned path (shown as dotted line). In each case the charge is at rest at the beginning. They all have paths of exactly equal lengths. Who must do the most positive work?



(A) Abhishek (B) Hritik

(C) Amir (D) John

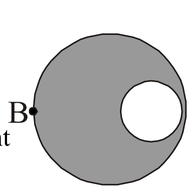
Q.13 Two point charges (Q each) are placed at (0, y) and (0, –y). A point charge q of the same polarity can move along x-axis. Then :

(A) the force on q is maximum at $x = \pm y/\sqrt{2}$
(B) the charge q is in equilibrium at the origin
(C) the charge q performs an oscillatory motion about the origin
(D) for any position of q other then origin the force is directed away from origin

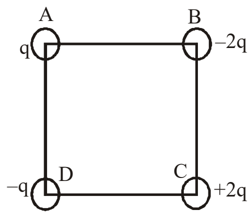
Q.14 A positively charged sphere of radius r_0 carries a volume charge density ρ_E (Figure). A spherical cavity of radius $r_0/2$ is then scooped out and left empty, as shown. What is the direction and magnitude of the electric field at point B?

(A) $\frac{17\rho r_0}{54 \in_0}$ left (B) $\frac{\rho r_0}{6 \in_0}$ left

(C) $\frac{17\rho r_0}{54 \in_0}$ right (D) $\frac{\rho r_0}{6 \in_0}$ right



Q.15 Four charges are placed on corners of a square as shown in figure having side 10cm. If q is 1μC then what will be electric field intensity at the centre of the square ?



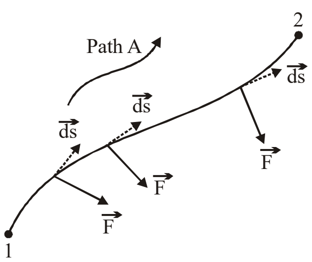
ANSWERS

(1) (a)	**(2)** 14 N/C,45°	**(3)** (D)
(4) (ABC)	**(5)** zero	**(6)** (A)
(7) (BC)	**(8)** 45°	**(9)** (C)
(10) $E = E_0$	**(11)** (D)	**(12)** (D)
(13) (ABD)	**(14)** (A)	**(15)** 2.54×10^6 N/C

CONCEPT OF ELECTRIC POTENTIAL

Potential Energy and Conservative Forces:

Electric force is a conservative force. Work done by the electric force $\vec{F}$ as a charge moves an infinitesimal distance $d\vec{s}$ along Path A = dW



$$dW = \vec{F} \,.\, d\vec{s}$$

Note: $d\vec{s}$ is in the tangent direction of the curve of Path A.

Total work done W by force $\vec{F}$ in moving the particle from Point 1 to Point 2

$$W = \int_{1\ \text{Path A}}^{2} \vec{F} \,.\, d\vec{s}$$

$\int_{1\ \text{Path A}}^{2}$ = Path Integral = Integration over Path A from Point 1 to Point 2.

* A force is conservative if the work done on a particle by the force is independent of the path taken.
* The work done by a conservative force on a particle when it moves around a closed path returning to its initial position is zero.
* Since the work done by a conservative force $\vec{F}$ is path-independent, we can define a quantity, potential energy, that depends only on the position of the particle.
* We define potential energy U such that

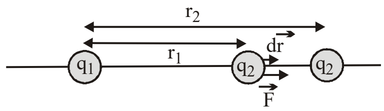
$$dU = -W = -\int \vec{F} \,.\, d\vec{s}$$

For particle moving from 1 to 2 :

$$\int_1^2 dU = U_2 - U_1 = -\int_1^2 \vec{F} \,.\, d\vec{s} \,,$$

where U_1, U_2 are potential energy at position 1, 2.

Suppose charge q_2 moves from point 1 to 2.



From definition :

$$U_2 - U_1 = -\int_1^2 \vec{F} \,.\, d\vec{r} = -\int_{r_1}^{r_2} F \,.\, dr \quad (\because \vec{F} \,||\, d\vec{r})$$

$$= -\int_{r_1}^{r_2} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} dr \quad \left(\int \frac{dr}{r^2} = -\frac{1}{r} + C\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_2}{r}\right]_{r_1}^{r_2} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$-\Delta W = \Delta U = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Note :

* If q_2 moves away from q_1, then $r_2 > r_1$, we have
  - If q_1, q_2 are of same sign, then $\Delta U < 0$, $\Delta W > 0$ (ΔW = Work done by electric repulsive force)
  - If q_1, q_2 are of different sign, then $\Delta U > 0$, $\Delta W < 0$ (ΔW = Work done by electric attractive force)
* If q_2 moves towards q_1, then $r_2 < r_1$, we have
  - If q_1, q_2 are of same sign, then $\Delta U > 0$, $\Delta W < 0$
  - If q_1, q_2 are of different sign, then $\Delta U < 0$, $\Delta W > 0$

* It is the difference in potential energy that is important.

Reference point :

$$U(r=\infty)=0 \quad U_\infty - U_1 = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{\underset{\infty}{\underset{\downarrow}{r_2}}} - \frac{1}{r_1}\right)$$

$$\boxed{U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

If q_1, q_2 same sign, then $U(r) > 0$ for all r
If q_1, q_2 opposite sign, then $U(r) < 0$ for all r

* Conservation of Mechanical Energy:
For a system of charges with no external force,

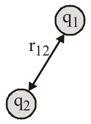
$$E = \underset{\text{(Kinetic Energy)}}{K} + \underset{\text{(Potential Energy)}}{U} = \text{Constant}$$

or $\Delta E = \Delta K + \Delta U = 0$

Potential energy of a system of charges

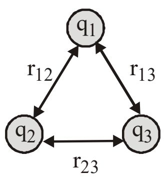
P.E. of 3 charges q_1, q_2, q_3
Start: q_1, q_2, q_3 all at $r=\infty; U=0$

Step1: q_1 Move q_1 from ∞ to its position
$\Rightarrow \quad U = 0$

Step2: 

Move q_2 from ∞ to new position

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Step3: 

Move q_3 from ∞ to new position $\Rightarrow$Total P.E.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

or $\quad U = U_{12} + U_{13} + U_{23}$

This can be generalized for any system containing n point charges q_1, q_2, q_3, ... q_n. The potential energy of the system will be written as

$$U = (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + ... + U_{2n}) + (U_{34} + ... + U_{3n}) + ... + U_{(n-1)n}$$

or $\quad U = \sum_{i<j} U_{ij} = \sum_{i<j} \frac{1}{4\pi\varepsilon_0}\frac{q_i\, q_j}{r_{ij}}$

Thus, the potential energy of a system of charges is the sum of the potential energies of all possible pairs of charges that can be formed by taking two charges from the system.

Electric potential : Consider a charge q at center, we consider its effect on test charge q_0.

Definition : We define electric potential V so that

$$\Delta V = \frac{\Delta U}{q_0} = \frac{-\Delta W}{q_0}$$

($\because$ V is the P.E. per unit charge)

* We take $V(r=\infty) = 0$.
* Electric Potential is a scalar.
* Unit: Volt (V) = Joules/Coulomb
* For a single point charge:

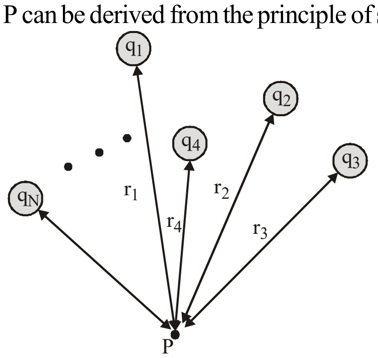
$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

* Energy Unit: $\Delta U = q\,\Delta V$

$$\text{electron – Volt (eV)} = \underbrace{1.6\times10^{-19}}_{\text{charge of electron}}\ \text{J}$$

Potential for a system of charges

For a total of N point charges, the potential V at any point P can be derived from the principle of superposition.



Recall that potential due to q_1 at point P:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

$\therefore$ Total potential at point P due to N charges:

$$V = V_1 + V_2 + ... + V_N$$

(principle of superposition)

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + ... + \frac{q_N}{r_N}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$$

NOTE

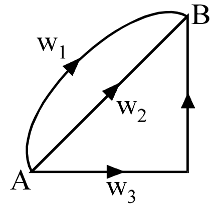
* For $\vec{E}, \vec{F}$, we have a sum of vectors.
For V, U, we have a sum of scalars.

POTENTIAL DIFFERENCE

(i) Definition : $V_{ab} = -\int_a^b \vec{E}.d\vec{r}$

(ii) Its value does not depend on the frame of reference, hence it is an absolute quantity.

(iii) As the electric field is conservative, work done and hence potential difference between to points is path independent and depends only on the position of points.



$W_1 = W_2 = W_3$.

(iv) Potential is theoretically zero at infinite & practically zero at earth's surface.

Relation Between Electric Field E and Electric Potential V :

(A) To get V from $\vec{E}$:

Recall our definition of the potential V:

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W_{12}}{q_0}$$

where ΔU is the change in P.E.; W_{12} is the work done in bringing charge q_0 from point 1 to 2.

$$\therefore \quad \Delta V = V_2 - V_1 = \frac{-\int_1^2 \vec{F}.d\vec{s}}{q_0}$$

However, the definition of E-field: $\vec{F} = q_0\vec{E}$

$$\Delta V = V_2 - V_1 = --\int_1^2 \vec{E}.d\vec{s}$$

Note: The integral on the right hand side of the above can be calculated along any path from point 1 to 2. (Path-Independent)

Convention: $V_\infty = 0 \Rightarrow V_P = -\int_\infty^P \vec{E}.d\vec{s}$

(B) To get $\vec{E}$ from V :

Again, use the definition of V :

$$\Delta U = q_0\Delta V = \underbrace{-W}_{\text{Work done}}$$

However, $W = \underbrace{q_0\vec{E}}_{\text{Electric force}}.\Delta\vec{s}$

$= q_0 E_s \Delta s$

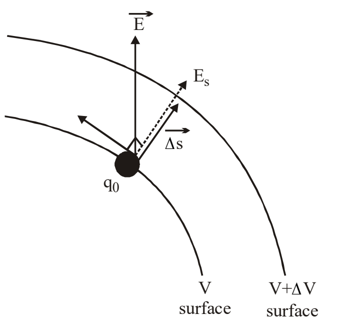
where Es is the E-field component along the path $\Delta\vec{s}$

$\therefore q_0\Delta V = -q_0E_s \Delta s$

$$\therefore \ E_s = -\frac{\Delta V}{\Delta s}$$

For infinitesimal Δs,

$$\therefore \ E_s = -\frac{dV}{ds}$$



(i.e. Potential = V on the surface)

Note:

(1) Therefore the E-field component along any direction is the negative derivative of the potential along the same direction. i.e. in direction of E-field potential decreases.

(2) If $d\vec{s} \perp \vec{E}$, then $\Delta V = 0$

(3) ΔV is biggest/smallest if $d\vec{s} \,||\, \vec{E}$

Generally, for a potential V (x; y; z), the relation between $\vec{E}\,(x, y, z)$ and V is

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ are partial derivatives

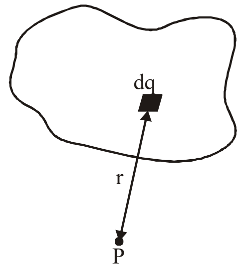
For $\frac{\partial V}{\partial x}$ (x, y, z), everything y, z are treated like a constant and we only take derivative with respect to x.

Electric potential of continuous charge distribution

For any charge distribution, we write the electrical potential dV due to infinitesimal charge dq:

$$dV = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r}$$

$$\therefore \ V = \int_{\substack{\text{charge} \\ \text{distribution}}} \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r}$$



Similar to the previous cases on E-field, for the case of uniform charge distribution:

1-D $\Rightarrow$ long rod $\Rightarrow$ $dq = \lambda\, dx$

2-D $\Rightarrow$ charge sheet $\Rightarrow$ $dq = \sigma\, dA$

3-D $\Rightarrow$ uniformly charged body $\Rightarrow$ $dq = \rho\, dV$

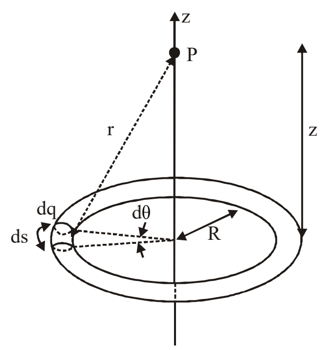
Let us consider some cases :

Case 1 : Uniformly-charged ring

Length of the infinitesimal ring element = $ds = Rd\theta$

$\therefore$ charge $dq = \lambda\, ds = \lambda\, R\, d\theta$

$$dV = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda R d\theta}{\sqrt{R^2 + z^2}}$$



Charge density λ

Total charge on the ring = $\lambda \cdot (2\pi R)$

After integration,

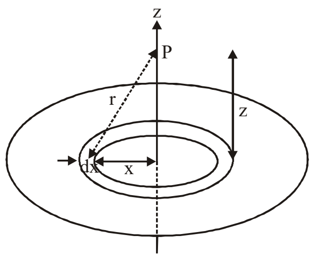
$$\boxed{V = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}}$$

Limiting case : $z >> R$

$$\Rightarrow V = \frac{Q}{4\pi\varepsilon_0\sqrt{z^2}} = \frac{Q}{4\pi\varepsilon_0 |z|}$$

Case 2 : Uniformly charged disk

Using the principle of superposition, we will find the potential of a disk of uniform charge density by integrating the potential of concentric rings.



Total charge = Q
Charge density = σ

$$\therefore \quad dV = \frac{1}{4\pi\varepsilon_0}\int_{disk}\frac{dq}{r}$$

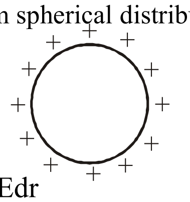
After integration, $V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2+R^2} - |z|)$

ELECTRIC POTENTIAL DUE TO SPHERICAL CHARGE DISTRIBUTION

Electric Potential Due to a Shell :

A shell of radius R has a charge Q uniformly distributed over its surface.

(a) At points outside a uniform spherical distribution, the electric field is



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r},$$

since $\vec{E}$ is radial, $\vec{E}.d\vec{r} = Edr$

since V (∞) = 0, we have

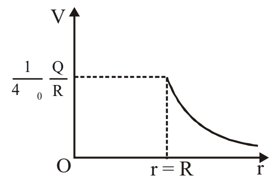
$$V(\infty) - V(r) = -\int \vec{E}.d\vec{r} \;;$$

$$0 - V = -\int_r^{\alpha}\frac{Q}{4\pi\varepsilon_0 r^2}dr \;\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

We see that the potential due to a uniformly charged shell is the same as that due to a point charge Q at the centre of the shell.

(b) **At an internal Point :** At points inside the shell, $E = 0$. So, the work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$



Variation of electric potential with the distance from the centre (r).

All the above results hold for a conducting sphere also whose charge lies entirely on the outer surface.

Electric potential due to a non-conducting charged sphere :

A charge Q is uniformly distributed throughout a non-conducting spherical volume of radius R.

Let us find expressions for the potential at an

(a) external point ($r > R$); (b) internal point ($r < R$) where r is the distance of the point from the centre of the sphere.

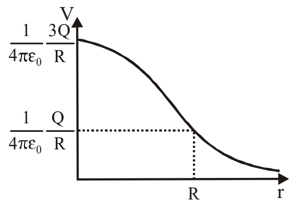
(a) **At an external point :**

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

(b) **Potential at an internal point :**

$$\boxed{V = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}}$$

But $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

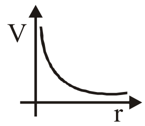
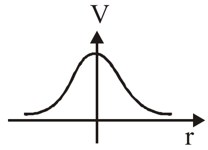
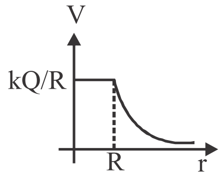
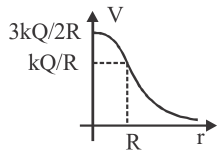


$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}[3R^2 - r^2]$$

Alternatively expression can be derived using

$$\int dV = -\int E\,dr \text{ and } E = \frac{\rho r}{3\varepsilon_0}$$

Table : Electric potential due to various charge distributions :

Name/type	Formula	Note	Graph
Point charge	$\dfrac{kq}{r}$	* q is source charge. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre : $\dfrac{kQ}{R}$ at the axis : $\dfrac{kQ}{\sqrt{R^2+x^2}}$	* Q is source charge. * x is the distance of the point on the axis from centre of ring.	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	For $r \geq R$, $V=\dfrac{kQ}{r}$ for $r \leq R$, $V=\dfrac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge = $\sigma 4\pi R^2$	
Uniformly charged solid nonconducting sphere.	For $r \geq R$, $V=\dfrac{kQ}{r}$ for $r \leq R$, $V=\dfrac{kQ\,(3R^2-r^2)}{2R^3}$ $=\dfrac{\rho}{6\varepsilon_0}(3R^2-r^2)$	* R is radius of sphere * r is distance from centre to the point * $V_{centre}=\dfrac{3}{2}V_{surface}$ * Q is total charge = $\rho\dfrac{4}{3}\pi R^3$ * Inside the sphere potential varies parabolically * Outside the sphere potential varies hyperbolically.	
Infinite line charge	* Absolute potential is not defined.	* Potential difference between two points is given by formula : $V_B - V_A = -2k\lambda \ln (r_B/r_A)$	
Infinite nonconducting thin sheet	* Absolute potential is not defined.	* Potential difference between two points is given by formula : $V_B - V_A = -\dfrac{\sigma}{\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	* Absolute potential is not defined.	* Potential difference between two points is given by formula : $V_B - V_A = -\dfrac{\sigma}{\varepsilon_0}(r_B - r_A)$	

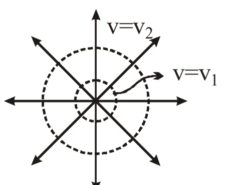
EQUIPOTENTIAL SURFACE

Equipotential surfaces are defined as surfaces over which the potential is constant

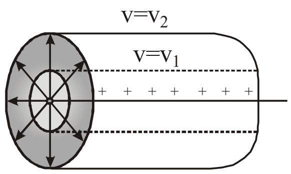
$$V(\vec{r}) = \text{constant}$$

At each point on the surface, the electric field is perpendicular to the surface since the electric field, being the gradient of potential, does not have component along a surface of constant potential.

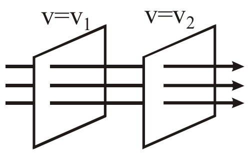
* Any charge on a conductor must reside on its surface. These charges would move along the surface if there were a tangential component of the electric field. The electric field must therefore be along the normal to the surface of a conductor. The conductor surface is, therefore, an equipotential surface.
* Electric field lines are perpendicular to equipotential surfaces (or curves) and point in the direction from higher potential to lower potential.
* In the region where the electric field is strong, the equipotentials are closely packed as the gradient is large.
* Work done in the moving a charge between any two points on an equipotential surface is zero.
* Equipotential surfaces can never cross each other because there will be two normals at the point of intersection giving two different directions of electric field which is absurd.
* For a point charge the equipotential surface is spherical. For a line charge equipotential surface is cylindrical and for uniform field the equipotential surface is planar.



Concentric spherical EPS for a point charge



Co-axial cylindrical EPS for a line charge



Coplanar plane EPS for uniform field

Properties of Equipotential surface

(i) Potential difference between two points in an equipotential surface is zero.

(ii) If a test charge q_0 is moved from one point to the other on such a surface, the electric potential energy q_0V remains constant.

(iii) No work is done by the electric force when the test charge is moved along this surface.

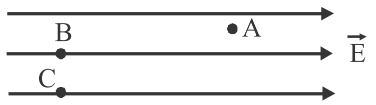
(iv) Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not acceptable.

(v) Field lines and equipotential surfaces are always mutually perpendicular.

(vi) Charged conductors is always equipotential.

NOTE :

A, B and C are three points in a uniform electric field then $V_B = V_C$; $V_A < V_B$.



Line perpendicular to electric field will be equipotential and in the direction of electric field potential decreases.

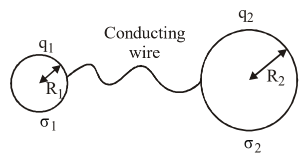
Connected conducting spheres

Two conductors connected can be seen as a single conductor.

$\therefore$ Potential everywhere is identical.

Potential of radius R_1 sphere $V_1 = \dfrac{q_1}{4\pi\varepsilon_0 R_1}$

Potential of radius R_2 sphere $V_2 = \dfrac{q_2}{4\pi\varepsilon_0 R_2}$



$$V_1 = V_2 \Rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

Surface charge density

$$\sigma_1 = \frac{q_1}{\underbrace{4\pi R_1^2}_{\text{Surface area of radius } R_1 \text{ sphere}}} \qquad \therefore \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2} \cdot \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}$$

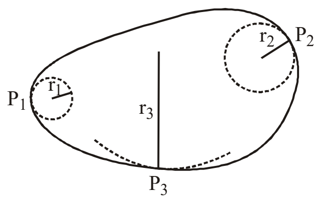
$\therefore$ If $R_1 < R_2$ then $\sigma_1 > \sigma_2$

And the surface electric field $E_1 > E_2$

For arbitrary shape conductor:

At every point on the conductor, we fit a circle.

The radius of this circle is the radius of curvature.



$E_3 < E_2 < E_1$

Note: Charge distribution on a conductor does not have to be uniform.

Example 25 :

At the mid point of a line joining an electron and a proton, the values of E and V will be.

(A) $E = 0, V \neq 0$ (B) $E \neq 0, V = 0$

(C) $E \neq 0, V \neq 0$ (D) $E = 0, V = 0$

Sol. **(B).** $E = \frac{Kq^2}{x^2} + \frac{Kq^2}{x^2} = \frac{2Kq^2}{x^2}$ towards electron

$V = \frac{Kq}{x} - \frac{Kq}{x} = 0 \quad \therefore$ At mid point, $E \neq 0, V = 0$

Example 26 :

A charge $2\mu C$ is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is $-40\mu J$, then find the potential at that point.

Sol. $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20V$

Example 27 :

In a region the electric field intensity E is given by $E = 100/x^2$ where x in metre. The potential difference between the points at x = 10 m and x = 20 m will be.

(A) 1V (B) 2V (C) 5 V (D) 10 V

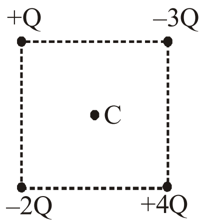
Sol. **(C).** $\vec{E} = \frac{100}{x^2}\hat{x}$; $V_B - V_A = \int_{10}^{20} \vec{E}.d\vec{a} = -\int_{10}^{20} \frac{100}{x^2} dx$

$= +100\left[\frac{1}{x}\right]_{10}^{20} = 100\left[\frac{1}{20} - \frac{1}{10}\right] = -5V$

$\therefore \quad |V_B - V_A| = 5V$

Example 28 :

Four point charges are placed at the corners of a square of side ℓ. Calculate potential at the centre of square.



Sol. V = 0 at C, [Use V = Kq/r]

Example 29 :

Determine the electric field strength vector if the potential of this field depends on x, y coordinates as

(a) $V = a(x^2 - y^2)$ and (b) $V = a\,x\,y$

Sol. (a) $V = a(x^2 - y^2)$

Hence, $E_x = -\frac{\partial V}{\partial x} = -2ax$, $E_y = -\frac{\partial V}{\partial y} = +2ay$

$\therefore \quad E = -2ax\,\hat{i} + 2ay\,\hat{j}$ or $E = -2a(x\,\hat{i} - y\hat{j})$

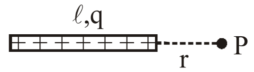
(b) $V = a\,x\,y$

Hence, $E_x = -\frac{\partial V}{\partial x} = -ay$, $E_y = -\frac{\partial V}{\partial y} = -ax$

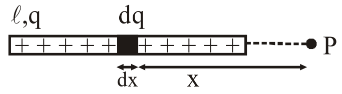
$\therefore \quad E = -ay\hat{i} - ax\hat{j} = -a[y\hat{i} + x\hat{j}]$

Example 30 :

A rod of length ℓ is uniformly charged with charge q. Calculate potential at point P.



Sol. Take a small element of length dx, at a distance x from left end. Potential due to this small element.



$dV = \frac{K\,(dq)}{x} \quad \therefore$ Total potential, $V = \int_{x=0}^{x=\ell} \frac{kdq}{x}$

where, $dq = \frac{q}{\ell}dx$

$\Rightarrow V = \int_{x=r}^{x=r+\ell} \frac{K\left(\frac{q}{\ell}dx\right)}{x} = \frac{Kq}{\ell}\log_e\left(\frac{\ell + r}{r}\right)$

Example 31 :

Eight charged water droplets, each with a radius of 1mm and a charge of 10^{-10} C, coalesce to form a single drop. Calculate the potential of the bigger drop.

Sol. Let R and r be the radii of the bigger drop and one droplet respectively.

Volume of 8 droplets = Volume of bigger drop

$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $R = (8)^{1/3} r = 2r$

or $R = 2 \times 1 \times 10^{-3} m = 2 \times 10^{-3} m$ $[\because r = 1\text{ mm}]$

Charge on bigger drop, $Q = 8 \times 10^{-10} C$

Potential of the bigger drop

$= \frac{1}{4\pi \in_0}\frac{Q}{R} = 9 \times 10^9 \frac{8 \times 10^{-10}}{2 \times 10^{-3}}$ volt = 3600 volt

Example 32 :

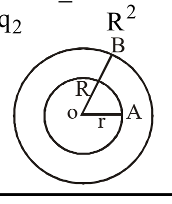
A charge Q is distributed over two concentric hollow spheres of radii r and R (> r) such that the surface densities are the same. Calculate the potential at the common centre of the two spheres.

Sol. If q_1 and q_2 are the charges on two spheres of radii r and R respectively, then the surface charge density is given

by $\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ or $\frac{q_1}{q_2} = \frac{r^2}{R^2}$

or $\frac{q_1}{q_2} + 1 = \frac{r^2}{R^2} + 1$ or $\frac{q_1 + q_2}{q_2} = \frac{r^2 + R^2}{R^2}$

But $q_1 + q_2$ = total charge Q



$\therefore \quad \frac{Q}{q_2} = \frac{r^2 + R^2}{R^2}$ or $\frac{QR}{q_2} = \frac{r^2 + R^2}{R}$

or $\frac{q_2}{R} = \frac{QR}{r^2 + R^2}$. Similarly, $\frac{q_1}{r} = \frac{Qr}{r^2 + R^2}$

Potential at the common centre

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Qr}{r^2 + R^2} + \frac{QR}{r^2 + R^2}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r + R}{r^2 + R^2}\right)$$

Alternative method : Let σ be the surface density of charge on both hollow spheres. Then charges on spheres will be

$$q_1 = 4\pi r^2\sigma \text{ and } q_2 = 4\pi R^2\sigma$$

$$Q = 4\pi r^2\sigma + 4\pi R^2\sigma = 4\pi\sigma\,(r^2 + R^2)$$

Potential at common centre = Potential due to first sphere + Potential due to second sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R} + \frac{1}{4\pi\varepsilon_0}\frac{4\pi r^2\sigma}{R}$$

$$V = \frac{1}{4\pi\varepsilon_0}4\pi\sigma\,(R + r)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2 + R^2}\right)(R + r) = \frac{Q}{4\pi\varepsilon_0}\frac{R + r}{R^2 + r^2}$$

Example 33 :

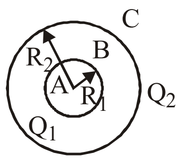
Two concentric spherical shells of radius R_1 and R_2 (R_2>R_1) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential

(i) at point A

(ii) at surface of smaller shell (i.e., at point B)

(iii) at surface of larger shell (i.e., at point C)

(iv) at $r \le R_1$ (v) at $R_1 \le r \le R_2$ (vi) at $r \ge R_2$



Sol. Using the result of hollow sphere,

(i) $V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$ (ii) $V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(iii) $V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$ (iv) at $\le R_1$, $V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

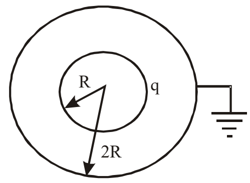
(v) at $R_1 \le r \le R_2$; $V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$

(vi) For $r \ge R_2$; $V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

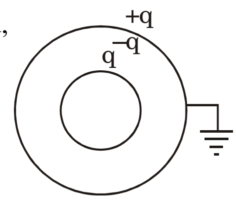
Example 34 :

A charge q is distributed uniformly on the surface of a sphere of radius R. It is covered by a concentric hollow conducting sphere of radius 2R. Find the charges on inner and outer surfaces of hollow sphere if it is earthed.

Sol. The charge on the inner surface should be –q, because if we draw a closed Gaussian surface through the material of the hollow sphere the total charge enclosed by this Gaussian surface should be zero. Let q' be the charge on the outer surface of the hollow sphere.



Since, the outer sphere is earthed, its potential should be zero. The potential on it is due to the charges q, –q and q', Hence,



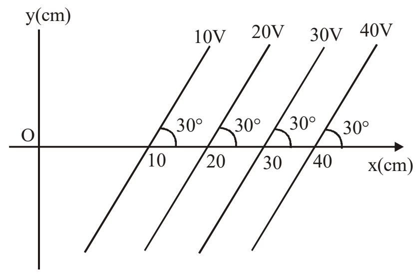
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} - \frac{q}{2R} + \frac{q'}{2R}\right] = 0$$

$\therefore$ q' = 0.

Therefore, there will be no charge on the outer surface of the hollow sphere.

Example 35 :

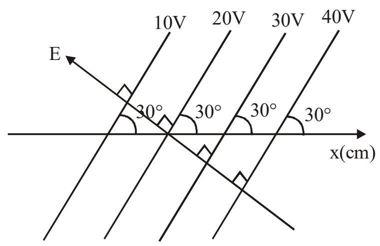
Some equipotential surfaces are shown in figure. What can you say about the magnitude and direction of electric field ?



Sol. (i) We know the direction of electric field is $\perp$ to the equipotential surfaces.

(ii) In the direction of electric field potential decrease.

Keeping these 2 things in mind we get direction of electric field is 120° from x-axis i.e. as shown in figure.



As we know, that, $|E| = \frac{dV}{dx}$ or for uniform electric field $E = \frac{\Delta V}{\Delta x}$; ΔV = potential difference

$\Delta x = \perp$ distance between equipotential surfaces

$$= \frac{20 - 10}{10\sin 30°} = \frac{10}{5} = 2 \text{ V/m}$$

Example 36 :

In nuclear fission, a Uranium-235 nucleus captures a neutron and splits apart into two lighter nuclei. Sometimes the two fission products are a barium nucleus (charge 56e) and a krypton nucleus. Assume that these nuclei are positive point charges separated by $r = 14.6 \times 10^{-15}$ m. Calculate the potential energy of this two charge system in electron volts.

Sol. The potential energy for two point charges separated by a distance r is $U = kq_1q_2/r$. To find this energy in electron volts we calculate the potential due to one of the charges kq_1/r in volts and multiply by the other charge.

(i) The potential energy of the two charges :

$$U = \frac{kq_1q_2}{r} = \frac{k(56e)\,(36e)}{r}$$

(ii) Factor out e and substitute the given values :

$$U = \frac{k(56e)\,(36e)}{r} = e\,\frac{ke(56)\,(36)}{r}$$

$$U = e\,\frac{(8.99 \times 10^9\,\text{N.m}^2/\text{C}^2)(1.6 \times 10^{-19}\,\text{C})(56)(36)}{14.6 \times 10^{-15}\,\text{m}}$$

$$U = e\,(1.99 \times 10^8\,\text{V}) = 199\ \text{MeV}.$$

Example 37 :

Two point charge of 8 μC and 12 μC are kept in air at a distance of 10 cm from each other. The work required to change the distance between them to 6 cm will be.

(A) 5.8 J (B) 4.8 J
(C) 3.8 J (D) 2.8 J

Sol. **(A).** Work= Increase in potential energy $= Kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$= 9 \times 10^9 \times 8 \times 10^{-6} \times 12 \times 10^{-6}\left(\frac{1}{6 \times 10^{-2}} - \frac{1}{10 \times 10^{-2}}\right)$$

$$= 9 \times 8 \times 12\left(\frac{1}{6} - \frac{1}{10}\right) \times 10^{-1} = 5.8\ \text{J}$$

Example 38 :

Maximum charge stored on a metal sphere of radius 15 cm may be 7.5 μC. The potential energy of the sphere in this case is.

(A) 1.69 J (B) 0.25 J
(C) 3.25 J (D) 9.67 J

Sol. **(A).** Potential energy $= \frac{1}{2}QV = \frac{1}{2}Q\left(\frac{KQ}{R}\right)$

$$V = \frac{1}{2}\,\frac{7.5 \times 10^{-6} \times 7.5 \times 10^{-6} \times 9 \times 10^9}{15 \times 10^{-2}} = 1.687\ \text{J} \approx 1.69\ \text{J}$$

Example 39 :

Four identical particles each of mass m and charge q are kept at the four corners of a square of length L. The final velocity of these particles after setting them free will be.

(A) $\left[\frac{Kq^2}{mL}(5.4)\right]^{1/2}$ (B) $\left[\frac{Kq^2}{mL}(1.35)\right]^{1/2}$

(C) $\left[\frac{Kq^2}{mL}(2.7)\right]^{1/2}$ (D) Zero

Sol. **(C).** Potential energy of the system

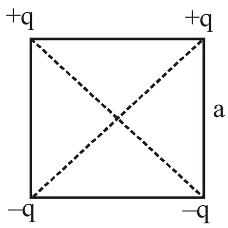
$$= 4 \times \frac{K}{2}\left(\frac{q^2}{L} + \frac{q^2}{L} + \frac{q^2}{L\sqrt{2}}\right) = 5.4\,\frac{Kq^2}{L}$$

= Final kinetic energy of the system

$$= 4 \times \frac{1}{2}mv^2 = 2\,mv^2 \quad \therefore\ v = \left[\frac{Kq^2}{mL}(2.7)\right]^{1/2}$$

Example 40 :

Calculate the electric potential energy of the system of charges shown in figure.



Sol. Taking zero of potential energy at infinity, we get P.E. of the system of charges

$$\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_jq_k}{r_{jk}}$$

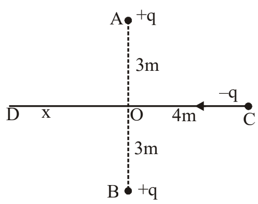
$$\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\left[\begin{array}{l}\frac{qq}{a} + \frac{q(-q)}{a} + \frac{(-q)(-q)}{a} \\ + \frac{(-q)\,(+q)}{a} + \frac{q\,(-q)}{a\sqrt{2}} + \frac{q\,(-q)}{a\sqrt{2}}\end{array}\right]$$

$$\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{a\sqrt{2}} - \frac{q^2}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{-2q^2}{a\sqrt{2}}\right)\ ;\ \text{P.E.} = \frac{q^2}{4\pi\varepsilon_0 a}(-\sqrt{2})$$

Example 41 :

Two fixed positive charges, each of magnitude 5×10^{-5} C are located at points A and B, separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of line AB.



The moving charge, when reaches the point C at a distance of 4m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.

Sol. The kinetic energy is lost and converted to electrostatic potential energy of the system as the negative charge goes from C to D and comes to rest at D instantaneously.

Loss of KE = gain in PE

$4 = U_f - U_i$

$$4 = \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+x^2}}\right] - \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+16}}\right]$$

$$4 = \frac{2q^2}{4\pi\varepsilon_0}\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

$$4 = 2(5\times10^{-5})^2(9\times10^9)\left(\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right)$$

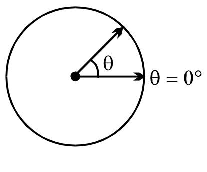
$$4 = 9 - \frac{45}{\sqrt{9+x^2}} \; ; \; x = \sqrt{72} = 8.48\text{m}$$

TRY IT YOURSELF - 3

Q.1 The linear charge density on a dielectric ring of radius R is varying with θ as $\lambda = \lambda_0 \cos(\theta/2)$. The potential at the centre of the ring is

(A) 0 (B) $\frac{\lambda_0}{2\pi\varepsilon_0}$

(C) $\frac{\lambda_0}{4\pi\varepsilon_0}$ (D) $\frac{\lambda_0}{\pi\varepsilon_0}$

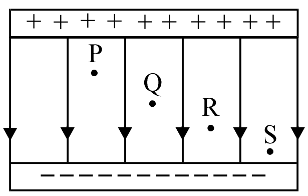


Q.2 In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is –

(A) constant (B) zero

(C) positive (D) negative

Q.3 The figure shows the electric field lines between two parallel plates that for all practical purposes extend an infinite distance both to the right and to the left and into and out of the paper. Four point P, Q, R and S are marked in this figure. At which point is the electric potential the largest



(A) P (B) Q

(C) R (D) S

Q 4 An electron at a potential of – 10 kV moves to a point where its potential is – 1 kV. Its potential energy has –

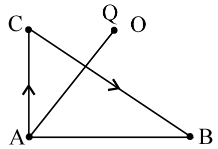
(A) decreased

(B) increased

(C) not changed

(D) one needs to know the distance b/w the points to say

Q.5 There is a fixed positive charge Q at O and A and B are points equidistant from O. A positive charge + q is taken slowly by an external agent from A to B along the line AC and then along the line CB.



(A) The total work done on the charge is zero.

(B) The work done by the electrostatic force from A to C is negative.

(C) The work done by the electrostatic force from C to B is positive.

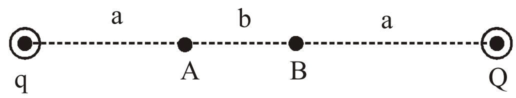
(D) The work done by electrostatic force in taking the charge from A to B is dependent on the actual path.

Q.6 A metallic rod of length l rotates at angular velocity ω about an axis passing through one end and perpendicular to the rod . If mass of electron is m and its charge is –e then the magnitude of potential difference between its two ends is :

(A) $m\omega^2 l^2/(2e)$ (B) $m\omega^2 l^2/e$

(C) $m\omega^2 l/e$ (D) none of these

Q.7 If a = 30 cm , b = 20 cm, q = + 2.0 nC, and Q = – 3.0 nC in the figure, what is the potential difference $V_A - V_B$?



(A) + 60 V (B) + 72 V

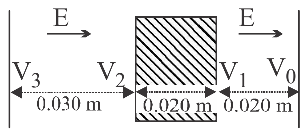
(C) + 84 V (D) + 96 V

Q.8 A spherical drop of mercury having an electric potential of 2.5 V is obtained as a result of merging 125 identical spherical droplets. The electric potential of each of the original small droplets is

(A) 0.1 V (B) 0.2 V

(C) 0.4 V (D) 0.5 V

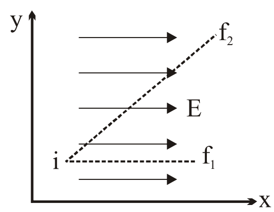
Q.9 An infinite conducting plate of thickness 0.0200 m is surrounded by a uniform field E = 400 V/m directed left to right. See the figure. Let the potential $V_0 = 0$ at a distance 0.0200 m to the right of the plate. What is V_3, the potential 0.0300 m to the left of the plate?

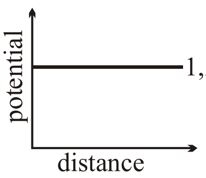


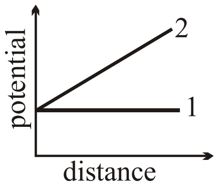
(A) –28 V (B) –20 V

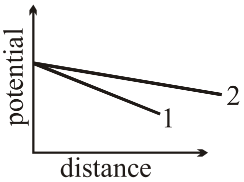
(C) +20 V (D) +28 V

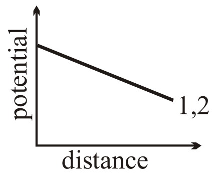
Q.10 A uniform electric field points in the positive x direction, as shown. Along the two lines f_1, f_2 , we plot the electric potentials as a function of distance. Choose the correct plot.



(A) 

(B) 

(C) 

(D) 

Q.11 A number of spherical shells of different radii are uniformly charged to same potential. The surface charge density of each shell is related with its radius as

(A) $\sigma \propto \frac{1}{R^2}$ (B) $\sigma \propto \frac{1}{R}$

(C) $\sigma \propto R$ (D) σ is same for all

Q.12 Two point charges + Q and – Q are kept at a distance d from each other. At the mid point of the line joining both the charges

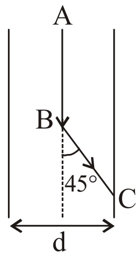
(A) Potential is zero but electric field is not zero

(B) Electric field is zero but potential is not zero

(C) Electric field as well as potential are zero

(D) Electric field as well as potential are non zero

Q.13 A sphere carrying a charge of Q having weight w falls under gravity between a pair of vertical plates at a distance of d from each other. When a potential difference V is applied between the plates the acceleration of sphere changes as shown in the figure, to along line BC. The value of Q is



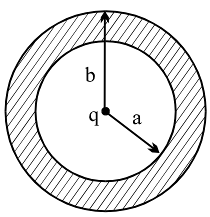
(A) $\frac{w}{V}$ (B) $\frac{w}{2V}$ (C) $\frac{wd}{V}$ (D) $\frac{\sqrt{2}wd}{V}$

Q.14 Two conducting, concentric, hollow spheres A and B have radii a and b respectively, with A inside B. They have the same potential V. A is now given some charge such that its potential becomes zero. The potential of B will now be–

(A) 0 (B) V (1–a/b)

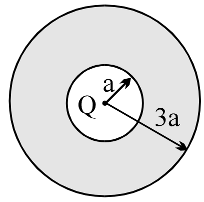
(C) Va/b (D) V (b – a) (b + a)

Q.15 In the figure shown the electric potential energy of the system is(q is at the centre of the conducting neutral spherical shell of inner radius a and outer radius b) Ignore self energy of point charge.



(A) 0 (B) $\frac{kq^2}{2b}$ (C) $\frac{kq^2}{2b} - \frac{kq^2}{2a}$ (D) $\frac{kq^2}{2a} - \frac{kq^2}{2b}$

Q16 A solid, uncharged conducting sphere of radius 3a contains a hollowed spherical region of radius a. A point charge +Q is placed at the common center of the spheres. Taking V = 0 as $r \to \infty$, the potential at position r = 2a from the center of the spheres is:



(A) 0 (B) kQ/3a

(C) kQ/2a (D) 2kQ/3a

Q.17 A neutral spherical conductor (radius r_2) has a concentric spherical cavity (radius r_1). A point charge Q is placed at a distance 'r' (less than r_1) from the centre. The potential at the centre is :

(A) $\frac{KQ}{r_2} - \frac{KQ}{r_1} + \frac{KQ}{r}$ (B) $\frac{2KQ}{r_2} - \frac{KQ}{r}$

(C) $\frac{KQ}{r}$ (D) Cannot be determined by given data

ANSWERS

(1) (A)	**(2)** (A)	**(3)** (A)
(4) (A)	**(5)** (ABC)	**(6)** (A)
(7) (A)	**(8)** (A)	**(9)** (C)
(10) (C)	**(11)** (B)	**(12)** (A)
(13) (C)	**(14)** (B)	**(15)** (C)
(16) (B)	**(17)** (A)	

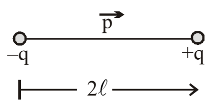
ELECTRIC DIPOLE

* In some molecules, the centre of +ve and -ve charge do not coincide. This results in the formation of electric dipole. Atom is non-polar because in it the centre of +ve and -ve charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

* An electric dipole is a system formed by two equal and opposite charges placed at a short distance apart. Product of one of the two charges and the distance between them is called "**electric dipole moment**" $\vec{p}$.

$$\vec{p} = q \times 2\vec{\ell}$$

- It is a vector quantity, directed from – ve to + ve charge.



- Dimension → [L T A], unit → Cb x mt.
- Practical unit is Debye ≡ $\vec{p}$ of two equal and opposite point charges each having charge 10^{-10} frankline and separation of 1 Å. i.e.
$1D = 10^{-10} \times 10^{-10} = 10^{-20}$ Fr × mt.

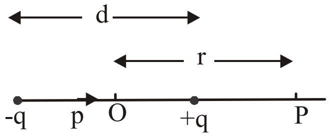
$$= \frac{10^{-20}}{3 \times 10^9} \text{ Cb} \times \text{mt.}$$

$\Rightarrow$ $1D \approx 3.3 \times 10^{-30}$ Cb × mt.

* **Electric field at an axial point :**

Electric field at P due to negative charge

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r+\frac{d}{2}\right)^2}(-\hat{r})$$



Electric field at P due to positive charge

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r-\frac{d}{2}\right)^2}(+\hat{r})$$

Total electric field at axial point P is

$$\vec{E}_a = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_a = \frac{q}{4\,\pi\,\varepsilon_0}\left(\frac{1}{(r-d/2)^2} - \frac{1}{(r+d/2)^2}\right)\hat{r}$$

or $$\vec{E}_a = \frac{q}{4\,\pi\,\varepsilon_0}\,\frac{2.r.d}{(r^2-d^2/4)^2}\,\hat{r}$$

$$= \frac{2pr}{4\,\pi\,\varepsilon_0(r^2-d^2/4)^2}\hat{r}$$

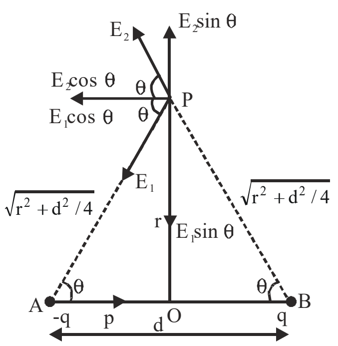
When r >> d , $\boxed{\vec{E}_a = \frac{1}{4\,\pi\,\varepsilon_0}\frac{2p}{r^3}\hat{r}}$,

The axial field is parallel to dipole moment.

Electric field at an equatorial point :

Electric field at P due to negative charge

$$E_1 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{(r^2+d^2/4)}$$



Electric field at P due to positive charge

$$E_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2+d^2/4}\right)$$

Fields E_1 and E_2 are equal in magnitude

Resolving E_1 and E_2 into two components one along O and other perpendicular to OP

We find $E_1\sin\theta = E_2\sin\theta$

Total field $E_\perp = E_1\cos\theta + E_2\cos\theta = 2E_1\cos\theta$
$= 2E_2\cos\theta$

$$E_\perp = 2\,\frac{q}{4\,\pi\,\varepsilon_0}\,\frac{1}{(r^2+d^2/4)}\,\frac{d/2}{\sqrt{r^2+d^2/4}}$$

$$= \frac{q.d}{4\pi\varepsilon_0(r^2+d^2/4)^{3/2}}$$

$$= \frac{p}{4\pi\varepsilon_0(r^2+d^2/4)^{3/2}}$$

If r >> d , $\boxed{\vec{E}_\perp = \frac{1}{4\pi\,\epsilon_0}\frac{p}{r^3}(-\hat{r})}$

i.e. field at equatorial point is antiparallel to dipole moment.

NOTE

1. Intensity due to a dipole varies as $(1/r^3)$ and can never be zero unless $r \to \infty$ or $p \to 0$.

2. E will be maximum for end on, axial or tan A position

$$E_{max} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

3. E will be minimum for broad on, equatorial or tan B position $E_{min} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

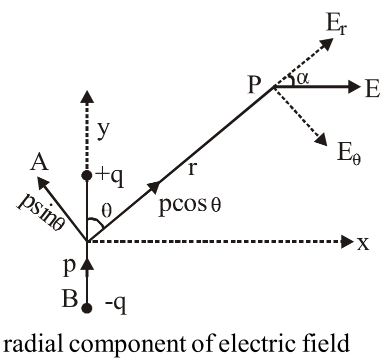
4. The electric field at axial point is parallel to dipole moment vector.

5. The electric field at equatorial point is antiparallel to dipole moment vector.

6. The ratio of field at axial point to field at equatorial point is $E_a : E_\perp = 2 : 1$.

Electric field at an arbitrary point :

We resolve dipole moment p in two components one along r and another perpendicular to r.



The radial component of electric field

$$E_r = \frac{1}{4\pi\,\epsilon_0}\,\frac{2p\cos\theta}{r^3}$$

The magnitude of resultant field is

$$E_\theta = \frac{1}{4\pi \epsilon_0} \frac{p \sin\theta}{r^3}$$

The magnitude of resultant field is

$$E = \sqrt{E_r^{\,2} + E_\theta^{\,2}} = \frac{1}{4\pi \epsilon_0} \frac{p}{r^3} \sqrt{1 + 3\cos^2\theta}$$

The direction of resultant field is

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2} \tan\theta$$

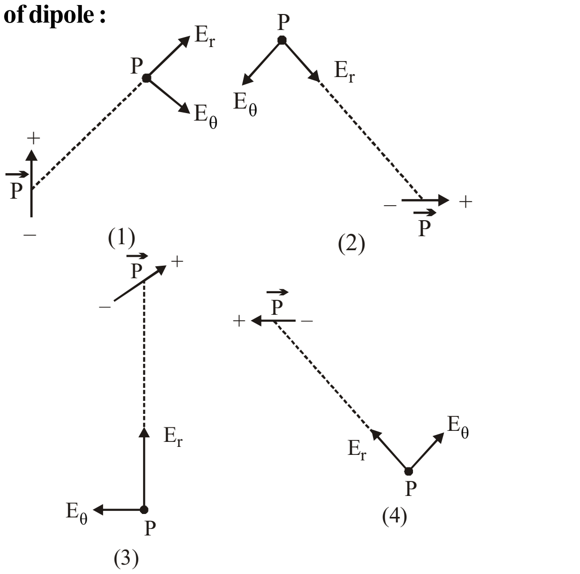
Case I at axial point $\theta = 0°$

$$\text{so } E_a = \frac{1}{4\pi \epsilon_0} \frac{p}{r^3} \sqrt{1 + 3\cos^2 0°} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}$$

Case II at equatorial point $\theta = \pi/2$

$$\text{so } E_\perp = \frac{1}{4\pi \epsilon_0} \frac{p}{r^3} \sqrt{1 + 3\cos^2 \pi/2} = \frac{1}{4\pi \epsilon_0} \frac{p}{r^3}$$

Direction of electric field due to various arrangement of dipole :



Electric Potential Due to a Dipole

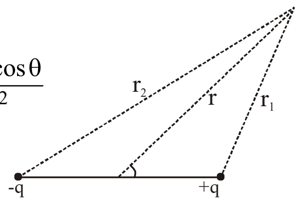
$$V_{(+)} = \frac{Kq}{r_1} \approx \frac{Kq}{r - d\cos\theta}, \quad [r_1 \approx r - d\cos\theta]$$

$$V_{(-)} = \frac{Kq}{r_2} \approx \frac{-Kq}{r + d\cos\theta}, \quad [r_2 \approx r + d\cos\theta]$$

$$V = V_1 + V_2 = Kq\left[\frac{1}{r - d\cos\theta} - \frac{1}{r + d\cos\theta}\right]$$

$$= Kq\left[\frac{r + d\cos\theta - r + d\cos\theta}{r^2 - d^2\cos^2\theta}\right]$$

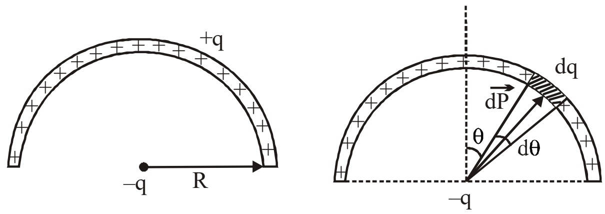
$$V = \frac{Kp\cos\theta}{r^2\left[1 - \frac{d^2\cos^2\theta}{r^2}\right]} \approx \frac{Kp\cos\theta}{r^2}$$



We can use $E = -\frac{dV}{dr}$ to calculate electric field from potential function.

The dipole moment of distributed system :

Consider a half ring with charge +q uniformly distributed and –q placed at centre of ring.



To find dipole moment consider a polar element of angular width $d\theta$ at an angle θ from the vertical axis as shown.

Charge on this element is $dq = \frac{q}{\pi} d\theta$

Now in the point charge –q, we consider an element charge – dq which forms a dipole with the polar element of charge +dq. This element dipole moment can be given as

$$dp = dqR = \frac{q}{\pi} R d\theta$$

Total dipole moment of system can be given by integrating all such element dipole moments as

$$p_{total} = \int dp\cos\theta \quad \text{[As dP sin θ cancels out]}$$

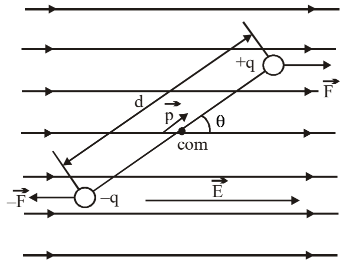
$$= \int_{-\pi/2}^{+\pi/2} \frac{q}{\pi} R\cos\theta \, d\theta = \frac{qR}{\pi}[\sin\theta]_{-\pi/2}^{+\pi/2} = \frac{2qR}{\pi}$$

Dipole in E-field : Consider the force exerted on the dipole in an external E-field:

Assumption: E-field from dipole doesn't affect the external E-field.

* Dipole moment : $\vec{p} = q\vec{d}$

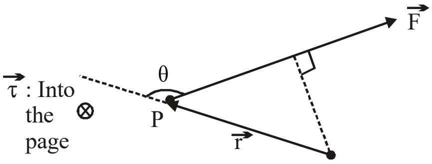
* Force due to the E-field on +ve and – ve charge are equal and opposite in direction. Total external force on dipole = 0.



BUT: There is an external torque on the center of the dipole.

Force $\vec{F}$ exerts at point P. The force exerts a torque $\vec{\tau} = \vec{r} \times \vec{F}$ on point P with respect to point O.

Direction of the torque vector $\vec{\tau}$ is determined from the right-hand rule.



Net torque $\vec{\tau}$; direction : clockwise torque

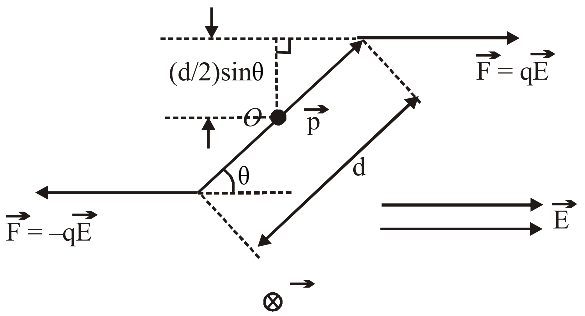
magnitude :

$$\tau = \vec{\tau}_{+ve} + \tau_{-ve} = F.\frac{d}{2}\sin\theta + F.\frac{d}{2}\sin\theta = qE.d\sin\theta$$
$$= pE\sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Energy consideration :

When the dipole $\vec{p}$ rotates dθ, the E-field does work.



Work done by external E-field on the dipole:

$dW = -\tau\, d\theta$

Negative sign here because torque by E-field acts to decrease θ.

BUT: Because E-field is a conservative force field , we can define a potential energy (U) for the system, so that

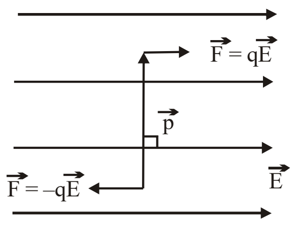
$dU = -dW$

For the dipole in external E-field: $dU = -dW = pE\sin\theta\, d\theta$

$\therefore U(\theta) = \int dU = \int pE\sin\theta\, d\theta = -pE\cos\theta + U_0$

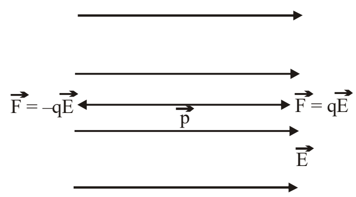
Set $U(\theta = 90°) = 0$, $\therefore\ 0 = -pE\cos 90° + U_0\ \therefore\ U_0 = 0$

$\therefore$ Potential energy : $\boxed{U = -pE\cos\theta = -\vec{p}.\vec{E}}$



$\theta = 90°$; Torque $|\vec{\tau}| = pE$; $U = 0$ (define)

(based on definition)



$\theta = 0°$; Torque $|\vec{\tau}| = 0$; $U = -pE$

(minimum energy configuration)

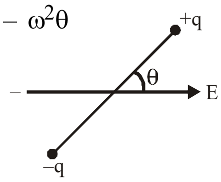
Angular SHM : When a dipole is suspended in uniform field, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its angular position, the restoring couple will be

$\tau = -pE\sin\theta$. if θ is small $\Rightarrow \sin\theta \approx \theta$.

$\Rightarrow \tau = -pE\,\theta \Rightarrow \tau \propto -\theta$ (Angular SHM).

for balanced condition : $\tau_{deflecting} = \tau_{restoring}$

$$I\alpha = -pE\,\theta \Rightarrow \alpha = -\left(\frac{pE}{I}\right)\theta = -\omega^2\theta$$



$$\Rightarrow \omega = \sqrt{\frac{pE}{I}}$$

$$\Rightarrow \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}}}$$ [I → moment of inertia]

Note : If dipole is placed in a non-uniform electric field, it preforms rotatory as well as translatory motion, because now a net force also acts on the dipole along with the torque.

In Uniform electric field, Total force = 0, Torque may or may not be zero. ((τ = 0 if θ = 0)

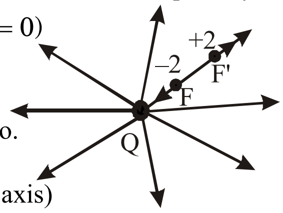
In Non-uniform electric field,

Total force ≠ 0,

Torque may or may not be zero.

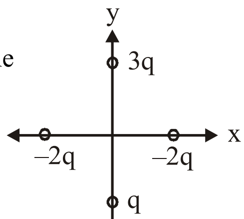
For situation shown in figure,

Torque = 0 (Force along same axis)



Example 42 :

4 charges are placed each at a distance 'a' from origin. The dipole moment of configuration is –



(A) $2qa\hat{j}$ (B) $3qa\hat{j}$

(C) $2aq[\hat{i} + \hat{j}]$ (D) None

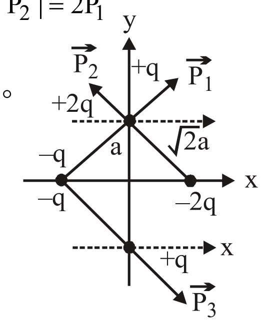
Sol. **(A).** $|\vec{P}_1| = |\vec{P}_3| = q\sqrt{2}a \Rightarrow |\vec{P}_2| = 2P_1$

$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$

$P_x = P_1\cos 45° - 2P_1\cos 45° + P_1\cos 45° = 0$

$P_y = P_1\sin 45° + 2P_1\sin 45° - P_3\sin 45° = \sqrt{2}P_1 = 2qa$

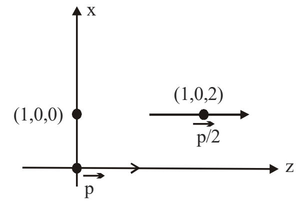
$\vec{P} = P_x\hat{i} + P_y\hat{j} = 2qa\,\hat{j}$



Example 43 :

Two point dipoles $p\hat{k}$ and $\frac{p}{2}\hat{k}$ are located at (0, 0, 0) and (1m, 0, 2m) respectively. Find the resultant electric field due to the two dipoles at the point (1m, 0, 0)

Sol.



The given point is at axis of $\frac{\vec{p}}{2}$ dipole and at equatorial line of $\vec{p}$ dipole so that field at given point.

$$\vec{E} = -\frac{k\vec{p}}{(1)^3} + \frac{2k(\vec{p}/2)}{(2)^3} = \frac{-7\vec{p}}{32\pi\varepsilon_0}$$

Example 44 :

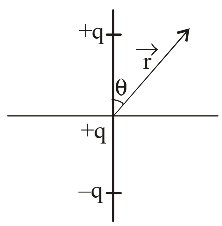
Three charges –q, +q and +q are situated in X-Y plane at points (0, –a), (0, 0) and (0, a) respectively. The potential at a point distant r (r >> a) in a direction making an angle θ from Y-axis will be

(A) $\frac{kq}{r}\left(1 - \frac{2a\cos\theta}{r}\right)$ (B) $\frac{2kq\cos\theta}{r^2}$

(C) $\frac{kq}{r}$ (D) $\frac{kq}{r}\left(1 + \frac{2a\cos\theta}{r}\right)$

Sol. (D). $V = \frac{kq.2a\ \cos\theta}{r^2} + \frac{kq}{r}$

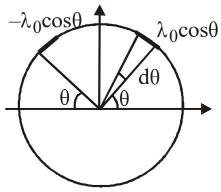
$= \frac{kq}{r}\left[\frac{2a\cos\theta}{r} + 1\right]$



Example 45 :

A nonunifom charge given on the ring according to the equation $\lambda = \lambda_0 \cos\theta$ (where θ is measured from x-axis). Find its dipole moment.

Sol.



In the figure shown the two symmetric elements are equal and opposite charge whose dipole moment will be

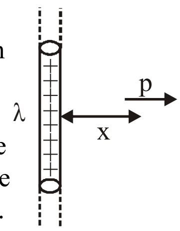
$dp = \{(\lambda_0 \cos\theta)(R\,d\theta)\}\ (2R\cos\theta\} = 2\lambda_0 R^2 \cos^2\theta d\theta$

$$\therefore \quad p = \int dp = 2\lambda_0 R^2 \int_{-\pi/2}^{+\pi/2} \cos^2\theta\, d\theta = \pi\lambda_0 R^2$$

Example 46 :

A dipole of mass m is placed in front of a line charge at a separation x (as shown in figure).

Find (i) interaction energy between point charge and dipole (ii) force acting on the dipole due to line charge (iii) what will be its velocity if it reaches as separation x_0.



Sol. (i) $U = -\vec{p}\cdot\vec{E} = -pE\cos 0° = -p\frac{\lambda}{2\pi\varepsilon_0 x}(1) = \frac{\lambda p}{2\pi\varepsilon_0 x}$

(ii) $F = -\frac{dU}{dx} = -\frac{\lambda p}{2\pi\varepsilon_0 x^2} \Rightarrow \vec{F} = \frac{\lambda p}{2\pi\varepsilon_0 x^2}$

directed along –ve x-axis.

(iii) Using conservation of energy

$K_f + U_f = K_i + U_i$

$$\frac{1}{2}mv^2 - \frac{\lambda p}{2\pi\varepsilon_0 x_0} = 0 - \frac{\lambda p}{2\pi\varepsilon_0 x}$$

$$\frac{1}{2}mv^2 = \frac{\lambda p}{2\pi\varepsilon_0}\left(\frac{1}{x_0} - \frac{1}{x}\right)$$

$$v = \sqrt{\frac{2\lambda p}{2\pi\varepsilon_0 m}\left(\frac{1}{x_0} - \frac{1}{x}\right)}$$

TRY IT YOURSELF - 4

Q.1 An electric dipole is placed in an electric field generated by a point charge

(A) The net force on the dipole must be zero.

(B) The net force on the dipole may be zero.

(C) Torque on the dipole due to the field must be zero.

(D) The torque on the dipole due to the field may be zero.

Q.2 An electric dipole is placed in an electric field generated by an infinitely long uniformly charged wire

(A) The net electric force on the dipole must be zero

(B) The net electric force on the dipole may be zero

(C) The torque on the dipole due to the field must be zero

(D) The torque on the dipole due to the field may be zero

Q.3 A dipole lies on the x axis, with the positive charge + q at x = + d/2, and –q at x = –d/2. The electric flux Φ_E through the yz plane midway between the charges

(A) is zero.

(B) depends only on d.

(C) depends only on q.

(D) depends on both q and d.

Q.4 An electric dipole is placed at the origin O such that its equator is y-axis. At a point P far away from dipole, the electric field direction is along y-direction. OP makes an angle α with the x-axis such that :

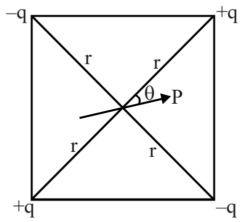
(A) $\tan\alpha = \sqrt{3}$ (B) $\tan\alpha = \sqrt{2}$

(C) $\tan\alpha = 1$ (D) $\tan\alpha = \frac{1}{\sqrt{2}}$

Q.5 An electric dipole of moment p is kept along an electric field E. The workdone by external agent in rotating it from stable equilibrium position by an angle θ, is

(A) $pE \sin\theta$ (B) $pE \cos\theta$
(C) $pE(1-\sin\theta)$ (D) $pE(1-\cos\theta)$

Q.6 Given a square frame of diagonal length 2r made of insulating wires. There is a short dipole, having dipole moment P, fixed in the plane of the figure lying at the center of the square, making an angle θ as shown in figure. Four identical particles having charges of magnitude q each and alternatively positive and negative sign are placed at the four corners of the square. Select the correct alternative(s).



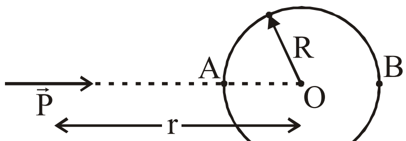
(A) Electrostatic force on the system of four charges due to dipole is $\frac{6kPq}{r^3}$.
(B) Electrostatics force on the system of four charges due to dipole is $\frac{6kPq}{r^3}\cos\theta$.
(C) Net torque on the system of four charges about the centre of the square due to dipole is zero.
(D) Net torque on the system of four charges about the centre of the square due to dipole is $\frac{3kPq}{r^2}$.

Q.7 The potential energies associated with four orientations of an electric dipole in an uniform electric field are

(i) $-V_0$ (ii) $-7V_0$
(iii) $3V_0$ (iv) $4V_0$

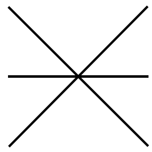
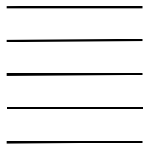
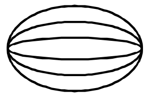
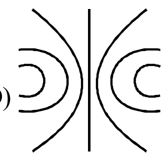
Choose correct statement if V_0 is positive.
(A) The angle between electric field and dipole is maximum in case (ii)
(B) The maximum torque is being experienced by the dipole in case (i)
(C) $V_0 = |P|\,|E|$ with usual notations
(D) The angle between $\vec{E}$ & $\vec{P}$ is acute in case (iii)

Q.8 An ideal dipole of dipole moment $\vec{P}$ is placed in front of an uncharged conducting sphere of radius R as shown.



(A) The potential at point A is $\frac{KP}{(r-R)^2}$.
(B) The potential at point A is $\frac{KP}{r^2}$.
(C) The potential due to dipole at point B is $\frac{KP}{(r+R)^2}$.
(D) The potential due to dipole at point B is $\frac{KP}{r^2}$

Q.9 Which of the following represents the equipotential lines of a dipole ?

(A) 
(B) 
(C) 
(D) 

Q.10 Two opposite charges each of magnitude 2 μC are 1cm apart. Find electric field at a distance of 5cm from the mid-point on axial line of the dipole. Also find the field on equatorial line at the same distance from mid-point.

ANSWERS

(1) (D)	**(2)** (BD)	**(3)** (C)
(4) (B)	**(5)** (D)	**(6)** (AC)
(7) (B)	**(8)** (BC)	**(9)** (D)

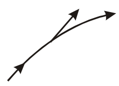
(10) 2.93×10^6 N/C, 1.41×10^6 N/C

ELECTRIC LINES OF FORCES

The concept of electric field was introduced by Michael Faraday.

The magnitude of electric field strength at any point is measured by the number of electric line of force passing per unit small area around that point normally and the direction of field at any point is given by the tangent to the line of force at the point.

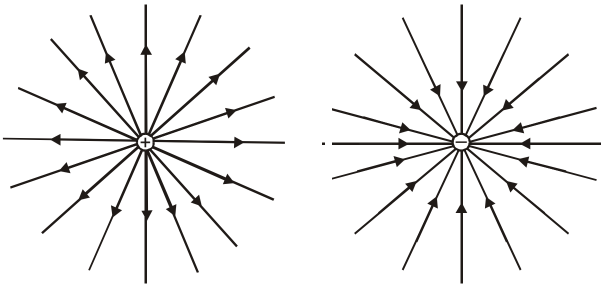
An electric line of force is that imaginary smooth curve drawn in an electric field along which a free isolated unit positive (initially at rest) charge moves.



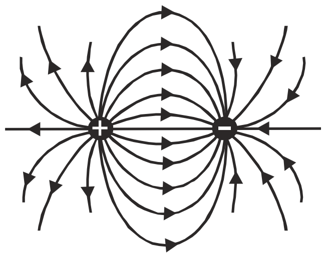
$\vec{E}$ at any point on a line of force.

Properties :

1. The lines of force diverge out radially from a +ve charge and converge at a – ve charge. More correctly the lines of force are always directed from higher to lower potential.

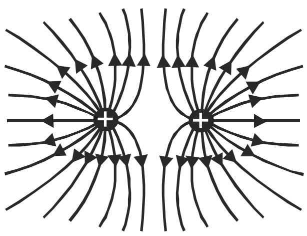


2. The tangent drawn at any point on line of force gives the direction of force acting on a positive charge placed at that point.
3. Two lines of force never intersect. If they are assumed to intersect. There will be two directions of electric field at the point of intersection : which is impossible.
4. These lines have a tendency to contract in tension like a stretched elastic strong. This actually explains attraction between opposite charges.



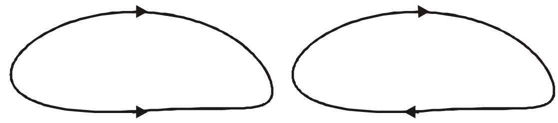
⟶ Attraction ⟵

5. These lines have a tendency to separate from each other in the direction perpendicular to their length. This explains repulsion between like charges.



⟵ Repulsion ⟶

6. The no. of lines originating or terminating on a charge is proportional to the magnitude of charge.
   There is no rule as to how many lines are to be shown. However, it is customary to draw number of lines proportional to the charge. Thus if N number of lines are drawn from or into a charge Q, 2N number of lines would be drawn for charge 2Q.
7. Total lines of force may be fractional as lines of force are imaginary.
8. Lines of force ends or strarts normally on the surface of a conductor.
9. If there is no electric field there will be no lines of force.
10. Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines represent weak field.
11. Electric lines of force differ from magnetic lines of force.
    (a) Electric lines of force never form closed loop while magnetic lines are always closed or extended to infinity.

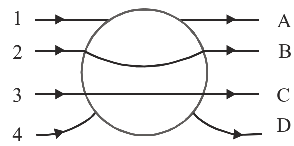


Electric line of force (A) | Magnetic line of force (B)

(b) Electric lines of force always emerge or terminate normally on the surface of charged conductor, while magnetic lines emerge or terminate on the surface of a magnetic material at any angle.
(c) Electric lines of force do not exist inside a conductor but magnetic lines of force may exist inside magnetic material.

NOTE

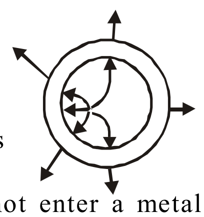
1. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure is 4.



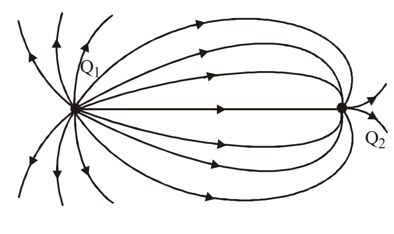
Electric field lines never enter a metallic conductor (E = 0, inside a conductor) and they fall normally on the surface of a metallic conductor.

2. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The possible resulting lines of force can be sketched as shown.

3. A metallic shell has a point charge q kept inside its cavity then the diagrams which can represent the electric lines of forces. Electric field is zero everywhere inside a metal (conductor) i.e, field lines do not enter a metal. Simultaneously these are perpendicular to a metal surface.



4. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that $|Q_1| > |Q_2|$ and at a finite distance to the right of Q_2 the electric field is zero.



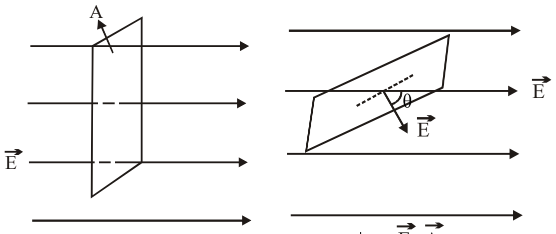
- From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.
  No. of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.
  So, $|Q_1| > |Q_2|$
  Electric field will be zero to the right of Q_2 as it has small magnitude & opposite sign to that of Q_1.

ELECTRIC FLUX

Electric flux (Latin word flux means "to flow") :

Graphically: Electric flux ϕ_E represents the number of E-field lines crossing a surface.

Mathematically:



$\phi_E = EA$ $\qquad$ $\phi_E = \vec{E}\cdot\vec{A}$

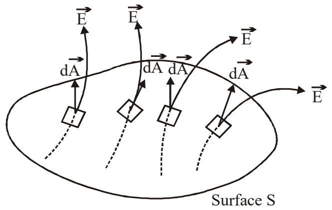
Vector of the area $\vec{A}$ is perpendicular to the area A. For non-uniform E-field & surface, direction of the area vector $\vec{A}$ is not uniform.

$d\vec{A}$ = Area vector for small area element dA.

$\therefore$ Electric flux $\boxed{d\phi_E = \vec{E}.d\vec{A}}$

Electric flux of $\vec{E}$ through surface S :

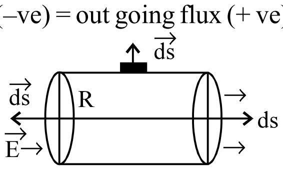
$$\phi_E = \int_S \vec{E}.d\vec{A}$$



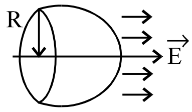
$\int_S$ = Surface integral over surface S

= Integration of integral over all area elements on surface S.

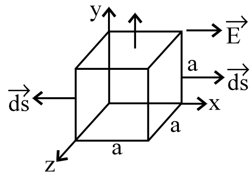
* Electric flux is a scalar quantity
* Units $\rightarrow$ (V – m) or (N – m^2/Cb)
  Dimensions : $[M^1 L^3 T^{-3} A^{-1}]$
* The value of ϕ is zero in the following circumstances :
  (a) If a dipole is enclosed by a closed surface.
  (b) Magnitude of +ve and – ve charges are equal inside a closed surface.
  (c) If no charge is enclosed by a closed surface.
  (d) In coming flux (–ve) = out going flux (+ ve).
* $|\phi_{in}| = \pi R^2 E$
  $|\phi_{out}| = \pi R^2 E$
  $\phi_{circular} = \phi_{curved} = \phi_{total} = 0$



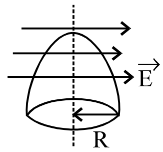
* $\phi_{out} = \phi_{in} = \pi R^2 E$



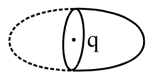
* $\phi_{in} = \phi_{out} = Ea^2$
  $\phi_{total} = 0.$



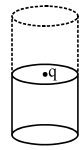
* $\phi_T = 0$



* $\phi_t = \dfrac{q}{\varepsilon_0}$, $\phi_{hemisphere} = \dfrac{q}{2\varepsilon_0}$

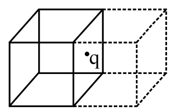


* $\phi_T = \dfrac{q}{\varepsilon_0}$, $\phi_{cyl.} = \dfrac{q}{2\varepsilon_0}$.

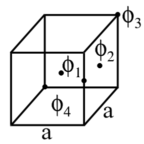


* $\phi_T = \dfrac{q}{\varepsilon_0}$

  $\phi_{cube} = \dfrac{q}{2\varepsilon_0}$



* **Charge position**
  Cube centre $\phi_1 = q/\varepsilon_0$.
  Face centre $\phi_2 = q/2\varepsilon_0$.
  At corner $\phi_3 = q/8\varepsilon_0$.
  At centre of edge, $\phi_4 = q/4\varepsilon_0$.



GAUSS'S LAW

The total flux linked with a closed surface is $\left(\dfrac{1}{\varepsilon_0}\right)$ times the charge enclosed by the closed surface (Gaussian surface). i.e. $\boxed{\oint \vec{E}.d\vec{s} = \dfrac{q}{\varepsilon_0}}$

Law is valid for symmetrical charge distribution and for all vector fields obeying inverse square law.

Gaussian surface :

(a) Imaginary surface
(b) Is spherical for a point charge, conducting and non–conducting spheres.
(c) Is cylindrical for infinite sheet of charge conducting charge surfaces, infinite line of charges, charged cylindrical conductors, etc.

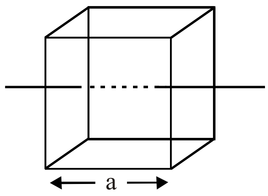
Note : If the flux emerging out of a Gaussian surface is zero then it is not necessary that the intensity of electric field is zero.

Example 47 :

A long string with charge per unit length λ on it passes through a cube of side a. The minimum flux through the cube is

(A) $\frac{\lambda a}{\varepsilon_0}$ (2) $\frac{\sqrt{2}\lambda a}{\varepsilon_0}$

(C) $\frac{\sqrt{3}\lambda a}{\varepsilon_0}$ (D) $\frac{2\lambda a}{\varepsilon_0}$

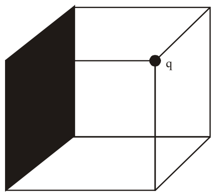


Sol. (A). The minimum length which can fit in the cube is 'a'.

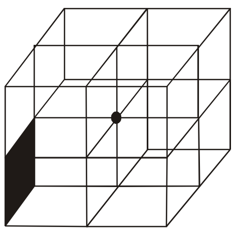
Hence, $\phi = \frac{\Sigma q}{\varepsilon_0} = \frac{\lambda a}{\varepsilon_0}$

Example 48 :

Calculate the flux through the shaded area (face of a cube of side a) when a charge q is located at one of the distant corners from the side.



Sol. If the charge were located at the centre of the cube instead of the corner, the flux would have been $q/6\varepsilon_0$, by symmetry. To use this symmetry consider the given cube to be a part of a bigger cube of side 2a × 2a, as shown, so that the charge q is in the centre of the bigger cube.



The flux through each face of the bigger cube is now $q/6\varepsilon_0$. Because the side of the bigger cube consists of four identical faces, the flux through one fourth of the face is clearly $q/24\varepsilon_0$.

Example 49 :

A gaussian surface encloses an object with a net charge of +2.0 C and there are 6 lines leaving the surface. Some charge is added to the object and now there are 18 lines entering the surface. How much charge was added ?

Sol. Since there are 6 lines when there is +2.0 C, therefore a charge of +1.0 C is equivalent to 3 lines. After charge is added, there are 18 lines entering.

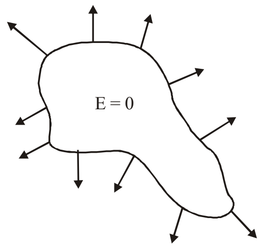
So the net charge is now $-\frac{18 \text{ lines}}{3 \text{ lines/coulomb}} = -6.0\text{ C}$

Therefore, the charge added was

$$\Delta Q = Q_f - Q_i = -6.0\text{ C} - 2.0\text{ C} = -8.0\text{C}$$

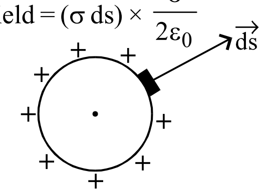
CONDUCTORS

* The electric field inside a conductor is zero.
* Free charges exist only on the surface of a conductor
* At the surface of a conductor, the electric field is normal to the surface



* Electric field intensity near the conducting surface is given by formula, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$
* **Electrostatic Pressure :** Force on small element $d\vec{s}$ of charged conductor.

$$dF = (\text{charge on ds}) \times \text{Electric field} = (\sigma\, ds) \times \frac{\sigma}{2\varepsilon_0}$$



$$\Rightarrow \vec{F} = \oint d\vec{F} = \oint \frac{\sigma^2}{2\varepsilon_0} ds$$

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$\boxed{P = \frac{dF}{dS} = \frac{\sigma^2}{2\varepsilon_0}}$$

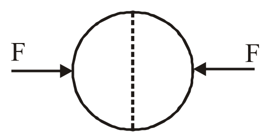
* The force is always directed normally outwards to the surface as $(\pm\sigma)^2$ is positive, i.e. whether charged positively or negatively, this force will try to expand the charged body.
* A soap bubble or rubber balloon expands on gives charge to it (charge of any kind +ve or – ve.)
 + ve charge $\Rightarrow$ M $\downarrow$; – ve charge $\Rightarrow$ M$\uparrow$.
* Energy associated per unit volume of electric field of intensity E is defined as energy density.

$$u = \frac{dw}{dv} = \frac{\varepsilon_0 E^2}{2} = \frac{\sigma^2}{2\varepsilon_0}\ \text{J/m}^3\ ; \quad U = \int u.dv = \frac{\varepsilon_0}{2}\int_v E^2 dv$$

v is the volume of electric field.

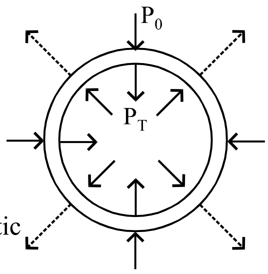
Example 50 :

A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). Find force F.



Sol. Pressure = $\frac{\sigma^2}{2\varepsilon_0}$ and force = $\frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$

SOAP BUBBLE



Pressures (forces) acts on a charged soap bubble, due to

(i) Surface tension of a soap bubble P_T (inward)

(ii) Air out side the bubble p_0 (inward)

(iii) Electric charges (electrostatic pressure) P_e (outward)

(iv) Air inside the soap bubble P_i (outward)

Hence, in state of equilibrium

Inward pressure = Outward pressure

$P_T + P_0 = P_i + P_e$

Excessive pressure $(P_{ex.}) = P_i - P_0 = P_T - P_e$

But $P_T = \dfrac{4T}{r}$, $P_e = \dfrac{\sigma^2}{2\varepsilon_0} \Rightarrow P_{ex.} = \dfrac{4T}{r} - \dfrac{\sigma^2}{2\varepsilon_0}$

If $P_i = P_0$, then $\boxed{\dfrac{4T}{r} = \dfrac{\sigma^2}{2\varepsilon_0}}$

Note :

1. A soap bubble always inflates whether it is charged with positive or negative electricity.

2. When a soap bubble is charged then during the charging process the volume of the air inside the bubble remains constant.

DIELECTRICS AND POLARISATION

* In dielectric materials, effectively there are no free electrons.
* **Polar dielectrics :**

(i) In absence of external field the centre of positive and negative charge do not coincide-due to asymmetric shape of molecules.

(ii) Each molecule has permanent dipole moment.

(iii) The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.

(iv) In presence of external field dipoles tends to align in direction of field.

(v) Ex. water, alcohol, CO_2, HCl, NH_3

* **Non polar dielectrics :**

(i) In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.

(ii) The dipole moment is zero in normal state.

(iii) In presence of external field they acquire induced dipole moment.

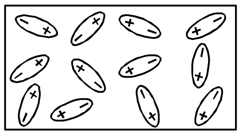
(iv) Ex. nitrogen, oxygen, benzene, methane

* **Polarisation :** The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.
* The dipole moment per unit volume is called polarisation and is denoted by **P**. For linear isotropic dielectrics,

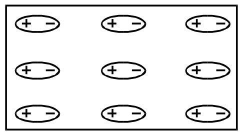
$$\boxed{\mathbf{P} = \chi_e \mathbf{E}}$$

where χ_e is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium.

* The charge appearing on the surface of a dielectric when placed in an electric field is called **induced charge.** As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called **bound charge.**



Dielectric in absence of electric field



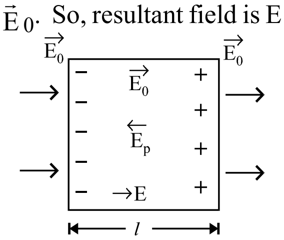
Dielectric in presence of electric field

Because of the induced charges, an extra electric field is produced inside the material. If $\vec{E}_0$ be the applied field due to external sources and $\vec{E}_P$ be the field due to polarization. The resultant field is $\vec{E} = \vec{E}_0 + \vec{E}_P$. For homogeneous and isotropic dielectrics, the direction of $\vec{E}_P$ is opposite to the direction of $\vec{E}_0$. The resultant field $\vec{E}$ is in the same direction as the applied field $\vec{E}_0$ but its magnitude is reduced. We can write $\vec{E} = \dfrac{\vec{E}_0}{K}$

where K is a constant for given dielectric which has a value greater than one. This constant K is called the dielectric constant or relative permittivity of the dielectric.

ELECTRIC FIELD IN DIELECTRIC

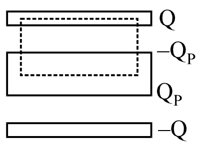
Let $\vec{E}_0$ be the applied field due to polarisation. The resultant field is $\vec{E} = \vec{E}_0 + \vec{E}_P$. For homogeneous and isotropic dielectric, the direction of $\vec{E}_p$ is opposite to the direction of $\vec{E}_0$. So, resultant field is $E = E_0 - E_P$.



(i) An alternative form of Gauass's law consider a parallel plate capacitor with a charge Q, the space between the plate is filled with a dielectric slab of dielectric constant K.

$$\oint \vec{E}.d\vec{s} = \frac{Q - Q_P}{\varepsilon_0} = \frac{1}{\varepsilon_0}\,[Q - (1 - 1/K)Q] = \frac{Q}{\varepsilon_0 K}$$

or $\oint \vec{E}.d\vec{s} = \dfrac{Q_{free}}{\varepsilon_0}$



(ii) Displacement vector : Field due to polarisation is

$$\vec{E}_P = \frac{\sigma_P}{\varepsilon_0} = \frac{-P}{\varepsilon_0}$$

Now, $\vec{E} = \vec{E}_0 + \vec{E}_P = \vec{E}_0 - \dfrac{\vec{P}}{\varepsilon_0}$

$\Rightarrow \varepsilon_0 \vec{E}_0 = \varepsilon_0 \vec{E} + \vec{P}$

so, $\oint (\varepsilon_0 \vec{E} + \vec{p}) . ds = \oint \varepsilon_0 \vec{E} \cdot d\vec{s} = Q_{free}$

or $\oint \vec{D} \cdot d\vec{s} = Q_{free}$, Where $\vec{D}$ is displacement

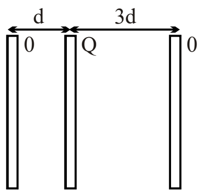
$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

TRY IT YOURSELF - 5

Q.1 An uncharged thick spherical conducting shell is surrounding a charge –q at the center of the shell. Then charge +3q is placed on a point outside of the shell. When static equilibrium is reached, the total charges on the inner and outer surfaces of the shell are respectively

(A) +q, –q (B) –q, +q
(C) +q, +2q (D) +2q, +q

Q.2 Three metallic plates out of which middle is given charge Q as shown in the figure. The area of each plate is same.



(A) The charge appearing on the outer surface of extreme left plate is Q/2.
(B) The charge appearing on the right surface of middle plate is Q/4.
(C) Each of the facing surfaces will bet equal and opposite.
(D) The charge on surface with separation 'd' is more than that on other two charge surfaces.

Q.3 If the flux of the electric field through a closed surface is zero,

(A) the electric field must be zero everywhere on the surface.
(B) the net electric field may be zero everywhere on the surface.
(C) the net charge inside the surface must be zero.
(D) the charge in the vicinity of the surface must be zero.

Q.4 A sphere of radius R carries charge density proportional to the square of the distance from the center: $\rho = Ar^2$, where A is a positive constant. At a distance of R/2 from the center, the magnitude of the electric field is

(A) $A/(4\pi\varepsilon_0)$ (B) $AR^3/(40\varepsilon_0)$
(C) $AR^3/(24\varepsilon_0)$ (D) $AR^3/(5\varepsilon_0)$

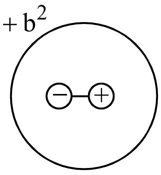
Q.5 Electric Flux is a measure of

(A) the rate at which moving electric charges are crossing an area.
(B) the no. of electric field lines passing through an area.
(C) the surface density of electric charge spread along the area.
(D) the rate at which electric field lines are spreading out in space as on moves further and further away from electric charges.

Q.6 A uniform electric field $\vec{E} = a\hat{i} + b\hat{j}$, intersects a surface of area A. What is the flux through this area if the surface lies in the yz plane?

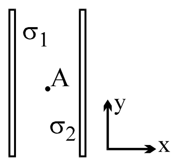
(A) a A (B) 0
(C) b A (D) $A\sqrt{a^2 + b^2}$

Q.7 Consider a gaussian spherical surface, covering a dipole of charge q and –q, then



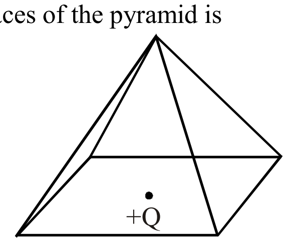
(A) $q_{in} = 0$ (Net charge enclosed by the spherical surface)
(B) $\phi_{net} = 0$ (Net flux coming out the spherical surface)
(C) E = 0 at all points on the spherical surface
(D) $\int \vec{E} \cdot d\vec{s} = 0$ (Surface integral of $\vec{E}$ over the spherical surface)

Q.8 Two large conducting sheets are kept parallel to each other as shown. In equilibrium, the charge density on facing surfaces is σ_1 and σ_2. The value of electric field at A is –



(A) $\dfrac{\sigma_1}{\varepsilon_0}\hat{i}$ (B) $-\dfrac{\sigma_2}{\varepsilon_0}\hat{i}$
(C) $\dfrac{\sigma_1 + \sigma_2}{2\varepsilon_0}\hat{i}$ (D) $\dfrac{\sigma_1 - \sigma_2}{2\varepsilon_0}\hat{i}$

Q.9 A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is



(A) $\dfrac{Q}{16\varepsilon_0}$ (B) $\dfrac{Q}{4\varepsilon_0}$
(C) $\dfrac{Q}{8\varepsilon_0}$ (D) None

Q.10 A 5.0 μC point charge is placed at the center of a cube. The electric flux in N-m^2 /C through one side of the cube is approximately :

(A) 0 (B) 7.1×10^4
(C) 9.4×10^4 (D) 1.4×10^5

Q.11 If a rectangular area is rotated in a uniform electric field from the position where the maximum electric flux goes through it to an orientation where only half the maximum flux goes through it, what has been the angle of rotation?

(A) 30° (B) 60°
(C) 45° (D) 26.6°

Q.12 Eight field lines emerge from a closed surface surrounding an isolated point charge. Would this fact change if a second identical charge were brought to a point just outside the surface?
(A) The number of lines would change but the shape of the lines remains the same.
(B) The number of lines would remain the same but the shape of the lines change.
(C) The number of lines as well as the shape of the lines remains the same.
(D) The number of lines as well as the shape of the lines would change.

Q.13 The electric field in a region is given by $\vec{E} = 200\hat{i}\ N/C$ for $x > 0$ and $-200\hat{i}\ N/C$ for $x < 0$. A closed cylinder of length 2m and cross-section area $10^2\ m^2$ is kept in such a way that the axis of cylinder is along X-axis and its centre coincides with origin. The total charge inside the cylinder is (Take $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 m^2.N$)
(A) zero (B) 1.86×10^{-5} C
(C) 1.77×10^{-11} C (D) 35.4×10^{-8} C

ANSWERS

(1) (A)	**(2)** (AC)	**(3)** (BC)
(4) (B)	**(5)** (B)	**(6)** (A)
(7) (ABD)	**(8)** (ABD)	**(9)** (C)
(10) (C)	**(11)** (B)	**(12)** (B)
(13) (D)		

CAPACITOR

CONCEPT OF CAPACITANCE

Capacitance of a conductor is a measure of ability of the conductor to store charge on it. When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor.

$Q \propto V$ or $Q = CV$

The constant C is known as the capacity of the conductor.

For Capacitance :

(i) Capacitance is a scalar quantity with dimension

$$C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$$

(ii) Unit : farad, coulomb/volt

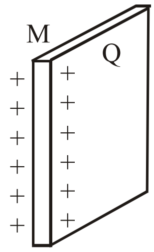
$$\text{If} = \frac{IC}{IV} = \frac{3\times10^9 \text{ esu of charge}}{(1/300) \text{ esu of potential}} = 9 \times 10^{11}$$

state farad (esu of capacity)

Capacitance depends upon :
(a) Size and Shape of Conductor
(b) Surrounding medium
(c) Presence of other conductors nearby

Capacitance of a conductor is independent of :
(a) Charge on conductor,
(b) Potential of conductor
(c) Nature of material and thickness of the conductor

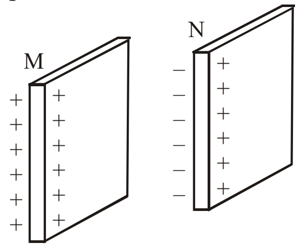
PRINCIPLE OF A CAPACITOR



The pair of conductor of opposite charges on which fixed quantity of charge may be accommodated is defined as capacitor. Consider a conducting plate M which is given a charge Q such that its potential rises to V, then $C = \frac{Q}{V}$

Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N,

$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

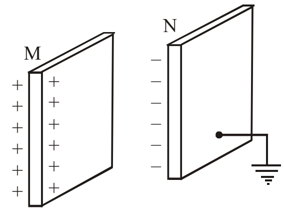


Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge).
$\Rightarrow$ C' > C

Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ ($\because$ the entire positive charge flows to the earth)

$$\therefore \quad C'' = \frac{Q}{V''} = \frac{Q}{V_+ - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.



TYPES OF CAPACITOR

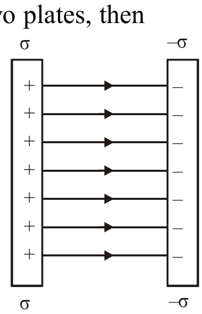
(A) Parallel Plate Capacitor

(i) Capacitance : It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then

I step : Finding electric field

$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_1} \qquad [\varepsilon = \varepsilon_0\,\varepsilon_r]$$



II step: Finding potential difference

$$V = Ed = \frac{\sigma}{\varepsilon_0 \varepsilon_r} d \; ; \quad V = \frac{q}{A\varepsilon_0\varepsilon_r} \qquad (\because \; E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A})$$

III step: Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

If medium between the plates is air or vacuum, then $\varepsilon_r = 1$

$\boxed{C_0 = \frac{\varepsilon_0 A}{d}}$ so $C = \varepsilon_r C_0 = KC_0$(where ε_r=K =dielectric const.)

(ii) Force between the plates : The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$$

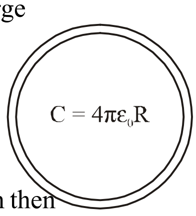
Force on negative charge –Q is $F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \frac{F}{A} = u = p = \frac{1}{2}\varepsilon_0 E^2$

(B) Spherical capacitor :

(i) Sperical conductor : When a charge Q is given to a isolated spherical conductor then its potential rises.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \; ; \; C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

If conductor is placed in a medium then

$C_{medium} = 4\pi\varepsilon R$ or $\boxed{C_{medium} = 4\pi\varepsilon_0\varepsilon_r R}$

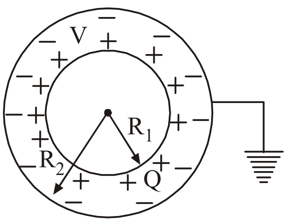
(ii) Outer sphere is earthed :

When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

E = 0 for $r < R_1$

and E = 0 for $r > R_2$

Potential of inner sphere



$$V_1 = \frac{Q}{4\pi\varepsilon_0 R_1} + \frac{-Q}{4\pi\varepsilon_0 R_2}$$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

As outer surface is earthed so potential $V_2 = 0$

Potential difference between plates

$$V = V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

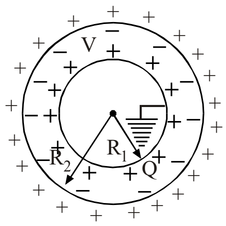
So $C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum}

In presence of medium between plate

$$\boxed{C = 4\pi\varepsilon_r\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}}$$

(iii) Inner sphere is earthed

Here the system is equivalent to a spherical capacitor of inner and outer radii R_1 and R_2 respectively and a spherical conductor of radius R_2 in parallel.

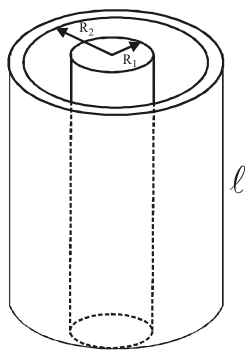


This is because charge Q given to outer sphere distributes in such a way that for the outer sphere charge on the inner side is $\frac{R_1}{R_2}Q$ and charge on the outer side is $Q - \frac{R_1}{R_2}Q = \frac{(R_2 - R_1)}{R_2}Q$

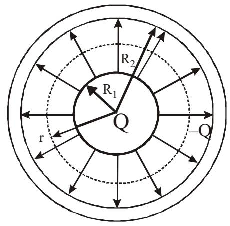
So total capacity of the system.

$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1} + 4\pi\varepsilon_0 R_2 \; ; \quad \boxed{C = \frac{4\pi\varepsilon_0 R_2^2}{R_2 - R_1}}$$

(C) Cylindrical Capacitor : When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded



(a)



(b)

(i) Electrical field between cylinders

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$$

(ii) Potential difference between plates

$$V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r\ell} dr = \frac{Q}{2\pi\varepsilon_0 \ell}\ \ell n\left(\frac{R_2}{R_1}\right)$$

(iii) Capacitance, $C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \ell}{\log_e(R_2/R_1)}$

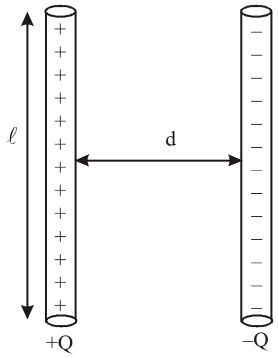
In presence of medium, $\boxed{C = \frac{2\pi\varepsilon_0\varepsilon_r \ell}{\log_e(R_2/R_1)}}$

(D) Two long parallel conductor

If $d >> a$ then

$$C = \frac{\pi\varepsilon_0 \ell}{\log_e(d/a)}$$

(where a is diameter of conductor)



Example 51 :

The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Sol. The capacitance of a spherical capacitor is

$$C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$$

b = radius of the top of stratosphere layer
$= 6400\text{ km} + 50\text{ km} = 6450\text{ km} = 6.45 \times 10^6$ m
a = radius of earth = 6400 km = 6.4×10^6 m

$$\therefore\quad C = \frac{1}{9\times10^9} \times \frac{6.45\times10^6 \times 6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092\text{ F}$$

Example 52 :

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

Sol. $\ell = 15\text{ cm} = 15 \times 10^{-2}$ m; $a = 14\text{ cm} = 1.4 \times 10^{-2}$ m ;
$b = 1.5\text{ cm} = 1.5 \times 10^{-2}$ m; $q = 3.5\text{ mC} = 3.5 \times 10^{-6}$ C

$$C = \frac{2\pi\varepsilon_0 \ell}{2.303\log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi\times8.854\times10^{-12}\times15\times10^{-2}}{2.303\log_{10}\frac{1.5\times10^{-2}}{1.4\times10^{-2}}}$$

$$= 1.21 \times 10^{-8}\text{ F}$$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder is

$$V = \frac{q}{C} = \frac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.89\times10^4\text{ V}$$

Example 53 :

The capacity of a condenser A is 1μF. It is filled with a medium of dielectric constant 15. The capacity of another condenser B is 1 μF. Both are separately charged by a battery of 100V. After charging the two condenser are connected in parallel without battery and without dielectric. Calculate the common potential.

Sol. $Q = Q_1 + Q_2 = C_1V_1 + C_2V_2 = 100[15+1] \times 10^{-6} = 1600\mu C$
$C_1 = \varepsilon_r C = 15 \times 1\mu F = 15\mu F$
$C'_1 = 1\mu F,\ C'_2 = 1\mu F$

$$C'_1 = \frac{C_1}{\varepsilon_r} = \frac{15}{15} = 1\mu F$$

$$V_{cm} = \frac{Q}{C'_1 + C'_2} = \frac{1600\times10^{-6}}{2\times10^{-6}} = 800\text{ V}$$

Example 54 :

If the distance between the plates of a capacitor is made half and the area of plates is doubled then what will be the capacitance.

Sol. $C = \frac{\varepsilon_0 A}{d} \Rightarrow \frac{C_1}{C_2} = \frac{A_1}{A_2}\frac{d_2}{d_1} = \frac{A_1}{2A_1} \times \left(\frac{1}{d_1}\right)\left(\frac{d_1}{2}\right) = \frac{1}{4}$

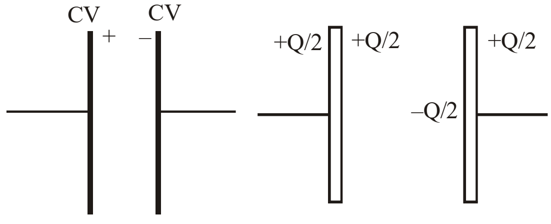
$\Rightarrow C_2 = 4C_1$

Example 55 :

A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now –

(A) V
(B) $V + \frac{Q}{C}$
(C) $V + \frac{Q}{2C}$
(D) $V - \frac{Q}{C}$, if $V < CV$

Sol. **(C).** After redistribution half of total charge remains on outer surface and then apply conservation of charge on each plate.



Charge on outer plates $= \frac{Q + CV - CV}{2} = \frac{Q}{2}$

Charge of innerface of first plate $= Q + CV - \frac{Q}{2} = \frac{Q}{2} + CV$

Charge on innerface of second plate $= -\left(\frac{Q}{2} + CV\right)$

$$V' = \frac{\frac{Q}{2} + CV}{C} = V + \frac{Q}{2C}$$

ENERGY STORED IN A CAPACITOR

Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV

Let battery supplies small amount of charge dq to the conductor at constant potential V.

Then small amount of work done by the battery against the force exerted by existing charge is

$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. this work done is stored as potential energy, so

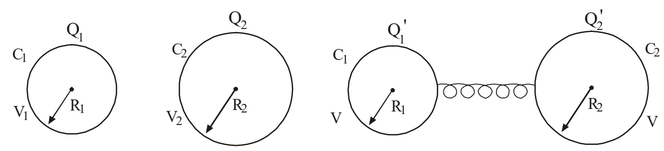
$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$$

$$\therefore \quad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

(When two charged conductors are mutually connected)

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal. Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



Common potential

According to law of Conservation of charge

$$Q_{\text{before connection}} = Q_{\text{after connection}}$$

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

Common potential after connection

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Charges after connection

$$Q_1' = C_1V = C_1\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_1}{C_1 + C_2}\right)Q$$

(Q : Total charge on system)

$$Q_2' = C_2V = C_2\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_2}{C_1 + C_2}\right)Q$$

Ratio of the charges after redistribution

$$\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2} \text{ (in case of spherical conductors)}$$

When charge flows through the conducting wire then energy is lost mainly on account of Joule effect, electrical energy is converted into heat energy, so change in energy of this system, $\Delta U = U_f - U_i$

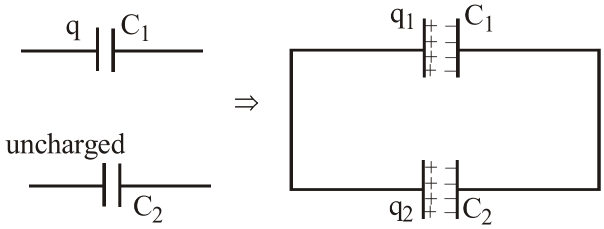
$$\Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right)$$

$$\Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

Here negative sign indicates that energy of the system decreases in the process.

In parallel plate capacitor we can consider 3 cases :

Case 1 : Charged capacitor (say C_1) connected to uncharged capacitor (say C_2)

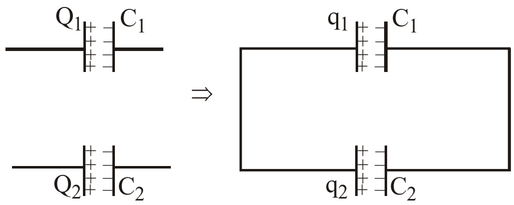


$q_1 + q_2 = q$

$q_1 = C_1V$ & $q_2 = C_2V$ (where V is common potential)

$$V = \frac{q}{C_1 + C_2}$$

Case 2 : Positive plate of capacitor C_1 connected to positive plate of another charged capacitor C_2.



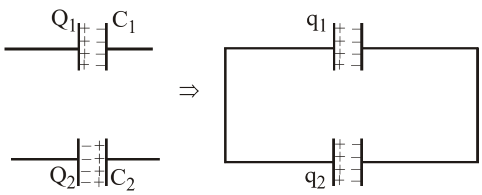
$Q_1 + Q_2 = q_1 + q_2$

$C_1V_1 + C_2V_2 = C_1V + C_2V$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{Q_1 + Q_2}{C_1 + C_2}$$

(Q_1 & Q_2 initial charge on capacitor, V final common potential)

Case 3 : Positive plate of C_1 connected to negative plate of C_2.



Assuming $Q_1 > Q_2$,

$$Q_1 - Q_2 = q_1 + q_2 = C_1V + C_2V$$

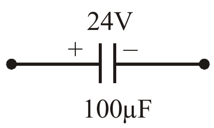
$$V = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

Example 56 :

A capacitor having a capacitance of 100μF is charged to a potential difference of 24V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12V with the positive plate of the capacitor joined with the positive terminal of the battery.

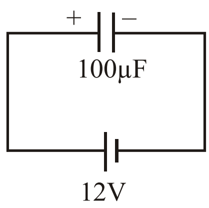
(a) Find the charges on the capacitor before and after the reconnection.
(b) Find the charge flown through the 12V battery
(c) Is work done by the battery or is it done on the battery? Find its magnitude.
(d) Find the decrease in electrostatic field energy.
(e) Find the heat developed during the flow of charge after reconnection.

Sol. (a) Charge on capacitor before connection



$Q_1 = CV_1 = 2400\ \mu C$

Charge on capacitor after connection



$Q_2 = CV_2 = 100 \times 12 = 1200\ \mu C$

(b) Charge flown through the 12V battery

$= 2400 - 1200 = 1200\ \mu C$

(c) Work is done on the battery

$= Q_2 \times V_2 = 1200 \times 12 = 14.4\ mJ$

(d) The decrease in electrostatic field energy

$$= U_i - U_f = \frac{1}{2}CV_1^2 - \frac{1}{2}CV_2^2$$

$$= \frac{1}{2} \times 100 \times (24)^2 - \frac{1}{2} \times 100 \times (12)^2 = 21.6mJ$$

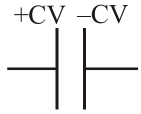
(e) $W = \Delta U + \Delta H$

$-14.4\ mJ = -21.6mJ + \Delta H\ [\Delta H = 7.2\ mJ]$

Example 57 :

A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V volt with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be –

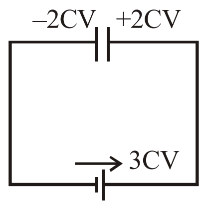
(A) 1.75 (B) 2.25
(C) 2.5 (D) 1/2



Sol. (B). $U_i = \frac{1}{2}CV^2$

$$U_f = \frac{1}{2}C\,(2V)^2 = 2CV^2$$

$W_{battery} = 3CV \times 2CV = 6\ CV^2$



Heat produced

$= W_{battery} - (U_f - U_i)$

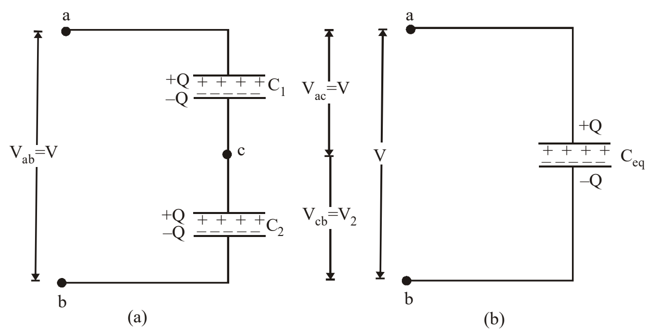
$$= 6CV^2 - \frac{3}{2}CV^2 = \frac{9}{2}CV^2$$

$$\frac{\text{Heat produced}}{U_f} = \frac{9}{4} = 2.25$$

COMBINATION OF CAPACITOR

Capacitor in series:

In this arrangement of capacitors the charge has no alternative path(s) to flow.



(i) The charges on each capacitor are equal i.e.

$Q = C_1V_1 = C_2V_2$

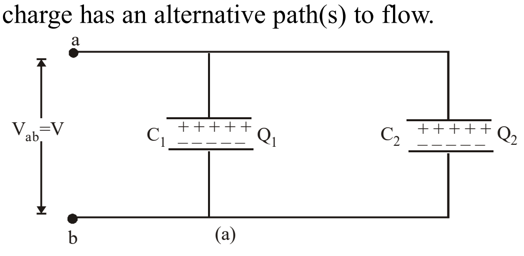
(ii) The total potential difference across ab is shared by the capacitors in the inverse ratio of the capacitances

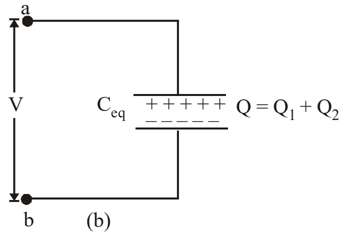
$V = V_1 + V_2$

If C_{eq} is the net capacitance of the series combination,

then $\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$; $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Capacitors in parallel : In such in arrangement of capacitors the charge has an alternative path(s) to flow.





(b)

(i) The potential difference across each capacitor is same and equal the total potential applied. i.e.

$$V = V_1 = V_2$$

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances.

$$Q = Q_1 + Q_2$$

If C_{eq} is the net capacitance for the parallel combination of capacitors :

$$C_{eq}V = C_1V + C_2V$$
$$C_{eq} = C_1 + C_2$$

Example 58 :

Capacitor C, 2C, 4C, ...∞ are connected in parallel, then what will be their effective capacitance?

Sol. Let the resultant capacitance be

$$C_{resultant} = C + 2C + 4C + ...\ \infty$$
$$= C[1 + 2 + 4 + ...\ \infty] = C \times \infty = \infty$$

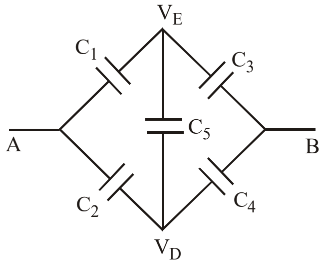
METHODS OF FINDING EQUIVALENT CAPACITANCE

We know that in series, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$ and

in parallel, $C_{eq} = C_1 + C_2 + ... + C_n$

Sometimes there are circuits in which capacitors are in mixed grouping. To find C_{eq} for such circuits few methods are suggested here which help you in finding C_{eq}.

1. **Method of same potential (Point Potential techniques) :** Give any arbitrary potentials (V_1, V_2 .... etc.) to all terminals of capacitors. But notice that the points connected directly by a conducting wire will have at the same potential. The capacitors having the same P.D are in parallel. Make a table corresponding to the figure. Now corresponding to this table a simplified figure can be formed and from this figure C_{eq} can be calculated.
2. **Infinite series problems :** This consists of an infinite series of identical loops. To find C_{eq} of such a series first we consider by ourself a value (say x) of C_{eq}. Then we break the chain in such a manner that only one loop is left with us and in place of the remaining portion we connect a capacitor x. Then we find the C_{eq} and put it equal to x. With this we get a quadratic equation in x. By solving this equation we can find the desired value of x.
3. **Symmetrical method :** Points having symmetrically located about initial and final points have same potential. So, the capacitors between these points can be ignored.
4. **Connection removal method :** This method is useful when the circuit diagram is symmetric except for the fact that the input and output are reversed. That is the flow of charge is a mirror image between input and output above a particular axis. In such cases some junctions are unnecessarily made. Even if we remove that junction there is no difference in the remaining circuit or charge distribution. But after removing the junction, the problem becomes very simple.
5. **Balanced Wheatstone's Bridge :**



If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, bridge is said to be balanced and in that case.

$V_E = V_D$ or $V_E - V_D = 0$ or $V_{ED} = 0$

i.e., no charge is stored in C_5. Hence, it can be removed from the circuit.

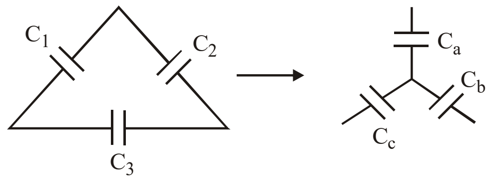
$$C_{AB} = (C_1 S\, C_3) \parallel (C_2 S\, C_4)$$

$$C_{AB} = \frac{C_1C_3}{C_1 + C_3} + \frac{C_2C_4}{C_2 + C_4}$$

6. **Unbalanced Wheatstone's Bridge :**

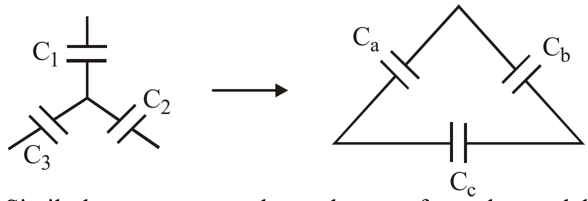
For solving unbalanced Wheatstone's bridge use Star-delta conversion method.

A delta network is transformation to a star-network according to the formula



$$C_a = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_3},\quad C_b = \frac{C_1C_2 + C_2C_3 + C_2C_1}{C_1}$$

$$C_c = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_2}$$



Similarly, a star network can be transformed to a delta network according to the formula.

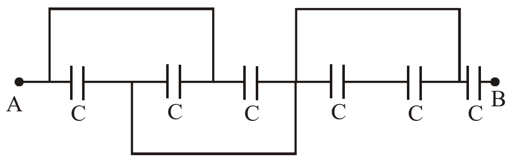
$$C_a = \frac{C_1C_3}{C_1 + C_2 + C_3},\quad C_b = \frac{C_1C_2}{C_1 + C_2 + C_3},$$

$$C_c = \frac{C_2C_3}{C_1 + C_2 + C_3}$$

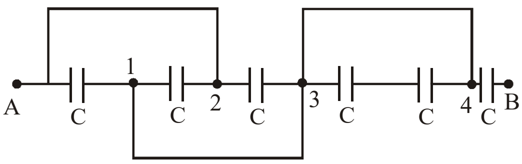
7. **By distributing charge :** In this method assume a main charge q. Distribute it in different capacitors as q_1, q_2 .... etc. Using Kirchhoff's laws (Refer Current electricity chapter) find q_1, q_2 .... etc. in terms of q. Then find the potential difference between starting and end point through any path and equate it with q/C_{net}. By doing so we can calculate C_{net}.

Example 59 :

Find equivalent capacitance between points A and B shown in figure.

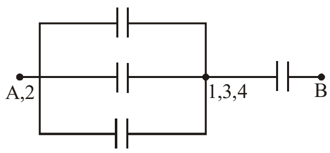


Sol. Number the junction as shown



$V_A = V_2$
$V_1 = V_3 = V_4$

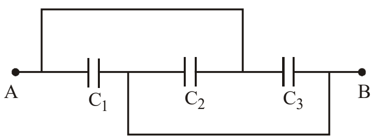
Redraw the network as



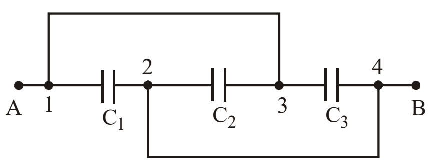
Now, 3C and C are in series and their equivalent capacitance is, $C_{eq} = \dfrac{(3C)\,(C)}{3C + C} = \dfrac{3}{4}C$

Example 60 :

Find equivalent capacitance between A and B.



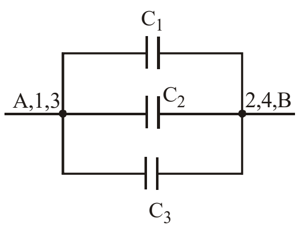
Sol. Number the junction as shown



$V_A = V_1 = V_3$
$V_2 = V_4 = V_B$

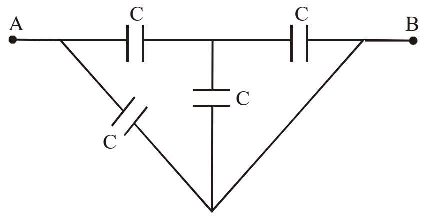
Redraw the network

$C_{eq} = C_1 + C_2 + C_3$

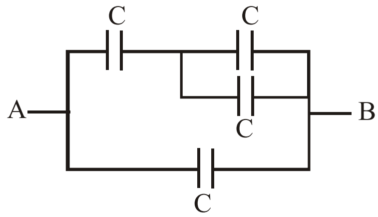


Example 61 :

Find equivalent capacitance between points A and B.



Sol. Circuit can be redrawn as



$\Rightarrow$ Parallel combination, $C_{eq} = C + C = 2C$

Series combination

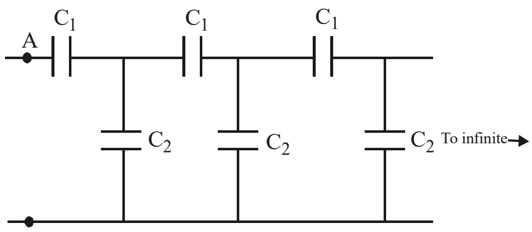
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C} \quad \text{So, } C_{eq} = \frac{2}{3}C$$

Parallel combination, $C_{eq} = \dfrac{2}{3}C + C = \dfrac{5}{3}C$

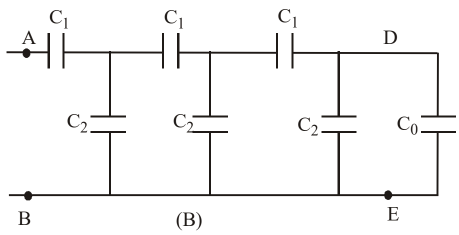
Example 62 :

(a) Find the effective capacitance between A and B of an infinite chain of capacitors joined as shown in Fig. (A).

(b) For what value of C_0 in the circuit shown in Fig. (B) will be net effective capacitance between A and B be independent of the number of capacitor in the chain ?

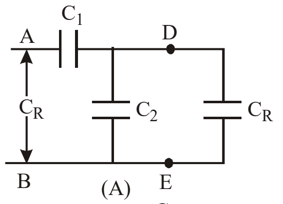


(A)

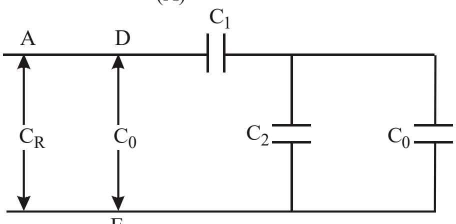


(B)

Sol. (a) Suppose the effective capacitance between A and B is C_R. Since the network is infinite, even if we remove one repeating unit from the chain remaining network would still have infinite cells, i.e., effective capacitance between DE would also be C_R.



(A)



(B)

In other words the given infinite chain is equivalent to capacity C_1 in series with combination of C_2 and C_R in parallel as shown in Fig. (A). So

$$C_R = C_1 \, S \, [C_2 + C_R]$$

i.e., $C_R = \dfrac{C_1(C_2 + C_R)}{C_1 + C_2 + C_R}$ or $C_R^2 + C_2C_R - C_1C_2 = 0$

i.e., $C_R = \dfrac{1}{2}\left[-C_2 \pm \sqrt{C_2^2 + 4C_1C_2}\right]$

And as capacitance cannot be negative, only permissible value of C_R is :

$$C_R = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

However, if $C_1 = C_2 = C$; $C_R = [\sqrt{5} - 1]C/2$

(b) Suppose there are n capacitor between A & B and the network is terminated by C_0 with equivalent capacitance C_R [Fig. (B)], Now if we add one more capacitor to the network between D and E, the equivalent capacitance of the network C_R will be independent of number of cells if the capacitance between D and E still remains C_0, i.e., $C_1 \, S \, [C_2 + C_0] = C_0$ or $\dfrac{C_1(C_2 + C_0)}{[C_1 + C_2 + C_0]} = C_0$

i.e., $C_0^2 + C_2C_0 - C_1C_2 = 0$

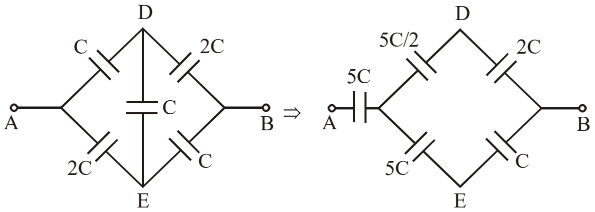
Which on simplification gives :

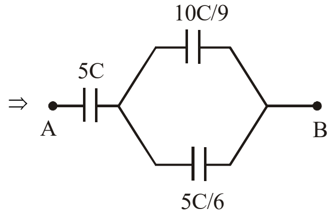
$$C_0 = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

Example 63 :

Find equivalent capacitance across AB.

Sol. Using Star-Delta conversion

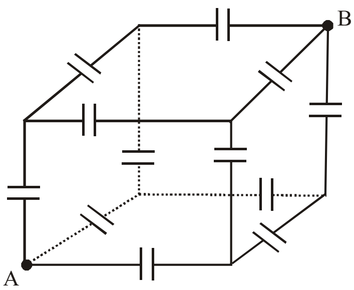




This implies that $C_{eff} = C_{AB} = \dfrac{(105C/54) \times 5C}{(375C/54)} = \dfrac{7C}{5}$

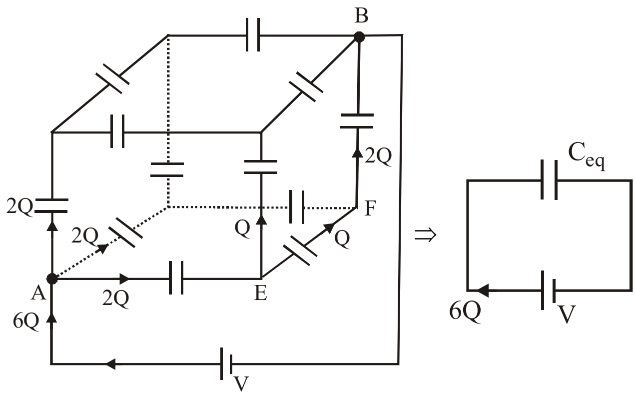
Example 64 :

Twelve capacitors, each having a capacitance C, are connected to form a cube. Find the equivalent capacitance between the diagonally opposite corners such as A and B.



Sol. Connect a battery between point A and B. Let say charge 6Q flows from battery.

Due to symmetry charge divides as shown in figure.



From Kirchhoff's rule for loop ABFBA

$$\frac{2Q}{C} + \frac{Q}{C} + \frac{2Q}{C} - V = 0 \qquad \text{....... (1)}$$

From equivalent capacitance concept

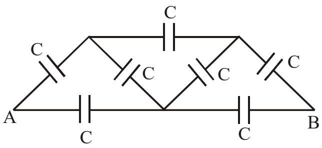
$$6Q = C_{eq}V \qquad \text{....... (2)}$$

From (1) and (2)

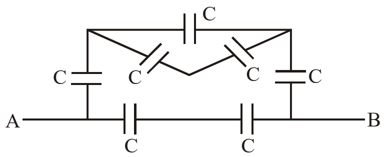
$$\frac{5Q}{C} = \frac{6Q}{C_{eq}} \; ; \quad C_{eq} = \frac{6C}{5}$$

Example 65 :

Find the equivalent capacitance between points A and B.

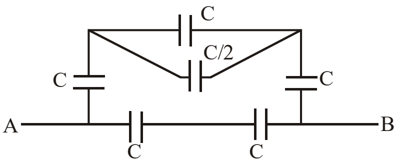


Sol.



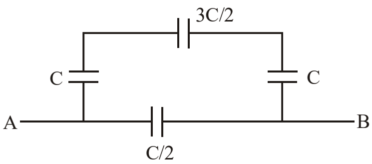
$$\frac{1}{C}+\frac{1}{C}=\frac{2}{C} \Rightarrow C_1=\frac{C}{2}$$

$\Rightarrow$



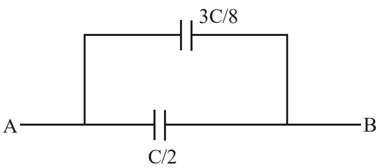
$$C+\frac{C}{2}=\frac{3C}{2} \Rightarrow C_2=\frac{3C}{2}$$

$\Rightarrow$



$$\frac{1}{C}+\frac{1}{C}+\frac{2}{3C}=\frac{8}{3C} \Rightarrow C_3=\frac{3C}{8}$$

$\Rightarrow$

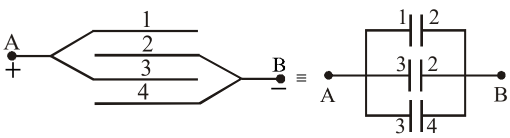


$$\frac{C}{2}+\frac{3C}{8}=\frac{7C}{8} \Rightarrow C_{eq}=\frac{7C}{8}$$

Example 66 :

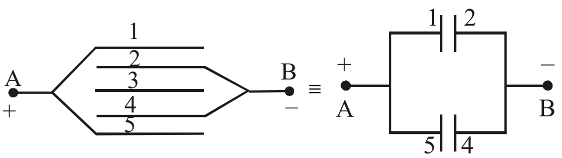
Find the equivalent capacitance between A and B.

(i)



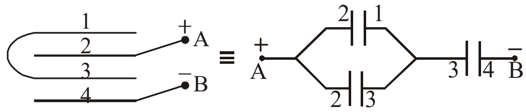
$\Rightarrow$ $C' = 3C_0$

(ii)



$\Rightarrow C' = 2C_0$

(iii)



$$\Rightarrow C' = \frac{2}{3}C_0$$

EFFECT OF DIELECTRIC

1. The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as dielectrics.

2. Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called dielectric strength they lose their insulating property and begin to conduct.

3. Dielectric strength is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre Dimensions $M^1L^1T^{-3}A^{-1}$.

4. **Polar dielectrics :**

(i) In absence of external field the centre of positive and negative charge do not coincide-due to asymmetric shape of molecules.

(ii) Each molecule has permanent dipole moment.

(iii) The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.

(iv) In presence of external field dipoles tends to align in direction of field.

(v) Ex. water, alcohol, CO_2, HCl, NH_3

5. **Non polar dielectrics :**

(i) In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.

(ii) The dipole moment is zero in normal state.

(iii) In presence of external field they acquire induced dipole moment.

(iv) Ex. nitrogen, oxygen, benzene, methane

6. **Polarisation :** The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

7. Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric

$$E_0=\frac{V_0}{d}=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 A}$$

Electric field in presence of dielectric

$$E=E_0-E_1=\frac{\sigma-\sigma_i}{\varepsilon_0}=\frac{Q-Q_i}{\varepsilon_0}=\frac{V}{d}$$

Capacitance in absence of dielectric, $C_0=\dfrac{Q}{V_0}$

Capacitance in presence of dielectric, $C = \dfrac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K

or $\varepsilon_r = \dfrac{E_0}{E} = \dfrac{V_0}{V} = \dfrac{C}{C_0} = \dfrac{Q}{Q - Q_i} = \dfrac{\sigma}{\sigma - \sigma_i} = \dfrac{\varepsilon}{\varepsilon_0}$

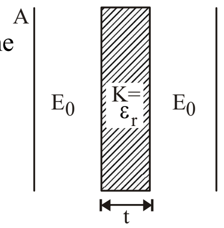
From $K = \dfrac{Q}{Q - Q_i}$

$$Q_1 = Q\left(1 - \frac{1}{K}\right) \text{ and } K = \frac{\sigma}{\sigma - \sigma_i} \quad \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

CAPACITY OF DIFFERENT CONFIGURATION

In case of parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$

If capacitor is partially filled with dielectric of thickness t (t < d)



If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \dfrac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \dfrac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space (d – t). If V is total potential then

$$V = E_0(d - t) + E\,t$$

$$\Rightarrow \quad V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right]$$

$$\because \quad \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right]$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \qquad \text{..........(1)}$$

If medium is fully present between the space. $\because$ t = d

Now from equation (1), $C_{\text{medium}} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

If capacitor is partially filled by a conducting slab of thickness (t < d).

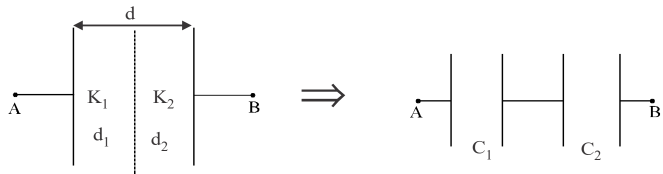
$\because \quad \varepsilon_r = \infty$ for conductor

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} \quad ; \quad C = \frac{\varepsilon_0 A}{(d - t)}$$

FORMING OF CAPACITOR

* **Distance division :**

Case 1 :



(i) Distance is divided and area remains same.

(ii) Capacitors are in series.

$$C = \frac{K\varepsilon_0 A}{d}, \quad C_1 = \frac{K_1\varepsilon_0 A}{d_1}, \quad C_2 = \frac{K_2\varepsilon_0 A}{d_2}$$

These two in series

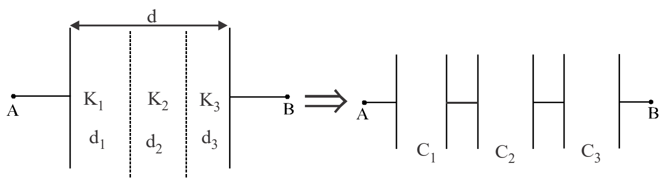
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{K_1\varepsilon_0 A} + \frac{d_2}{K_2\varepsilon_0 A}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{K_1K_2}{K_2d_1 + K_1d_2}\right]$$

$$\text{If } d_1 = d_2 = \frac{d}{2} \quad \Rightarrow \quad C = \frac{\varepsilon_0 A}{d}\left[\frac{2K_1K_2}{K_1 + K_2}\right]$$

$$\therefore \quad \boxed{K_{eq} = \frac{2K_1K_2}{K_1 + K_2}}$$

Case 2 : Similarly if three dielectric slabs are arranged as shown in figure.



$$C_1 = \frac{K_1\varepsilon_0 A}{d_1}, \quad C_2 = \frac{K_2\varepsilon_0 A}{d_2}, \quad C_3 = \frac{K_3\varepsilon_0 A}{d_3}$$

They are in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$= \frac{d_1}{K_1\varepsilon_0 A} + \frac{d_2}{K_2\varepsilon_0 A} + \frac{d_3}{K_3\varepsilon_0 A}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right]$$

$$\frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1K_2K_3 + d_2K_1K_3 + d_3K_1K_2}{K_1K_2K_3}\right]$$

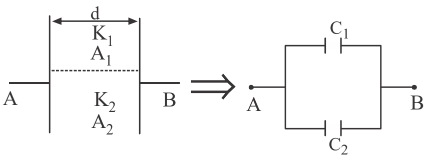
$$C = \varepsilon_0 A\left[\frac{K_1K_2K_3}{d_1K_2K_3 + d_2K_1K_3 + d_3K_1K_2}\right]$$

If $d_1 = d_2 = d_3 = \dfrac{d}{3}$

$$C = \frac{\varepsilon_0 A}{d}\left[\frac{3K_1K_2K_3}{K_1K_2 + K_2K_3 + K_3K_1}\right]$$

$$\therefore \quad \boxed{K_{eq} = \frac{3K_1K_2K_3}{K_1K_2 + K_2K_3 + K_3K_1}}$$

* **Area division :**

Case 3 : 

(i) Area is divided and distance remains same.
(ii) Capacitors are in parallel.

$$C_1 = \frac{K_1\varepsilon_0 A_1}{d}, \quad C_2 = \frac{K_2\varepsilon_0 A_2}{d}$$

C_1 and C_2 are in parallel

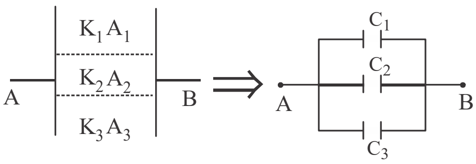
$$C = C_1 + C_2 = \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d}$$

$$C = \frac{\varepsilon_0}{d}(K_1A_1 + K_2A_2)$$

If $A_1 = A_2 = \dfrac{A}{2}$; $\quad C = \dfrac{\varepsilon_0 A}{d}\left(\dfrac{K_1 + K_2}{2}\right)$

$$\therefore \quad \boxed{K_{eq} = \frac{K_1 + K_2}{2}}$$

Similarly for three dielectrics



$$C = C_1 + C_2 + C_3$$

$$= \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d} + \frac{K_3\varepsilon_0 A_3}{d}$$

$$C = \frac{\varepsilon_0}{d}(K_1A_1 + K_2A_2 + K_3A_3)$$

If $A_1 = A_2 = A_3 = \dfrac{A}{3}$; $\quad C = \dfrac{\varepsilon_0 A}{d}\left(\dfrac{K_1 + K_2 + K_3}{3}\right)$

$$\therefore \quad \boxed{K_{eq} = \frac{K_1 + K_2 + K_3}{3}}$$

Example 67 :

For making a parallel plate capacitor two plates of Cu, which of the following is the most appropriate dielectric? Make calculations.
(a) A sheet of mica (thickness = 0.10 mm, K = 5.4)
(b) A sheet of glass (thickness = 0.20 mm, K = 7)
(c) A sheet of paraffin (thickness = 1 cm, K = 2)

Sol. $C = \dfrac{K\varepsilon_0 A}{d}$ as A = area of Cu plates which is constant

$\therefore$ For maximum C, K/d should be maximum

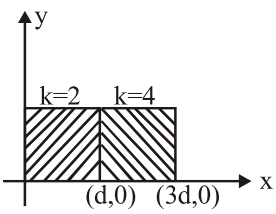
For mica $= \dfrac{K}{d} = \dfrac{5.4}{0.1} = 54$,

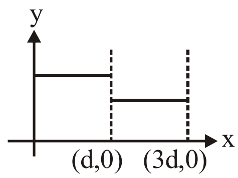
For glass $= \dfrac{K}{d} = \dfrac{7}{0.2} = 35$

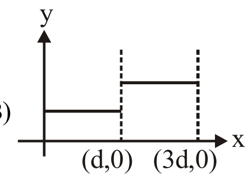
and For paraffin $= \dfrac{K}{d} = \dfrac{2}{10} = 0.2$

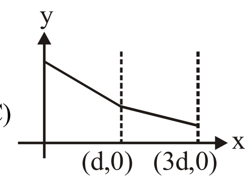
Example 68 :

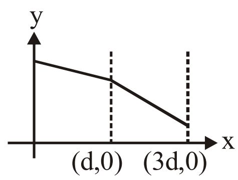
A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



(A) 

(B) 

(C) 

(D) 

Sol. **(A).** Field in dielectric is E/K when E is the field in air.

Example 69 :

A parallel plate capacitor with air between the plates has a capacitance of 8 pF. Calculate the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant. ($\varepsilon_r = 6$)

Sol. Capacity of parallel plate capacitor

$C = \dfrac{\varepsilon_r\varepsilon_0 A}{d}$ (For air ε_r = I). So, $\dfrac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$

If $d \to d/2$ and $\varepsilon_r \to 6$, then new capacitance

$$C' = 6 \times \frac{\varepsilon_0 A}{d/2} = 12\frac{\varepsilon_0 A}{d} = 12 \times 8\ \text{pF} = 96\ \text{pF}$$

Example 70 :

A capacitor has two circular plates whose radius are 8cm and distance between them is 1mm. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Sol. Area of plate $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201\ m^2$

and $d = 1mm = 1 \times 10^{-3}$ m

Capacity of capacitor

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 0.0201}{1 \times 10^{-3}} = 1.068 \times 10^{-9}\ F$$

Potential difference, V = 150 volt

Energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}\ J$$

VARIATION IN PPC UNDER TWO CONDITIONS

(a) Battery disconnected (after charging) $\rightarrow$ Q = const.

(b) Battery remains connected $\Rightarrow$ V $\rightarrow$ const.

$$Q = CV,\ C = \frac{\varepsilon_0 \varepsilon_r A}{d},\ \frac{Q}{A\varepsilon_0 \varepsilon_r} = \frac{V}{d} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$$

(a) Battery removed (Q $\rightarrow$ const.)

(i) Initially	Q_0	V_0	E_0	C_0	U_0
(ii) Insert a dielectric slab.	$\rightarrow$	$\downarrow K$	$\downarrow K$	$\uparrow K$	$\downarrow K$
(iii) Separation d$\uparrow$	$\rightarrow$	$\uparrow$	$\rightarrow \left(\frac{\sigma}{\varepsilon_0}\right)$	$\downarrow$	$\uparrow$
(iv) Eff. area A'$\uparrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$

(b) Battery remains connected (V $\rightarrow$ const.)

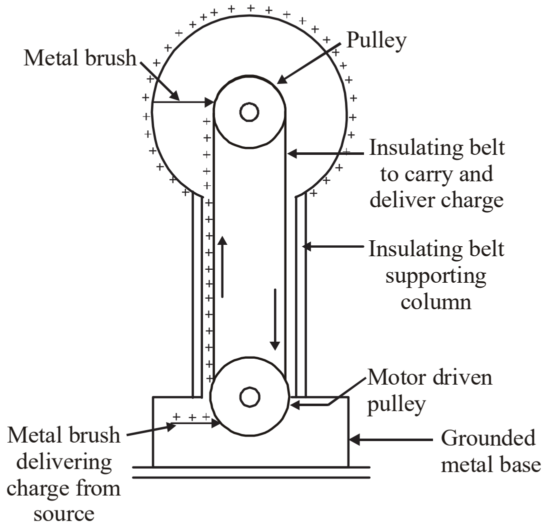
(i) Initially	Q_0	V_0	E_0	C_0	U_0
(ii) Insert a dielectric slab.	$\uparrow K$	$\rightarrow$	$\rightarrow$	$\uparrow K$	$\uparrow K$
(iii) Separation d$\uparrow$	$\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\downarrow$
(iv) Eff. area A'$\uparrow$	$\uparrow$	$\rightarrow$	$\rightarrow$	$\uparrow$	$\uparrow$

VAN DE GRAAFF GENERATOR

* It is a machine capable of building up potential difference of a few million volts, and fields close to the breakdown field of airwhich is about 3×10^6 V/m.
* **Principle :** It is based on the following principles:

(1) Action of sharp points: Charges are leaked from pointed ends of charged conductors. This creates an electric wind (as moving air is ionized) which moves away from the conductor.

(2) The property that the charge given to a hollow conductor is transferred to the outer surface and is distributed uniformly on it.



Working : A pulley drives an insulating belt by a sharply pointed metal comb which has been given a positive charge by a power supply. Electrons are removed from the belt, leaving it positively charged. A similar comb at the top allows the net positive charge to spread to the dome. The uncharged belt again collects the positive charge and the process continues. Thus, the positive charge goes on accumulating and hence the potential goes on increasing.

Example 71 :

A parallel plate air capacitor is made using two plates 0.2 m square, spaced 1 cm apart. It is connected to a 50 V battery.

(a) What is the capacitance ?

(b) What is the charge on each plate ?

(c) What is the energy stored in the capacitor ?

(d) What is the electric field between the plates ?

(e) If the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts ?

Sol. (a) $C_0 = \frac{\varepsilon_0 A}{d_0} = \frac{8.85 \times 10^{-12} \times 0.2 \times 0.2}{0.01} = 3.54 \times 10^{-5}\ \mu F$

(b) $Q_0 = C_0 V_0 = (3.54 \times 10^{-5} \times 50) = 1.77 \times 10^{-3}\ \mu C$

(c) $U_0 = (1/2)\, C_0 V_0^2 = 1/2\, (3.54 \times 10^{-11})\, (50)^2 = 4.42 \times 10^{-8}$ J

(d) $E_0 = \frac{V_0}{d_0} = \frac{50}{0.01} = 5000 V/m.$

(e) If the battery is disconnected, the charge on the capacitor plates remains constant while the potential difference between plates can change.

$$C = \frac{\varepsilon_0 A}{2d} = 1.77 \times 10^{-5}\ \mu F;\ Q = Q_0 = 1.77 \times 10^{-3}\ \mu C$$

$$V = \frac{Q}{C} = \frac{Q_0}{C_0 / 2} = 2V_0 = 100 \text{ volts}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{(C_0/2)} = 2U_0 = 8.84 \times 10^{-8}\ J$$

$$E = \frac{2V_0}{2d_0} = E_0 = 5000\ V/m$$

work has to be done against the attraction of plates when they are separated. This gets stored in the energy of the capacitor.

Example 72 :

A parallel plate capacitor is charged and then battery is disconnected. If the plates of the capacitor are moved further apart by means of insulating handles, then how will the charge, voltage across the plates and capacitance change.

Sol. When the battery is disconnected, the charge will remains same in any case.

Capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

When d is increased, capacitance will decreases and because the charge remains the same. So hence the electrostatic energy stored in the capacitor will increases.

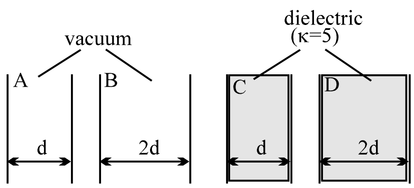
Change In stored energy $= \frac{1}{2}Q(V_2 - V_1) = +ve$

(We know that $V_2 > V_1$)

TRY IT YOURSELF - 6

For Q 1-2

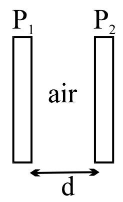
The figure below shows four parallel plate capacitors : A, B, C and D. Each capacitor carries the same charge q and has the same plate area A. As suggested by the figure, the plates of capacitors A and C are separated by a distance d while those of B and D are separated by a distance 2d. Capacitors A and B are maintained in vacuum while capacitors C and D contain dielectrics with constant $\kappa = 5$.



Q.1 Which list below places the capacitors in order of increasing capacitance?
(A) A, B, C, D (B) B, A, C, D
(C) B, A, D, C (D) A, B, D, C

Q.2 Which capacitor has the largest potential difference between its plates?
(A) A (B) B (C) D
(D) A and D are the same and larger than B or C

Q.3 Two conducting large plates P_1 & P_2 are placed parallel to each other at very small separation 'd'. The plate area of either face of plate is A. A charge +2Q is given to plate P_1 & –Q to the plate P_2 (neglect ends effects). If plate P_1 & P_2 are now connected by conducting wire, then total amount of heat produced is

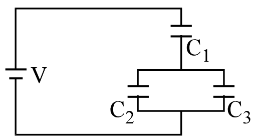


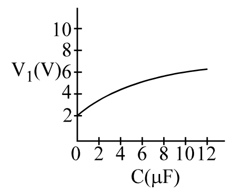
(A) $\frac{4Q^2 d}{3 \in_0 A}$ (B) $\frac{9}{8}\frac{Q^2 d}{\in_0 A}$

(C) $\frac{3Q^2 d}{4 \in_0 A}$ (D) None of these

For Q.4-Q.5

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymtote of 10 V as $C_3 \to \infty$.



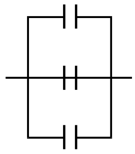


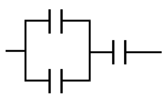
Q.4 The electric potential V across the battery is equal to:
(A) 10 V (B) 12 V
(C) 16 V (D) 20 V

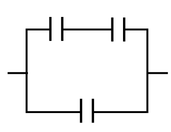
Q.5 C_1/C_2 has value:
(A) 4 (B) 1/4
(C) 2 (D) 1/2

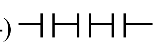
Q.6 The plates of a parallel-plate capacitor are separated by a solid dielectric. This capacitor and a resistor are connected in series across the terminals of a battery. Now the plates of the capacitor are pulled slightly farther apart. When equilibrium is restored in the circuit.
(A) the potential difference across the plates has increased
(B) the energy stored in the capacitor has increased
(C) the capacitance of the capacitor has increased
(D) the charge on the plates of the capacitor has decreased

Q.7 In four options below, all the four circuits are arranged in order of equivalent capacitance. Select the correct order. Assume all capacitors are of equal capacitance.

(1) 

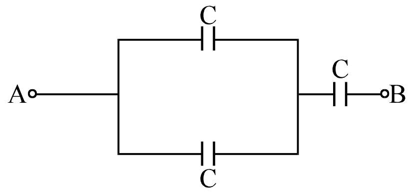
(2) 

(3) 

(4) 

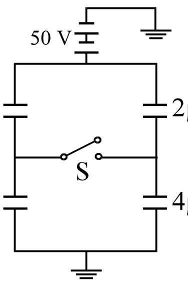
(A) $C_1 > C_2 > C_3 > C_4$ (B) $C_1 > C_3 > C_2 > C_4$
(C) $C_1 < C_2 < C_3 < C_4$ (D) $C_1 < C_3 < C_2 < C_4$

Q.8 In the network shown we have three identical capacitors. Each of them can withstand a maximum 100 V p.d. What maximum voltage can be applied across A and B so that no capacitor gets spoiled?



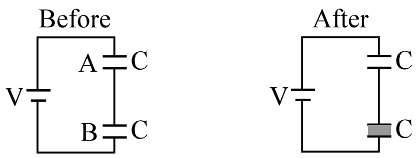
(A) 150 V (B) 120 V
(C) 180 V (D) 200 V

Q.9 The circuit was in the shown state from a long time. Now the switch S is closed. The charge that flows through the switch is



(A) $\frac{400}{3}\mu C$
(B) 100 μC
(C) 50 μC
(D) $\frac{100}{3}\mu C$

Q.10 Two identical capacitors are connected in series as shown in the figure. A dielectric slab ($\kappa > 1$) is placed between the plates of the capacitor B and the battery remains connected. Which of the following statement(s) is/are correct following the insertion of the dielectric?



(A) The charge supplied by the battery increases.
(B) The capacitance of the system increases.
(C) The electric field in the capacitor B increases.
(D) The electrostatic potential energy decreases.

Q.11 The plates of a parallel plate capacitor are charged upto 100 volt. A 2 mm thick plate is inserted between the plates, then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. The dielectric constant of the plate is
(A) 5 (B) 1.25
(C) 4 (D) 2.5

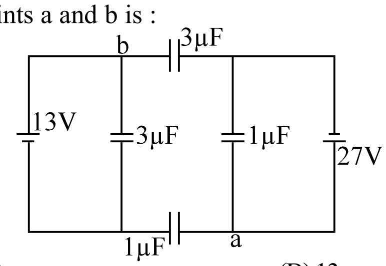
Q.12 A capacitor of capacitance of 2 μF is charged to a potential difference of 200 V, after disconnecting from the battery, it is connected in parallel with another uncharged capacitor. The final common potential is 20V, the capacitance of second capacitor is:
(A) 2 μF (B) 4 μF
(C) 18 μF (D) 16 μF

Q.13 The potential across a 3 μF capacitor is 12 V when it is not connected to anything. It is then connected in parallel with an uncharged 6 μF capacitor. At equilibrium, the charge q on the 3 μF capacitor and the potential difference V across it are
(A) q = 12μC, V = 4V (B) q = 24μC, V = 8 V
(C) q = 36μC, V = 12 V (D) q = 12μC, V = 6V

Q.14 A parallel plate capacitor is charged using a battery, and the battery is then removed. The plates of the capacitor are then brought closer together. Which of the following statements is false ?
(A) electric field inside the capacitor remains the same
(B) The capacitance of the capacitor increases
(C) The charge on the capacitor remains the same
(D) The energy stored in the capacitor increases

Q.15 Seven capacitors, each of capacitance 2μF are to be connected to obtain a capacitance of 10/11 μF. Which of the following combinations is possible?
(A) 5 in parallel 2 in series (B) 4 in parallel 3 in series
(C) 3 in parallel 4 in series (D) 2 in parallel 5 in series

Q.16 Four capacitors and two sources of e.m.f. are connected as shown in the figure. The p.d. in volts between the points a and b is :

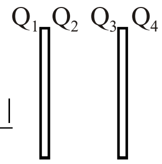


(A) zero (B) 13
(C) 17 (D) 27

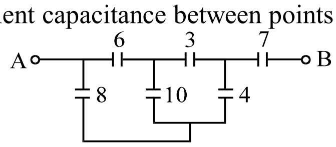
Q.17 In an isolated charged capacitor of capacitance 'C', the four surfaces have charges Q_1, Q_2, Q_3 and Q_4 as shown. Potential difference between the plates of the capacitor is

(A) $\frac{Q_1 + Q_2 + Q_3 + Q_4}{C}$ (B) $\frac{Q_2 + Q_3}{C}$

(C) $\frac{|Q_2 - Q_3|}{2C}$ (D) $\frac{|Q_1 - Q_4|}{2C}$



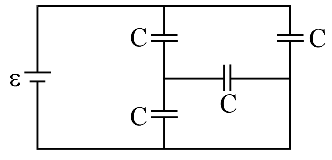
Q.18 In the circuit diagram shown all the capacitors are in μF. The equivalent capacitance between points A and B is (in μF)



(A) 14/5 (B) 7/5
(C) 3/7 (D) None of these

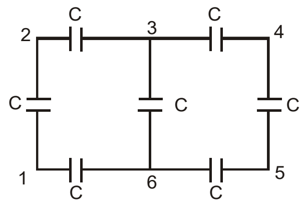
Q.19 Two capacitors C_1 and C_2 are connected in series, assume that $C_1 < C_2$. The equivalent capacitance of this arrangement is C, where
(A) $C < C_1/2$ (B) $C_1/2 < C < C_1$
(C) $C_1 < C < C_2$ (D) $C_2 < C < 2C_2$

Q.20 For the arrangement of identical capacitors shown, what is the equivalent capacitance ?



(A) 5C/2 (B) 5C/3
(C) 2C/3 (D) 3C/5

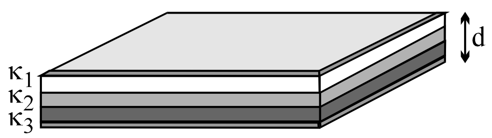
Q.21 For the given circuit, select the correct alternative(s)



(A) The equivalent capacitance between points 1 & 2 is (15C/11).
(B) The equivalent capacitance between points 3 & 6 is (5C/3).
(C) The equivalent capacitance between points 1 & 3 is (15C/14).
(D) The equivalent capacitance between points 3 & 5 is (14C/15).

Q.22 Three capacitors each of capacity 4μF are to be connected in such a away that the effective capacitance becomes 6μF. This can be done by connecting
(A) all of them in series
(B) all of them in parallel
(C) two in series and the third parallel to the combination
(D) two in parallel and the third in series with the combination.

Q.23 A parallel plate capacitor is filled with 3 dielectric materials of same thickness, as shown in the sketch. The dielectric constants are such that $\kappa_3 > \kappa_2 > \kappa_1$. Let the magnitudes of the electric field in and potential drops across each dielectric be $E_3, E_2, E_1, \Delta V_3, \Delta V_2,$ and ΔV_1, respectively. Which one of the following statements is true?



(A) $E_3 < E_2 < E_1$ and $\Delta V_3 < \Delta V_2 < \Delta V_1$
(B) $E_3 > E_2 > E_1$ and $\Delta V_3 > \Delta V_2 > \Delta V_1$
(C) $E_3 < E_2 < E_1$ and $\Delta V_3 > \Delta V_2 > \Delta V_1$
(D) $E_3 > E_2 > E_1$ and $\Delta V_3 < \Delta V_2 < \Delta V_1$

Q.24 A capacitor of capacity C_0 is connected to a battery of emf V_0. When steady state is attained a dielectric slab of dielectric constant K is slowly introduced in the capacitor to fill the capacitor completely. Mark the correct statement(s), in final stady state.
(A) Magnitude of induced charge on the each surface of slab is $C_0V_0(K-1)$
(B) Electric force due to induced charges on any plate is zero.
(C) Force of attraction between plates of capacitor is

$$\frac{K(C_0V_0)^2}{2\in_0 A}$$

(D) Field due to induced charges in dielectric slab is

$$\frac{(K-1)\,C_0V_0}{\in_0 A}$$

Q.25 Dielectric slab fills the space between the plates of a parallel-plate capacitor. The magnitude of the bound charge on the slab is 75% of the magnitude of the free charge on the plates. The capacitance is 480μF and the maximum charge that can be stored on the capacitor is $240\varepsilon_0 L^2 E_{max}$, where E_{max} is the breakdown field.
(A) the dielectric constant for the dielectric slab is 4
(B) without the dielectric, the capacitance of the capacitor would be 360μF.
(C) the plate area is $60\ L^2$.
(D) if the dielectric slab is having the same area as the capacitor plate but the width half that of the capacitor, the capacitance would be 192μF.

Q.26 In a spherical capacitor, we have two concentric spherical shells, the inner one carrying a charge Q and outer one carrying a charge of –Q. If the inner shell is displaced from the center without touching the outer, shell,
(A) the capacitance of the capacitor will increase
(B) the capacitance of the capacitor will remain same.
(C) the energy of capacitor with decrease.
(D) the potential difference between inner and outershell will increase.

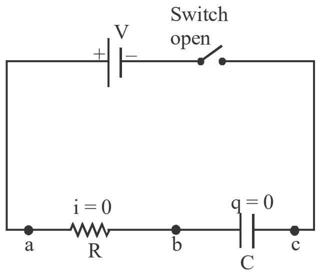
ANSWERS

(1) (C)	**(2)** (B)	**(3)** (B)
(4) (A)	**(5)** (A)	**(6)** (D)
(7) (B)	**(8)** (A)	**(9)** (C)
(10) (AB)	**(11)** (A)	**(12)** (C)
(13) (A)	**(14)** (D)	**(15)** (A)
(16) (C)	**(17)** (C)	**(18)** (A)
(19) (B)	**(20)** (B)	**(21)** (ABC)
(22) (C)	**(23)** (A)	**(24)** (ABD)
(25) (ACD)	**(26)** (AC)	

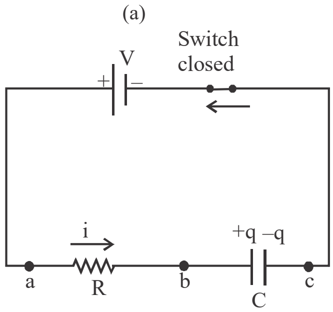
CHARGING & DISCHARGING OF A CAPACITOR

Charging

When a capacitor, resistance, batter, and key is connected in series and key is closed



(a)



(b)

(a) Charged at any instant

$V = V_C + V_R$

$$V = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$$

$$Q = CV\left[1 - e^{-t/RC}\right]\ ;\ \ Q = Q_0\left[1 - e^{-t/RC}\right]$$

Special case : At $t = \tau = RC$ = time constant

$Q = Q_0[1 - e^{-1}] = 0.632\ Q_0$

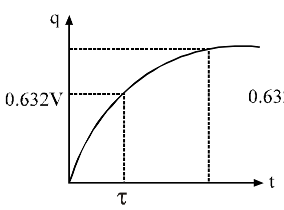
So, in charging charge increases to 63.2% of the initial charge in the time equal to τ.

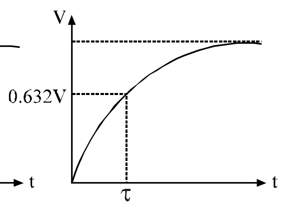
(b) Current at any instant

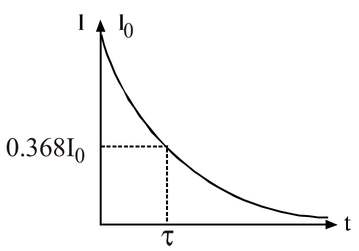
$i = dQ/dt\ ;\ \ i = i_0 e^{-t/RC}$

(c) Potential at any instant

$V = V_0(1 - e^{-t/RC})$

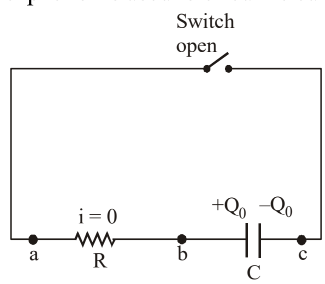




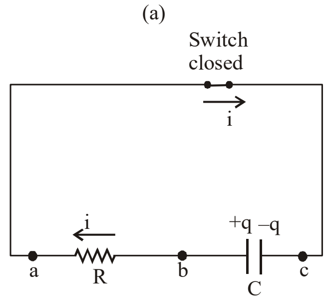


Discharging

When a charged capacitor, resistance and keys is connected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.



(a)



(b)

(b) Charge at any instant

$V_C + V_R = 0$

$$Q = Q_0 e^{-t/RC}$$

Special case : At $t = \tau = RC$ = time constant

$Q = Q_0\,e^{-1} = 0.368\ Q_0$

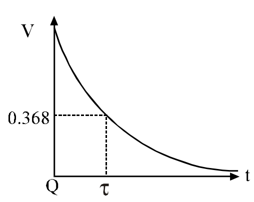
So, in discharging charge decreases to 36.8% of charge in the time equal to τ.

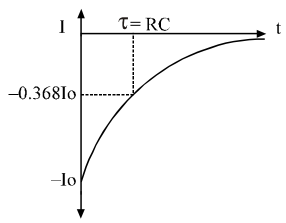
(b) Current at any instant

$i = dQ/dt\ ;\ \ i = -i_0 e^{-t/RC}\quad \{i_0 = Q_0/RC\}$

(c) Potential at any instant

$V = V_0 e^{-t/RC}$

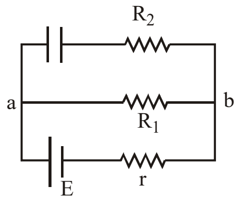




Solving RC circuit in steady state :

* Steady state condition means that all capacitors connected in different branched of circuit are fully charged to their capacity and further more there is no charge flow in these branches and resistances connected in series to capacitor have no importance.

* The current flowing in circuit is called steady state current which can be find out by Ohm's law and Kirchhoff's Law. For circuit shown :



(i) $I_{steady} = \dfrac{E}{R_1 + r}$

(ii) Potential difference across capacitor

$= V_a - V_b = IR_1 \Rightarrow \text{p.d.} = V = \dfrac{ER_1}{R_1 + r}$.

(iii) Charge on capacitor plates

$= CV = C\left(\dfrac{ER_1}{R_1 + r}\right)$; $U_{capacitor} = \dfrac{1}{2}\left(\dfrac{ER_1}{R_1 + r}\right)^2$

Example 73 :

A capacitor of $1\,\mu F$ capacity is connected to a battery of 2V through a resistance of $10^4\,\Omega$. Calculate initial current and current after 0.02 second.

Sol. The current is given by $I = I_0\, e^{-t/RC}$

at $t = 0$, $I = I_0 = \dfrac{E}{R} = \dfrac{2}{10^4} = 2 \times 10^{-4}\,A$

Current after 0.02 sec. $I = I_0\, e^{-t/RC} = I_0\, e^{-0.02/10^4 \times 10^{-6}}$

$= I_0\, e^{-2} = 2 \times 10^{-4}\,(2.718)^{-2} = 27 \times 10^{-6}\,A$

Example 74 :

A capacitor charges from a cell through a series resistance. The time constant of the circuit is τ. Find the time taken by the capacitor to collect 10% of its final charge.

Sol. At any time t, the charge on the capacitor is given as

$q = q_0(1 - e^{-t/\tau})$

Here, $q = 0.1\, q_0$. Therefore, $0.1\, q_0 = q_0(1 - e^{-t/\tau})$

or $e^{-t/\tau} = 0.9 \Rightarrow e^{t/\tau} = \dfrac{10}{9}$ $\quad\therefore\ t = \tau \ln(10/9)$

Example 75 :

A $2500\,\mu F$ capacitor is charged through a $1\,k\Omega$ resistor by a 12V d.c. source. What is the voltage across the capacitor after 5s.

Sol. The time constant of the circuit is

$\tau = RC = 10^3 \times 2500 \times 10^{-6} = 2.5$ s

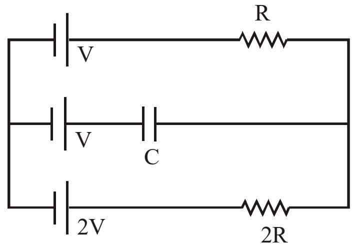
For charging,

$V = V_0(1 - e^{-t/RC}) = 12(1 - e^{-2}) = 12\,(1 - 0.135)$

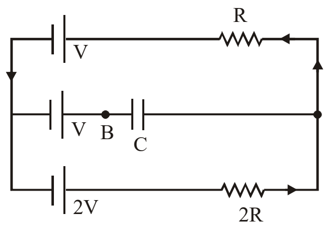
$= 12\,(1 - 0.135) = 10.38$ volt

Example 76 :

In the given circuit, with steady current, find the potential drop across the capacitor.



Sol. In steady state, the current I flowing in the circuit will be



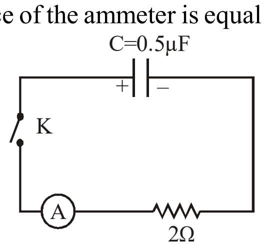
$I = \dfrac{2V - V}{R + 2R} = \dfrac{V}{3R}$

Next, $V_A - \dfrac{V}{3R}.R - V + V = V_B \Rightarrow V_A - V_B = \dfrac{V}{3}$

Example 77 :

A charged capacitor is allowed to discharge through a resistor by closing the key at the instant t = 0. At the instant t = (ln 4) μs, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to

(A) $1M\Omega$
(B) 1Ω
(C) 2Ω
(D) $2M\Omega$



Sol. **(C).** $I = I_0\, e^{-t/\tau}$

$2 = e^{t/\tau}$

$\Rightarrow \ln 2 = \dfrac{t}{\tau} \Rightarrow \tau = \dfrac{\ln 4}{\ln 2}$

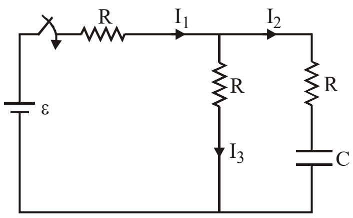
$\Rightarrow \tau = 2\mu S$

$RC = 2 \times 10^{-6} \Rightarrow (2 + R_A) \times 0.5 \times 10^{-6} = 2 \times 10^{-6}$

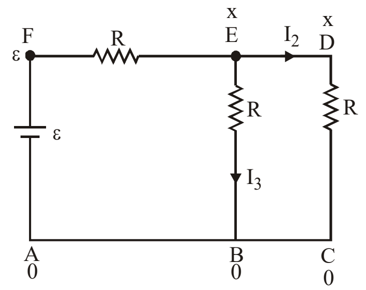
$2 + R_A = 4 \Rightarrow R_A = 2\Omega$

Example 78 :

Find out I_1, I_2, I_3, charge on capacitor and dQ/dt of capacitor in the circuit which is initially uncharged in the following situations: (a) Just after the switch is closed. (b) After a long time when switch is closed.



Sol. (a) Initially the capacitor is uncharged so its behaviour is like a conductor. Let potential at A is zero so at B and C also zero and at F it is ε.



Let potential at E is x so at D also x. Apply Kirchhoff's first law at point E :

$$\frac{x-\varepsilon}{R}+\frac{x-0}{R}+\frac{x-0}{R}=0 \Rightarrow \frac{3x}{R}=\frac{\varepsilon}{R}\ ;\ x=\frac{\varepsilon}{3}\ ;\ Q_c=0$$

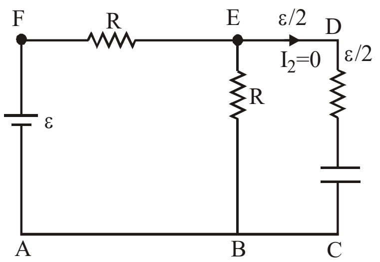
$$I_1=\frac{-\varepsilon/3+\varepsilon}{R}=\frac{2\varepsilon}{3R}\Rightarrow I_2=\frac{dQ}{dt}=\frac{\varepsilon}{3R}\ \text{and}\ I_3=\frac{\varepsilon}{3R}$$

$$\text{Alternatively, } i_1=\frac{\varepsilon}{R_{eq}}=\frac{\varepsilon}{R+\frac{R}{2}}=\frac{2\varepsilon}{3R}$$

$$\Rightarrow\ i_2=i_3=\frac{i_1}{2}=\frac{\varepsilon}{3R}\ \text{and}\ \frac{dQ}{dt}=i_2=\frac{\varepsilon}{3R}$$

(b) At $t=\infty$ (finally)

Capacitor completely charged so their will be no current through it.



$$I_2=0, I_1=I_3=\frac{\varepsilon}{3R}$$

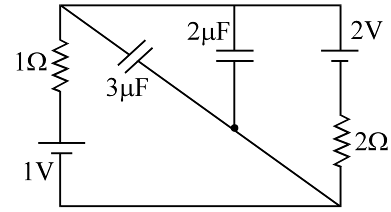
$$V_E-V_B=V_D-V_C=(\varepsilon/2R)\,R=\varepsilon/2$$

$$\Rightarrow\ Q_C=\frac{\varepsilon C}{2}\ ;\ \frac{dQ}{dt}=I_2=0$$

	I_1	I_2	I_3	Q	dQ / dt
Time t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t = ∞	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

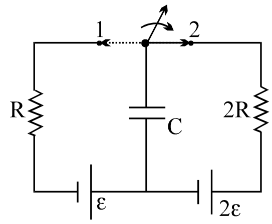
TRY IT YOURSELF - 7

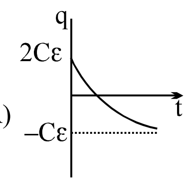
Q.1 In the circuit shown, the charge on the 3μF capacitor at steady state will be

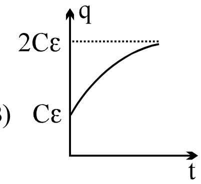


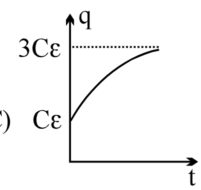
(A) 6 μC (B) 4 μC
(C) 2/3 μC (D) 3 μC

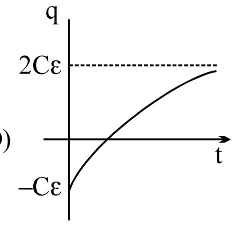
Q.2 In the circuit shown, the switch is shifted from position $1 \rightarrow 2$ at $t = 0$. The switch was initially in position 1 since a long time. The graph between charge on upper plate of capacitor C and time 't' is



(A)


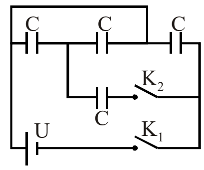
(B)


(C)


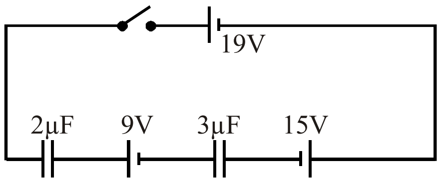
(D)


Q.3 How does the total energy stored in the capacitors in the circuit shown in the figure change when first switch K_1 is closed (process-1) and then switch K_2 is also closed (process-2). Assume that all capacitor were initially uncharged ?

(A) Increases in process-1
(B) Increases in process-2
(C) Decreases in process-2
(D) Magnitude of change in process-2 is less than that in process-1

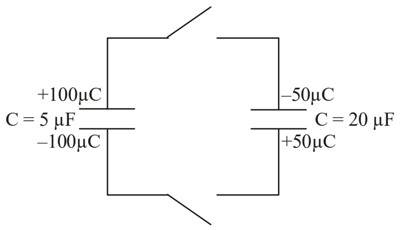


Q.4 Both capacitors are initially uncharged and then connected as shown and switch is closed. What is the potential difference across the 3μF capacitor ?



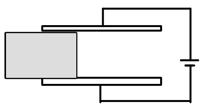
(A) 30V (B) 10V
(C) 25V (D) None of these

Q.5 For the configuration of capacitors shown, both switches are closed simultaneously. After equilibrium is established, what is the charge on the top plate of the 5μF capacitor ?

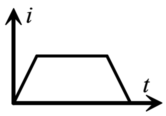


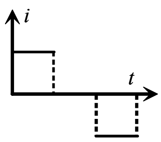
(A) 100μC (B) 90μC
(C) 10 μC (D) None of these

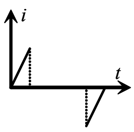
Q.6 A dielectric slab of area A passes between the capacitor plates of area 2A with a constant speed v.

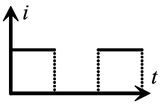


The variation of current (i) through the circuit as function of time (t) can be qualitatively represented as

(A) 

(B) 

(C) 

(D) 

Q.7 The capacitor shown in figure 1 is charged by connecting switch S to contact a. If switch S is thrown to contact b at time t = 0, which of the curves in figure 2 represents the magnitude of the current through the resistor R as a function of time?

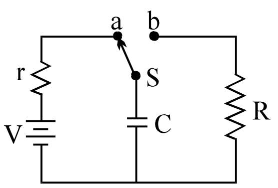


Figure 1

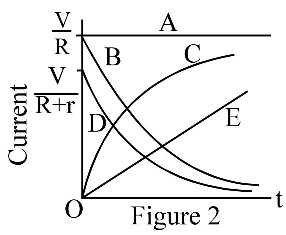
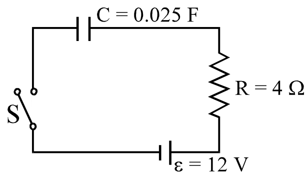


Figure 2

(A) A (B) B
(C) C (D) D

Question No. Q.8-Q.10

A circuit contains a battery, a capacitor and a resistor as shown. The capacitor is initially uncharged and the switch is closed at time t = 0.



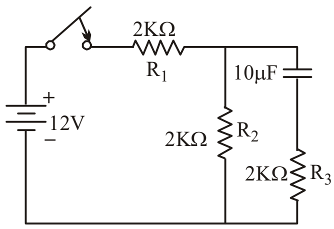
Q.8 At t = 0, the current in this circuit is:
(A) zero
(B) its maximum possible value (ε/R)
(C) somewhere between zero and maximum, depending on R, C

Q.9 At t = 0, the voltage drop across the capacitor is:
(A) zero
(B) its maximum possible value (Q_{max}/C)
(C) somewhere between zero and maximum, depending on R, C

Q.10 At what time (in secs) does the charge on the capacitor reach 75% of its maximum possible value?
(A) 0.2 ln 2s (B) 0.1 ln 2s
(C) 0.4 ln 2s (D) 0.8 ln 2s

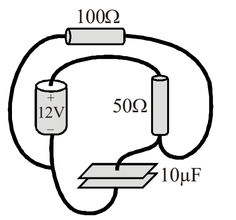
Q.11 When the charge on the capacitor is at 75% of its maximum possible value, what is the voltage drop across the resistor?
(A) V (B) 2V
(C) 3V (D) 4V

Q.12 In the circuit shown, capacitor is initially uncharged the switch is turned on at t = 0. Then,



(A) at t = 0, current supplied by battery is 4 mA
(B) at t = 0, current in R_3 is 2 mA
(C)in the steady state current supplied by battery is 3mA
(D) in the steady state current in R_3 is zero

Q.13 In the circuit shown in the figure,



(A) In steady state, there is no current in the 100 Ω resistor.
(B) In steady state, the current in 100 Ω resistor is 0.08A.
(C) In steady state, there is no current in the 50 Ω resistor.
(D) In steady state, the current in 50 Ω resistor is 0.04 A.

ANSWERS

(1) (B) **(2)** (D) **(3)** (ABD)
(4) (B) **(5)** (C) **(6)** (B)
(7) (B) **(8)** (B) **(9)** (A)
(10) (A) **(11)** (C) **(12)** (ABCD)
(13) (B)

USEFUL TIPS

1. The electric field inside a charged conductor is zero.

2. The electric potential is uniform everywhere on the surface and inside a charged conductor.

3. Potential of earth is considered to be zero.

4. The electric field vanishes in a cavity made in a conductor (i.e. E = 0 in the cavity). This is called electrostatic shielding and it implies that the instrument can be protected from outside electric fields by placing it in a box made of a good conducting material.

5. Potential depends on the charge on the conductor.

6. If E = 0 at any point then it is not necessary that the electrostatics potential at that point will also be zero. It may be finite, as in case of the interior point of a uniformly charged conducting sphere. E = 0 but $V \neq 0$.

7. If V = 0 at any point, then it is not necessary that the intensity of electric field at that point will also be zero, as in case of broad side on position of a dipole in which V= 0 but $E \neq 0$.

8. If a small charged conductor is placed inside another big and hollow charged conductor and the two are joined by a wire then the charge flows from smaller conductor to bigger conductor because the potential of smaller conductor is more than that of bigger conductor.

9. If two like charges are placed at some distance from each other, then the intensity of field will be zero at a point on the line joining the two charges, somewhere between the charges.

10. If two unlike charges are placed at some distance from each other, then the intensity of field will be zero at any point lying on the line joining the charges but outside the charges. The neutral point is situated on the side of charge of smaller magnitude.

11. If + q and – q charges are placed at the ends of a diagonal of a rectangle of side a & b, then potential difference between the ends of other diagonal will be $V = \frac{2kq(a-b)}{ab}$.

12. Charge density and intensity of electric field at pointed ends is more while electric potential is same as that of other points.

13. The electric potential due to a unipolar charge $V \propto 1/x$, due to dipolar charge $V \propto 1/x^2$.

14. If n small drops, each of surface density of charge σ Coalesce to form a big drop then the surface density of charge of the big drop is $\sigma' = \sigma n^{1/3}$.

15. A sphere of 1cm. radius, cannot be given charge of 1cb ; because the electric field intensity at the surface of air will be 9×10^{11}. In air the electric field intensity greater than 3×10^6 V/m. ionizes the air, and the charge of sphere starts leaking.

16. Self potential energy of a uniformly charged conducting sphere of radius R is $U = \frac{Q^2}{8\pi\varepsilon_0 R}$.

17. Capacitance of parallel plate capacitor with dielectric of thickness t filled in, $C = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{k}}$

18. Energy density $u = \frac{1}{2}\varepsilon_0 E^2$

19. If n identical charged liquid drops are combined to form a big drop then

S.N.	Quantity	For each charged small drop	For big drop
1.	Radius	r	$R = n^{1/3} r$
2.	Charge	q	$Q = nq$
3.	Capacitance	C	$C' = n^{1/3} C$
4.	Potential	V	$V' = n^{2/3} V$
5.	Energy	U	$U' = n^{5/3} U$
6.	Surface charge density	σ	$\sigma' = n^{1/3} \sigma$

ADDITIONAL EXAMPLES

Example 1 :

Two parallel conducting plates 5mm apart are held horizontally one above the other. The upper plate is maintained at a positive potential of 15 kV while the lower plate is earthed. If a small oil drop of relative density 0.92 and of radius 5 μm remains stationary between the plates, then the charge on the drop will be.

(A) 10e (B) 8e
(C) 5e (D) 3e

Sol. **(A).** Electric field intensity between plates

$$E = \frac{V}{d} = \frac{15 \times 10^3}{5 \times 10^{-3}} = 3 \times 10^6 \text{ V/m}$$

Weight of the oil drop $= \frac{4}{3}\pi a^3 \rho g$

$$= \frac{4}{3}\pi (5 \times 10^{-6})^3 \times (0.92 \times 10^3)(9.8)$$

For equilibrium qE = mg.

$$\therefore \quad q = \frac{mg}{E} = \frac{4 \times \pi \times 125 \times 0.92 \times 9.8 \times 10^{-15}}{3 \times 3 \times 10^6} C$$

$$= \frac{4 \times \pi \times 125 \times 0.92 \times 9.8 \times 10^{-15}}{9 \times 10^6 \times 1.6 \times 10^{-19}}$$

= 10 e electronic charge

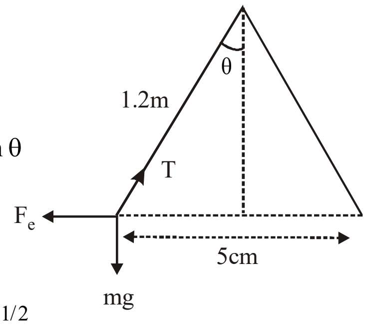
Example 2 :

Two equally charged pith-balls each of mass 10 gm are suspended from the same point by two silk threads each of length 1.2 m. As a result of mutual repulsion the balls are separated by 5 cm. The charge on each ball will be.

(A) 2.38×10^{-10} C (B) 2.38×10^{-6} C

(C) 2.38×10^{-8} C (D) 1.19×10^{-8} C

Sol. **(C).** For equilibrium mg = T cos θ



$$\frac{Kq^2}{d^2} = T \sin\theta$$

$$\therefore \quad \frac{Kq^2}{d^2} \times \frac{1}{mg} = \tan\theta$$

$$\text{or} \quad q^2 = \frac{d^2\, mg \tan\theta}{K}$$

$$\text{or} \quad q = d\left(\frac{mg \tan\theta}{K}\right)^{1/2}$$

$$= 5 \times 10^{-2}\left[\frac{10\times10^{-3}\times10\times(2.5\times10^{-2}/1.2}{9\times10^{9}}\right]^{1/2}$$

$$= 5 \times 10^{-2}\left(\frac{25\times10^{-3}}{1.2\times9\times10^{9}}\right)^{1/2} \approx 2.4 \times 10^{-8}\text{ C}$$

Example 3 :

Three charges each of charge + q are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of gravity of the triangle. If Q = – q, then –

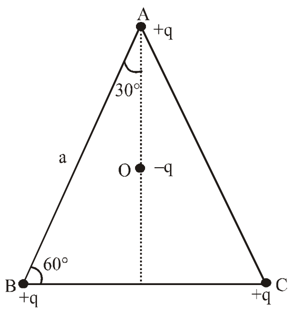
(A) The charges placed at A, B & C will move towards centre

(B) The charges placed at A, B & C will move away from the centre.

(C) The charge at A will move away and the charges at B and C will move towards the centre.

(D) All will remain in equilibrium

Sol. **(A).** $AO = \frac{2}{3}$ a sin 60º



$$= \frac{2}{3}\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

Resultant force on charge q placed at A (outwards)

$$F = \frac{2Kq^2}{a^2}\cos 30^\circ - \frac{3Kq^2}{a^2}$$

$$= \frac{Kq^2}{a^2}(\sqrt{3} - 3) = -1.27\frac{Kq^2}{a^2}$$

Therefore every charge situated at corners will experience a force towards O. So all the three charges will move in the inward direction.

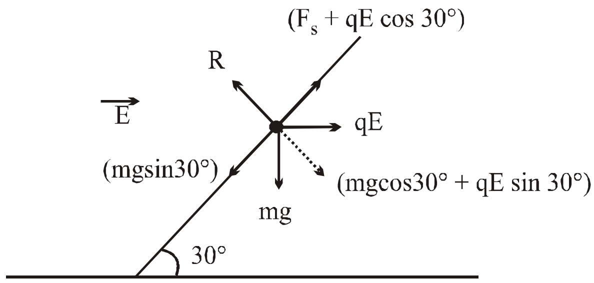
Example 4 :

An inclined plane making an angle of 30º with the horizontal is placed in a uniform electric field of intensity 100 V/m. A particle of mass 1 kg and charge 0.01 C is allowed to slide down from rest on the plane from a height of 1m. If the coefficient of friction is 0.2, then the time taken by the particle to reach the bottom will be.

(A) 1.40 s (B) 1.44 s

(C) 1.32 s (D) 2.34 s

Sol. **(C).** Form the figure



R = mg cos 30º + qE sin 30º

$$= \frac{10\sqrt{3}}{2} + \frac{0.01 \times 100}{2} = 5\sqrt{3} + 0.5 = 9.16\text{N}$$

Frictional force $F_s = \mu R = 0.2 \times 9.16 = 1.832$ N

Resultant force along the plane in the downward direction

$F = mg \sin 30^\circ - (F_s + qE \cos 30^\circ)$

$$= 5 - \left(1.832 + 0.01 \times 100 \times \frac{1.732}{2}\right) = 5 - 2.698 = 2.3\text{N}$$

$$\therefore \text{ Acceleration along the plane } f = \frac{F}{m} = 2.3\text{ m/sec}^2$$

Distance along the plane = 1 × cosec 30° = 2m

$s = ut + (1/2)\, ft^2$, u = 0

$$\therefore\ t = \left(\frac{2s}{f}\right)^{1/2} = \left(\frac{2 \times 2}{2.3}\right)^{1/2} = 1.319\text{ sec} \approx 1.32\text{ sec.}$$

Example 5 :

An infinite number of charges (each of magnitude 1 μc) are placed along the X-axis at x = 1, 2, 4, 8....metre. If the charges are alternately of opposite sign, then the potential at the point x = 0 due to these charges will be.

(A) 6×10^3 V (B) 9×10^3 V

(C) 1.8×10^4 V (D) 1.2×10^4 V

Sol. **(A).** Potential due to different charges at x = 0

$$V = K \times 10^{-6}\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...\right]$$

$$= 9 \times 10^9 \times 10^{-6}\left(\frac{1}{1 + 1/2}\right) = 6 \times 10^3\text{ volt}$$

Example 6 :

Three charges q, 2q and 8q are to be placed on a 9 cm long straight line. Where the charges should be placed so that the potential energy of this system is minimum ?

(A) q charge between 2q and 8q charges and 3 cm from charge 2q.

(B) q charge between 2q and 8q charges and 5 cm from the charge 2q.

(C) 2q charge between q and 8q charges and 7 cm from the charge q.

(D) 2q charge between q and 8q charges and 9 cm from the charge q.

Sol. (A). For minimum potential energy firstly charges of larger magnitude should be kept at largest distance. Therefore charges 2q and 8q should be at the ends of the line of length 9 cm. If charge q is placed at a distance x from the charge 2q then for minimum potential energy the resultant force on it must be zero.

$$\therefore \quad \frac{K(2q)q}{x^2} = \frac{K(8q)q}{(9\times10^{-2}-x)^2}$$

or $(9\times10^{-2}-x)^2 = 4x^2$ or $9\times10^{-2}-x = 2x$

$\therefore \quad x = 3\times10^{-2}\,m = 3$ cm

Example 7 :

A proton and an α-particle are at a distance r from each other. After letting them free if they move to infinity, the kinetic energy of the proton will be.

(A) $\frac{2Ke^2}{r}$ (B) $\frac{8Ke^2}{r}$ (C) $\frac{Ke^2}{r}$ (D) $\frac{8}{5}\frac{Ke^2}{r}$

Sol. (D). Potential energy in the stationary state = $\frac{K(2e)(e)}{r}$

Let velocity of proton be v_p and velocity of α particle be v_α then for conservation of momentum

$$mv_p = -4mv_\alpha \quad \therefore v_\alpha = \frac{-v_p}{4}$$

Increase in kinetic energy = Loss in potential energy

$$\therefore \quad \frac{1}{2}mv_p^2 + \frac{1}{2}(4m)v_\alpha^2 = K\frac{2e^2}{r}$$

$$\text{or} \quad mv_p^2 + 4m\left(\frac{v_p^2}{16}\right) = K\frac{4e^2}{r} \quad \text{or} \quad \frac{5mv_p^2}{4} = K\frac{4e^2}{r}$$

Thus kinetic energy of proton = $\frac{1}{2}mv_p^2 = \frac{8Ke^2}{5r}$

Example 8 :

The intensity of electric field due to a uniformly charged non-conducting sphere at a distance x from the centre inside the sphere is proportional to –

(A) 1/x (B) x

(C) $1/x^2$ (D) x^2

Sol. (B). $E_i = \frac{kq}{R^3}x$ or $E_i \propto x$

Example 9 :

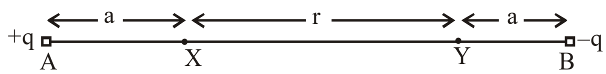
A charged particle of mass m and charge q is released in an electric field of magnitude E. Its kinetic energy after time t will be.

(A) $\frac{2E^2t^2}{mq}$ (B) $\frac{E^2q^2t^2}{2m}$ (C) $\frac{Eq^2m}{2t^2}$ (D) $\frac{Eqm}{2t}$

Sol. (B). $E_k = \frac{1}{2}mv^2 = \frac{1}{2}ma^2t^2 = \frac{1}{2}m\left(\frac{qE}{m}\right)^2 t^2 = \frac{q^2E^2t^2}{2m}$

Example 10 :

Two charges +q and – q are arranged as shown in the figure. The work done in carrying a test charge q' from X to Y will be



(A) $\frac{kqq'}{r+2a}$ (B) $\frac{kqq'}{r}$

(C) $\frac{2kqq'a}{r(r+a)}$ (D) $\frac{2kqq'r}{a(a+r)}$

Sol. (D). $W = q'(V_2 - V_1) = q'k\left[\frac{q}{a+r} - \frac{q}{a} - \frac{q}{a} + \frac{q}{a+r}\right]$

$$= 2Kqq'\left[\frac{1}{a+r} - \frac{1}{a}\right] \quad \text{or} \quad |W| = \frac{2Kqq'r}{a(a+r)}$$

Example 11 :

A particle of mass m and charge q is lying at the mid point of two stationary particles distant 2ℓ and each carrying a charge q. If the free charged particle is displaced from its equilibrium position through distance $x(x \ll \ell)$, then the particle will

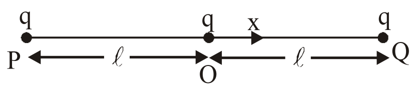
(A) move in the direction of displacement

(B) stop at its equilibrium position

(C) oscillate about its equilibrium position

(D) execute S.H.M. about its equilibrium position

Sol. (D). When the charge q is displaced through small displacement x, then the resultant force acting on it is



$$F = \frac{kq^2}{(\ell-x)^2} - \frac{kq^2}{(\ell+x)^2} \approx \frac{4kq^2}{\ell^3}x = ma$$

$$\text{or} \quad a = \left(\frac{4kq^2}{m\ell^3}\right)x = \omega^2 x \quad \text{or } a \propto x$$

$\therefore$ The motion of the particle will be S.H.M.

Example 12 :

The radius of a soap bubble is r and its surface tension is T. If the surface charge density on the bubble is σ and the excess of pressure inside it is p, then the value of σ will be

(A) $\left[2\varepsilon_0\left(\frac{4T}{r}+p\right)\right]^{1/2}$ (B) $\left[\frac{\varepsilon_0}{2}\left(\frac{4T}{r}-p\right)\right]^{1/2}$

(C) $\left[2\varepsilon_0\left(\frac{4T}{r}-p\right)\right]^{1/2}$ (D) $\left[\varepsilon_0\left(\frac{4T}{r}+p\right)\right]^{1/2}$

Sol. **(C).** In the state of equilibrium

$p_{ex} + p_{el} = p_{ST}$

or $p + \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{r}$ or $\sigma = \left[2\varepsilon_0\left(\frac{4T}{r}-p\right)\right]^{1/2}$

Example 13 :

A ball of mass 1g and charge 10^{-7} C moves from a point A whose potential is 500V to a point B whose potential is zero. If the speed of the ball at A is 0.51 m/s, its speed at point B will be

(A) 0.4 m/s (B) 0.2 m/s

(C) 0.6 m/s (D) 1.0 m/s

Sol. **(C).** $q(V_A - V_B) = \frac{1}{2}m(v_B^2 - v_A^2)$

or $10^{-7} \times (500-0) = \frac{1}{2} \times 10^{-3} \times (v_B^2 - 0.51^2)$

or $v_B^2 = 0.1 + 0.26 = 0.36 \therefore v_B = 0.6$ m/s

Example 14 :

Two conductors having capacities $2\mu F$ and $5\mu F$ and potentials 2 volt and 10 volt respectively. Calculate the ratio of their charges after connecting by a wire.

Sol. $\frac{Q_1'}{Q_2'} = \frac{C_1 V_{com}}{C_2 V_{com}} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}} = \frac{2}{5}$

Example 15 :

A parallel plate capacity of capacity C is connected to a battery and is charged to potential V, another condenser of capacity 2C is connected to another battery and is charged to potential 2V the charging batteries are removed and now the condensers are connected in parallel in such a way that the positive plate of one is connected to negative plate of another. Calculate final energy of the system.

Sol. Total charge on system

$Q = Q_1 - Q_2$

$V_{com} = \frac{Q}{C_{net}} = \frac{|Q_1 - Q_2|}{C + 2C} = \frac{3CV}{3C} = V$

$\therefore U = \frac{1}{2}(C + 2C)V_{com}^2 = \frac{3}{2}CV^2$

Example 16 :

An infinite number of capacitors of capacitance C, 4C, 16C ....∞ are connected in series then what will be their resultant capacitance ?

Sol. Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq.}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + ...\infty = \left[1 + \frac{1}{4} + \frac{1}{16} + ...\infty\right]\frac{1}{C}$$

(this is G.P. series)

$S_\infty = \frac{a}{1-r}$ [first term a = 1, common ratio r = 1/4]

$\frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$

Example 17 :

A parallel plate capacitor has potential 20 kV and capacitance 2×10^{-4} μF. If area of plate is 0.01 m^2 and distance between the plates is 2mm then find

(a) potential gradient (b) dielectric constant of medium

(c) energy

Sol. (a) Potential gradient $= \frac{V}{d} = \frac{2000}{0.002} = 10^7$ V/m

(b) $C = \frac{\varepsilon_0 \varepsilon_1 A}{d} \Rightarrow \varepsilon_r = \frac{Cd}{\varepsilon_0 A} = \frac{2 \times 10^{-10} \times 2 \times 10^{-3}}{8.85 \times 10^{-12} \times 0.01} = 4.52$

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-10} \times (20000)^2 = 4 \times 10^{-2}$ J

Example 18 :

A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any ,dielectric breakdown.

Sol. If the charge on the capacitor = Q

The surface charge density $\sigma = Q/A$

and the electric field $= \frac{Q}{KA\varepsilon_0}$

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

If the maximum charge which can be given is Q

then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m

$A = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

$Q = (5.0)(10^{-2})(8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}$ C.

Example 19 :

The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted. What would be the potential difference if a plate of dielectric constant K = 2 is introduced in place of metal plate?

Sol. In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ kV}$$

Now as after charging battery is removed , capacitor is isolated so q = constant. If C' and V are the capacity and potential after the introduction of plate.

$$q = CV = C'V' \quad \text{i.e.} \quad V' = \frac{C}{C'}V$$

And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$

$$V' = \frac{(d-t)+(t/K)}{d} \times V$$

So in case of metal plate as K = ∞

$$V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2 \text{ kV}$$

And if instead of metal plate, dielectric with K = 2 is introduced

$$V_D = \left[\frac{(0.05-0.01)+(0.01/2)}{0.05}\right] \times 1.5 = 1.35 \text{ kV}$$

Example 20 :

The plate separation in a parallel plate capacitor is d and plate area is A. If it is charged to V volt then calculate the work done in increasing the plate separation to 2d.

Sol. $W = U_F - U_I = \frac{Q^2}{2C'} - \frac{Q^2}{2C}$ (as Q is constant)

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2}$$

$$W = \frac{Q^2}{C} - \frac{Q^2}{2C} = \frac{Q^2}{2C}.\text{ But } Q = CV;\ W = \frac{CV^2}{2} = \frac{\varepsilon_0 AV^2}{2d}$$

Example 21 :

Two parallel plate capacitor of capacitances C and 2C are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. Determine the potential difference across the capacitors.

Sol. $C_{eq} = (C+2C) = 3C,\ \ q = C_{eq}V = 3CV$

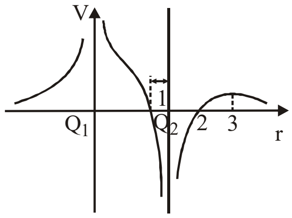
∴ as battery disconnected so

$$q' = q = 3CV \text{ and } C'_{eq} = KC + 2C = (K+2)C$$

$$\therefore \quad V' = \frac{q'}{C'_{eq}} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$$

Example 22 :

Two point charge Q_1 and Q_2 lie along a line, at a distance from each other. Figure shows the potential variation along the line of charge. At which points 1, 2 and 3 is the electric field zero? Choose the correct statement



(A) $|Q_1| > |Q_2|$ (B) $Q_1 \to$ +ve, $Q_2 \to$ –ve

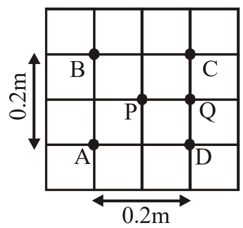
(C) $Q_1 \to$ –ve, $Q_2 \to$ +ve (D) Both (A) and (B)

Sol. **(D).** The electric field vector is zero at point 3. As $-\frac{dV}{dr} = E_r$, the negative of slop of V vs r curve represents component of electric field along r. Slope of curve is zero only at 3.

Near positive charge net potential is positive and negative near a negative charge. Thus charge Q_1 is positive and Q_2 negative. From the graph it can be seen that net potential due to two charge is positive in the region left of charge Q_1 is greater than due to Q_2. Therefore absolute value of charge Q_1 is greater than due to Q_2. Secondly, the point 1 where potential due to two charge is zero, is nearer to charge Q_2 thereby implying the Q_1 has greater absolute value.

Example 23 :

A, B, C, D, P and Q are points in a uniform electric field. The potentials a these points are V (A) = 2 volt. V (P) = V (B) = V (D) = 5 volt. V (C) = 8 volt. The electric field at P is –



(A) 10 Vm^{-1} along PQ (B) $15\sqrt{2}$ Vm^{-1} along PA

(C) 5 Vm^{-1} along PC (D) 5 Vm^{-1} along PA

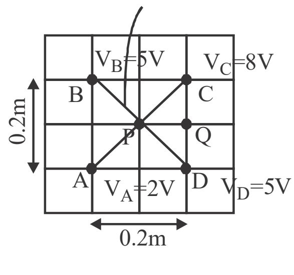
Sol. **(B).** $V_C - V_A = E \times 0.2\sqrt{2}$

$$6 = E \times 0.2\sqrt{2}$$

$$E = 15\sqrt{2} \text{ V/m}$$

along PA

Equipotential line field will be ⊥ to it



QUESTION BANK

CHAPTER 1 : ELECTROSTATICS

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : ELECTRIC CHARGE

Q.1 When a glass rod is rubbed with silk, the rod acquires one kind of charge and silk acquires the second kind of charge. Now, if the electrified rod is brought in contact with silk, with which it was rubbed, they
(A) no longer attract each other.
(B) they attract other light objects.
(C) they repel other light objects.
(D) attract each other very strongly and stick together.

Q.2 A glass rod rubbed with silk is used to charge a gold leaf electroscope and the leaves are observed to diverge. The electroscope thus charged is exposed to X-rays for a short period. Then,
(A) the divergence of leaves will not be affected.
(B) the leaves will diverge further.
(C) the leaves will collapse
(D) the leaves will melt.

Q.3 The minimum charge on an object is
(A) 1 C (B) 1 stat C
(C) 1.6×10^{-19} C (D) 3.2×10^{-19} C

Q.4 Which of these are properties of charge?
I. Charges are additive in nature.
II. Charges are conservative in nature.
III. Charges are quantised in nature.
IV. Charges can be transformed from one type to another.
(A) I, II and III (B) I, II and IV
(C) 1, III and IV (D) II, III and IV

Q.5 Two bodies are rubbed and one of them is negatively charged. For this body, if m_i = initial mass, m_f = mass after charging, then
(A) $m_i = m_f$ (B) $m_i < m_f$
(C) $m_i > m_f$ (D) $m_i + m_f = 2m_f$

Q.6 If two bodies are rubbed and these pairs are
I. Glass $\leftrightarrow$ Silk
II. Wool $\leftrightarrow$ Ebonite or Plastic
III. Ebonite $\leftrightarrow$ Polythene
IV. Dry hair $\leftrightarrow$ Comb
Then charges appearing on first member and second member of list are respectively
(A) positive, positive (B) positive, negative
(C) negative, negative (D) negative, positive

Q.7 If a charge on the body is 1 nC, then how many electrons are present on the body?
(A) 1.6×10^{19} (B) 6.25×10^{9}
(C) 6.25×10^{27} (D) 6.25×10^{28}

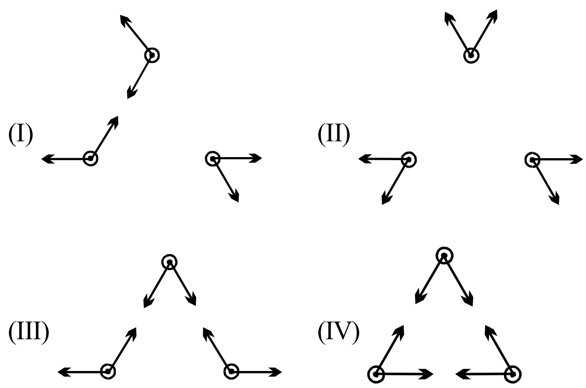
Q.8 When 10^{14} electrons are removed from a neutral metal sphere, the charge on the sphere becomes
(A) 16μC (B) – 16μC
(C) 32μC (D) – 32μC

Q.9 A conductor has 14.4×10^{-19} coulombs positive charge. The conductor has (Charge on electron 1.6×10^{-19} C)
(A) 9 electrons in excess (B) 27 electrons in short
(C) 27 electrons in excess (D) 9 electrons in short

Q.10 The electric charge in uniform motion produces
(A) An electric field only
(B) A magnetic field only
(C) Both electric and magnetic field
(D) Neither electric nor magnetic field

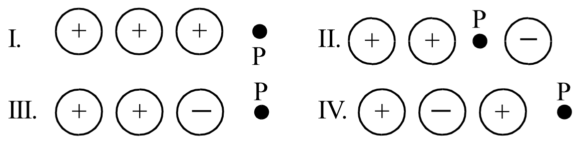
PART - 2 : COULOMB'S LAW

Q.11 The figure below shows the forces that three charged particles exert on each other. Which of the four situations shown can be correct.



(A) all of the above (B) none of the above
(C) II, III (D) II, III & IV

Q.12 Given are four arrangements of three fixed electric charges. In each arrangement, a point labeled P is also identified –test charge, +q, is placed at point P. All of the charges are of the same magnitude, Q, but they can be either positive or negative as indicated. The charges and point P all lie on a straight line. The distances between adjacent charges, either between two charges or between a charge and point P, are all the same.



Correct order of choices in a decreasing order of magnitude of force on P is –
(A) II > I > III > IV (B) I > II > III > IV
(C) II > I > IV > III (D) III > IV > I > II

Q.13 Two electrons are a certain distance apart from one another. What is the order of magnitude of the ratio of the electric force between them to the gravitational force between them?
(A) $10^{8} : 1$ (B) $10^{28} : 1$
(C) $10^{31} : 1$ (D) $10^{42} : 1$

Q.14 Nucleus $_{92}U^{238}$ emits α-particle ($_2He^4$). α-particle has atomic number 2 and mass number 4. At any instant α-particle is at distance of 9×10^{-15} m from the centre of nucleus of uranium. What is the force on α-particle at this instant ? $_{92}U^{238} \rightarrow {}_2He^4 + {}_{90}Th^{234}$

(A) 512 N (B) 412 N

(C) 325 N (D) 612 N

Q.15 A force F acts between sodium and chlorine ions of salt (sodium chloride) when put 1cm apart in air. The permittivity of air and dielectric constant of water are ε_0 and K respectively. When a piece of salt is put in water, electrical force acting between sodium and chlorine ions 1cm apart is

(A) $\frac{F}{K}$ (B) $\frac{FK}{\varepsilon_0}$

(C) $\frac{F}{K\varepsilon_0}$ (D) $\frac{F\varepsilon_0}{K}$

Q.16 A charge Q is divided into two parts of q and Q – q . If the coulomb repulsion between them when they are separated is to be maximum, the ratio of Q/q should be

(A) 2 (B) 1/2

(C) 4 (D) 1/4

Q.17 The force between two charges 0.06m apart is 5N. If each charge is moved towards the other by 0.01m, then the force between them will become

(A) 7.20 N (B) 11.25 N

(C) 22.50 N (D) 45.00 N

Q.18 Two point charges placed at a certain distance r in air exert a force F on each other. Then the distance r' at which these charges will exert the same force in a medium of dielectric constant K is given by

(A) r (B) r/K

(C) $r/\sqrt{K}$ (D) $r\sqrt{K}$

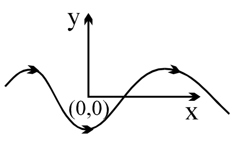
Q.19 The charges on two sphere are +7μC and – 5μC respectively. They experience a force F. If each of them is given an additional charge of – 2μC, the new force of attraction will be

(A) F (B) F / 2

(C) $F/\sqrt{3}$ (D) 2F

PART - 3 : ELECTRIC FIELD

Q.20 The field of an electric field is a cosine function in xy-plane as shown in the diagram, then the representation of electric feld can be



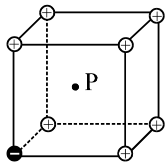
(A) $\vec{E}(x, y) = \hat{i} + \sin(x)\hat{j}$

(B) $\vec{E}(x, y) = \hat{i} - \cos(x)\hat{j}$

(C) $\vec{E}(x, y) = \hat{i} - \sin(x)\hat{j}$

(D) $\vec{E}(x, y) = \hat{i} + \cos(x)\hat{j}$

Q.21 Consider a regular cube with positive point charge +Q in all corners except for one which has a negative point charge –Q. Let the distance from any corner to the center of the cube be r. What is the magnitude of electric field at point P, the center of the cube?



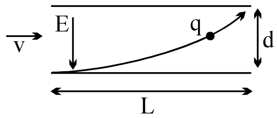
(A) $E = 7k_eQ/r^2$

(B) $E = 1k_eQ/r^2$

(C) $E = 2k_eQ/r^2$

(D) $E = 6k_eQ/r$

Q.22 In an ink-jet printer, an ink droplet of mass m is given a negative charge q by a computer -controlled charging unit, and then enters at speed v the region between two deflecting parallel plates of length Lseparated by distance d (see figure).

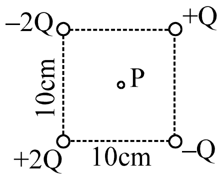


All over this region exists a downward electric field which you can assume to be uniform. Neglecting the gravitational force on the droplet, the maximum charge that it can be given so that it will not hit a plate is most closely approximated by

(A) $\frac{mv^2E}{dL^2}$ (B) $\frac{mv^2d}{EL^2}$

(C) $\frac{2dmv^2}{EL^2}$ (D) none

Q.23 Four electrical charges are arranged on the corners of a 10cm square as shown. What would be the direction of the resulting electric field at the center point P



(A) $\longrightarrow$ (B) $\uparrow$

(C) $\longleftarrow$ (D) $\downarrow$

Q.24 If the nucleus of a hydrogen atom is considered to be a sphere of radius 10^{-15} m, then the electric field on its surface will be

(A) 14.4 V/m (B) 14.4×10^{11} V/m

(C) 14.4×10^{15} V/m (D) 14.4×10^{20} V/m

Q.25 The intensity of electric field due to a uniformly charged non-conducting sphere at a distance x from the centre inside the sphere is proportional to –

(A) 1/x (B) x

(C) $1/x^2$ (D) x^2

Q.26 A charged particle of mass m and charge q is released in an electric field of magnitude E. Its kinetic energy after time t will be.

(A) $\frac{2E^2t^2}{mq}$ (B) $\frac{E^2q^2t^2}{2m}$

(C) $\frac{Eq^2m}{2t^2}$ (D) $\frac{Eqm}{2t}$

Q.27 An electron and a proton are in a uniform electric field, the ratio of their accelerations will be –

(A) Zero

(B) Unity

(C) The ratio of the masses of proton & electron

(D) The ratio of the masses of electron and proton

Q.28 If an insulated non-conducting sphere of radius R has charge density ρ. The electric field at a distance r from the centre of sphere (r < R) will

(A) $\frac{\rho R}{3\varepsilon_0}$ (B) $\frac{\rho r}{\varepsilon_0}$

(C) $\frac{\rho r}{3\varepsilon_0}$ (D) $\frac{3\rho R}{\varepsilon_0}$

Q.29 The electric potential inside a conducting sphere

(A) Increases from centre to surface

(B) Decreases from centre to surface

(C) Remains constant from centre to surface

(D) Is zero at every point inside

Q.30 The number of electrons to be put on a spherical conductor of radius 0.1m to produce an electric field of 0.036 N/C just above its surface is

(A) 2.7×10^5 (B) 2.6×10^5

(C) 2.5×10^5 (D) 2.4×10^5

Q.31 An electron falls a distance of 4 cm in a uniform electric field of magnitude 5×10^4 N/C. The time taken by electron in falling will be-

(A) 2.99×10^{-7} s (B) 2.99×10^{-8} s

(C) 2.99×10^{-9} s (D) 2.99×10^{-10} s

Q.32 A sphere of radius 5 cm has electric field 5×10^6 V/m on its surface. What will be the force acting on a charge of 5×10^{-8}C placed at distance of 20cm from the centre of sphere-

(A) 1.5×10^{-2} N (B) 40 N

(C) 4 N (D) 0 N

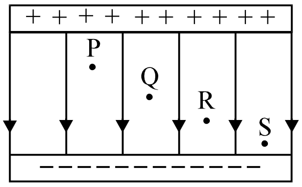
PART - 4 : ELECTRIC POTENTIAL AND EQUIPOTENTIAL SURFACE

Q.33 In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is –

(A) constant (B) zero

(C) positive (D) negative

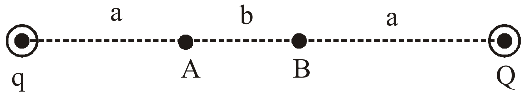
Q.34 The figure shows the electric field lines between two parallel plates that for all practical purposes extend an infinite distance both to the right and to the left and into and out of the paper. Four point P, Q, R and S are marked in this figure. At which point is the electric potential the largest



(A) P (B) Q

(C) R (D) S

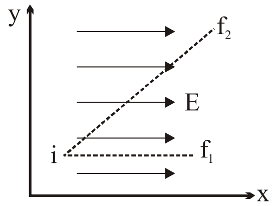
Q.35 If a = 30 cm , b = 20 cm, q = + 2.0 nC, and Q = – 3.0 nC in the figure, what is the potential difference $V_A - V_B$?

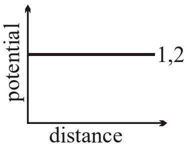


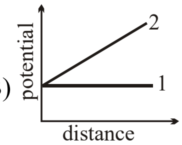
(A) + 60 V (B) + 72 V

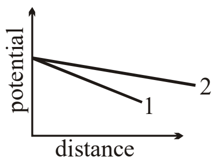
(C) + 84 V (D) + 96 V

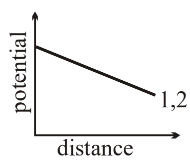
Q.36 A uniform electric field points in the positive x direction, as shown.Along the two lines f_1, f_2 , we plot the electric potentials as a function of distance. Choose the correct plot.



(A)


(B)


(C)


(D)


Q.37 A number of spherical shells of different radii are uniformly charged to same potential. The surface charge density of each shell is related with its radius as

(A) $\sigma \propto \frac{1}{R^2}$ (B) $\sigma \propto \frac{1}{R}$

(C) $\sigma \propto R$ (D) σ is same for all

Q.38 Two conducting, concentric, hollow spheres A and B have radii a and b respectively, with A inside B. They have the same potential V. A is now given some charge such that its potential becomes zero. The potential of B will now be–

(A) 0 (B) V (1–a/b)

(C) Va/b (D) V (b – a) (b + a)

Q.39 A hollow conducting sphere of radius R has a charge (+Q) on its surface. What is the electric potential within the sphere at a distance r = R/3 from its centre –

(A) Zero (B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

Q.40 A spherical conductor of radius 2m is charged to a potential of 120 V. It is now placed inside another hollow spherical conductor of radius 6m. Calculate the potential to which the bigger sphere would be raised –

(A) 20 V (B) 60 V

(C) 80 V (D) 40 V

Q.41 125 identical drops each charged to the same potential of 50 volts are combined to form a single drop. The potential of the new drop will

(A) 50V (B) 250V

(C) 500V (D) 1250V

PART - 5 : ELECTRIC POTENTIAL ENERGY

Q.42 An electron at a potential of – 10 kV moves to a point where its potential is – 1 kV. Its potential energy has –

(A) decreased

(B) increased

(C) not changed

(D) one needs to know the distance between the points to say

Q.43 A charge (–q) and another charge (+Q) are kept at two points A and B respectively. Keeping the charge (+Q) fixed at B, the charge (–q) at A is moved to another point C such that ABC forms an equilateral triangle of side ℓ. The net work done in moving the charge (–q) is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\ell}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$

(C) $\frac{1}{4\pi\varepsilon_0}Qq\ell$ (D) Zero

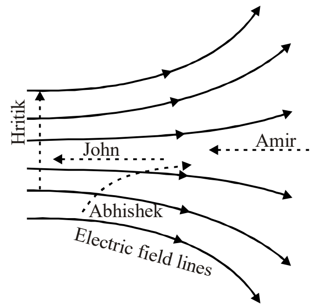
PART - 6 : ELECTRIC FIELD LINES

Q.44 Electric lines of force about negative point charge are

(A) Circular, anticlockwise (B) Circular, clockwise

(C) Radial, inward (D) Radial, outward

Q.45 Abhishek, Hritik, John, and Amir are assigned the tasks of moving equal positive charges slowly through an electric field, along assigned path (shown as dotted line). In each case the charge is at rest at the beginning. They all have paths of exactly equal lengths. Who must do the most positive work?



(A) Abhishek (B) Hritik

(C) Amir (D) John

PART - 7 : ELECTRIC FLUX

Q.46 Electric Flux is a measure of

(A) the rate at which moving electric charges are crossing an area.

(B) the number of electric field lines passing through an area.

(C) the surface density of electric charge spread along the area.

(D) the rate at which electric field lines are spreading out in space and moves further and further away from electric charges.

Q.47 A uniform electric field $\vec{E} = a\hat{i} + b\hat{j}$, intersects a surface of area A. What is the flux through this area if the surface lies in the yz plane?

(A) a A (B) 0

(C) b A (D) $A\sqrt{a^2 + b^2}$

Q.48 A uniform electric field E = 2 × 10^3 NC^{-1} is acting along the positive x-axis. The flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane is

(A) 20 NC^{-1} m^2 (B) 30 NC^{-1} m^2

(C) 10 NC^{-1} m^2 (D) 40 NC^{-1} m^2

PART - 8 : GAUSS'S LAW

Q.49 Which of the following statements is not true about Gauss's law?

(A) Gauss's law is true for any closed surface.

(B) The term q on the right side of Gauss's law includes the sum of all charges enclosed by the surface.

(C) Gauss's law is not much useful in calculating electrostatic field when the system has some symmetry.

(D) Gauss's law is based on the inverse square dependence on distance contained in the coulomb's law.

Q.50 A 5.0 μC point charge is placed at the center of a cube. The electric flux in N-m^2 /C through one side of the cube is approximately :

(A) 0 (B) 7.1 × 10^4

(C) 9.4 × 10^4 (D) 1.4 × 10^5

PART - 9 : ELECTRIC DIPOLE

Q.51 An electric dipole is placed in an electric field generated by a point charge
(A) The net force on the dipole must be zero.
(B) The net force on the dipole may be zero.
(C) The torque on the dipole due to the field must be zero.
(D) The torque on the dipole due to the field may be zero.

Q.52 An electric dipole of moment p is kept along an electric field E. The work done by external agent in rotating it from stable equilibrium position by an angle θ, is
(A) pE sin θ (B) pE cos θ
(C) pE $(1-\sin\theta)$ (D) pE $(1-\cos\theta)$

Q.53 An electric dipole of moment $\vec{p}$ is placed in the position of stable equilibrium in uniform electric field of intensity $\vec{E}$. It is rotated through an angle θ from the initial position. The potential energy of electric dipole in the final position is
(A) pE cos θ (B) pE sin θ
(C) pE $(1-\cos\theta)$ (D $-$pE cos θ

Q.54 An electric dipole is kept in non-uniform electric field. It experiences –
(A) A force and a torque
(B) A force but not a torque
(C) A torque but not a force
(D) Neither a force nor a torque

Q.55 An electric dipole consisting of two opposite charges of 2×10^{-6}C each separated by a distance of 3cm is placed in an electric field of 2×10^5 N/C. The maximum torque on the dipole will be
(A) 12×10^{-1} Nm (B) 12×10^{-3} Nm
(C) 24×10^{-1} Nm (D) 24×10^{-1} Nm

Q.56 Two opposite and equal charges 4×10^{-8}C when placed 2×10^{-2}cm away, form a dipole. If this dipole is placed in an external electric field 4×10^8N/C, the value of maximum torque and the work done in rotating it through 180° will be
(A) 64×10^{-4} Nm and 64×10^{-4}J
(B) 32×10^{-4} Nm and 32×10^{-4}J
(C) 64×10^{-4} Nm and 32×10^{-4}J
(D) 32×10^{-4} Nm and 64×10^{-4}J

Q.57 If E_a be the electric field strength of a short dipole at a point on its axial line and E_e that on the equatorial line at the same distance, then
(A) $E_e = 2E_a$ (B) $E_a = 2E_e$
(C) $E_a = E_e$ (D) None of the above

PART - 10 : CONDUCTORS, DIELECTRICS AND POLARISATION

Q.58 Which of the following statements is false for a perfect conductor?
(A) The surface of the conductor is an equipotential surface.
(B) The electric field just outside the surface of a conductor is perpendicular to the surface.
(C) The charge carried by a conductor is always uniformly distributed over the surface of the conductor.
(D) None of these

Q.59 I. The molecules of a substance may be polar or non-polar.
II. In a non-polar molecule, the centres of positive and negative charge coincide.
III. The non-polar molecule has no permanent (or intrinsic) dipole moment.
(A) I, II are correct, III may be correct.
(B) I and III are correct, II may be correct.
(C) II and III are correct, I is incorrect.
(D) I, II and II are correct.

Q.60 Dielectric constant for a metal is
(A) zero (B) infinite
(C) 1 (D) 10

Q.61 For linear isotropic dielectric, the polarisation is
(A) $P = \chi_e E$ (B) $P = -\chi_e E$
(C) $P = 2\chi_e E$ (D) $P = \chi_e / E$

Q.62 Which among the following is an example of polar molecule?
(A) O_2 (B) H_2
(C) N_2 (D) HCl

PART - 11 : CAPACITORS AND CAPACITANCE

Q.63 Two insulated conductors are charged by transferring from one conductor to another. The potential difference of 100V was produced on transferring 6.25×10^{15} electrons from one conductor to another. Find the capacitor of the system.
(A) 5 μF (B) 10 μF
(C) 15 μF (D) 20 μF

Q.64 The potentials of the two plates of capacitor are +10V and –10 V. The charge on one of the plates is 40 C. The capacitance of the capacitor is
(A) 2 F (B) 4 F
(C) 0.5 F (D) 0.25 F

Q.65 One plate of parallel plate capacitor is smaller than other, then charge on smaller plate will be
(A) Less than other
(B) More than other
(C) Equal to other
(D) Will depend upon the medium between them

PART - 12 : PARALLEL PLATE CAPACITOR

Q.66 The capacity of a parallel plate condenser is C. Its capacity when the separation between the plates is halved will be
(A) 4C (B) 2C
(C) C/2 (D) C/4

Q.67 Force of attraction between the plates of a parallel plate capacitor is

(A) $\frac{q^2}{2\varepsilon_0 AK}$ (B) $\frac{q^2}{\varepsilon_0 AK}$ (C) $\frac{q}{2\varepsilon_0 A}$ (D) $\frac{q^2}{2\varepsilon_0 A^2 K}$

Q.68 A parallel plate capacitor has a capacity C. The separation between the plates is doubled and a dielectric medium is introduced between the plates. If the capacity now becomes 2C, the dielectric constant of the medium is

(A) 2 (B) 1
(C) 4 (D) 8

Q.69 A parallel plate condenser with oil between the plates (dielectric constant of oil K = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes

(A) $\sqrt{2}C$ (B) 2C

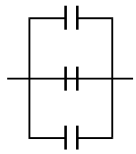
(C) $\frac{C}{\sqrt{2}}$ (D) $\frac{C}{2}$

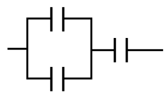
Q.70 When a dielectric material is introduced between the plates of a charged condenser, then electric field between the plates

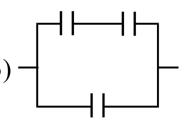
(A) Remain constant
(B) Decreases
(C) Increases
(D) First increases then decreases

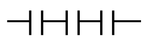
PART - 13 : COMBINATION OF CAPACITORS

Q.71 In four options below, all the four circuits are arranged in order of equivalent capacitance. Select the correct order. Assume all capacitors are of equal capacitance.

(1) 

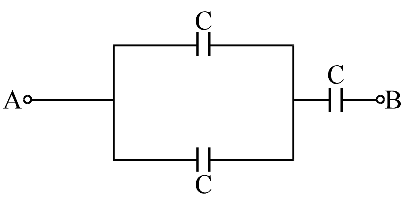
(2) 

(3) 

(4) 

(A) $C_1 > C_2 > C_3 > C_4$ (B) $C_1 > C_3 > C_2 > C_4$
(C) $C_1 < C_2 < C_3 < C_4$ (D) $C_1 < C_3 < C_2 < C_4$

Q.72 In the network shown we have three identical capacitors. Each of them can withstand a maximum 100 V p.d. What maximum voltage can be applied across A and B so that no capacitor gets spoiled?

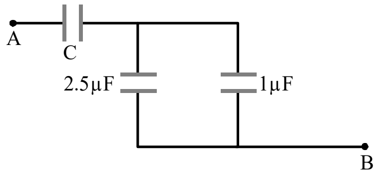


(A) 150 V (B) 120 V
(C) 180 V (D) 200 V

Q.73 Two capacitors C_1 and C_2 are connected in series, assume that $C_1 < C_2$. The equivalent capacitance of this arrangement is C, where

(A) $C < C_1/2$ (B) $C < C_2/2$
(C) $C_1 < C < C_2$ (D) $C_2 < C < 2C_2$

Q.74 The equivalent capacitance between A and B in the figure is 1μF. Then the value of capacitance C is



(A) 1.4μF (B) 2.5μF
(C) 3.5μF (D) 1.2μF

Q.75 Between the plates of a parallel plate condenser, a plate of thickness t_1 and dielectric constant k_1 is placed. In the rest of the space, there is another plate of thickness t_1 and dielectric constant k_2. The potential difference across the condenser will

(A) $\frac{Q}{A\varepsilon_0}\left(\frac{t_1}{k_1}+\frac{t_2}{k_2}\right)$ (B) $\frac{\varepsilon_0 Q}{A}\left(\frac{t_1}{k_1}+\frac{t_2}{k_2}\right)$

(C) $\frac{Q}{A\varepsilon_0}\left(\frac{k_1}{t_1}+\frac{k_2}{t_2}\right)$ (D) $\frac{\varepsilon_0 Q}{A}(k_1t_1+k_2t_2)$

Q.76 The capacitor of capacitance 4μFand 6μFare connected in series. A potential difference of 500V applied to the outer plates of the two capacitor system. Then the charge on each capacitor is numerically

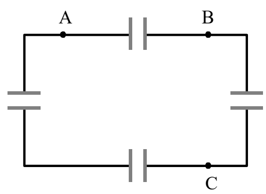
(A) 6000 C (B) 1200 C
(C) 1200 μC (D) 6000 μC

Q.77 Two capacitances of capacity C_1 and C_2 are connected in series and potential difference V is applied across it. Then the potential difference across C_1 will be

(A) $V\frac{C_2}{C_1}$ (B) $V\frac{C_1+C_2}{C_1}$

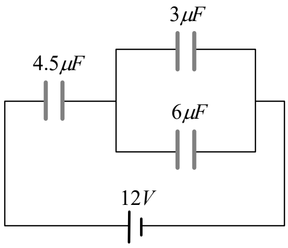
(C) $V\frac{C_2}{C_1+C_2}$ (D) $V\frac{C_1}{C_1+C_2}$

Q.78 Four capacitors of each of capacity 3μFare connected as shown in the adjoining figure. The ratio of equivalent capacitance between A & B and between A & C will be



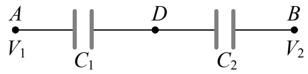
(A) 4 : 3 (B) 3 : 4
(C) 2 : 3 (D) 3 : 2

Q.79 In the circuit shown in the figure, the potential difference across the 4.5μF capacitor is



(A) 8/3 volts (B) 4 volts
(C) 6 volts (D) 8 volts

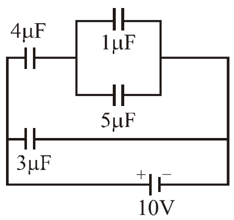
Q.80 Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be



(A) $\frac{1}{2}(V_1 + V_2)$ (B) $\frac{C_2V_1 + C_1V_2}{C_1 + C_2}$

(C) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (D) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

Q.81 For the circuit shown in the figure, the charge on 4μF capacitor is :



(A) 30 μC (B) 40 μC
(C) 24 μC (D) 54 μC

PART - 14 : ENERGY STORED IN A CAPACITOR

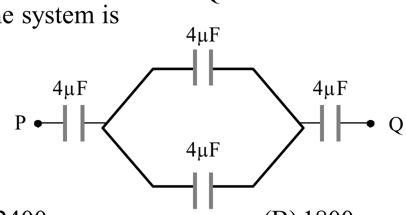
Q.82 The energy stored in a condenser of capacity C which has been raised to a potential V is given

(A) $\frac{1}{2}CV$ (B) $\frac{1}{2}CV^2$ (C) CV (D) $\frac{1}{2VC}$

Q.83 A capacitor 4μF charged to 50V is connected to another capacitor of 2μF charged to 100V with plates of like charges connected together. The total energy before and after connection in multiples of $(10^{-2}$ J) is
(A) 1.5 and 1.33 (B) 1.33 and 1.5
(C) 3.0 and 2.67 (D) 2.67 and 3.0

Q.84 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor
(A) 1.5×10^{-8}J (B) 2.5×10^{-7}J
(C) 3.5×10^{-5}J (D) 4.5×10^{-2}J

Q.85 Four condensers each of capacity 4μF are connected as shown in figure. $V_P - V_Q$ = 15 volts. The energy stored in the system is



(A) 2400 ergs (B) 1800 ergs
(C) 3600 ergs (D) 5400 ergs

PART - 15 : VANDE GRAAFF GENERATOR

Q.86 Van de Graaff generator is used to –
(A) store electrical energy
(B) build up high voltages of few million volts.
(C) decelerate charged particle like electrons
(D) Both (A) and (B) are correct.

Q.87 Van de Graaff generator is a machine capable of building up potential difference of a few million volts and fields close to the breakdown field of air which is about –
(A) 3×10^5 V/m (B) 3×10^6 V/m
(C) 3×10^{-6} V/m (D) 3×10^4 V/m

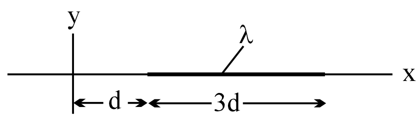
EXERCISE - 2 (LEVEL-2)

Choose one correct response for each question.

Q.1 Number of electrons in 10 g of water is
(A) 3.344×10^{24} (B) 5.35×10^5
(C) 3.344×10^{23} (D) 5.35×10^{23}

Q.2 Three point charges 3nC, 6nC and 9nC are placed at the corners of an equilateral triangle of side 0.1 m. The potential energy of the system is
(A) 9910×10^{-9} J (B) 8910×10^{-9} J
(C) 99100×10^{-9} J (D) 89100×10^{-9} J

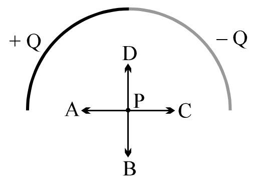
Q.3 A continuous line of charge of length 3d lies along the x-axis, extending from x = d to x = + 4d. the line carries a uniform linear charge density λ.



In terms of d, λ and any necessary physical constants, find the magnitude of the electric field at the origin.
(A) $\lambda / 5\pi\varepsilon_0 d$ (B) $\lambda / 4\pi\varepsilon_0 d$
(C) $3\lambda / 16\pi\varepsilon_0 d$ (D) $3\lambda / 8\pi\varepsilon_0 d$

Q.4 As shown in the figure, an insulating rod is net into the shape of a semicircle. The left half of the rod has a charge of + Q uniformly distributed along its length, and the right half of the rod has a charge of –Q uniformly distributed along its length.

What vector shows the correct direction of the electric field at point P, the centre of the semicircle ?

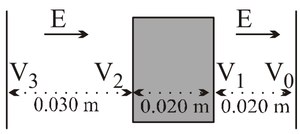


(A) A (B) B
(C) C (D) D

Q.5 The maximum electric field at a point on the axis of a uniformly charged ring is E_0. At how many points on the axis will the magnitude of electric field be $E_0/2$
(A) 1 (B) 2
(C) 3 (D) 4

Q.6 A spherical drop of mercury having an electric potential of 2.5 V is obtained as a result of merging 125 identical spherical droplets. The electric potential of each of the original small droplets is
(A) 0.1 V (B) 0.2 V
(C) 0.4 V (D) 0.5 V

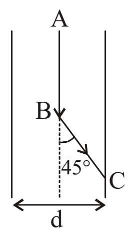
Q.7 An infinite conducting plate of thickness 0.0200m is surrounded by a uniform field E = 400 V/m directed left to right. See the figure. Let the potential V_0 = 0 at a distance 0.0200 m to the right of the plate. What is V_3, the potential 0.0300 m to the left of the plate?



(A) –28 V (B) –20 V
(C) +20 V (D) +28 V

Q.8 Two point charges + Q and – Q are kept at a distance d from each other. At the mid point of the line joining both the charges
(A) Potential is zero but electric field is not zero
(B) Electric field is zero but potential is not zero
(C) Electric field as well as potential are zero
(D) Electric field as well as potential are non zero

Q.9 A sphere carrying a charge of Q having weight w falls under gravity between a pair of vertical plates at a distance of d from each other. When a potential difference V is applied between the plates the acceleration of sphere changes as shown in the figure, to along line BC. The value of Q is



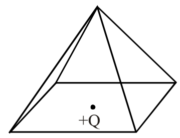
(A) w/V (B) w/2V
(C) $\frac{wd}{V}$ (D) $\frac{\sqrt{2}wd}{V}$

Q.10 A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential and capacitance respectively are
(A) constant, decreases, increases
(B) constant, decreases, decreases
(C) constant, increases, decreases
(D) increases, decreases, decreases

Q.11 A spherical shell of radius 10 cm is carrying a charge q. if the electric potential at distances 5 cm, 10 cm and 15 cm from the centre of the spherical shell is V_1, V_2 and V_3 respectively,
(A) $V_1 = V_2 > V_3$
(B) $V_1 > V_2 > V_3$
(C) $V_1 = V_2 < V_3$ (D) $V_1 < V_2 < V_3$

Q.12 An electric dipole is placed at the origin O such that its equator is y-axis. At a point P far away from dipole, the electric field direction is along y-direction. OP makes an angle α with the x-axis such that :
(A) $\tan\alpha = \sqrt{3}$ (B) $\tan\alpha = \sqrt{2}$
(C) $\tan\alpha = 1$ (D) $\tan\alpha = \frac{1}{\sqrt{2}}$

Q.13 A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is

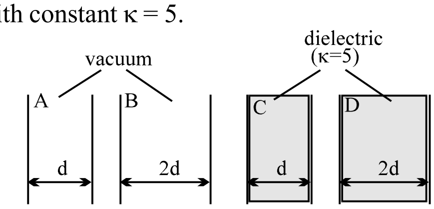


(A) $\frac{Q}{16\varepsilon_0}$ (B) $\frac{Q}{4\varepsilon_0}$ (C) $\frac{Q}{8\varepsilon_0}$ (D) None

Q.14 If a rectangular area is rotated in a uniform electric field from the position where the maximum electric flux goes through it to an orientation where only half the maximum flux goes through it, what has been the angle of rotation?
(A) 30° (B) 60°
(C) 45° (D) 26.6°

For Q 15-16

The figure below shows four parallel plate capacitors : A, B, C and D. Each capacitor carries the same charge q and has the same plate area A. As suggested by the figure, the plates of capacitors A and C are separated by a distance d while those of B and D are separated by a distance 2d. Capacitors A and B are maintained in vacuum while capacitors C and D contain dielectrics with constant κ = 5.



Q.15 Which list below places the capacitors in order of increasing capacitance?
(A) A, B, C, D (B) B, A, C, D
(C) B, A, D, C (D) A, B, D, C

Q.16 Which capacitor has the largest potential difference between its plates?
(A) A
(B) B
(C) D
(D) A and D are the same and larger than B or C

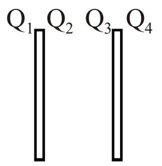
Q.17 The plates of a parallel-plate capacitor are separated by a solid dielectric. This capacitor and a resistor are connected in series across the terminals of a battery. Now the plates of the capacitor are pulled slightly farther apart. When equilibrium is restored in the circuit.
(A) the potential difference across the plates has increased.
(B) the energy stored in the capacitor has increased.
(C) the capacitance of the capacitor has increased.
(D) the charge on the plates of the capacitor has decreased.

Q.18 The plates of a parallel plate capacitor are charged upto 100 volt. A 2 mm thick plate is inserted between the plates, then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. The dielectric constant of the plate is
(A) 5 (B) 1.25
(C) 4 (D) 2.5

Q.19 A capacitor of capacitance of 2 μF is charged to a potential difference of 200 V, after disconnecting from the battery, it is connected in parallel with another uncharged capacitor. The final common potential is 20V, the capacitance of second capacitor is:
(A) 2 μF (B) 4 μF
(C) 18 μF (D) 16 μF

Q.20 Two spheres carrying charges +6μC and +9μC, separated by a distance d, experiences a force of repulsion F. When a charge of –3μC is given to both the sphere and kept at the same distance as before, the new force of repulsion is
(A) F/3 (B) F
(C) F/9 (D) 3 F

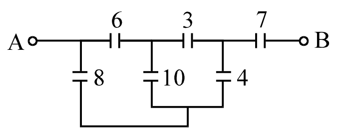
Q.21 In an isolated charged capacitor of capacitance 'C', the four surfaces have charges Q_1, Q_2, Q_3 and Q_4 as shown. Potential difference between the plates of the capacitor is



(A) $\frac{Q_1+Q_2+Q_3+Q_4}{C}$ (B) $\frac{Q_2+Q_3}{C}$

(C) $\frac{|Q_2-Q_3|}{2C}$ (D) $\frac{|Q_1-Q_4|}{2C}$

Q.22 In the circuit diagram shown all the capacitors are in μF. The equivalent capacitance between points A & B is (in μF)

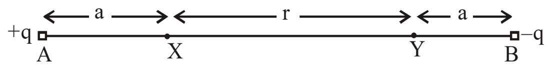


(A) 14/5 (B) 7/5
(C) 3/7 (D) None of these

Q.23 The angle between the dipole moment and electric field at any point on the equatorial plane is
(A) 180° (B) 0°
(C) 45° (D) 90°

Q.24 Pick out the statement which is incorrect.
(A) A negative test charge experiences a force opposite to the direction of the field.
(B) The tangent drawn to a line of force represents the direction of electric field.
(C) Field lines never intersect.
(D) The electric field lines forms closed loop.

Q.25 Two charges +q and – q are arranged as shown in the figure. The work done in carrying a test charge q' from X to Y will be



(A) $\frac{kqq'}{r+2a}$ (B) $\frac{kqq'}{r}$

(C) $\frac{2kqq'a}{r(r+a)}$ (D) $\frac{2kqq'r}{a(a+r)}$

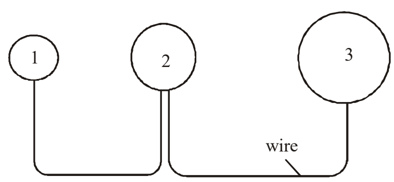
Q.26 Two capacitors of 10 μF and 20 μF are connected to 200 V and 100 V sources respectively. If they are connected by the wire, what is the common potential of the capacitors?
(A) 133.3 volt (B) 150 volt
(C) 300 volt (D) 400 volt

Q.27 A capacitor with air as the dielectric is charged to a potential of 100 volts. If the space between the plates is now filled with a dielectric of dielectric constant 10, the potential difference between the plates will be
(A) 1000 volts (B) 100 volts
(C) 10 volts (D) Zero

Q.28 Two conducting spheres of radii 5 cm and 10 cm are given a charge of 15μC each. After the two spheres are joined by a conducting wire, the charge on the smaller sphere is
(A) 5μC (B) 10μC
(C) 15μC (D) 20μC

Q.29 A 10 μF capacitor is charged to a potential difference of 1000 V. The terminals of the charged capacitor are disconnected from the power supply and connected to the terminals of an uncharged 6μF capacitor. What is the final potential difference across each capacitor
(A) 167 V (B) 100 V
(C) 625 V (D) 250 V

Q.30 Three charged, metal spheres of different radii are connected by a thin metal wire. The potential and electric field at the surface of each sphere are V and E. Which of the following is true?



(A) $V_1 = V_2 = V_3$ and $E_1 = E_2 = E_3$
(B) $V_1 = V_2 = V_3$ and $E_1 > E_2 > E_3$
(C) $V_1 > V_2 > V_3$ and $E_1 = E_2 = E_3$
(D) $V_1 = V_2 = V_3$ and $E_1 < E_2 < E_3$

Q.31 A voltmeter reads 4V when connected to a parallel plate capacitor with air as a dielectric. When a dielectric slab is introduced between plates for the same configuration, voltmeter reads 2V. What is the dielectric constant of the material?
(A) 0.5 (B) 2
(C) 8 (D) 10

Q.32 A spherical conductor of radius 2 cm is uniformly charged with 3 nC. What is the electric field at a distance of 3 cm from the centre of the sphere?
(A) 3×20^6 V m^{-1} (B) 3 V m^{-1}
(C) 3×10^4 V m^{-1} (D) 3×10^{-4} V m^{-1}

Q.33 Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the sphere are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is –
(A) 2 : 1 (B) 1 : 4
(C) 4 : 1 (D) 1 : 2

Q.34 An electric charge 10^{-3} μC is placed at the origin (0,0) of X - Y co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be -
(A) 9 volt (B) zero
(C) 2 volt (D) 4.5 volt

Q.35 What is the nature of Gaussian surface involved in Gauss law of electrostatic?
(A) Scalar (B) Electrical
(C) Magnetic (D) Vector

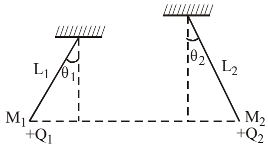
Q.36 What is the electric potential at a distance of 9 cm from 3nC?
(A) 270 V (B) 3 V
(C) 300 V (D) 30 V

Q.37 The polarised dielectric is equivalent to –
(A) two charged surface with induced surface
(B) only single charged surface with induced surface
(C) either (A) or (B)
(D) neither (A) nor (B)

Q.38 When air is replaced by a dielectric medium of constant K, the maximum force of attraction between two charges separated by same distance –
(A) increases K times (B) remains unchanged
(C) decreases K times (D) increases K^{-1} times

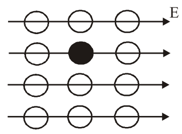
Q.39 I. In an external electric field, the positive and negative charges of a non-polar molecule are displaced in opposite directions.
II. In non-polar molecules displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force.
III. The non-polar molecule develops an induced dipole moment.
(A) I, II are III are correct.
(B) I, II and III are incorrect.
(C) I and II are correct, III is incorrect.
(D) I and III are correct, II is incorrect.

Q.40 Two small spheres of masses M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2. The spheres carry charges of Q_1 and Q_2 respectively. The spheres are suspended such that they are in level with one another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown. Which of the following conditions is essential, if $\theta_1=\theta_2$



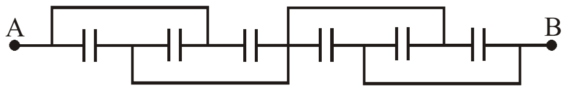
(A) $M_1 \neq M_2$, but $Q_1 = Q_2$ (B) $M_1 = M_2$
(C) $Q_1 = Q_2$ (D) $L_1 = L_2$

Q.41 There is a uniform electric field of intensity E which is as shown. How many labelled points have the same electric potential as the fully shaded point?



(A) 2 (B) 3
(C) 8 (D) 11

Q.42 All capacitors used in the diagram are identical and each is of capacitance C. Then the effective capacitance between the point A and B is –



(A) 1.5C (B) 6C
(C) C (D) 3C

Q.43 Two identical conducting balls A and B have positive charges q_1 and q_2 respectively. But $q_1 \neq q_2$. The balls are brought together so that they touch each other and then kept in their original positions. The force between them is–
(A) less than that before the balls touched
(B) greater than that before the balls touched
(C) same as that before the balls touched
(D) zero

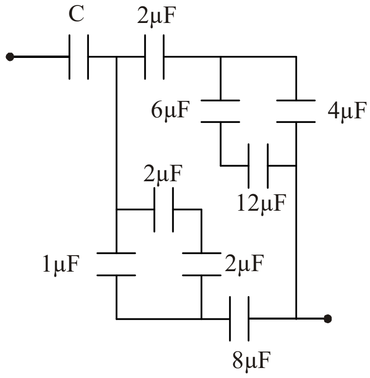
Q.44 Two identical charged spheres of material density ρ, suspended from the same point by inextensible strings of equal length make an angle θ between the strings. When suspended in a liquid of density σ the angle θ remains the same. Dielectric constant K of the liquid is –

(A) $\frac{\rho+\sigma}{\rho}$ (B) $\frac{\rho}{\rho+\sigma}$ (C) $\frac{\rho-\sigma}{\rho}$ (D) $\frac{\rho}{\rho-\sigma}$

Q.45 Two equal and opposite charges of masses m_1 and m_2 are accelerated in an uniform electric field through the same distance. What is the ratio of their accelerations if their ratio of masses is $m_1/m_2 = 0.5 = 0.5$?

(A) $\frac{a_1}{a_2} = 0.5$ (B) $\frac{a_1}{a_2} = 1$ (C) $\frac{a_1}{a_2} = 2$ (D) $\frac{a_1}{a_2} = 3$

Q.46 In the given network, the value of C, so that an equivalent capacitance between A and B is 3μF,



(A) 36μF (B) 48μF
(C) (31/5) μF (D) (1/5) μF

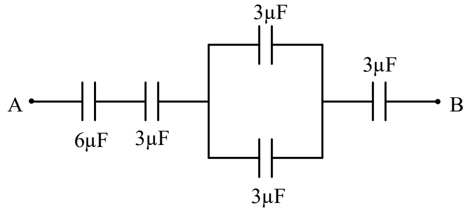
Q.47 Acceleration of a charged particle of charge 'q' and mass 'm' moving in a uniform electric field of strength 'E' is –
(A) m/qE (B) mqE
(C) q/mE (D) qE/m

Q.48 Two fixed charges A and B of 5μC each are separated by a distance of 6m. C is the mid point of the line joining A and B. A charge 'Q' of –5μC is shot perpendicular to the line joining A and B through C with a kinetic energy of 0.06 J. The charge 'Q' comes to rest at a point D. The distance CD is –
(A) $\sqrt{3}$m (B) $3\sqrt{3}$m
(C) 4m (D) 3 m

Q.49 A capacitor of capacitance 10μF is charged to 10 V. The energy stored in it is –
(A) 500 μ J (B) 1000 μ J
(C) 1 μ J (D) 100 μ J

Q.50 In the uniform electric field of $E = 1 \times 10^4$ N/C, an electron is accelerated from rest. The velocity of the electron when it has travelled a distance of 2×10^{-2} m is nearly ___ms^{-1} (e/m of electron = 1.8×10^{11} C kg^{-1})
(A) 0.85×10^6 (B) 0.425×10^6
(C) 8.5×10^6 (D) 1.6×10^6

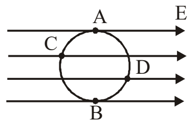
Q.51 In this diagram, the P.D between A and B is 60V, the P.D across 6 μF capacitor is –



(A) 5V (B) 20V
(C) 4V (D) 10V

Q.52 A small oil drop of mass 10^{-6} kg is hanging in at rest between two plates separated by 1 mm having a potential difference of 500 V. The charge on the drop is ($g = 10ms^{-2}$)
(A) 2×10^{-9} C (B) 2×10^{-11} C
(C) 2×10^{-6} C (D) 2×10^{-8} C

Q.53 A uniform electric field in the plane of the paper as shown. Here A, B, C, D are the points on the circle. V_1, V_2, V_3, V_4 are the potentials at those points respectively. Then



(A) $V_A = V_C, V_B = V_D$ (B) $V_A = V_C, V_B > V_D$
(C) $V_A > V_C, V_B > V_D$ (D) $V_A = V_B, V_C > V_D$

Q.54 Two metal spheres of radii 0.01 m and 0.02 m are given a charge of 15 mC and 45 mC respectively. They are then connected by a wire. The final charge on the first sphere is
(A) 40×10^{-3} C (B) 30×10^{-3} C
(C) 20×10^{-3} C (D) 10×10^{-3} C

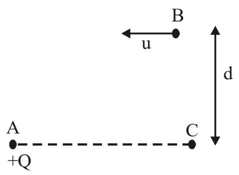
EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

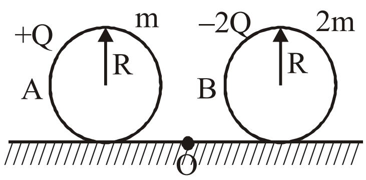
Q.1 A conducting disc of radius R about its axis with an angular velocity ω. Then the potential difference between the centre of the disc and its edge is $\frac{m_e\omega^2R^2}{Ae}$ (no magnetic field is present). Find the value of A.

Q.2 A positive charge +Q is fixed at a point A. Another positively charged particle of mass m and charge +q is projected from a point B with velocity u as shown in the figure. The point B is at the large distance from A and at distance d from the line AC. The initial velocity is parallel to the line AC. The point C is at very large distance from A. The minimum distance (in meter) of +q from +Q during the motion is $d(1+\sqrt{A})$. Find the value of A.

[Take $Qq = 4\pi\varepsilon_0 mu^2 d$ and $d = (\sqrt{2}-1)$ meter.]



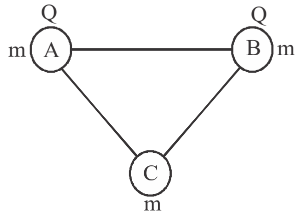
Q.4 A hollow non-conducting sphere A and a solid non conducting sphere B of equal radius R and masses m and 2m are kept a large distance apart on a rough horizontal surface. Charge on the two spheres A and B are Q and –2Q respectively. Charges are distributed uniformly and remain constant and uniform as the spheres come closer. Friction is sufficient to support pure rolling and the kinetic energy of the two spheres just before collision is K_A and K_B. Find $100 \times \frac{K_A}{K_B}$.



Q.5 A particle is uncharged and is thrown vertically upward from ground level with a speed of $5\sqrt{5}$ m/s. As a result, it attains a maximum height h. The particle is then given a positive charge +q and reaches the same maximum height h when thrown vertically upward with a speed of 13 m/s. Finally, the particle is given a negative charge –q. Ignoring air resistance, determine the speed (in m/s) with which the negatively charged particle must be thrown vertically upward, so that it attains exactly the same maximum height h.

Q.6 Three balls of equal mass m are connected by light insulating inextensible threads of length ℓ each and kept on a level smooth non conducting ground. The balls A and B are given charge Q each. The strings are all taut. The string A and B suddenly snaps. What is the maximum speed (in m/s) of C during the resulting motion ?

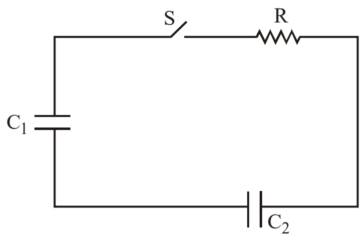
$Q = 1\,\mu C$, $\ell = 1.5$m, mass m = 1 gm.



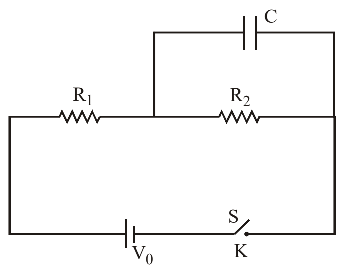
Q.7 A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges, there are two places where the total potential is zero. The reference is assumed to be at infinity. The first place is between the charges and is 4.00 cm to the left of the negative charge. The second place is 7.00 cm to the right of the negative charge. If $q_2 = -12\,\mu C$, what is the value of charge q_1 in μC.

Q.8 A capacitor of capacity $C_1 = 2F$ is fully charged by using a cell of emf E = 4V. At an instant t = 0, it is connected to an uncharged capacitor of capacity $C_2 = 3F$ through a resistance R = 1Ω. If charge on the capacitor C_2 grows with time t as $\frac{x}{5}(1-e^{-5t/6})$ C, then find the value of x.

Q.9 Consider the shown network, the capacitor C_1 (= 6μF) has an initial charge $q_0 = \frac{30e}{e-1}\mu C$, $C_2 = 4\mu F$ and R = 80Ω. Initially C_2 is uncharged. At t = 0, the switch S is closed. Obtain the charge on C_2 (in μC) at t = 192 μs.

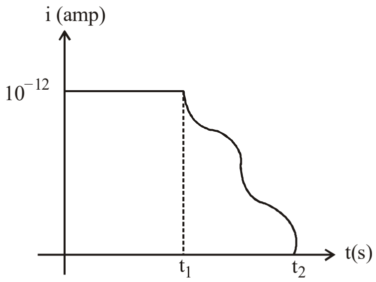


Q.10 In the connection shown in the figure the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge up to the maximum and open the switch again. Then –

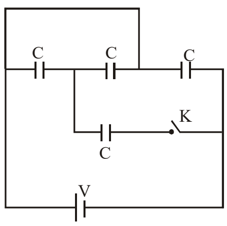


(a) the current through R_1 be I_1 immediately after closing the switch; (b) the current through R_2 be I_2 a long time after the switch was closed; (c) the current through R_2 be I_3 immediately after reopening the switch. Find $\frac{I_1}{I_2 I_3}$ (in ampere^{-1}) (Use the following data : $V_0 = 30V$, $R_1 = 10k\Omega$, $R_2 = 5k\Omega$)

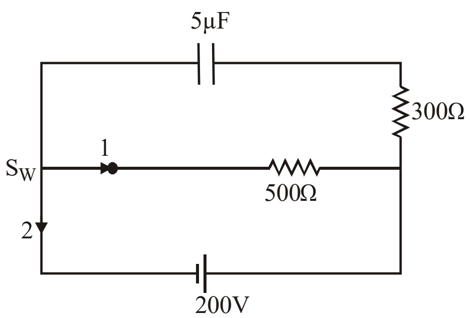
Q.11 A parallel plate capacitor of capacitance 100μF and a separation of 1cm. is charged with a battery to a potential difference of 10V. The battery is then disconnected. Electromagnetic wave is now incident on negatively charged plate which emits electrons with kinetic energies ranging from 0 to 1.5eV. The electrons are attracted to the positive plate. The current which flows between the two plates varies with time t as shown in figure. The numerical value of t_1 is $n \times 10^6$ s. Find n.



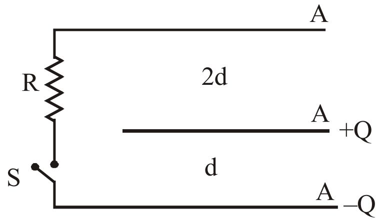
Q.12 Find the amount by which the total energy stored in the capacitors will increase (in μJ) in the circuit shown in the figure after switch K is closed ? [C = 3 μF, V = 10 volt]



Q.13 A capacitor of capacitance 5μF is connected to a source of constant emf of 200V for a long time, then the switch was shifted to contact 1 from contact 2. The amount of heat generated in the 500Ω resistance is H. Find 3200 H (in joule)



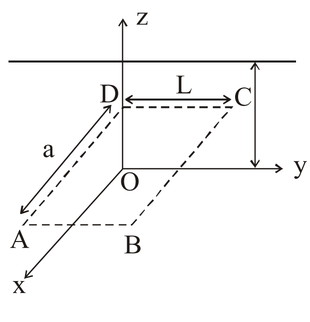
Q.14 Three identical large metal plates of area A are arranged at distances d and 2d from other. Top metal plate is uncharged, while other metal plates have charges +Q and – Q. Top and bottom metal plates are connected by switch S through a resistor of unknown resistance. 1×10^{-x} energy (in mJ) is dissipated in the resistor when switch is closed. Find the value of x. (Given : $\frac{\epsilon_0 A}{d} = 6\mu F$, $Q = 60\mu C$)



Q.15 A parallel-plate capacitor is filled by a dielectric whose permittivity varies with the applied voltage according to the law $\varepsilon = \alpha V$, where $\alpha = 1\ V^{-1}$. The same (but containing no dielectric) capacitor charged to a voltage $V_0 = 156V$ is connected in parallel to the first "nonlinear" uncharged capacitor. Determine the final voltage V across the capacitors.

Q.16 An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the yz plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$.

(ε_0 = permittivity of free space), then the value of n is :

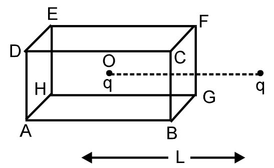


EXERCISE - 4 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 When two charges are placed at a distance apart. Find the magnitude of third charge which is placed at mid point the line joining the charge. So that system is in equilibrium - **[AIEEE-2002]**
(A) –Q/4 (B) –Q/2
(C) –Q/3 (D) – Q_1

Q.2 On moving a charge of 20 coulombs by 2 cm, 2J of work is done, then the potential difference between the points is – **[AIEEE-2002]**
(A) 0.1 V (B) 8 V
(C) 2 V (D) 0.5 V

Q.3 A charged particle q is placed at the centre O of cube of length L (ABCDEFGH). Another same charge q is placed at a distance L from O. Then the electric flux through BCFG is – **[AIEEE-2002]**



(A) $\frac{q}{4\pi\varepsilon_0}L$ (B) zero

(C) $\frac{q}{2\pi\varepsilon_0}L$ (D) $\frac{q}{3\pi\varepsilon_0 L}$

Q.4 If n capacitor connected in series with a cell of emf V volt. The energy of system is – **[AIEEE-2002]**

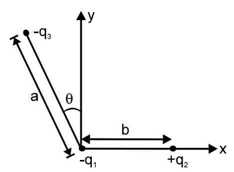
(A) $\frac{1}{2}n\,CV^2$ (B) $\frac{1}{2}\frac{CV^2}{n}$ (C) $\frac{1}{2}CV^2$ (D) none

Q.5 Capacitance (in F) of a spherical conductor with radius 1 m is – **[AIEEE-2002]**
(A) 1.1×10^{-10} (B) 10^{-6}
(C) 9×10^{-9} (D) 10^{-3}

Q.6 A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor – **[AIEEE-2003]**
(A) Remains unchanged (B) Becomes infinite
(C) Increases (D) Decreases

Q.7 The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is – **[AIEEE-2003]**
(A) 3.1×10^{-26} J (B) 4×10^{-10} J
(C) 32×10^{-32} J (D) 16×10^{-32} J

Q.8 Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in figure. The x-component of the force on $-q_1$ is proportional to – **[AIEEE-2003]**



(A) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$ (B) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$

(C) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$ (D) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$

Q.9 If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be – **[AIEEE-2003]**
(A) $(\phi_1 + \phi_2)/\varepsilon_0$ (B) $(\phi_2 - \phi_1)/\varepsilon_0$
(C) $(\phi_1 + \phi_2)\varepsilon_0$ (D) $(\phi_2 - \phi_1)\varepsilon_0$

Q.10 A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P a distance R/2 from the centre of the shell is – **[AIEEE-2003]**

(A) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$ (B) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$

(C) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$ (D) $\frac{2Q}{4\pi\varepsilon_0 R}$

Q.11 Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[AIEEE-2004]**
(A) F/4 (B) 3F/4
(C) F/8 (D) 3F/8

Q.12 A charged particle 'q' is shot towards another charged particle 'Q', which is fixed, with a speed 'v'. It approaches 'Q' upto a closest distance r and then returns. If q were given a speed of '2v', the closest distances of approach would be – **[AIEEE-2004]**
(A) r (B) 2 r
(C) r/2 (D) r/4

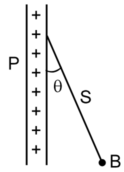
Q.13 Four charges equal to – Q are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is – **[AIEEE-2004]**

(A) $-\frac{Q}{4}(1+2\sqrt{2})$ (B) $\frac{Q}{4}(1+2\sqrt{2})$

(C) $-\frac{Q}{2}(1+2\sqrt{2})$ (D) $\frac{Q}{2}(1+2\sqrt{2})$

Q.14 A charged oil drop is suspended in a uniform field of 3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass of the charge = 9.9×10^{-15} kg and g = 10 m/s^2) – **[AIEEE-2004]**
(A) 3.3×10^{-18} C (B) 3.2×10^{-18} C
(C) 1.6×10^{-18} C (D) 4.8×10^{-18} C

Q.15 A charged ball B hangs from a silk thread S which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to - **[AIEEE-2005]**



(A) $\cos\theta$ (B) $\cot\theta$
(C) $\sin\theta$ (D) $\tan\theta$

Q.16 Two point charges +8q and –2q are located at x = 0 and x=L respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is **[AIEEE-2005]**
(A) 2 L (B) L/4
(C) 8 L (D) 4 L

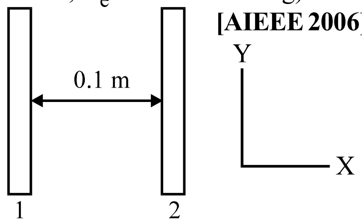
Q.17 Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are +q and –q. The potential difference between the centres of the two rings is **[AIEEE-2005]**
(A) $QR/4\pi\epsilon_0 d^2$ (B) $\frac{Q}{2\pi\epsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$
(C) zero (D) $\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

Q.18 A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is – **[AIEEE-2005]**
(A) $\sqrt{\frac{2mC\Delta T}{s}}$ (B) $\frac{mC\Delta T}{s}$
(C) $\frac{ms\Delta T}{C}$ (D) $\sqrt{\frac{2ms\Delta T}{C}}$

Q.19 A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C' then the resultant capacitance is – **[AIEEE-2005]**
(A) (n – 1)C (B) (n + 1)C
(C) C (D) nC

Q.20 An electric dipole is placed at an angle of 30º to a non-uniform electric field. the dipole will experience – **[AIEEE 2006]**
(A) a torque as well as a translational force
(B) a torque only.
(C) a translational force only in the direction of the field.
(D) a translational force only in a directin normal to the direction of the field.

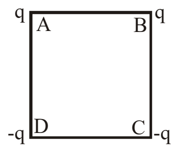
Q.21 Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 – V_1$ = 20 V. (i.e. plate 2 is at a higher potential). The plates are separated by d = 0.1 m and can be treated as infinitely large. An electron is relaeased from rest on the inner surface of plate 1. What is its speed when it hits plate 2 ? (e = 1.6×10^{-19} C, $m_e = 9.11 \times 10^{-31}$ kg) – **[AIEEE 2006]**
(A) 1.87×10^6 m/s
(B) 32×10^{-19} m/s
(C) 2.65×10^6 m/s
(D) 7.02×10^{12} m/s



Q.22 Two spherical conducors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the sphere are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is – **[AIEEE 2006]**
(A) 2 : 1 (B) 1 : 4
(C) 4 : 1 (D) 1 : 2

Q.23 An electric charge 10^{-3} μC is placed at the origin (0,0) of X - Y co-ordinate system. Two points A and B are situated at ($\sqrt{2}$, $\sqrt{2}$) and (2, 0) respectively. The potential difference between the points A and B will be - **[AIEEE 2007]**
(A) 9 volt (B) zero
(C) 2 volt (D) 4.5 volt

Q.24 Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **[AIEEE 2007]**



(A) $\vec{E}$ remains unchanged, V changes
(B) Both $\vec{E}$ and V change
(C) $\vec{E}$ and V remain unchanged
(D) $\vec{E}$ changes, V remains unchanged

Q.25 The potential at a point x (measured in μm) due to some changes situated on the x-axis is given by
V (x) = 20 /(x^2- 4) volts. The electric field E at x = 4 μm is given by **[AIEEE 2007]**
(A) 5/3 Volt/μm and in the –ve x direction
(B) 5/3 Volt/μm and in the +ve x direction
(C) 10/9 Volt/μm and in the -ve x direction
(D) 10/9 Volt/μm and in the +ve x direction

Q.26 If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio (electronic charge on the moon/ electronic charge on the earth) to be
(A) 1 (B) 0 **[AIEEE 2007]**
(C) g_E/g_M (D) g_M/g_E

Q.27 A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be - **[AIEEE-2007]**

(A) 1 (B) 2

(C) 1/4 (D) 1/2

Q.28 A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is – **[AIEEE-2007]**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$

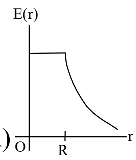
(C) $(K-1)CV^2$ (D) zero

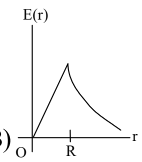
Q.29 A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness d/3 while the other one has dielectric constant $k_2 = 6$ and thickness 2d/3. Capacitance of the capacitor is now – **[AIEEE-2008]**

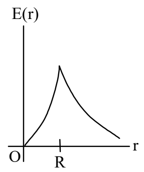
(A) 45 pF (B) 40.5 pF

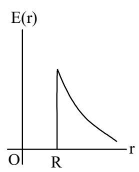
(C) 20.25 pF (D) 1.8 pF

Q.30 A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field E (r) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[AIEEE 2008]**

(A) 

(B) 

(C) 

(D) 

Q.31 This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\pi GM$. and

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. **[AIEEE-2008]**

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true. Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

Q.32 **Statement-1** : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2 : The net work done by a conservative force on an object moving along a closed loop is zero. **[AIEEE-2009]**

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true. Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

Q.33 Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is –**[AIEEE-2009]**

(A) 0 (B) $\frac{Q}{4\pi \epsilon_0 r_1^2}$

(C) $\frac{Qr_1^2}{4\pi \epsilon_0 R^4}$ (D) $\frac{Qr_1^2}{3\pi \epsilon_0 R^4}$

Q.34 Two points P and Q are maintained at the potentials of 10V and –4V, respectively. The work done in moving 100 electrons from P to Q is – **[AIEEE-2009]**

(A) -9.60×10^{-17} J (B) 9.60×10^{-17} J

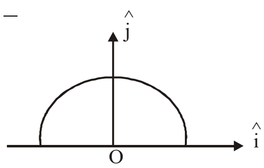
(C) -2.24×10^{-16} J (D) 2.24×10^{-16} J

Q.35 A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals – **[AIEEE-2009]**

(A) $-2\sqrt{2}$ (B) – 1

(C) 1 (D) $-\frac{1}{\sqrt{2}}$

Q.36 A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is – **[AIEEE 2010]**



(A) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (B) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(C) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (D) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Q.37 Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by – **[AIEEE 2010]**

(A) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (B) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(C) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (D) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Q.38 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 16gm^{-3}, the dielectric constant of the liquid is – **[AIEEE 2010]**

(A) 4 (B) 3
(C) 2 (D) 1

Q.39 Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be – **[AIEEE 2010]**

(A) 1 (B) 1/2
(C) 1/4 (D) 2

Q.40 A resistor R and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10}2.5 = 0.4$) **[AIEEE 2011]**

(A) $1.3 \times 10^4\Omega$ (B) $1.7 \times 10^5\,\Omega$
(C) $2.7 \times 10^6\Omega$ (D) $3.3 \times 10^7\Omega$

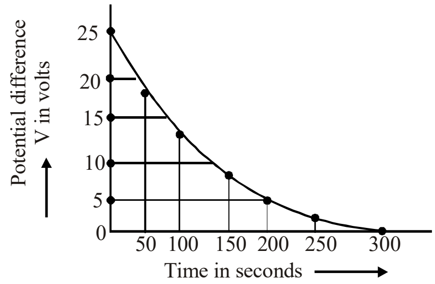
Q.41 The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a,b are constants. Then the charge density inside the ball is : **[AIEEE 2011]**

(A) $-24\pi a\varepsilon_0 r$ (B) $-6\pi a\varepsilon_0 r$
(C) $-24\pi a\varepsilon_0$ (D) $-6a\varepsilon_0$

Q.42 Two identical charged spheres suspended from a common point by two massless strings of length ℓ are initially a distance d ($d << \ell$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate.As a result the charges approach each other with a velocity v. Then as a function of distance x between them – **[AIEEE 2011]**

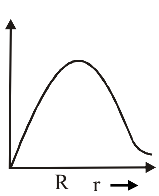
(A) $v \propto x^{-1/2}$ (B) $v \propto x^{-1}$
(C) $v \propto x^{1/2}$ (D) $v \propto x$

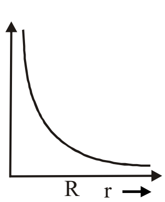
Q.43 The figure shows an experimental plot discharging of capacitor in an RC circuit. The time constant τ of this circuit lies between– **[AIEEE 2012]**

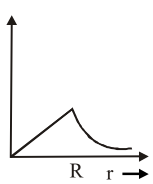


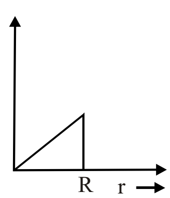
(A) 150 sec and 200 sec (B) 0 and 50 sec
(C) 50 sec and 100 sec (D) 100 sec and 150 sec

Q.44 In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be : **[AIEEE 2012]**

(A) 

(B) 

(C) 

(D) 

Q.45 An insulating solid sphere of radius R has a uniformly positive charge density ρ.As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero. **[AIEEE 2012]**

Statement-1 : When a charge 'q' is take from the centre of the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$.

Statement-2 : The electric field at a distance r (r < R) from the centre of the sphere is $\rho r/3\varepsilon_0$.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.
(B) Statement 1 is true Statement 2 is false.
(C) Statement 1 is false Statement 2 is true.
(D) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

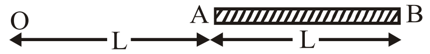
Q.46 Two capacitors C_1 and C_2 are charged to 120V and 200V respectively. It is found that by connecting them together the potential on each one can be made zero. Then – **[JEE MAIN 2013]**

(A) $5C_1 = 3C_2$ (B) $3C_1 = 5C_2$
(C) $3C_1 + 5C_2 = 0$ (D) $9C_1 = 4C_2$

Q.47 Two charges, each equal to q, are kept at x = –a and x= a on the x-axis. A particle of mass m and charge $q_0 = q/2$. is placed at the origin. If charge q_0 is given a small displacement (y << a) along the y-axis, the net force acting on the particle is proportional to **[JEE MAIN 2013]**

(A) y (B) –y
(C) 1/y (D) –1/y

Q.48 A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is –



[JEE MAIN 2013]

(A) $\frac{Q}{8\pi\varepsilon_0 L}$ (B) $\frac{3Q}{4\pi\varepsilon_0 L}$

(C) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (D) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$

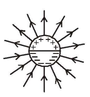
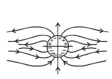
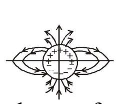
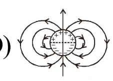
Q.49 Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is – **[JEE MAIN 2014]**

(A) –80 V (B) 80 V
(C) 120 V (D) –120 V

Q.50 A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to – **[JEE MAIN 2014]**

(A) 3×10^4 C/m^2 (B) 6×10^4 C/m^2
(C) 6×10^{-7} C/m^2 (D) 3×10^{-7} C/m^2

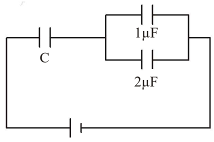
Q.51 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge –σ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **[JEE MAIN 2015]**

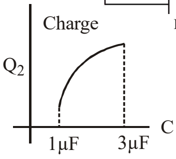
(A)  (B)  (C)  (D) 

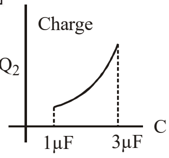
Q.52 A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE MAIN 2015]**

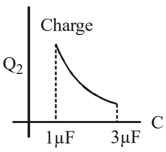
(A) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(B) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(C) $2R < R_4$
(D) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

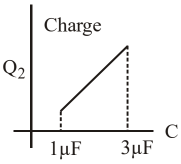
Q.53 In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of C is given properly by : (Figures are drawn schematically and are not to scale) **[JEE MAIN 2015]**



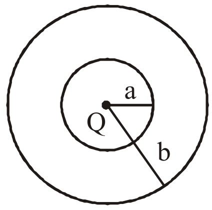
(A) 

(B) 

(C) 

(D) 

Q.54 The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$,

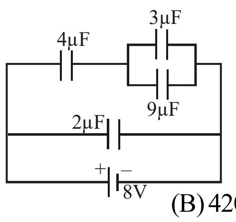


where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, **[JEE MAIN 2016]**

(A) $\frac{Q}{2\pi(b^2 - a^2)}$ (B) $\frac{2Q}{\pi(b^2 - a^2)}$

(C) $\frac{2Q}{\pi a^2}$ (D) $\frac{Q}{2\pi a^2}$

Q.55 A combination of capacitor is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9μF capacitors), at a point distant 30 m from it, would equal : **[JEE MAIN 2016]**



(A) 360 N/C (B) 420 N/C
(C) 480 N/C (D) 240 N/C

Q.56 A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is: **[JEE MAIN 2017]**

(A) 16 (B) 24
(C) 32 (D) 2

Q.57 An electric dipole has a fixed dipole moment $\vec{P}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\,\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\,\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is: **[JEE MAIN 2017]**

(A) 45º (B) 60º
(C) 90º (D) 30º

Q.58 A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant K = 5/3 is inserted between the plates, the magnitude of the induced charge will be : **[JEE MAIN 2018]**

(A) 2.4 nC (B) 0.9 nC
(C) 1.2 nC (D) 0.3 nC

Q.59 Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is: **[JEE MAIN 2018]**

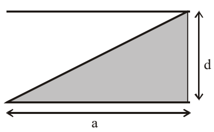
(A) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (B) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

(C) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (D) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

Q.60 Three charges +Q, q, +Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x=0, is zero, then value of q is **[JEE MAIN 2019 (JAN)]**

(A) +Q/2 (B) –Q/2
(C) –Q/4 (D) +Q/4

Q.61 A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant k, as shown in the figure. Capacitance of this capacitor is : **[JEE MAIN 2019 (JAN)]**



(A) $\frac{1}{2}\frac{k\varepsilon_0 a^2}{d}$ (B) $\frac{k\varepsilon_0 a^2}{d}\ln k$

(C) $\frac{k\varepsilon_0 a^2}{d\,(k-1)}\ln k$ (D) $\frac{k\varepsilon_0 a^2}{2d\,(k+1)}$

Q.62 For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is : **[JEE MAIN 2019 (JAN)]**

(A) $R/\sqrt{5}$ (B) R
(C) $R/\sqrt{2}$ (D) $R\sqrt{2}$

Q.63 The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : (take g = 10 m/s^2) **[JEE MAIN 2019 (APRIL)]**

(A) $\tan^{-1}$ (5.0) (B) $\tan^{-1}$ (2.0)
(C) $\tan^{-1}$ (0.5) (D) $\tan^{-1}$ (0.2)

Q.64 Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant is the capacitance is 15 pF?
(Given $\varepsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2) **[JEE MAIN 2019 (APRIL)]**

(A) 3.8 (B) 4.5
(C) 6.2 (D) 8.5

Q.65 A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of – 4Q, the new potential difference between the same two surfaces is **[JEE MAIN 2019 (APRIL)]**

(A) V (B) 2V
(C) –2V (D) 4V

Q.66 An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is :- **[JEE MAIN 2019 (APRIL)]**

(A) $\sqrt{\frac{qE}{2md}}$ (B) $2\sqrt{\frac{qE}{md}}$

(C) $\sqrt{\frac{2qE}{md}}$ (D) $\sqrt{\frac{qE}{md}}$

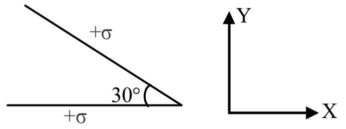
Q.67 The electric field in a region is given by $\vec{E} = (Ax + B)\,\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is : **[JEE MAIN 2019 (APRIL)]**

(A) – 48 V (B) –520 V
(C) 180 V (D) 320 V

Q.68 A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is: **[JEE MAIN 2019 (APRIL)]**

(A) 5V (B) 2V
(C) 3V (D) 1V

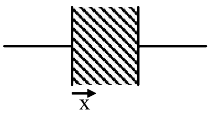
Q.69 There are two infinite plane sheets each having uniform surface charge density $+\sigma$ C/m^2. They are inclined to each other at an angle 30° as shown in the figure. The electric field in the region shown between them is given by: **[JEE MAIN 2020 (JAN)]**



(A) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{1}{2}\hat{x}\right]$ (B) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{1}{2}\hat{x}\right]$

(C) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{1}{2}\hat{x}\right]$ (D) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{1}{2}\hat{x}\right]$

Q.70 A parallel plate capacitor with plate area A & plate separation d is filled with a dielectric material of dielectric constant given by $k = k_0(1+\alpha x)$, where 'x' is the distance measured from one of the plates. Calculate capacitance of system: (given $\alpha d << 1$)



[JEE MAIN 2020 (JAN)]

(A) $\frac{k_0\varepsilon_0 A}{d}(1+\alpha^2 d^2)$ (B) $\frac{k_0\varepsilon_0 A}{d}\left(1+\frac{\alpha d}{2}\right)$

(C) $\frac{k_0\varepsilon_0 A}{2d}(1+\alpha d)$ (D) $\frac{k_0\varepsilon_0 A}{2d}\left(1+\frac{\alpha d}{2}\right)$

Q.71 A capacitor of 60 pF charged to 20 volt. Now battery is removed and then this capacitor is connected to another identical uncharged capacitor. Find heat loss in nJ. **[JEE MAIN 2020 (JAN)]**

Q.72 In finding the electric field using Gauss Law the formula $|\vec{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. The equation can be used in which of the following situation? **[JEE MAIN 2020 (JAN)]**

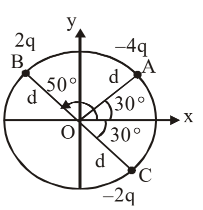
(A) Only when the Gaussian surface is an equipotential surface.

(B) Only when $|\vec{E}|$ = constant on the surface.

(C) For any choice of Gaussian surface.

(D) Only when the Gaussian surface is an equipotential surface and $|\vec{E}|$ is constant on the surface.

Q.73 Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is : **[JEE MAIN 2020 (JAN)]**



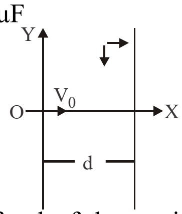
(A) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (B) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (C) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (D) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

Q.74 Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be : **[JEE MAIN 2020 (JAN)]**

(A) 3.2 μF (B) 8.4 μF

(C) 1.6 μF (D) 4.2 μF

Q.75 A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is : **[JEE MAIN 2020 (SEPT)]**

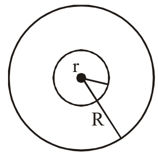


(A) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$ (B) $y = \frac{qEd}{mV_0^2}(x-d)$

(C) $y = \frac{qEd}{mV_0^2}x$ (D) $y = \frac{qEd^2}{mV_0^2}x$

Q.76 A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is (X/100) J then value of X to the nearest integer is___. **[JEE MAIN 2020 (SEPT)]**

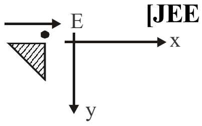
Q.77 A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R>r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is : **[JEE MAIN 2020 (SEPT)]**

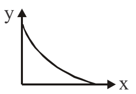


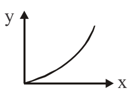
(A) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)Q}{2(R^2+r^2)}$

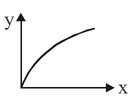
(C) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)Q}{(R^2+r^2)}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)Q}{(R^2+r^2)}$

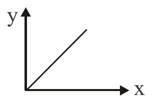
Q.78 A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale). **[JEE MAIN 2020 (SEPT)]**



(A) 

(B) 

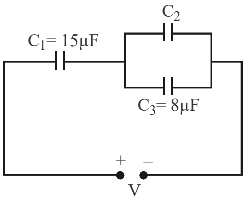
(C) 

(D) 

Q.79 A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is: **[JEE MAIN 2020 (SEPT)]**

(A) 10 μF (B) 15 μF
(C) 20 μF (D) 30 μF

Q.80 In the circuit shown in the figure, the total charge in 750 μC and the voltage across capacitor C_2 is 20V. Then the charge on capacitor C_2 is : **[JEE MAIN 2020 (SEPT)]**



(A) 590 μC (B) 450 μC
(C) 650 μC (D) 160 μC

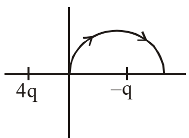
Q.81 Two isolated conducting spheres S_1 and S_2 of radius (2/3) R and (1/3) R have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively **[JEE MAIN 2020 (SEPT)]**

(A) 6 μC and 3 μC (B) +4.5μC and –4.5μC
(C) 3 μC and 6 μC (D) 4.5 μC on both

Q.82 Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is:
[JEE MAIN 2020 (SEPT)]

(A) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (B) $\frac{Q_2}{4\pi\varepsilon_0 R}$ (C) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ (D) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

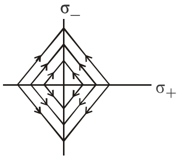
Q.83 A two point charges 4q and –q are fixed on the x-axis at x = –d/2 and x = d/2, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will : **[JEE MAIN 2020 (SEPT)]**

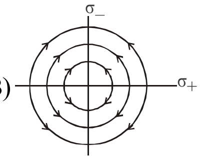


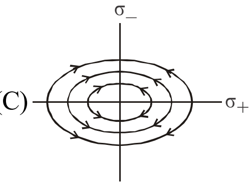
(A) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$ (B) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

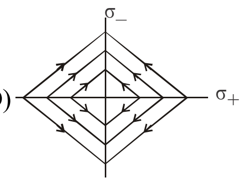
(C) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$ (D) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

Q.84 Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_- where $|\sigma_+| > |\sigma_-|$ intersect at right angle. Which of the following best represents the electric field lines for this system :
[JEE MAIN 2020 (SEPT)]

(A) 

(B) 

(C) 

(D) 

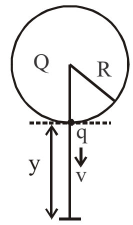
Q.85 A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance C/2. The energy loss in the process after the charge is distributed between the two capacitors is :
[JEE MAIN 2020 (SEPT)]

(A) $\frac{1}{6}CV_0^2$ (B) $\frac{1}{2}CV_0^2$ (C) $\frac{1}{3}CV_0^2$ (D) $\frac{1}{4}CV_0^2$

Q.86 A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is **[JEE MAIN 2020 (SEPT)]**

(A) $\sqrt{2/a}$ (B) $\sqrt{1/a}$
(C) a (D) $\sqrt{3/a}$

Q.87 A solid sphere of radius R carries a charge (Q + q) distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).



[JEE MAIN 2020 (SEPT)]

(A) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(B) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(C) $v^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

(D) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

Q.88 Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is: **[JEE MAIN 2020 (SEPT)]**

(A) $\frac{9}{2}CV^2$ (B) $\frac{25}{6}CV^2$ (C) zero (D) $\frac{3}{2}CV^2$

Q.89 A parallel plate capacitor has plate of length 'ℓ', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will be energy stored in the capacitor be two times the initial energy stored? **[JEE MAIN 2020 (SEPT)]**

(A) $\ell / 4$ (B) $\ell / 2$

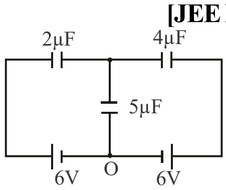
(C) $\ell / 3$ (D) $2\ell / 3$

Q.90 Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively: (Take V = 0 at infinity) **[JEE MAIN 2020 (SEPT)]**

(A) $V = \frac{10q}{4\pi\varepsilon_0 R}, E = \frac{10q}{4\pi\varepsilon_0 R^2}$ (B) $V = 0, E = \frac{10q}{4\pi\varepsilon_0 R^2}$

(C) $V = 0, E = 0$ (D) $V = \frac{10q}{4\pi\varepsilon_0 R}, E = 0$

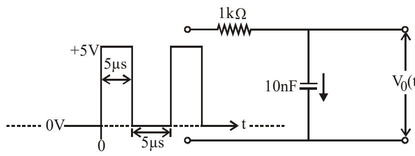
Q.91 In the circuit shown, charge on the 5 μF capacitor is : **[JEE MAIN 2020 (SEPT)]**

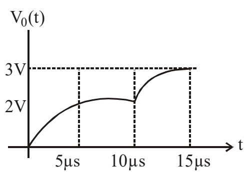


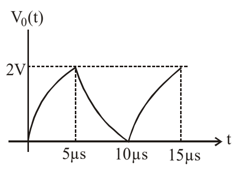
(A) 5.45 μC (B) 16.36 μC

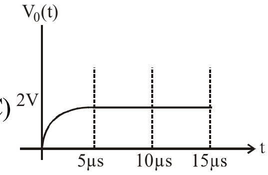
(C) 10.90 μC (D) 18.00 μC

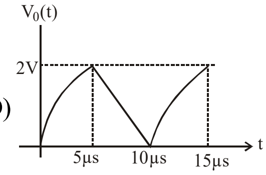
Q.92 For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by: **[JEE MAIN 2020 (SEPT)]**



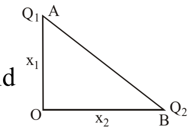
(A) 

(B) 

(C) 

(D) 

Q.93 Charges Q_1 and Q_2 arc at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to : **[JEE MAIN 2020 (SEPT)]**



(A) $\frac{x_2^2}{x_1^2}$ (B) $\frac{x_1^3}{x_2^3}$ (C) $\frac{x_1}{x_2}$ (D) $\frac{x_2}{x_1}$

Q.94 Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell ? **[JEE MAIN 2020 (SEPT)]**

(A) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for r > R

(B) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} > F > 0$ for r < R

(C) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for all r

(D) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for r < R

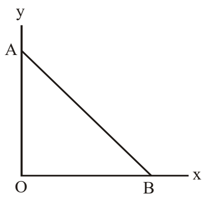
Q.95 Two identical electric point dipoles have dipole moments $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is'm', their speed when they arc infinitely far apart is: **[JEE MAIN 2020 (SEPT)]**

(A) $\frac{p}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (B) $\frac{p}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

(C) $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ (D) $\frac{p}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$

EXERCISE - 5 [PREVIOUS YEARS AIPMT / NEET QUESTIONS]

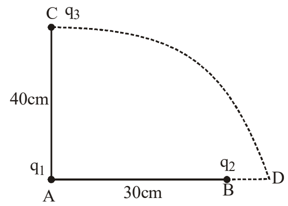
Q.1 As per the diagram, a point charge +q is placed at the origin O. Work done in taking another point charge –Q from the point A [coordinates (0, a)] to another point P [coordinates (a, 0)] along the straight path AB is – **[AIPMT 2005]**



(A) zero (B) $\left(\frac{-qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\sqrt{2}a$

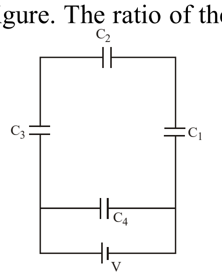
(C) $\left(\frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\frac{a}{\sqrt{2}}$ (D) $\left(\frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\sqrt{2}a$

Q.2 Two charges q_1 and q_2 are placed 30cm. apart, as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40cm. from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is – **[AIPMT 2005]**



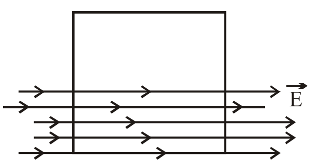
(A) $8q_1$ (B) $6q_1$
(C) $8q_2$ (D) $6q_2$

Q.3 A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in figure. The ratio of the charges on C_2 and C_4 is – **[AIPMT 2005]**



(A) 4/7 (B) 3/22
(C) 7/4 (D) 22/3

Q.4 A square surface of side L metres is in the plane of the paper. A uniform electric field $\vec{E}$ (volt/m), also in the plane of the paper, is limited only to the lower half of the square surface (see figure). The electric field flux in SI units associated with the surface is – **[AIPMT 2006]**

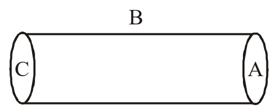


(A) $EL^2/2$ (B) zero
(C) EL^2 (D) $EL^2/2\varepsilon_0$

Q.5 An electric dipole of moment $\vec{p}$ is lying along a uniform electric field $\vec{E}$. The work done in rotating the dipole by 90° is – **[AIPMT 2006]**
(A) pE/2 (B) 2pE
(C) pE (D) $\sqrt{2}$pE

Q.6 A parallel plate air capacitor is charged to a potential difference of V volts. After disconnecting the charging battery the distance between the plates of the capacitor is increased using an insulating handle. As a result the potential difference between the plates **[AIPMT 2006]**
(A) does not change (B) becomes zero
(C) increases (D) decreases

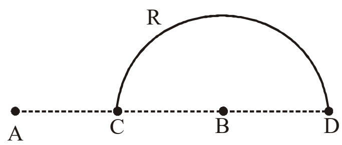
Q.7 A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in units of voltmeter associated with the curved surface B, the flux linked with the plane surface A in units of voltmeter will be – **[AIPMT 2007]**



(A) $\frac{q}{2\varepsilon_0}$ (B) $\frac{\phi}{3}$

(C) $\frac{q}{\varepsilon_0} - \phi$ (D) $\frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$

Q.8 Charges +q and –q are placed at points A and B respectively which are a distance 2L apart, C is the midpoint between A and B. The work done in moving a charge +Q along the semicircle CRD is – **[AIPMT 2007]**

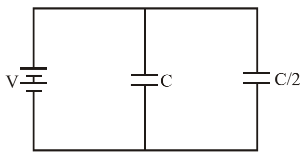


(A) $\frac{qQ}{2\pi\varepsilon_0 L}$ (B) $\frac{qQ}{6\pi\varepsilon_0 L}$

(C) $-\frac{qQ}{6\pi\varepsilon_0 L}$ (D) $\frac{qQ}{4\pi\varepsilon_0 L}$

Q.9 Three point charges +q, –q and +q are placed at points (x = 0, y = a, z = 0), (x = 0, y= 0, z = 0) and (x= a, y = 0, z = 0) respectively. Magnitude and direction of the electric dipole moment vector of this charge assembly are –
(A) $\sqrt{2}$qa along the line joining points (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0) **[AIPMT 2007]**
(B) qa long the line joining points (x = 0, y = 0, z = 0) and (x = a, y = a, z = 0)
(C) $\sqrt{2}$qa along +ve x direction
(D) $\sqrt{2}$qa along +ve y direction

Q.10 Two condensers, one of capacity C and other of capacity C/2 are connected to a V-volt battery, as shown. The work done in charging fully both the condensers is **[AIPMT 2007]**



(A) $\frac{1}{4}CV^2$ (B) $\frac{3}{4}CV^2$

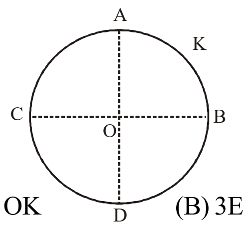
(C) $\frac{1}{2}CV^2$ (D) $2CV^2$

Q.11 The energy required to charge a parallel plate condenser of plate separation (d) and plate area of cross-section (A) such that the uniform electric field between the plates is E, is **[AIPMT 2008]**

(A) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (B) $\frac{1}{2}\varepsilon_0 E^2 / Ad$

(C) $\varepsilon_0 E^2/Ad$ (D) $\varepsilon_0 E^2 Ad$

Q.12 A Thin conducting ring of radius R is given a charge +Q. The electric field at the centre O of the ring due to the charge on the part AKB of the ring is E. The electric field at the centre due to the charge on the part ACDB of the ring is – **[AIPMT 2008]**



(A) 3E along OK (B) 3E along KO

(C) E along OK (D) E along KO

Q.13 The electric potential at a point in free space due to a charge Q coulomb is $Q \times 10^{11}$ volts. The electric field at the point is **[AIPMT 2008]**

(A) $12\pi\varepsilon_0 Q \times 10^{22}$ volt/m (B) $4\pi\varepsilon_0 Q \times 10^{22}$ volt/m

(C) $12\pi\varepsilon_0 Q \times 10^{20}$ volt/m (D) $4\pi\varepsilon_0 Q \times 10^{20}$ volt/m

Q.14 Three concentric spherical shells have radii a, b and c (a < b < c) and have surface charge densities σ, $-\sigma$ and σ respectively. If V_A, V_B and V_C denote the potentials of the three shells, then for c = a + b, we have **[AIPMT 2009]**

(A) $V_C = V_B \neq V_A$ (B) $V_C \neq V_B \neq V_A$

(C) $V_C = V_B = V_A$ (D) $V_C = V_A \neq V_B$

Q.15 Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be:

(A) 3C, V/3 (B) C/3, 3V **[AIPMT 2009]**

(C) 3C, 3V (D) C/3, V/3

Q.16 The electric potential at a point (x, y, z) is given by $V = -x^2y - xz^3 + 4$. The electric field $\vec{E}$ at that point is –

(A) $\vec{E} = \hat{i}\,2xy + \hat{j}\,(x^2 + y^2) + \hat{k}\,(3xz - y^2)$ **[AIPMT 2009]**

(B) $\vec{E} = \hat{i}z^3 + \hat{j}\,xyz + \hat{k}z^2$

(C) $\vec{E} = \hat{i}\,(2xy - z^3) + \hat{j}\,xy^2 + \hat{k}3z^2x$

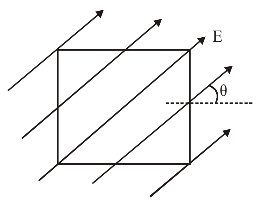
(D) $\vec{E} = \hat{i}\,(2xy + z^3) + \hat{j}\,x^2 + \hat{k}3xz^2$

Q.17 Two positive ions, each carrying a charge q, are separated by a distance d. If F is the force of repulsion between the ions, the number of electrons missing from each ion will be (e being the charge on an electron) **[AIPMT (PRE) 2010]**

(A) $\frac{4\pi\varepsilon_0 Fd^2}{e^2}$ (B) $\sqrt{\frac{4\pi\varepsilon_0 Fe^2}{d^2}}$

(C) $\sqrt{\frac{4\pi\varepsilon_0 Fd^2}{e^2}}$ (D) $\frac{4\pi\varepsilon_0 Fd^2}{q^2}$

Q.18 A square surface of side L meter in the plane of the paper is placed in a uniform electric field E (volt/m) acting along the same plane at an angle θ with the horizontal side of the square as shown in figure. The electric flux linked to the surface in units of volt × meter is **[AIPMT (PRE) 2010]**



(A) EL^2 (B) $EL^2 \cos\theta$

(C) $EL^2 \sin\theta$ (D) zero

Q.19 A series combination of n_1 capacitors, each of value C_1 is charged by a source of potential difference 4V. When another parallel combination of n_2 capacitors, each of value C_2, is charged by a source of potential difference V, it has the same (total) energy stored in it, as the first combination has. The value of C_2, in terms of C_1, is then **[AIPMT (PRE) 2010]**

(A) $\frac{2C_1}{n_1 n_2}$ (B) $16\frac{n_2}{n_1}C_1$ (C) $2\frac{n_2}{n_1}C_1$ (D) $\frac{16C_1}{n_1 n_2}$

Q.20 Two parallel metal plates having charges +Q and –Q face each other at a certain distance between them. If the plates are now dipped in kerosene oil tank, the electric field between the plates will **[AIPMT (MAINS) 2010]**

(A) become zero (B) increase

(C) decrease (D) remains same

Q.21 The electric field at a distance 3R/2 from the centre of a charged conducting spherical shell of radius R is E. The electric field at a distance R/2 from the centre of the sphere is **[AIPMT (MAINS) 2010]**

(A) zero (B) E

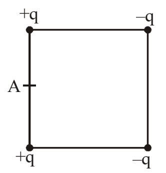
(C) E/2 (D) E/3

Q.22 A charge Q is enclosed by a Gaussian spherical surface of radius R. If the radius is doubled, then the outward electric flux will **[AIPMT (PRE) 2011]**

(A) Be doubled (B) Increase four times

(C) Be reduced to half (D) Remain the same

Q.23 Four electric charges +q, +q, –q and –q are placed at the corners of a square of side 2L (see figure). The electric potential at point A, midway between the two charges +q and +q, is – **[AIPMT (PRE) 2011]**



(A) zero (B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}(1+\sqrt{5})$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$

Q.24 A parallel plate condenser has a uniform electric field E (V/m) in the space between the plates. If the distance between the plates is d (m) and area of each plate is A (m^2) the energy (joules) stored in the condenser is – **[AIPMT (PRE) 2011]**

(A) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (B) E^2Ad/ε_0

(C) $\frac{1}{2}\varepsilon_0 E^2$ (D) $\varepsilon_0 EAd$

Q.25 The electric potential V at any point (x, y, z), all in meters in space is given by $V = 4x^2$ volt. The electric field at the point (1, 0, 2) in volt/meter is **[AIPMT (MAINS) 2011]**

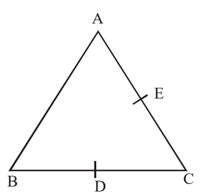
(A) 8 along positive X-axis

(B) 16 along negative X-axis

(C) 16 along positive X-axis

(D) 8 along negative X-axis

Q.26 Three charges, each +q, are placed at the corners of an isosceles triangle ABC of sides BC and AC, D and E are the mid points of BC and CA. The work done in taking a charge Q from D to E is: **[AIPMT (MAINS) 2011]**



(A) $\frac{qQ}{8\pi\varepsilon_0 a}$ (B) $\frac{qQ}{4\pi\varepsilon_0 a}$ (C) zero (D) $\frac{3qQ}{4\pi\varepsilon_0 a}$

Q.27 An electric dipole of moment 'p' is placed in an electric field of intensity 'E'. The dipole acquires a position such that the axis of the dipole makes an angle θ with the direction of the field. Assuming that the potential energy of the dipole to be zero when θ = 90°, the torque and the potential energy of the dipole will respectively **[AIPMT (PRE) 2012]**

(A) p E sin θ, – p E cos θ (B) p E sin θ, – 2 pE cos θ

(C) p E sin θ, 2 p Ecos θ (D) p E cos θ, – p Ecos θ

Q.28 Four point charges –Q, –q, 2q and 2Q are placed, one at each corner of the square. The relation between Q and q for which the potential at the centre of the square is zero is: **[AIPMT (PRE) 2012]**

(A) Q = –q (B) Q = q

(C) Q = q (D) Q = 1/q

Q.29 What is the flux through a cube of side 'a' if a point charge of q is at one of its corner : **[AIPMT (PRE) 2012]**

(A) $\frac{2q}{\varepsilon_0}$ (B) $\frac{q}{8\varepsilon_0}$ (C) $\frac{q}{\varepsilon_0}$ (D) $\frac{q}{2\varepsilon_0}6a^2$

Q.30 A parallel plate capacitor has a uniform electric field E in the space between the plates. If the distance between the plates is d and area of each plate is A,the energy stored in the capacitor is : **[AIPMT (MAINS) 2012]**

(A) $\frac{1}{2}\varepsilon_0 E^2$ (B) $\frac{E^2Ad}{\varepsilon_0}$

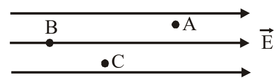
(C) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (D) $\varepsilon_0 EAd$

Q.31 Two metallic spheres of radii 1 cm and 3cm are given charges of -1×10^{-2} C and 5×10^{-2} C, respectively . If these are connected by a conducting wire, the final charge on the bigger sphere is – **[AIPMT (MAINS) 2012]**

(A) 2×10^{-2} C (B) 3×10^{-2} C

(C) 4×10^{-2} C (D) 1×10^{-2} C

Q.32 A, B and C are three points in a uniform electric field. The electric potential is – **[NEET 2013]**



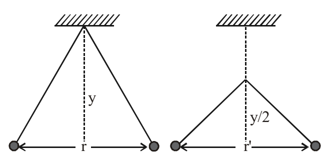
(A) Same at all the three points A, B and C

(B) Maximum at A

(C) Maximum at B

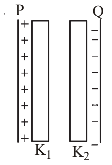
(D) Maximum at C

Q.33 Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r. Now the strings are rigidly clamped at half the height. The equilibrium separation between the balls now become **[NEET 2013]**

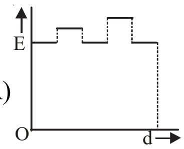


(A) $\left(\frac{2r}{3}\right)$ (B) $\left(\frac{r}{\sqrt{2}}\right)^2$ (C) $\left(\frac{r}{\sqrt[3]{2}}\right)$ (D) $\left(\frac{2r}{\sqrt{3}}\right)$

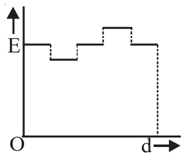
Q.34 Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between plates of a parallel plate capacitor, as shown in the figure. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by **[AIPMT 2014]**



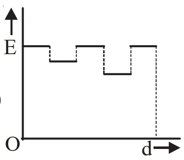
(A)



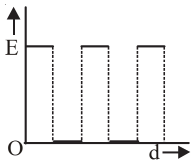
(B)



(C)



(D)



Q.35 A conducting sphere of radius R is given a charge Q. The electric potential and the electric field at the centre of the sphere respectively are – **[AIPMT 2014]**

(A) Zero and $\dfrac{Q}{4\pi\varepsilon_0 R^2}$ (B) $\dfrac{Q}{4\pi\varepsilon_0 R}$ and zero

(C) $\dfrac{Q}{4\pi\varepsilon_0 R}$ and $\dfrac{Q}{4\pi\varepsilon_0 R^2}$ (D) Both are zero

Q.36 In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where V is in volts and x, y, z are in metres. The electric force experienced by a charge of 2 coulomb situated at point (1, 1, 1) is **[AIPMT 2014]**

(A) $6\sqrt{5}$ N (B) 30 N

(C) 24 N (D) $4\sqrt{35}$ N

Q.37 A parallel plate air capacitor of capacitance C is connected to a cell of emf V and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect? **[AIPMT 2015]**

(A) The energy stored in the capacitor decreases K times.

(B) The change in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K}-1\right)$

(C) The charge on the capacitor is not conserved.

(D) The potential difference between the plates decreases K times.

Q.38 The electric field in a certain region is acting radially outward and is given by E = Ar. A charge contained in a sphere of radius 'a' centred at the origin of the field, will given by : **[AIPMT 2015]**

(A) $A\varepsilon_0 a^2$ (B) $4\pi \varepsilon_0 Aa^3$

(C) $\varepsilon_0 Aa^3$ (D) $4\pi \varepsilon_0 Aa^2$

Q.39 A parallel plate air capacitor has capacity 'C' distance of separation between plates is 'd' and potential difference 'V' is applied between the plates force of attraction between the plates of the parallel plate air capacitor is : **[RE-AIPMT 2015]**

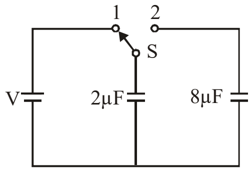
(A) $\dfrac{C^2V^2}{2d^2}$ (B) $\dfrac{C^2V^2}{2d}$ (C) $\dfrac{CV^2}{2d}$ (D) $\dfrac{CV^2}{d}$

Q.40 If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1, 1, 0) is : **[RE-AIPMT 2015]**

(A) $-(6\hat{i}+9\hat{j}+\hat{k})$ (B) $-(3\hat{i}+5\hat{j}+3\hat{k})$

(C) $-(6\hat{i}+5\hat{j}+2\hat{k})$ (D) $-(2\hat{i}+3\hat{j}+\hat{k})$

Q.41 A capacitor of 2 μF is charged as shown in the diagram. When the switch S is turned to position 2, the percentage of its stored energy dissipated is **[NEET PHASE 1 - 2016]**



(A) 0% (B) 20%

(C) 75% (D) 80%

Q.42 Two identical charged spheres suspended from a common point by two massless strings of lengths ℓ, are initially at a distance d ($d << \ell$) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then v varies as a function of the distance x between the spheres, as **[NEET PHASE 1 - 2016]**

(A) $v \propto x^{1/2}$ (B) $v \propto x$

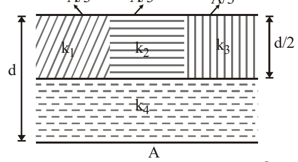
(C) $v \propto x^{-1/2}$ (D) $v \propto x^{-1}$

Q.43 An electric dipole is placed at an angle of 30° with an electric field intensity 2×10^5 N/C. It experiences a torque equal to 4 N m. The charge on the dipole, if the dipole length is 2 cm, is **[NEET PHASE 2 - 2016]**

(A) 8 mC (B) 2 mC

(C) 5 mC (D) 7 μC

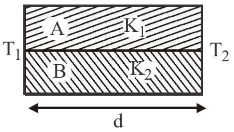
Q.44 A parallel-plate capacitor of area A, plate separation d and capacitance C is filled with four dielectric materials having dielectric constants k_1, k_2, k_3 and k_4 as shown in the figure below. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by **[NEET PHASE 2 - 2016]**



(A) $k = k_1 + k_2 + k_3 + 3k_4$ (B) $k = \frac{2}{3}(k_1 + k_2 + k_3) + 2k_4$

(C) $\dfrac{2}{k} = \dfrac{3}{k_1 + k_2 + k_3} + \dfrac{1}{k_4}$ (D) $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_3} + \dfrac{3}{2k_4}$

Q.45 Two rods A and B of different materials are welded together as shown in figure. Their thermal conductivities are K_1 and K_2. The thermal conductivity of the composite rod will be – **[NEET 2017]**



(A) $\frac{3(K_1+K_2)}{2}$ (B) K_1+K_2

(C) $2(K_1+K_2)$ (D) $\frac{K_1+K_2}{2}$

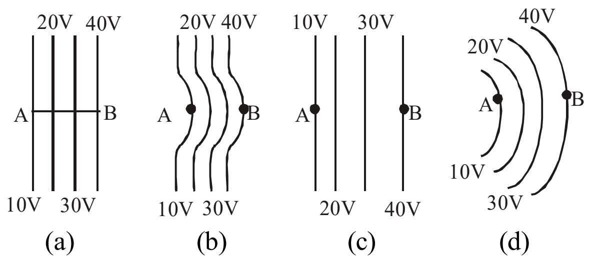
Q.46 A capacitor is charged by a battery. The battery removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system– **[NEET 2017]**

(A) Decreases by a factor of 2
(B) Remains the same
(C) Increases by a factor of 2
(D) Increases by a factor of 4

Q.47 Suppose the charge of a proton and an electron differ slightly. One of them is –e, the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order of [Mass of hydrogen $m_h = 1.67 \times 10^{-27}$ kg] **[NEET 2017]**

(A) 10^{-23} C (B) 10^{-37} C
(C) 10^{-47} C (D) 10^{-20} C

Q.48 The diagrams show regions of equipotentials



A positive charge is moved from A to B in each diagram. **[NEET 2017]**

(A) In all the four cases the work done is the same
(B) Minimum work is required to move q in fig.(a)
(C) Maximum work is required to move q in fig. (b)
(D) Maximum work is required to move q in figure (c)

Q.49 An electron falls from rest through a vertical distance h in a uniform and vertically upward directed electric field E. The direction of electric field is now reversed, keeping its magnitude the same. A proton is allowed to fall from rest in it through the same vertical distance h. The time of fall of the electron, in comparison to the time of fall of the proton is **[NEET 2018]**

(A) 10 times greater (B) 5 times greater
(C) Smaller (D) Equal

Q.50 The electrostatic force between the metal plates of an isolated parallel plate capacitor C having a charge Q and area A, is **[NEET 2018]**

(A) Proportional to the square root of the distance between the plates.
(B) Linearly proportional to the distance between the plates.
(C) Independent of the distance between the plates.
(D) Inversely proportional to the distance between the plates.

Q.51 A hollow metal sphere of radius R is uniformly charged. The electric field due to the sphere at a distance r from the centre **[NEET 2019]**

(A) Increases as r increases for r < R and for r > R.
(B) Zero as r increases for r < R, decreases as r increases for r > R.
(C) Zero as r increases for r < R, increases as r increases for r > R.
(D) Decreases as r increases for r < R and for r > R.

Q.52 Two parallel infinite line charges with linear charge densities $+\lambda$ C/m and $-\lambda$ C/m are placed at a distance of 2R in free space. What is the electric field mid-way between the two line charges? **[NEET 2019]**

(A) Zero (B) $\frac{2\lambda}{\pi\varepsilon_0 R}$ N / C

(C) $\frac{\lambda}{\pi\varepsilon_0 R}$ N / C (D) $\frac{\lambda}{2\pi\varepsilon_0 R}$ N / C

Q.53 Two point charges A and B, having charges +Q & –Q respectively, are placed at certain distance apart and force acting between them is F. If 25% charge of A is transferred to B, then force between the charges becomes : **[NEET 2019]**

(A) F (B) 9F/16
(C) 16F/9 (D) 4F/3

Q.54 A parallel plate capacitor of capacitance 20 μF is being charged by a voltage source whose potential is changing at the rate of 3 V/s. The conduction current through the connecting wires, and the displacement current through the plates of the capacitor, would be, respectively. **[NEET 2019]**

(A) Zero, 60 μA (B) 60 μA, 60 μA
(C) 60 μA, zero (D) Zero, zero

Q.55 A short electric dipole has a dipole moment of 16×10^{-9} Cm. The electric potential due to the dipole at a point at a distance of 0.6 m from the centre of the dipole, situated on a line making an angle of 60° with the dipole axis is :

$\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2/\text{C}^2\right)$ **[NEET 2020]**

(A) zero (B) 50 V
(C) 200 V (D) 400 V

Q.56 In a certain region of space with volume 0.2 m^3 the electric potential is found to be 5 V throughout. The magnitude of electric field in this region is : **[NEET 2020]**

(A) 5 N/C (B) Zero
(C) 0.5 N/C (D) 1 N/C

Q.57 A spherical conductor of radius 10 cm has a charge of 3.2×10^{-7} C distributed uniformly. What is the magnitude of electric field at a point 15 cm from the centre of the sphere ? **[NEET 2020]**

(A) 1.28×10^{7} N/C
(B) 1.28×10^{4} N/C
(C) 1.28×10^{5} N/C
(D) 1.28×10^{6} N/C

Q.58 The capacitance of a parallel plate capacitor with air as medium is 6 μF. With the introduction of a dielectric medium, the capacitance becomes 30 μF. The permittivity of the medium is: ($\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$) **[NEET 2020]**

(A) $5.00\ C^2 N^{-1} m^{-2}$
(B) $0.44 \times 10^{-13}\ C^2 N^{-1} m^{-2}$
(C) $1.77 \times 10^{-12}\ C^2 N^{-1} m^{-2}$
(D) $0.44 \times 10^{-10}\ C^2 N^{-1} m^{-2}$

ANSWER KEY

EXERCISE - 1

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	A	A	C	A	B	B	B	A	D	C	C	C	D	A	A	A	B	C	A	A	C	C	B	D	B
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	B	C	C	C	C	C	A	A	A	A	C	B	B	C	D	D	A	D	C	D	B	A	A	C	C
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
A	D	D	D	A	B	D	B	D	D	B	A	D	B	A	C	B	A	C	D	B	B	A	B	A	A
Q	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**													
A	C	C	A	D	C	C	B	A	A	B	B	B													

EXERCISE - 2

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	A	B	C	C	D	A	C	A	C	C	A	B	C	B	C	B	D	A	C	A	C	A	A	D	D
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	A	C	B	C	B	B	C	A	B	D	C	A	C	A	B	B	A	B	D	C	B	D	C	A	C
Q	**51**	**52**	**53**	**54**																					
A	D	B	D	C																					

EXERCISE - 3

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	2	2	2	168	9	2	44	24	12	750	1000	100	200	4	12	6

EXERCISE - 4

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	A	A	B	B	A	A	C	A	D	B	D	D	B	A	D	A	B	D	A	A	C	A	B	D	D
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	A	D	D	B	D	A	A	C	D	A	C	B	C	C	C	D	A	D	C	C	B	A	D	A	C
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
A	D	BC	A	D	B	C	B	C	D	C	C	C	C	D	A	C	C	D	A	B	6	D	D	C	A
Q	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**					
A	4	C	D	B	A	A	A	C	A	A	D	A	D	C	C	B	A	C	A	C					

EXERCISE - 5

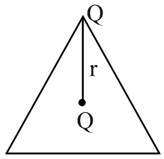
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	A	C	B	B	C	C	D	C	A	B	D	C	B	D	B	D	C	D	D	C	A	D	D	A	D	C	A	A	B	C
Q	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**		
A	B	C	C	C	B	D	C	B	C	C	D	C	B	C	D	A	B	A	C	C	B	C	B	B	C	B	C	D		

ELECTROSTATICS - SOLUTIONS
TRY IT YOURSELF - 1

(1) **(b).** The amount of charge present in the isolated system after rubbing is the same as that before because charge is conserved; it is just distributed differently.

(2) **(a, c, e).** The experiment shows that A and B have charges of the same sign, as do objects B and C. Thus, all three objects have charges of the same sign. We cannot determine from this information, however, whether the charges are positive or negative.

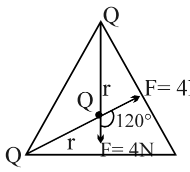
(3) **(B).**



$$F = k\frac{Q^2}{r^2} = 4N$$

In II case : Charge at centre experiences two forces each of magnitude 4N at 120°.

$$F = \sqrt{4^2 + 4^2 + 2\times 4\times 4\cos 120°} = 4N$$



(4) **(D).** $F_e = \frac{kq_1q_2}{r^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{r^2}$

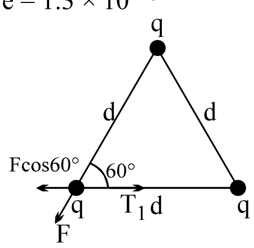
$$\approx \frac{9\times1.6\times1.6\times10^{-29}}{r^2}$$

$$F_e = \frac{Gm_1m_2}{r^2} = \frac{6.7\times10^{-11}\times(9.1\times10^{-31})^2}{r^2}$$

$$\approx \frac{6.7\times9.1\times9.1\times10^{-73}}{r^2} \quad ; \quad \frac{F_e}{F_g} \approx 10^{42} \text{ Ans.}$$

(5) **(A).** $3.9\times10^{-19} = n_1e,\ 6.5\times10^{-19} = n_2e,\ 9.1\times10^{-19} = n_3e$
n_1, n_2 and n_3 are integers for $e = 1.3\times10^{-19}$

(6) **(C). First case :**



$$T = \frac{kq^2}{d^2} \quad ...(1)$$

$$T_2 = \frac{kq^2}{d^2} + \frac{kq^2}{d^2}\cos 60°$$

$$= \frac{3kq^2}{2d} \ ...(2)$$

$T_2 = 3T/2$

(7) **(B).**

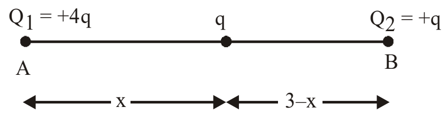
(8) **(D).**

(9) **(C).**

(10) **(C).**

(11) **(e).** In the first experiment, objects A and B may have charges with opposite signs, or one of the objects may be neutral. The second experiment shows that B and C have charges with the same signs, so that B must be charged. But we still do not know if A is charged or neutral.

(12) For quick answer remember if Q_1 and Q_2 are of same nature (means both positive or both negative) then the third charge should be put between (not necessarily at midpoint) Q_1 and Q_2 on the straight line joining Q_1 and Q_2. But if Q_1 and Q_2 are of opposite nature, then the third charge will be put outside and close to that charge which is lesser in magnitude.



Since q is in equilibrium, so net force on it must be zero. We can see, the forces applied by Q_1 and Q_2 on q are in opposite direction so just balance their magnitude.

Force on q by $Q_1 = \frac{kQ_1q}{x^2}$ and force on q by $Q_2 = \frac{kQ_2q}{(3-x)^2}$

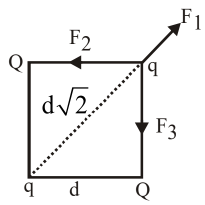
$$\frac{kQ_1q}{x^2} = \frac{kQ_2q}{(3-x)^2} \text{ or } \frac{Q_1}{x^2} = \frac{Q_2}{(3-x)^2} \text{ or } \frac{4}{x^2} = \frac{1}{(3-x)^2}$$

$$\frac{2}{x} = \frac{1}{(3-x)} \text{ or } 6-2x = x \text{ or, } x = 2\text{ m}$$

So, q will be placed at a distance 2 m from Q_1 and at 1 m from Q_2.

(13) Note here both the 'q' will have same sign either positive or negative. Similarly both the Q will have same sign. Let us make the force on upper right corner q equal to zero.
Lower 'q' will apply a repelling force F_1 on upper q (because both the charges have same sign).
To balance this force both 'Q' must apply attractive forces $\vec{F}_2$ & $\vec{F}_3$ of equal magnitude (So, Q and q will have opposite signs). Now the resultant of $\vec{F}_2$ & $\vec{F}_3$ will be $F_2\sqrt{2}$ (Pythagoras theorem) and it will be exactly opposite to F_1 and same in magnitude. So, $F_1 = F_2\sqrt{2}$

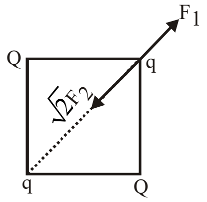
From Coulomb's Law



$$F_1 = \frac{kq^2}{(d\sqrt{2})^2}$$

$$F_2 = \frac{kQq}{d^2}$$

and $\because$ $F_1 = \sqrt{2}F_2$

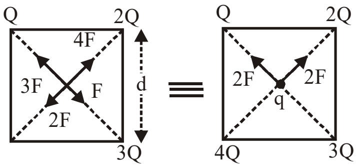


$$\frac{q^2}{(d\sqrt{2})^2} = \frac{\sqrt{2}Qq}{d^2}$$

$$Q = \frac{q}{2\sqrt{2}}$$

But as we said Q & q have opposite sign so, $q = -2\sqrt{2}\ Q$

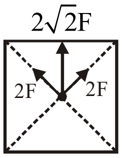
(14) If Q applies a force F, then 2Q will apply 2F on q. Because as per Coulomb's law the force is proportional to charge, if rest of the factors are same.
Similarly 3Q will apply 3F and 4Q will apply 4F on q. The four forces along with its direction are shown below.



3F and F are exactly opposite to each other so its net effect will be 2F towards Q and 4F and 2F are exactly opposite to each other so its effect will be 2F towards 2 as shown in the figure. So resultant force will be (Pythagoras) of 2F and 2F equal to $2\sqrt{2}$ F , where $F = \frac{Qq}{4\pi\varepsilon_0 (d/\sqrt{2})^2}$.

Note that the distance between Q and q is $(d/\sqrt{2})$ as the side of the square is d.

So, final answer is $2\sqrt{2}F = \frac{2\sqrt{2}Qq}{4\pi\varepsilon_0 (d/\sqrt{2})^2} = \frac{4\sqrt{2}Qq}{4\pi\varepsilon_0 d^2}$



Direction is upwards as shown above.

TRY IT YOURSELF - 2

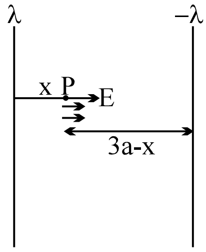
(1) **(a).** There is no effect on the electric field if we assume that the source charge producing the field is not disturbed by our actions. Remember that the electric field is created by source charge(s) (unseen in this case), not the test charge(s).

(2) Since these field contributions are in perpendicular directions, the resultant magnitude is

$$E = \sqrt{10^2 + 10^2}\,N/C = \sqrt{2} \times 10\ N/C$$

= 14 N/C; since the two perpendicular components are equal, the direction of the resultant is at 45° , i.e., precisely in the northeast direction.

(3) **(D).** Electric field at P



$$E = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{3a - x}\right)$$

$$W = \int_a^{2a} q_0 E\,dx = \frac{\lambda q_0}{2\pi\varepsilon_0}\left[\int_a^{2a}\frac{dx}{x} + \int_a^{2a}\frac{dx}{3a - x}\right] = \frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$$

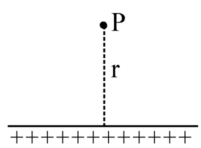
(4) **(ABC).** Diagonally opposite charges will produce field in z-axis, but fields due to +ve & -ve charges will cancel

(5) At the center of the square, the contribution to the electric field from each point charge points away from that point charge. Since the charges are equal and are equidistant from the center, the electric fields due to charges diagonally across from each other exactly cancel, and the net field is zero.

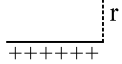
(6) **(A).** $S = \frac{1}{2}at^2 \Rightarrow S = \frac{1}{2}\frac{qE}{m}t^2 \Rightarrow \frac{Se}{Sp} = \frac{mp}{Se}$

$$\Rightarrow Sp \simeq \frac{d}{2000}$$

(7) **(BC).**



$E_y = \frac{2k\lambda}{r}$, $E_x = 0$

how 

$E_x = \frac{k\lambda}{r} = \frac{E}{2}$; $E_y = \frac{k\lambda}{r} = \frac{E}{2}$

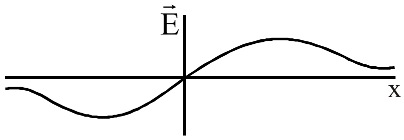
(8) The contribution to the electric field from each positive sheet of charge points perpendicularly out from the corresponding sheet and is independent of distance from the sheet. For two perpendicular equivalent sheets, these two vectors are equal in magnitude and at right angles; they add to produce a field at 45° to the sheets (with magnitude $\sqrt{2}$ times as large as each individual sheet field).

(9) **(C).** $E = \int_a^{4d} \frac{K\lambda\, dx}{x^2} = K\lambda \int_a^{4d} x^{-2}\, dx = \frac{\lambda}{4\pi\varepsilon_0} \int_d^{4d} \frac{dx}{x^2}$

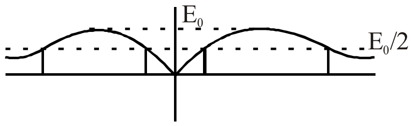
$$E = \frac{3\lambda}{16\pi\varepsilon_0 d}$$

(10) For an infinite sheet, the electric field is independent of distance. So $E = E_0$ at both 2 m and 4 m from the sheet..

(11) **(D).** $\vec{E}$ -x graph:



$|\vec{E}|$-x graph



Hence 4 times.

(12) **(D).** Most positive work is done when positive charge is displaced maximum against strongest electric field.

(13) **(ABD).** At a point with coordinates (x, 0) the force is

$$F = \frac{2Qq}{4\pi\varepsilon_0} \frac{x}{(x^2 + y^2)^{3/2}}.$$

For F to be maximum, equating $\frac{dF}{dx}$ to zero gives

$x \pm \frac{y}{\sqrt{2}}$. The charge is obviously in equilibrium at the origin. However, the equilibrium is not stable since the force is repulsive and hence will not be able to restore the charge at the origin. The charge therefore cannot perform oscillatory motion.

(14) **(A).** Electric field on surface of a uniformly charged square is given by $\frac{Q}{4\pi\varepsilon_0 R^3} = \frac{\rho R}{3\varepsilon_0}$

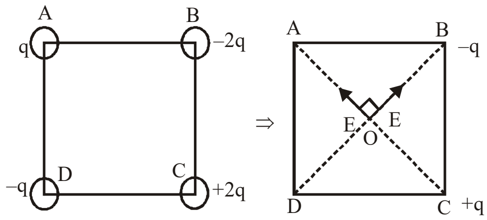
Electric field at outside point is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$|\vec{E}_r| = \frac{\rho r_0}{3\varepsilon_0} - \frac{\rho\left(\frac{r_0}{2}\right)^3}{3\varepsilon_0\left(\frac{3r_0}{2}\right)^2} = \frac{17\rho r_0}{54\varepsilon_0}$$

(15) Side = 0.1m ; $q = 1 \times 10^{-6}$ C

Half of diagonal $= \frac{a}{\sqrt{2}} = \frac{0.1}{\sqrt{2}}$



Electric field due to charge q

$$= \frac{9 \times 10^9 \times 1 \times 10^{-6}}{(0.1/\sqrt{2})^2} = \frac{9 \times 10^3 \times 2}{0.01} = 18 \times 10^5 \text{ N/C}$$

At centre there are two electric field which are perpendicular to each other, so net electric field $E_0 = \sqrt{E^2 + E^2} = E\sqrt{2}$

$= 18 \times 10^5 \times \sqrt{2} = 2.54 \times 10^6$ N/C

TRY IT YOURSELF - 3

(1) **(A).** Net charge on the ring

$$Q = \int_0^{2\pi} \lambda R\, d\theta = \lambda_0 R \int_0^{2\pi} \cos(\theta/2)\, d\theta = 0$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right) = 0$$

(2) **(A).** $E = -\frac{dV}{dr}$

$$E = 0 \Rightarrow \frac{dV}{dr} = 0 \Rightarrow V = \text{constant}$$

(3) **(A).** Potential will be largest at P as we move towards (–) plate potential will reduce.

(4) **(A).** Change in P.E. $\Delta U = q(\Delta V)$

$= -1.6 \times 10^{-19} [(-1) - (10)] \times 10^3$

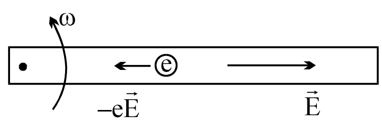
$\Delta U \longrightarrow$ – ve i.e. decrease in P.E.

(5) **(ABC).**

Point A and B are equipotential, work done by electric field is path independent and $V_C > V_A$

(6) **(A).** When rod rotates the centripetal acceleration of electron comes from electric field

$$eE = mr\omega^2 \; ; \; E = \frac{mr\omega^2}{e}$$



Thus, $\Delta V = -\int \vec{E}\cdot dr = -\int_0^{l} \frac{mr\omega^2}{e}\,dr = \frac{m\omega^2 l^2}{2e}$

(7) **(A).** $V_A = \frac{kq}{a} + \frac{kQ}{a+b}$; $\quad V_B = \frac{kq}{a+b} + \frac{kQ}{a}$

$$V_A - V_B = \frac{kqb}{a\,(a+b)} - \frac{kQb}{a\,(a+b)}$$

$$= \frac{kb}{a\,(a+b)}(q-Q) = 60\text{ V}$$

(8) **(A).** $n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow \frac{r^3}{R^3} = \frac{1}{n}$ ... (1)

$v_o = \frac{kq}{r}$ ...(2)

$v = \frac{knq}{R}$ ...(3)

$v = \frac{nv_o r}{R} = nv_o \frac{1}{n^{1/3}}$

$\therefore \quad v_o = \frac{v}{n^{2/3}} = \frac{2.5}{(125)^{2/3}} = 0.1\text{ V}$ **Ans**

(9) **(C).** E goes from higher potential to lower potential

$E = \frac{\Delta V}{\Delta x} = \frac{V_3 - V_0}{0.05} \Rightarrow V_3 = 400 \times 0.05 = 20\text{V}$

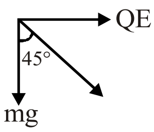
(10) **(C).** Since electric field lines point in the direction of decreasing electric potential, the potential decreases as one travels from initial point to final point on each line. Since the electric field is uniform and in the x-direction, equipotential lines will be perpendicular to the x-axis. Therefore, since the final position of each line has the same x-coordinate, each will have the same final potential. Plot (C) best describes this situation.

(11) **(B).** $\frac{kQ}{R} = V$; $Q = \frac{VR}{k}$

$\sigma = \frac{Q}{4\pi R^2} = \frac{VR}{4\pi k R^2} \Rightarrow \sigma \times \frac{1}{R}$

(12) **(A).** Q ------→------ –Q $\quad E \neq 0 ; V = 0$

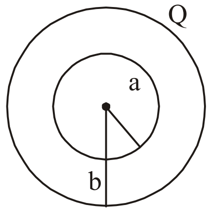
(13) **(C).** $QE = Mg$



$\frac{QV}{d} = w$

$Q = \frac{wd}{V}$

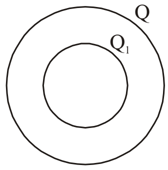
(14) **(B).** Charge must be on outer surface only



$v = \frac{kQ}{b}$ ... (1)

$\frac{kQ}{b} + \frac{kQ'}{a} = 0$

$Q' = -\frac{Qa}{b}$ ... (2)



$v_B = \frac{kQ}{b} + \frac{kQ'}{b}$

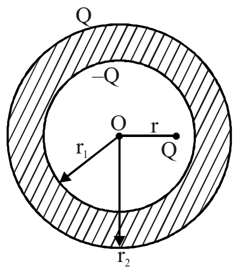
$v_B = v\left[1 - \frac{a}{b}\right]$

(15) **(C).** $u = \frac{k\,(q)\,(-q)}{a} + \frac{kqq}{b} + \frac{kq\,(-q)}{b} + \frac{k\,(-q)^2}{2a} + \frac{kq^2}{2b}$

$u = \frac{kq^2}{2b} - \frac{kq^2}{2a}$

(16) **(B).** $E = \frac{kQ}{r^2}$; $V = \int_{\infty}^{3a} -\frac{kQ}{r^2}\,dr = \frac{kQ}{3a}$

(17) **(A).**



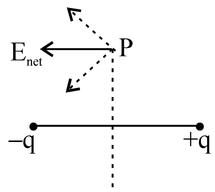
$V_0 = \frac{kQ}{r} - \frac{kQ}{r_1} + \frac{kQ}{r_2}$

TRY IT YOURSELF - 4

(1) **(D).** Net force on dipole can never be zero in a nonuniform field. But torque can be zero if forces on charges are along axis of dipole.

(2) **(BD).** Since electric field due a uniformly charged wire is non-uniform so force and torque on a dipole may or may not be zero.

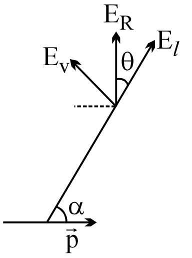
(3) **(C).** Direction of net electric field will be along x-axis.



(4) **(B).** Given E_R along y-axis
thus $\theta = 90° - \alpha$

Also $\tan\theta = \frac{1}{2}\tan\alpha$

or $\tan\alpha = \sqrt{2}$

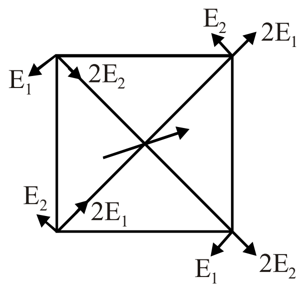


(5) **(D).** $U_{(\theta)} = -\vec{P}\cdot\vec{E}$

$W = \Delta U = U_{(\theta)} - U_0 = -PE\cos\theta - (-PE)$

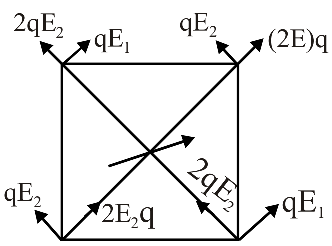
$= PE(1-\cos\theta)$

(6) **(AC).** Electric field diagram due to dipole



$E_1 = \frac{kP\cos\theta}{r^3}$; $E_2 = \frac{kP\sin\theta}{r^3}$

Force diagram on the system system



Net force $= \sqrt{(6qE_1)^2 + (6qE_2)^2}$

$= \frac{6kPq}{r^3}\sqrt{\sin^2\theta + \cos^2\theta}$

Net force $= \frac{6kPq}{r^3}$

The net torque on the system = 0

(7) **(B).** $U = -pE\cos\theta$

θ is max. is cosθ is min. in case 3 & 4 U is +ve so angle is obtuse

$\tau = pE\sin\theta$ max. is angle is closet to 90º

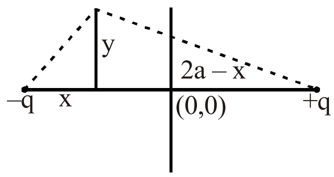
$-pE\cos\theta = -V_0$

$\cos\theta_1 = \frac{V_0}{PE}$, $\cos\theta_2 = \frac{7V_0}{PE}$, $\cos\theta_3 = -\frac{3V_0}{pE}$

$\cos\theta_4 = \frac{4V_0}{pE}$

(8) **(BC).** Potential at center due to induced charger on the conductor is zero (by symmetry), and net potential at each point of conductor is same as that at center, i.e. $\left(\frac{KP}{r^2}\right)$. Potential due to dipole at A is $\frac{KP}{(r-R)^2}$ and at B is $\frac{KP}{(r+R)^2}$

(9) **(D).**



$$V_p = \frac{-kq}{\sqrt{y^2+x^2}} + \frac{kq}{\sqrt{y^2+(2a-x)^2}} = 0$$

(10) $E_{\text{on axial line}} = \frac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2}$

$$= \frac{9\times10^9\times2\,(2\times10^{-6}\times10^{-2})\times5\times10^{-2}}{[(5\times10^{-2})^2-(0.5\times10^{-2})^2]^2}$$

$= 2.93\times10^6$ N/C

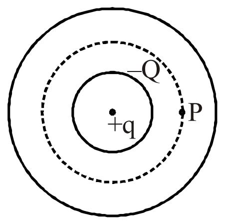
Electric field on equatorial line is given as

$$E = \frac{p}{4\pi\varepsilon_0(r^2+a^2)^{3/2}} = \frac{9\times10^9\times(2\times10^{-6}\times10^{-2})}{[(5\times10^{-2})^2+(0.5\times10^{-2})^2]^{3/2}}$$

$= 1.41\times10^6$ N/C

TRY IT YOURSELF - 5

(1) **(A).**



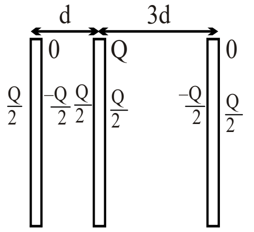
· 3q (outside)

Apply Gauss law at P

$\phi = 0$ $\frac{+q+Q_{in}}{\varepsilon_0} = 0$; $Q_{in} = -q$

$Q_{in} + Q_{out} = 0$; $Q_{out} = q$

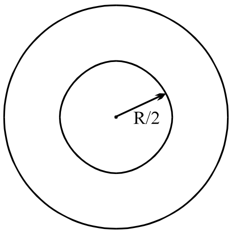
(2) **(AC).**



(3) **(BC).**

(4) **(B).** $\rho(r) = A(r)^2$

Charge enclosed for sphere of radius R/2



$$Q = \int (4\pi r^2)\, dr\, \rho(r)$$

$$= 4\pi A \int_0^{R/2} r^4\, dr$$

$$= 4\pi A \left[\frac{v^5}{5}\right]_0^{R/2} = \frac{4\pi A}{5\times 32}(R^5) = \frac{\pi A}{40} R^5$$

Applying Gauss's law for this sphere

$$4\pi(R/2)^2 E = Q/\varepsilon_0 = \frac{\pi A}{40} R^5 \Rightarrow E = \frac{AR^3}{40\varepsilon_0}$$

(5) **(B).**

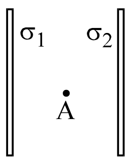
(6) **(A).** Unit vector normal to x-y plane is $\hat{i}$, thus

$$Q = \vec{E}\cdot\vec{A} = aA$$

(7) **(ABD).**

(8) **(ABD).**

Two conducting surfaces facing each other have equal and opposite charges



$$\varepsilon_A = \frac{\sigma_1 - \sigma_2}{2\varepsilon_0}$$

So $\varepsilon_A = \dfrac{\sigma_1}{\varepsilon_0} = -\dfrac{\sigma_2}{\varepsilon_0}$

Since, $\sigma_1 = -\sigma_2$

(9) **(C).** Flux going in pyramid $= \dfrac{Q}{2\varepsilon_0}$

Which is devided equally among all 4 faces

$\therefore$ Flux through one face $= \dfrac{Q}{8\varepsilon_0}$

(10) **(C).** By symmetry, $\phi = \dfrac{q}{6\varepsilon_0} \Rightarrow \dfrac{5\times10^{-6}}{6} \times 4\pi \times 10^9 \times 9$

$\approx 9.4 \times 10^9$

(11) **(B).** ϕ = Flux = $EA\cos\theta$, where θ is the angle between $\vec{E}$ and $\vec{A}$.

For maximum flux, $\theta = 0 \Rightarrow \cos 0 = 1$

$\therefore$ $\phi_{max} = EA$

$\phi_{max}/2 = EA\cos\theta \Rightarrow \cos\theta = 1/2 \;\therefore\; \theta = 60°$

$\therefore$ Angle of rotation is 60°

(12) **(B).** The second charge is kept just outside surface.

$\therefore$ Flux will not change but another carge influences the shape.

(13) **(D).** $Q_{ene} = 100 \times \sigma$

$$E = \frac{\sigma}{2\in_0} \Rightarrow \sigma = 200 \times 2\in_0 = 4\times10^4 \in_0 = 35.4\times10^{-8}C$$

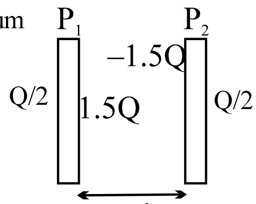
TRY IT YOURSELF - 6

(1) **(C).** $C_A = \dfrac{\in_0 A}{d}$; $C_B = \dfrac{\in_0 A}{2d}$; $C_C = \dfrac{5\in_0 A}{d}$

$C_D = \dfrac{5\in_0 A}{2d}$; $C_B < C_A < C_D < C_C$

(2) **(B).** $q = CV$ = constant ; $V \propto 1/C$

$V \to$ maximum ; $C \to$ minimum



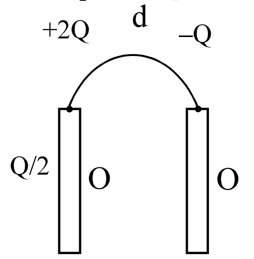
(3) **(B).** $C = \dfrac{\varepsilon_0 A}{d}$;

$$U_i = \frac{(1.5\,Q)^2}{2C} = \frac{2.25\,Q^2 d}{2C}$$

$$\Rightarrow U_i = \frac{9Q^2 d}{8\varepsilon_0 A}$$

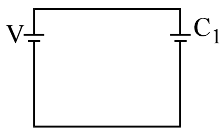
After connecting, $U_f = 0$

$$\text{Heat} = U_i - U_f = \frac{9Q^2}{d\,\varepsilon_0 A}$$



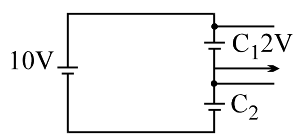
(4) **(A), (5) (A).**

$C_3 \to \infty \Rightarrow$ shorted



$\Rightarrow V = 10$ ... (1)

for $C_3 = 0$; $V = \dfrac{Q}{C}$



$$\frac{2}{8}=\frac{C_2}{C_1}\ ;\ C_1=4C_2 \qquad \dots(2)$$

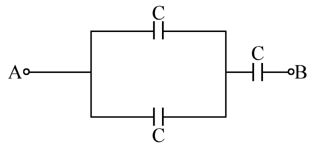
$\Rightarrow\ C_2=2\ \&\ C_1=8\mu F$ out of options given

(6) **(D).** Due to increase in distance between the plates, capacitance increase.

(7) **(B).** $C_1=3C, C_2=2C/3, C_3=3C/2, C_4=C/3$

(8) **(A).** When applied p.d. is V across A & B

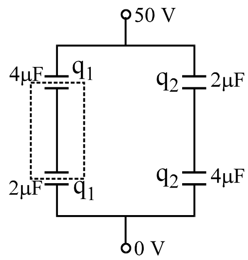
Assuming $V_{AC}=V_1 \quad \& V_{CB}=V_2$



We have, $V_1=\dfrac{V_2}{2}$ & hence $V_1=\dfrac{V}{3}$ & $V_2=\dfrac{2V}{3}$

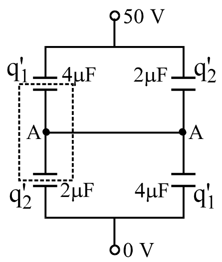
As V_1 & V_2 both must not exceed 100 V, the maximum value of applied p.d. across A & B would be 150 V.

(9) **(C).**



Net charge under dotted box shown $=-q_1+q_1=0$

Finally : $V_A=25V$



$q'_1=25(4)=100\ \mu C\ ;\ q'_2=25(2)=50\ \mu C$

Net charge under the dotted box shown $=-q'_1+q'_2$

$=-100+50=-50\ \mu C$

$\therefore$ The charge which flows $=50\ \mu C$

(10) **(AB).** On insertion of dielectric, capacitance increased k times, thus, for system :

$$C_{before}=\frac{C\times C}{C+C}=\frac{C}{2}$$

$$C_{after}=\frac{C\times(kC)}{C+(kC)}=\frac{kC}{k+1} \qquad (k>1)$$

$\therefore\ C_{after}>C_{before}$ [C increased]

& $Q_{after}>Q_{after}$ [Q increased]

$Q_{before}=CV/2$ or $(V_B)_{before}=V/2$

$Q_{after}=\dfrac{kCV}{k+1}$ or $(V_B)_{after}=\dfrac{V}{k+1}$ $(k>1)$

$(V_B)_{before}>(V_B)_{after}$ [V decreased]

& $E=\dfrac{V}{d}$

hence, $(E_B)_{before}>(E_B)_{after}$ (E decreased)

$U=\dfrac{1}{2}CV^2$ thus $(U)_{after}>(U)_{before}$ (U increased)

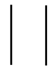
(11) **(A).** Potential difference between plates remains same. Decrease in potential difference is counteracted by potential difference due to the extra distance.

$$t\left(E-\frac{E}{k}\right)=Ed\Rightarrow t\left(1-\frac{1}{k}\right)=d\Rightarrow k=\frac{t}{t-d}$$

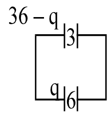
E is original electric field, k dielectric constant of plate, t thickness of plate & d extra distance.

(12) **(C).** $V'=\dfrac{CV}{C+C'}$

(13) **(A).**



3μf $\quad Q=CV=3\times10^{-6}\times12=36\mu C$



$\dfrac{36-q}{3}=\dfrac{q}{6}\ ;\ 72-2q=q\ ;q=\dfrac{72}{3}=24\ \mu C$

$\therefore$ Charge on $3\mu F\Rightarrow 36-q=12\ \mu C$

$$V=\frac{36-q}{C}=\frac{12}{3}=4\text{ volt}$$

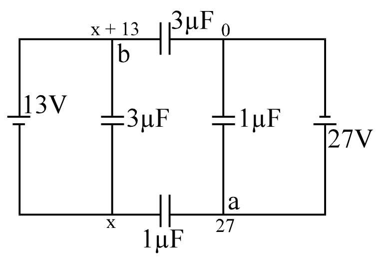
(14) **(D).** Plates are brought closer capacity will increase. As battery is removed charge remain constant.

$$U=\frac{1}{2}\frac{Q^2}{C}\Rightarrow U\propto 1/C.$$

Hence stored energy will decrease.

(15) **(A).**

(16) **(C).** $(x+13)\times3=(27-x)\times1$



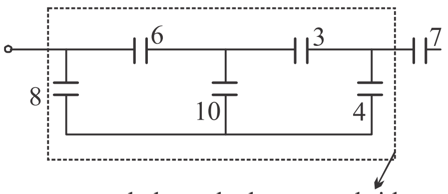
$3x+39=-x+27\ ;\ x=-3$

So $V_a-V_b=27-(x+13)=17$

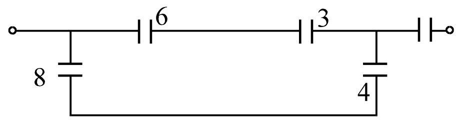
(17) **(C).** From charged is tribution $Q_1=Q_4$ net elctric field between plates is $E\times d$

$$\text{Potential Difference}=\frac{Q_2-Q_3}{2A\varepsilon_0}d=\frac{Q_2-Q_3}{2C}$$

(18) **(A).**



balanced wheatstone bridge



$C_{eq} = 14/5$

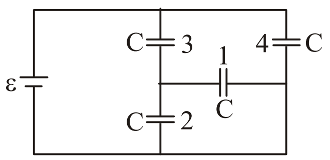
(19) **(B).** $C_1 < C_2$

$$\therefore \quad \frac{C_1}{C_1 + C_2} < \frac{1}{2} \text{ and } \frac{C_2}{C_1 + C_2} > \frac{1}{2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = C_1 \cdot \frac{C_2}{C_1 + C_2} > \frac{C_1}{2}$$

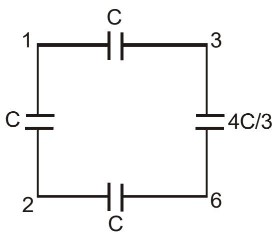
Similarly, $C < \dfrac{C_2}{2}$

(20) **(B).**

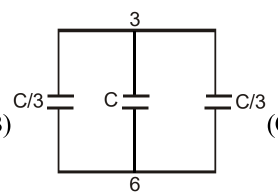


1 & 2 parallel and in series with 3 and combination is parallel with 4.

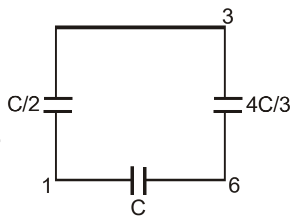
(21) **(ABC).** (A)



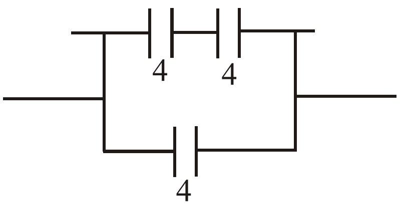
(B)



(C)



(22) **(C).**



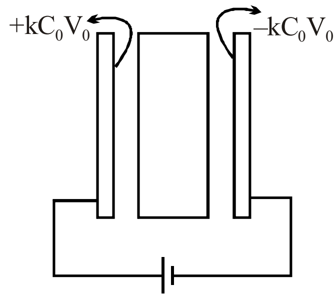
(23) **(A).** $E \times \dfrac{1}{k} \Rightarrow \Delta V \times \dfrac{1}{k}$

$\Rightarrow E_3 < E_2 < E_1$ and $\Delta V_3 < \Delta V_2 < \Delta V_1$

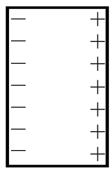
(24) **(ABD).**

(A) Charged induced on the surface of dielectric slab

$$= kC_0V_0 \left(1 - \frac{1}{k}\right) = (k-1)\, C_0V_0$$



(B) $-(k-1)\,C_0V_0$



$+(k-1)\,C_0V_0$

$E- \longleftarrow \bullet \longrightarrow E+$

Resultant field is zero due to charge on both the surfaces.

(C) Force of attraction $= \left(\dfrac{kC_0V_0}{2 \epsilon_0 A}\right) (kC_0V_0) = \dfrac{k^2 C_0 V_0}{2 \epsilon_0 A}$

(D) $\dfrac{(k-1)C_0V_0}{\epsilon_0 A}$

(25) **(ACD).** $\dfrac{Q}{A \epsilon_0} = E_{max}$; $\dfrac{240 \epsilon_0 L^2}{kA \epsilon_0}$; $A = 60L^2$

$$\frac{4 \epsilon_0 A}{d} = 480$$

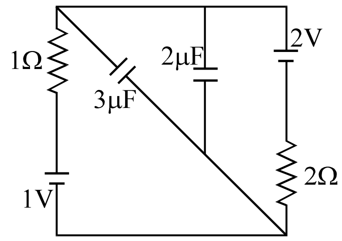
$$\frac{\epsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{4}\right)} = 120 \times \frac{8}{5} = 192$$

(26) **(AC).** Inner shell is in unstable equillibrium

$$\Rightarrow U \downarrow \text{ \& } U = \frac{Q^2}{2C} \Rightarrow C \uparrow$$

TRY IT YOURSELF - 7

(1) **(B).** At steady state, there will be no current in the branches having capacitor only thus equivalent circuit diagram will be as shown in the figure.

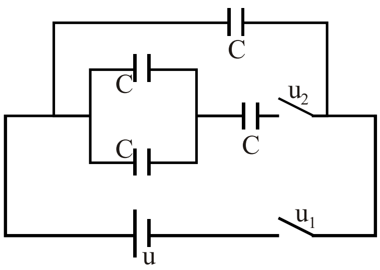


$$V_{AB} - 1 + \frac{V_{AB} - 2}{2} = 0 \Rightarrow V_{AB} = \frac{4}{3}V$$

Thus $q = CV_{AB} = 4\ \mu C$

(2) **(D).** After shifting the switch, charge on the upper plate of capacitor at t = 0 is $-C\varepsilon$ & after a long time from switching, charge on the same plate will be $2C\varepsilon$.

(3) **(ABD).** Equivalent circuit



Equivalent capacitance $= \frac{5C}{3}$

When k_2 is closed no increasing in energy

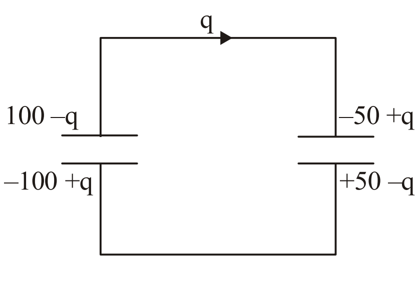
When k_1 is closed $E = \frac{1}{2}cqV^2 = \frac{1}{2}\frac{5C}{3}V^2 = \frac{5CV^2}{6}$

(4) **(B).** Let q charge flow through the circuit then using Kirchoff's law

$$19 + 15 + \frac{q}{3} - 9 + \frac{q}{2} = 0 \Rightarrow 25 + \frac{5q}{6} = 0 \Rightarrow q = 30\mu C$$

P.D. Across $3\mu F$ is $V = \frac{30\mu C}{3\mu F} = 10\ V$

(5) **(C).**



$$\frac{100 - q}{5 \times 10^{-6}} + \frac{50 - q}{20 \times 10^{-6}} = 0 \Rightarrow q = 90 \times 10^{-6}\ C$$

$\therefore$ Final charge on $5\mu F$ top plate is $10\mu C$

(6) **(B).** $\left(\frac{\epsilon_0\ klx}{d} + \frac{\epsilon_0\ (2A - lx)}{d}\right)V = Q$

$$i = \left(\frac{\epsilon_0\ klx}{d} + \frac{-\epsilon_0\ l}{d}\right)\frac{dx}{dt} = \text{constant}$$

As the dielectric leaves $c\downarrow \Rightarrow Q\downarrow \Rightarrow$ i is -ve.

(7) **(B).** When stwich is shifted to position b discharging will take place & $q = q_0 e^{-t/RC} = C\Sigma\ e^{-t/RC}$

and $i = -\frac{dq}{dt}$; $\frac{C\Sigma}{RC}e^{-t/RC} = \frac{\Sigma}{R}e^{-t/RC}$

At $t = 0$; $i = \frac{\Sigma}{R}$

(8) **(B).**

(9) **(A).**

(10) **(A).** $\frac{Q}{Q_{max}} = 0.75 = /- e^{-t/RC} = -0.25 = -e^{-t/RC}$

$\Rightarrow\ ln\,0.25 = -\frac{t}{RC}$

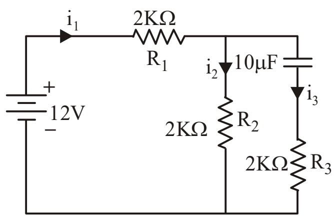
$t = -RC\ ln0.25 = -(4.0)\,(0.025)\ ln\,0.25 = 0.2\ ln\,2$ sec

(11) **(C).** $\Delta V_C = Q/C = \frac{0.75Q_{max}}{C} = 0.75\ \Delta V_{max} = 9\ V$

$\Delta V_C + \Delta V_R - \varepsilon = 0 \Rightarrow \Delta V_R = 12 - 9 = 3\ V$

(12) **(ABCD).**

t = 0 uncharged capacitor has zero potential diff.



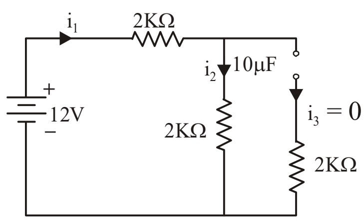
$R_{eq} = 3K\Omega$

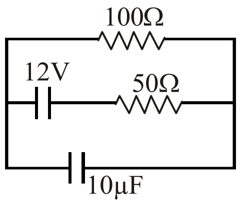
$i_1 = \frac{12V}{3K\Omega} = 4mA$

$i_2 = i_3 = \frac{i_1}{2} = 2mA$

Steady state, capacitor does not allow current flow, so

$i_1 = i_2 = \frac{12V}{4K\Omega} = 3mA$ and $i_3 = 0$

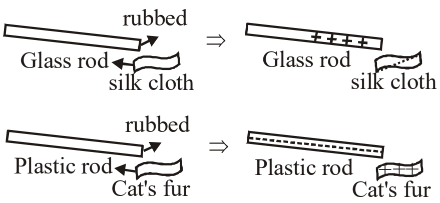


(13) **(B).** 

$$i = \frac{12}{150} = \frac{4}{50} = 0.08\,A$$

CHAPTER-1 : ELECTROSTATICS
EXERCISE-1

(1) **(A).**



It is convention to take charge on glass rod as positive charge and silk cloth as negative charge. When charged glass rod touches charged silk cloth, afterwards they do not attract paper pieces because their charges neutralise each other.

(2) **(A).** As X-rays are electromagnetic waves and they are neutral photons like light photons, they do not have charge. So, divergence of leaves will not be affected because divergence will change only when leaves get some positive or negative charge.

(3) **(C).** $q = ne, n = 1$
$q_{min} = e = 1.6 \times 10^{-19}$ C
Charge = (integer) × (1.6×10^{-19} C)
For minimum charge (integer) should be minimum. We know that 1, 2, 3, 4, ... are integers, minimum integer is 1.
Minimum charge
$= (1) \times (1.6 \times 10^{-19}\,C) = 1.6 \times 10^{-19}C$
Minimum negative charge $= -1.6 \times 10^{-19}C$
Permissible charge quantities are
$\pm 1.6 \times 10^{-19}\,C, \pm 3.2 \times 10^{-19}C,$
$\pm 4.8 \times 10^{-19}\,C \pm N \times 1.6 \times 10^{-19}\,C.$
Here, N is natural number
N = 1, 2, 3, 4, ...

(4) **(A).** Charges are additive, conservative and quantised, in nature.

(5) **(B).** A negatively charged body acquires some electrons, so its mass is more than its neutral mass.

(6) **(B).** First members acquires positive charge and IInd member have negative charge.

(7) **(B).** $q = ne \Rightarrow n = \dfrac{q}{e} = \dfrac{1 \times 10^{-9}}{1.6 \times 10^{-19}} = 6.25 \times 10^9$

(8) **(A).** $Q = ne = 10^{14} \times 1.6 \times 10^{-19}$

$\Rightarrow \quad Q = 1.6 \times 10^{-5}\,C = 16\mu C$

Electrons are removed, so charge will be positive.

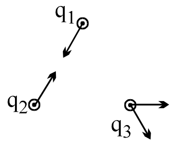
(9) **(D).** Positive charge shows the deficiency of electrons.

$$\text{Number of electrons } = \frac{14.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 9$$

(10) **(C).** A movable charge produces electric field and magnetic field both.

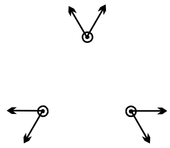
(11) **(C).**

(I)



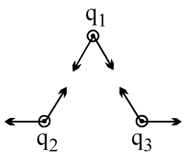
q_1 & q_2 are opposite,
q_1 & q_3 are like, q_2 & q_3 are like
$\Rightarrow$ not possible

(II)



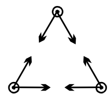
all three like charges
$\Rightarrow$ possible

(III)



q_1 & q_2 are unlike,
q_2 & q_3 are like, q_1 & q_3 are unlike
$\Rightarrow$ possible

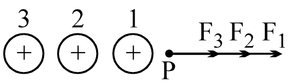
(IV)



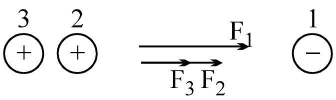
All three unlike charges
$\Rightarrow$ not possible

(12) **(C).**

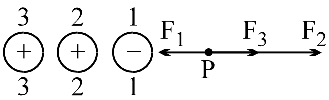
(I)



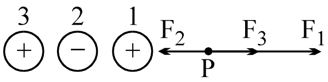
(II)



(III)



(IV)



(13) **(D).** $F_e = \dfrac{kq_1q_2}{r^2} \approx \dfrac{9 \times 1.6 \times 1.6 \times 10^{-29}}{r^2}$

$$F_g = \frac{Gm_1m_2}{r^2} \approx \frac{6.7 \times 9.1 \times 9.1 \times 10^{-73}}{r^2}$$

$$\frac{F_e}{F_g} \approx 10^{42}$$

(14) **(A).** $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{R^2}$

${}_{92}U^{238}$ has charge 92e. When α-particle is emitted, charge on residual nucleus is
$92e - 2e = 90e$
$\therefore \quad q_1 = 90e, q_2 = 2e$, and $R = 9 \times 10^{-15}$m

$\therefore \quad F = 9\times10^9 \times \dfrac{(90e)\,(2e)}{(9\times10^{-15})^2}$

$= \dfrac{9\times10^9 \times 90\times(1.6\times10^{-19})^2\times 2}{(9\times10^{-15})^2} = 512\text{ N}$

(15) **(A).** When put 1 cm apart in air, the force between Na and Cl ions = F.
When put in water, the force between Na and Cl ions = F/K

(16) **(A).** Let separation between two parts be

$r \Rightarrow F = k.q\dfrac{(Q-q)}{r^2}$

For F to be maximum

$\dfrac{dF}{dq} = 0 \Rightarrow \dfrac{Q}{q} = \dfrac{2}{1}$

(17) **(B).** $F \propto \dfrac{1}{r^2} \Rightarrow \dfrac{F_1}{F_2} = \left(\dfrac{r_2}{r_1}\right)^2$

$\Rightarrow \dfrac{5}{F_2} = \left(\dfrac{0.04}{0.06}\right)^2 = F_2 = 11.25\,N$

(18) **(C).** F = F' or $\dfrac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 r'^2\,K} \Rightarrow r' = \dfrac{r}{\sqrt{K}}$

(19) **(A).** $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(+7\times10^{-6})(-5\times10^{-6})}{r^2}$

$= -\dfrac{1}{4\pi\varepsilon_0}\dfrac{35\times10^{12}}{r^2}N$

$F' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(+5\times10^{-6})(-7\times10^{-6})}{r^2} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{35\times10^{12}}{r^2}N$

(20) **(A).** y = –cos x

$E_x = \dfrac{\partial y}{\partial x} = \sin x$ and $E_y = \dfrac{\partial y}{\partial y} = 1$

When x ∈ (0, π), slope of given graph is positive thus x & y component of E should positive in this domain.

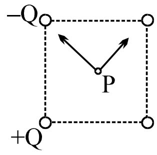
(21) **(C).** Due to symmetry field due to one pair will be forming the resultant field strength and equal to

$E = 2\times\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2} = \dfrac{2k_eQ}{r^2}$

(22) **(C).** $d = \dfrac{1}{2}at^2,\ t = \dfrac{L}{v}$

$d = \dfrac{1}{2}\times\left(\dfrac{Eq}{m}\right)\times\left(\dfrac{L}{v}\right)^2 \;;\; q = \dfrac{2dmv^2}{EL^2}$

(23) **(B).** The given system can be reduced as



. So, resultant is ↑.

(24) **(D).** $E = \dfrac{kQ}{r^2} = \dfrac{9\times10^9\times1.6\times10^{-19}}{(10^{-15})^2} = 14.4\times10^{20}\text{ V/m}$

(25) **(B).** $E_i = \dfrac{kq}{R^3}\,x$ or $E_i \propto x$

(26) **(B).** $E_k = \dfrac{1}{2}mv^2 = \dfrac{1}{2}ma^2t^2 = \dfrac{1}{2}m\left(\dfrac{qE}{m}\right)^2t^2 = \dfrac{q^2E^2t^2}{2m}$

(27) **(C).** $a = \dfrac{qE}{m} \Rightarrow \dfrac{a_e}{a_p} = \dfrac{m_p}{m_e}$

(28) **(C).** For non-conducting sphere $E_{in} = \dfrac{k.Qr}{R^3} = \dfrac{\rho r}{3\varepsilon_0}$

(29) **(C).** Electric potential inside a conductor is constant and it is equal to that on the surface of conductor.

(30) **(C).** $E = \dfrac{1}{4\pi\varepsilon_0}.\dfrac{ne}{r^2} \Rightarrow n = \dfrac{Er^2}{e}.4\pi\varepsilon_0$

$\Rightarrow \quad n = \dfrac{0.036\times0.1\times0.1}{9\times10^9\times1.6\times10^{-19}} = \dfrac{360}{144}\times10^5$

$= 2.5\times10^5$ N/C.

(31) **(C).** $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m}t^2 \;\left[\because a = \dfrac{F_e}{m} = \dfrac{eE}{m}\right]$

$t = \sqrt{\dfrac{2ym}{e.E}} = 3\times10^{-9}$ s [Putting $y = 4\times10^{-2}$ m, $m = 9.1\times10^{-31}$ kg, $e = 1.6\times10^{-19}$C, $E = 5\times10^4$ N/C]

(32) **(A).** $E_0 = \dfrac{kq}{r^2},\ E_s = \dfrac{kq}{R^2} \Rightarrow F = qE_0 = \dfrac{qE_sR^2}{r^2} = 1.5\times10^{-2}N$

[$q = 5\times10^{-8}$ C, $E_s = 5\times10^6$ v/m, R = 0.05 m, r = 0.20 cm]

(33) **(A).** $E = -\dfrac{dV}{dr}\;;\; E = 0 \Rightarrow \dfrac{dV}{dr} = 0 \Rightarrow V = \text{constant}$

(34) **(A).** Potential will be largest at P as we move towards (–) plate potential will reduce.

(35) **(A).** $V_A = \dfrac{kq}{a} + \dfrac{kQ}{a+b}\;;\; V_B = \dfrac{kq}{a+b} + \dfrac{kQ}{a}$

$V_A - V_B = \dfrac{kqb}{a\,(a+b)} - \dfrac{kQb}{a\,(a+b)}$

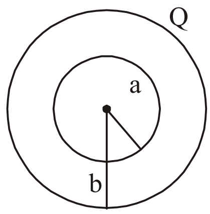
$= \dfrac{kb}{a\,(a+b)}(q-Q) = 60\text{ V}$

(36) **(C).** Since electric field lines point in the direction of decreasing electric potential, the potential decreases as one travels from initial point to final point on each line. Since the electric field is uniform and in the x-direction, equipotential lines will be perpendicular to the x-axis. Therefore, since the final position of each line has the same x-coordinate, each will have the same final potential. Plot (C) best describes this situation.

(37) **(B).** $\frac{kQ}{R} = V$; $Q = \frac{VR}{k}$; $\sigma = \frac{Q}{4\pi R^2} = \frac{VR}{4\pi kR^2} \Rightarrow \sigma \propto \frac{1}{R}$

(38) **(B).** Charge must be on outer surface only

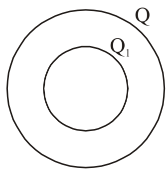
$$v = \frac{kQ}{b} \quad \text{... (1)}$$



$$\frac{kQ}{b} + \frac{kQ'}{a} = 0$$

$$Q' = -\frac{Qa}{b} \quad \text{... (2)}$$

$$v_B = \frac{kQ}{b} + \frac{kQ'}{b}$$



$$v_B = v\left[1 - \frac{a}{b}\right]$$

(39) **(C).** Inside a conducting body, potential is same everywhere and equals to the potential of it's surface.

(40) **(D).** If charge acquired by the smaller sphere is Q then it's

potential $120 = \frac{kQ}{2}$ ..... (i)

Also potential of the outer sphere

$$V = \frac{kQ}{6} \quad \text{.....(ii)}$$

From equation (i) and (ii) V = 40 volt

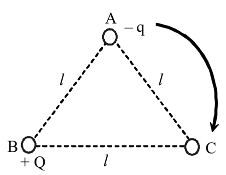
(41) **(D).** $V = n^{2/3} v \Rightarrow V = (125)^{2/3} \times 50 = 1250$ V

(42) **(A).** Change in P.E. $\Delta U = q(\Delta V)$

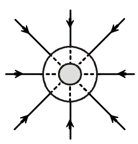
$= -1.6 \times 10^{-19} [(-1) - (10)] \times 10^3$

$\Delta U \rightarrow -$ ve i.e. decrease in P.E.

(43) **(D).** According to figure, potential at A and C are equal. Hence work done in moving – q charge from A to C is zero.



(44) **(C).** Electric lines force due to negative charge are radially inward.



(45) **(D).** Most positive work is done when positive charge is displaced maximum against strongest electric field.

(46) **(B).** Electric Flux is a measure of the number of electric field lines passing through an area.

(47) **(A).** Unit vector normal to x-y plane is $\hat{i}$, thus

$Q = \vec{E} \cdot \vec{A} = aA$

(48) **(A).** $\phi = E\, s \cos\theta = 2 \times 10^3 \times 10^{-2} \cos 0°$

$= 20\ NC^{-1}\ m^2$

(49) **(C).** Gauss's law is often useful towards a much easier calculation of electrostatic field when the system has some symmetry. This is facilitated by the choice of a suitable Gaussian surface.

(50) **(C).** By symmetry, $\phi = \frac{q}{6\varepsilon_0}$

$\Rightarrow \phi = \frac{5 \times 10^{-6}}{6} \times 4\pi \times 10^9 \times 9 = 9.4 \times 10^4$

(51) **(D).** Net force on dipole can never be zero in a nonuniform field. But torque can be zero if forces on charges are along axis of dipole.

(52) **(D).** $U_{(\theta)} = -\vec{p}.\vec{E}$

$W = \Delta U = U_{(\theta)} - U_0$

$= -pE\cos\theta - (-pE) = pE(1 - \cos\theta)$

(53) **(D).** Potential energy of dipole in electric field

$U = -pE\cos\theta$, where θ is the angle between electric field and dipole.

(54) **(A).** As the dipole will feel two forces which are although opposite but not equal.

$\therefore$ A net force will be there and as these forces act at different points of a body, a torque is also there.

(55) **(B).** Maximum torque = pE

$= 2 \times 10^{-6} \times 3 \times 10^{-2} \times 2 \times 10^5 = 12 \times 10^{-3}$ N-m.

(56) **(D).** Dipole moment $p = 4 \times 10^{-8} \times 2 \times 10^{-4}$

$= 8 \times 10^{-12}$ m

Maximum torque = pE

$= 8 \times 10^{-12} \times 4 \times 10^8 = 32 \times 10^{-4}$ Nm

Work done in rotating through $180^o = 2pE$

$= 2 \times 32 \times 10^{-4} = 64 \times 10^{-4}$ J

(57) **(B).** $E_a = \frac{2kp}{r^3}$ and $E_e = \frac{kp}{r^3}$; $\therefore E_a = 2E_e$

(58) **(D).** For a perfect conductor :

* The surface of the conductor is an equipotential surface.
* The electric field just outside the surface of a conductor is perpendicular to the surface.
* The charge carried by a conductor is always uniformly distributed over the surface of the conductor.

(59) **(D).** The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule then has no permanent (or intrinsic) dipole moment.

Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules, because of their symmetry, have no dipole moment.

(60) **(B).** Permittivity of metals is very high comparable to permittivity of free space. So dielectric constant for metal is infinite.

(61) **(A).** For linear isotropic dielectric $P = \chi_e E$
where, χ_e is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium and it is possible to relate χ_e to the molecular properties of the substance.

(62) **(D).** In polar molecule the centres of positive and negative charges are separated even when there is no external field. Such molecule have a permanent dipole moment. Ionic molecule like HCl is an example of polar molecule.

(63) **(B).** $Q = CV \Rightarrow C = \frac{Q}{V} = \frac{ne}{V}$

Given V = 100 volt; $n = 6.25 \times 10^{15}$

$$\therefore \quad C = \frac{6.25 \times 10^{15} \times 1.6 \times 10^{-19}}{100} = 10\mu F$$

(64) **(A).** The potential difference across the parallel plate capacitor is $10V - (-10V) = 20V$.

Capacitance $= \frac{Q}{V} = \frac{40}{20} = 2F.$

(65) **(C).** Because the charges are produced due to induction and moreover the net charge of the condenser should be zero.

(66) **(B).** $C = \frac{\varepsilon_0 A}{d}.C' = \frac{\varepsilon_0 A}{d/2} \Rightarrow C' = 2C$

(67) **(A).** Force on one plate due to another is

$$F = qE = q \times \frac{\sigma}{2\varepsilon_0 K} = q\left(\frac{q}{2AK\varepsilon_0}\right) = \frac{q^2}{2AK\varepsilon_0}$$

(where $\frac{\sigma}{2\varepsilon_0 K}$ is the electric field produced by one plate at the location of other).

(68) **(C).** $C_1 = \varepsilon_0 \frac{A}{d_1}$ and $C_2 = K\varepsilon_0 \frac{A}{d_2}$

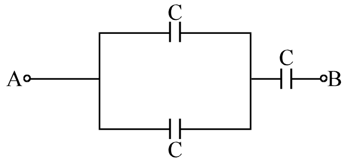
$$\therefore \quad \frac{C_1}{C_2} = \frac{1}{K} \times \frac{d_2}{d_1} = \frac{C}{2C} = \frac{1}{K} \times \frac{2d}{d} \Rightarrow K = 4$$

(69) **(D).** $C_{air} = \frac{C_{medium}}{K} = \frac{C}{2}$

(70) **(B).** In general electric field between the plates of a charged parallel plate capacitor is given by $E = \frac{\sigma}{\varepsilon_0 K}$

(71) **(B).** $C_1 = 3C, C_2 = 2C/3, C_3 = 3C/2,$
$C_4 = C/3$

(72) **(A).** When applied p.d. is V across A & B
Assuming $V_{AC} = V_1$ & $V_{CB} = V_2$



We have, $V_1 = V_2/2$
hence $V_1 = V/3$ & $V_2 = 2V/3$
As V_1 & V_2 both must not exceed 100 V, the maximum value of applied p.d. across A & B would be 150 V.

(73) **(B).** $C_1 < C_2$

$$\therefore \quad \frac{C_1}{C_1 + C_2} < \frac{1}{2} \text{ and } \frac{C_2}{C_1 + C_2} > \frac{1}{2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = C_1 \cdot \frac{C_2}{C_1 + C_2} > \frac{C_1}{2}$$

Similarly, $C < \frac{C_2}{2}$

(74) **(A).** From the given figure, total capacitance is

$$\frac{1}{1} = \frac{1}{C} + \frac{1}{(1+2.5)} \Rightarrow 1 = \frac{1}{C} + \frac{1}{3.5}$$

$$\Rightarrow \quad C = \frac{3.5}{2.5} = 1.4\,\mu F$$

(75) **(A).** Potential difference across the condenser

$$V = V_1 + V_2 = E_1 t_1 + E_2 t_2 = \frac{\sigma}{K_1 \varepsilon_0} t_1 + \frac{\sigma}{K_2 \varepsilon_0} t_2$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left(\frac{t_1}{K_1} + \frac{t_2}{K_2}\right) = \frac{Q}{A\varepsilon_0}\left(\frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

(76) **(C).** $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2.4\mu F.$

Charge flown $= 2.4 \times 500 \times 10^{-6}$ C
$= 1200\ \mu C.$

(77) **(C).** Charge flowing $= \frac{C_1 C_2}{C_1 + C_2} V$.

So potential difference across

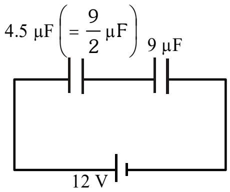
$$C_1 = \frac{C_1 C_2 V}{C_1 + C_2} \times \frac{1}{C_1} = \frac{C_2 V}{C_1 + C_2}$$

(78) **(A).** $C_{AB} = 3 + \frac{3}{3} = 4\mu F$

$C_{AC} = \frac{3}{2} + \frac{3}{2} = 3\mu F \quad \therefore \qquad C_{AB} : C_{AC} = 4 : 3$

(79) **(D).** The given circuit can be redrawn as follows potential difference across 4.5 μF capacitor

$V = \dfrac{9}{\left(\dfrac{9}{2}+9\right)} \times 12 = 8V$



(80) **(C).** Charge on C_1 = charge on C_2

$\Rightarrow \; C_1(V_A - V_D) = C_2(V_D - V_B)$

$\Rightarrow \; C_1(V_1 - V_D) = C_2(V_D - V_2)$

$\Rightarrow \; V_D = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

(81) **(C).** Potential on 4μF capacitor $= \dfrac{6}{10} \times 10 = 6V$

Hence charge q = (4μF) × (6V) = 24 μC

(82) **(B).** $U = \int_0^V CV\,dV = \dfrac{1}{2}CV^2$

(83) **(A).** The total energy before connection

$= \dfrac{1}{2} \times 4 \times 10^{-6} \times (50)^2 + \dfrac{1}{2} \times 2 \times 10^{-6} \times (100)^2$

$= 1.5 \times 10^{-2}$ J

When connected in parallel

$4 \times 50 + 2 \times 100 = 6 \times V \;\Rightarrow V = \dfrac{200}{3}$

Total energy after connection

$= \dfrac{1}{2} \times 6 \times 10^{-6} \times \left(\dfrac{200}{3}\right)^2 = 1.33 \times 10^{-2}$ J

(84) **(A).** $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = 1.5 \times 10^{-8}$J

(85) **(B).** Total capacitance of given system

$C_{eq} = \dfrac{8}{5}\mu F$

$U = \dfrac{1}{2}C_{eq}V^2 = \dfrac{1}{2} \times \dfrac{8}{5} \times 10^{-6} \times 225 = 180 \times 10^{-6}$ J

$= 180 \times 10^{-6} \times 10^7 \text{ erg} = 1800 \text{ erg}$

(86) **(B).** Van de Graaff generator is used to build up high voltages of few million volts.

(87) **(B).** The breakdown field of air which is about 3×10^6 V/m

EXERCISE-2

(1) **(A).** Molar mass of water (H_2O) = 2 × 1 + 16 = 18g.

$\therefore$ Number of mole of water in 10 g $= \dfrac{10}{18}$ mole

1 Mole contain $N_A = 6.023 \times 10^{23}$ number of molecules.

$\therefore$ Number of molecules in 10 g

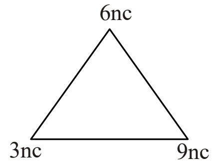
$= \dfrac{10}{18} \times 6.023 \times 10^{23}$ molecules

Each molecule contains 10 electrons

Number of electrons $= 10 \times \dfrac{10}{18} \times 6.023 \times 10^{23}$

$= 3.344 \times 10^{24}$ electrons.

(2) **(B).** $PE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{d}$



$= \dfrac{9 \times 10^9}{0.1}[18 + 54 + 27] \times 10^{-18}$

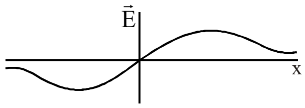
$= 90 \times 99 \times 10^{-9} = 8910 \times 10^{-9}$ J

(3) **(C).** $E = \int_d^{4d} \dfrac{k\lambda\,dx}{x^2} = k\lambda \int_d^{4d} x^{-2}\,dx$

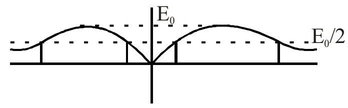
$= \dfrac{-\lambda}{4\pi\varepsilon_0}\left[\dfrac{1}{x}\right]_d^{4d} = \dfrac{3\lambda}{16\pi\varepsilon_0 d}$

(4) **(C).** Direction of field will be along C. Vertical components of field will be cancelled out net field will be towards C.

(5) **(D).** $\vec{E}$ -x graph:



$|\vec{E}|$-x graph



Hence 4 times.

(6) **(A).** $n\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3 \Rightarrow \dfrac{r^3}{R^3} = \dfrac{1}{n}$ ... (1)

$v_o = \dfrac{kq}{r}$ ...(2) ; $v = \dfrac{knq}{R}$ ...(3)

$v = \dfrac{nv_o r}{R} = nv_o \dfrac{1}{n^{1/3}}$

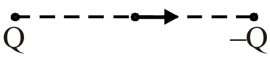
$\therefore \; v_o = \dfrac{v}{n^{2/3}} = \dfrac{2.5}{(125)^{2/3}} = 0.1$ V

(7) **(C).** E goes from higher potential to lower potential

$E = \dfrac{\Delta V}{\Delta x} = \dfrac{V_3 - V_0}{0.05}$

$\Rightarrow V_3 = 400 \times 0.05 = 20V$

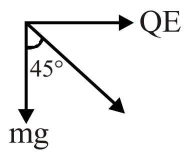
(8) **(A).**



$E \neq 0$; $V = 0$

(9) **(C).** QE = Mg

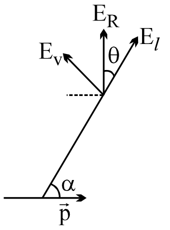
$\dfrac{QV}{d} = w \Rightarrow Q = \dfrac{wd}{V}$



(10) **(C).** After separation charge = constant
Capacity [C] = E_0A/d
Capacity decreases with increase in distance
$V = Q/C$
Potential increases

(11) **(A).** Inside the spherical shell potential remains same
$V_1 = V_2$; $V \propto 1/d$ $\therefore V_1 = V_2 > V_3$

(12) **(B).** Given E_R along y-axis
thus $\theta = 90° - \alpha$



Also $\tan\theta = \frac{1}{2}\tan\alpha$

or $\tan\alpha = \sqrt{2}$

(13) **(C).** Flux going in pyramid $= \frac{Q}{2\varepsilon_0}$ which is devided equally among all 4 faces

$\therefore$ Flux through one face $= \frac{Q}{8\varepsilon_0}$

(14) **(B).** $\phi = \text{Flux} = EA\cos\theta$,
where θ is the angle between $\vec{E}$ and $\vec{A}$.
For maximum flux, $\theta = 0 \Rightarrow \cos 0 = 1$
$\therefore \phi_{max} = EA$
$\phi_{max}/2 = EA\cos\theta \Rightarrow \cos\theta = 1/2$
$\therefore \theta = 60°$
$\therefore$ Angle of rotation is 60°

(15) **(C).** $C_A = \frac{\epsilon_0 A}{d}$; $C_B = \frac{\epsilon_0 A}{2d}$; $C_C = \frac{5\epsilon_0 A}{d}$

$C_D = \frac{5\epsilon_0 A}{2d}$; $C_B < C_A < C_D < C_C$

(16) **(B).** $q = CV = \text{constant}$; $V \propto 1/C$
$V \to$ maximum ; $C \to$ minimum

(17) **(D).** Due to increase in distance between the plates, capacitance decreases.
As battery remains connected
$V = \text{constant}$; $Q = CV$; $C \downarrow Q \downarrow$

(18) **(A).** Potential difference between plates remains same. Decrease in potential difference is counteracted by potential difference due to the extra distance.

$$t\left(E - \frac{E}{k}\right) = Ed \Rightarrow t\left(1 - \frac{1}{k}\right) = d \Rightarrow k = \frac{t}{t-d}$$

E is original electric field, k dielectric constant of plate, t thickness of plate & d extra distance.

(19) **(C).** $V' = \frac{CV}{C+C'}$; $20 = \frac{2\times 200}{2+C'}$; $C' = 18\mu F$

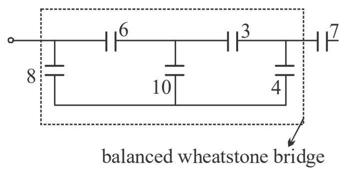
(20) **(A).** $F \propto Q_1Q_2$

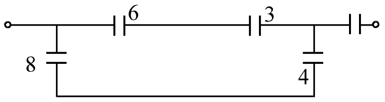
$$\frac{F}{F_1} = \frac{6\times 9}{3\times 6} \Rightarrow F' = \frac{F}{3}$$

(21) **(C).** From charged is tribution $Q_1 = Q_4$ net elctric field between plates is $E \times d$

Potential Difference $= \frac{Q_2 - Q_3}{2A\varepsilon_0}d = \frac{Q_2 - Q_3}{2C}$

(22) **(A).**





$C_{eq} = 14/5$

(23) **(A).** Dipole moment and electric field are opposite to each other on equatorial line.

(24) **(D).** Electric lines never form closed loops.

(25) **(D).** $W = q'(V_2 - V_1)$

$$= q'k\left[\frac{q}{a+r} - \frac{q}{a} - \frac{q}{a} + \frac{q}{a+r}\right] = 2Kqq'\left[\frac{1}{a+r} - \frac{1}{a}\right]$$

or $|W| = \frac{2Kqq'r}{a(a+r)}$

(26) **(A).** $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{10\times 200 + 20\times 100}{10+20} = 133.3\text{ V}$

(27) **(C).** New potential difference $= \frac{V}{K} = \frac{100}{10} = 10V$

(28) **(B).** Charge on smaller sphere

$= \text{Total charge}\left(\frac{r_1}{r_1 + r_2}\right) = 30\left(\frac{5}{5+10}\right) = 10\mu C$

(29) **(C).** After charging, total charge on the capacitor $Q = CV$
$= 10\times 10^{-6}\text{ F} \times 1000\text{ V} = 10^{-2}\text{ C}$.
Common potential

$$V = \frac{C_1V_1}{C_1 + C_2} = \frac{10^{-2}}{16\times 10^{-6}} = 625V.$$

(30) **(B).** When spheres are connected by a wire then charge will flow untill they have same potential $V_1 = V_2 = V_3$

and for sphere $V = \frac{KQ}{R} = ER$

So for same potential $E \propto 1/R$. So, $E_1 > E_2 > E_3$

(31) **(B).** $\epsilon_r = \frac{V_a}{V_m} = \frac{4}{2} = 2$

(32) **(C).** $E = 9\times 10^9 \times \frac{q}{r^2} = \frac{9\times 10^9 \times 3\times 10^{-9}}{(3\times 10^{-2})^2}$
$= 3\times 10^4\text{ V m}^{-1}$

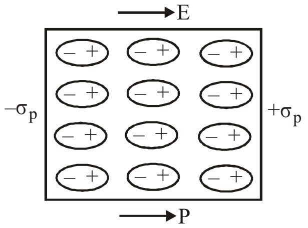
(33) **(A).** For equipotential spheres, $E \propto \frac{1}{\text{Radius}}$

(34) **(B).** V at $(\sqrt{2}, \sqrt{2})$ = V at $(2, 0)$
$\therefore$ Potential difference = 0

(35) **(D).** Area vector

(36) **(C).** $v = 9\times10^9 \times \frac{q}{r} = \frac{9\times10^9\times3\times10^{-9}}{9\times10^{-2}} = 300V$

(37) **(A).** The polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say σ_p and $-\sigma_p$.



A uniformly polarised dielectric amount to induced surface charge density but not volume charge density.

(38) **(C).** As $F_m = \frac{F_0}{K}$

$\therefore$ The maximum force decreases by K times.

(39) **(A).** In an external electric field, the positive and negative charges of a non-polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule).

The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the external field.

(40) **(B).** $\tan\theta = \frac{F}{Mg}$

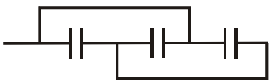
F is same on both the charges;

θ is same only if M is equal

(41) **(B).** Take vertical plane through the shaded circle, which is equilpotential.

($\because$ lines of force and equipotential surface are perpendicular)

(42) **(A).** Capacitors are in parallel, $C_p = 3C$



Another such circuit in series

$\therefore$ $C_{AB} = C_p/2 = 1.5\,C$

(43) **(B).** When two charged spheres are brought in contact they attain equal charge $= \frac{q_1+q_2}{2}$.

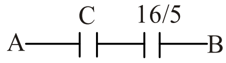
For a given total charge $q_1 + q_2$, the force between the charges is maximum when charges are equal.

(44) **(D).** $\varepsilon_r = \frac{F_a}{F_m} = \frac{mg}{(m-m_l)\,g} = \frac{\rho V}{(\rho-\sigma)\,V} = \frac{\rho}{\rho-\sigma}$

(45) **(C).** Force is same in magnitude for both

$$\frac{a_1}{a_2} = \frac{F/m_1}{F/m_2} = \frac{m_2}{m_1} = 2$$

(46) **(B).** $3 = \frac{\frac{16}{5}C}{\frac{16}{5}+C} \Rightarrow C = 48\mu F$



(47) **(D).** $a = \frac{F}{m} = \frac{qE}{m}$

(48) **(C).** Total energy at C = Potential energy at D

$$0.06 + \frac{9\times10^9\times5\times10^{-6}}{3}\times2\times5\times10^{-6}$$

$$= \frac{9\times10^9\times5\times10^{-6}\times2}{\sqrt{9+x^2}}\ ;\quad x = 4m$$

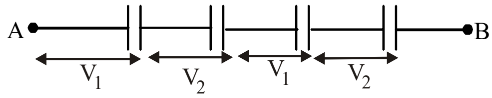
(49) **(A).** $U = \frac{1}{2}CV^2 = \frac{1}{2}\times10\times10^{-6}\times100 = 500\mu J$

(50) **(C).** $v^2 = u^2 + 2as \Rightarrow v = \sqrt{2as}$

$$= \sqrt{2\times\left(\frac{Ee}{m}\right)s} = \sqrt{2\times10^4\times1.8\times10^{11}\times2\times10^{-2}}$$

$$= 2\sqrt{1.8\times10^{13}} = 2\times4.25\times10^6 = 8.5\times10^6\ m/s$$

(51) **(D).**



$2V_1 + 2V_2 = 60$ and $V_2 = 2V_1$ ($\because V \propto 1/C$)

$\therefore$ $6V_1 = 60 \Rightarrow V_1 = 10V$

(52) **(B).** $Eq = mg\ ;\ \frac{Vq}{r} = mg \Rightarrow q = \frac{mgr}{V}$

(53) **(D).** Line perpendicular to E-field : Equipotential

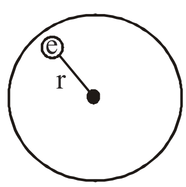
In direction of E-field potential decreases.

(54) **(C).** $\frac{R_1}{R_2} = \frac{0.01}{0.02} = \frac{1}{2}\ ;\ Q \propto R\ ;\ Q_1 = \left(\frac{1}{3}\right)Q = 20mC$

EXERCISE-3

(2) **2.** $eE = m_e\omega^2 r$

$$\Rightarrow \int E\,dr = \frac{m_e\omega^2}{e}\int_0^R r\,dr$$



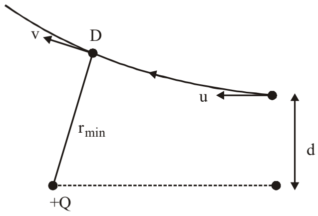
$$\Rightarrow V = \frac{m_e\omega^2R^2}{2e}$$

(2) **2.** The path of the particle will be as shown in figure. At the point of minimum distance (D) the velocity of the particle will be $\perp$ to its position vector w.r.t. +Q.

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + \frac{KQq}{r_{min}} \qquad ....(1)$$

$\because$ Torque on q about Q is zero hence angular momentum about Q will be conserved

$\Rightarrow mvr_{min} = mud$ ....(2)



By eq. (2) in eq. (1)

$$\frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{ud}{r_{min}}\right)^2 + \frac{KQq}{r_{min}}$$

$$\Rightarrow \frac{1}{2}mu^2\left(1-\frac{d^2}{r_{min}^2}\right) = \frac{mu^2d}{r_{min}} \qquad \{\because KQq = mu^2d\}$$

$$\Rightarrow r_{min}^2 - 2r_{min}d - d^2 = 0$$

$$\Rightarrow r_{min} = \frac{2d \pm \sqrt{4d^2+4d^2}}{2} = d\,(1\pm\sqrt{2})$$

$\because$ distance cannot be negative

$\therefore\ r_{min} = d\,(1+\sqrt{2})$

(3) **2.** Consider a Gausian surface of radius r ($R_1 < r < R_2$)
The net charge enclose is

$$q_{net} = q + \int_{R_1}^{r} (4\pi r^2)\frac{b}{r}dr \quad \text{or} \quad q_{net} = q + 2\pi b\,(r^2 - R_1^2)$$

Using Gauss's law

$$E\,(4\pi r^2) = \frac{q_{net}}{\varepsilon_0} = \frac{q + 2\pi b\,(r^2 - R_1^2)}{\varepsilon_0}$$

$$\text{or} \quad E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{b}{2}\left(1 - \frac{R_1^2}{r^2}\right)$$

For E = constant, $b = \dfrac{q}{2\pi R_1^2} = 2C/m^2$

(4) **168.** Let v_1 and v_2 be the speeds of sphere A and B just before collision, then from conservation of energy,

$$0 = \frac{1}{2}mv_1^2\left(1+\frac{2}{3}\right) + \frac{1}{2}(2m)v_2^2\left(1+\frac{2}{5}\right) + \frac{K.Q(-2Q)}{2R}$$

$$0 = \frac{5}{6}mv_1^2 + \frac{7}{5}mv_2^2 - \frac{K.Q^2}{R}$$

$$\frac{5}{6}mv_1^2 + \frac{7}{5}mv_2^2 = \frac{K.Q^2}{R} \qquad \text{..........}(1)$$

From conservation of angular momentum about any point O

$$0 = mv_1R + \frac{2}{3}mR^2\left(\frac{v_1}{R}\right) - 2mv_2R - \frac{2}{5}(2m)\,r^2\left(\frac{v^2}{R}\right)$$

$$v_1 = \frac{42}{25}v_2 \qquad \text{..........}(2)$$

Solving eq. (1) and (2), $K_B = \dfrac{25KQ^2}{67R}$; $K_A = \dfrac{42KQ^2}{67R}$

(5) **9.** In uniform electric in vertical direction if (+ve) charge feels extra acceleration in downward direction, then (–ve) charge will feel acceleration in upward direction.

$v_{uncharged} = 5\sqrt{5}$ m/sec
v = 0, h = height
$v^2 - u^2 = -2(g)\,h$

$-(5\sqrt{5})^2 = -2gh$; $u_{q+} = 13$ m/sec
v = 0, h = h

$$v^2 - u^2 = 2\left(g + \frac{F_E}{m}\right)h \ ; \quad 0 - (13)^2 = -2\left(g + \frac{F_E}{m}\right)h$$

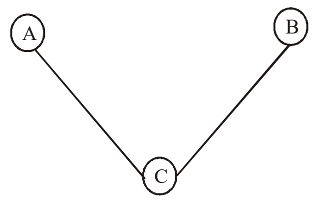
Let $u_{q-} = u$ (say)
v = 0 h = ht

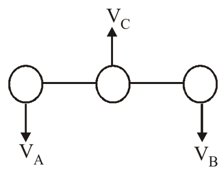
$$v^2 - u^2 = -2\left(g - \frac{F_E}{m}\right)h \ ; \quad -u^2 = -2\left(g - \frac{F_E}{m}\right)h$$

u = 9 m/sec

(6) **2.** Takes all three bodies as system electrostatic force
(1) Apply law of conservation of momentum in direction
(2) Apply law of conservation of energy,

So, $m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C = 0$
$\Rightarrow v_C = -2v_A$ (as $V_A = V_B$)





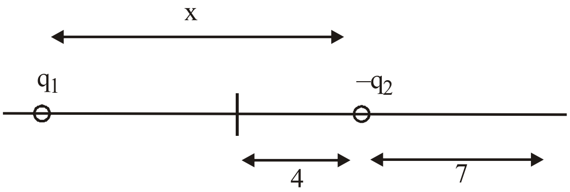
Change in electrostatic P.E. = Increase in KE

$$\frac{KQ^2}{\ell} - \frac{KQ^2}{2\ell} = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 + \frac{1}{2}m_Cv_C^2$$

$[v_A = v_B, v_C = -2v_A]$
So, $v_C = 2$ m/s

(7) **44.** $\dfrac{q_1}{x-4} - \dfrac{q_2}{4} = 0$; $\dfrac{q_1}{q_2} = \dfrac{x-4}{4}$

$$\frac{q_1}{x+7} - \frac{q_2}{7} = 0 \ ; \quad \frac{q_1}{q_2} = \frac{x+7}{7} \ ; \quad \frac{x-4}{4} = \frac{x+7}{7}$$



$7x - 28 = 4x + 28$; $3x = 56 \Rightarrow x = 56/3$

$$\frac{q_1}{q_2} = \frac{\frac{56}{3}+7}{7} = \frac{11}{3}; |q_2| = +12\ \mu c \ \Rightarrow q_1 = 12 \times \frac{11}{3} = 44\ \mu c$$

(8) **24.** Let total charge on capacitor $C_1 = EC_1 = Q$

Let q be the charge on second one $\frac{q}{C_2} + Ri = \frac{Q-q}{C_1}$

$$R\frac{dq}{dt} = \frac{Q}{C_1} - q\left[\frac{1}{C_1} + \frac{1}{C_2}\right]$$

Integrating, $\frac{C_1C_2}{C_1+C_2}\ln\left[QC_2 - q\left(C_1+C_2\right)\right] = -\frac{t}{R} + A$

At t = 0, q = 0 $\therefore$ $A = \frac{C_1C_2}{C_1+C_2}\ln QC_2$

$$\frac{C_1C_2}{C_1+C_2}\ln\left[1 - \frac{q\left(C_1+C_2\right)}{QC_2}\right] = -\frac{t}{R}$$

$$1 - \frac{q\left(C_1+C_2\right)}{QC_2} = e^{-\frac{(C_1+C_2)t}{C_1C_2R}}$$

$$q = \frac{QC_2}{C_1+C_2}\left[1 - e^{-\frac{(C_1+C_2)}{C_1C_2}\frac{t}{R}}\right]$$

$$q = \frac{EC_1C_2}{C_1+C_2}\left[1 - e^{-\frac{(C_1+C_2)}{C_1C_2}\frac{t}{R}}\right] = \frac{24}{5}\left(1 - e^{-5t/6}\right)$$

$\therefore$ x = 24

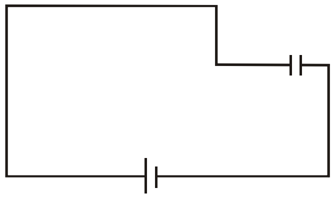
(9) **12.** $q_2 = \frac{q_0C_2}{(C_1+C_2)(1-e^{-t/\tau})}$ where τ is $\frac{RC_1C_2}{C_1+C_2}$

(10) **750.** Just after closing switch no current flows through R_2 so $I_1 = 3mA$
Long time after closing switch no current flows through C so $I_2 = 2mA$
Directly after re-opening the switch no current flows through R_1 and the capacitor will discharge through R_2 so $I_3 = 2mA$.

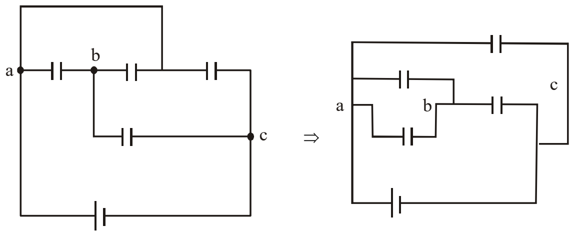
(11) **1000.** $Q = CV = \int_0^{t_1} I\,dt = \int_0^{t_1} (10^{-12})\,dt \Rightarrow t_1 = 10^9$ s

$\therefore$ n = 1000

(12) **100.** With key open circuit is



With key closed



$$u_f = \frac{1}{2}\left(\frac{5}{3}C\right)V^2$$

$$u_f - u_i = \frac{1}{2}\frac{2}{3}CV^2 = \frac{1}{2}\times\frac{2}{3}\times 3\times 10^{-6}\times 10^2 = 100\mu J$$

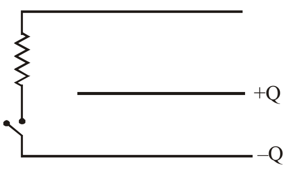
(13) **200.** Charge on capacitor = $5 \times 10^{-6} \times 200 = 10^{-3}$

Total heat generated = $\frac{10^{-6}}{2\times 5\times 10^{-6}} = 10^{-1}$

The fraction lost in 500Ω is $\frac{5}{8}$ of total so H = $\frac{5}{8}\times\frac{1}{10} = \frac{1}{16}$ J

$$3200 \times \frac{1}{16} = 200$$

(14) **4.** $U_i = \frac{Q^2d}{2\varepsilon_0 A}$ then $V_C = V_A$ (At equilibrium condition)

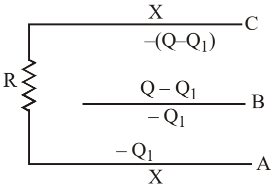


$V_{AB} = V_{BC}$

$$\frac{Q_1d}{\varepsilon_0 A} = \frac{(Q-Q_1)(2d)}{\varepsilon_0 A}$$

$Q_1 = 2(Q - Q_1)$
$3Q_1 = 2Q$

$$U_f = \frac{Q_1^2 d}{2\varepsilon_0 A} + \frac{(Q-Q_1)^2 2d}{2\varepsilon_0 A} = \frac{Q^2 d}{3\varepsilon_0 A}$$



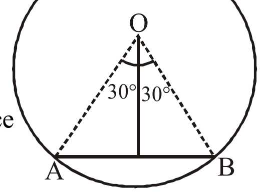
$$\Delta U = \frac{Q^2 d}{2\varepsilon_0 A} - \frac{Q^2 d}{3\varepsilon_0 A} = \frac{Q^2 d}{6\varepsilon_0 A} = \frac{(60)^2}{6(6)} = 0.1 \text{ mJ}$$

(15) **12.** The capacitance of the nonlinear capacitor is $C = \varepsilon C_0 = \alpha V C_0$,
where C_0 is the capacitance of the capacitor without a dielectric. The charge on the nonlinear capacitor is $q_n = CV = \alpha C_0 V^2$, while the charge on the normal capacitor is $q_0 = C_0 V$. It follows from the charge conservation law $q_n + q_0 = C_0 V_0$

that the required voltage is $V = \dfrac{\sqrt{4\alpha V_0 + 1} - 1}{2\alpha} = 12V$.

(16) **6.** Flux from total cylindrical surface (angle = 2π)

$= \dfrac{Q_{in}}{\varepsilon_0}$



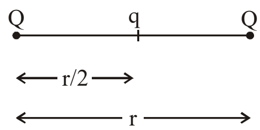
Flux from cylindrical surface AB = flux from the given

surface $= \dfrac{Q_{in}}{6\varepsilon_0} = \dfrac{\lambda \ell}{6\varepsilon_0}$; n = 6

EXERCISE-4

(1) **(A).** For system equilibrium

$\vec{F}_{Q,Q} + \vec{F}_{Q,q} = 0$



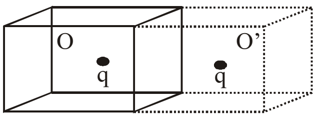
$\dfrac{KQ^2}{r^2} + \dfrac{KQq}{(r/2)^2} = 0$

$q = -Q/4$

(2) **(A).** $W = q(\Delta V)$

$2 = 20(\Delta V) \Rightarrow \Delta V = 0.1$ volt.

(3) **(B).**



Flux due to charge at point O $= \dfrac{q}{6\varepsilon_0}$ (Rightward)

Flux due to charge at point O' $= \dfrac{q}{6\varepsilon_0}$ (Leftward)

$\therefore$ Flux through BC FG is zero.

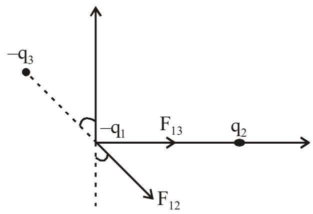
(4) **(B).** $U_{Sys} = \dfrac{1}{2} C_{Sys} V_{Sys}^2 = \dfrac{1}{2}\left(\dfrac{C}{n}\right) V^2$

(5) **(A).** $C = 4\pi\varepsilon_0 R$

(6) **(A).** Remains unchanged

(7) **(C).** $W = \dfrac{Q^2}{2C}$

(8) **(A).**



x component of force on $q_1 = F_{13} + F_{12} \sin q$

$= \dfrac{Kq_1q_2}{b^2} + \dfrac{Kq_1q_3}{a^2} \sin\theta \propto \left(\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2} \sin\theta\right)$

(9) **(D).** By Gauss Theorem $\phi_2 - \phi_1 = \dfrac{\Sigma q}{\varepsilon_0}$

(10) **(B).** $V = V_{shell} + V_Q = \dfrac{kq}{R} + \dfrac{KQ}{(R/2)}$

(11) **(D).** Let $q_B = q_C = Q$

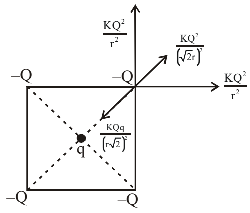
After conduction with sphere A

$q_B = \dfrac{Q}{2}$; $q_C = \dfrac{3Q}{4}$; $F \propto q_B q_C$

(12) **(D).** At closest distance K.E. converts into P.E.

$\dfrac{KQq}{r} = \dfrac{1}{2}mv^2 \Rightarrow r \propto \dfrac{1}{v^2}$

(13) **(B).**



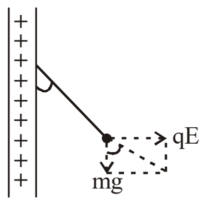
For system equilibrium, $F_{net} = 0$

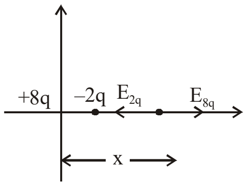
$\Rightarrow \sqrt{2}\left(\dfrac{KQ^2}{r^2}\right) + \dfrac{KQ^2}{2r^2} - \dfrac{2KQq}{r^2} = 0$

$\Rightarrow q = \dfrac{Q}{4}\left(1 + 2\sqrt{2}\right)$

(14) **(A).** $mg = qE \Rightarrow q = \dfrac{mg}{E}$

(15) **(D).** $\tan\theta = \dfrac{qE}{mg} = \dfrac{q\left(\dfrac{\sigma}{\varepsilon_0}\right)}{mg} \Rightarrow \sigma \propto \tan\theta$



(16) **(A).** 

For net electric field to be zero

$$E_{2q} = E_{8q} \Rightarrow \frac{K(2q)}{(x-L)^2} = \frac{K(8q)}{x^2} \Rightarrow x = 2L$$

(17) **(B).** Potential difference = $V_1 - V_2$

(18) **(D).** $\frac{1}{2}CV^2 = ms\Delta T$

(19) **(A).** This is (n – 1) capacitors in parallel.
So $C_{eq} = (n-1)\,C$

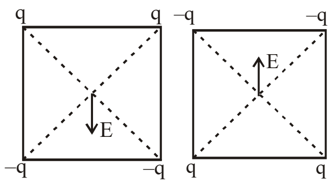
(20) **(A).** Non uniform field so torque as well as translational force.

(21) **(C).** $e(V_2 - V_1) = \frac{1}{2}mv^2$

(22) **(A).** For equipotential spheres, $E \propto \frac{1}{\text{Radius}}$

(23) **(B).** V at $\left(\sqrt{2}, \sqrt{2}\right)$ = V at (2, 0)

$\therefore$ Potential difference = 0

(24) **(D).** 

(25) **(D).** $E = -\frac{dV}{dx}$

(26) **(A).** Electron charge is same everywhere.

(27) **(D).** $\frac{\text{Energy stored}}{\text{Work by battery}} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2}$

(28) **(D).** Once slab is removed and then reinserted
So no change in energy $\Rightarrow$ W = 0

(29) **(B).** $C_{\text{Air ppc}} = 9\mu F = \frac{\varepsilon_0 A}{d}$

$$C' = \frac{\varepsilon_0 A}{\frac{t_1}{\varepsilon r_1} + \frac{t_2}{\varepsilon r_2}} = 4.5\,C_{Air} = 40.5\ \mu F$$

(30) **(D).** Inside shell E = 0

(31) **(A).** Gauss Theorem

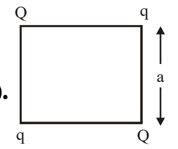
(32) **(A)** **(33)** **(C).**

Charge enclosed in a radius $r_1 = \int_0^{r_1} \frac{Q}{\pi R^4} r.4\pi r^2 dr = \frac{Qr_1^4}{R^4}$

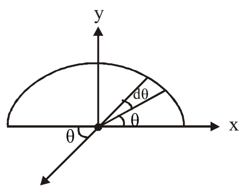
Using Gauss's law,

$$E.4\pi r_1^2 = \left(\frac{Qr_1^4}{R^4}\right)\frac{1}{\varepsilon_0} \quad ; \quad E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

(34) **(D).** $W_{ext} = q(V_f - V_i) = (-100 \times 1.6 \times 10^{-19})(-4-10)$
$= -1.6 \times 10^{-17} \times (-14) = 2.24 \times 10^{-16}$ J

(35) **(A).** 

$$\frac{Q^2}{4\pi\varepsilon_0(2a^2)} + \frac{qQ\sqrt{2}}{4\pi\varepsilon_0 a^2} = 0 \quad ; \quad \frac{Q}{q} = -2\sqrt{2}$$

(36) **(C).** 

Linear charge density $\lambda = \left(\frac{q}{\pi r}\right)$

$$E = \int dE \sin\theta\,(-\hat{j}) = \int \frac{K.dq}{r^2}\sin\theta\,(-\hat{j})$$

$$E = \frac{K}{r^2}\int \frac{qr}{\pi r} d\theta \sin\theta\,(-\hat{j}) = \frac{K}{r^2}\frac{q}{\pi}\int_0^{\pi}\sin\theta\,(-\hat{j}) = \frac{q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$$

(37) **(B).** Apply shell theorem the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2.dr.\rho = 4\pi r^2.dr.\rho_0\left[\frac{5}{4} - \frac{r}{R}\right]$$

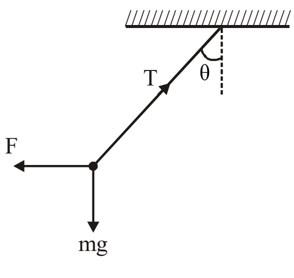
$$= \int dq = q = 4\pi\rho_0 \int_0^r \left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$

$$= 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

$$E = \frac{kQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}.4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

(38) **(C).** From F.B.D of sphere, using Lami's theorem



$$\frac{F}{mg} = \tan\theta \qquad \text{....... (1)}$$

when suspended in liquid,
as θ remains same,

$$\therefore \quad \frac{F'}{mg\left(1-\frac{\rho}{d}\right)} = \tan\theta \qquad ....(2)$$

Using eq. (1) and (2)

$$\frac{F}{mg} = \frac{F'}{mg\left(1-\frac{\rho}{d}\right)}, \text{ where } F' = \frac{F}{K}$$

$$\therefore \quad \frac{F}{mg} = \frac{F'}{mg\,K\left(1-\frac{\rho}{d}\right)} \quad \text{or } K = \frac{1}{1-\frac{\rho}{d}} = 2$$

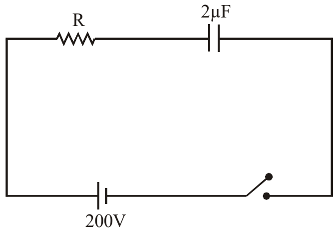
(39) (C). $U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}(q_0 e^{-t/T})^2 = \frac{q_0^2}{2C}e^{-2t/T}$ (where t = CR)

$$U = U_i e^{-2t/\tau}$$

$$\frac{1}{2}U_i = U_i e^{-2t_1/\tau} \Rightarrow t_1 = \frac{T}{2}\ln 2$$

Now, $q = q_0\, e^{-t/T}$; $\frac{1}{4}q_0 = q_0 e^{-t/2T}$

$t_2 = T\ln 4 = 2T\ln 2 \qquad \therefore \frac{t_1}{t_2} = \frac{1}{4}$

(40) (C). 

$v = 200\,(1 - e^{-t/\tau})$
$120 = 200\,(1 - e^{-t/\tau})$

$$e^{-t/\tau} = \frac{200-120}{200} = \frac{80}{200}$$

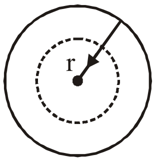
$t/\tau = \log(2.5) = 0.4$
$5 = (0.4) \times R \times 2 \times 10^{-6}$

$$R = \frac{5}{(0.4)\times 2\times 10^{-6}} = 2.7\times 10^6\Omega$$

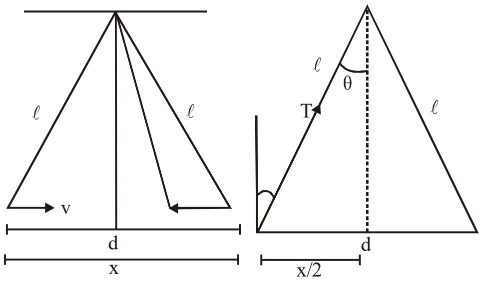
(41) (D). $\phi = ar^2 + b$

$$E = -\frac{d\phi}{dt} = -2ar \;;\; \oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

$$-2ar.4\pi r^2 = \frac{q}{\varepsilon_0}$$



$q = -8\varepsilon_0 ar^3$; $\rho = \frac{q}{\frac{4}{3}\pi r^3} = -6\,a\varepsilon_0$

(42) (A). 

$$\sin\theta = \frac{kq^2}{d^2}$$

$$\cos\theta = mg \;;\; \tan\theta = \frac{k}{mg}.\frac{q^2}{x^2} \;;\; \frac{x}{2\ell} = \frac{k}{mg}.\frac{q^2}{x^2}$$

$$x^3 = \frac{2k\ell}{mg}q^2 \;;\; q^2 \propto x^3 \;;\; q \propto x^{3/2}$$

$$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt} \qquad \text{(dq/dt is constant)}$$

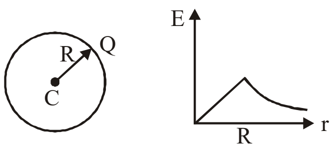
$c \propto x^{1/2}v$; $v \propto x^{-1/2}$

(43) (D). $Q = C\varepsilon_0 e^{-t/cR}$; $4\varepsilon = 4\varepsilon_0\varepsilon^{-t/\tau}$
$\varepsilon = \varepsilon_0\varepsilon^{-t/\tau}$
When $t = 0 \Rightarrow \varepsilon_0 = 25$

$$5 = 25e^{\frac{-200}{\tau}} \;;\; \ln 5 = \frac{200}{\tau}$$

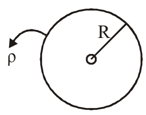
$$\tau = \frac{200}{\ln 5} = \frac{200}{\ln 10 - \ln 2} = \frac{200}{\ln 10 - 0.693}$$

Alternative : Time constant is the time in which 63% discharging is completed.
So remaining charge = 0.37 × 25 = 9.25 V
Which time in 100 < t < 150 sec.

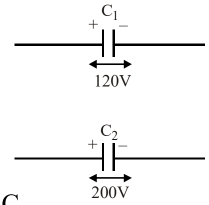
(44) (C). 

(45) (C). $U_c = \frac{3}{2}\frac{KQ}{R}q$; $U_s = \frac{KQ}{R}q$

$$\therefore \quad \Delta U = \frac{KQ}{2R}q = \frac{1}{4\pi\varepsilon_0}.\frac{1}{2R}\rho\frac{4\pi R^3}{3}q = \frac{\rho R^2 q}{6\varepsilon_0}$$



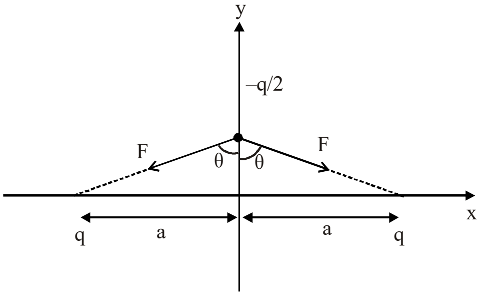
(46) **(B).** For potential to be made zero, after connection Common voltage



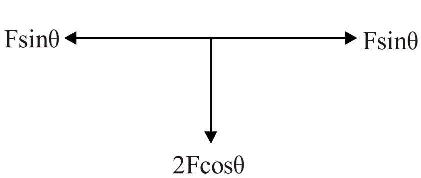
$$= \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$120\ C_1 = 200\ C_2 \Rightarrow\ 3C_1 = 5C_2$

(47) **(A).**



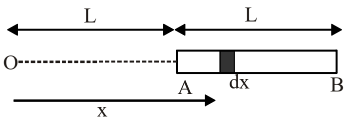
$\Rightarrow$



$$F_{net} = \frac{2kq\ (q/2)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{net} = \frac{2kq\ (q/2)\ y}{(y^2 + a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3} \propto y$$

(48) **(D).**



$$V = \int_L^{2L} \frac{kdq}{x} = \int_L^{2L} \frac{1}{4\pi\varepsilon_0} \frac{(q/L)\ dx}{x} = \frac{q}{4\pi\varepsilon_0 L} \ln (2)$$

(49) **(A).** $\vec{E} = 30x^2\hat{i}$

$$dV = -\int E.dx\ ;\ \int_{v_0}^{v_A} dV = -\int_0^2 30x^2\ dx$$

$V_A - V_O = -80$ volt

(50) **(C).** By formula of electric field between the plates of a capacitor $E = \sigma/K\varepsilon_0$

$\Rightarrow\ \sigma = EK\varepsilon_0 = 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12}$
$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7}$
$= 6 \times 10^{-7}\ C/m^2$

(51) **(D).** (A) and (B) is not possible since field lines should originate from positive and terminate to negative charge. (C) is not possible since field lines must be smooth. (D) satisfies all required condition.

(52) **(BC).** $V_0 = k\dfrac{Q}{R}$ ...... (1) ; $V_1 = \dfrac{kQ}{2R^3}(3R^2 - r^2)$

$$V = \frac{3}{2}V_0 \Rightarrow R_1 = 0$$

$$\frac{5}{4}\frac{kQ}{R} = kQ\frac{(3R^2 - r^2)}{2R^3} \Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

$$\frac{3}{4}\frac{kQ}{R} = \frac{kQ}{R_3} \Rightarrow R_3 = \frac{4R}{3}\ ;\ \frac{1}{4}\frac{kQ}{R} = \frac{kQ}{R_4}$$

$\Rightarrow\ R_4 = 4R \Rightarrow R_4 > 2R$

(53) **(A).** $C_{aq} = \dfrac{3C}{3 + C}$ ...... (1)

Total charges $q = \left(\dfrac{3C}{3 + C}\right) E$ ...... (2)

Charge upon capacitor $2\mu F$,

$$q' = \frac{2}{3} \times \frac{3CE}{(3 + C)} = \frac{2CE}{3 + C} = \frac{2E}{1 + \frac{3}{C}}\ ;\quad \frac{dQ}{dC} > 0,\ \frac{dQ^2}{dC^2} < 0$$

(54) **(D).** Charge in the region between a and r is calculated as follows : $Q_1 = \int_a^r 4\pi r^2 dr \dfrac{A}{r}$ and $Q_1 = 2\pi A\ (r^2 - a^2)$

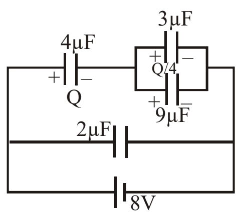
$$\Rightarrow\ E\ (r) = \frac{K}{r^2}[Q + Q_1] = \frac{K}{r^2}[Q + 2\pi A\ (r^2 - a^2)]$$

$$= K\ 2\pi A + \frac{KQ}{r^2} - \frac{2\pi A a^2}{r^2}K$$

For uniform E_1 last two terms should cancel

$\therefore\ \ KQ = 2\pi A \times a^2 K \Rightarrow\ A = \dfrac{Q}{2\pi a^2}$

(55) **(B).**

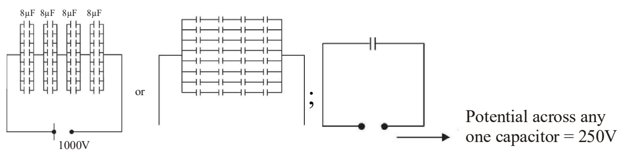


$Q = 3 \times 8 = 24\ \mu C$

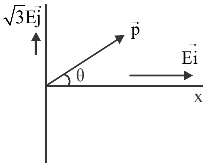
$\therefore$ The charges on $4\mu F$ and $9\mu F$ capacitors are $24\mu C$ and $18\mu C$ respectively. Hence,

$$E = \frac{9 \times 10^9 \times (24 + 18)\ \mu}{30 \times 30} = 420\ N/C$$

(56) **(C).** $C_{eq} = \dfrac{8\mu F}{4} = 2\mu F$



(57) (B).



$$\vec{\tau} = \vec{p} \times \vec{E} \quad ; \quad \vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$$

$$pE\sin\theta = \tau \quad \text{and} \quad p\sqrt{3}E\cos\theta = \tau$$

$$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

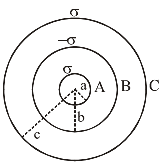
(58) (C). $q_i = CV$

$q_f = KCV$

$q_{induced} = q_f - q_i = (K-1)\,CV$

$$= \left(\frac{5}{3} - 1\right) \times 90 \times 10^{-12} \times 20 = 1.2\ \text{nC}$$

(59) (D).

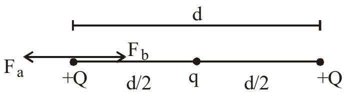


$$V_B = \frac{kQ_A}{b} + \frac{kQ_B}{b} + \frac{kQ_C}{c}$$

$$= k\left[\frac{(+\sigma)(4\pi a^2)}{b} + \frac{(-\sigma)(4\pi b^2)}{b} + \frac{(+\sigma)(4\pi c^2)}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\sigma 4\pi\left[\frac{a^2}{b} - \frac{b^2}{b} + c\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

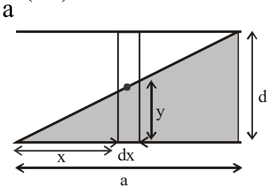
(60) (C).



For equilibrium, $\vec{F}_a + \vec{F}_b = 0$; $\vec{F}_a = -\vec{F}_b$

$$\frac{kQQ}{d^2} = -\frac{kQq}{(d/2)^2} \Rightarrow q = -\frac{Q}{4}$$

(61) (C). $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$; $dy = \frac{d}{a}(dx)$



$$\frac{1}{dc} = \frac{y}{k\varepsilon_0 \cdot a\,dx} + \frac{(d-y)}{\varepsilon_0 a\,dx}$$

$$\frac{1}{dc} = \frac{1}{\varepsilon_0 a\,dx}\left(\frac{y}{k} + d - y\right)$$

$$\int dc = \int \frac{\varepsilon_0 a\,dx}{\frac{y}{k} + d - y} \quad ; \quad c = \varepsilon_0 a \cdot \frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$$

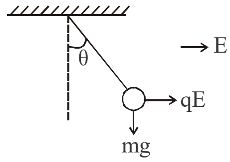
$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ln\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k\varepsilon_0 a^2}{(1-k)\,d}\ln\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right) = \frac{k\varepsilon_0 a^2}{(1-k)\,d}\ln\left(\frac{1}{k}\right) = \frac{k\varepsilon_0 a^2 \ln k}{(k-1)\,d}$$

(62) (C). Electric field on axis of ring, $E = \frac{kQh}{(h^2 + R^2)^{3/2}}$

For maximum electric field, $\frac{dE}{dh} = 0 \Rightarrow h = \frac{R}{\sqrt{2}}$

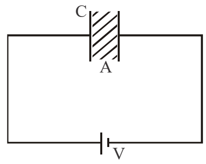
(63) (C). $\tan\theta = \frac{qE}{mg} = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10}$



$$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$$

(64) (D). $A = 10^{-4}\ \text{m}^2$

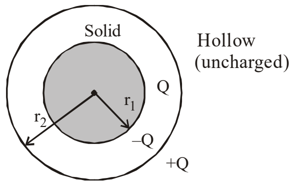
$E_{max} = 10^6$ V/m, $C = 15\ \mu F$



$$C = \frac{k\varepsilon_0 A}{d} \quad ; \quad \frac{Cd}{\varepsilon_0 A} = k$$

$$k = \frac{15 \times 10^{-12} \times 500 \times 10^{-6}}{8.86 \times 10^{-12} \times 10^4} \simeq 8.5$$

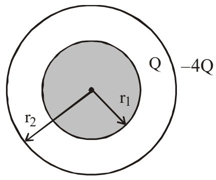
(65) (A). As given in the first condition :



Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right) = kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

In the second condition :



Shell is now given charge – 4Q.

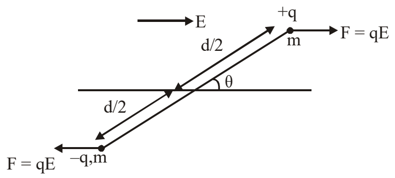
$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2} = kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

Hence, we also obtain that potential difference does not depend on charge of outer sphere.

$\therefore$ P.d. remains same

(66) (C).



Moment of inertia (I) = $m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by $\tau = I\,\alpha$

$(qE)\,(d\sin\theta) = \frac{md^2}{2}\cdot\alpha$; $\alpha = \left(\frac{2qE}{md}\right)\sin\theta$

for small θ, $\alpha = \left(\frac{2qE}{md}\right)\theta$

Angular frequency $\omega = \sqrt{\frac{2qE}{md}}$

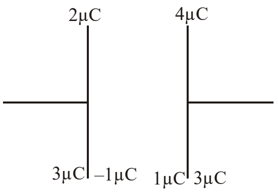
(67) (C). $\vec{E} = (20x + 10)\,\hat{i}$

$$V_1 - V_2 = -\int_{-5}^{1} (20x + 10)\,dx$$

$$V_1 - V_2 = -(10x^2 + 10x)_{-5}^{1}$$

$$V_1 - V_2 = 10\,(25 - 5 - 1 - 1) = 180\text{ V}$$

(68) (D).

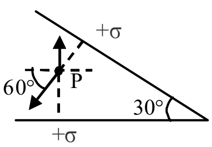


Charges at inner plates are $1\,\mu C$ and $-1\,\mu C$

$\therefore$ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\,\mu C}{1\,\mu F} = \frac{1\times10^{-6}C}{1\times10^{-6}\text{ Farad}} = 1V$$

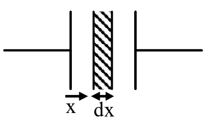
(69) (A).



$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\cos 60°\,(-\hat{x}) + \left[\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\sin 60°\right](\hat{y})$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$$

(70) (B). Capacitance of element $= \frac{k\varepsilon_0 A}{dx}$



Capacitance of element

$$C' = \frac{k_0\,(1+\alpha x)\,\varepsilon_0 A}{dx}\ ;\quad \sum\frac{1}{C'} = \int_0^d \frac{dx}{k_0\varepsilon_0 A\,(1+\alpha x)}$$

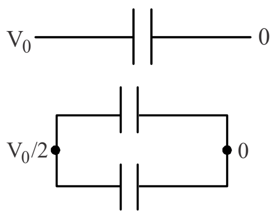
$$\frac{1}{C} = \frac{1}{k_0\varepsilon_0 A\alpha}\ln(1+\alpha d)$$

Given : $\alpha d << 1$

$$\frac{1}{C} = \frac{1}{k_0\varepsilon_0 A\alpha}\left(\alpha d - \frac{\alpha^2 d^2}{2}\right);\quad \frac{1}{C} = \frac{d}{k_0\varepsilon_0 A}\left(1 - \frac{\alpha d}{2}\right)$$

$$C = \frac{k_0\varepsilon_0 A}{d}\left(1 + \frac{\alpha d}{2}\right)$$

(71) 6.



$V_0 = 20\text{ V}$

Heat loss = $U_i - U_f$

$$= \frac{1}{2}CV_0^2 - 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right] = \frac{CV_0^2}{4} = \frac{(60\times10^{-12})\,(20)^2}{4}\text{ J}$$

$= 6\times10^{-9}\text{ J} = 6\text{ nJ}$

(72) (D). $|\vec{E}|$ should be constant on the surface and the surface should be equipotential.

(73) (D). $E_x = \frac{K\,(4q)}{R^2}\cos 30° + \frac{K\,(2q)}{R^2}\cos 30° + \frac{K\,(2q)}{R^2}\cos 30°$

(74) (C). $C_1 + C_2 = 10$ ....(i)

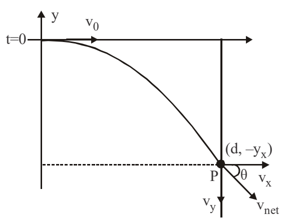
$\frac{1}{2}C_2V^2 = 4\times\frac{1}{2}C_1V^2$

$C_2 = 4C_1$ ....(ii)

$C_1 = 2\ \&\ C_2 = 8$

For series combination, $C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6$

(75) (A).



Let particle have charge q and mass 'm'

Solve for (q, m) mathematically

$F_x = 0$, $a_x = 0$, $(v)_x$ = constant

Time taken to reach at 'P' = $\frac{d}{v_0} = t_0$ (let) ...(1)

(Along –y), $y_0 = 0 + \frac{1}{2}\frac{qE}{m}t_0^2$ ....(2)

$v_x = v_0$

$v = u + at$ (along -ve 'y')

Speed $v_{y_0} = \frac{qE}{m}t_0$

$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{mv_0}, \left(t_0 = \frac{d}{v_0}\right)$

$\tan\theta = \frac{qEd}{mv_0^2}$; Slope $= \frac{-qEd}{mv_0^2}$

Now we have to find eqn of straight line whose slope is $\frac{-qEd}{mv_0^2}$ and it pass through point $\rightarrow (d, -y_0)$

Because after x > d

No electric field $\Rightarrow F_{net} = 0, \vec{v} = \text{const.}$

$y = mx + c, \left\{\begin{matrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix}\right\}$

$-y_0 = \frac{-qEd}{mv_0^2}d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$

Put the value, $y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$

$y_0 = \frac{1}{2}\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEd}{mv_0^2}x + \frac{1}{2}\frac{qEd^2}{mv_0^2} = \frac{qEd^2}{mv_0^2}\left(\frac{d}{2} - x\right)$

(76) **4.** $u_i = \frac{1}{2} \times 5 \times 10^{-6}(220)^2$

Final common potential

$v = \frac{220 \times 5 + 0 \times 2.5}{5 + 2.5} = 220 \times \frac{2}{3}$

$u_f = \frac{1}{2}(5 + 2.5) \times 10^{-6}\left(220 \times \frac{2}{3}\right)^2$

$\Delta u = u_f - u_i = -403.33 \times 10^{-4}$

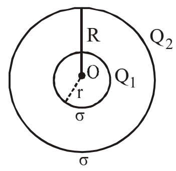
$\Rightarrow -403.33 \times 10^{-4} = \frac{X}{100} \Rightarrow X = -4.03$

or magnitude or value of X is approximate 4.

(77) **(C).** Let the charges on inner and outer spheres are Q_1 and Q_2.

Since charge density 'σ' is same for both spheres, so

$\sigma = \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r^2}{R^2}$



$Q_1 + Q_2 = Q \Rightarrow \frac{Q_2 r^2}{R^2} + Q_2 = Q$

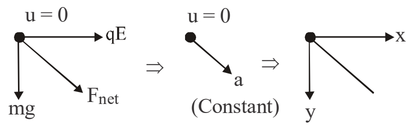
$Q_2 = \frac{Q_2 R^2}{r^2 + R^2}$; $Q_1 = \frac{r^2}{R^2} \cdot \frac{QR^2}{R^2 + r^2} = \frac{Qr^2}{R^2 + r^2}$

Potential at centre 'O' $= \frac{kQ_1}{r} + \frac{kQ_2}{R}$

$= k\left[\frac{Qr^2}{r(R^2 + r^2)} + \frac{QR^2}{R(R^2 + r^2)}\right]$

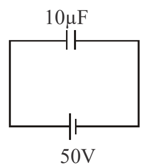
$= \frac{kQ(r + R)}{R^2 + r^2} = \frac{1}{4\pi\varepsilon_0}\frac{R + r}{R^2 + r^2}Q$

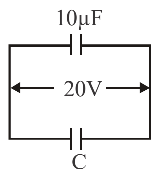
(78) **(D).**



Since initial velocity is zero and acceleration of particle will be constant, so particle will travel on a straight line path.

(79) **(B).** Initially





Charge on capacitor 10 μF

$Q = CV = (10\ \mu F)(50V) = 500\ \mu C$

Final Charge on 10 μF capacitor

$Q = CV = (10\ \mu F)(20V) = 200\ \mu C$

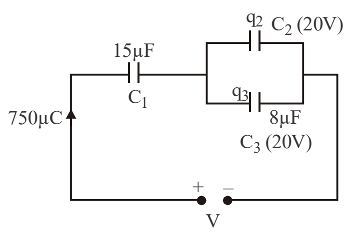
From charge conservation,

Charge on unknown capacitor

$C = 500\ \mu C - 200\ \mu C = 300\ \mu C$

Capacitance, $C = \frac{Q}{V} = \frac{300\ \mu C}{20V} = 15\ \mu F$

(80) **(A).**



$q_3 = 20 \times 8 = 160\ \mu C$

$q_2 = 750 - 160 = 590\ \mu C$

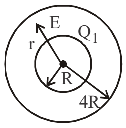
(81) **(A).** $Q_1 + Q_2 = Q'_1 + Q'_2 = 12\mu C - 3\mu C = 9\mu C$

$V_1 = V_2 \Rightarrow \frac{KQ_1'}{2R/3} = \frac{KQ_2'}{R/3}$

$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$

$\Rightarrow Q'_2 = 3\ \mu C\ \ \&\ \ Q'_1 = 6\ \mu C$

(82) **(A).** $E = \frac{KQ_1}{r^2}$; $\Delta V = \int_R^{4R} E\, dr = \frac{3KQ_1}{4R}$



(83) **(C).** Potential of –q is same as initial and final point of the path therefore potential due to 4q will only change and as potential is decreasing the energy will decrease

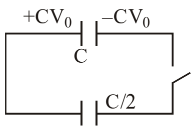
Decrease in potential energy = q $(V_i - V_f)$

Decrease in potential energy

$$= q\left[\frac{k4q}{d/2} - \frac{k4q}{3d/2}\right] = \frac{4q^2}{3\pi\varepsilon_0 d}$$

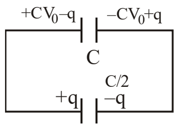
(84) **(A).** Thin infinite uniformly charged planes produces uniform electric field therefore option 2 and option 3 are obviously wrong. And as positive charge density is bigger in magnitude so its field along Y direction will be bigger than field of negative charge in X direction and this is evident in option 1 so it is correct.

(85) **(A).** $\frac{CV_0 - q}{C} = \frac{q}{C/2} = \frac{2q}{C}$



$V_0 = \frac{3q}{C} \Rightarrow q = \frac{CV_0}{3}$

$U_i = \frac{1}{2}CV_0^2$



$$U_f = \frac{\left(\frac{2CV_0}{3}\right)^2}{2C} + \frac{\left(\frac{CV_0}{3}\right)^2}{2\,(C/2)}$$

$$= \frac{1}{2}CV_0^2\left[\frac{4}{9} + \frac{2}{9}\right] = \frac{1}{2}CV_0^2\left(\frac{2}{3}\right)$$

$$\text{Heat loss} = \frac{1}{2}CV_0^2 - \left(\frac{2}{3}\right)\left(\frac{1}{2}CV_0^2\right) = \frac{1}{6}CV_0^2$$

(86) **(D).** $E = E_0\,(1 - ax^2)$

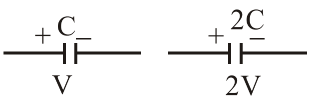
$$W = \int qE\,dx = qE_0 \int_0^{x_0} (1 - ax^2)\,dx = qE_0\left[x_0 - \frac{ax_0^3}{3}\right]$$

For Δ KE = 0, W = 0. Hence, $x_0 = \sqrt{3/a}$

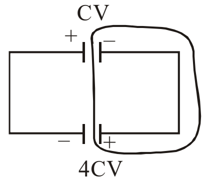
(87) **(A).** $\frac{kQq}{R} + mgy = \frac{kQq}{R+y} + \frac{1}{2}mv^2$

$$v^2 = 2gy + \frac{2kQqy}{mR\,(R+y)}$$

(88) **(D).**



$Q_1 = CV$ $\quad Q_2 = 2C \times 2V = 4CV$

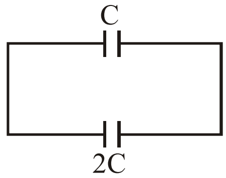


By conservation of charge,

$q_i = q_f$

$Q_1 + Q_2 = q_1 + q_2$

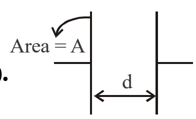
$4CV - CV = (C + 2C)\ V_C$

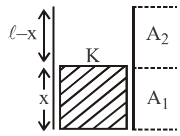


$V_C = \frac{3CV}{3C} = V$

$\frac{1}{2} \times (3C) \times V_C^2 = \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}CV^2$

(89) **(C).**





Before inserting slab	After inserting dielectric slab
$C_i = \frac{\varepsilon_0 A}{d}$	$C_f = C_1 + C_2$
$C_i = \frac{\varepsilon_0 \ell w}{d}$	$C_f = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$
	$C_f = \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d}$

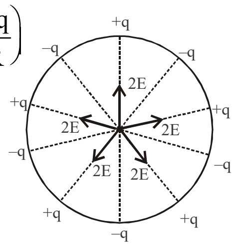
$C_f = 2C_i$

$\Rightarrow \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d} = \frac{2\varepsilon_0 \ell w}{d}$

$4x + \ell - x = 2\ell$

$x = \frac{\ell}{3}$

(90) **(C).** Potential of centre $V = \Sigma\left(\frac{kq}{R}\right)$

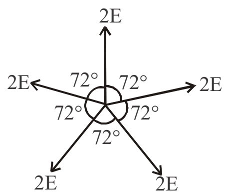


$V_C = \frac{K\,(\Sigma q)}{R} = \frac{K\,(0)}{R} = 0$

Electric field at centre

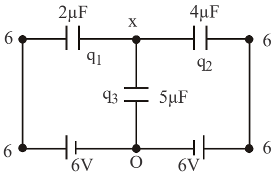
$\vec{E}_B = \Sigma\,\vec{E}$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C = 0$.



Since equal electric field vectors are acting at equal angle so their resultant is equal to zero.

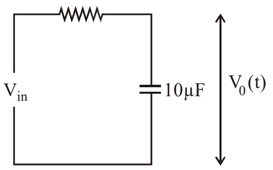
(91) (B).



Let potential of point O : $v_0 = 0$
Now, using junction analysis
We can say, $q_1 + q_2 + q_3 = 0$
$2(x-6)+4(x-6)+5(x)=0$

$$x = \frac{36}{11},\ q_3 = \frac{36(5)}{11} = \frac{180}{11} = 16.36\ \mu C$$

(92) (A).

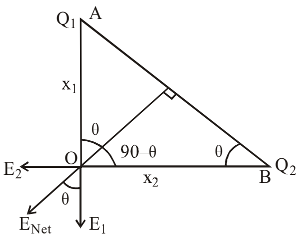


$V_0(t) = V_{in}(1-e^{-t/RC})$
At t = 5μs

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right) = 5(1-e^{-0.5}) = 2V$$

Now $V_{in} = 0$ means discharging
$V_0(t) = 2e^{-t/RC} = 2e^{-0.5} = 1.21$ V
Now for next 5 μs, $V_0(t) = 5 - 3.79e^{-t/RC}$
After 5 μs again, $V_0(t) = 2.79$ Volt $\approx$ 3V
Most approperiate Ans. (A)

(93) (C).

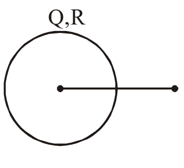


$$E_2 = \text{electric field due to } Q_2 = \frac{kQ_2}{x_2^2}\ ;\ E_1 = \frac{kQ_1}{x_1^2}$$

From diagram, $\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$

$$\frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}\ ;\ \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2}\ ;\ \frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

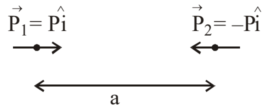
(94) (A). Inside the shell E = 0
hence F = 0
Oustside the shell



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\ \text{for } r > R$$

(95) (C). Using energy conservation:
$KE_i + PE_i = KE_f + PE_f$



$$0 + \frac{2KP}{a^3}\times P = \frac{1}{2}mv^2 \times 2 + 0$$

$$V = \sqrt{\frac{2P^2}{4\pi\varepsilon_0 a^3 m}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 am}}$$

EXERCISE-5

(1) (A). We know that potential energy of two charge system is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

According to question,

$$U_A = \frac{1}{4\pi\varepsilon_0}\frac{(+q)(-Q)}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}$$

and $U_B = \frac{1}{4\pi\varepsilon_0}\frac{(+q)(-Q)}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}$

$\Delta U = U_B - U_A = 0$
We know that for conservative force
$W = -\Delta U = 0$

(2) (C). We know that potential energy of discrete system of charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right)$$

According to question,

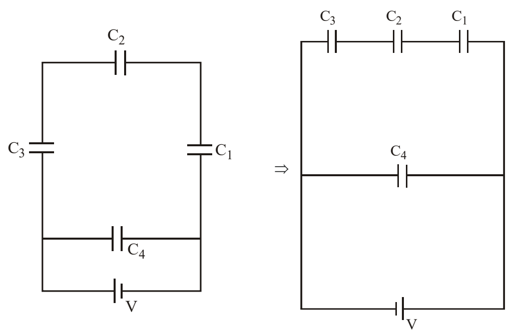
$$U_{initial} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{0.3} + \frac{q_2q_3}{0.5} + \frac{q_3q_1}{0.4}\right)$$

$$U_{final} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{0.3} + \frac{q_2q_3}{0.1} + \frac{q_3q_1}{0.4}\right)$$

$$U_{final} - U_{initial} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{0.1} - \frac{q_2q_3}{0.5}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}[10q_2q_3 - 2q_2q_3] = \frac{q_3}{4\pi\varepsilon_0}(8q_2)$$

(3) (B).



Equivalent capacitance for three capacitors (C_1, C_2 & C_3) in series is given by

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_2C_3+C_3C_1+C_1C_2}{C_1C_2C_3}$$

$$C_{eq}=\frac{C_1C_2C_3}{C_1C_2+C_2C_3+C_3C_1}$$

$$C_{eq}=\frac{C_1C_2C_3}{C(2C)+(2C)(3C)+(3C)C}=\frac{6}{11}C$$

Charge on capacitors (C_1, C_2 & C_3) in series

$$=C_{eq}V=\frac{6C}{11}$$

Charge on capacitor $C_4=C_4V=4CV$

$$\frac{\text{Charge on } C_2}{\text{Charge on } C_4}=\frac{\frac{6C}{11}V}{4CV}=\frac{6}{11}\times\frac{1}{4}=\frac{3}{22}$$

(4) **(B).** Flux $=\vec{E}.\vec{A}$

$\vec{E}$ is electric field vector and $\vec{A}$ is area vector. Here, angle between $\vec{E}$ & $\vec{A}$ is 90°.

(5) **(C).** Work done in rotating a dipole = pE (1 – cos θ)
If θ = 90°, work done = pE (1 – 0) = pE

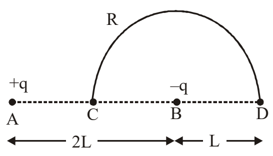
(6) **(C).** If we increase the distance between the plates its capacity decreases resulting in higher potential as we know Q = CV.
Since Q is constant (battery has been disconnected), on decreasing C, V will increase.

(7) **(D).** Since $\phi_{total}=\phi_A+\phi_B+\phi_C=\frac{q}{\varepsilon_0}$

where q is the total charge.
Flux associated with the curved surface B is $\phi=\phi_B$.
Let us assume flux linked with the plane surfaces A and C be $\phi_A=\phi_C=\phi'$.

$$\frac{q}{\varepsilon_0}=2\phi'+\phi_B=2\phi'+\phi \Rightarrow \phi'=\frac{1}{2}\left(\frac{q}{\varepsilon_0}-\phi\right)$$

(8) **(C).** Potential at C = $V_C=0$
Potential at D = V_D



$$K\left(\frac{-q}{L}\right)+\frac{Kq}{3L}=-\frac{2}{3}\frac{Kq}{L}$$

Potential difference

$$V_D-V_C=\frac{-2}{3}\frac{Kq}{L}=\frac{1}{4\pi\varepsilon_0}\left(-\frac{2}{3}\frac{q}{L}\right)$$

Work done = Q (V_D-V_C)

$$=-\frac{2}{3}\times\frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}=\frac{-qQ}{6\pi\varepsilon_0 L}$$

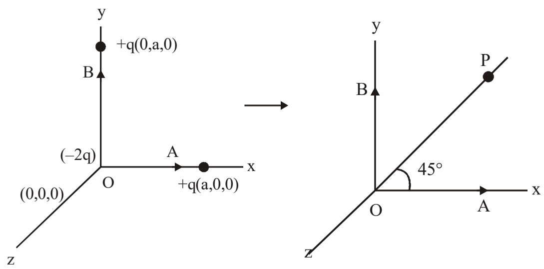
(9) **(A).** Three point charges +q, –2q and +q are placed at points B (x = 0, y = a, z = 0), O (x = 0, y = 0, z = 0) and A (x = a, y = 0, z = 0).

The system consists of two dipole moment vectors due to (+q and –q) and again due to (+q and –q) charges having equal magnitudes q units – one along $\overrightarrow{OA}$ and other along $\overrightarrow{OB}$.

Hence, net dipole moment,

$$p_{net}=\sqrt{(qa)^2+(qa)^2}=\sqrt{2}qa$$

along $\overrightarrow{OP}$ at an angle 45° with positive X-axis.



(10) **(B).** Work done = Change in energy

$$=\frac{1}{2}\left(C+\frac{C}{2}\right)V^2=\frac{1}{2}\left(\frac{3C}{2}\right)V^2=\frac{3CV^2}{4}$$

(11) **(D).** Energy of charged capacitor $=\frac{1}{2}CV^2$

Energy given by cell = CV^2

$$=\frac{A\varepsilon_0}{d}\times(Ed)^2=\varepsilon_0E^2Ad$$

(12) **(C).** $\vec{E}_{total}=0$; $\vec{E}_{AKB}+\vec{E}_{ACDB}=0$

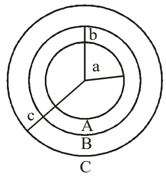
$\vec{E}_{ACDB}=-\vec{E}_{AKB}$; $\vec{E}_{ACDB}=E$ along OK

(13) **(B).** $V=\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{r}$; $E=\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{r^2}$

$$E=\frac{4\pi\varepsilon_0V^2}{Q}=4\pi\varepsilon_0\times\frac{Q^2\times10^{22}}{Q}$$

$E=4\pi\varepsilon_0Q\times10^{22}$ volt/m

(14) **(D).**



c = a + b

$$q_A=\sigma\,4\pi a^2,\ q_B=-\sigma\,4\pi b^2,\ q_C=\sigma\,4\pi c^2$$

$$V_A=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right];\ V_A=\frac{\sigma}{\varepsilon_0}[a-b+c]$$

$$V_B=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b}+\frac{q_B}{b}+\frac{q_C}{c}\right];\ V_B=\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c}+\frac{q_B}{c}+\frac{q_C}{c}\right]$$

$$V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2+c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[c-\frac{(b^2-a^2)}{c}\right] = \frac{\sigma}{\varepsilon_0}[c-(b-a)] = \frac{\sigma}{\varepsilon_0}[a-b+c]$$

$V_A = V_C \neq V_B$

(15) **(B).** $Q = CV$; $V_{eff} = V+V+V = 3V$

$$\frac{1}{C_{eff}} = \frac{1}{C}+\frac{1}{C}+\frac{1}{C} \Rightarrow C_{eff} = \frac{C}{3} \;;\; \left(\frac{C}{3}, 3V\right)$$

(16) **(D).** $\vec{E} = -\frac{\partial V}{\partial r} = \left[-\frac{\partial V}{\partial r}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}\right]$

$$= [(2xy+z^3)\,\hat{i} + \hat{j}\,x^2 + \hat{k}3xz^2]$$

(17) **(C).** $F = \frac{1}{4\pi\varepsilon_0}\frac{(q)\,(q)}{d^2} = \frac{q^2}{4\pi\varepsilon_0 d^2}$

$q^2 = 4\pi\varepsilon_0 F d^2$ ...(i)

Since, q = ne, where, n = number of electrons missing from each ion, e = magnitude of charge on electron.

$$\therefore \quad n = \frac{q}{e} = \frac{\sqrt{4\pi\varepsilon_0 F d^2}}{e} = \sqrt{\frac{4\pi\varepsilon_0 F d^2}{e^2}}$$

(18) **(D).** $\phi = E.A\cos\theta$; $\phi = \vec{E}.\vec{A} = 0$

lines are parallel to the surfaces

(19) **(D).** Total capacitance (series combination) $C_s = \frac{C_1}{n_1}$

Total energy (series combination)

$$u_s = \frac{1}{2}C_s(4V)^2 = \frac{1}{2}\left(\frac{C_1}{n_1}\right)(4V)^2$$

Total capacitance (parallel combination)

$C_p = C_2 + C_2 + \ldots\ldots + \text{upto } n_2$ terms

$= n_2C_2$

Total energy (parallel combination)

$$u_p = \frac{1}{2}C_pV^2 = \frac{1}{2}(n_2C_2)\,(V)^2$$

$$U_s = U_p \;;\; \frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}(n_2C_2)\,(V)^2$$

or $C_2 = \frac{16C_1}{n_1n_2}$

(20) **(C).** In a medium of dielectric constant K, $E' = \frac{\sigma}{\varepsilon_0 K}$

(21) **(A).** Electric field insided charged conductor is always zero.

(22) **(D).** $\phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$, $Q_{enclosed}$ remains unchanged

(23) **(D).** $V = \frac{2q}{4\pi\varepsilon_0 L} - \frac{2q}{4\pi\varepsilon_0 L\sqrt{5}} = \frac{2q}{4\pi\varepsilon_0 L}\left(1-\frac{1}{\sqrt{5}}\right)$ volt

(24) **(A).** Energy density = $\frac{1}{2}\varepsilon_0E^2$ × volume

(25) **(D).** $\vec{E} = -\frac{dV}{dx}\hat{i} = -8x\hat{i}$ volt/meter

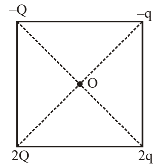
$$\vec{E}_{(1,\,0,\,2)} = -8\hat{i}\ \text{V/m}$$

(26) **(C).** AC = BC ; $V_D = V_E$; $W = Q\,(V_E - V_D)$

(27) **(A).** $\tau = PE\sin\theta$

$U = -PE\cos\theta$

(28) **(A).** Let the side length of square be 'a' then potential at centre O is



$$V = \frac{k\,(-Q)}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{k\,(-q)}{\frac{a}{\sqrt{2}}} + \frac{k\,(2q)}{\frac{a}{\sqrt{2}}} + \frac{k\,(2Q)}{\frac{a}{\sqrt{2}}} = 0$$

$= -Q - q + 2q + 2Q = 0 = Q + q = 0 = Q = -q$

(29) **(B).** Eight identical cubes are required to arrange so that this charge is at centre of the cube formed so flux. $\phi = q/8\varepsilon_0$

(30) **(C).** $U = \frac{1}{2}CV^2$; $U = \frac{1}{2}\frac{A\varepsilon_0}{d}(Ed)^2 = \frac{1}{2}A\varepsilon_0E^2d$

(31) **(B).** At equilibrium potential of both sphere becomes same

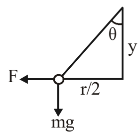
$$\frac{kx}{1\text{ cm}} = \frac{k\,(Q-x)}{3\text{ cm}} \;;\; 3x = Q - x \;;\; 4x = Q$$

$$x = \frac{Q}{4} = \frac{4\times10^{-2}}{4}C = 1\times10^{-2}$$

$Q' = Q - x = 3\times10^{-2}$ C

(32) **(C).** Electric potential dec. in the direction of electric field.

(33) **(C).**



$$\tan\theta = \frac{F}{mg} \Rightarrow \frac{r/2}{y} = \frac{kq^2}{r^2mg} \Rightarrow y \propto r^3$$

$$\left(\frac{r'}{r}\right)^3 = -\frac{y/2}{y} \Rightarrow r' = r\left(\frac{1}{2}\right)^{1/3}$$

(34) **(C).** Electric field inside parallel plate capacitor having charge Q at place where dielectric is absent = $\frac{Q}{A\varepsilon_0}$

where dielectric is present = $\frac{Q}{KA\varepsilon_0}$

(35) **(B).** Electric potential, $V = \frac{Q}{4\pi\varepsilon_0 R}$; Electric field E = 0.

(36) **(D).** $V = 6x - 8xy - 8y + 6yz$

$$E_x = -\frac{\partial V}{\partial x} = -(6-8y) = 2$$

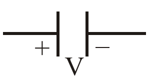
$$E_y = -\frac{\partial V}{\partial y} = -(8x-8+6z) = 10$$

$$E_z = -\frac{\partial V}{\partial z} = -6y = -6$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$= \sqrt{4+100+36} = \sqrt{140} = 2\sqrt{35}\ \text{N/C}$$

$$F = qE = 4\sqrt{35}\ \text{N}$$

(37) **(C).** 

$U_i = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$; $Q = CV$

$$U_f = \frac{Q^2}{2C} = \frac{Q^2}{2KC} = \frac{C^2V^2}{2KC} = \frac{U_i}{K}$$

$$\Delta U = U_f - U_i = \frac{1}{2}CV^2\left\{\frac{1}{K} - 1\right\}$$

As the capacitor is isolated, so charge will remain conserved. $PD = \frac{Q}{C_f} = \frac{Q}{KC} = \frac{V}{K}$

(38) **(B).** Net flux emmited from a spherical surface of radius a is $\phi_{net} = \frac{q_{in}}{\varepsilon_0}$; $(Aa)(4\pi a^2) = \frac{q_{in}}{\varepsilon_0}$

So, $q_{in} = 4\pi\varepsilon_0 Aa^3$

(39) **(C).** $F = \frac{Q^2}{2\varepsilon_0 A}$; $Q = CV$ and $C = \frac{\varepsilon_0 A}{d} \Rightarrow \varepsilon_0 A = Cd$

So, $F = \frac{C^2V^2}{2Cd} = \frac{CV^2}{2d}$

(40) **(C).** $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$\vec{E} = -(6y)\,\hat{i} - (6x - 1 + 2z)\,\hat{j} - (2y)\,\hat{k}$$

At point (1, 1, 0)

$$\vec{E} = -6\hat{i} - 5\hat{j} - 2\hat{k} = -(6\hat{i} + 5\hat{j} + 2\hat{k})$$

(41) **(D).** Initial energy stored $= \frac{1}{2}(2\mu F) \times V^2$

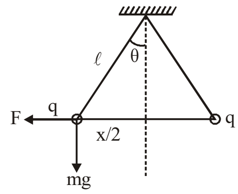
Energy dissipated on connection across $8\mu F$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}V^2 = \frac{1}{2} \times \frac{2\mu F \times 8\mu F}{10\mu F} \times V^2$$

$$= \frac{1}{2} \times (1.6\mu F)\ V^2$$

% loss of energy $= \frac{1.6}{2} \times 100 = 80\%$

(42) **(C).** $\frac{F}{mg} = \tan\theta$



$$\frac{Kq^2}{x^2mg} = \frac{\frac{x}{2}}{\sqrt{\ell^2 - \frac{x^2}{4}}}$$

$$\frac{Kq^2}{x^2mg} = \frac{x}{2\ell}\ ;\ q^2 \propto x^3 \Rightarrow q^2 \propto x^{3/2}$$

$$\Rightarrow \frac{dq}{dx} \propto \frac{d\,(x^{3/2})}{dx}\frac{dx}{dt} \quad \Rightarrow \frac{dq}{dx} \propto x^{1/2}v \Rightarrow v \propto \frac{1}{x^{1/2}}$$

(43) **(B).** $\tau = PE\sin\theta \Rightarrow \tau = qdE\sin\theta$

$$\Rightarrow q = \frac{\tau}{dE\sin\theta} = \frac{4}{2\times10^{-2}\times0.5\times2\times10^5} = 2mC$$

(44) **(C).** k_1, k_2 and k_3 are in parallel so Arithmetic mean.

$$k_{eq} = \frac{k_1 + k_2 + k_3}{3}$$

k_{eq} is in series with k_4. So harmonic mean

$$\Rightarrow \frac{2}{k} = \frac{1}{k_{eq}} + \frac{1}{k_4} \Rightarrow \frac{2}{k} = \frac{3}{k_1 + k_2 + k_3} + \frac{1}{k_4}$$

(45) **(D).** In parallel $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{K_{eq}(2A)}{\ell} = \frac{K_1A}{\ell} + \frac{K_2A}{\ell};\ K_{eq} = \frac{K_1 + K_2}{2}$$

(46) **(A).** $U_i = \frac{1}{2}CV^2$; $U_f = \frac{1}{2}[2C]\left[\frac{V}{2}\right]^2 = \frac{1}{2}U_i$

Decrease by a factor of 2.

(47) **(B).** $\frac{K \times (\Delta e)^2}{r^2} = \frac{GM^2}{r^2}$

$$\Delta e = m\sqrt{\frac{G}{K}} = 1.67\times10^{-27}\sqrt{\frac{6.67\times10^{-11}}{9\times10^9}}C$$

$$= 1.436 \times 10^{-37}\ C$$

(48) **(A).** $W = q\Delta V$

As ΔV is same in all conditions, work will be same.

(49) **(C).** $H = \frac{1}{2}\frac{eE}{m}t^2 \quad \therefore\ t = \sqrt{\frac{2hm}{eE}}$

$\therefore\ t \propto \sqrt{m}$ as ‘e’ is same for electron and proton.

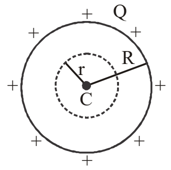
$\because$ Electron has smaller mass so it will take smaller time.

(50) **(C).** For isolated capacitor Q = Constant

$$F_{plate} = \frac{Q^2}{2A\varepsilon_0}$$

F is Independent of the distance between plates.

(51) (B).



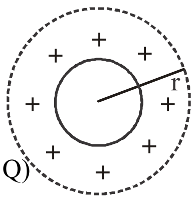
Charge Q will be distributed over the surfaceof hollow metal sphere.

(i) For r < R (inside)

By Gauss law, $\oint \vec{E}_{in} \cdot \overrightarrow{dS} = \frac{q_{en}}{\varepsilon_0} = 0$

$E_{in} = 0 \quad (\because q_{en} = 0)$

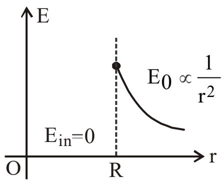
(ii) For r > R (outside)



$\oint \vec{E}_0 \cdot \overrightarrow{dS} = \frac{q_{en}}{\varepsilon_0}$

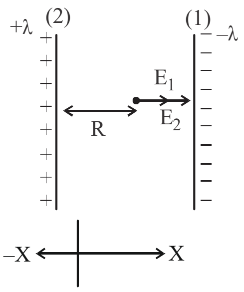
Here, $q_{en} = Q \quad (\because q_{en} = Q)$

$\therefore \quad E_0 \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \therefore E_0 \propto 1/r^2$



(52) (C). Electric field due to line charge 1

$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$ N / C



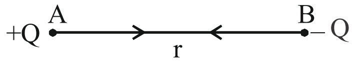
Electric field due to line charge 2

$\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$ N / C

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{i} + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$

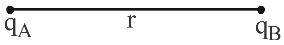
$= \frac{\lambda}{\pi\varepsilon_0 R}\hat{i}$ N / C

(53) (B). $F = \frac{kQ^2}{r^2}$



If 25% of charge of A transferred to B then

$q_A = Q - \frac{Q}{4} = \frac{3Q}{4}$ and $q_B = -Q + \frac{Q}{4} = \frac{-3Q}{4}$



$F_1 = \frac{kq_Aq_B}{r^2}$; $F_1 = \frac{k(3Q/4)^2}{r^2} = \frac{9}{16}\frac{kQ}{r^2} = \frac{9F}{16}$

(54) (B). Capacitance of capacitor C = 20 μF = 20 × 10^{-6} F

Rate of change of potential (dV/dt) = 3 v/s

q = CV

$\frac{dq}{dt} = C\frac{dV}{dt}$

$i_c = 20 \times 10^{-6} \times 3 = 60 \times 10^{-6}$ A = 60 μA

As we know that $i_d = i_c$ = 60 μA

(55) (C). $V = \frac{kP\cos\theta}{r^2} = \frac{9\times10^9 \times 16\times10^{-9}}{(0.6)^2} \times \frac{1}{2} = 200\,V$

(56) (B). Potential is constant throughout the volume. Electric field is zero.

(57) (C). $E = \frac{kQ}{r^2} = \frac{9\times10^9 \times 3.2\times10^{-7}}{(15\times10^{-2})^2} = 1.28\times10^5$ N/C

(58) (D). $C_m = \varepsilon_r C_0$; $\varepsilon_r = \frac{30}{6} = 5$

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r = 8.85 \times 10^{-12} \times 5 = 0.44 \times 10^{-10}$