CLASS - XI

PHYSICS MODULE -1

A BIRD SITTING ON A TREE IS NEVER AFRAID OF THE BRANCH BREAKING, BECAUSE HER TRUST IS NOT ON THE BRANCH BUT ON IT'S OWN WINGS.

ALWAYS BELIEVE IN YOURSELF.

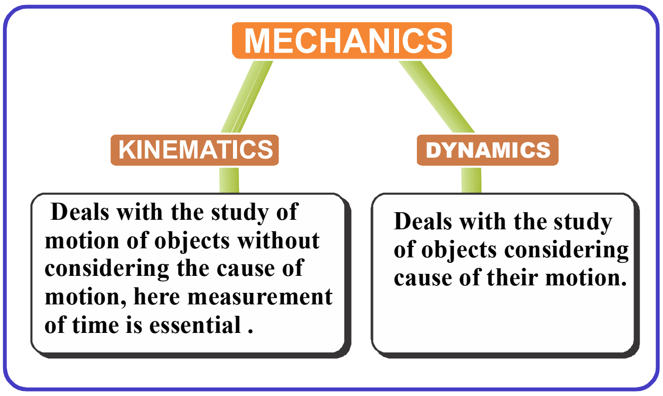
XI / 3 MOTION IN A STRAIGHT LINE

LEARNING OBJECTIVES

1. Frame of Reference
2. Distance and Displacement.
3. Speed and Velocity.
4. Average Speed and Instantaneous Velocity.
5. Uniform and Non-uniform Motion.
6. Uniformly Accelerated Motion, Velocity-Time and Position-Time Graphs.
7. For Uniformly Accelerated Motion (Graphical Treatment).
8. Relative Velocity.

3.1 INTRODUCTION

* Motion is the most fundamental observation about nature at large.
* To describe motion we require terms like time interval, distance, displacement, speed, velocity and acceleration.
* To study the motion, branch of physics called Mechanics is defined.



* Generally motion we observe in practical life are 2 or 3-dimensional to analyse them we have to break them into single dimension. Hence, we need to study one dimension motion.
* **Concept of point object :** An object is said to be a point object if its dimensions (i.e. length, breadth, thickness etc.) are negligible as compared to the distance travelled by it. For example, an aeroplane, which flies from Delhi to London, a train moving over long distances etc. All these objects i.e. an aeroplane, a train etc. are treated as point objects as their size or dimensions are negligible as compared to the distance travelled by them.
* **Rest and Motion are relative terms :** When we say that an object is at rest or in motion, then this statement is incomplete and meaningless. Basically, rest and motion are relative terms. An object which is at rest can also be in motion simultaneously. For example, the passengers sitting in a moving bus are at rest with respect to each other but they are also in motion at the same time with respect to the objects like trees, buildings on the road side. So the motion and rest are relative terms.
* **Frame of Reference :** To locate the position of a particle we need a reference frame. A commonly used reference frame is cartesian coordinate system or simply coordinate system. The coordinates (x, y, z) of a particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes, it means the particle is at rest w.r.t. this frame. The reference frame is chosen according to the problems.

NOTE

* We will consider all object as point object for considering one dimensional motion. We will also neglect air resistance if not specified. In analysing any motion consider time as time interval i.e. think initial and final situation according to time interval in which you have to solve the problem.

3.2 DISTANCE AND DISPLACEMENT

Distance

* The length of the actual path between initial and final positions of a particle in a given time interval is called distance covered by the particle.

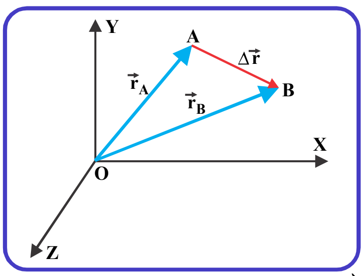
Characteristics of Distance

* It is a scalar quantity.
* It depends on the path
* It never reduces with time.
* Distance covered by a particle is always positive and can never be negative or zero.

Displacement

The shortest distance from the initial position to the final position of the particle is called displacement. Position vector of A w.r.t. O = $\overrightarrow{OA}$

$\Rightarrow \quad \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$



Position Vector of B w.r.t. O = $\overrightarrow{OB}$

$\Rightarrow \quad \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Displacement

$= \overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

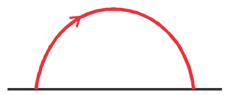
$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

Characteristics of Displacement

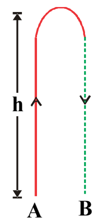
* It is a vector quantity.
* The displacement of a particle between any two points is equal to the shortest distance between them.
* The displacement of an object in a given time interval may be +ve, –ve or zero.
* The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e. Distance ≥ | Displacement |

Golden Tips

* For motion between two points displacement is single-valued while distance depends on actual path and so can have many values.
* For a moving particle distance can never decrease with time while displacement can.
* For a moving particle distance can never be negative or zero while displacement can be.
* An object is said to be in uniform motion in a straight line if its displacement is equal in equal interval of time, however small the interval may be. Otherwise the motion is said to be non-uniform.
* Distance v/s Displacement



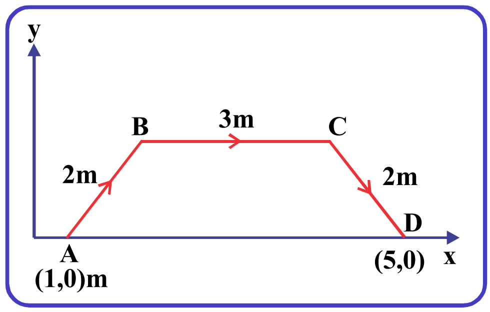
Distance = πr
| Displacement | = 2r



Distance = 2h
| Displacement | = 0

EXAMPLE 1

Suppose a particle moves from position A to B as shown after travelling from A to B to C to D.Find distance and displacement.



SOLUTION:

Displacement $\vec{S} = \overrightarrow{AD} = 5\hat{i} - \hat{i} = 4\hat{i}$ m

$\therefore |\text{Displacement}| = 4\text{m}$

Distance covered

$\ell = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}| = 2 + 3 + 2 = 7$ m

EXAMPLE 2

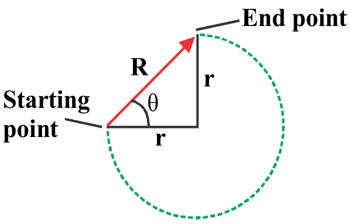
One afternoon, a couple walks three-fourths of the way around a circular lake, the radius of which is 1.50 km. They start at the west side of the lake and head due south to begin with. (a) What is the distance they travel? (b) What are the magnitude and direction (relative to due east) of the couple's displacement?

SOLUTION:

(a) The distance traveled is equal to three-fourths of the circumference of the circular lake. The circumference of a circle is $2\pi r$, where r is the radius of the circle. Thus, the distance d that the couple travels is

$$d = \frac{3}{4}(2\pi r) = \frac{3}{4}[2\pi\,(1.50\text{ km})] = 7.07\text{ km}$$

(b) The couple's displacement is the hypotenuse of a right triangle with sides equal to the radius of the circle (see the drawing).



The magnitude R of the displacement can be obtained with the aid of the Pythagorean theorem:

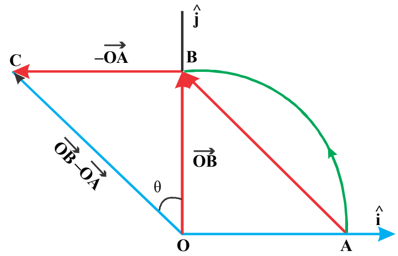
$$R = \sqrt{r^2 + r^2} = \sqrt{2\,(1.50\text{ km})^2} = 2.12\text{ km}$$

The angle θ that the displacement makes with due east is

$\theta = \tan^{-1}(r/r) = \tan^{-1}(1) = 45.0°$ north of east

EXAMPLE 3

A particle goes along a quadrant from A to B is a circle radius 10m as shown in figure. Find the direction and magnitude of displacement and distance along path AB.



SOLUTION:

Displacement $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\hat{j} - 10\hat{i}$

$|\overrightarrow{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}\text{m}$

From Δ OBC,

$$\tan\theta = \frac{OA}{OB} = \frac{10}{10} = 1 \Rightarrow \theta = 45°$$

Angle between displacement vector $\overline{OC}$ and x-axis $= 90° + 45° = 135°$

Distance of path AB

$$= \frac{1}{4}(\text{Circumference}) = \frac{1}{4}(2\pi R)\text{ m} = (5\pi)\text{ m}$$

3.3 SPEED

* It is measured by the distance travelled by the object in unit time in any direction. i.e.,

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

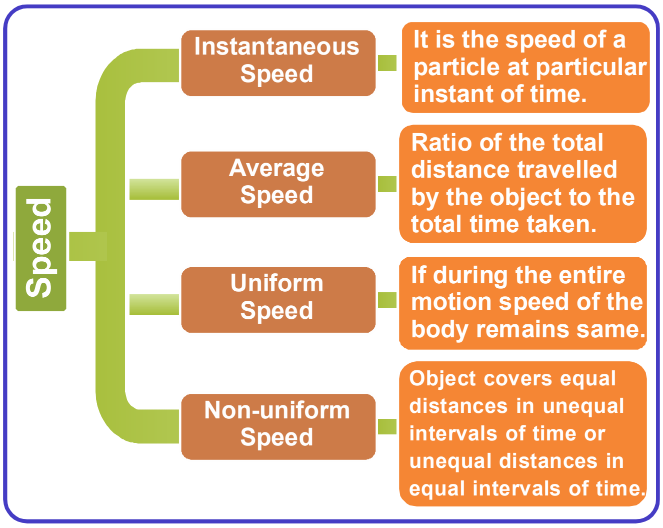
Characteristics of Speed

* It is a scalar quantity
* It gives no idea about the direction of motion of the object.
* It can be zero or positive but never negative.
* Unit : C.G.S cm/sec, S.I. m/sec,

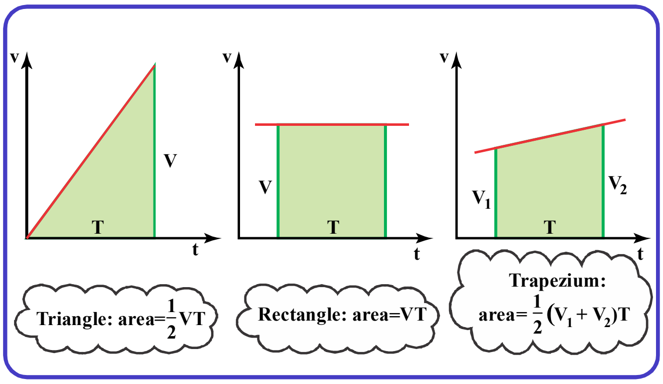
$$1\text{ km/h} = \frac{1000}{60 \times 60} = \frac{5}{18}\text{ m/s}$$

* Instantaneous speed $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

* Average speed, $\bar{V} = \frac{\text{total distance travelled}}{\text{total time taken}}$



* The area between a speed–time graph and the time axis represents the distance travelled. Many of these graphs consist of straight-line sections. The area is easily found by splitting it up into triangles, rectangles or trapezia.



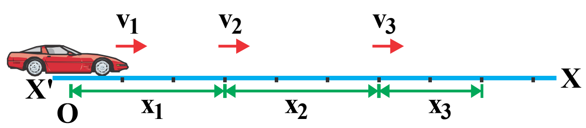
Golden Tips

* If any object covers distance x_1, x_2,...... in the time intervals t_1, t_2,....... then.

$$\bar{V} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{t_1 + t_2 + \ldots\ldots\ldots + t_n}$$

* **Important cases related to average speed :**

Case : 1



If car covers distances x_1, x_2, and x_3 with speeds v_1, v_2, and v_3 respectively in same direction then average speed of car.

$$\bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3}$$

$$t_1 = \frac{x_1}{v_1},\ t_2 = \frac{x_2}{v_2},\ t_3 = \frac{x_3}{v_3}$$

$$\bar{V} = \frac{x_1 + x_2 + x_3}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3}}$$

If car covers equal distances with different speeds then, $x_1 = x_2 = x_3 = x$

$$\bar{V} = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}$$

$$= \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Case : 2 : If any body travels with speeds v_1, v_2, v_3 during time intervals t_1, t_2, t_3 respectively then the average speed of the body will be.

Average speed

$$\bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} = \frac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3}$$

If $t_1 = t_2 = t_3 = t$

$$= \frac{(v_1 + v_2 + v_3) \times t}{3 \times t} = \frac{(v_1 + v_2 + v_3)}{3}$$

* Speedometer of the vehicle measures its instantaneous speed.

* In case of a uniform motion of an object, the instantaneous speed and average speed is equal to its uniform speed.

EXAMPLE 4

A train 150 m long is moving with a speed of 90km/h. In what time shall it cross a bridge 850m long ?

SOLUTION:

Total distance to be covered

= 850 + 150 = 1000 m

Speed = 90 km/h = 90 × (5/18) m/s = 25 m/s

$$\text{Now, time} = \frac{1000}{25}\ s = 40\ s$$

EXAMPLE 5

The distance travelled by a particle $x = 10t^2$ (m). Find the value of instantaneous speed at t = 2 sec

SOLUTION:

$$v = \frac{dx}{dt} = \frac{d}{dt}(10t^2) = 10\,(2t) = 20\,t$$

Put t = 2 sec.

$v = 20 \times 2 = 40$ m/s.

EXAMPLE 6

If the speed of a particle is $v = 10\,t^2$ m/s. Then find out covered distance from t = 2s to t = 5s.

SOLUTION:

$$s = \int v\,dt = \int_2^5 10\,t^2 dt = 10\int_2^5 t^2\,dt$$

$$= \frac{10}{3}(t^3)_2^5 = \frac{10}{3}(5^3 - 2^3) = 390\text{m}$$

EXAMPLE 7

A woman and her dog are out for a morning run to the river, which is located 4.0 km away. The woman runs at 2.5 m/s in a straight line. The dog is unleashed and runs back and forth at 4.5 m/s between his owner and the river, until the woman reaches the river. What is the total distance run by the dog?

SOLUTION:

The time required for the woman to reach the water is $\text{Elapsed time} = \frac{d_{woman}}{v_{woman}}$

$$= \left(\frac{4.0\text{ km}}{2.5\text{ m/s}}\right)\left(\frac{1000\text{ m}}{1.0\text{ km}}\right) = 1600\text{ s}$$

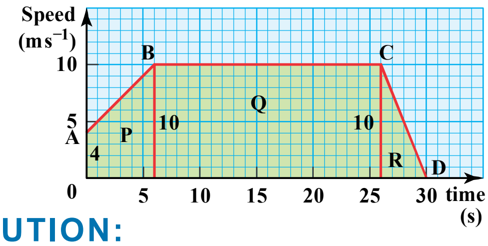
In 1600 s, the dog travels a total distance of

$d_{dog} = v_{dog}\,t = (4.5\text{ m/s})\,(1600\text{ s})$

$= 7.2 \times 10^3$ m

EXAMPLE 8

The graph shows Hinesh's journey from the time he turns on to the main road until he arrives home. How far does Hinesh cycle?



SOLUTION:

Split the area under the speed–time graph into three regions.

P - Trapezium : Area $= \frac{1}{2}(4 + 10) \times 6 = 42$ m

Q - Rectangle : Area $= 10 \times 20 = 200$ m

R - Triangle : Area $= \frac{1}{2} \times 10 \times 4 = 20$ m

Total area = 262 m

Hinesh cycles 262 m.

3.4 VELOCITY

* The rate of change of displacement of a particle with time is called the velocity of the particle.

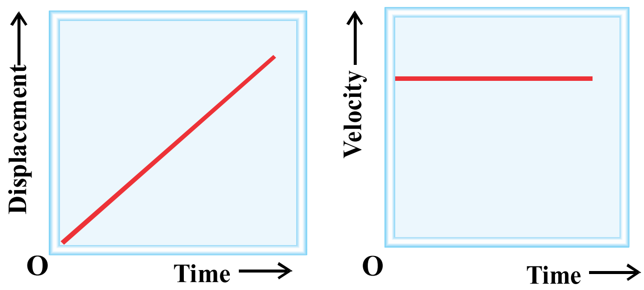
i.e. $\text{Velocity} = \frac{\text{Displacement}}{\text{Time interval}}$

* It is a vector quantity.
* The velocity of an object can be positive, zero and negative.
* The area between a velocity–time graph and the time axis represents the displacement.
* Slope of velocity-time graph shows acceleration.

Types of Velocity

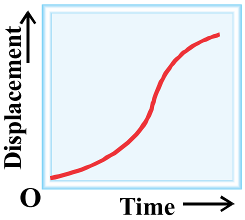
1. Uniform Velocity

* A body is said to move with uniform velocity, if it covers equal displacements in equal intervals of time.



2. Non-uniform Velocity

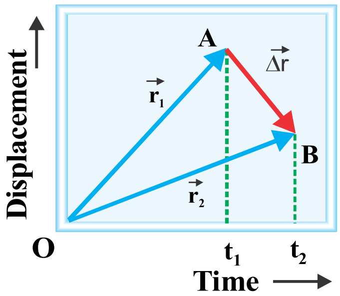
* The particle is said to have non-uniform motion if it covers unequal displacements in equal intervals of time. In this type of motion velocity does not remain constant.



3. Average Velocity

* The average velocity of an object is equal to the ratio of the displacement, to the time interval for which the motion takes place i.e.,

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}}$$



If the initial and final position of a particle are $\vec{r}_1$ and $\vec{r}_2$ at time t_1 and t_2 respectively,

Then displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

and elapsed time $\Delta t = t_2 - t_1$

$$\therefore \text{ Average velocity } \vec{V}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

4. Instantaneous Velocity

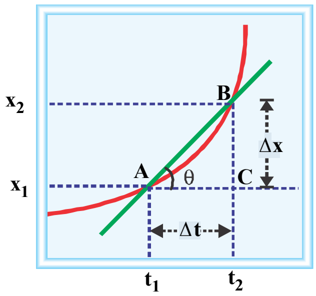
* The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity.
* The average velocity between points A and B

$$\vec{V}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{\Delta\vec{x}}{\Delta t}$$

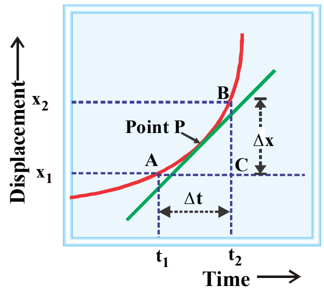
If time interval is small i.e. $t_2 - t_1 = \Delta t$

and $\vec{x}_2 - \vec{x}_1 = \Delta\vec{x}$, then $V_{av} = \frac{\Delta x}{\Delta t} = \tan\theta$

from graph (A).



Graph (A)



Graph (B)

* Average velocity is equal to slope of straight line joining two points on displacement time graph.
* If $\Delta t \to 0$, then average velocity becomes instantaneous velocity instantaneous velocity,

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} \quad ; \quad \vec{V} = \tan\alpha$$

(Slope of tangent at point P, graph B)

Golden Tips

* Average speed of a particle in a given time is never less than the magnitude of the average velocity because distance follows scalar addition while displacement follows vector addition.
* The magnitude of average velocity in an interval need not be equal to its average speed in that interval.
* An object may have varying velocity without having varying speed as in case of a uniform circular motion because velocity can change even by changing direction.
* If velocity is constant then speed will also be constant but if the speed is constant then velocity may or may not be constant as in case of uniform circular motion.
* It is not possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.
* It is not possible to have a situation in which the speed of a particle is always zero but the average speed is not zero.
* Average speed or velocity depends on time interval over which it is defined.

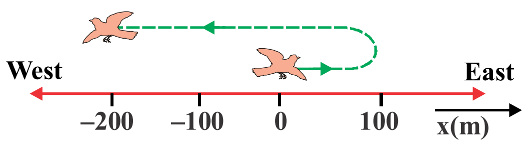
* For a given time interval average velocity has single value while average speed can have many values depending on path followed.
* If after motion the body comes back to its initial position then average velocity is zero but the average speed is greater than zero and finite.
* For a moving body average speed can never be negative or zero while average velocity can be zero.
* If the instantaneous velocity remains constant throughout an interval of time (i.e. in uniform motion) then it is also equal to the average velocity. But the converse may or may not be true, that means, if the instantaneous velocity at a moment is equal to average velocity then the motion may or may not be uniform.
* For a particle moving with constant velocity its average velocity and instantaneous velocity are always equal.

EXAMPLE 9

A bird flies east at 10 m/s for 100 m. It then turns around flies at 20 m/s for 15 s. Neglect time taken for turning, find
(a) its average speed, (b) its average velocity

SOLUTION:

Let us take the x axis to point east.
The first part of the journey took



$\Delta t_1 = (100\text{ m}) / (10\text{ m/s}) = 10\text{s}$,
and we are given $\Delta t_2 = 15$ s for the second part. Hence the total time interval is

$\Delta t = \Delta t_1 + \Delta t_2 = 25$ s

The bird flies 100 m east and then

$(20\text{ m/s})(15\text{s}) = 300$ m west

(a) Average speed

$$= \frac{\text{Distance}}{\Delta t} = \frac{100\text{m} + 300\text{m}}{25\text{s}} = 16\text{ m/s}$$

(b) The net displacement is $\Delta x = -200$ m

So that $v_{av} = \frac{\Delta x}{\Delta t} = \frac{-200}{25\text{s}} = -8\text{ m/s}$

The negative sign means that v_{av} is directed toward the west.

EXAMPLE 10

In reaching her destination, a backpacker walks with an average velocity of 1.34 m/s, due west. This average velocity results because she hikes for 6.44km with an average velocity of 2.68m/s, due west, turns around, and hikes with an average velocity of 0.447 m/s, due east. How far east did she walk?

SOLUTION:

Let west be the positive direction. The average velocity of the backpacker is

$$v = \frac{x_w + x_e}{t_w + t_e} \text{ where } t_w = \frac{x_w}{v_w} \text{ and } t_e = \frac{x_e}{v_e}$$

Combining these equations and solving for x_e (suppressing the units) gives

$$x_e = \frac{-(1 - v/v_w)\, x_w}{(1 - v/v_e)}$$

$$= \frac{-[1 - (1.34\text{ m/s})/(2.68\text{ m/s})]\,(6.44\text{ km})}{1 - (1.34\text{ m/s})/(0.447\text{ m/s})}$$

$= -0.81$ km

The distance traveled is the magnitude of x_e, or 0.81 km.

3.5 ACCELERATION

* The rate of change of velocity of an object with time is called acceleration of the object.
* Let v and v' be the velocity of the object at time t and t' respectively, then acceleration of the body is given by

$$\text{Acceleration} = \vec{a} = \frac{\text{Change in velocity}}{\text{Time interval}} = \frac{\vec{v}' - \vec{v}}{t' - t}$$

* Acceleration is a vector quantity.
* The term "decelerating" means that the acceleration vector points opposite to the velocity vector.
* In problem solving decide at the start which directions are to be called positive (+) and negative (–) relative to a conveniently chosen coordinate origin. This decision is arbitrary, but important because displacement, velocity, and acceleration are vectors, and their directions must always be taken into account.

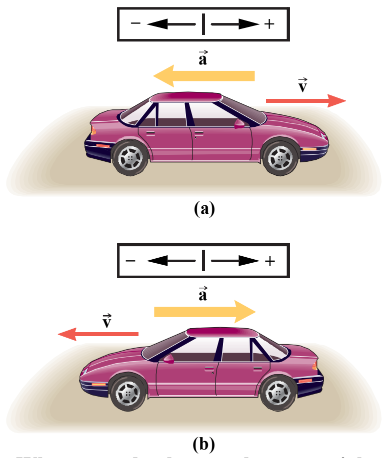
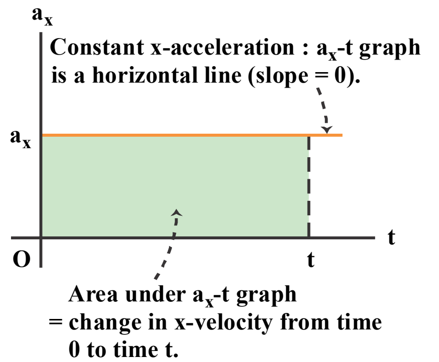
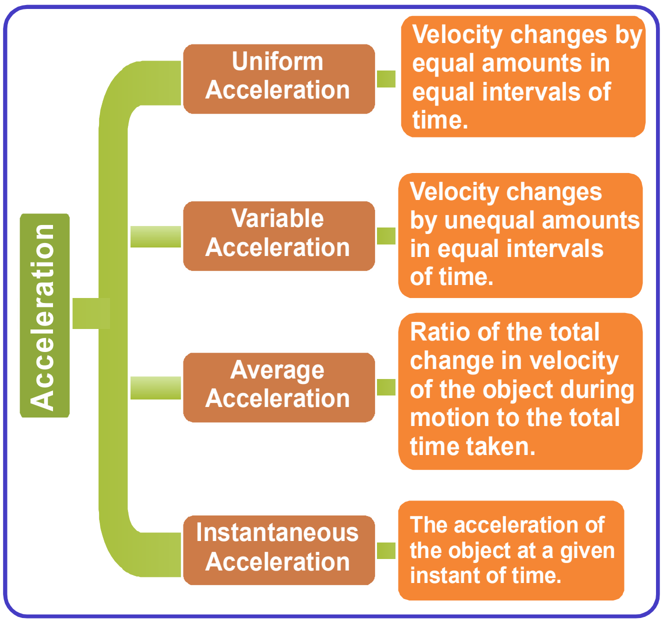


Figure : When a car decelerates along a straight road, the acceleration vector points opposite to the velocity vector.

* Area under acceleration-time graph shows change in velocity.





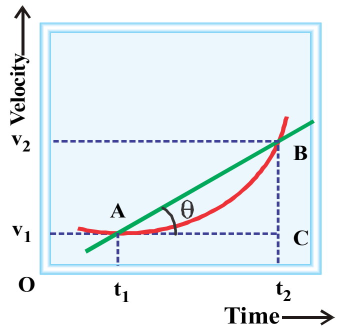
Average Acceleration

* Average acceleration

$$= \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \;;\quad \vec{a}_{av} = \frac{BC}{AC} = \tan\theta$$

= the slope of chord of v-t graph is average acceleration.

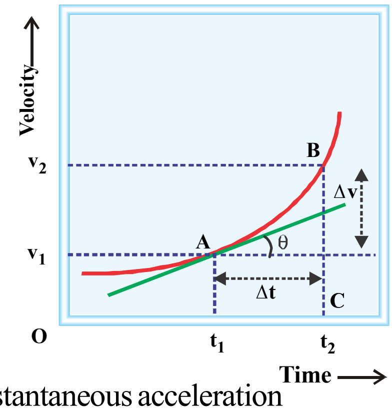


Instantaneous Acceleration

* Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes $\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time

$t_2 = t + \Delta t$, Then, $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration.



* Instantaneous acceleration

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

* $\vec{v} = \dfrac{d\vec{x}}{dt}$, therefore, $\vec{a} = \dfrac{d}{dt}\left(\dfrac{d\vec{x}}{dt}\right) = \dfrac{d^2\vec{x}}{dt^2}$

* Instantaneous acceleration of an object is equal to the second time derivative of the position of the object at the given instant.

Golden Tips

* If any body is accelerated with acceleration $\vec{a}_1$ till time t_1 and acceleration $\vec{a}_2$ up to time t_2 then average acceleration will $\vec{a}_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$
* It is not essential that when velocity is zero acceleration must be zero. e.g. In vertical motion at the top point v = 0 but $a \neq 0$.
* The acceleration may vary but v may be constant e.g. In uniform circular motion.
* If a particle has non-zero acceleration its velocity has to vary. (either in magnitude or in direction or in both)
* It is possible that an object can be increasing in speed when its acceleration is decreasing as in case of a raindrop.
* The velocity of an object can reverse direction even when the acceleration of the object is constant. For example, in case of motion under gravity.
* For a particle moving in straight line, its acceleration must be along the same line.
* It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d|\vec{v}|}{dt} = 0$ as in the case of a uniform circular motion. $\frac{d|\vec{v}|}{dt}$ means the time rate of change of speed and $\left|\frac{d\vec{v}}{dt}\right|$ means the magnitude of acceleration. Note that these two quantities are equal when a particle moves with uniform velocity or when a particle moves with constant acceleration along a straight line.

EXAMPLE 11

The position of a particle moving along x-axis is given by $x = (5t^2 - 4t + 20)$ meter, where t is in second.

(a) Find average velocity between 1s & 3s

(b) Find velocity as a function of time v(t) and its value at t = 3s

(c) Find acceleration at t = 2 sec.

(d) When is the particle at rest ?

SOLUTION:

(a) At t = 1s; $x_{in} = 5(1)^2 - 4(1) + 20 = 21$ m

At t = 3s; $x_f = 5(3)^2 - 4(3) + 20 = 53$ m

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{53 - 21}{3 - 1} = 16 \text{ m/s}$$

(b) $v(t) = \frac{dx}{dt} = (10t - 4)$ m/s

At t = 3s, v = 10(3) – 4 = 26 m/s

(c) $a = \frac{dv}{dt} = 10 \text{ m/s}^2$ (constant at any instant)

(d) Particle at rest i.e. v = 0 = 10t – 4

$\Rightarrow t = 0.4$s

NOTE

(i) Here we can observe at t = 0.4s, particle has zero velocity but acceleration of 10m/s^2. Thus particle having zero velocity need not have zero acceleration.

(ii) For t < 0.4s, velocity is negative and for t > 0.4 s, velocity is in positive direction i.e. its velocity changes its direction at t = 0.4 sec.

EXAMPLE 12

(a) Suppose that a NASCAR race car is moving to the right with a constant velocity of +82 m/s. What is the average acceleration of the car?

(b) Twelve seconds later, the car is halfway around the track and traveling in the opposite direction with the same speed. What is the average acceleration of the car?

SOLUTION:

The average acceleration as $\bar{a} = \frac{v - v_0}{t - t_0}$

(a) The average acceleration is

$\bar{a} = (82 \text{ m/s} - 82 \text{ m/s}) / (t - t_0) = 0 \text{ m/s}^2$

(b) The initial velocity is +82 m/s, and the final velocity is –82 m/s. The average acceleration is

$\bar{a} = (-82 \text{ m/s} - 82 \text{ m/s}) / (12 \text{ s})$

$= -14 \text{ m/s}^2$

EXAMPLE 13

In 1998, NASA launched Deep Space1(DS-1), a spacecraft that successfully flew by the asteroid named 1992 KD (which orbits the sun millions of miles from the earth). The propulsion system of DS-1 worked by ejecting high-speed argon ions out the rear of the engine. The engine slowly increased the velocity of DS-1 by about +9.0 m/s per day. (a) How much time (in days) would it take to increase the velocity of DS-1 by +2700 m/s? (b) What was the acceleration of DS-1 (in m/s^2)?

SOLUTION:

(a) The time Δt that it takes for the spacecraft to change its velocity by an amount $\Delta v = +2700$ m/s is

$$\Delta t = \frac{\Delta v}{a} = \frac{+2700\ \text{m/s}}{+9.0\,\dfrac{\text{m/s}}{\text{day}}} = 3.0 \times 10^2\ \text{days}$$

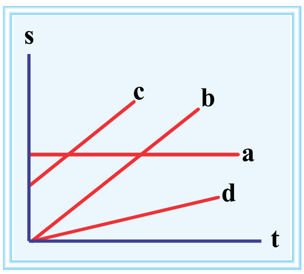
(b) Since 24 hr = 1 day and 3600 s = 1 hr, the acceleration of the spacecraft (in m/s^2) is

$$a = \frac{\Delta v}{t} = \frac{+9.0\ \text{m/s}}{(1\ \text{day})\left(\dfrac{24\ \text{hr}}{1\ \text{day}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{hr}}\right)}$$

$$= +1.04 \times 10^{-4}\ \text{m/s}^2$$

Checkup 1

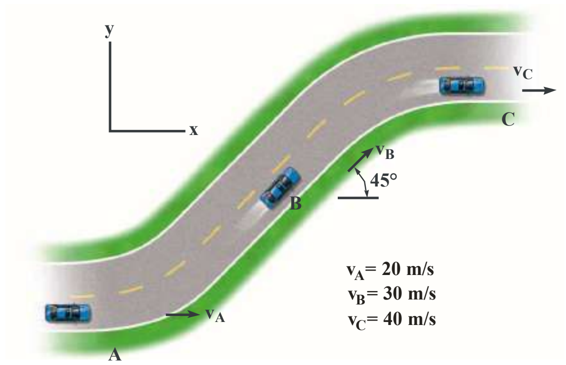
Q.1 For each of the position-time graphs shown in figure.



Rank the average velocities from largest to smallest.

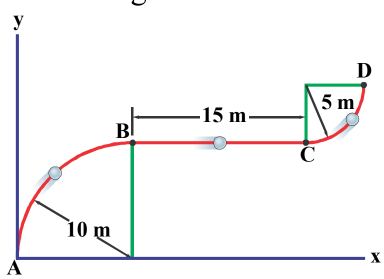
(A) b = c > d > a (B) a = b > c > d
(C) d = c > b > a (D) a = b > d > c

Q.2 A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3s to go from A to B, and then 5s to go from B to C, determine the average acceleration between points A and C.



(A) $(2.50\,\hat{i})\ m/s^2$ (B) $(5.0\,\hat{i})\ m/s^2$
(C) $(1.50\,\hat{i})\ m/s^2$ (D) $(6.50\,\hat{i})\ m/s^2$

Q.3 A particle travels along the curve from A to B in 2s. It takes 4s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



(A) 2.28 m/s (B) 0.28 m/s
(C) 4.28 m/s (D) 5.28 m/s

Q.4 An 18-year-old runner can complete a 10.0-km course with an average speed of 4.39 m/s. A 50-year-old runner can cover the same distance with an average speed of 4.27 m/s. How much later (in seconds) should the younger runner start in order to finish the course at the same time as the older runner?

(A) 32 s (B) 64 s
(C) 108 s (D) 120 s

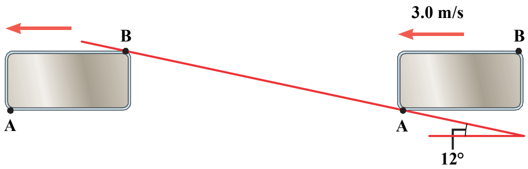
Q.5 A tourist being chased by an angry bear is running in a straight line toward his car at a speed of 4.0m/s. The car is a distance d away. The bear is 26 m behind the tourist and running at 6.0 m/s.

The tourist reaches the car safely. What is the maximum possible value for d?

(A) 32 m (B) 42 m

(C) 52 m (D) 62 m

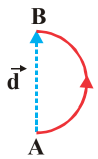
Q.6 You are on a train that is traveling at 3.0 m/s along a level straight track. Very near and parallel to the track is a wall that slopes upward at a 12° angle with the horizontal. As you face the window (0.90 m high, 2.0 m wide) in your compartment, the train is moving to the left, as the drawing indicates. The top edge of the wall first appears at window corner A and eventually disappears at window corner B. How much time passes between appearance and disappearance of the upper edge of the wall? [tan 12° = 0.212]



(A) 0.1 s (B) 2.1 s

(C) 1.1 s (D) 3.1 s

Q.7 A person moves on semicircular track of radius 40 m. If he starts at one end of the track and reaches at the other end. Find the distance covered and magnitude of displacement of the person.

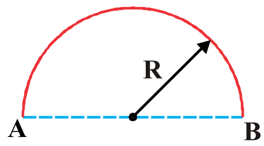


Q.8 A man has to go home 50m due north, 40m due east and 20m due south to reach a cafe from his home. (a) What distance he has to walk to reach the cafe. (b) What is his displacement from his home to the cafe.

Q.9 Straight distance between a hotel and a railway station is 10 km, but circular route is followed by a taxi covering 23 km in 28 minute. What is average speed and magnitude of average velocity,

Q.10 Position of a particle moving along a straight live is given by $x = (t^2 - 4t)$ meters. (t is in sec.) Find displacement and distance travelled between t = 0 and t = 3sec

Q.11 If a particle traverses on a semicircular path of radius R from A to B as shown in time T, find average speed and average velocity.

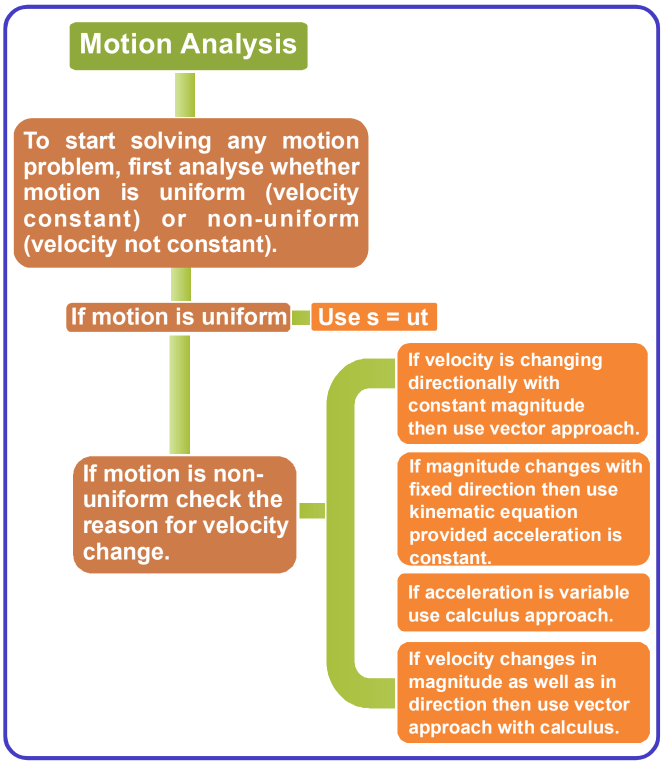


Q.12 A man runs for first 120 m at 6m/s and then next 120m at 3m/s in the same direction. Find (a) Total time of run, (b) Average velocity

Q.13 Position of particle moving along x-axis is given by $x = (3t^2 - 2t^3)$ m (t is in sec).
Find (a) its average velocity form t= 0 s to t= 2s
(b) v (t) and a (t)
(c) The time at which its acceleration is zero and find velocity at the instant.

Q.14 Position of a particle moving along straight line is given by $x = (-t^2 + 6t + 5)$ m (t is in sec). Find
(a) The time at which velocity of particle is zero.
(b) Average velocity from t = 0 to t = 4 sec
(c) Average speed from t = 0 to t = 4sec

3.6 MOTION ANALYSIS



3.7 KINEMATIC EQUATIONS

Uniformly Acceleration Motion

Let $\vec{u}$ = Initial velocity (at t = 0),

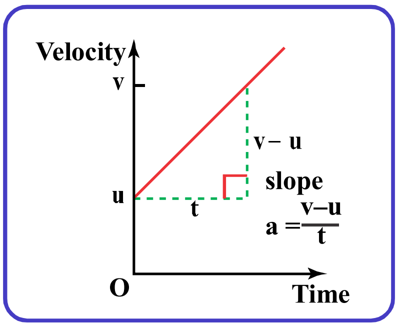
$\vec{v}$ = Velocity of the particle after time t,

$\vec{a}$ = Acceleration (uniform)

$\vec{s}$ = Displacement of the particle during time t

(a) Acceleration, $\vec{a} = \dfrac{\vec{v}-\vec{u}}{t}$

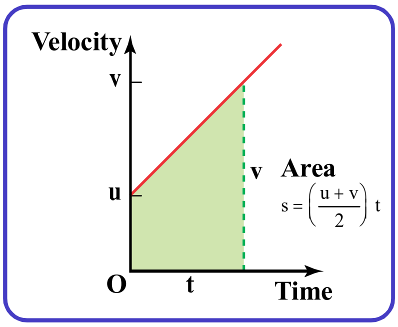
$$\boxed{\vec{v} = \vec{u} + \vec{a}t} \qquad \text{........ (1)}$$



(b) Displacement $\vec{s}$ = Average velocity × time.

$$\boxed{\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right) \times t} \qquad \text{........ (2)}$$

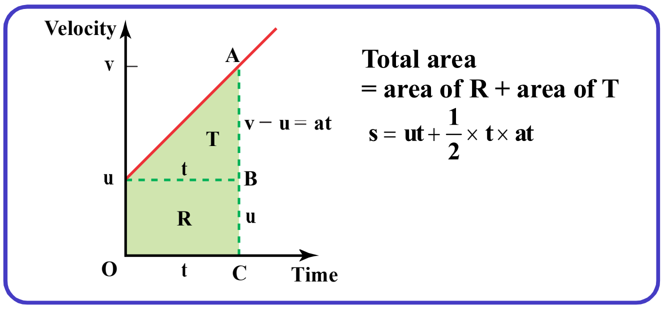
[This is very useful equation, when acceleration is not given]



(c) From (1) & (2), $\boxed{\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2}$ .... (3)

[$\vec{v} = \vec{u} + \vec{a}t$, $\dfrac{d\vec{s}}{dt} = \vec{u} + \vec{a}t$

$\Rightarrow \int d\vec{s} = \int (\vec{u} + \vec{a}t)\, dt \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$]



(d) $\boxed{v^2 = u^2 + 2\vec{a}\cdot\vec{s}}$ .... (4)

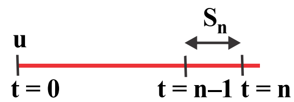
$a = \dfrac{dv}{dt} = \dfrac{dv}{ds}\dfrac{ds}{dt} = v\dfrac{dv}{ds}$; $\int v\, dv = \int \vec{a}.d\vec{s}$

$\dfrac{v^2}{2} = \vec{a}.\vec{s} + c$, At s = 0, v = u, $\dfrac{u^2}{2} = c$

$\therefore \quad \dfrac{v^2}{2} = \vec{a}.\vec{s} + \dfrac{u^2}{2} \Rightarrow v^2 = u^2 + 2\vec{a}.\vec{s}$

(e) $\vec{s}_n$ = displacement of particle in n^{th} second

$\vec{s}_n = \vec{s}_n - \vec{s}_{n-1}$



$= \left\{\vec{u}(n) + \frac{1}{2}an^2\right\} - \left\{\vec{u}(n-1) + \frac{1}{2}\vec{a}(n-1)^2\right\}$

$$\boxed{\vec{s}_n = \vec{u} + \frac{1}{2}\vec{a}\,(2n-1)}$$

* If the velocity and acceleration are collinear, we conventionally take the direction of motion to be positive, so equation of motion becomes.

$v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$

* If the velocity and acceleration are antiparallel then body retards and equation of motion becomes $v = u - at$

$s = ut - \frac{1}{2}at^2$; $v^2 = u^2 - 2as$

* In x-coordinate form :

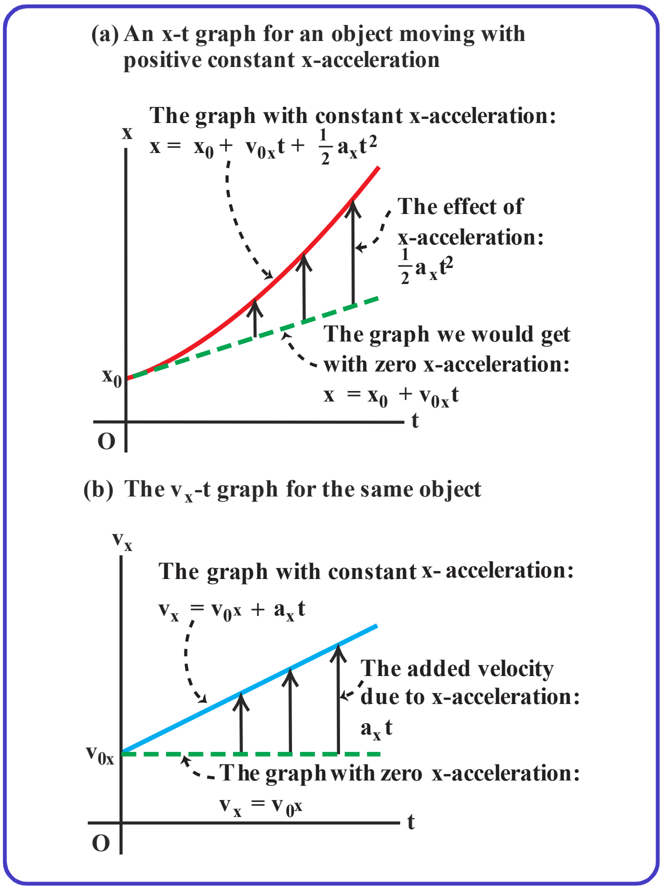
$x = x_0 + ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2a\,(x - x_0)$

$x = x_0 + \frac{1}{2}(u + v)\, t$

In these equations we mean that particle has position x_0 and velocity u at time t = 0, it has position x and velocity v at time t.

Instead of u we can use v_0 also for initial velocity, while solving the problems extract the meaning of symbols.

* **How a constant x-acceleration affects a body's (a) x-t graph and (b) v_x-t graph.**



Golden Tips

* In equation $s = ut + \frac{1}{2}at^2$, u is initial speed for time interval t while in

$$s_{nth} = u + \frac{a}{2}(2n - 1), \text{ u is speed at } t = 0$$

EXAMPLE 14

[Apply the basic kinematics equations]

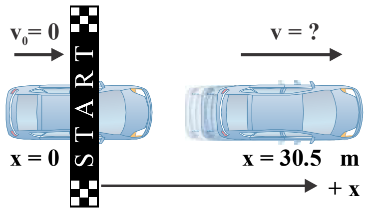
(a) A race car starting from rest accelerates at a constant rate of 5.00 m/s^2. What is the velocity of the car after it has traveled 1.00×10^2 ft?

(b) How much time has elapsed?

SOLUTION:

(a) Convert units of Δx to SI.

$$1.00 \times 10^2 \text{ft} = (1.00 \times 10^2 \text{ ft})\left(\frac{1\text{m}}{3.28 \text{ ft}}\right) = 30.5\text{m}$$



$$v^2 = v_0^2 + 2a\,\Delta x$$

$$v = \sqrt{v_0^2 + 2a\Delta x}$$

$$= \sqrt{(0)^2 + 2\,(5.00 \text{ m/s}^2)\,(30.5\text{m})}$$

$$= 17.5 \text{ m/s}$$

(b) Apply the equation, $v = at + v_0$

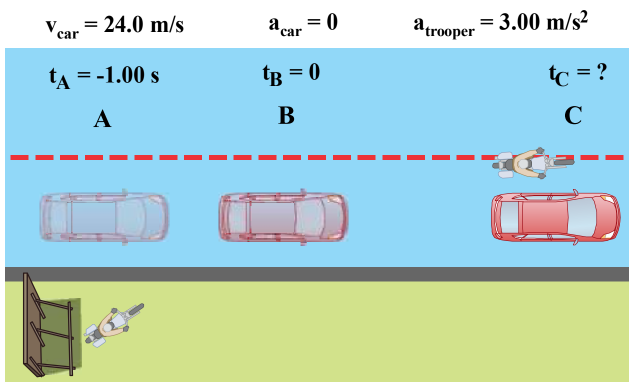
$17.5 \text{ m/s} = (5.00 \text{ m/s}^2)\, t$

$$t = \frac{17.5 \text{ m/s}}{5.0 \text{ m/s}^2} = 3.50 \text{ s}$$

EXAMPLE 15

[Solve a problem involving two objects, one moving at constant acceleration and the other at constant velocity].

A car traveling at a constant speed of 24.0 m/s passes trooper hidden behind a billboard, as in figure. One second after the speeding car passes the billboard, the trooper sets off in chase with a constant acceleration of 3.00 m/s^2. (a) How long does it take the trooper to overtake the speeding car? (b) How fast is the trooper going at that time?



A Speeding car passes a hidden trooper. when does the trooper catch up to the car ?

SOLUTION:

(a) $$\Delta x_{car} = x_{car} - x_0 = v_0 t + \frac{1}{2} a_{car} t^2$$

Take $x_0 = 24.0$m, $v_0 = 24.0$ m/s and $a_{car} = 0$.

$$x_{car} = x_0 + vt = 24.0\text{m} + (24.0 \text{ m/s})\, t$$

Write the equation for the trooper's position, taking $x_0 = 0$, $v_0 = 0$ and $a_{trooper} = 3.00$ m/s^2

$$x_{trooper} = \frac{1}{2} a_{trooper} t^2 = \frac{1}{2}(3.00 \text{ m/s}^2)\, t^2$$

$$= (1.50 \text{ m/s}^2)\, t^2$$

Set $x_{trooper} = x_{car}$, solve the quadratic equation

$$(1.50 \text{ m/s}^2)t^2 = 24.0\text{m} + (24.0 \text{ m/s})\, t$$

$$(1.50 \text{ m/s}^2)t^2 - (24.0 \text{ m/s})\, t - 24.0 \text{ m} = 0$$

$$t = 16.9 \text{ s}$$

(b) Find the trooper's speed at this time. Substitute the time into the trooper's velocity equation :

$$v_{trooper} = v_0 + a_{trooper}\, t$$

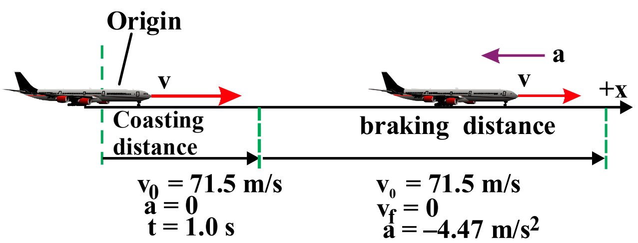
$$= 0 + (3.00 \text{ m/s}^2)(16.9) = 50.7 \text{ m/s}$$

EXAMPLE 16

[Apply kinematics to horizontal motion with two phases]

A typical jetliner lands at a speed 160 mi/h and decelerates at the rate of (10 mi/h)/s. If the plane travels at a constant speed of 160mi/h for 1.0s after landing before applying the brakes, what is the total displacement of the aircraft between touch down on the runway and coming to rest?

SOLUTION:



Coasting and braking distances for a landing jetliner

$$v_0 = (160 \text{ mi/h})\left(\frac{0.447 \text{ m/s}}{1.00 \text{ mi/h}}\right) = 71.5 \text{ m/s}$$

$$a = (-10.0 \text{ (mi/h)/s})\left(\frac{0.447 \text{ m/s}}{1.00 \text{ mi/h}}\right)$$

$$= -4.47 \text{ m/s}^2$$

Taking $a = 0$, $v_0 = 71.5$ m/s, and $t = 1.00$ s, find the displacement while the plane is coasting:

$$\Delta x_{coasting} = v_0 t + \frac{1}{2} a t^2$$

$$= (71.5 \text{ m/s})(1.00 \text{ s}) + 0 = 71.5 \text{ m}$$

$$v^2 = v_0^2 + 2a\, \Delta x_{braking}$$

Take $a = -4.47$ m/s^2 and $v_0 = 71.5$ m/s.

The negative sign on a means that the plane is slowing down.

$$\Delta x_{braking} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (71.5 \text{ m/s})^2}{2.00(-4.47 \text{ m/s}^2)}$$

$$= 572 \text{ m}$$

Sum the two results to find the total displacement

$$\Delta x_{coasting} + \Delta x_{braking} = 72\text{m} + 572\text{m} = 644 \text{ m}$$

EXAMPLE 17

On seeing a board of speed limit, you brake a car from speed of 108 km/h to a speed of 72 km/h. covering a distance of 100m at a constant acceleration. (a) What is that acceleration ? (b) How much time is required for the given decrease in speed ?

SOLUTION:

Initial speed, $u = 108$ km/h

$$= 108 \times \frac{5}{18} \text{ m/s} = 30 \text{ m/s}$$

Final speed, $v = 72 \text{ km/h} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

(a) $v^2 = u^2 + 2as$

$$\therefore \quad a = \frac{v^2 - u^2}{2s} = \frac{(20)^2 - (30)^2}{2 \times 100}$$

$$\therefore \quad a = -2.5 \text{ m/s}^2$$

(b) $$v = u + at \Rightarrow t = \frac{v - u}{a} = \frac{20 - 30}{-2.5} = 4\text{sec}$$

EXAMPLE 18

A body moving with uniform acceleration has a velocity of 12.0 cm/s when its x coordinate is 3.00cm. If its x coordinate 2.s later is –5cm, what is the magnitude of its acceleration?

SOLUTION:

In this problem we are given the initial coordinate (x = 3 cm), the initial velocity ($v_0 = 12$ cm/s).

The final x coordinate (x = –5cm) and the elapsed time (2s).

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow \frac{1}{2}at^2 = x - x_0 - v_0 t$$

$$\frac{1}{2}at^2 = -5\text{ cm} - 3\text{ cm} - (12\text{ cm/s})(2.00\text{s})$$
$$= -32\text{ cm}$$

Solve for a: $a = \frac{2(-32.0\text{ cm})}{t^2} = \frac{2(-32.0\text{ cm})}{(2.00\text{s})^2}$

$$= -16.0\text{ cm/s}^2$$

The x-acceleration of the object is –16 cm/s^2.

EXAMPLE 19

A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

SOLUTION:

$$s = \frac{1}{2}at^2 \;;\; \frac{s^2}{t^2} = \frac{1}{2}as \;;\; V_{av} = \frac{s}{t} = \sqrt{\frac{as}{2}}$$

EXAMPLE 20

A train is moving with 108 km/h. On a straight track. receiving red signal its brakes are applied and it retards at the rate of 3m/s^2. Find its displacement and average velocity for next 15sec.

SOLUTION:

Initial velocity, u = 100 km/h = 30 m/s

Let time required for the velocity to become zero is t. $V_{final} = u + at$

$\therefore$ $30 - 3t = 0 \Rightarrow t = 10$ sec. < 15 sec.

i.e., it covers no distance after t = 10 sec.

Displacement till 15sec = displacement till 10 sec

$$= 30(10) + \frac{1}{2}(-3)(10)^2 = 150\text{ m}$$

$$V_{av} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{150}{15} = 10\text{ m/s}$$

NOTE

In above example, for finding V_{av}, we have taken total time of 15 sec, which actually was required. If we have to find V_{av} for 10 sec, it would be

$$V_{av} = \frac{150}{10} = 15\text{ m/s}$$

Although displacement in 15 sec = Displacement in 10 sec., but times are different. Thus V_{av} for 15 sec. is not same as V_{av} for 10 sec.

EXAMPLE 21

A car is traveling at 20.0 m/s, and the driver sees a traffic light turn red. After 0.530 s **(the reaction time)**, the driver applies the brakes, and the car decelerates at 7.00 m/s^2. What is the stopping distance of the car, as measured from the point where the driver first sees the red light?

SOLUTION:

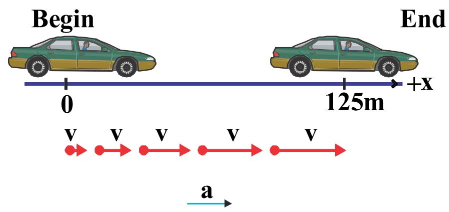
$$\Delta x_1 = x_1 = vt_1 = (20.0\text{ m/s})(0.530\text{ s})$$

$$x_2 = \frac{v^2 - v_0^2}{2a} = \frac{(0\text{ m/s})^2 - (20.0\text{ m/s})^2}{2(-7.00\text{ m/s}^2)}$$

$$x_{Stopping} = x_1 + x_2 = (20.0\text{ m/s})(0.530\text{ s}) + \frac{(0\text{ m/s})^2 - (20.0\text{ m/s})^2}{2(-7.00\text{ m/s}^2)} = 39.2\text{ m}$$

Checkup 2

Q.1 A car accelerates at a constant rate from 15 m/s to 25 m/s while it travels 125 m. How long does it take to achieve this speed?

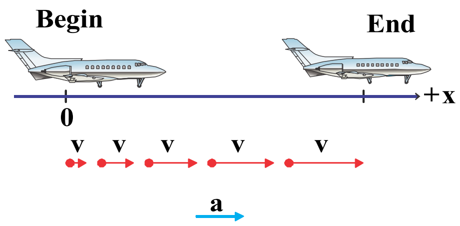


(A) 3.3 (B) 4.3
(C) 5.3 (D) 6.3

Q.2 A bike rider accelerates constantly to a velocity of 7.5 m/s during 4.5 s. The bike's displacement during the acceleration was 19 m.
What was the initial velocity of the bike?

(A) 0.54 m/s (B) 0.94 m/s
(C) 1.94 m/s (D) 2.94 m/s

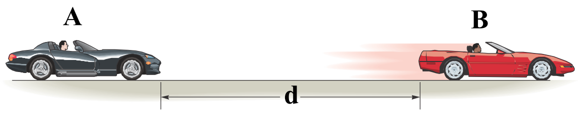
Q.3 An airplane starts from rest and accelerates at a constant 3.00 m/s^2 for 30.0 s before leaving the ground. How far did it move?



(A) 1.35×10^2 m (B) 2.7×10^3 m
(C) 1.35×10^6 m (D) 1.35×10^3 m

Q.4 In a quarter-mile drag (402 m) race, two cars start simultaneously from rest, and each accelerates at a constant rate until it either reaches its maximum speed or crosses the finish line. Car A has an acceleration of 11.0 m/s^2 and a maximum speed of 106 m/s. Car B has an acceleration of 11.6 m/s^2 and a maximum speed of 92.4 m/s. Which car wins the race, and by how many seconds?
(A) Car A wins the race by 0.22 s.
(B) Car B wins the race by 0.11 s.
(C) Car A wins the race by 0.11 s.
(D) Car B wins the race by 0.22 s.

Q.5 When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



(A) $\left|\dfrac{v_A v_B + v_A^2}{2a_A}\right|$ (B) $\left|\dfrac{2v_A v_B - v_A^2}{a_A}\right|$

(C) $\left|\dfrac{2v_A v_B - v_A^2}{2a_A}\right|$ (D) $\left|\dfrac{3v_A v_B - v_A^2}{a_A}\right|$

Q.6 A sprinter explodes out of the starting block with an acceleration of +2.3 m/s^2, which she sustains for 1.2 s. Then, her acceleration drops to zero for the rest of the race. What is her velocity (a) at t = 1.2 s and (b) at the end of the race?

Q.7 (a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0 m/s when going down a slope for 5.0 s? (b) How far does the skier travel in this time?

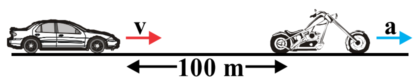
Q.8 The muzzle velocity of a gun is the velocity of the bullet when it leaves the barrel. The muzzle velocity of one rifle with a short barrel is greater than the muzzle velocity of another rifle that has a longer barrel. In which rifle is the acceleration of the bullet larger?

Q.9 The leader of a bicycle race is traveling with a constant velocity of +11.10 m/s and is 10.0 m ahead of the second-place cyclist. The second-place cyclist has a velocity of +9.50 m/s and an acceleration of +1.20 m/s^2. How much time elapses before he catches the leader?

Q.10 A football player, starting from rest at the line of scrimmage, accelerates along a straight line for a time of 1.5 s. Then, during a negligible amount of time, he changes the magnitude of his acceleration to a value of 1.1 m/s^2. With this acceleration, he continues in the same direction for another 1.2 s, until he reaches a speed of 3.4 m/s. What is the value of his acceleration (assumed to be constant) during the initial 1.5-s period?

Q.11 A body is started from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retards to rest with 3 m/s^2. If total time taken is 10 second then maximum speed attained is
(A) 12 m/s (B) 8 m/s
(C) 6 m/s (D) 4 m/s

Q.12 A man travelling in car with a maximum constant speed of 20 m/s watches the friend start off at a distance 100 m ahead on a motor cycle with constant acceleration 'a'. The maximum value of 'a' for which the man in the car can reach his friend is :



(A) 2 m/s^2 (B) 1 m/s^2
(C) 4 m/s^2 (D) None of these

3.8 MOTION UNDER GRAVITY

* The most important example of motion in a straight line with constant acceleration is motion under gravity.
* The acceleration is constant, i.e. $a = g = 9.8 m/s^2$ and directed vertically downwards.
* The motion is in vacuum, i.e., viscous force or thrust of the medium has no effect on the motion.
* Figure (a) shows the well-known phenomenon of a rock falling faster than a sheet of paper. The effect of air resistance is responsible for the slower fall of the paper, when air is removed from the tube, as in Figure (b), the rock and the paper have exactly the same acceleration due to gravity. In the absence of air, the rock and the paper both exhibit free-fall motion.

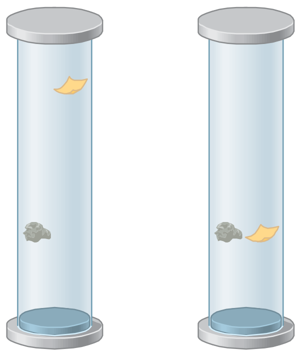


Figure : (a) In the presence of air resistance, the acceleration of the rock is greater than that of the paper. (b) In the absence of air resistance, both the rock and the paper have the same acceleration.

Body Falling Freely Under Gravity

* Taking initial position as origin and downward direction of motion as positive, we have
$u = 0$ [as body starts from rest]
$a = +g$ [as acceleration is in the direction of motion]
So if the body acquires velocity v after falling a distance h in time t, equations of motion, viz.

$$v = u + at\ ;\ s = ut + \frac{1}{2}at^2 \text{ and } v^2 = u^2 + 2as$$

reduces to $v = gt$ ....(1),

$$h = \frac{1}{2}gt^2 \text{ ....(2) and } v^2 = 2gh \text{ ....(3)}$$

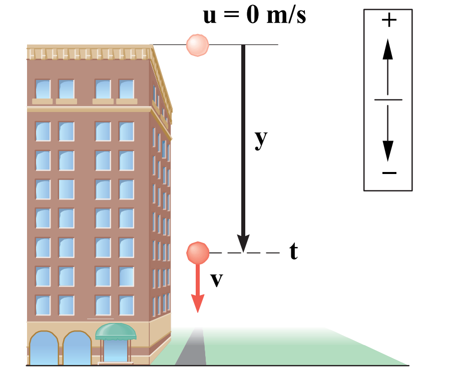
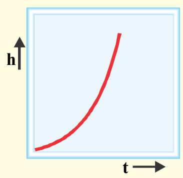
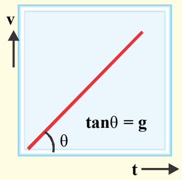
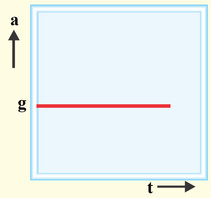


Figure : The stone, starting with zero velocity at the top of the building, is accelerated downward by gravity.

These equations can be used to solve most of the problems of freely falling bodies as if.

If t is given use equation. (1) and equation (2)	If h is given use equation (2) and equation (3)	If v is given use equation (3) and equation (1)
$v = gt$ and $h = \frac{1}{2}gt^2$ 	$t = \sqrt{\frac{2h}{g}}$ $v = \sqrt{2gh}$ 	$t = \frac{v}{g}$ $h = \frac{v^2}{2g}$ 

Body Projected Vertically Up

* Taking initial position as origin and direction of motion (i.e., vertically up) as positive, here we have $v = 0$ [at highest point velocity = 0]
$a = -g$ [as acceleration is downwards while motion upwards].

* If the body is projected with velocity u and reaches the highest point at a distance h above the ground in time t, the equations of motion viz.,

$$v = u + at,\quad s = ut + \frac{1}{2}at^2 \text{ and } v^2 = u^2 + 2as$$

reduces to $0 = u - gt$, $h = ut - \frac{1}{2}gt^2$ and

$$0 = u^2 - 2gh$$

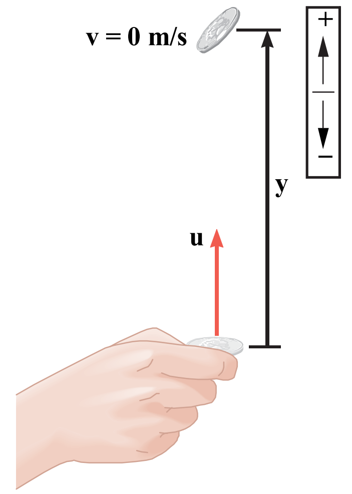
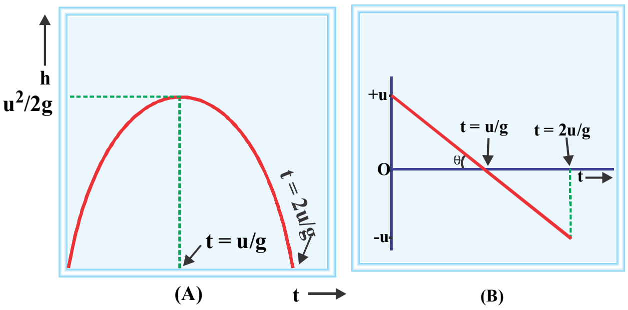


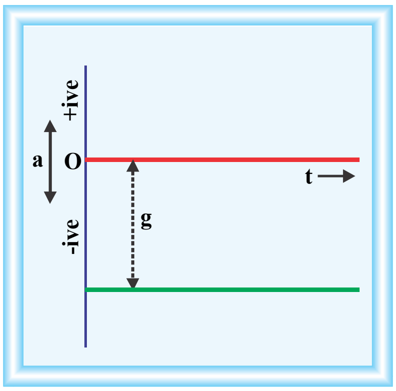
Figure : At the start of a football game, a referee tosses a coin upward with an initial velocity of u. The velocity of the coin is momentarily zero when the coin reaches its maximum height.

Substituting the value of u from first equation in second and rearranging these,

$u = gt$ ....(1) ; $h = \frac{1}{2}gt^2$ ....(2)

and $u^2 = 2gh$ ....(3)





(C)

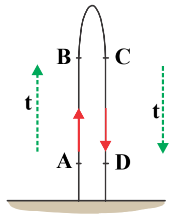
Some Results

* Maximum height reached, $h = \frac{u^2}{2g}$

* Time of ascent = Time of descent = u/g

* Total time of flight = 2u/g

* Time of ascent = Time of descent for motion between two points at same horizontal level for example between A & B and between C & D shown in the figure.



* A particle has the same speed at a point on the path. While going vertically up and down.

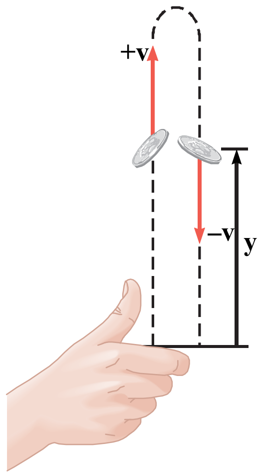


Figure : For a given displacement along the motional path, the upward speed of the coin is equal to its downward speed, but the two velocities point in opposite directions.

Motion Under Gravity with Constant Air Resistance

* Air resistance always acts in opposite direction to motion. Let say ball is thrown with speed u in upward direction.
Assume constant air retardation = a

$$h = \frac{1}{2}(g+a)\,t_{up}^2 = \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)\,t_{down}^2$$

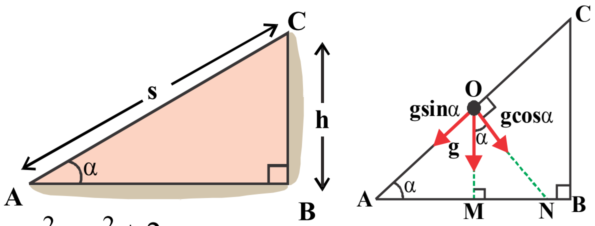
$$\frac{t_{up}}{t_{down}} = \sqrt{\frac{g-a}{g+a}} < 1$$

$t_{up} < t_{down}$ surprising result for students who think without keeping mathematics in mind.

Motion Along Smooth Inclined Plane

* Acceleration due to gravity being a vector quantity can be resolved, along and perpendicular to the inclined plane. The component of g along the plane is g sin α and perpendicular to the plane is g cos α.

Particle start sliding from point C



$v^2 = u^2 + 2as$

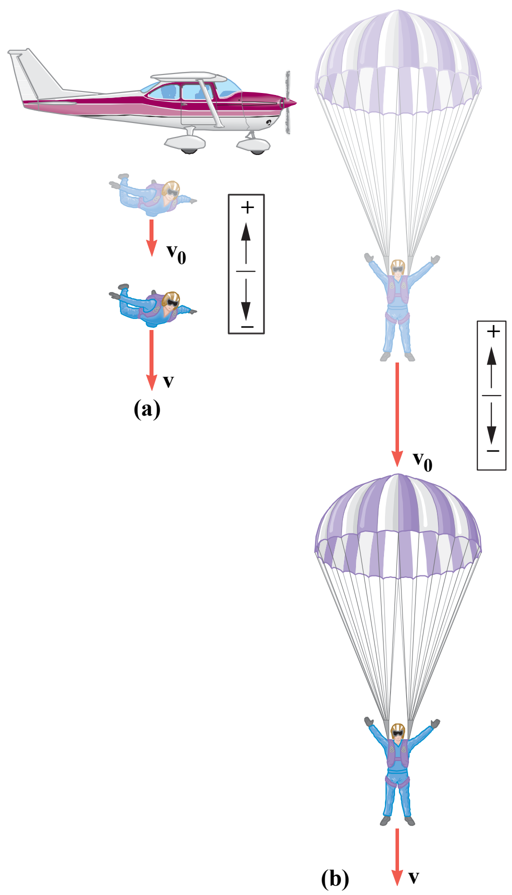
$v_1^2 = 2g\sin\alpha.s = 2g\left[\frac{h}{s}\right]s = 2gh$

[If α be the angle of inclination then, $\sin\alpha = \frac{h}{s}$]

$\therefore\ v_1 = \sqrt{2gh}$, where v_1 = velocity at point A

EXAMPLE 22

A skydiver is falling straight down, along the negative y direction. (a) During the initial part of the fall, her speed increases from 16 to 28 m/s in 1.5 s, as in Figure a. (b) Later, her parachute opens, and her speed decreases from 48 to 26 m/s in 11 s, as in part b of the drawing.



In both instances, determine the magnitude and direction of her average acceleration.

SOLUTION:

(a) Since the skydiver is moving in the negative y direction, her initial velocity is v_0 = –16 m/s and her final velocity is v = –28 m/s.

Her average acceleration $\bar{a}$ is the change in the velocity divided by the elapsed time:

$$\bar{a} = \frac{v - v_0}{t} = \frac{-28\,m/s - (-16\,m/s)}{1.5s} = -8.0\ m/s^2$$

As expected, her average acceleration is negative. Note that her acceleration is not that due to gravity (– 9.8m/s^2) because of air resistance.

(b) Now the skydiver is slowing down, but still falling along the negative y direction. Her initial and final velocities are v_0 = – 48 m/s and v = –26 m/s, respectively. The average acceleration for this phase of the motion is

$$\bar{a} = \frac{v - v_0}{t} = \frac{-26\,m/s - (-48\ m/s)}{11\,s} = +2.0\ m/s^2$$

Now, as anticipated, her average acceleration is positive.

EXAMPLE 23

A ball is released from the top of a building. It travels 25 m in last second of its motion before striking the ground. Find height of the building.

SOLUTION:

Let it takes 't' time to strike the ground.

|Δy in t sec| – |Δy in (t – 1) sec| = 25

$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = 25$$

On solving , t = 3 sec

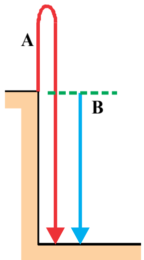
∴ Height of the building, h = (1/2) g (3)^2 = 45 m

EXAMPLE 24

Two identical pellet guns are fired simultaneously from the edge of a cliff. These guns impart an initial speed of 30.0 m/s to each pellet. Gun A is fired straight upward, with the pellet going up and then falling back down, eventually hitting the ground beneath the cliff.

Gun B is fired straight downward. In the absence of air resistance, how long after pellet B hits the ground does pellet A hit the ground?

SOLUTION:



The flight time of pellet **A** will be greater than that of **B** by the amount of time that it takes for pellet **A** to cover the extra distance. The time required for pellet **A** to return to the cliff edge after being fired can be found from $v = v_0 + at$. If "up" is taken as the positive direction then $v_0 = +30.0$ m/s and $v = -30.0$m/s. Solving Equation for t gives

$$t = \frac{v - v_0}{a} = \frac{(-30.0\ m/s) - (+30.0\ m/s)}{-9.80\ m/s^2}$$

$= 6.12$ s

Notice that this result is independent of the height of the cliff.

EXAMPLE 25

A woman on a bridge 75.0 m high sees a raft floating at a constant speed on the river below. Trying to hit the raft, she drops a stone from rest when the raft has 7.00 m more to travel before passing under the bridge. The stone hits the water 4.00m in front of the raft. Find the speed of the raft.

SOLUTION:

During the time t that it takes the stone to fall, the raft travels a distance of 7.00 m – 4.00m = 3.00m,

and its speed is, $\text{Speed} = \frac{3.00\ m}{t}$

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-75.0\ m)}{-9.80\ m/s^2}} = 3.91\ s$$

Therefore, the speed of the raft is

$$\text{Speed} = \frac{3.00\ m}{3.91\ s} = 0.767\ m/s$$

EXAMPLE 26

Two arrows are shot vertically upward. The second arrow is shot after the first one, but while the first is still on its way up. The initial speeds are such that both arrows reach their maximum heights at the same instant, although these heights are different. Suppose that the initial speed of the first arrow is 25.0 m/s and that the second arrow is fired 1.20 s after the first. Determine the initial speed of the second arrow.

SOLUTION:

The time required for the first arrow to reach its maximum height can be determined from $v = v_0 + at$. Taking upward as the positive direction, we have

$$t = \frac{v - v_0}{a} = \frac{0\ m/s - 25.0\ m/s}{-9.80\ m/s^2} = 2.55\ s$$

Note that the second arrow is shot 1.20 s after the first arrow. Therefore, since both arrows reach their maximum height at the same time, the second arrow reaches its maximum height

2.55 s – 1.20 s = 1.35 s

after being fired. The initial speed of the second arrow can then be found from :

$v_0 = v - at = 0\ m/s - (-9.80\ m/s^2)(1.35\ s)$

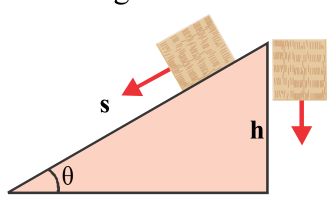
$= 13.2$ m/s

EXAMPLE 27

A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground: (a) earlier (b) with greater speed?

SOLUTION:

In case of sliding motion on an inclined plane.



$$\frac{h}{s} = \sin\theta \Rightarrow s = \frac{h}{\sin\theta},\ a = g\sin\theta$$

$$t_s = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2}{g\sin\theta} \times \frac{h}{\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}} = \frac{t_F}{\sin\theta}$$

$$v_s = \sqrt{2as} = \sqrt{2g\sin\theta \times \frac{h}{\sin\theta}} = \sqrt{2gh} = v_F$$

In case of free fall

$$t_F = \sqrt{\frac{2h}{g}} \text{ and } v_F = \sqrt{2gh}\ ;\ v_s = v_p$$

(a) $\sin\theta < 1$, $t_F < t_s$, i.e., falling body reaches the ground first.

(b) $v_F = v_s$, i.e., both reach the ground with same speed.

Note : Same speed but not same velocity, as for falling body direction is vertical while for sliding body along the plane downwards).

Checkup 3

Q.1 A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?

(A) 80 m (B) 100 m
(C) 60 m (D) 40m

Q.2 An astronaut on a distant planet wants to determine its acceleration due to gravity. The astronaut throws a rock straight up with a velocity of +15 m/s and measures a time of 20.0 s before the rock returns to his hand. What is the acceleration due to gravity on this planet?

Q.3 A ball is thrown straight upward with a velocity $\vec{v}_0$ and in a time t reaches the top of its flight path, which is a displacement $\vec{y}$ above the launch point. With a launch velocity of $2\vec{v}_0$, what would be the time required to reach the top of its flight path and what would be the displacement of the top point above the launch point?

(A) 4t and $2\vec{y}$ (B) 2t and $4\vec{y}$
(C) 2t and $2\vec{y}$ (D) 4t and $4\vec{y}$

Q.4 Two objects are thrown vertically upward, first one, and then, a bit later, the other. Is it (a) possible or (b) impossible that both objects reach the same maximum height at the same instant of time?

Q.5 A ball is dropped from rest from the top of a building and strikes the ground with a speed v_f. From ground level, a second ball is thrown straight upward at the same instant that the first ball is dropped. The initial speed of the second ball is $v_0 = v_f$, the same speed with which the first ball eventually strikes the ground. Ignoring air resistance,decide whether the balls cross paths

(A) at half the height of the building
(B) above the halfway point
(C) below the halfway point.

Q.6 An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(A)On the way up $a_y>0$, on the way down $a_y>0$
(B)On the way up $a_y<0$, on the way down $a_y>0$
(C)On the way up $a_y>0$, on the way down $a_y<0$
(D)On the way up $a_y<0$, on the way down $a_y<0$

Q.7 A particle is projected vertically upwards from a point A on the ground. It takes t_1 time to reach a point B but it still continues to move up. If it takes further t_2 time to reach the ground from point B then height of point B from the ground is

(A) $\frac{1}{2}g(t_1+t_2)^2$ (B) $g\,t_1\,t_2$
(C) $\frac{1}{8}g(t_1+t_2)^2$ (D) $\frac{1}{2}\,g\,t_1\,t_2$

Q.8 Balls are thrown vertically upward in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5m, the number of balls thrown per minute will be

(A) 40 (B) 50
(C) 60 (D) 120

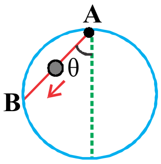
Q.9 A particle is projected vertically upward with initial velocity 25 m/s. For its motion during third second, which statement is correct?

(A) Displacement of the particle is 30 m
(B) Distance covered by the particle is 30 m.
(C) Distance covered by the particle is 2.5 m
(D) None of these

Q.10 A man holds four balls 180 m above the ground and drops them at regular intervals of time so that when the first ball hits ground, the fourth ball is just leaving his hand. At this time, the second and third balls from the ground are at the positions

(A) 160 m and 100 m respectively
(B) 80 m and 20 m respectively
(C) 20 m and 80 respectively
(D) 100 m and 160 m respectively

Q.11 A bead is free to slide down a smooth wire tightly stretched between points A and B on a vertical circle. If the bead starts from rest at A, the highest point on the circle



(A) its velocity v on arriving at B is proportional to sec θ.
(B) its velocity v on arriving at B is proportional to tan θ.
(C) time to arrive at B is proportional to cosθ.
(D) time to arrive at B is independent of θ

3.9 MOTION WITH NON-UNIFORM ACCELERATION

* If acceleration depends on velocity v or time t, then by definition of acceleration, we have

$$a = \frac{dv}{dt}$$

* If a is in terms of t, $\int_{v_0}^{v} dv = \int_0^t a(t)\, dt$.

* If a is in terms of v, $\int_0^t \frac{dv}{a(v)} = \int_0^t dt$

On integrating, we get a relation between v and t, and then using $\int_{x_0}^{x} dx = \int_0^t v(t)\, dt$, x and t can also be related.

* If acceleration depends on velocity v or position x then $a = \frac{dv}{dt} \Rightarrow a = \frac{dx}{dt}\frac{dv}{dx} \Rightarrow a = v\frac{dv}{dx}$. This is another important expression for acceleration.

* If a is in terms of x, $\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\, dx$.

* If a is in terms of v, $\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x & v.

Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

EXAMPLE 28

The acceleration of a particle is given by $a = 2t^2\ m/s^2$. If it is at rest at the origin at time t = 0, find its position, velocity, and acceleration at time t = 1s.

SOLUTION:

$a = 2t^2 \quad \therefore\ a = 2 \times 1^2 = 2\ m/s^2$ (at t = 1 sec.)

Formula for v, $\frac{dv}{dt} = 2t^2$

or, $\int_0^v dv = \int_0^t 2t^2 dt$ or $v = \frac{2t^3}{3}$

At t = 1 sec, $v = \frac{2 \times 1^3}{3} = \frac{2}{3}$ m/s

Formula for x, $\frac{dx}{dt} = \frac{2}{3}t^3$ or $\int_0^x dx = \int_0^t \frac{2}{3}t^3 dt$

or, $x = \frac{t^4}{6}$. At t = 1 sec, $x = \frac{1}{6}$ m

EXAMPLE 29

A particle start with initial velocity v_0 and acceleration a = kt, where k is constant. Find velocity and displacement after time t.

SOLUTION:

(i) Given : $a = kt \Rightarrow \frac{dv}{dt} = kt$

$\Rightarrow \quad dv = kt\, dt$

$\Rightarrow \quad \int_{v_0}^{v} dv = k\int_0^t t\, dt$

$\Rightarrow \quad v - v_0 = \frac{kt^2}{2}$

$\Rightarrow \quad v = v_0 + \frac{k}{2}t^2$

(ii) $v = \frac{ds}{dt} = v_0 + \frac{k}{2}t^2$

$\Rightarrow \quad ds = (v_0 + \frac{k}{2}t^2)\, dt$

$\Rightarrow \quad \int_0^s ds = \int_0^t v_0 dt + \frac{k}{2}\int_0^t t^2 dt$

$\Rightarrow \quad s = v_0 t + \frac{k}{2}\frac{t^3}{3} \Rightarrow s = v_0 t + \frac{k}{6}t^3$

Checkup 4

Q.1 The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Q.2 If a particle starting from rest has an acceleration that increases linearly with time as a = 2t, then the distance travelled in third sec will be

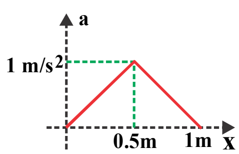
(A) 9 m (B) (8/3) m
(C) (19/3) m (D) (11/3) m

Q.3 A particle starts with velocity v_0 at at time t = 0 and is decelerated at a rate proportional to the square root of its speed at time t with constant of proportionality α. The total time for which it will move before coming to rest is

(A) $\sqrt{v_0}$ (B) $\frac{v_0^{3/2}}{\alpha}$

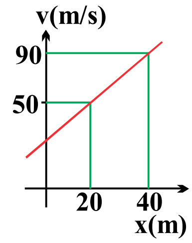
(C) $\frac{2v_0^{3/2}}{\alpha}$ (D) $\frac{2\sqrt{v_0}}{\alpha}$

Q.4 A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is



(A) 1 m/ s (B) 6 m/s
(C) 2 m/s (D) none

Q.5 From the given graph find acceleration at x = 20m.



(A) 25 m/s^2 (B) 50 m/s^2
(C) 75 m/s^2 (D) 100 m/s^2

3.10 GRAPHICAL ANALYSIS

* Graphical techniques are helpful in understanding the concepts of position, velocity and acceleration.

* You should know :
(a) How to plot the graph
(b) Mathematical interpretations
(c) To convert one graph to other graph.
(d) To solve problems with the help of graph.

To Plot the Graph

* Think physical quantities to be consider on x & y axis, find the relation between quantities see if it is : Linear relation → Graph is straight line
Quadratic relation → Parabola
$y \propto 1/x$ → Rectangular hyperbola

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ → Ellipse (if a = b → circle)

* Locate two points on graph by analysing x = 0, y = ?, y = 0, x = ?

Mathematical Interpretation

* Check slope $\left(= \frac{dy}{dx}\right)$ variation (increasing, decreasing or constant).

* If graph passes through origin interpret its meaning.

* If graph is continuous or discrete interpret its meaning.

Graph Conversion

* To convert given graph into other graph develop the relation among quantities on both the graph and observe their variation.

1. $\frac{dy}{dx} = z$: Physical quantity obtain by dividing y with x. $\frac{dy}{dx}$ shows slope of the tangent on the graph between y and x. **Example,**

Slope of displacement - time graph	**Velocity** $\left(v = \frac{ds}{dt}\right)$
Slope of velocity - time graph	**Acceleration** $\left(a = \frac{dv}{dt}\right)$
Slope momentum-time graph	**Force** $\left(F = \frac{dp}{dt}\right)$

2. $\int_{x_1}^{x_2} y\,dx = z_0$: Physical quantity obtain by multiplying y with x.

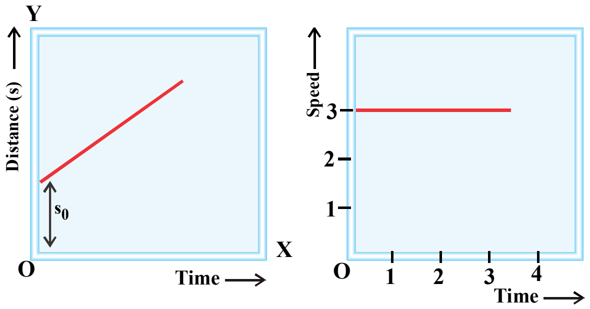
$\int_{x_1}^{x_2} y\,dx$ shows area under the graph between x_1 and x_2. **Example,**

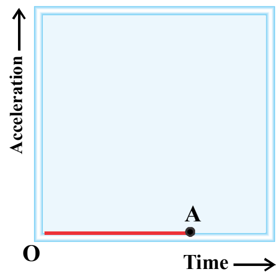
Area under v-t graph gives displacement	$\int ds = \int v\,dt$
Area under a-t graph gives change in velocity	$\int dv = \int a\,dt$
Area under F-t graph gives impulse (change in momentum)	$\int dP = \int F\,dt$
Area under F-s graph gives work	$\int dW = \int F\,ds$

Study Following Graphs

1. For a particle travelling with uniform speed

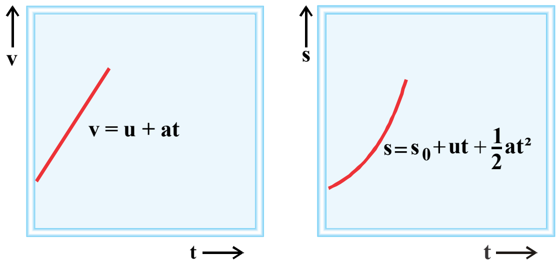
(Distance = speed × time, v = constant, a = 0)





2. For a particle travelling with constant acceleration. (one-dimensional motion)

(a) Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$

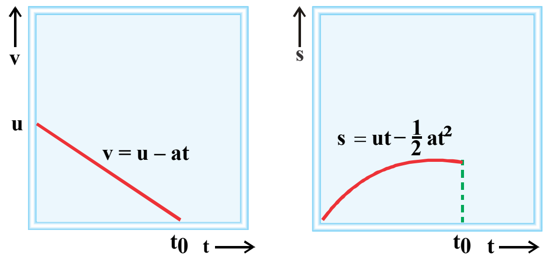


(b) Uniformly retarded motion till velocity becomes zero.

(i) Slope of s-t graph at t = 0 gives u.

(ii) Velocity decreasing, slope of s-t graph decreases,

$s = ut - \frac{1}{2}at^2$ quadratic relation hence graph is parabola between s and t.

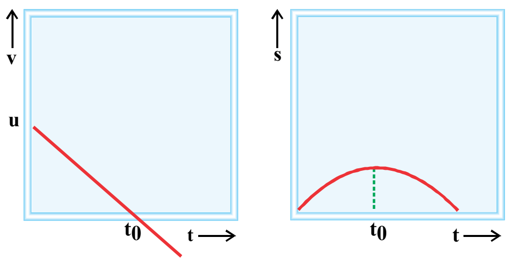


(c) Uniformly retarded then accelerated in opposite direction

(i) At time $t = t_0$, v = 0 or slope of s-t graph is zero

(ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

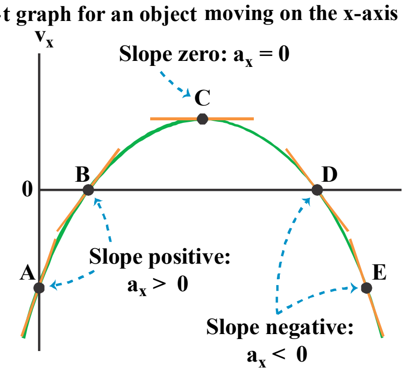
(iii) At $t = t_0$ velocity = 0, $a \neq 0$ hence it is a turning point. Slope of s-t graph decreases from 0 to t_0 & then increases.



Observe the Following Graphs Carefully

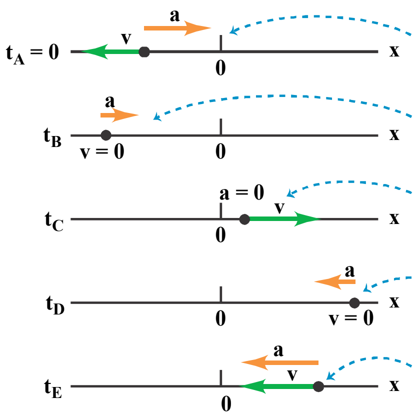
1

(a) v_x-t graph for an object moving on the x-axis



The steeper the slope (positive or negative) of an object's v_x-t graph, the greater is the object's acceleration in the positive or negative x-direction.

(b) Object's position, velocity, and acceleration on the x-axis



Object is at $x < 0$, moving in the –x-direction ($v_x < 0$), and slowing down (v_x and a_x have opposite signs).

Object is at $x < 0$, instantaneously at rest ($v_x = 0$), and about to move in the +x-direction ($a_x > 0$).

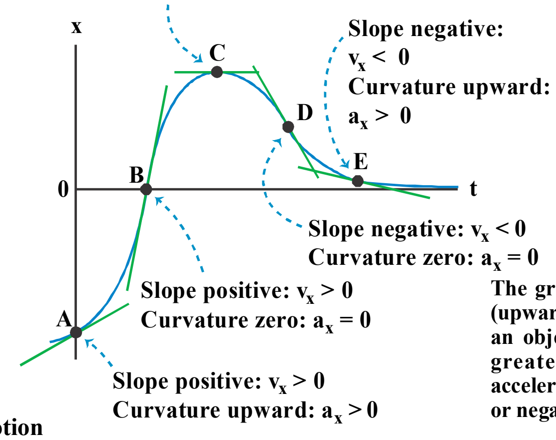
Object is at $x > 0$, moving in the +x-direction ($v_x > 0$); its speed is instantaneously not changing ($a_x = 0$).

Object is at $x > 0$, instantaneously at rest ($v_x = 0$), and about to move in the –x-direction ($a_x < 0$).

Object is at $x > 0$, moving in the –x-direction ($v_x < 0$), and speeding up (v_x and a_x have the same sign).

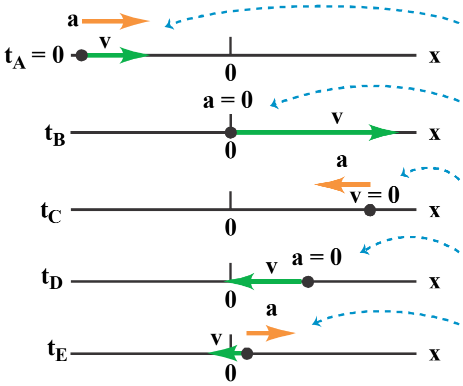
2

(a) x-t graph



The greater the curvature (upward or downward) of an object's x-t graph, the greater is the object's acceleration in the positive or negative x-direction.

(b) Object's motion



Object is at $x < 0$, moving in the +x-direction ($v_x > 0$), and speeding up (v_x and a_x have the same sign).

Object is at $x = 0$, moving in the +x-direction ($v_x > 0$); speed is instantaneously not changing ($a_x = 0$).

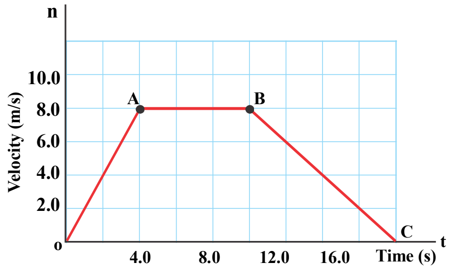
Object is at $x > 0$, instantaneously at rest ($v_x = 0$), and about to move in the –x-direction ($a_x < 0$).

Object is at $x > 0$, moving in the –x-direction ($v_x < 0$); speed is instantaneously not changing ($a_x = 0$).

Object is at $x > 0$, moving in the –x-direction ($v_x < 0$), and slowing down (v_x and a_x have opposite signs).

EXAMPLE 30

What is the average acceleration for each graph segment in fig? Describe the motion of the object over the total time interval. Also calculate displacement.



SOLUTION:

Segment OA; $a = \frac{8-0}{4-0} = 2\ m/s^2$

Segment AB; graph horizontal i.e., slope zero i.e., a = 0

Segment BC; $a = \frac{0-8}{18-10} = -1 m/s^2$

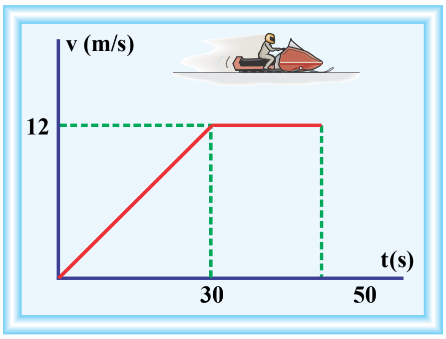
The graph is trapezium. Its area between t = 0 to t = 18s is displacement

$$\text{Area} = \text{displacement} = \frac{1}{2}(18+6)\times 8 = 96\text{m}$$

Particle accelerates uniformly for first 4 sec., then moves uniformly for 6 sec. and then retards uniformly to come to rest in next 8 sec.

EXAMPLE 31

The snowmobile moves along a straight course according to the v–t graph. Construct the s-t and a–t graphs for the same 50s time interval. When t = 0, s = 0.



SOLUTION:

s–t Graph : For time interval $0s \le t < 30s$;

$$v = \frac{12}{30}t = \left(\frac{2}{5}t\right) m/s;\ s = \frac{1}{2}at^2 = \left(\frac{1}{5}t^2\right) m$$

At t = 30s, $s = \frac{1}{5}(30^2) = 180\text{m}$

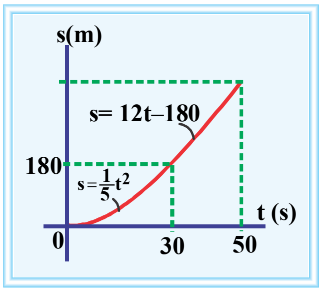
For time interval $30s < t \le 50$ s,

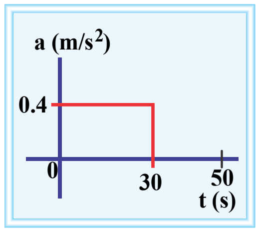
s = (12t – 180) m

At t = 50 s, s = 12 (50) – 180 = 420 m

a–t Graph: For time interval $0\ s \le t < 30$ s and $30s < t \le 50s$,

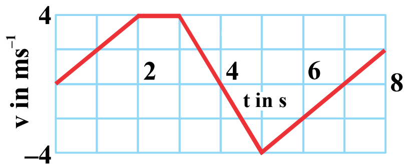
$$a = \frac{dv}{dt} = \frac{2}{5} = 0.4\ m/s^2\ ;\ a = \frac{dv}{dt} = 0\ \text{, respectively.}$$





EXAMPLE 32

Figure here gives the velocity time graph for a body. Find the displacement and distance travelled between t = 0s and t = 7.0.



SOLUTION:

Area between t = 0 sec. to t = 4 sec.

$$= \frac{1}{2}\times(4+1)\times 4 = 10\text{ m}$$

Area between t = 4 sec. to t = 7 sec.

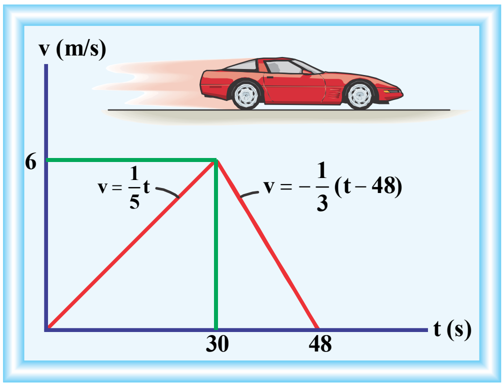
$$= \frac{1}{2}\times 3\times(-4) = -6$$

Net displacement = total area = 10 – 6 = 4 m

Distance = |10| + |– 6| = 16 m

EXAMPLE 33

A car travels along a straight road with the speed shown by the v–t graph. Plot the a–t graph.



SOLUTION:

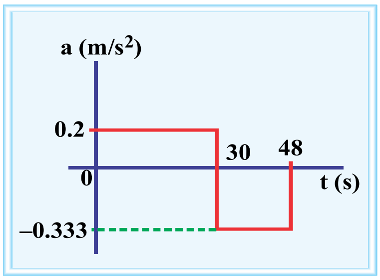
a–t graph : For $0 \le t < 30s$,

$$v = \frac{1}{5}t \;;\quad a = \frac{dv}{dt} = \frac{1}{5} = 0.2 \text{ m/s}^2$$

For $30s < t \le 48$ s

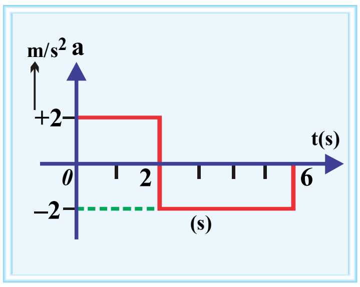
$$v = -\frac{1}{3}(t-48)\;;\; a = \frac{dv}{dt} = -\frac{1}{3}(1)$$

$$= -0.333 \text{ m/s}^2$$



EXAMPLE 34

At t = 0, a particle is at rest at origin. Its acceleration is 2m/s^2 for first 2 sec. and -2 m/s^2 for next 4 sec as shown in a versus t graph.



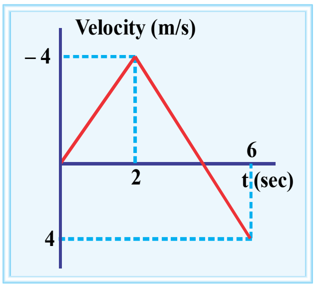
Plot graphs for velocity versus time and speed versus time.

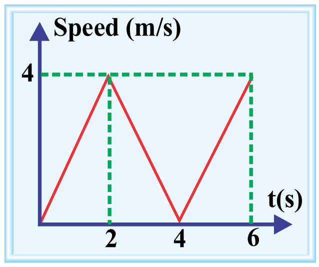
SOLUTION:

$V_2 - V_0$ = Area of a v/s t graph for t = 0 to t = 2 sec

$V_2 - 0 = 2 \times 2 \Rightarrow V_2 = +4$ m/s

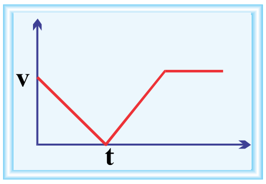
Now, $V_6 - V_2 = -2 \times 4 \Rightarrow V_6 = -4$ m/s



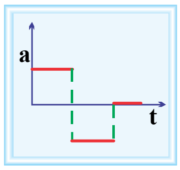


Checkup 5

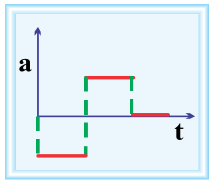
Q.1 The velocity of a particle moving in straight line is given by the graph shown here. Then its acceleration is best represented by



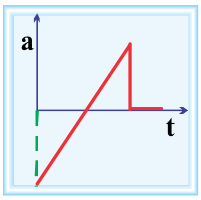
(A)



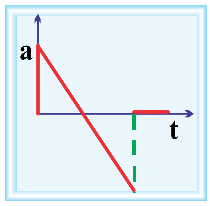
(B)



(C)

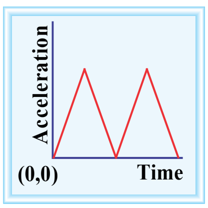


(D)

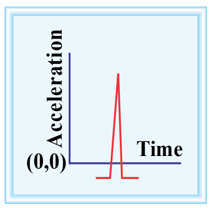


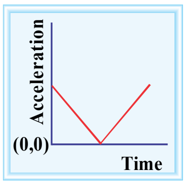
Q.2 Which of the following graphs would best represent acceleration versus time for the bouncing ball?

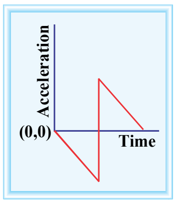
(A)



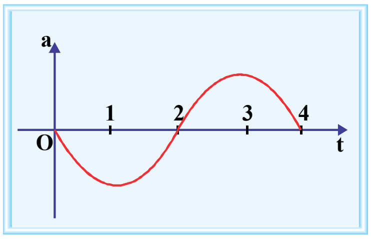
(B)



(C) 

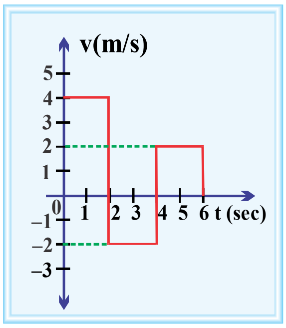
(D) 

Q.3 Acceleration(a)-time(t) graph for a particle starting from rest at t = 0 is as given below. The particle has maximum speed at :



(A) 1s (B) 2s
(C) 3s (D) 4s

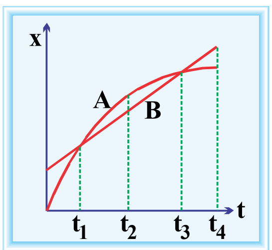
Q.4 The velocity-time graph of a body moving in a straight line is shown in the figure. The ratio of displacement to the distance travelled by the body in first 6 seconds is



(A) 1 : 1 (B) 1 : 2
(C) 1 : 3 (D) 1 : 4

For Q.5-Q.8

The graph given shows the **POSITION** of two cars, A and B, as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding.



Q.5 At which instant in time is car-A overtaking the car-B ?
(A) t_1 (B) t_2
(C) t_3 (D) t_4

Q.6 At time t_3, which car is moving faster?
(A) car A (B) car B
(C) same speed (D) None of these

Q.7 At which instant do the two cars have the same velocity ?
(A) t_1 (B) t_2
(C) t_3 (D) t_4

Q.8 Which one of the following best describes the motion of car A as shown on the graphs?
(A) speeding up
(B) constant velocity
(C) slowing down
(D) first speeding up, then slowing down

3.11 RELATIVE VELOCITY

* To understand the concept of relative velocity observed the figure.

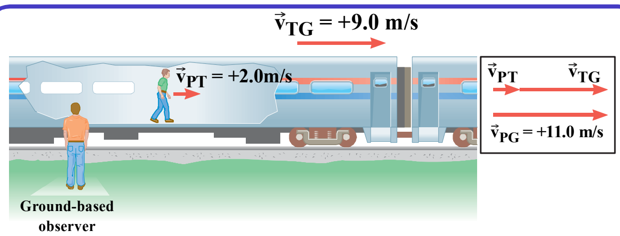


Figure : The velocity of the passenger relative to the ground-based observer is $\vec{v}_{PG}$. It is the vector sum of the velocity $\vec{v}_{PT}$ of the passenger relative to the train and the velocity $\vec{v}_{TG}$ of the train relative to the ground: $\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$.

* A passenger is walking toward the front of a moving train. The people sitting on the train see the passenger walking with a velocity of +2.0 m/s, where the plus sign denotes a direction to the right.
* Suppose the train is moving with a velocity of +9.0 m/s relative to an observer standing on the ground.

* Then the ground-based observer would see the passenger moving with a velocity of +11 m/s, due in part to the walking motion and in part to the train's motion.

* As an aid in describing relative velocity, let us define the following symbols:

$\vec{v}_{PT}$ = velocity of the **Passenger** relative to the **Train** = +2.0 m/s

$\vec{v}_{TG}$ = velocity of the **Train** relative to the **Ground** = +9.0 m/s

$\vec{v}_{PG}$ = velocity of the **Passenger** relative to the **Ground** = +11 m/s

In terms of these symbols, the situation in Figure can be summarized as follows:

$$\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG} \qquad \ldots\ldots (1)$$

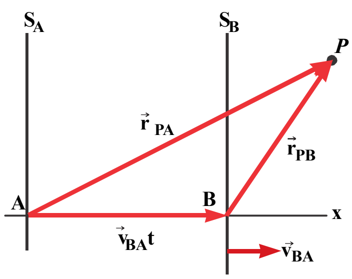
or $\vec{v}_{PG} = (2.0\ m/s) + (9.0\ m/s) = +11\ m/s$

According to Equation (1), $\vec{v}_{PG}$ is the vector sum of $\vec{v}_{PT}$ and $\vec{v}_{TG}$ and this sum is shown in the drawing. Had the passenger been walking toward the rear of the train, rather than toward the front, the velocity relative to the ground-based observer would have been

$$\vec{v}_{PG} = (-2.0\ m/s) + (9.0\ m/s) = +7.0\ m/s.$$

Mathematics behind relative velocity expression

* A particle located at P is described by two observers, one in the fixed frame of reference S_A and the other in the frame S_B, which moves to the right with a constant velocity $\vec{v}_{BA}$. The vector $\vec{r}_{PA}$ is the particle's position vector relative to S_A, and $\vec{r}_{PB}$ is its position vector relative to S_B.



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{v}_{BA}t$$

By differentiating with respect to time

$$\frac{d\vec{r}_{PA}}{dt} = \frac{d\vec{r}_{PB}}{dt} + \vec{v}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

where $\vec{v}_{PA}$ is the velocity of the particle at P measured by observer A and $\vec{v}_{PB}$ is its velocity measured by B.

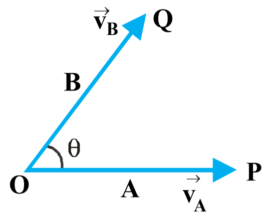
* If $\vec{v}_A$ and $\vec{v}_B$ be the respective velocity of object A and B (wrt ground) then relative velocity of A w.r.t. B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Relative velocity of B w.r.t. A $(\vec{v}_{BA}$ or $\vec{v}_{B/A})$

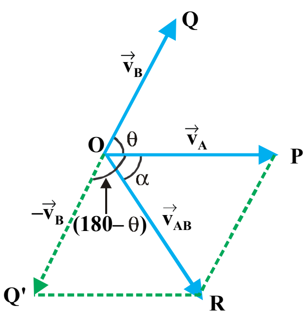
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

The bodies moving in directions inclined to each other : Relative velocity of A w.r.t B

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$



The relative velocity of A with respect to B is given by the diagonal OR of the parallelogram OPRQ' as shown in fig.



The magnitude of the relative velocity v_{AB} is given by $v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180-\theta)}$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

Let α be the angle made by v_{AB} with v_A, then

$$\tan\alpha = \frac{v_B \sin(180-\theta)}{v_A + v_B \cos(180-\theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

$\angle\alpha$ gives the direction of the relative velocity with $\vec{v}_A$.

(i) $\theta = 0^\circ : v_{AB} = v_A - v_B$

(ii) $\theta = 180^\circ : v_{AB} = v_A + v_B$

Application of Relative Velocity

(i) Motion Analysis

(a) **Relative position :** If position of A w.r.t. to origin is x_A and that of B w.r.t. origin is x_B then "position of A w.r.t. B" x_{AB} or $x_{A/B}$ is

$$x_{AB} = x_A - x_B$$

(b) **Relative velocity :** $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

(c) **Relative acceleration :** $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

(d) **Kinematic equations:(constant acceleration)**

$$v_{rel} = u_{rel} + a_{rel}\, t$$

$$s_{rel} = u_{rel}\, t + \frac{1}{2}\, {}_{rel}\, t^2$$

$$v^2{}_{rel} = u^2{}_{rel} + 2a_{rel}\, s_{rel}$$

(e) **Velocity of approach/separation :**
It is the component of relative velocity of one particle w.r.t. another, along the line joining them. If the separation is decreasing, we say it is velocity of approach and it separation is increasing, then we say it is velocity of separation.
In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach/separation is simply equal to magnitude of relative velocity of A w.r.t. B.

EXAMPLE 35

Assume two cars A and B each 5 m long. Car A is travelling at 84 km/h and overtakes another car B which travelling at low speed of 12 km/h. Find out the time taken for overtaking.

SOLUTION:
To analyse the motion in case of overtaking we need relative velocity of object which overtakes w.r.t. the other object. Therefore, we need to find relative velocity of car A w.r.t car B which is $84 - 12 = 72$ km/h $= 20$ ms^{-1}
Total relative distance covered with this velocity = sum of lengths of car A and car B = 5+ 5 =10m.

$$\text{The time taken} = \frac{\text{Distance covered}}{\text{Relative velocity}} = \frac{10}{20}$$

$$= 0.5 \text{ s}$$

EXAMPLE 36

Two trains, one travelling at 54kph and the other at 72kph, are headed towards each other on a level track. When they are two kilometers apart, both drivers simultaneously apply their brakes. If their brakes produces equal retardation in both the trains at a rate of 0.15 m/s^2, determine whether there is a collision or not.

SOLUTION:
Speed of first train is = 54 kph = 15m/s.
Speed of second train is = 72kph = 20 m/s
As both the trains are heated towards each other, relative velocity of one train with respect to other is given as: v_r =15 + 20 = 35 m/s
Both trains are retarded by acceleration of 0.15 m/s^2, relative retardation is

$$a_r = 0.15 + 0.15 = 0.3 \text{ m/s}^2$$

Now we assume one train is at rest and other is coming at 35m/s retarded by 0.3 m/s^2 is at a distance of two kilometer.
The maximum distance travelled by the moving train while retarding is

$$s_{max} = \frac{v_r^2}{2a_r} = \frac{(35)^2}{2 \times 0.3} = 2041.66\text{m}$$

It it more than 2km, which shows that it will hit the second train.

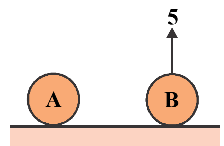
EXAMPLE 37

Balls A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s^2). Find separation between them after one second.

SOLUTION:
Relative acceleration,

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$$

Also, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s



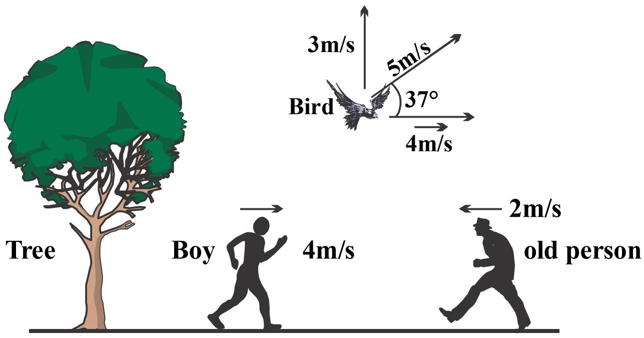
As relative acceleration is zero we can use

$$\vec{s}_{BA} \text{ (in 1 sec)} = \vec{v}_{BA} \times t = 5 \times 1 = 5\text{m}$$

Distance between A and B after 1 sec = 5m

EXAMPLE 38

(a) Find velocity of tree, bird and old man as seen by boy.
(b) Find velocity of tree, bird, boy as seen by old man
(c) Find velocity of tree, boy and old man as seen by bird.



SOLUTION:

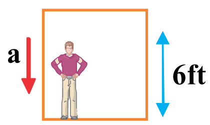
(a) With respect to boy :
$V_{tree} = 4$ m/s (←)
$V_{bird} = 3$ m/s (↑) & 0 m/s (→)
$V_{old\ man} = 6$ m/s (←)
(b) With respect to old man :
$V_{boy} = 6$ m/s (→)
$V_{tree} = 2$ m/s (→)
$V_{bird} = 6$ m/s (→) and 3 m/s (↑)
(c) With respect to Bird :
$V_{tree} = 3$ m/s (↓) and 4 m/s (←)
$V_{old\ man} = 6$ m/s (←) and 3 m/s (↓)
$V_{boy} = 3$ m/s (↓)

EXAMPLE 39

An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator.

SOLUTION:

Acceleration of coin wrt lift



$$\vec{a}_{coin/lift} = (g - a)\,\hat{j}$$

$$S = \frac{1}{2}(g-a)\,t^2 \;;\; 6 = \frac{1}{2}(32-a)\,(1)^2$$

$a = 20$ ft /s^2

EXAMPLE 40

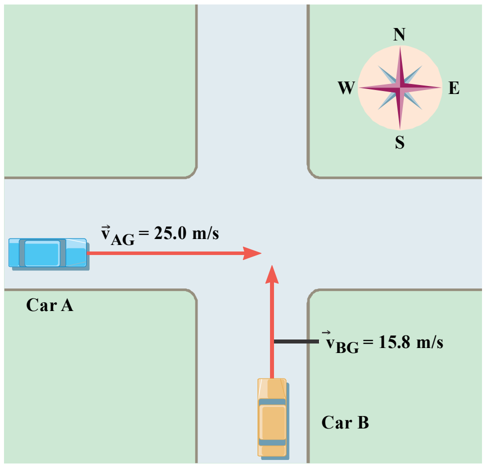
Figure 'a' shows two cars approaching an intersection along perpendicular roads. The cars have the following velocities:

$\vec{v}_{AG}$ = velocity of car A relative to the Ground = 25.0 m/s, eastward

$\vec{v}_{BG}$ = velocity of car B relative to the Ground = 15.8 m/s, northward

Find the magnitude and direction of $\vec{v}_{AB}$, where

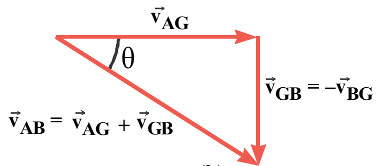
$\vec{v}_{AB}$ = velocity of car A as measured by a passenger in car B



(a)

SOLUTION:

$$\vec{v}_{AB} = \vec{v}_{AG} + \vec{v}_{GB}$$



(b)

$\vec{v}_{GB} = -\vec{v}_{BG}$

From the vector triangle in Figure b, the magnitude and direction of $\vec{v}_{AB}$ can be calculated as $v_{AB} = \sqrt{(v_{AG})^2 + (v_{GB})^2}$

$$= \sqrt{(25.0\text{ m/s})^2 + (-15.8\text{ m/s})^2} = 29.6\text{ m/s}$$

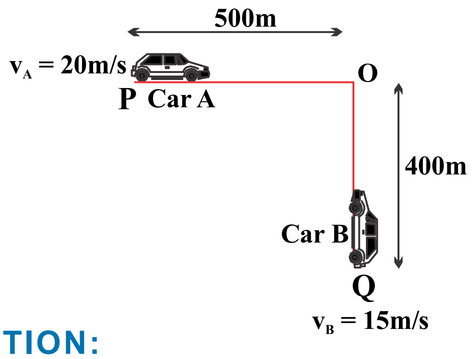
$$\cos\theta = \frac{v_{AG}}{v_{AB}} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{v_{AG}}{v_{AB}}\right)$$

$$= \cos^{-1}\left(\frac{25.0\text{ m/s}}{29.6\text{ m/s}}\right) = 32.4°$$

EXAMPLE 41

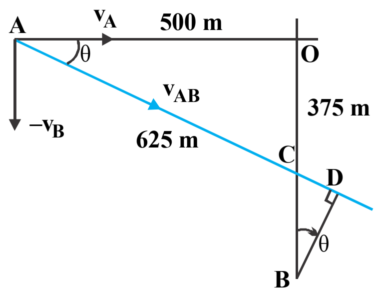
[Closest distance of approach between two bodies]

Two roads intersect at right angles. Car A is situated at P which is 500m from the intersection O on one of the roads. Car B is situated at Q which is 400m from the intersection on the other road. They start out at the same time and travel towards the intersection at 20m/s and 15m/s respectively. What is the minimum distance between them ? How long do they take to reach it.



SOLUTION:

First we find out the velocity of car A relative to B.



$$\vec{v}_{AB} = 20\hat{i} - 15\hat{j}$$

$$v_{AB} = \sqrt{20^2 + 15^2} = 25 \text{ m/s}$$

Mathematically v_{AB} means if B is assumed to be at rest A is moving with v_{AB}.

$$\tan\theta = \frac{15}{20} = \frac{3}{4}; \quad \cos\theta = \frac{4}{5}; \quad \sin\theta = \frac{3}{5}$$

$$OC = AO\tan\theta = 500 \times \frac{3}{4} = 375 \text{ m}$$

$$BC = OB - OC = 400 - 375 = 25 \text{ m}$$

$$BD = BC(\cos\theta) = 25 \times \frac{4}{5} = 20 \text{ m}$$

Shortest distance = 20 m

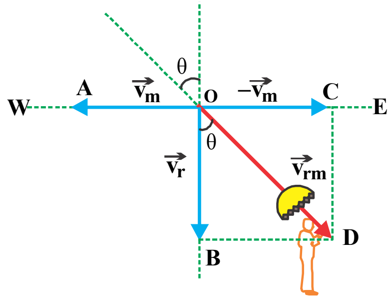
AD = AC + CD = 625 + 15 = 640

$$|\vec{v}_{AB}| = 25 \text{ m/s}$$

$$t = \frac{640}{25} = 25.6 \text{ sec.}$$

(ii) Rain Based Problems

* A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in fig.



* The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$, will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore \; v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ}$$

$$= \sqrt{v_r^2 + v_m^2}$$

* If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

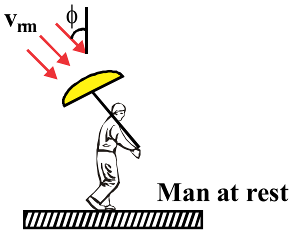
Here angle θ is from vertical towards west and is written as θ, west of vertical.

NOTE

* In the above case if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain w.r.t. man i.e. the umbrella should be hold making an angle $\theta\left(= \tan^{-1}\frac{v_m}{v_r}\right)$ west of vertical.

* **How man has to changes orientation of umbrella to prevent himself from rain**

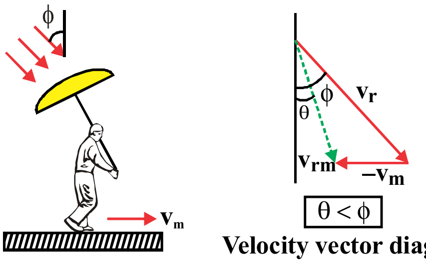
The man is stationary and the rain is falling at his back to an angle ϕ with the vertical



$v_{rm} = v_r$; $v_m = 0$; $\boxed{\theta = \phi}$

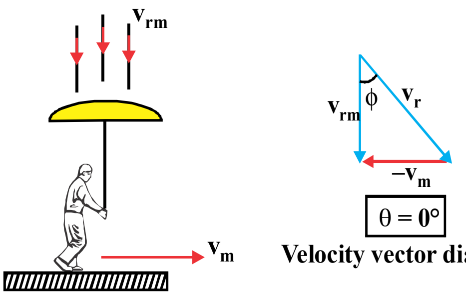
Here θ = Angle at which rain appears to man.

Now man starts moving forward with speed v_m. The relative velocity of rain w.r.t. man shifts towards vertical direction.



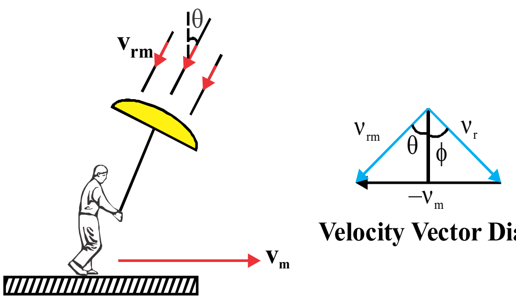
Velocity vector diagram

As the man further increase his speed, then at a particular value the rain appears to be falling vertically.



Velocity vector diagram

If the man increases his speed further more, then rain appear to be falling from the forward direction.



Velocity Vector Diagram

EXAMPLE 42

A person standing on a road has to hold his umbrella at 60° with the vertical to keep the rain away. He throws the umbrella and starts running at 20 ms^{-1}. He finds that rain drops are falling on him vertically. Find the speed of the rain drops with respect to (i) the road, and (ii) the moving person.

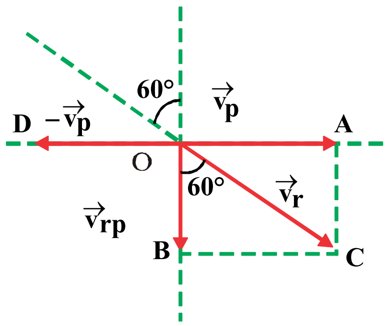
SOLUTION:

Given $\theta = 60°$ and velocity person

$\vec{v}_P = \overrightarrow{OA} = 20 \text{ ms}^{-1}$.

This velocity is the same as the velocity of person w.r.t ground. First of all let's see how the diagram works out.

$\vec{v}_{rP} = \overrightarrow{OB}$ = velocity of rain w.r.t. the person.



$\vec{v}_r = \overrightarrow{OC}$ = velocity of rain w.r.t. earth $\vec{v}_{rP}$ is along $\overrightarrow{OB}$

(i) Speed of rain drops w.r.t. earth = $\vec{v}_r = \overrightarrow{OC}$

From ΔOCB, $\dfrac{CB}{OC} = \sin 60°$

$$OC = \frac{CB}{\sin 60°} = \frac{20}{\sqrt{3}/2} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ ms}^{-1}$$

(ii) Speed of rain w.r.t. the person $\vec{v}_{rP} = \overrightarrow{OB}$

From ΔOCB, $\dfrac{OB}{CB} = \cot 60°$

$$\Rightarrow \quad OB = CB \cot 60° = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{ ms}^{-1}$$

* **Interesting Observation**
While driving a car, sometime we noticed that the rear window remains dry, even though rain is falling. This phenomenon is a consequence of relative velocity, as figure helps to explain.

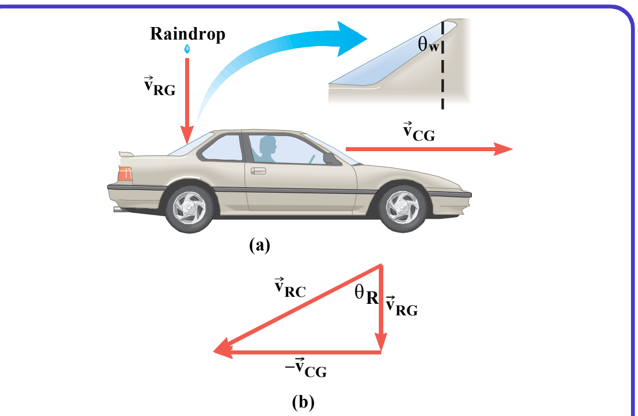


Figure : (a) With respect to the ground, a car is traveling at a velocity of $\vec{v}_{CG}$ and a raindrop is falling at a velocity of $\vec{v}_{RG}$. The rear window of the car is inclined at an angle θ_W with respect to the vertical. (b) This tail-to-head arrangement of vectors corresponds to the equation $\vec{v}_{RC} = \vec{v}_{RG} - \vec{v}_{CG}$

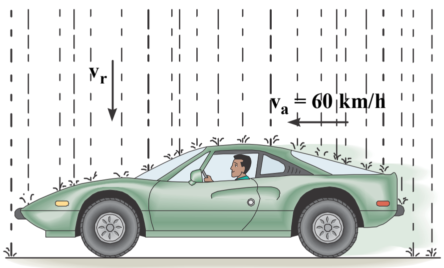
Part (a) shows a car traveling horizontally with a velocity of $\vec{v}_{CG}$ and a raindrop falling vertically with a velocity of $\vec{v}_{RG}$. Both velocities are measured relative to the ground. To determine whether the raindrop hits the window, however, we need to consider the velocity of the raindrop relative to the car, not to the ground. This velocity is $\vec{v}_{RC}$, and we know that

$$\vec{v}_{RC} = \vec{v}_{RG} + \vec{v}_{GC} = \vec{v}_{RG} - \vec{v}_{CG}$$

Here, we have used the fact that $\vec{v}_{GC} = -\vec{v}_{CG}$. Part (b) of the drawing shows the tail-to-head arrangement corresponding to this vector subtraction and indicates that the direction of $\vec{v}_{RC}$ is given by the angle θ_R. In comparison, the rear window is inclined at an angle θ_W with respect to the vertical (see the blowup in part a). When θ_R is greater than θ_W, the raindrop will miss the window. However, θ_R is determined by the speed v_{RG} of the raindrop and the speed v_{CG} of the car, according to $\theta_R = \tan^{-1}(v_{CG}/v_{RG})$. At higher car speeds, the angle θ_R becomes too large for the drop to hit the window. At a high enough speed, then, the car simply drives out from under each falling drop.

EXAMPLE 43

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60km/h. Compute the terminal (constant) velocity $\vec{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION:

$$v_r = v_a + v_{r/a}$$

$$-v_r\hat{j} = -60\hat{i} + v_{r/a}\cos 30°\hat{i} - v_{r/a}\sin 30°\hat{j}$$

$(\overset{+}{\rightarrow})$ $\quad 0 = -60 + v_{r/a}\cos 30°$

$(+\uparrow)$ $\quad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

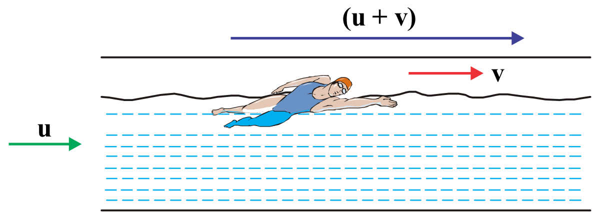
$v_r = 34.6$ km/h

(iii) River Based Problems

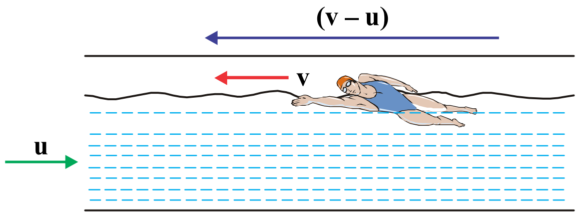
* When a man is swimming in water, he generates a velocity relative to water (v m/s) by his own efforts.

* Actual velocity of man in water will be a resultant of man's effort and the river velocity (u m/s).

* **Down stream :** Man makes efforts in direction of flow, the velocity of man w.r.t. ground is (u + v) m/s as shown below.



* **Up stream :** Man makes efforts opposite to the direction of flow, the velocity of man w.r.t. ground is (v – u) m/s as shown below.



River-Boat Problems

* In river-boat problems we come across the following three terms :

$\vec{v}_r$ = absolute velocity of river.

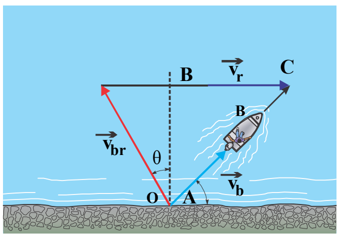
$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

and $\vec{v}_b$ = absolute velocity of boatman.

* Hence, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground. Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$.

* Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in figure. River is flowing along positive x-direction



with velocity $\vec{v}_r$ width of the river is w. Then

$$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$ and $v_{by} = v_{by} + v_{bry} = 0 + v_{br}\cos\theta = v_{br}\cos\theta$

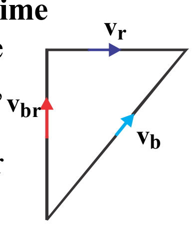
Now, time taken by the boatman to cross the river is :

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br}\cos\theta} \quad \text{or} \quad t = \frac{w}{v_{br}\cos\theta} \quad \text{..... (i)}$$

Further, displacement along x-axis when he reaches on the other bank (also called drift) (= BC) is

$$x = v_{bx}t = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \quad \text{..... (ii)}$$

* **Condition when the boatman crosses the river in shortest interval of time**



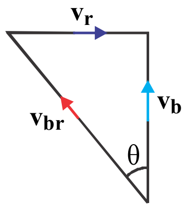
From eq. (i) we can see that time (t) will be minimum when $\theta = 0^o$, i.e., the boatman should steer his boat perpendicular to the river current. $t = \frac{w}{v_{br}}$

* **Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started (shortest distance)**

In this case, the drift (x) should be zero.

$\therefore \quad x = 0$

$$(v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} = 0$$



$$v_r = v_{br}\sin\theta \text{ or } \sin\theta = \frac{v_r}{v_{br}}$$

or $\quad \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

$$t = \frac{w}{v_b} = \frac{w}{\sqrt{v_{br}^2 - v_r^2}} \qquad \text{Since, } \sin\theta \not> 1$$

So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90^o$ and it is just impossible to reach at B if $\theta = 90^o$. Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

* **Crossing of the river with Minimum drift if $v_{br} < v_r$**

To minimise x in eq. (ii), $\frac{dx}{d\theta} = 0$

$$\frac{dx}{d\theta}=\left(\frac{v_r}{v_{br}}\sec\theta\tan\theta-\sec^2\theta\right)w=0$$

$$\frac{v_r}{v_{br}}\tan\theta=(\sec\theta)\Rightarrow\sin\theta=\frac{v_{br}}{v_r}$$

$$\Rightarrow\quad\theta=\sin^{-1}\left(\frac{v_{br}}{v_r}\right).$$

EXAMPLE 44

A man swims at an angle $\theta = 120^o$ to the direction of water flow with a speed $v_{mw} = 5$ km/hr relative to water. If the speed of water $v_w = 3$ km/hr, find the speed of the man.

Sol. Using relative velocity concept :

$$\vec{v}_{mw}=\vec{v}_m-\vec{v}_w$$

$$\vec{v}_m=\vec{v}_{mw}+\vec{v}_w$$

$$v_m=|\vec{v}_{mw}+\vec{v}_w|$$

$$=\sqrt{v_{mw}^2+v_w^2+2v_{mw}.v_w\cos\theta}$$

$$=\sqrt{5^2+3^2+2(5)(3)\cos120^\circ}$$

$$=\sqrt{25+9-15}=\sqrt{19}\text{ m / sec}$$

EXAMPLE 45

A man crosses the river in shortest time at an angle $\theta = 60^o$ to the direction of flow of water. If the speed of water is $v_w = 5$km/hr, find the speed of the man.

SOLUTION:

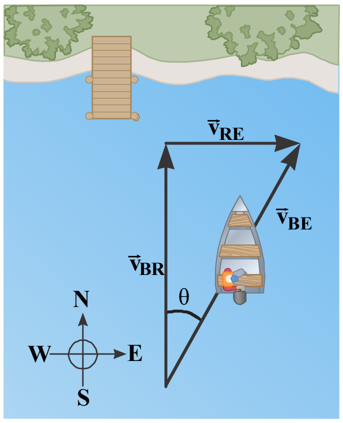
For minimum time of crossing the man should head perpendicular to the shore $\vec{v}_{mw}\perp\vec{v}_w$

Therefore, $\cos\theta=\frac{v_w}{v_m}\Rightarrow\cos60^o=\frac{4}{v_m}$

$\Rightarrow\quad v_m = 8$ km/hr

EXAMPLE 46

The boat in figure is heading due north as it crosses a wide river with a velocity of 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the velocity of the boat with respect to an observer on the river bank.



SOLUTION:

Arrange the three quantities into the proper relative velocity equation: $\vec{v}_{BR}=\vec{v}_{BE}-\vec{v}_{RE}$

Vector	x-component (km/h)	y-component (km/h)
$\vec{v}_{BR}$	0	10.0
$\vec{v}_{BE}$	v_x	v_y
$\vec{v}_{RE}$	5.00	0

Find the x-component of velocity :

$0 = v_x - 5.00$ km/h $\Rightarrow v_x = 5.00$ km/h

Find the y-component of velocity :

10.0 km/h $= v_y - 0 \Rightarrow v_y = 10.0$ km/h

Find the magnitude of $\vec{v}_{BE}$:

$$v_{BE}=\sqrt{v_x^2+v_y^2}$$

$$=\sqrt{(5.00\text{ km / h})^2+(10.0\text{ km / h})^2}$$

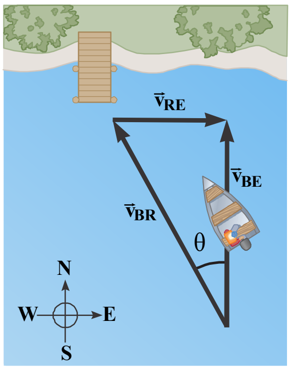
$$=11.2\text{ km/h}$$

Find the direction of $\vec{v}_{BE}$:

$$\theta=\tan^{-1}\left(\frac{v_x}{v_y}\right)=\tan^{-1}\left(\frac{5.00\text{ m / s}}{10.0\text{ m / s}}\right)=26.6^\circ$$

EXAMPLE 47

If the skipper of the boat of previous example moves with the same speed of 10.0 km/h relative to the water but now wants to travel due north, as in figure, in what direction should be head? What is the speed of the boat, according to an observer on the shore? The river is flowing east at 5.00 km/h.



SOLUTION:

Arrange the three quantities, $\vec{v}_{BR} = \vec{v}_{BE} - \vec{v}_{RE}$

Vector	x-component (km/h)	y-component (km/h)
$\vec{v}_{BR}$	$-(10.0 \text{ km/h}) \sin\theta$	$(10.0 \text{ km/h}) \cos\theta$
$\vec{v}_{BE}$	0	v
$\vec{v}_{RE}$	5.00 km / h	0

The x-component of the relative velocity equation can be used to find θ :

$$-(10.0 \text{ m/s}) \sin\theta = 0 - 5.00 \text{ km/h}$$

$$\sin\theta = \frac{5.00 \text{ km/h}}{10.0 \text{ km/h}} = \frac{1.00}{2.00}$$

Apply the inverse sine function and find θ, which is the boat's heading, east of north :

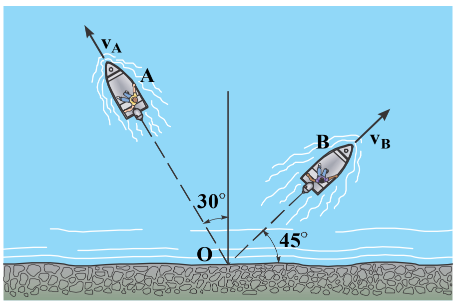
$$\theta = \sin^{-1}\left(\frac{1.00}{2.00}\right) = 30.0°$$

The y-component of the relative velocity equation can be used to find v :

$$(10.0 \text{ km/h}) \cos\theta = v \Rightarrow v = 8.66 \text{ km/h}$$

EXAMPLE 48

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$m/s and $v_B = 15$ m/s, determine the velocity of boat A with respect to boat B.



SOLUTION:

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$-20\sin 30°\hat{i} + 20\cos 30°\hat{j}$$

$$= 15\cos 45°\ \hat{i} + 15\sin 45°\ \hat{j} + \vec{v}_{A/B}$$

$$\vec{v}_{A/B} = (-20.61\hat{i} + 6.714\hat{j}) \text{ m/s}$$

EXAMPLE 49

A river flows due south with a speed of 2.0 m/s. A man steers a motorboat across the river: his velocity relative to the water is 4 m/s due east. The river is 800 m wide.

(i) What is his velocity (magnitude direction) relative to the earth ?

(ii) How much time is required to cross the river ?

(iii) How far south of his starting point will be reach the opposite bank ?

SOLUTION:

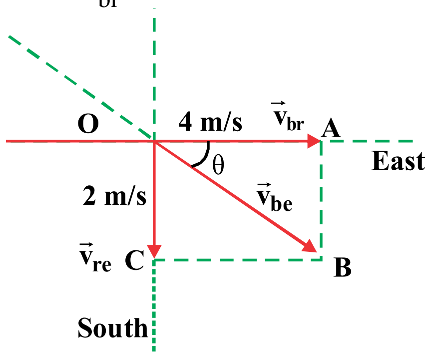
Velocity of river (i.e., speed of river w.r.t. earth)

$\vec{v}_{re} = 2$ m/s, Width of the river = 800 m

$$\vec{v}_{br} = \vec{v}_{be} - \vec{v}_{re}\ ;\ \vec{v}_{be} = \vec{v}_{br} + \vec{v}_{re}$$

(i) $\vec{v}_{be} = \sqrt{v_{br}^2 + v_{re}^2} = \sqrt{16+4} = \sqrt{20} = 4.6$ m/s

$$\tan\theta = \frac{v_{re}}{v_{br}} = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$



(ii) Time taken to cross the river

$$= \frac{\text{Displacement of boat w.r.t. river}}{\text{Velocity of boat w.r.t river}} = \frac{800}{4}$$

= 200 s

(iii) Desired position on other side is A, but due to current of river boat is drifted to position B. To find out this drift we need time taken in all to cross the river (200s) and speed of current (2 ms^{-1}).

The distance AB
= Time taken × speed of current = 200 × 2
= 400 m.
Hence, the boat is drifted by 400 m away from position A.

(iv) Aeroplane Based problems

* This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.

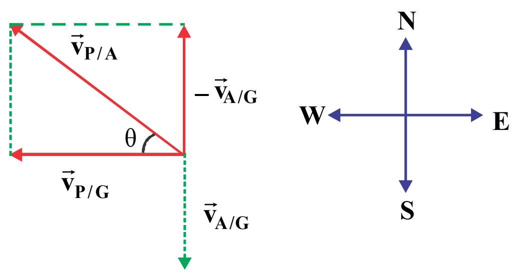
EXAMPLE 50

An aeroplane pilot wishes to fly due west. A wind of 100 km/h is blowing toward the south

(i) What is the speed of the plane with respect to ground ?

(ii) If the airspeed of the plane (its speed in still air) is 300 km/h, in which direction should the pilot head ?

SOLUTION:



(i) Velocity of air wrt ground $\vec{v}_{A/G}$ = 100 km/hr.

Velocity of plane wrt air $\vec{v}_{P/A}$ = 300 km/hr.

(ii) As the plane is to move towards west, due to air in south direction, air will try drift the plane in south direction., air will try to drift the plane in south direction. Hence, the plane has to make an angle θ towards north-west, south west direction, in order to reach at point on west.

$\vec{v}_{P/A} = \vec{v}_{P/G} - \vec{v}_{A/G}$ and $v_{P/A} \sin\theta = v_{AG}$
$\sin\theta = 1/3$

Checkup 6

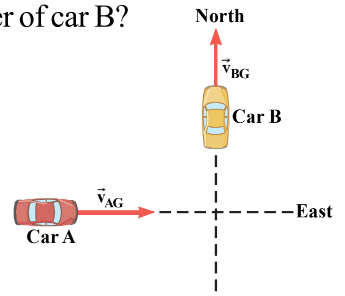
Q.1 Three cars, A, B, and C, are moving along a straight section of a highway. The velocity of A relative to B is $\vec{v}_{AB}$, the velocity of A relative to C is $\vec{v}_{AC}$, and the velocity of C relative to B is $\vec{v}_{CB}$. Fill in the missing velocities in the table.

	$\vec{v}_{AB}$	$\vec{v}_{AC}$	$\vec{v}_{CB}$
(a)	?	+40 m/s	+30 m/s
(b)	?	+50 m/s	–20 m/s
(c)	+60 m/s	+20 m/s	?
(d)	–50 m/s	?	+10 m/s

Q.2 The drawing shows two cars traveling in different directions with different speeds. Their velocities are: $\vec{v}_{AG}$ = velocity of car A relative to the ground = 27.0 m/s, due east

$\vec{v}_{BG}$ = velocity of car B relative to the ground = 21.0 m/s, due north

The passenger of car B looks out the window and sees car A. What is the velocity (magnitude and direction) of car A as observed by the passenger of car B?



Q.3 A man whose velocity in still water is 5m/s swims from point A to B (100m downstream of A) and back to A. Velocity of river is 3m/s. Find the time taken in going down stream and up stream and the average speed of the man during the motion?

Q.4 An observer standing near the shore notes that Ram is swimming upstream at 0.9 m/s and Shyam is swimming downstream at 1.5 m/s. If velocity of both in still water is the same, what is velocity of river?
(A) 0.1 m/s (B) 0.2 m/s
(C) 0.3 m/s (D) 0.4 m/s

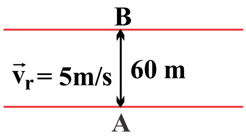
Q.5 A swimmer crosses a flowing stream of breadth b to and fro in time T_1. The time taken to cover the same distance up and down the stream is T_2. If T_3 is the time the swimmer would take to swim a distance 2b in still water, then
(A) $T_3 = T_1 + T_2$ (B) $T_1^2 = T_2T_3$
(C) $T_2^2 = T_1T_3$ (D) $T_3^2 = T_1T_2$

Q.6 Two boats were going down stream with different velocities. When one overtook the other a plastic ball was dropped from one of the boats. Some time later both boats turned back simultaneously & went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first?
(A) the boat which has greater velocity (relative to water).
(B) the boat which has lesser velocity (relative to water).
(C) both will reach the ball simultaneously.
(D) cannot be decided unless we know the actual values of the velocities and the time after which they turned around.

Q.7 Man A sitting in a car moving at 54 km/hr observes a man B in front of the car crossing perpendicularly the road of width 15 m in three seconds. Then the velocity of man B will be
(A) $5\sqrt{10}$ towards the car
(B) $5\sqrt{10}$ away from the car
(C) 5 m/s perpendicular to the road
(D) None

Q.8 A river is flowing from west to east at a speed of 5 meters per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction
(A) due north (B) 30° east of north
(C) 30° north of west (D) 60° east of north

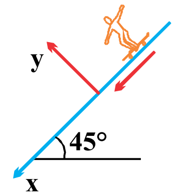
Q.9 A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 m in 5 sec. His velocity in still water should be



(A) 12 m/s (B) 13 m/s
(C) 5 m/s (D) 10 m/s

Q.10 To man running at a speed of 5 m/sec, the rain drops appear to be falling at an angle of 45° from the vertical. If the rain drops are actually falling vertically downwards, then velocity in m/sec is
(A) 5 (B) $5\sqrt{3}$
(C) $5\sqrt{2}$ (D) 4

Q.11 A boy on skateboard is coming down on a smooth incline. He throws a ball such that he catches it back What should be unit vector of the ball's velocity relative to him.

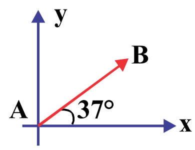


(A) $-\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$ (B) $\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$
(C) $\hat{j}$ (D) None of these

Q.12 Three ships A, B & C are in motion. The motion of A as seen by B is with speed v towards north -east. The motion of B as seen by C is with speed v towards the north-west. Then as seen by A, C will be moving towards
(A) north (B) south
(C) east (D) west

Q.13 A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction (as observed from car).
(A) East (B) North - East
(C) South - East (D) South - West

Q.14 A butterfly is flying with velocity $10\hat{i}+12\hat{j}$ m/s and wind is blowing along x axis with velocity u. If butterfly starts motion from A and after some time reaches point B, find the value of u.



IMPORTANT POINTS

* If particle travels first half distance between two places with a speed of v_1 and the rest half with a speed v_2. Average speed $=\frac{2v_1v_2}{v_1+v_2}$.

* If particle travels for first half distance of the journey with speed v_1 and for half of the remaining time with speed v_2 and then with v_3 then average speed is $\frac{2v_1(v_2+v_3)}{2v_1+v_2+v_3}$.

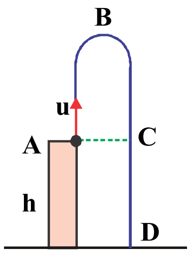
* If particle covers each 1/3 of the total distance with speed v_1, v_2 and v_3 respectively then the average speed $= \dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$.

* If particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively and C is the mid point between A and B then the velocity of the particle at C is $\sqrt{\dfrac{v_1^2 + v_2^2}{2}}$.

* If the angle between $\vec{a}$ and $\vec{v}$ is 0° or 180° the path of the particle is a straight line.

* If the angle between $\vec{a}$ and $\vec{v}$ is other than 0° or 180° the path of the particle is a curve.

* If ball is thrown upward with speed u from some reference level (h from ground) then total time taken to reach the ground is $\dfrac{u + \sqrt{u^2 + 2gh}}{g}$.



* If the body is dropped from a height H, as in time t is has fallen a distance h from its initial position, the height of the body from the ground will be $h' = H - h$ with $h = \dfrac{1}{2} gt^2$

As $h = \dfrac{1}{2} gt^2$, i.e., $h \propto t^2$,

distance fallen in time t, 2t, 3t etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

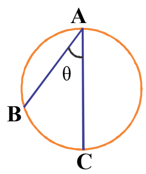
The distance fallen in the n^{th} sec

$$= h_{(n)} - h_{(n-1)} = \frac{1}{2} g(n)^2 - \frac{1}{2} g(n-1)^2$$

$$= \frac{1}{2} g(2n - 1)$$

So distances fallen in 1^{st}, 2^{nd}, 3^{rd} sec etc. will be in the ratio of 1 : 3 : 5 i.e., odd integers only.

* Time to slide along AB and AC (diameter) of circle is same.



* For a particle having zero initial velocity if $v \propto t$, $s \propto t^2$ and $v^2 \propto s$ then acceleration of particle must be constant i.e. particle is moving rectilinearly with uniform acceleration.

* For a particle having zero initial velocity if $s \propto t^\alpha$ where $\alpha > 2$, then particle's acceleration increases with time.

* For a particle having zero initial velocity if $s \propto t^\alpha$ where $\alpha < 0$, then particle's acceleration decreases with time.

* Relative velocity : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

* **Relationships between the variables describing motion**

Position →	**Velocity** →	**Acceleration**
	differentiate →	
s	$v = \dfrac{ds}{dt}$	$a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

Acceleration →	**Velocity** →	**Position**
	← **integrate**	
a	$v = \int a\,dt$	$s = \int v\,dt$

* **Acceleration may be due to change in direction or change in speed or both.**

SOLVED EXAMPLES

EXAMPLE 1

A ball is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2 s. If acceleration due to gravity is 9.8 m/s² (a) What is the height of the bridge? (b) With which velocity does the ball strike the water?

SOLUTION:

Taking the point of projection as origin and downward direction as positive,

(a) Using $s = ut + \dfrac{1}{2} at^2$ we have

$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times 2^2 = 9.8 \text{ m}$$

(u is taken to be negative as it is upwards.)

(b) Using, $v = u + at$

$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

EXAMPLE 2

A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies after 2 sec the release of the second body, if $g = 9.8\ m/s^2$.

SOLUTION:

The 2nd body falls for 2s,

so $h_2 = \frac{1}{2} g (2)^2$ ....(1)

while 1st has fallen for 2 + 1 = 3 sec so

$h_1 = \frac{1}{2} g (3)^2$ ....(2)

Separation between two bodies after 2 sec the release of 2nd body,

$$d = h_1 - h_2 = \frac{1}{2} g(3^2 - 2^2) = 4.9 \times 5 = 24.5\ m$$

EXAMPLE 3

A stone is dropped into a well and the sound of impact of stone on the water is heard after 2.056sec. of the release of stone from the top. If acc. due to gravity is 980 cm/sec² and velocity of sound in air is 350 m/s, calculate the depth of the well.

SOLUTION:

If the depth of well is h and time taken by stone to reach the bottom is t_1, then

$h = \frac{1}{2} g t_1^2$ ....(1)

time taken by sound to reach surface

$t_2 = \frac{h}{350}$ ....(2)

But $t_1 + t_2 = 2.056$ ....(3)

$$t_1 + \frac{g t_1^2}{700} = 2.056$$

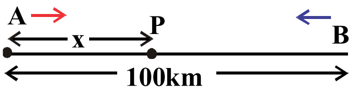
Solving quadratic equation with positive root

$t_1 = 2$ sec

the depth of well $h = \frac{1}{2} \times 9.8 \times 2^2 = 19.6\ m$

EXAMPLE 4

Two trains A and B, 100 km. apart, are travelling towards each other with starting speeds of 50 km/hr. for both. The train A is accelerating at 18 km/hr² and B is decelerating at 18 km/hr². Find the distance from the initial position of A of the point when the engines cross each other.



SOLUTION:

Let P be the point, where the two engines cross each other. If t hr be the time to occur this event, then total distance covered by the two trains should be equal to 100 km.(fig.)

i.e., AP + BP = 100

$$\Rightarrow \quad 50t + \frac{1}{2} \times 18t^2 + 50t - \frac{1}{2} \times 18t^2 = 100$$

$\Rightarrow$ 100t = 100 $\Rightarrow$ t = 1 hr.

$$\therefore \quad x = AP = 50\,(1) + \frac{1}{2} \times 18\,(1)$$

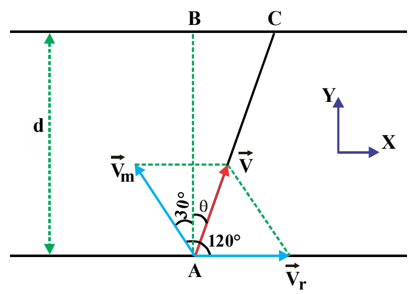
$\Rightarrow$ x = 50 + 9 = 59 km.

EXAMPLE 5

A man can swim in still water at a speed of 3km/h. He wants to, cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming. (a) Find the time he takes to cross the river. (b) At what point on the opposite bank will he arrive.

SOLUTION:

Width of river AB = d = 500 m = 1/2 km.



$V_m = 3$ km/hr velocity of man in still water

$V_r = 2$ km/hr velocity of river

V = resultant velocity of man in flowing river

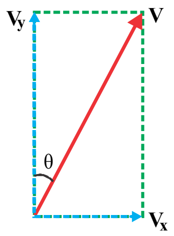
$$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j}$$

$$V_x = V_r - V_m \sin 30° = 2 - 3 \times \frac{1}{2} = \frac{1}{2} \text{ km/hr}$$

$$V_y = V_m \cos 30° = \frac{3\sqrt{3}}{2} \text{ km/hr}$$

Displacement along Y-axis,

$d = V_y \times t$



$$t = \frac{d}{V_y} = \frac{\frac{1}{2}}{\frac{3\sqrt{3}}{2}} = \frac{1}{3\sqrt{3}} \text{hr.}$$

Displacement along X-axis,

$$BC = V_x \times t = \frac{1}{2} \times \frac{1}{3\sqrt{3}} = \frac{1}{6\sqrt{3}} \text{ km.}$$

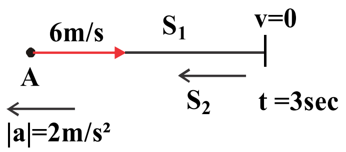
EXAMPLE 6

A particle is moving along a straight line with a constant acceleration of $-2ms^{-2}$. It passes through a point A on the line with a velocity of $6ms^{-1}$. Find the displacement from A of the particle after 5 seconds and the distance travelled by the particle in this time.

Sol. As velocity and acceleration are opposite, particle will retard and stops at some time and then return back due to acceleration.

$v = 0 - at$

$0 = 6 - 2t \Rightarrow t = 3$ sec.



$$S_1 = 6 \times 3 - \frac{1}{2} \times 2 \times (3)^2 = 18 - 9 = 9\text{m}$$

$$S_2 = \frac{1}{2} \times 2 \times (2)^2 = 4\text{m}$$

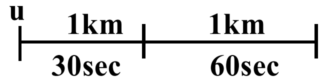
Displacement = $S_1 - S_2 = 9 - 4 = 5$m,

Total distance = 9 + 4 = 13m

EXAMPLE 7

A train travelling along a straight line with constant acceleration is observed to travel consecutive distances of 1 km in times of 30s and 60s respectively. Find the initial velocity of the train.

Sol.



$$1000 = u \times 30 + \frac{1}{2} a (30)^2 \quad \text{......... (1)}$$

$$2000 = u \times 90 + \frac{1}{2} a (90)^2 \quad \text{......... (2)}$$

Multiply eq(1) by 9 both side & subtract eq. (2)

$$7000 = 180 u \Rightarrow u = \frac{700}{18} \text{m/s} = \frac{350}{9} \text{m/s}$$

EXAMPLE 8

A particle is moving in a straight line with initial velocity u and uniform acceleration f. If the sum of the distances travelled in t^{th} and $(t + 1)^{th}$ seconds is 100 cm, then find its velocity after t seconds, in cm/s.

Sol. Let distance travelled in t^{th} second = s_1 and in $(t + 1)^{th}$ seconds = s_2 then

$$S_1 = u + \frac{f}{2}(2t - 1) \; ; \; S_2 = u + \frac{f}{2}[2(t+1) - 1]$$

$$S_1 + S_2 = 100$$

$$2u + \frac{f}{2}(2t - 1 + 2t + 2 - 1) = 100$$

$\Rightarrow 2u + 2ft = 100 \Rightarrow u + ft = 50$

$\Rightarrow v = u + ft = 50$ cm/s

EXAMPLE 9

The velocity of a particle moving in the positive direction of x-axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0, find

(i) the time dependence of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

SOLUTION:

(i) Given that $v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}$

$$\therefore \quad \frac{dx}{\sqrt{x}} = \alpha \, dt \quad \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha \, dt$$

$$2\sqrt{x} = \alpha t \Rightarrow x = (\alpha^2 t^2 / 4)$$

Velocity, $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$

and acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres

$$s = \frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$$

Average velocity

$$= \frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}\,\alpha$$

EXAMPLE 10

A person moves due east at speed 6 m/s and feels the wind is blowing to south at speed 6m/s. (a) Find the actual velocity of wind blow. (b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

SOLUTION:

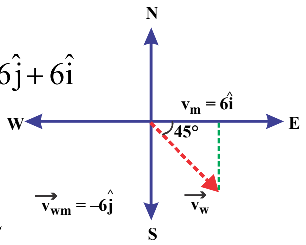
(a) $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6\hat{j} + 6\hat{i}$

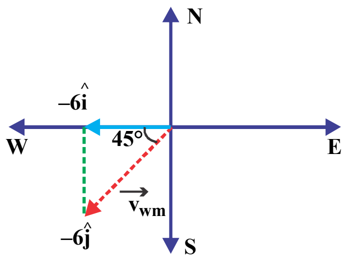
$\vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}\ m/s$

and it blowing to S–E



(b) Person doubles its velocity then $\vec{v}_m = 12\hat{i}$



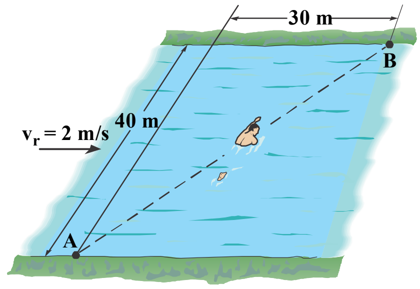
but actual wind velocity remain unchanged.

$$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i} = -6\hat{i} - 6\hat{j}$$

Relative velocity of wind is $6\sqrt{2}\ m/s$ to S-W.

EXAMPLE 11

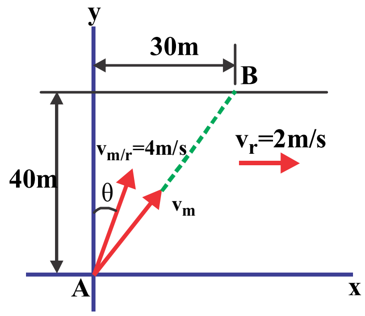
A man can swim at 4 m/s in still water. He wishes to cross the 40m-wide river to point B, 30m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the man and the time needed to make the crossing.



SOLUTION:

Relative Velocity : $v_m = v_r + v_{m/r}$

$$\frac{3}{5}v_m\hat{i} + \frac{4}{5}v_m\hat{j} = 2\hat{i} + 4\sin\theta\,\hat{i} + 4\cos\theta\,\hat{j}$$



Equating the $\hat{i}$ and $\hat{j}$ components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \quad \text{....... (1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \quad \text{....... (2)}$$

Solving Eqs. (1) and (2) yields

$v_m = 4.866$ m/s $= 4.87$ m/s

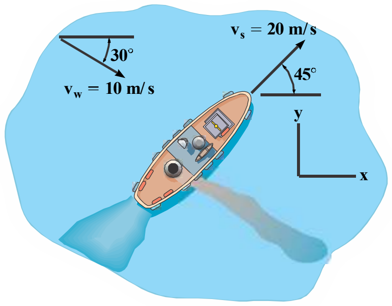
Thus, the time t required by the boat to travel from points A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3\ s$$

In order for the man to reached point B, the man has to direct himself at an angle θ with y-axis.

EXAMPLE 12

The ship travels at a constant speed of $v_s = 20$m/s and the wind is blowing at a speed of $v_w = 10$m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION:

$$\vec{v}_s = [20\cos 45°\hat{i} + 20\sin 45°\hat{j}]\ m/s$$

$$= [14.14\,\hat{i} + 14.14\,\hat{j}]\ m/s$$

and $\vec{v}_w = [10\cos 30°\hat{i} - 10\sin 30°\hat{j}]$

$$= [8.660\hat{i} - 5\hat{j}]\ m/s.$$

Applying the relative velocity equation,

$$\vec{v}_w = \vec{v}_s + \vec{v}_{w/s}$$

$$8.660\hat{i} - 5\hat{j} = 14.14\hat{i} + 14.14\hat{j} + \vec{v}_{w/s}$$

$$\vec{v}_{w/s} = [-5.482\hat{i} - 19.14\hat{j}]\ m/s$$

Thus, the magnitude of $\vec{v}_{w/s}$ is given by

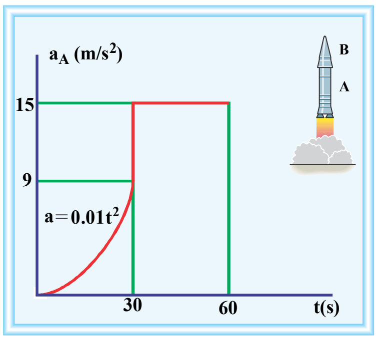
$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2}$$

$$= 19.9\ m/s$$

and the direction angle θ that $\vec{v}_{w/s}$ makes with the x axis is $\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0°$

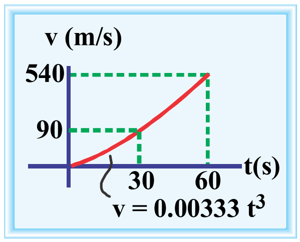
EXAMPLE 13

A two-stage rocket is fired vertically from rest at s = 0 with an acceleration as shown. After 30s the first stage A burns out and the second stage B ignites. Plot the v-t and s-t graphs which describe the motion of the second stage for $0 \le t \le 60s$.



SOLUTION:

For $0 \le t \le 30s$.



$$\int_0^v dv = \int_0^t 0.01\,t^2 dt$$

$v = 0.00333\,t^3$

When t = 30 s, v = 90 m/s

For $30s \le t \le 60s$

$$\int_{90}^{v} dv = \int_{30}^{t} 15\,dt\ ;\quad v = 15t - 360$$

When t = 60s, v = 540 m/s

EXAMPLE 14

A car moves between two sets of traffic lights, stopping at both. Its speed v m/s at time t s is modelled by $v = \frac{1}{20}t(40 - t),\ 0 \le t \le 40.$

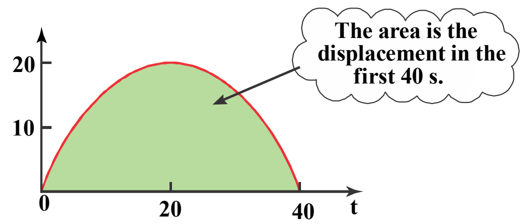
Find the times at which the car is stationary and the distance between the two sets of traffic lights.

SOLUTION:

The car is stationary when v = 0. Substituting this into the expression for the speed gives

$$0 = \frac{1}{20}t(40 - t) \Rightarrow t = 0 \text{ or } t = 40.$$

These are the times when the car starts to move away from the first set of traffic lights and stops at the second set.



The distance between the two sets of lights is given by

$$= \int_0^{40} \frac{1}{20}t(40 - t)\,dt = \frac{1}{20}\int_0^{40}(40t - t^2)\,dt$$

$$= \frac{1}{20}\left[20t^2 - \frac{t^3}{3}\right]_0^{40} = 533.3\ m$$

QUESTION BANK

HOW AND WHY?

Q.1 The average velocity for a trip has a positive value. Is it possible for the instantaneous velocity at a point during the trip to have a negative value?

Q.2 At one instant of time, a car and a truck are traveling side by side in adjacent lanes of a highway. The car has a greater velocity than the truck has. Does the car necessarily have the greater acceleration?

Q.3 One of the following statements is incorrect. (a) The car traveled around the circular track at a constant velocity. (b) The car traveled around the circular track at a constant speed. Which statement is incorrect?

Q.4 A runner runs half the remaining distance to the finish line every ten seconds. She runs in a straight line and does not ever reverse her direction. Does her acceleration have a constant magnitude?

Q.5 On a riverboat cruise, a plastic bottle is accidentally dropped overboard. A passenger on the boat estimates that the boat pulls ahead of the bottle by 5 meters each second. Is it possible to conclude that the magnitude of the velocity of the boat with respect to the shore is 5 m/s?

Q.6 Can an object have zero velocity and non-zero acceleration at the same time? Give examples.

Q.7 Can an object have zero acceleration and non-zero velocity at the same time? Give examples.

Q.8 If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

Q.9 If you stand motionless under an umbrella in a rainstorm where the drops fall vertically, you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

Q.10 Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first? Explain.

EXERCISE-1 (LEVEL-1)

SECTION - 1 (VOCABULARY BUILDER)

Choose one correct response for each question.
For Q.1-Q.6 : Match the column I with column II.

Q.1

Column I	Column II
(a) Average velocity	(i) Distance divided by the time interval.
(b) Average speed	(ii) Displacement divided by the time interval.
(c) Instantaneous velocity	(iii) The time rate of change of the position.
(d) Average acceleration	(iv) Change in velocity divided by the time interval .

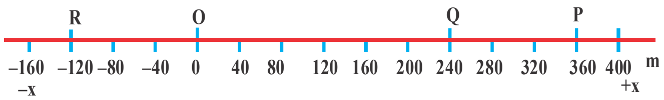
(A) (a) - ii, (b)-i, (c)-iii, (d)-iv
(B) (a) - i, (b)-ii, (c)-iii, (d)-iv
(C) (a) - ii, (b)-i, (c)-iv, (d)-iii
(D) (a) - i, (b)-iii, (c)-iv, (d)-ii

Q.2

Column I		Column II
(a) Instantaneous acceleration	(i)	equals the derivative of the velocity with respect to time.
(b) Instantaneous velocity	(ii)	slope of the velocity-time graph.
(c) Displacement	(iii)	slope of the displacement-time graph.
(d) Change in velocity	(iv)	Area under velocity-time graph
	(v)	Area under acceleration-time graph

(A) (a) - i, ii ; (b)-ii ; (c)-iv ; (d)-iii, v
(B) (a) - i, (b)-ii, (c)-iii, (d)-iv
(C) (a) - i, ii ; (b)-iii ; (c)-iv ; (d)-v
(D) (a) - iii, (b)-ii, (c)-i, (d)-iv

Q.3 Let P, Q and R represent the positions of the car at different instants of time.



Column I		Column II
(a) Distance moved by the car from O to P	(i)	360 m
(b) If the car moves from O to P and then moves back from P to Q, the path length traversed is	(ii)	480 m
(c) Displacement of car in moving from O to P	(iii)	240 m
(d) The car goes from O to P and returns back to O, the displacement is	(iv)	zero

(A) (a) - i, (b)-ii, (c)-iii, (d)-iv
(B) (a) - ii, (b)-iii, (c)-iv, (d)-i
(C) (a) - i, (b)-ii, (c)-i, (d)-iv
(D) (a) - iv, (b)-iii, (c)-i, (d)-ii

Q.4 Consider the following combinations (in column I) of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis. Column II describe what a particle is doing in each case, for an automobile on an east-west one-dimensional axis, with east considered the positive direction.

Column I Velocity	Acceleration	Column II
(a) + ve	+ve	(i) Accelerating East
(b) + ve	–ve	(ii) Braking West
(c) + ve	0	(iii) Cruising East
(d) – ve	+ve	(iv) Braking East

(A) a-i, b-ii, c-iii, d-iv
(B) a-i, b-iv, c-iii, d-ii
(C) a-ii, b-i, c-iv, d-iii
(D) a-i, b-iii, c-iv, d-ii

Q.5

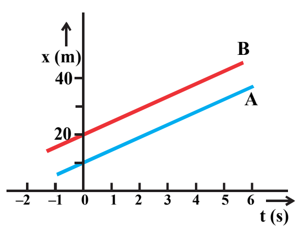
Column I (Equation)		Column II (Information given by equation)
(a) $v = u + at$	(i)	Position as a function of velocity and time
(b) $s = \frac{1}{2}(u+v)\times t$	(ii)	Velocity as a function of time
(c) $s = ut + \frac{1}{2}at^2$	(iii)	Position as a function of time
(d) $v^2 = u^2 + 2as$	(iv)	Velocity as a function of position

(A) a-ii, b-i, c-iii, d-iv
(B) a-i, b-ii, c-iii, d-iv
(C) a-ii, b-iii, c-iv, d-i
(D) a-iii, b-i, c-iv, d-ii

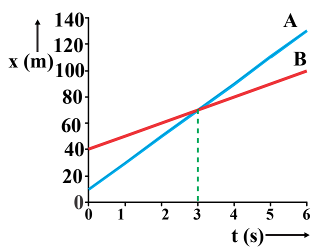
Q.6 **Column I**

(a) Position-time graphs of two objects with equal velocities.
(b) Position-time graphs of two objects with unequal velocities, showing the time of meeting.
(c) Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting.

Column II

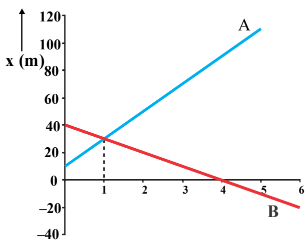


(i)



(ii)

(iii)



(A) (a) - i, (b)-iii, (c)-ii
(B) (a) - i, (b)-ii, (c)-iii
(C) (a) - ii, (b)-iii, (c)-i
(D) (a)- iii, (b)-ii, (c)-i

SECTION - 2 (BASIC CONCEPTS BUILDER)

For Q.7 to Q.18 : Choose one word for the given statement from the list.

Same, Below, Different, Zero, Decreasing, Velocity, Yes, Displacement, Above, No, Increasing, At, $a^2/4b$, $a^2/2b$.

Q.7 Can an object have a varying speed if its velocity is constant? **[Yes / No]**

Q.8 When an object moves with constant velocity, its average velocity during any time interval differ from its instantaneous velocity at any instant? **[Yes / No]**

Q.9 Can an object have a northward velocity and a southward acceleration? **[Yes / No]**

Q.10 Can the velocity of an object be negative when its acceleration is postive ? **[Yes / No]**

Q.11 Can an object be increasing in speed as its acceleration decreases ? **[Yes / No]**

Q.12 Can the equations of kinematics be used in a situation where the acceleration varies in time? **[Yes / No]**

Q.13 A student at the top of a building of height h throws one ball upward with a speed of v_i and then throws a second ball downward with the same initial speed, v_i. Velocities of the balls when they reach the ground are _____

Q.14 For uniform motion, acceleration is ____

Q.15 The ______ at a particular instant is equal to the slope of the tangent drawn on position-time graph at that instant.

Q.16 The area under the velocity-time curve between times t_1 and t_2 is equal to the _____ of the object during that interval of time.

Q.17 If a particle speed is ____ acceleration is in the direction of velocity; if its speed is _________, acceleration is in the direction opposite to that of the velocity.

Q.18 Velocity of A w.r.t. to B is always same as velocity of B w.r.t. to A. **[Yes / No]**

SECTION - 3 (ENHANCE PROBLEM SOLVING SKILLS)

Choose one correct response for each question.

PART 1 POSITION, PATH LENGTH AND DISPLACEMENT

Q.19 The numerical ratio of distance to displacement is
(A) always equal to one
(B) always less than one
(C) always greater than one
(D) equal to or more than one

Q.20 An athlete is running on a circular track of radius 50 meter. Calculate the displacement (in m) of the athlete after completing 5 rounds of the track.
(A) 0 (B) 50
(C) 100 (D) 75

Q.21 A monkey is moving on circular path of radius 80m. Calculate the distance covered by the monkey in one round.
(A) 160.00 m (B) 542.40m
(C) 502.40m (D) 602.40m

Q.22 Which of the following statements is incorrect?
(A) Displacement is independent of the choice of origin of the axis.
(B) Displacement may or may not be equal to the distance travelled.
(C) When a particle returns to its starting point, its displacement is not zero.
(D) Displacement does not tell the nature of the actual motion of a particle between the points.

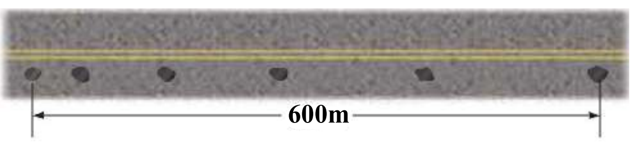
Q.23 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Determine how long the drunkard takes to fall in a pit 13 m away from the start.

(A) 29s (B) 32s

(C) 37s (D) 24s

PART 2 AVERAGE VELOCITY AND AVERAGE SPEED

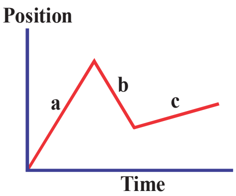
Q.24 One drop of oil falls straight down onto the road from the engine of a moving car every 5 s. Figure shows the pattern of the drops left behind on the pavement. What is the average speed of the car over this section of its motion?



(A) 20 m/s (B) 24 m/s

(C) 30 m/s (D) 100 m/s

Q.25 The graph accompanying this problem shows a three-part motion. For each of the three parts, a, b, and c, identify the direction of the motion. A positive velocity denotes motion to the right.



(A) a right, b left, c right (B) a right, b right, c left

(C) a right, b left, c left (D) a left, b right, c left

Q.26 A jogger runs along a straight and level road for a distance of 8.0 km and then runs back to her starting point. The time for this round-trip is 2.0h. Which one of the following statements is true?

(A) Her average speed is 8.0 km/h, but there is not enough information to determine her average velocity.

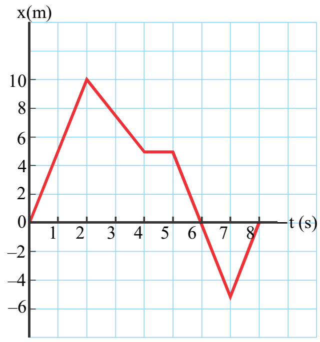
(B) Her average speed is 8.0 km/h, and her average velocity is 8.0 km/h.

(C) Her average speed is 8.0 km/h, and her average velocity is 0 km/h.

(D) None of these

For Q.27-Q.31

The position versus time for a certain particle moving along the x axis is shown in Figure.



Q.27 Find the average velocity in the time intervals 0 to 2 s.

(A) 3 m/s (B) 4 m/s

(C) 5 m/s (D) 2 m/s

Q.28 Find the average velocity in the time intervals 0 to 4s.

(A) 1.2 m/s (B) 3.2 m/s

(C) 4.2 m/s (D) 5.2 m/s

Q.29 Find the average velocity in the time intervals 2 s to 4 s.

(A) – 0.5 m/s (B) – 1.5 m/s

(C) – 2.5 m/s (D) – 3.5 m/s

Q.30 Find the average velocity in the time intervals 4 s to 7 s

(A) – 1.3 m/s (B) – 2.5 m/s

(C) – 6.1 m/s (D) – 3.3 m/s

Q.31 Find the average velocity in the time intervals 0 to 8 s.

(A) 1 m/s (B) 0 m/s

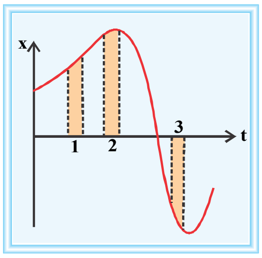
(C) 5 m/s (D) 2 m/s

Q.32 A bicyclist is travelling along a straight road for the first half time with speed v_1 and for second half time with speed v_2. What is the average speed of the bicyclist?

(A) $\frac{v_1 + v_2}{2}$ (B) $\frac{v_1 - v_2}{2}$

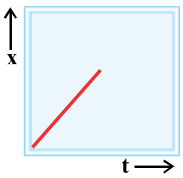
(C) $\frac{2v_1v_2}{v_1 + v_2}$ (D) None of these

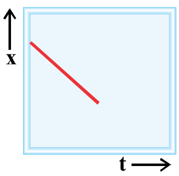
Q.33 Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown.

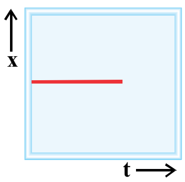


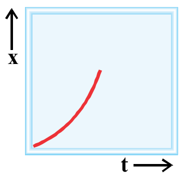
Choose the correct statement –
(A) Average speed is greatest in interval 3.
(B) Average speed is least in interval 2.
(C) Average speed is greatest in interval 1.
(D) Both (A) and (B)

Q.34 Which of the following graphs represents the position time graph of a particle moving with negative velocity?

(A) 

(B) 

(C) 

(D) 

Q.35 The area under velocity-time graph for a particle in a given interval of time represents
(A) velocity (B) acceleration
(C) work done (D) displacement

Q.36 A table clock has its minute hand 4 cm long. Choose the correct statement
(A) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 a.m. is 4.4×10^{-3} cm/s
(B) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 p.m is 1.8×10^{-4} cm/s
(C) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 p.m is 4.4×10^{-4} cm/s
(D) Both (A) and (B)

Q.37 Which of the following changes when a particle is moving with uniform velocity?
(A) Position (B) Speed
(C) Velocity (D) Acceleration

For Q.38-Q.39

The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m and $b = 2.5$ m s^{-2} and t is measured in seconds.

Q.38 The average velocity of the object between t = 2 s and t = 4 s is
(A) 5 m s^{-1} (B) 10 m s^{-1}
(C) 15 m s^{-1} (D) 20 m s^{-1}

Q.39 The velocity of the object at t = 2s is
(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s

Q.40 A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is –
(A) $\frac{v_1 + v_2}{2}$ (B) $\frac{2v_1 + v_2}{v_1 + v_2}$
(C) $\frac{2v_1v_2}{v_1 + v_2}$ (D) $\frac{L(v_1 + v_2)}{v_1v_2}$

For Q.41-Q.42

A particle moves according to the equation $x = 10t^2$ where x is in meters and t is in seconds.

Q.41 Find the average velocity for the time interval from 2.00 s to 3.00 s.
(A) 50.0 m/s (B) 31.0 m/s
(C) 41.0 m/s (D) 20.0 m/s

Q.42 Find the average velocity for the time interval from 2.00 to 2.10 s.
(A) 50.0 m/s (B) 31.0 m/s
(C) 41.0 m/s (D) 20.0 m/s

Q.43 A cyclist moving on a circular track of radius 40 m completes half a revolution in 40 s. His average velocity is
(A) zero (B) 4π m/s
(C) 2 m/s (D) 8π m/s

For Q.44-Q.45

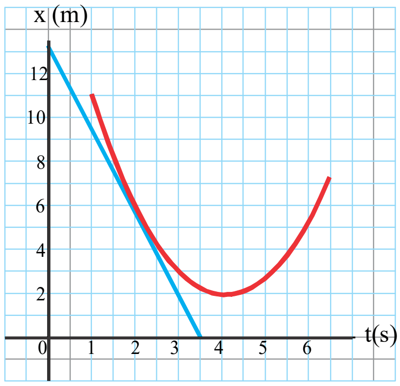
A person walks first at a constant speed of 5m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s.

Q.44 What is her average speed over the entire trip?
(A) 1.25 m/s (B) 3.75 m/s
(C) 4.15 m/s (D) 5.75 m/s

Q.45 What is her average velocity over the entire trip?
(A) 0 m/s (B) 1 m/s
(C) 2 m/s (D) 3 m/s

PART 3 INSTANTANEOUS VELOCITY AND SPEED

For Q.46-Q.48



A position-time graph for a particle moving along the x axis is shown in figure.

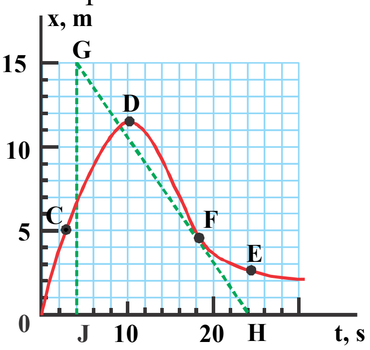
Q.46 Find the average velocity in the time interval t = 1.50 s to t = 4.00 s.
(A) – 1.2 m/s (B) – 2.4 m/s
(C) – 3.8 m/s (D) – 4.2 m/s

Q.47 Determine the instantaneous velocity at t=2.00s by measuring the slope of the tangent line shown in the graph.
(A) – 1.2 m/s (B) – 2.4 m/s
(C) – 3.8 m/s (D) – 4.2 m/s

Q.48 At what value of t is the velocity zero?
(A) 4s (B) 2s
(C) 6s (D) 8s

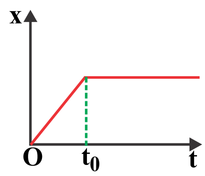
Q.49 A particle moves with uniform velocity. Which of the following statements about the motion of the particle is true?
(A) Its speed is zero. (B) Its acceleration is zero.
(C) Its acceleration is opposite to the velocity.
(D) Its speed may be variable.

Q.50 With the help of given fig. find the instantaneous velocity at point F for the object whose motion the curve represents.



(A) – 0.25 m/s (B) – 0.5 m/s
(C) – 0.75 m/s (D) – 0.1 m/s

Q.51 Figure shows the displacement (x)-time (t) graph of the particle moving on the x-axis.



(A) The particle is at rest.
(B) The particle is continuously going along x-direction.
(C) The velocity of the particle increases upto time t_0 and then becomes constant.
(D) The particle moves at a constant velocity up to a time t_0 and then stops.

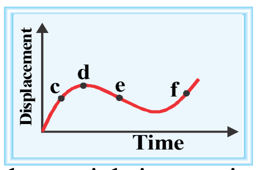
For Q.52-Q.53

The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds.

Q.52 Evaluate its position at t = 3 s
(A) 7.0 m (B) 17.0 m
(C) 27.0 m (D) 20.0 m

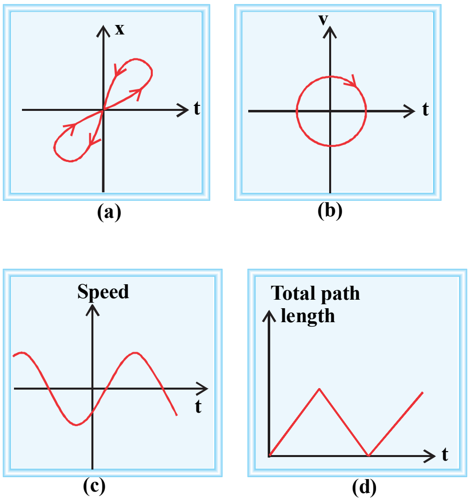
Q.53 Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero, to find the velocity at t = 3 s.
(A) 7.0 m/s (B) 20.0 m/s
(C) 27.0 m/s (D) 18.0 m/s

Q.54 The displacement-time graph of a moving particle is as shown in the figure. The instantaneous velocity of the particle is negative at the point



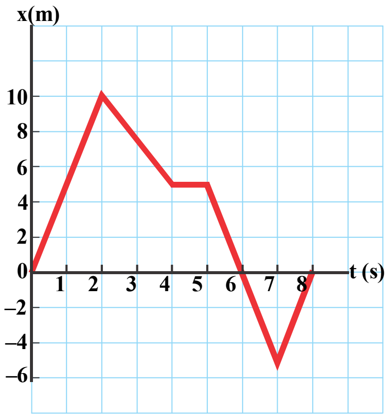
(A) c (B) e
(C) d (D) f

Q.55 Look at the graphs (a) to (d) (Fig.) carefully which of these cannot possibly represent one-dimensional motion of a particle.



(A) a, b (B) c, d
(C) a, b, c (D) all of these

For Q.56-Q.59
Find the instantaneous velocity of the particle described in figure at the following times:



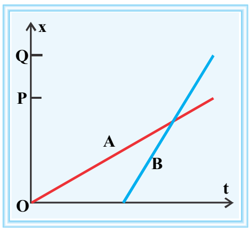
Q.56 t = 1.0 s,
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.57 t = 3.0 s,
(A) –1.5 m/s (B) –3.5 m/s
(C) –2.5 m/s (D) –4.5 m/s

Q.58 t = 4.5 s
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.59 t = 7.5 s.
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.60 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig.



Choose the INCORRECT statement –
(A) A lives closer to the school than B.
(B) A starts from the school earlier than B.
(C) A walks faster than B.
(D) A and B reach home at the same time.

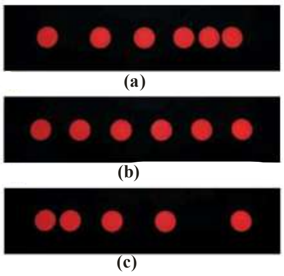
PART 4 ACCELERATION

Q.61 The velocity of a train is 80.0 km/h, due west. One and a half hours later its velocity is 65.0 km/h, due west. What is the train's average acceleration?
(A) 10 km/h^2,due west (B) 43.3km/h^2, due west
(C) 10 km/h^2, due east (D) 43.3 km/h^2, due east

Q.62 When the pilot reverses the propeller in a boat moving north, the boat moves with an acceleration directed south. Assume the acceleration of the boat remains constant in magnitude and direction. What happens to the boat?
(A) It eventually stops and remains stopped.
(B) It eventually stops and then speeds up in the forward direction.
(C) It eventually stops and then speeds up in the reverse direction.
(D) It never stops but loses speed more and more slowly forever.

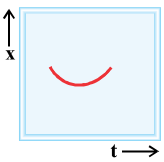
Q.63 As an object moves along the x axis, many measurements are made of its position, enough to generate a smooth, accurate graph of x versus t. Which of the following quantities for the object cannot be obtained from this graph alone?
(A) the velocity at any instant.
(B) the acceleration at any instant.
(C) the displacement during some time interval
(D) the average velocity during some time interval.

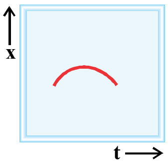
Q.64 Each of the strobe photographs (a), (b), and (c) in Figure was taken of a single disk moving toward the right, which we take as the positive direction. Within each photograph, the time interval between images is constant.

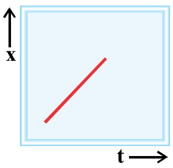


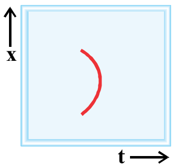
Choose the correct option –
(A) Photograph (b) shows motion with zero acceleration.
(B) Photograph (c) shows motion with positive acceleration.
(C) Photograph (a) shows motion with negative acceleration.
(D) All of these

Q.65 Position-time graph for motion with zero acceleration is

(A) 

(B) 

(C) 

(D) 

Q.66 An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acceleration ?
(A) 1.5 m/s^2 (B) 2.5 m/s^2
(C) 3.5 m/s^2 (D) 0.5 m/s^2

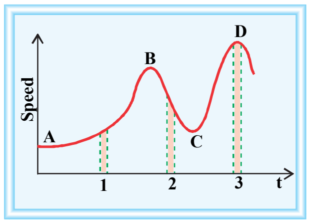
Q.67 A car starts from rest and acquires velocity equal to 10 m/s after 5 sec. Find the acceleration of the car.
(A) 1.5 m/s^2 (B) 2.5 m/s^2
(C) 3.5 m/s^2 (D) 2.0 m/s^2

Q.68 The position x of a particle varies with time 't' as $x = at^2 - bt^3$. When will the acceleration of the particle become zero?
(A) $t = a/3b$ (B) $t = a/2b$
(C) $t = a/b$ (D) $t = 2a/b$

Q.69 A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval?
(Note: 1 ms = 10^{-3} s.)
(A) 0.34×10^4 m/s^2 (B) 1.34×10^6 m/s^2
(C) 2.17×10^4 m/s^2 (D) 1.34×10^4 m/s^2

Q.70 The area under acceleration-time graph represents the –
(A) initial velocity (B) final velocity
(C) change in velocity (D) distance travelled

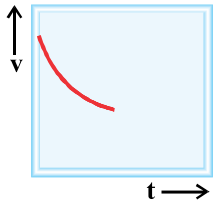
Q.71 Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown.

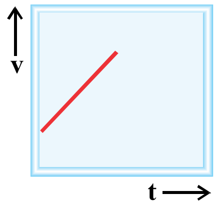


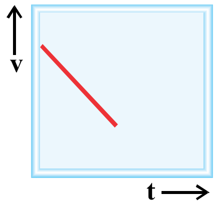
Choose the correct statement –
(a) Average acceleration is greatest in interval 2
(b) Average speed is greatest in interval 2
(c) Velocity is positive only in interval 3
(d) Acceleration is positive in intervals 1 and 3 and negative in interval 2
(A) a, b (B) c, d
(C) b, c (D) a, d

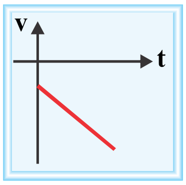
Q.72 The slope of the tangent drawn on velocity-time graph at any instant of time is equal to the instantaneous
(A) acceleration (B) velocity
(C) impulse (D) momentum

Q.73 Given below are four curves describing variation of velocity with time of a particle. Which one of these describe the motion of a particle initially in positive direction with constant negative acceleration?

(A) 

(B) 

(C) 

(D) 

PART 5 KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Q.74 In which one of the following situations can the equations of kinematics not be used?
(A) When the velocity changes from moment to moment.
(B) When the velocity remains constant.
(C) When the acceleration changes from moment to moment.
(D) When the acceleration remains constant.

Q.75 In a race two horses, Silver Bullet and Shotgun, start from rest and each maintains a constant acceleration. In the same elapsed time Silver Bullet runs 1.20 times farther than Shotgun. According to the equations of kinematics, which one of the following is true concerning the accelerations of the horses?
(A) $a_{\text{Silver Bullet}} = 1.44\ a_{\text{Shotgun}}$
(B) $a_{\text{Silver Bullet}} = a_{\text{Shotgun}}$
(C) $a_{\text{Silver Bullet}} = 2.40\ a_{\text{Shotgun}}$
(D) $a_{\text{Silver Bullet}} = 1.20\ a_{\text{Shotgun}}$

Q.76 A skateboarder starts from rest and moves down a hill with constant acceleration in a straight line, traveling for 6 s. In a second trial, he starts from rest and moves along the same straight line with the same acceleration for only 2 s. How does his displacement from his starting point in this second trial compare with that from the first trial?
(A) one-third as large (B) three times larger
(C) one-ninth as large (D) nine times larger

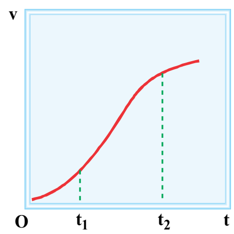
Q.77 A racing car starts from rest at t = 0 and reaches a final speed v at time t. If the acceleration of the car is constant during this time, which of the following statements are true?
(a) The car travels a distance vt.
(b) The average speed of the car is v/2.
(c) The magnitude of the acceleration of the car is v/t.
(d) The velocity of the car remains constant.
(A) a, b (B) b, c
(C) a, d (D) c, d

Q.78 The velocity of a particle (moving with uniform acceleration) at an instant is 10m/s. After 3s its velocity will becomes 16 m/s. The velocity at 2s, before the given instant will be
(A) 6 m/s (B) 4 m/s
(C) 2 m/s (D) 1 m/s

Q.79 A particle starts moving from the position of rest under a constant acc. If it covers a distance x in t sec, what distance will it travel in next t sec?
(A) y = 3x (B) y = x
(C) y = 2x (D) y = 4x

Q.80 Which of the following statements is not correct?
(A) The zero velocity of a body at any instant does not necessarily imply zero acceleration at that instant.
(B) The kinematic equation of motions are true only for motion in which the magnitude and the direction of acceleration are constants during the course of motion.
(C) The sign of acceleration tells us whether the particle's speed is increasing or decreasing.
(D) All of these

Q.81 The velocity-time graph of a particle in one-dimensional motion is shown in figure :



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2:

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (½)a(t_2 - t_1)^2$
(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$
(c) $v_{average} = (x(t_2) - x(t_1))/(t_2 - t_1)$
(d) $a_{average} = (v(t_2) - v(t_1))/(t_2 - t_1)$
(e) $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + (1/2)\, a_{average}(t_2 - t_1)^2$
(f) $x(t_2) - x(t_1)$ = area under the v-t curve bounded by the t-axis and the dotted line shown.

(A) (c), (d) and (f) (B) (a), (b) and (e)
(C) (b), (c) and (d) (D) (d), (e) and (f)

Q.82 Stopping distance of a moving vehicle is directly proportional to (Assume uniform retardation)
(A) square of the initial velocity
(B) square of the initial acceleration
(C) the initial velocity
(D) the initial acceleration

Q.83 A car moving along a straight road with speed 144 km h^{-1} is brought to a stop within a distance 200m. How long does it take for the car to stop?
(A) 5s (B) 10s
(C) 15s (D) 20s

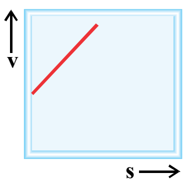
Q.84 Which of the following equations does not represent the kinematic equations of motion?

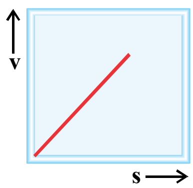
(A) $v = u + at$ (B) $S = ut + \frac{1}{2}at^2$

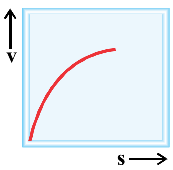
(C) $S = vt + \frac{1}{2}at^2$ (D) $v^2 - u^2 = 2aS$

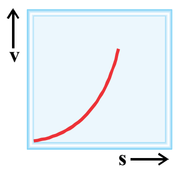
where, u = initial velocity of a body
v = final velocity of the body
a = uniform acceleration of the body
S = distance travelled by the body in time t

Q.85 A body starting from rest moves along a straight line with a constant acceleration. The variation of speed (v) with distance (s) is given by

(A)


(B)


(C)


(D)


Q.86 A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.
(A) 50 m/s^2 (B) 15 m/s^2
(C) 30 m/s^2 (D) 40 m/s^2

Q.87 A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping ?
(A) 12m (B) 18m
(C) 16m (D) 24m

For Q.88-Q.90

A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2.

Q.88 Find the distance travelled by it in the first two seconds.
(A) 2.0 m (B) 4.0 m
(C) 6.0 m (D) 8.0 m

Q.89 How much time does it take to reach the velocity 7.5 m/s?
(A) 2s (B) 5s
(C) 7s (D) 10s

Q.90 How much distance will it cover in reaching the velocity 7.5 m/s?
(A) 40 m (B) 50 m
(C) 30 m (D) 20 m

Q.91 A particle starts from rest with constant acceleration=2m/s^2.Find displacement in 5^{th} sec.
(A) 9 m (B) 18 m
(C) 25 m (D) 20 m

PART 6 MOTION UNDER GRAVITY

Q.92 A juggler throws a bowling pin straight up in the air. After the pin leaves his hand and while it is in the air, which statement is true?
(A) The velocity of the pin is always in the same direction as its acceleration.
(B) The velocity of the pin is never in the same direction as its acceleration.
(C) The acceleration of the pin is zero.
(D) The velocity of the pin is opposite its acceleration on the way up.

Q.93 A rocket is sitting on the launch pad. The engines ignite, and the rocket begins to rise straight upward, picking up speed as it goes. At about 1000 m above the ground the engines shut down, but the rocket continues straight upward, losing speed as it goes. It reaches the top of its flight path and then falls back to earth. Ignoring air resistance, decide which one of the following statements is true.

(A) All of the rocket's motion, from the moment the engines ignite until just before the rocket lands, is free-fall.

(B) Only part of the rocket's motion, from just after the engines shut down until just before it lands, is free-fall.

(C) Only the rocket's motion while the engines are firing is free-fall.

(D) Only the rocket's motion from the top of its flight path until just before landing is free-fall.

Q.94 The top of a cliff is located a distance H above the ground. At a distance H/2 there is a branch that juts out from the side of the cliff, and on this branch a bird's nest is located. Two children throw stones at the nest with the same initial speed, one stone straight downward from the top of the cliff and the other stone straight upward from the ground. In the absence of air resistance, which stone hits the nest in the least amount of time?

(A) There is insufficient information for an answer.

(B) Both stones hit the nest in the same amount of time.

(C) The stone thrown from the ground.

(D) The stone thrown from the top of the cliff.

Q.95 A rock is thrown downward from the top of a 40.0-m-tall tower with an initial speed of 12m/s. Assuming negligible air resistance, what is the speed of the rock just before hitting the ground?

(A) 28 m/s (B) 30 m/s
(C) 56 m/s (D) 784 m/s

Q.96 On another planet, a marble is released from rest at the top of a high cliff. It falls 4.00 m in the first 1 s of its motion. Through what additional distance does it fall in the next 1 s?

(A) 4.00 m (B) 8.00 m
(C) 12.0 m (D) 16.0 m

Q.97 A pebble is dropped from rest from the top of a tall cliff and falls 4.9 m after 1.0 s has elapsed. How much farther does it drop in the next 2.0 s?

(A) 9.8 m (B) 19.6 m
(C) 39 m (D) 44 m

Q.98 A cannon shell is fired straight up from the ground at an initial speed of 225 m/s. After how much time is the shell at a height of 6.20×10^2 m above the ground and moving downward?

(A) 2.96 s (B) 17.3 s
(C) 25.4 s (D) 43.0 s

Q.99 A player throws a ball vertically upwards with velocity u. At highest point,

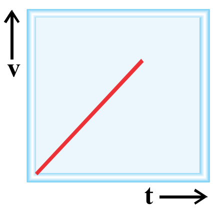
(A) both the velocity and acceleration of the ball are zero.

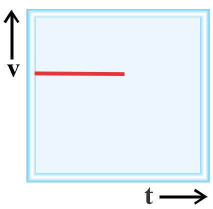
(B) the velocity of the ball is u but its acceleration zero.

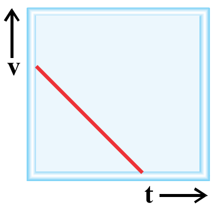
(C) the velocity of the ball is zero but its acceleration g.

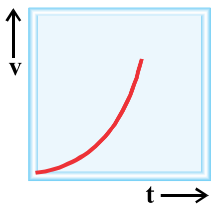
(D) the velocity of the ball is u but its acceleration g.

Q.100 Which of the following graphs represents the velocity-time variation of an object falls freely under gravity?

(A) 

(B) 

(C) 

(D) 

Q.101 A girl standing on a stationary lift (open from above) throws a ball upwards with initial speed 50 m/s. The time taken by the ball to return to her hands is (Take $g = 10\ m\ s^{-2}$)

(A) 5 s (B) 10 s
(C) 15 s (D) 20 s

Q.102 A body falling freely under gravity passes two points 30 m apart in 1 s. From what point above the upper point it began to fall?
(A) 32.1 m (B) 16.0 m
(C) 8.6 m (D) 4.0 m

Q.103 Free fall of an object in vacuum is a case of motion with
(A) uniform velocity (B) uniform acceleration
(C) variable acceleration (D) uniform speed

Q.104 The distances traversed during equal intervals of time by a body falling from rest stand to one another in the same ratio as the odd numbers beginning with unity that is, 1 : 3 : 5 : 7 : . This law was established
(A) Galileo Galilei (B) Isaac Newton
(C) Johannes Kepler (D) Albert Einstein

Q.105 A player throws a ball upwards with an initial speed of 30 m/s. How long does the ball take to return to the player's hands?
(Take $g = 10\ m\ s^{-2}$)
(A) 3s (B) 6s
(C) 9s (D) 12s

Q.106 A ball is dropped from height 'h' in the last second it travels $\frac{9h}{25}$. Find h.
(A) (25/2) g (B) (15/2) g
(C) (5/2) g (D) (35/2) g

Q.107 A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of –
(A) 3s (B) 5s
(C) 7s (D) 9s

Q.108 Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant –
(A) 2.50 m (B) 3.75 m
(C) 4.00 m (D) 1.25 m

Q.109 A stone is shot straight upward with a speed of 20m/sec from a tower 200 m high. The speed with which it strikes the ground is approximately–
(A) 60 m/sec (B) 65 m/sec
(C) 70 m/sec (D) 75 m/sec

PART 7 VARIABLE ACCELERATION

Q.110 A particle moves along a straight line in such a way that its acceleration is increasing at the rate of $2\ m/s^3$. Its initial acceleration and velocity were zero. Then, the distance which it will cover in the 3^{rd} second is:
(A) 19/3 m (B) 12/5 m
(C) 17/5 m (D) 19/4 m

Q.111 A particle of unit mass moves along a smooth horizontal surface under the action of a horizontal force of magnitude $[5 - K\sqrt{S}\,]$ N, where S is its distance from a fixed point O on the surface. The force is directed always away from O. If it starts from rest at O and comes to instantaneous rest again 400 m from O, the value of the constant K will be
(A) 3/8 (B) 3/5
(C) 2/3 (D) 2/7

For Q.112-Q.114
The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ m/s^2$, where t is in seconds. If s = 1 m and v = 2 m/s when t = 0,

Q.112 Determine the particle's velocity when t = 6 s.
(A) 32 m/s (B) 16 m/s
(C) 20 m/s (D) 24 m/s

Q.113 Determine the position when t = 6 s.
(A) 66 m (B) 67 m
(C) 68 m (D) 69 m

Q.114 Determine the total distance the particle travels during this time period.
(A) 66 m (B) 67 m
(C) 68 m (D) 69 m

Q.115 A freight train travels at $v = 60\,(1 - e^{-t})$ m/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds.

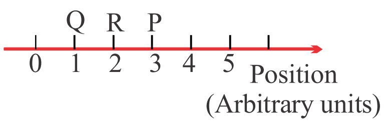
(A) 123 m (B) 160 m
(C) 140 m (D) 150 m

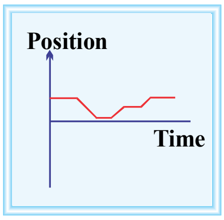
Q.116 A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$m when $t = 0$.

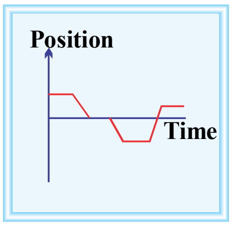
(A) 2.87 m (B) 17.87 m
(C) 7.87 m (D) 27.87 m

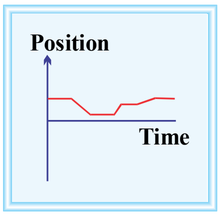
PART 8 GRAPHICAL ANALYSIS

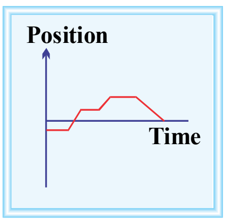
Q.117 A person initially at point P in the illustration stays there for some time and then moves along the axis to Q and stays there for some time. She then returns quickly to R, stays there for some time, and then strolls slowly back to P. The position vs. time graph below that best represents this motion is –



(A) 

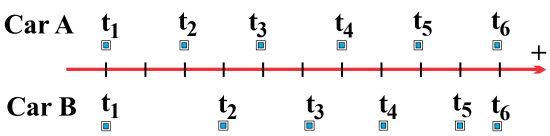
(B) 

(C) 

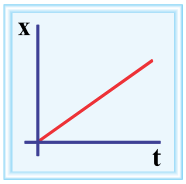
(D) 

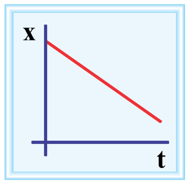
For Q.118-119

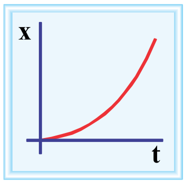
Suppose you are looking down from a helicopter at two cars traveling in the same direction along the freeway. The positions of the two cars every 2 seconds are represented by dots.

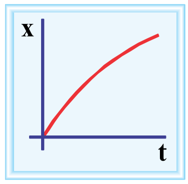


Q.118 Which graph below best represents the position-versus-time graph of Car A?

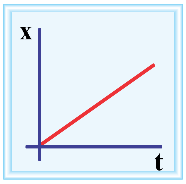
(A) 

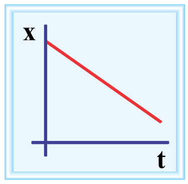
(B) 

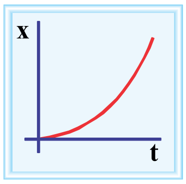
(C) 

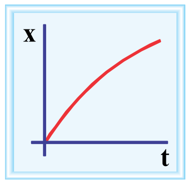
(D) 

Q.119 Which graph below best represents the position-versus-time graph of Car B?

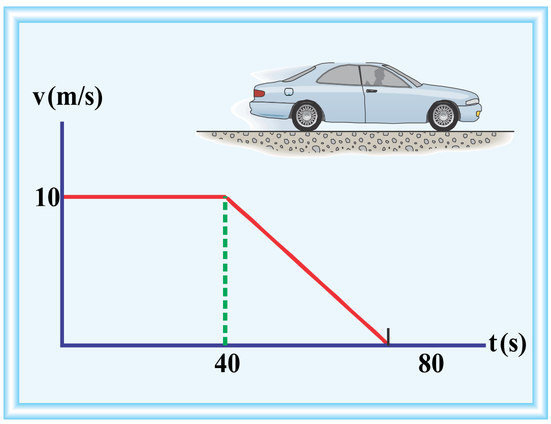
(A) 

(B) 

(C) 

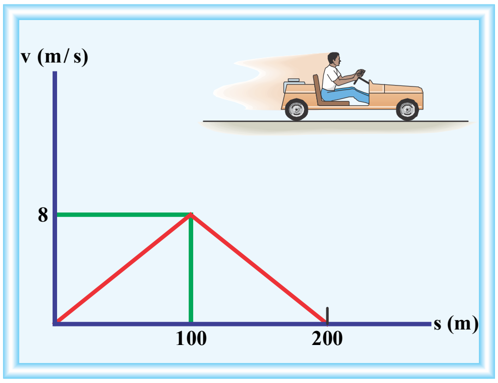
(D) 

Q.120 The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s).



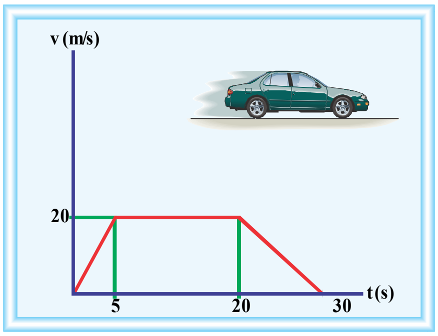
(A) 300 m (B) 400 m
(C) 500 m (D) 600 m

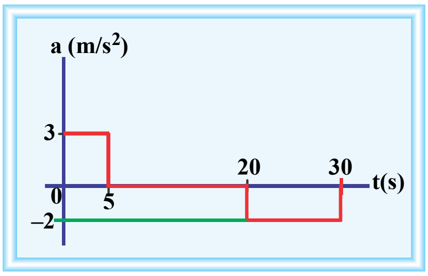
Q.121 The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at s = 50 m.

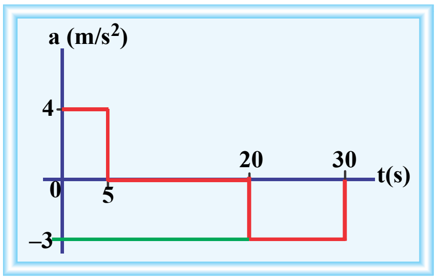


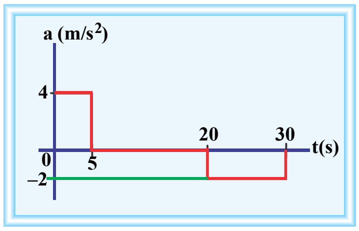
(A) 0.16 m/s^2 (B) 0.32 m/s^2
(C) 0.48 m/s^2 (D) 0.62 m/s^2

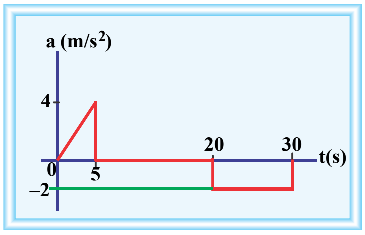
Q.122 The v–t graph of a car while traveling along a road is shown. The a–t graphs for the motion is



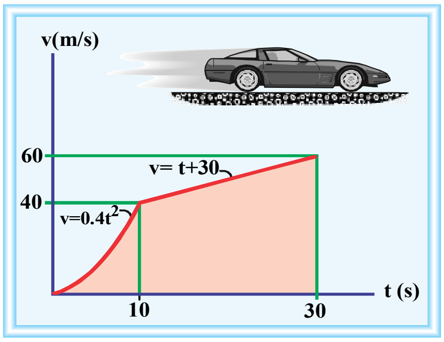
(A)


(B)


(C)


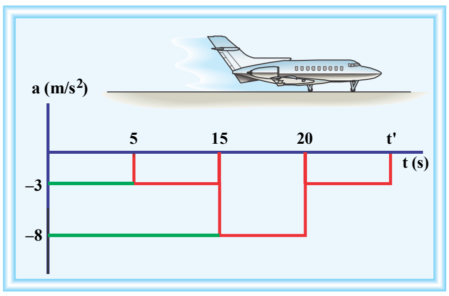
(D)


Q.123 The v–t graph for the motion of a car as it moves along a straight road is shown. Determine the maximum acceleration during the 30-s time interval. The car starts from rest at s = 0.



(A) 4 m/s^2 (B) 5 m/s^2
(C) 6 m/s^2 (D) 8 m/s^2

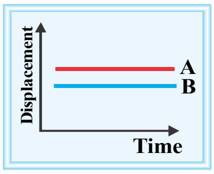
Q.124 An airplane lands on the straight runway, originally traveling at 110 m/s when s = 0. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane.

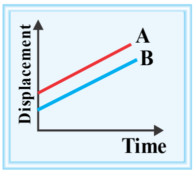


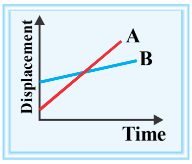
(A) 13.3 s (B) 23.3 s
(C) 33.3 s (D) 43.3 s

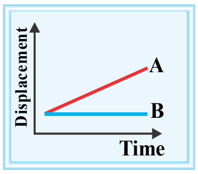
PART 9 RELATIVE VELOCITY

Q.125 Which one of the following represents displacement time graph of two objects A and B moving with zero relative velocity?

(A) 

(B) 

(C) 

(D) 

For Q.126-Q.127

Two cars A and B are running at velocities of 60 km h^{-1} and 45 km h^{-1}.

Q.126 What is the relative velocity of car A with respect to car B, if both are moving eastward?

(A) 15 km h^{-1} (B) 45 km h^{-1}
(C) 60 km h^{-1} (D) 105 km h^{-1}

Q.127 What is the relative velocity of a car A with respect to car B, if car A is moving eastward and car B is moving westward?

(A) 15 km h^{-1} (B) 45 km h^{-1}
(C) 60 km h^{-1} (D) 105 km h^{-1}

Q.128 A jet airplane travelling at the speed of 500km/h ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the combustion with respect to an observer on the ground ?

(A) – 500 km h^{-1}. (B) – 1000 km h^{-1}.
(C) – 1500 km h^{-1}. (D) – 2000 km h^{-1}.

For Q.129-Q.131

Two parallel rail tracks run north-south. On one track train A moves north with a speed of 54 km/h and on the other track train B moves south with a speed of 90 km/h.

Q.129 The velocity of train A with respect to train B is

(A) 10 m/s (B) 15 m/s
(C) 25 m/s (D) 40 m/s

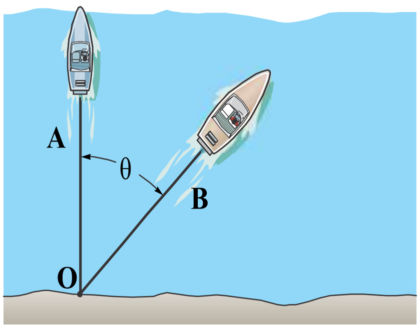
Q.130 What is the velocity of a monkey running on the roof of the train A against its motion with a velocity of 18 km/h with respect to the train A as observed by a man standing on the ground?

(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s

Q.131 On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

(A) 1 ms^{-2} (B) 2 ms^{-2}
(C) 3 ms^{-2} (D) 4 ms^{-2}

Q.132 Consider boats A and B leaving the shore at O at the same time. If A travels at v_A and B travels at v_B. General expression to determine the velocity of A with respect to B is –



(A) $\sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta}$

(B) $\sqrt{v_A^2 + v_B^2 + 2v_Av_B\cos\theta}$

(C) $\sqrt{v_A^2 - v_B^2 - 2v_Av_B\cos\theta}$

(D) $\sqrt{v_A^2 - v_B^2 + 2v_Av_B\cos\theta}$

EXERCISE-2 (LEVEL-2)

Choose one correct response for each question.

Q.1 The average velocity of a particle moving with constant acceleration a and initial velocity u in a straight line in first t seconds is

(A) $u+\frac{1}{2}at$ (B) $\frac{u}{2}$

(C) $u+at$ (D) $\frac{u+at}{2}$

Q.2 The velocity of any particle is related with its displacement as $x=\sqrt{v+1}$, Calculate acceleration at x = 5 cm.

(A) 140 m/s^2 (B) 240 m/s^2

(C) 40 m/s^2 (D) 340 m/s^2

Q.3 The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is

(A) increasing with time

(B) decreasing with time

(C) constant but not zero

(D) zero

Q.4 The velocity of the particle at any time t is given by v = 2t (3 – t) m/s. At what time is its velocity maximum?

(A) 2 s (B) 3 s

(C) (2/3) s (D) (3/2) s

Q.5 Which of the following statements is not correct regarding the motion of a particle in a straight line?

(A) x-t graph is a parabola, if motion is uniformly accelerated.

(B) v-t is a straight line inclined to the time axis, if motion is uniformly accelerated.

(C) x-t graph is a straight line inclined to the time axis if motion is uniform and acceleration is zero.

(D) v-t graph is a parabola if motion is uniform and acceleration is zero.

Q.6 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20km/h in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min. in the opposite direction. What is the time period T of the bus service. Assume buses ply on the road with constant speed.

(A) 5 mins (B) 9 mins

(C) 18 mins (D) 27 mins

Q.7 A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has speed of 27 km/h while the other has the speed of 18 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h when the two cars were separated by 36 km. The total distance covered by the bird is –

(A) 28.8 km (B) 38.8 km

(C) 48.8 km (D) 58.8 km

Q.8 It is a common observation that rain clouds can be at about 1 km altitude above the ground. If a rain drop falls from such a height freely under gravity, then what will be its speed in km h^{-1}? (Take g = 10 m s^{-2})

(A) 510 (B) 610

(C) 710 (D) 910

Q.9 In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(A) The displacement in time T must always take non-negative values.

(B) The displacement x in time T satisfies $-v_0T < x < v_0T$.

(C) The acceleration is always a non-negative number.

(D) The motion has no turning points.

Q.10 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car ? (Obtain that speed which is relevant for damaging the thief's car).

(A) 125 m/s (B) 160 m/s

(C) 95 m/s (D) 105 m/s

Q.11 A boy walks on a straight road from his home to a market 2.5 km with a speed of 5 km h^{-1}. Finding the market closed he instantly turns and walks back with a speed of 7.5 km h^{-1}. What is the average speed and average velocity of the boy between t = 0 to t = 50 min?

(A) 0, 0 (B) 6 km h^{-1}, 0
(C) 0, 6 km h^{-1} (D) 6 km h^{-1}, 6 km/h

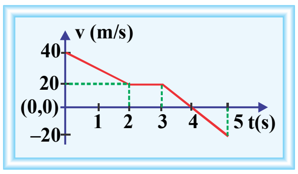
Q.12 A particle moving with uniform acceleration has average velocities v_1, v_2 and v_3 over the successive intervals of time t_1, t_2 and t_3 respectively. The value of $\frac{v_1 - v_2}{v_2 - v_3}$ will be –

(A) $\frac{t_1 - t_2}{t_2 - t_3}$ (B) $\frac{t_1 - t_2}{t_2 + t_3}$
(C) $\frac{t_1 + t_2}{t_2 - t_3}$ (D) $\frac{t_1 + t_2}{t_2 + t_3}$

Q.13 An auto travelling along a straight road increases its speed from 30.0 m/s to 50.0 m/s in a distance of 180m. If the acceleration is constant, how much time elapses while the auto moves this distance?

(A) 6.0 s (B) 4.5 s
(C) 3.6 s (B) 7.0 s

Q.14 In the given v-t graph the distance travelled by the body in 5 sec. will be



(A) 100 m (B) 80 m
(C) 40 m (D) 20 m

Q.15 Which of the following statements may be correct?

(i) Average velocity is path length divided by time interval.
(ii) In general, speed is greater than the magnitude of the velocity.
(iii) A particle moving in a given direction with a nonzero velocity can have zero speed.
(iv) The magnitude of average velocity is the average speed.

(A) (ii) and (iii) (B) (ii) and (iv)
(C) (i), (iii) and (iv) (D) (iv)

Q.16 For the one-dimensional motion, described by $x = t - \sin t$

(A) x (t) > 0 for all t > 0
(B) v (t) > 0 for all t > 0
(C) a (t) > 0 for all t > 0
(D) all of these

Q.17 A body A starts from rest with an acceleration a_1. After 2 seconds, another body B starts from rest with an acceleration a_2. If they travel equal distances in the 5^{th} second, after the start of A, then the ratio $a_1 : a_2$ is equal to

(A) 5 : 9 (B) 5 : 7
(C) 9 : 5 (D) 9 : 7

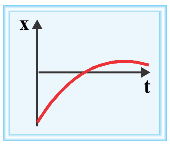
Q.18 A bus is moving with a speed of 10 m/s on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist with what speed should the scooterist chase the bus?

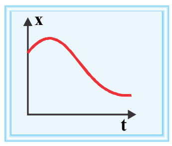
(A) 40 m/s (B) 25 m/s
(C) 10 m/s (D) 20 m/s

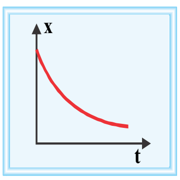
Q.19 A ball A is thrown vertically upwards with speed u. At the same instant another ball B is released from rest at height h. At time t, the speed of A relative to B is

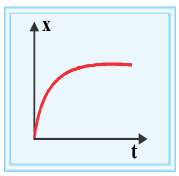
(A) u (B) u – 2gt
(C) $\sqrt{u^2 - 2gh}$ (D) u – gt

Q.20 Among the four graphs, there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?

(A)


(B)


(C)


(D)


Q.21 At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be

(A) $\frac{t_1 + t_2}{2}$ (B) $\frac{t_1 t_2}{t_2 - t_1}$

(C) $\frac{t_1 t_2}{t_2 + t_1}$ (D) $t_1 - t_2$

For Q.22-Q.23

A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building of 25 m high. (Take g = 10 m s^{-2})

Q.22 How high will the ball rise?

(A) 10 m (B) 15 m
(C) 20 m (D) 25 m

Q.23 Time taken by the ball to reach the ground is

(A) 2s (B) 3s
(C) 5s (D) 4s

Q.24 A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 second is

(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$

(C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(n+1)}{n}$

Q.25 An object falling through a fluid is observed to have acceleration given by a = g – bv where g = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. The value of constant speed is

(A) g/b (B) b/g
(C) bg (D) b

Q.26 A body covers a distance of 4 m in 3rd second and 12m in 5th second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds?

(A) 10 m (B) 30 m
(C) 40 m (D) 60 m

Q.27 A ball A is dropped from a building of height 45m. Simultaneously another identical ball B is thrown up with a speed 50 m/s. The relative speed of ball B w.r.t ball A at any instant of time is (Take g = 10 m/s^2)

(A) 0 m/s (B) 10 m/s
(C) 25 m/s (D) 50 m/s

Q.28 Two cars A and B are travelling in the same direction with velocities v_1 and v_2 ($v_1 > v_2$). When the car A is at a distance d ahead of the car B, the driver of the car A applied the brake producing a uniform retardation a. There will be no collision when

(A) $d < \frac{(v_1 - v_2)^2}{2a}$ (B) $d < \frac{v_1^2 - v_2^2}{2a}$

(C) $d > \frac{(v_1 - v_2)^2}{2a}$ (D) $d > \frac{v_1^2 - v_2^2}{2a}$

Q.29 The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in metres and t in sec. The displacement, when velocity is zero, is

(A) 24 metres (B) 12 metres
(C) 5 metres (D) Zero

Q.30 If the velocity of a particle is given by $v = (180-16x)^{1/2}$ m/s, then its acceleration will

(A) Zero (B) 8 m/s^2
(C) – 8 m/s^2 (D) 4 m/s^2

Q.31 If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is

(A) $\frac{1}{2}\sqrt{v_1 v_2}$ (B) $\frac{v_1 + v_2}{2}$

(C) $\frac{2v_1 v_2}{v_1 + v_2}$ (D) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

Q.32 A ball is projected upwards from a height h above the surface of the earth with velocity v. The time at which the ball strikes the ground is

(A) $\frac{v}{g} + \frac{2hg}{\sqrt{2}}$ (B) $\frac{v}{g}\left[1 - \sqrt{1 + \frac{2h}{g}}\right]$

(C) $\frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$ (D) $\frac{v}{g}\left[1 + \sqrt{v^2 + \frac{2g}{h}}\right]$

Q.33 A man throws ball with the same speed vertically upwards one after the other at an interval of 2seconds. What should be the speed of the throw so that more than two balls are in the sky at any time? (Given $g = 9.8\ m/s^2$)
(A) More than 19.6 m/s
(B) At least 9.8 m/s
(C) Any speed less than 19.6 m/s
(D) Only with speed 19.6 m/s

Q.34 A particle moves in a straight line with a constant acceleration. It changes its velocity from 10m/s to 20 m/s while passing through a distance 135m in t second. The value of t is –
(A) 12 (B) 9
(C) 10 (D) 1.8

Q.35 Balls A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively ($g = 10m/s^2$). Find separation between them after one second.
(A) 2m (B) 3m
(C) 5m (D) 6m

Q.36 An electron starting from rest has a velocity that increases linearly with the time that is $v = kt$, where $k = 2m/sec^2$. The distance travelled in the first 3seconds will be
(A) 9m (B) 16 m
(C) 27m (D) 36m

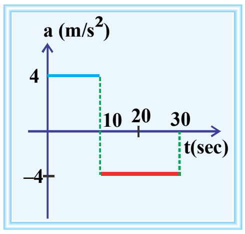
Q.37 The initial velocity of a body moving along a straight line is 7m/s. It has a uniform acceleration of $4\ m/s^2$. The distance covered by the body in the 5^{th} second of its motion is –
(A) 25 m (B) 35 m
(C) 50 m (D) 85 m

Q.38 If a body starts from rest and travels 120 cm in the 6^{th} second, then what is the acceleration
(A) $0.20\ m/s^2$ (B) $0.027\ m/s^2$
(C) $0.218\ m/s^2$ (D) $0.03\ m/s^2$

Q.39 A particle starts from rest, accelerates at $2\ m/s^2$ for 10s and then goes for constant speed for 30s and then decelerates at $4\ m/s^2$ till it stops. What is the distance travelled by it
(A) 750 m (B) 800 m
(C) 700 m (D) 850 m

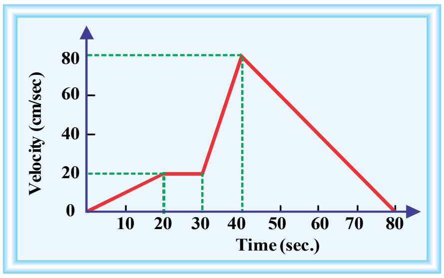
Q.40 A particle moves along X-axis in such a way that its coordinate X varies with time t according to the equation $x = (2 - 5t + 6t^2)$ m. The initial velocity of the particle is
(A) – 5m/s (B) 6 m/s
(C) – 3m/s (D) 3 m/s

Q.41 The acceleration versus time graph for a particle moving along a straight line is shown in the figure. If the particle starts from rest at t = 0, then its speed at t = 30 sec. will be–



(A) 20m/sec (B) 0 m/sec
(C) – 40 m/sec. (D) 40 m/sec.

Q.42 The v-t graph of a moving object is given in figure. The maximum acceleration is –



(A) $1\ cm/sec^2$ (B) $2\ cm/sec^2$
(C) $3\ cm/sec^2$ (D) $6\ cm/sec^2$

Q.43 A train is moving east at a speed of 5m/s. A bullet fired westward with a velocity of 10m/s crosses the train in 8sec. The length of train is –
(A) 120m (B) 60m
(C) 30m (D) 15m

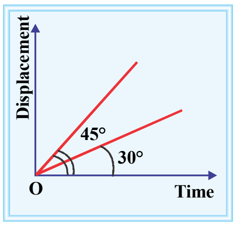
Q.44 A ball takes t second to fall from height h_1 and 2t second to fall from a height h_2. The h_1/h_2 is –
(A) 2 (B) 4
(C) 0.5 (D) 0.25

Q.45 A body starts from rest with uniform acceleration. Its velocity after 2n seconds is v_0. The displacement of the body in last n seconds of this 2n seconds interval is –

(A) $\dfrac{v_0(2n-3)}{6}$ (B) $\dfrac{v_0}{4n}(2n-1)$

(C) $\dfrac{2v_0 n}{3}$ (D) $\dfrac{3v_0 n}{4}$

Q.46 The displacement-time graphs of two moving particles make angles of 30° and 45° with the X-axis. The ratio of their velocities is –



(A) $1 : \sqrt{3}$ (B) 1: 2

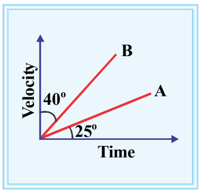
(C) 1 : 1 (D) $\sqrt{3}$: 2

Q.47 A person throws balls into air vertically upward in regular intervals of time of one second. The next ball is thrown when the velocity of the ball thrown earlier becomes zero. The height to which the balls rise is (Assume, $g = 10\ ms^{-2}$)

(A) 10m (B) 7.5m

(C) 20m (D) 5m

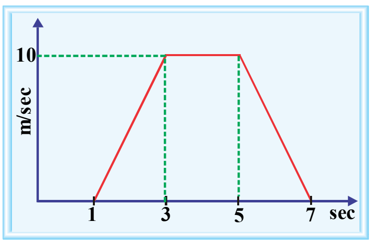
Q.48 The velocity-time graph for two bodies A and B are shown. Then the acceleration of A and B are in the ratio –



(A) sin 25° to sin 50° (B) tan 25° to tan 40°

(C) cos 25° to cos 50° (D) tan 25° to tan 50°

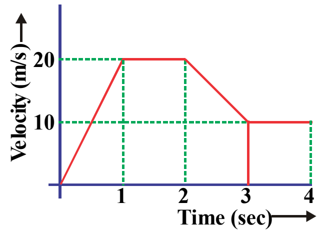
Q.49 A particle moves according to velocity time graph. Then what is the ratio between distance travelled in last 2 seconds and 7 second-



(A) 1/4 (B) 1/2

(C) 1/8 (D) 1/6

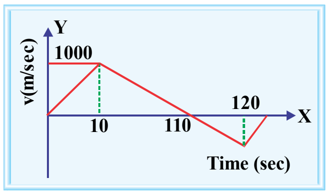
Q.50 The variation of velocity of a particle moving along straight line is shown in figure. The distance travelled by the body in 4 seconds is-



(A) 70 m (B) 60 m

(C) 40 m (D) 55 m

Q.51 Adjacent graph shows the variation of velocity of a rocket with time. Find the time of burning of fuel from the graph-



(A) 10 sec

(B) 110 sec

(C) 120 sec

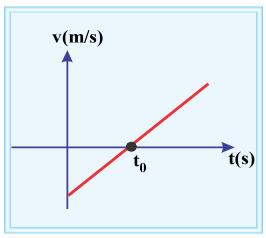
(D) Cannot be estimated from the graph

Q.52 A car starts from rest from point 'A' with constant acceleration $4\ ms^{-2}$. A passenger arrives at 'A' 10s after car has left 'A'. What is least constant speed passenger must run to catch the car.

(A) 40 m/s (B) 25 m/s

(C) 80 m/s (D) 60 m/s

Q.53 The velocity time graph of a particle moving along a straight line in a given time interval is as shown in figure. Then the particle (with increase in time starting from t = 0 sec.)



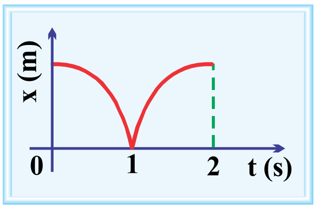
(A) speeds up continuously

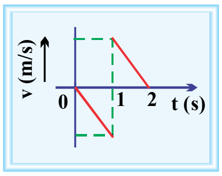
(B) first speeds down and then speeds up

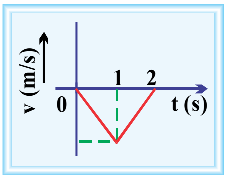
(C) Speeds down continuously

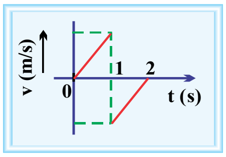
(D) first speeds up and then speeds down

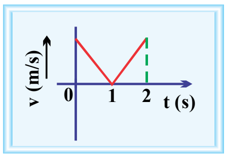
Q.54 The displacement-time graph of a moving particle with constant acceleration is shown in the figure. The velocity-time graph is best given by



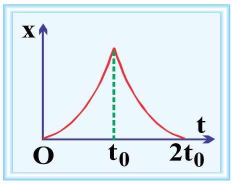
(A)


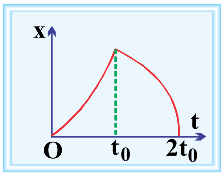
(B)


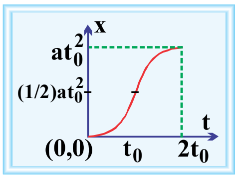
(C)


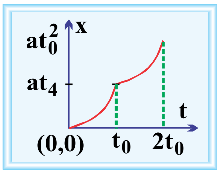
(D)


Q.55 A particle starts from origin accelerates for t_0 sec then decelerates with same acceleration till $2t_0$ sec along the x-direction. The graph representing variation of displacement (x) with time(t) is

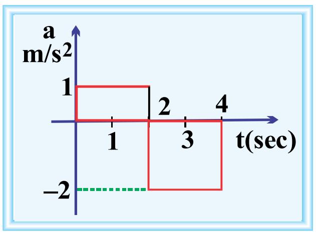
(A)


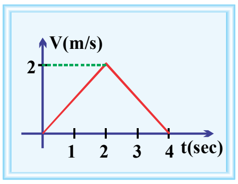
(B)


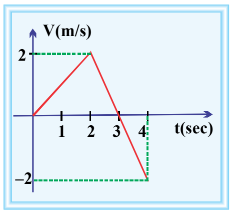
(C)


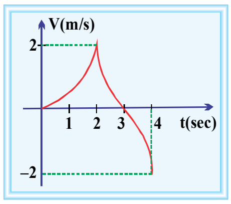
(D)


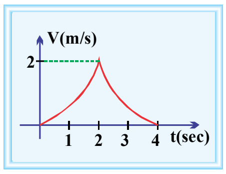
Q.56 A particle moves in one dimension. Its velocity is given by V and its acceleration by a. The figure shows its measured acceleration versus time graph. Which of the graph (V vs t) below are consistent with measured acceleration?



(A)


(B)


(C)


(D)


Q.57 Equation of motion of a body moving along x-axis at an instant t second is given by $x = 40 + 12t - t^3$ m. Displacement of the particle before coming to rest is

(A) 16 m (B) 56 m
(C) 24 m (D) 40 m

Q.58 A particle starts moving rectilinearly at time $t = 0$ such that its velocity 'v' changes with time 't' according to the equation $v = t^2 - t$ where t is in seconds and v is in m/s. The time interval for which the particle retards is

(A) $t < 1/2$ (B) $1/2 < t < 1$
(C) $t > 1$ (D) $t < 1/2$ and $t > 1$

Q.59 A ball is thrown vertically upwards from the ground. It crosses a point at the height of 25 m twice at an interval of 4 sec. With what velocity the ball was thrown ?

(A) 30 m/sec (B) 10 m/sec
(C) 20 m/sec (D) 40 m/sec

Q.60 A man in a balloon rising vertically with an acceleration of 5 m/s^2 releases a ball 2 seconds after the balloon is let go from the ground. Find the greatest height above the ground reached by the ball.

(A) 14 m (B) 15 m
(C) 5 m (D) 25 m

EXERCISE-3 (LEVEL-3)

Choose one correct response for each question.

Q.1 A body moving along a straight line travels one third of the total distance with a speed of 3 m/s. The remaining distance is covered with a speed of 4.0 m/s for half the time and 5.0 m/s for the other half of the time. The average speed during the motion is

(A) 4.0 m/s (B) 6.0 m/s
(C) 3.8 m/s (D) 2.4 m/s

Q.2 A particle starts from point A moves along a straight line path with an acceleration given by $a = p - qx$ where p, q are constants and x is distance from point A. The particle stops at point B. The maximum velocity of the particle is

(A) p/q (B) $p/\sqrt{q}$
(C) q/p (D) $\sqrt{q}/p$

Q.3 A man is standing on top of a building 100 m high. He throws two balls vertically, one at t = 0 and other after a time interval (less than 2 s). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is 15 m at t = 2 s. The gap is found to remain constant. The velocity with which the balls were thrown are – (Take $g = 10\ m\ s^{-2}$)

(A) 20 m/s, 10 m/s (B) 10 m/s, 5 m/s
(C) 16 m/s, 8 m/s (D) 30 m/s, 15 m/s

Q.4 A particle moving along a straight line has a velocity v m/s, when it cleared a distance of x m. These two are connected by the relation $v = \sqrt{49 + x}$. When its velocity is 1 m/s, its acceleration is –

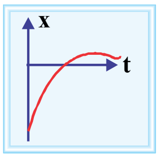
(A) $2\ m/s^2$ (B) $7\ m/s^2$
(C) $1\ m/s^2$ (D) $0.5\ m/s^2$

Q.5 The motion of a particle is described by $x = x_0(1 - e^{-kt})$; $t \geq 0$, $x_0 > 0$, $k > 0$. With what velocity does the particle start?

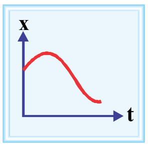
(A) x_0/k (B) x_0k
(C) k/x_0 (B) $2x_0k$

Q.6 Match the Column I with Column II.

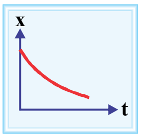
Column I — **Graph** | **Column II** — **Characteristic**

(a) 

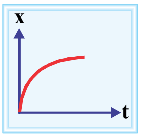
(p) Has v > 0 and a < 0 throughout.

(b) 

(q) Has x > 0 throughout and has a point with v = 0 and a point with a = 0.

(c) 

(r) Has a point with zero displacement for t > 0.

(d) 

(s) Has v < 0 and a > 0.

(A) a-p, b-q, c-s, d-r (B) a-q, b-p, c-r, d-s
(C) a-s, b-r, c-q, d-p (D) a-r, b-q, c-s, d-p

Q.7 The displacement of a particle is given by $x = (t - 2)^2$ where x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is –

(A) 4m (B) 8m
(C) 12m (D) 16 m

Q.8 A motorboat covers the distance between two spots on the river in 8h and 12h downstream and upstream respectively. The time required by the boat to cover this distance in still water is

(A) 6.3 h (B) 9.6 h
(C) 3.2 h (D) 18.12 h

Q.9 The motion of a body is given by the equation $\frac{dv}{dt} = 6 - 3v$, where v is the speed in m/s and t is time in s. The body is at rest at t = 0. The speed varies with time as

(A) $v = (1 - e^{-3t})$ (B) $v = 2(1 - e^{-3t})$
(C) $v = (1 + e^{-2t})$ (D) $v = 2(1 + e^{-2t})$

Q.10 A particle moves rectilinearly. Its displacement x at time t is given by $x^2 = at^2 + b$ where a & b are constants. Its acceleration at time t is proportional to –

(A) $\frac{1}{x^3}$ (B) $\frac{1}{x} - \frac{1}{x^2}$

(C) $-\frac{t}{x^2}$ (D) $\frac{1}{x} - \frac{t^2}{x^3}$

Q.11 A particle is released from rest from a tower of height 3h. The ratio of the intervals of time to cover three equal heights h is

(A) $t_1 : t_2 : t_3 = 3 : 2 : 1$

(B) $t_1 : t_2 : t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - 2)$

(C) $t_1 : t_2 : t_3 = \sqrt{3} : \sqrt{2} : 1$

(D) $t_1 : t_2 : t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

Q.12 An elevator car, whose floor to ceiling distance is equal to 2.7 m, starts ascending with constant acceleration of $1.2 ms^{-2}$. 2 sec after the start, a bolt begins fallings from the ceiling of the car. The free fall time of the bolt is –

(A) $\sqrt{0.54}$ s (B) $\sqrt{6}$ s

(C) 0.7 s (D) 1 s

Q.13 The velocity of a bullet is reduced from 200m/s to 100m/s while travelling through a wooden block of thickness 10cm. The retardation, assuming it to be uniform, will be

(A) 10×10^4 m/s^2 (B) 12×10^4 m/s^2

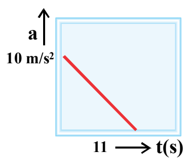
(C) 13.5×10^4 m/s^2 (D) 15×10^4 m/s^2

Q.14 A stone is dropped from a certain height which can reach the ground in 5 second. If the stone is stopped after 3second of its fall and then allowed to fall again, then the time taken by the stone to reach the ground for the remaining distance is

(A) 2 sec (B) 3 sec

(C) 4 sec (D) None of these

Q.15 A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be –



(A) 110 m/s (B) 55 m/s

(C) 550 m/s (D) 660 m/s

Q.16 A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then the maximum velocity acquired by the car

(A) $\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)t$ (B) $\left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)t$

(C) $\frac{(\alpha + \beta)t}{\alpha\beta}$ (D) $\frac{\alpha\beta t}{\alpha + \beta}$

Q.17 A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Find speed after 10s.

(A) $7\sqrt{2}$ m/s (B) $5\sqrt{2}$ m/s

(C) $3\sqrt{2}$ m/s (D) $2\sqrt{2}$ m/s

Q.18 A body travelling along a straight line traversed one-third of the total distance with a velocity v_1. The remaining part of the distance was covered with a velocity v_2 for half the time and with velocity v_3 for the other half of time. The mean velocity averaged over the whole time of motion is ?

(A) $\frac{3v_1(v_2 + v_3)}{2v_1 + v_2 + v_3}$ (B) $\frac{3v_1(v_2 + v_3)}{4v_1 + v_2 + v_3}$

(C) $\frac{v_1(v_2 + v_3)}{4v_1 + v_2 + v_3}$ (D) $\frac{v_1(v_2 + v_3)}{v_1 + v_2 + v_3}$

Q.19 The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(A) $v_0 t + \frac{1}{3}bt^2$ (B) $v_0 t + \frac{1}{3}bt^3$

(C) $v_0 t + \frac{1}{6}bt^3$ (D) $v_0 t + \frac{1}{2}bt^2$

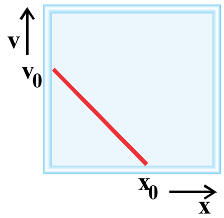
Q.20 Two trains travelling on the same track are approaching each other with equal speeds of 40m/s. The drivers of the trains begin to decelerate simultaneously when they are just 2.0km apart. Assuming the decelerations to be uniform and equal, the value of the deceleration to barely avoid collision should be

(A) 11.8 m/s^2 (B) 11.0 m/s^2
(C) 2.1 m/s^2 (D) 0.8 m/s^2

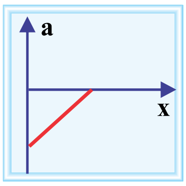
Q.21 Starting from rest, acceleration of a particle is $a = 2(t-1)$. Velocity of the particle at $t = 5s$ is

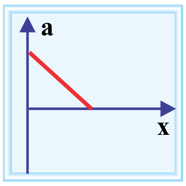
(A) 15 m/sec (B) 25 m/sec
(C) 5 m/sec (D) None of these

Q.22 The given graph shows the variation of velocity with displacement.



Which one of the graph given correctly represents the variation
of acceleration with displacement

(A)


(B)


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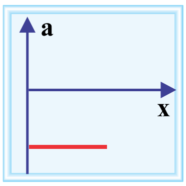
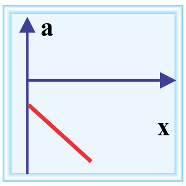


(D)


Q.23 A bird flies with a speed of 10 km/h and a car moves with uniform speed of 8 km/h. Both start from B towards A (BA = 40km) at the same instant. The bird having reached A, flies back immediately to meet the approaching car. As soon as it reaches the car, it flies back to A. The bird repeats this till both the car and the bird reach. A simultaneously. The total distance flown by the bird is –

(A) 80 km (B) 40 km
(C) 50 km (D) None of these

Q.24 A ball is thrown vertically upwards. It was observed, at a height h twice with a time interval Δt. The initial velocity of the ball is-

(A) $\sqrt{8gh + g^2(\Delta t)^2}$ (B) $\sqrt{8gh + \left(\frac{g\Delta t}{2}\right)^2}$

(C) $\frac{1}{2}\sqrt{8gh + g^2(\Delta t)^2}$ (D) $\sqrt{8gh + 4g^2(\Delta t)^2}$

Q.25 A body A begins to move with initial velocity 2 m/s and continues to move at a constant acceleration 'a'. In $\Delta t = 10$ seconds after the body A begins to move a body B departs from the same point with a initial velocity 12 m/sec. and moves with the same acceleration 'a'. What is the maximum acceleration 'a' at which the body B can overtake A ?

(A) 1 m/s^2 (B) 2 m/s^2
(C) 1/2 m/s^2 (D) 3 m/s^2

Q.26 The velocity of an object moving rectilinearly is given as a function of time by $v = 4t - 3t^2$, where v is in m/s and t is in seconds. The average velocity of particle between t = 0 to t = 2 sec is

(A) 0 (B) –2m/s
(C) – 4m/s (D) None of these

Q.27 A particle starts from rest and travel a distance x with uniform acceleration, then moves uniformly a distance 2x and finally comes to rest after moving further 5x with uniform retardation. Find the ratio of maximum speed to average speed.

(A) 4/7 (B) 3/7
(C) 5/7 (D) 6/7

For Q.28-Q.29

A particle moving with uniform acceleration along a straight line passes three successive points A, B and C where the distances AB : BC is 3 : 5 & the time taken from A to B is 40 sec. If the velocities at A & C are 5 m/s & 15 m/s respectively.

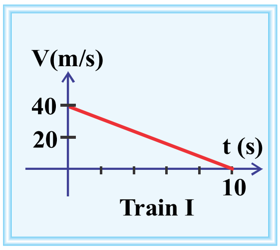
Q.28 The velocity of the particle at B is –

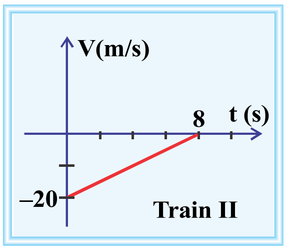
(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s

Q.29 Acceleration of the particle is –

(A) 1/5 m/s^2 (B) 1/6 m/s^2
(C) 1/8 m/s^2 (D) 1/9 m/s^2

Q.30 Two trains, which are moving along different tracks in opposite directions, are put on the same track due to a mistake. Their drivers, on noticing the mistake, start slowing down the trains when the trains are 300 m apart. Graphs given below show their velocities as function of time as the trains slow down. Find the separation, between the trains when both have stopped.





(A) 10m (B) 15m
(C) 20m (D) 25m

Q.31 The greatest acceleration or deceleration that a train may have is a. Find the minimum time in which the train may reach from one station to the other separated by a distance d.

(A) $2\sqrt{d/a}$ (B) $2\sqrt{a/d}$
(C) $\sqrt{a/d}$ (D) $3\sqrt{d/a}$

Q.32 Two cars A and B are traveling with same speed 108 km/hr on a road. The car A is 3m behind the car B. The driver of the car B suddenly applies brake causing it to decelerate at a constant rate of 3 m/s^2. One sec. later the driver of the car A applies brake and just manage to avoid a rear end collision. Find the constant deceleration of the car A.

(A) – 6 m/s^2 (B) – 3 m/s^2
(C) – 2 m/s^2 (D) – 8 m/s^2

Q.33 A driver traveling at 60 km/h sees a chicken dash out onto the road and slams on the brakes. Accelerating at –7m/s^2, the car stops just in time 23.3m down the road. What was the driver's reaction time (i.e., the time that elapsed before he engaged the brake) ?

(A) 0.2 sec. (B) 0.3 sec.
(C) 0.4 sec. (D) 0.5 sec.

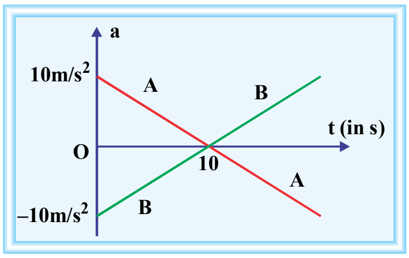
Q.34 A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively. If C is the mid point between A and B then determine the velocity of the particle at C.

(A) $\sqrt{\frac{v_1 + v_2}{2}}$ (B) $\sqrt{\frac{v_1 \times v_2}{2}}$

(C) $\sqrt{\frac{v_1^2 + v_2^2}{2}}$ (D) $\sqrt{\frac{v_1^2 - v_2^2}{2}}$

For Q.35-Q.36

Two particles A and B start moving from rest from x = 0 at t = 0 along x-axis with variable acceleration as shown in graph.



Q.35 Separation between the two particles –

(A) Decreases for first 10sec. and then increases.
(B) Increases for first 10sec. and then decreases.
(C) Increases for first 20sec.
(D) Increases for first 20sec. then continuously decreases.

Q.36 Relative velocity of A with respect to B is 21 m/s along negative x-axis after

(A) 11 sec. (B) 21 sec.
(C) 31 sec. (D) None of these

Directions : Assertion-Reason type questions.

(A) S-1 is True, S-2 is True, S-2 is a correct explanation for S-1
(B) S-1 is True, S-2 is True; S-2 is NOT a correct explanation for S-1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.37 **Statement-1 :** Velocity-time graph for an object in uniform motion along a straight path is a straight line parallel to the time axis.

Statement-2 : In uniform motion of an object velocity increases as the square of time elapsed.

Q.38 **Statement 1 :** Magnitude of average velocity is equal to average speed.
Statement 2 : Magnitude of instantaneous velocity is equal to instantaneous speed.

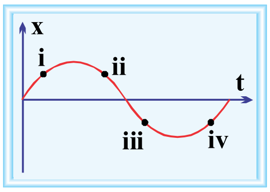
Q.39 **Statement 1 :** When velocity of a particle is zero then acceleration of particle is also zero.
Statement 2 : Acceleration is equal to rate of change of velocity.

Q.40 A car is moving along a straight line. It's displacement (x) – time (t) graph is shown in column II. Match the entries in column I with points on graph.

Column I

(a) $x \to$ negative, $v \to$ positive, $a \to$ positive
(b) $x \to$ positive, $v \to$ negative, $a \to$ negative
(c) $x \to$ negative, $v \to$ negative, $a \to$ positive
(d) $x \to$ positive, $v \to$ positive, $a \to$ negative

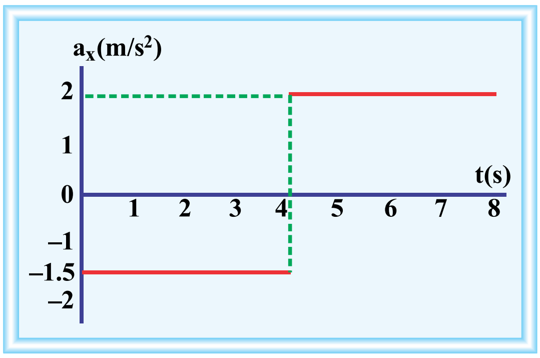
Column II



(A) a-iv, b-ii, c-iii, d-i (B) a-i, b-iv, c-iii, d-ii
(C) a-ii, b-i, c-iv, d-iii (D) a-i, b-iii, c-iv, d-ii

For Q.41-Q.42

An object has the acceleration v/s time graph shown.

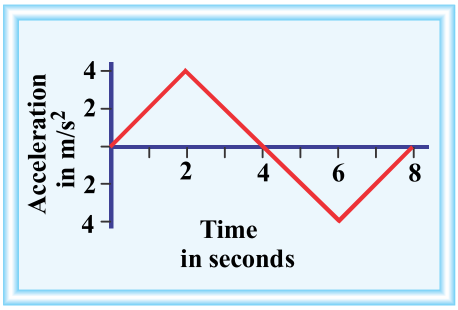


Q.41 When does the object return to it's initial velocity?
(A) At t = 4 s
(B) At t = 7 s
(C) At t = 8 s
(D) Impossible to determine from the given information.

Q.42 When is the object at rest?
(A) At t = 0 s
(B) At t = 4 s
(C) Between t = 4 s and t = 8 s
(D) Impossible to determine from the given information

For Q.43-Q.44

The next two questions will refer to the following situation. A car starts from rest and accelerates as shown in the accompanying diagram.



Q.43 At what time would the car be moving with the greatest velocity ?
(A) 2 seconds (B) 4 seconds
(C) 6 seconds (D) 8 seconds

Q.44 At what time would the car be farthest from its original starting position ?
(A) 2 seconds (B) 4 seconds
(C) 6 seconds (D) 8 seconds

ANSWER KEY

CHECK UP 1

(1) (A) (2) (A) (3) (C) (4) (B)
(5) (C) (6) (B)
(7) (40π) m, 80 m from A to B
(8) (a) 110m, (b) 50m, $\theta = \tan^{-1}(3/4) = 37°$
(9) 49.3 kmh^{-1}, 21.4 kmh^{-1} (10) – 3m, 5m
(11) $\frac{\pi R}{T}$; $\frac{2R}{T}$ (from A to B)
(12) (a) 60s (b) 4 m/s
(13) (a) – 2m/s, (b) $v(t) = 6t - 6t^2$ m/s
(c) (1/2) s; (3/2) m/s
(14) (a) 3 sec, (b) –2 m/s, (c) 2.5 m/s.

CHECK UP 2

(1) (D) (2) (B) (3) (D) (4) (D)
(5) (C) (6) (a) + 2.8 m/s, (b) + 2.8 m/s.
(7) (a) 1.6 m/s^2 (b) 2.0×10^1 m
(8) The rifle with the short barrel
(9) 5.63 s. (10) 1.4 m/s^2
(11) (A). (12) (A).

CHECK UP 3

(1) (D) (2) – 1.5m/s^2 (3) (B) (4) (b)
(5) (B) (6) (D) (7) (D) (8) (C)
(9) (C) (10) (D) (11) (D)

CHECK UP 4

(1) $s\sqrt{2}$ (2) (C) (3) (D) (4) (A)
(5) (D)

CHECK UP 5

(1) (B) (2) (B) (3) (B) (4) (B)
(5) (A) (6) (B) (7) (B) (8) (C)

CHECK UP 6

(1) (a) +70 m/s, (b) +30 m/s, (c) +40 m/s
(d) –60 m/s
(2) 34.2 m/s, $\theta = 37.9°$ south of east.
(3) 12.5 sec, 50 sec, 3.2 m/s (4) (C)
(5) (B) (6) (C) (7) (B) (8) (A)
(9) (B) (10) (A) (11) (C) (12) (B)
(13) (C) (14) 6 m/s

HOW AND WHY ?

(1) Yes (2) No (3) (a) (4) No
(5) No (6) Yes
(7) Yes (8) Due to relative reference frame
(9) Due to relative reference frame
(10) The rower heading straight across will reach the other side first

EXERCISE - 1

SECTION - 1 & 2

(1) (A) (2) (C) (3) (C)
(4) (B) (5) (A) (6) (B)
(7) No (8) No (9) Yes
(10) Yes (11) Yes (12) No
(13) Same (14) Zero (15) Velocity
(16) Displacement
(17) Increasing, Decreasing (18) No

EXERCISE 1 (SECTION–3)

Q	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
A	D	A	C	C	C	B	A	C	C	A	C	D	B	A	D	B	D	D	A	C	B	C	A	C	C
Q	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
A	B	A	B	C	A	B	C	D	C	D	B	D	A	C	C	A	C	C	C	B	D	C	B	D	A
Q	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
A	D	C	D	A	C	C	D	C	B	A	A	C	A	A	B	C	C	A	A	C	D	B	A	D	B
Q	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
A	D	B	C	C	D	C	A	D	A	B	A	B	A	B	B	B	A	A	A	B	A	A	C	C	A
Q	119	120	121	122	123	124	125	126	127	128	129	130	131	132											
A	D	D	B	C	D	C	B	A	D	B	D	B	A	A											

EXERCISE–2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	B	A	D	D	B	A	A	B	D	B	D	B	A	D	A	A	D	A	B	C	C	C	A	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	D	C	D	C	D	C	A	B	C	A	A	C	A	A	C	D	A	D	D	A	D	D	A	D
Q	51	52	53	54	55	56	57	58	59	60															
A	A	C	B	A	C	B	A	B	A	B															

EXERCISE–3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	B	A	D	B	D	B	B	B	A	D	C	D	C	B	D	A	B	C	D	A	A	C	C	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44						
A	A	A	B	C	C	A	A	A	C	C	B	C	D	D	A	B	D	B	D						