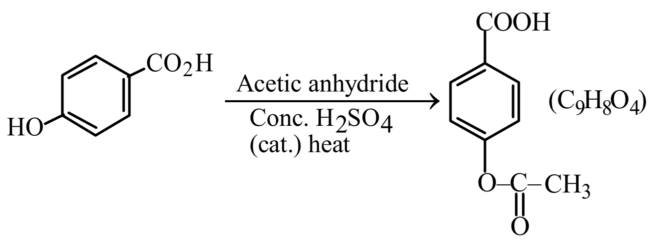
JEE MAIN 2020
FULL TEST-2 SOLUTIONS

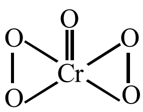
STANDARD ANSWER KEY

Q	1	2	3	4	5	6	7	8	9	10	11
A	1	1	3	2	4	1	2	3	4	4	1
Q	12	13	14	15	16	17	18	19	20	21	22
A	1	2	2	4	1	2	3	1	2	5	6
Q	23	24	25	26	27	28	29	30	31	32	33
A	7	5	2	1	3	3	1	4	2	1	3
Q	34	35	36	37	38	39	40	41	42	43	44
A	1	3	1	4	1	4	4	4	1	2	3
Q	45	46	47	48	49	50	51	52	53	54	55
A	3	5	3	6	4	4	3	1	4	3	2
Q	56	57	58	59	60	61	62	63	64	65	66
A	2	4	1	3	2	3	2	3	1	2	4
Q	67	68	69	70	71	72	73	74	75		
A	3	1	4	3	4	8	1	3	2		

(1) **(1).**



(2) **(1).**

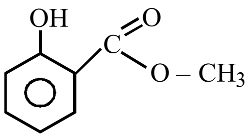
(a) O.N. of grim CrO_5  = + 6

(b) $N_2O_4\,(g) \rightarrow 2NO_2\,(g)$.
$\Delta H = \Delta U + \Delta n_g RT$
$\Delta n_g = 2 - 1$ so $\Delta H > \Delta U$

(c) pH of 0.1 N $H_2SO_4 \Rightarrow [H^+]$ 0.1 N
$pH = -\log [H^+] = 10\,g\,(10^{-1})$
pH of 0.1 N HCl $= [H^+] = 0.1$ N
$pH = \log [H^+] = -\log (10^{-1}) = (1)$

(d) $\frac{RT}{F} = \frac{8.314 \times 298}{96500} = 0.256$

$$\frac{2.303RT}{F} = \frac{2.303 \times 8.314 \times 298}{96500} = 0.0591$$

(3) **(3).** 

Merthyl-2-Hydroxy benzoate.

(4) **(2).** Ist electron affinity order : N < C < O
According to electronic configuration
$N = 1s^2\, 2s^2\, 2p^3$
Half-filled orbital are more stable

EA. kJ/mole $\begin{pmatrix} C = 121.77 \\ N = -6.8 \\ O = 140 \end{pmatrix}$

(5) **(4).** For Lyman : $\bar{\nu}_{max} = R_H\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R_H$

$$\bar{\nu}_{min} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R_H$$

$$\Delta\bar{\nu}_{Lyman} = \frac{R_H}{4}$$

For Balmer

$$\bar{\nu}_{max} = R_H\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R_H}{4}$$

$$\bar{v}_{min} = R_H\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \frac{5}{36}R_H$$

$$\Delta\bar{v}_{Balmer} = \frac{R_H}{4}-\frac{5R_H}{36} = \frac{4R_H}{36} = \frac{R_H}{9}$$

$$\frac{\Delta\bar{v}_{Lyman}}{\Delta\bar{v}_{Balmer}} = \frac{R_H/4}{R_H/9} = \frac{9}{4}$$

(6) **(1).** $CH_3\underset{}{CH}(OH)CH_2CH_2NH_2 \xrightarrow[\text{Triethylamine}]{\text{Ethyl formate (1 equivalent)}}$

$CH_3-CH(OH)-CH_2-CH_2-NH-\overset{O}{\overset{\|}{C}}-H$

as NH_2 is a better nucleophile than OH.

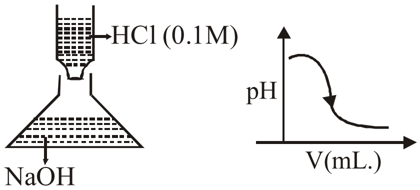
(7) **(2).**
(1) Zincite is ZnO.
(2) Aniline is the forth stablizer.
(3) Zone refining process is not used for refining of 'Ti'.
(4) Sodium cyanide is used in the metallurgy of silver.

(8) **(3).**
- Gas A and C have same value of 'b' but different value of 'a' so gas having higher value of 'a' have more force of attraction so molecules will be more closer hence occupy less volume.
- Gas B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

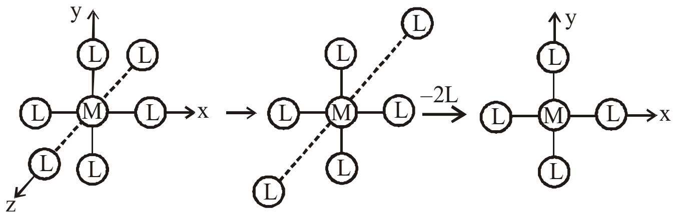
(9) **(4).** $E^o_{Red}\uparrow \Rightarrow$ oxidizing power $\uparrow$

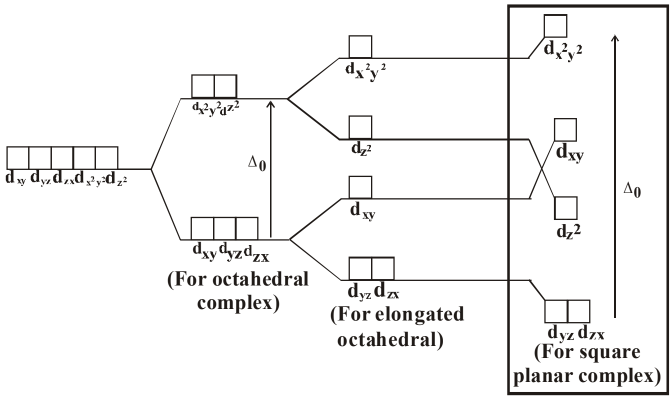
(10) **(4).** High density polythene : Ziegler-Natta catalyst
Polyacrylonitrile : Peroxide catalyst
Novolac : Acid or base catalyst
Nylon 6 : Condensation at high temperature & pressure.

(11) **(1).**



(12) **(1).** If both ligands present along z-axis removed from octachedral field and converted into square planar field, then



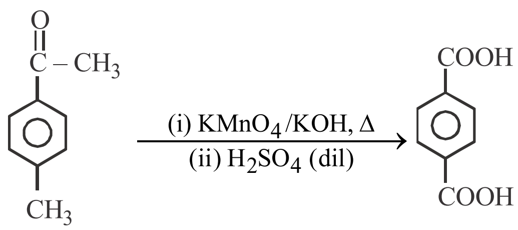


(13) **(2).** $y_A = \frac{P_A}{P_{Total}} = \frac{P_A^0 x_A}{P_A^0 x_A \times P_B^0 x_B}$

$$= \frac{7\times10^3\times0.4}{7\times10^3\times0.4+12\times10^3\times0.6} = \frac{2.8}{10} = 0.28$$

$y_B = 0.72$

(14) **(2).**



(15) **(4).** Catenation is not shown by lead.

(16) **(1).** For zero order
$[A_0]-[A_t] = kt$
$0.2 - 0.1 = k \times 6$

$k = \frac{1}{60}$ M/hr and $0.5 - 0.2 = \frac{1}{60}\times t$

t = 18 hrs.

(17) **(2).**

$\underset{\text{(Terminal alkyne)}}{CH_3-C\equiv CH} \xrightarrow[\Delta]{Ag_2O} \underset{\text{(ppt)}}{CH_3-C\equiv C-Ag\downarrow}$

$$CH_3-C\equiv CH \xrightarrow[\text{dil. } H_2SO_4 \text{ (Kucherov reaction)}]{Hg^{2+}}$$
(Terminal alkyne)

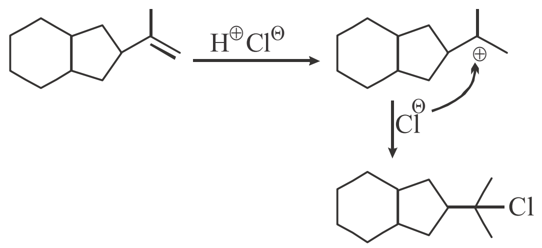
$$CH_3-\underset{}{\overset{OH}{\overset{|}{C}}}-CH_2 \rightleftharpoons CH_3-\overset{O}{\overset{||}{C}}-CH_3$$

$$\xrightarrow{NaBH_4} CH_3-\underset{OH}{\underset{|}{CH}}-CH_3$$
(2° alcohol)

$$\xrightarrow{\text{Conc. HCl+ZnCl}_2} \text{Turbidity within 5 min.}$$
(Lucas reagent test)

(18) **(3).** Main principle of column chromatography is differential adsorption of the substance on the solid phase.

(19) **(1).**



(20) **(2).** $Na_2CO_3.10H_2O \rightarrow$ Solvay process
$Mg(HCO_3)_2 \rightarrow$ Temporary hardness
$NaOH \rightarrow$ Castner-kellner cell
$Ca_3Al_2O_6 \rightarrow$ Portland cement

(21) **5.** $[Pt(NH_3)Cl(H_2O)Br]$, $[Cu(NH_3)_4]^{2+}$, $[Co(en_3)]^{3+}$, XeF_4, XeO_6^{4-}.

$d_{x^2-y^2}$ orbital involved in dsp^2 & sp^3d^2/d^2sp^3 hybridisation.

(1) dsp^2 : $[Pt(NH_3)Cl(H_2O)Br] \rightarrow$ Square planar
(2) sp^3d : $SF_4 \rightarrow$ See-Saw sp^3d
(3) dsp^2 : $[Cu(NH_3)_4]^{2+} \rightarrow$ Square planar
(4) sp^3d : $[XeO_3F_2] \rightarrow$ Trigonal bipyramidal
(5) sp^3d : $[XeO_2F_2] \rightarrow$ See-Saw
(6) d^2sp^3 : $[Co(en_3)]^{3+} \rightarrow$ Octahedral
(7) dsp^3 : $[Fe(CO)_5] \rightarrow$ Square pyramidal
(8) sp^3 : $POCl_3 \rightarrow$ Tetrahedral
(9) sp^3d^2 : $XeF_4 \rightarrow$ Square planar
(10) sp^3d^2 : $XeO_6^{4-} \rightarrow$ Square bipyramidal

(22) **6.** $A(g) + B(g) \rightleftharpoons C(g) + D$
$= \Delta G^o_{reaction}$
$= (\Delta G^o_{fC} + \Delta G^o_{fD}) - (\Delta G^o_{fA} + \Delta G^o_{fB})$
$= (-50 \text{ kJ} + 100 \text{ kJ}) - (30 \text{ kJ} + 20 \text{ kJ})$
$\Delta G^\circ = 0$
$\Delta G^\circ = -RT/\ln k_{eq}$
$k_{eq} = 1$
$D(\ell) \rightleftharpoons D(g)$
[Partial pressure of D gas = Vapour pressure]

$A(g)$ +	$B(g)$	$\rightleftharpoons C(g)$ +	$D(g)$
1 bar	1 bar	1 bar	1/6 bar
(1 – x) bar	(1 – x) bar	(1 + x) bar	(1/6) bar

$$K_{eq} = 1 = \frac{1/6 \times [1+x]}{(1-x)^2}$$

$$1 + x^2 - 2x = \frac{1}{6} + \frac{x}{6} \;;\; x^2 - 2x - \frac{x}{6} + \frac{5}{6} = 0$$

$$x^2 - \frac{13x}{6} + \frac{5}{6} = 0$$

$6x^2 - 13x + 5$; $x = 1/2$
$P_A = 1/2$, $P_B = 1/2$, $P_A = 3/2$
a = 1, b = 1, c = 1, d = 3

(23) **7.** Molar mass of salt = 120 + 18x
Mass of water present in the salt

$$= \frac{18x}{120+18x} \times 32g$$

Molality of the solution

$$= \frac{32}{120+18x} \times \left[\frac{1000}{84 + \frac{18 \times 32x}{120+18x}}\right] = \frac{4000}{1260+261x}$$

$$\Delta T_f = 4.836 = 2 \times 1.86 \times \frac{4000}{1260+261x}$$

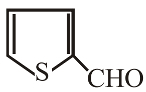
$x = 6.9 \approx 7$

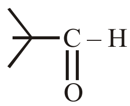
(24) **5.** Compound with α–H absent will give Cannizaro reaction.

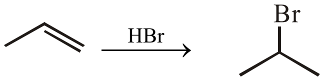
(i) Ph – CHO

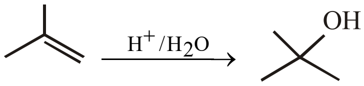
(ii) $Cl_3C-\overset{}{\underset{O}{\underset{||}{C}}}-H$

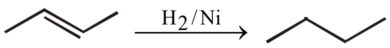
(iii) $H-\underset{O}{\underset{||}{C}}-\underset{O}{\underset{||}{C}}-H$

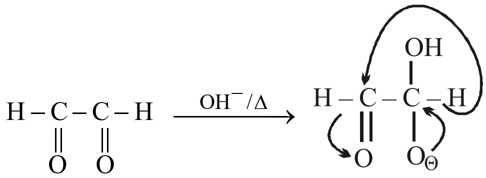
(iv) 

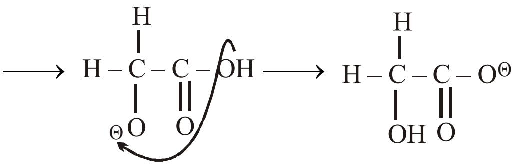
(v) 

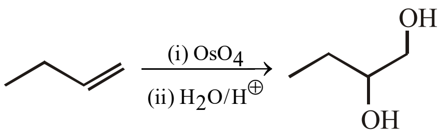
(25) **2.** (i) 

(ii) 

(iii) 

(iv) 



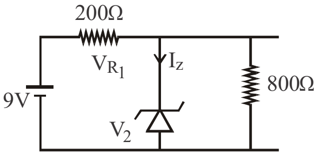
(v) 

(vi) $H_3C-Br \xrightarrow{\overset{\ominus}{OH}} CH_3OH$

(vii) $H_3C-CH_2-OH \xrightarrow{SOCl_2} CH_3-CH_2-Cl$

(26) **(1).** $T^2 \propto r^3,\ \frac{r_E}{r_P} = \left(\frac{T_E}{T_P}\right)^{2/3} = \left(\frac{\omega_P}{\omega_E}\right)^{2/3}$

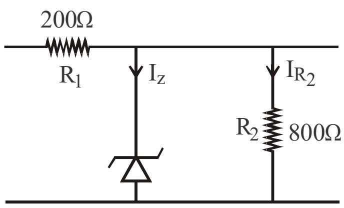
$\frac{R}{r_P} = (2)^{2/3}\ ;\ r_p = (2)^{-2/3}R$

(27) **(3).** 

$9 = V_Z + V_{R_1}\ ;\ V_Z = 5.6\ V$

$V_{R_1} = 9 - 5.6 = 3.4$

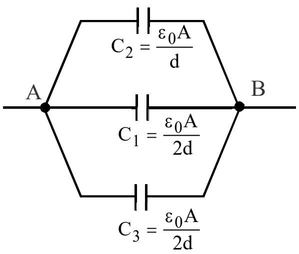
$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200} = 17\ mA$



$V_Z = V_{R_2} = I_{R_2}(R_2)$

$\frac{5.6}{800} = I_{R_2}\ ;\ I_{R_2} = 7mA$

$I_Z = (17-7)\ mA = 10\ mA$

(28) **(3).** 

$C_{eq} = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{2d} = \frac{2\varepsilon_0 A}{d}$

(29) **(1).** $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)\ ;\ \frac{\lambda_1}{\lambda} = \frac{\frac{3}{4}}{\frac{7}{16\times 9}}$

$\lambda_1 = \frac{3}{4} \times \frac{16\times 9}{7}\lambda = \frac{108}{7}\lambda$

(30) **(4).** Magnification is 2
If image is real, $x_1 = 3f/2$
If image is virtual, $x_2 = f/2$

$\frac{x_1}{x_2} = 3:1$

(31) **(2).** $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q\omega}{2R \times 2\pi}$

$\Rightarrow\ q = 3 \times 10^{-5}\ C$

(32) **(1).** $\beta = \frac{\lambda D}{d}$

$\frac{\Delta\beta}{\beta} \times 100 = \frac{\Delta D}{D} \times 100 - \frac{\Delta d}{d} \times 100$

$= 0.5 - (-0.3) = 0.8\%$

Fringe width increases by 0.8%.

(33) **(3).** The physical size of antenna of receiver and transmitter both inversely proportional to carrier frequency.

(34) **(1).** $\frac{F}{A} = \text{Stress}$; $\frac{400 \times 4}{\pi d^2} = 379 \times 10^6$

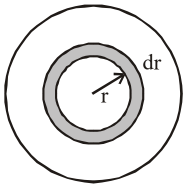
$$d^2 = \frac{1600}{\pi \times 379 \times 10^6} = 1.34 \times 10^{-6}$$

$$d = \sqrt{1.34} \times 10^{-3} = 1.15 \times 10^{-3}\,m$$

(35) **(3).** $I_{Disc} = \int_0^R (dm)\, r^2 = \int_0^R (\sigma\, 2\pi r\, dr)\, r^2$

$$I_{Disc} = \int_0^R (kr^2\, 2\pi r\, dr)\, r^2 = 2\pi k \int_0^R r^5\, dr$$

$$= 2\pi k \left(\frac{r^6}{6}\right)_0^R = 2\pi k \frac{R^6}{6}$$



Mass of disc, $M = \int_0^R 2\pi r\, dr\, kr^2$

$$= 2\pi k \int_0^R r^3\, dr = 2\pi k \left.\frac{r^4}{4}\right|_0^R = 2\pi k \frac{R^4}{4}$$

$$I_{disc} = \frac{2}{3} MR^2$$

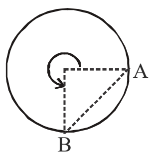
(36) **(1).** $M = NIA$; $dq = \lambda\, dx$ & $A = \pi x^2$

$$\int dm = \int (n) \frac{\rho_0 x}{\ell} dx \cdot \pi x^2$$

$$M = \frac{n\rho_0 \pi}{\ell} \int_0^\ell x^3 \cdot dx = \frac{n\rho_0 \pi}{\ell} \left[\frac{\ell^4}{4}\right]$$

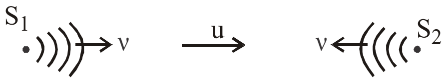
$$M = \frac{n\rho_0 \pi \ell^3}{4}$$

(37) **(4).**



$$|\bar{V}| = \frac{R\sqrt{2}}{\left(\frac{3\pi R/2}{v}\right)} = \frac{2\sqrt{2}v}{3\pi} = \frac{4}{3\pi} m/s$$

(38) **(1).** $f = 660$ Hz, $\nu = 330$ m/s



$$f_1 = f\left(\frac{\nu - u}{\nu}\right) \;;\quad f_2 = f\left(\frac{\nu + u}{\nu}\right)$$

$$f_2 - f_1 = \frac{f}{\nu}[\nu + u - (\nu - u)]$$

$$10 = f_2 - f_1 = \frac{f}{\nu}[2u]$$

$u = 2.5$ m/s

(39) **(4).**

$\frac{h}{\lambda_1} = P_1$

$P_2 = \frac{h}{\lambda_1}$

$$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i} \;\;\&\;\; \vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$$

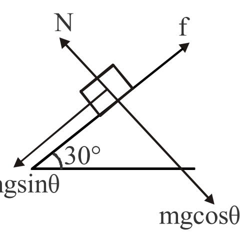
Using momentum conservation

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$$

$$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2} \;;\quad \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

(40) **(4).**



Since block is at rest

$N = mg\cos\theta$ & $f = mg\sin\theta$

Force exerted by the surface on the block will be $= \sqrt{N^2 + f^2} = mg = 80$ N

(41) **(4).** $M = M_0 e^{-\lambda t}$; Given $t = 2\left(\frac{1}{\lambda}\right)$

$\Rightarrow \quad M = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 \quad (e = 2.71)$

$\Rightarrow \quad M = 1.35$ g

(42) **(1).** State function $(\Delta U)_{cycle} = 0$
and (work) = –ve [anticlockwise]
$\therefore$ Using FLOT
$Q = \Delta U + W$
Also (Q = –ve)

(43) **(2).** Let distance moved by plank = x
$3Mx = M(\ell - x)$
$x = \ell / 4$

Distance moved by man $= \ell - \frac{\ell}{4} = \frac{3\ell}{4}$

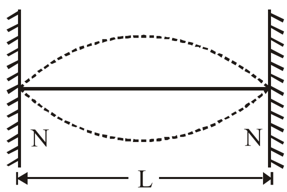
(44) **(3).** L.C. = 1 MSD – 1 VSD

$= 1 \text{ MSD} - \frac{9}{10} \text{ MSD}$

$= \left(1 - \frac{9}{10}\right) \times 1 \text{ MSD}$ $\left[\begin{array}{l}\text{Also} \\ 10 \text{ VSD} = 9 \text{ MSD} \\ 1 \text{ VSD} = \frac{9}{10} \text{ MSD}\end{array}\right]$

$= 0.1 \times 0.1$ cm
$= 0.01$ cm
Diameter = MSR + LC × VSR
$= 1.3 + 0.01 \times 2 = 1.3 + 0.02 = 1.32$ cm

(45) **(3).** Here, mass of the wire,
$M = 30\text{g} = 30 \times 10^{-3}$ kg
Mass per unit length of the wire,
$\mu = 4 \times 10^{-2}$ kg m^{-1}



$\therefore$ Length of the wire,

$$L = \frac{M}{\mu} = \frac{30 \times 10^{-3} \text{kg}}{4 \times 10^{-2} \text{kg m}^{-1}} = 0.75 \text{ m}$$

For the fundamental mode $\frac{\lambda}{2} = L$

$\Rightarrow \quad \lambda = 2L = 2 \times 0.75 = 1.5$ m
Speed of the transverse wave,
$v = n\lambda = (50 \text{ s}^{-1})(1.5 \text{ m}) = 75 \text{ m s}^{-1}$

(46) **5.** $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta W = -\Delta U = \frac{f}{2} nR\Delta T$

$147 = \frac{f}{2} \times 8.4 \times 7 \Rightarrow f = \frac{147}{7 \times 4.2} = 5$

(47) **3.** Magnification, $m = \frac{-f}{-f + \frac{3f}{2}} = -2$

Taking the direction in the right hand side
$V_{im} = -m^2 V_{om}$
$V_{im} \to$ velocity of image w.r.t. mirror
$V_{om} \to$ velocity of object w.r.t mirror
$\Rightarrow \quad V_{im} = -4u$

$V_m = \frac{-mu}{m + 2m} = -\frac{u}{3}$

$\to$ Velocity of platform (minor)
$V_{im} = V_i - V_m$
$V_i \to$ velocity of image w.r.t ground

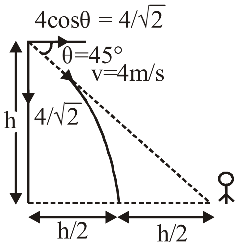
$\Rightarrow \quad V_i = -4u - \frac{u}{3} \Rightarrow V_i = -\frac{13}{3}u$

$\Rightarrow \quad V_i = 3$ m/s

(48) **6.** After a large no. of time, the potential of each capacitor will be equal to the potential of the reservoir.
$\Rightarrow \quad Q_2 = C_2 V$
$Q_2 = 6\mu C$

(49) **4.** $\frac{4}{\sqrt{2}} \cdot t = \frac{h}{2}$ .... (1)

$h = \left(\frac{4}{\sqrt{2}}\right) \cdot t + \frac{1}{2} g t^2$ .... (2)



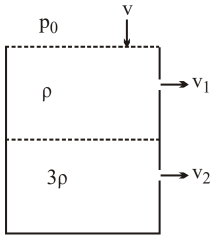
From eq. (1) & (2)

$\frac{h}{2} = \frac{1}{2} g \frac{4^2}{2} = \frac{1}{2} g \left(\frac{h}{4\sqrt{2}}\right)^2$

$h = \frac{16 \times 2}{10} = 3.2\text{m}$

(50) **4.** $\rho_0 + \frac{\rho g h}{3} + \frac{1}{2}\rho v^2 = p_0 + 0 + \frac{1}{2}\rho v_1^2$

$v_1 = \sqrt{\frac{2gh}{3}}$



$\rho_0 + \frac{3\rho g h}{3} + \rho g h + \frac{1}{2}\rho v^2$

$= p_0 + 0 + \frac{1}{2} 3\rho v_2^2$

$v_2 = \sqrt{\frac{4gh}{3}}$

(51) **(3).** Equation of required plane is

$(3 - 1)x + (4 - 2)y + (5 - 3)z = k$

which passes through (2, 3, 4)

$\Rightarrow$ k = 9

$\Rightarrow$ Equation of plane is $x + y + z = 9$

(52) **(1).** $|A^2| = 25 \Rightarrow |A| = 5$ or -5

$|A| = 25x = 5$ or $-5 \Rightarrow x = 1/5, -1/5$

$\Rightarrow$ $|x| = 1/5$

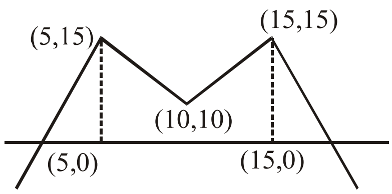
(53) **(4).** $\int_{\alpha}^{\alpha+1} \frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx$

$= (\ln|x+\alpha| - \ln|x+\alpha+1|)|_{\alpha}^{\alpha+1}$

$= \ln\left|\frac{2\alpha+1}{2\alpha+2} \times \frac{2\alpha+1}{2\alpha}\right| = \ln\frac{9}{8}$

$\Rightarrow$ $\alpha = -2, 1$

(54) **(3).**



$f(x) = 15 - |x - 10|, x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$

x = 5, 10, 15 are points of non differentiability

Aliter :

At x = 10 f(x) is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow$ x = 5, 15

$\therefore$ Non-differentiability points are {5, 10, 15}

(55) **(2).** If AB = AC $\Rightarrow \angle ABC = \angle ACB$

$\Rightarrow$ $\tan(\angle ABC) = \tan(\angle ACB)$

If let slope of AC is m

$\therefore$ $\left|\frac{m + \frac{1}{4}}{1 - \frac{m}{4}}\right| = \left|\frac{-\frac{1}{4} - 1}{1 - \frac{1}{4}}\right|$

$\Rightarrow$ $m = \frac{-23}{7}, 1$ (rejected)

$\therefore$ Equation of line is $23x + 7y + 3 = 0$

(56) **(2).** $\lim_{x \to 0} \frac{\left(\frac{\sin^2 x}{x^2}\right)(\sqrt{2} + \sqrt{1 + \cos x})}{\left(\frac{1 - \cos x}{x^2}\right)}$

$= \frac{(1)^2 \cdot (2\sqrt{2})}{1/2} = 4\sqrt{2}$

(57) **(4).**

(1) $(p \vee q) \wedge (\sim p \vee \sim q)$

$\equiv (p \vee q) \wedge \sim(p \wedge q)$

$\rightarrow$ Not tautology (Take both p and q as T)

(2) $(p \wedge q) \vee (p \wedge \sim q)$

$\equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$

(3) $(p \vee q) \wedge (p \vee \sim q)$

$\equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$

(4) $(p \vee q) \vee (p \vee \sim q)$

$\equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$

(58) **(1).** $\frac{34 + x}{2} = 35 \Rightarrow x = 36$

$42 = \frac{10 + 22 + 26 + 29 + 34 + 36 + 42 + 67 + 70 + y}{10}$

$420 - 336 = y \Rightarrow y = 84$

$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

(59) **(3).** $R_1 \rightarrow R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1 + \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$\Rightarrow$ $(1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$
$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$$6\theta = \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$$

(60) **(2).** $\frac{dy}{dx} = \frac{b^2x}{a^2y}$

$$-\frac{dx}{dy}\bigg]_{(4,3)} = -\frac{3a^2}{4b^2} = \frac{3-0}{4-16}$$

$$\frac{a^2}{b^2} = \frac{1}{3} \Rightarrow \frac{b^2}{a^2} = 3$$

$$e = \sqrt{\frac{1+b^2}{a^2}} = \sqrt{1+3} = 2$$

(61) **(3).** Given $y^2 = 4x$ ...(1)
and $x^2 + y^2 = 5$ ...(2)
By (1) and (2)
$\Rightarrow$ $x = 1$ and $y = 2$
Equation of tangent at (1, 2) to $y^2 = 4x$ is $y = x + 1$

(62) **(2).** $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 \cdot (x^{\log_8 x})^3$

$$20 \times 8^7 = \frac{160}{x^3} \cdot x^{3\log_8 x}$$

$$8^6 = x^{\log_2 x} - 3$$

$$2^{18} = x^{\log_2 x - 3}$$

$\Rightarrow$ $18 = (\log_2 x - 3)(\log_2 x)$
Let $\log_2 x = t$
$\Rightarrow$ $t^2 - 3t - 18 = 0$
$\Rightarrow$ $(t - 6)(t + 3) = 0$
$\Rightarrow$ $t = 6, -3$
$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$
$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

(63) **(3).** Probability that no student solve the problem is

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5} = \frac{1}{5}$$

Probability that the problem will be solved by at least one student is equal to $1 - \frac{1}{5} = \frac{4}{5}$.

(64) **(1).** ${}^6C_2 \times {}^6C_4 + {}^6C_3 \times {}^6C_3 + {}^6C_4 \times {}^6C_2$
$= (12)^2 + (20)^2 + (15)^2$
$= 225 + 400 + 225 = 850$

(65) **(2).** S_A = sum of numbers between 100 & 200 which are divisible by 7.
$\Rightarrow$ $S_A = 105 + 112 + ... + 196$

$$S_A = \frac{14}{2}[105 + 196] = 2107$$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.
$S_B = 104 + 117 + ... + 195$

$$= \frac{8}{2}[104 + 195] = 1196$$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.
$S_C = 182$
$\Rightarrow$ H.C.F. $(91, n) > 1 = S_A + S_B - S_C = 3121$

(66) **(4).** $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$\int \cos\left(2\left(\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right)dx = \int \frac{1-\left(\frac{1-x}{1+x}\right)}{1+\frac{1-x}{1+x}}dx$$

$$= \int x\,dx = \frac{x^2}{2} + c$$

$y = \frac{x^2}{2} + c$ are family of parabolas.

(67) **(3).** $a < b < c$ are in A.P.
$\angle C = 2\angle A$ (Given)
$\Rightarrow$ $\sin C = \sin 2A$
$\Rightarrow$ $\sin C = 2\sin A \cdot \cos A$
$\Rightarrow$ $\frac{\sin C}{\sin A} = 2\cos A$

$\Rightarrow \quad \frac{c}{a} = 2\frac{b^2+c^2-a^2}{2bc}$

Put $a = b - \lambda,\ c = b + \lambda,\ \lambda > 0$

$\Rightarrow \quad \lambda = \frac{b}{5}$

$\Rightarrow \quad a = b - \frac{b}{5} = \frac{4}{5}b,\ c = b + \frac{b}{5} = \frac{6b}{5}$

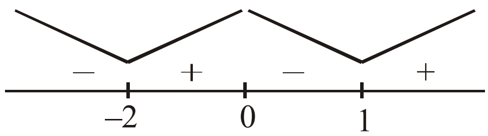
$\Rightarrow$ Required ratio = 4 : 5 : 6

(68) (1). $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x-1)(x+2)$



Points of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$

(69) (4). $\vec{x} = \frac{6(2\hat{i} - 2\hat{j} + \hat{k})}{3},\ \vec{y} = \frac{\sqrt{3}(\hat{i} + \hat{j} - \hat{k})}{\sqrt{3}}$

$|\vec{x} + 2\vec{y}| = |6\hat{i} - 2\hat{j}| = \sqrt{40} = 2\sqrt{10}$

(70) (3). R is reflexive if it contains (1, 1) (2, 2) (3, 3)

$\because$ $(1, 2) \in R,\ (2, 3) \in R$

$\therefore$ R is symmetric if $(2, 1), (3, 2) \in R$

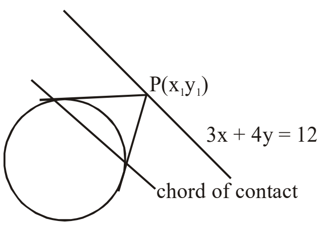
Now, R = {(1, 1), (2, 2),(3, 3),(2, 1),(3, 2), (2, 3), (1, 2)}

R will be transitive if $(3, 1)\ ;\ (1, 3) \in R$.

Thus, R becomes and equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1), (3, 2), (1, 3), (1, 2). Hence, the total number of ordered pairs is 7.

(71) 4. Let $P(x_1, y_1)$ be a point on the line $3x + 4y = 12$.

Equation of variable chord of contact of $P(x_1, y_1)$ wrt circle $x^2 + y^2 = 4$ is



$xx_1 + yy_1 - 4 = 0$ .....(1)

Also $3x_1 + 4y_1 - 12 = 0$

$x_1 + \frac{4}{3}y_1 - 4 = 0$ ......(2)

Comparing (1) & (2), x = 1, y = 4/3

$\therefore$ Variable chord of contact always passes through (1, 4/3).

(72) 8. Let n (A) = number of students opted Mathematics = 70,

n (B) = number of students opted Physics = 46,

n(C) = number of students opted Chemistry = 28,

$n(A \cap B) = 23,\ n(B \cap C) = 9,$

$n(A \cap C) = 14,\ n(A \cap B \cap C) = 4,$

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B)$

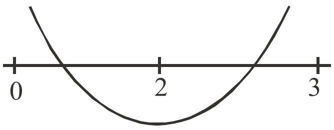
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

Number of students not opted for any course

$=$ Total $- n(A \cup B \cup C) = 140 - 102 = 38$

(73) 1.



Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore$ $f(0)\,f(2) < 0$ .....(1)

& $f(2)\,f(3) < 0$ .....(2)

From (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$

$\Rightarrow \quad \frac{49}{4} < c < 49$

$\therefore$ s = {13, 14, 15, ... 23}

Number of elements in set S = 11

(74) 3. $\cot\left(\sum_{n=1}^{19} \cot^{-1}(1 + n(n+1))\right)$

$= \cot\left(\sum_{n=1}^{19} \cot^{-1}(n^2 + n + 1)\right)$

$= \cot\left(\sum_{n=1}^{19} \tan^{-1}\frac{1}{1 + n(n+1)}\right)$

$= \sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1} n)$

$$= \cot(\tan^{-1} 20 - \tan^{-1} 1) = \frac{\cot A \cot\beta + 1}{\cot\beta - \cot A}$$

(Where $\tan A = 20$, $\tan B = 1$)

$$= \frac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}} = \frac{21}{19}$$

(75) **2.** $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$,

$$\text{I.F.} = e^{\int\left(\frac{2x+1}{x}\right)dx} = e^{\int\left(2+\frac{1}{x}\right)dx}$$

$$= e^{2x+\ln x} = e^{2x} \cdot x$$

$$y\,(xe^{2x}) = \int e^{-2x} \cdot x\, e^{2x} + C$$

$$\Rightarrow \quad xye^{2x} = \int x\, dx + C$$

$$\Rightarrow \quad 2xye^{2x} = x^2 + 2C$$

It passes through $\left(1, \frac{1}{2}e^{-2}\right)$ we get $C = 0$

$$y = \frac{xe^{-2x}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$$

$\Rightarrow$ $f(x)$ is decreasing in (1/2, 1).