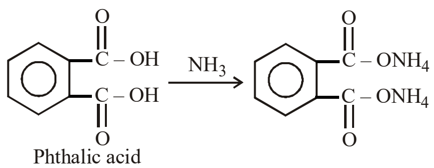
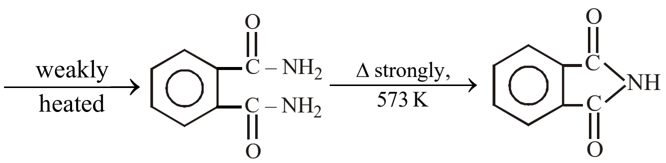
JEE MAIN 2020
FULL TEST-6 SOLUTIONS

STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	4	1	3	2	4	1	2	3	3	4	2
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	2	4	1	4	3	3	4	4	4	2	3
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	4	8	3	1	2	2	2	2	4	3	3
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	4	3	4	3	4	2	4	3	2	4	3
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	2	1	6	3	5	5	1	4	4	1	1
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	1	4	1	2	3	2	2	2	4	2	1
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**		
A	2	1	4	4	3	8	0	1	4		

(1) **(4).**





(2) **(1).** $X_2 \quad + \quad Y_2 \quad \rightleftharpoons \quad 2XY$

$$t = 0 \qquad \frac{1\text{ mol}}{1L} \qquad \frac{2\text{ mol}}{2L} \qquad 0$$

$$\text{At eqm.} \qquad \frac{1-x}{3} \qquad \frac{2-x}{3} \qquad \frac{2x}{3}$$

$\because \dfrac{2x}{3} = 0.6\text{ mol / L}$. Therefore, $x = 0.9$

$$[X_2] = \frac{1-0.9}{3} = \frac{1}{3} - 0.3$$

$$[Y_2] = \frac{2-0.9}{3} = \frac{2}{3} - 0.3$$

(3) **(3).** $pH = 7 + \dfrac{pK_a - pK_b}{2} = 7 + \dfrac{0.02}{2} = 7.01$

(4) **(2).** $C_2H_5OH\,(\ell) + 3O_2\,(g) \rightarrow 2CO_2(g) + 3H_2O\,(\ell)$

In Bomb calorimeter

$\Delta E = -670.48\text{ kCal mol}^{-1}$

$\Delta H = \Delta E + \Delta n_g RT$

$\Delta H = -670.48 - 1 \times 2 \times 10^{-3} \times 298$

(5) **(4).** $SO_4^{2-} + BaCl_2 \rightarrow BaSO_4 + 2Cl^-$

white ppt

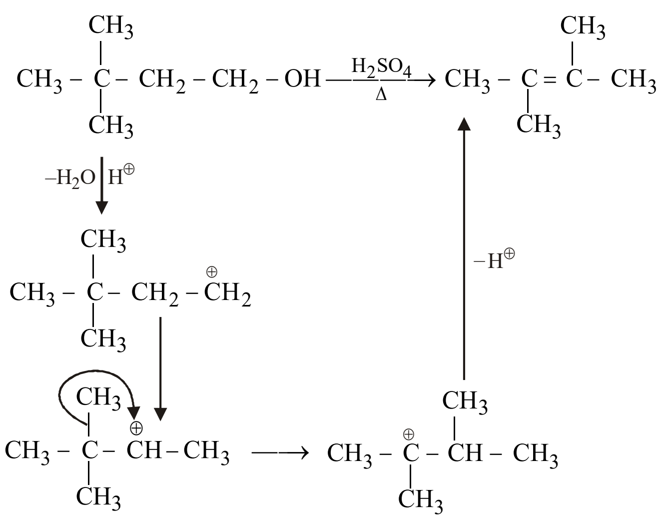
$\downarrow$ dil. HCl

Not dissolves

(6) **(1).**

The ℓ.p. present on N(II) is involved in resonance and is the part of aromaticity with two pi bonds in ring so to maintain aromaticity the compound does not have tendency to donate this ℓ.p. while ℓ.p. of N (I) is not involved in resonance and is not the part of aromaticity so compound can easily donate this ℓ.p. Protonation is possible on N (I) not on N (II).

(7) **(2).**



(8) **(3).** Given:

(i) $OCl^- + 2H^+ + 2e^- \to Cl^- + H_2O$
; $E_1° = 0.94$ V ; $\Delta G_1° = -2FE_1°$

(ii) $Cl^- \to \frac{1}{2}Cl_2 + e^-$; $E_2° = -1.36$ V
; $\Delta G_2° = -1F\,E_2°$

(iii) $OCl^- + 2H^+ + 2e^- \to \frac{1}{2}Cl_2 + H_2O$;
; $E_3° = ?$; $\Delta G_3° = -1F\,E_3°$

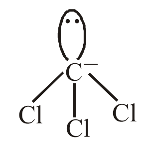
Equation (i) + equation (ii) will gives eq. (iii)

$-2FE_1° - F\,E_2° = -1F\,E_3°$

$\therefore$ $E_3° = 2E_1° + E_2°$

(9) **(3).** $CHF_3 \xrightarrow{-H^+} CF_3^{-1}$

$CHCl_3 \xrightarrow{-H^+} CCl_3^{-1}$



(Most acidic)

(P_π-d_π) back bonding more stable.

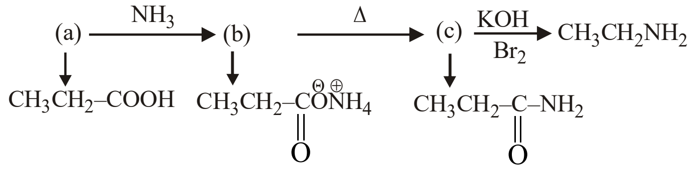
(10) **(4).** SF_6: 'S' atom is strength protested by six atoms so hydrolysis is not possible.
NF_3 : Vacant orbital absent.

(11) **(2).** ZnO turns yellow on heating as Zn^{2+} ions move in interstitial sites and electrons also get entrapped in nearby interstitial sites to maintain electrical neutrality. As extra Zn^{2+} ions are present in interstitial sites thus, it is metal excess defect.

(12) **(2).** $CuSO_4 + \underset{\substack{(CN^-\text{ act as}\\ \text{reducing agent)}\\ \text{(Pseudo halide)}}}{KCN} \to CuCN + \underset{\text{(excess)}}{KCN}$

$\to K_3[Cu(CN)_4]$

(13) **(4).**



(14) **(1).** $Ca(NH) + 2H_2O \longrightarrow Ca(OH)_2 + \underset{B}{NH_3(g)}$

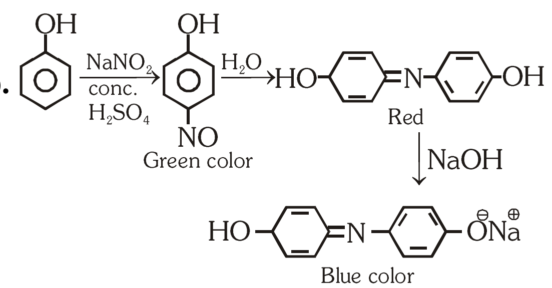
$\underset{B}{2NH_3} + 3CaOCl_2 \longrightarrow \underset{C}{N_2(g)} + 3CaCl_2 + 3H_2O$

$\underset{C}{N_2(g)} + 3Mg \longrightarrow \underset{D}{Mg_3N_2}$

$\underset{D}{Mg_3N_2} + 6H_2O \longrightarrow 3Mg(OH)_2 + \underset{B}{2NH_3}$

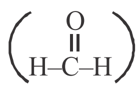
(15) **(4).**

(16) **(3).**



This is known as Liebermann nitroso reaction.

(17) **(3).** Aldol condensation is not given by compounds which does not have α-H i.e. methanal

.

$\overset{\alpha}{CH_3}-\overset{\overset{O}{||}}{C}-H \longrightarrow 3\alpha-H$
Ehtanal

$CH_3-\overset{\alpha}{CH_2}-\overset{\overset{O}{||}}{C}-H \longrightarrow 2\alpha-H$

$CH_3-CH_2-\overset{\alpha}{CH_2}-\overset{\overset{O}{||}}{C}-H \to 2\alpha-H$

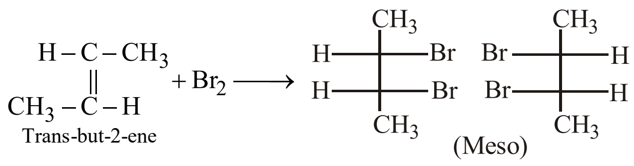
all give aldol condensation

(18) **(4).** $[Co(py)_3(NH_3)_3]^{3+}$: 2G.I.
$[Ni(en)(NH_3)_4]^{2+}$: No G.I.
$[Fe(C_2O)(en)_2]^{2-}$: 2 optical
$[Cr(NO_2)_2(NH_3)_4]^-$: 3 set of 2 G.I. (total 6 isomer)

$\because$ NO_2 can show linkage isomerism

(19) **(4).** Larger the value of pK_a, smaller will be its acidity. Out of the four groups, –COOH, $-NO_2$ and – CN are e^- withdrawing which makes benzoic acid more acidic whereas $-OCH_3$ is e^- donating which reduces the acidity (makes H^+ less easily available). pK_a value increases if $-OCH_3$ is present at para-position of benzoic acid.

(20) **(4).** With trans-but-2-ene, the product of Br_2 addition is optically inactive due to the formation of symmetric meso-compounds.



(21) **2.** Volume of hydrogen = 1.12 L

$\Rightarrow \dfrac{1.12}{22.4} \times 2g = 0.1\ g$

0.1 gm H_2 is displaced by

$\rightarrow$ 1.2 gm of metal

$\therefore$ 1 gm H_2 is displaced by

$\rightarrow$ 12 gm = eq. wt. of metal

(22) **3.** $O_2^{-1} \rightarrow 17\ e^-$

$\sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2\, \sigma^* 2s^2\, \sigma 2p_z^{\ 2}\, \pi 2p_x^{\ 2}$

$= \pi 2p_y^{\ 2}\, \pi^* 2p_x^{\ 2} = \pi^* 2p_y^{\ 1}\, \sigma^* 2p_z$

[Electron pair in ABMO = 3]

(23) **4.** $\log\left(\dfrac{K_2}{K_1}\right)$

or $\log K_2 - \log K_1 = \dfrac{Ea}{2.303R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

$1.1 - 2.9 = \dfrac{Ea}{2.303 \times 2}\ (1.3 \times 10^{-3} - 1.5 \times 10^{-3})$

(24) **8.** $T_b^{\,o} = 353.23$ K, $W_B = 1.8$ g,

$W_A = 90$ g, $T_b = 354.11$ K,

$K_b = 2.53$ kg mol^{-1}

$\Delta T_b = T_b - T_b^{\,o} = 354.11 - 353.23 = 0.88$ K

$$M_B = \frac{W_B \times K_b \times 1000}{\Delta T_b \times W_A} = \frac{1.8 \times 2.53 \times 1000}{0.88 \times 90}$$

$= 57.5 \approx 58$ g mol^{-1}

(25) **3.** $Cr_2O_7^{2-} + I^- \rightarrow Cr^{+3} + I_2$

(26) **(1).** From figure :

For time interval t = 0 to t = 1 sec

Slope of x-t graph is negative and increasing, so velocity increases in negative direction.

For t = 1 to 2 sec.

The slope is +ve and decreasing, so velocity is decreasing in +ve direction and become zero at t = 2.

(27) **(2).** Maximum heat will be generated when the distance travelled by the block with respect to the belt is maximum this will be the case when the block attains zero velocity after covering a distance ℓ and then come ϕ back.

$a = \mu g$

$v^2 = u^2 - 2a$; $\quad 0 = v_0^2 - 2 \times (\mu g)\ell$

$v_0 = \sqrt{2\mu g \ell}$

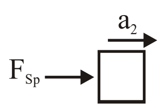
(28) **(2).** Spring force does not change instantaneously. Thus for m_1 ; $a_1 = a_0$

For m_2 : $F_{S_p} = m_2 a_2$ .....(i)

Instantaneously after F_2 is withdrawn

Initially $F_{S_p} - F_2 = m_2 a_0$

$F_{S_p} = F_2 + m_2 a_0$ ......(ii)



From (i) and (ii) $a_2 = \dfrac{F_2}{m_2} + a_0$

(29) **(2).** For equilibrium F = 0 and from F – x graph it is clear at x = 4 F = 0.

From V – x graph,

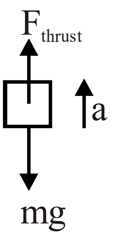
It is clear that $F = \dfrac{dV}{dx} = 0$ at x = z

(30) **(2).** $F_{thrust} - mg = ma$

m = 5000 kg, a = 20 m/s^2

$\Rightarrow F_{thrust} = 150000$ N

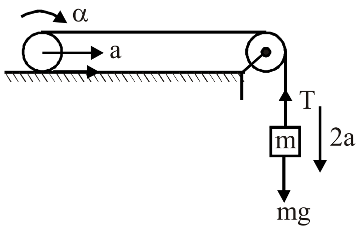
$F_{thrust} = U_{rel} \times \dfrac{dm}{dt}$



$\Rightarrow (-800) \times \dfrac{dm}{dt} = 150000$

$\Rightarrow \dfrac{dm}{dt} \approx -187.5$ kg/s

(31) **(4).**

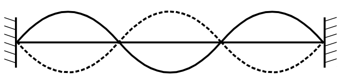


Applying Newton's II law on
block ; $mg - T = 2ma$ ...(i)
considering force and torque on the ring;
$T \times r - f \times r = mr\alpha$ ...(ii)
$a = r\alpha$ ...(iii)
$T + f = ma$ ...(iv)
Solving (ii), (iii) and (iv); $f = 0$; $T = ma$
$\therefore$ From (i), $a = g/3$

(32) **(3).** $x = a\cos\omega t$; $v = -a\omega\sin\omega t$

$$<v> = \frac{\int_0^{T/6}(-a\omega\sin\omega t)}{\int_0^{T/6}dt} = \frac{3a}{T}$$

(33) **(3).**

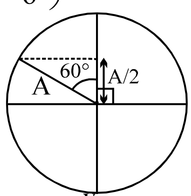


$$3\frac{\lambda}{2} = 3L \Rightarrow \lambda = 2L$$

$$y = A_0\sin kx\sin\left(\omega t + \frac{\pi}{2} + \frac{\pi}{3}\right)$$

$$= A_0\sin\left(\frac{2\pi}{2L}\right) \times \left(\frac{L}{2}\right)\sin\left(\omega t + \frac{5\pi}{6}\right)$$

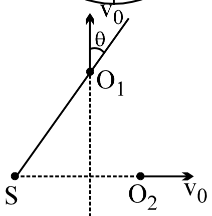
$$= A_0\sin\left(\omega t + \frac{5\pi}{6}\right)$$



(34) **(4).** $f_1 = f\left[\frac{v - v_0\cos\theta}{v}\right]$ ...(1)

$f_2 = f\left[\frac{v - v_0}{v}\right]$ ...(2)

$$\therefore \frac{f_1}{f_2} = \frac{v - v_0\cos\theta}{v - v_0} > 1$$



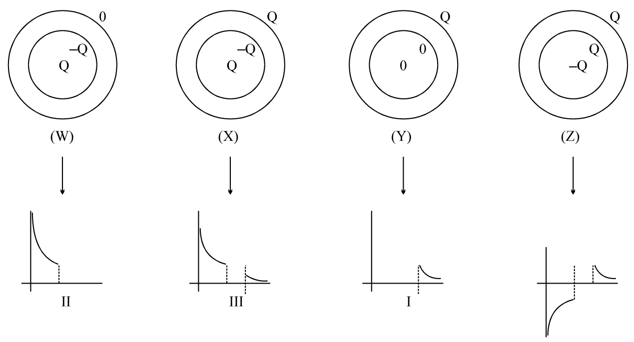
(35) **(3).** For ring just slides on to the steel rod the diameter of rod and ring should be equal to each other and suppose due to $\Delta\theta$ increment in temperature the diameter of both are equal then $4(1 + \alpha_s\Delta\theta) = 3.992(1 + \alpha_{Brass}\Delta\theta)$

$4 + 4 \times 11 \times 10^{-6} \times \Delta\theta$
$= 3.992 + 3.992 \times 20 \times 10^{-6} \times \Delta\theta$
$4 + 44 \times 10^{-6}\Delta\theta$
$= 3.992 + 79.84 \times 10^{-6} \times \Delta\theta$
$0.008 = 35.84 \times 10^{-6}\Delta\theta$

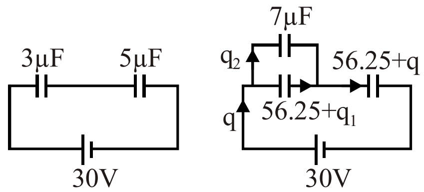
$$\frac{8\times10^3}{35.84} = \Delta\theta \;\; ; \;\; \Delta\theta = \frac{8000}{35.84} = 283$$

So if temperature increased by 223°C then ring will start to slide and this temperature will equal to
$\theta = 30° + \Delta\theta = 30 + 253 = 283°C$
$\theta = 283°C \approx 280°C$

(36) **(4).**



(37) **(3).**



$q_i = 56.25\ \mu C$

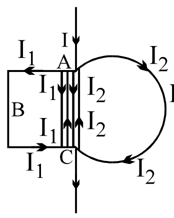
$$\frac{q_2}{7} = \frac{56.25 + q_1}{3} \quad ..... (1)$$

$$\frac{q_2}{7} + \frac{56.25 + q_1 + q_2}{5} = 30 \quad ..... (2)$$

$\therefore$ $q_2 = 70$; $q_1 = -26.25$ $\therefore$ $q = 43.75$

(38) **(4).** $(1680 + r)\,I = 20$
$(2930 + r)\,I = 30$
$2 \times 2930 + 2r = 3 \times 1680 + 3r$; $r = 820$

(39) **(2).** Introducing two equal and opposite current I_1 and also I_2 between A & C.



Force on ABCA closed loop zero
Force on ADCA closed loop zero
Force on extra I_1 & I_2
$F = (I_1 + I_2)\, lB = IlB$

(40) **(4).** $\phi = \int \frac{\mu_0 i}{2\pi x} x \ adx = M_x i$

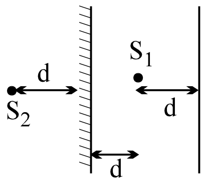
$M = \frac{\mu_0 a}{2\pi} \ell n \frac{(b+c)}{c}$

(41) **(3).** $P_{av} = \frac{v^2}{2z_1^2} R = \frac{v^2 R}{2z_2^2}$

$z_1 = \pm z_2 \Rightarrow \omega^2 L^2 + R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 + R^2$

$-\frac{2L}{C} + \frac{1}{\omega^2 C^2} = 0 \Rightarrow C = \frac{1}{2\omega^2 L} = 25\ \mu F$

(42) **(2).** $I_1 = I_0 = \frac{P}{4\pi d^2}$; $I_2 = \frac{P}{4\pi(3d)^2}$



$I = I_1 + I_2 = \frac{P}{4\pi d^2}\left[1 + \frac{1}{9}\right] = \frac{10}{9} I_0$

(43) **(4).** $D_1 \Rightarrow$ Reverse biased
$D_2 \Rightarrow$ Forward biased

$i = \frac{12}{4+2} = 2$

(44) **(3).** We know that
$h\nu = K.E. + \phi$ ......(1)
$\therefore$ $h\nu_1 = K_1 + \phi$ ......(2)
and $h\nu_2 = K_2 + \phi$

Dividing $\frac{\nu_2}{\nu_1} = \frac{K_2 + \phi}{K_1 + \phi} \Rightarrow \frac{2\nu_1}{\nu_1} = \frac{K_2 + \phi}{K_1 + \phi}$

$\Rightarrow$ $2(K_1 + \phi) = K_2 + \phi \Rightarrow K_2 = \phi + 2K_1$
Hence K.E. will become slightly more than double.

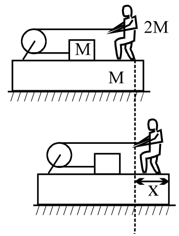
(45) **(2).** $\because$ $T_a = \frac{1}{\lambda}$

$N = N_0 e^{-\lambda T_a} = N_0 e^{-\frac{1}{T_a} \times T_a} = N_0\, e^{-1} = \frac{N_0}{e}$

$N = \frac{N_0}{2.718} = 0.37\ N_0$

Then number of decayed nuclei
$N' = N_0 - N \quad \Rightarrow N' = N_0 - 0.37\ N_0$
$N' = 0.63\ N_0 \Rightarrow$ 63 % of N_0
or About 2/3 of substance disintegrate.

(46) **1.** Since there is no external horizontal force on whole system C.M. of whole system need move,



$\Delta r_{CM} = \frac{m_1 \Delta r_1 + m_2 \Delta r_2}{m_1 + m_2}$

$O = \frac{M\ (x-2) + 3Mx}{4M}$

$Mx - 2M + 3Mx = 0$; $4x = 2$; $x = 0.5$ m

(47) **6.** $w_S - w_E = \frac{2\pi}{T}$; $w_S - \frac{2\pi}{24} = \frac{2\pi}{8}$

$w_S = 2\pi\left[\frac{1}{24} + \frac{1}{8}\right]$; $w_S = 2\pi\left[\frac{1+3}{24}\right] = \frac{2\pi}{6}$

$T_{satellite} = 6$ hr

(48) **3.** Let volume of block $= V_1$
Volume of concrete $= V_2$
$\therefore$ Displaced volume of water $= (V_1 + V_2)$
Weight of the combination = Buoyant force
$\therefore$ $0.5 \times V_1 g + 0.25 \times V_2 g = 1 \times (V_1 + V_2)\, g$

$\therefore$ $\frac{V_1}{V_2} = \frac{3}{1}$ $\therefore$ $\frac{m_1}{m_2} = \frac{0.5 \times V_1}{0.25 \times V_2} = 3/5$

(49) **5.** $F - T = 3a$; $T = 2a$
$T = 2.5 \times 10^9 \times 4 \times 10^{-8}$; $T = 100$ N
$T = 2a$; $100 = 2a$; $a = 50$ N
$F = 5 \times 50$; $F = 250$ N

(50) **5.** $W = \frac{1}{2} \times 3v_0 \times P_0 + 3v_0 \times P_0$

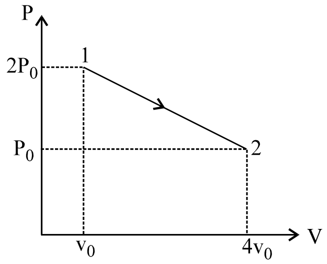
$W = \frac{3}{2} P_0v_0 + 3v_0P_0$; $W = \frac{a}{2} P_0v_0$

$\Delta U = nC_v\Delta T = n\left(\frac{3R}{2}\right)(T_f - T_i)$

$\Delta U = \frac{3}{2} nR(T_f - T_i) = \frac{3}{2}[P_fV_f - P_iV_i]$

$\Delta U = \frac{3}{2}[4P_0V_0 - 2P_0V_0] = 3P_0V_0$

$\Delta Q = \Delta U + W = \frac{9}{2} P_0V_0 + 3P_0V_0 = \frac{15}{2} P_0V_0$

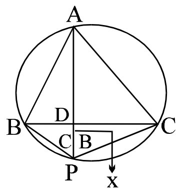


$P_0 = \frac{10^6}{2}$, $V_0 = 0.1$

$\Delta Q = \frac{15}{2} \times \frac{10^6}{2} \times 0.1 = 375000$ J

$\Delta Q = 375$ kJ

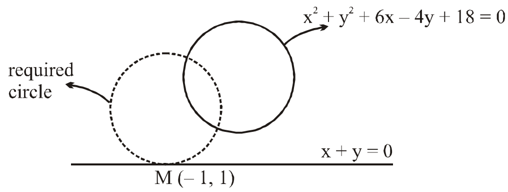
(51) **(1).** $BD = x\tan C$ in ΔPDB
and $DC = x\tan B$ for ΔPDC
$\therefore$ $BD + DC = a = x(\tan B + \tan C)$



$\frac{a}{x} = \tan B + \tan C \Rightarrow$ result

(52) **(4).** Clearly, equation of required circle, is
$(x+1)^2 + (y-1)^2 + \lambda(x+y) = 0$
$\Rightarrow$ $x^2 + y^2 + (\lambda+2)x + (\lambda-2)y + 2 = 0$ .....(1)
As circle (1) intersects the circle
$x^2 + y^2 + 6x - 4y + 18 = 0$ orthogonally,
so using orthogonality condition, we get

$$2\left(\left(\frac{\lambda+2}{2}\right)3 - 2\left(\frac{\lambda-2}{2}\right)\right) = 18 + 2$$



$\Rightarrow$ $(3\lambda + 6) - (2\lambda - 4) = 20 \Rightarrow \lambda = 10$
So, putting $\lambda = 10$ in equation, we get
$x^2 + y^2 + 12x + 8y + 2 = 0$.

Clearly, radius $= \sqrt{(6)^2 + (4)^2 - 2}$

$= \sqrt{36+16-2} = \sqrt{50} = 5\sqrt{2}$

(53) **(4).**
(1) $f(x) = x^4 + 2x^3 - x^2 + 1 \rightarrow$ A polynomial of degree even will always be into say
$f(x) = a_0x^{2n} + a_1x^{2n-1} + a_2x^{2n-2} + ... + a_{2n}$

$\underset{x\to\pm\infty}{\text{Limit}} f(x)$

$= \underset{x\to\pm\infty}{\text{Limit}} [x^{2n}\left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ... + \frac{a_{2n}}{x^{2n}}\right)$

$= \begin{bmatrix} \infty & \text{if } a_0 > 0 \\ -\infty & \text{if } a_0 < 0 \end{bmatrix}$

Hence it will never approach $\infty / -\infty$
(2) $f(x) = x^3 + x + 1 \Rightarrow f'(x) = 3x^2 + 1$
$\Rightarrow$ Injective as well as surjective
(3) $f(x) = \sqrt{1+x^2}$ – neither injective nor surjective (minimum value = 1)
(4) $f(x) = x^3 + 2x^2 - x + 1$
$\Rightarrow$ $f'(x) = 3x^2 + 4x - 1 \Rightarrow D > 0$
Hence f (x) is surjective but not injective.

(54) **(1).** $k(x) = \sqrt{1+\text{sgn}\,x} = \begin{cases} 0; & x < 0 \\ 1; & x = 0 \\ 2; & x > 0 \end{cases}$

So, k(x) is discontinuous at x = 0

(55) **(1).** $\underset{x\to\infty}{\text{Limit}} \sqrt{x+1} - \sqrt{x} = 0 \Rightarrow \cot^{-1}(0) = \pi/2$

$\underset{x\to\infty}{\text{Limit}} \left(\frac{2x+1}{x-1}\right)^x \to \infty \Rightarrow \sec^{-1}(\infty) = \pi/2$

$\therefore$ $l = 1$

(56) **(1).** $u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right) dx$

$u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\cos^2 x\right) dx$ (using King)

$2u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right) + \cos\left(\frac{2\pi}{3}\cdot\cos^2 x\right) dx$

On adding

$2u = \int_0^{\pi/2} 2\cos\frac{\pi}{3}\cdot\cos\left(\frac{\pi}{3}\cos 2x\right) dx$

(using cos C + cos D)

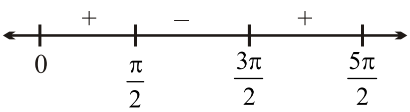
$= \frac{1}{2}\int_0^{\pi} \cos\left(\frac{\pi}{3}\cos t\right) dt$

[Put 2x = t]

$= \int_0^{\pi/2} \cos\left(\frac{\pi}{3}\cos t\right) dt = \int_0^{\pi/2} \cos\left(\frac{\pi}{3}\sin t\right) dt$

$= v$

(57) **(4).** We have $f'(x) = \sqrt{x}\ \cos x$, $x \in \left(0, \frac{5\pi}{2}\right)$



sign of $f'(x)$

∴ f (x) has local maximum at π/2 and local minimum at 3π/2

(58) **(1).** $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$; $|\vec{a}| = |\vec{b}| = 1$; $|\vec{c}| = 2$

$\vec{a} \times \vec{d} = -\vec{b} \Rightarrow (\vec{a} \times \vec{d})^2 = \vec{b}^2 = 1$

or $|\vec{a}|^2 |\vec{d}|^2 - (\vec{a}\,.\,\vec{d})^2 = 1$

or $(\vec{a} \times \vec{c})^2 - 0 = 1 \Rightarrow |\vec{a}|^2 |\vec{c}|^2 - (\vec{a}\,.\,\vec{c})^2 = 1$

$\Rightarrow 4 - 2\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{3}{4}$; $\theta = \pi/6$]

Alternative: $(\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})\vec{c} = -\vec{b}$

$(\lambda\vec{a} - \vec{c})^2 = 1$ or $\lambda^2 \vec{a}^2 + \vec{c}^2 - 2\lambda\vec{a}.\vec{c} = 1$

(where $\lambda = \vec{a}\cdot\vec{c}$)

$\Rightarrow \lambda^2 + 4 - 2\lambda^2 = 1$ or $\lambda^2 = 3$

$\vec{a}^2\ \vec{c}^2 \cos^2\theta = 3$; $\cos^2\theta = 3/4$; $\theta = \pi/6$

(59) **(2).** The equation of any plane through the intersection of P_1 and P_2 is

$P_1 + \lambda P_2 = 0$

$\Rightarrow (2x - y + z - 2) + \lambda(x + 2y - z - 3) = 0$

Also it passes through (3, 2, 1), then $\lambda = -1$

∴ Equation of plane is

$x - 3y + 2z + 1 = 0$

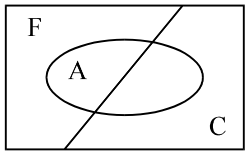
(60) **(3).** $P(F/F) = 0.9$; $P(C/F) = 0.1$; $P(C/C) = 0.8$

$P(F/C) = 0.2$

$P(F) = \frac{3}{10}$; $P(C) = \frac{7}{10}$

A : Wine tasted was French

B_1 : It is a Californian wine ; $P(B_1) = \frac{7}{10}$



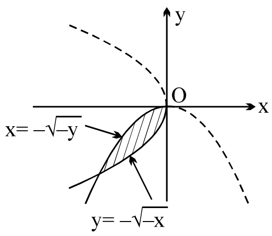
B_2 : It is a French wine ; $P(B_2) = \frac{3}{10}$

$P(A/B_1) = 0.2$; $P(A/B_2) = 0.9$

$P(B_1/A) = \frac{0.7 \times 0.2}{0.7 \times 0.2 + 0.3 \times 0.9}$

$= \frac{0.14}{0.14 + 0.27} = \frac{14}{41}$

(61) **(2).**



$y = -\sqrt{-x} \Rightarrow y^2 = -x$

where x & y both (–) ve

$x = -\sqrt{-y} \Rightarrow x^2 = -y$

where x & y both (–) ve

Hence $A = \frac{16ab}{3}$, where $a = b = \frac{1}{4}$

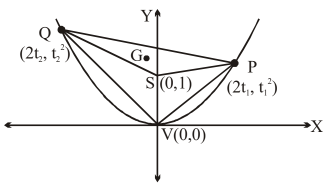
∴ $A = 1/3$

(62) **(2).** Let $P(2t_1, t_1^2)$; $Q(2t_2, t_2^2)$

Clearly, $t_1t_2 = -4$ ....(1)

Also, $h = \dfrac{2t_1 + 2t_2}{3}$ ....(2)

$\Rightarrow \dfrac{3h}{2} = t_1 + t_2$



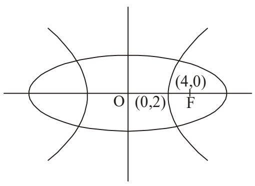
and $k = \dfrac{1 + t_1^2 + t_2^2}{3}$ ....(3)

$\Rightarrow 3k = 1 + t_1^2 + t_2^2$

$\therefore$ On eliminating we get t_1, t_2 from (1), (2) &(3)

, $\dfrac{9h^2}{4} = 3k - 9 \Rightarrow x^2 = \dfrac{4}{3}(y-3)$

(63) **(2).**



For the ellipse, $a^2 = 25,\ b^2 = 9$

$\therefore\ 9 = 25(1 - e^2) \Rightarrow e^2 = \dfrac{16}{25} \Rightarrow e = 4/5$

$\therefore$ One of the foci is (ae, 0) i.e. (4, 0)

$\therefore$ For the hyperbola

$a'e' = 4 \Rightarrow 2a' = 4 \Rightarrow a' = 2$

and $b'^2 = 4(e'^2 - 1) = 4 \times 3 = 12$

$\therefore$ Equation of the hyperbola is $\dfrac{x^2}{4} - \dfrac{y^2}{12} = 1$

(64) **(4).** $z = (3p - 7q) + i(3q + 7p)$

for purely imaginary $3p = 7q \Rightarrow p = 7$

or $q = 3$ (for least value)

$|z| = |3 + 7i|\,|p + iq|$

$\Rightarrow |z|^2 = 58\,(p^2 + q^2) = 58\,[7^2 + 9] = 58^2$

(65) **(2).** On the set N of natural number, $R = \{(x, y) : x, y \in N \text{ and } 2x + y = 41\}$

Here $(1, 1) \notin R$ as $2 \cdot 1 + 1 = 3 \neq 41$. So, R is not reflexive. $(1, 39) \in R$ but $(39, 1) \notin R$. So, R is not symmetric. $(20, 1), (1, 39) \in R$. But $(20, 39) \notin R$. So, R is not transitive]

(66) **(1).** $\sigma_x^2 = \left\{\dfrac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left(\dfrac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2\right\}$

$$\sigma_x^2 = \left(\frac{1}{f_1 + f_2}\right)\left\{f_1x_1^2 + f_2x_2^2 - \frac{(f_1x_1 + f_2x_2)^2}{f_1 + f_2}\right\}$$

$$\sigma_x^2 = \frac{1}{(f_1 + f_2)^2}\begin{pmatrix} f_1^2x_1^2 + f_1f_2x_1^2 + f_2f_1x_2^2 + f_2^2x^2 \\ -f_1^2x_1^2 - f_2^2x_2^2 - 2f_1f_2x_1x_2 \end{pmatrix}$$

$$\therefore\ \sigma_x^2 = \frac{f_1f_2}{(f_1 + f_2)}(x_1 - x_2)^2$$

(67) **(2).** $\sim p$: Ram does not work hard

Use '$\to$' symbol for then

$(\sim p \to q)$ means, if Ram does not work hard, then he gets good grade.

(68) **(1).** Put $\ell n\, x = t$

$\dfrac{dx}{x} = dt$; $\displaystyle\int_1^{2010}\left(1 + \frac{1-t}{t(t - \ell n\, t)}\right)dt$

$2009 + \displaystyle\int_1^{2010}\frac{\frac{1}{t} - 1}{t - \ell n\, t}dt$

$2009 - [\ell n(t - \ell n\, t)]_1^{2010}$

$2009 - [\ell n\,(2010 - \ell n\, 2010)]$

(69) **(4).** $y = \alpha x + \beta$ is a tangent

Then $\beta^2 = 4\alpha^2 - 9$

the locus of P is $4x^2 - y^2 = 9$ which is hyperbola.

(70) **(4).** $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$,

$$\text{Now, } [\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}$$

$$= 1\left(1 - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{2}\right) = \frac{3}{4} - \frac{1}{8} - \frac{1}{8} = \frac{1}{2}$$

$$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}] = \pm\frac{1}{\sqrt{2}}$$

$$\text{Now, } \vec{a} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c}) = p + \frac{q}{2} + \frac{r}{2}$$

$$\pm\frac{1}{\sqrt{2}} = p + \frac{q}{2} + \frac{r}{2}\ ;\ 2p + q + r = \pm\sqrt{2}\ ...\ (1)$$

$$\vec{b} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c}) = \frac{p}{2} + q + \frac{r}{2} \Rightarrow p + 2q + r = 0 ...\ (2)$$

$\vec{c}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c})=\frac{p}{2}+\frac{q}{2}+r$; $p+q+2r=\pm\sqrt{2}$ ... (3)

Now, $p=r=-q$

$$p=r=\pm\frac{1}{\sqrt{2}},\ q=\mp\frac{1}{\sqrt{2}}$$

$$\frac{p^2+2q^2+r^2}{q^2}=4$$

(71) **3.** $(x-c)^2=1 \Rightarrow x-c=\pm 1$

$\Rightarrow$ $x=c+1, c-1$

$\therefore$ $c-1>-2$ and $c+1<4 \Rightarrow -1<c<3$

$\therefore$ Number of integral values of c are 3 i.e., $c=0, 1, 2$

(72) **8.** $P_n = {}^{n-2}C_3$; $P_{n+1} = {}^{n-1}C_3$

Hence ${}^{n-1}C_3 - {}^{n-2}C_3 = 15$

${}^{n-2}C_3 + {}^{n-2}C_2 - {}^{n-2}C_3 = 15$

or ${}^{n-2}C_2 = 15 \Rightarrow n=8$

(73) **0.** $2\tan^{-1}(1/2)+\tan^{-1}(4/3)$

$$=\tan^{-1}\frac{2.\frac{1}{2}}{1-\frac{1}{4}}+\tan^{-1}\frac{4}{3}=2\tan^{-1}\frac{4}{3}>\frac{\pi}{2}$$

but $\operatorname{cosec}^{-1}x\in\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

$\Rightarrow$ no solution

(74) **1.** $A+A^T=\mathbf{O} \Rightarrow \begin{bmatrix}2\sin\theta & 0\\ 0 & -2\sin\theta\end{bmatrix}=\mathbf{O}$

$\Rightarrow$ $\sin\theta=0 \Rightarrow \theta=n\pi, n\in I$

$\therefore$ $\theta=\pi\in(0, 6)$

(75) **4.** We have, $\frac{dy}{dx}+\left(\frac{-1}{x}\right)y=x\,(xe^x+e^x-1)$

[Linear differential equation]

$\therefore$ I.F. $= e^{-\int\frac{dx}{x}} = e^{-\ln x} = \frac{1}{x}$

Now, general solution is

$$y\left(\frac{1}{x}\right)=\int\left(e^x(x+1)-1\right)dx+C$$

$\Rightarrow$ $\frac{y}{x}=x\,e^x-x+C$

As, $y\,(x=1)=e-1 \Rightarrow \frac{e-1}{1}=e-1+C$

$\Rightarrow$ $C=0$ $\quad\therefore\ x+\frac{y}{x}=xe^x$

Now, $2+\frac{y(2)}{2}=2e^2 \Rightarrow y(2)=4e^2-4$