JEE MAIN 2020
FULL TEST-8 SOLUTIONS

STANDARD ANSWER KEY											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	2	3	2	4	1	2	1	1	3	2	4
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	1	1	3	4	1	2	1	3	2	9	8
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	5	1	5	3	4	3	1	3	1	2	1
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
A	2	4	2	3	3	2	4	1	1	2	1
Q	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
A	2	6	5	2	5	3	1	4	3	1	2
Q	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
A	3	4	2	3	4	3	1	4	2	3	4
Q	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**		
A	3	1	3	1	8	2	7	7	6		

(1) **(2).** Let atomic weight of metal = x

Oxide : I

M O

$\frac{50}{x} : \frac{50}{16}$

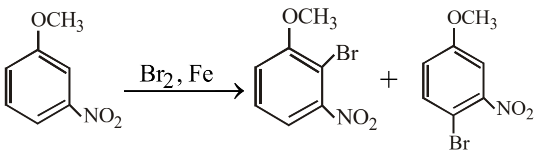
Formula = MO, Hence x = 16

In oxide II : M O

Formula of II oxide M_2O_3 $\frac{40}{16}$ $\frac{60}{16}$

2 : 3

(2) **(3).**



Minor (Y) Major (X)

Strong electron releasing group ($-OCH_3$) generally win over the deactivating group. Thus, o and p-products will be formed. Due to steric hindrance ortho product will be formed in lesser amount than para product.

(3) **(2).**

V	Cr	Mn	Fe
$[Ar]4s^23d^2$	$4s^23d^5$	$4s^23d^5$	$4s^23d^6$
$\downarrow -2e^-$	$\downarrow -2e^-$	$\downarrow -2e^-$	$\downarrow -2e^-$
$3d^2$	$3d^4$	$3d^5$	$3d^6$

Along the period $Z_{eff}\uparrow$, IP$\uparrow$, but after removing $2e^-$ from Mn, it has half filled of configuration, so having maximum ionisation enthalpy among above 4 elements.

(4) **(4).** Li^+ Na^+ K^+ Rb^+ Cs^+

$\longrightarrow$

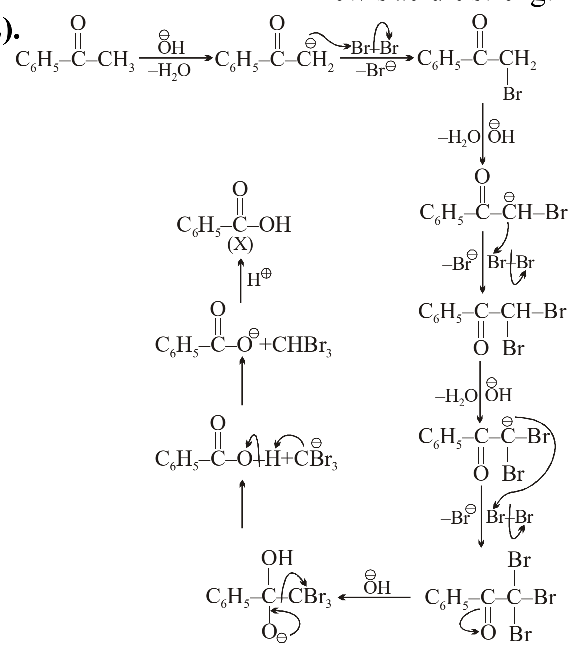
$r^+(\uparrow)$, Hydrated radii $(\downarrow)$, ionic conductivity $(\uparrow)$

(5) **(1).** Halide of 13[th] group act as Lewis acid

$BCl_3 > AlCl_3 > GaCl_3 > InCl_3$

– Lewis acidic strength

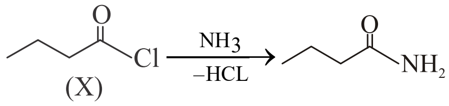
(6) **(2).**



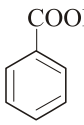
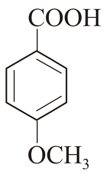
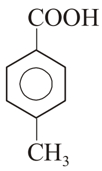
(7) **(1).** Electron withdrawing power increases the reactivity of the molecule towards hydrolysis reaction.

Electron withdrawing power will be following

$-NO_2 > -CHO > -C_6H_5 > -CH_3$.

(8) **(1).** 

$\xrightarrow[KOH]{Br_2} CH_3CH_2CH_2NH_2 + KBr + K_2CO_3 + H_2O$

(9) **(3).** (1)  (3)  (4) 

Induced +M + M + H

(10) **(2).** In CCl_4, I_2 in molecular from, hence colour in violet.

In KI, form complex I_3^-

$I_2 + I^- \rightarrow I_3^-$ (brown in colour)

In ether also I_2 in the polymeric form.

In starch solution I_2 form deep blue complex.

(11) **(4).**

(12) **(1).** $T_f^0 = 5.45°C$, $T_f = 3.55°C$

$$\Delta T_f = 5.45°C \; ; \; \Delta T_f = 5.45 - 3.55 = 1.90$$

$$\Delta T_f = k_f \times m \; ; \; 1.90 = 0.374 \times m$$

$$m = \frac{1.90}{0.374} = \frac{5.08 Kkg}{mol}$$

(13) **(1).** Let initial concentration of $[Zn^{2+}]$, be 1 M after 10 times dilution concentration of

$$[Zn^{2+}] = \frac{1}{10} = 0.1$$

For the potential of Zn/Zn^{2+}

$Zn \longrightarrow Zn^{2+} + 2e^-$

$$E = E° - \frac{0.0591}{2} \log [Zn^{2+}]$$

When $[Zn^{2+}]_1 = 1$ M

When $[Zn^{2+}]_2 = 0.1$ M

$E_1 = E°$

$$E_2 = E° - \frac{0.0591}{2} \log [0.1] = (E° + 0.03)$$

$\therefore$ $E°_{cell} = 0.5400$ V

$\therefore$ Potential of Zn/Zn^{2+} is increased by 0.03V.

(14) **(3).**

Order	Rate of Rx.
1	$r_1 = K[A]^1$
2	$r_2 = K[A]^2$
3	$r_3 = K[A]^3$

K is numerically same & $[A] > 1M$

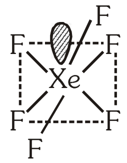
$\therefore$ $r_1 < r_2 < r_3$

(15) **(4).** $AgNO_3 + KI \longrightarrow AgI + KNO_3$

If milli moles of $KI > AgNO_3$;

Negatively charge colloidal is formed.

(16) **(1).** XeF_6 Hyb. = sp^3d^3



Shape distorted octahedral

(17) **(2).** Nb_{41} & Ta_{72} have almost single size due to lanthanide contraction.

(18) **(1).** $Cu^{2+} \xrightarrow[excess]{\bar{C}N} \underset{\text{Stable}}{[Cu(CN)_4]^{3-}} \xrightarrow{H_2S}$ No reaction

$Cd^{+2} \xrightarrow[excess]{\bar{C}N} \underset{\text{Unstable}}{[Cd(CN)_4]^{2-}} \xrightarrow{H_2S} \underset{\text{Yellow ppt}}{CdS}$

(19) **(3).** In catalytic dehydration carbocation is formed as an intermediate.

$\therefore$ Rate of dehydration of alcohol $\propto$ Stability of carbocation

(1) $(CH_3)_2CH - CH_2 - \overset{+}{C}H_2$ [1°C⁺]

(2) $(CH_3)_2\overset{+}{C} - CH_2 - CH_3$ [3°C⁺]

(3) $(CH_3)_2CH - \overset{+}{C}H - CH_3$ [2°C⁺]

(20) **(2).**

(21) **9.**

	N_2O_4	$\rightleftharpoons 2NO_2$
At t = 0	1 mol	0
At eqm	1 – x	2x

Total moles at equilibrium = 1 + x

$$K_p = 0.66\frac{\left(\frac{2x}{1+x}\times 0.5\right)^2}{\left(\frac{1-x}{1+x}\times 0.5\right)} = \frac{2x^2}{1-x^2}$$

or $0.66 - 0.66x^2 = 2x^2$; $2.66\,x^2 = 0.66$

$$x^2 = \frac{0.66}{2.66} = 0.25 \quad \text{or} \quad x = 0.5$$

$$P_{N_2O_4} = \frac{1-x}{1+x}\cdot P = \frac{1-0.5}{1+0.5}\times 0.5 = \frac{0.5}{3}$$

$= 0.168$

(22) **8.** For $Y(OH)_2$ $[OH^-] = C\alpha_1 + C\alpha_1\alpha_2$

$$[OH^-] = 4\times 10^{-3}\times 4\times 10^{-3}\times\frac{50}{100}$$

$= 6\times 10^{-3}$

$\therefore$ $p^{OH} = -\log[OH^-] = 3 - \log 6 = 2.22$

and $pH = 14 - 2.22 = 11.78$

(23) **5.** Deduct the mass of SiO_2 present in Kaolin, from the total mass of SiO_2 in the rock.
Let we have 100 g of rock

Moles of Al_2O_3 present $= \frac{0.816}{102} = 8\times 10^{-3}$

$\therefore$ Mass of SiO_2 present in Kaolin
$= 8\times 10^{-3}\times 2\times 60 = 0.96$ g

$\therefore$ Percentage of free SiO_2 in the rock
$= 1.22 - 0.96 = 0.26$ %

(24) **1.** First calculate $\Delta_C H$ per mol and then convert it into per gm.
The required thermochemical equation is

$$6x\,C(s) + 5x\,H_2(g) + \frac{5x}{2}\,O_2(g) \rightarrow (C_6H_{10}O_5)_x\ ;\ \Delta_f H = ?$$

$$\Delta_f H = \left[6x\times\Sigma\Delta_C H_{C(s)} + 5x\times\Sigma\Delta_C H_{H_2(g)}\right] - \left[\Delta_C H_{(C_6H_{10}O_5)_x}\right]$$

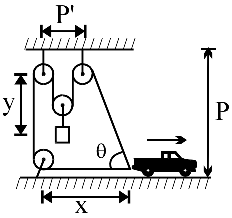
$= 6x\times(-94.05) + 5x\times(-68.32) - (-4.18)\times 162\,x$

$= -228.74\,x$ Kcal/mol $= -\frac{228.74x}{162x}$

$= -1.41$ Kcal/ gm

(25) **5.** In ketone for position isomerism minimum carbon required = 5.
2-pentanone and 3-pentanone.

(26) **(3).** Length of rope = constant



$$x + P + 2y + \sqrt{(x-P')^2 + P^2} = \text{const.}$$

$$\frac{dx}{dt} + 0 + \frac{2dy}{dt} + \frac{1}{2}\sqrt{(x-P')^2+P^2}\times 2(x-P')\frac{dx}{dt} = 0$$

$v - 2v' + \cos\theta\, v = 0$; $2v' = v(1+\cos\theta)$

$\therefore$ $v' = \frac{v}{2}(1+\cos\theta) = \frac{v}{2}(1+\cos 60°)$

$v' = 3v/4$

(27) **(4).** For observer on cart

$$\vec{v}_{rel} = 0\ ;\ \vec{S}_{rel} = \frac{1}{2}\vec{a}_{rel}t^2$$

Trajectory is straight line along $\vec{a}_{rel}$ for observer on ground trajectory is parabola because $\vec{v}_0$ and $\vec{g}$ are at angle θ initially.

(28) **(3).** Work done in cyclic process for $\vec{F}$, is zero therefore $\vec{F}_1$ is conservative (all uniform forces are conservative)

Work done in a cyclic process in case $\vec{F}_2$ is non-zero

$w_{ABCD} = w_{AB} + w_{BC} + w_{CD} + w_{DA}$

$w_{AB} = -F_{AB}\,\ell$

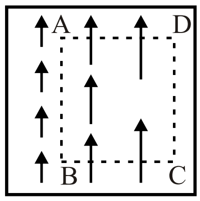
$w_{BC} = 0$

$w_{CD} = +F_{CD}\,\ell$

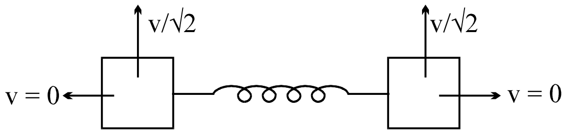
$w_{BA} = 0$

$F_{BA} \neq F_{CD}$

$w_{ABCDA} \neq 0$



(29) **(1).** There is no change in velocity ⊥ to the spring, at the moment of maximum extension relative velocity along the spring is zero. Due to symmetry velocity of blocks is zero along spring at the moment of maximum extension.



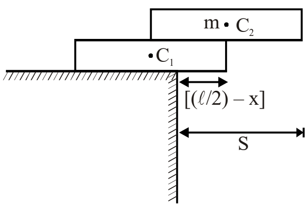
Let extension be x

Applying conservation of energy

$$2\left[\frac{1}{2}m\left(\frac{v}{\sqrt{2}}\right)^2\right]+\frac{1}{2}kx^2=2\left(\frac{1}{2}mv^2\right)$$

$$kx^2=mv^2 \Rightarrow \quad x=v\sqrt{\frac{m}{k}}$$

(30) **(3).** $mx=m\left(\frac{\ell}{2}-x\right)$

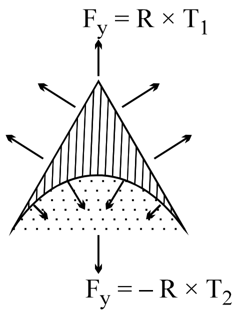


$$x=\frac{\ell}{4}$$

$$S=\frac{\ell}{4}+\frac{\ell}{2}=\frac{3\ell}{4}$$

(31) **(1).** v_z-0 because the slope of the position graph is zero. The negative value of x shows that the particle is left of the equilibrium position, so the restoring force is to the right.

(32) **(2).** $F_y = T \times$ Projected length in y direction



$$F=(T_1-T_2)R$$

(33) **(1).** The distance between source and receiver is not changing, so there is no change in frequency. (1)]

(34) **(2).** As temperature increases spectral intensity corresponding to all wavelengths increases but the wavelength with maximum spectral intensity decreases according to Wein's displacement Law.

(35) **(4).** $PV^{\gamma}=P'V'^{\gamma}$; $\rho=\frac{M}{V}$; $V=\frac{M}{\rho}$

$$P\left(\frac{M}{\rho}\right)^{\gamma}=P'\left(\frac{M}{\rho'}\right)^{\gamma} ; \left(\frac{\rho'}{\rho}\right)^{\gamma}=\frac{P'}{P}$$

$$\frac{P'}{P}=(32)^{1.5}=(2^5)^{3/2}=2^{15/2}$$

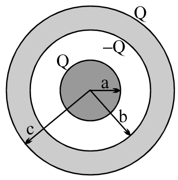
(36) **(2).** $T_S=3mg$

$$\frac{3mg}{A_s}=y_S\times\frac{\Delta\ell_S}{\ell_S} ; \frac{2mg}{A_b}=y_b\times\frac{\Delta\ell_b}{\ell_b}$$

$$\frac{\Delta\ell_s}{\Delta\ell_b}=\frac{3}{2}\times\frac{\ell_s}{\ell_b}\times\frac{A_b}{A_S}\times\frac{y_b}{y_S}=\frac{3}{2}\times\frac{a}{b^2c}$$

(37) **(3).** **For r < a**

$$V=\frac{KQ(3a^2-r^2)}{2a^3}+\frac{K(-Q)}{b}+\frac{KQ}{c}$$



For a < r < b

$$V=\frac{KQ}{r}+\frac{K(-Q)}{b}+\frac{KQ}{c}$$

For b < r < c

$$V=\frac{KQ}{r}+\frac{K(-Q)}{b}+\frac{KQ}{c}=\frac{KQ}{c}$$

For r > c

$$V=\frac{KQ}{r}+\frac{k(-Q)}{r}+\frac{kQ}{r}=\frac{kQ}{r}$$

(38) **(3).** $\frac{1}{2}Li^2=\frac{1}{2}C\varepsilon^2$

$$i=\varepsilon\sqrt{\frac{C}{L}}=12\sqrt{\frac{9\times10^{-12}}{2.5\times10^{-3}}}=\frac{12\times3}{5}\sqrt{10^{-8}}$$

$$i=7.2\times10^{-4}\,A$$

(39) **(2).**

$$P=\frac{a^3b^2}{cd}\Rightarrow\frac{\Delta P}{P}=\pm\left(3\frac{\Delta a}{a}+2\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}\right)$$

$$=\pm(3\times1+2\times2+3+4)=\pm14\%$$

(40) **(4).** $F=[M\,V\,T^{-1}] \Rightarrow M=[F\,V^{-1}\,T]$

(41) **(1).** When 100 V DC applied,

$X_L = 0$ (because f=0)

From, V = I Z

$V = I R\ (\because Z = R)\ ;\ R = \frac{V}{I} = \frac{100}{1}$

$R = 100\Omega$

When 100 V AC applied

$V = I Z\ ;\ Z = \frac{V}{I} = \frac{100}{0.5}\ ;\ Z = 200\Omega$

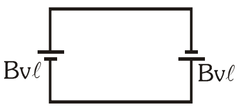
Now, $Z^2 = R^2 + X^2{}_L$

$X^2{}_L = Z^2 - R^2\ ;\ X_L = 100\sqrt{3}\ \Omega$

and $X_L = \omega L$

So $\omega L = 100\sqrt{3}\ ;\ L = \frac{100\sqrt{3}}{\omega} = \frac{100\sqrt{3}}{2\pi f}$

$L = \frac{100\sqrt{3}}{2\pi \times 50} \Rightarrow L = 0.55\ H$

(42) **(1).** 

$e_{net} = Bv\ell + Bv\ell = 2Bv\ell$

(43) **(2).** Power = 10×10^3 W = 10^4 J/s

Amount of U^{235} to operate 10 kW reactor is

$= \frac{10^4 \times 235}{6.02 \times 10^{23} \times 200 \times 10^6 \times 1.6 \times 10^{-19}}$

$= 1.22 \times 10^{-7}$ g/s

(44) **(1).** $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = 12400\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) eV - Å$

$= 12400\left(\frac{1}{1800} - \frac{1}{2300}\right) eV$

$= \frac{12400 \times 500}{1800 \times 2300} = 1.497\ eV \approx 1.5\ eV$

(45) **(2).** Energy of electron $E = mc^2$ and $\lambda_e = \frac{h}{mv}$

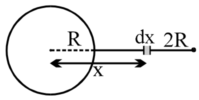
$\Rightarrow\ m = \frac{h}{\lambda_e v}$ so $E = \frac{h}{\lambda_e v}c^2$

and energy of photon $E = \frac{hc}{\lambda_{ph}}$ given that

energy of electron is equal to photon

so $\frac{h}{\lambda_e v}c^2 = \frac{hc}{\lambda_{ph}}\ \therefore\ \frac{\lambda_e}{\lambda_{ph}} = \frac{c}{v}$

$\because\ c > v \qquad \therefore \lambda_e > \lambda_{ph}$

(46) **6.** 

Force on this element, $dF = \frac{GM(\lambda dx)}{x^2}$

$\therefore$ Total force on the stick, $\int_0^F dF = GM\lambda \int_R^{3R} \frac{dx}{x^2}$

$F = GM\lambda \left[\frac{-1}{x}\right]_R^{3R} = GM\lambda\left[\frac{1}{R} - \frac{1}{3R}\right]$

$= GM\lambda\left[\frac{2}{3R}\right]$

CM of stick is rotating in a circle of radius 2R

$\therefore\ GM\left[\frac{2\lambda}{3R}\right] = \lambda(2R)(2R)\omega^2\ ;\ \omega^2 = \frac{GM}{6R^3}$

$T = 2\pi\sqrt{6}\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{6}\sqrt{\frac{R}{g}}$

(47) **5.** $I = \frac{24}{\frac{RG}{R+G} + 1}$

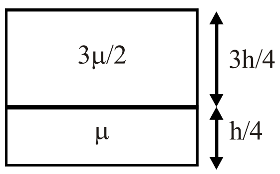
V = 24 – Potential difference across 1 Ω

= 24 – 1 × I

$V = 24 - \frac{24}{\left(\frac{1}{1/R} + \frac{1}{G}\right) + 1}$

For $G \to \infty \Rightarrow \frac{1}{G} \to 0$ & V = 20 V

$\Rightarrow\ 20 = 24 - \frac{24}{R+1} \Rightarrow R = 5\Omega$

(48) **2.** 

Apparent depth $= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}$

$$\frac{h}{4\mu} + \frac{3h}{4 \times \frac{3\mu}{2}} = \frac{h}{2} \; ; \; \mu = \frac{3}{2}$$

(49) **5.** Energy of photon

$$= \frac{hc}{\lambda} = hcR\left(\frac{1}{1^2} - \frac{1}{5^2}\right) = \frac{24hcR}{25}$$

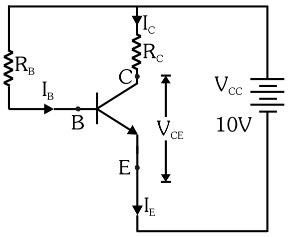
Momentum of photon

$$= \frac{E}{c} = \frac{24hR}{25} = \text{Momentum of atom}$$

Velocity of atom $= \frac{24hR}{25m}$

where m = mass of atom.

(50) **3.**



Here, $\beta = 100$, $V_{CE} = 5V$

$R_C = 1k\Omega$, $V_{BE} = 0$

$V_{CC} = V_{CE} + I_C R_C$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10-5}{1k\Omega} = 5mA$$

$$\because \quad \beta = \frac{I_C}{I_B} \; ; \; I_B = \frac{I_C}{\beta} = \frac{5}{100} mA$$

$V_{CC} = I_B R_B + V_{BE}$

$$R_B = \frac{V_{CC}}{I_B} = \frac{10}{0.05mA} = 200 \times 10^3 \,\Omega$$

(51) **(1).** $y = \frac{x^2}{8} + x\cos x + 2\cos^2 x - 1$

$$= \frac{1}{8}[x^2 + 8x\cos x + 16\cos^2 x] - 1$$

$$= \frac{(x + 4\cos x)^2}{8} - 1$$

(52) **(4).** We have, $x^2 + x - 2 = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$\therefore \quad \alpha + \beta = -1; \alpha\beta = -2$

$$\frac{\alpha\beta^4(\beta+1)^4 + \beta\alpha^4(\alpha+1)^4}{\alpha^2 + \beta^2 + \alpha + \beta}$$

$$= \frac{\alpha^5\beta^4 + \beta^5\alpha^4}{(\alpha+\beta)^2 - 2\alpha\beta + \alpha + \beta}$$

$$= \frac{(\alpha\beta)^4(\alpha+\beta)}{-2\alpha\beta} = \frac{(\alpha\beta)^4}{2\alpha\beta} = \frac{16}{-4} = -4$$

Aliter: $\frac{\alpha\beta^4(\beta+1)^4 + \beta\alpha^4(\alpha+1)^4}{\alpha^2 + \beta^2 + \alpha + \beta}$

$= \frac{16(\alpha+\beta)}{4}$ [Using $\alpha^2 + \alpha = \beta^2 + \beta = 2$ and

$\alpha + 1 = \frac{2}{\alpha}$ and $\beta + 1 = \frac{2}{\beta}$]

$$= \frac{-16}{4} = -4$$

(53) **(3).** For the G.P. a, ar, ar^2, ...

$P_n = a(ar)(ar^2)$ ....$(ar^{n-1}) = a^n r^{n(n-1)/2}$

$$\therefore \quad S = \sum_{n=1}^{\infty} \sqrt[n]{P_n} = \sum_{n=1}^{\infty} ar^{(n-1)/2}$$

$$\sum_{n=1}^{\infty} ar^{(n-1)/2} = a[1 + \sqrt{r} + r + r\sqrt{r} + ... + \infty] = \frac{a}{1-\sqrt{r}}$$

Given a = 16 and r = 1/4

$$\therefore \quad S = \frac{16}{1-(1/2)} = 32$$

(54) **(1).** Group 6 persons can be divided into 3 equal groups in $\frac{6!}{2!\cdot 2!\cdot 2!\cdot 3!}$ ways

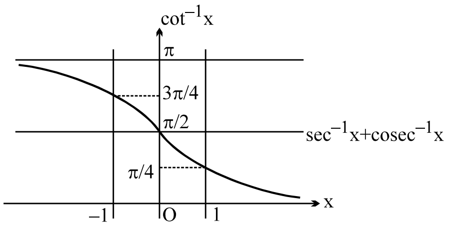
P_1	P_2	P_3
P_4	P_5	P_6

say P_1P_4; P_2P_5; P_3P_6

Now each elements of a group can be arranged in 3! ways.

Total ways = $\dfrac{6!\cdot 3!}{2!\cdot 2!\cdot 2!\cdot 3!} = \dfrac{720}{8} = 90$

(55) (2).



Domain of $f(x)$ is $|x| \geq 1$

From the graph for $x \geq 1$ f(x) can attain values from $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]$

Also for $x \leq -1$, f(x) can attain values from $\left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right)$

(56) (3). $f(x) = \begin{cases} 3\left(1+|\tan x|\right)^{\frac{\alpha}{|\tan x|}} ; & \dfrac{-1}{2} < x < 0 \\ \beta ; & x = 0 \\ 3\left(1+\left|\dfrac{\sin x}{3}\right|\right)^{\frac{6}{|\sin x|}} ; & 0 < x < \dfrac{2}{3} \end{cases}$

Now, R.H.L. = $f(0^+)$

$= \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} 3\left(1+\dfrac{|\sin x|}{3}\right)^{\frac{6}{|\sin x|}} = 3e^2.$

Similarly, L.H.L = $f(0^-) = \lim\limits_{x\to 0^-} f(x) = 3e^{\alpha}$

For the function f(x) to be continuous at x = 0, we must have $f(0^+) = f(0^-) = f(0)$

$\Rightarrow 3e^{\alpha} = 3e^2 = \beta$. So, $\alpha = 2, \beta = 3e^2$.

(57) (4). $a = (x+1)^2 + 2 \Rightarrow a = 2$

$b = \lim\limits_{x\to 0} \dfrac{\sin 2x}{\dfrac{2(e^{2x}-1).2x}{2x}} = \dfrac{1}{2}$

$\sum\limits_{r=0}^{n} a^r b^{n-r} = \sum 2^r \left(\dfrac{1}{2}\right)^{n-r} = \dfrac{1}{2^n}\sum\limits_{r=0}^{n} 2^{2r}$

$= \dfrac{1}{2^n}\sum\limits_{r=0}^{n} 4^r = \dfrac{1}{2^n}[1 + 4 + 4^2 + ... + 4^n]$

$= \dfrac{1}{2^n}\left[\dfrac{4^{n+1}-1}{3}\right] = \dfrac{4^{n+1}-1}{3.2^n}$

(58) (2). Differentiate both sides w.r.t. x

$f'(x) = \cos x + f'(x)(2\sin x - \sin^2 x)$

$\therefore (1 + \sin^2 x - 2\sin x) f'(x) = \cos x$

$f'(x) = \dfrac{\cos x}{1+\sin^2 x - 2\sin x} = \dfrac{\cos x}{(1-\sin x)^2}$

Integrating $f(x) = \displaystyle\int \dfrac{\cos x\, dx}{(1-\sin x)^2}$

(Put $1 - \sin x = t$);

$f(x) = -\displaystyle\int \dfrac{dt}{t^2} = \dfrac{1}{t} = \dfrac{1}{1-\sin x} + C$

Also $f(0) = 0$, hence $C = -1$

$f(x) = \dfrac{1}{1-\sin x} - 1 = \dfrac{1-1+\sin x}{1-\sin x}$

$= \dfrac{\sin x}{1-\sin x}$

(59) (3). Let $f(x) = \left(\dfrac{a^2-4}{a^2+2}\right)x^3 - 3x + \sin 3$

$\therefore f'(x) = 3\cdot\left(\dfrac{a^2-4}{a^2+2}\right)x^2 - 3$

As, f is strictly decreasing on R

So, $\left(\dfrac{a^2-4}{a^2+2}\right) \leq 0 \Rightarrow a^2 - 4 \leq 0 \Rightarrow a \in [-2, 2]$.

(60) (4). $A^3 = B^3$ ....(1)

and $A^2B = B^2A$ ....(2)

(1) – (2) gives, $A^3 - A^2B = B^3 - B^2A$

$A^2(A-B) = -B^2(A-B)$

$\Rightarrow (A^2 + B^2)(A - B) = 0$

$\det (A^2 + B^2)(A - B) = 0$

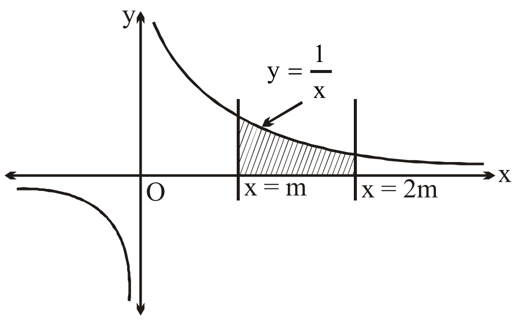
$\det (A^2 + B^2) \cdot \det (A - B) = 0$

(61) **(3).** 5 → D D / G G G

P (Second test) = P (D D) = $\frac{2}{5}\cdot\frac{1}{4}=\frac{1}{10}$

(62) **(1).** Area = $\int_{m}^{2m}\frac{1}{x}dx = ln\ x \Big|_{m}^{2m}$

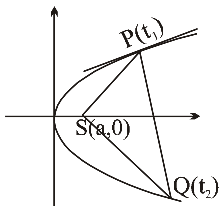
$= \ln(2m) - \ln(m) = \ln 2,$



So the area is independent of m.

(63) **(4).** $a t_1^2 = 2at_1 \Rightarrow t_1 = 2$; P(4a , 4a)

$t_2 = -t_1 - \frac{2}{t_1} = -3$



∴ Q (9a ,– 6a)

∴ $m_{SP} = \frac{4a}{4a-a} = \frac{4}{3}$

$m_{SQ} = \frac{-6a}{9a-a} = -\frac{3}{4}$

$\angle PSQ = 90^0$

(64) **(2).** Let A = {1, 2, 3}

Let the two transitive relations on set A be

R = {(1, 1), (1, 2)}and S = {(2, 2), (2, 3)}

Now, R ∪ S = {(1, 1), (1, 2), (2, 2), (2, 3)}

Here, (1, 2), (2, 3) ∈ R ∪ S

⇒ (1, 3) ∉ R ∪ S

∴ R ∪ S is not transitive.

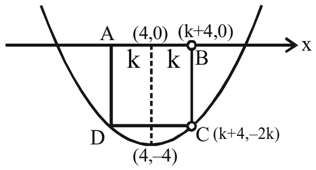
(65) **(3).**

p	q	p ⇒ q	~ (p ⇒ q)	~ p	~ q	~ p∨ ~ q	~ (p ⇒ q) ⇔ ~ p∨ ~ q
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	F
F	F	T	F	T	T	T	F

The last column shows that the result is neither a tautology and a contradiction.

(66) **(4).** Let the side length be 2k

Now point ((k + 4), –2k) lies on parabola.



$y = x^2 - 8x + 12$

$(-2k) = (k+4)^2 - 8(k+4) + 12$

⇒ $k^2 + 2k - 4 = 0$

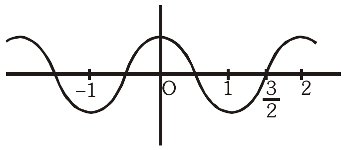
⇒ $k = \frac{-2+\sqrt{4+16}}{2} = 1+\sqrt{5}$

So area = $4k^2 = 4(-1+\sqrt{5})^2 = 4(6-2\sqrt{5})$

(67) **(3).** $f(x) = \cos(2\pi[x] + \pi|x|) = \cos(\pi|x|)$

$f(-x) = f(x)$ hence function is even.

It is a periodic function



From graph $f(x) = |f(x)|$ is not possible for all x.

(68) **(1).** Let $P \equiv (2\sqrt{2}\cos\theta,\ 2\sin\theta)$

$Q \equiv (-2\sqrt{2}\sin\theta,\ 2\cos\theta)$

$(PQ)^2 = 8(\cos\theta + \sin\theta)^2 + 4(\sin\theta - \cos\theta)^2$

$= 12 + 4\sin 2\theta \le 16$

$(PQ)_{max} = 4$

(69) **(3).** $(\vec{a}.\vec{b})(\vec{b}.\vec{c}) - (\vec{a}.\vec{c})(\vec{b}.\vec{b}) + (\vec{b}.\vec{c})(\vec{c}.\vec{a})$

$-(\vec{b}.\vec{a})(\vec{c}.\vec{c}) + (\vec{c}.\vec{a})(\vec{a}.\vec{b}) - (\vec{c}.\vec{a})(\vec{a}.\vec{b}) - (\vec{c}.\vec{b})(\vec{a}.\vec{a})$

$= 0 - (\vec{a}.\vec{c}) + (\vec{b}.\vec{c})(\vec{c}.\vec{a}) - 0 + 0 - (\vec{c}.\vec{b})\ (\because \vec{a}.\vec{b} = 0)$

$= (\vec{b}.\vec{c})(\vec{c}.\vec{a}) - (\vec{c}.\vec{a}) - (\vec{b}.\vec{c}) + 1 - 1$

$= ((\vec{b}.\vec{c}) - 1)((\vec{c}.\vec{a}) - 1) - 1$

$= (1-1)(1-1) - 1 = -1\ (\because \vec{c} = \vec{a} + \vec{b})$

(70) **(1).** Let plane is Ax + Bz = 1

Passes through (1,0,1) & (3,2,–1)

⇒ A + B = 1 & 3A – B = 1 ⇒ A = 1/2, B=1/2

Plane is x + z = 2

Point P(λ + 1, 2λ + 2, 3λ + 5) lie on plane

⇒ (λ + 1) + (3λ + 5) = 2 ⇒ λ = –1

$\Rightarrow$ Point is (0,0,2)

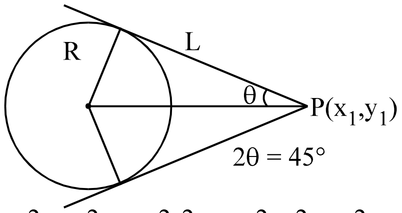
(71) **8.** $\tan\theta = \dfrac{R}{L}$ where $2\theta = 45°$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(note that $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$)

$$1 = \frac{2(R/L)}{1-(R^2/L^2)} = \frac{2RL}{L^2-R^2}$$

$$= \frac{2a\sqrt{x_1^2+y_1^2-a^2}}{x_1^2+y_1^2-a^2-a^2}$$



$(x^2+y^2-2a^2)^2 = 4a^2(x^2+y^2-a^2)$

$(x^2+y^2)^2 + 4a^4 - 4a^2(x^2+y^2)$

$= 4a^2(x^2+y^2-a^2)$

$(x^2+y^2)^2 + 8a^4 = 8a^2(x^2+y^2)$

$(x^2+y^2)^2 = 8a^2(x^2+y^2-a^2) \Rightarrow \lambda = 8$

(72) **2.** $g(x) = f^{-1}(x)$

$g(f(x)) = f^{-1}(f(x)) = x$

$\Rightarrow g'(f(x)) \cdot f'(x) = 1 \Rightarrow g'(f(x)) = \dfrac{1}{f'(x)}$

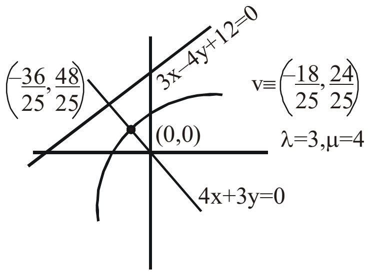
Put $x = 0 \Rightarrow g'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{1/2} = 2$

(73) **7.** $\lim\limits_{n\to\infty} \dfrac{\Sigma r^2 \Sigma r^4}{\Sigma r^7} = \dfrac{K+1}{15}$

$$\frac{\int_0^1 x^2dx\int_0^1 x^4dx}{\int_0^1 x^7\,dx} = \frac{K+1}{15}\ ;\ \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{8}} = \frac{K+1}{15}\ ;\ K = 7$$

(74) **7.** $\sqrt{x^2+y^2} = \left|\dfrac{3x-4y+12}{5}\right|$

$\Rightarrow$ focus is (0,0) and directrix is $3x - 4y + 12 = 0$



(75) **6.** $z^{11} + 2z^{10} + 3z^9 + \ldots\ldots\ldots + 2z^2 + z = 0$

$\Rightarrow z(z^{10} + 2z^9 + 3z^8 + \ldots\ldots\ldots + 2z + 1) = 0$

$\Rightarrow z(1 + z + z^2 + z^3 + z^4 + z^5)^2 = 0$

$\Rightarrow z\left(\dfrac{1-z^6}{1-z}\right)^2 = 0 \qquad (z \neq 1)$

$\Rightarrow z(1-z^6)^2 = 0$

$\therefore$ $z = 0$ and every root of the equation $z^6 - 1 = 0$ (except $z = 1$) two times repeated are the 11 roots of the given equation.

$$z = (1)^{1/6} = \cos\frac{2m\pi}{6} + i\sin\frac{2m\pi}{6}$$

$m = 1, 2, 3, 4, 5$

$$\therefore \sum_{n=1}^{11} |a_n| = 0 + 2\left(\sum_{m=1}^{5}\left|\cos\frac{m\pi}{3}\right|\right)$$

$$= 2\left(\left|\cos\frac{\pi}{3}\right| + \left|\cos\frac{2\pi}{3}\right| + |\cos\pi| + \left|\cos\frac{4\pi}{3}\right| + \left|\cos\frac{5\pi}{3}\right|\right)$$

$$= 2\left(\frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{2}\right) = 6$$