JEE MAIN 2020
FULL TEST-7 SOLUTIONS

STANDARD ANSWER KEY

Q	1	2	3	4	5	6	7	8	9	10	11
A	2	3	2	2	2	4	2	4	1	1	3
Q	12	13	14	15	16	17	18	19	20	21	22
A	3	3	2	4	3	2	2	1	4	6	9
Q	23	24	25	26	27	28	29	30	31	32	33
A	5	4	2	4	1	3	2	1	1	3	3
Q	34	35	36	37	38	39	40	41	42	43	44
A	1	3	3	1	1	1	4	3	3	3	4
Q	45	46	47	48	49	50	51	52	53	54	55
A	4	2	3	5	4	2	3	2	2	2	4
Q	56	57	58	59	60	61	62	63	64	65	66
A	4	4	1	1	1	1	4	2	1	2	1
Q	67	68	69	70	71	72	73	74	75		
A	1	3	4	3	2	2	6	6	0		

(1) **(2).** $CaC_2 + 2H_2O \rightarrow HC \equiv CH + CaO$

$\downarrow$ $\qquad$ $\downarrow$

1 mol $\Rightarrow$ 64 gm. $\quad$ 1 mol. $\Rightarrow$ 22.4 L

64 gm. of CaC_2 produces

= 22.4 L gas at NTP

$\therefore$ 100 gm. of CaC_2 will produce

$$= \frac{22.4}{64} \times 100 \text{ L gas at NTP}$$

(2) **(3).** Along the period, basic nature of oxides decreases while on moving down the group, basic nature of oxides increases.

$Na_2O > MgO > Al_2O_3$

so $Al_2O_3 < MgO < Na_2O < K_2O$

(3) **(2).** $N_2 \xrightarrow{\text{electron}} N_2^-$

$$(\sigma 2P_z)^2 < (\pi 2P_x)^0 \rightarrow (\sigma 2P_z)^2 < (\pi 2P_x)^1$$

Diamagnetic $\qquad$ Paramagnetic

$O_2 \xrightarrow{\text{electron}} O_2^-$

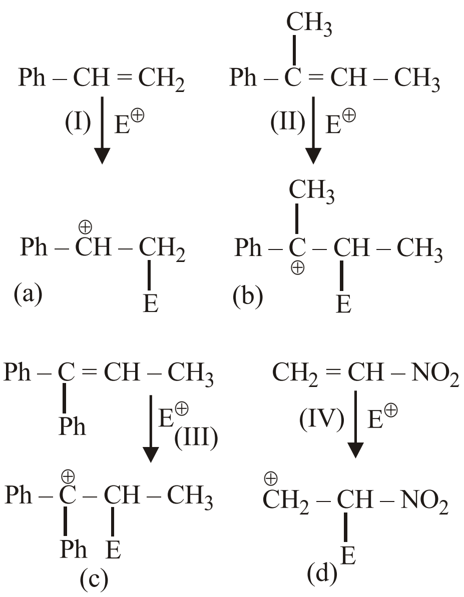
$$(\sigma 2P_x*)^1 = (\pi 2P_y*)^1 \rightarrow (\pi 2P_x*)^2 = (\pi 2P_y*)^1$$

Diamagnetic $\qquad$ Paramagnetic

(4) **(2).** Lassaigne's test is given by those nitrogenous compounds in which carbon is also present along with nitrogen.

In $NH_2 - NH_2 \cdot HCl$, carbon is absent, so it does not give Lassaigne's test.

(5) **(2).** Reactivity $\propto$ Stability of intermediate carbocation



Stability order : c > b > a > d

Reactivity order : III > II > I > IV

(6) (4).

$$\underset{(a)}{CH_3-\overset{\overset{H}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_2-Br} \xrightarrow[-HBr]{} \underset{(I)}{CH_3-\underset{\underset{CH_3}{|}}{C}=CH_2}$$

$$\underset{(b)}{CH_3-CH_2-Br} \xrightarrow[-HBr]{} \underset{(II)}{CH_2=CH_2}$$

$$\underset{(c)}{CH_3-CH_2-CH_2-Br} \xrightarrow[-HBr]{} \underset{(III)}{CH_3-CH=CH_2}$$

Reactivity of alkyl halide $\propto$ Stability of alkene
Stability of alkene : I > III > II
Reactivity : a > c > b

(7) (2).

$$\underset{Alkane}{C_nH_{2n+2}}+\left(\frac{3n+1}{2}\right)O_2 \longrightarrow nCO_2+(n+1)H_2O$$

1 mole of an alkane required

$$=\frac{3n+1}{2}\text{ mole of }O_2$$

10 L of alkane required = 35 L of O_2

$$1:\frac{3n+1}{2}::10:35$$

$$\left(\frac{1}{\frac{3n+1}{2}}\right)=\frac{10}{35}\ ;\ \frac{2}{3n+1}=\frac{10}{35}$$

30n + 10 = 70 ; 30n = 60 ; n = 2
So, alkane is CH_3-CH_3 (X)

$$\underset{(X)}{CH_3-CH_3} \xrightarrow{Cl_2} \underset{(Y)}{CH_3-\underset{\underset{Cl}{|}}{CH_2}} \xrightarrow{Alc.\ KOH} CH_2=CH_2$$

(8) (4).

(i) $AgI+e^- \to Ag+I^-,\quad E°=-0.152V$
(ii) $Ag \to Ag^+ + e^-\quad ,\quad E°=-0.80\ V$

Cell reaction $AgI \rightleftharpoons Ag^+ + I^-$
$E°_{cell}=-0.152-0.80=-0.952V$

$$E_{cell}=E°_{cell}-\frac{0.0591}{1}\log_{10}\frac{[Ag^+][I^-]}{[AgI]}$$

[$E_{cell}=0$ at equilibrium)
$\therefore\ E°_{cell}=0.0591\log_{10}K_{sp}$

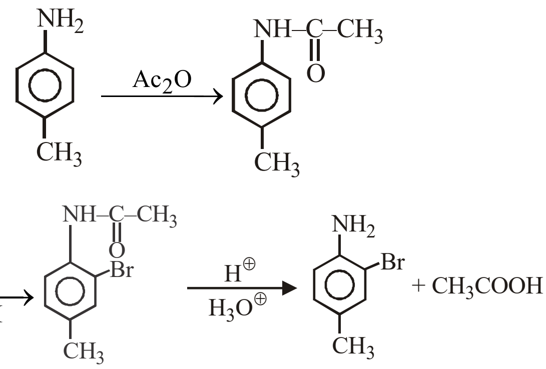
or or $\log K_{sp}=-\dfrac{0.952}{0.0591}=-16.11$

(9) (1). $Cl_2O_7 > SO_2 > P_4O_{10}$
$SO_3 > N_2O_5 > CO_2$

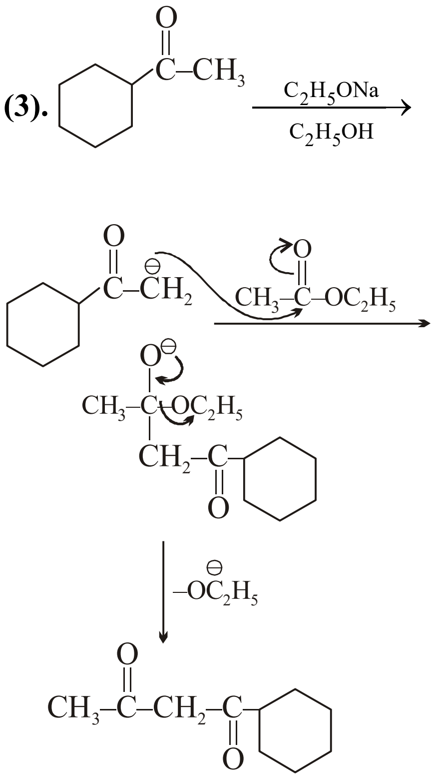
(10) (1). H_3BO_3 is a weak monobasic acid.
$H_3BO_3 + H_2O \rightleftharpoons [B(OH)_4]^- + H^+$

(11) (3). Due to Lanthanide contraction size of 4d & 5d series elements approx. same.

(12) (3).



(13) (3).



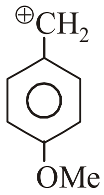
(14) **(2).** More is the stability of benzylic carbocation less will be the bond energy.

(I) No extra effect

(II) +M of –I

(III) –I

(IV) +M and –I but –I is less than (II)

$\therefore$ Most stable will be 

$\therefore$ III > I > II > IV

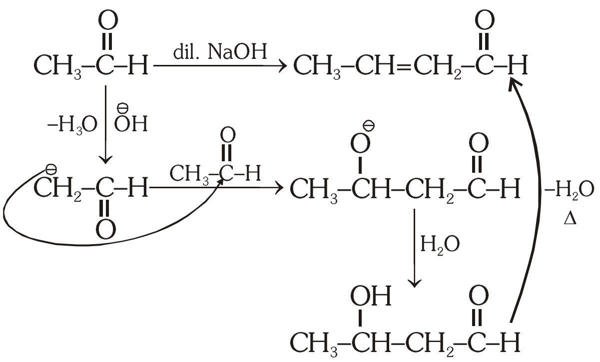
(15) **(4).** $CaCO_3 + 2CH_3COOH \rightarrow (CH_3COO)_2Ca + CO_2 + H_2O$

$(CH_3COO)_2Ca + (NH_4)_2C_2O_4 \rightarrow CaC_2O_4\downarrow + 2CH_3COONH_4$

(white ppt.)

(16) **(3)** **(17) (2).**

(18) **(2).**



(19) **(1).**

* $\underset{(A)}{Na_2SO_3} + 2HCl \longrightarrow H_2SO_3 + 2NaCl$
  $\quad \hookrightarrow H_2O + \underset{(Y)}{SO_2}\uparrow$

$\underbrace{K_2Cr_2O_7 + H_2SO_4 +}_{\text{Orange}} SO_2 \longrightarrow \underbrace{Cr_2(SO_4)_3}_{\text{Green}}$

* $\underset{(B)}{Na_2S} + 2HCl \longrightarrow \underset{(Z)}{H_2S}\uparrow + 2NaCl$

$H_2S + (CH_3COO)_2Pb \longrightarrow \underset{\text{Black}}{PbS} + 2CH_3COOH$

(20) **(4).**

Molecule	B.O.
N_2	3
N_2^-	2.5
N_2^+	2.5
N_2^{2-}	1

$\left[\text{B.O.} \propto \dfrac{1}{\text{B.L.}} \propto \text{B.E.}\right]$

(21) **6.** $Fe^{3+} + SCN^- \rightleftharpoons FeSCN^{2+}$

At t = 0: 3.1   3.2   0

At eqm.: 3.1 – x   3.2 – x   x

x = 3.0 mol

$$K_c = \frac{[FeSCN^{+2}]}{[Fe^{+3}][SCN^-]} = \frac{3}{0.1 \times 0.2} = 150$$

(22) **9.** $N_1V_1 = N_2V_2$

$10^{-1} \times 1 = 10^{-2} \times V_2$

$V_2 = 10$ L

so $V_2 - V_1 = 10 - 1 = 9$ L

(23) **5.** $\frac{1}{2}A_2(g) + \frac{3}{2}B_2(g) \rightarrow AB_3(g);\ \Delta H = -20KJ$

$\Delta S° = \Sigma (S°)_P - \Sigma (S°)_R$

$$= 50 - \left(\frac{1}{2} \times 60 + \frac{3}{2} \times 40\right)$$

$\Delta S° = -40\ JK^{-1}\ mol^{-1}$

Ae equilibrium $\Delta G = 0\ \therefore\ \Delta H = T\Delta S$

$$T = \frac{+20 \times 1000}{-40} = 500\ K$$

(24) **4.** Given: $w_2 = 0.2$ g, $w_1 = 20$ g, $\Delta T_f = 0.45°C$

$$\Delta T_f = \frac{1000 \times K_f \times w_2}{w_1 \times M}$$

$$0.45 = \frac{1000 \times 5.12 \times 0.2}{20 \times M}$$

$\therefore$ $M_{(observed)} = 113.78$ (acetic acid)

$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$

Before association

1   0

After association

$1 - \alpha$ $\alpha / 2$

(where α is degree of association)

Molecular weight of acetic acid = 60

$$i = \frac{\text{Normal molecular mass}}{\text{Observed molecular mass}}$$

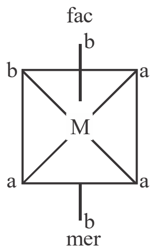
$$\therefore \quad \frac{M_{(normal)}}{M_{(observed)}} = 1 - \alpha + \frac{\alpha}{2}$$

or $\quad \frac{60}{113.78} = 1 - \alpha + \frac{\alpha}{2}$

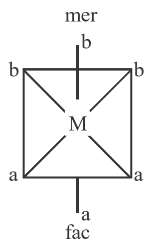
$\therefore \quad \alpha = 0.945$ or 94.5%

(25) **2.** In Ma_3b_3 has 2 gI

1.



2.



(26) **(4).** $\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$

$\Delta u = \Delta v$ for optical bench

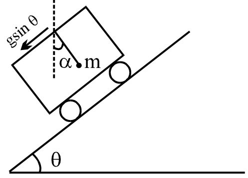
$$\Rightarrow \frac{1}{f^2} \times d(\Delta f) = \Delta u\left[\frac{-2}{u^3} - \frac{2}{v^3} \times \frac{dv}{du}\right]_{\text{for min error}}$$

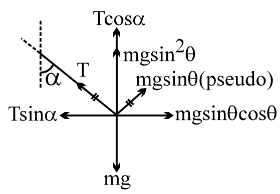
$$= 0$$

$$\Rightarrow \frac{dv}{du} = -\frac{v^3}{u^3} = -\frac{v^2}{u^2} \Rightarrow v = u$$

for $u = 2f$, error is minimum.

(27) **(1).**





$T\sin\alpha = mg\sin\theta\cos\theta \qquad ...(1)$

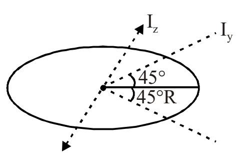
$T\cos\alpha = mg\cos^2\theta \qquad ...(2)$

$\tan\alpha = \tan\theta \Rightarrow \alpha = \theta$

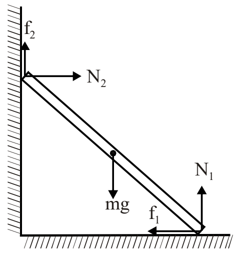
(28) **(3).** Assume 3perpendicular axis I_x, I_y and I_z where I_z is diameter of the ring and $I_x = I_y$

$$2I + \frac{MR^2}{2} = 2MR^2$$

$$I = \frac{3}{4}MR^2$$



(29) **(2).**



N_2 can only be balanced by f_1

$\therefore \quad \mu_1 \neq 0$

But f_2 can be zero, as in vertical mg can be balanced by N alone.

(30) **(1).** $\frac{1}{2}m\left(\frac{v_e}{2}\right)^2 - \frac{GM_em}{R_e} = 0 - \frac{GM_em}{(R_e + h)}$

$$\frac{1}{2}m\left(\frac{2gR_e}{4}\right) - mgR_e = -\frac{mgR_e^2}{(R_e + h)}$$

$$\frac{mgR_e}{4} - mgR_e = -\frac{mgR_e^2}{R_e + h}$$

$$-\frac{3mgR_e}{4} = -\frac{mgR_e^2}{R_e + h}$$

$3R_e + 3h = 4R_e \quad ; \quad h = R_e/3$

(31) **(1).** $y = f(x \pm c \cdot t)$ is the general wave equation

At $t = 0$, $y = f(x) \Rightarrow y = \frac{1}{\sqrt{1 + x^2}}$

$$y = \frac{1}{\sqrt{2 - 2x + x^2}} = \frac{1}{\sqrt{1 + (x-1)^2}} = f(x - 1)$$

$\Rightarrow f(x - ct) = f(x - 1)$ at $t = 1 \Rightarrow c = 1$ m/s.

(32) **(3).** $\Delta\phi = 2n\pi$

$$\Rightarrow \frac{\pi}{2} + \frac{2\pi}{\lambda} d\sin\theta = 2n\pi$$

$$\frac{2\pi}{\lambda} d\sin\theta = \left(2n - \frac{1}{2}\right)\pi$$

$$\sin\theta = \left(2n - \frac{1}{2}\right)\frac{\lambda}{2d} = \frac{1}{2} \times \frac{\lambda}{2 \times 3\lambda} = \frac{1}{12}$$

$$\Rightarrow \frac{y}{\sqrt{(100\lambda)^2}} = \frac{1}{12}$$

$$144y^2 = (100\lambda)^2 \quad ; \quad y \approx \frac{100\lambda}{12} = \frac{25\lambda}{3}$$

(33) **(3).** Rate of absorption = Rate of emission

$P_{ab} + P_{ab} = P_{em.}$

$eA\,\sigma\,T_0^4 = eA\,\sigma\,T_1^4 = eA\,\sigma\,T_B^4$

T_B = Remains constant as (T_0 and T_1) are constant.

(34) **(1).** Let x mole of the gas dissociate at 1000 K

No. of mole of diatomic gas molecule = 1 – x

No. of moles of monatomic gas molecules = 2 × x

Energy of diatomic molecules = energy of monatomic molecule

$$\Rightarrow\ (1-x)\frac{5}{2}RT = 2x \times \frac{3}{2}RT \Rightarrow x = 5/11$$

Now new no. of moles = (1 – x) + 2x = 1 + x = (16/11)

$$P = \frac{nRT}{V}$$

Pressure initially at 300 K = $P_i = \dfrac{300R}{V}$

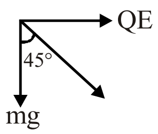
Pressure finally at 1000K

$$= P_f = \frac{(1+x)R\times 1000}{V} = \frac{16}{11}\times 1000\left(\frac{R}{V}\right)$$

(35) **(3).** QE = Mg

$$\frac{QV}{d} = w$$

$$Q = \frac{wd}{V}$$



(36) **(3).** $E = \rho\,\dfrac{i}{A}$; $E = \dfrac{kr^2}{R}\,\dfrac{i}{\pi r^2}$; $E = \dfrac{ki}{\pi r}$

(37) **(1).** Suppose in 1st region radius of circular path is r_1 & in region 2 this is r_2.

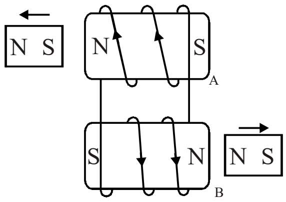
$\therefore\ r_1 > 5$ & $r_2 > 5$

$r = mv/(qB)$, so, $v_{min} = \dfrac{rqB_{min}}{m}$

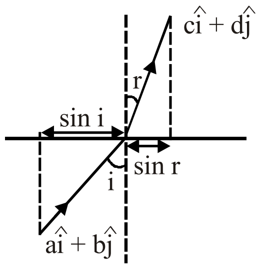
$$\therefore\ v_{min} = \frac{5\times 10^{-2}\times 1.6\times 10^{-19}\times 0.01}{9\times 10^{-31}}$$

$$= \frac{8}{9}\times 10^7 \text{ m/s}$$

(38) **(1).**



(39) **(1).**



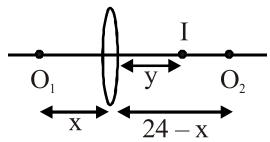
As $a\hat{i} + b\hat{j}$ and $c\hat{i} + d\hat{j}$ are unit vectors their x components represent sin θ

1.5 sin i = 2 sin r

$a = \times 1.5 = 2 \times c$; $\dfrac{a}{c} = \dfrac{4}{3}$

(40) **(4).** Let the image distance from lens be y

$$\frac{1}{y} - \frac{1}{-(24-x)} = \frac{1}{9}$$



$$\frac{1}{-y} - \frac{1}{-x} = \frac{1}{9}\ ;\ \frac{1}{x} + \frac{1}{(24-x)} = \frac{2}{9}$$

$$\frac{24x - x^2}{24} = \frac{9}{2}$$

$x^2 - 24x + 108 = 0$; x = 6 cm, 18cm.

(41) **(3).** $\lambda_1 = \dfrac{1}{30}$; $\lambda_2 = \dfrac{1}{60}$; $\lambda = \lambda_1 + \lambda_2$; $\lambda = \dfrac{1}{20}$

$$N = \frac{N_0}{2^{t/t_{1/2}}}\ ;\ \frac{N_0}{4} = \frac{N_0}{(2)^{t/t_{1/2}}}$$

$$\therefore\ t = 2t_{1/2} = 2\times\frac{0.693}{\lambda}$$

t = 2×0.693×20 yrs = 27.72 yrs.

(42) **(3).**

A	B	X
0	0	0
1	0	1
0	1	1
1	1	1

(43) **(3).** B.E. of A = 240 × 7.6 = 1824 MeV

B.E. of B = 100 × 8.1 = 810 MeV

B.E. of C = 140 × 8.1 = 1134 MeV

So Q = (810 + 1134) MeV – 1824 MeV

= 120 MeV

(44) **(4).** Wattless power $= VI\sin\phi$,

Wattless power

$$= \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}} \times \sin\frac{\pi}{6} \quad \begin{cases} V = \dfrac{100}{\sqrt{2}} V \\ I = \dfrac{100}{\sqrt{2}} A \\ \phi = \dfrac{\pi}{6} \end{cases}$$

$= 2.5 \times 10^3$ Watt

(45) **(4).** At same spot so fringes coin side

$n_1\lambda_1 = n_2\lambda_2$

$3 \times 700\text{ nm} = 5 \times \lambda_2$; $\lambda_2 = 420$ nm

(46) **2.** $k_{eq} = \frac{100 \times 150}{250} = 60$ N/m

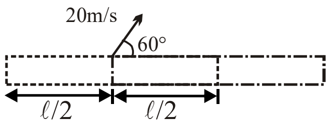
$F = k_{eq}\, x = 60 \times \frac{2.5}{100} = \frac{3}{2}$ N

For left spring $x_1 = \frac{3}{2(100)}$

For right spring $x_2 = \frac{3}{2(150)}$

$$\frac{\frac{1}{2}(100)\left(\frac{3}{2}\right)^2\left(\frac{1}{100}\right)^2}{\frac{1}{2}(150)\left(\frac{3}{2}\right)^2\left(\frac{1}{150}\right)^2} = \frac{150}{100} = \frac{3}{2}$$

(47) **3.**

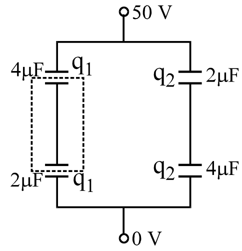


As net external horizontal force is zero, and initial velocity of system is also zero,

∴ $m_{ball} \times x_{ball} = m_{plank} \times x_{plank}$ (backward) distance travelled by the ball is its

Range, $R = \frac{u^2 \sin 2\theta}{g}$; $\frac{\ell}{2} = \frac{(20)^2}{10} \times \frac{\sqrt{3}}{2}$

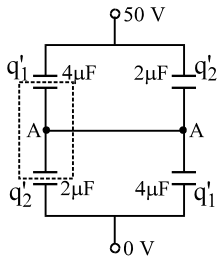
(48) **5.**



Net charge under dotted box shown

$= -q_1 + q_1 = 0$

<u>Finally :</u> $V_A = 25V$



$q'_1 = 25(4) = 100\ \mu C$

$q'_2 = 25(2) = 50\ \mu C$

Net charge under the dotted box shown

$= -q'_1 + q'_2 = -100 + 50 = -50\ \mu C$

The charge which flows = 50 μC

(49) **4.** $\frac{hc}{\lambda} = eV_0 + \phi_0 = 10\text{ eV} + 2.75\text{ eV} = 12.75\text{eV}$

But $\frac{hc}{\lambda} = 13.6\left[\frac{1}{1^2} - \frac{1}{n^2}\right]\text{eV} \Rightarrow 1 - \frac{1}{n^2} = \frac{12.75}{13.6}$

$\Rightarrow \frac{1}{n^2} = 0.0625 \Rightarrow n^2 = \frac{10000}{625} = 16 \Rightarrow n = 4$

(50) **2.** $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Here $\frac{1}{\lambda_{L_1}}$ (Hydrogen) $= \frac{1}{\lambda_{B_1}}$ (other)

$R \times 1^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R \times Z^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$\Rightarrow Z = 2$

(51) **(3).** $y = (7\cos\theta + 24\sin\theta) \times (7\sin\theta - 24\cos\theta)$

$r\cos\phi = 7$; $r\sin\phi = 24$

$r^2 = 625$; $\tan\phi = \frac{24}{7}$

$y = r\cos(\theta - \phi) \,.\, r\sin(\theta - \phi)$

$= \frac{r^2}{2} \,.\, 2\sin(\theta - \phi)\cos(\theta - \phi)$

$= \frac{r^2}{2} \,.\, (\sin 2(\theta - \phi))$; $y_{max} = \frac{25^2}{2} = \frac{625}{2}$

(52) **(2).** $\frac{1}{3}$, H and x are in H.P. ; 3, $\left(\frac{1}{H} - 6\right)$ and x are in G.P. $\Rightarrow \left(\frac{1}{H} - 6\right)^2 = \frac{3}{x}$ ....(1)

Also $H = \frac{\frac{2x}{3}}{\frac{1}{3} + x} = \frac{2x}{3x+1}$; Hence, $\frac{1}{H} = \frac{3x+1}{2x}$

$\Rightarrow \frac{1}{H} - 6 = \frac{3x+1}{2x} - 6 = \frac{1-9x}{2x}$

$\therefore \frac{(1-9x)^2}{4x^2} = \frac{3}{x} \Rightarrow (9x-1)^2 = 12x,$

now verify.

(53) **(2).** Let P (x_1, y_1) be any point on required locus. So equation of chord of contact w.r.t. circle $x^2 + y^2 = 4$, is $xx_1 + yy_1 = 4$ ...(1)

Also, equation of common chord between two circles is,

$4 + (k+1)x - (k-2)y - 1 = 0$

or $(k+1)x - (k-2)y + 3 = 0$ ...(2)

As equation (1) and (2) are identical, so on comparing, we get

$\frac{x_1}{k+1} = \frac{y_1}{2-k} = \frac{-4}{3} = \frac{x_1 + y_1}{3}$

So, locus of (x_1, y_1) is $x + y = -4$

(54) **(2).** ${}^6C_2 \cdot {}^5C_1 \cdot 4! = 1800$

S_1 S_2 S_3 S_4 S_5 × × × × ×

Note that at least one of the subject has to be repeated two periods in which one subject is to be repeated ${}^5C_1 \cdot 4!$ one subject

(55) **(4).** $4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7$

$= 4\cos^4 x - 2(2\cos^2 x - 1)$

$- \frac{1}{2}(2\cos^2 2x - 1) - x^7$

$= 4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2$

$+ \frac{1}{2} - x^7 = \left(\frac{3}{2} - x^7\right)$

We get $g(x) = \left(\frac{3}{2} - x^7\right)^{1/7}$

$\Rightarrow g(g(x)) = \left(\frac{3}{2} - (g(x))^7\right)^{1/7}$

$= \left(\frac{3}{2} - \left(\frac{3}{2} - x^7\right)\right)^{1/7} = x$

Hence $g(g(100)) = 100$

(56) **(4).** Let θ = arc cos (x – 1)

Now, $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

So, $4y^3 - 3y = 0$, where $y = x - 1$

$\therefore y = \pm\frac{\sqrt{3}}{2}, 0 \Rightarrow x = 1 \pm \frac{\sqrt{3}}{2}, 1$

Hence three values of x

Aliter : $\cos(3\cos^{-1}(x-1)) = 0$

$\Rightarrow 3\cos^{-1}(x-1) = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore \cos^{-1}(x-1) = (2n+1)\frac{\pi}{6}, n \in I$

$\Rightarrow \cos^{-1}(x-1) = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

$\Rightarrow x - 1 = \frac{\sqrt{3}}{2}, 0, \frac{-\sqrt{3}}{2}$

$\therefore x = 1 + \frac{\sqrt{3}}{2}, 1, 1 - \frac{\sqrt{3}}{2}.$

(57) **(4).** f(x) will be continuous when $\cos^2 x = -\cos^2 x$

$\Rightarrow \cos^2 x = 0 \Rightarrow \cos x = 0$

$\Rightarrow x = (2k+1)\frac{\pi}{2}, k \in I.$

(58) **(1).** $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{r^2}{n^3 + n^2 + r}$

$= \frac{1^2}{n^3 + n^2 + 1} + \frac{2^2}{n^3 + n^2 + 2}$

$+ \ldots\ldots + \frac{n^2}{n^3 + n^2 + n}$

$\because \quad \frac{1^2+2^2+......+n^2}{n^3+n^2+n} < \sum_{r=1}^{n} \frac{r^2}{n^3+n^2+r}$

$< \frac{1^2+2^2+......+n^2}{n^3+n^2+1}$

$\Rightarrow \quad \frac{n(n+1)(2n+1)}{6(n^3+n^2+n)} < \sum_{r=1}^{n} \frac{r^2}{n^3+n^2+r}$

$< \frac{n(n+1)(2n+1)}{6(n^3+n^2+1)}$

$\because \quad \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6(n^3+n^2+n)} = \frac{1}{3}$

$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6(n^3+n^2+1)} = \frac{1}{3}$

$\therefore$ According to Sandwitch theorem, so given limit = 1/3.

(59) **(1).** $I = \int_0^{\pi/3} \ln\left(\frac{\cos x+\sqrt{3}\sin x}{\cos x}\right)dx$

$= \underbrace{\int_0^{\pi/3} \ln 2\cos\left(x-\frac{\pi}{3}\right)dx}_{\text{use King}} - \int_0^{\pi/3} \ln \cos x\, dx$

$= \int_0^{\pi/3} \ln(2\cos x)\,dx - \int_0^{\pi/3} \ln(\cos x)\,dx$

$= \int_0^{\pi/3} \ln 2\,dx + \int_0^{\pi/3} \ln(\cos x)\,dx - \int_0^{\pi/3} \ln(\cos x)\,dx$

$= \frac{\pi}{3}\ln 2$

Alternatively: $I = \int_0^{\pi/3} \ln\left(1+\sqrt{3}\tan\left(\frac{\pi}{3}-x\right)\right)dx$

$= \int_0^{\pi/3} \ln\left(1+\sqrt{3}\left(\frac{\sqrt{3}-\tan x}{1+\sqrt{3}\tan x}\right)\right)dx$

$= \int_0^{\pi/3} \ln\left(\frac{1+\sqrt{3}\tan x+\sqrt{3}-\sqrt{3}\tan x}{1+\sqrt{3}\tan x}\right)dx$

$= \int_0^{\pi/3} \ln\left(\frac{4}{(1+\sqrt{3}\tan x)}\right)dx$

$2I = \left(\frac{\pi}{3}\right)\cdot 2\ln 2 \Rightarrow I = \frac{\pi}{3}\ln 2$

(60) **(1).** We have $f(x) = x^3 - 3\alpha x^2 + 3(\alpha^2-1)x + 1$

So, $f'(x) = 3(x^2 - 2\alpha x + \alpha^2 - 1)$
$= 3(x-\alpha+1)(x-\alpha-1)$

Clearly, $\alpha - 1 > -2$ and $\alpha + 1 < 4$ must be satisfied simultaneously, so $\alpha \in (-1, 3)$

(61) **(1).** A is involutary $\Rightarrow A^2 = I \Rightarrow A = A^{-1}$

$A^2 = \left(\frac{A}{2}\right)(2A) = I \Rightarrow 2A = \left(\frac{A}{2}\right)^{-1}$

(62) **(4).** $\vec{r}_1 + 2\vec{r}_2 = (p\vec{a}+q\vec{b}+\vec{c}) + 2(\vec{a}+p\vec{b}+q\vec{c})$

$\vec{r}_1 + 2\vec{r}_2 = (p+2)\vec{a} + (q+2p)\vec{b} + (1+2q)\vec{c}$

$2\vec{r}_1 + \vec{r}_2 = (2p+1)\vec{a} + (2q+p)\vec{b} + (2+q)\vec{c}$

$\frac{p+2}{2p+1} = \frac{q+2p}{2q+p} = \frac{1+2q}{2+q}$

$= \frac{p+q+2p+2q+3}{p+q+2p+2q+3} = 1$

$\Rightarrow$ p = 1 & q = 1]

(63) **(2).** P (number chosen is odd) = 3/5

P (number chosen is even) = 2/5

ab + c is even

< a,b,c, are all odd
< c is even and atleast a or b is even

E : (ab + c) is even ;

Note that event E can occurs in two cases

E_1 : All the three number a, b and c are odd;

$P(E_1) = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$

E_2 : c is even and atleast one of a or b is even

$P(E_2) = \frac{2}{5}\cdot\left(1-\frac{9}{25}\right) = \frac{2}{5}\cdot\frac{16}{25} = \frac{32}{125}$

$P(E) = P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \frac{59}{125}$

(64) **(1).** $x\dfrac{dy}{dx} + y\,(ln\,y) = 0$

$\Rightarrow \int \dfrac{dx}{x} + \int \dfrac{dy}{y\,(ln\,y)} = C$;

$ln\,(x\,ln\,y) = C$. If $x = 1$ then $y = e$

$\Rightarrow ln(ln\,e) = C \Rightarrow C = 0$

(65) **(2).** The equation of normal at $(at^2, 2at)$ is

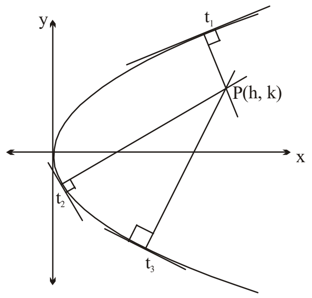
$y + tx = 2at + at^3$ ....(1)

As (1) passes through P(h, k), so

$at^3 + t(2a - h) - k = 0$ ⟨ t_1, t_2, t_3 ....(2)

Here, a = 1

$t_1 + t_2 + t_3 = 0$ ....(3)



Given $\dfrac{2}{t_1 + t_2} = 2 \Rightarrow t_1 + t_2 = 1$ ....(4)

From (3) and (4) $\Rightarrow t_3 = -1$

Put $t_3 = -1$ in (2), we get

$-1 - 1\,(2 - h) - k = 0$

$\Rightarrow -1 - 2 + h - k = 0$

$\therefore$ Locus of P(h, k), is $x - y = 3$

(66) **(1).** $|a - a| = 0 < 1$

$\therefore$ a R a for all $a \in R$

$\therefore$ R is reflexive

Again aRb $\Rightarrow |a - b| \le 1$ and $|b - a| \le 1$

$\Rightarrow$ b R a $\therefore$ R is symmetric

Again, $1\,R\dfrac{1}{2}$ and $\dfrac{1}{2}R\,1$ but $\dfrac{1}{2} \ne 1$

$\therefore$ R is not anti-symmetric.

Further, 1R2 and 2R3 but not 1R3.

[$\because$ $|1 - 3| = 2 > 1$]

$\therefore$ R is not transitive.

(67) **(1).** Since deviation about point c is

$\dfrac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$ (given)

$\therefore$ Deviation about –2 is

$\dfrac{1}{n}\sum_{i=1}^{n}\{x_i - (2)\}^2 = 18$ ……(i)

Also, deviation about 2 is

$\dfrac{1}{n}\sum_{i=1}^{n}(x_i - 2)^2 = 10$ ……(ii)

Adding and subtracting (i) & (ii), we obtain

$\dfrac{2}{n}\sum_{i=1}^{n}\left(x_1^2 + 2^2\right) = 28$ or $\dfrac{1}{n}\sum_{i=1}^{n}x_i^2 = 10$

and $\dfrac{8}{n}\sum_{i=1}^{n}x_i = 8$ or $\dfrac{1}{n}\sum_{i=1}^{n}x_i = 1$

$\sigma^2 = \dfrac{1}{n}\left\{\sum_{i=1}^{n}x_i^2 - \left(\sum_{i=1}^{n}x_i\right)^2\right\} = 10 - 1 = 9$

$\therefore$ S.D. $= \sigma = 3$

(68) **(3).** $(x - 1)^2 + (y + 1)^2 = (x - y + 1)^2$

$2xy - 4x + 4y + 1 = 0$

(69) **(4).** $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{\vec{b}}{3} + \dfrac{\vec{c}}{2}$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \dfrac{\vec{b}}{3} + \dfrac{\vec{c}}{2}$

$\vec{a}.\vec{c} = \dfrac{1}{3}$ & $\vec{a}.\vec{b} = -\dfrac{1}{2}$; $\vec{b} \times (\vec{c} \times \vec{a}) = -\dfrac{\vec{c}}{2}$

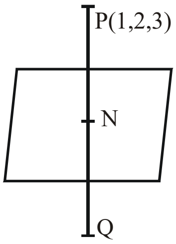
$\Rightarrow (\vec{b}\ \vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a} = -\dfrac{\vec{c}}{2} \Rightarrow \vec{b}.\vec{c} = 0$

Volume of parallelopiped $= \left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$

$$[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} & \frac{1}{3} \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 1 \end{vmatrix} = \frac{23}{36}$$

$\therefore$ Volume $= \dfrac{\sqrt{23}}{6}$

(70) **(3).** Equation of PN



$\vec{r} = (1,2,3) + \lambda(1,1,1)$

N $(1 + \lambda, 2 + \lambda, 3 + \lambda)$

Lies on plane $x + y + z = 12$

$\Rightarrow 1 + \lambda + 2 + \lambda + 3 + \lambda = 12$

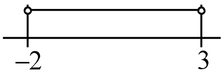
$\Rightarrow \lambda = 2$

N (3, 4, 5) so Q (5, 6, 7)

(71) **2.** $\dfrac{2x-1}{x+2} < 1 \Rightarrow \dfrac{2x-1}{x+2} - 1 < 0$

or $\dfrac{2x-1-x-2}{x+2} < 0 \Rightarrow \dfrac{x-3}{x+2} < 0;$

Hence $-2 < x < 3$ ... (1)



also $|x - k| < 2 \Rightarrow -2 < x - k < 2$

$k - 2 < x < k + 2$ ....(2)

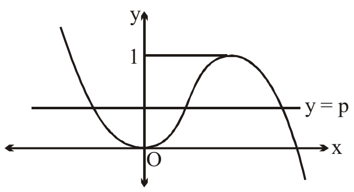
Hence, $k + 2 \le 3 \Rightarrow k \le 1$

and $k - 2 \ge -2 \Rightarrow k \ge 0$

Hence $k \in [0, 1] \Rightarrow$ Number of integral values of k is {0, 1}

(72) **2.** Draw graph of $y = p$ and $y = 3x^2 - 2x^3$

Hence, for three roots $p \in [0, 1]$.



Aliter : Let $f(x) = 2x^3 - 3x^2 + p$

$f'(x) = 6x(x-1)$

$\therefore f'(x) = 0 <$ 0, 1

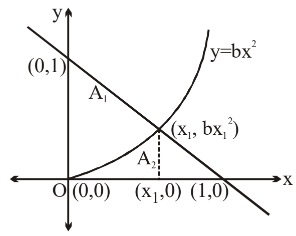
Now, $f(0) \cdot f(1) \le 0 \Rightarrow p(p-1) \le 0$

$\Rightarrow p \in [0, 1]$

(73) **6.** $A_2 = \int_0^{x_1} (bx^2)\,dx + \frac{1}{2}(1 - x_1) \cdot bx_1^2$

$= \dfrac{bx_1^2}{6} \cdot (3 - x_1) = \dfrac{(1-x_1)(3-x_1)}{6}$

(As, $bx_1^2 = 1 - x_1$)



$\dfrac{A_1}{A_2} = \dfrac{11}{16} \Rightarrow \dfrac{A_1 + A_2}{A_2} = \dfrac{27}{16} \Rightarrow A_2 = \dfrac{8}{27}.$

$\left(\text{As, } A_1 + A_2 = \frac{1}{2}(1)(1) = \frac{1}{2}\right)$

$\therefore \dfrac{(1-x_1)(3-x_1)}{6} = \dfrac{8}{27} \Rightarrow x_1 = \dfrac{1}{3} \Rightarrow b = 6.$

(74) **6.** A circle and a parabola can meet at most in four points. Thus maximum number of common chords in ${}^4C_2$ i.e. 6

(75) **0.** $\dfrac{3}{|z_2 - z_3|} = \dfrac{4}{|z_3 - z_1|} = \dfrac{5}{|z_1 - z_2|} = k$ (let)

$\Rightarrow \dfrac{9}{|z_2 - z_3|^2} = \dfrac{16}{|z_3 - z_1|^2} = \dfrac{25}{|z_1 - z_2|^2} = k^2$

$\dfrac{9}{|z_2 - z_3|^2} = k^2 \Rightarrow \dfrac{9}{z_2 - z_3} = k^2(\bar{z}_2 - \bar{z}_3)$ ....(1)

[As $|z|^2 = z\bar{z}$]

|||ly $\dfrac{16}{|z_3 - z_1|^2} = k^2$

$\Rightarrow \dfrac{16}{z_3 - z_1} = k^2(\bar{z}_3 - \bar{z}_1)$ ....(2)

|||ly $\dfrac{25}{|z_1 - z_2|^2} = k^2$

$\Rightarrow \dfrac{25}{z_1 - z_2} = k^2(\bar{z}_1 - \bar{z}_2)$ ....(3)

$\therefore$ On adding (1), (2) and (3), we get

$\dfrac{9}{z_2 - z_3} + \dfrac{16}{z_3 - z_1} + \dfrac{25}{z_1 - z_2}$

$= k^2(\bar{z}_2 - \bar{z}_3 + \bar{z}_3 - \bar{z}_1 + \bar{z}_1 - \bar{z}_2) = 0$