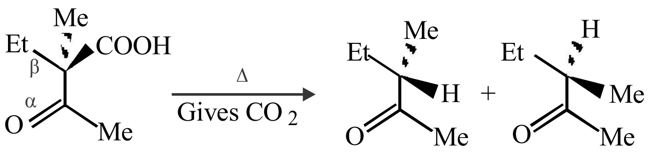
JEE MAIN 2020
FULL TEST-4 SOLUTIONS

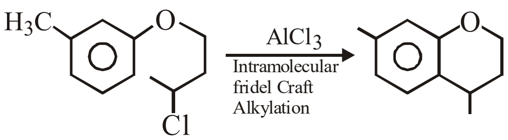
STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	3	4	1	2	4	3	1	4	4	2	3
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	3	1	4	3	4	2	3	2	3	7	2
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	6	9	5	1	4	2	4	2	2	1	1
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	4	4	1	4	4	1	2	1	3	2	3
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	1	7	2	6	2	6	3	1	1	3	4
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	2	4	4	3	2	3	1	1	3	3	1
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**		
A	2	2	2	1	2	2	4	5	5		

(1) **(3).**



β-keto acid   Achiral mixture

(2) **(4).**



(3) **(1).** Glycogen is an animal starch.
It consists of α-amylose and amylopectin.
Amylopectin is branched chain polysaccharide
Hence statement (1) is incorrect.

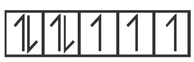
(4) **(2).** $V^{2+} \rightarrow [V(H_2O)_6]Cl_2$;

$[Ar]_{18}$ 

$3d^3$

3 unpaired e^-, spin only
Magnetic moment = 3.89 B.M.

$Co^{2+} \rightarrow [Co(H_2O)_6]Cl_2$

$[Ar]_{18}$ 

$3d^7$

3 unpaired e^-, spin only
Magnetic moment = 3.89 B.M.

(5) **(4).**

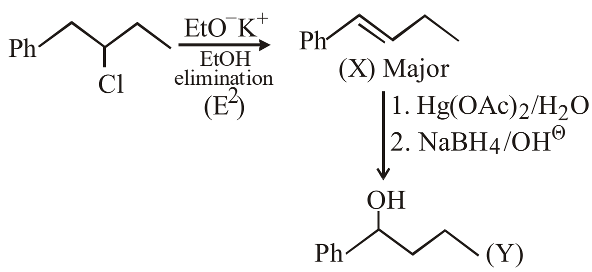
	Dispersed Phase	Dispersion Medium
Cheese	Liquid	Solid
Milk	Liquid	Liquid
Smoke	Solid	Gas

(6) **(3).** Siderite : $FeCO_3$
Kaolinite : $Al_2(OH)_4Si_2O_5$
Malachite : $Cu(OH)_2.CuCO_3$
Calamine : $ZnCO_3$

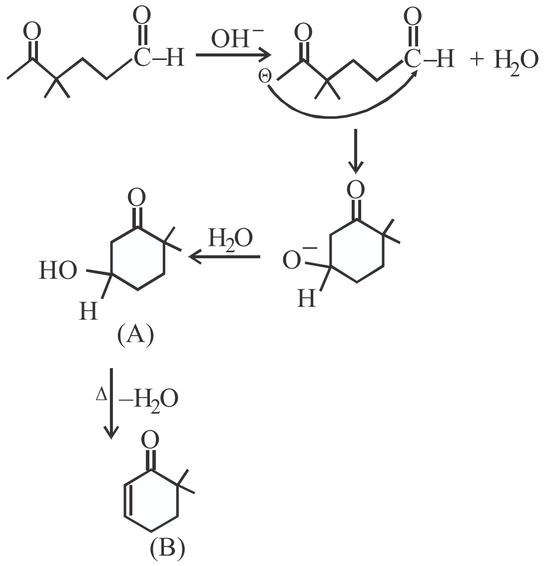
(7) **(1).** Rate constant (K) = 0.05 μg/year
means zero order reaction

$$t_{1/2} = \frac{a_0}{K} = \frac{5\mu g}{2 \times 0.05\ \mu g / year} = 50 \text{ year}$$

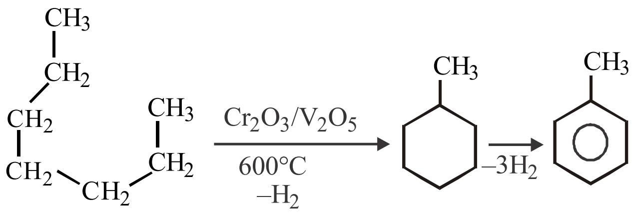
(8) **(4).**



(9) **(4).**



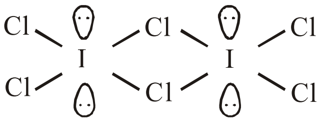
(10) **(2).**



(11) **(3).** CO_2 – sp linear.
SiO_2 – sp^3 – tetrahedral giant structure.

(12) **(3).**
(a) $PCl_5 + HOH \rightarrow H_3PO_4 + HCl$

(c)



Planar but Non-polar
(d) AlF_3 – Ionic compound No hybridisation

(13) **(1).** $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

At t = 0	3 mol	0	0
at equil.	3 – x	x	x

$$K_c = \frac{[PCl_3][Cl_2]}{[PCl_5]}$$

$$1.80 = \frac{x \times x}{3 - x}$$

$5.4 - 1.8\,x = x^2$
$x^2 + 1.8x - 5.4 = 0$
$D = b^2 - 4ac = (1.8)^2 - 4\times1\times(-5.4) = 24.84$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1.8 + \sqrt{24.84}}{2\times1} = \frac{4.98 - 1.8}{2}$$

$$= \frac{3.18}{2} = 1.59\ M$$

$[PCl_3] = 1.59$ M, $[Cl_2] = 1.59$ M

(14) **(4).** Molar of Cr^{+++} electrolysed
$= 250 \times 10^{-3}\,(0.20 - 0.10)$
$= 0.250 \times 0.10 = 0.025$ molar

$$m = \frac{E}{96500} \times i \times t$$

$$0.025 \times 52 = \frac{52}{3\times96500} \times 96.5 \times t$$

t = 75 s

(15) **(3).** $IE_1 \Rightarrow$

Li	Be	B	C
$2s^1$	$2s^2$	$2p^1$	$2p^2$

Order Li < B < Be < C

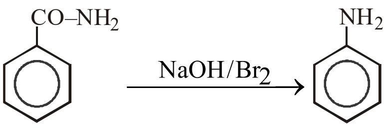
in gaseous form reducing power $\propto \dfrac{1}{I.E.}$

In aq. medium size $\Rightarrow Li^+ > .... > Rb^+ > Cs^+$

$\therefore$ Ionic mobility $\propto \dfrac{1}{H.R.}$

EA $\Rightarrow$ as per data
S > Se > Te > Po > O

(16) **(4).**



(17) **(2).** $H_2^{+1}O^{-2} \rightarrow H_2^0 + \frac{1}{2}O_2^0$

{from 1 molecule exchange of electrons = 2 = valency factor}

$$\text{mole} \times f = \frac{W}{E} = \frac{I \times t}{96500}\ ;\quad 4 \times 2 = \frac{4 \times t}{96500}$$

$t = 96500 \times 2 = 1.93 \times 10^5$ seconds.

(18) **(3).**

$CH_2 - CN$
|
$CH - CN$
|
$CH_2 - CN$

Propane–1,2,3–tricarbonitrile.

(19) **(2).** $\pi = C \times RT$

$hdg = C\,RT$

$h_{Hg} = \dfrac{11 \times 0.9}{13/6}\,cm$

$\dfrac{11 \times 0.9}{13.6 \times 76} = \dfrac{10}{M} \times 0.0821 \times 300$

$M = 25.2 \times 10^3$ g/mol

(20) **(3).** $Mn\,|\,Mg + \text{v. dil. } HNO_3 \rightarrow H_2 \uparrow$
(2%)

(21) **7.** $q = 0 \Rightarrow \Delta U = w$

$nC_V (T_2 - T_1) = -P_{ext.} (V_2 - V_1)$

$C_V (T_2 - 300) = -1\left(\dfrac{R \times T_2}{1.2} - \dfrac{R \times 300}{6}\right)$

$\dfrac{5}{2}(T_2 - 300) = \dfrac{300}{6} - \dfrac{T_2}{1.2}$

$5T_2 - 1500 = 100 - \dfrac{5T_2}{3}$

$\dfrac{20T_2}{3} = 1600 \qquad \therefore\ T_2 = 240$ K

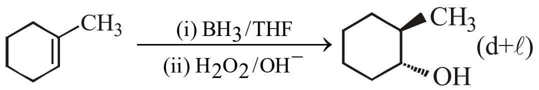
$\Delta U = nC_V\,\Delta T = 10 \times \dfrac{5R}{2}\,(240 - 300)$

$= -1500$ R

$\Delta H = \gamma\,\Delta U = \dfrac{7}{5} \times (-1500R) = -2100$ R

$\therefore\ X \times 300\,R = 2100\,R \qquad \therefore X = 7.$

(22) **2.**

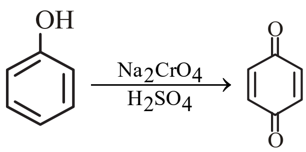


Reaction takes place through syn addition and boron goes at less crowded carbon in the major product.

(23) **6.** Magnesite, Fluorspar, Chalcocite, Argentite, Calamine, Barytes.

(24) **9.** $(NH_4)_3AsMo_{12}O_{40}$ (N in –3 and Mo in +6)
So, 12 – 3 = 9.

(25) **5.**



(D.U. = 5)

(26) **(1).** $W_{bb} = \Delta U + H$

$\Rightarrow (CE)\,E = \left(\dfrac{1}{2}CE^2 - 0\right) + H \quad \therefore\ H = \dfrac{1}{2}CE^2$

(27) **(4).** $\dfrac{1}{15} = (1.5 - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\dfrac{1}{f} = \left(\dfrac{1.5}{4/3} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\dfrac{f}{15} = \dfrac{1/2}{1/8}$; $f = 4 \times 15 = 60$ cm

(28) **(2).** $T_0 = 2\pi\sqrt{\dfrac{m}{k}} = \dfrac{2\pi}{\sqrt{10}}$

$A = A_0 e^{-t/\gamma}$

For $A = \dfrac{A_0}{e}$, $t = \gamma$

$t = \gamma = \dfrac{2m}{b} = \dfrac{2m}{\dfrac{B^2\ell^2}{R}} = 10^4\,s$

No. of oscillation $\dfrac{t}{T_0} = \dfrac{10^4}{2\pi/\sqrt{10}} \approx 5000$

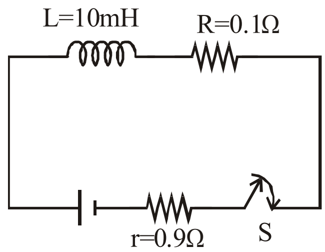
(29) **(4).** $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

$\dfrac{1}{\lambda_1} = R\left(\dfrac{1}{3^2} - \dfrac{1}{4^2}\right) = R\left(\dfrac{7}{9 \times 16}\right)$

$\dfrac{1}{\lambda_2} = R\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = R\left(\dfrac{5}{4 \times 9}\right)$

$\dfrac{\lambda_1}{\lambda_2} = \dfrac{\dfrac{5}{36}}{\dfrac{7}{9 \times 16}} = \dfrac{20}{7}$

(30) **(2).**



$i = i_0 (1 - e^{-t/\tau})$

$\frac{80}{100} i_0 = i_0 (1 - e^{-t/\tau})$

$0.8 = 1 - e^{-t/\tau}$

$e^{-t/\tau} = 0.2 = 1/5$

$-\frac{t}{\tau} = \ln\left(\frac{1}{5}\right)$; $-\frac{t}{\tau} = -\ln(5)$

$t = \tau \ln(5) = \frac{L}{R_{eq}} \cdot \ln(5) = \frac{10 \times 10^{-3}}{(0.1 + 0.9)} \times 1.6$

$t = 1.6 \times 10^{-2} = 0.016$ s

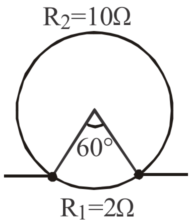
(31) (2). $R = \frac{\rho \ell^2}{A\ell D} d = \frac{\rho d \ell^2}{m}$

$R \propto \ell^2$

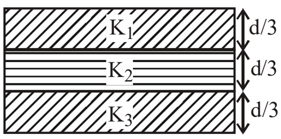
$R = 12\Omega$ (new resistance of wire)

$R_1 = 2\Omega, R_2 = 10\Omega$

$R_{eq} = \frac{10 \times 2}{10 + 2} = \frac{5}{3}\Omega$



(32) (1). I.

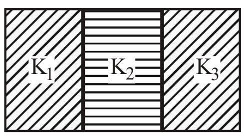


$C_1 = \frac{3\varepsilon_0 AK_1}{d}, C_2 = \frac{3\varepsilon_0 AK_2}{d}, C_3 = \frac{3\varepsilon_0 AK_3}{d}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$ ..... (1)

II.



$C_1 = \frac{\varepsilon_0 K_1 A}{3d}, C_2 = \frac{\varepsilon_0 K_2 A}{3d}, C_3 = \frac{\varepsilon_0 K_3 A}{3d}$

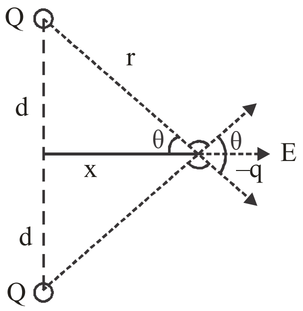
$C'_{eq} = C_1 + C_2 + C_3$

$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3)$ ..... (2)

$\frac{E_1}{E_2} = \frac{\frac{1}{2} C_{eq} V^2}{\frac{1}{2} C'_{eq} V^2}$

$= \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(33) (1).



$E = \frac{2KQ}{r^2} \cos\theta = \frac{2KQ}{r^2} \times \frac{x}{r}$

$F = \frac{2KQx}{r^3}$; $E = \frac{2KQx}{(d^2 + x^2)^{3/2}}$

$F = \frac{-2KQqx}{(d^2 + x^2)^{3/2}}$

for $x << d$, x^2 can be neglected

$F = \frac{-2KQqx}{d^3}$; $F \propto x$

(34) (4). $f_m = 100$ MHz $= 10^8$ Hz, $(V_m)_0 = 100$ V

$f_c = 300$ GHz, $(V_c)_0 = 400$V

Modulation Index $= \frac{(V_m)_0}{(V_c)_0} = \frac{100}{400} = \frac{1}{4}$

$= 0.25$

Upper band frequency (UBF) $= f_c + f_m$

Lower band frequency (LBF) $= f_c - f_m$

$\therefore$ UBF – LBF $= 2f_m = 2 \times 10^8$ Hz

(35) (4). $m = \int_0^R \rho\, 4\pi r^2 dr$

$m = 4\pi KR$

$v \propto \sqrt{4\pi K}$

$\frac{T}{R} = \frac{2\pi}{\sqrt{4\pi K}}$

(36) **(1).** $R = \frac{V}{I} = \frac{100}{1} = 100\,\Omega$

$V = IZ \Rightarrow Z = \frac{100}{0.5} = 200\,\Omega$

$Z = \sqrt{R^2 + X_L^2}$

$200 = \sqrt{(100)^2 + X_L^2} \Rightarrow X_L = 173.2\,\Omega$

$X_L = \omega_L$

$L = \frac{173.2}{100\pi} = 0.55\text{ H}$

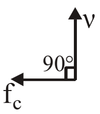
(37) **(4).** $a_t = K^2rt^2$. As, $\frac{dv}{dt} = a_t$

So $dv = K^2rt^2\,dt \Rightarrow \int dv = \int k^2 r\, t^2 dt$

$v = k^2 r \frac{t^3}{3}$; $v \propto t^3$

So, centripetal acceleration is variable

$\begin{bmatrix} \text{as } a_c \propto v^2 \\ \text{so } a_c \propto t^6 \end{bmatrix}$



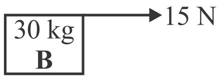
$P_{f_t} = \vec{f}_t \cdot \vec{v} \neq 0$; $P_{f_c} = \vec{f}_c \cdot \vec{v} = 0$ as

$\vec{f}_c$ is always perpendicular to $\vec{v}$

(38) **(4).** Pressure difference $P_2 - P_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$

(According to Bernoulli's theorem)

(39) **(1).**



Since both block will move separately, therefore $15 = 30\,a \Rightarrow a = 0.5\text{ m/s}^2$

(40) **(2).** Impulse $= \Delta p = m\,(v_f - v_i)$

Impulse received by m,

$= m\,[-2\hat{i} + \hat{j} - (3\hat{i} + 2\hat{j}) = m\,[-5\hat{i} - \hat{j}]$

Impulse received by

M = – (impulse received by m)

$= m\,[5\hat{i} + \hat{j}]$

(41) **(1).** $\frac{\alpha\, z}{k\theta} = 1$ and $\frac{\alpha}{\beta} = P$

$\alpha = \frac{k\theta}{z} = \frac{ML^2T^{-2}}{L} = MLT^{-2}$ and $\frac{\alpha}{\beta} = P$

$\beta = \frac{\alpha}{P} = \frac{ML^2T^{-2}}{ML^{-1}T^{-2}} = [M^0L^2T^0]$

(42) **(3).** Frequency of light $= 6 \times 10^{14}$ Hz

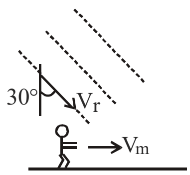
Energy in electron volt

$= \frac{6 \times 10^{14} \times 6.63 \times 10^{-34}}{1.6 \times 10^{-19}} = 2.49\text{ eV}$

Work function of the metal = 2eV

∴ Maximum kinetic energy of photoelectrons $= 0.49$ eV

(43) **(2).**



Given $V_r \cos 30° = 10$ m/s ; $V_r = \frac{20}{\sqrt{3}}$ m/s

(44) **(3).** Process $A \to B$ and $C \to D$ are isochoric

so $W_{AB} = 0$ and $W_{CD} = 0$

Process BC and DA are isobaric

So total work $W = 0 + W_{BC} + 0 + W_{DA}$

$= \mu R\,(T_C - T_B) + \mu R\,(T_A - T_D)$

$= \mu R[T_C - T_B + T_A - T_D]$

$= 6 \times \frac{25}{3}[2200 - 800 + 600 - 1200]$

$= 2 \times 25 \times 800 = 40$ kJ

(45) **(1).** In case of potentiometer, $E_1/E_2 = \ell_1/\ell_2$.

As given that $E_1 > E_2$, therefore $\ell_1 > \ell_2$.

Therefore, the null point for the cell of emf E_2 must be at shortest length than that of cell E_1. Thus, the null point on potentiometer wire should shift towards left of C.

(46) **7.** $V_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$0 = \frac{(200)\,(2) + (80)\,V_m}{200 + 80}$

$\Rightarrow V_m = -5$ m/s

Now, $V_{mw} = V_m - V_w = (-5) - (2)$

$= -7$ m/s

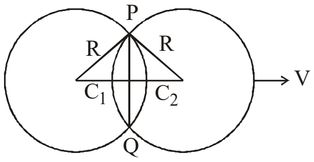
(47) **2.** K.E. $= 2E_0 - E_0 = E_0$ (for $0 \le x \le 1$)

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

KE $= 2E_0$ (for $x > 1$);

$$\lambda_2 = \frac{h}{\sqrt{4mE_0}} \ ; \ \frac{\lambda_1}{\lambda_2} = \sqrt{2} \Rightarrow \left(\frac{\lambda_1}{\lambda_2}\right)^2 = 2$$

(48) **6.** $\varepsilon = |(\vec{v} \times \vec{B}) \cdot \vec{\ell}\,|$



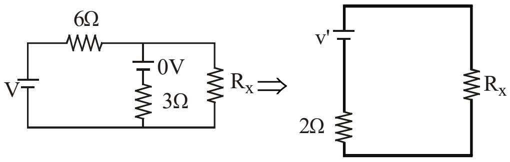
$\varepsilon = VB\,(PQ)$

$$= VB\,2\sqrt{R^2 - \left(\frac{vt}{2}\right)^2} = VB\sqrt{4R^2 - V^2t^2}$$

$$= 4 \times 0.25\sqrt{4 \times 25 - 16 \times 4} = 6 \text{ volt}$$

$(V_Q > V_P)$

(49) **2.**



$R_x = 2$ (max. power transfer theorem)

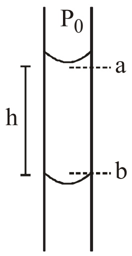
(50) **6.** $P_a = P_0 - \frac{2T}{r}$; $P_b = P_0 + \frac{2T}{r}$

Also, $P_b = P_a + dgh$

Substituting values

$$h = \frac{4T}{rdg} = \frac{4 \times 0.75}{\left(\frac{1}{2} \times 10^{-3}\right) \times (1 \times 10^{-3}) \times 10}$$

$= 6 \times 10^{-2}$ m $= 6$ cm

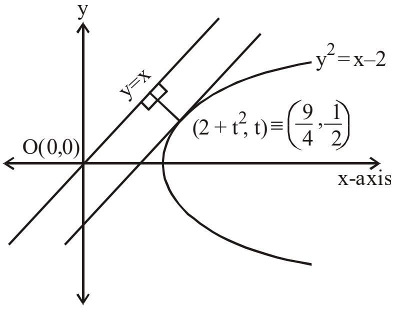


(51) **(3).** $\frac{dy}{dt} = \frac{-10}{x^2}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{-10}{x^2}$

$\Rightarrow$ $\frac{dy}{dt}$ at $x = 5$ equal to $\frac{-10}{25}$

$\Rightarrow$ Ordinate deceases at rate 2/5 unit per second

(52) **(1).**



$$2y\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right]_{P\,(2+t^2,t)} = \frac{1}{2t} = 1$$

$\Rightarrow$ $t = 1/2$

$\therefore$ P (9/4, 1/2)

$$\text{Shortest distance} = \frac{\left|\frac{9}{4} - \frac{2}{4}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$$

(53) **(1).** Let us Suppose equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through $(4,6) \Rightarrow a^2 = 4, b^2 = 12$

$\Rightarrow$ Equation of tangent

$$x - \frac{y}{2} = 1 \Rightarrow 2x - y - 2 = 0$$

(54) **(3).** Let f(x) is periodic with period equal to T then $\cos(x+T)^2 = \cos x^2 \ \forall\, x \in R$

$$\Rightarrow -2\sin\left(\frac{(x+T)^2 - x^2}{2}\right)\sin\left(\frac{(x+T)^2 + x^2}{2}\right) = 0$$

$\forall\, x \in R$

$\Rightarrow$ $(x+T)^2 - x^2 = n\pi$

or $(x+T)^2 + x^2 = n\pi \ \forall\, x \in R$

which is false because these equation are quadratic equation but not identity

$\Rightarrow$ f(x) is not periodic.

(55) **(4).** Vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ = Coplanar

$[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ 0 & 0 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (-4-\lambda)(-1+2) = 0 \Rightarrow \lambda = -4$

(56) **(2).** $y = x^3 + ax - b$

$(1, -5)$ lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'(1, -5) = 3 + a$ (slope of tangent)

This tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

By (i) and (ii) : $a = -4$, $b = 2$

$\therefore y = x^3 - 4x - 2.$

$(2, -2)$ lies on this curve.

(57) **(4).** $\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{1/x}$ (1^∞ form)

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(2-x)-f(3)+f(2)}{x(1+f(2-x)-f(2))}}$$

Using L'Hopital

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f'(3+x)+f'(2-x)}{f'(2-x)+(1+f(2-x)-f(2))}}$$

$$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}} = 1$$

(58) **(4).** $\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{2}{15}$; $\dfrac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \dfrac{2}{15}$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$15r = 2n - 2r + 2$

$17r = 2n + 2$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$$\frac{r+1}{n-r} = \frac{3}{14}$$

$14r + 14 = 3n - 3r$

$3n - 17r = 14$

$2n - 17r = -2$

$n = 16$

$17r = 34$, $r = 2$

${}^{16}C_1, {}^{16}C_2, {}^{16}C_3$

$$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$$

$$\frac{680+16}{3} = \frac{696}{3} = 232$$

(59) **(3).** $P \to (\sim p \vee r)$

$\sim p \vee (\sim q \vee r)$

$$\left.\begin{matrix} \sim p \to F \\ \sim q \to F \\ r \to F \end{matrix}\right\} \Rightarrow \left.\begin{matrix} p \to T \\ q \to T \\ r \to F \end{matrix}\right\}$$

(60) **(2).** $\det(A - \lambda I_3) = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0$

$\Rightarrow (3-\lambda)[(3-\lambda)^2 - 3^2] = 0$

$\Rightarrow 3 - \lambda = 0$ or $3 - \lambda - 3 = 0$ or $3 - \lambda + 3 = 0$

$\Rightarrow \lambda = 0, 3$ or 6

(61) **(3).** $I = \displaystyle\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}\,dx$

$$I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx$$

$$= \int_0^{\pi/4} (1 - \sin x \cos x)\,dx$$

$$= \left(x - \frac{\sin^2 x}{2}\right)_0^{\pi/4} = \frac{\pi}{4} - \frac{1}{4} = \frac{\pi - 1}{4}$$

(62) **(1).** $\Sigma f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = 2.8$$

(63) **(1).** Let other end is (h, k), then centre equal to

$$\left(\frac{p+h}{2}, \frac{q+k}{2}\right)$$

Because circle touches x-axis hence radius

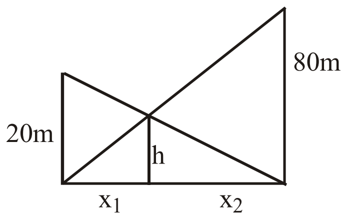
$$= \left|\frac{q+k}{2}\right|$$

$$\Rightarrow \sqrt{(h-p)^2 + (k-q)^2} = 2\left|\frac{q+k}{2}\right|$$

$\Rightarrow (x-p)^2 = (y+q)^2 - (y-q)^2$

$\Rightarrow (x-p)^2 = 4qy$

(64) **(3).**



By similar triangle

$$\frac{h}{x_1} = \frac{80}{x_1 + x_2} \qquad ...(1)$$

$$\frac{h}{x_2} = \frac{20}{x_1 + x_2} \qquad ...(2)$$

By (1) and (2)

$$\frac{x_2}{x_1} = 4 \text{ or } x_2 = 4x_1$$

$$\frac{h}{x_1} = \frac{80}{5x_1} \text{ or } h = 16\text{m}$$

(65) **(3).** $dx + e^{x/y}dx + e^{x/y}dy - \frac{x}{y}e^{x/y}dy = 0$

$$\Rightarrow dx + e^{x/y}dy + e^{x/y}\frac{(ydx - xdy)}{y} = 0$$

$\Rightarrow dx + e^{x/y}dy + yd(e^{x/y}) = 0$

$\Rightarrow dx + d(ye^{x/y}) = 0$

$\Rightarrow x + ye^{x/y} = c$

(66) **(1).** Let $\frac{\alpha + i}{\alpha - i} = z$

$$\Rightarrow \frac{|\alpha + i|}{|\alpha - i|} = |z| \Rightarrow 1 = |z|$$

$\Rightarrow$ Circle of radius 1.

(67) **(2).** ${}^7C_4 \times$ (De-arrangement of 3 things)

$35 \times 2 = 70 = {}^7C_3 \times 2$

(68) **(2).** np = 6, npq = 3

$\Rightarrow q = 1/2, p = 1/2, n = 12$

So required probability

$$= {}^{12}C_0\, p^0 q^{12} + {}^{12}C_1\, p^1 q^{11} = \frac{13}{2^{12}} = \frac{13}{4096}$$

(69) **(2).** Let $\cos^{-1}\frac{4}{5} = A \Rightarrow \cos A = \frac{4}{5}$

$$\tan^{-1}\frac{2}{3} = B \Rightarrow \tan B = \frac{2}{3}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}\frac{2}{3}} = \frac{17}{6}$$

(70) **(1).**

$$f(x) = \left[\frac{2(\sin x - \sin^3 x) + (\sin x - \sin^3 x)}{2(\sin x - \sin^3 x) - (\sin x - \sin^3 x)}\right];\ x \neq \frac{\pi}{2}$$

$(\because \sin x > \sin^3 x \text{ in } (0, \pi))$

$$= 3;\qquad x = \frac{\pi}{2}$$

$$\text{Now } f(x) = 3;\qquad x \neq \frac{\pi}{2}$$

$$= 3;\qquad x = \frac{\pi}{2}$$

Hence f(x) is continuous & differentiable at $x = \frac{\pi}{2}$

(71) **2.** $f(x) = \tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$

$$= \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right) = \tan^{-1}\left(\tan\left(x - \frac{\pi}{4}\right)\right)$$

$\therefore \quad x-\frac{\pi}{4}\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right) \quad \therefore \quad f(x)=x-\frac{\pi}{4}$

$\therefore$ Its derivative w.r.t. x $\frac{x}{2}$ is $\frac{1}{1/2}=2$

(72) **2.** $\frac{a}{1-r}=3$

$\frac{a^3}{1-r^3}=\frac{27}{19}\Rightarrow\frac{27\,(1-r^3)}{1-r^3}=\frac{27}{19}$

$\Rightarrow$ $6r^2-13r+6=0$

$\Rightarrow$ $r=2/3$ as $|r|<1$

(73) **4.** $|A|^2.\,|B|=8$ and $\frac{|A|}{|B|}=8\Rightarrow|A|=4$

and $|B|=1/2$

$\therefore$ $\det(BA^{-1}.B^T)=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

(74) **5.** $\frac{17-\beta}{-8}\times\frac{2}{3}=-1$; $\beta=5$

(75) **5.** DR's of line are 2, 1, –2

Normal vector of plane is $\hat{i}-2\hat{j}-k\hat{k}$

$\sin\alpha=\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-k\hat{k})}{3\sqrt{1+4+k^2}}$

$\sin\alpha=\frac{2k}{3\sqrt{k^2+5}}$ ....(1)

$\cos\alpha=\frac{2\sqrt{2}}{3}$ ....(2)

$(1)^2+(2)^2=1\Rightarrow k^2=5/3$