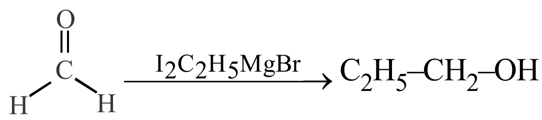
JEE MAIN 2020
FULL TEST-1 SOLUTIONS

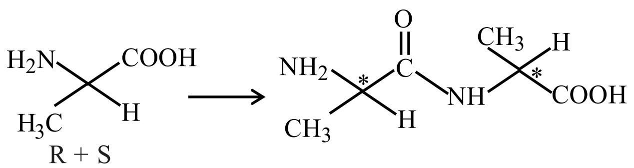
STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	4	3	4	2	4	3	2	1	3	4	3
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	2	2	2	4	3	2	3	3	2	9	7
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	3	4	5	4	1	1	3	1	3	4	3
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	3	2	2	1	2	2	2	1	3	2	2
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	3	4	4	2	3	2	2	3	1	3	2
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	2	3	4	4	3	4	4	3	2	2	1
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**		
A	1	2	2	4	1	3	5	4	2		

(1) **(4).** Pentaammine chloride cobalt (III) chloride
$[Co(NH_3)_5Cl]Cl_2$ $[Co(NH_3)_5Cl^-]^{+2} + 2Cl^-$
Gives 3 ions in aqueous solution.

(2) **(3).**



(3) **(4).**

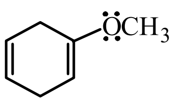


Dipeptide has two chiral carbon and both side unsymmetrical hence RR, RS, SR and SS is possible.

(4) **(2).** Charge on 1 milimole M^{n+} ions = 193 cb

$$= \frac{n \times 96500}{1000}$$

$$n = \frac{193 \times 1000}{96500} = 2$$

(5) **(4).** In  , the lone pair of oxygen get delocalised on the π bond located on the next carbon.

(6) **(3).** Quantum number set $n = 2, \ell = 0, m = -1$ it is not possible (not valid).
(Value of m : $+\ell$ to $-\ell$]

(7) **(2).** $n_{eq.}$ $KMnO_4$
$= n_{eq.}[FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$
or $n \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$
$\therefore$ $n = 2$

(8) **(1).**

$$CH_3 - \underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}} - Cl + AgNO_3 \rightarrow (CH_3)C^{\oplus} + \underset{\text{White ppt}}{AgCl(s)\downarrow}$$

Reason : Due to most stable carbocation formation tert-butyl chloride given the ppt immediately.

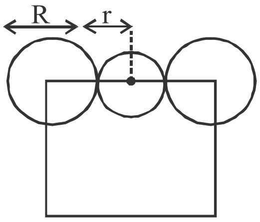
(9) **(3).** $\Delta T_b = K_b \times m$

$$\therefore \quad \frac{\Delta T_{b\,(A)}}{\Delta T_{b\,(B)}} = \frac{K_{b\,(A)}}{K_{b\,(B)}} \text{ as } m_A = m_B$$

$$\therefore \quad \frac{\Delta T_{b\,(A)}}{\Delta T_{b\,(B)}} = \frac{1}{5}$$

(10) **(4).** Colligative properties of colloidal solution are smaller than true solution.

(11) **(3).** $a = 2(R + r)$



$\frac{a}{2} = (R + r)$ ...(1)

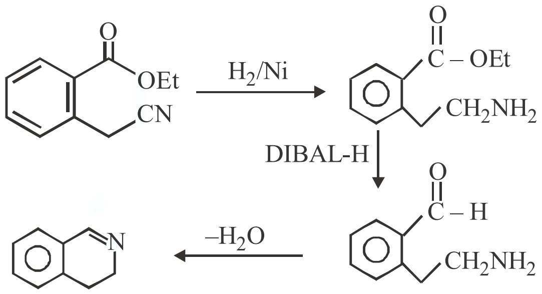
$a\sqrt{3} = 4R$ ...(2)

Using (1) & (2)

$\frac{a}{2} = \frac{a\sqrt{3}}{4} = r$; $a\left(\frac{2-\sqrt{3}}{4}\right) = r$

$r = 0.067\ a$

(12) **(2).**



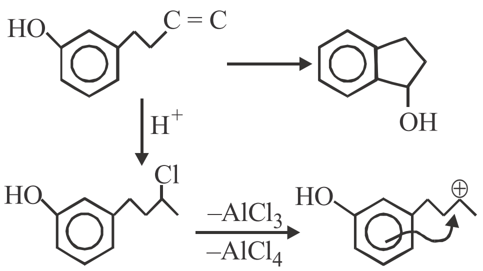
(13) **(2).** A complex having strong field ligand has tendency to absorb light of highest energy. Among the three complexes $[Co(NH_3)_6]^{+3}$ will absorb radiation of highest energy and least wavelength.

$[Co(NH_3)_5H_2O]^{3+}$ has field weaker than the above compound and therefore absorb radiation of lesser energy and more wavelength.

$[CoCl(NH_3)_5]^{2+}$ has the weakest field and therefore will absorb light of least energy and highest wavelength.

Strength of ligand $NH_3 > H_2O > Cl$.

(14) **(2).**



(15) **(4).** Nitrogen oxides and hydrocarbons (unburnt fuel) are primary pollutant that leads to photochemical smog.

(16) **(3).** NaH is an example of ionic hydride which is also known as saline hydride.

(17) **(2).** $H_2(g) + I_2(g) \rightarrow 2HI\ (g)$

Apply Arrhenius equation

$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{600} - \frac{1}{800}\right)$$

$$\log\frac{1}{2.5\times10^{-4}} = \frac{E_a}{2.303\times8.31}\left(\frac{200}{600\times800}\right)$$

$\therefore$ $E_a \approx 166$ kJ/mol

(18) **(3).** Polarisation power $\propto$ + ive charge

$\propto 1$ / Size of ion

(19) **(3).** $(CH_3)_2CHBr \xrightarrow[\text{(ii) CuI}]{\text{(i) Li}} [(CH_3)_2CH]_2CuLi$

$\xrightarrow{(CH_3)_2CHCH_2Br} (CH_3)_2CH–CH_2–CH(CH_3)_2$

(20) **(2).** $MnO_4^- + Fe^{2+} + 8H^+ \rightarrow Mn^{2+} + Fe^{3+}$

0.1M	1.72M	
0.01 M	1.0M	0.09 M

$E^0_{RP\ (MnO_4^-/Mn^{2+})} > E^0_{RP\ (H^+/H_2)}$

So, permangnate electrode behave as cathode

$E°_{cell} = 1.51$ V

Using Nernsts equation : (n = 5)

$$E^0_{MnO_4^-/Mn^{2+}} = 1.51 + \frac{0.059}{5}\log\frac{[MnO_4^-][H^+]^8}{[Mn^{2+}]}$$

$= 1.51 – 0.011 = 1.499 \approx 1.5V$

(21) **9.** Initial moles of $H_2O_2 = 2 \times 2 = 4$

Moles of H_2O_2 left after 20 min.

$= 4 \times \frac{1}{2^2} = 1$ (2 half lives)

Moles of H_2O_2 decomposed 20 min.

$= 4 – 1 = 3$

$H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$

Moles of O_2 formed during 20 min. = 3/2

Volume change = $\frac{\frac{3}{2}\times R\times T}{P}$

$q = 0$; $\Delta U = W$

$W = –P_{ext}(V_2 – V_1) = – P_{ext}\Delta V$

$= -\frac{3}{2} \times 2 \times 300 = – 900$ Cal.

(22) **7.** (i) Planar molecules :

XeF_2, ClF_3, H_2O, $[XeF_5]^-$, I_3^-, BCl_3, XeF_4.

(ii) SF_4 – See- Saw shape
PCl_5 – Trigonal bipyramidal
SF_6 – Square bipyramidal
IF_7 – Pentagonal bipyramidal

(23) **3.** (P) $\xleftarrow[(1)]{CF_3CH_2O^{\ominus}}$ $\boxed{CH_3-CH_2-Br}$ $\xrightarrow[(2)]{CH_3CH_2O^{\ominus}}$

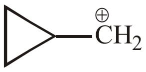
In reaction 2nd Nucleophile is stronger.

(Q) (Alkene) $\xleftarrow[(CH_3)_3CBr/\Delta]{(1)}$ $\boxed{C_2H_5O^{\ominus}}$

$\xrightarrow[(CH_3)_2CHCH_2Br/\Delta]{(2)}$ (Alkene)

In reaction 1st 3° alkyl halide
$1 > 2$

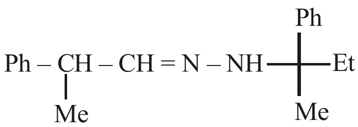
(R) In reaction 1st stable carbocation is formed



(Cyclopropyl methyl $C^{\oplus}$)
$1 > 2$

(S) In reaction 1st reagent in HI.
HI is more ionised, gives sufficient concentration of $H^{\oplus}$ ion and $I^{\ominus}$ is a good Nucleophile also.
$1 > 2$

(24) **4.** The product is



I = (+, +) (E)
II = (+, +) (Z)
III = (–, +) (E)
IV = (–, +) (Z)

(25) **5.** $Na_2SO_3 \xrightarrow{AgNO_3} Ag_2SO_3 \leftarrow$ white ppt.

$NaCl \xrightarrow{AgNO_3} AgCl \leftarrow$ white ppt.

$Na_2C_2O_4 \xrightarrow{AgNO_3} Ag_2C_2O_4 \leftarrow$ white ppt.

$Na_2HPO_4 \xrightarrow{AgNO_3} Ag_3PO_4 \leftarrow$ yellow ppt.

$Na_2CrO_4 \xrightarrow{AgNO_3} Ag_2CrO_4 \leftarrow$ red ppt.

$NaNO_2 \xrightarrow{AgNO_3} AgNO_2 \leftarrow$ white ppt.

$CH_3COONa \xrightarrow{AgNO_3} CH_3COOAg \leftarrow$ white ppt.

(26) **(4).** $L = mr^2\omega = \dfrac{nh}{2\pi}, \quad \omega \propto \dfrac{n}{r^2}$

$$\omega \propto \frac{n}{(r_0 n^2)^2}, \quad \omega \propto \frac{1}{n^3}$$

(27) **(1).** $\vec{a}_A = -a\,\hat{i},\ \vec{a}_B = a\,\hat{j}, \vec{a}_C = a\,\hat{i}, \vec{a}_D = -a\,\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})$$

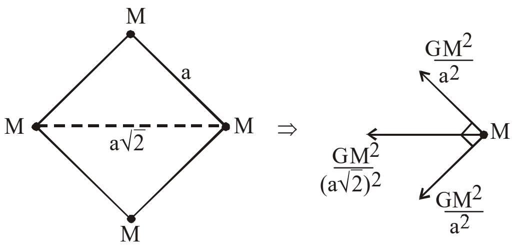
(28) **(1).**

$$A = \frac{5}{\sqrt{21}} \times 10^{-3}\,m^2$$

$$\frac{dv}{dt} = Av = \frac{1}{10}, \quad v = \frac{1}{10A}, \quad \frac{1}{2}mv^2 = ms\,\Delta\theta$$

$$\Delta\theta = \frac{v^2}{2s}$$

(29) **(3).**



Net force on particle towards centre of circle is

$$F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.
Distance of particle from centre of circle is $\dfrac{a}{\sqrt{2}}$

$$r = \frac{a}{\sqrt{2}}, \quad F_C = \frac{mv^2}{r}$$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}}+1\right) = \frac{GM}{a}(1.35)$$

$$v = 1.16\sqrt{\frac{GM}{a}}$$

(30) (1). $\tau = \vec{M}\times\vec{B}$

$C\theta = i\,N\,A\,B$

$$10^{-6}\times\frac{\pi}{180} = 10^{-3}\times10^{-4}\times175\times B$$

$B = 10^{-3}$ Tesla

(31) (3). $v_{rms} = \sqrt{\frac{3RT}{m}}$; $v_{escape} = \sqrt{2gR_e}$

$v_{rms} = v_{escape}$

$$\frac{3RT}{m} = 2gR_e$$

$$\frac{3\times1.38\times10^{-23}\times6.02\times10^{26}}{2}\times T$$

$= 2\times10\times6.4\times10^6$

$$T = \frac{4\times10\times6.4\times10^6}{3\times1.38\times6.02\times10^3} = 10\times10^3 = 10^4\,k$$

(32) (4). By law of conservation of energy

$$\frac{1}{2}kx^2 = (m_1s_1+m_2s_2)\,\Delta T$$

$$\Delta T = \frac{16\times10^{-2}}{4384} = 3.65\times10^{-5}$$

(33) (3). Magnetic field when electromagnetic wave propagates in +z direction

$B = B_0\sin(kz-\omega t)$

where, $B_0 = \frac{60}{3\times10^8} = 2\times10^{-7}$

$$k = \frac{2\pi}{\lambda} = 0.5\times10^3$$

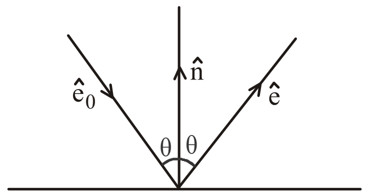
$\omega = 2\pi f = 1.5\times10^{11}$

(34) (3). $\lambda = \frac{h}{P}$; $P = \frac{h}{\lambda}$ $(\lambda = 7.5\times10^{-12}\text{ m})$

$$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left(\frac{6.6\times10^{-34}}{7.5\times10^{-12}}\right)^2}{2\times9.1\times10^{-31}}\,J$$

KE = 25 KeV

(35) (2). $\hat{e}_0\cdot\hat{n} = 1\times1\cos(180-\theta)$



$\hat{e}_0\cdot\hat{n} = -\cos\theta$ ...... (i)

$\hat{e}\cdot\hat{n} = 1\times1\times\cos\theta$ ...... (ii)

$\hat{e}\cdot\hat{n} - \hat{e}_0\cdot\hat{n} = 2\cos\theta$

$(\hat{e}-\hat{e}_0)\cdot\hat{n} = 2(-\hat{e}_0\cdot\hat{n})$

$(\hat{e}-\hat{e}_0)\,\hat{n}\cdot\hat{n} = -2(\hat{e}_0\cdot\hat{n})\,\hat{n}$

$\hat{e} = \hat{e}_0 - 2(\hat{e}_0\cdot\hat{n})\,\hat{n}$

(36) (2). $m = N\,I\,A = 1\times I\times a^2$

Here a = side of square

Now, $4a = 2\pi r \Rightarrow r = 2a/\pi$

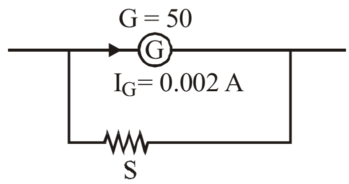
For circular loop

$m' = 1\times I\times\pi r^2$

$= 1\times I\times\pi\times(2a/\pi)^2$

$m' = 4m/\pi$

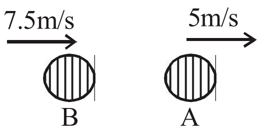
(37) (1).



S (0.5 – 0.002) = 50 × 0.002

$$S = \frac{50\times0.002}{(0.5-0.002)} = \frac{0.1}{0.498} = 0.2$$

(38) **(2).**



$f_0 = 500$ Hz

Frequency received by A

$\Rightarrow \left(\frac{1500-5}{1500-7.5}\right) f_0 = f_1$

and frequency received by B again

$f_2 = \left(\frac{1500+7.5}{1500+5}\right) \times \left(\frac{1500-5}{1500-7.5}\right) f_0$

$= 502$ Hz.

(39) **(2).** Given $\frac{Y_A}{Y_B} = \frac{7}{4}$

$L_A = 2m, \;\; A_A = \pi R^2$

$L_B = 1.5$ m, $A_B = \pi\,(2mm)^2$

$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$

Given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$

$\frac{(\pi R^2)\left(\frac{7}{4} Y_B\right)}{2} = \frac{\pi\,(2mm)^2 \cdot Y_B}{1.5}$

$R = 1.74$ mm

(40) **(2).** Dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

Dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{1/2}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2} = [M^{-1}L^{-2}T^3A^2]$

(41) **(1).** $E = \frac{KQ}{r^2}$ towards the missing corner.

$\vec{E}_1 + \vec{E}_2 + ... + \vec{E}_{10} + \vec{E}_{11} + \vec{E}_{12} = 0$

$\therefore \;\; \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_9 + \vec{E}_{11} + \vec{E}_{12} = -\vec{E}_{10}$

(42) **(3).** The initial velocity of the stone is 12 m/s downward under gravity.

$h = ut + \frac{1}{2} gt^2$

$= 12 \times 10 + \frac{1}{2} \times 10 \times 10^2 = 120 + 500$

$= 620$ m

(43) **(2).** $\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{16K}{K}} = 4$

$\frac{\lambda_1}{\lambda_2} = 4 \Rightarrow \lambda_2 = \frac{\lambda_1}{4}$; $\lambda_2 = 0.25\,\lambda_1$

$\frac{\lambda_1 - \lambda_2}{\lambda_1} \times 100\% = \frac{\lambda_1 - 0.25\lambda_1}{\lambda_1} \times 100\%$

$= 0.75 \times 100 = 75\%$

(44) **(2).** $\frac{hc}{\lambda_A} = K_A + W$

$\frac{hc}{2\lambda_B} = K_A + W \Rightarrow \frac{hc}{\lambda_B} = 2K_A + 2W$ ..(1)

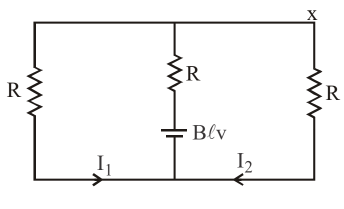
and $\frac{hc}{\lambda_B} = K_B + W$ ..(2)

by eq. (1) and eq. (2)

$2K_A + 2W = K_B + W$

$\Rightarrow \frac{K_B}{2} - K_A = \frac{W}{2} \;\; \therefore \;\; \frac{K_B}{2} > K_A$

(45) **(3).**



$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$

$I = 2\frac{B\ell v}{3R}$; $I_1 = I_2 = \frac{B\ell v}{3R}$

(46) **4.** $M.A = \frac{Load}{Effort} = \frac{Mg}{T}$

$Mg = 4$ T

$\Rightarrow$ M.A. = 4

(47) **4.** For element of mean life 5τ as t = 5τ

$N_1 = N_0 e^{-1} \Rightarrow N_1 = N_0 / e$

and for element of mean life τ

$$N_2 = N_0 e^{-5\lambda/\tau} = \frac{N_0}{e^5}$$

So, active nuclei in sample = $N_1 + N_2$

$$= \frac{N_0}{e} + \frac{N_0}{e^5} = \frac{N_0\,(e^4+1)}{e^5}$$

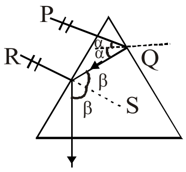
(48) **2.** PQ & RS are parallel (normal to face)

Then, $\beta = 2\alpha$

But, $\alpha + 2\beta = 180°$

$\alpha = 36°$

Then, x = 2.



(49) **3.** The FBD of any one rod is

$T = \mu N$ ...(1)

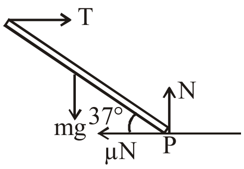
$mg = N$ ...(2)

Taking torque about point 'P'

$$mg\frac{L}{2}\cos 37° = TL\sin 37°$$

$$T = \frac{mg}{2}\frac{4}{3} = \frac{2mg}{3}$$

$$\frac{2mg}{3} = \mu mg \Rightarrow \mu = \frac{2}{3}$$



(50) **2.** $P = V^2 / R$

20V peak ac is equivalent to $\frac{20}{\sqrt{2}}$ dc

i.e., 14.14V dc power

$$\frac{\text{dc power}}{\text{ac power}} = \frac{(20^2 / R)}{[(20^2/\sqrt{2})^2 / R]}$$

$$= \frac{20^2}{(20^2/\sqrt{2})^2} = 2$$

(51) **(2).** Normal vector of plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1+1 & 2+2 & -3-1 \end{vmatrix} = -28\hat{i} + 16\hat{j} + 2\hat{k}$$

Direction ratios of the normal to plane can be (14, –8, –1).

(52) **(3).** $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

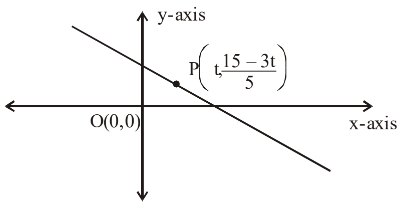
$$A - 3I_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$\det(A - 3I_3) = -2(3) + 1(3) + 1(3) = 0$

$\Rightarrow$ $A - 3I_3$ is non-invertible.

(53) **(1).**



$$\left|\frac{15-3t}{5}\right| = |t|$$

$$\frac{15-3t}{5} = t \ \text{ or } \ \frac{15-3t}{5} = -t$$

$$t = \frac{15}{8} \ \text{ or } t = \frac{-15}{2}$$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{nd}$ quadrant

(54) **(3).** If arg z = θ > 0

then arg (–z) = θ – π

Now, arg z – arg (–z) = θ – (θ – π) = π

(55) **(2).** $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$

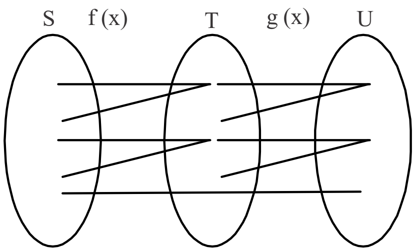
$$= \sum_{r=0}^{20} (3r+2)\ {}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r\cdot {}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r\left(\frac{20}{r}\right) {}^{19}C_{r-1} + 2\cdot 2^{20}$$

$$= 60.2^{19} + 2.2^{20} = 2^{25}$$

(56) **(2).** Obvious g is surjective otherwise gof cannot be surjective but there is no need of f to be surjective. See example.



Hence f(x) is not surjective still gof is surjective.

(57) **(3).** $b = \frac{\sqrt{3}}{2}(2ae)$; $b^2 = 3\ (a^2 - b^2)$

$4b^2 = 3a^2$

$$\frac{b^2}{a^2} = \frac{3}{4}\ ;\ e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow e = \frac{1}{2}$$

(58) **(4).** $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}\left(\left([x] - \left[\frac{x}{4}\right]\right)\right) = 4 - 1 = 3$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}\left([x] - \frac{x}{4}\right) = 3 - 0 = 3$$

$f(x) = 3$

$\therefore$ Continuous at x = 4

(59) **(4).** $\frac{dy}{dx} = \frac{a^2}{(x+y)^2}$

Put $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow \frac{dt}{dx} = 1 + \frac{a^2}{t^2} \Rightarrow \frac{t^2}{t^2 + a^2} dt = dx$$

$$\Rightarrow dt - \frac{a^2 dt}{t^2 + a^2} = dx \Rightarrow t - a\tan^{-1}\frac{t}{a} + c = x$$

$$\Rightarrow y + c = a\tan^{-1}\frac{x+y}{a}$$

$$\Rightarrow \tan\left(\frac{y+c}{a}\right) = \frac{x+y}{a}$$

(60) **(3).** Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\overline{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56$ ......(1)

Also, $\sigma^2 = 16$

$$16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\overline{x})^2$$

$$16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$$

$\left(\sum_{i=1}^{7} x_i^2\right) = 560$ ......(2)

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$ (from (1))

$x_6{}^2 + x_7{}^2 = 100$ (from (2))

$\therefore x_6{}^2 + x_7{}^2 = (x_6 + x_7)^2 - 2x_6x_7$

$\Rightarrow x_6x_7 = 48$

(61) **(4).** $(\hat{\alpha}\cdot\hat{\gamma})\,\hat{\beta} - (\hat{\alpha}\cdot\hat{\beta})\,\hat{\gamma} = \frac{1}{2}\hat{\beta} + \frac{1}{2}\hat{\gamma}$

because $\hat{\beta}$ is not parallel to $\hat{\gamma}$ so

$$\hat{\alpha}\cdot\hat{\beta} = -\frac{1}{2}$$

Angle between $\hat{\alpha}$ and $\hat{\beta}$ is $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

(62) **(4).** $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q)$

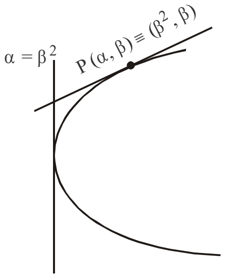
$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

(63) **(3).** $T : y(\beta) = \frac{1}{2}(x + \beta^2)$



$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$

$m = \frac{1}{2\beta}$; $C = \frac{\beta}{2}$

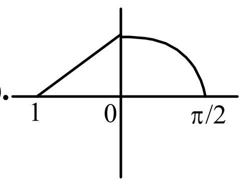
$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$; $\frac{\beta^2}{4} = \frac{1 + 2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$(\beta^2 - 1)^2 = 2$; $\beta^2 - 1 = \sqrt{2}$; $\beta^2 = \sqrt{2} + 1$

(64) **(2).**



$\text{Area} = \frac{1}{2} \times 1 \times 1 + \int_0^{\pi/2} \cos x \, dx = \frac{1}{2} + 1 = \frac{3}{2}$

(65) **(2).** $\lim_{x \to 0^+} (x^n \ln x)$

$= \lim_{x \to 0^+} \left(\frac{\ln x}{1/x^n}\right) = \lim_{x \to 0^+} \left(\frac{1/x}{\frac{-n}{x^{n+1}}}\right)$

$= \lim_{x \to 0^+} \left(\frac{x^n}{-n}\right) = 0$ (Using L hospital rule)

(66) **(1).** $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n-4)$

$d = T_n - T_{n-1}$

$= 50 + A(n-4) - 50 - A(n-5) = A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50 + 46A)$

(67) **(1).** $I = \int_0^1 x \tan\left(\frac{1}{1 + x^2(x^2 - 1)}\right) dx$

$I = \int_0^1 x \tan(\tan^{-1} x^2 - \tan^{-1}(x^2 - 1)) \, dx$

$x^2 = t \Rightarrow 2x\,dx = dt$

$I = \frac{1}{2}\int_0^1 (\tan^{-1} t - \tan^{-1}(t-1)) \, dx$

$= \frac{1}{2}\int_0^1 \tan^{-1} t \, dt - \frac{1}{2}\int_0^1 \tan^{-1}(t-1) \, dt$

$= \frac{1}{2}\int_0^1 \tan^{-1} t \, dt - \frac{1}{2}\int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt$

$\tan^{-1} t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta \, d\theta$

$\int_0^{\pi/4} \theta \cdot \sec^2\theta \, d\theta$

$1 = (\theta \cdot \tan\theta)\Big|_0^{\pi/4} - \int_0^{\pi/4} \tan\theta \, d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - (\ln\sqrt{2} - 0) = \frac{\pi}{4} - \frac{1}{2}\ln 2$

(68) **(2).** Let (h, k) satisfies $x^2 + y^2 = 1$ then

$h^2 + k^2 = 1$. Now $h^8 + k^8 = h^8 + (1 - h^2)^4$

$= 2h^8 - 4h^6 + 6h^4 - 4h^2 + 1$

$= 2h^2 (h^2 - 1)(h^4 - h^2 + 2) + 1$

$= -2h^2k^2 (h^4 - h^2 + 2) + 1 < 1$

$\forall\, h > 0, k > 0$

$\Rightarrow$ All solution of $x^2 + y^2 = 1$ satisfies

$x^8 + y^8 < 1 \Rightarrow P \cap T = P$

(69) **(2).** $1 - {}^3C_0 (0.6)^0 (0.4)^3 = 0.936$

(70) **(4).** $|y_1 m| = a$

$$y\frac{dy}{dx} = \pm a \Rightarrow \int y\,dy = \int \pm a\,dx$$

$\Rightarrow \dfrac{y^2}{2} = \pm ax + c$, it is a parabola so e = 1

(71) **1.** $(x^2 + 4)^2 = (2x - 3)^2$

$\Rightarrow x^2 + 4 = \pm (2x - 3)$

$\Rightarrow x^2 + 2x + 1 = 0$ or $\underbrace{x^2 - 2x + 7}_{D<0} = 0$

$\Rightarrow (x + 1)^2 = 0$ or No solution

$\Rightarrow x = -1$

Have only one solution.

(72) **3.** $\tan^2 x - \sec^{10} x + 1 = 0$

$\Rightarrow \sec^2 x - 1 - \sec^{10} x + 1 = 0$

$\Rightarrow \sec^2 x (1 - \sec^8 x) = 0$

$\Rightarrow \sec^8 x = 1 \Rightarrow \cos x = \pm 1$

$\Rightarrow x = \pi, 2\pi, 3\pi$

(73) **5.** $x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda (3x - 4y - 7) = 0$

It passes through (4, 0) :

$(16 + 16 - 12) + \lambda (12 - 7) = 0$

$20 + \lambda (5) = 0$

$\Rightarrow \lambda = -4$

$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius $= \sqrt{16 + 25 - 16} = 5$

(74) **4.** $f'(x) = 7 - \dfrac{3}{4}\dfrac{f(x)}{x}, (x > 0)$

Given $f(1) \neq 4$; $\lim\limits_{x \to 0^+} x f\left(\dfrac{1}{x}\right) = ?$

$\dfrac{dy}{dx} + \dfrac{3}{4}\dfrac{y}{x} = 7$ (This is LDE)

$$IF = e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln|x|} = x^{3/4}$$

$$y \cdot x^{3/4} = \int 7 \cdot x^{3/4} dx$$

$$y \cdot x^{3/4} = 7 \cdot \frac{x^{7/4}}{7/4} + C$$

$f(x) = 4x + cx^{-3/4}$

$$f\left(\frac{1}{x}\right) = \frac{4}{x} + C \cdot x^{3/4}$$

$$\lim_{x \to 0^+} x f\left(\frac{1}{x}\right) = \lim_{x \to 0^+} (4 + C \cdot x^{7/4}) = 4$$

(75) **2.** $\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$

$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$

$14 - 7\sin 3\theta - 14\cos 2\theta = 0$

$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$

$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$

$7\sin\theta[-3 + 4\sin^2\theta + 4\sin\theta] = 0$

$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = -3/2$; $\sin\theta = 1/2$

Hence, 2 solutions in $(0, \pi)$.