JEE MAIN 2020
FULL TEST-5 SOLUTIONS

STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	1	2	3	1	3	3	1	3	3	2	1
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	2	4	2	3	2	2	4	1	2	4	3
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	4	6	0	2	2	2	4	4	1	2	4
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	1	3	3	2	4	2	4	2	3	1	2
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	4	4	2	7	4	7	3	3	3	4	1
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	3	2	3	3	3	2	1	4	1	2	2
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**		
A	1	1	1	3	2	1	2	3	1		

(1) **(1).** $CH_3COOH + NaOH \rightleftharpoons CH_3COONa + H_2O$

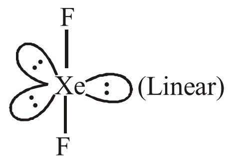
On addition of NaOH to CH_3COOH solution, 60% of the acid is neutralised i.e. after reaction 40% of acid & 60% of salt are present which is an acidic buffer solution.

$$pH = pK_a + \log\frac{[Salt]}{[Acid]} = 4.7 + \log\frac{60}{40}$$
$$= 4.88$$

(2) **(2).** A = 56, D = 58

(3) **(3).** $NO_2^+ : O = N^+ = O$ (sp, linear)

$CO_2 : O = C = O$ (linear)

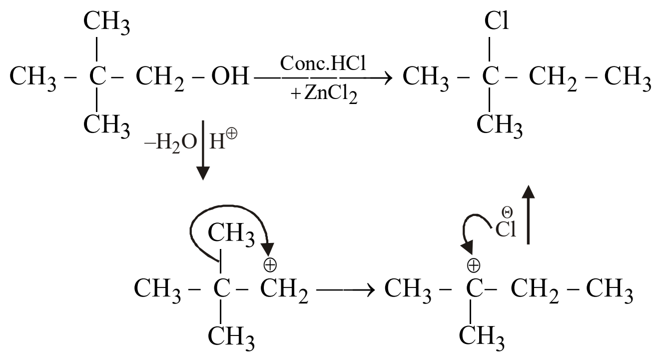
XeF_2 :


(4) **(1).** $(\sigma\,1s)^2 < (\sigma\,1s^*)^2 < (\sigma\,2s^*)^2 < (\sigma\,2s^*)^2 < (\pi\,1p_x)^2 = (\pi\,2p_y^*)^2$

All electrons are paired in C_2 molecules hence C_2 will be diamagnetic.

(5) **(3).** Option three have –OH group gauche (stabilised by H-bonding) and $-CH_3$ group anti (minimum repulsion). So, it is most stable.

(6) **(3).**



(7) **(1).** $Me - CH_2 - C \equiv CH \xrightarrow{NH_3/NaNH_2}$

$Me - CH_2 - C \equiv \overset{\ominus\,\oplus}{CNa}$ (a) $\xrightarrow{Et-Br}$

$Me - CH_2 - C \equiv C - Et$ (b)

(8) **(3).** **Flask I :**

1 Lit of 1 M solution of A

$\downarrow t_{1/2}$

0.5 M   8 hrs.

$\downarrow t_{1/2}$

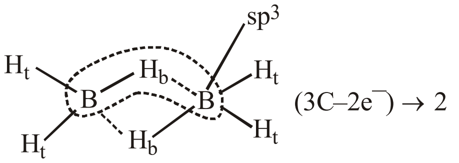
0.25 M

$\therefore\; 2t_{1/2} = 8\;;\; t_{1/2} = 4$

Flask II :
100 ml of 0.6 M solution of A
$\downarrow \quad t_{1/2} = ?$
0.3 M
$t_{1/2} = 4$ hrs.
[For 1st order reaction $t_{1/2}$ does not depends on concentration]

(9) **(3).** Given:
(i) $Cu^{2+} + 2e^- \rightarrow Cu$; $E_1° = 0.337$ V ; $\Delta G_1° = -2F\, E_1°$
(ii) $Cu^{2+} + e^- \rightarrow Cu^+$; $E_2° = 0.153$ V ; $\Delta G_2° = -1F\, E_2°$
(iii) $Cu^+ + e^- \rightarrow Cu$; $E_3° = ?$; $\Delta G_3° = -1F\, E_3°$
Equation (i) – equation (ii) will gives equation (iii)
$\Delta G_1° - \Delta G_2° = \Delta G_3°$
$-2F\, E_1° + F\, E_2° = -F\, E_3°$
$-F\,(2E_1° - E_2°) = -F\, E_3°$
$\therefore \quad E_3° = 2E_1° - E_2°$

(10) **(2).** B_2H_6 (diborane)

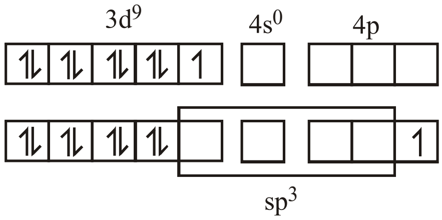


All the hydrogen atoms are not in the same plane.

(11) **(1).** At. no. $\underset{\text{(III-B)}}{58} \xrightarrow{{}^4_2He\,(\alpha)} \underset{\text{(II-A)}}{56}$

At. no. $\underset{\text{(III-B)}}{90} \xrightarrow{{}^4_2He\,(\alpha)} \underset{\text{(II-A)}}{88}$

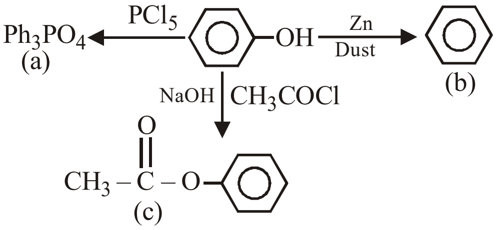
(12) **(2).** $[Cu(NH_3)_2]^+ \Rightarrow Cu^{+2}$



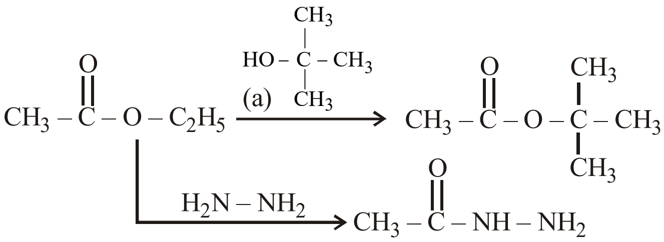
NH_3 = SFL so pairing possible.

(13) **(4).** $CH_3 - C \equiv N + CH_3 - Mg - I \xrightarrow{H_2O} CH_3 - \overset{\overset{O}{||}}{C} - CH_3$

(14) **(2).**



(15) **(3).**



(16) **(2).** $R - C \equiv N \xrightarrow{\text{Reduction}} \underset{(a)}{R - CH_2 - NH_2}$

$R - C \equiv N \xrightarrow[\text{(ii) } H_2O]{\text{(i) } CH_3MgBr} \underset{(b)}{R - \underset{\underset{O}{||}}{C} - CH_3}$

$RNC \xrightarrow{\text{Hydrolysis}} \underset{(c)}{R - NH_2} + HCOOH$

$R - NH_2 \xrightarrow{HNO_2} \underset{(d)}{R - OH} + N_2$

(17) **(2).** Sucrose is a disaccharide of α-D-Glucopyranose and β-D-fructofuranose.

(18) **(4).**

$\underset{\text{Chloroprene}}{nCH_2 = \underset{\underset{Cl}{|}}{C} - CH = CH_2} \xrightarrow{\text{Polymersation}} \underset{\substack{\text{Neoprene} \\ \text{[Artificial rubber]}}}{\left[CH_2 - \underset{\underset{Cl}{|}}{C} = CH - CH_2 \right]_n}$

(19) **(1).** $H_2SO_4 + 2NH_3 \rightarrow (NH_4)_2SO_4$
10mL of 1M H_2SO_4 = 10m mol
[$\because M \times V_{(mL)}$ = m mol]
NH_3 consumed = 20m mol
Acid used for the absorption of ammonia
= 20 – 10 m mol
= 10 mL of 2N (or 1 M) H_2SO_4

$$\%N = \frac{1.4 \times N \times V}{w} = \frac{1.4 \times 10 \times 2}{0.75} = 37.33\%$$

(20) **(2).** Acidic strength of hydrides increase with increase in molecular mass.
Thus order of acidic strength :
HF < HCl < HBr < HI
$H_2O < H_2S < H_2Se < H_2Te$
$NH_3 < PH_3 < AsH_3 < SbH_3$
Acidic strength increases, pK_a decreases.
Order of pK_a : $H_2O > H_2S > H_2Se > H_2Te$

(21) **4.**

$$\begin{array}{cccc} CH_3COOH + & C_2H_5OH \rightleftharpoons & CH_3COOC_2H_5 + & H_2O \\ 4 & 4 & 0 & 0 \\ 4-x & 4-x & x & x \end{array}$$

$$K_c = \frac{x \times x}{(4-x)\times(4-x)} \Rightarrow 4 = \frac{x^2}{(4-x)^2}$$

Taking root

$$2 = \frac{x}{4-x} \Rightarrow x = \frac{8}{3} \Rightarrow CH_3COOH = 4 - \frac{8}{3} = \frac{4}{3}$$

(22) **3.**

$$\overset{+7}{MnO_4^-} + Fe\overset{+2}{}\overset{+3}{C_2O_4} \longrightarrow \overset{+2}{Mn^{+2}} + \overset{+3}{Fe^{+3}} + \overset{+4}{CO_2}$$

v.f.$_1$ = 5 ; v.f.$_2$ = 1 + 2 = 3

$n_1vf_1 = n_2vf_2$; $n \times 5 = 1 \times 3$; $n_1 = 3/5$

(23) **4.** $$\frac{r_{H_2}}{r_{O_2}} = \frac{n_{H_2}/t}{n_{O_2}/t} = \sqrt{\frac{Mw_{O_2}}{Mw_{H_2}}}$$

$$\frac{w_{H_2}/2}{w_{O_2}/32} = \sqrt{\frac{32}{2}} = 4$$

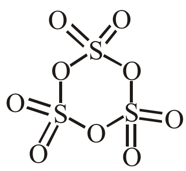
(24) **6.** $d = 6.8\ g\ cm^{-3}$
$a = 290\ pm = 2.9 \times 10^{-8}\ cm$

$$d = \frac{Z \times m}{N \times a^3}$$

$$N = \frac{Z \times m}{d \times a^3} = \frac{2 \times 200}{6.8 \times (2.9 \times 10^{-8})^3}$$

$= 2.4 \times 10^{24}$

(25) **0.** $SO_3 \rightarrow S_3O_9$
6 (S = O) bonds
three (S–O–S) bonds



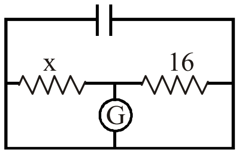
(26) **(2).** $[a] = T^2$; $[x] = L$

$$[P] = ML^{-1}T^{-2} = \frac{T^2}{[b]L}$$

$$[b] = \frac{T^2}{ML^{-1}T^{-2}L} = M^{-1}T^4$$

$$\therefore \frac{[a]}{[b]} = \frac{T^2}{M^{-1}T^4} = MT^{-2}$$

(27) **(2).** $\frac{x}{16} = \frac{36}{64}$; $x = 9\Omega$



$$\frac{dx}{x} = \frac{d\ell}{\ell(1-\ell)} = \frac{0.5 \times 10^{-1}}{36 \times 64}$$

$$dx = \frac{0.05 \times 9}{36 \times 64} = \frac{1}{5120}\Omega$$

$$x = 9 \pm \frac{1}{5120}\Omega$$

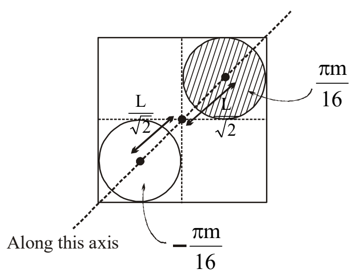
(28) **(2).** $$R = \frac{u^2 \sin 2\theta}{g} = \frac{100^2 \times 1}{10} = 1000\ m$$

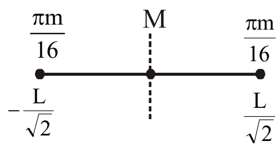
$$\frac{\Delta R}{R} = \frac{2\Delta u}{u} + \frac{\cos 2\theta}{\sin 2\theta} \times \Delta\theta$$

$$\Rightarrow \frac{\Delta R}{1000} = \frac{2 \times 1}{100} \Rightarrow \Delta R = 20\ m$$

$980\ m < R < 1020\ m$

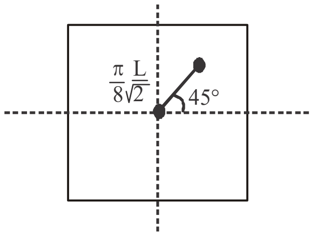
(29) **(4).**





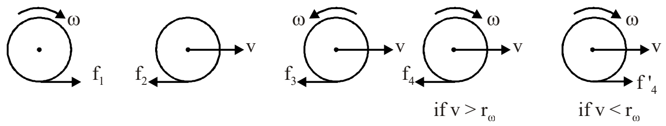
$$X_{Cr} = \frac{\left(\frac{\pi m}{16}\right)\frac{L}{\sqrt{2}} - \frac{\pi m}{16}\left(-\frac{L}{\sqrt{2}}\right)}{m} \Rightarrow \frac{\pi}{8}\left(\frac{L}{\sqrt{2}}\right)$$

from origin



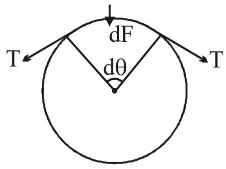
$$\vec{r} = \frac{\pi}{8}\left(\frac{L}{\sqrt{2}}\right)(\cos 45\,\hat{i} + \sin 45\,\hat{j}) = \frac{\pi L}{16}\left(\hat{i}+\hat{j}\right)$$

(30) (4). In this case, friction would oppose the motion of lowest point (point of contact)



(31) (1). $dF = 2T\sin\left(\frac{d\theta}{2}\right)$

$$a = \frac{dF}{dm} = \frac{2\left(\frac{AY}{\ell}\Delta\ell\right)\left(\frac{d\theta}{2}\right)}{\left(\frac{m}{2\pi}\cdot d\theta\right)}$$



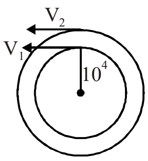
$$= \frac{2\pi AY\Delta\ell}{m\ell} = \frac{2\pi AY(\Delta R)}{m\cdot R} = \left(\frac{AY2\pi}{mR}\right)\Delta R$$

$= (\omega^2)\,\Delta R$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mR}{AY(2\pi)}}$$

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{mR}{AY(2\pi)}}\ ;\ \sqrt{\frac{\pi mR}{8YA}}$$

(32) (2).



$$\omega_1 = \frac{2\pi}{1}\text{rad/hr}\ ;\ \omega_2 = \frac{2\pi}{8}\text{ rad/hr}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_3}\right)^3 \Rightarrow \frac{R_2}{R_1} = 4$$

$\Rightarrow R_2 = 4 \times 10^4$ km

$$V_1 = \frac{2\pi R_1}{1h} = 2\pi \times 10^4 \text{ km/hr}$$

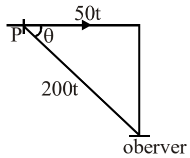
$$V_2 = \frac{2\pi R_2}{8h} = \pi \times 10^4 \text{ km/hr}$$

At closest separation

$$\omega = \frac{V_{rel} \perp \text{ to line joining}}{\text{length of line journing}}$$

$$= \frac{\pi \times 10^4 \text{km/hr}}{3\times 10^4 \text{km}} = \frac{\pi}{3}\text{rad/hr.}$$

(33) (4). When source is at origin, the observer receives the sound emitted by the source, when it was at P.



Such that $\cos\theta = \frac{50t}{200t} = \frac{1}{4}$

$$\nu = \frac{\nu_0(v)}{v - v_s\cos\theta} = \frac{90\times 200}{200 - \frac{50}{4}} = 96 \text{ Hz}$$

(34) (1). For chamber : $\frac{dQ}{dt} = k(\theta_1 - \theta_0) = k(\theta_2 - \theta_0)$

$\Rightarrow \theta_1 = \theta_2$

For heater

$$\frac{dQ}{dt} = e_1A\sigma(T_1^4 - \theta_1^4) = e_2A\sigma(T_2^4 - \theta_2^4)$$

$e_1 > e_2 \Rightarrow T_1 < T_2$

(35) (3). For isothermal process $V_f = 2V_0$

$\therefore\ P_f = P_0/2$

For isobaric process

$$V_f = V_0/2\ ;\ T_f = \frac{V_0}{2\times 2V_0}\bullet T_0 = \frac{T_0}{4}$$

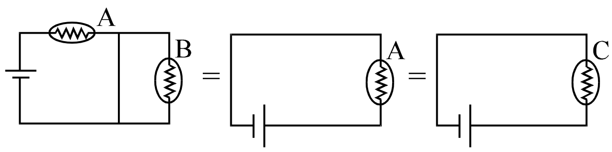
For $P \propto V$ process

P–V must be straight line

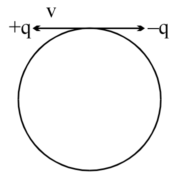
$T \propto V^2 \quad \Rightarrow \quad$ V–T must be parabolic

$P^2 \propto T \quad \Rightarrow \quad$ P–T must be parabolic

(36) (3). In the first diagram where A & B are there B is short circuit only A in the circuit.



(37) (2). $R = \frac{mv}{qB}$



$q_1 = -q_2$

$m_1v_1 = m_2v_2$

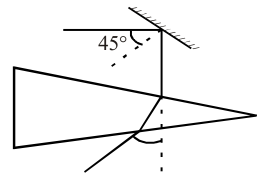
$\Rightarrow$ $R_1 = R_2$

But $m_1 \neq m_2 \Rightarrow \omega_1 = \dfrac{qB}{m_1} \neq \dfrac{qB}{m_2}$

$\Rightarrow$ ω is not equal. So collision does not occur at diametrically opposite point.

(38) **(4).** Consider the expression for the current rising exponentially in the LR circuit. The time constant is (L/R). In this case the curve (1) is rising faster than curve (2) indicating that $(L_1 / R_1) < (L_2 / R_2)$. However, in both the cases the maximum current is the same and equal to (V/R_1) or (V/R_2), which means $R_1 = R_2$

(39) **(2).**



$\rho = A(H - 1) = 4 \times \dfrac{1}{2} = 2°$

$\therefore$ Total deviation = 90° (due to reflection) + 2° (due to prism) = 92°

but net deviation should be 90°

$\therefore$ Due to reflection = 88° = $\pi - 2i \Rightarrow i = 46°$

i.e. Mirror must rotated by 1° anticlockwise.

(40) **(4).** Position of 10^{th} maxima = $\dfrac{10\lambda D}{d} = 3$ cm

(w.r. to central maxima)

$\dfrac{\lambda D}{d} = \dfrac{3}{10}$ cm

New fringe width = $\dfrac{3}{10 \times \mu}$

New position of 10^{th} maxima

$= \dfrac{3}{10 \times 1.5} \times 10 = 2$cm

$\therefore$ Position of central maxima = 2cm

$\therefore$ 10^{th} maxima = 4 cm

(41) **(2).** For reverse bias : N end of PN junction should be connected to high potential wrt P end.

(42) **(3).** Modulation factor determines both the strength and quality of the signal.

(43) **(1).** $\lambda = \dfrac{h}{p}$; $\dfrac{d\lambda}{\lambda} = -\dfrac{dp}{p}$; $\dfrac{0.5}{100} = \dfrac{p}{p'}$

$\Rightarrow$ p' = 200p

(44) **(2).** $N_1 = N_0 e^{-\lambda t}$; $N_1 = \dfrac{1}{3} N_0$

$\dfrac{N_0}{3} = N_0 e^{-\lambda t_2}$ ...... (1)

$N_2 = \dfrac{2}{3} N_0$; $\dfrac{2}{3} N_0 = N_0 e^{-\lambda t_1}$ ...... (2)

From eq. (1) and (2)

$\dfrac{1}{2} = e^{-\lambda (t_2 - t_1)}$; $\lambda (t_2 - t_1) = \ln 2$

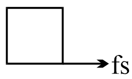
$t_2 - t_1 = \dfrac{\ln 2}{\lambda} = T_{1/2} = 50$ days

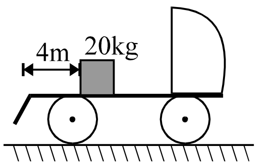
(45) **(4).** Q = 2 (BE of He) – (BE of Li)

$= 2 \times (4 \times 7.06) - (7 \times 5.60)$

$= 56.48 - 39.2 = 17.3$ MeV

(46) **4.** $fs_{max} = \mu mg = 0.15 \times 20 \times 10 = 30$ N





$a_{block} = \dfrac{30}{20} = \dfrac{3}{2}$ m/s^2

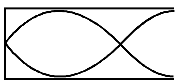
$a_{truck} = 2$ m/s^2 ; $a_{rel} = \dfrac{1}{2}$ m/s^2

$d = \dfrac{1}{2} a_{rel} t^2$; $4 = \dfrac{1}{2} \times \dfrac{1}{2} \times t^2$; t = 4 sec.

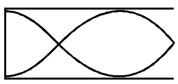
Distance travelled by truck

$= \dfrac{1}{2} \times 2 (4)^2 = 16$ m

(47) **2.**



displacement variation



pressure variation

It is clear from the figure that L/3 path difference represent $\pi/2$ phase difference.

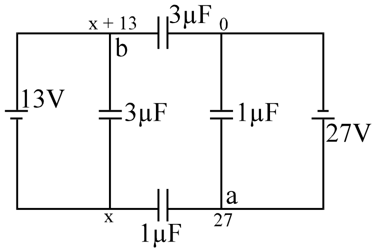
$\therefore$ 7L/9 path difference represents $7\pi/6$ phase difference.

Lets say amplitude of pressure variation be A

then amplitude at $\frac{7L}{9}$ will be $\left|A\sin\frac{7\pi}{6}\right|$

The ratio of pressure amplitude at Q to the maximum pressure amplitude is 1 : 2.

(48) **7.** $(x + 13) \times 3 = (27 - x) \times 1$



$3x + 39 = -x + 27$; $x = -3$

So $V_a - V_b = 27 - (x + 13) = 17$

(49) **4.** $\phi = 2eV$; $\frac{hc}{\lambda_1} = 8eV$; $T_2 = 2T_1$

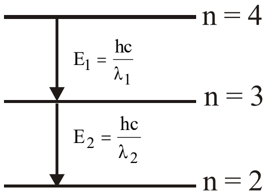
If λ_{11} is the wavelength corresponding to maximum intensity at T_1 & T_2 at T_2;

Then $\lambda_2 = \lambda_1/2$ (by wein's displacement Law)

$$\frac{hc}{\lambda_2} = \frac{2hc}{\lambda_1} = 16eV$$

$$\phi = 2eV \quad \therefore K.E._{max} = \frac{hc}{\lambda} - \phi = 14eV$$

(50) **7.**



$$E = \frac{hc}{\lambda_1} = 13.6\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right] \quad ...... (1)$$

$$E = \frac{hc}{\lambda_2} = 13.6\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] \quad ...... (2)$$

Dividing eq. (2) by (1),

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{9} - \frac{1}{16}} = \frac{20}{7}$$

(51) **(3).** $\pi \log_3\left(\frac{1}{x}\right) = k\pi, \ k \in I;$

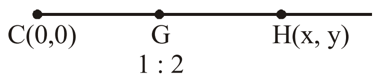
$\log_3\left(\frac{1}{x}\right) = k \Rightarrow x = 3^{-k}$

Possible values of k are – 1, 0, 1, 2, 3, ...

$$S = (3 + 1) + \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\infty\right)$$

$$= 4 + \frac{(1/3)}{1 - (1/3)} = 4 + \frac{1}{2} = \frac{9}{2}$$

(52) **(3).** $\left(\frac{1 + 2\cos\theta + 2\sin\theta}{3}, \frac{\sqrt{3} + 2\sin\theta - 2\cos\theta}{3}\right)$



$$\frac{x}{3} = \frac{1 + 2\cos\theta + 2\sin\theta}{3}$$

$\Rightarrow$ $x = 1 + 2\cos\theta + 2\sin\theta$

$$\frac{y}{3} = \frac{\sqrt{3} + 2\sin\theta - 2\cos\theta}{3}$$

$\Rightarrow$ $y = \sqrt{3} + 2\sin\theta - 2\cos\theta$

$(x - 1)^2 + (y - \sqrt{3})^2 = 8$

(53) **(3).** $3x^2 + 4xy + 4y^2 + 2x - 2y + 1 + \alpha = 0$

$\Rightarrow$ $x\underbrace{(3x + 2y + 1)}_{\text{vanishes}} + y\underbrace{(2x + 4y - 1)}_{\text{vanishes}}$

$- y + x + 1 + \alpha = 0$

$\Rightarrow$ $x - y + 1 + \alpha = 0$

$\therefore$ equations are

$3x + 2y + 1 = 0$, $2x + 4y - 1 = 0$ and

$x - y + (1 + \alpha) = 0$

So, they will admit a unique solution, if

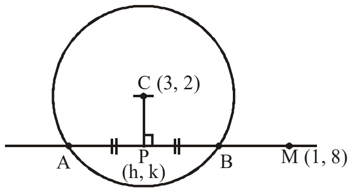
$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & -1 \\ 1 & -1 & 1+\alpha \end{vmatrix} = 0 \Rightarrow \alpha = \frac{3}{8}$$

(54) **(4).** Clearly, $m_{CP} \times m_{AB} = -1$

$\Rightarrow$ $\left(\frac{k - 2}{h - 3}\right) \times \left(\frac{k - 8}{h - 1}\right) = -1$

$\therefore$ Locus of (h, k) is

$(x - 1)(x - 3) + (y - 2)(y - 8) = 0$



i.e., $x^2 + y^2 - 4x - 10y + 19 = 0$

(55) **(1).** Any element, is of the form of

$$\frac{8!}{t_1!\,t_2!\,t_3!\,t_4!}\left(x^2\right)^{t_1}\left(\frac{1}{x^2}\right)^{t_2}(y)^{t_3}\left(\frac{1}{y}\right)^{t_4}$$

where $t_1 + t_2 + t_3 + t_4 = 8$; $t_1 \geq 0$

The constant term occur when $t_1 = t_2$ and $t_3 = t_4$

So, $t_1 + t_3 = 4 \Rightarrow (0, 4); (4, 0) ; (1, 3), (3, 1); (2, 2) \Rightarrow$ constant term:

$$\frac{8!}{0!\,0!\,4!\,4!}+\frac{8!}{4!\,4!\,0!\,0!}+\frac{8!}{1!\,1!\,3!\,3!}+\frac{8!}{3!\,3!\,1!\,1!}+\frac{8!}{2!\,2!\,2!\,2!}$$

$= (2 \times 70) + (2 \times 1120) + 2520 = 4900$

(56) **(3).** Clearly, $(x, x) \in R \ \forall \ x \in W$,

So, R is reflexive.

Let $(x, y) \in R$ then $(y, x) \in R$ as x and y have atleast one letter in common.

So R is also symmetric.

But R is not transitive

e.g. let x = MILK

y = LIME

and z = ENERGY

then $(x, y) \in R$ and $(y, z) \in R$

but $(x, z) \notin R$.]

(57) **(2).** $f'(x) = -\frac{2}{\sqrt{1-x^2}} \cdot \frac{x}{|x|}$

$\Rightarrow$ not differentiable at x = 0,

$$\text{now } f'(x) = \begin{cases} -\frac{2}{\sqrt{1-x^2}} & \text{for } x > 0 \\ \frac{2}{\sqrt{1-x^2}} & \text{for } x < 0 \end{cases}$$

$f''(x) = (1 - x)^{-3/2} \cdot (-2x) < 0$.

Also not differentiable at x = 0

(58) **(3).** We know that if $k(x) = f(x) \cdot g(x)$, where f(x) is differentiable at x = a and f(a) = 0 but g(x) is continuous at x = a then k (x) is also derivable at x = a.

$\therefore$ In options (1), (4), functions are differentiable at x = 2.

For option (2),

$$f'(2^+) = \lim_{h\to 0}\frac{\sin h - h}{h} = 0$$

$$f'(2^-) = \lim_{h\to 0}\frac{\sin h - h}{-h} = 0$$

$\Rightarrow f(x) = \sin\left(|x-2|\right) - |x - 2|$ is derivable at x = 2.

For option (3)

$$f'(2^+) = \lim_{h\to 0}\frac{\sin h + h}{h} = 2$$

$$\text{and } f'(2^-) = \lim_{h\to 0}\frac{\sin h + h}{-h} = -2$$

$\Rightarrow f(x) = \sin\left(|x-2|\right) + |x - 2|$

is non-derivable at x = 2.

(59) **(3).** $\because\ g(x) = \left(f(3f(x)+6)\right)^3$

$\Rightarrow g'(x) = 3\left(f(3f(x)+6)\right)^2 f'\left(3f(x)+6\right)\cdot 3f'(x)$

$\therefore g'(0) = 3\left(f(3f(0)+6)\right)^2 \cdot f'(3f(0)+6)\cdot 3f'(0)$

$= 9\left(f(-6+6)\right)^2 f'(-6+6) f'(0)$

$= 9\left(f(0)\right)^2 \left(f'(0)\right)^2 = 9 \times 4 \times 1 = 36$]

(60) **(3).** We have, $\lim_{x\to 0}\frac{\int_0^{x^2}\left(t^2\cos^5 t\right)dt}{x^3(x-\sin x)}\quad \left(\frac{0}{0}\right)$

$$= \lim_{x\to 0}\frac{\int_0^{x^2}(t^2\cos^5 t)\,dt}{x^6\left(\frac{x-\sin x}{x^3}\right)} = 6\lim_{x\to 0}\frac{\int_0^{x^2}(t^2\cos^5 t)\,dt}{x^6}$$

$$\left(\text{As, } \lim_{x\to 0}\left(\frac{x-\sin x}{x^3}\right) = \lim_{x\to 0}\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+.......\infty\right)}{x^3} = \frac{1}{6}\right)$$

$$= 6\left(\frac{2x \cdot x^4 \cdot \cos^5(x^2)}{6x^5}\right)$$

$$= \lim_{x\to 0} 2\cos^5(x^2) = 2$$

(61) **(2).** $y = f(x) \Rightarrow x = f^{-1}(y) \Rightarrow x = g(y)$

Given $y = f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^3}} \Rightarrow \frac{dx}{dy} = \sqrt{1+x^3}$$

$$g'(y) = \sqrt{1+g^3(y)}$$

$$g''(y) = \frac{3g^2(y)g'(y)}{2\sqrt{1+g^3(y)}}$$

$$\Rightarrow 2g''(y) = 3g^2(y)\frac{g'(y)}{\sqrt{1+g^3(y)}}$$

$$= 3g^2(y)\frac{\sqrt{1+g^3y}}{\sqrt{1+g^3(y)}} = 3g^2(y)$$

$\Rightarrow 2g''(y) = 3g^2(y)$

(62) **(1).** put $\ln x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$$I = \int_{-\infty}^{\infty} f(e^t + e^{-t})\frac{t}{e^t}e^t\,dt$$

$$= \int_{-\infty}^{\infty} f(e^t + e^{-t})t\,dt = 0$$

(as the function is odd)

Alternatively-1: put $x = \tan\theta$;

$$\int_0^{\pi/2} f(\tan\theta + \frac{1}{\tan\theta})\frac{\ln\tan\theta}{\tan\theta}\cdot\sec^2\theta\,d\theta$$

$$= \int_0^{\pi/2} f(\tan\theta + \frac{1}{\tan\theta})\frac{\ln\tan\theta}{\sin\theta\cos\theta}d\theta$$

Alternatively-2:

Put $x = 1/t \Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$

(63) **(4).** A is non singular $\det A \neq 0$

Given $AB - BA = A$

Hence $AB = A + BA = A(I + B)$

det. A · det. B = det. A · det. (I + B)

(det A ≠ 0)

det. B = det. (I + B) ....(1)

(as A is non singular)

Again $AB - A = BA$

$A(B - I) = BA$

(det. A) · det.(B – I) = det. B · det. A

⇒ det. (B – I) = det. (B) ....(2)

From (1) and (2)

det. (B – I) = det. (B + I)

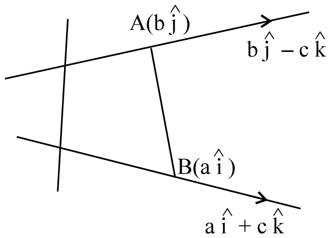
(64) **(1).** Note that the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ is in y-z plane while the lines $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is in the x-z plane

1st line intersecting the y and z axis at (0, b, 0) and (0, 0, c) respectively. Hence its equation is $\vec{r} = b\hat{j} + \lambda(b\hat{j} - c\hat{k})$ ....(1)

|||ly 2nd line intersecting the x and z axis at (a, 0, 0) and (0, 0, – c) respectively. Hence its equation is $\vec{r} = a\hat{i} + \mu(a\hat{i} + c\hat{k})$ ....(2)

A vector perpendicular to both $b\hat{j} - c\hat{k}$ and

$$a\hat{i} + c\hat{k} \text{ is} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & b & -c \\ a & 0 & c \end{vmatrix}$$



$$\vec{v} = bc\hat{i} - ac\hat{j} - ab\hat{k}$$

$$\text{S.D.} = 2d = \left|\text{Projection of } \overrightarrow{AB} \text{ on } \vec{v}\right|$$

$$= \left|\frac{\overrightarrow{AB}\cdot\vec{v}}{|\vec{v}|}\right| = \left|\frac{(a\hat{i} - b\hat{j})\cdot(bc\hat{i} - ac\hat{j} - ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}\right|$$

$$2d = \frac{abc + bac}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$$

$d = \dfrac{abc}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$

$\therefore \quad d^2 (a^2b^2 + b^2c^2 + c^2a^2) = a^2 b^2 c^2]$

$\dfrac{1}{d^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}$

(65) (2). A (1, a, b) ; B (a, 2, b) ; C (a, b, 3)

$\overrightarrow{AB} = (a-1)\,\hat{i} + (2-a)\,\hat{j} + 0\,\hat{k}$;

$\overrightarrow{BC} = 0\hat{i} + (b-2)\hat{j} + (3-b)\hat{k}$

$\overrightarrow{AB} = \lambda\,\overrightarrow{BC} = \lambda\left(0\hat{i} + (b-2)\hat{j} + (3-b)\hat{k}\right)$

where $\lambda \neq 0$

Hence $a - 1 = 0 \Rightarrow a = 1$ ....(1)

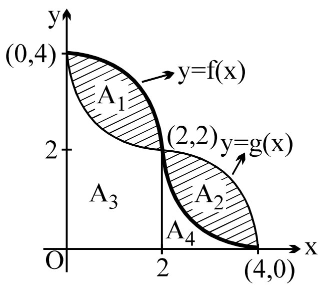
$2 - a = \lambda(b-2)$ ....(2)

and $3 - b = 0 \Rightarrow b = 3$ ....(3)

with $a = 1$ and $b = 3$, $\lambda = 1$

Hence $a + b = 4$

(66) (2).



Given $\int_0^4 f(x)\,dx - \int_0^4 g(x)\,dx = 10$

$(A_1 + A_3 + A_4) - (A_2 + A_3 + A_4) = 10$

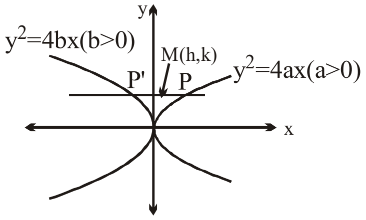
$A_1 - A_2 = 10$ ....(1)

Again $\int_2^4 g(x)\,dx - \int_2^4 f(x)\,dx = 5$

$(A_2 + A_4) - A_4 = 5$; $A_2 = 5$ .....(2)

$\therefore$ (1) + (2), $A_1 = 15$

(67) (1).



$P(at_1^2, 2at_1)$; $P'(-bt_2^2, 2bt_2)$

$2at_1 = 2bt_2 = k$; $at_1^2 - bt_2^2 = 2h$

$a\left(\dfrac{k^2}{4a^2}\right) - b\left(\dfrac{k^2}{4b^2}\right) = 2h$

$y^2\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = 8x \Rightarrow y^2 = \left(\dfrac{8ab}{b-a}\right)x$

(68) (1). $P\left(\dfrac{a}{\sqrt{2}}, \dfrac{b}{\sqrt{2}}\right)$ $\quad p_1 = \dfrac{\sqrt{2}\,ab}{a^2 + b^2}$

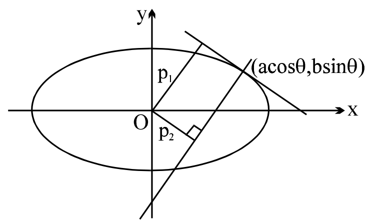
$p_2 = \dfrac{a^2 - b^2}{\sqrt{2}\left(a^2 + b^2\right)} \Rightarrow p_1p_2 = \text{result}$

$T : \dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{b} = 1$

$p_1 = \left|\dfrac{ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right|$ ....(1)

$N_1 : \dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = a^2 - b^2$

$p_2 = \left|\dfrac{(a^2 - b^2)\sin\theta\cos\theta}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}\right|$ ....(2)



$p_1p_2 = \dfrac{ab(a^2 - b^2)}{2\left(\dfrac{a^2}{2} + \dfrac{b^2}{2}\right)}$ when $\theta = \pi/4$;

$p_1p_2 = \dfrac{ab(a^2 - b^2)}{a^2 + b^2}$

(69) (1). $e_1^2 = 1 + \dfrac{b^2}{a^2} = 1 + \dfrac{12}{4} = 4 \quad \Rightarrow \quad e_1 = 2$

Now $\dfrac{1}{e_1^2} + \dfrac{1}{e_2^2} = 1$; $\dfrac{1}{e_2^2} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\Rightarrow \quad e_2^2 = \dfrac{4}{3} \quad \Rightarrow \quad e_2 = \dfrac{2}{\sqrt{3}}$

(70) **(3).** $p \Rightarrow q$ is false only when p is true and q is false.

$p \Rightarrow q$ is false when p is true and $q \vee r$ is false, and $q \vee r$ is false when both q and r are false.

(71) **2.** $\sin x = \sin 2y$ .....(1)

and $\cos x = \sin y$ .....(2)

$\therefore$ $(1)^2 + (2)^2 \Rightarrow 1 = \sin^2 y\,(1 + 4\cos^2 y)$

$\therefore$ $\cos^2 y = 4\sin^2 y \cdot \cos^2 y$

$\Rightarrow$ $\cos^2 y\,(4\sin^2 y - 1) = 0$

$\therefore$ $y = \frac{\pi}{2}$ or $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

$$\left.\begin{array}{l} \text{if } y = \frac{\pi}{2} \text{ then } x = 0 \\ \text{if } y = \frac{\pi}{6} \text{ then } x = \frac{\pi}{3} \end{array}\right]$$

$\Rightarrow$ 2 ordered pairs. i.e.,

$\left(x = 0, y = \frac{\pi}{2}\right)$ or $\left(x = \frac{\pi}{3}, y = \frac{\pi}{6}\right)$.

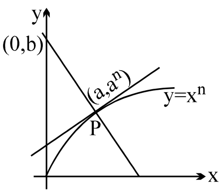
(72) **1.** $P_n = {}^{n-2}C_3$

$Q_n = {}^nC_3 - [n + n(n-4)]$

or $Q_n = \frac{{}^nC_1 \cdot {}^{n-4}C_2}{3}$

$P_n - Q_n = 6 \Rightarrow n = 10$

(73) **2.**



$y = x^n$; $\frac{dy}{dx} = n\,x^{n-1} = na^{n-1}$

slope of normal $= -\frac{1}{na^{n-1}}$

Equation of normal $y - a^n = -\frac{1}{na^{n-1}}(x - a)$

Put $x = 0$ to get y-intercept

$y = a^n + \frac{1}{na^{n-2}}$; Hence $b = a^n + \frac{1}{na^{n-2}}$

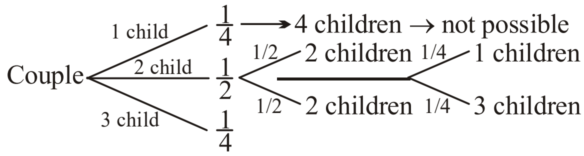
$$\lim_{a\to 0} b = \left[\begin{array}{ll} 0 & \text{if } n < 2 \\ \frac{1}{2} & \text{if } n = 2 \\ \infty & \text{if } n > 2 \end{array}\right.$$

(74) **3.** A : exactly one child

B : exactly two children

C : exactly 3 children

$P(A) = \frac{1}{4}$; $P(B) = \frac{1}{2}$; $P(C) = \frac{1}{4}$

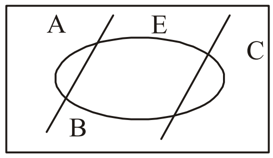


E : couple has exactly 4 grandchildren

$P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$

$$= \underbrace{\frac{1}{4} \cdot 0}_{\substack{\text{one child} \\ \text{and have} \\ \text{4 children} \\ \text{(not possible)}}} + \frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^2}_{2/2} + \underbrace{\left(\frac{1}{4} \cdot \frac{1}{4}\right) \cdot 2}_{(1,3) \text{ or } (3,1)}\right]$$

$$+ \frac{1}{4}\left[3\,\underbrace{\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{2}\right)}_{1 \quad 1 \quad 2}\right]$$



$= \frac{1}{8} + \frac{1}{16} + \frac{3}{128} = \frac{27}{128}$

|||ly 2 / 2 denotes each child having two children; '0' indicated that the child can have a maximum of 3 children $2 \cdot \frac{1}{4} \cdot \frac{1}{4}$ denotes each child having 1 and 3 or 3 and 1 children

$= \frac{16}{128} + \frac{8}{128} + \frac{3}{128} = \frac{27}{128}$

(75) **1.** We know that

If $y = \frac{x}{h}$ then $\sigma_y = \frac{\sigma_x}{|h|}$

Since each observation is divided by 4

$\therefore$ The S.D. of new set of observations will be

$\frac{4}{4} = 1$