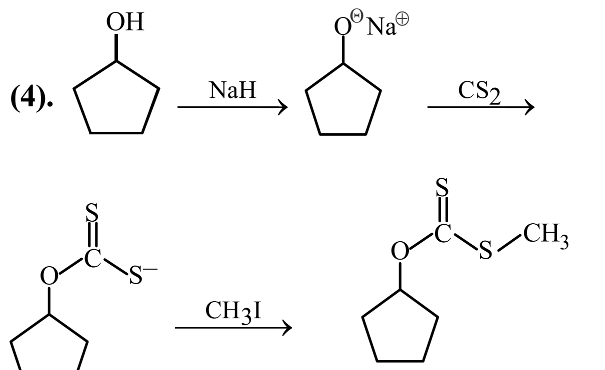
JEE MAIN 2020
FULL TEST-3 SOLUTIONS

STANDARD ANSWER KEY

Q	1	2	3	4	5	6	7	8	9	10	11
A	4	1	1	3	2	2	2	3	3	1	1
Q	12	13	14	15	16	17	18	19	20	21	22
A	2	3	1	1	3	1	3	4	1	5	4
Q	23	24	25	26	27	28	29	30	31	32	33
A	8	2	4	3	2	3	3	3	3	2	2
Q	34	35	36	37	38	39	40	41	42	43	44
A	2	2	1	4	4	4	3	4	1	4	1
Q	45	46	47	48	49	50	51	52	53	54	55
A	1	2	2	7	4	6	1	2	2	3	2
Q	56	57	58	59	60	61	62	63	64	65	66
A	3	2	2	2	4	2	3	1	2	4	1
Q	67	68	69	70	71	72	73	74	75		
A	1	1	4	1	8	4	0	2	2		

(1) **(4).**



Methyl cyclopentylxanthate

(2) **(1).** $[Fe(H_2O)_6]Cl_2$, $Fe^{2+} \rightarrow 3d^6 \rightarrow (t_{2g})^4(e_g)^2$

C.F.S.E. $= 4 \times (-0.4\ \Delta_0) + 2 \times 0.6\Delta_0$

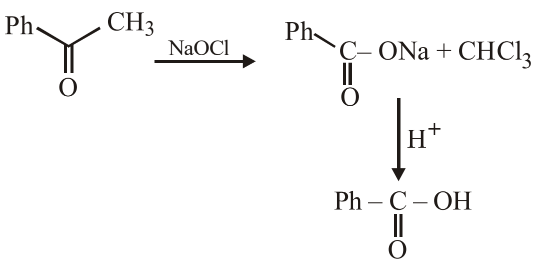
$= -0.4\Delta_0$

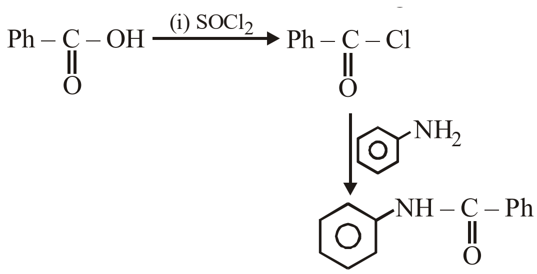
$K_2[NiCl_4]$, $Ni^{2+} \rightarrow 3d^8 \rightarrow (e)^4(t_2)^4$

C.F.S.E. $= 4 \times (-0.6\Delta_t) + 4 \times (0.4\Delta_t)$

$= -0.8\ \Delta_t$

(3) **(1).**





(4) **(3).** $h\nu = \phi + h\nu_0$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$h\nu = \phi + \frac{1}{2}mv^2$$

$$\phi = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} - \frac{1}{2} \times 9 \times 10^{-31} \times (6 \times 10^5)^2$$

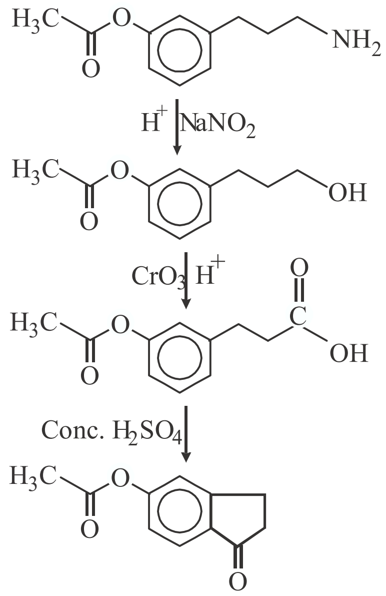
$\phi = 3.35 \times 10^{-19}$ J $\Rightarrow \phi \approx 2.1$ eV

(5) **(2).** A_2B_3 has HCP lattice.

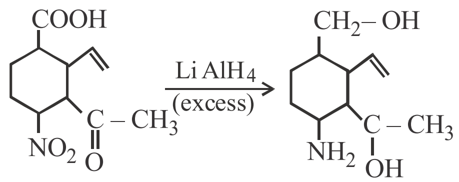
If A form HCP, then $(3/4)^{th}$ of THV must occupied by B to form A_2B_3.

If B form HCP, then $(1/3)^{th}$ of THV must occupied by A to form A_2B_3.

(6) **(2).**

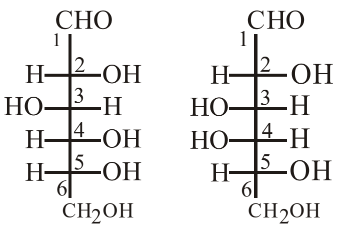


(7) **(2).**



$LiAlH_4$ will not affect C = C in this compound.

(8) **(3).** Glucose and galactose are C-4 Epimer's



Glucose Galactose

(9) **(3).** Z = 120

Its general electronic configuration may be represented as [Nobal gas] ns^2 , like other alkaline earth metals.

(10) **(1).** Pt(s) | H_2 (g, 1 bar | HCl (aq) | AgCl (s) | Ag (s) | Pt (s)

10^{-6} m

Anode : $H_2 \rightarrow 2H^+ + 2e \times 1$

Cathode : $e^- + AgCl(s) \rightarrow Ag(s) + Cl^-(aq) \times 2$

$$H_2(g) + AgCl\,(s) \rightarrow 2H^+ + 2Ag(s) + 2Cl^-(aq)$$

$$E_{cell} = E^\circ_{cell} - \frac{0.06}{2}\log_{10}((H^+)^2 \cdot (Cl^{-1})^2)$$

$$0.92 = (E^0_{H_2/H^+} + E^0_{AgCl/Ag,Cl^-}) - \frac{0.06}{2}\log_{10}((10^{-6})^2(10^{-6})^2)$$

$$0.92 = 0 + E^0_{AgCl/Ag,Cl^-} - 0.03\log_{10}(10^{-6})^4$$

$$E^0_{AgCl/Ag,Cl^-} = 0.92 + 0.03 \times -24 = 0.2\ V$$

(11) **(1).** $Ag_2S + 2\,NaCN \xrightarrow{O_2}$

$Na\,[Ag\,(CN)_2] + Na_2SO_4$

$\downarrow Zn$

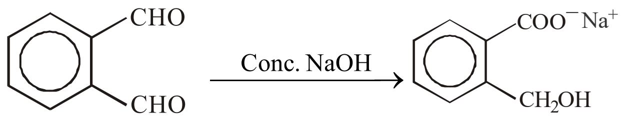
$Na_2\,[Zn\,(CN)_4] + 2Ag$

(12) **(2).** (a) $2NaN_3(s) \xrightarrow{\Delta} Na + 3H_2$

(b) $[Ni(CO)_4] \xrightarrow{250°C} Ni + 4CO$

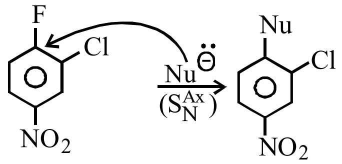
(d) $HgO(s) \xrightarrow{\Delta} Hg + O_2$

(13) **(3).**



(14) **(1).** $H_3BO_3 + HOH \rightarrow [B(OH)_4]^- + H^+$

(15) **(1).**



(16) **(3).** Rate of S_N1 reaction $\propto$ stability of carbocation

(17) **(1).**

$H_2O(g) + C(s) \rightarrow CO(g) + H_2(g);\ \Delta H = 131$ kJ ..... (1)

$CO(g) + ½\,O_2(g) \rightarrow CO_2(g);\ \Delta H = –282$ kJ ..... (2)

$H_2(g) + ½\,O_2(g) \rightarrow H_2O(g);\ \Delta H = –242$ kJ ..... (3)

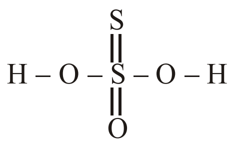
$C(s) + O_2(g) \rightarrow CO_2(g);\ \Delta H = x$ kJ ..... (4)

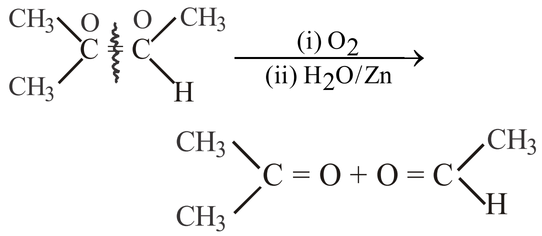
(1) + (2) + (3) = 4

131 – 282 – 242 = x

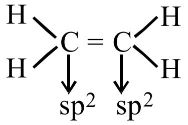
x = – 393 kJ

(18) **(3).** $H_2S_2O_3$



(19) **(4).** 

(20) **(1).** C_2H_4 : s = sp^2–sp^2 ; π = $2p\pi$–$2p\pi$



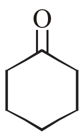
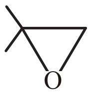
(21) **5.** 1 litre colloidal solution contains = $10^{-3} \times N_A$ Molecule of $C_{12}H_{25}SO_4Na$.
1 mm^3 colloidal solution contains

$$= \frac{10^{-3} \times N_A \times 1}{10^6}$$

= 6×10^{14} molecule of $C_{12}H_{25}SO_4Na$.
10^{13} colloidal particles = 6×10^{14} molecule of $C_{12}H_{25}SO_4Na$.

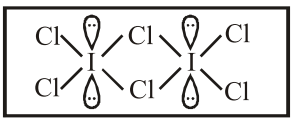
1 colloidal particle = $\frac{6 \times 10^{14}}{10^{13}}$ molecule of

$C_{12}H_{25}SO_4Na$ = 60 molecules.
Hence, a = 5

(22) **4.** , $CH_3 - COOC_2H_5$, ,

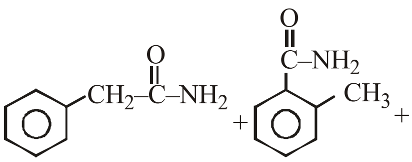
$H - \underset{\underset{O}{\|}}{C} - O - CMe_3$ gives 3° alcohol.

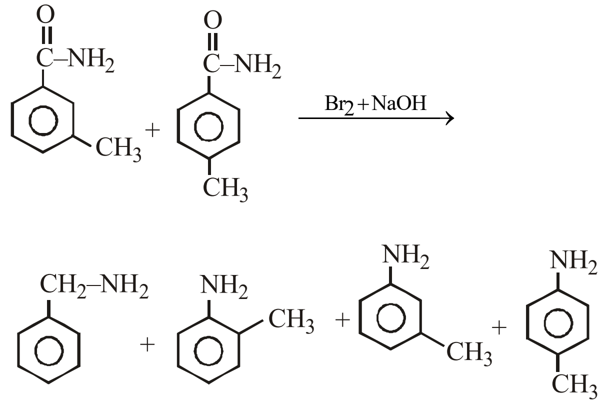
(23) **8.** I_2Cl_6 is a planar molecule.



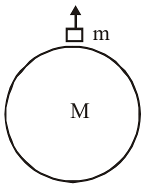
(24) **2.** $\Delta T_f = i \times K_f \times$ molality

$$\Rightarrow \quad 0.5 \times 1.72 \times \frac{20}{172} \times \frac{1000}{50} = 2$$

(25) **4.** 



(26) **(3).** $V_{terminal} \Rightarrow F_{net} = 0$

(27) **(2).** 

Minimum energy required (E)
= – (Potential energy of object at surface of earth)
Now $M_{earth} = 64\, M_{moon}$

$$\rho \cdot \frac{4}{3}\pi R_e^3 = 64 \cdot \frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$$

$$\text{Now, } \frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$$

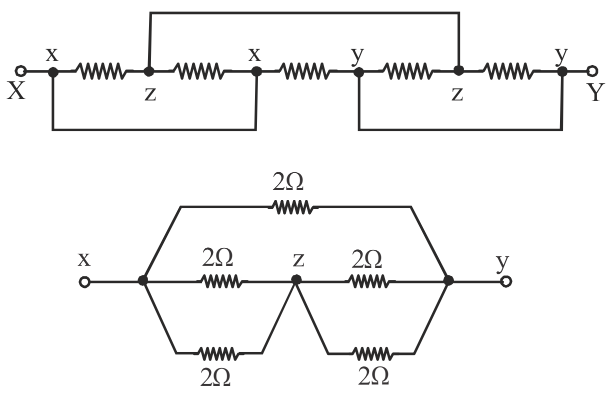
$$\Rightarrow \quad E_{moon} = \frac{E}{16}$$

(28) **(3).** $X = 5\,YZ^2$

$$Y = \frac{X}{5Z^2} \; ; \; [Y] = \frac{[X]}{[Z^2]} = \frac{A^2 \cdot M^{-1}L^{-2}T^4}{(MA^{-1}T^{-2})^2}$$

$$= M^{-3}\, L^{-2}\, T^8\, A^4$$

(29) **(3).**



$R_{xy} = 1\ \Omega$

(30) **(3).** 

By momentum conservation

$P_x - P_y = P_P$

$$\frac{h}{\lambda_x} - \frac{h}{\lambda_y} = \frac{h}{\lambda_P}$$

$$\lambda_P = \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$$

(31) **(3).** $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad 4\pi^2 \times \frac{1250 \times 1250}{\pi^2} = \frac{1}{20 \times 10^{-3} \times C}$$

$$\therefore \quad C = \frac{1000}{20 \times 4 \times 1250 \times 1250} = 8 \times 10^{-6} F$$

$\therefore \quad Q = CV = 8 \times 10^{-6} \times 25 = 0.2m\ C$

(32) **(2).** Angular momentum conservation.

$mV_0 L = mV_1 (L - \ell)$

$$V_1 = V_0 \left(\frac{L}{L-\ell}\right)$$

$w_g + w_p = \Delta\ KE$

$$-mg\ell + w_p = \frac{1}{2} m (V_1^2 - V_0^2)$$

$$w_p = mg\ell + \frac{1}{2} mV_0^2 \left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2} mV_0^2 \left(\left(1 - \frac{\ell}{L}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2} mV_0^2 \left(\left(1 + \frac{2\ell}{L}\right) - 1\right)$$

$$= mg\ell + \frac{1}{2} mV_0^2 \left(\frac{2\ell}{L}\right)$$

$$w_p = mg\ell + mV_0^2 \left(\frac{\ell}{L}\right)$$

Here, V_0 = maximum velocity = $\omega \times A$

$$= \left(\sqrt{\frac{g}{L}}\right) (\theta_0 L)$$

$$V_0 = \theta_0 \sqrt{gL}$$

$$w_p = mg\ell + m(\theta_0 \sqrt{gL})^2 \frac{\ell}{L}$$

$$= mg\ell (1 + \theta_0^2)$$

(33) **(2).** $\frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{2(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{R}$$

$2\mu_1 - \mu_2 = 1$

(34) **(2).** Maximum current will flow from zener if input voltage is maximum.

When zener diode works in breakdown state, voltage across the zener will remain same.

$\therefore \quad V_{across\ 4k\Omega} = 6V$

$\therefore$ Current through $4K\Omega = \frac{6}{4000} A = \frac{6}{4} mA$

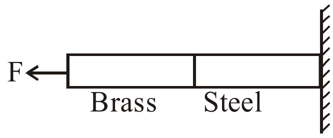
Since input voltage = 16V

$\therefore$ Potential difference across $2K\Omega$ = 10V

$\therefore$ Current through $2k\Omega = \frac{10}{2000} = 5mA$

$\therefore$ Current through zener diode

$= (I_S - I_L) = 3.5$ mA

(35) **(2).** 

$$k_1 = \frac{y_1 A_1}{\ell_1} = \frac{120 \times 10^9 \times A}{I}$$

$$k_2 = \frac{y_2 A_2}{\ell_2} = \frac{60 \times 10^9 \times A}{I}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{120 \times 60 \times 10^9 \times A}{180}$$

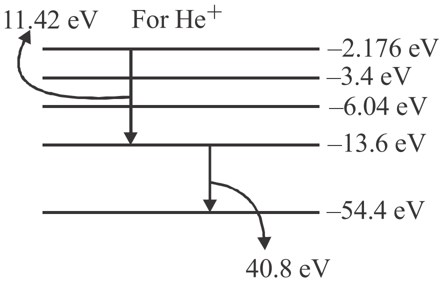
$k_{eq} = 40 \times 10^9 \times A$

$F = k_{eq}(x)$

$F = (40 \times 10^9)A \,.\, (0.2 \times 10^{-3})$

$\frac{F}{A} = 8 \times 10^6 \text{ N/m}^2$

(36) (1).



For 108.5nm, $\Delta E = 11.42$ eV

For 30.4 nm, $\Delta E = 40.8$ eV

(37) (4).

(38) (4). $\frac{1}{2}mV^2 = -q(V_f - V_i)$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$; $\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right)$

$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right)$

$v \propto \sqrt{\ln\left(\frac{r_0}{r}\right)}$

(39) (4). $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; $1 - \frac{v}{u} = \frac{v}{f}$

$1 - m = \frac{v}{f}$; $m = 1 - \frac{v}{f}$

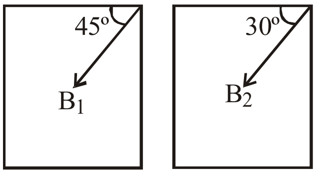
At $v = a$, $m_1 = 1 - \frac{a}{f}$

At $v = a + b$, $m_2 = 1 - \frac{a+b}{f}$

$m_2 - m_1 = c = \left[1 - \frac{a+b}{f}\right] - \left[1 - \frac{a}{f}\right]$

$c = \frac{b}{f}$; $f = \frac{b}{c}$

(40) (3).



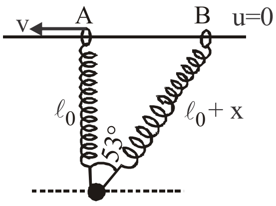
$f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1 \cos 45^\circ}{I}}$

$f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2 \cos 30^\circ}{I}}$

$\frac{f_1}{f_2} = \sqrt{\frac{B_1 \cos 45^\circ}{B_2 \cos 30^\circ}}$; $\frac{B_1}{B_2} = 0.7$

(41) (4). $R = \frac{mV}{qB}$ $\therefore R \propto$ momentum

(42) (1).



$\cos 53^\circ = \frac{\ell_0}{\ell_0 + x}$

$\frac{3}{5} = \frac{\ell_0}{\ell_0 + x}$; $x = \frac{2}{3}\ell_0$

From $B \to A$

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$; $v = x\sqrt{\frac{k}{m}} = \frac{2}{3}\ell_0\sqrt{\frac{k}{m}}$

(43) (4). $P = \frac{\vec{F}\cdot\vec{S}}{t} = \frac{(2\hat{i}+3\hat{j}+4\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{4}$

$= \frac{6+12+20}{4} = 9.5$ Watt

(44) (1). $g_{eff} = g - a = 10 - 2 = 8 \text{ m/s}^2$

$T = \frac{2m_1m_2}{m_1+m_2}g_{eff} = \frac{2(2)(4)(8)}{6} = \frac{64}{3}$ N

(45) (1). Let v be velocity acquired by the charged particle when accelerated through the potential difference V.

$\therefore \quad \frac{1}{2}mv^2 = qV \quad \text{or} \quad v = \sqrt{\frac{2qV}{m}}$

As the charged particle describes a circular path of radius R in the uniform magnetic field.

$\therefore \quad \frac{mv^2}{R} = qvB$

or $\quad R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{\sqrt{m}}{B}\sqrt{\frac{2V}{q}}$

As q, B and V remain the same.

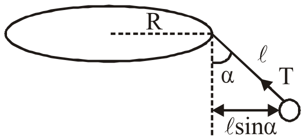
$R \propto \sqrt{m}$

$\frac{R_A}{R_B} = \sqrt{\frac{m_A}{m_B}}$

$\frac{m_A}{m_B} = \left(\frac{R_A}{R_B}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(46) **2.** $T\cos\alpha = mg$

$T\sin\alpha = m\omega^2 r$



$\tan\alpha = \frac{\omega^2 r}{g}$

$\omega = \sqrt{\frac{g\tan\alpha}{r}} = \sqrt{\frac{10\times\frac{3}{4}}{\frac{35}{24}\times\frac{3}{5}+1}} = \sqrt{\frac{\frac{15}{2}}{\frac{15}{8}}} = 2$

(47) **2.** $B = \mu_0 nI = \mu_0 nI_0 \sin\omega t$

$\phi = B\pi r^2 = \mu_0 nI_0 \pi r^2 \sin\omega t$

$2\pi rE = -\frac{d\phi}{dt} = -\omega\pi r^2\mu_0 nI_0\cos\omega t$

$E = \frac{1}{2}\omega\mu_0 nI_0 r\cos\omega t$

(48) **7.** $I_1 \propto A^2$

$I_1 = kA_1^2\ ;\ I_2 = kA_2^2$

$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \Rightarrow \frac{A_1}{A_2} = \frac{4}{3}$

$\frac{I_{max}}{I_{min}} = \frac{(A_1+A_2)^2}{(A_1-A_2)^2} = 49$

(49) **4.** $x = R\frac{\ell}{100-\ell}\ ;\ \frac{\Delta x}{x} = \frac{\Delta\ell}{\ell} + \frac{\Delta\ell}{100-\ell}$

$\frac{\Delta x}{x}\times 100 = \left(\frac{1}{50}+\frac{1}{50}\right)\times 100 = 4\%$

(50) **6.** $\frac{A_1}{A_0} = \left(\frac{1}{2}\right)^{t/\tau}$

$\Rightarrow \frac{100/20}{141/20} = \frac{1}{\sqrt{2}} = \left(\frac{1}{2}\right)^{t/\tau} \Rightarrow \frac{t}{\tau} = \frac{1}{2}$

$\Rightarrow \tau = 2t = 2 = 2\times 3$ days $= 6$ days.

(51) **(1).** $f'(x)\times h'(f(x)) = 1$

$\Rightarrow f'(x) = \frac{1}{h'(f(x))} = 1 + \log f(x)$

(52) **(2).** $h(x) = f(g(x))$

$\Rightarrow h'(x) = f'(g(x)).\ g'(x)$ and $f'(x) = e^x - 1$

$\Rightarrow h'(x) = (e^{g(x)}-1)\ g'(x)$

$\Rightarrow h'(x) = (e^{x^2-x}-1)\ (2x-1) \ge 0$

Case-I : $e^{x^2-x} \ge 1$ and $(2x-1) \ge 0$

$\Rightarrow x \in [1, \infty)$ ......(1)

Case-II : $e^{x^2-x} \le 1$ and $(2x-1) \le 0$

$x \in \left[0, \frac{1}{2}\right]$ ......(2)

Hence, $x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

(53) **(2).** $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. $= e^{\ln(x^2+1)} = x^2+1$

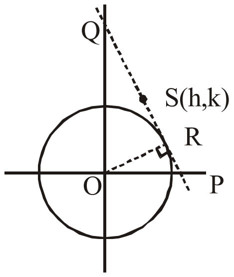
So, general solution is $y.(x^2+1) = \tan^{-1}x + c$

As $y(0) = 0 \Rightarrow c = 0$

$y(x) = \frac{\tan^{-1}x}{x^2+1}$. As $\sqrt{a}\cdot y(1) = \frac{\pi}{32}$

$\Rightarrow \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

(54) **(3).**



Let the mid point be S (h, k)
P(2h, 0) and Q (0, 2k)

Equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\frac{1}{4h^2}+\frac{1}{4k^2}=1$

$\Rightarrow x^2+y^2-4x^2y^2=0$

Aliter : Tangent to circle
$x\cos\theta+y\sin\theta=1$
P : $(\sec\theta, 0)$
Q : $(0, \text{cosec}\,\theta)$

$2h=\sec\theta \Rightarrow \cos\theta=\frac{1}{2h}$ & $\sin\theta=\frac{1}{2k}$

$\frac{1}{(2x)^2}+\frac{1}{(2y)^2}=1$

(55) **(2).** $\sim(\sim s\vee(\sim r\wedge s))$
$s\wedge(r\vee\sim s)$
$(s\wedge r)\vee(s\wedge\sim s)$
$(s\wedge r)\vee(c)$
$(s\wedge r)$

(56) **(3).** $a_1+a_4+a_7+a_{10}+a_{13}+a_{16}=114$

$\Rightarrow \frac{6}{2}(a_1+a_{16})=114$

$\Rightarrow a_1+a_{16}=38$

So, $a_1+a_6+a_{11}+a_{16}=4$

$=\frac{4}{2}(a_1+a_{16})=2\times 38=76$

(57) **(2).** $(3^{1/8}+5^{1/4})^{84}=(5^{1/4}+3^{1/8})^{84}$

Now $T_n={}^{84}C_n\left(5^{\frac{84-n}{4}}\right)(3^{n/8})$

If T_n = rational $\Rightarrow$ n is multiple of 8

$\Rightarrow$ n = 0, 8, 16,...... 80 $\Rightarrow$ n can take 11 terms

$\Rightarrow$ Number of rational terms = 11

$\Rightarrow$ Number of irrational terms = 85 – 11 = 74

(58) **(2).** $4a^2+b^2=8$ ....(1)

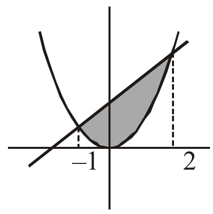
Also, $\left.\frac{dy}{dx}\right)_{(1,2)}=-\frac{4x}{y}=-2$

$\Rightarrow -\frac{4a}{b}=\frac{1}{2} \Rightarrow b=-8a$

$\Rightarrow b^2=64a^2 \Rightarrow 68a^2=8$

$\Rightarrow a^2=2/17$

(59) **(2).** $x^2\le y\le x+2$
$x^2=y\,;\,y=x+2$
$x^2=x+2$
$x^2-x-2=0$
$(x-2)(x-1)=0$
$x=2,-1$



Area = $\int_{-1}^{2}(x+2)-x^2dx=\frac{9}{2}$

(60) **(4).** $I=\int 2^{2^x}\cdot 2^x dx=\int 2^{2^x}\cdot(2^x\ln 2)\times\frac{(\ln 2)}{(\ln 2)^2}dx$

Put $2^{2^x}=t \Rightarrow (2^{2^x}\ln 2)(2^x\ln 2)=dt$

$\Rightarrow I=\int\frac{dt}{(\ln 2)^2}=\frac{t}{(\ln 2)^2}+c=\frac{2^{2^x}}{(\ln 2)^2}+C$

$\Rightarrow A=\frac{1}{(\log 2)^2}$

(61) **(2).** Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is parallel

to vector = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$

$\therefore$ Required magnitude of projection

$$= \frac{|(2\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+\hat{k}|}$$

$$= \frac{|2-6+1|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

(62) (3). $\lim\limits_{x\to 0^+} (e^x + x)^{1/x} = e^{\lim\limits_{x\to 0^+}\left(\frac{e^x - 1 + x}{x}\right)} = e^{1+1} = e^2$

(63) (1). $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$$z^5 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}+i}{2}$$

$$z^8 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\left(\frac{1+i\sqrt{3}}{2}\right)$$

$$(1 + iz + z^5 + iz^8)^9$$

$$= \left(1 + \frac{i\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{i}{2} - \frac{i}{2} + \frac{\sqrt{3}}{2}\right)^9$$

$$= \left(\frac{1+i\sqrt{3}}{2}\right)^9 = \cos 3\pi + i\sin 3\pi = -1$$

(64) (2). $S.D = \sqrt{\frac{\Sigma (x-\overline{x})^2}{n}}$

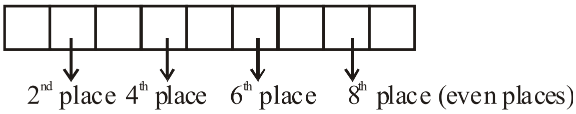
$$\overline{x} = \frac{\Sigma x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$$

$$\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$$

$$\Rightarrow \quad 5\times 4 = 2\left(1+\frac{k}{16}\right)^2 + \frac{5k^2}{8}$$

$$\Rightarrow \quad 18 = \frac{3k^2}{4} \Rightarrow k^2 = 24 \Rightarrow k = 2\sqrt{6}$$

(65) (4).



Number of such numbers

$$= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!\,4!} = 180$$

(66) (1). $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$\therefore$ $f'(x) = 6x^2 - 18ax + 12a^2$

$f''(x) = 12x - 18a$

For max./min value $6x^2 - 18ax + 12a^2 = 0$

$6(x-a)(x-2a) = 0$, thus $x = a$, $x = 2a$

$f''(a) = 12a - 18a = -6a < 0$

$f''(2a) = 24a - 18a = 6a < 0$

$\therefore$ f(x) is max at x = a and minimum at x = 2a.

$\Rightarrow$ p = a and q = 2a

Given that $p^2 = q$ so $a^2 = 2a$

a = 2

(67) (1). $P\left(\frac{\overline{A}}{B}\right) = \frac{P(\overline{A}\cap B)}{P(B)} = \frac{P(B) - P(\overline{A}\cap B)}{P(B)}$

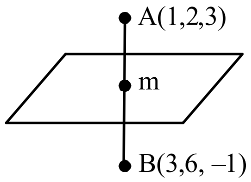
$$= 1 - \frac{1/14}{1/8} = \frac{3}{7}$$

(68) (1). $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 2x}{1+25^x} dx$

$$\Rightarrow \int_0^{\pi/2} \frac{\cos^2 2x}{1+25^x} + \frac{\cos^2 2x}{1+25^{-x}} dx = \int_0^{\pi/2} \cos^2 2x\, dx$$

$$= \int_0^{\pi/2} \frac{1+\cos 4x}{2} dx = \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$$

(69) (4).



M (mid point) = (2, 4, 1)

DR (2, 4, –4)

Equation of plane

$2(x-2) + 4(y-4) - 4(z-1) = 0$

$\Rightarrow$ $x - 2 + 2y - 8 - 2z + 2 = 0$

$\Rightarrow$ $x + 2y - 2z - 8 = 0$

(70) **(1).** Let the point be (h, k)

Now equation of tangent to the parabola $y^2 = 4ax$ whose slope is m is

$$y = mx + \frac{a}{m}$$

as it passes through (h, k)

$\therefore$ $k = mh + \frac{a}{m} \Rightarrow m^2h - mk + a = 0$

It has two roots $m_1, 2m_1$

$\therefore$ $m_1 + 2m_1 = \frac{k}{h}, 2m_1.m_1 = \frac{a}{h}$

$m_1 = \frac{k}{3h}$ ... (i)

$m_1^2 = \frac{a}{2h}$ ... (ii) from (i) & (ii)

$\Rightarrow$ $\frac{k^2}{(3h)^2} = \frac{a}{2h} \Rightarrow k^2 = \frac{9a}{2}h$

Thus locus of point is $y^2 = \frac{9}{2}ax$.

(71) **8.** $|A_r| = 2r - 1$

$$\sum_{r=1}^{100}(2r-1) = 1 + 3 + \ldots + 199 = 10000 = 10^{k/2}$$

$\Rightarrow$ $\frac{k}{2} = 4 \Rightarrow k = 8$

(72) **4.** $2^{\sin^2 x} + 4.2^{\cos^2 x} = 6$

Let $2^{\sin^2 x} = t$

$\therefore$ $t + \frac{8}{t} = 6 \Rightarrow (t-4)(t-2) = 0$

$2^{\sin^2 x} = 4$ & $2^{\sin^2 x} = 2$

$\Rightarrow$ $\sin x = \pm\sqrt{2}$ (reject) and $\sin x = \pm 1$

$\therefore$ in $(-2\pi, 2\pi)$ there are 4 solutions.

(73) **0.** $\lim_{h\to 0} |f(x+h) - f(x)| \le (x + h - x)^2$

$\Rightarrow$ $\lim_{h\to 0} |f(x+h) - f(x)| \le |h|^2$

$\Rightarrow$ $\lim_{h\to 0} \left|\frac{f(x+h) - f(x)}{h}\right| \le 0 \Rightarrow f'(x) = 0$

$\Rightarrow$ f(x) is constant function.

$\Rightarrow$ $f(1) = 0$.

(74) **2.** $f(0) = 0$ & f(x) is odd.

Further, if $x > 0$ then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

(75) **2.** $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$

$B = \{0, 1, 2, 3, 4\}$

$A \times B$ has is 15 elements so number of subsets of $A \times B$ is 2^{15}.